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# INTERACTIONS BETWEEN HEAT FLOW AND GROUNDWATER FLOW - A REVIEW

G. van der Kamp

Department of Earth Sciences
University of Waterloo

Earth Physics Branch Open File Number 82-19
Ottawa, Canada, 1982

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## Abstract

The review presented in this report provides a synthesis of results from various fields of research related to the interaction of groundwater flow and heat flow. Emphasis has been placed on the basic physical processes and their relevance to a variety of real-world phenomena.

Throughout this study an important underlying theme is the problem of applying theoretical findings to the irregular real-world, and the reverse problem, all too often lightly regarded, of verifying theory by means of empirical results from the real world.

## Resumé

Ce rapport présente une synthèse des résultâts obtenus dans différents champs de recherche et qui sont relié à l'interaction entre le flux d'eau souterrain et le flux de chaleur. On y met l'accent sur les processus physiques de base et leurs applicabilité à une variété de phénomênes réels.

Un thême sous-jaçent important à travers cette étude est le problème de l'application des résultâts théoriques ou monde réel irrégulier, et le problême inverse, de la vérification de la théorie à la lumière de résultats empiriques obtenus dans le monde réel.

University of
**Waterloo Research Institute**

# Interactions Between Heat Flow and Groundwater Flow – A Review

**Final Report**

May 1982

University of Waterloo

Waterloo Research Institute

Project No. 109-17

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Table of Contents

# Interactions between Heat Flow and Groundwater Flow - A Review

## 1. Introduction

Interactions between heat flow and groundwater flow are an important factor for many subsurface processes of interest to the earth sciences. Among these are terrestrial heat flow, geothermal phenomena, circulation of fluids in the subsea crust, storage or extraction of thermal energy, subsurface isolation of nuclear wastes, and permafrost phenomena.

The theory of interactions between heat and fluid flow is unavoidably complex, especially in its applications to naturally occurring geologic formations, whose properties are generally not well-known and difficult to describe mathematically. Additional complexity is introduced if phase changes of the fluid are considered. In view of the complexity and variety of problems involved, there is an on-going need for review and synthesis of the theoretical and empirical results that appear in the literature.

The present study is essentially a critical review of the literature concerning interactions of heat and fluid flow. The emphasis is on the basic theoretical concepts and how these relate to the available observations. Basic equations are given, but for detailed mathematical derivations the reader is referred to the relevant papers or texts.

A review covering as wide a subject area as the present one must almost of necessity be limited in its scope by perhaps rather arbitrary bounds. The discussion concentrates on how the groundwater literature relates to the heat flow literature. Only physical interactions of groundwater and heat flow in saturated media are considered and transport of heat and fluid in the unsaturated zone is not treated. Chemical interactions and the effects of dissolved constituents are only briefly mentioned. The water-steam phase change is not treated. The water-ice phase change is considered, but only in terms of subpermafrost phenomena. Mechanical effects relating to problems of subsidence and soil mechanics are only briefly mentioned.

A wide-ranging reference list is included, but no attempt at completeness or at establishing precedence has been made. Whenever possible the reader is referred to existing reviews. The emphasis has been placed on the most recent relevant papers. For any particular topic a more complete list of references can be assembled from the references given in the recent papers.

A study of this nature cannot help but be subjective in its selection of topics on which to concentrate. No doubt some relevant topics have been treated rather summarily or missed altogether. The aim has been not so much to achieve complete coverage of the subject, but to inquire how the basic physical concepts apply in a variety of real-world situations.

## 2. The Constitutive Equations

### 2.1 The Basic Equations

Any discussion about the interaction of heat flow and fluid flow in a porous medium must be based on the constitutive equations describing the interaction between solids, fluid and heat. The phenomena involved are highly complex so that the use of fully general equations is both virtually impossible and impractical. The equations presented here describe the principal effects that are observed in nature. More subtle secondary effects may occur, but the available data rarely allow for more than a speculative theoretical description.

The following presentation of the equations essentially follows that of Bear (1972, p641), but with considerable modification and reference to other sources. Bean treats heat and solute transport together, however here only heat effects will be considered. Other presentations of the basic equations are given by Brownell et al., (1977) and Morland (1978). These authors concentrate on geothermal and on thermoelastic phenomena respectively.

The basic equations may be written as follows:

a) Conservation of fluid mass:

$$\frac{\partial}{\partial t}(n\rho_f) + \frac{\partial}{\partial x_i}(n\rho_f v_i) = 0 \qquad 1$$

where $n$ - porosity of the porous medium

$\rho_f$ - density of the fluid

$v_i$ - average interstitial velocity of the fluid

$x_i$ - spatial co-ordinate on the i direction

Repeated indices denote summation, as in the second term in equation 1:

$$\frac{\partial}{\partial x_i}(n\rho_f v_i) \equiv \frac{\partial}{\partial x_1}(n\rho_f v_1) + \frac{\partial}{\partial x_2}(n\rho_f v_2) + \frac{\partial}{\partial x_3}(n\rho_f v_3)$$

b) Conservation of linear momentum of fluid:

$$nv_i = -\frac{k_{ij}}{\mu}\left(\frac{\partial p}{\partial x_j} + \rho_f g \delta_{i3}\right) \qquad 2$$

where: $k_{ij}$ - permeability tensor

$\mu$ - fluid viscosity

$p$ - fluid pressure

$g$ - acceleration of gravity

$\delta_{j3} = 1$ when $j = 3$, $\delta_{j3} = 0$ when $j = 1$ and $j = 2$

In equation 2 second-order terms involving fluid inertia and time or space variations of porosity have been neglected (see Brownell et al., 1977).

c) Conservation of fluid energy

$$\rho_f c_f \left(\frac{\partial T_f}{\partial t} + v_i \frac{\partial T_f}{\partial x_i}\right) = \frac{\partial}{\partial x_i} (\lambda_f')_{ij} \frac{\partial T_f}{\partial x_j} + \qquad 3$$

$$+ \frac{\partial}{\partial x_i}\left(\frac{E_{ij}}{n}\frac{\partial T_f}{\partial x_j}\right) + h_f\,(T_f, T_s)$$

$c_f$ - specific heat capacity of fluid

$T_f$ - fluid temperature

$(\lambda_f)_{ij}$ - effective thermal conductivity tensor of fluid

$E_{ij}$ - dispersion tensor for heat flux

$T_s$ - solid temperature

$h_f$ - coefficient of heat transfer from solid to fluid.

In equation (3) terms involving pressure-work and viscous energy dissipation are neglected (see Bear, 1972; Garg and Pritchett, 1977).

d) Conservation of solid energy:

$$\rho_s c_s \frac{\partial T_s}{\partial t} = \frac{\partial}{\partial x_i} (\lambda_s{}')_{ij} \frac{\partial T_s}{\partial x_j} + h_s (T_x, T_f) \qquad 4$$

where: $\rho_s$ - density of solid

$C_s$ - specific heat capacity of solid

$T_s$ - temperature of solid

$(\lambda_s)_{ij}$ - effective thermal conductivity tensor of solid

$h_s$ - coefficient of heat transfer from fluid to solid

In equation 4 mechanical energy of the solid is neglected (see Brownell et al 1977). Other possible heat-producing processes such as diagenesis of sediments (Shvetsov, 1973) and chemical processes such as oxidation of sulfides (Kappelmeyer and Haenel, 1974) have also been neglected in equations 3 and 4.

e) Density of fluid:

Equations for the dependence of the density of water on temperature and pressure are summarized by Bear and Corapcioglu (1981). An expression for the temperature dependence is given by Wooding (1957):

$$\rho_f = \rho_f^o [1 - \beta_T (T_f - T_o) - \beta_T{}' (T_f - T_o)^2] \qquad 5$$

where: $\rho_f^o$ - density of water at reference state

$T_o$ - reference temperature

$\beta_T, \beta_T{}'$ - coefficients for thermal expension of water

Detailed data for density of water are given in various handbooks (e.g. Weast, 1975) and by Dorsey (1968).

f) Changes of porosity:

Equations 1 through 5 are expressed in terms of six variables: $n$, $\rho_f$, $v_i$, $p$, $T_f$ and $T_s$. To complete this set of equations an expression relating the porosity n to the other five variables is required. Such an expression will in general be extremely complex, involving also the three-dimensional stress-strain relationships of the solid medium, its thermal expansion properties, and the interelation of these with fluid pressure. General equations for fluid flow in a thermoelastic matrix are given by Morland (1978). Bear and Corapcioglu (1981) derive equations for consolidation of a thermoelastic aquifer and Brownell et al (1977) show that for practical problems of geothermal reservoirs the rock matrix behaves essentially in a quasi-static manner. Garg and Kassoy (1981) give an expression for the change of porosity, assuming uniaxial compaction and constant overburden stress:

$$\frac{\partial n}{\partial t} = (1 - n)\, c_m \frac{\partial p}{\partial t} + 3\,(c_m \eta k_r - \eta_s)\, \frac{\partial T}{\partial t} \qquad 6$$

where: $C_m$ - uniaxial compaction coefficient

$\eta$ - coefficient of linear thermal expansion for porous rock

$\eta_s$ - coefficient of linear thermal expansion for rock grain

$k_r$ - bulk modulus of porous rock

Equations such as 6 inevitably involve numerous assumptions concerning the deformation behaviour of the formation. The occurrence of viscoelastic and plastic effects, neglected in equation 6, is particularly difficult to deal with, especially in the presence of temperature changes. Finnemore and Gillam (1977) review the problems involved in analyzing compaction and subsidence in geothermal areas.

Equations 1-6 all refer to averages over a representative elementary volume, REV, (Bear 1972) which contains a sufficient number of pores so that the averages are meaningful. The dimensions of the heterogeneities (pores and solid grains) must therefore be much smaller than the dimensions of the REV and the dimensions of the REV must be much smaller than the dimensions of the volume of formation to which the equations are applied. In practice most natural geologic formations contain heterogeneities at all length scales and the question of defining an REV of adequate dimensions is far from trivial. In the context of heat and water flow this point is of particular importance with regard to hydrodynamic dispersion and thermal equilibrium between fluid and solid.

## 2.2 The Generalized Form of Darcy's Law

Equation 2 is frequently referred to in the literature as Darcy's law, but it would be preferable to consider Darcy's law as the special case when fluid density in a function of pressure only (Corey, 1977). For this special case the fluid flow can be written as proportional to the gradient of a potential:

$$nv_i = -\frac{k_{ij}\rho_f g}{\eta}\frac{\partial h}{\partial x_j} \qquad 7$$

where (e.g. Freeze and Cherry, 1979)

$$h = \frac{1}{g}\int_{p_o}^{p}\frac{1}{\rho(p')}\,dp' + z \qquad 8$$

(Here and subsequently the subscript "f" is dropped from $\rho_f$).

For the more general case where density changes due to temperature are significant, the flow depends on two variables, pressure and density. This dependency is brought out more clearly by writing equation 2 in the form (Bear, 1972):

$$nv_i = -\frac{\rho_o g k_{ij}}{\mu}\left[\frac{\partial}{\partial x_j}\left(\frac{p}{\rho_o g} + z\right) + \left(\frac{\rho}{\rho o} - 1\right)\delta_{i3}\right] \qquad 9$$

The driving forces are now separated into the gradient of a piezometric potential and a vertical buoyancy force. Equation 9 shows clearly that to determine the flow in a formation it is necessary to measure both pressure and density throughout the system if buoyancy forces are expected to be significant. By contrast in most groundwater studies only piezometric head is considered. It should be noted that the piezometric potential introduced in equation 9, $p/\rho_o g+z$, is defined in terms of pressure and corresponds to water level in a piezometer only for the case where density may be assumed constant.

## 2.3 Thermal Equilibrium of Fluid and Solid

The significance of equations 3 and 4, which express the conservation of thermal energy of fluid and solid, is dependent on the dimensions of the pores and solid particles. If the dimensions are very small then the fluid and solid may be assumed to be in thermodynamic equilibrium. The characteristic time required for a volume of material to achieve thermal equilibrium with the surrounding fluid by conduction only is given by (e.g. Lachenbruch and Sass, 1977):

$$\tau = \frac{L^2}{4D_h} \qquad 10$$

where L is a characteristic dimension for the volume and $D_h$ is the thermal diffusivity of the material. For a solid particle the diffusivity is given by:

$$D_{hs} = \frac{\lambda_s}{\rho_s c_s} \qquad 11$$

With a typical value for $D_{hs}$ of $10^{-6}$ $m^2$/sec, the characteristic time is 0.25 seconds for a particle of 1 millimetre diameter and 250,000 seconds, or about 3 days, for solid volume of 1 meter diameter. Thus for a porous medium viewed at the microscopic grain-size scale it may safely be assumed that solid and fluid are in thermal equilibrium. On the other hand if macroscopic heterogeneities with dimensions of meters are considered, such as clay lenses or unfractured rock blocks, then thermal equilibrium may only be assumed for processes with characteristic times of many days.

Practically all naturally occurring formations are porous at the grain-size scale and can generally be considered as locally homogeneous with an REV that includes many pores and grains. The solid and fluid temperatures may then be assumed equal and equations 3 and 4 can be combined, using the relation

$$nh_f(T_f,T_s) = -(n - 1)\ h_s(T_s,T_f) \qquad 12$$

to give:

$$\rho_e c_e \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_i} [(\lambda_e)_{ij} + E_{ij}] \frac{\partial T}{\partial x_j} - \rho_f c_f n v_i \frac{\partial T}{\partial x_i} \qquad 13$$

where

$$\rho_e c_e = n\rho_f c_f + (n - 1)\, \rho_s c_s \qquad 14$$

$$(\lambda_e)_{ij} = n(\lambda_f')_{ij} + (n - 1)(\lambda_s')_{ij} \qquad 15$$

The effective thermal conductivities $\lambda_f$ and $\lambda_s$ are in fact defined by equation 15. The appearance of these quantities in equations 3 and 4 is somewhat artificial since in practice they can only be measured through equation 15 which defines the effective thermal conductivity of the saturated medium. $\lambda_f$ and $\lambda_s$ depend on the thermal conductivities and the geometry of the pores and of the solid grains (Chan et al, 1981). Bear (1972) and Sass et al (1971) review various methods of calculating effective thermal conductivity if the porosity and thermal conductivities of the fluid and solid are known, but in practice reliable determination of $\lambda_e$ generally requires direct measurement.

## 2.4 Microscale Dispersion

The theory of hydrodynamic dispersion in the transport of mass or heat is at present a topic of active discussion in the groundwater literature (e.g. Sauty, 1980; Pickens and Grisak, 1981; Sudicky et al, 1982). For heat transport the dispersion tensor can be written in the form (Bear, 1979, Sauty et al.,1978)

$$E = n\rho_f c_f \begin{vmatrix} \alpha_L & 0 & 0 \\ 0 & \alpha_T & 0 \\ 0 & 0 & \alpha_T \end{vmatrix} |v_1| \qquad 16$$

where $E$ - dispersion tensor

$v_I$ - groundwater velocity (in $x_1$ direction)

$\alpha_L$ - longitudinal dispersivity

$\alpha_T$ - transverse dispersivity

Equation 16 is written in its simplest form, which applies when the groundwater motion is parallel to the $x_1$ axis. For the more general case when the velocity is not parallel to one of the reference axes, the appropriate rotation matrices must be included in equation 16 (Bear, 1979).

The dispersivities have units of length and the longitudinal dispersivity has magnitudes roughly reflecting the dimensions of the largest heterogeneities along the flow path. The transverse dispersivities are generally smaller than the longitudinal dispersivity by factor of 5 or 10 or more, depending on the geometry of the heterogeneities.

The dispersion processes are due to mixing at intersections of flow paths and thermal conduction within the fluid and the solid. Solute dispersion processes differ from heat dispersion processes in that solute diffusion (equivalent to conduction) occur only in the fluid.

A major problem with equation 16 or other forms thereof is that in actual situations the dispersivities tend to increase with length of the flow path in a rather unpredictable manner, apparently depending on the structure of the formation heterogeneities. Field measurements have yielded dispersivity values varying from .01m to 100 meters. Thus equation 16 has low predictive value and even the validity of its general form is questionable.

Equation 13 shows that dispersion essentially leads to an apparently enhanced thermal conductivity or diffusivity. We can write:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x_i} [(D_h')_{ij}] \frac{\partial T}{\partial x_j} - n v_i \frac{\rho_f c_f}{\rho_e c_e} \frac{\partial T}{\partial x_i} \qquad 17$$

where

$$(D_h')_{ij} = \frac{1}{\rho_e c_e} [(\lambda_e)_{ij} + E_{ij}] \qquad 18$$

If the medium is isotropic with respect to thermal conductivity and the flow is parallel to the $x_1$-axis (equation 16) then equation 18 becomes

$$(D_h')_{ij} = [D_h + \frac{n\rho_f c_f |v_1|}{\rho_e c_e} \alpha_i]\delta_{ij} \qquad 19$$

Values of the thermal diffusivity for geological materials generally lie within a fairly narrow range around $10^{-6}$ $m^2$/sec. If microscale dispersion at the grainsize scale is considered, then the dispersivity values generally lie on the range $10^{-3}$ - $10^{-2}$ m, as born out by laboratory experiments. As discussed by Bean (1972) thermal dispersion on this scale will usually be negligible relative to conduction. If, for instance, $\alpha_1$ ($=\alpha_L$) has a high value (for microscale dispersion) of $10^{-2}$ m and $n|v_1|$ has a high value of $10^{-5}$ m/sec then the dispersion term in equation 19 is still small compared to the conduction term. Thus thermal dispersion at the microscale is likely to be negligible except for cases of very rapid flow through coarse-grained materials.

## 2.5 Macroscale Dispersion

Detailed field studies of heat transport by moving groundwater suggest strongly that dispersion at the macroscale can be a significant factor. Longitudinal dispersivities of 0.10 m to 7 m for flow path lengths of 10-100 m have been determined. (Andrews and Anderson, 1978, Kinmark and Voss, 1980, Sauty et al, 1978). With dispersivities of this magnitude dispersion effects may be as large as or larger than conduction effects and the net result is an enhanced apparent thermal conductivity (see equation 19).

The large thermal dispersivities that have been calculated are similar in magnitude to dispersivities for solute transport and can probably be ascribed to the existence of permeability heterogeneities within the formation. The values of dispersivity are obtained by matching an equation such as 17 to the observations. However, since dispersivity tends to increase in an unpredictable manner with length of flow path, equation 17 has little predictive value and in fact its general validity for macroscale dispersion is questionable.

Macroscale dispersion is essentially a combination of two processes: 1) conduction (or diffusion) into and through the less permeable heterogeneities and 2) mixing by microdispersion at the intersections of flow paths. These two processes can lead to a variety of delaying and "smearing out" phenomena and it is to be expected that equation 17 is applicable for at most some special cases. In the present state of the theory "dispersivity" serves as a lumped parameter which depends on the degree of detail to which the heterogeneity of the formation is modelled.

Thermal dispersion differs from solute dispersion in two important ways: 1) in contrast to solute movement, heat is conducted through both the solid and the fluid, and 2) the thermal diffusivity of saturated geological materials has values of about $10^{-6}$ $m^2$/sec whereas solute diffusivity has values of about $10^{-9}$ $m^2$/sec. Thus for dispersion processes which are dominated by diffusion into and through the heterogeneities it is to be expected that thermal dispersion may well be very different from solute dispersion. On the other hand kinematic dispersion processes, which are dominated by mixing at flow path intersections may be expected to be the same for heat as for solute transport. The fact that large thermal dispersivities, roughly equal to solute dispersivities, have been observed (Sauty et al 1978) implies that for these instances mixing was probably the dominant dispersion process.

There is another inherent problem with using equation 17 to describe macroscale thermal dispersion. Dispersivities of meters or more imply heterogeneities with dimensions of meters. But for heterogeneities of this size thermal equilibrium with the fluid in the permeable zones may require many days. Thus the assumption of a common temperature for fluid and solid on which equation 17 is based may be inconsistent with macroscale dispersion during transient processes (Sorey, 1978). The large dispersivities that have been calculated for some field studies therefore may, amongst other factors, reflect lack of thermal equilibrium in the formation.

For very slowly-changing or steady-state processes the assumption of local thermal equilibrium is valid and equation 17 may be applicable. An important example of such a process is the transport of heat by groundwater flow systems. Thermal dispersion at the macroscale during such processes essentially result in an enhanced apparent thermal conductivity both parallel and normal to the flow, as shown by equation 19.

A different approach to the problem of formation heterogeneity is to consider the various segments of the formation separately. Heat transfer within each part of the formation can then be considered at the microscale. Dispersion at this scale is generally negligible and the governing equation, derived from equation 13, becomes:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x_i} (D_h)_{ij} \frac{\partial T}{\partial x_j} - \frac{\rho_f c_f n v_i}{\rho_e c_e} \frac{\partial T}{\partial x_i} \qquad 20$$

In the less permeable segments the convection term will often be negligible.

This approach generally requires a highly simplified model of the geometry of the heterogeneities, but it has the advantage that equation 20 does not involve the questionable assumptions of the macroscale approach. Similar approaches for pressure diffusion (Streltsova, 1978) and for solute transport in fractured rock (Neretnieks, 1980; Smith and Schwartz, 1980; Tang et al 1981) are leading to valuable insights. In the context of heat transport such an approach has been used for fractured rock by Bodvarsson (1974) and Goguel (1976). Examination of the processes of macrodispersion by this approach is in the early stages, and may lead to improved descriptions of formation heterogeneity and dispersion processes.

In conclusion, macroscopic dispersion is an important factor in many heat transport processes, but as yet a soundly-based theory is not available. For many practical problems a qualitative understanding of the effects of dispersion may be obtained by treating them as an enhanced thermal conductivity roughly proportional to the product of flow velocity times a characteristic dimension of the heterogeneities.

## 2.6 Properties of Water and Porous Media

The variations of the density and viscosity of water with temperature have important consequences for the interaction of water and heat flow. Some of the most important mechanical properties of water in the range -10°C to 110°C are summarized in Table 1 (after Weast, 1975).

Huyakorn and Pinder (1978) give the following expression for the variation of viscosity:

$$\mu = 239.4 \times 10^{-7} \times 10^{\left(\frac{248.37}{T + 133.15}\right)} \qquad 21$$

where $\mu$ is in kg $m^{-1}$ $sec^{-1}$, T is in °C. Due to the strong variation of viscosity the hydraulic conductivity $K$ as normally used in groundwater studies ($K = \rho g k/\mu$) is temperature dependent and not a suitable parameter for use where large temperature changes of the pore fluid may occur. Values of hydraulic conductivity are often quoted without the temperature at which the measurement was carried out so that comparison with other measurements may be unreliable.

The decrease of viscosity with temperature can in principle result in "fingering" of an advancing warm-water front at the most permeable zones. Although this possibility is recognized in the literature there appears to be no detailed theoretical analysis or empirical data available. The problem is one of stability since the fingering effect is counteracted by conduction and dispersion. Fingering may well be one of the causes of the macroscopic dispersion effects that have been observed in field tests of hot water injection.

Data given by Dorsey (1968) show that viscosity is only slightly dependent on pressure and salinity. At 10°C viscosity of water with a salt content of 40,000 ppm is 7.5% greater than that of pure water and at 30°C it is 11% greater. Temperature effects will therefore generally predominate with respect to variations of viscosity.

The changes of water density with temperature are highly significant for buoyancy effects. The coefficient of thermal expansion, $\beta_T$, given in Table 1 is defined by:

$$\frac{\partial \rho}{\partial T} = -\beta_T(T)\rho \qquad 22$$

$\beta_T$ is itself strongly temperature-dependent and therefore a linear approximation for the variation of density with temperature, such as used by many authors (e.g. Combarnous and Bories, 1975; Bear 1972) is only valid for very small temperature changes (see equation 5).

The isothermal bulk modulus of water, $K_w$, as given in Table 1, is defined by the equation

$$\frac{\partial \rho}{\partial p} = \frac{1}{K_w}\rho \qquad 23$$

The significance of temperature effects versus pressure effects with respect to buoyancy are discussed in a following section.

Thermal properties of water and ice are summarized in Table 2, together with a few typical values for common rock types. In general the thermal diffusivity of saturated rock is about an order of magnitude larger than that of water.

A variety of formulas for calculating the thermal conductivity of saturated rocks are available, varying from the "parallel" model to the "series" model (Bear, 1972). Sass et al (1971) suggest that the best estimate of thermal conductivity is given by the geometric mean, weighted by volume fraction, of the thermal conductivities of the rock components including the pore fluid.

The permeability of a porous medium depends on the shape and interconnections of the pores, but is not in any simple manner a function of porosity. Temperature increases can lead to an increase of permeability by micro-cracking and pore expansion (Heard, 1980), but this effect may easily be offset by chemical deposition effects (Summers et al., 1978; Morrow et al., 1981). Clogging of pores due to mobilization of clay particles can result in very marked permeability decreases during hot water injection (Molz et al., 1979).

An increase of total stress or decrease of fluid pressure in a formation should in principle lead to decrease of permeability through decreasing the pore sizes. This effect should be most marked in fractured rock since fractures are generally relatively deformable and fracture permeability in a sensitive formation of fracture aperture. Numerous laboratory studies of flow in deformable fractures have been carried out (see for instance Witherspoon et al 1980, Walsh, 1981), but there appears to be little information in the literature concerning changes of in-sites permeability due to fluid pressure changes.

# 3. Theory of Buoyancy Effects and Free Convection

## 3.1 Introduction

The variation of water density with temperature, leads to a variety of complex buoyancy and free convection effects. These are most evident in geothermal systems, driven by intense heating from below, but they may also be of importance in normal groundwater flow systems and whenever there is artificial heating or cooling in the subsurface.

Buoyancy effects are an important or governing factor for almost all types of interactions between heat and groundwater flow and therefore require special consideration. The theory of free convection is based on the constitutive equations introduced in the foregoing section, but requires the special methods of stability analysis.

## 3.2 The nature of the buoyancy force

Two recent reviews of free convection effects (Combarnous and Bories, 1975, and Cheng, 1978) provide detailed discussions of theoretical and empirical results. The following discussion presents only some of the highlights.

Equation 2, or its equivalent version, equation 9, shows how the buoyancy force acts:

$$nv_i = -\frac{\rho_o g k}{\eta}\left[\frac{\partial h}{\partial x_i} + \left(\frac{\rho}{\rho_o} - 1\right)\delta_{i3}\right] \qquad 24$$

where h is the piezometric head defined by:

$$h = \frac{p}{\rho_o g} + z \qquad 25$$

The buoyancy force is directed vertically and depends on the density difference across the vertical interval in question. The operative density differences are almost entirely those due to temperature variations. Pressure differences also lead to density differences by compression of the fluid, but these lead to only very slight buoyancy effects because they are small and because pressure in natural systems is largely hydrostatic. In equations 24 and 25 the reference density $\rho_o$ can be interpreted as incorporating density changes due to pressure, in accordance with the usual approach for isothermal groundwater flow (e.g. Freeze and Cherry, 1979). Most theoretical treatments of buoyancy assume that density changes due to pressure have no effect. Sorey (1978) found that including pressure-dependency of density in a numerical model had very little effect on the calculated free convection.

To bring out the principal elements of the buoyancy effects, consider a vertical column of the formation of length H, with a vertical temperature difference $\Delta T$ and a piezometric head difference $\Delta h$. Equation 13 combined with 24 gives:

$$\rho_e c_e \frac{\partial T}{\partial t} = \lambda_e' \frac{\partial^2 T}{\partial z^2} + \frac{\rho_f c_f k \rho_o g}{\mu} \left[\frac{\partial h}{\partial z} + \frac{\Delta\rho}{\rho}\right] \frac{\partial T}{\partial z} \qquad 26$$

Here horizontal gradients are assumed to be negligible and the subscript "o" refers to the cool top surface. The right side of equation 26 consists of a conductive heat flux term, a forced convection term and a buoyancy term. Scaling Z,h and T by H,$\Delta$h and $\Delta T$ respectively, the following dimensionless number are obtained for the relative magnitudes of the terms:

$$\frac{\text{Free Convection}}{\text{Conduction}}: \quad \frac{\rho_f c_f g \Delta\rho H k/\mu}{\lambda_e'} = Ra \qquad 27$$

This number is the Rayleigh number, which is basic to discussions of free convection effects.

$$\frac{\text{Forced Convection}}{\text{Conduction}}: \quad \frac{\rho^2 c_f g \Delta h k/\mu}{\lambda_e'} = P_1 \tag{28}$$

This number, the Peclet number, scales the relative importance of forced convective heat transfer to conductive heat transfer.

$$\frac{\text{Free Convection}}{\text{Forced Convection}}: \quad \frac{\Delta\rho/\rho}{\Delta h/H} = F_1 \tag{29}$$

This number gives the relative importance of buoyancy-aided heat flux to forced convective flux (Bear, 1972). Another dimensionless number that is often used in heat transfer studies is the Nusselt number, which is the ratio of the convective heat flux across a surface (usually taken as the top surface) to the conductive flux that would occur across the system by conduction alone (e.g. Garg and Kassoy, 1981):

$$Nu = \frac{\left.\frac{\partial T}{\partial z}\right|_{z=0}}{\frac{\Delta T}{H}} \tag{30}$$

It provides a measure of the relative magnitudes of convective and conductive heat transfer across a sealed system. If there is no convection the Nusselt number equals one.

As shown by Combarnous and Bories (1975) a necessary, but not sufficient, condition for stability is that the temperature gradient must be everywhere vertical. It follows that whenever horizontal temperature variations occur there is free convection flow. On the other hand, even when the temperature gradient is vertical, buoyancy flow will be initiated if the Rayleigh number is large enough. Thus free convection can be considered as due to two distinct effects: flow due to externally imposed horizontal temperature gradients and flow due to the inherent instability of a cold fluid overlying a warm less dense fluid.

### 3.3 Stability Criteria for Free Convection

If the temperature gradient is vertical, free convection may or may not occur depending on the combination of physical variables expressed in the Rayleigh number.

The stability criteria for the onset of free convection have been theoretically studied for many different types of boundary condition (see the reviews by Cheng, 1978 and Combarnous and Bories, 1975). These may be viewed as variations of the basic result for an infinite horizontal system between two flat isothermal plates. For such a system free convection sets in when the Rayleigh number, as defined by equation 27 satisfies the condition:

$$Ra > 4\pi^2 \qquad 31$$

As suggested by equation 27, the Rayleigh number is essentially the ratio of heat flux by free convection to the heat flux by conduction. The physical meaning of condition 31 is that the increase of temperature at some point due to a rising buoyancy flow is larger than the opposing decrease of temperature due to conductive heat loss. The situation is then unstable and convection cells form (see Goguel 1976, p.42).

The viscosity and the coefficient of thermal expansion for water change markedly with temperature. Thus the definition of the Rayleigh number and its critical value depend on which reference values are chosen. This problem has been analyzed by Morland et al (1977) Straus and Schubert (1977), and Horne and O'Sullivan (1978). Their results essentially show that the critical value of the Rayleigh number can be considerably smaller than $4\pi^2$ if the reference value of viscosity is taken to be the value at the "cold" top surface. This result is indicated by equation 27, since the average value of $\mu$ may be considerably less than its highest values at $T_0$.

As noted by Wooding (1960) and Homsy and Sherwood (1976) the decrease of viscosity with increasing temperature also leads to an additional unstability due to "fingering" of the rising hot and less viscous fluid. This effect increases the effective Rayleigh number for rising hot fluid, but decreases it for sinking cold fluid. The asymetry suggests that convection would start with the growth of hot fluid perturbations.

The effective thermal conductivity $\lambda_e'$ in equations 26 and 27 includes the possible effects of dispersion (see equation 13 and the following discussion). Thus if there are significant horizontal or vertical flows superimposed on the buoyancy effects, and if the formation has large dispersivities due to macroscale heterogeneities, then the value of $\lambda_e'$ may be considerably higher than the strict conductive value in the absence of superimposed flow. The result, as indicated by equation 27, should be that the Rayleigh number becomes smaller and thus the system is stabilized as compared to the no-flow case. Rubin (1981) and Tyvand (1981) have given detailed analyses of the effects of dispersion on the onset of free convection, and come to similar conclusion. In view of the present uncertainty in the theory of thermal dispersion, such detailed analyses using particular theoretical models of dispersion have only limited applicability. It is interesting to note that Prats (1966) found no effect of horizontal fluid flow on the onset of free convection, but he neglected dispersion effects. Combarnous and Bories (1975 p.294) give experimental data which show no results of horizontal flow, presumably because the experiments were carried out in a laboratory setting with only (negligible) microscale dispersion. They mention the need for research concerning the effects of nonhomogeneity. Aside from the effects of dispersion, a superimposed vertical flow has a stabilizing effect because it tends to concentrate the vertical temperature variations within a portion of the total vertical thickness of the layer (Wooding 1960, Hornsy and Sherwood, 1976), thereby decreasing the effective value of H with respect to the Rayleigh number.

Many natural formations are strongly anistropic with regard to permeability. The effect of horizontal-vertical anistrophy has been studied by Castinel and Combarnous, (1974). If the permeability k in the definition of the Rayleigh number (equation 27) is taken to be the horizontal permeability, then in the case of horizontal-vertical anisotrophy the Rayleigh number is given by:

$$Ra_{anis.} = Ra_{is.} \frac{4}{[(\frac{k_h}{k_v})^{1/2} + 1]^2} \qquad 32$$

If, for instance, $K_h/K_v = 10$, a typical value for sedimentary formations, then $Ra_{anis} = 0.23\ Ra_{is}$. Thus low vertical permeability has an inhibiting effect on the onset of free convection, as could be expected since the vertical flow due to density differences would be reduced.

### 3.4 Free Convection due to Horizontal Temperature Gradients:

Free convection due to horizontal temperature gradients occur at any value of the gradients so that the question of stability does not arise. In many actual systems free convection may be due to a combination of horizontal and vertical gradients and stability considerations do enter.

Perhaps the clearest case of free convection due to horizontal temperature gradients is that of a point heat source in an infinite medium, analyzed by Bejan (1978), Hickox and Watts (1980), and Hickox (1981). These authors develop the theory for heat and fluid flow due to such a source in terms of the Rayleigh number for a point heat source:

$$Ra' = \frac{\rho_f^{\,2} c_f g \beta_T Q k/\mu}{2\pi\lambda_e^{\,2}} \qquad 33$$

where Q is the rate of heat production by the source. This expression for the Rayleigh number is equivalent to that given by equation 27 if L is taken to be the characteristic dimension of the conductively heated volume around the source, given by equation 10, and the density change is an average value over the heated volume.

For low values of Ra (less than 1.0) the temperature field around the source is essentially conductive, with a minor vertical displacement due to vertical convective motion of the fluid. The flow lines for this case are in the form of a horizontal vortex roll centered on the source. For high values of Ra (greater than 100) the temperature field is dominated by convection and the flow lines take the form of a plume above the source.

For a point source the transition from the conduction-dominated to the convection-dominated case is smooth. The fluid flow velocity at the centre is roughly proportional to the Rayleigh number for all values of Ra, but the velocity may be large even at low values of Ra. The absence of a marked transition in the flow pattern at some critical value of Ra distinguishes the point heat source from that of horizontal isothermal planes.

A noteworthy aspect of the expression for the Rayleigh number as defined for a point source (equation 33) is that it is essentially independent of time. Thus the convective flow velocity is predicted to remain relatively constant even though the dimensions of the heated volume and the temperature at the centre increase with time. (The Rayleigh number may change slightly, due to the change of the values of $\beta_T$ and $\mu$ as the temperature increases.) This somewhat surprising conclusion is due to the fact that as the size of the heated volume increases, the temperature change (and the corresponding density change), averaged over the volume, decreases.

An intermediate case analyzed by Combarnous and Bories (1975) is that of the flow between inclined isothermal planes. In this case a slow free convection occurs even at low Rayleigh numbers due to the horizontal temperature gradient. However, at the critical value of Ra instability sets in and the fluid flow becomes similar to that for unstable free convection between horizontal plates. Laboratory studies described by Combarnous and Bories (1975) and Kanako et al (1974) show good agreement with the theoretical prediction. This result is interesting because it indicates that stability criteria for the overall vertical temperature gradient are applicable even in the presence of free convective flow due to horizontal temperature gradients. In essence the stabilizing mechanism of conductive heat flow continues to operate. Most naturally occurring free convection systems are associated with both concentrated heat sources and an overall vertical temperature gradient and either or both may be driving the convection.

## 3.5 Patterns of Free Convective Flow

The reviews by Combarnous and Bories (1975) and Cheng (1978) describe the various types of flow patterns that arise due to thermal instability. The basic pattern for free convection between horizontal isothermal planes consists of hexagonal cells with approximately equal height and width. Hot water rises at the centre of the cell and cooled water sinks down at the edges.

For Rayleigh number greater than about 280-320 the flow pattern itself becomes unstable and the flow becomes oscillatory with a period of the order of 0.001 to 0.01 of the thermal diffusion time defined by equation 10 (Cheng, 1978, Schubert and Strauss, 1979). The mechanism for the onset of this oscillatory behaviour is not clearly established, but appears to involve the boundary layer conditions.

When horizontal forces affect the convective flow, for instance horizontal temperature gradients (Weber, 1974), inclined planes (Combarnous and Bories, 1975), or dispersion effects of a superimposed horizontal flow (Rubin, 1981), then the most stable pattern may be convective rolls.

The decrease of viscosity with temperature tends to speed the upward flow of the hot fluid and to concentrate it in a narrow plume (Horne and O'Sullivan, 1978).

It is interesting to note that laboratory observations of convective cells always show hot fluid rising at the centre. For a fluid with constant viscosity and a constant coefficient of thermal expansion it would seem that the reverse configuration with cold fluid sinking at the centre should be equally possible. The observed configuration may be related to the greater instability of rising hot fluid due to the fingering caused by viscosity contracts. Thus the lesser viscosity of the hot fluid causes it to rise in narrow plumes whereas the downward flow of cold fluid is diffused over a relatively large area around the hot plume.

The boundary conditions occuring in nature are different from those used for most theoretical and laboratory studies. The lower boundary of a natural convective system may be viewed as a thick slab in which heat flow is only conductive. The thermal conditions at the upper surface may be largely controlled by the complex and poorly understood processes of heat transfer at the ground surface. The onset of free convection in a permeable layer increases the heat transport across the layer, reduces the temperature difference and will tend to cool the bottom surface and heat the top surface. Thus naturally-occurring free convection would tend to be self-quenching and possibly periodic with a period dependent in part on the heat transfer processes above and below the layer. Virtually all natural systems are also affected by horizontal temperature gradients and by fairly localized heat sources which would tend to force cell formation.

## 4. Forced Convection in Groundwater Flow Systems

### 4.1 Groundwater Flow and Subsurface Temperatures

Both theory and observations show clearly that groundwater movements can have a major effect on subsurface temperatures and heat flow, even when buoyancy effects are negligible. In natural flow systems groundwater flows of centimeters per year may be sufficient to produce large convective heat flow.

The governing parameter is the Peclet number, the ratio between forced convective and conductive heat flow. If the heat and water flows are vertical then the Peclet number as defined by equation 27 can be used:

$$P_1 = \frac{\rho c_f q_w}{\lambda_e'/H} \qquad 34$$

Here $q_w$ is the groundwater flow per unit area given by:

$$q_w = nv_3 = \rho g \frac{k}{\mu} \frac{\Delta h}{H} \qquad 35$$

Sass and Lachenbruch (1977, p.642) show that for such a vertical column with no horizontal flow the ratio of the conduction heat flows at the top and bottom of the column is given by:

$$q_h(o)/q_h(-H) = e^{P_1} \qquad 36$$

where $P_1$ is defined by equation 34. The temperature in the column is given by (Bredehoeft and Papadopulos, 1965: Lubimova et al, 1965):

$$T(z) = (T_H - T_o) \frac{(1 - e^{P_1 z/H})}{(1 - e^{-P_1})} + T_o \qquad 37$$

Equations 35 and 37 illustrate how the heat flow and temperature are dependent on the Peclet number. The quantity $\lambda e/\rho c_f$ generally has a magnitude of about $10^{-6}$ $m^2$/sec. Thus for a formation with a thickness of 100 a vertical groundwater flow of $10^{-9}$ m/sec (or about 3 cm/year) would suffice to give a Peclet number of about 0.1, which would indicate a 10% change of the conductive heat flow at the top as compared to the bottom of the formation, (equation 36).

Equation 37 has been used to determine vertical groundwater flow from temperature measurements, as suggested by Bredchoeft and Papadopulos (1965). This procedure is subject to considerable difficulties as discussed in a following section.

For considering the effect of groundwater flow systems on heat flow and temperatures, a two-dimensional approach is more suitable. A number of theoretical studies of these effects have been described in the literature. Palciauskas and Domenico (1973) derive an analytical solution for a simple isotropic flow system of length L and depth H. Their solution is only approximate since it neglects horizontal heat convection, but nevertheless, it is very useful since it allows a first-order consideration of the parameters governing the forced convection of heat by the groundwater flow. According to these authors the effects are governed by the parameter

$$\chi = \frac{\rho c_f K \Delta h / L}{\lambda_e / H} \qquad 38$$

where $\Delta h$ is the water table elevation at the midpoint of the flow system and K is the hydraulic conductivity ($K = \rho g k/\mu$). This parameter may be viewed as a Peclet number, representing the ratio of convective heat flux per unit area to the conductive heat flux per unit area. A similar definition of the Peclet number was given by Kilty and Chapman (1980).

Close inspection of the analytical solution given by Domenico and Palciauskas indicates that the parameter $\chi$ does not adequately characterize the forced convection effects if the system is fairly flat, i.e. if L/H is large as in the case for virtually all natural flow systems. In analogy with equation 34 the Peclet number for a two-dimensional flow system can be defined by:

$$P_2 = \frac{\rho c_f Q_w}{\lambda_e' L/H} \qquad 39$$

where $Q_w$ is the total water flow per unit width of the system at the boundary between the recharge and discharge areas. $P_2$ represents the ratio of the total convective heat flux to the total vertical conductive heat flux. For the solution given by Domenico and Palciauskas $P_2 = (\pi H/L)X$, and it turns out that if H/L is small the ratio of the convective to conductive discharge at the top surface, and the thermal gradients at the top surface can be concisely given in terms of $P_2$. Essentially the Peclet number as defined by equation 39 allows for the "flatness" of the system.

Domenico and Palciauskas present some figures, but a more detailed description of the water and heat flow interaction is provided by the work of Betcher (1977) who carried out a numerical modelling study of temperature distributions in groundwater flow systems. Some of Betcher's results are reproduced in Figures 1-5. These results are all based on $\lambda_e = 1.65$ $w/m^{-1}\,°C^{-1}$, a geothermal flux of 41.9 $mWm^{-2}$, and an isothermal boundary condition at the top surface.

The temperature patterns given in Figures 1 and 2 give an indication of how the subsurface temperatures can be effected by groundwater flow. The governing parameter in the dimensionless Peclet number $P_2$ as defined by equation 39. Figure 3 shows how the geothermal gradients are affected by groundwater flow. Near the top surface the gradients vary by a factor of 2 for $P_2 = 0.32$ and for $P_2 = 3.2$ the gradient varies from practically isothermal at the recharge end to about 3.6 times the average geothermal gradient at the discharge end.

The overall effect may be summarized by the statement that the groundwater flow tends to "sweep up" the terrestrial heat flow and eject it by convection and conduction over a relatively small discharge area. At high Peclet numbers most of the system is cooler than it would be if no convective effects occurred, but at the discharge end temperatures may be much higher. The characteristic magnitudes of the quantities involved in the definition of the Peclet number $P_2$ are such that significant forced convection effects may be expected to occur frequently in nature.

The theoretical predictions of forced convection effects, summarized above, are borne out by the results of many field observations. Kilty and Chapman (1981) carried out modelling studies of three different areas and show that in each case forced convection plays a major role with regard to the subsurface temperature distribution and conductive heat flow at the surface.

Brott et al (1981) in a study of the Eastern Snake River Plain, Idaho, concluded that the average heat flux by forced convection is about 6 times the average conductive heat flow. An estimate of the Peclet number using data provided by these authors gives a value for $P_2$ of about 4.0, which corresponds roughly to the estimated ratio of convective to conductive heat flux.

Benoit (1978), Blackwell and Morgan (1976) and Chapman et al (1981) describe instances of shallow subsurface temperature and heat flow being influenced by convective heat flow at greater depth. In such "blind" areas convection by groundwater flow may result in important thermal anomalies with little or no surface expression.

In an earlier detailed study Parsons (1970) concluded that the subsurface temperatures in a shallow groundwater flow system was controlled by complex interactions between terrestrial heat flow, convection by groundwater, and heat transfer processes at the ground surface. Lewis and Beck (1977) found significant variations of conductive heat flow within a small area of the Canadian Shield and attribute these to groundwater convection in major fractures.

The results of Betcher's work, discussed earlier, assumed isothermal conditions at the top surface. The ground surface temperature is largely controlled by radiation and evaporation at the ground surface, but the groundwater temperature at the top surface may vary significantly from the average ground temperature. Groundwater discharge areas will generally tend to have warmer groundwater temperatures, the extreme case being hot springs. Recharge areas may have colder groundwater temperatures, depending on the temperature of the infiltrating water. Brott et al (1981) observed anomalously low temperatures and negative near-surface heat flow in the recharge area of the Eastern Snake River Plain. They ascribe these phenomena to recharge by cold snow-melt water. Cartwright (1970, 1974) used deep and shallow thermal anomalies to trace groundwater flow systems.

## 4.2 Heat Flow Measurements and Forced Convection

The discussion of the foregoing section show that the heat carried by forced convection of groundwater may often constitute a significant portion of the total terrestrial heat flow near the surface. This possibility has long been recognized (see for instance Jaeger (1965) and the review by Lachenbruch and Sass, 1977). Detailed field studies frequently encounter thermal anomalies due to convection, to the extent that in areas of active groundwater flow such anomalies may be expected to be the rule rather than the exception.

The study of heat flow in the Eastern Snake River Plain by Brott et al (1981) is a prime example of convection effects, but more subtle effects are also common. Conductive heat flow at the surface may be strongly perturbed by groundwater circulation, perhaps at considerable depth and with little or no surface expression (Benoit, 1978; Chapman et al 1981). The detailed study by Lewis and Beck (1977) showed that even in a shield area conductive heat flow may be affected by groundwater convection through fractures. Majorowicz and Jessop (1981) suggest that regional heat flow anomalies in the western Canadian sedimentary basin may be due at least in part to groundwater movements in deep flow systems extending over the full width of the basin. Eckstein and Simmons (1978) report on many heat flow measurements in Israel and attribute the variability of the conductive heat flow to active groundwater circulation at shallow depth.

The modelling results obtained by Betcher (1977), shown in Figures 1-3, illustrate the nature of the convection effects on conductive heat flows. As the Peclet number $P_2$ approaches values of 1.0 and more, much of the total heat flux is discharged by convection. The thermal gradients near the surface are smaller than they would be without convection over most of the extent of the flow system. Only at the bottom of the discharge area is the conductive heat flow enhanced by convection. These results show clearly that regional heat flow determinations based on measurements of conductive heat flow only will in general tend to underestimate the total heat flow. This discrepancy may be the greater if, as is often the practice, heat flow measurements are discarded if the temperature gradients show evidence of vertical groundwater flow. Such a selective procedure may result in neglect of the strong conductive heat flow in groundwater discharge areas. A case in point is the study by Eckstein and Simmons (1978) who selected only the lowest values of conductive heat flow to represent the regional heat flow. They recognize that the regional heat flow value which they arrive at in this manner, 39.5 $mW/m^2$, is rather low for the tectonic setting of Israel. Inclusion of convective heat flux would no doubt have considerably increased their value for the regional heat flow.

Both theory and observations suggest that convective heat flow effects are common in nature. Groundwater flows, perhaps at great depths, can significantly affect conductive heat flow near the ground surface. Neither linear temperature gradients nor consistency of measurements in neighbouring boreholes can guarantee that the total heat flow is conductive. In general, heat flow measurements based on conduction only will tend to underestimate regional heat flows. To be reliable heat flow measurements must consider all possible groundwater flow systems and the associated groundwater heat budgets.

Groundwater flow could also have an effect on heat flow through the dispersion mechanism. Equation 17 shows that dispersion leads to an apparently enhanced thermal conductivity, both parallel and perpendicular to the flow. Thus for instance, a strong horizontal flow in a formation with large-scale heterogeneities could result in an enhanced vertical thermal conductivity, a reduced vertical temperature gradient and an enhanced heat flow by conduction. Laboratory measurements of thermal conductivity would not include the effects of dispersion and consequently heat flow would be underestimated. Macroscale dispersion, particularly for flow systems of very large extent, is poorly understood (see the discussion following equation 17), but the effect could be important for strong flow systems. For instance, in the case of the Eastern Snake River Plain (Brott et al 1981) the average horizontal discharge is about $2 \times 10^{-6}$ m/sec. A low transverse dispersivity of 0.10 meters would increase the thermal conductivity (about 2 $Wm^{-1}$ $k^{-}$) by a dispersive term equal to $\rho c_f \, q \, \alpha = (4.2 \times 10^{6}) \times (2 \times 10^{-6}) \times (0.10) = 0.84 \ W \, m^{-1} K^{-1}$ (see equation 19). Thus the conductive heat flow may well have been seriously underestimated in this case. Much larger transverse dispersivities, say, 1.0 meters, or even 10 meters, are conceivable and therefore even much slower flows may give rise to significant dispersion effects. This particular aspect of dispersion effects does not appear to have been treated explicitly in the literature.

Some numerical modelling studies of heat and fluid flow account for dispersion effects by using an enhanced value of $\lambda_e$. Sorey (1978) suggest such and approach and Mercer and Faust (1979) use $\lambda_e' = 10\,\lambda_e$ in modelling the Wairakei geothermal system.

## 4.3 Determination of groundwater flow from temperature measurements

Since groundwater flow can lead to significant anomalies of subsurface temperatures and heat flow, it follows that measurement of such anomalies can yield information concerning groundwater flow. Such information requires considerable interpretation, however and therefore the most effective studies would include direct measurements of both water and heat flow.

Vertical groundwater flow can in principle be determined from a measurement of the variation of temperature with depth. This method was apparently first proposed by Bredehoeft and Papadopulos (1965). The theory is represented by equation 37:

$$T(z) = (T_H - T_o)\,\frac{(1 - e^{P_1 z/H})}{(1 - e^{-P_1})} + T_o \qquad 40$$

where $P_1$ is the Peclet number for vertical flow.

$$P_1 = \frac{\rho c_f q_w H}{\lambda_e'} \qquad 41$$

This equation assumes vertical flow only of water and heat, an assumption which is not always valid since the water must move horizontally at some depth and this flow will give rise to horizontal temperature gradients (Lachenbruch and Sass, 1977).

Sorey (1971) describes applications of this method for several areas. His resulting values for groundwater flow are consistent with the values obtained by other methods. Even with careful measurements values of $P_1$ less than about 0.1 cannot be reliably determined. Since $\lambda_e/\rho c_f$ generally equals about $10^{-6}$

$m^2$/sec, the value of $q_wH$ should exceed about $10^{-7}$ $m^2$/sec for the method to be practicable. Thus the method is most useful for thick layers. Cartwright (1974, 1979) also reports applications of the method, with partial success.

A major limitation on this method is that the temperatures at the top and bottom are assumed constant in time. Ground-surface temperatures however generally have changed due to changes of climate and of surface cover. Boyle and Saleem (1979) applied the method with some apparent success, but the overall negative temperature gradients which they encountered may well be due to recent surface-temperature changes and the validity of their results is therefore questionable. Similar reversals of the geothermal gradient at shallow depths are often encountered (e.g. Hyndman et al, 1979) and constitute an important limitation for the determination of heat flow or groundwater flow from temperature measurements (Beck, 1977).

The propagation of the annual temperature wave into the subsurface is also affected by vertical water flow. In principle determination of the lag and attenuation of the temperature wave can allow a calculation of thermal diffusivity and of the vertical groundwater flow (Stallman, 1965). The sensivity of the method is such that a minimum flow of about $10^{-8}$ m/sec can be detected so that its usefulness is limited to cases of strong vertical flow, such as might occur in the discharge areas of strong flow systems or under impounded water (e.g. Nightingale, 1975). Watts and Adams (1978) report the use of annual temperature variations to determine thermal diffusivity.

Horizontal groundwater flows can also lead to measurable temperature anomalies. Such anomalies may be used to obtain information about the groundwater flow systems (Cartwright, 1970, 1974; Kilty and Chapman, 1980; Chapman et al, 1981; Lewis and Beck, 1977). However, unless detailed information concerning the hydrogeology is available such interpretations in terms of groundwater flow tend to be qualitative only and of limited usefulness.

## 4.4 Buoyancy Effects and Groundwater Flow

In the foregoing discussion of forced convection it is assumed that the heat flow does not affect the groundwater flow, in other words, that buoyancy effects are negligible. In nature many examples are known of free convection systems where buoyancy effects dominate the flow. It is to be expected therefore that there are also many flow systems which are driven by a combination of buoyancy effects and by piezometric conditions due to recharge and discharge of water. It is also likely that in some cases one or the other of the driving forces has not been recognized.

A first approach for evaluating the relative magnitudes of the two driving forces is suggested by Bear (1972). It is expressed through the dimensionless ratio of free versus forced flow defined by equation 30:

$$F_1 = \frac{\Delta\rho/\rho}{\Delta h/H} \qquad 42$$

where $\Delta\rho$ is the density difference due to temperature differences, and $\Delta h$ is the driving piezometric head. However, this equation applies for one-dimensional flow in a vertical column only. In a groundwater flow system the buoyancy effects will oppose the piezometric flow at the recharge end and aid it at the discharge end. The operative buoyancy force is thus related to the horizontal density differences along the system. An analysis of this problem does not appear to be available in the literature. The ratio of the net buoyancy forces to the piezometric forces for two-dimensional flow can be characterized by:

$$F_2 = \frac{(\Delta\rho)_{hor.}/\rho}{\Delta h/H} \qquad 43$$

where $(\Delta p)_h$ is the horizontal density difference along the system. For the system shown in Figure 1 $\Delta h/H = 200/2000 = 0.1$. For the case $P_2 = 3.2$, the density difference is about 0.01 giving $F_2 = 0.10$. These rough calculations would suggest that for such relatively deep systems buoyancy forces may contribute to the overall flow. If the system were 200 meters deep rather than 2000 meters the buoyancy forces would be reduced by at least a factor of 10 and would be negligible.

The detailed study of heat and groundwater flow in the Long Valley Caldera, described by Sorey et al., (1978) provides a good example of thermal effects in a deep flow system. With H = 2 km, L = 20 km and other parameter values estimated from the data given by the authors, the values of $F_2$ comes to about 0.2. Thus the fluid flow is apparently dominated by forced convection, but with a significant free convection effect. The value of $P_2$ comes to about 7, indicating that the temperatures should be strongly influenced by groundwater flow. These conclusions are borne out by the data and modelling studies reported by the authors.

In general the horizontal temperature gradients caused by forced convection may give rise to buoyancy forces, especially if the flow system is deep and the formations highly permeable. Thus free convection may occur in the absence of enhanced heat sources. Whenever significant buoyancy effects are to be expected the groundwater flow is governed by both pressure gradients and density differences, as expressed by equation 2, and a description of the flow in terms of piezometric head only (equation 7) is not possible.

## 5. Observations of Natural Free Convection

### 5.1 General Remarks

Numerous instances are known of flow systems which are apparently dominated by the free convection of fluid and heat. The growth of interest in geothermal energy has sparked many investigations of such systems. Detailed reviews of principles and case histories are given by Garg and Kassoy (1981), Rybach (1981) and others in the review volume edited by Rybach and Muffler (1981). Cheng (1978) presents an extensive review of heat transfer in geothermal systems and detailed descriptions of recent findings are also given by Lackenbruch and Sass (1977) and others in the geophysical monograph edited by Heacock (1977).

In general continental geothermal systems appear to occur as more or less isolated plumes or lines of rising heated water, surrounded by a relatively cooler environment. These systems probably are due to enhanced heat flow from fairly concentrated sources. Concentration of the rising plumes within small areas may be enhanced by viscosity effects, the occurrence of narrow permeable zones, and forced convection. The free convection systems which have been encountered under the ocean floor at the midocean ridges are more akin to the theory and laboratory observations for convection due to uniform heating from below. Subsea heat flow measurements indicate periodic variations of heat flow such as would be caused by a system of convective cells or rolls.

Data for natural free convection systems are generally sparse. Even for the most well-known geothermal systems there is considerable uncertainty concerning flow patterns and permeability, especially at the deep end of the system. The observed temperatures, pressures and flows can be accounted for and to some extent predicted by theoretical models, however, the agreement between theory and observations is semi-quantitative at best. There is no obvious reason to question the validity of the basic theory, but it is certainly possible that important effects can be overlooked through fitting an inadequate theoretical model to limited data. The predictive value of theoretical models is generally low due to the uncertainties with regard to formation parameters and the governing processes.

## 5.2 The Relevance of Stability Criteria

For free convection due to a concentrated source of heat, such as appears to be the case for most continental geothermal systems, the transition from conduction to convection-dominated heat flow is gradual. The relevant Rayleigh number for such systems can be defined by equation 33 (Bejan 1978). Model studies by Elder (1967) suggest that a single convective cell will be set up as long as the width of the heat source is less than twice the thickness of the overlying formation in which convection occurs.

In practice the Rayleigh number is inferred from the observed flow pattern and temperature gradients. A reliable independent determination of the Rayleigh number is generally not possible because the overall permeability of the system is not known. The problem is complicated by the fact that virtually all natural formations are anisotropic and heterogeneous. Comparison between observations and theory is based on numerical modelling results which roughly reproduce the observations (e.g. Ribando et al, 1976; Turcotte et al, 1977; Sorey, 1978; Fehn and Cathless, 1979; Goyal and Kassoy, 1980). Thus, strictly speaking, the validity of criteria for the onset of free convection cannot be said to have been verified by observations of natural systems.

In principle free convection effects could occur in shallow highly permeable aquifers with normal terrestrial heat flow. Hamza et al (1978) report measurements of isothermal conditions in aquifers and ascribe these to free convection. Eckstein (1982) also reports near-isothermal conditions in a shallow aquifer above and isolated from a geothermal system. He attributes these to mixing due to strong horizontal flow, but free convection could also be responsible. Detailed studies of free convection in shallow aquifers due to uniform terrestrial heat flow could provide an interesting test of the theory for onset of free convection because independent determination of permeability would be feasible. However, no such studies appear to be reported in the literature.

The problem of the criteria for the onset and extinction of free convection are particularly relevant for subsea circulation near mid-ocean ridges (Anderson et al, 1977; Fehn and Cathless, 1979; Ribando et al, 1976). Observations show clearly that free convection occurs in the crustal material near the ridges in a pattern corresponding to that for uniform heating from below. This result is in general agreement with the theory in view of the high heat flow and relatively high permeability of the material. However, the extinction of free convection further away from the ridge is less well understood and could be due to reduced permeability, blanketing by sediments, reduced heat flow, or some combination of these effects.

The stability criterion for uniform heating from below (equations 27 and 31) also suggest that free convection may occur in thick formations at depths of kilometers or more under the continents, especially if the rock is fractured and terrestrial heat flow is high (Norton, 1977). Such convective systems may find surface impressions as geothermal systems, but could conceivably go unrecognized other than as slightly anomalous near-surface heat flows and "hot spots". In past earth history geothermal gradients and temperatures probably were higher than at present and free convection of pore fluids may well have been much more common.

The significance with respect to stability criteria of such factors as dispersion, formation heterogeneity, anisotropy, complex boundary conditions and temperature-dependent viscosity are only understood, if at all, from theory and laboratory experiments. The paucity of detailed information for naturally occurring free convection in effect means that such second-order effects have not been empirically tested for natural geological formations. A careful scrutiny of natural hydrogeological systems characterized by near-critical Rayleigh number would serve to verify the theory and might well show up interesting anomalies with respect to the onset or extinction of free convection. Experiments involving artificial heating or cooling could also prove useful in this regard.

## 5.3 Observed Patterns of Free Convection

Continental free convection systems in general occur as relatively isolated plumes or lines of rising hot fluids. The location of the hot-water plumes appears to be strongly forced by localized heat sources and by areas of highest vertical permeability. The convective cells occur more or less in isolation. The ideal pattern of multiple convective cells or rolls, which would occur for an extensive horizontal layer heated uniformly from below, apparently has not yet been encountered, at least not in easily recognizable form.

The observed patterns of free convective circulation correspond more or less to the theoretical patterns predicted by means of modelling studies. A large number of such studies are described in the literature (e.g. Donaldson, 1970; Lachenbruch et al, 1976; Sorey et al, 1978; Kilty and Chapman, 1980; Garg and Kassoy, 1981; Goyal and Kassoy, 1981; Fradkin et al, 1981; Wooding, 1981). Typical features of convective cells which are commonly encountered are: high temperatures and near-isothermal conditions in the rising plume, temperature inversions around the plume, and high conductive heat flow and/or hot water discharge at the ground surface in the area of the plume. Pore pressures should be higher on the rising plume than in the down-going part of the system, but little data to corroborate this prediction is available. The hot-water plumes are generally concentrated in fairly small areas and are often associated with faults or other indicators of high vertical permeability. This characteristic feature is probably due to localized heat sources and to the effects of temperature-dependent viscosity which would tend to concentrate the plume and favour rising rather than sinking in the most permeable zones (Horne and O'Sullivan, 1978).

Theoretical studies predict that the convective flow pattern should be unstable for high Rayleigh numbers (e.g. Schubert and Strauss, 1979). However, the dimension of the free convection systems that have been studied are generally so large that the characteristic times of the transients would be hundreds or thousands of years so that transient effects would not be directly observable, and in fact no such transient effects appear to have been documented. In this regard it would be of considerable interest to locate and study shallow and small free convection systems such as could exist in highly permeable aquifers (e.g. Hamza et al 1978).

Hydrothermal systems driven by conductive heat flow from below may exhaust the available sensible heat because the heat flux through the system is enhanced (Lachenbruch and Sass, 1977). The system may stabilize or quench itself in a relatively short time of up to $10^6$ years depending on the thickness of the system. In the latter case the conductive heat flow after quenching would be reduced below its regional value for long afterward and near-surface temperatures would initially be high. Eventually the convective systems could start up again. Such transient effects due to limited supply of heat should lead to heat flow anomalies and the occurrence of ______ extinct systems which could be identified from hydrothermal deposits and alterations (e.g. Norton and Knight, 1977). Heat flow anomalies of this type would not be easily distinguishable and none appear to have been documented as yet, although the association of high temperatures with low gradients reported by Carron et al (1980) could be explained by invoking recently extinguished convective systems. Quenching of a free convection system by hydrothermal sealing of the rock would similarly lead to high temperatures and low temperature gradients.

Measurements of heat flow at the mid-ocean spreading centres provide perhaps the only clearly documented instances of naturally occurring free convection due to uniform heating from below. Detailed measurements have shown periodically varying conductive heat flow with a wavelength of 5-20 km (Williams et al, 1974; Davis et al, 1980). The convective motions presumably occur in rolls parallel to the axis of the ridge, induced by horizontal gradients of temperature and permeability, perpendicular to the ridge axis. Heat flow measurements on a two-dimensional grid do not appear to be available so that the possible occurrence of cells rather than rolls cannot be excluded.

The occurrence of free convection in the subsea crust near the mid-ocean ridges has been confirmed by other observations. Anderson and Zoback (1982) measured pore pressures varying from 8 to 12 bars below hydrostatic in the crust of the Costa Rica Rift, indicating strong downward flow of pore water. They also determined relatively high permeabilities of 2 to 40 millidarcies (2 to 40 x $10^{-15}$ $m^2$ ). Hyndman et al (1976) and Kutas et al (1979) measured anomalously low heat flows at the ocean floor and attribute these to water circulation in the crust and the sediment cover.

The patterns of heat and fluid flow in the oceanic crust have been modelled by Ribando et al (1976) and Fehn and Cathles (1979). Anderson et al, (1977) show that the measured conductive heat flows are anomalously low near the ridges and that convective heat flux can account for this anomaly.

It seems that heat flow and permeability in the oceanic crust are uniform enough to give rise to an overall pattern of multiple convective rolls. Clearly it would be of considerable interest if similar systems could be located and studied on land. The sub-ocean systems are of course different in that the top boundary consists of approximately isothermal ocean-bottom water. Thus part of the convective circulation can take place through the ocean above the crust. At the same time forced convection is not a factor since the piezometric head is constant at the top boundary.

## 6. Artificial Changes of Subsurface Temperature and Pressure

### 6.1 Thermal Energy Storage in Aquifers

The possibility of storing heated or cooled water in quifers for subsequent re-use has received considerable attention in recent years. Several experimental studies have been reported in the literature and others are no doubt being carried out at present. One of the principal concerns is the question of how much of the thermal energy can be recovered, but the experiments also have yielded interesting information on the mechanisms of the interaction of heat and fluid.

Detailed descriptions of a heat storage experiment have been given by Molz et al (1979) and Molz et al (1981). The heated water was injected into a confined aquifer and thermal energy recovery was high, ranging from 66% to 76% in the temperature range from 55°C to 33°C. Modelling of the experiment (Tsang et al, 1981) indicated that forced convection in the aquifer and heat conduction in the overlying aquitard were the principal heat transport mechanisms. The model did not include the effects of dispersion, but comparison with the data indicated a slight amount of dispersion. Free convection led to a certain degree of "mushrooming" of the hot water. It proved possible to simulate the final production temperatures very closely.'

Mathey (1977) and Werner and Kley (1977) report experiments involving heat storage in shallow water-table aquifers. The results show that heat losses to the ground surface can be important for shallow aquifers, especially if aided by free convection in the acquifer. Displacement of the hot water "bubble" by regional flow can also greatly reduce heat recovery (Mathey, 1977). Sauty et al (1978) report on a number of hot-water injection experiments. Their results indicate important macrodispersion effects leading to an apparent thermal conductivity of up to 20 times the purely conductive value.

Andrews and Anderson (1979) modelled the thermal effects of water seeping from a cooling pond. They found that longitudinal dispersivity of 0.10 m gave the best match to data. This value is rather low in view of the path length of about 100 m, but the low value may be due to the detailed modelling of the subsurface which the authors found necessary. By way of contrast Kinmark and Voss (1980?) found thermal dispersivities of 2 to 7 meters for a path length of about 40 m. Werner and Kley (1977) used high thermal diffusivities of 3.2 to $6.4 \times 10^{-6}$ $m^2$/sec to account for dispersion in modelling of their thermal energy storage data.

The results of the thermal energy storage experiments show that the efficiency of thermal energy recovery is influenced by four main factors: a) buoyancy effects leading to "mushrooming" of the hot water body, b) hydrodynamic dispersion leading to increased diffusion of the heat, c) conductive and convective losses through upper and lower aquifer surfaces, d) displacement of the heated water by background flow. These factors put strong limits on the suitability of aquifers for thermal energy storage. The aquifer should have low vertical permeability to avoid strong buoyancy effects. The aquifer should be relatively homogeneous to reduce dispersion effects. The aquifer should be relatively thick to minimize the relative importance of conductive losses through confining layers. The confining layers should be highly impermeable to minimize convective losses through these layers. In addition, the aquifer should be deep and have relatively high horizontal permeability so that sufficient quantities of water can be injected or withdrawn without undue pressure changes. In summary, the ideal aquifer for thermal energy storage would be deep-lying, well-confined, thick, homogeneous, have strong horizontal-vertical anistrophy with regard to permeability, and have low regional flow velocities.

Although it falls outside the scope of this discussion it should be noted that chemical and physical changes of the aquifer material are also important considerations, especially with regard to clogging of the aquifer during injection.

The theory of thermal energy storage in aquifers is closely analogous to that of solute transport by groundwater and many of the analytical and numerical models for solute transport can be adapted for heat storage. The principle difference is that the thermal diffusivity is about three orders of magnitude larger than solute diffusivity. In addition, changes of viscosity and density must be taken into account for thermal problems. It should be noted that fluid pressure diffusivity is again orders of magnitude larger than thermal diffusivity so that virtual pressure equilibrium will be achieved very quickly during injection and withdrawal.

On the whole the theoretical and experimental results suggest that the injection and recovery of thermal energy in aquifers is a fairly predictable process. There is a need for further theoretical work aimed at defining dimensionless numbers, such as Peclet and Rayleigh numbers, to characterize the relative importance of the various heat transport processes of conduction, free convection and forced convection in the aquifer and the overlying and underlying formations. Sauty et al (1978) propose two such numbers to describe forced convection versus conduction and Hellström et al., (1979) give a theoretical consideration of buoyancy effects in the aquifer. Such expressions for generally applicable characteristic numbers would greatly aid in the comparison of various experiments and in the site selection and design of thermal energy storage projects.

To date the study of thermal disturbances in shallow aquifers have yielded the only reliable data on the nature and importance of thermal dispersion. Analyses of data have yielded dispersivities ranging from practically nil to large values equivalent to experimental values for solute dispersivities. The phenomena of thermal dispersion are as yet poorly understood but they are clearly of considerable potential significance for a wide variety of thermal processes involving fluid movements.

## 6.2 Heat Extraction from the Subsurface

Deep hot aquifers represent a potentially significant source of energy (Jessop, 1976). Projects investigating the feasibility of extracting heat from such aquifers are underway at present (Menjoz and Santy, 1982; Vandenberghe and Vigrass, 1979) but as yet data for a functioning system do not appear to be available in the literature. The theory of single and double well systems has been treated by Gringarten and Santy (1975), Gringarten (1977), and Menjoz and Santy (1982).

Theoretical considerations show that a system of one of more pairs of doublet wells is much more efficient than a single well in terms of total energy extraction. A doublet well system, with no net fluid extraction also has only a localized influence on fluid pressures in the aquifer. On the other hand a doublet well system has a finite life-time due to breakthrough aided by dispersion, of cold water from the injection well. Heat flow by conduction from over and underlying formations can greatly increase the total recoverable energy and lifetime of the system. If heat is extracted during part of the year only the total lifetime of the system will be further enhanced since heat conduction into the aquifer will carry on while the system is not in use.

As for the case of thermal energy storage the processes should be fairly predictable. Characteristic numbers can be defined (e.g. Sauty et al, 1978) which can aid in comparison, site selection and design of the systems.

Experiments aimed at determining system parameters have been described by Sauty et al (1978), Hosanski and Ledoux (1982) and Vandenberghe and Vigrass (1979).

Single-hole heat extraction could be carried out by pumping out of the fluid if disposal on the surface of the fluid (usually a brine) and lowered pressures in the acquifer are acceptable. Single-well systems would have the advantage of predictability and avoidance of clogging problems in the injection well, although encrustation in and around the pumping well could also prove to be a problem.

Other variants of the process include the use of water from shallow aquifers for cooling purposes and waste heat storage (Jury et al, 1979; Lippmann and Tsang, 1980; Kley and Heckmann, 1981).

Heat extraction from hot "dry" rock is another potential source of energy. The technical problem is essentially to induce fractures such that sufficient heat exchange surface is available while maintaining appropriate fluid pressure and circulation rate (Harlow and Pracht, 1971; Tester and Smith, 1977). The process is based on recirculation through a doublet well system interconnected in the subsurface by induced fractures. A prototype experiment at Los Alamos has been carried out with reasonable success (Smith, 1978). It appears that the method may become economically viable, but considerable technical difficulties and uncertainties remain especially with regard to the processes of fracturing due to thermal and hydraulic stresses. A single-hole recirculation variant of the method is described by Ratigan and Lindblom, (1978). It has the advantage of good predictability since heat flux from the rock would be essentially by conduction only.

Donaldson and Grant (1981) review the problems involved with heat extraction from hot geothermal systems. They suggest that the useful lifetime of a geothermal field is largely dependent on cold-water inflow from the sides and top. Formation heterogeneity is crucial in this regard. Inflow of cold water along large fractures may quench a system rather quickly, the more so if the flow is so fast that little heat is absorbed from the surrounding rock.

The useful lifetime of the system can therefore be much shortened if the exploitation rate is too high.

In any reservoir a large amount of the total heat energy is in the solid rock and must be extracted by fluid circulation at a rate which strikes the appropriate balance between fluid temperature and total energy extraction. The difficulties are most acute for fractured-rock reservoirs in which fluid flow may be concentrated in a few dominant fractures. Field and theoretical methods for characterizing the behaviour of such reservoirs are still in the initial stages of development (e.g. Bodvarson and Tsang, 1980; Wang et al, 1981).

## 6.3 Subsurface Nuclear Waste Disposal

The heat given off by high-level nuclear wastes buried underground will lead to decreased travel times for the flow of water to the surface, both through the reduced viscosity of the heated water and through buoyancy affects. This effect is obviously of considerable concern with respect to underground disposal of nuclear wastes.

As a first approximation for the effects of a concentrated heat source on fluid flow, the analyses by Bejan (1978), Hickox and Watts (1980) and Hickox (1981) are useful. They show that the heat and fluid flow patterns are governed by the Rayleigh number for a concentrated heat source (equation 33):

$$Ra' = \frac{\rho_f^{\,2} c_f g \beta_T Q k/\mu}{2\pi\lambda_e^{\,2}} \qquad 44$$

Free convection of the fluid occurs for all values of Ra', but the heat flow is conduction-dominated for small values of Ra' and it is convection-dominated for high values of Ra'. An estimate of the value of Ra' allows a preliminary judgment of the effects that are to be expected.

Witherspoon et al (1981) report on the results of in-situ heater experiments. The observed temperature field around the heaters corresponded to conduction only with no indication of convective heat transfer. Using values of the various parameters reported by the authors ($Q = 5$ k W, $k = 10^{-11}$ m/sec) gives a value for Ra' of about $10^{-6}$. Thus convection effects were not to be expected unless the hydraulic conductivity near the heaters were about seven orders of magnitude larger than the overall rock mass value of $10^{-11}$ m/sec.

Runchal and Maini carried out a numerical modelling study of the impact of high level nuclear wastes on the regional groundwater flow. They show that the heat given off by the wastes can lead to strong convective motion of the groundwater and shortened travel times for movement of water from a repository to the ground surface. Calculation of Ra' for the cases they study ($Q = 2 \times 10^5$ kW, $k = 5 \times 10^{-10}$ m/sec to $2 \times 10^{-5}$ m/sec) gives values for Ra' ranging from 6.5 to 260,000. For such values of Q and K strong convective effects are therefore to be expected.

Wang et al (1981) report a theoretical study of thermohydraulic effects due to nuclear waste disposal in hard rock. They use a model in which water circulation is through horizontal and vertical faults only. They show that buoyancy effects may be significant, but they assume that the temperature regime is due to conduction only, in other words that the Rayleigh number is very small. For the case they consider the Rayleigh number would have to be defined in terms of fluid moving in a vertical plane fracture.

The Rayleigh number for a concentrated heat source, Ra', provides a useful guide for the design of heater experiments or of an eventual repository. If convection effects are to be kept small the value of Ra' should be less than 1.0. This restriction implies a maximum value for the product QK of about 0.01 J.m/sec$^2$. On the other hand if heater experiments are to produce significant free convection effects the value of QK should be much greater than 0.01 J.m/sec$^2$. (An important consideration is that fluid flow in the vicinity of a drained vault is likely to be dominated by the large fluid pressure drawdown due to the vault. Also, the permeability of fractures near a drained vault my be much reduced because the low fluid pressure implies a high effective stress on the fractures.)

As the analysis by Wang et al (1981) suggests, if fluid movement near an underground repository is mostly through a few large fractures than a different approach must be used. For such a case thermal equilibrium between the fluid and the solid cannot be assumed.

The value of Ra' as defined by equation 44 is essentially constant in time for a constant heat production rate Q. Thus even though the heated volume increases with time the flow pattern of fluid and heat around a repository should be fairly stable in time. A dramatic onset of strong free convection is not to be expected.

Barring large permeability changes, it would appear that the convective effects of heating due to nuclear wastes should be fairly stable and predictable. Indications are that the magnitudes of such convection effects can be limited to acceptable values on the basis of relatively straightforward design criteria.

## 6.4 Well tests

Well tests for determining formation permeability and storage coefficient (or compressibility-porosity product) are a useful tool in groundwater studies (e.g. Weeks, 1977) and oil reservoir engineering (Earlougher, 1977). Similar tests have been used for geothermal systems (Ramey, 1975; Witherspoon et al, 1978); Narasinhan and Witherspoon, 1979). Aquifer models have also been used by Wooding (1981) to analyze pressure drawdown in the Warakei geothermal region.

The successful application of well tests to geothermal systems, in the presence of free convection flows, and even small fractions of steam, can be viewed as a consequence of the high diffusivity for pressure variations. Even for highly impermeable formations such as solid granite the ratio of permeability to storage coefficient is about $10^{-5}$ to $10^{-2}$ $m^2$/sec. Thus pressure changes are transmitted much more rapidly than temperature changes and the rapid pressure changes due to well testing should be only slightly influenced by the slow accompanying temperature changes.

## 7. Permafrost, Heat Flow and Groundwater Flow

### 7.1 General principles

When a porous water-filled material freezes or thaws its properties with respect to heat and fluid flow change and the freezing boundary also acts as a source of heat and of fluid mass. In the context of permafrost these phenomena lead to a variety of important effects with respect to heat flow and groundwater flow.

As may be seen from Table 2 ice at 0°C has half the heat capacity of water, three times the thermal conductivity and almost seven times as great a thermal diffusivity. Thus a porous material may undergo a large change of its thermal properties when it freezes, depending on its water content and pore structure. The permeability of a porous material decreases drastically when it freezes. Burt and Williams (1974) found that the permeability of a silt decreased from about $10^{-6}$ m/s at °C to about $10^{-11}$ m/sec at −0.4°C. The water in the smallest pores may remain unfrozen at temperatures several degrees below 0°C (e.g. Patterson and Smith, 1981), but the permeability is mostly a function of the largest pores, which freeze first. For practically all cases except fine-grained materials near 0°C, permafrost may be assumed to be impermeable. If the pore fluid has a high solute content its freezing point may of course be significantly lower than 0°C. For instance, a pure sodium chloride solution of 30 parts per thousand has a freezing point of −1.8°C.

A moving freezing boundary acts as a source or sink of heat and fluid because of the latent heat of ice ($L = 3.4 \times 10^5$ J/kg) and the 9 per cent increase of specific volume of water upon freezing. The change of heat and fluid flux (assuming that the frozen material is impermeable) may be written as:

$$(\lambda_e)_p \left(\frac{\partial T}{\partial \ell}\right)_p - (\lambda_e)_u \left(\frac{\partial T}{\partial \ell}\right)_u = Lnw \qquad 45$$

$$(q_w)_u = -K_u \left(\frac{\partial h}{\partial \ell}\right)_u = 0.09\ nw \qquad 46$$

where w is the rate of movement of the freezing front in direction of increasing $\ell$ and the subscripts "p" and "u" refer to the frozen and unfrozen material respectively.

The thermodynamics of the ice-water interface in porous materials leads to important stress effects. McRoberts and Morgenstern (1975) consider this phenomenon in terms of a capillary model. Gilpin (1980) provides an analytical model using a thermodynamic approach and relates the results to ice lensing, frost heave, and permeability of frozen soils. The simple capillary model states that at equilibrium the ice surface in a pore curves outward and this curvature leads to an excess stress in the ice as compared to the fluid pressure. As an equation this condition is expressed by:

$$\sigma_i = p + c \qquad 47$$

where $\sigma_i$ is the stress in the ice. The excess stress C is given approximately by:

$$c = \frac{2\,\sigma_{iw}}{r} \qquad 48$$

where $\sigma_{iw}$ is the surface tension of the ice-water interface and r is the radius of curvature of the interface, approximately equal to the pore radius. Taking $\sigma_{iw}$ = 0.03 N.m (Everett and Haynes, 1965) and r = 2 μm gives a typical value for C of 30 KPa. Measurements on Devon silt gave values of C ranging from 30 to 215 KPa (McRoberts and Morgenstern, 1975; Arvidson and Morgenstern, 1977). In view of the plastic nature of ice $\sigma_i$ may be assumed isotropic.

If p + c exceeds the overburden stress ice lensing will tend to occur because the ice stress is then sufficient to lift the overlying material. (If soil adhesion and the rigidity of the overburden are significant p + c would have to be proportionately larger for lens formation to take place.) More sophisticated models have been developed (e.g. Gilpin, 1980), but this simple model suffices to account for the principal features of ice lens formation.

The following discussion of permafrost phenomena concentrates on processes occurring at the base of the permafrost where it interacts with subpermafrost groundwater. Processes in the active layer at the top of the permafrost often involve moisture migration in unsaturated or partially frozen soils under the influence of thermal and hydraulic gradients (Harlan, 1973; Guymon and Luthin, 1974; Mageau and Morgenstern, 1980). These extend beyond the scope of the present study.

## 7.2 Distribution and Thickness of Permafrost

The distribution of permafrost near the ground surface is largely controlled by surface heat exchange processes (Brown, 1969). Groundwater flow and terrestrial heat flow have only a minor effect in this regard because ground temperature is almost entirely controlled by atmospheric processes and by the absorption and emission of radiation. Convection of heat by groundwater flow can lead to slightly lowered soil temperatures in recharge areas and higher temperatures in discharge areas. In the zone of discontinuous permafrost groundwater flow systems should therefore have some effect on permafrost distribution. Th more extreme cases are well-documented; for example icings due to groundwater discharge (Williams and van Everdingen, 1973) and taliks under rivers and lakes, kept open in part by upward flow of warm groundwater (Pinneker, 1973; Tolstikhin, 1973). Gold and Lachenbruch (1973) suggest that thawback of permafrost in the discontinuous zone, due to recent climatic warming, may be strongly influenced by moving groundwater. As discussed in chapter 4, active flow systems can have a marked effect on the subsurface temperatures through forced convection of heat.

The thickness of permafrost is largely determined by a dynamic balance between terrestrial heat flow and the present and past ground surface temperatures (Jessop, 1972). The thicker the permafrost zone, the more slowly it responds to changes of surface temperature. In general permafrost is probably rarely in a steady-state condition, but is either aggrading or degrading in response to long-term surface temperature changes. Thus in North America thick permafrost is probably in general aggrading due to the lowering of surface temperature following deglaciation. In northern Asia and Europe the thick permafrost is generally degrading slowly due to climatic warming since the last glacial period (Tolstikhin, 1973).

If steady-state conditions can be assumed, the thickness of permafrost depends on surface temperature, thermal conductivity of the frozen zone and terrestrial heat flow (Jessop, 1973; Judge, 1973). Measured permafrost thicknesses correspond roughly to the steady-state thickness calculated on the basis of present-day conditions. The rate of growth of permafrost thickness can be predicted by means of Neuman's equation for depth of frost penetration (Brown, 1964) in response to an abrupt introduction of a freezing temperature at the surface:

where $\varepsilon$ is the thickness of the frozen zone. The proportionality factor B is a function of thermal conductivity and diffusivity, initial and final temperature, latent heat, and water content. This equation is basically limited to shallow processes since it neglects terrestrial heat flow. A general treatment of permafrost thickness including both surface temperature changes and terrestrial heat flow does not yet appear to have been presented in the literature, but the necessary theoretical and numerical tools are available (e.g. Taylor, 1979).

If the subpermafrost groundwater has a high dissolved minerals contents, as is often the case, (Brandon, 1965) its freezing temperature is lowered and the permafrost thickness will be correspondingly reduced (Williams and van Everdingen, 1973; Tolstikhin (1973). Even a very slow movement of groundwater beneath the permafrost could affect the permafrost thickness due to the efficacy of moving groundwater in transporting heat by convection. The effect should be particularly pronounced near discharge areas and may even lead to taliks in an otherwise continous permafrost zone, the more so if the discharging water is highly mineralized.

## 7.3 Mechanical and Thermal Effects of Permafrost Aggradation or Degradation

The changes of heat and fluid flux across a moving freezing boundary, as expressed by equations 45 and 46, can lead to a variety of significant effects.

With respect to heat flow it may be noted that the latent heat released or absorbed at the boundary may be significant as compared to terrestrial heat flow even for a very slowly-moving boundary. Thus if the porosity is 0.10 and the boundary is moving at 1 cm per year the change of heat flux across the boundary would be 10.9 $mW/m^2$. Gold and Lackenbruch (1973) suggest that the change of heat flux across a freezing boundary could be used to determine in-situ porosity. However, such a determination would only be feasible if the rate of boundary movement and the thermal conductivities can be determined.

The expulsion or absorption of pore water at a moving boundary can also lead to significant changes of pore pressure. The flow of water to or from the freezing boundary sets up a hydraulic gradient as expressed by equation 6. If the boundary is located on a material with low permeability such as a silt or clay the resulting pore pressure change at the boundary may be large. If the boundary is for instance advancing at 1 meter per year through a clay with porosity of 0.30 and permeability of $10^{-9}$ m/sec, then the gradient would be 0.86. If the unfrozen part of the layer was 10 meters thick the excess pore pressure would be 8.6 meters of water. This effect may be important with respect to ice lens formation.

The expulsion or absorption of water can also lead to large pore pressure anomalies if the underlying hydrogeologic system is closed or semi-closed. This effect is best-known from studies of talik infreezing, for example beneath a recently drained lake (Mackay, 1973, 1977). Beneath a continuous and extensive permafrost layer large pore pressure anomalies may also develop. Tolstikhin (1973) and Pinneker (1973) report anomalously low pore pressures of as much as 333 meters below ground level under permafrost in parts of Siberia. They attribute these to slow degradation of the permafrost. In the North American Arctic region, where permafrost is generally aggrading, the same effect should lead to high pore pressures beneath the permafrost. The simple analytical model presented by McRoberts and Morgenstern for pore pressure due to pore water expulsion can be used to estimate the size of the effects. Very little is known of subpermafrost conditions in the Arctic, but sample calculations suggest that excess pore pressures of several MPa, such as observed in Siberia, are a distinct possibility.

It would be of considerable benefit if a transient two-dimensional model of permafrost formation involving both heat and subpermafrost fluid flow, were available. Such a model would inevitably be rather complex, but the analytical and numerical tools are available. The model would allow parametric studies of the interrelation between permafrost thickness, heat flow and fluid flow; and it could guide further field studies.

Another problem of considerable interest concerns the mechanical and thermal effects of wells drilled through permafrost. Taylor (1979) describes a model for calculating the thermal effects of drilling and producing. Goodman and Wood (1975) analyzed the possible freeze-back pressures around a well and concluded that the maximum stresses are limited by the elastic and plastic properties of the frozen material.

### 7.4 Formation of ice lenses and pingos

The previous discussion of anomalous pore pressures due to movement of the freezing boundary assumed that the porosity is constant. However, if pore pressures are sufficiently high massive ice may form by uplifting of the overlying material. The critical phenomenom is described by equation 46 which states that the stress in the ice exceeds the fluid pressure by an amount C dependent on the size of the pores and the thermodynamics of the freezing process.

Essentially three different conditions are possible at an advancing freezing front. If p + c is less than the overburden stress water will be expelled from the pores. If p + c exceeds the overburden stress but p is less than the overburden stress water will move towards the freezing front and ice lenses will form by segregation. If p exceeds the overburden stress the overburden may heave or repture due to the fluid pressure and massive ice may form by injection. Due to the dynamics of heat and fluid flow at the front the critical conditions may vary from one state to another as the front advances. Soil adhesion and resistance of the overburden to heaving or rupture will also affect the transition from one state to another.

The critical parameter is the fluid pressure. Before the onset of the freezing process the fluid pressures will be controlled by the piezometric conditions of the flow system, being generally higher in the discharge areas. These piezometric pressures may change as freezing commences, going down as the recharge areas become sealed off by freezing and going up as the discharge areas freeze. Since freezing at the surface is largely controlled by heat exchange processes at the surface it would generally be difficult to predict how a flow system will react to the onset of freezing. As the freezing front advances water is expelled from the pores leading to a build-up of pressure near the front by Darcy's law, especially if the material has low permeability. Finally, as the subpermafrost groundwater becomes confined by the overlying permafrost, fluid pressures will build up in the entire system and may reach high enough levels to rupture the overburden.

These general principles suffice to account for the principal oserved features of massive ice in the subsurface, as described for example by Washburn (1973) and in series of papers by MacKay (1971, 1973, 1977, 1978). To start with, ice lenses are rarely encountered at depths of more than a few tens of meters. This general observation can be explained by the fact that at depth p + c can only exceed the overburden stress if p is considerably greater than the hydrostatic pressure. Artesian pressures are often encountered under permafrost (e.g. Williams and van Everdingen, 1973) but as depth increases it becomes increasingly unlikely that fluid pressures could build up high enough for ice lens formation. In addition the adhesive strength of the formation and its resistance to heaving are likely to increase with depth.

The condition for ice lens formation by segregation implies that this process is likely to occur mostly in fine-grained formations for which the value of C (equation 48) will be high. In the interior of such beds ice lenses are likely to occur as multiple thin bands. An ice lens starts to form when the pore pressure is high enough, but as soon as it starts to form the flow of water towards the freezing front will reduce the fluid pressure at the front by Darcy's law, thereby tending to inhibit further lens growth. The formation of thin bands of ice is aided by soil adhesion, or illustrated by the modelling calculations described by Gilpin (1980). The thickness and spacing of the bands should be controlled by the rate of advance of the freezing front and the hydraulic conductivity. In the extreme case of fast advance rate and very low permeability the lenses would appear as interstitial ice.

Massive ice lenses, such as are frequently encountered in permafrost regions, are most likely to occur near the base of a fine-grained formation, overlying a more permeable layer. The flow of water towards the freezing front will then result in only a slight pressure drop and not reduce the fluid pressure below the critical value for lens formation. For such cases the fluid pressure in the underlying formation in the controlling factor and must be high enough for lens formation. The growth of the lens itself adds to the overburden stress at the freezing front. The lens will therefore cease to grow when the total overburden stress becomes equal to p + c, and at that stage the freezing front would advance into the coarser material.

The detailed shape and locations of ice lenses depends on a complex interaction between the nature and geometry of the formations and the processes of heat and fluid flux. Thus for instance massive ice lenses would tend to form where the fine-grained overburden is thinnest. Other processes such as regelation and the occurrence of a frozen fringe (Gilpin, 1980) also have a bearing on the formation of ice lenses and of ice veins (Mackay, 1974).

When the fluid pressure exceeds the overburden stress other processes may come into play. Under such circumstances the overburden may heave and water lenses would form at the top of permeable zones. Massive ice may then be formed by injection. Marked plastic deformation or rupture of the overburden may occur at points where the overburden is weakest and pore pressure highest.

The marked contrast between ice lenses and pingos probably reflects the difference between injection and segregation as the formative process. Observations of water lenses and high artesian pressures under pingos (Mackay, 1977, 1978) strongly indicate that high water pressure is an essential factor in pingo formation. Piezometric heads of the water under the pingo exceeding the height of the pingo have been measured. Since the centre of the pingo consists of massive ice this fluid pressure exceeds the overburden weight at the centre of the pingo and therefore further growth of the pingo by plastic deformation of the ice can occur. For the closed-system pingos studied by Mackay the high fluid pressure is probably due to expulsion of water at the freezing front into a closed system, entirely sealed by permafrost. Other causes of high fluid pressure could also lead to pingo formation, for example, aggradation of an extensive permafrost zone, or freezing-in of the discharge end of a groundwater flow system.

As mentioned by French (1976) ice diaparism has sometimes been proposed as a possible mechanism for pingo formation. It seems unlikely that this process is a factor for the formation of pingos inland, although plastic deformation of ice due to high fluid pressures clearly is an essential factor. Possibly the occurrence of subsea pingos (Mackay, 1972) could be related to diapir mechanisms acting on the thick lenses of ice which presumably constitute part of subsea permafrost.

## 8. Final Remarks

The review presented in this report aims at providing a synthesis of results from various fields of research related to the interaction of groundwater flow and heat flow. Emphasis has been placed on the basic physical processes and their relevance to a variety of real-world phenomena.

Throughout this study an important underlying theme is the problem of applying theoretical findings to the irregular real world, and the reverse problem, all too often lightly regarded, of verifying theory by means of empirical results from the real world. Often only semi-quantitative comparisons of theory and observations are possible because the data is so limited as to leave much room for interpretation.

In view of the complexity of the processes involved in heat and fluid flow through natural formations qualitative and more or less subjective decisions are unavoidable with regard to which processes are assumed negligible and which theoretical models are most suitable. The correctness and objectiveness of such decisions can be much enhanced by careful use of the appropriate dimensionless numbers, which characterize the relative importance of various processes. To take an example from a related subject: one of the most useful quantities in fluid mechanics is the Reynold's number, which is essentially the ratio of inertial to viscous forces. For heat and fluid flow the Rayleigh, Peclet and

Nussselt numbers are often used, but many other characteristic numbers have been defined or remain to be identified.

In the course of this review a variety of as yet unresolved problems and research needs have been mentioned. The most interesting ones are summarized here.

One of the most important concerns with respect to heat and fluid flow in natural formations is the problem posed by heterogeneity of the formations. Macroscale heterogeneities with dimensions of meters or more may lead to large dispersion effects and lack of thermodynamic equilibrium between fluid and solid. Macroscale dispersion particularly is as yet poorly understood. The problem of developing adequate theoretical means for dealing with heterogeneities is a subject of active research in groundwater flow, solute transport, rock mechanics, geothermal energy and convective heat transport. The most useful information on thermal dispersion is presently coming from studies of artificial temperature changes in shallow aquifers, but dispersion probably plays an important role in many other processes involving fluid and heat flow.

Sophisticated theory is available for dealing with the elastic and plastic response of the solid medium to changes of temperature and fluid pressure. However, in most applications little is known about the actual mechanical properties of the medium and calculations are often reduced to crude approximations with little predictive value. Field experiments designed to study the mechanical properties of various media are needed to put the theory on a more solid footing.

Stability criteria for the onset of free convection can be expressed in terms of a suitably defined Rayleigh number. However, for many practical configurations appropriate expressions for the Rayleigh number are not yet available. Examples include the case of a vertical warm water front in aquifers and the case of hot fluid rising through an inclined fracture or fault zone. Few if any tests of stability criteria have been carried out in natural systems where heterogeneity, anisotropy, irregular boundaries and background flow may all play a role. In this regard it would be particularly useful to identify and study shallow aquifer systems in which free convection can be induced or where it occurs naturally due to terrestrial heat flow.

Theoretical and field studies have shown that there are important interactions between groundwater and heat flow even in relatively cool flow systems. There appears to be a need for more field studies in which groundwater and heat flow are dealt with in an integrated manner. The heat flow determined by heat flow studies is generally only conductive heat flow. Convective heat flow is often an important part of the total regional heat flow and integrated studies would lead to a better understanding of the total heat flow budgets in various geologic settings.

The present understanding of the distribution and thickness of permafrost would be enhanced by further studies viewing permafrost in the context of regional heat flow and groundwater flow. Ideally transient two-dimensional models should be developed which allow for variations of surface temperature. Such models would lead to better understanding of permafrost thickness, pingo and ice lens formation, regional heat flow, subpermafrost pore pressures and past climatic changes. They would be of great use in the selection, design and interpretation of field observations.

Many applications of the theory to actual situations require judgments which should, if possible, be based on the magnitudes of appropriate dimensionless numbers. The Rayleigh number for instance characterizes the relative importance of heat flux by free convection as compared to conduction. In the course of this review it was shown that a bulk Peclet number, $P_2$, (equation 39) characterizes the effect of forced convection on subsurface temperatures. Similarly the Rayleigh number for a concentrated heat source, Ra', (equation 33) is useful for characterizing the effects of subsurface nuclear waste disposal on groundwater flow. A variety of other such numbers appear in the literature, but for many practical problems the appropriate numbers have not yet been defined. Further theoretical and numerical analyses are needed; aimed at identifying useful dimensionless quantities and verifying their relevance. Such quantities should then be tested by application to field data.

In the context of heat and fluid flow the careful use of dimensionless quantities in the selection, design, execution and evaluation of experiments or practical applications can save a great deal of effort and lead to a better understanding of the physical processes.

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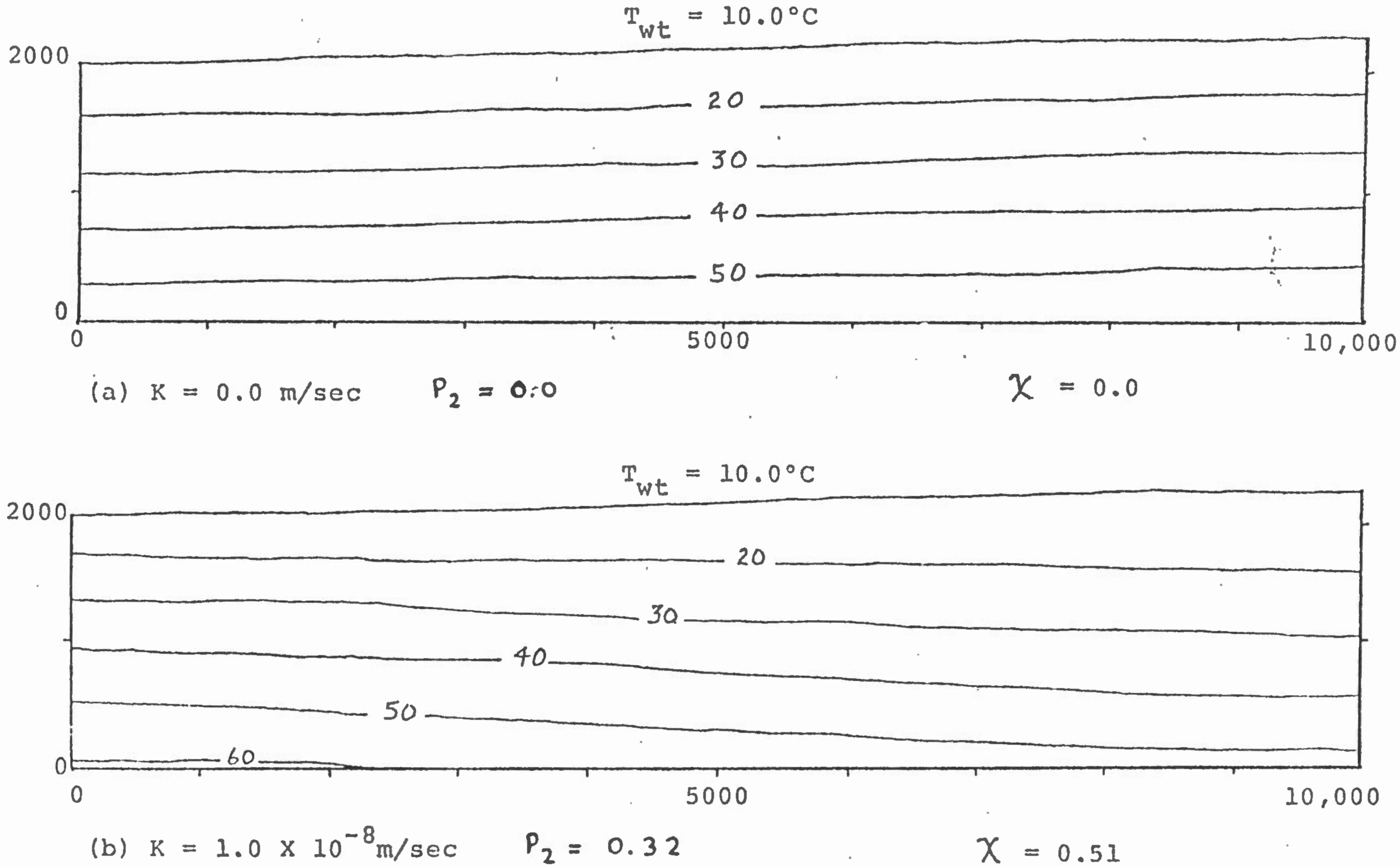


Figure 1. Effect of forced convection on subsurface temperatures: variations of K (after Betcher, 1977)

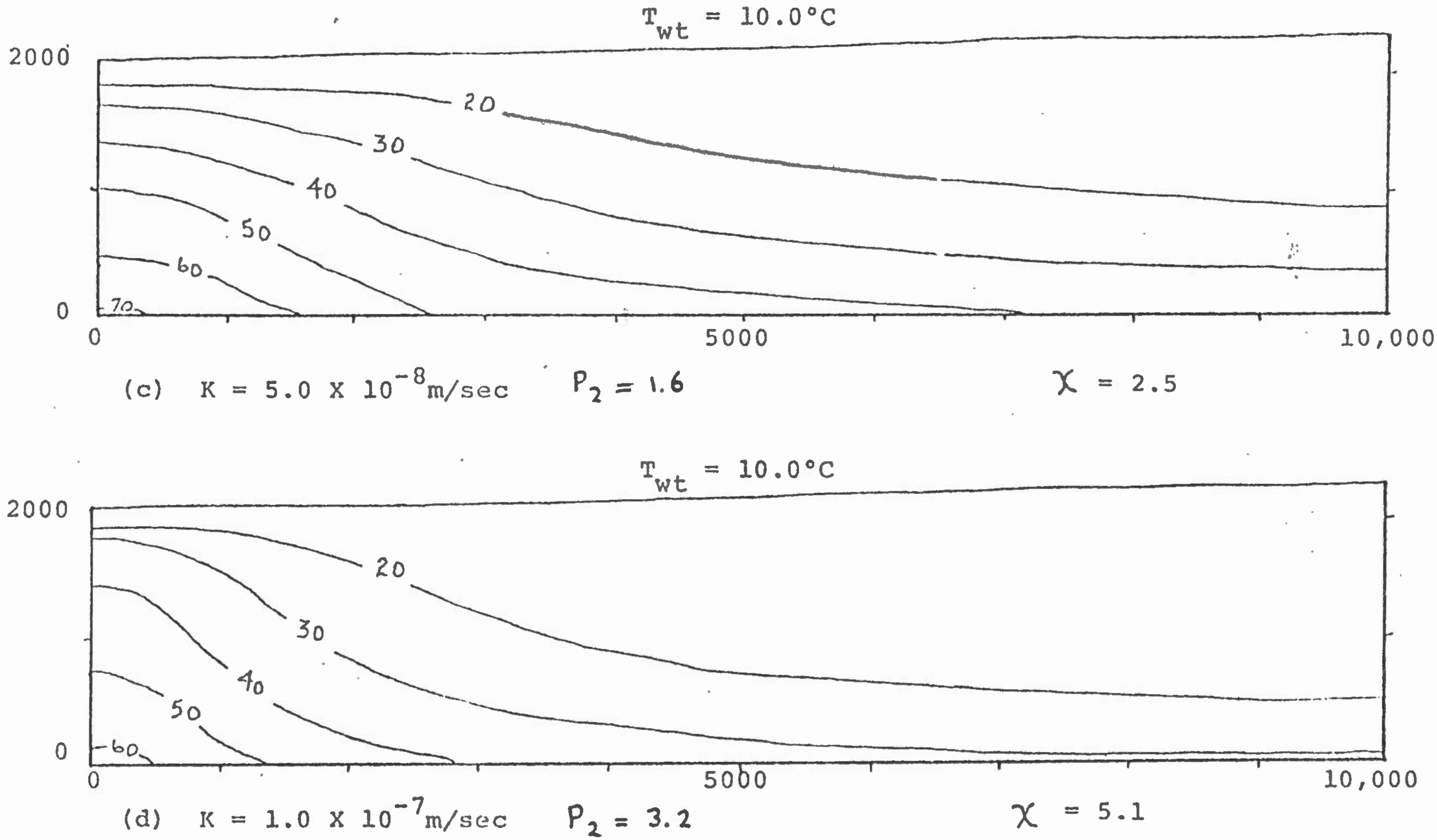


Figure 1. (continued) Effect of forced convection on subsurface temperatures: variation of K (after Betcher, 1977).

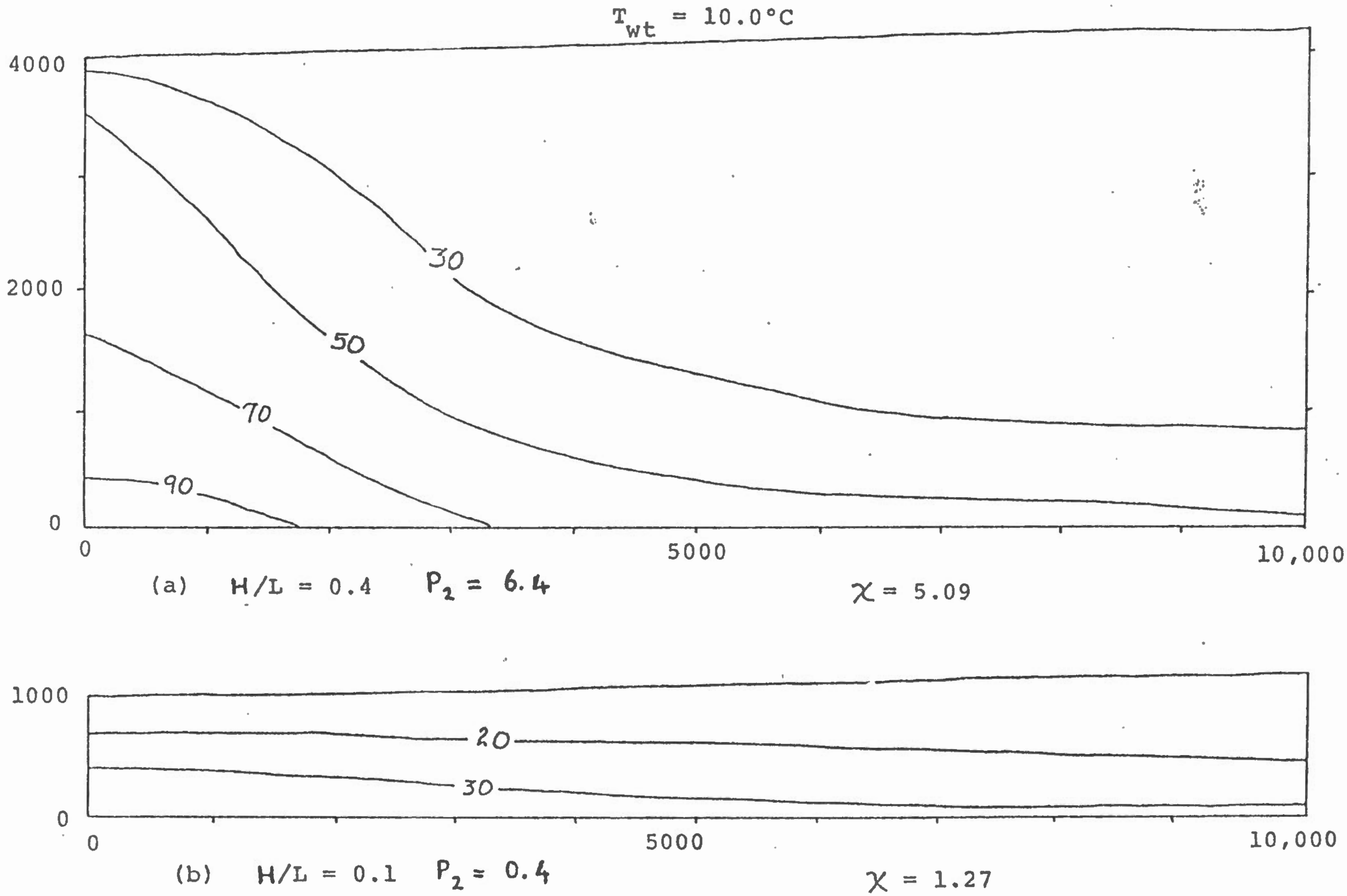


Figure 2. Effect of forced convection on subsurface temperatures: variation of H/L with K = 5.0 x $10^{-8}$ m/sec (after Betcher, 1977)

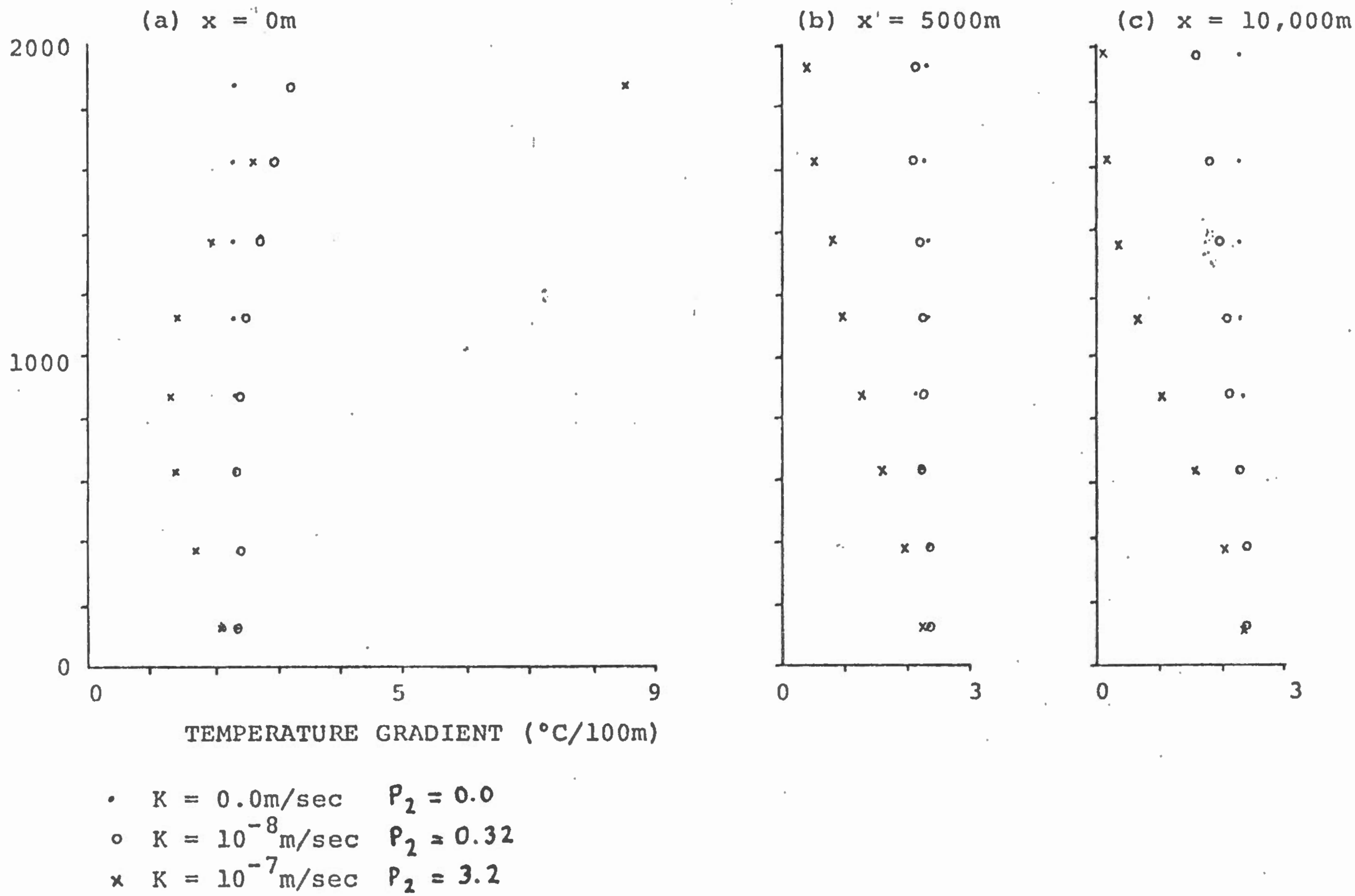


Figure 3. Effect of forced convection on vertical temperature gradients, cf. Figure 1. (after Betcher, 1977)

| Temperature (°C) | Viscosity ($10^{-3}$ $\frac{kg}{m.\ sec}$) | Density ($10^3$ $kg/m^3$) | Coeff. of thermal expansion ($10^{-6}$ °C $^{-1}$) | Isothermal Bulk modulus ($10^{10}$ Pa) |
|---|---|---|---|---|
| -10 | 2.6 | 0.99814 | -294.7 | 0.181 |
| 0 | 1.787 | 0.99987 | - 68.14 | 0.196 |
| 4 | 1.567 | 1.00000 | 0.26 | 0.202 |
| 10 | 1.307 | 0.99973 | 87.90 | 0.209 |
| 20 | 1.002 | 0.99823 | 206.6 | 0.218 |
| 30 | 0.7975 | 0.99567 | 303.14 | 0.223 |
| 40 | 0.6529 | 0.99224 | 385.36 | 0.226 |
| 50 | 0.5378 | 0.98807 | 457.81 | 0.226 |
| 60 | 0.4665 | 0.98324 | 523.38 | 0.225 |
| 70 | 0.4042 | 0.97781 | 584.04 | 0.221 |
| 80 | 0.3547 | 0.97183 | 641.27 | 0.217 |
| 90 | 0.3147 | 0.96534 | 696.26 | 0.211 |
| 100 | 0.2818 | 0.95838 | 750.01 | 0.204 |
| 110 | - | 0.95097 | 803.42 | 0.197 |

Table 1. Mechanical Properties of water between -10°C and 110°C. (Source: Weast, 1975).

| Substance | Temperature (°C) | Specific Heat Capacity (Jowles $kg^{-1}$ $°C^{-1}$) | Thermal Conductivity (W $m^{-1}$ $°C^{-1}$) | Thermal Diffusivity ($10^{-6}$ $m^2$ $sec^{-1}$) |
|---|---|---|---|---|
| Water | 0 | 4218 | 0.553 | 0.131 |
| Water | 50 | 4181 | 0.650 | 0.158 |
| Water | 100 | 4216 | 0.682 | 0.169 |
| Ice | 0 | 2100 | 1.67 | 0.867 |
| Granite | 0 | 800 | 3.51 | 1.66 |
| Shale | 0 | 710 | 1.92 | 1.02 |
| Rock salt | 0 | - | 6.3 | - |
| Quartzitic sandstone | 100 | 1090 | 4.44 | 1.54 |

Table 2. Thermal properties of water and some common rock types. The thermal diffusivity of the rocks is calculated assuming a density of 2650 $kg/m^3$. (Source: Weast, 1975).