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AN IMPROVED BRONZE PENDULUM APPARATUS FOR RELATIVE GRAVITY DETERMINATIONS

BY

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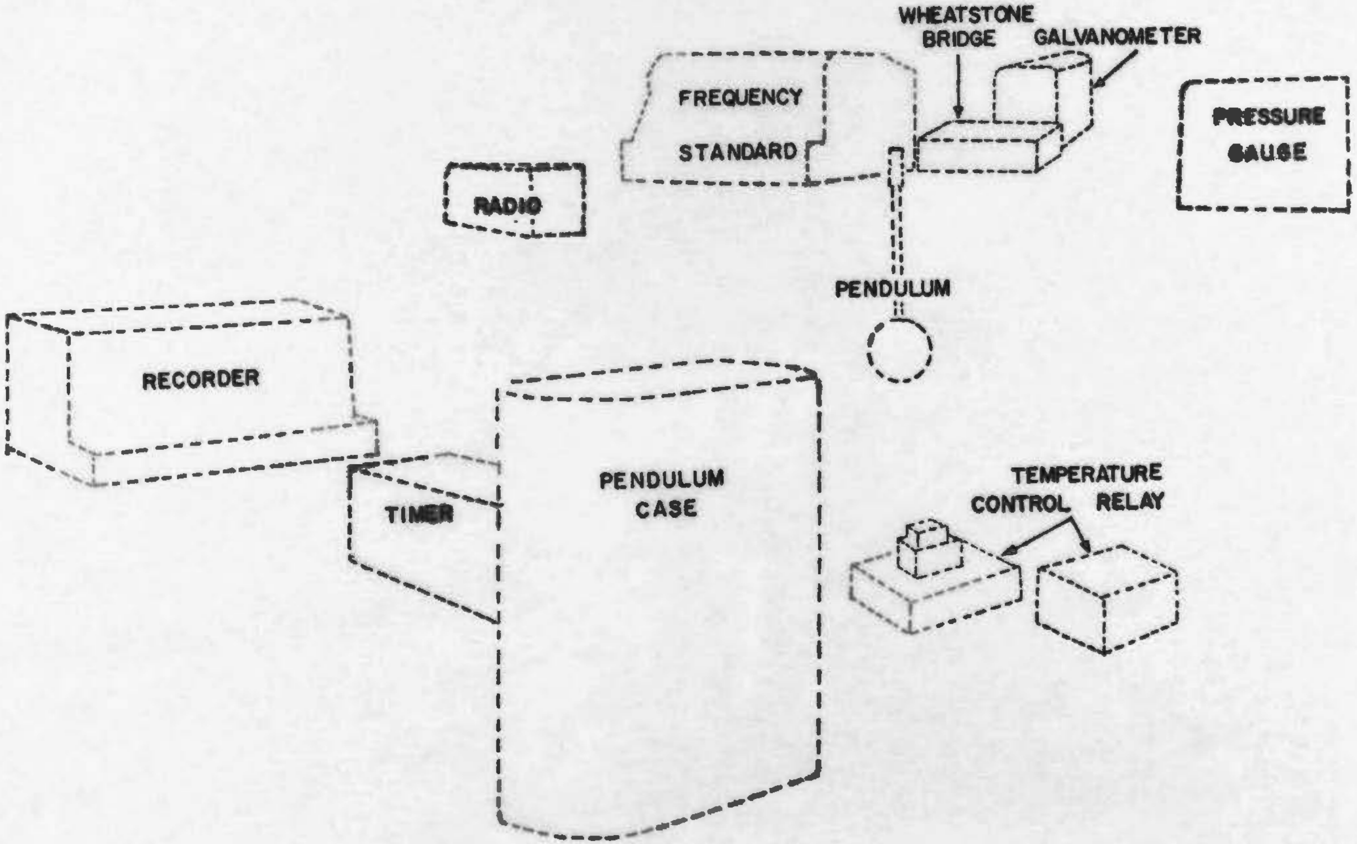
The pendulum apparatus set up on the new pendulum pier in the Geophysical Laboratory of the Dominion Observatory. The author is holding one of the gilded bronze pendulums.


WHEATSTONE BRIDGE
GALVANOMETER
FREQUENCY STANDARD
PRESSURE GAUGE
RADIO
PENDULUM
RECORDER
PENDULUM CASE
TIMER
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An Improved Bronze Pendulum Apparatus for Relative Gravity Determinations

BY

L. G. D. THOMPSON

ABSTRACT

A bi-pendulum apparatus which uses bronze half-second pendulums has recently been developed by the Dominion Observatory for relative gravity measurements. Variable factors such as temperature, pressure, arc, time, etc., are precisely controlled so that all observations are made under identical conditions and no corrections need be applied to the observed periods. Results show that repeated swings with one pair of pendulums give observed periods (the mean of two pendulums) which are consistent to about $\pm 8 \times 10^{-8}$ seconds. For repeat occupations of the same site the mean period for 3 or more swings with one pair of pendulums is consistent to about 5×10^{-8} seconds (± 0.2 milligal). Observations with several pairs of pendulums give gravity differences that are accurate to the order of ± 0.2 milligal.

Gravity values obtained with this apparatus in 1957 are in close agreement with other pendulum and gravimeter values. The difference in gravity between the national reference piers in Ottawa, Canada, and Washington, D.C. has been evaluated at 501.50 milligals with a standard deviation of 0.08 milligal. Relative to the adopted value of 980.62200 cm./sec^2 for the Ottawa reference pier, the following gravity values have been obtained for important sites in the United States and Canada:

Washington (Commerce pier):	$g = 980.12050$ cm./sec.^{2}
Vancouver (U.B.C. Physics Bldg.):	$g = 980.93676$ cm./sec^2.
Winnipeg (Dominion Public Bldg.):	$g = 980.99483$ cm./sec^2.

INTRODUCTION

The pendulum method has been used for relative gravity measurements since early in the nineteenth century and while it was common opinion that this method became obsolete, it is a fact that accurate pendulum determinations still constitute an essential part of gravity work. The present needs for these observations are as follows:

(*a*) to provide a regional network of fundamental gravity values for the control and adjustment of gravimeter surveys in Canada.

(*b*) for the precise calibration of gravimeters.

(*c*) for the measurement of gravity differences of the order of 500 milligals and over, which is considered too large an interval to be accurately measured by gravimeters.

(*d*) for determining gravity at places great distances apart where long travel times are necessary.

(*e*) to provide accurate measurement of the gravity differences between international sites and ensure that the gravity standards in Canada are consistent with the world network.

The first gravity measurements performed by the Dominion Observatory in Canada were made with a Mendenhall pendulum apparatus just after the turn of the century. This apparatus, which has been fully described by Swick (1921), had an accuracy of about 2 milligals and was used with few modifications until as recently as 1948 (Beer, 1950). With the introduction of sensitive gravimeters after the second world war, the pendulum apparatus fell into disuse for gravity measurements and existing pendulum values were used to check the calibration of the gravimeters. After gravimeters had been in use for several years, sufficient data was available to cast suspicion on the reliability of their calibrations. In 1952 a gravimeter base network was established (Innes and Thompson, 1953) which showed that many of the existing pendulum values were in error by 2 milligals or over. Continued investigation (Gilbert, 1958) has definitely shown that gravimeter calibration constants are subject to change and require constant checking. Since the Mendenhall pendulum values are not accurate enough for present gravity standards and do not permit accurate calibration of gravimeters over a reasonable range, it was obvious that a program for obtaining more accurate pendulum values was necessary.

A start on this program was made in 1952 and 1953 when G. D. Garland used the Cambridge pendulum apparatus to establish a series of pendulum values in North America from Mexico City, Mexico, to Fairbanks, Alaska (Garland 1953, 1955), and to make a connection from Ottawa to sites of absolute gravity measurements at Washington, D.C. and Teddington, England (Garland and Cook, 1955). Following this work the development of a new pendulum apparatus designed to use the original Mendenhall bronze pendulums was begun by the Dominion Observatory. In this apparatus as in other equipment currently in use, it was specified that the pendulums would swing in pairs in antiphase and a quartz crystal frequency standard would be used as a time reference. In addition, the pendulum case was to be precisely temperature-controlled and other variable factors such as pressure, arc, etc., were to be controlled so that identical conditions could be maintained for all observations and no corrections to the observed periods would be necessary. It was felt that if these conditions could be realized, the apparatus would be as accurate as other types of pendulums in use today and also it would be informative to compare the results obtained with bronze pendulums, with those already obtained with invar and quartz pendulums.

Initial swings with the new apparatus were made in March, 1956. Since that time many tests have been made and as the results are of value to other workers in this field, they are included in this report. Exhaustive field trials with the apparatus were begun in October, 1956 with observations at sites from Ottawa to Washington. The object was two-fold: firstly to assess the performance of the apparatus and secondly, to measure the gravity difference between these two important sites. After overcoming initial difficulties, it was evident that the apparatus was giving very consistent results and attention was then given to a program of observations for gravity determinations. From January to June, 1957, three trips were made to Washington, and observations were also made at Prescott, Ontario, and Ithaca, N.Y. (Cornell University). From June to August of the same year observations were made at Vancouver, B.C., and Winnipeg, Manitoba.

A description of the apparatus and the results of miscellaneous tests are given in Part I of this report. Part II contains the field observations made with the apparatus up to the end of 1957, followed by a general discussion of the results.

PART I—THE PENDULUM APPARATUS

The Pendulums and Knife Edges

The pendulums and knife edges used with this apparatus are those originally made for the Mendenhall type apparatus (Swick, 1921) which was designed by the United States Coast and Geodetic Survey around the turn of the century. The quarter-metre pendulums are made of bronze and have agate flats mounted on them instead of knife edges. Two agate knife edges, on which the pendulums swing, are mounted in the same plane on separate brass blocks and each pendulum can be placed on either knife edge.

Three pendulums and two knife edges from the Dominion Observatory Mendenhall apparatus have been carefully examined and found to be in excellent condition. In fact, the agate flats and knife edges are as well ground and polished as might be expected if made by present day methods, and even after some 50 years of service show no signs of wear. The decision to continue using these pendulums and knife edges eliminated the need to design, manufacture, and test new pendulums, knife edges, and flats. Also, since these pendulums gave good results in the past, it was believed that even better results could be obtained under the more favourable conditions in the new apparatus.

Six pendulums have been prepared for use with this apparatus. Three are those already mentioned from the Dominion Observatory Mendenhall apparatus; they are numbered 1, 2, and 3. The other three pendulums were obtained on loan from the U.S. Coast and Geodetic Survey and are numbered 4, 5, and 6. All of the pendulums have been cleaned, gilded to prevent oxidation, and fitted with new aluminized glass mirrors. Pendulums 1, 2, and 3 have been kept as one set and matched to within 5 parts in the 6th decimal place of the half period. This is as close a match as practical since the period of a pendulum may differ by this amount on different knife edges. The period of these pendulums at Ottawa is 0.5004 second to four significant figures. The other set of pendulums 4, 5, and 6 has been matched to the same accuracy but the period is 0.4994 second. This difference in period between the two sets has been selected to investigate the possibility of any systematic differences in gravity values determined by pendulums having fundamentally different periods.

Design Features of the Apparatus

Since the original pendulums and knife edges from the Mendenhall apparatus were used, many limitations were placed on the design of the new apparatus. Also since provision had to be made for making many tests and experiments (such as heat distribution, temperature measurements, etc.), the final product differs considerably from an ideal arrangement.

The pendulum case, which serves as a vacuum chamber, accommodates two pendulums and is precisely temperature-controlled. The two pendulums swing in antiphase with equal arcs to eliminate the effects of flexure and horizontal accelerations of the support according to the method of Vening-Meinesz (1923). As the case is large and bulky, an optical timing system is used which is separate from the case. The period of each pendulum is determined separately to observe variations in individual periods. Since several pairs of pendulums can be used, the case is not permanently sealed but has a readily removable top. Continual opening of the case makes rapid pumping to a high vacuum very difficult, therefore observations are made at a reasonable pressure, usually 1 mm. of mercury.

In determining the limits to which each influencing factor must be controlled (temperature, pressure, arc, time, etc.) a required accuracy of 1 x 10^{-7} seconds in the pendulum period (the true half period of 0.5 second) has been selected for design purposes. This corresponds to a gravity determination accurate to 0.4 milligal. In the text the figure of 1 x 10^{-7} seconds (or a similar figure) is always used with reference to a half-second period and is equivalent to saying 1 in the 7th decimal place of the pendulum period.

The Pendulum Case

The case is made from a casting of an aluminum alloy, Alcan 135, with all walls one inch thick. It is made in two parts, a lower section forming the main body, and an upper section forming a lid or cover. As seen in Figure 1, the horizontal cross-section is oval in shape and the overall dimensions are approximately 7 inches x 16 inches x 18 inches high.

The main features are best seen in Figures 2 and 3 which show the case before thermal insulation and a plastic jacket were added. The two original knife edges and carriages from the Mendenhall apparatus are mounted in the same plane on the supporting surface of the lower section (Figure 2). A lens and removable mirror system are also mounted on the same surface for use with the optical timing method. Devices to raise and lower the pendulums individually are mounted at either end of the case. A mechanism for deflecting, releasing, and stopping the pendulums is contained in an aluminum box at the rear of the lower section. Below this and to the left is mounted a vacuum valve for a hose connection. Electrical leads for temperature measurement are passed through the case via Cannon G S receptacles (glass-to-metal seal) which are mounted on both the upper and lower sections (Figure 3). The upper section also contains a small window which centres in front of the lens previously mentioned and permits light beams to enter the case for the external optical timing system. The case is also equipped with 90-second levels, levelling legs, and handles. The levels are for coarse adjustment only, the final levelling being done with a stride level resting on knife edge No. 1. All of the fittings have been mounted on one-inch-thick blocks of bakelite for thermal insulation. The windings of the heater coils may be seen around the case. To maintain a vacuum tight chamber, O-ring seals have been used throughout.

In the final assembly (frontispiece) the entire case is thermally insulated with a one-inch-thick layer of glass wool and covered with a plastic jacket to prevent variable heat

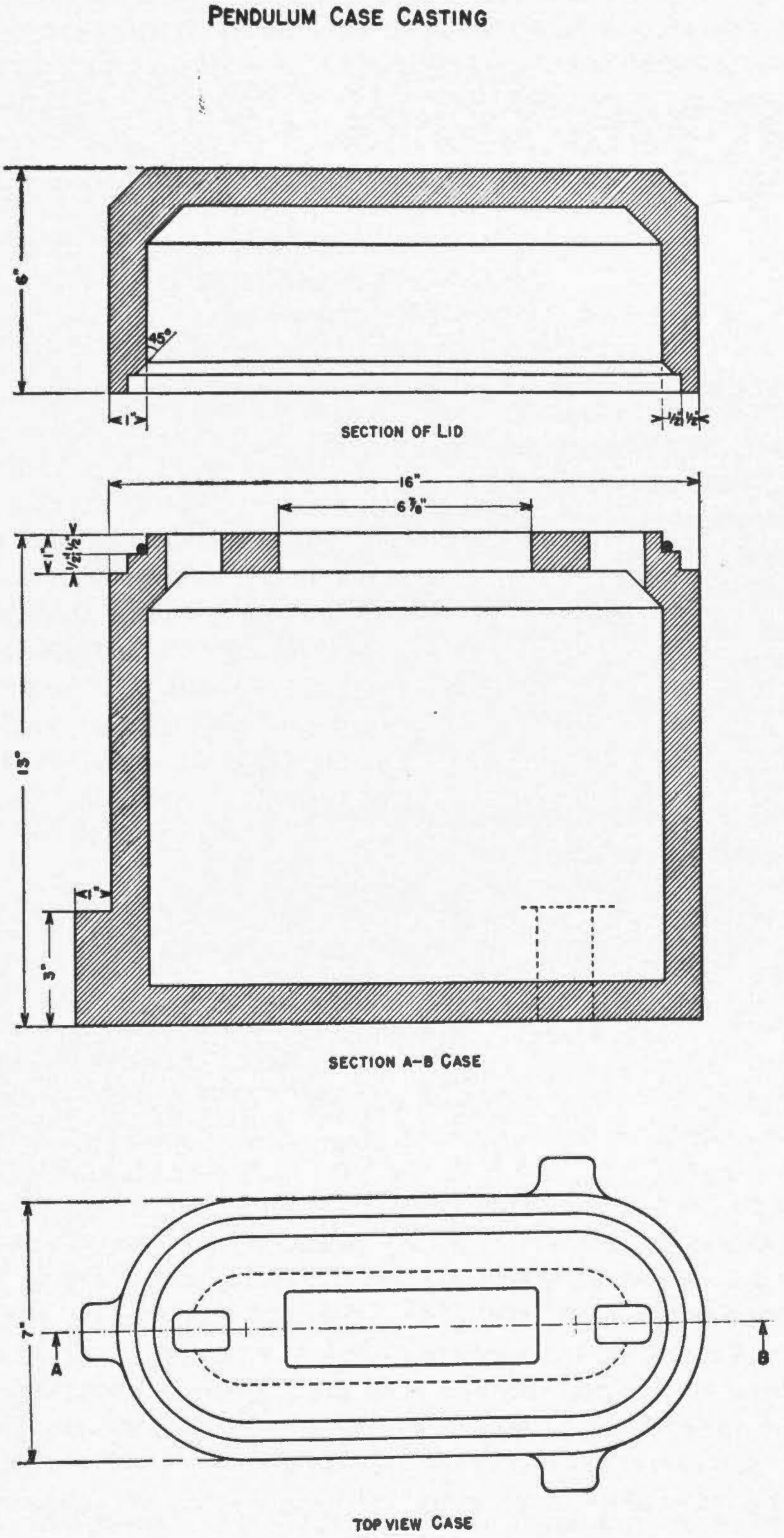

FIGURE 1—Sectional view of the pendulum case casting, showing dimensions.

losses caused by air currents or draughts. To reduce heat loss via the steel levelling legs the case is placed on three glass discs having the usual hole, slot, and plane arrangement. This picture also shows a different type vacuum valve on the case with an attached electrical vacuum gauge detector.

THE TEMPERATURE CONTROL PROBLEM

The most serious problem in developing the new apparatus was controlling the temperature of the pendulums within the necessary limits. Since a change of 0.02°C in the temperature of a bronze pendulum will change its half-period by nearly 1×10^{-7} seconds (the required accuracy), the pendulum must be maintained within ±0.01°C of the selected operating temperature.

To provide suitable temperature control a sensitive electronic relay was developed which employs a thermistor as the sensing element. The thermistor is embedded in the outer surface of the pendulum case giving single point control. This unit is so sensitive that a temperature differential of only about 0.002°C operates the relay to control the heat supplied to the case. While this control is a simple on-off system, the actual temperature variation as indicated by the change in resistance of other thermistors mounted on the inside wall of the case (discussed later) is less than 0.01°C and the temperature of a pendulum inside remains steady to much better than 0.01°C. The actual operating temperature is arbitrary, depending on circuit parameters but is very nearly 40°C. The sensing thermistor has shown no sign of aging; had it aged, this would have been indicated by a change in resistance of all temperature-measuring thermistors inside the case. Indeed, the same control point has been recovered repeatedly for all observations made to date and is still the same after more than a year of operation.

The case is wound with five separate heater coils operating in parallel. Each coil has a resistance of nearly 5 ohms so that the parallel resistance is 1 ohm. Three coils are wound on the bottom section and two on the top, each being wrapped around the case to cover a symmetrical portion of the outside area from top to bottom. They are carefully wound to compensate for anticipated heat losses from the fittings, levelling feet, etc. Figures 2 and 3 show how extra turns have been added near the bottom of the case and around the various fittings and levelling leg lugs. The power to each coil has been adjusted by means of a special multi-tapped, 12-volt transformer until the temperature at all points on the inside wall is within ±0.05°C of the mean value. Allowing for calibration and reading errors of the thermistor thermometers mounted inside the case, and variable heat losses through the fittings on the case, it is believed that the temperature distribution obtained is the best that might be expected. To allow for varying line voltages at different locations and to maintain the same proportional voltages to the heater coils, a constant voltage transformer is used. The total power to all heaters can be varied and is usually increased when the case is being heated from a cold condition. Once heated the case requires only about 70 watts of power on a 50-50 duty cycle.

To measure the temperature of the inside wall of the case 23 rod thermistors (used as resistance thermometers) were mounted at various points on the wall. These were

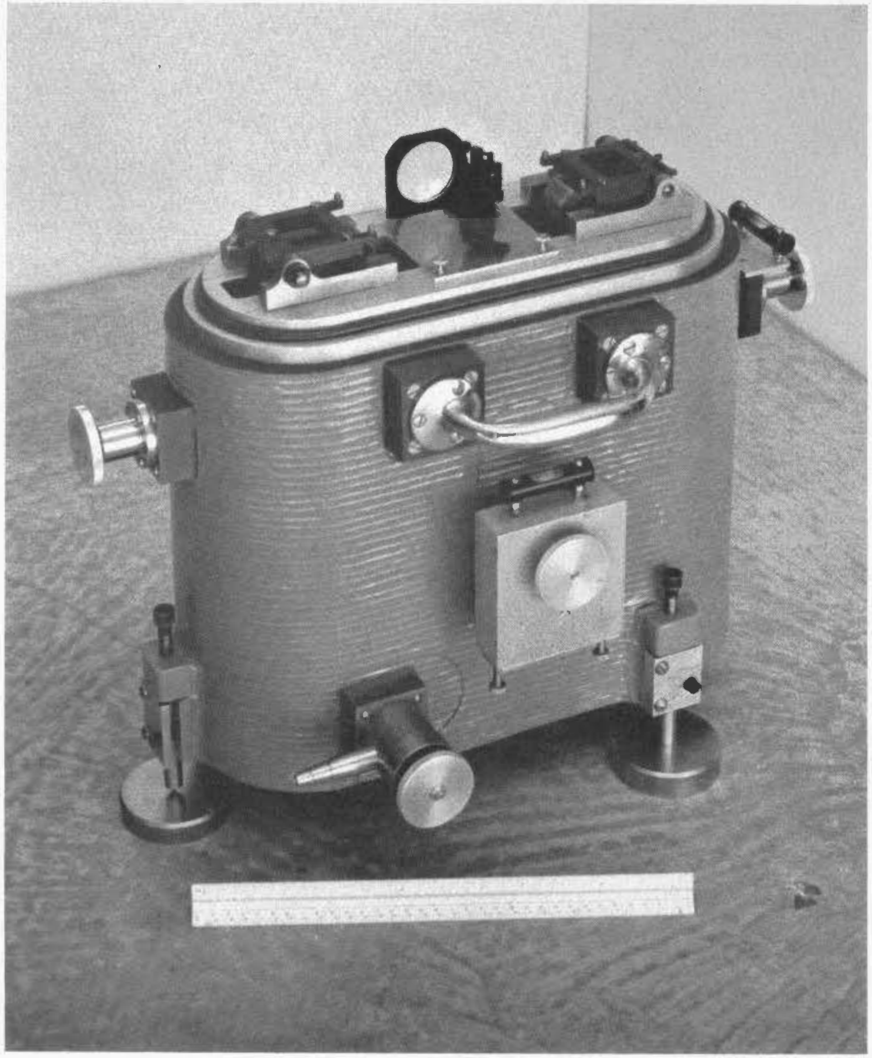

FIGURE 2—Rear view of pendulum case with top removed showing accessories and fittings before the addition of fibreglass insulation and a plastic jacket.

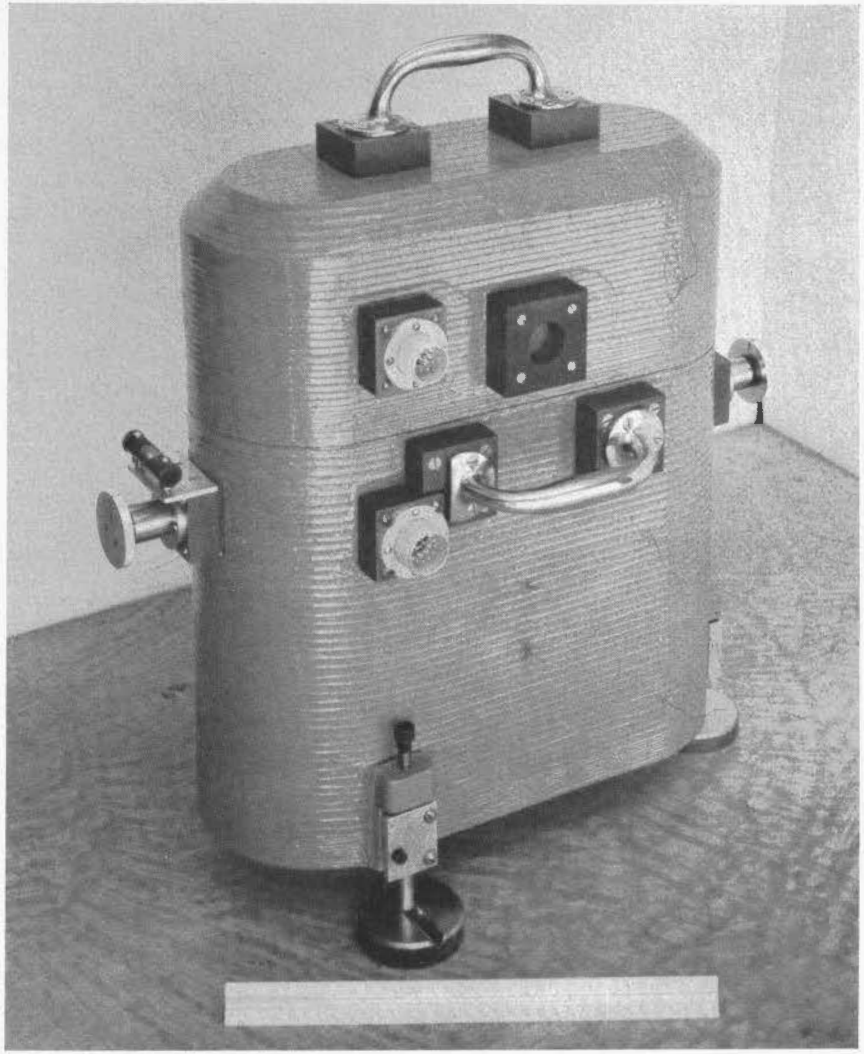

FIGURE 3—Front view of pendulum case with top in place showing fittings before the addition of fibreglass insulation and a plastic jacket.

calibrated by the National Research Council, Ottawa, to an accuracy of ±0.01°C. During the initial tests the thermistor resistances were measured with an accurate Wheatstone bridge. For the more recent field observations this has been replaced with a special compact bridge and thermistor selector circuit. Since thermistors have a very large temperature coefficient of resistance, very sensitive temperature measurements are possible with these simple measuring circuits. For the type of rod thermistor in use, a change in temperature of 0.01°C produces a change in resistance at 40°C of approximately 2 ohms in 5,000. Resistances are easily measurable to less than 1 ohm so that temperature changes of 0.005°C are detectable and meaningful. Of all the thermistors used, relatively few showed any signs of aging (amounting to a change in resistance of only 1 to 2 ohms) during the first few months of operation.

With the case in its equilibrium condition, tests showed that the temperature was essentially the same in any one horizontal plane controlled by one heating coil, which indicates symmetrical heating. In view of this only 7 thermistors spaced vertically from top to bottom are currently used to monitor the temperature. Observations during the past year show that the "standard" temperature distribution is always reproduced at different sites and the mean temperature does not vary by as much as 0.01°C.

In order to check the stability of the temperature of a pendulum, four tiny head thermistors calibrated to an accuracy of ±0.01°C were mounted on a dummy pendulum which was placed in position inside the evacuated, temperature controlled case. Exhaustive tests showed that tbe temperature of the pendulum as recorded at any one point remained constant to within 0.01°C, which is adequate.

A mechanical Fenwall thermostat is also mounted on the outside of the case and the cables to the heating coils are arranged so that this thermostat can be used to keep the case at about 40°C when no observations are being made and precise temperature control is not required. If 110 volts AC is available, a 12-volt transformer is used to provide power to heat the case. When the apparatus is being transported, the pendulum case is carried in an insulated box, and in a car the 12-volt battery can be used as a power supply. In this manner the pendulum case can be kept hot at all times which eliminates the time needed to heat the case to its operating temperature from a cold state each time a new site is occupied.

THE PENDULUM CARRYING BOX

The pendulums are kept in a temperature-controlled aluminum box $8\frac{1}{2}$ inches x 9 inches x 13 inches high which is divided into 6 vertical compartments, each of which holds a pendulum vertically in its normal swinging position. They are rigidly held in place by chamois covered blocks which conform to the shape of the pendulum bob and stirrup. The aluminum box is wound with a heater wire requiring about 70 watts from a 12-volt power supply, and a Fenwall thermostat controls the temperature at about 40°C. This box is shock-mounted with pads of foam rubber inside a larger wooden box and the intervening space of about 2 inches is filled with glass wool insulation. Either a 12-volt transformer or a 12-volt battery can be used as a power supply.

In transit the carrying box (wooden and aluminum) can be shock-mounted in various ways. At present it is placed inside another large box which is lined with 4 inches of foam rubber. In a car or truck this box is placed on additional resilient padding on the floor and with careful driving very little movement of the box is noticed. The results to date show that no pendulum has received any severe shock while in transit.

THE TIMING METHOD

The Frequency Standard

The source of mean time for determining the pendulum periods is a portable frequency standard which has special features for this application. The signal from a temperature-controlled 100-kilocycle crystal is divided down to 60 cycles per second and fed to a 25-watt amplifier which is used to drive synchronous motors. Provision is made to tune audibly the crystal frequency to that of the precise WWV carrier wave, to an accuracy of the order of 2 parts in 10^8 seconds, so that no time correction for the rate of the crystal frequency is necessary.

The frequency standard has proven to be quite satisfactory for an accuracy of 1 part in 10^7 seconds. While the unit is quite stable after a warm-up period of 3 to 4 weeks, it is natural that some frequency drift should occur during the first few days of warm-up, and re-tuning the crystal is necessary. As this is always the case when new observations are being made, frequent checks on the frequency are made although experience has shown that after one day of operation the drift is usually less than 1 part in 10^7 seconds for a 6-hour period during which time 3 or 4 observations can be made.

To tune the crystal to 100 kc, the signal from a harmonic generator driven by the crystal is fed into the aerial of a radio receiving WWV, usually on 10 mc. If the crystal frequency is very nearly 100 kc, the proper harmonic will be superimposed on the WWV carrier wave to produce a beat frequency which is detectable in the audible tone from the receiver. By adjusting a small tuning capacitor in the crystal drive circuit, the crystal frequency is changed until the audible beat is eliminated and the crystal is then oscillating at 100 kc. The greatest difficulty in tuning the frequency standard is poor radio reception of the WWV signal. It is important to tune the crystal only when a good radio signal is being received since fading and other atmospheric disturbances tend to cause false "beats" which lead to improper tuning.

The Method of Measuring Pendulum Periods

A simple optical and photographic method of measuring the pendulum period has been selected for preliminary observations (*see* frontispiece). The signal from the frequency standard controls a synchronous motor in a timer which rotates a series of slotted discs in front of a light source so that flashes of light occur every tenth of a second. The flashes are coded with time, in that every half-second flash is less intense, and every half-minute flash is blanked out. These flashes are focussed on a horizontal slit in a recorder or camera which contains an ordinary chronograph with photographic paper on the drum. Light passing through the slit is focussed by a condensing lens to a point

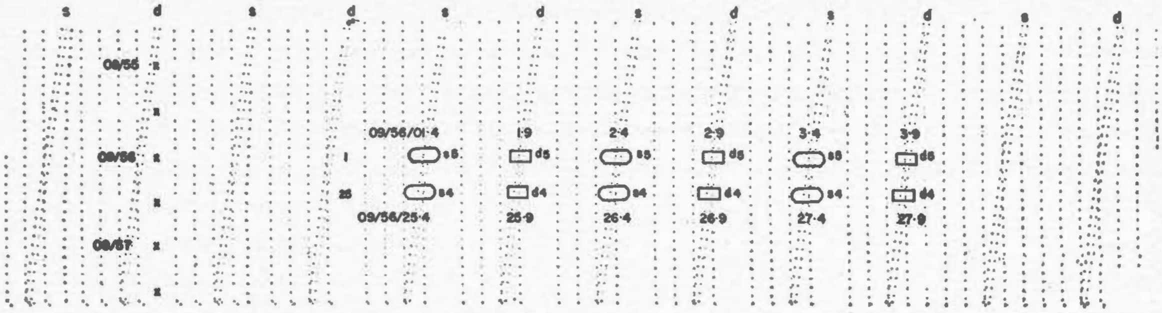

Figure 4—Photograph of an actual pendulum record with time and pendulum marks identified.

on the photographic paper. Two line filament sources on the recorder emit beams which are reflected from mirrors on the pendulums back to the recorder, so that horizontal images of the filament fall exactly on the slit when the pendulums are in their vertical positions. As the pendulums swing in antiphase, the two images move together in an up-and-down motion and each time they pass the slit a mark is recorded on the photographic paper. In this manner a photographic record is obtained containing both time and pendulum marks. One record lasting 3 minutes is taken at the start of a swing and another is taken at the end after 3,000 seconds (50 minutes). From these records the pendulum periods are determined.

Scaling of the Photographic Records

A photographic record is about 16 inches long and 5 inches wide. A copy of an actual record taken at the end of an observation, using pendulums 4 and 5, is shown in Figure 4. The time marks are the small dots that form a regular pattern at right angles to the long side of the record. The pendulum marks are the larger dots which occur in pairs and progress across the time lines since the pendulum periods are not exactly one half second. The start of the record is indicated at the top by a pair of heavy black dots and the end of the record is noted by the repetition of dots as the chronograph drum revolves at the end of its travel. The time increases from left to right and top to bottom, each line being 6 seconds, the time of one revolution of the drum. The half-second marks are very faint compared with the regular tenth-second marks and the half-minute marks are blanked out so that they are readily identified. One pendulum mark is more intense than the other so that the pendulums can be identified.

The method of assigning times to the pendulum marks is clearly indicated on the sample record. The record is started immediately after the second hand of the observing clock on the front of the frequency standard passes a half-minute mark. The first half-minute break on the record is then assigned the time of the next whole minute and is recorded in hours, minutes, and zero seconds; for example 09 hours, 55 minutes, 00 second on the sample record. Thus the proper time can be assigned to any time mark on the record.

Pendulum marks are made each time the reflected light beams move up or down past the slit and are identified as "s" (for sharp) and "d" (for diffuse). Since there is little difference between the two types of marks, three consecutive "s" marks (up swings) are usually manually blanked out for easy identification on the record.

The precise time of a pendulum mark is obtained by scaling the distance to the nearest time mark with a travelling microscope and converting to time by a factor for the paper speed. For greatest accuracy and ease of measurement, pendulum marks are selected where they are very close to time marks. Measurements are made to 0.5 millisecond in time, and as long as the distances measured are less than 1 mm, errors from varying paper speed and paper shrinkage are negligible. Three different near coincidences of pendulum and time marks are selected for both "s" swings and "d" swings, usually in consecutive order of occurrence. The times of corresponding coincidences on both the first and last records are subtracted to give total time intervals. The three "s" intervals are averaged and recorded to 0.1 millisecond as are the three "d" intervals and the mean of these two averages is taken to compute the pendulum period. The approximate period can be initially computed from the first three minutes of a record and this in turn can be used to calculate the number of swings in the total time interval. The final pendulum period is then computed and recorded to 1 x 10^{-8} seconds.

While the pendulum periods are computed separately, it is the mean of the two periods which is used for gravity computations, since this is the period which is free from the effect of horizontal accelerations.

MISCELLANEOUS TESTS

Sway of the Case

As a matter of interest, flexure tests were made using the new case and swinging both one and two pendulums. An interferometer with one mirror on the case and a sodium lamp were used to make the observations. For one pendulum swinging on either knife edge no sway was observed for small arcs up to about 1 degree. It is, of course, quite likely that a small sway was present but it was masked by the tremors of the fringes caused by ground vibrations. The maximum sway which would be undetectable is of the order of 0.01 of a fringe width. Only when the arc was increased to about 1.5 degrees was the sway noticeable at all and for about 2 degrees of arc it was quite definite with a movement of about 0.02 of a fringe. This indicates that the case with legs is a very solid and rigid support.

With two matched pendulums swinging in antiphase no sway was detected even for large arcs. This observation verified that the bi-pendulum method is highly effective in eliminating sway of the support.

These tests were performed with the case resting on glass discs one inch thick firmly cemented to the concrete floor with plaster of Paris. From the above results it is apparent that the glass is quite rigid and makes little or no contribution to the sway of the case. The glass feet have therefore been used in preference to metal ones to reduce the heat loss from the case via the hard steel levelling legs.

Pressure

This type of vacuum apparatus which is continually being opened, and so contaminated, is very difficult to evacuate and maintain at a high vacuum, hence operation at a reasonable pressure is more desirable. A Pirani gauge is used to measure the pressure of the case and a pressure of 1 mm. of mercury has been selected as a suitable operating pressure. In this range pressure variations of the order of hundredths of millimeters are indicated by the gauge.

The first observations were made with the case evacuated to about 0.8 mm., the vacuum pump off, and the pressure slowly increasing. Variations in the mean pressure of 0.4 mm. showed no change in the pendulum periods and as long as the mean pressure was within ±0.2 mm. of 1 mm. on the gauge scale no correction was applied. This method involved pumping down the case after each swing and allowing sufficient time for temperature disturbances from adiabatic cooling to be eliminated. Also long pumping periods were necessary to outgas the case after every opening to change pendulums.

To eliminate this waste of time tests were made with the vacuum pump running continuously. The pump was shock mounted on rubber blocks and connected to the case by lengths of copper tubing which included three metal bellows to isolate the case from vibrations from the pump. The pressure in the case was controlled by a needle valve in the vacuum line and was easily maintained at 1 ± 0.05 mm. at all times. Test swings under these conditions showed no detectable change in period from those performed with the pump off. All subsequent observations have therefore been made with the pump running and the case at a pressure of 1 ± 0.05 mm. This method considerably reduces the time taken between observations and eliminates critical pressure measurements and the need for any pressure correction.

A water vapour trap containing phosphorous pentoxide is included in the vacuum line ahead of the pump. While a check on the moisture content is not made during an observation, it is felt that after a long pumping period the moisture content of the remaining air in the case may be more nearly the same from one set of observations to the next than if no vapour trap were used at all. The results of repeated observations, chiefly restandardization swings at Ottawa, indicate that by this method variations in pendulum periods caused by moisture content are very small and less than 5×10^{-8} seconds.

Arc

The arc through which a swinging pendulum moves is determined by the displacement of a beam of light on a millimetre scale 1 metre in front of the pendulum case. The light beam, whose source is on the recorder, is reflected from a mirror on the pendulum to the millimetre scale which is also on the recorder. Thus the angular displacement of the light beam is twice the displacement of the pendulum. For the length of the light path in this apparatus an angular displacement of the pendulum of 1 degree produces a linear displacement of the light beam of 42 mm. on the scale. In practice a total arc of 30 mm. is used which corresponds to less than 0.5 degree amplitude of swing.

The change in pendulum period with arc has been determined for the range from 20 mm. to 40 mm. measured on the scale. The curve plotted from the results and referred to zero amplitude gives the equation:

$$T = 2.0 \times a^2 \times 10^{-7} \text{ seconds}$$

where "a" is the double amplitude or arc measured in centimetres on a scale 1 metre in front of the pendulum case. This is the same equation as found with the Cambridge pendulum apparatus (Cook, 1950). Since the pendulums are given constant starting arcs, a correction to zero amplitude is not necessary. Instead, it is more important to know the effect of small changes in arc about one selected value. For a 30-mm. arc a change of 1 mm. will change the pendulum period by 1.4 x 10^{-7} seconds. Thus to keep errors less than ±5 x 10^{-8} seconds, the starting arc must always be maintained to ±0.3 mm. Provision has been made to adjust the pendulums independently to equal starting amplitudes at any chosen value and in practice the starting arcs can be repeated to ±0.3 mm. with proper care.

During early observations difficulty was experienced in obtaining constant starting arcs because the deflecting mechanism was sticking. The trouble was soon remedied but as a precaution it became the practice to take a photographic record of the pendulum arcs by placing a slip of photographic paper in front of the scale on the recorder and exposing first one part to the beam from one pendulum and then the other part to the beam from the other pendulum. The exposed arc on the photographic paper is measured under a travelling microscope to 0.1 mm. and a small correction (usually less than 0.5 x 10^{-8} seconds) is applied to reduce the pendulum period to its value for a 30-mm. arc. While this may not be necessary for present accuracies, the practice has been continued since it is very little trouble to make the correction and it eliminates one source of error.

Heat Losses

The pendulum case as shown in the frontispiece is quite satisfactory, thermally speaking, for use under normal conditions. The heat supply and heat losses are balanced to give an equal on-off heating cycle. If an air current or draught is allowed to blow across the case the effect is to increase the heat loss from the top of the case and decrease its temperature slightly. If this occurs for a short time only, the temperature of the pendulums is not altered, but if the air currents are sustained for long periods such as overnight, the temperature of the upper part of the pendulums may be decreased slightly. The only occurrence of this behaviour was the first visit to the gravity room in the Commerce building in Washington. Here, the ventilating system was exhausting air from the room via a register in the ceiling and cool air was being drawn into the room through the open door and past the pendulum case. The situation was temporarily remedied by closing the door and the exhaust vent and adding some insulation around the fittings on the case.

To eliminate similar situations in future work, on return to Ottawa an insulated box or housing was placed around the case to ensure that it would be in the same environment at all sites. Since the heat losses were obviously decreased, the power to the heating coils had to be readjusted to give the same temperature distribution as before and the heating cycle was no longer symmetrical but remained "off" longer than "on". The overall effect of these changes was to increase the pendulum periods by about 1.5 x 10^{-7} seconds.

Knife Edge Effects

In this work three matched pendulums, such as 4, 5, and 6, form a set which are combined in three pairs; 4-5, 4-6, and 5-6. For each swing with one pair of pendulums the individual period for each pendulum is computed and the two periods averaged to give the observed period of the fictitious pendulum. A series of three or more swings are made with each pair and the observed periods are averaged to give a mean period of the fictitious pendulum. A set of swings constitutes at least three swings with each of the three pairs of pendulums.

From the many observations made to date a study of the behaviour of the pendulums with reference to the knife edges is possible. In each set of swings pendulum 5 is used on both knife edges when using pendulums 4, 5, and 6. The individual period of pendulum 5 is on the average 48.3 x 10^{-7} seconds greater on knife edge no. 1 than on knife edge no. 2 but this difference is never repeated exactly. The extreme differences observed have been about 46.0 x 10^{-7} seconds as a minimum and 50.5 x 10^{-7} seconds as a maximum but the value is usually within 1.5 x 10^{-7} seconds of the mean. This variation is probably caused by the exchange of pendulums and the raising and lowering of the pendulums on the knife edges for each swing; therefore, it may indicate the practical limit of reproducability of a pendulum period. In the bi-pendulum method where the mean of the two individual periods is used, the effect of a variation in one period is halved and also there is the chance that the other period has changed in the opposite direction thus neutralizing the effect. This is indicated by recent results which show that repeat observations have mean periods which lie within a range of about 1.0 x 10^{-7} seconds.

When a pendulum is reversed on a knife edge, changes in period up to about 2.5 x 10^{-7} seconds are noted, which is similar to the scatter noted above; thus the effect of reversal, while uncertain, appears to be insignificant.

Time Integration of Disturbing Accelerations

To check the effect of ground movement on the pendulum periods and establish a criterion for the time necessary for a good observation, a series of swings were made in which records were taken after 30 minutes of swinging and also after 50 minutes. The individual periods for each pendulum for the 50-minute swings varied through a range of 0.9 x 10^{-7} seconds, while those for the 30-minute swings varied through 2.0 x 10^{-7} seconds. Hence, the corresponding changes from the mean of the individual period for the same pendulum were twice as great for the 30-minute swings as for the 50-minute swings. It follows that the integration of disturbing accelerations is twice as effective for 50-minute swings as for 30-minute swings.

The effect of ground movement is also noted in the fact that if the individual period of one pendulum decreases the other usually increases the same amount. This shows the advantage of the bi-pendulum method in that the mean of the two individual periods (the observed period of the fictitious pendulum) is usually very consistent. Even though 30-minute swings show variations in individual periods that are twice as large as for 50-minute swings, the observed period of the fictitious pendulum is just as consistent for each case. Hence, while 50-minute or 1-hour swings are more desirable if stability of individual periods is wanted, there is no particular reason why 30-minute swings should not be used, if only the mean period of the fictitious pendulum is considered.

PART II—FIELD OBSERVATIONS

1957

GENERAL

In the fall of 1956 a preliminary test was made at Prescott, Ontario. From February to April, 1957 two tests were made at Washington, with observations also being made at Prescott and Ithaca. This work was in the nature of a field test program but even before it was completed it was evident that the apparatus was performing extremely well, and the completed results provided precise gravity determinations. A third trip to Washington was made in June, 1957, for the purpose of a precise gravity measurement (rather than a test) to check the previous results. The program of gravity determinations with the new apparatus was continued in July 1957 at Vancouver and Winnipeg.

For all this work only pendulums 4, 5, and 6 were used, combined in three pairs of 4-5, 4-6, and 5-6, although comparative tests were carried out with pendulums 1, 2, and 3 at the close of the field season over a 40-milligal gravity difference in the vicinity of Ottawa. The lowest numbered pendulum was swung on knife edge no. 1 and the highest on knife edge no. 2. The pendulums were always placed on the knife edges in the same manner with the number on the pendulum towards the end of the case. For about the first half of the work, four independent swings were made with each pair of pendulums while three swings were made during the latter half. The pendulums were raised and lowered again for each swing. The same starting amplitude was used for all swings by deflecting the pendulums until the light beams were at 15.5 mm. on a millimeter scale 1 metre in front of the pendulums. On release the starting arcs would consistently be in the range 30.2 to 30.6 mm. After Ottawa (3)—*see* Table I—, photographic records of the starting arcs were always made and the periods corrected to the value for 30.0 mm. initial arc. The pressure in the pendulum case was maintained at 1.0 ± 0.5 mm. of mercury as measured by a Pirani gauge. The time of one observation was 50 minutes (or 3,000 seconds).

For the work in 1957 the pendulums were kept hot at a nominal 40°C when not in use, including in transit (*see* Part I). The pendulum case was also kept hot for the trips to Washington but for the trip to Vancouver this was impractical and it was shipped cold by rail.

The photographic records were scaled and the periods computed immediately after the swings were completed so that any irregularities could be detected and remedied before leaving the site.

In order to avoid confusion arising from terminology involving the words "swings", "periods", and "sets", it is well at this point to give a list of definitions which will explain the various terms used in reference to this apparatus. These are as follow:

One swing: is a single observation in which 2 pendulums are lowered onto the knife edges and oscillate together in antiphase for the required observation time (50 minutes).

Individual period: is the computed average time interval required for one swing of an individual pendulum.

Observed period: is the mean value of the individual periods of two pendulums swung together (i.e. the period of the fictitious pendulum).

Series of swings: is any number of consecutive swings made with the same pair of pendulums; usually three or four in the same day.

Mean period: is the average of the observed periods of a series of swings.

Set of swings: is a specified number of swings which constitute observational requirements at one site for one set of three matched pendulums; usually three pendulums are combined into three pairs and each pair is given three swings (e.g. set of pendulums 4, 5, and 6 are combined into three pairs 4-5, 4-6, and 5-6 and each pair is given three swings giving a total of nine swings or observations).

DESCRIPTION OF SITES

Ottawa: Dominion Observatory, Geophysical Laboratory, pendulum Room 8; site on concrete pendulum pier at floor level.

Elevation: 257 ft.

$g = 980.62295$ cm./sec.^{2} (adopted reference value)

Prescott, Ont.: R.C.A. Victor plant, employee training room immediately north of main entrance hall; site is on concrete floor 6 feet 9 inches from west wall, 1 foot 4 inches from north wall.

Elevation: 309 ft.

Ithaca, N.Y.: Cornell University, Rockfeller Hall, Room 5 (basement); site is on concrete floor 3 feet from east wall, 4 feet from north wall, and marked by a bronze tablet.

Elevation: 848 ft.

Washington: Department of Commerce Building, U.S. Coast and Geodetic Survey gravity room (basement); site is on the west concrete pier at floor level.

Elevation: -1 ft.

Vancouver: University of British Columbia, Physics Bldg., Room 120 (basement, Optics laboratory); site is on concrete floor 10 feet 6 inches from north wall, 9 feet from east wall, 3 feet 7 inches north of pillar in centre of room, and marked by a bronze tablet.

Elevation: 277 ft.

Winnipeg: Dominion Public Building, Room 2 (basement); site is on concrete floor 10 feet from westerly wall, 5 feet 3 inches from northerly wall.

Elevation: 753 ft.

THE OBSERVATIONS

The sequence in which observations were made is given in Table I. The first observations from Ottawa (1) to Ottawa (2) are considered experimental only. During this work many difficulties, chiefly mechanical and electrical failures of the accessory equipment, were encountered. Modifications, mainly to minimize shocks during transport of the equipment, also resulted in some delay. For this work the pendulums were kept cold between swings and the standardization periods (Table III) at Ottawa are in very poor agreement. This is in contrast to the observations at Prescott (2) and later, when the pendulums were kept hot between swings, the periods repeated very consistently for different reoccupations of each site. It is therefore considered that the first three observations do not belong to the same set and are of no value in assessing the final performance of the apparatus. The later observations from Prescott (2) appear to form a very good set which is suitable for analysis.

TABLE I

Sequence of Observations

Date	Station	REMARKS	
1956			
Oct. 23-27	Ottawa(1)	Some difficulties, delays and modifications Pendulums kept cold between swings	
Nov. 1–4	Prescott(1)		
Dec. 3–Jan. 18	Ottawa(2)		
		Pendulums now kept hot at all times	
1957			
Feb. 11–13	Prescott(2)		Group I
Feb. 19–21	Cornell(1)	Poor radio reception. Difficulty with arc adjustments	
		Repaired sticking deflecting mechanism	
Feb. 26–28	Washington(1)	Difficulty with flow of cold air Apparent discontinuity in period of pendulums (Pendulums and case too cold in Washington(1))	
Mar. 1– 2	Washington(2)	4 swings per pair	
Mar. 12–14	Ottawa(3)		
		Following observations corrected to equal arcs	
1957			
Mar. 19–21	Ottawa(4)	4 swings per pair	Group II
Apr. 1– 3	Washington(3)		
Apr. 7– 9	Cornell(2)		
Aprl 12–14	Prescott(3)		
Apr. 16–18	Ottawa(5)		
		Insulated box around pendulum case, changed thermostat control point slightly	
May 23–30	Ottawa(6)	3 swings per pair	Group III
June 4– 6	Washington(4)		
June 15–21	Ottawa(7)		
June 24–28	Ottawa(8)		Group IV
July 13–18	Vancouver(1)	2 sets of 3 swings per pair	
July 24–26	Winnipeg(1)		
Aug. 2– 4	Ottawa(9)		

Various conditions and modifications separate these observations into several distinct groups. From Prescott (2) to Ottawa (5) at least four swings were made with each pair of pendulums and of these observations only Cornell (1) and Washington (1) presented any difficulties. At Cornell (1) poor radio reception made it difficult to tune the frequency standard with WWV and it is felt that a time error could have been introduced into some of the swings. Also a sticking deflecting mechanism made it difficult to get precise starting arcs; this trouble was repaired after Cornell (1). For Ottawa (4) and subsequent observations, photographic records of the starting arcs were made and measured, and small corrections were applied to reduce the pendulum periods to the equivalent of a 30 mm. arc. Observations from Prescott (2) to Ottawa (3) form one group (I) in which no arc correction was applied.

At Washington (1) difficulty was experienced from cold air moving past the case and cooling it excessively. This cooling was detected by the temperature measuring thermistors inside the case and verified by the mean periods in Table III which show that the periods at Prescott (2) and (3) agree, and those at Washington (2) and (3) agree while those for Washington (1) do not. It is therefore concluded that the swings for Washington (1) were performed when the pendulums were too cold and the results do not fit into the same consistent set as the rest of the observations and the mean periods for Washington (2) are the correct ones for this occupation.

To prevent similar situations arising, an insulated housing was put around the case prior to Ottawa (6). This had an overall effect of increasing the period of the pendulums by about 1.5×10^{-7} seconds thus giving new standardization periods. Therefore observations from Ottawa (6) to Ottawa (7) form a group (III) leaving those from Ottawa (4) to Ottawa (5) as a separate group (II). The Vancouver-Winnipeg observations are obviously another group (IV).

After completing Ottawa (5), analysis of the results showed that the standard deviation of the mean period for three swings was about the same as that for four swings. The additional swing was not considered essential and for all work from Ottawa (6) and later only three swings were made with each pair of pendulums.

At Vancouver a radio blackout existed and WWV (or WWVH) could only be received on rare occasions. In this instance the frequency standard could not be kept tuned to zero beat for the observations. Instead, the crystal was rated against WWV or WWVH whenever possible and a time correction was applied in computing the pendulum periods. The corrections were quite small since the rate of the crystal was usually less than 2 parts in 10^7 seconds. To allow for the additional error by this method, two sets of observations were made (three swings with each pair of pendulums per set) and the mean period for six swings was treated with the same weight as the mean period of the usual three swings at other sites.

The individual, observed, and mean periods are given in Table II according to the sequence of observations and a summary of the mean periods for each site is given in Table III. While values for Ottawa (1) and (2) and Prescott (1) have been omitted in Table II, the mean periods are included in Table III for reference purposes. When comparing results of repeated occupations of one site, it must be remembered that they are divided into groups and adjustments must be made according to the restandardization periods at Ottawa.

TABLE II

Observed Pendulum Periods

Date	Station	Pendulum Number				$\frac{4+5}{2}$	$\frac{4+6}{2}$	$\frac{5+6}{2}$
		4 Knife Edge 1	5 Knife Edge 2	5 Knife Edge 1	6 Knife Edge 2			
1957 Feb. 11	PRESCOTT(2)	.499,462,91	.499,458,60			.499,460,76		
		462,85	458,86			460,86		
		462,82	458,83			460,83		
		462,65	458,76			460,71		
Feb. 12		.499,462,93			.499,458,29		.499,460,61	
		462,95			458,59		460,77	
		463,12			458,72		460,92	
		462,96			458,66		460,81	
Feb. 13				.499,463,36	.499,458,43			.499,460,90
				30	458,66			460,98
				38	458,43			460,91
				34	458,39			460,87
MEAN PERIODS						.499,460,79	.499,460,78	.499,460,92
Feb. 19	CORNELL(1)			.499,531,14	.499,525,95			.499,528,55
				531,13	526,05			528,59
				530,68	526,23			528,46
				530,90	526,17			528,54
Feb. 20		.499,530,58			.499,526,30		.499,528,44	
		530,75			526,10		528,43	
		530,72			526,37		528,55	
		530,66			526,58		528,62	
Feb. 21		.499,530,74	.499,526,55			.499,528,65		
		530,69	526,58			528,64		
		530,70	526,47			528,59		
MEAN PERIODS						.499,528,63	.499,528,51	.499,528,54
Feb. 26	WASHINGTON(1)	.499,576,32	,499,571,96			.499,574,14		
		576,39	572,13			574,26		
		576,42	572,32			574,37		
		576,30	572,22			574,26		
Feb. 27		.499,576,70			.499,570,25		.499,573,48	
		576,59			572,63		574,61	
		576,61			571,60		574,11	
		576,55			571,75		574,15	
Feb. 28				.499,576,56	.499,571,56			.499,574,06
				576,78	571,77			574,28
				576,93	572,11			574,52
				577,18	572,09			574,64
MEAN PERIODS						.499,574,26	.499,574,09	.499,574,37

Mar. 1	WASHINGTON(2)			.499,577,14	.499,572,17			.499,574,66
				577,03	572,17			574,60
				577,31	572,29			574,80
				577,14	572,44			574,79
Mar. 2		.499,576,82			.499,572,13		.499,574,48	
		576,56			572,28		574,42	
		576,58			572,46		574,52	
		576,68			572,24		574,46	
	MEAN PERIODS						.499,574,47	.499,574,71
1957 Mar. 12	OTTAWA(3)	.499,448,51	.499,444,31			.499,446,41		
		448,66	444,44			446,55		
		448,45	444,12			446,29		
		448,33	444,42			446,38		
Mar. 13		.499,448,87			.499,444,12		.499,446,50	
		448,70			444,19		446,45	
		448,51			444,21		446,36	
		448,51			444,34		446,43	
Mar. 14				.499,449,13	.499,444,31			.499,446,72
				449,47	444,39			446,93
				449,25	444,39			446,82
				449,29	444,37			446,83
	MEAN PERIODS					.499,446,41	.499,446,44	.499,446,83
Mar. 19	OTTAWA(4)			.499,449,19	.499,444,50			.499,446,85
				449,27	444,56			446,92
				449,30	444,44			446,87
				449,39	444,44			446,92
		.499,448,58			.499,444,50		.499,446,44	
		448,67			444,49		446,58	
		448,38			444,44		446,41	
		448,47			444,30		446,39	
Mar. 21		.499,448,51	.499,444,19			.499,446,35		
		448,50	444,32			446,41		
		448,58	444,44			446,51		
		448,68	444,40			446,54		
	MEAN PERIODS					.499,446,45	.499,446,46	.499,446,89
1957 Apr. 1	WASHINGTON(3)	.499,576,79	.499,572,38			.499,574,59		
		576,67	572,40			574,54		
		576,50	572,37			574,44		
		576,79	572,49			574,64		
Apr. 2		.499,576,78			.499,572,12		.499,574,45	
		576,74			572,28		574,51	
		576,68			572,30		574,49	
		576,95			572,06		574,51	
Apr. 3				.499,577,14	.499,572,21			.499,574,68
				577,14	572,36			574,75
				577,19	572,39			574,79
				577,08	572,42			574,75
	MEAN PERIODS					.499,574,56	.499,574,49	.499,574,74

TABLE II (Continued)

Observed Pendulum Periods

Date	Station	Pendulum Number				$\frac{4+5}{2}$	$\frac{4+6}{2}$	$\frac{5+6}{2}$
		4 Knife Edge 1	5 Knife Edge 2	5 Knife Edge 1	6 Knife Edge 2			
Apr. 7	CORNELL(2)			.499,531,18	.499,526,21			.499,528,70
				531,16	526,32			528,74
				531,19	526,17			528,68
				531,18	526,23			528,71
Apr. 8		.499,530,23			.499,526,22		.499,528,23	
		530,52			526,16		528,34	
		530,45			526,05		528,25	
		530,13			526,27		528,20	
Apr. 9		.499,530,46	.499,526,06			.499,528,26		
		530,35	526,38			528,37		
		530,59	526,23			528,41		
		520,24	526,09			528,17		
		MEAN PERIODS				.499,528,30	.499,528,26	.499,528,71
1957 Apr. 12	PRESCOTT(3)	.499,462,84	.499,458,65			.499,460,75		
		462,89	458,87			460,87		
		462,78	458,87			460,83		
		462,77	458,67			460,72		
Apr. 13		.499,462,77			.499,458,64		.499,460,71	
		462,66			458,43		460,55	
		462,93			458,62		460,78	
		463,02			458,65		460,84	
Apr. 14				.499,463,53	.499,458,65			.499,461,09
				463,46	458,69			461,08
				463,33	458,72			461,03
				463,65	458,86			461,26
		MEAN PERIODS				.499,460,79	.499.460,72	.499,461,11
Apr. 16	OTTAWA(5)			.499,449,00	.499,444,52			.499,446,76
				449,12	444,43			446,78
				449,02	444,10			446,65
				449,26	444,55			446,91
Apr. 17		.499,448,49			.499,444,21		.499,446,35	
		448,76			444,42		446,59	
		448,58			444,59		446,59	
		448,64			444,41		446,51	
Apr. 18		.499,448,83	.499,444,44			.499,446,64		
		448,48	444,68			446,58		
		448,77	444,53			446,65		
		448,88	444,62			446,75		
		448,69	444,41			446,55		
		MEAN PERIODS				.499,446,63	.499,446,53	.499,446,78

1957 May 23	OTTAWA(6)	.499,448,64	.499,444,61			.499,446,63		
		448,74	444,53			446,63		
		448,73	444,57			446,65		
May 26		.499,449,30			.499,444,50		.499,446,90	
		449,09			444,30		446,70	
		448,94			444,47		446,70	
May 27				.499,449,23	.499,444,32			.499,446,78
				449,46	444,36			446,91
				449,50	444,59			447,04
May 30				.499,449,53	.499,444,40			446,96
				449,47	444,66			447,06
				449,39	444,32			446,80
	MEAN PERIODS					.499,446,64	.499,446,77	.499,446,93
June 4	WASHINGTON(4)			.499,577,22	.499,572,31			.499,574,76
				577,50	572,36			574,93
				577,54	572,49			575,02
June 5		.499,576,64			.499,572,43		.499,574,54	
		576,70			572,64		574,67	
		576,86			572,57		574,72	
June 6		.499,576,89	.499,572,54			.499,574,71		
		576,91	572,69			574,80		
		576,71	572,65			574,68		
	MEAN PERIODS					.499,574,73	.499,574,64	.499,574,90
1957 June 15	OTTAWA(7)			.499,449,32	.499,444,51			.499,446,92
				449,61	444,67			447,14
				449,57	444,48			447,03
June 18		.499,449,04			.499,444,57		.499,446,81	
June 19		.499,449,26			.499,444,51		.499,446,88	
		448,91			444,61		446,76	
		448,98			444,49		446,73	
June 20		.499,448,75			.499,444,71		.499,446,73	
		448,96			444,53		446,75	
		448,97			444,65		446,81	
June 21		.499,449,11	.499,444,77			.499,446,94		
		448,93	444,53			446,73		
		449,06	444,69			446,88		
	MEAN PERIODS					.499,446,85	.499,446,78	.499,447,03
1957 June 24	OTTAWA(8)			.499,449,66	.499,444,70			.499,447,18
				449,58	444,57			447,08
				449,57	444,59			447,08
June 25		.499,448,75	.499,444,55			.499,446,65		
		448,91	444,65			446,78		
		448,72	444,52			446,57		
June 26				.499,449,37	.499,444,18			.499,446,78
				449,65	444,21			446,93
June 27				.499,449,54	.499,444,23			.499,446,89
				449,74	444,23			446,99
June 28		.499,448,43	.499,444,52			.499,446,48		
		448,42	444,52			446,47		
		448,73	444,57			446,65		
	MEAN PERIODS					.499,446,60		.499,446,99

TABLE II (Concluded)

Observed Pendulum Periods

Date	Station	Pendulum Number				$\frac{4+5}{2}$	$\frac{4+6}{2}$	$\frac{5+6}{2}$
		4 Knife Edge 1	5 Knife Edge 2	5 Knife Edge 1	6 Knife Edge 2			
1957 July 13	VANCOUVER(1)	.499,368,82	.499,364,72			.499,366,77		
		368,96	364,53			366,75		
		368,99	364,66			366,83		
July 14		.499,368,85			.499,364,13		.499,366,49	
		369,01			364,68		366,85	
		368,88			364,52		366,70	
July 15		.499,368,51	.499,365,09			.499,366,80		
		369,37	364,30			366,70		
		369,03	364,67			366,85		
July 16		.499,368,57			.499,364,79		.499,366,68	
		368,90			364,67		366,79	
		368,78			364,79		366,79	
July 17				.499,369,46	.499,364,78			.499,367,12
				369,43	365,01			367,22
				369,31	364,93			367,12
July 18				.499,369,39	.499,364,78			.499,367,09
				369,22	365,05			367,14
				369,39	364,99			367,19
		MEAN PERIODS				.499,366,78	.499,366,72	.499,367,15
1957 July 24	WINNIPEG(1)			.499,354,61	.499,349,79			.499,352,20
				355,12	349,70			352,41
				355,02	349,76			352,39
July 25		.499,354,21			.499,349,76		.499,351,98	
		354,09			349,79		351,92	
		354,13			349,68		351,91	
July 26		.499,354,26	.499,349,89			.499,352,08		
		354,24	349,75			352,00		
		354,25	349,83			352,04		
		MEAN PERIODS				.499,352,04	.499,351,94	.499,352,33
1957 Aug. 2	OTTAWA(9)	.499,449,03	.499,444,46			.499,446,75		
		448,86	444,51			446,69		
		448,75	444,44			446,60		
Aug. 3		.499,448,89			.499,444,33		.499,446,61	
		449,04			444,37		446,70	
		449,00			444,30		446,65	
Aug. 4				.499,449,61	.499,444,58			.499,447,10
				449,62	444,58			447,10
				449,74	444,44			447,09
		MEAN PERIODS				.449,446,68	.499,446,65	.499,447,10

TABLE III

Summary of Mean Periods

—	PENDULUM PAIRS			Group	Remarks
	4–5	4–6	5–6		
OTTAWA					
1956					
Oct. 23–27 (1)		.499,445,73	.499,446,01		—not suitable for gravity computations
1957					
Dec. 3–Jan. 18 (2)	.499,446,20	.499,446,16	.499,446,32		—not suitable for gravity computations
Mar. 12–14 (3)	446,41	446,44	446,83	I	—not corrected to equal arcs
Mar. 19–21 (4)	446,45	446,46	446,89	II	—corrected to equal arcs
Apr. 16–18 (5)	446,63	446,53	446,78	II	
Average of (4) and (5)	.499,446,54	.499,446,50	.499,446,84		
May 23–30 (6)	446,64	446,77	446,93	III	—insulated housing around case
June 15–21 (7)	446,85	446,78	447,03	III	
Average of (5) and (6)	.499,446,75	.499,446,78	.499,446,98		
June 24–28 (8)	446,60	—	446,99	IV	
Aug. 2– 4 (9)	446,68	446,65	447,10	IV	
Average of (8) and (9)	.499,446,64	.499,446,65	.499,447,10		
PRESCOTT					
1956					
Nov. 1– 4 (1)	.499,460,36	.499,460,25	.499,460,44		—not suitable for gravity computations
1957					
Feb. 11–13 (2)	.499,460,79	.499,460,78	.499,460,92	I	—not corrected to equal arcs
Apr. 12–14 (3)	460,79	460,72	461,11	II	—corrected to equal arcs
CORNELL					
1957					
Feb. 19–21 (1)	.499,528,63	.499,528,51	.499,528,54	I	—poor radio reception, difficulty with arc adjustment; not corrected to equal arcs
Apr. 7– 9 (2)	528,30	528,26	528,71	II	— corrected to equal arcs
WASHINGTON					
1957					
Mar. 1– 2 (2)		.499,574,47	.499,574,71	I	—not corrected to equal arcs
Apr. 1– 3 (3)	.499,574,56	574,49	574 74	II	—corrected to equal arcs
June 4– 6 (4)	574,73	574,64	574,90	III	—insulated house around case
VANCOUVER					
1957					
July 13–18 (1)	.499,366,78	.499,366,72	.499,367,15	IV	
WINNIPEG					
1957					
July 24–26 (1)	.499,352,04	.499,351,94	.499,352,33	IV	

Performance of the Apparatus

From the original design specifications it was expected that observed periods for a series of swings should agree to $\pm 1 \times 10^{-7}$ seconds (i.e. lie within a range of 3×10^{-7} seconds) and the mean periods for restandardization swings would vary by slightly less. The results in Table II show that the spread in the observed period of a pair of pendulums for three or four swings is about 1.5×10^{-7} seconds. Allowing for changes in the periods at restandardization swings at Ottawa (4) and (6), the mean periods in Table III show variations of only 1.0×10^{-7} seconds and less for reoccupations such as at Ottawa or Washington. These results show that the apparatus is capable of very precise gravity measurements of roughly twice the expected accuracy.

Since the mean periods of reoccupation swings vary so little, it is evident that the pendulums have not received any severe shocks while in transit. It is also apparent that the observation conditions are repeated precisely for different occupations and each condition is more rigidly controlled than originally specified so that the sum of the errors is less than anticipated. This verifies that the temperature control is very good and that the pendulums are maintained at the same temperature for all observations to better than 0.01°C (0.2 milligal). The control of pressure and moisture content of the air seems adequate and since no serious effect is noted by running the vacuum pump continuously during observations, this method of pressure control is suitable. The use of an arc correction virtually eliminates error from this source; however, by comparing results at Washington (2) and Washington (3) (Table II) it is noted that the same scatter in periods is obtained with or without the correction. This indicates that the starting arcs can be reproduced with sufficient precision to make a correction unnecessary. Time errors in scaling records must be kept small. A systematic method of scaling and the effect of averaging at least six measurements reduces the error to a few parts in 10^8 seconds.

Practically all the variation in pendulum periods can be accounted for in the uncertainty of the frequency standard. While it is possible audibly to tune the crystal frequency to better than 1 part in 10^7 seconds (a 1-second beat at 10 megacycles) it is difficult to say how much better, since a beat longer than 5 seconds (2 parts in 10^8) is difficult to detect even under good conditions of radio reception. Experience has shown that once the crystal is tuned to "zero" beat (which may be a 5-second beat or longer) the crystal will not drift enough in one day to present anything worse than a 2- or 3-second beat. Thus while one can say that the frequency is accurate to better than 1 in 10^7, it is quite possible that the crystal rate may wander in one case too slow and in another too fast by about 5 parts in 10^8 (a 2-second beat). This effect would be most prominent in considering the mean periods for one occupation and may be the cause of the small changes in the restandardization periods at Ottawa. It is suspected that the total variation of either observed periods or mean periods is, however, of the same order of magnitude as that caused by repeatedly lowering a pendulum on the knife edge. It seems therefore that a limit has been reached where the uncertainty in the stability of the pendulums is of the same order of accuracy as the uncertainty in the time standard. To investigate further the effects from these sources of error it is evident that a more precise frequency standard must be used to eliminate the critical time error.

COMPUTATION OF GRAVITY DIFFERENCES

The gravity differences relative to the Ottawa site have been computed using the relation:

$$\Delta g = g_x - g_0 = g_0 \left(\frac{T_0^2}{T_x^2} - 1 \right)$$

where g_x, T_x are the gravity and period respectively at the unknown location and g_0, T_0 are the gravity and period at the Ottawa site ($g_0 = 980.62295$ cm./sec^2). The differences given in Table IV have been computed from the mean periods in Table III according to the appropriate group of observations and restandardization periods at Ottawa. Where applicable, differences have been computed relative to the initial standardization period and also relative to the final restandardization period, and the two results averaged. For example, Cornell (2) is taken relative to Ottawa (4) and also Ottawa (5) and the two values averaged. This is equivalent to the case where the two standardization periods, Ottawa (4) and (5), are averaged and only one difference is computed. The standard deviations have been computed using the weighted average differences.

ACCURACY OF MEASUREMENTS

The accuracy of the results depends a great deal on the restandardization periods which in turn depend on the stability of the site and other disturbing factors. The instability of a site (ground movement) is indicated by the difference in the individual periods of the pendulums from swing to swing (see Table II). If the disturbance is not severe, it is quite common in repeated swings that if one pendulum period increases by 1×10^{-7} seconds the other decreases by about the same amount, so that the observed period (mean of the two) is unaffected. If the disturbance is severe, one pendulum period may increase by 2×10^{-7} seconds while the other decreases by about 4×10^{-7} seconds, thereby decreasing the observed period by 1×10^{-7} seconds. At Cornell, which is a good basement site, the observed periods vary by about 1×10^{-7} seconds while at Ottawa the periods vary by about 2×10^{-7} seconds. The greater variation at Ottawa is not completely understood but may be partly explained by the recent construction work in widening a main traffic thoroughfare near the site.

In the present results the restandardization periods at Ottawa are primarily responsible for the uncertainties in the measured gravity differences. Allowing for changes in the restandardization periods at Ottawa, Table III shows that the mean periods for different reoccupations at Washington and Prescott are essentially the same to about 3×10^{-8} seconds. For Ottawa, variations in the mean period of about 1×10^{-7} seconds (0.4 milligal) are common and these are especially significant for Ottawa (6) and (7) which affect the results for Washington (4), and for Ottawa (8) and (9) which control the Vancouver and Winnipeg results. The resulting gravity differences in Table IV are in very close agreement for any one site. Nearly all of the values are within ± 0.4 milligal of the mean, which is the scatter that is to be expected from the variations of 1×10^{-7} seconds in the mean periods discussed above. Considering the weighted results in Table IV, a single determination of Δg with one pair of pendulums has a standard deviation of about 0.3 milligal, whereas with three pairs of pendulums the deviation is reduced to about 0.1 milligal.

TABLE IV

Gravity Differences (Δg) Relative to Ottawa

(in milligals)

—	Group	PENDULUM PARTS			Weight
		4—5	4—6	5—6	
PRESCOTT					
Prescott(2)—Ottawa(3)..............	I	−56.47	−56.31	−55.33	
		Mean = −56.04			1
Prescott(3)—Ottawa(4)...............	II	−56.31	−55.99	−55.83	
Prescott(3)—Ottawa(5)...............	II	−55.60	−55.72	−56.27	
	Average	−55.96	−55.86	−56.05	
		Mean = −55.96			2
		Weighted Mean Δg = −55.98 ± 0.13			
CORNELL					
Cornell(1)—Ottawa(3)................	I	−322.79	−322.20	−320.78	
		Mean = −321.92			1
Cornell(2)—Ottawa(4)................	II	−321.33	−321.14	−321.22	
Cornell(2)—Ottawa(5)................	II	−320.63	−320.86	−321.65	
	Average	−320.98	−321.00	−321.44	
		Mean = −321.14			3
		Weighted Mean Δg = −321.34 ± 0.26			
WASHINGTON					
Washington(2)—Ottawa(3)............	I	—	−502.56	−501.97	
		Mean = −502.27			1
Washington(3)—Ottawa(4)............	II	−502.87	−502.56	−501.85	
Washington(3)—Ottawa(5)............	II	−502.17	−502.28	−502.28	
	Average	−502.52	−502.42	−502.07	
		Mean = −502.34			2
Washington(4)—Ottawa(6)............	III	−502.80	−501.93	−502.32	
Washington(4)—Ottawa(7)............	III	−501.97	−501.89	−501.93	
	Average	−502.39	−501.91	−502.13	
		Mean = −502.14			2
		Weighted Mean Δg −502.25 ± 0.09			
VANCOUVER					
Vancouver(1)—Ottawa(8).............	IV	313.51	—	313.59	
Vancouver(1)—Ottawa(9).............	IV	313.83	313.95	314.02	
	Average	313.67	313.95	313.81	
		Mean Δg = 313.81 ± 0.14			
Winnipeg(1)—Ottawa(8)..............	IV	371.43	—	371.82	
Winnipeg(1)—Ottawa(9)..............	IV	371.74	372.01	372.25	
	Average	371.59	372.01	372.04	
		Mean Δg = 371.88 ± 0.25			

VALUES OF GRAVITY

The final values of gravity for the sites occupied, relative to the value of 980.62200 cm./sec.^{2} for the national reference pier in Ottawa, are given in Table V. The gravity differences shown in column 2 of this table are different from those in Table IV by a factor of 0.95 milligal, which is the difference in gravity between the site of the pendulum observations in Ottawa and the national reference pier (Innes, 1958). A correction of -0.2 milligal has also been applied to the value determined for the Washington site at floor level to give a value for the top of the national reference pier.

COMPARISON WITH OTHER RESULTS

Gravity determinations at these sites by gravimeters and other pendulums are also given in Table V. The gravimeter values for Prescott, Cornell, and Washington are determined from a line of gravimeter stations between Ottawa and Washington established by the Dominion Observatory as a calibration standard for gravimeters (Innes, 1958). At Prescott the gravimeter station is at the CNR station and $g = 980.56656$ cm./sec^2. From here to the pendulum site the gravity difference as measured with a Worden gravimeter is -0.32 milligal, which gives $g = 980.56624$ cm./sec.^{2} at the pendulum site. The nearest gravimeter station to Cornell University is Cortland where $g = 980.29989$ cm./sec.^{2} A gravimeter connection from Cortland to the pendulum site at Cornell gives a gravity difference of $+1.51$ milligals, therefore $g = 980.30140$ cm./sec^2. The Washington value at the national reference pier is the direct result of the gravimeter connections from Ottawa. The total gravity difference between the national reference piers at Ottawa and Washington as determined by the gravimeter connection is 501.44 ± 0.25 milligals which gives a value at Washington relative to Ottawa of 980.12056 cm./sec.^{2}.

In Vancouver a gravimeter station has been established at Brockton Point (Garland, 1957) with $g = 980.9597$ cm./sec.^{2} This value is relative to a pendulum determination at Edmonton with the Cambridge pendulum apparatus (Garland, 1955). A further connection from Brockton Point, to a site near the flagpole on the University of British Columbia campus (Garland, 1957) gives $g = 980.93665$ cm./sec^2. A good connection by the author from this flagpole site to the pendulum site gives a gravity difference of $+0.15$ milligal which gives a gravimeter value at the pendulum site of 980.93680 cm./sec.^{2}.

With the exception of the Prescott determination, which differs by 0.7 milligal, the bronze pendulum values and the gravimeter results are in splendid agreement. The reason for this discrepancy is not fully known. It was suspected that sustained vibrations from the boiler room beside the pendulum site might have consistently affected the pendulums in the same manner for each occupation. Recent tests with a vibrating pump on the pendulum pier indicate that minor vibrations could not be responsible for such a large change in the pendulum periods (nearly 2×10^{-7} seconds). A recent gravimeter check has verified the gravimeter value at the Prescott pendulum site. Recent pendulum observations at Kemptville, Ontario, on the Ottawa-Washington calibration line, give a gravity value which agrees within experimental error with that established by gravimeters.

TABLE V

Gravity Values

Site	Δg(mgals) to National Reference	g(cm./sec^2) Bronze Pendulums	g(cm./sec^2) Gravimeter	g(cm./sec^2) Other Pendulums
OTTAWA (National Reference Pier)	0	980.62200 (adopted)	980.6220 (adopted)	980.6218 (Mendenhall Apparatus; relative to Potsdam, Miller, 1928)
OTTAWA (Pendulum Pier, Geophysical Lab.)	0.95		980.62295	
PRESCOTT (R.C.A. Plant)	−55.03	980.56697	980.56624 (Dominion Observatory Calibration Line; Innes, 1958)	
ITHACA, N.Y., (Cornell University, Rockfeller Hall)	−320.39	980.30161	980.30140 (Calibration Line)	
WASHINGTON, D.C. (Commerce Floor)	−501.30	980.12070		
WASHINGTON, D.C. (Commerce Pier)	−501.50	980.12050	980.12056 (Calibration Line)	980.1191 (Cambridge Apparatus; relative to Ottawa, Garland, 1953) 980.1220 (Cambridge Apparatus; relative to Ottawa, Garland and Cook, 1955)
VANCOUVER (Physics Building)	314.76	980.93676	980.93680 (Relative to Edmonton Pendulum Value, Garland and Tanner, 1957)	
WINNIPEG (Dominion Public Building)	372.83	980.99483	980.99494 (Primary Gravimeter Network; Innes and Thompson, 1953)	980.994 (Mendenhall Apparatus relative to Ottawa, 1946) 980.9952 (Cambridge Apparatus relative to Beloit, Kansas; relative to Ottawa, Garland 1953)

This indicates that there is nothing inherently wrong in the observational procedure with the pendulum apparatus, and that there must have been some other factor influencing the Prescott observations.

Other pendulum observations by Canadian observers are shown (Table V) with self-explanatory notes. Of most interest are the values for the reference pier in Washington. With the Cambridge pendulum apparatus, Garland (1953) obtained a gravity difference from Ottawa of 502.9 milligals and a later connection by Garland and Cook (1955) between Ottawa, Washington and Teddington gave this difference as 500.0 milligals. Measurements with the Gulf pendulums (Rose and Woollard, 1955) give the difference between these reference piers as 501.7 milligals. The present value of 501.5 milligals therefore agrees with the average of the two measurements by Garland (501.5 milligals), the value by Rose, and also the gravimeter determinations.

At Winnipeg the pendulum observations were made in the same room as previous observations by Innes (1946) and Garland (1953). The difference of 0.4 milligal between the bronze pendulum value and the Cambridge pendulum value is not considered unduly large since the Cambridge value is relative to Beloit, Kansas and is therefore subject to a summation of errors at the two places.

DISCUSSION OF RESULTS

There is no doubt that the results obtained with this apparatus compare favourably with recent observations made with other equipment, such as the Cambridge pendulum apparatus (Cook, 1950; Garland, 1953, 1955; Garland and Cook, 1955; Jelstrup, 1957), the G.S.I. (Japan) pendulum apparatus (Muto, 1953; Okuda *et al.*, 1956), and the Gulf pendulum apparatus Woollard *et al.*, 1953). Escept for an occasional value, the gravity differences ($\triangle g$) for repeat occupations of the various sites, vary only about ±0.4 milligal from the mean value and at least half of the variations are less than ±0.2 milligal. It has been demonstrated that the restandardization periods at Ottawa alone can account for many of the 0.4 milligal variations.

A feature to note is that the gravity differences obtained with all 3 pairs of pendulums are essentially the same indicating consistent behaviour for all three pendulums (4, 5, 6). Recent observations with pendulums 1, 2 and 3 which have a fundamentally greater period than pendulums 4, 5, and 6 (*see* Part I), show that gravity differences determined with these pendulums (1, 2, 3) have the same values within experimental error as those determined with pendulums 4, 5, and 6.

In the observations no corrections for tidal effect have been applied. If variations in pendulum periods equivalent to gravity changes of ±0.4 milligal are consistently encountered, such a correction would be of doubtful value. However, this apparatus appears to be capable of gravity determinations consistent to ±0.2 milligal in which case a tide correction might prove to be significant. If this accuracy is maintained, a tide correction should be considered and even the present results might benefit from its application.

These results have been obtained with bronze pendulums used under controlled conditions so that no corrections need be applied to the observed periods. Bronze pendulums have always been difficult to use because of the large temperature coefficient and were generally discarded in favour of invar pendulums early in the century. In the present apparatus, however, they seem to be very stable and have given good results. Both the experimental procedures and the reduction of results are simplified. It is likely that some of the scatter in the periods of pendulums in other equipment, such as the Cambridge apparatus (Gilbert, 1957) may be due to the corrections applied for temperature, arc, etc.

Using six pendulums successively in pairs has several advantages as each pair can be considered to give a separate gravity determination. The pairs of each set of three pendulums are not actually independent and a change in period of one pendulum is readily detected. The use of several pairs virtually eliminates any false results which could arise from spurious periods of a single pair, and a reliable mean gravity difference, determined by the several pairs, can be obtained from one occupation of a site. Even with this type of apparatus, it is still desirable to repeat observations. The advantage of such reoccupations has been recognized in discussions on the results obtained with the Cambridge apparatus (Gilbert, 1957).

Although there appears to be some discrepancy in the period when a pendulum is lowered onto the knife edge for different swings, the irregularity is very small. The limit of stability is, at the moment, doubtful because of the physical nature of the pendulum, the method of raising and lowering, and the accuracy of the present time standard. With a more precise time standard, and more precise methods of identically raising and lowering pendulums onto the knife edges, and perhaps a different type of pendulum, it may be possible to extend the limit of reproducability of a period to less than 1×10^{-7} seconds. Thus the pendulum and knife edge method of measuring gravity, while now capable of gravity determinations to the order of a tenth milligal, is still worthy of further investigation.

SUMMARY

1. A bronze pendulum apparatus has been developed which is capable of gravity determinations consistent to the order of ±0.2 milligal. Repeated swings with one pair of pendulums give mean periods which agree to about $\pm 8 \times 10^{-8}$ seconds. For repeat occupations or restandardizations the final mean periods agree to about $\pm 5 \times 10^{-8}$ seconds. This leads to computed gravity differences which usually agree to ±0.2 milligal so that the average gravity difference of 3 to 6 determinations is accurate to the order of ±0.1 milligal.

2. The apparatus has demonstrated that variable factors such as temperature, pressure, arc, time, etc., can be controlled so that all observations are made under adequately similar conditions. Hence no corrections need be applied to the observed periods, and all the work and errors associated with the corrections are eliminated. The temperature control of the pendulum case is so precise that bronze pendulums can be used satisfactorily.

3. The method of observation has shown that a vacuum pump, properly shock mounted, can be left running near the pendulum case without affecting the pendulum periods. This permits observations to be made rapidly.

4. Relative to the adopted value of 980.62200 cm./sec.^{2} for the national reference pier in Ottawa, observations in 1957 yielded the following values:

Prescott: $g = 980.56697 \pm 0.00013$ cm./sec.^{2}
Ithaca: $g = 980.30161 \pm 0.00026$ cm./sec.^{2}
Washington (Commerce Pier): $g = 980.12050 \pm 0.00009$ cm./sec.^{2}
Vancouver: $g = 980.93676 \pm 0.00014$ cm./sec.^{2}
Winnipeg: $g = 980.99483 \pm 0.00025$ cm./sec.^{2}

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REFERENCES

Beer, M. Measurements of gravity in the Canadian Arctic and Greenland, *Can. J. Res.*, A, Vol. 28, Sept., 1950.

Cook, A. H. Measurements of gravity in Ireland, *Geophys. Mem.*, Dublin, No. 2, Part I, 1950.

Garland, G. D. Gravity measurements in North America with the Cambridge Pendulum Apparatus, *Proc. Roy. Soc.*, A, Vol. 219, 1953.

Garland, G. D. Gravity measurements in North America with the Cambridge Pendulum Apparatus, II, *Proc. Roy. Soc.*, A, Vol. 233, 1955.

Garland G. D. and Tanner, J. G. Investigations of gravity and isostasy in the southern Canadian Cordillera, *Pub. D. O.*, Vol. XIX, No. 5, 1957.

Garland G. D. and Cook, A. H. A determination of the differences in gravity between the National Physical Laboratory, Teddington, the Dominion Observatory, Ottawa, and the National Bureau of Standards, Washington, *Proc. Roy. Soc.*, A, Vol. 229, 1955.

Gilbert, R. L. G. Some comments on the results obtained with the Cambridge Pendulum Apparatus in North America, *Trans. Am. Geophys. Union*, Vol. 38, No. 4, Aug., 1957.

Gilbert, R. L. G. Investigations into the calibration of gravimeters, *Geophys. J., RAS*, Vol. 1, 1958.

Innes, M. J. S. and Thompson, L. G. D. The establishment of primary gravimeter bases in Canada, *Pub. D. O.*, Vol. XVI, No. 8, 1953.

Innes, M. J. S. The establishment of a calibration standard for gravimeters in eastern Canada and United States, *Trans. Am. Geophys. Union*, Vol. 39, No. 2, 1958.

Jelstrup, G. Observations on the gravimetric calibration base Hammerfest-Munich with Cambridge pendulum apparatus *Geograph. Surv. Norway*, Geodetic Pub. No. 7, 1957.

Miller, A. H. A determination of the relative values of gravity at Potsdam, Greenwich, Ottawa, and Washington, *Pub. D. O.*, Vol. II, No. 2, 1931.

Muto, K. A GSI Pendulum apparatus for gravity measurements, *Proc. Japan Acad.*, Vol. 29, No. 8, 1953.

Okuda, T., Inoue, E., and Suzuki, H. Determination of the differences in gravity values at the Coast and Geodetic Survey, the National Bureau of Standards, Washington, D.C., and the Geographical Survey Institute, Chiba, Japan. *Bull. Geograph. Surv. Inst.*, Vol. 5, Part I, July, 1956.

Rose, J. C. and Woollard, G. P. Gravity values obtained with the Gulf Quartz pendulums over the North American gravity range, *Pub. Woods Hole Oceanographic Inst.*, Ref. No. 55-44, 1955.

Swick, C. H. Modern methods for measuring the intensity of gravity, Spec. Pub., *U.S. Coast and Geod. Surv.*, No. 69, 1921.

Vening-Meinesz, F. A. Observations de Pendule dans les pays bas, *Pub. Meth. Geod. Comm.* Delft, 1923.

Woollard, G. P., Bonini, W. E., Black, W. A., Rose, J. C., and Ault, W., A study of methods for measuring large changes in gravity on an inter-continental basis. *Tech. Rept. Pub. Woods Hole Oceanographic Inst.* No. 53-36, 1953.