CANADA
DEPARTMENT OF MINES AND TECHNICAL SURVEYS
Dominion Observatories

PUBLICATIONS

of the

DOMINION OBSERVATORY

OTTAWA

Volume XXVII · No. 6

THE SEISMIC INVESTIGATION OF THE MANICOUAGAN-MUSHALAGAN LAKE AREA IN THE PROVINCE OF QUEBEC

P. L. Willmore

Price 25 cents

ROGER DUHAMEL, F.R.S.C.
QUEEN'S PRINTER AND CONTROLLER OF STATIONERY
OTTAWA, 1963

The Seismic Investigation of the Manicouagan - Mushalagan Lake Area in the Province of Quebec

P. L. Willmore*

INTRODUCTION

Mushalagan and Manicouagan lakes, in central Quebec, constitute a striking topographical feature in the form of an approximately circular trench, some thirty miles in diameter. (Figure 1.) The Dominion Observatory has a continuing interest in circular features, which stems from the fact that many of them have geometrical proportions closely similar to those of lunar craters of equivalent size. This fact, in turn, suggests a common origin, believed to be meteorite impact, for both the lunar and terrestrial features. Geophysical surveys, followed by deep drilling, have supported the meteorite hypothesis in a number of Canadian examples, notably in the case of the Brent Crater (Millman et al., 1960).

In the case of the Manicouagan-Mushalagan feature, the topographical analogy is weaker than usual, for here we have a circular trench in place of the ridge and basin which is typical of meteorite impact. The geologists who were consulted felt that differential erosion was the most likely explanation of the topography, but it was agreed that the feature, whatever its origin, was sufficiently remarkable to warrant further investigation. A team of field workers from the Dominion Observatory under the leadership of Dr. M. J. S. Innes entered the area in the summer of 1954, accompanied by Dr. E. R. Rose of the Geological Survey of Canada. The party remained in the field for a little more than a month. Gravitational, seismic and magnetic studies were conducted, and a geological survey was carried out. The geological results were reported in the form of a map with marginal notes (Rose 1955), and the gravity data were included in a regional report (Innes 1957).

In the case of the seismic work, the equipment and operational plan were designed to meet a difficulty which is inherent in seismic work in deep basins. The problem is that the horizontal length of a refraction spread must be several times greater than the depth of the structure under investigation, so if the basin is a deep one, there is no guarantee that a spread inside it will ever yield observations of the bottom. Even if the problem does not take this extreme form, it is unlikely that the spread can be made short enough to permit simple computations, based on the assumption of plane stratification beneath the spread, to be carried out in a valid way. The alternative of using vertical reflections is hardly acceptable because, in the absence of velocity control from borehole or refraction studies, the results do not include a vertical scale factor. Apart from the possible distortions of the structural picture, the scale uncertainty requires continuous profiling to ensure that all reflections are associated with the appropriate boundaries.

In the Brent crater which had been investigated in the previous year, the problem had been solved by setting up two seismometers outside the crater on opposite ends of a diameter, and by firing a set of charges at intervals along the line between them. In this way, the refracted energy travelled down through the sediments into the basement rock at the bottom of the crater, and was then refracted outwards towards one of the detectors. As only one end of the refraction path had to be inside the feature, the difficulty in providing the necessary length of spread was eliminated, and each shot provided a measure of the time lag introduced by the sediments between the shot point and the nearest point of the basin floor.

The Brent observations had proved capable of revealing the existence of a deep basin, filled with materials having a much lower velocity of propagation than those outside, but it was felt that the observations would have been more definitive if a greater number of detectors could have been used. This would have enabled the shots inside the crater to have been observed from a number of different distances, and thereby yielded estimates of crustal velocities for the materials at various distances outside the crater. Furthermore, if the longer-range observations had yielded evidence of crustal stratification, it would have been possible to obtain several independent estimates of the delays introduced by the materials inside the crater, as these could have been considered as residuals from travel-time curves representing propagation through several different "marker layers".

By the time of the operation in the Mushalagan-Manicouagan area a 12-channel seismograph system had been completed. Each channel consisted of a Willmore seismometer, having a natural period of about 1 second, operating through a low-frequency vacuum tube amplifier and a frequency-modulated oscillator to produce an audio-frequency tone. The oscillator output was

* Now at Royal Observatory, Edinburgh, Scotland.

delivered to the modulator of a 5-watt v.h.f. radio transmitter, which transmitted it to a central recorder. It was intended that the seismographs should have been deployed as closely as possible along two perpendicular diameters of the crater, and that the explosions would have been set off in lakes on the same lines. The extreme range of the radio links was about 30 miles, so that by using a central recording site it was hoped to obtain a range of the order of 60 miles from one end of each line to the other. With such a range, the seismic waves would have been expected to travel deep within the earth's crust, so that even if the disturbed region associated with the topographic feature had extended downwards to a depth comparable to the radius of the feature, the waves observed at maximum range might have been expected to pass beneath it. Travel times for waves passing under the feature at somewhat shorter ranges would have been compared with those for waves travelling a comparable distance outside the feature, between the ends of the perpendicular profiles.

In the event, the operating conditions were found to be less favourable than had been expected. The only way of delivering equipment to the operating sites was by means of float-equipped aircraft landing on the lakes, but in many cases the lakes which were suitable for the aircraft did not provide access to bedrock exposures suitable for setting up the seismographs. When such exposures did exist, the radio propagation path to the central recorder was sometimes blocked by intervening high ground. Such difficulties soon made it clear that the concept of intersecting linear profiles was impracticable, and it was therefore decided that the instruments should be set up wherever it was possible to gain access to a site giving tolerable foundation and communication conditions, without regard to the preconceived geometrical pattern.

The consequence of the new policy was that there was no prospect of collecting a body of data which could be reduced by the methods that were available at the time. Nevertheless it was believed that the possibility of solving a set of travel-time equations could not logically be expected to depend on the conformity of the layout of shots and detectors with any arbitrary pattern. In fact the only obvious mathematical requirement was that the number of observations should exceed the number of unknowns. Thus, in the simple case of m recording stations observing waves from n sources, distributed over an area in which a surface layer of one material overlies an undulating basement of another material, one could attempt to describe the structure in terms of the thickness of the upper material under each shot point and detector, together with the propagation velocities in the upper and lower media. This would give a total of $n + m + 2$ unknowns, whereas there would be nm possible travel-time observations, even if the reduction was to be restricted to the times of the first arrivals. Clearly, this could yield a comfortable excess of observations over unknowns, and the optimum policy was to aim for the maximum possible number of connections between the available survey points. This policy was carried out within the limits of the time and resources available. In all, 16 recording stations were occupied and 20 shots (including some small short-range ones) were fired. Some stations failed completely to receive seismic energy or to communicate to base, and the equipment of these was moved after a few shots. After eliminating doubtful results, 50 reliable observations were obtained, representing connections between the 12 most favourable seismic stations, or shots close to them. A preliminary plot of the data showed that practically all the first-arrival times lay within a quarter of a second of a travel-time line which had an intercept of half a second and indicated a propagation velocity of a little over 6 km/sec. This immediately excluded the possibility that the circular feature contained a great thickness of low-velocity material, and it became clear that a quite sensitive statistical procedure would be needed to reveal any structure at all.

After the completion of the field operation the necessary theory gradually developed (Scheidegger and Willmore 1957, Willmore and Bancroft 1961). The former of these will be referred to later as Paper I. It is now clear that the Manicouagan-Mushalagan readings did, in fact, constitute a coherent body of data, and that the reduction involves a number of interesting features which might not have been brought out in a more favourable situation.

FIELD PROCEDURE

The main base camp for the field operation was sited on a peninsula, projecting into a large lake, to which the name of Observatory Lake was assigned during the operation. (Figure 1). This lake provided an excellent landing area for large supply aircraft, and its central position seemed to be favourable in relation to the proposed layout of seismic stations and shot points.

The first step in the seismic work was to set up the central receiving station at the base camp, a seismograph at T_1, on Mushalagan Lake and another at T_3 on Manicouagan Lake. After a test shot had been fired, a number of other seismograph stations were established, including one near each end of Observatory Lake. Other shots were then fired, the first nine being in Observatory Lake, where they constituted a body of short-range data, covering the range of distance from 0 to 5 km. The remaining shots were fired wherever a suitable depth of water could be found close to one of the seismic transmitting stations, the station nearest to each shot being used in each case to determine the shot time.

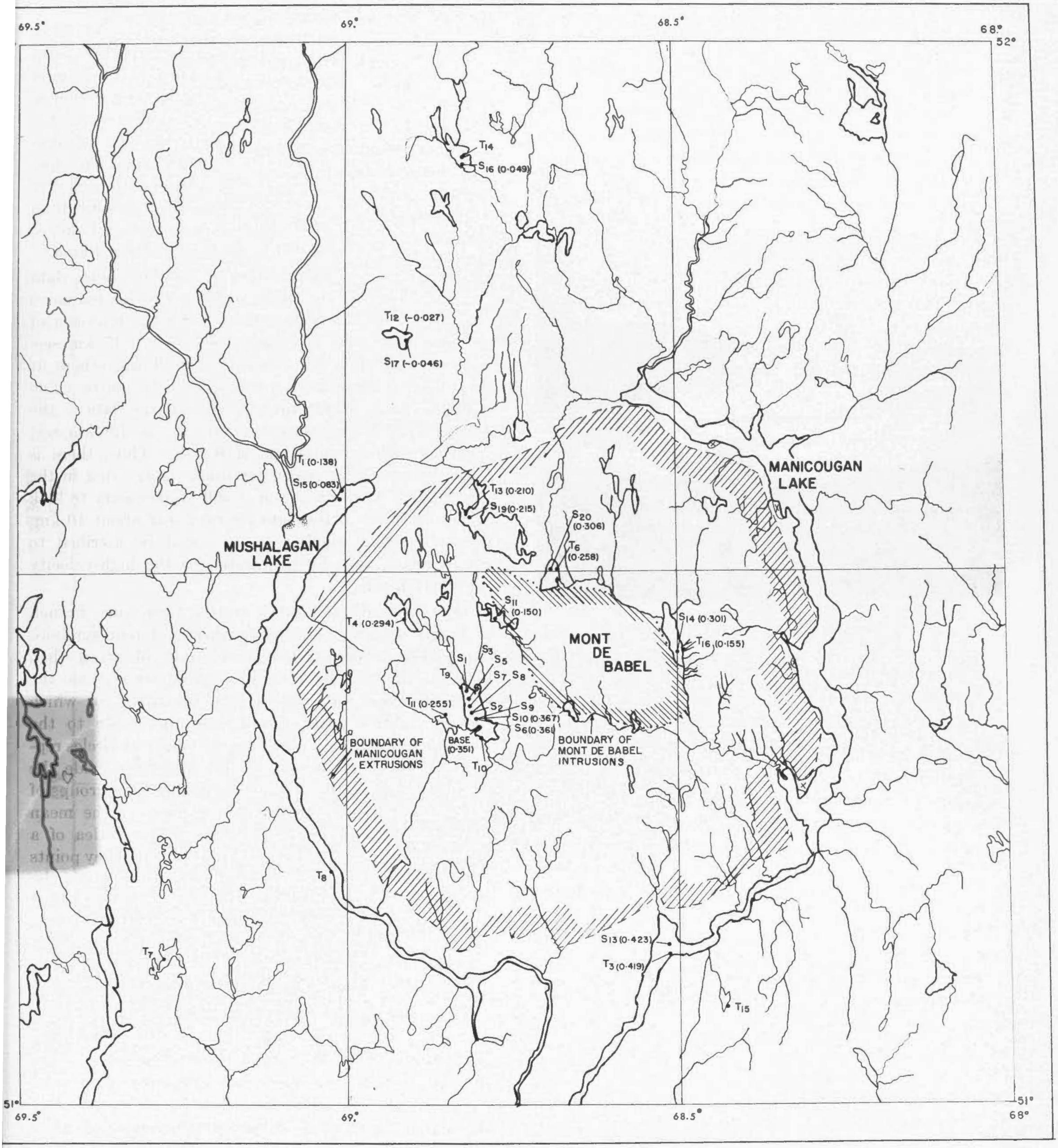

69.5°
69.°
68.5°
68.°
52°
T14
S16 (0.049)
T12 (−0.027)
S17 (−0.046)
T1 (0.138)
S15 (0.083)
T13 (0.210)
S19 (0.215)
S20 (0.306)
T6 (0.258)
MUSHALAGAN LAKE
MANICOUGAN LAKE
T4 (0.294)
S11 (0.150)
S14 (0.301)
T16 (0.155)
MONT DE BABEL
S1
S3
S5
S7
S8
T9
T11 (0.255)
S2
S9
S10 (0.367)
S6 (0.360)
BASE (0.351)
T10
BOUNDARY OF MANICOUGAN EXTRUSIONS
BOUNDARY OF MONT DE BABEL INTRUSIONS
T8
S13 (0.423)
T3 (0.419)
T7
T15
51°
69.5°
69.°
68.5°
51°
68°

FIGURE 1

The explosives had been acquired in 5-lb sticks, in tubular cardboard cases which could be coupled together at the ends. Large charges were made up by coupling the tubes together in twos or threes, and lashing a number of such units to a pole. The assembled bundle, up to eight feet long and two feet in diameter, could be lashed either to the underside of an outrigger on a canoe (Figure 2) or to the float of an aeroplane. On being released into the water, the bundle had a high enough density to sink slowly to the bottom.

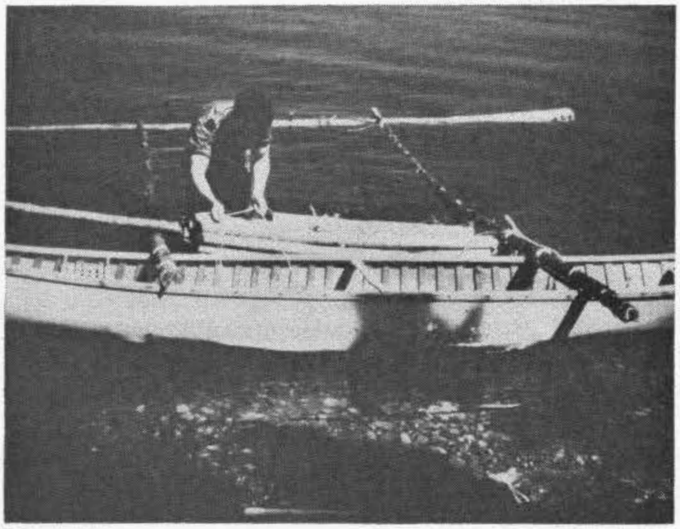


FIGURE 2

The charges were detonated electrically, using the firing circuit of a small seismic prospecting recorder. The normal procedure was to set up the recorder beside one of the field seismographs, and to feed the seismic signal into the local recorder as well as to the radio transmitter. Thus the seismic impulse was recorded both by the shot-firing unit and at base, whereas the interval between the seismic impulse and the detonator break was recorded by the firing unit. In Observatory Lake, where some of the shots were quite a long way from shore, the charges were fired from a canoe, and a radio in the canoe was used to transmit the shot instant. Using a few hundred feet of shot cable, it was found that charges of up to 250 lb could be fired from the canoe without causing discomfort to the occupants.

The positions of stations and shot-points were pinpointed on air photographs, being fixed in relation to small local landmarks. Larger features on the photographs were then used to enable positions to be transferred to a map, based on a trimetragon survey, on the scale of 1″ to the mile. Long distances were then measured from the map. This procedure avoided the cumulative errors which were present in the uncontrolled mosaic, which was also assembled from the photographs.

THE PRELIMINARY DATA-PLOT

The preliminary travel-time curve is shown in Figure 3. The first arrivals suggest a two-layered structure, with a propagation velocity of about 4.5 km/sec in the upper layer, and about 6.4 km/sec below. The intersection of the two branches of the travel-time curve occurs at a range of about 6 km from the origin, which suggests that the observation at a distance of 8.49 km, and those for all longer ranges, should be associated with the lower layer. Moreover, all the connections for ranges less than 6 km were inside the boundaries of Observatory Lake, so that these data referred to a single, well-defined area.

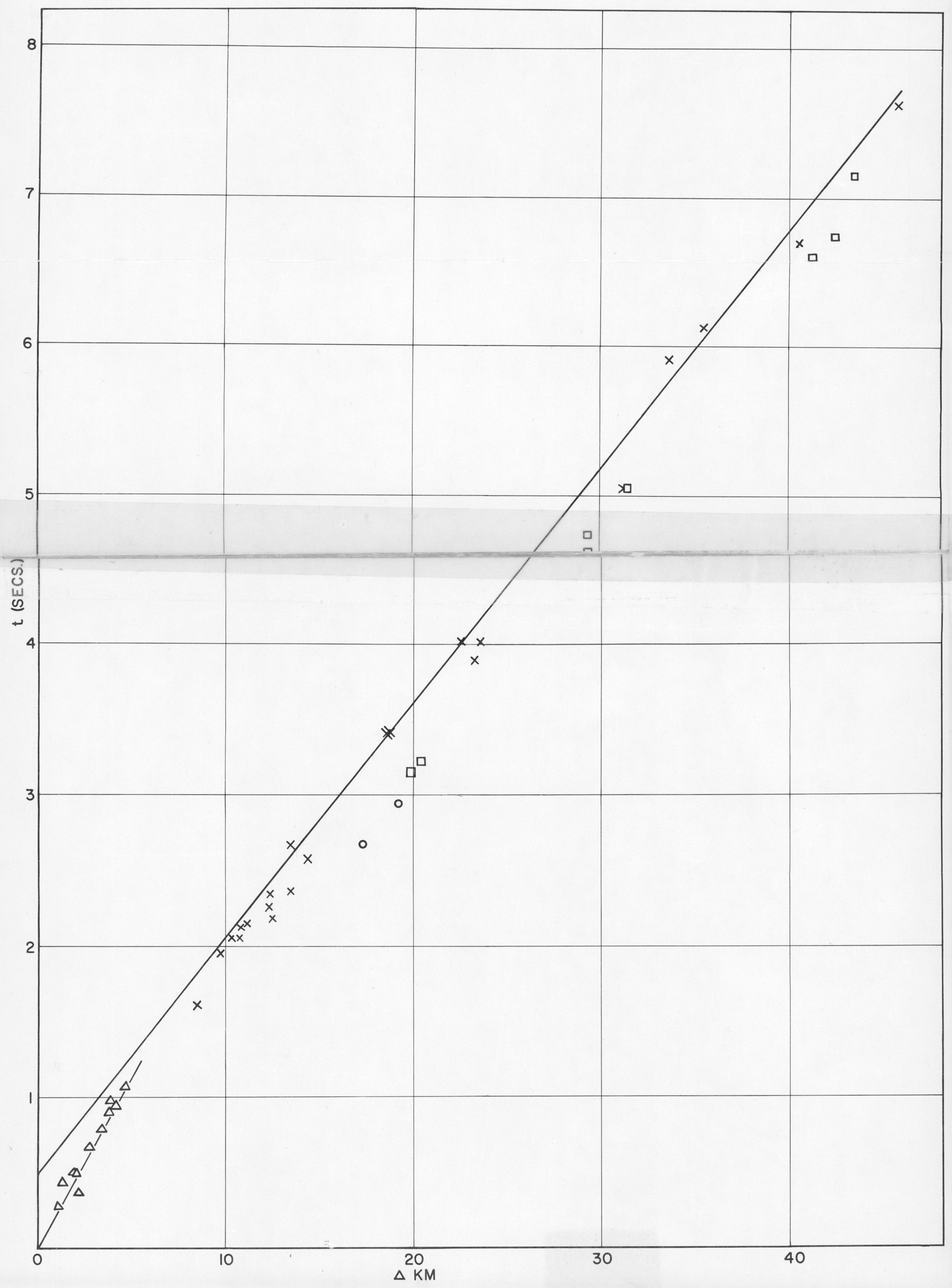
A more detailed examination of the short-range data throws doubt on this simple picture. The least-squares solution for all the observations (after the rejection of one late one) gives a velocity of 4.45 ± 0.17 km/sec. However, the late observation, and all the others at distances less than 2.5 km would fit better on to a line with a velocity of 4.0 km/sec, whereas the data in the range of 2.5-5 km suggest a velocity of 4.9 km/sec, together with an intercept of 0.1 sec. Thus, there is some evidence of a velocity gradient or layering in the Observatory Lake area, and it will be necessary to look rather carefully at the data for ranges of about 10 km, before deciding whether these should be ascribed to waves which actually penetrated to the high-velocity "marker" layer.

The observations plotted on the long-range branch of Figure 3 are represented by three different symbols. The crosses represent the travel times observed when both the shot and the recording station were inside the circle of lakes, the squares give the times for which the propagation path crossed from the inside to the outside of the topographical feature and the circles represent the connections which lay entirely outside the feature. The fact that the means of the three groups of data get progressively earlier in relation to the mean travel-time line gives some support for the idea of a crater-like structure, but the distribution of survey points is such that this preliminary observation would be equally consistent with the existence of a wedge-shaped layer of low-velocity material, thickening towards the south and east. In fact, the more detailed reduction will show that the latter type of structure is more consistent with the data.

THE TIME-TERM SOLUTION

The method which has been developed for determining the form of non-plane structures, starts with the ordinary equation for the travel time of a refracted wave, which is written in the form

$$a_i + b_j = t_{ij} - \frac{\Delta_{ij}}{v_2} \tag{1}$$



8
7
6
5
4
3
2
1
t (SECS.)
0
10
20
30
40
Δ KM

Where t_{ij} is the travel time between the i^{th} shot point and the j^{th} detector, a_i and b_j are "time terms" characteristic of the shot point and the detector, respectively, Δ_{ij} is, to a first approximation, the distance between the shot point and the detector, and v_2 is the velocity in the marker layer. Corrections can be applied if v_2 is a function of depth below the surface of the marker layer, or if a preliminary solution indicates steeply dipping parts of the structure. In the present study, the first approximation indicates a relatively shallow structure, so that the steep-dip corrections are not required.

In more general terms, equation (1) may be written in the form

$$p_{st}\, x_t = m_s\,. \tag{2}$$

Where x_t can represent any of the time terms a_i or b_j, and m_s represents the right hand side of equation (2) for any one observation of travel time and distance. The coefficients of p_{st} will be unity when the value of t refers to a shot point or seismometer for which the shot-point appears in the s^{th} equation, and will otherwise be zero.

In this notation, the entire body of travel-time data is set out in Table I observations being listed in order of increasing distance.

The first 12 entries in the table are the short-range observations which have already been discussed, and the remainder are those which, for the time being, are assumed to represent waves refracted through the marker

TABLE I

Connection No.	RECORDING POINTS										SHOT-POINTS																		
	B	T_1	T_3	T_4	T_6	T_{10}	T_{11}	T_{12}	T_{13}	T_{16}	S_1	S_2	S_3	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{19}	S_{20}	t_{ij}	Δ_{ij}
1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	0.29	1.18
2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	0.32	1.28
3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	0.35	1.38
4	1	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0.51	2.04
5	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	0.51	2.15
6	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	0.69	2.83
7	1	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	0.81	3.50
8	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0.92	3.86
9	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	0.93	3.89
10	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0.99	3.97
11	—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	0.96	4.19
12	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1.10	4.74
13	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1.64	8.49
14	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1.99	9.78
15	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2.10	10.29
16	—	—	—	1	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2.10	10.67
17	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2.17	10.80
18	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2.20	11.19
19	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	2.31	12.35
20	—	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2.44	12.47
21	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	2.23	12.67
22	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2.74	13.44
23	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2.42	13.50
24	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	2.63	14.40
25	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	3.10	17.04
26	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	2.73	17.36
27	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	3.48	18.62
28	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	3.50	18.71
29	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	3.00	19.16
30	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	3.22	19.74
31	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	3.29	20.38
32	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	4.11	22.51
33	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	4.10	22.77
34	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	3.96	23.28
35	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	4.11	23.47
36	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	4.70	26.75
37	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	4.91	27.78
38	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	4.82	29.19
39	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	4.73	29.26
40	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	5.22	29.90
41	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	5.62	31.18
42	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	5.16*	31.44
43	—	—	1	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6.03	33.69
44	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6.26	35.49
45	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	6.84	40.57
46	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	6.75	41.20
47	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	6.88	42.48
48	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	7.04	43.52
49	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	7.79	45.85
50	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	9.65	59.80

*Could have been read as 5.01 sec.

layer. The connections to T_{16} and S_{14} are, however, suspect, for these are connections to points on the east flank of Mont de Babel. Geologically, this mountain is a large intrusive mass, which might, it seemed, have been associated with disturbance of the underlying basement. The suspect connections all passed through this region, and to include them in the same mathematical scheme as the others might have led to a distorted view of the entire structure. It was considered safer to reserve them for separate discussion. The connections from S_{20} and T_6 into Observatory Lake are also potentially suspect, but these can be checked against connections which run outwards to other survey points.

After isolating T_{16} and S_{14}, we see that the columns headed by T_{11}, S_5 and S_6 have only a single entry, which means the corresponding survey points each featured in only one travel time equation. The travel times which appeared in these equations can therefore be explained exactly by postulating an appropriate time term for the singly-connected survey point, and there is no need to include these equations in the network solution.

After eliminating the surplus or suspect connections, we are left with 29 equations connecting 17 survey points, and the time terms of some of the survey points are determined by only two connections. If one of such a pair of observations were to be faulty, the solution would produce a time term such that the error of the faulty observation would apparently be shared with the other, giving equal and opposite residuals. Moreover, the error in the calculated time term would also be partially transferred to the time terms of other survey points. The general statement of this situation is that, in a weakly controlled network, the errors arising from a single faulty observation are widely distributed throughout the solution, and the source of the trouble becomes very difficult to recognise.

In view of this unsatisfactory property of the data, it was decided to introduce the assumption that the time terms would not vary much within areas a few kilometres in extent. Acting on this assumption, we can equate the time terms of all the shot points in Observatory Lake to that of the base camp seismometer. Further, the time term of any other shot point which was near to a successful recording station is equated to the

TABLE II

Condensed Matrix, Residuals and Marker-Wave Lead

Connection No.	B	T_1	T_3	T_4	T_6	T_{12}	T_{13}	S_{11}	S_{16}	t_{ij}	Δ_{ij}	1st Residual	Marker Wave Lead	2nd Residual
13	—	—	—	—	1	—	—	1	—	1.64	8.49	−.107	.030	—
14	—	—	—	1	—	—	—	1	—	1.99	9.78	+.079	.125	—
15	—	—	—	—	1	—	1	—	—	2.10	10.29	+.037	.194	+.029
16	1	—	—	1	—	—	—	—	—	2.10	10.67	−.105	.028	—
17	—	—	—	—	1	—	1	—	—	2.17	10.80	+.029	.151	−.005
18	1	—	—	1	—	—	—	—	—	2.20	11.19	−.086	.055	—
20	1	—	—	1	—	—	—	—	—	2.44	12.47	−.041	.122	—
21	—	1	—	—	—	—	1	—	—	2.23	12.67	−.054	.397	−.008
22	1	—	—	1	—	—	—	—	—	2.74	13.44	+.104	.172	—
23	—	1	—	—	—	—	1	—	—	2.42	13.50	0.000	.441	−.033
24	—	1	—	1	—	—	—	—	—	2.63	14.40	+.053	.433	+.019
26	—	1	—	—	—	1	—	—	—	2.73	17.36	−.046	.852	−.016
27	1	—	—	—	1	—	—	—	—	3.48	18.62	+.011	.416	−.025
28	1	—	—	—	1	—	—	—	—	3.50	18.71	+.008	.421	−.057
29	—	—	—	—	—	1	—	—	1	3.00	19.16	+.051	.990	+.016
30	—	—	—	—	—	1	1	—	—	3.22	19.74	−.029	.876	+.014
31	—	—	—	—	—	1	1	—	—	3.29	20.38	−.056	.909	−.053
32	1	—	—	—	—	—	1	—	—	4.11	22.51	+.073	.666	+.056
34	—	1	—	—	1	—	—	—	—	3.96	23.28	−.037	.869	+.009
35	—	1	—	—	1	—	—	—	—	4.11	23.47	+.087	.879	+.032
38	—	—	—	—	1	1	—	—	—	4.82	29.19	+.043	1.320	+.027
39	—	—	—	—	1	1	—	—	—	4.73	29.26	−.055	1.323	−.011
41	1	—	1	—	—	—	—	—	—	5.62	31.18	+.032	0.924	+.018
42	—	—	—	—	—	1	—	1	—	5.16*	31.44	+.112	1.518	—
43	1	—	1	—	—	—	—	—	—	6.03	33.69	+.046	1.055	—
44	1	—	1	—	—	—	—	—	—	6.26	35.49	−.001	1.149	—
45	—	—	1	—	—	—	—	1	—	6.84	40.57	−.081	1.556	—
46	1	—	—	—	—	1	—	—	—	6.75	41.20	+.024	1.884	+.029
47	1	—	—	—	—	1	—	—	—	6.88	42.48	−.044	1.950	−.005
48	—	—	—	—	1	—	—	—	1	7.04	43.52	−.020	1.960	−.004
49	—	—	1	1	—	—	—	—	—	7.79	45.85	−.002	1.777	−.019
50	1	—	—	—	—	—	—	—	1	9.65	59.80	−.030	2.757	−.013

*Adopt 5.01 instead of 5.16, in view of large first residual.

time term of the station. As a result, the following approximations are introduced:—

Time terms for $S_1 - S_{10}$ = Time term for base
" $S_{13} = T_3$
" $S_{15} = T_1$
" $S_{17} = T_{12}$
" $S_{19} = T_{13}$
" $S_{20} = T_6$

After this condensation, it was possible to re-introduce some of the connections which had been dropped because they were the only ones between survey points, and we now have 32 equations involving only 9 time terms. The matrix representation of these equations is set out in the first part of Table II.

A quick check on the order of magnitude of the errors introduced by the condensation process can be derived from the fact that some of the connections listed in Table II involve the same pairs of time terms. These are not genuinely repeated observations, for if the first of two such equations represents the connection from a shot-point in a lake to a distant station, the other will represent the travel time from a shot-point near the first detector to a detector near the first shot-point. Thus there will be a small difference in the range of the two connections, and a further difference arising from the fact that the time terms for the shots and the nearby detectors are not identical.

The matched pairs of equations are listed in the first column of Table III. The differences in travel time and range are shown in the next two columns. The last column of the table gives the residual, formed by subtracting the increment of travel time which would correspond to the actual difference in range, from the difference of travel time observed:—

TABLE III

Connection Number	δt (sec)	$\delta\Delta$ (km)	$\delta t - \frac{\delta\Delta}{6.4}$
18–20	+0.24	1.27	+0.05
27–28	+0.02	0.10	0.00
30–31	+0.07	0.64	−0.03
34–35	+0.15	0.19	+0.12
38–39	−0.09	0.07	−0.10
41–43	+0.41	2.51	+0.01
46–47	+0.13	1.28	−0.07

The root-mean-square value of the residuals listed in the last column of Table III is 0.07 secs. This figure represents the r.m.s. error resulting from the condensation process, compounded with the r.m.s. error of one observation, and is low enough to justify the use of the condensation process for a preliminary reduction.

Returning to Table II, we reduce the 32 observational equations to nine normal equations, following the method which was described in Paper I. These yield the following preliminary solution for the time terms and the basement velocity:—

Time term for base = $0.309 \pm .026$
$T_1 = 0.098 \pm .033$
$T_3 = 0.393 \pm .036$
$T_4 = 0.220 \pm .032$
$T_6 = 0.246 \pm .026$
$T_{12} = -0.045 \pm .027$
$T_{13} = 0.198 \pm .031$
$S_{11} = 0.165 \pm .041$
$S_{16} = 0.052 \pm .044$

Standard deviation for 1 time observation = 0.073 secs

$$\text{Velocity} = 6.365 \pm 0.065 \text{ km/sec}$$

On entering the time terms and the velocity into the original travel-time equations we obtain the "first residuals" of Table II. Looking down the list of first residuals, we immediately note the concentration of negative values near the beginning of the table; indeed, the three largest negative residuals occur in the first six entries of the table. This fact suggests that the solution has been carried too close to the origin, so that waves travelling in the upper layer, arriving earlier than those refracted through the marker, have been erroneously included. In this simple form, the hypothesis is weakened by the fact that connection No. 21 gives almost the largest positive residual of the whole solution, even though the range of observation is only slightly greater than that of the connections numbered 18, 20 and 21, all of which seem clearly to be members of the negatively anomalous group. A more satisfactory form of the hypothesis is given in the next paragraph, in which the positive residual ceases to create any difficulty. Before treating this point we note that the largest of all the positive residuals is that for connection No. 42, for which the record had an emergent start, which could have been read 0.15 seconds ahead of the time listed in Tables I and II. The positive residual strongly suggests that the earlier movement is the true beginning, so that a propagation time of 5.01 seconds is adopted.

Returning now to the validity of the short-range connections which were included in the condensed matrix, we note that the ability of the wave refracted through the marker layer to arrive before any of the waves travelling nearer the surface does not depend only on the range of observation and the velocity contrast between the upper and lower media. Another variable which must be taken into account is the time lost by the refracted waves in penetrating to the marker layer, and in getting back to the surface. For any given connection, this is the sum of the time terms. In fact, the condition for the refracted wave to arrive first in the connection i, j is simply

$$a_i + b_j + \frac{\Delta_{ij}}{v_2} < \frac{\Delta_{ij}}{v_1}$$

Where v_1 is the velocity in the upper medium.

In practice, we do not know v_1 and v_2 exactly, and there will therefore always be some uncertainty in the identity of the first arrival when the difference between the two sides of the inequality is small. It is therefore convenient to define the quantity ϕ which we call the "marker-wave lead" such that

$$\phi = \Delta_{ij} \left\{ \frac{1}{v_1} - \frac{1}{v_2} \right\} - (a_i + b_j)$$

In view of the uncertainty of the short-range solution we have to guess at the average value of v_1, but taking $v_1 = 4.8$ km/sec makes $(1/v_1 - 1/v_2)$ about 0.0522 sec/km, and yields the values of ϕ which are given in Table II.

The application of this concept is illustrated in Figure 4. In section (a) of the figure, the first residuals from Table II are plotted as a function of distance. The concentration of negative residuals at short distances is noticeable, but does not lead to a clear pattern. In Figure 4b, the residuals are plotted against marker-wave lead, and it is immediately clear that the onsets for values of ϕ less than 0.13 (which are marked as circles in both sections of the figure) form a different population from the rest.

Evidently, our estimate of the average velocity in the upper media was too low, probably because the tendency for velocity to increase with depth continued beyond the points which were originally considered in the short-range profile. The sharpness of the break in Figure 4b does, however, set a limit to this trend, and enables us to feel that, if we exclude the circled points, the others definitely represent waves propagated below the marker. After rejecting these observations, the velocity which was derived above ceases to give the best fit to the surviving data. A rough indication of the required change can be obtained by fitting a least-squares line to the residuals which remain, and this indicates that the velocity should be increased by about .05 km/sec. This, however, is a rough procedure, and the only proper method is to re-work the entire solution.

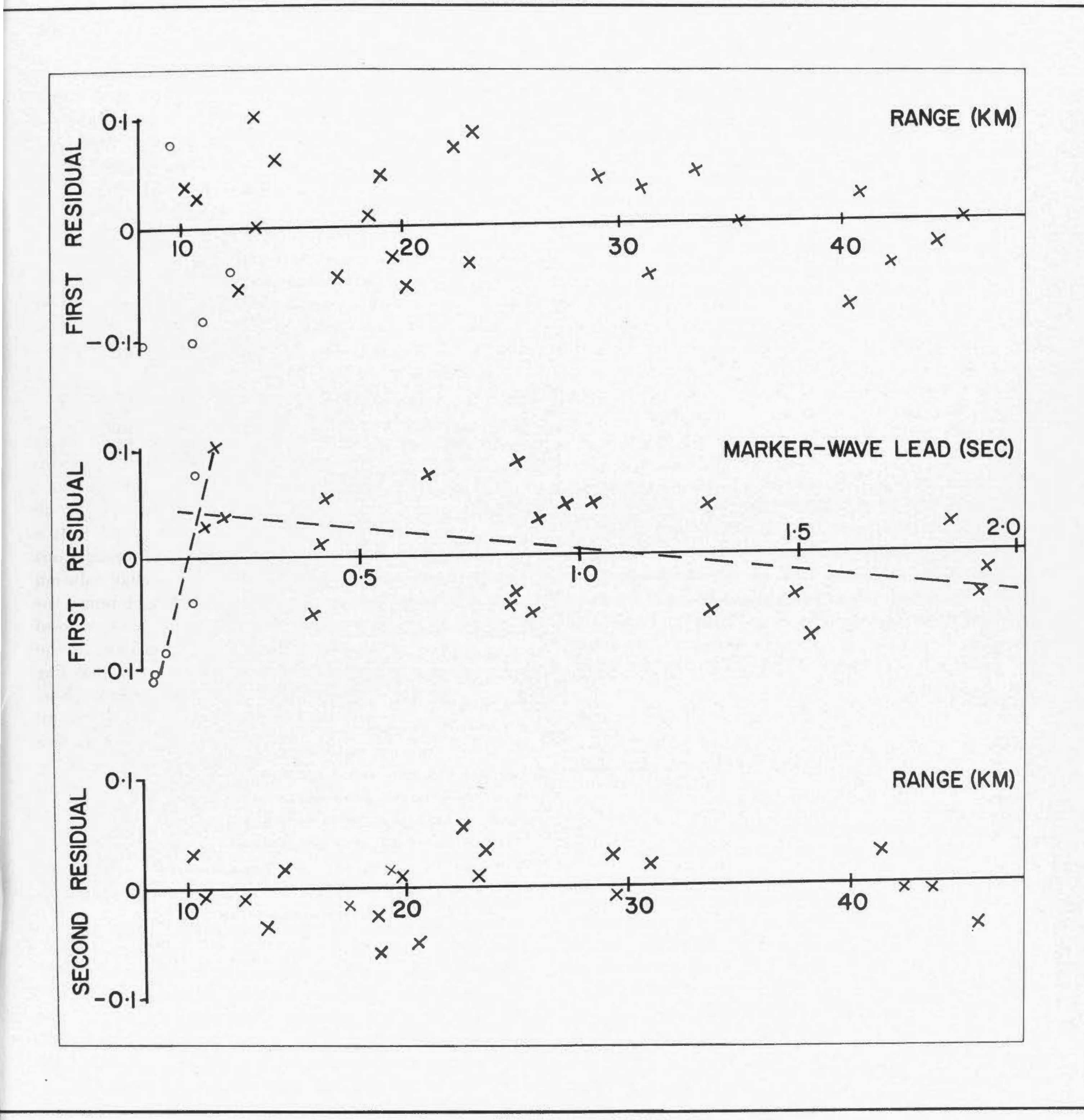
At this stage, the condensed matrix has served its purpose. We are therefore ready to proceed to the final solution, in which we assign separate time terms to each shot-point and detector. The rejection of the short connections has further depleted our stock of data, but we still have 22 observations to control 13 time terms. Furthermore, the loss of data is concentrated in the central part of the survey, where connections are still fairly plentiful, and where the significance of the observations in controlling v_2 is small.

By assigning separate time terms to shot-points and detectors, we have introduced an ambiguity in the solution which arises from the fact that it would now be possible to add an arbitrary constant α to the time terms of all the shots, and to subtract the same constant from the time terms of all the detectors, without altering any of the travel-time relations. The method of dealing with this arbitrary term is to perform the reduction with one time term set arbitrarily equal to zero, and then to choose α to give the simplest structure over the survey area. In the present case, we choose α so as to make the mean of the time terms for all shots which were fired close to successful detectors equal to the mean time term for those detectors.

The solution is given in Table IV. The individual residuals are listed in Table II and Figure 4c, for comparison with those of the reduced solution. Table IV includes time terms estimated from single or doubtful connections, in addition to those which followed directly from the matrix. The standard deviations following the time terms are those which arise directly from the observed scatter of the residuals, after making appropriate allowances for the number of constants fitted to the data, and for the small size of the remaining sample. These will slightly underestimate the true uncertainties in the data, because errors in shot-timing will all be absorbed into the time terms of the shots, and will not contribute to the residuals.

No uncertainty is assigned to the time term for the base, as this has been set equal to the arbitrary constant α. An error in the choice of α would have introduced a constant error in the whole set of shot-point time terms, and an opposite error in all the detector terms. The hypothesis used, and the distribution of survey points, are such that many of these opposite errors would have occurred at nearby points of the map, and hence would have little effect on any regional picture.

Other time terms for which no standard deviations are given are those which involve additional assumptions. Thus the time term for T_3 depends on a value for S_2 and S_3, which were not connected to any other survey points. Here we assume a time term equal to the mean of the values for the other two long-range shots in Observatory Lake, and list the separate estimates for T_3 which arise from this hypothesis. There is a weak closure to this loop from the fact that T_3 is connected through S_{11} to T_{12}, although the final link in this chain is the doubtful observation No. 42. In spite of the weakness of these connections, the individual estimates listed are all within the standard deviations to be expected from the main body of data.



RANGE (KM)
FIRST RESIDUAL
0·1
0
-0·1
10
20
30
40
MARKER-WAVE LEAD (SEC)
FIRST RESIDUAL
0·1
0
-0·1
0·5
1·0
1·5
2·0
RANGE (KM)
SECOND RESIDUAL
0·1
0
-0·1
10
20
30
40

FIGURE 4

TABLE IV

FINAL SOLUTION

Survey Point	Connection	Time Terms	Adopted Value
Base	Matrix	α	0.351
T_1	Matrix	$-.213 + \alpha$	$0.138 \pm .044$
T_3	S_2 and S_3	$+.092 + \alpha, \ +.043 + \alpha$	0.419
T_4	Matrix	$-.057 + \alpha$	$0.294 \pm .047$
T_6	Matrix	$-.093 + \alpha$	$0.258 \pm .034$
T_{11}	S_{11}	$-.096 + \alpha$	0.255
T_{12}	Matrix	$-.378 + \alpha$	$-0.027 \pm .034$
T_{13}	Matrix	$-.141 + \alpha$	$0.210 \pm .039$
T_{16}	S_{20}	$-.196 + \alpha$	0.155
S_2	T_3	Adopt mean of S_6 and S_{10}	0.365
S_3	T_3		
S_6	T_4	$+.712 - \alpha$	$0.361 \pm .068$
S_{10}	Matrix	$-.718 - \alpha$	$0.367 \pm .046$
S_{11}	T_3 and T_{12}	$+.485 - \alpha, \ +.516 - \alpha$	0.150
S_{13}	Matrix	$+.774 - \alpha$	$0.423 \pm .042$
S_{14}	base T_4, T_{11}, T_{13}	$+.571 - \alpha, \ +.662 - \alpha$ $+.650 - \alpha, \ +.745 - \alpha$	0.301
S_{15}	Matrix	$+.434 - \alpha$	$0.083 \pm .037$
S_{16}	Matrix	$+.400 - \alpha$	$0.049 \pm .035$
S_{17}	Matrix	$+.305 - \alpha$	$-0.046 \pm .035$
S_{19}	Matrix	$+.566 - \alpha$	$0.215 \pm .033$
S_{20}	Matrix	$+.655 - \alpha$	$0.304 \pm .033$

Standard deviation of 1 observation = .049 secs. V = 6.452 ± .056 km/sec...

The other doubtful connections are those relating to T_{16} and S_{14}, the discussion of which was deferred because of the possibility of anomalous propagation under Mont de Babel. The individual estimates of the time terms constitute a group with a standard error of 0.09 secs, whereas a value of about .07 sec would have been expected if the data had been of the same quality as the rest. Another indication of possible disturbance under Mont de Babel comes from connections No. 27 and 28, which represent propagation from T_6 and S_{20} into Observatory Lake. The second residuals for these equations are $-.025$ and $-.057$ which are respectively the fourth largest and the largest negative residual in a sample of 22 members. Thus, whilst the deviations are not enough to justify rejection of the data, the connections under Mont de Babel do appear to be somewhat less consistent than the others, and there is a suggestion that propagation times under parts of the mountain may be a few hundredths of a second shorter than those over corresponding distances of the normal marker layer.

The time terms from Table IV are entered alongside the survey points in Figure 1, and a measure of the success of the whole solution may be derived from the fact that the time-term differences for close shot-station pairs are all well within the indicated standard deviations. This means not only that the basic hypothesis of a uniform marker layer is supported, but also that local variations in overburden (which might have produced strong contrasts between lake shores and nearby lake bed) do not, in fact, interfere much with the run of the time terms.

Finally, we may ask whether the five observations which were rejected from the final long-range solution could be combined with some of the longer range observations in Observatory Lake, so as to yield a local time-term solution, and perhaps throw more light on the possibility of stratification within the upper media. The three longest connections within Observatory Lake (Nos. 10, 11 and 12) combine with the five rejected points to give eight equations in all. There are, however, five shot-points and four stations involved which, when the velocity in the hypothetical layer is counted, give rise to 10 unknowns. Hence no complete solution is possible, and, even if one attempts to group some of the survey points to form a condensed matrix, it will be found that the situation cannot be improved.

INTERPRETATION AND CONCLUSIONS

The one conclusion, which emerges immediately from the distribution of time terms on Figure 1, is that the trend is from low values to the north and west of the map towards larger values to the south and east. Thus the pronouncedly circular symmetry of the topography and surface geology is not reflected in the time terms, and we must conclude that the surface indications do not coincide with a major buried structure.

On comparing the seismic results with the gravity data (Innes, 1957) we note that Manicouagan and Mushalagan lakes lie on the southern side of a trough of strongly negative anomalies running from southwest to northeast. In the Manicouagan-Mushalagan area itself, the general trend is distorted, and the gravity anomaly contour for −50 milligals runs across the circular feature from northwest to southeast.

The trend of the seismic time terms implies the existence of a surface body of low-velocity material thickening towards the south and east. Generally speaking, low propagation velocities for seismic waves are associated with low density, so that large time terms are normally to be expected in association with negative gravity anomalies. Thus both the seismic and gravity patterns are consistent with the existence of a deep-seated ridge of low-density material (Innes suggests a large intrusive mass of granite) to produce the large-scale gravity anomaly, and an overlying wedge, also of relatively low density, to produce the distorted trend in the Mushalagan-Manicouagan area. Between these two masses, we have to postulate a stratum of high-velocity material, to provide the "marker layer".

It is very difficult to make suggestions on the rock types which could correspond to the main elements of this structure. The marker layer has a velocity substantially higher than the average for the Canadian Shield. Neither of the rock types which predominate within the feature can satisfy the requirements of either the high-velocity or the low-velocity material, because the trend of time terms shows no tendency to follow the surface exposures of these materials. Outside the circle of lakes, Rose's map shows the entire area covered by "Grenville type rocks—mainly granitic gneiss and garnetiferous gneiss", and yet the largest difference in time terms occurs between T_{12} and S_{17} on the Grenville rocks in the north, and S_{13} and T_3, ostensibly on the same rock province in the south. The only straightforward interpretation of this pattern is that the Grenville rocks provide the low-velocity cover, and that the marker is a high-velocity stratum, of which no exposures have been recognized.

Estimation of the vertical scale of the structure is hampered by the inadequacy of our knowledge of the velocity distribution near the surface. Our best method of estimating the average velocity above the marker layer will be to find the value of v_1 which gives zero marker-wave lead at the break-point between the short-range and long-range branches of the travel-time curve. This we can do by setting up the equations for the marker-wave lead for connection No. 17 (which was the first member of the long-range branch) and for connection No. 14, which was the last of the short-range group. Equating the lead to zero in each case yields estimates of v_1 of 4.95 and 4.99 km/sec respectively, so that a value of 4.97 km/sec will be adopted. As expected, this turns out to be a little higher than our first estimate of v_1, for the need to reject observations between the first and second attempts at a long-range solution had indicated that the first estimate of v_1 had been too low.

We are now in a position to set up a relationship between time term and depth to the marker, using the equation

$$t = \frac{h}{v_1}\sqrt{1 - \frac{v_1^2}{v_2^2}}$$

Inserting the appropriate values, we find h = 6.45 km/sec of time term, so that the time term of 0.42 sec at T_3 corresponds to a depth of about 2.7 km.

The proposed structure leads to suggestions for further geological or geophysical work, aimed at establishing the reality of the marker layer, and determining the velocity distribution in the rocks above it. The negative time terms for T_{12} and S_{17} indicate that the marker must be very near the surface in that area, and probably outcrops nearby. In any case, quite a short refraction profile should establish its presence. South of the feature, refraction profiles should be capable of determining the near-surface velocities, and waves through the marker layer should be expected to appear as first arrivals at distances in excess of about 18 km.

References

INNES, M. J. S. Gravity and isostasy in central Quebec, *Trans A.G.U.* 38, 156–165, 1957.

MILLMAN, P. M., LIBERTY, B. A., CLARK, J. F., WILLMORE, P. L. and INNES, M. J. S. *The Brent Crater Publ. Dom. Obs.* v. XXIV, no. 1, 1960.

ROSE, E. R. Manicouagan Lake-Mushalagan Lake area, Quebec, Map with marginal notes, *Geol. Surv. Can.*, Paper 55–2, 1955.

SCHEIDEGGER, A. E. and WILLMORE, P. L. The use of a least-squares method for the interpretation of data from seismic surveys, *Geophysics*, v. XXII, 9–22, 1957.

WILLMORE, P. L. and BANCROFT, A. M. The time term approach to refraction seismology *Geophys J., R. Astron. Soc.* 4, 419–432, 1961.

Note Added in Proof

Since the preparation of the main part of this paper, time-term theory has been developed to a stage at which we can begin to study variations in propagation velocity below the marker layer (Willmore, Herrin and Meyer, 1963). The procedure is to select pairs of survey points (either recording stations or shot-points) which are approximately in line with a third survey point to which they are both connected. By subtracting the time term for each of the first two points from the travel time to the third point, we obtain the times at which the refracted seismic wave reaches the foot of the perpendicular drawn from each survey point down to the marker layer. The difference in range for the two travel-time observations, divided by the difference in arrival time at the two points on the marker layer, yields an estimate for the local basement velocity.

The method assumes that the time terms are known independently of the observations used in the local velocity check, so that the ideal procedure would be to re-compute the entire set of time terms, dropping out the critical observations in each case. As an approximate, and much less laborious alternative, we note that if a given time term arises from n connections, and that if an observation which we desire to drop out has a residual r, then the mean residual for all observations except the unwanted one is $-r/(n-1)$. This quantity was applied as a correction to the time terms in the calculations.

The data available from the present survey contained ten pairs of points suitable for the work, which are listed in the table below. Each connection is identified by giving first the common point, followed by the pair which determine the interval over which the calculation is made:—

Connection	$\delta\Delta/\delta t$ (km/sec)	Connection	$\delta\Delta/\delta t$ (km/sec)
Base, $S_{17}-S_{19}$	6.632	S_{13}, T_4-Base	6.590
T_1, $S_{20}-S_{19}$	6.147	S_{16}, Base$-T_{12}$	6.487
T_6, $S_{15}-S_{19}$	6.544	S_{17}, T_6-T_{13}	6.556
T_{12}, $S_{20}-S_{19}$	6.001	S_{20}, T_1-T_{13}	6.190
T_{12}, $S_{10}-S_{19}$	6.206	S_{20}, $T_{12}-T_{13}$	6.348

The mean of the ten estimates of local velocity is $6.37 \pm .08$ km/sec, which is fully consistent with the value of 6.452 km/sec which was derived for the whole survey. The standard deviation of one velocity estimate from the mean is 0.23 km/sec, and the distribution could have arisen wholly or partly from errors of observation. There is no significant tendency for the velocity estimates to group themselves in a manner which would suggest the existence of major underground anomalies, so that the quoted standard deviation may be regarded as setting an upper limit to the true range of underground variations within the survey area.

Reference

Willmore, P. L., Herrin, E., and Meyer, R. P. Investigation of irregular sub-surface structures by seismic refraction methods. *Nature*, v. 197, 1094–1095, 1963.