CANADA
DEPARTMENT OF ENERGY, MINES AND RESOURCES
Dominion Observatories

PUBLICATIONS
of the
DOMINION OBSERVATORY
OTTAWA

Volume XXXV • No. 2

A TAYLOR EXPANSION OF THE GEOMAGNETIC FIELD IN THE CANADIAN ARCTIC

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Price 50 cents

ROGER DUHAMEL, F.R.S.C.
QUEEN'S PRINTER AND CONTROLLER OF STATIONERY
OTTAWA, 1966

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A TAYLOR EXPANSION OF THE GEOMAGNETIC FIELD IN THE CANADIAN ARCTIC

by

G. V. Haines
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Abstract—An aeromagnetic survey of an area of 1.6 million square miles in the Canadian Arctic was carried out in November 1963. Data from this area were used to obtain non-orthogonal Taylor series expansions of 1st, 2nd, and 3rd degrees by the method of least squares. The basic analysis was carried out in mutually perpendicular components U, V, and Z, where U and V are horizontal components referred to the Greenwich Grid system and Z is the vertical component. The condition that the vertical component of curl H be close to zero was satisfied by means of an approximation. The standard errors, of the order of 300γ in the 1st degree solution, decreased about 80γ in carrying the expansions from 1st to 2nd degree, and 20γ from 2nd to 3rd. Standard coefficients, showing the relative importance of the terms, are listed. All but two coefficients tested significant at the 5 per cent level. A comparison with the Canadian Magnetic Charts showed that the R.M.S. differences in D, H, and Z were less than the standard errors of estimate. The magnetic pole position determined from the 3rd degree expansion is 13 miles from the accepted position for 1964.0. A correction for disturbance, over an area of .16 million square miles, resulted in a 20γ decrease in the standard error of U but no significant change in the errors of V and Z. Maps of U, V, Z, G, D, and H are given, where G is the magnetic variation referred to the Greenwich grid co-ordinate system. The residuals of U, V, Z, D, and H from the 3rd degree expansions are plotted.

Résumé—La Direction des observatoires fédéraux a effectué en novembre 1963 le levé aéromagnétique d'une région de 1,600,000 milles carrés dans l'Arctique canadien. Les données recueillies ont servi à dresser des développements non orthogonaux de séries de Taylor du 1er, du 2e et du 3e degré par la méthode des moindres carrés. L'analyse de base a été faite à partir des composantes mutuellement perpendiculaires U, V et Z, dont U et V sont les composantes horizontales reportées au quadrillage de Greenwich, et Z la composante verticale. On s'est servi d'une approximation pour satisfaire à la condition que la composante verticale de rot H soit presque égale à zéro. Les erreurs standards d'environ 300γ dans la solution du 1er degré ont été réduites d'environ 80γ en poursuivant le développement du 1er au 2e degré, et de 20γ en passant du 2e au 3e degré. L'auteur énumère les coefficients réguliers indiquant l'importance relative des termes. Tous les coefficients, sauf deux, ont paru significatifs au niveau de 5 p. 100. La comparaison avec les cartes magnétiques canadiennes montre que les erreurs moyennes quadratiques pour D, H et Z étaient inférieures aux erreurs standards des estimations. La position du pôle magnétique déterminée par le développement de la formule au 3e degré se trouve à 13 milles de la position acceptée pour 1964.0. Une correction effectuée pour tenir compte des perturbations dans une région de 160,000 milles carrés, a réduit de 20 γ l'erreur normale de U mais a peu modifié les erreurs relatives à V et à Z. Les cartes pour U, V, Z, G, D et H sont données, et G représente la variation magnétique reportée au quadrillage de Greenwich. Les résiduelles de U, V, Z, D et H des développements du 3e degré sont présentées en graphique.

Introduction

An aeromagnetic survey of the Canadian Arctic was conducted by the Dominion Observatory from November 1 to 14, 1963. The area above latitude 70° and from Greenland to Alaska was covered by 16 flight lines, 90 miles apart, run parallel to the 90° longitude line. In addition, a flight was made around the magnetic dip pole area on a 300-mile square (Figure 1). Flight altitudes ranged from 15,000 to 20,000 feet.

The magnetometer consists of three orthogonal detectors of the saturated transformer type mechanically linked to a gyro-stabilized platform (Serson, *et al.*, 1957; Serson, 1960). The data are presented in digital form as 5-minute averages of D, H, and Z, representing approximately 20-mile intervals of flight path.

More than 1,000 averages (which will be referred to as observations) were obtained on this survey and corrected for altitude. These observations were used to fit, by least-squares methods, polynomials up to the 3rd degree in U, V, and Z, where U and V are horizontal components in the Greenwich grid co-ordinate system and Z is the vertical component. A condition ensuring that the vertical component of curl H be zero requires many of the coefficients in V to be related to those in U. Thus the regression in U and V was done simultaneously, while that in Z was done separately.

A non-orthogonal Taylor expansion of the 1st degree in U and V, over an area of 30,000 square miles, was done by Dawson and Loomer (1963). In the present analysis, the expansion was carried to the 2nd and

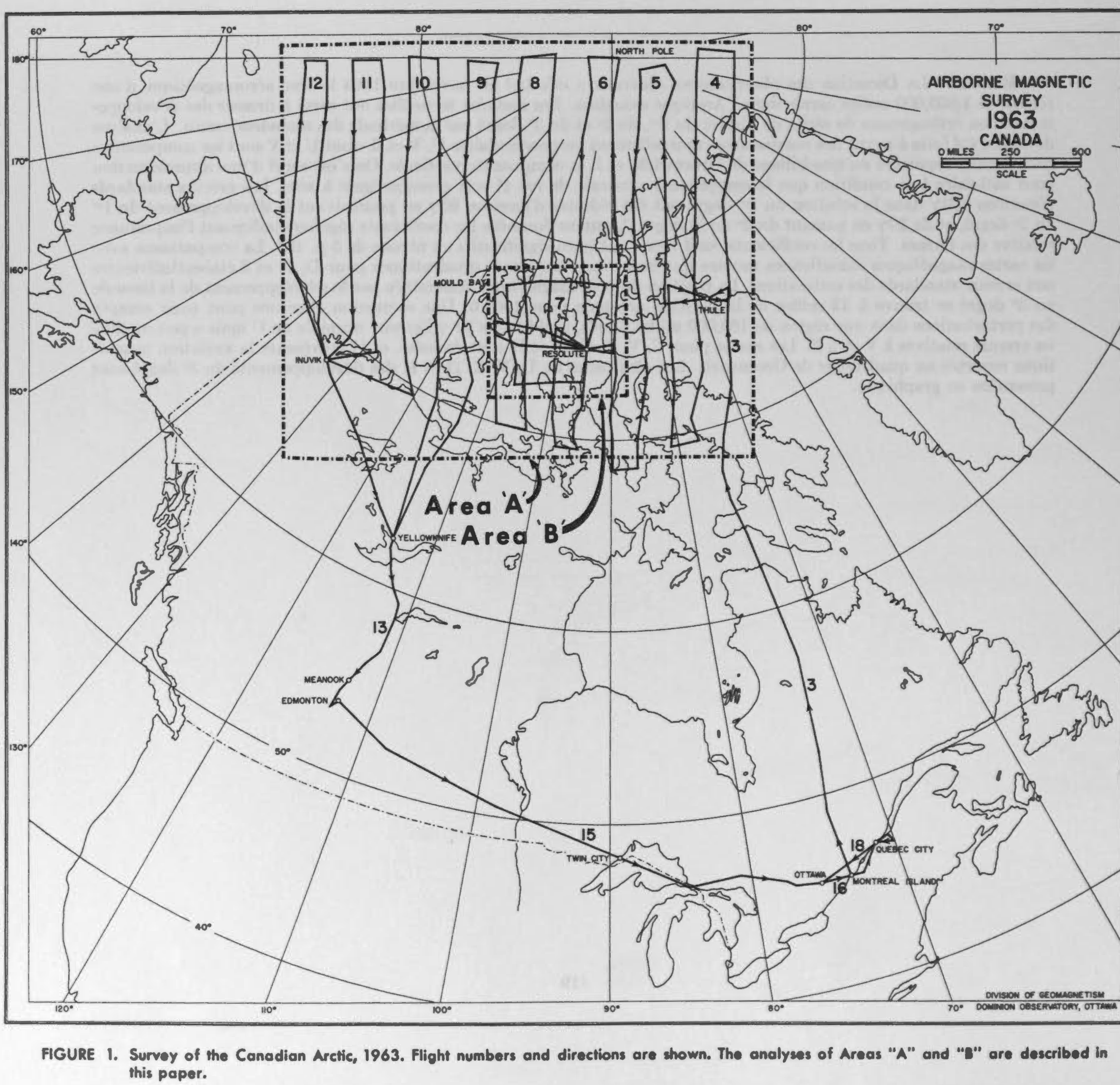

FIGURE 1. Survey of the Canadian Arctic, 1963. Flight numbers and directions are shown. The analyses of Areas "A" and "B" are described in this paper.

3rd degrees, and covered an area of 1.6 million square miles. The computations were done on an IBM 1620 Model II computer.

The least-squares solution of the coefficients was done with simple non-orthogonal terms. The use of the more complex orthogonal functions has been described by Fougere (1964).

Greenwich Grid System

Directions along great-circle flight tracks in the Arctic may change very rapidly if taken relative to the north pole, but are practically constant if taken relative to a meridian. By measuring directions relative to a meridian, the mathematical difficulty due to the singularity at the reference point is avoided also.

When the Greenwich meridian is chosen as a reference, "Grid North" is defined as the direction of the north geographic pole at Greenwich, and positive angles are measured clockwise. Then an angle D measured clockwise from true north, at a position whose east longitude is λ, may be expressed in the Greenwich grid system as a grid angle G defined by $G = D - \lambda$.

If the angle D is the east declination of the magnetic field, G is referred to as the "grivation". Lines of equal magnetic grivation thus have a singularity only at the magnetic dip pole, whereas those of equal declination have a singularity at the geographic pole as well.

Similarly the true north (X) and true east (Y) components have singularities at the geographic pole, which singularities can be avoided by using the grid north (U) and grid east (V) components:

$$U = H \cos G = X \cos \lambda + Y \sin \lambda$$

$$V = H \sin G = - X \sin \lambda + Y \cos \lambda$$

The expressions for the true components X and Y, in terms of the grid components U and V, are

$$X = U \cos \lambda - V \sin \lambda \qquad (1)$$

$$Y = U \sin \lambda + V \cos \lambda \qquad (2)$$

Polar Stereographic Projection

Given a position on the earth's surface whose co-latitude is φ and east longitude is λ, the transformation

$$u = -\tan \left(\frac{\varphi}{2}\right) \cos \lambda \qquad (3)$$

$$v = \tan \left(\frac{\varphi}{2}\right) \sin \lambda \qquad (4)$$

is a stereographic projection describing a co-ordinate system whose u-axis points grid north and whose v-axis points grid east. The transformation is one to one, its inverse being

$$\varphi = 2 \arctan \sqrt{u^2+v^2} \qquad (5)$$

$$\lambda = \pi - \arctan \left(\frac{v}{u}\right) \qquad (6)$$

where the arctangents are taken in the quadrants appropriate to the signs of u and v.

A flight line of constant grid heading through the Arctic region is, for all practical purposes, an arc of a great circle, and all lines of constant grid heading are stereographically projected as parallel straight lines. Since the stereographic projection is conformal, geometrical operations on these lines may be performed as in a regular 2-dimensional Cartesian co-ordinate system.

Maxwell's Curl Condition

It is assumed that the vertical component of the electric current density near the surface of the earth is negligible (Chapman and Bartels, (1940) vol. 1, p. 112). The rate of change of electric flux density also may be neglected. Maxwell's equation then requires the radial component of curl F to be zero, where F is the total magnetic field intensity. This results in a condition to be satisfied by the horizontal component H over the earth's surface, that is, the vertical component of curl H (the component in the direction of Z) must be zero:

$$(\text{curl H})_z = 0$$

Expressed in terms of true north (X) and true east (Y) components, the vertical downward component of curl H may be written as

$$(\text{curl H})_z = -\frac{1}{a \sin \varphi}\left[Y \cos \varphi + \frac{\partial Y}{\partial \varphi} \sin \varphi + \frac{\partial X}{\partial \lambda}\right]$$

where a is the earth's radius. (See Chapman and Bartels, (1940) vol. 2, p. 632).

Transforming X and Y to U and V by equations 1 and 2, this becomes

$$(\text{curl H})_z = -\frac{1}{a \sin \varphi}\Big[(\cos \varphi - 1)(U \sin \lambda + V \cos \lambda) + \sin \lambda \left(\sin \varphi \frac{\partial U}{\partial \varphi} - \frac{\partial V}{\partial \lambda}\right) + \cos \lambda \left(\sin \varphi \frac{\partial V}{\partial \varphi} + \frac{\partial U}{\partial \lambda}\right) \Big] \qquad (7)$$

Now, by the familiar chain rule

$$\frac{\partial U}{\partial \varphi} = \frac{\partial U}{\partial u}\frac{\partial u}{\partial \varphi} + \frac{\partial U}{\partial v}\frac{\partial v}{\partial \varphi}$$

with a similar relationship for $\frac{\partial U}{\partial \lambda}$.

After differentiating equations 3 and 4 to obtain the partial derivatives of u and v with respect to φ and λ, the following relationships are obtained:

$$\sin \varphi \frac{\partial U}{\partial \varphi} = \left(\sin \lambda \frac{\partial U}{\partial v} - \cos \lambda \frac{\partial U}{\partial u}\right) \tan \left(\frac{\varphi}{2}\right)$$

$$\frac{\partial U}{\partial \lambda} = (\sin \lambda \frac{\partial U}{\partial u} + \cos \lambda \frac{\partial U}{\partial v}) \tan \left(\frac{\varphi}{2}\right)$$

Those in V are, of course, similar. Simply replace the U in the above equations with a V.

Substituting these values into equation 7, the following result is obtained:

$$(\text{curl H})_z = \frac{1}{2a} \sec^2 \left(\frac{\varphi}{2}\right) \left[\frac{\partial V}{\partial u} - \frac{\partial U}{\partial v} + \sin \varphi (U \sin \lambda + V \cos \lambda)\right]$$

Substitution of φ and λ from equations 5 and 6 yields

$$(\text{curl H})_z = \frac{1}{2a} (1+u^2+v^2)(\frac{\partial V}{\partial u} - \frac{\partial U}{\partial v}) - \frac{1}{a}(uV-vU) \quad (8)$$

Hence a condition to be satisfied by the components U and V in the (u, v) co-ordinate system, if the vertical component of curl H is zero, is

$$\frac{\partial V}{\partial u} = \frac{\partial U}{\partial v} + \frac{2(uV-vU)}{1+u^2+v^2} \quad (9)$$

This result was obtained by Hutchison (1949) by using the conformal properties of the stereographic transformation.

Taylor Expansion of Magnetic Field

A function f(u, v) which has continuous partial derivatives of all orders may be expanded about a point (u_o, v_o) by Taylor's theorem as follows:

$$f(u, v) = f(u_o, v_o)$$
$$+ [(u - u_o) \frac{\partial}{\partial u} + (v - v_o) \frac{\partial}{\partial v}] f(u_o, v_o)$$
$$+ \frac{1}{2!} [(u - u_o) \frac{\partial}{\partial u} + (v - v_o) \frac{\partial}{\partial v}]^2 f(u_o, v_o)$$
$$+ \ldots\ldots$$

Thus the field components U, V, and Z may be expanded in a similar way. Upon replacing $(u - u_o)$ and $(v - v_o)$ by α and β, respectively, the expansions are in the form

$$U = U_o + U_u\alpha + U_v\beta + \tfrac{1}{2}U_{uu}\alpha^2 + U_{uv}\alpha\beta + \tfrac{1}{2}U_{vv}\beta^2 + \ldots.$$
$$V = V_o + V_u\alpha + V_v\beta + \tfrac{1}{2}V_{uu}\alpha^2 + V_{uv}\alpha\beta + \tfrac{1}{2}V_{vv}\beta^2 + \ldots.$$
$$Z = Z_o + Z_u\alpha + Z_v\beta + \tfrac{1}{2}Z_{uu}\alpha^2 + Z_{uv}\alpha\beta + \tfrac{1}{2}Z_{vv}\beta^2 + \ldots.$$

where the coefficients U_o, U_u, U_v, etc are understood to be the partial derivatives evaluated at (u_o, v_o).

If the expansions are taken close to the $(uV - vU) = 0$ isoline, then

$$\frac{2(uV - vU)}{1 + u^2 + v^2} << \frac{\partial U}{\partial v}$$

and equation 9 may be approximated by

$$\frac{\partial V}{\partial u} = \frac{\partial U}{\partial v}$$

Within the area covered by this survey, the $\frac{2(uV - vU)}{1 + u^2 + v^2}$ term has an average absolute value of less than 15 per cent of the $\frac{\partial U}{\partial v}$ term, and a maximum value (occurring on the edge of the data area) of about 35 per cent.

Conditions on higher order derivatives are obtained by differentiating equation 9. After applying approximations similar to that for the 1st order derivative, these conditions become

$$V_u = U_v$$
$$V_{uu} = U_{uv}$$
$$V_{uv} = U_{vv}$$
$$V_{uuu} = U_{uuv}$$
$$V_{uuv} = U_{uvv}$$
$$V_{uvv} = U_{vvv}$$

The average absolute value of the error terms in these approximations is about 20 per cent. This estimate was obtained by calculating the error terms for each derivative at 18 points in the survey area, and contouring. To do this, estimates must be available for all derivatives up to an order one less than that of the derivative whose error term is being calculated. Since the U and V isolines are roughly parallel and equispaced over the survey area, the derivatives at the magnetic pole were taken as estimates of the derivatives at all points in the area. The resulting average absolute values of the error term estimates, as a percentage of their derivatives, are 30 per cent for the two 2nd-order terms and 15 per cent for the three 3rd-order terms.

From equation 8, the effect of the approximations on the $(\text{curl H})_z = 0$ condition is $\frac{1}{a}(uV - vU)$, the average absolute value of which is about 50 ma/km² and the maximum absolute value of which is about 125 ma/km². However, Chapman (1942) shows that the average estimation error of $(\text{curl H})_z$ from isomagnetic charts is also about 125 ma/km². Hence the U and V charts are mutually consistent to this degree of accuracy, using the above approximations.

It may be noted that the above equations, relating derivatives of U to those of V, are necessary, but not sufficient, conditions for the orthogonality of the U and V isolines.

After applying these conditions to the equation in V, the expansions of U, V, and Z are as follows:

$$U = U_o + U_u\alpha + U_v\beta + \tfrac{1}{2}U_{uu}\alpha^2 + U_{uv}\alpha\beta + \tfrac{1}{2}U_{vv}\beta^2 + \ldots\ldots \quad (10)$$

$$V = V_o + U_v\alpha + V_v\beta + \tfrac{1}{2}U_{uv}\alpha^2 + U_{vv}\alpha\beta + \tfrac{1}{2}V_{vv}\beta^2 + \ldots\ldots \quad (11)$$

$$Z = Z_o + Z_u\alpha + Z_v\beta + \tfrac{1}{2}Z_{uu}\alpha^2 + Z_{uv}\alpha\beta + \tfrac{1}{2}Z_{vv}\beta^2 + \ldots\ldots \quad (12)$$

There are (n + 1) coefficients in the n^{th} order of U but only one additional one in the n^{th} order of V after applying Maxwell's $(\text{curl } H)_z = 0$ condition.

The coefficients of the U and V system may be obtained easily by the method of least squares by considering the equations

$$U = U_o.1 + V_o \cdot 0 + U_u . \alpha + U_v . \beta + V_v . 0 + \ldots\ldots$$

$$V = U_o.0 + V_o \cdot 1 + U_u .0 + U_v . \alpha + V_v . \beta + \ldots\ldots$$

The solution of the Z system is straightforward.

A 1st degree expansion in U and V requires a least-squares solution on 5 unknowns, a 2nd degree on 9, and a 3rd degree on 14. The expansions in Z require solutions on 3, 6, and 10 unknowns. That is, the n^{th} degree expansion contains $\sum_0^n (i + 2)$ unknowns in the case of U and V, and $\sum_0^n (i + 1)$ in the case of Z.

Regression Analysis

A variable y may have a frequency distribution when other variables $x_1, x_2, \ldots, x_p$ are held constant. If the mean, denoted by μ, of the y distribution at the given x - values depends linearly on the $x_1, x_2, \ldots, x_p$ the expression for μ is

$$\mu = \beta_o + \beta_1 x_1 + \ldots + \beta_p x_p$$

This is called the true regression equation. If, in addition, the variance σ^2 of the y distribution is constant for all values of $x_1, x_2, \ldots, x_p$, the method of least squares may be used to estimate $\beta_1, \beta_2, \ldots\ldots, \beta_p$. The resulting equation, called the least squares regression equation, is written

$$y' = b_o + b_1 x_1 + \ldots + b_p x_p$$

Associated with each b_i is a standard error s_{b_i} from which the 100 $(1 - \alpha)$ per cent confidence interval may be determined for the corresponding β_i. It is given by

$$b_i - t_{\frac{1}{2}\alpha}\, s_{b_i} < \beta_i < b_i + t_{\frac{1}{2}\alpha}\, s_{b_i} \quad (13)$$

where $t_{\frac{1}{2}\alpha}$ is the student's statistic at the significance level α for N − p − 1 degrees of freedom, N being the number of observations used in determining the regression. To test the hypothesis that β_i has the value B the t-value is computed as

$$t = \frac{b_i - B}{s_{b_i}} \quad (14)$$

The hypothesis is rejected at the significance level α if $|t|$ exceeds $t_{\frac{1}{2}\alpha}$ with N − p − 1 degrees of freedom.

The estimate of σ^2 is given by the error mean square s^2, the square root of which is called the standard error of estimate. A confidence inverval for σ^2, at the 100 $(1 - \alpha)$ per cent level, is given by

$$\frac{s^2}{\chi^2_{\frac{1}{2}\alpha}/df} < \sigma^2 < \frac{s^2}{\chi^2_{1-\frac{1}{2}\alpha}/df}$$

where χ^2 is the chi square statistic with N − p − 1 degrees of freedom.

The partial regression coefficients of a regression equation do not always present a clear picture of their relative importance, since the variables $y, x_1, x_2, \ldots, x_p$ usually are not in a common measure (see Steel and Torrie (1960) p. 284). When each variable is expressed in units of its standard deviation, the least squares regression equation becomes

$$\frac{y'}{s_{y'}} = c_o + c_1 \frac{x_1}{s_1} + \ldots + c_p \frac{x_p}{s_p}$$

This is referred to as the standard regression equation, and the coefficients c_i are called standard regression coefficients. It is clear that

$$c_o = \frac{b_o}{s_{y'}}$$

and, for i = 1, 2, . . . , p

$$c_i = \left(\frac{s_i}{s_{y'}}\right) b_i$$

Fitting the Data

The data from an area in the Canadian Arctic of 1.6 million square miles, where $-.04 < u < .15$ and $-.18 < v < .00$, denoted as Area A in Figure 1, were used to determine a least-squares solution to 1st, 2nd, and 3rd degree Taylor expansions of U, V, and Z. A total of 1636 observations were used in solving for the partial derivatives of U and V (that is, 818 equations in U and 818 in V) and a total of 1029 in solving for the derivatives of Z. The point of expansion was taken at $u_o = .0254$ and $v_o = -.1247$, the estimated magnetic pole position. Table I lists the partial derivatives, in gammas, evaluated at that point. To actually estimate U, V, and Z the derivative must still be multiplied by the appropriate fractions as found in equations 10, 11 and 12.

Since $-.07 < \alpha < .12$ and $-.05 < \beta < .13$, the the α^2, $\alpha\beta$, and β^2 are smaller than the α and β by a factor of 10, and the α^3, $\alpha^2\beta$, and $\alpha\beta^2$, and β^3 smaller by a factor of 10^2. Hence, although the coefficients increase by a factor of 10 from one order to the next, the contributions from the entire terms do not. In fact, the equations converge, the higher order terms contributing much less than the lower order terms.

Confidence intervals for the true regression coefficients are given by the standard errors and equation 13. For 95 per cent confidence, $t_{.025} = 2.0$ and the confidence interval is given by the coefficient ± twice its standard error. An interval of a coefficient ± its standard error corresponds to 68 per cent confidence.

Whether or not a particular coefficient contributes significantly to the regression may be seen by the "t" value—the ratio of the coefficient to its standard error. This is a test of the hypothesis $\beta = 0$ in equation 14.

At the 5 per cent level, a coefficient contributes significantly when the absolute value of t is greater than 2.0. A significant contribution at the one per cent level would require the absolute value of t to be greater than 2.6. The coefficients U_{uuv} and Z_{uuv}, for example, have t values of − 1.7 and − 1.5, respectively, and thus do not contribute significantly to the regression at the 5 per cent level.

The standard errors of U, V, Z, D, and H for the three degrees are given in Table II, with the number of observations used in determining the estimates. A high standard error of estimate of D is to be expected when working around the dip pole area. Similarly, the high error of estimate of Z for the 1st degree reflects the fact that over such a large area the Z surface cannot be approximated by a plane. In all cases there is a considerable improvement in going to the 2nd degree, but only an additional 20γ improvement in V and Z in going to the 3rd degree.

The standard regression coefficients (which are dimensionless) for the 3 degrees of U, V, and Z are listed in Table III. Here the fractions found in equations 10, 11 and 12 disappear since $kx/s_{kx} = x/s_x$ when k is a constant. Hence,

$$\frac{U}{s_U} = c_0 + c_1 \frac{\alpha}{s_\alpha} + c_2 \frac{\beta}{s_\beta} + c_3 \frac{\alpha^2}{s_{\alpha 2}} + \ldots$$

with similar equations in V and Z. The standard coefficients show immediately the relative importance of the various terms in estimating U, V, or Z. In estimating U from the 3rd degree equation, for example, the α term is 12 times as important as the α^2 term, and 29 times as important as the α^3 term.

Isolines of U and V for the 3 degrees were obtained by finding the roots of their equations and are shown in Figures 2 and 3. Those of Z for the 2nd and 3rd degrees were obtained similarly and shown in Figure 4. Isolines of G, D, and H for the 3rd degree were contoured from calculated values and are shown in Figures 5, 6, and 7.

The 3rd degree residuals of U, V, Z, D, and H for all observations in the area were plotted perpendicular to the flight paths (Figures 8 to 12). The residual is the observed value of the component minus the value obtained by least squares, plotted to the right of the flight track when positive and to the left when negative. An interval of no observations is designated by a dashed line drawn between the residuals on either side.

It must be emphasized that the observations used in determining the residuals are 5-minute averages, covering distances along the flight path of 20 to 25 miles; also, these observations were taken at altitudes ranging from 15,000 to 20,000 feet. Thus the residual profiles have been smoothed greatly by these effects.

All observed values with high residuals were checked for errors. Four D values were of doubtful validity and were consequently rejected.

The Z residuals, in Figure 10, are interesting geologically; and several characteristics are quite evident. The profiles over the Sverdrup Basin and Arctic Lowlands, for example, are particularly flat, implying very deep sediments. The Canadian Shield is not extremely disturbed magnetically, but it does not have the quiescent character of the synclinal areas of the archipelago. A large anomaly in this geological province occurs over Jones Sound, between Devon Island and Ellesmere Island, and is about 400γ in amplitude. The Boothia Arch and Minto Arch have little effect on the residuals.

The area in eastern Victoria Island and western Prince of Wales Islands, where the residuals are 200γ to 500γ higher than those over the rest of the lowlands, is one where the regional field is not adequately represented by the 3rd degree polynomial, this area being very small in relation to the entire area of expansion.

The ocean area between Alaska and Axel Heiberg Island, including the Beaufort Sea, is generally undisturbed. Some regional field is still present in the residuals of this area, however, as in the residuals over Victoria and Prince of Wales.

The area around the Chukchi Cap and Arctic Ocean is extremely anomalous, residuals of 400γ and 500γ being quite numerous.

A most interesting pattern emerges over the Alpha Rise. Two parallel lines of large positive anomalies describe a linear feature striking about 40° east of grid north, continuing along the Rise to the northwestern coast of Ellesmere Island. The feature disappears over eastern Ellesmere and the western coast of Greenland, but seems to reappear about 150 miles northeast of Thule, Greenland. The amplitudes of these anomalies are between 400γ and 600γ, and the wavelengths about 100 miles.

Many of these geomagnetic characteristics were noted by Ostenso, *et al.*, (1961) and Ostenso (1963).

Because H is small in an elliptical region about the pole, the D residuals in this region are very large. Hence the scale at the centre of Figure 11, where the residuals are in solid blue, was made one fourth as large as that at either side, where the residuals are cross-hatched. Figure 11 is a plot of the G residuals as well as the D, since G differs from D only by the longitude.

As a comparison with the Canadian 1965.0 Magnetic Charts, field values were generated for D, H, and Z at intervals along their given contour lines. The R.M.S. differences for the 3 degrees are listed in Table II. They are in all cases less than the standard errors of estimate. Dawson and Dalgetty (1966) estimate the chart errors as 0.6° in D, 128γ in H, and 122γ in Z.

The dip pole positions for the 3 degrees, obtained from the intersections of the U and V zero-gamma isolines, are as follows:

	Lat (°*N*)	*Long* (°*W*)
1st degree	77.2	101.5
2nd degree	76.3	101.7
3rd degree	75.6	101.3

These are plotted in Figure 3. The Dominion Observatory estimate for 1964.0, from all available Canadian data, is

	75.4	100.8

The 3rd-degree position agrees quite well with this estimate, being 13 miles to the northwest.

Effect of Correcting Data for Disturbance

A least-squares analysis was also done of an area around the magnetic dip pole, denoted in Figure 1 as Area B. Here $-.03 < \alpha < .03$ and $-.03 < \beta < .03$ for $u_o = .0254$ and $v_o = -.1247$. The area covers about .16 million square miles.

Area B has a greater density of observations than Area A, 296 observations being used for the analysis of U and V, and 168 for the analysis of Z. Table IV lists the resulting partial derivatives evaluated at (u_o, v_o), their standard errors, and the t-values.

Since many of the 3rd-degree coefficients are insignificant at the 5 per-cent level, they were eliminated from the equations in a stepwise fashion. The resulting coefficients, standard errors, and t values are shown in the last 3 columns of Table IV. Standard coefficients for the first 3 degrees are listed in Table V.

The disturbance field at Resolute is highly correlated, over the applicable frequency range, with that at Mould Bay (the locations of these 2 stations are shown in Figure 1); a high correlation can therefore be expected between Resolute and all points within Area B. Magnetic records from Resolute were used to correct all values in the area to the annual means centered at 1964.0. The corrections ranged from -50γ to $+40\gamma$ in X, from -140γ to $+90\gamma$ in Y, and from -150γ to -20γ in Z. The least squares solution for this corrected set is shown in Tables VI and VII, analagous to the solution in Tables IV and V of the uncorrected data.

The standard errors of estimate of U, V, Z, D, and H for both the corrected and uncorrected cases are listed in Table VIII. The standard error in U dropped by about 30γ as a result of the corrections, but there was no significant change in the standard errors of V and Z.

Acknowledgments

The author wishes to thank Dr. P. H. Serson for the suggestions, advice, and guidance he gave during this project, and Mr. W. Hannaford for his help and encouragement. Mr. Don MacLulich, a student assistant, helped a great deal in computations.

The survey data were gathered by Dominion Observatory personnel P. H. Serson, W. Hannaford, J. L. Roy, G. L. Carr, and the author.

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TABLE I

Partial derivatives of U, V, and Z, for Area A, evaluated at $u_o = .0254$ and $v_o = -.1247$, obtained by method of least squares. Value of t is coefficient divided by standard error.

	1st Degree			2nd Degree			3rd Degree		
	Coeff. (γ)	St. Error (γ)	t	Coeff. (γ)	St. Error (γ)	t	Coeff. (γ)	St. Error (γ)	t
U_o..................	−2.702+2	0.113+2	−23.9	−0.938+2	0.100+2	−9.4	−0.702+2	0.109+2	−6.4
V_o..................	2.294+2	0.124+2	18.6	1.423+2	0.097+2	14.7	0.295+2	0.116+2	2.5
U_u..................	−71.686+3	0.186+3	−386.1	−69.175+3	0.182+3	−379.8	−70.288+3	0.284+3	−247.4
U_v..................	3.781+3	0.135+3	28.0	5.009+3	0.142+3	35.3	4.908+3	0.163+3	30.2
V_v..................	−14.992+3	0.195+3	−77.0	−20.935+3	0.270+3	−77.5	−19.668+3	0.264+3	−74.5
U_{uu}..................				−14.606+4	0.491+4	−29.8	−17.993+4	0.725+4	−24.8
Uu_v..................				0.526+4	0.240+4	2.2	3.903+4	0.380+4	10.3
U_{vv}..................				−2.340+4	0.247+4	−9.5	−2.692+4	0.465+4	−5.8
V_{vv}..................				15.527+4	0.590+4	26.3	30.290+4	1.362+4	22.2
U_{uuu}..................							1.799+6	0.282+6	6.4
U_{uuv}..................							−0.157+6	0.091+6	−1.7
U_{uvv}..................							−0.731+6	0.071+6	−10.3
U_{vvv}..................							0.290+6	0.099+6	2.9
V_{vvv}..................							−3.962+6	0.339+6	−11.7
Z_o..................	57.795+3	0.024+3	2418.0	58.079+3	0.010+3	5951.1	58.042+3	0.011+3	5332.2
Z_u..................	8.385+3	0.338+3	24.8	13.226+3	0.153+3	86.7	16.106+3	0.227+3	71.1
Z_v..................	−16.689+3	0.371+3	−45.0	−24.244+3	0.208+3	−116.6	−24.492+3	0.217+3	−113.1
Z_{uu}..................				−31.734+4	0.386+4	−82.3	−25.367+4	0.558+4	−45.5
Z_{uv}..................				3.066+4	0.210+4	14.6	4.810+4	0.352+4	13.7
Z_{vv}..................				22.207+4	0.462+4	48.1	18.842+4	0.871+4	21.6
Z_{uuu}..................							−3.302+6	0.197+6	−16.8
Z_{uuv}..................							−0.101+6	0.070+6	−1.5
Z_{uvv}..................							−0.294+6	0.073+6	−4.0
Z_{vvv}..................							1.034+6	0.232+6	4.5

NOTE: Coefficients and standard errors are in floating-point notation, a decimal fraction followed by a power of ten. For example, $-2.701 + 2 = -2.701 \times 10^{-2}$

TABLE II

Standard errors of estimate from least-square analysis of Area A, and R.M.S. differences when compared to 1965.0 Canadian Magnetic Charts

	Standard Errors of Estimate					R.M.S. Differences		
	U(γ)	V(γ)	Z(γ)	D(°)	H(γ)	D(°)	H(γ)	Z(γ)
Observations..................	816	816	1029	889	912	59	97	63
1st degree..................	279	277	616	12.1	309	9.5	300	475
2nd degree..................	188	207	191	7.6	189	4.9	188	94
3rd degree..................	181	185	168	7.5	178	6.2	171	58

TABLE III

Standard coefficients (dimensionless) for Area A

	1st Degree			2nd Degree			3rd Degree		
	U	V	Z	U	V	Z	U	V	Z
1	−.0717	.2754	50.7668	−.0249	.1709	51.0163	−.0187	.0354	50.9835
α	−1.0036	.2394	.4202	−.9683	.3172	.6640	−.9841	.3109	.8086
β	.0505	−.9056	−.7633	.0669	−1.2651	−1.1089	.0656	−1.1886	−1.1202
α^2				−.0676	.0110	−.5301	−.0832	.0817	−.4237
$\alpha\beta$				.0050	−.1008	.1040	.0371	−.1158	.1632
β^2				−.0144	.4310	.4551	−.0165	.8411	.3861
α^3							.0341	−.0134	−.2344
$\alpha^2\beta$							−.0062	−.1458	−.0139
$\alpha\beta^2$							−.0342	.0614	−.0489
β^3							.0071	−.4386	.0862

TABLE IV

Partial derivatives of U, V, and Z for Area B, from uncorrected data. "Significant" terms taken at 5% level.

	1st Degree			2nd Degree			3rd Degree			3rd Degree, Significant		
	Coeff. (γ)	St. Error (γ)	t	Coeff. (γ)	St. Error (γ)	t	Coeff. (γ)	St. Error (γ)	t	Coeff. (γ)	St. Error (γ)	t
U_o	−0.415+2	0.132+2	−3.1	−0.035+2	0.191+2	−0.2	0.021+2	0.196+2	0.1			
V_o	0.721+2	0.129+2	5.6	0.366+2	0.192+2	1.9	0.482+2	0.199+2	2.4	0.551+2	0.137+2	4.0
U_u	−67.576+3	0.766+3	−88.2	−67.309+3	0.779+3	−86.5	−69.708+3	1.820+3	−38.3	−67.310+3	0.775+3	−86.9
U_v	7.099+3	0.543+3	13.1	6.902+3	0.549+3	12.6	5.699+3	1,040+3	5.5	6.906+3	0.544+3	12.7
V_v	−22.461+3	0.770+3	−29.2	−22.121+3	0.775+3	−28.5	−19.047+3	2.064+3	−9.2	−18.701+3	1.935+3	−9.7
U_{uu}				−19.234+4	9.398+4	−2.0	−23.471+4	9.869+4	−2.4	−20.465+4	6.905+4	−3.0
U_{uv}				10.764+4	4.093+4	2.6	9.769+4	4.215+4	2.3	10.460+4	4.073+4	2.6
U_{vv}				−7.612+4	4.125+4	−1.8	−7.167+4	4.226+4	−1.7	−7.763+4	3.919+4	−2.0
V_{vv}				15.155+4	9.994+4	1.5	6.677+4	10.689+4	0.6			
U_{uuu}							27.613+6	18.123+6	1.5			
U_{uuv}							3.133+6	5.073+6	0.6			
U_{uvv}							−0.047+6	4.083+6	−0.0			
U_{vvv}							8.446+6	5.801+6	1.5			
V_{vvv}							−37.953+6	22.069+6	−1.7	−42.753+6	21.486+6	−2.0
Z_o	57.972+3	0.011+3	5303.2	57.935+3	0.019+3	3086.4	57.941+3	0.019+3	3048.6	57.928+3	.014+3	4000.3
Z_u	17.407+3	0.619+3	28.1	17.758+3	0.604+3	29.4	18.348+3	1.566+3	11.7	17.586+3	.575+3	30.6
Z_v	−28.808+3	0.669+3	−43.1	−28.520+3	0.648+3	−44.0	−24.097+3	1.799+3	−13.4	−26.531+3	.894+3	−30.0
Z_{uu}				−5.936+4	7.382+4	−0.8	−5.785+4	7.415+4	−0.8			
Z_{uv}				−1.445+4	3.748+4	0.4	0.354+4	3.705+4	0.1			
Z_{vv}				35.990+4	8.317+4	4.3	31.776+4	8.704+4	3.7	35.858+4	8.058+4	4.4
Z_{uuu}							−8.937+6	14.495+6	−0.6			
Z_{uuv}							−14.494+6	4.424+6	−3.3	−13.858+6	4.344+6	−3.2
Z_{uvv}							1.373+6	4.933+6	0.3			
Z_{vvv}							−28.775+6	18.613+6	−1.5			

Note: Coefficients and standard errors are in floating-point notation. See note to Table I.

TABLE V

Standard coefficients for Area B, from uncorrected data

	1st Degree			2nd Degree			3rd Degree		
	U	V	Z	U	V	Z	U	V	Z
1	−.0368	.0171	102.54	−.0031	.0870	102.48	.0019	.1146	102.49
α	−.9859	.2774	.5293	−.9820	.2697	.5401	−1.0170	.2227	.5580
β	.1030	−.8732	−.8102	.1002	−.8600	−.8021	.0827	−.7405	−.6778
α^2				−.0233	.0349	−.0142	−.0284	.0317	−.0144
$\alpha\beta$				.0266	−.0505	−.0070	.0242	−.0476	.0017
β^2				−.0084	.0447	.0782	−.0079	.0197	.0691
α^3							.0408	.0124	−.0279
$\alpha^2\beta$							.0093	−.0004	−.0839
$\alpha\beta^2$							−.0001	.0607	.0072
β^3							.0110	−.1323	−.0718

TABLE VI

Partial derivatives of U, V, and Z for Area B, from corrected data. "Significant" terms taken at 5% level

	1st Degree			2nd Degree			3rd Degree			3rd Degree, Significant		
	Coeff. (γ)	St. Error (γ)	t	Coeff. (γ)	St. Error (γ)	t	Coeff. (γ)	St. Error (γ)	t	Coeff. (γ)	St. Error (γ)	t
U_o	−0.738+2	0.123+2	−6.0	−0.341+2	0.177+2	−1.9	−0.286+2	0.177+2	−1.6			
V_o	0.777+2	0.120+2	6.4	0.690+2	0.180+2	3.8	0.765+2	0.183+2	4.2	0.749+2	0.116+2	6.4
U_u	−65.491+3	0.709+3	−92.4	−65.269+3	0.714+3	−91.4	−69.816+3	1.583+3	−44.1	−70.084+3	1.507+3	−46.5
U_v	7.188+3	0.505+3	14.2	7.253+3	0.510+3	14.2	6.377+3	0.936+3	6.8	7.278+3	0.487+3	15.0
V_v	−22.732+3	0.720+3	−31.6	−22.252+3	0.728+3	−30.6	−19.115+3	1.899+3	−10.1	−18.800+3	1.790+3	−10.5
U_{uu}				−17.189+4	8.345+4	−2.1	−18.735+4	8.328+4	−2.2	−28.093+4	6.089+3	−4.6
U_{uv}				−0.562+4	3.715+4	−0.2	0.557+4	3.716+4	0.1			
U_{vv}				−11.197+4	3.807+4	−2.9	−10.463+4	3.810+4	−2.7	−12.635+4	3.570+4	−3.5
V_{vv}				6.735+4	9.439+4	0.7	−1.015+4	9.889+4	−0.1			
U_{uuu}							46.126+6	14.699+6	3.1	49.702+6	14.128+6	3.5
U_{uuv}							0.850+6	4.228+6	0.2			
U_{uvv}							0.944+6	3.524+6	0.3			
U_{vvv}							6.738+6	5.212+6	1.3			
V_{vvv}							−41.100+6	20.529+6	−2.0	−41.923+6	19.928+6	−2.1
Z_o	57.915+3	0.012+3	4862.2	57.905+3	0.020+3	2888.2	57.930+3	0.019+3	3066.3	57.952+3	0.015+3	3862.7
Z_u	16.844+3	0.670+3	25.1	17.299+3	0.646+3	26.8	16.423+3	1.466+3	11.2	15.746+3	0.849+3	18.6
Z_v	−29.033+3	0.731+3	−39.7	−29.015+3	0.702+3	−41.4	−20.985+3	1.796+3	−11.7	−20.269+3	1.763+3	−11.5
Z_{uu}				−18.312+4	7.680+4	−2.4	−22.806+4	7.197+4	−3.2	−23.062+4	7.192+4	−3.2
Z_{uv}				9.015+4	3.980+4	2.3	10.485+4	3.677+4	2.9	10.980+4	3.677+4	3.0
Z_{vv}				29.770+4	9.044+4	3.3	16.285+4	8.868+4	1.8			
Z_{uuu}							−3.047+6	13.369+6	−0.2			
Z_{uuv}							−20.387+6	4.130+6	−4.9	−21.061+6	4.097+6	−5.1
Z_{uvv}							10.145+6	4.992+6	2.0	12.350+6	4.766+6	2.6
Z_{vvv}							−61.293+6	18.922+6	−3.2	−70.287+6	18.381+6	−3.8

NOTE: Coefficients and standard errors are in floating-point notation. See note to Table I.

TABLE VII

Standard coefficients for Area B, from corrected data

	1st Degree			2nd Degree			3rd Degree		
	U	V	Z	U	V	Z	U	V	Z
1	−.0671	.1818	102.37	−.0310	.1614	102.36	−.0260	.1789	102.40
α	−.9892	.2792	.5158	−.9860	.2818	.5297	−1.0545	.2477	.5029
β	.1068	−.8692	−.8152	.1078	−.8509	−.8147	.0948	−.7309	−.5892
α^2				−.0222	−.0019	−.0469	−.0242	.0019	−.0585
$\alpha\beta$				−.0015	−.0752	.0445	.0014	−.0703	.0517
β^2				−.0126	.0195	.0630	−.0118	−.0294	.0351
α^3							.0749	.0036	−.0100
$\alpha^2\beta$							.0009	.0025	−.1257
$\alpha\beta^2$							.0026	.0493	.0549
β^3							.0090	−.1406	−.1579

TABLE VIII

Standard errors of estimate for uncorrected and corrected data of Area B

		U(γ)	V(γ)	Z(γ)	D(°)	H(γ)
	Observations	144	144	168	144	144
Uncorrected Data	1st Degree	167	137	138	13.6	161
	2nd Degree	167	129	131	13.9	160
	3rd Degree	168	130	128	13.3	161
Corrected Data	1st Degree	140	141	150	11.2	142
	2nd Degree	141	136	142	11.1	139
	3rd Degree	136	136	130	10.4	137

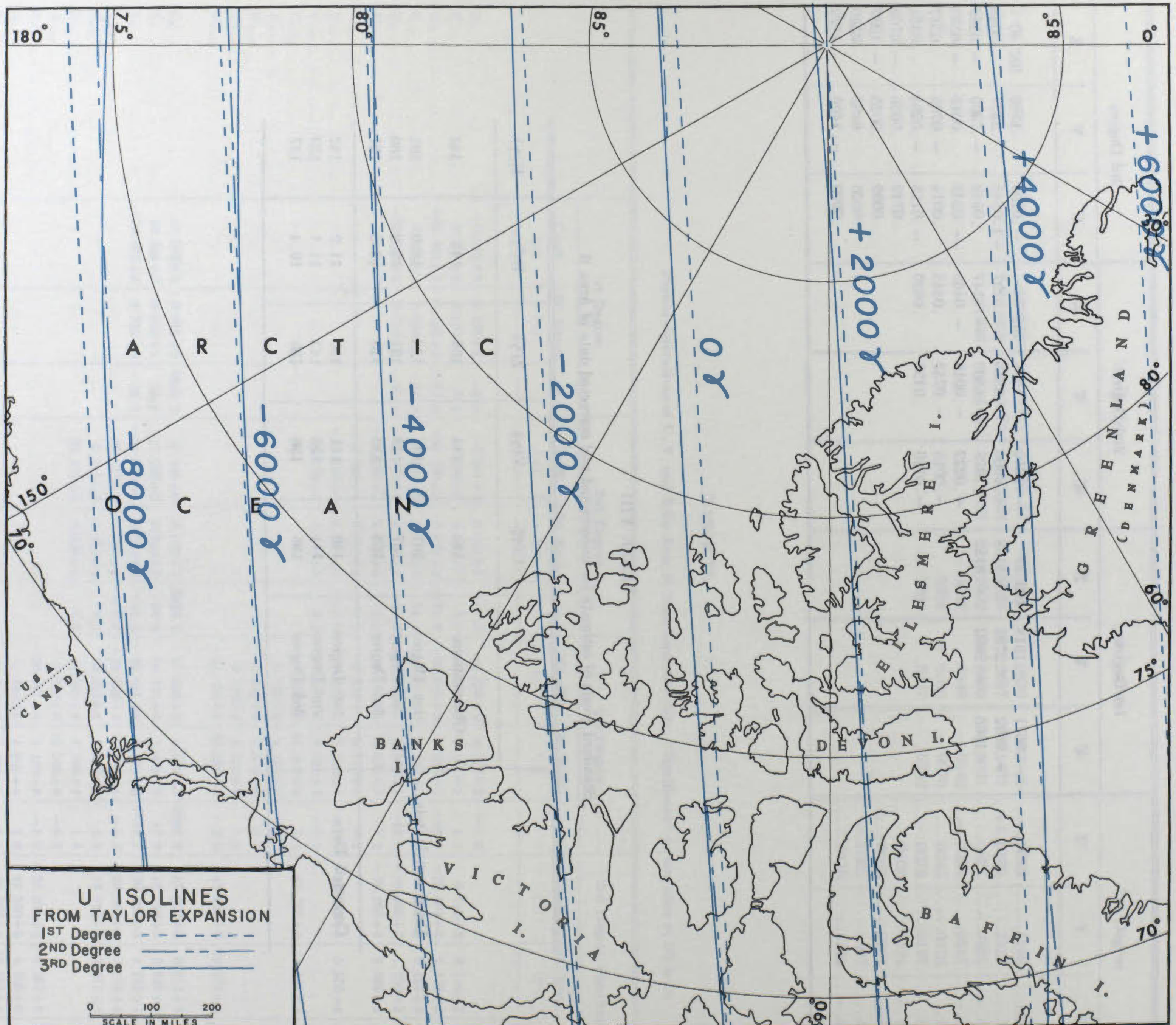

FIGURE 2. Isolines of grid north component of magnetic field, from 1st, 2nd, and 3rd degree Taylor expansions. Projections of all maps in this paper are polar stereographic.

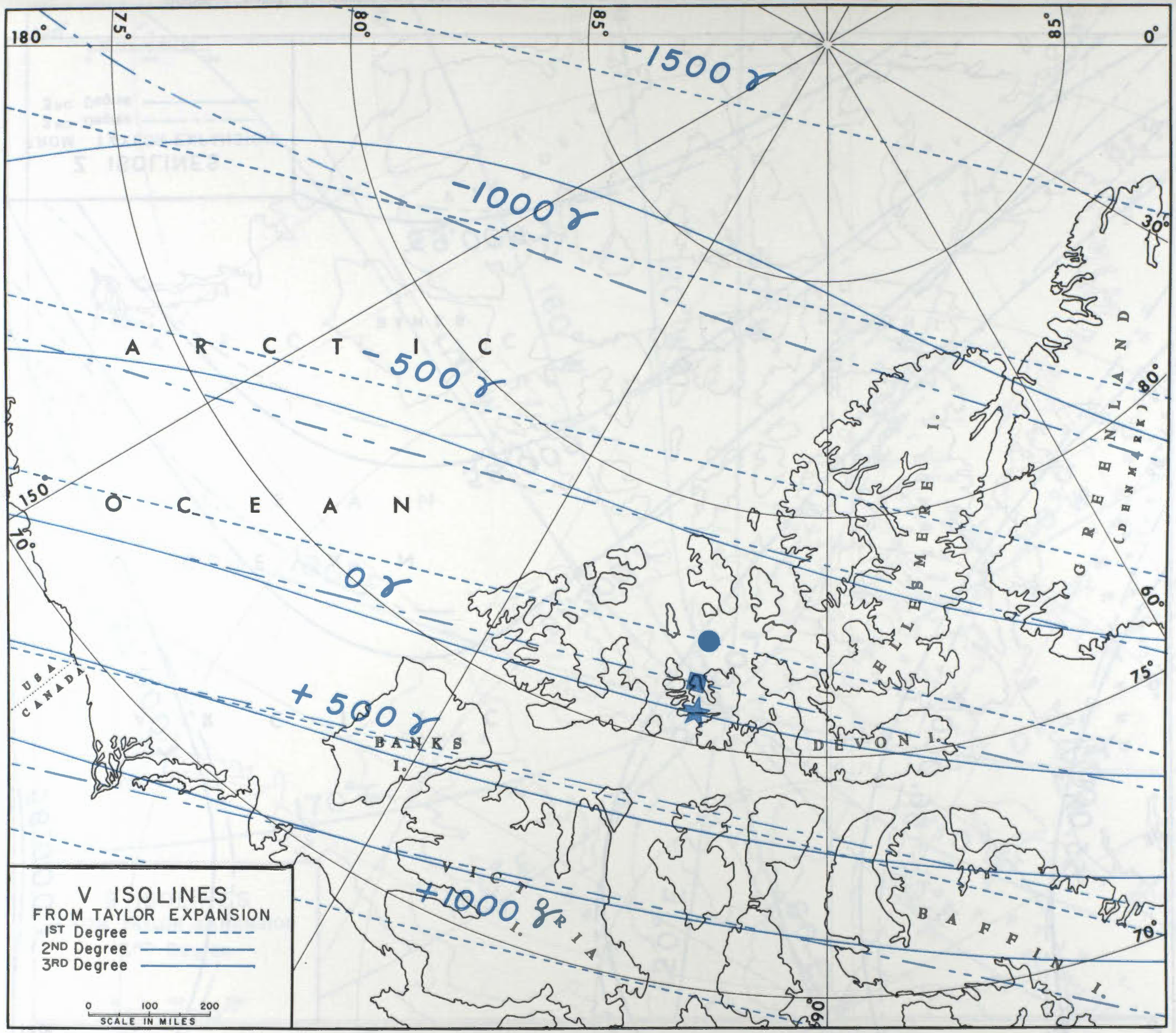

FIGURE 3. Isolines of grid east component of magnetic field. Positions of the dip pole corresponding to 1st, 2nd, and 3rd degree expansions are denoted by a circle, square, and star, respectively.

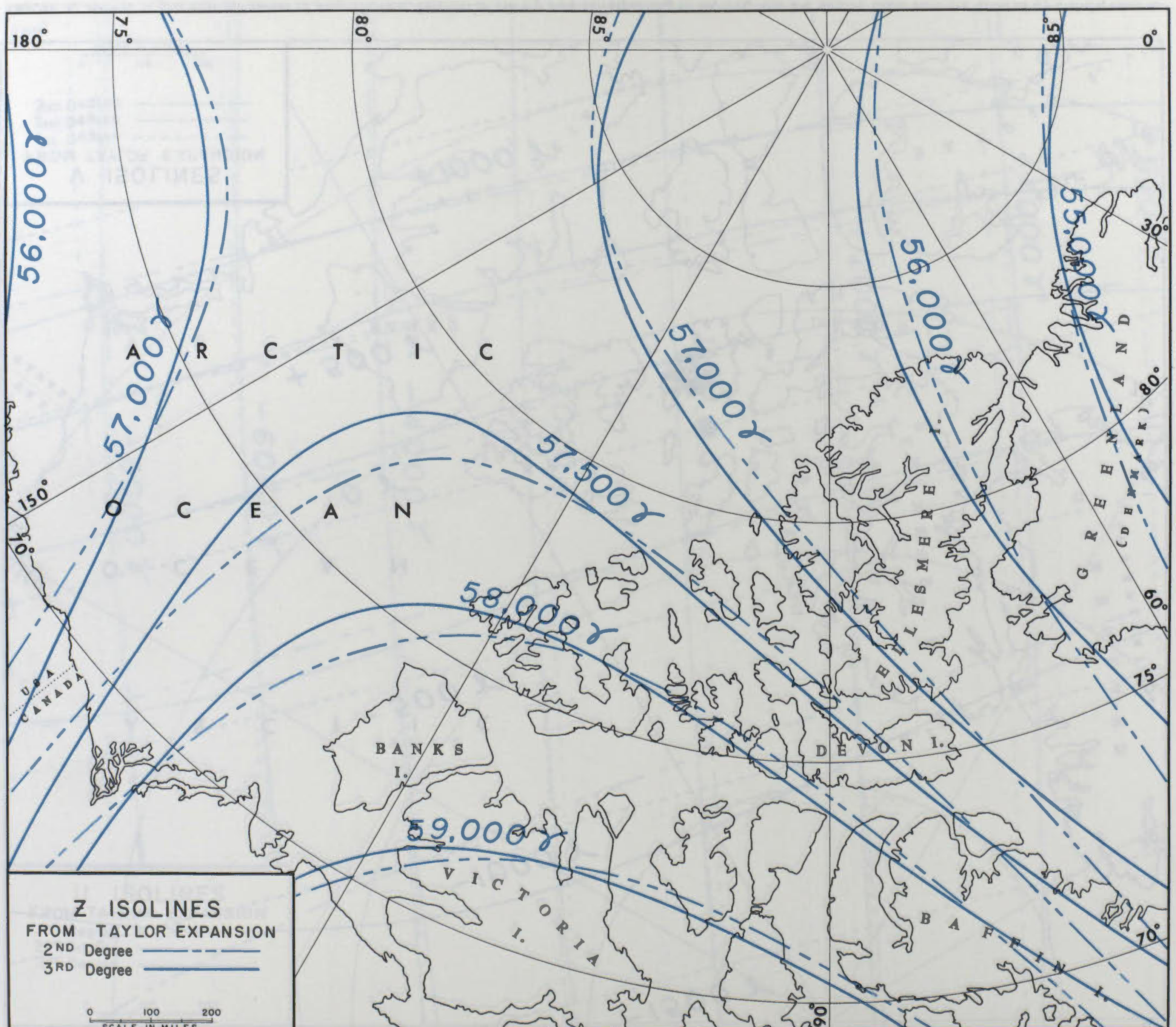

FIGURE 4. Isolines of vertical downward component of magnetic field, obtained from 2nd and 3rd degree Taylor expansions.

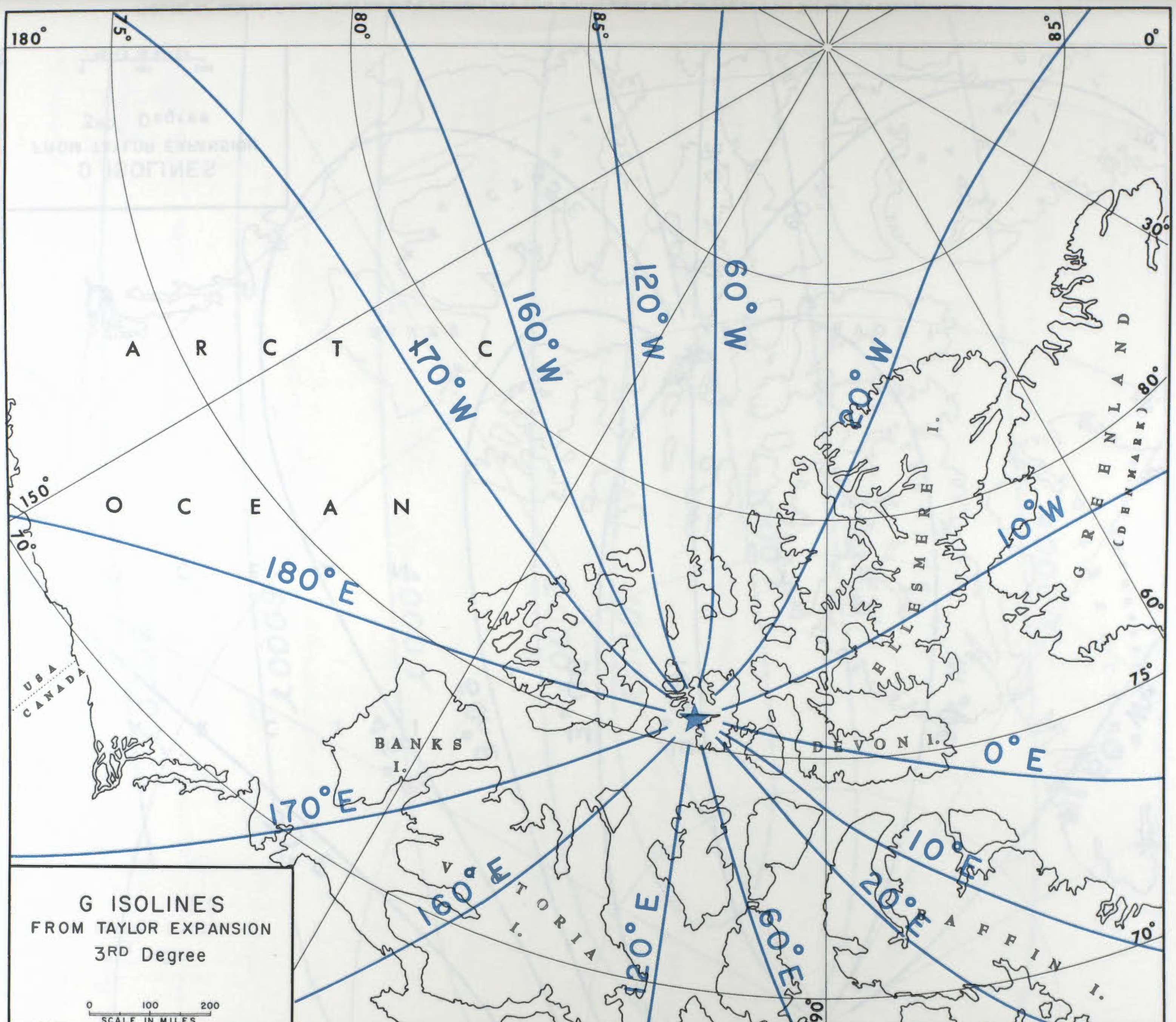

FIGURE 5. Isolines of grivation, the angle between grid north and magnetic north, contoured from 3rd degree expansions of U and V. Dip pole position denoted by star.

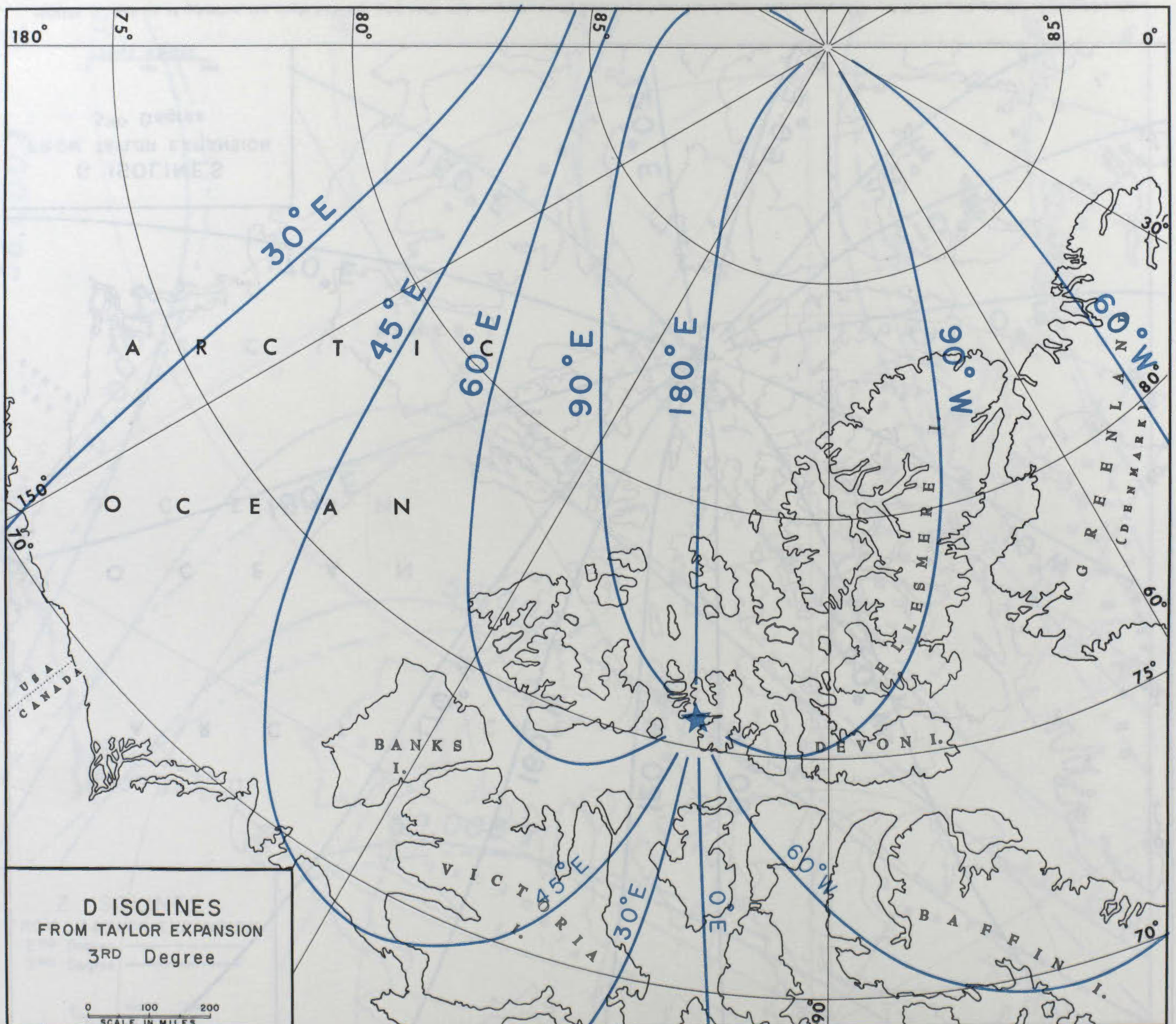

FIGURE 6. Isolines of declination, the angle between true north and magnetic north, contoured from 3rd degree expansions of U and V.

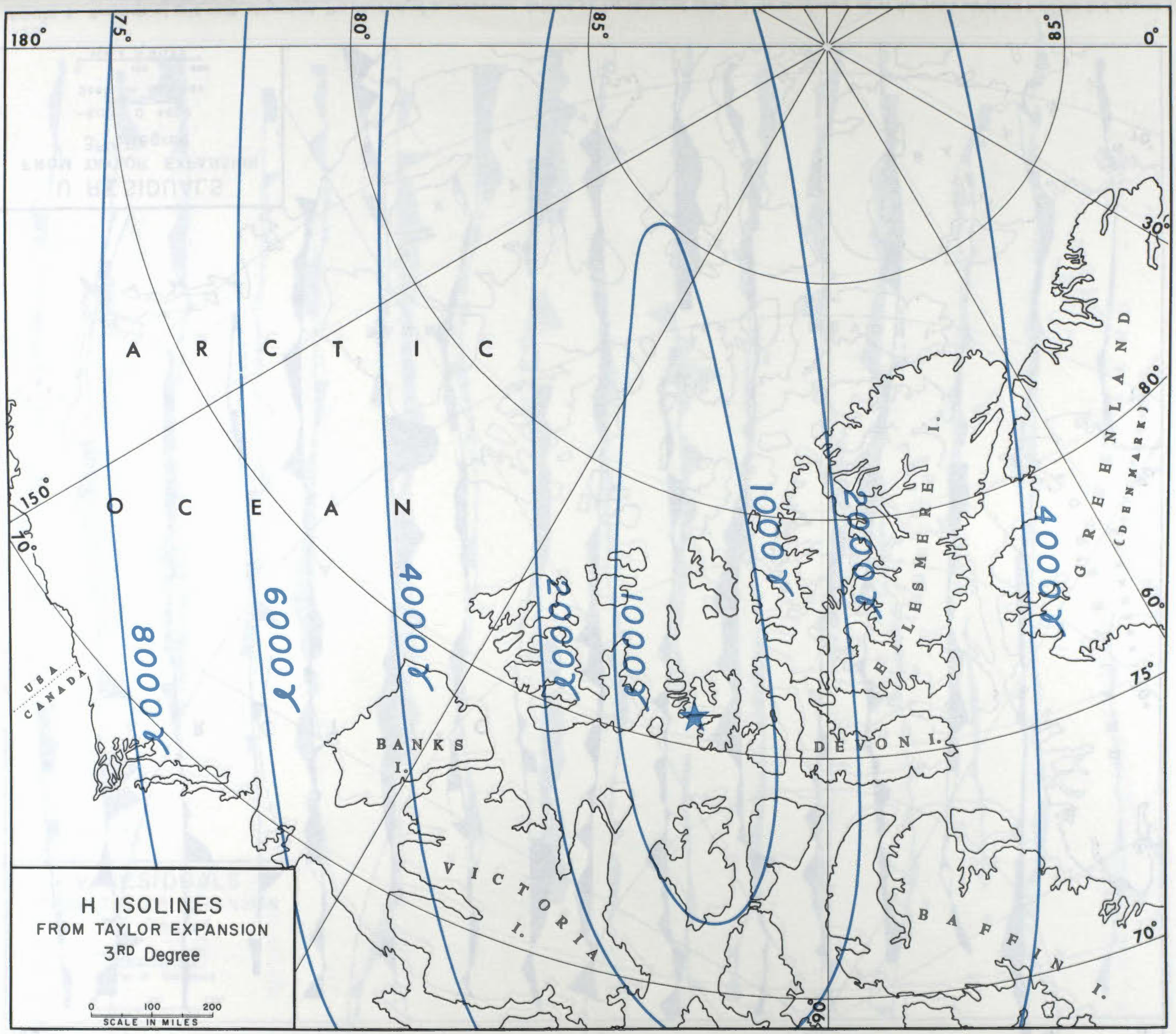

FIGURE 7. Isolines of horizontal component of magnetic field, contoured from 3rd degree expansions of U and V. Dip pole position, where horizontal component is zero, is denoted by star.

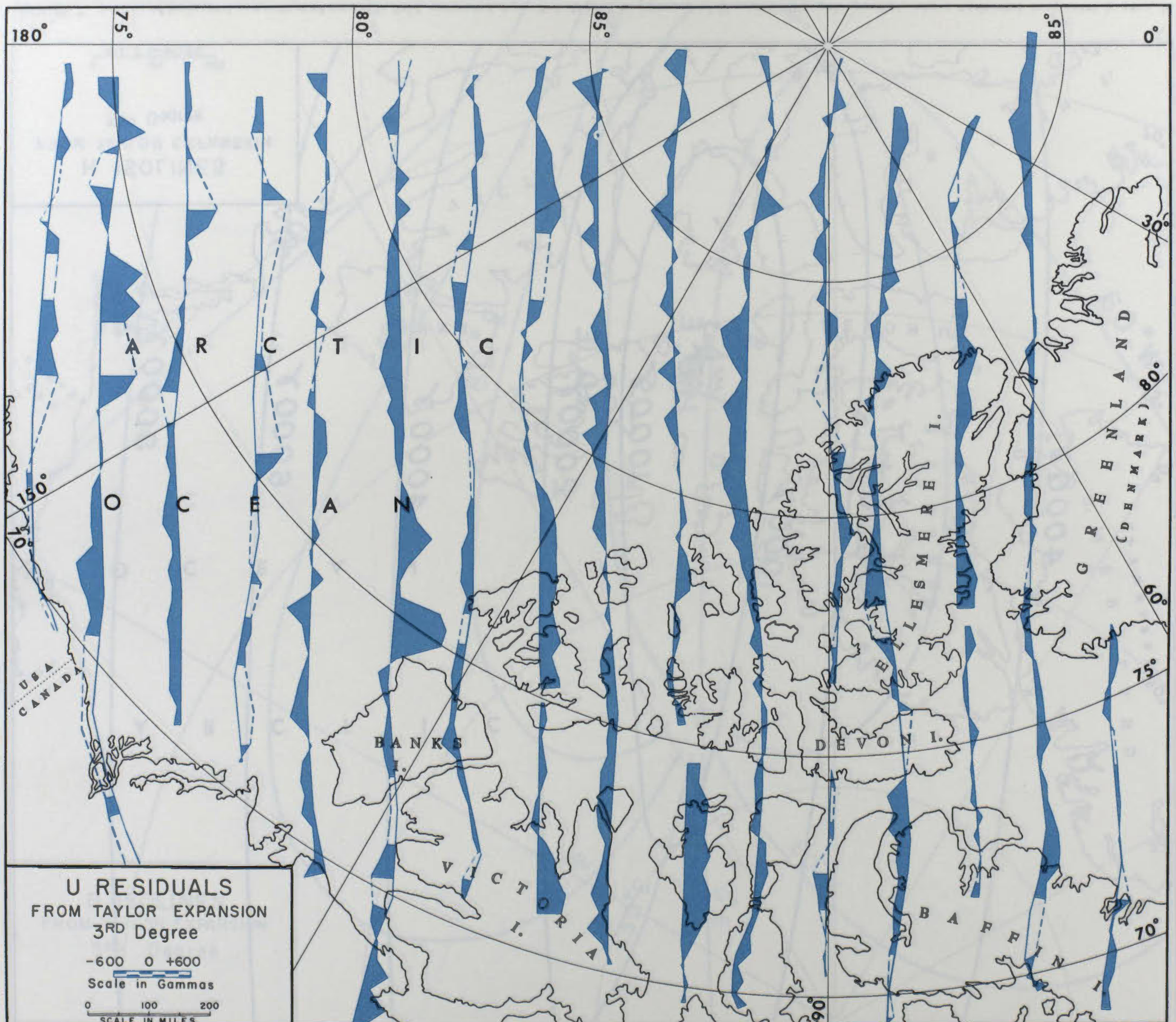

FIGURE 8. Residuals of grid north component. The residual of a component defined as the observed value of the component minus the value obtained from the 3rd degree expansion. Plotted to the right of the flight track when positive, to the left when negative. Dashed line indicates no observations.

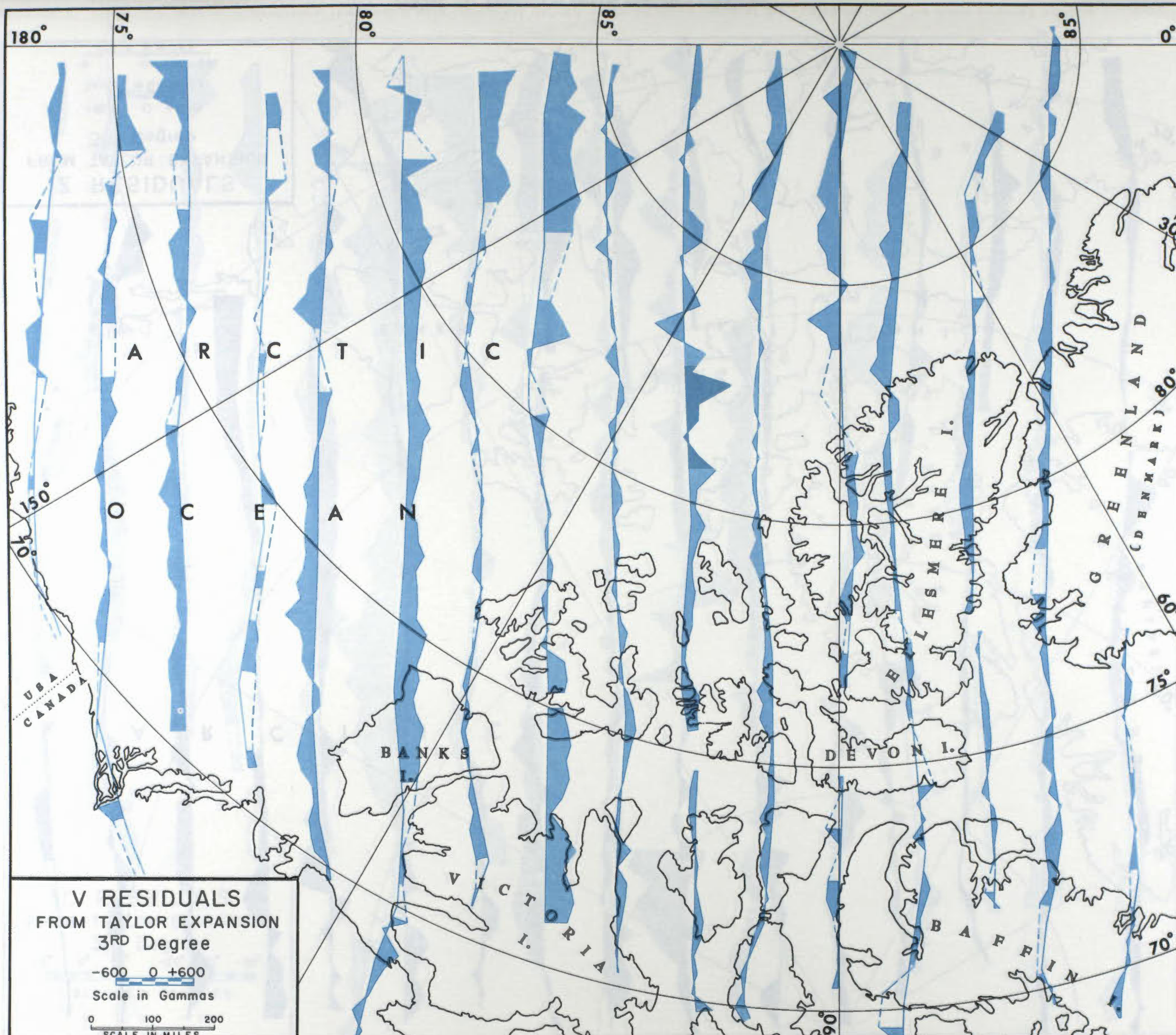

FIGURE 9. Residuals of grid east component.

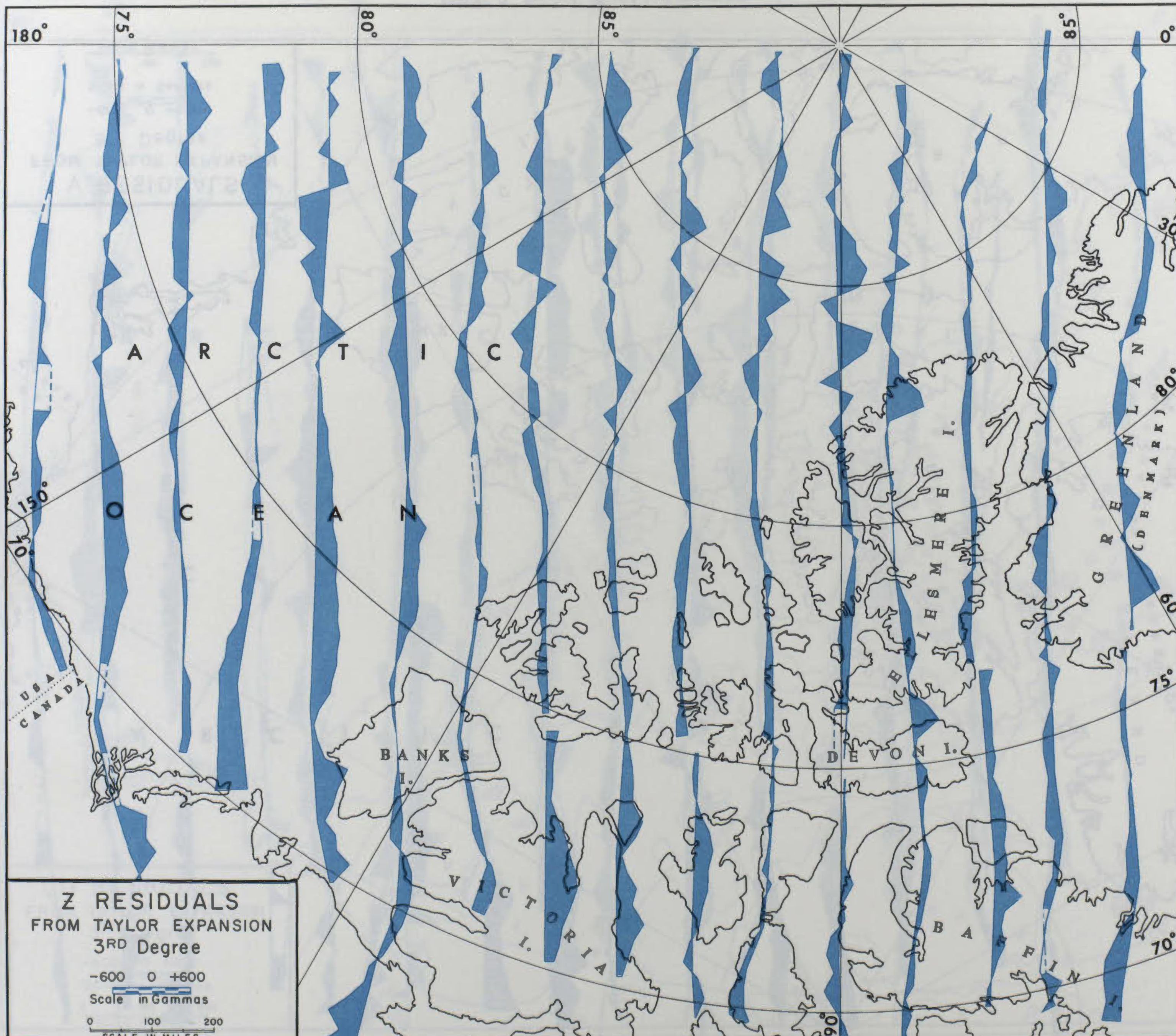

FIGURE 10. Residuals of vertical downward component.

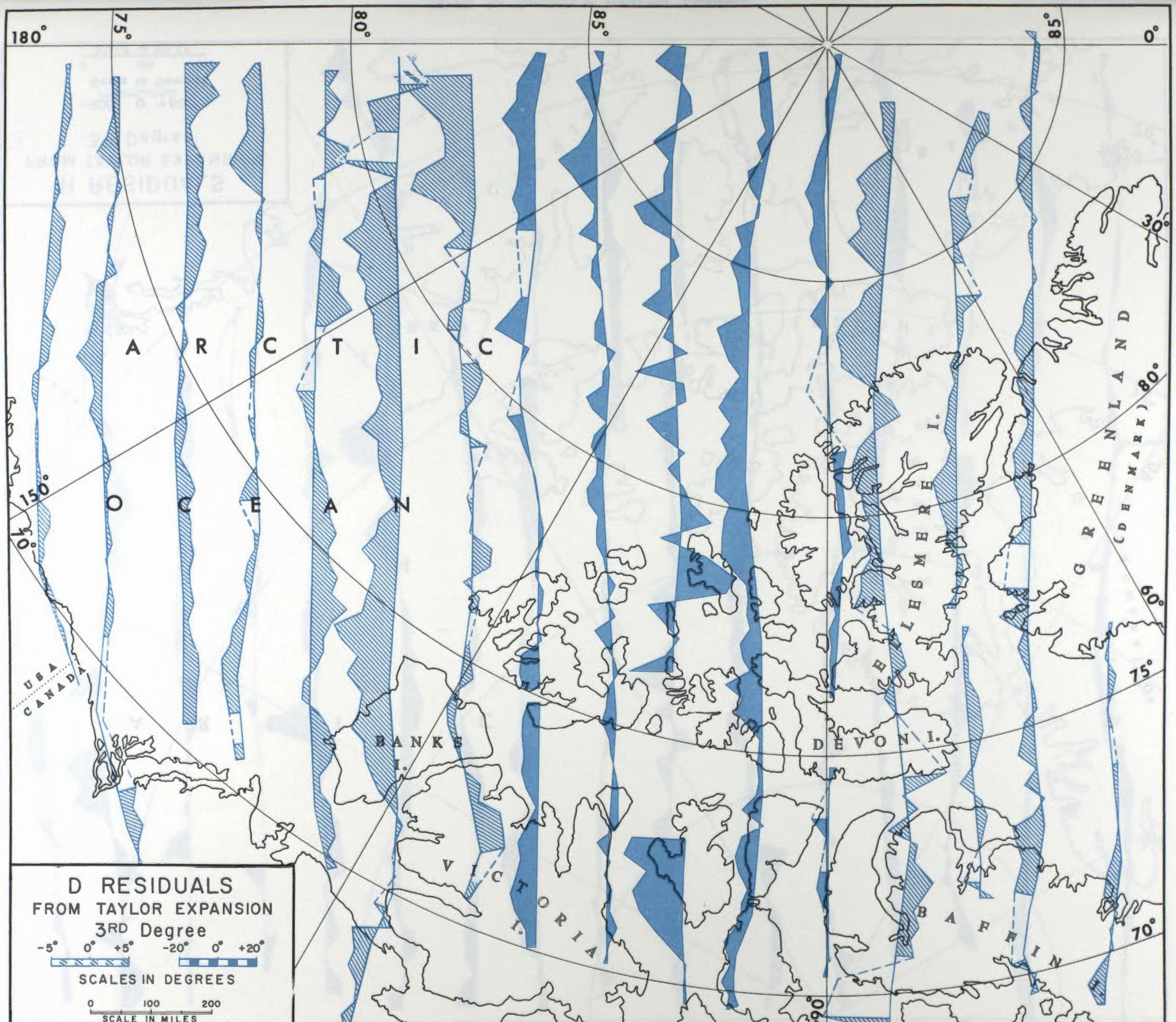

FIGURE 11. Residuals of declination, which are the same as residuals of grivation. Note that scale at centre is one fourth the size of the scale at either side.

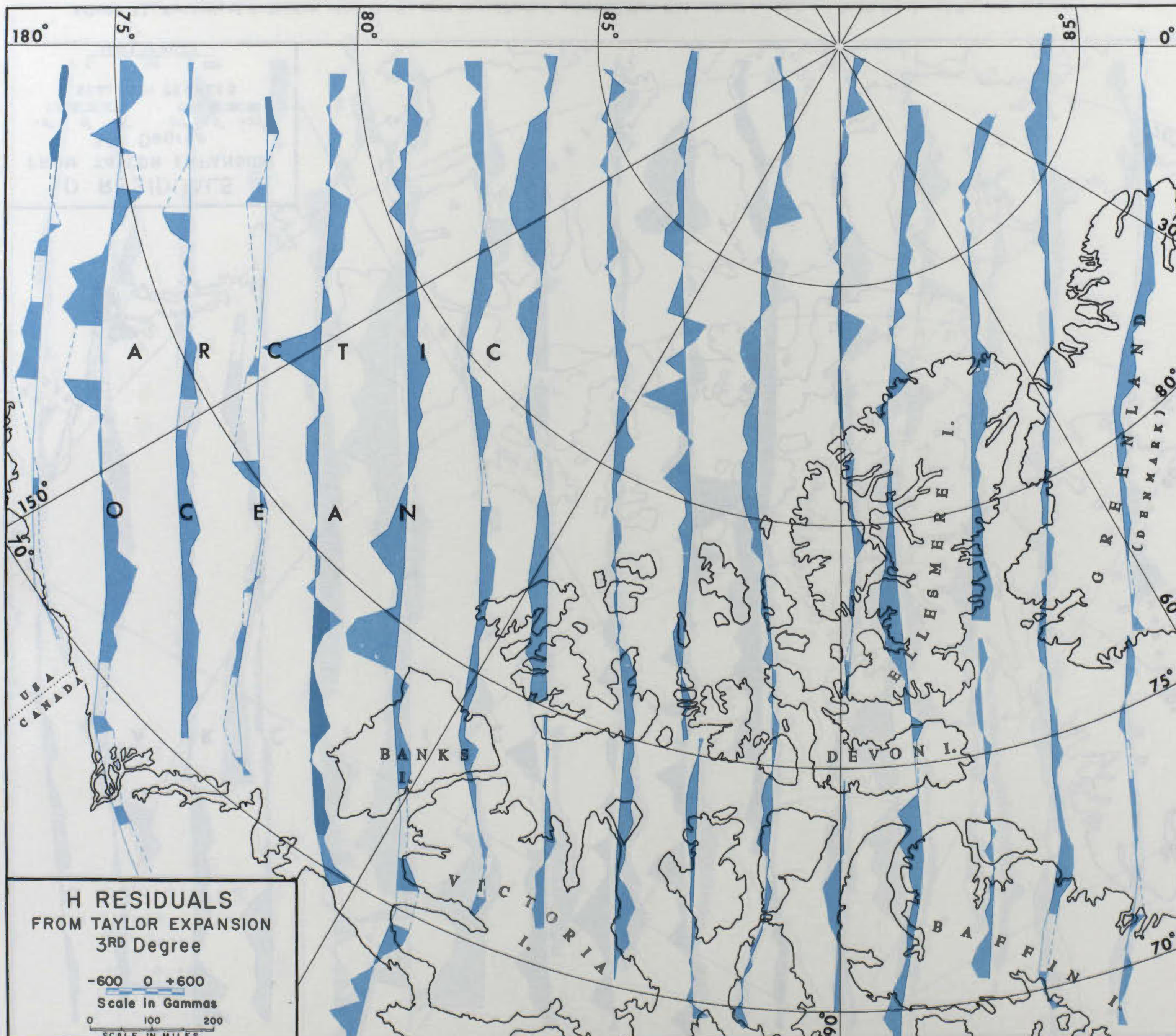

FIGURE 12. Residuals of horizontal component.