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the Canadian pendulum apparatus, design and operation

H. D. VALLIANT

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Contents

Frontispiece. Author holding pendulum prior to opening pendulum case.

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Abstract. A bi-pendulum apparatus which uses bronze quarter-metre pendulums has recently been developed by the Earth Physics Branch (formerly the Dominion Observatory) for relative gravity measurements. Variable factors such as temperature, pressure, humidity, amplitude, and time, are precisely controlled to minimize the corrections required to reduce the periods to identical conditions.

Rigorous testing of the equipment under laboratory and simulated field conditions disclosed that systematic errors in measuring gravity differences would not exceed 0.06 mgal and random errors would not exceed 0.20 mgal r.m.s.

Tests under complete field conditions along the North American Calibration Line resulted in a mean difference between gravimeter and pendulum values of 0.04 mgal and an r.m.s. difference of 0.06 mgal. It was expected that the random error would decrease, as a result of the effects of desiccating the pendulum storage box. Agreement with absolute measurements is within the error bounds estimated for these two instruments.

Résumé. Un dispositif à double pendule, constitué de pendules en bronze d'un quart de mètre, a été mis au point par la Direction de la physique du Globe (anciennement l'Observatoire fédéral) pour effectuer des mesures de gravité relative. Les facteurs variables, comme la température, la pression, l'humidité, l'amplitude et le temps, sont contrôlés avec une grande précision afin de réduire au minimum les corrections requises pour amener les périodes à des conditions identiques.

Des essais très précis de ce dispositif, effectués en laboratoire et dans des conditions simulées, ont révélé que les erreurs systématiques dans la mesure des différences de gravité ne dépasseraient pas 0.06 mgal et les erreurs aléatoires 0.20 mgal (erreur quadratique moyenne).

Les essais effectués sur le terrain même, le long de la Ligne d'étalonnage nord-américaine, ont révélé une différence moyenne de 0.04 mgal entre les valeurs indiquées par le gravimètre et le pendule, et une différence de 0.06 mgal dans l'erreur quadratique moyenne. On pense pouvoir réduire l'erreur aléatoire par assèchement de la boîte où est conservé le pendule. La comparaison avec les mesures absolues donne un résultat qui ne dépasse pas la marge d'erreur prévue pour ces deux instruments.

Introduction

General remarks. The Canadian pendulum apparatus is an instrument for the precise measurement of differences in gravity between two or more locations consisting of six bronze pendulums and ancillary equipment. This apparatus has recently undergone extensive redevelopment to increase its precision to a level equal to or exceeding present day standards. Descriptions of specific aspects of the design, construction and testing of this new apparatus has been published in appropriate engineering or geophysical journals (Valliant, 1965, 1967a and b, 1969a and b). The purpose of this paper is to co-ordinate the individual research reports, to provide a complete description of the Canadian pendulum apparatus and to summarize the results of nearly two years of testing.

History of the Canadian pendulum apparatus. The history of the Canadian pendulum apparatus dates back to the turn of the century when Mendenhall (Swick, 1921) designed a pendulum apparatus for the United States Coast and Geodetic Survey (USCGS). In 1902 the Dominion Observatory procured a pendulum apparatus and three bronze quarter-metre pendulums which were modelled after the 1890 Mendenhall design. Also included with the equipment was an air-tight chamber housing a single pendulum and "flash apparatus" for measuring the periods.

Gravity measurements were made with the original apparatus by Miller, Innes and other observers until around 1948 when pendulums were largely supplanted by gravimeters for geophysical surveying.

In 1954 Thompson (1959) began reconstructing the equipment with a view to converting it to a bi-pendulum apparatus with a high level of environmental control. In a bi-pendulum apparatus two pendulums are swung exactly out of phase in the same plane to eliminate motions of the support induced by the pendulums. At this time Thompson also acquired a second set of three bronze pendulums from USCGS.

Subsequent to the completion of these modifications, the equipment underwent three seasons of field observations from 1957 to 1959 (Thompson, 1959; Winter and Valliant, 1960; Winter, Valliant and Hamilton, 1961). Analyses of these results indicated that the apparatus only performed satisfactorily under stable laboratory conditions at Ottawa. Field measurements were subject to large systematic errors, affecting all six pendulums at any one location, and discrepancies as large as two milligals were sometimes noted.

Consequently a second attempt to develop a high precision pendulum apparatus using the original pendulums was begun late in 1959. By April 1965 the new apparatus which is the subject of this paper was essentially completed. The period from April 1965 until June 1966 was occupied by testing and improving the overall system, in measuring the stability of the temperature control and generally in assuring that all parts of the apparatus were functioning according to design parameters. During this time field procedures for the new apparatus were also established. From June 1966 until September 1967 a series of measurements of Δg between a field station at Almonte, Ontario and the base station at Ottawa, Ontario was made to determine the overall accuracy of the apparatus. The Almonte tests disclosed that the length of the pendulums (Valliant, 1969a) were altered by changes in humidity and steps were taken to control this effect.

In a dual effort to make a contribution to the establishment of the First Order World Gravity Network and to compare the accuracy of the Canadian pendulums with those of other countries, measurements along the North American calibration line at College, Alaska; Edmonton, Alberta; Denver, Colorado and Mexico City, Mexico were undertaken between September 1967 and September 1968 (Valliant, 1969b). At the present time the Canadian pendulum apparatus is being deployed to establish a network of pendulum stations at selected sites throughout Canada (see Appendix I).

The new Canadian pendulum apparatus

Description. The heart of the Canadian pendulum apparatus is a group of six bronze pendulums which are allowed to swing two at a time on agate knife edges. The knife edges are mounted on a common support so that the two pendulums oscillate in the same plane. While in use the pendulums are housed inside a vacuum chamber, where pressure and temperature may be maintained at a constant value. In operation the pendulums are first deflected by a mechanical device protruding through the wall of the vacuum chamber and suddenly released so that they oscillate freely in opposing phase. Anti-phase operation essentially decouples the pendulums from their support (Vening-Meinesz, 1929). The mean period of two nearly identical pendulums swinging in anti-phase on a common support may be thought of as the period of a fictitious pendulum (hereafter called the "fictitious period" for simplicity) swinging on a stationary support.

The fictitious period, which is approximately 1 second, must be determined to within 100 ns in order to achieve a 0.2 mgal accuracy in gravity. This precision is achieved by obtaining the average period for a large number of oscillations of the pendulum.

To simplify the description, the apparatus may be subdivided into the following sub-systems:

1. Pendulums.
2. Pendulum case.
3. Environmental control systems:
   a) Temperature control.
   b) Pressure control.
   c) Humidity control.
4. Data acquisition system:
   a) To measure and record pendulum periods.
   b) To measure and record phase and amplitude of oscillation.
   c) To measure and record environmental conditions such as pendulum temperature and operating pressure.

Pendulums. The pendulums approximate simple pendulums having a lenticular bob attached to a thin stem approximately 25 cm long. They are constructed from aluminum bronze and were gold plated in 1954 to prevent tarnishing. A complete description of the original form of the apparatus and the pendulums was given by Swick (1921). For relative gravity measurements accurate to 0.1 mgal it is required that the lengths of the pendulums remain constant to within 250 Å. The achievement of such a high degree of dimensional stability is hampered by the relatively large coefficient of linear expansion (17 ppm/degree C), the arrangement of the knife edges and the construction of the pendulums which are made from five separate pieces rivetted together. Because the knife edges are fixed to the pendulum case, it is essential that they engage the flats on the pendulums at the same location for each observation. Otherwise, errors are introduced because the effective length of the pendulum is altered. A rotation of the pendulum about its vertical axis also generates errors in the amplitude determination as described later.

On the other hand it is to be expected that the age of the pendulums may contribute significantly to the dimensional stability of the material of their construction. As it was felt that the advantages of using the old pendulums outweigh the disadvantages, in particular the disadvantage of providing a constant temperature environment, Thompson (1959) decided in 1954 to build a pendulum apparatus using the old Mendenhall pendulums. It should also be emphasized that the characteristics of the old pendulums were well established whereas the construction of new pendulums would create a completely new set of characteristics and problems some of which might have proved to be as severe as the limitations with the bronze pendulum.

Pendulum case. Since the pendulum case functions as an integral part of each of the sub-systems it is possibly the most critical component of the apparatus. It provides:

1. Support for two pendulums.
2. Primary insulation and heat source for the temperature control system.
3. The optical bench for the data acquisition system.
4. Means to accurately level the knife edges.
5. Means to deflect and release, raise and lower the pendulums.
6. The temperature sensors for the data acquisition system.
7. A vacuum chamber for pressure control and thermal insulation.

The construction details of the pendulum case are shown in Figure 1. The base of the pendulum case consists of a sixteen-sided aluminum vacuum "collar" constructed with a solid bottom. Twelve ports through the side of the base plate provide entry to the vacuum chamber for external mechanical and electrical controls. Three adjustable legs and two levels are mounted on the outside of the case to permit initial levelling of the pendulum apparatus. Precise levelling is accomplished with the aid of the adjustable legs and a striding-level resting on one of the knife edges. A glass bell jar, gold plated on its inner surface surmounts the base and completes the vacuum chamber. The gold plating helps prevent heat loss through radiation.

The oven which is located directly inside the vacuum chamber is constructed from thin sheet aluminum and has a highly polished outer surface to further reduce heat loss. A heating element is attached to the oven's inner surface with silicone rubber and arranged to supply an

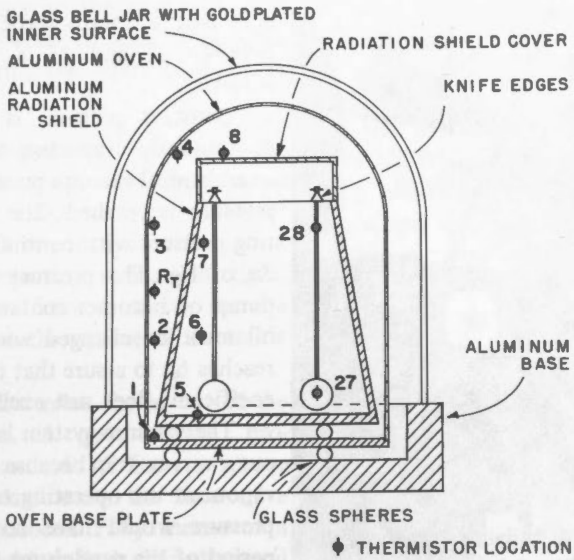

Figure 1. Schematic diagram of pendulum case showing approximate location of thermistors (1-28). To minimize heat loss, the major components are separated by glass spheres arranged in a kinematic mounting system.

even distribution of heat over its surface. Thin material is used in the construction of the oven to permit its temperature to respond quickly to changes in ambient temperature. The oven may be easily separated from its base to provide access to insert or remove the pendulums. The radiation shield which is located inside the oven carries the knife edges. It also prevents the direct transfer of radiant energy from the heaters to the pendulums. The surfaces of the radiation shield are dull to encourage heat transfer from the oven through the shield to the pendulums. As the mounting for the knife edges, the radiation shield ideally must provide a stationary support. Although two pendulums are operated in anti-phase, their periods cannot be perfectly matched, nor can they be started exactly out-of-phase so that some force is, in practice, coupled to the support. To completely eliminate forced vibrations the support must be either perfectly rigid or its mass must be infinite. Consequently the radiation shield was made as heavy as practical by constructing it from $^3/_4$-in.-thick aluminum.

The massive nature of the radiation shield is also compatible with the requirements of the temperature control system. The radiation shield must have a large thermal capacity to average out rapid fluctuations in the oven temperature. Also the thickness of its walls permits rapid heat flow to all parts preventing the establishment of temperature gradients.

Conductive heat losses are minimized by supporting the oven on $^3/_8$-in.-diameter glass spheres. These spheres are held in position by three conoidal depressions located in the oven base plate and by a "slot-cone-plane" arrangement in the base of the vacuum chamber. The radiation shield is separated from and supported by the oven base plate in a similar manner. As the temperature of the radiation shield is nominally the same as the oven temperature little or no heat is conducted from the oven to the radiation shield by the supports thereby eliminating the establishment of temperature gradients in the radiation shield because of thermal heat sinking. Evacuating the entire assembly to below 10μ of Hg virtually eliminates heat loss through conduction and convection. All shafts leading outside the vacuum chamber are either retractable or insulated with nylon sections to prevent heat flow along the shafts.

Operation of the pendulum case as part of the temperature control and measuring systems may be stated as follows. The pendulums are completely surrounded by the radiation shield which functions as a black-body enclosure having a uniform wall temperature. Under these conditions radiant energy flows between the pendulums and the heat shield until any temperature differential is eliminated. The temperature of the pendulums, which can not be measured directly, may be obtained after equilibrium is established by measuring the radiation shield temperature.

Figure 2 is a view of the completed pendulum case showing some of the mechanical and electrical devices attached to the outer base plate. The mechanism for deflecting and releasing both pendulums simultaneously is contained in the small aluminum box attached at the left side of the case. One of the two mechanisms for raising and lowering the pendulums is operated by the retractable knob located at the right. An interior view of the pendulum case is given in Figure 3 showing the radiation shield after the bell jar and oven have been removed.

The success of the pendulum case has been proven by three years of continuous trouble free operation. Tests have shown (Valliant, 1967a) that the temperature gradient in the radiation shield does not exceed 0.02°C and that changes in the pendulum's temperature may be measured with an accuracy of 0.01°C. While the flexibility of the pendulum case is considerably larger than for either the Gulf or Cambridge pendulum apparatus the resulting flexure is within acceptable limits.

The environmental control system. Three facets of environment control–temperature, pressure and humidity–are applied to the Canadian pendulums.

The function of the pendulum case as part of the temperature control system has already been described. The electronic thermostat which maintains a nominal temperature of 40°C and the electronic thermometer for measuring the temperature of the radiation shield have been described in detail (Valliant, 1967a). Tests indicate that the maximum temperature variation over a wide range of ambient conditions is about 0.025°C even when the pendulum case is opened at regular intervals to change pendulums

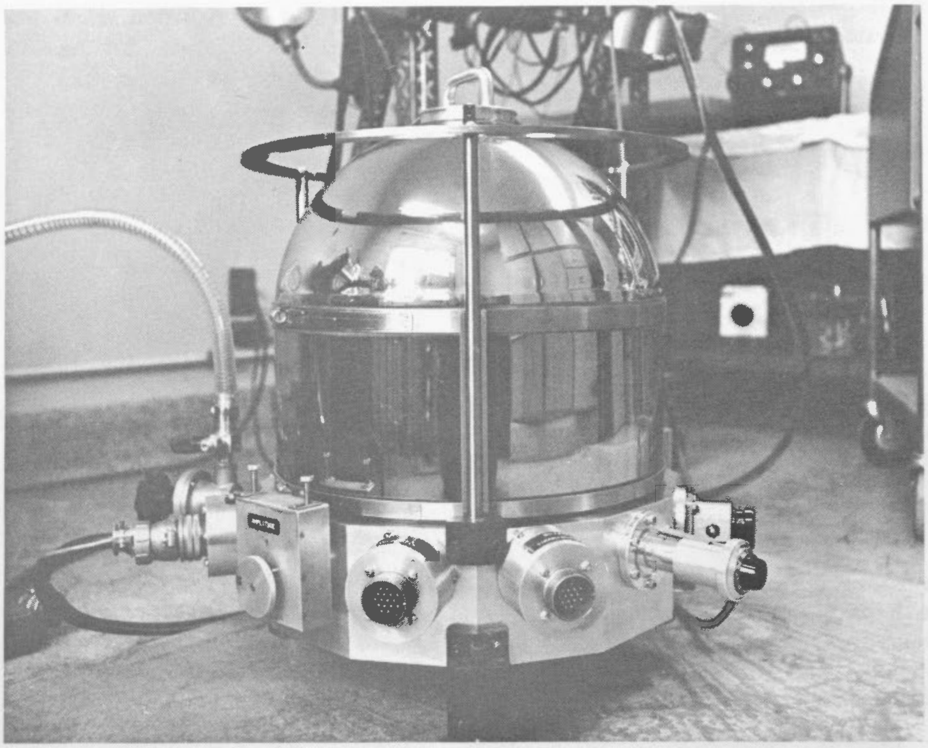

Figure 2. View of pendulum case showing vacuum sealed parts for electrical connections and mechanical devices for starting and lifting the pendulums.

and that a thermostatic coefficient of between 22 and 25 $\times$ 10^{-4} has been achieved.

Constant pressure is maintained by continuously evacuating the bell jar with a mechanical vacuum pump until terminal pressure is reached. The nominal operating pressure with continuous pumping is 3μ of Hg. The pressure increases if the pump oil becomes contaminated and the oil must be changed when the pressure reaches 6μ to assure that the thermostatic coefficient does not exceed 25 $\times$ 10^{-4}.

The vacuum system is operated without a vapour trap because even pure water vapour at the operating temperature and pressure would have no effect on the period of the pendulums. However, water absorbed by the agate flats and absorbed on the surface of the pendulums, while they are being stored, influences the effective length of the pendulums (Valliant, 1969a). The storage box for the pendulums must therefore be desiccated to maintain a constant humidity. By changing desiccant every two weeks the

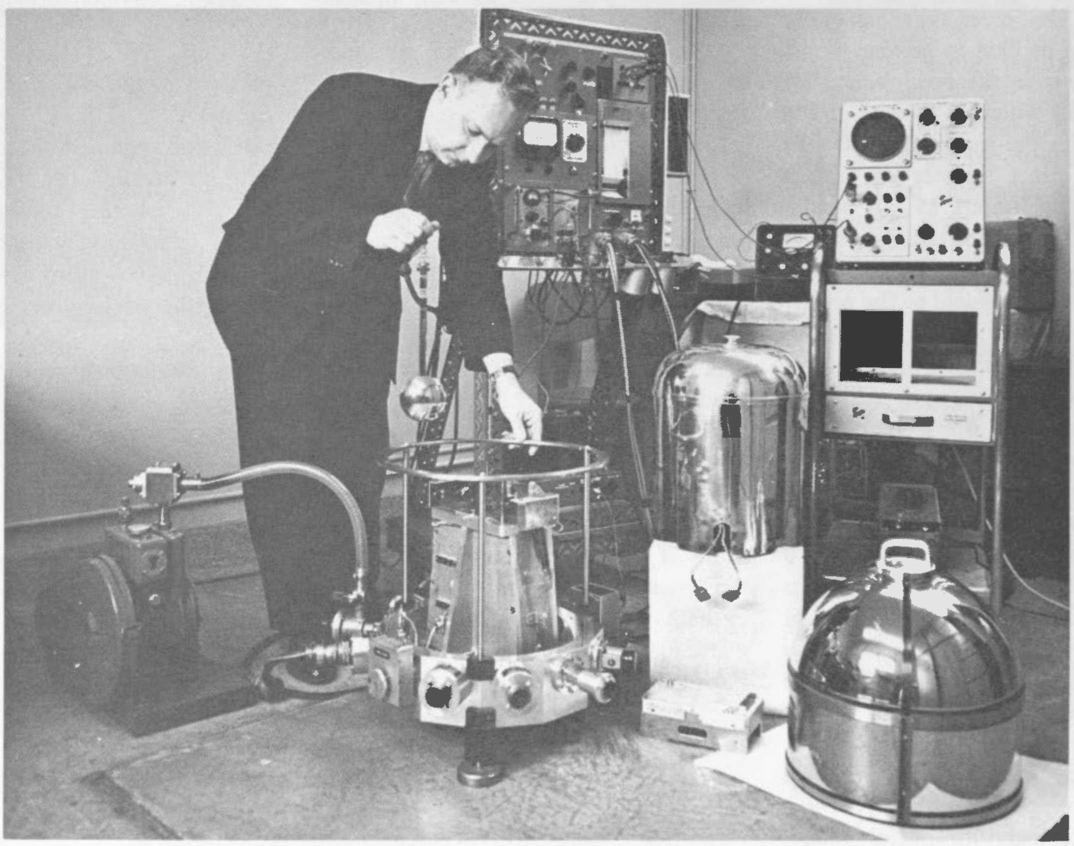

Figure 3. View of the pendulum apparatus, with the bell jar and oven-cover removed, showing the radiation shield.

relative humidity in the storage box is maintained between 10 and 20 per cent thereby rendering the effect of moisture negligible.

Data acquisition system

The optics. The purpose of the optics is to generate an exact electrical analog of the pendulum's motion. This is accomplished by projecting a beam of light, after reflection from a mirror on the pendulum, onto the surface of a suitable photodetector.

Figure 4 illustrates the layout of the optical system. A quartz-iodine, projection lamp and slit S form a source which is focussed onto the face of a differential photocell P by means of a lens L. The light path is folded several times by reflection from mirrors M_1, M_2, M_3, M_p, M_3, M_2, and M_4 in that order. The mirror M_P is attached to and rotates with the pendulum so that the beam plies back and forth across the face of the photocell as the pendulum oscillates.

The photodetector consists of two selenium photocells each measuring 0.5 X 0.75 in., which are mounted closely together end to end. The optical slit is adjusted so that the width of the image is 0.75 in. When the pendulum is vertical both photocells are 50 per cent illuminated and the difference in their output voltage is zero. As the light traverses the face of the photocell arrangement, the differential output is a one Hz sinusoid which is the exact electrical analog of the pendulum's motion.

As the optical system also provides amplification it functions as a low noise preamplifier for the data acquisition system. The optical leverage or gain of this system is such that a displacement of the pendulum of 0.003 radian displaces the light beam 0.375 in. providing the full output from one photocell.

Amplitude and phase determination. The amplitude of the pendulum's motion is measured directly by comparing the amplified photodetector output with a variable calibrated voltage. In practice this device is calibrated before and after each day's observations by allowing the pendulum to swing with sufficient amplitude for the light beam to deflect past the ends of the photodetector. The photodetector output is then a truncated sinusoid whose peak-to-peak amplitude corresponds to a pendulum rotation of .00625 radian. After calibration the amplitude of the pendulum is reduced to normal and the output from the photodetector is again determined. The pendulum's amplitude is then given by the ratio of the normal to truncated amplitudes times .00625 radian.

Correction to the period for variations in amplitude are applied according to the following equation:

$$T = To\left(1 - \frac{\alpha^2}{16}\right)$$

α = half amplitude in radians

To = observed period.

The magnitude of the amplitude correction varies between observations but usually amounts to 50 X 10^{-8} sec and never exceeds 200 X 10^{-8} sec. Thus an accuracy of 1 per cent in the amplitude measurement is required for a 0.1 mgal accuracy in gravity. This implies that the reference (amplitude of the truncated waveform) must be obtained with equal accuracy. While the voltage measurement is performed with an accuracy of 0.1 per cent the reference value varies with time as well as with the position of the pendulum on the knife edge.

Long term or daily drift in the value of the reference measurement is primarily due to variations in the light intensity because of aging of the lamp. Recent installation of a quartz-iodine lamp powered from a regulated supply has much alleviated this problem. Total variation in the reference value during the course of a day's observations rarely exceeds 10 per cent. To minimize this problem the mean of two determinations of the reference, one taken at the beginning and one at the end of each day's

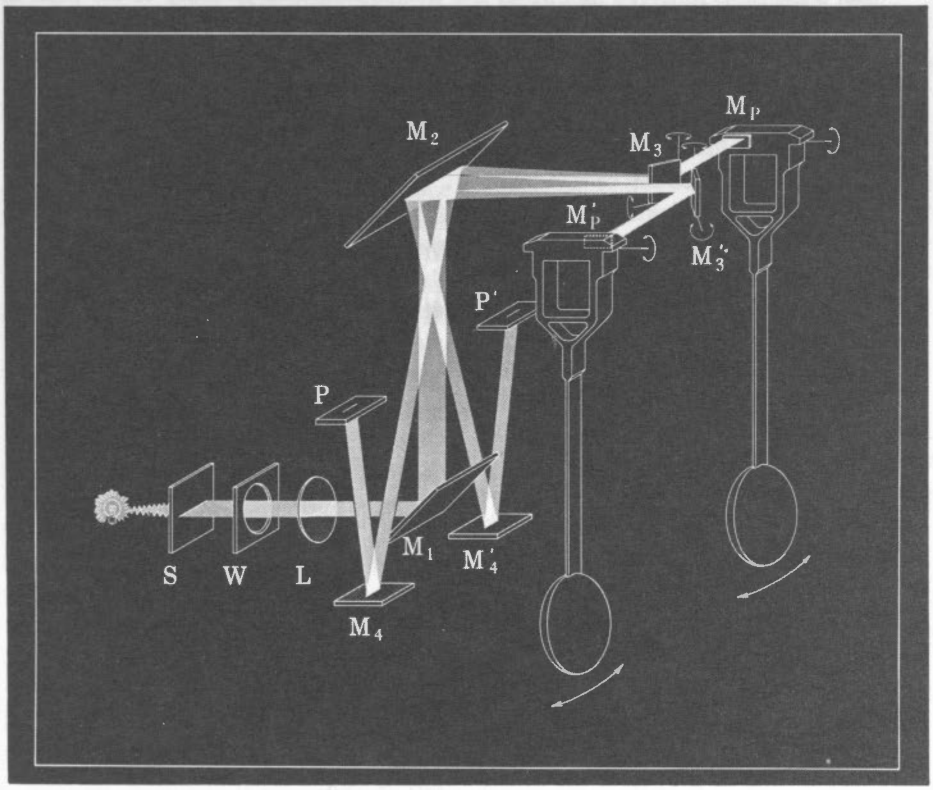

Figure 4. Optical system. A beam of light generated by a quartz-iodine lamp and slit (S) enters the vacuum chamber through a vacuum sealed window (W) and is focused on two photocells (P and P') by a lens (L). Two adjustable mirrors M_3 and M_3' divide the beam in two, one half for each pendulum.

observation, is used. If the long term drift in illumination is linear the error in the mean fictitious period for a day's observations is unaffected by the drift in the reference value.

An additional problem involves the rotation of the pendulum about a vertical axis as it is lowered onto the knife edge. Such a rotation causes a transverse displacement of the light beam across the photodetector. As the illumination of the light beam is not uniform along its length, but is brightest at the centre, lateral displacement of the beam alters the illumination of the photodetector and changes the reference value. The amount of rotation of the pendulum varies for each individual observation and depends on the degree of gentleness and uniformity, from observation to observation, with which the pendulum is lowered onto the knife edge. Changes in the reference of up to 20 per cent from this cause have been noted when a pendulum was rapidly lowered onto a knife edge.

The obvious solution–to measure the reference before and after each 15-minute observation–proves unwieldy in practice. It is more practical for the observer to *gently* lower the pendulum to the knife edge using a consistent technique to avoid rotating the pendulum about a vertical axis. Also if the amplitude of a pendulum is noted to change significantly after lowering to the knife edge, the pendulum is raised and lowered a second time before the observation is made. With these precautions the magnitude of this effect can be kept below 1 per cent of the total amplitude correction. Furthermore, as rotations of the pendulum in this sense are random, the effect on the mean fictitious period would be reduced by the square-root of the number of observations.

To observe the phase relation between the two pendulums concurrently with the observations, the amplified output from the two photodetectors, one for each pendulum, are displayed with a double beam oscilloscope. The pendulums are started so that their initial phase difference never falls outside the range of $\pi \pm 0.03$ radian. The maximum phase change for the poorest matched pair of pendulums during a 900-second observation is 0.125 radian.

Period measurement. A block diagram of the sub-system for measuring the pendulum periods is given in Figure 5. The primary time source is a James Knight frequency standard having a drift rate of better than 5 parts in 10^{10} per day. A "clock" pulse, 10 microseconds wide, is formed by the control and clock pulse generator every 20 seconds. The clock pulses are stored in a binary accumulator where they provide a measure of elapsed time and the resolution of a time interval measurement is ±1 count or 20 microseconds. A series of control pulses each spaced exactly between two clock pulses is also provided by the pulse generator. The control pulses regulate the operation of the print-out circuit so that the binary accumulator is interrogated only when it is dormant.

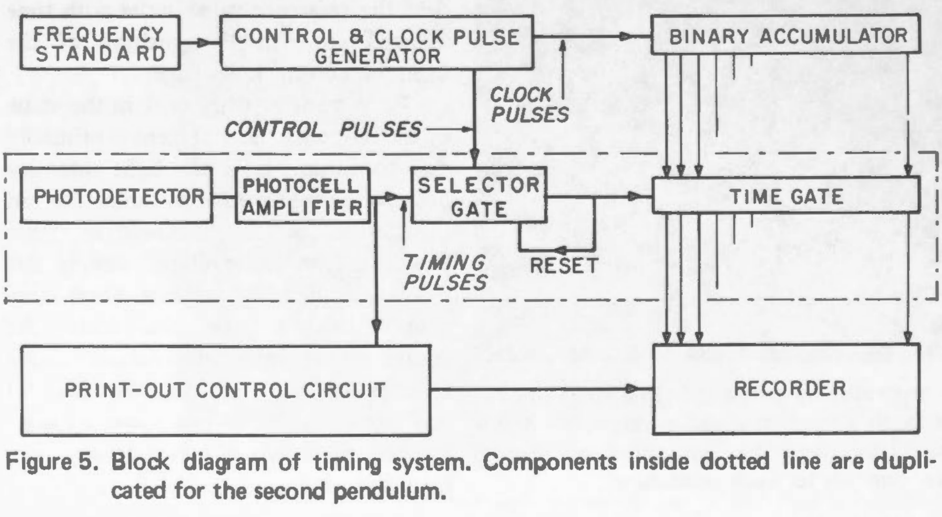

Figure 5. Block diagram of timing system. Components inside dotted line are duplicated for the second pendulum.

A timing pulse, generated at the zero-crossings of the photodetector signal, opens the selector gate so that a control pulse is applied to the time gate which consists of a series of transmission gates. As this control pulse also closes the selector gate only a single pulse is chosen. The transmission gates are arranged to provide a non-destructive readout of the instantaneous state of the binary accumulator. All gates connected to binaries in the one state are open, while those connected to binaries in the zero state are closed. The selected control pulse is then transmitted through the open gates to a specially designed digital recorder (Valliant, 1965).

Operation of the Canadian pendulum apparatus

Limitations of the temperature control system

Data acquisition rate. It has been pointed out that the environmental control system provides sufficient temperature control to assure an accuracy of 0.3 mgal in measuring g. Furthermore by measuring the temperature and applying a small correction to the pendulum's period errors because of temperature variations are reduced to below 0.1 mgal.

The chief drawback with this system is the excessive time required to establish temperature equilibrium after a pendulum change. Figure 6 shows empirically determined heating curves for both gold plated and tarnished bronze pendulums. These curves include the influence of heat transfer by radiation from the oven through the radiation shield to the pendulum, of pumping out the vacuum system concurrently with heating the pendulums, and heat transfer from the radiation shield through the pendulums via the lifting mechanism which supports the pendulums in their raised position. It is evident from the temperature gradient in the pendulum before temperature equilibrium is established that much of the heat transfer is through the lifting mechanism.

Approximately 35 hours are required to achieve temperature equilibrium with the gold plated pendulums. Since five hours are normally required to make the

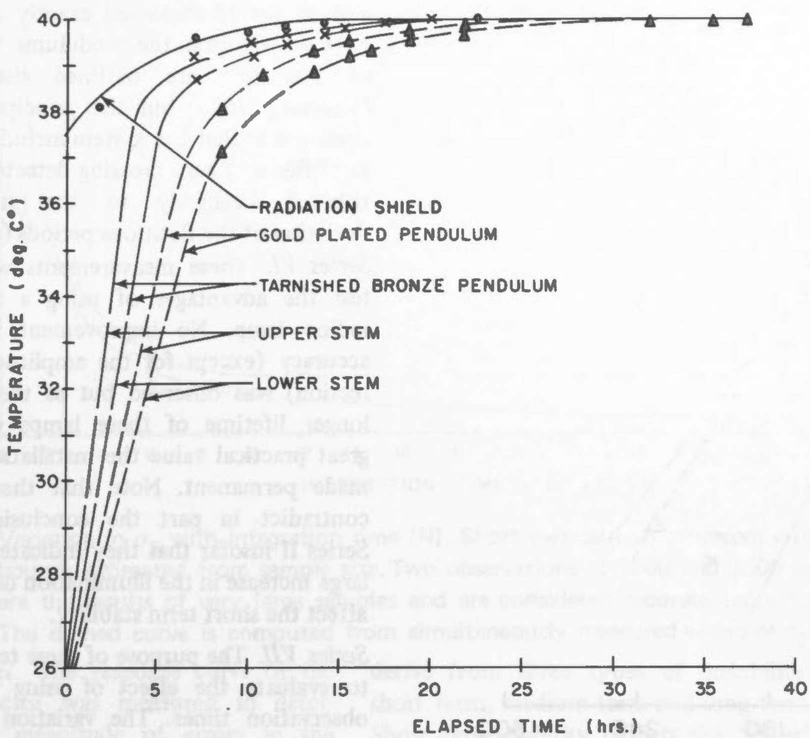

Figure 6. Pendulum heating curves. The temperature gradient between the upper and lower stem of the pendulum indicates downward heat flow from the lifting mechanism.

observations the turnaround time for one set of observations with one pair of pendulums amounts to 40 hours. As a 40-hour day is inconvenient the normal practice is to observe on a 48-hour schedule. The data acquisition rate could be doubled by the simple expedient of blackening the surface of the pendulums; equilibrium is established in under 20 hours with tarnished pendulums permitting a 24-hour schedule to be adopted. The pendulums have not as yet been blackened however, for fear of destroying their dimensional stability in so doing. Such a major modification of the pendulums would also introduce a singularity into the analysis of long-term drift in pendulum periods for which data has been accumulated since 1966.

Testing the pendulum apparatus

Tests were conducted essentially in three phases:

1. Laboratory tests to determine an upper limit to the accuracy under stable environmental conditions.
2. Simulated field tests to determine the reduction in accuracy as a result of transporting the apparatus.
3. Actual field tests to determine the accuracy of measurements under complete field conditions.

Laboratory tests. Eight series of tests were performed under a variety of conditions to determine the operational accuracy that could be achieved with stable conditions. Initially, the tests were concerned with adjusting the data acquisition system for optimum performance. As the tests proceeded more emphasis was placed on the determination of the performance of the complete system. In addition a series of temperature tests were carried out to insure adequate operation of the environmental control system. As field tests indicated that further knowledge of the influence of magnetic fields was required, this effect was briefly investigated in the laboratory.

Series I. These tests consist of the first observations made with the pendulum apparatus and were mainly to determine if the instrument functioned properly and to optimize the photodetector and associated amplifier circuitry. From these tests it was decided to use photo-voltaic cells rather than photo-conductive cells and to employ a.c. coupled electronics to alleviate the effect of drift in the photodetector response, amplifier d.c. levels and variations in illumination.

Series II. This series constituted a study of 60 samples of 100 individually measured pendulum periods which were observed during a 5-day interval. Various parameters were altered throughout the test in an attempt to determine the influence of these parameters on the short term stability (i.e. σ_S – the standard deviation of individual periods) of the measurements.

It was found that σ_S for knife edge 2 is always between 34 and 100 microseconds smaller than knife edge 1 (see Figure 20). As interchanging pendulums and/or the amplifiers has no effect it was concluded that this difference was due to differences in the behaviour of the two photodetectors. An attempt was made to differentiate between pendulum and photocell derived noise with three sets of observations. Firstly, the change in σ_S was observed as the pendulum amplitude was altered while the illumination remained constant. Secondly, the change in σ_S was observed as the illumination was reduced while the pendulum amplitude remained constant. Thirdly, the change in σ_S was observed as the pendulum amplitude was reduced with the output of the photocell maintained constant by increasing the illumination. The results of these tests are given in Figure 7. Interpretation of these data proves difficult. The lateral displacement between curves A and B appears to represent the reduction of noise with nighttime observations. The slope of these curves might be explained as the increase in error with reduced signal level due to the zero-crossing detector. It might also be explained if the pendulums and photocells contribute equally to the noise level. As curve C was derived with a constant signal level, zero crossing errors are excluded from these measurements and as the slope nearly equals that for curves A and B one might suggest that the pendulum is the sole contributor to the short term noise level. Obviously these two conclusions are contradictory and the contributions to the overall short

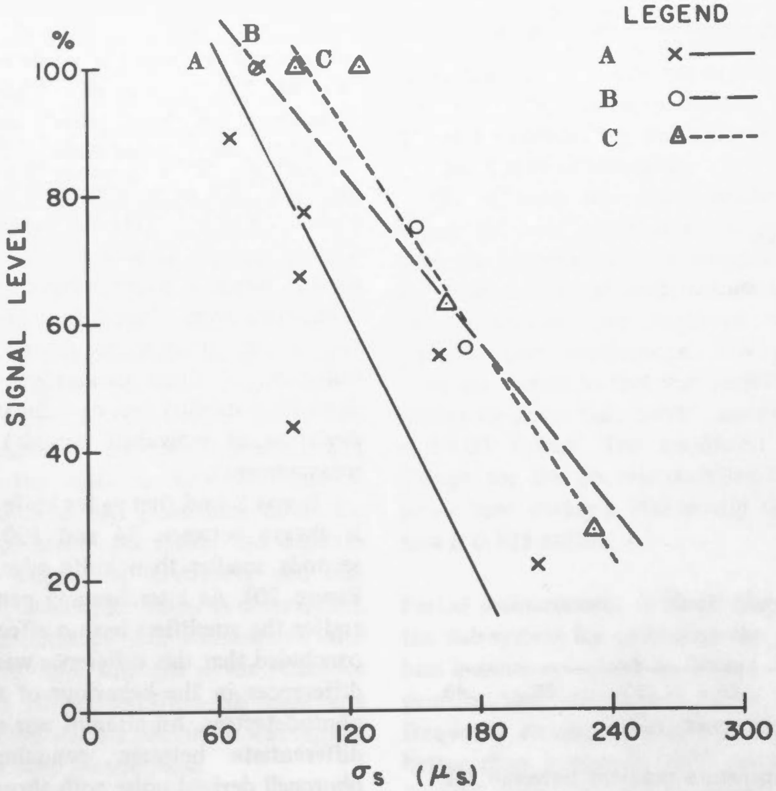

Figure 7. Series II test results.

term noise level of these two sources are not adequately explained. Although further tests made during Series VII also suggest that the photocells are not contributing significantly to the noise level a change of photocells during a field trip in 1969 resulted in a significant increase in σ_S. It will be shown later, however, that the existing short term noise level does not contribute significantly to the overall error for observation times exceeding approximately 200 seconds.

Series III. Following Series II a matched set of selenium photodetectors was permanently installed. The operational amplifier was reconnected to provide a differential input with an amplification of approximately 200. A device to permit the gain to be automatically increased in the vicinity of the zero crossing was included but discarded later as a result of these tests.

Series III was essentially a repeat of Series I to test the new amplifier configuration. The effect of changing pendulums on a routine basis was also observed and it was ascertained that a 24-hour warm up was insufficient. These tests further disclosed that the first of each day's observation was in error by approximately −10 ppm and that this error could be reduced or nearly eliminated by allowing the pendulums to swing for a 10-minute warm-up period before starting observations. This warm up has subsequently been increased to 20 minutes for routine observations. Tests to determine the time required to establish temperature equilibrium were begun as a result of these observations.

Series IV. These tests were performed to study the feasibility of using a photo-pot* instead of the selenium photodetectors. It was found that self heating of the photo-pot disrupted the temperature control system and this device could not as a result be employed.

Series V. Series V was conducted to test the complete electronic system exclusive of photocells. A 65 mv triangular waveform was generated by integrating the output of the binary accumulator to simulate the output of the photodetector. This signal was applied to both amplifiers and its period measured exactly as if it were derived from the pendulums. Details of this test are outlined elsewhere (Valliant, 1967b) but the principal conclusion was that the system including the amplifier and zero crossing detector contributed negligibly to the standard deviation of the fictitious periods (σ_F).

Series VI. These measurements were to test the advantages of using a quartz-iodine lamp. No improvement in the accuracy (except for the amplitude correction) was observed but as the much longer lifetime of these lamps was of great practical value the installation was made permanent. Note that these tests contradict in part the conclusions of Series II insofar that they indicate that a large increase in the illumination does not affect the short term stability.

Series VII. The purpose of these tests was to evaluate the effect of using various observation times. The variation in σ_F with observation time is plotted in Figure 8. The value of σ_F can be calculated from a knowledge σ_S as follows:

$$\sigma_F = \sigma_S/N\sqrt{7}$$

$$= \sigma_S'/\sqrt{7}$$

N = number of oscillations of pendulum.

It is interesting to note that the measured value of σ_F does not vary as 1/N but is nearly flat from 1,200 to 3,000 sec observations. Thus σ_F is influenced by sources of error other than σ_S such as the relative location of the knife edge and flat, errors in amplitude correction and errors because of seismic activity.

The integration time may be chosen to minimize the standard deviation of the mean fictitious period ($\sigma_{\bar{F}}$):

$$\sigma_{\bar{F}} = \sigma_F/\sqrt{n}$$

n = number of observations contributing to the mean.

Allowing five minutes to raise and lower the pendulums between observations and remembering that a total time limit of four hours is imposed by the heating effect of the projection lamp the optimum integration time is found to be 900 seconds. This allows 12 observations to be taken in the four-hour time interval and leads to a value of σ_F of approximately 350 ns.

*Registered trademark, Gianni Corp.

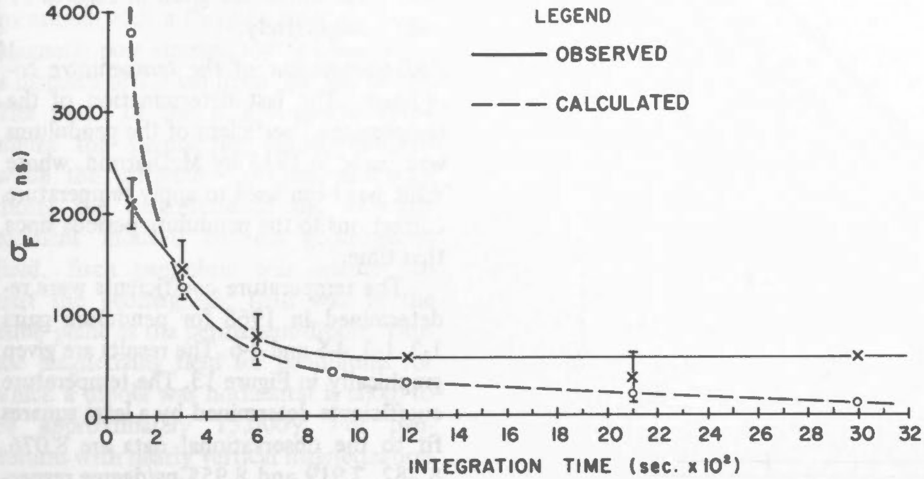

Figure 8. Variation in σ_F with integration time (N). Short vertical bars represent error bounds estimated from sample size. Two observations at 1200 and 3000 sec are the results of very large samples and are considered accurate estimates. The dashed curve is computed from simultaneously measured values of $\sigma_S{}'$.

Series VIII. The response curve of the photodetector was measured to determine the magnitude of errors in the amplitude determination because of non-linearity in the photodetector response. In Figure 9 the output of the photodetector is plotted versus the angle of elongation of the pendulum. The peak-to-peak amplitude, however, is used to compute the amplitude correction factor; the response to peak-to-peak measurements is shown in Figure 10. Comparison of Figures 9 and 10 indicate that for peak-to-peak measurements the non-linearities of each half of the photodetector compensate improving the accuracy of the amplitude measurement. Also included in Figure 10 is the experimentally determined linear approximation to the response curve which is used to calculate the pendulum amplitude from the reference amplitude. The error in this approximation which amounts to about 13 per cent is systematic for all stations and tends to be cancelled out when differences in gravity are measured. Of greater importance is the change in this error with different amplitudes giving rise to a random error in the amplitude correction. This variation amounts to about 3 per cent over the range of amplitudes normally employed.

Sources of error. The factors contributing to the errors may be considered to derive from three types of instability: short term, medium term and long term. Short term stability reflects the "jitter" in the analog signal from the photodetector and is characterized by the distribution of individual period measurements. Medium term stability is characterized by the distribution of the fictitious periods on a daily basis. This distribution is found to be normally distributed with an expected standard deviation (σ_F) of 400 ns. Long term stability is characterized by the distribution of mean fictitious periods (averaged on a daily basis). The mean fictitious periods are found to be normally distributed about a

Table I. Sources of error

A. Factors affecting short term stability (characterized by σ_S – standard deviation of individual periods).

1. Phase stability of pendulums, photocells, and amplifiers.
2. Electronic noise.
3. Zero-crossing detector errors.
4. Rapid changes in illumination.

B. Factors affecting medium term stability (characterized by σ_F – standard deviation of fictitious period on daily basis).

1. Ground motion.
2. Temperature errors.
3. Amplitude measurement errors.
4. Photocell and amplifier drift.
5. Rapid variation in pendulum's apparent length.
6. Knife edge effects.
7. Pressure changes.
8. Errors in flexure correction.

C. Factors affecting long term stability (characterized by σ_M – standard deviation of mean fictitious periods).

1. Temperature drift.
2. Changes in pendulum's length.
3. Levelling errors.
4. Humidity.
5. Secular change in gravity.

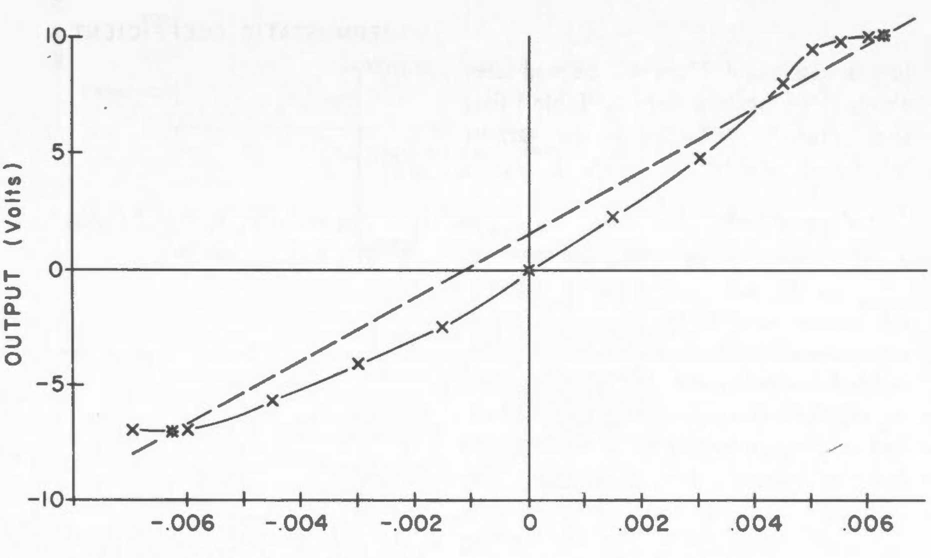

Figure 9. Photodetector calibration curve. The dotted line represents the empirical linear fit.

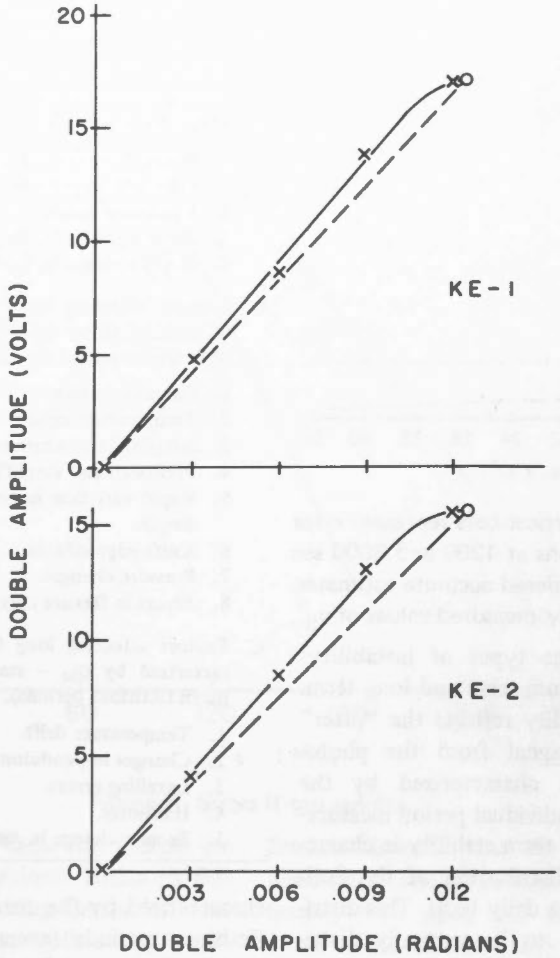

Figure 10. Photodetector peak-to-peak calibration curve. The dotted line represents the empirical linear fit.

line of regression. More will be said later about the long term trends. Table I lists the factors contributing to the various levels of instability.

Temperature tests

Temperature control. Temperature tests were carried out concurrently with the laboratory tests of the data acquisition system to determine if the temperature control was adequate, the time required to establish thermal equilibrium, the effect of pressure variations on temperature and to calibrate the thermostat. The highlights of these tests are given elsewhere (Valliant, 1967a) and the heating curve of the pendulums are reproduced in Figure 6. The variation of the thermostatic coefficient with pressure and the calibration curve are given in Figures 11 and 12 respectively.

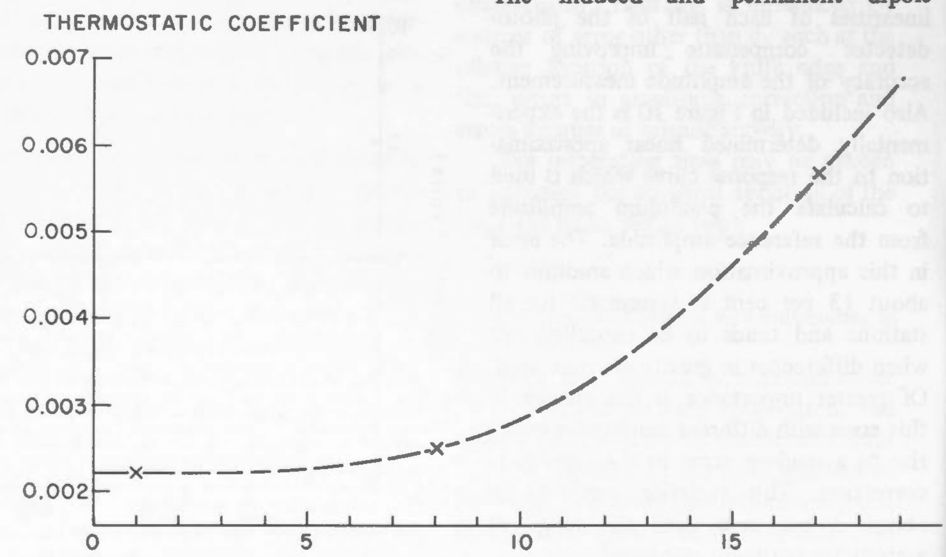

Figure 11. Variation of thermostatic coefficient with pressure.

Redetermination of the temperature coefficient. The last determination of the temperature coefficient of the pendulums was made in 1915 by McDiarmid, whose value has been used to apply temperature corrections to the pendulum periods since that time.

The temperature coefficients were redetermined in 1968 for pendulum pairs 1-2, 1-3, 4-5 and 4-6. The results are given graphically in Figure 13. The temperature coefficients determined by a least squares fit to the observational data are 8,076, 8,282, 7,919 and 8,955 ns/degree respectively. The mean coefficient for pendulum pairs 1-2 and 1-3 (8,179 ns/degree) agrees to within 3 per cent of McDiarmid's value of 8,380 ns/degree. Temperature corrections for all measurements subsequent to the NACL observations will be computed using the value 8,179 for the set comprising pendulums 1, 2 and 3 and 8,440 for the set comprising pendulums 4, 5 and 6. The use of McDiarmid's coefficient does not introduce appreciable error as it is within 2 per cent of the mean temperature coefficient for all pendulums and the maximum temperature correction seldom exceeds 800 ns.

Magnetic tests

Magnetic properties of the pendulums. The induced and permanent dipole moments of each pendulum was measured with a fluxgate magnetometer. Magnetic pole strengths were observed at a distance of 1.9 cm from the pendulums. The mean of the pole strengths determines the permanent dipole moment while the difference in these values indicate the magnitude of the magnetic moment induced by the geomagnetic field. Each pendulum was oriented so that the geomagnetic field was in the same plane as the permanent dipole. Thus the magnetizing field for pendulums for which a dipole was horizontal is taken to be approximately 15,000γ. For pendulums with nearly vertical magnetization the magnetic field is considered to be 55,000γ.

Although no magnetic field could be observed in the vicinity of the stem or stirrup, the bobs of all the pendulums were found to be slightly magnetized. The observed dipole always lay in a plane perpendicular to the plane of oscillation of the pendulums. The dipole for different pendulums was found to be oriented within this plane through an angle ranging from nearly horizontal to nearly vertical. The permanent dipole moment, induced dipole moment, the pole separation and direction of magnetization are tabulated in Table II. It can be seen that the permanent and induced moments appear to be from 40 to 100 times larger for pendulums 4, 5 and 6.

Effect of the observed properties. The effect of a dipole moment on the pendulums period was determined by Bullard (1933):

$$\delta s = -\frac{So}{2mgh}(MoZ + aZ^2)$$

where:

So = unperturbed half period

h = distance from centroid to fulcrum

m = mass of pendulum

z = magnetic field strength

Mo = permanent magnetic movement

aZ = induced magnetic movement

The first term in the brackets represents the effect of the permanent dipole whereas the second represents the effect of the

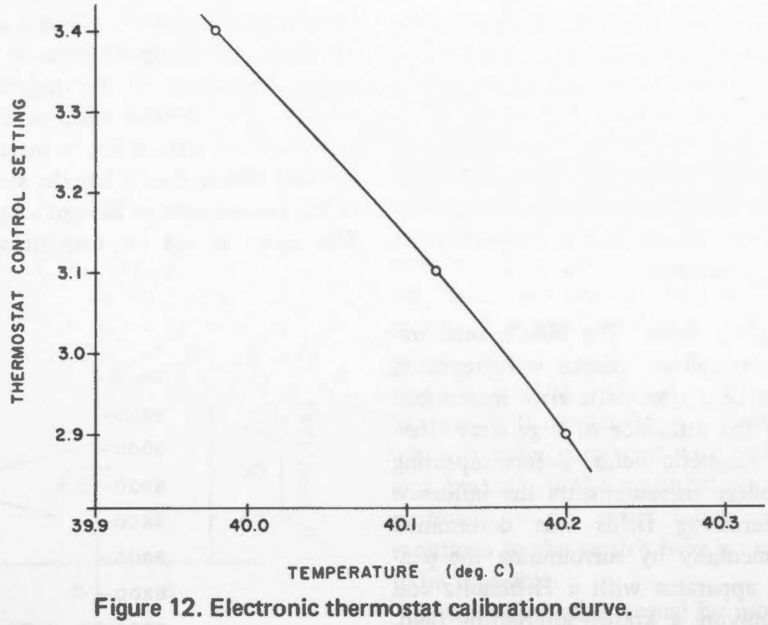

Figure 12. Electronic thermostat calibration curve.

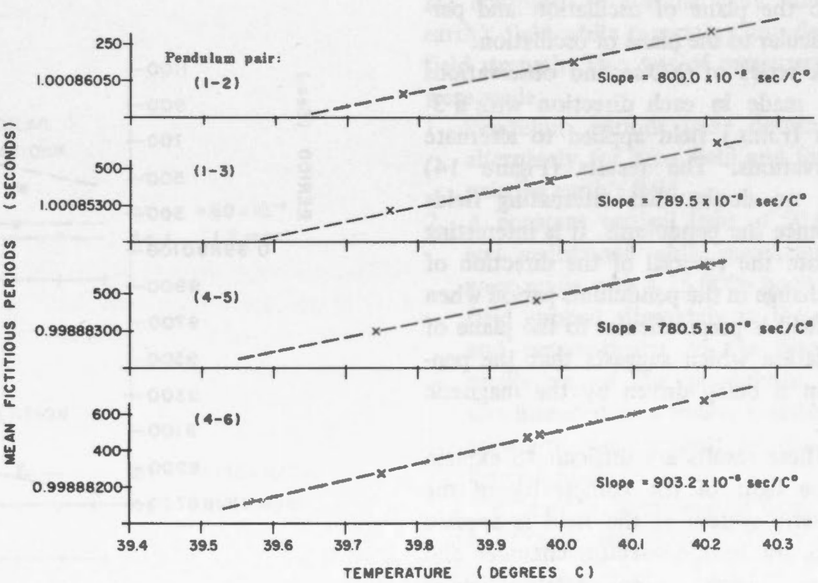

Figure 13. Effect of temperature change on the periods of the fictitious pendulums. The dotted lines represent a least-squares linear fit.

Table II. Magnetic properties of the pendulums

Pendulum	Permanent Moment (amp. m^2)	Induced Moment (amp. m^2)	Approximate Pole Separation (metres)	Orientation of Dipole
1	0.04×10^{-4}	0.03×10^{-4}	0.05	15° from horizontal
2	0.06×10^{-4}	0.01×10^{-4}	0.08	near vertical
3	0.07×10^{-4}	0.01×10^{-4}	0.04	horizontal
4	3.5×10^{-4}	2.4×10^{-4}	0.04	horizontal
5	2.7×10^{-4}	2.0×10^{-4}	0.04	horizontal
6	2.6×10^{-4}	0.3×10^{-4}	0.08	near vertical

induced dipole. Applying values representative of the Canadian pendulums we find that the magnetic moment either permanent or induced may not exceed 60 X 10^{-4} amp. m^2 if the error is to be maintained below 0.1 mgal. Table II makes it clear that any observed magnetic effects can not be due to magnetization of the pendulums.

Alternating fields. The NACL measurements at College, Alaska were repeated because of a systematic error introduced due to the influence of large stray alternating magnetic fields. Before repeating the College measurements the influence of alternating fields was determined experimentally by surrounding the pendulum apparatus with a Helmholtz coil and applying a known alternating field. The field was applied sequentially in the three principal directions–vertical, parallel to the plane of oscillation and perpendicular to the plane of oscillation.

A series of 900-second observations were made in each direction with a 3-gauss (r.m.s.) field applied to alternate observations. The results (Figure 14) leave no doubt that alternating fields influence the pendulums. It is interesting to note the reversal of the direction of the change in the pendulums period when the field is perpendicular to the plane of oscillation which suggests that the pendulum is being driven by the magnetic field.

These results are difficult to explain in the light of the complexity of the magnetic system as the field is applied externally to the vacuum chamber and the pendulums are completely shielded by three metal chambers including the gold plated bell jar. Also complicating the analysis is the irregular shape of the pendulums themselves. The increase in period for two field directions might well be due to eddy-current braking. As the power line frequency is about 120 ppm greater than the 60th harmonic of the pendulum's frequency the decrease in the period when the field is perpendicular may possibly be because of a resonance phenomenon.

The effect in changing the amplitude of the applied field in a plane parallel to the plane of oscillation is shown in Figure 15 where the observed error is plotted against the square of the magnetic field strength. The obvious linear relation suggests that the error is due to magnetically induced effects (i.e. it has the same form as the second term in Bullard's equation). This result is not inconsistent with the idea that eddy currents are the causative factor.

Obviously a much more extensive program is required to determine the cause of the anomalous behaviour due to alternating magnetic fields. However, these measurements indicate the existence and magnitude of such errors and

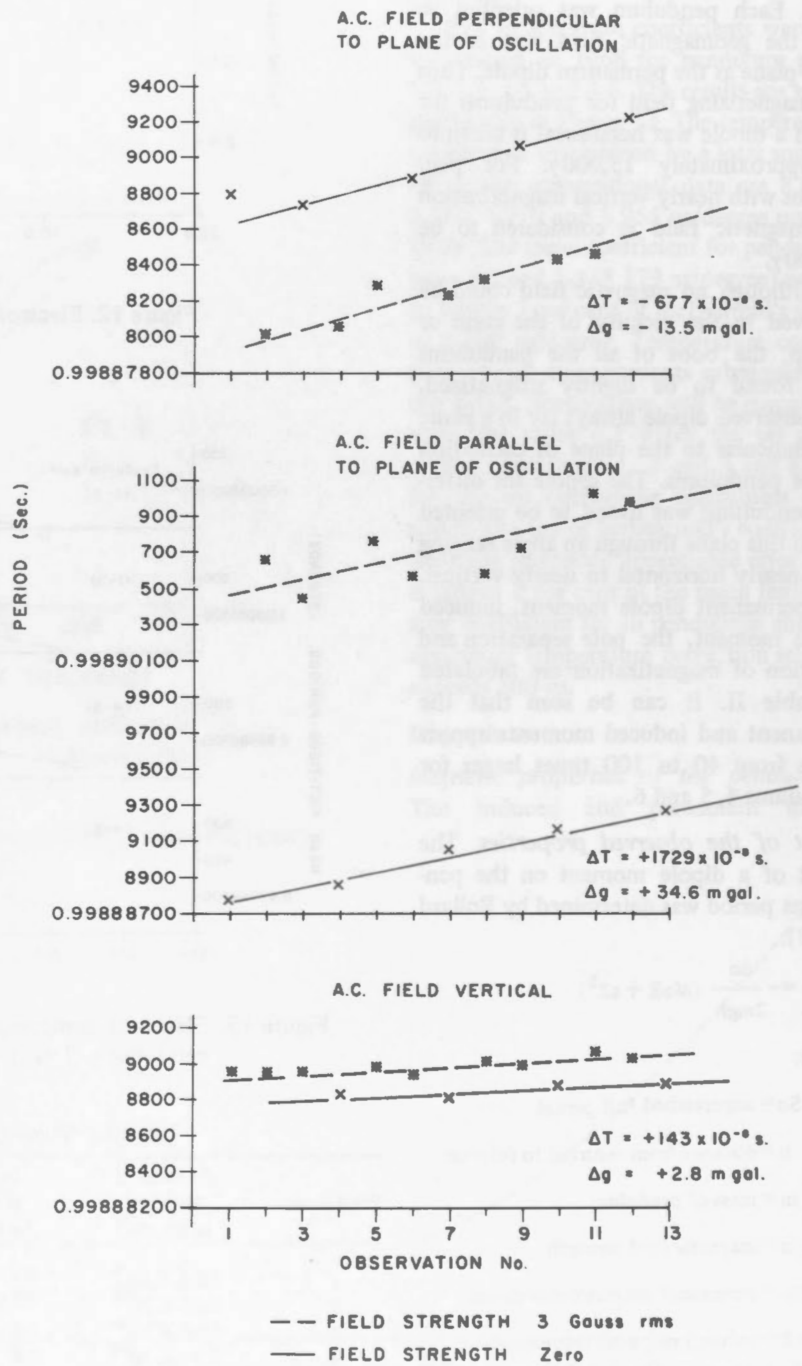

Figure 14. Effect of alternating magnetic fields applied in the three principle planes.

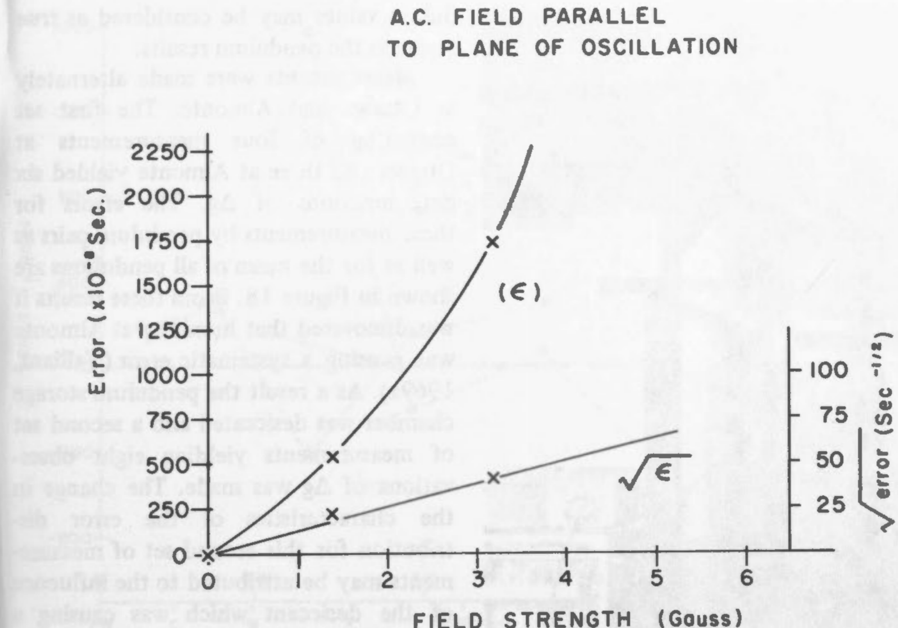

Figure 15. Variation of the effect of alternating magnetic fields with field strength.

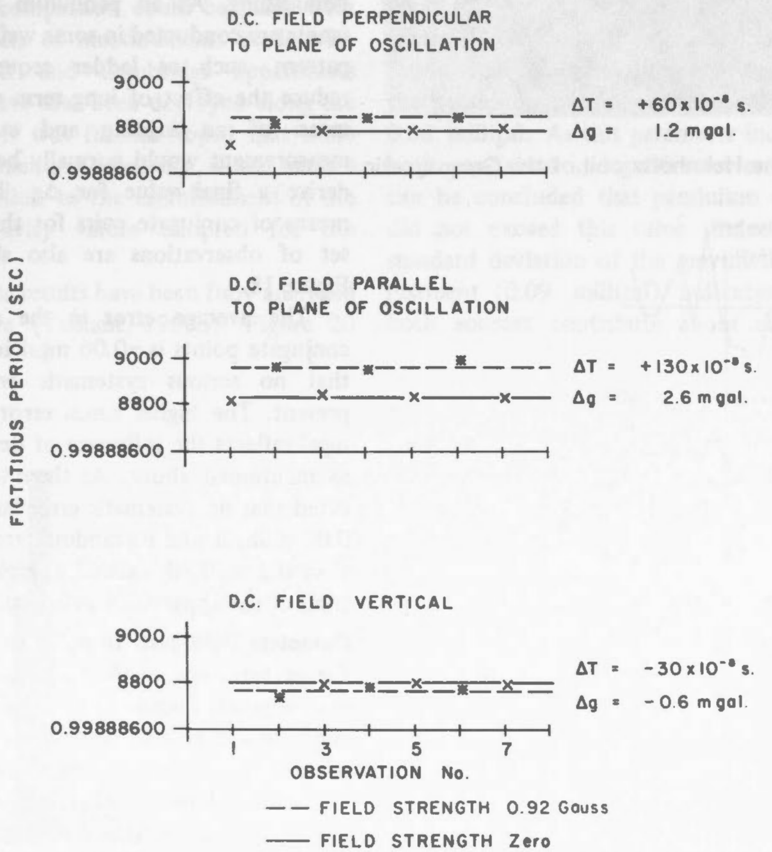

Figure 16. Effect of steady-state magnetic fields applied in the three principle planes.

the solution to this problem is to avoid locating the apparatus in areas subject to large alternating fields.

Direct fields. The effect of a direct or steady state field was also briefly investigated using the same Helmholtz coils. A field of nearly one gauss was applied in three principal directions as before. The net field, influencing the pendulum's motion, is of course the vector sum of the geomagnetic and applied fields. In every case (Figure 16) the error was found to increase with increasing field strength, which is again consistent with eddy current braking. If the perturbation is proportional to β^2 errors introduced by variations in the earth's field is expected to be negligible.

This was further tested by mounting the vacuum case inside large Helmholtz coils as shown in Figure 17. These coils are equipped to automatically null the earth's field while generating any desired field strength. Two sets of measurements were made:

1. Pendulum periods were determined alternately for zero field and for the existing earth's field.
2. A constant vertical field of 50,000γ was maintained while measurements were made with a 15,000γ horizontal field applied alternately in the plane and perpendicular to the plane of oscillation of the pendulums. This was intended to simulate rotating the apparatus with respect to the geomagnetic field.

No detectable effect was observed for either of these tests.

Simulated field tests. The effects of moving the apparatus during field operations was tested by making a series of measurements between the national reference pier in Ottawa and a typical field site located at Almonte, Ontario (a town 35 miles distant from Ottawa). As environmental conditions were carefully controlled at Ottawa, errors in the measured Δg would reflect the combined effects of transportation and environment at Almonte. The Δg was also measured with gravimeters and since this value is small (Δg = 32.15 milligal) differences between the gravimeter and pen-

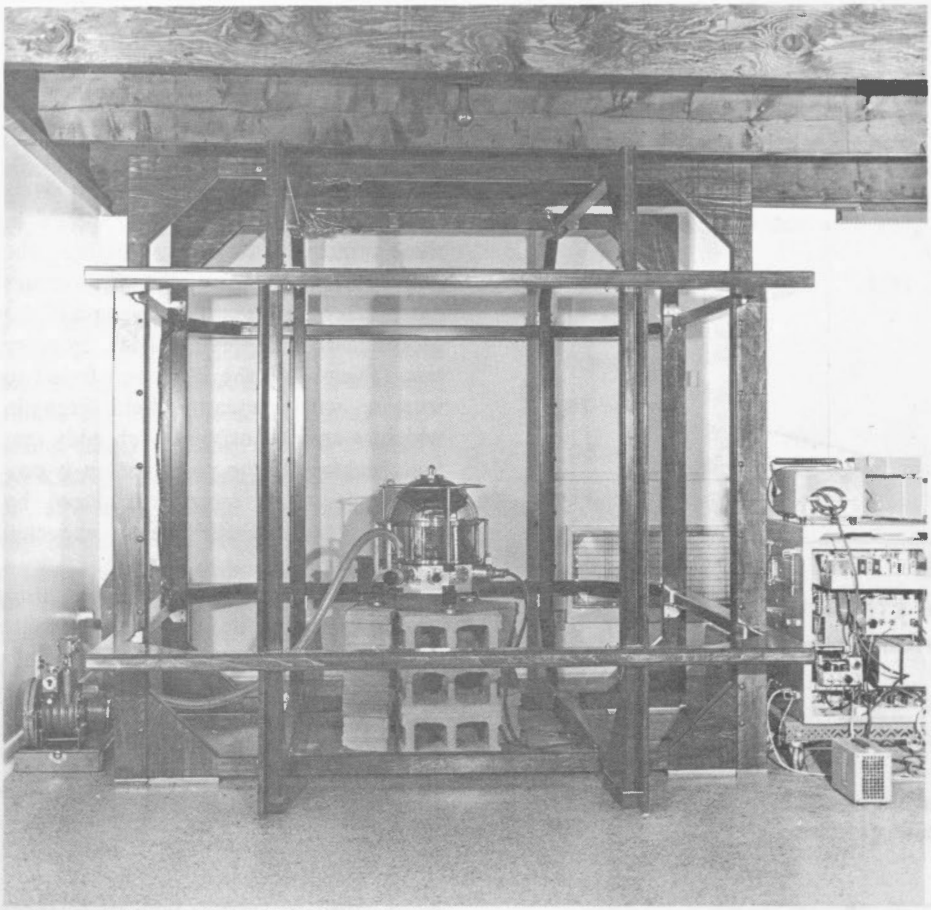

Figure 17. The pendulum apparatus installed in the Helmholtz coil of the Geomagnetic Division of the Earth Physics Branch.

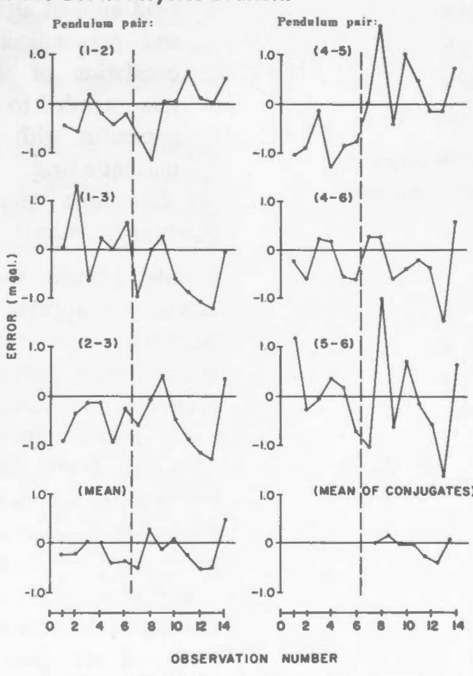

Figure 18. Observed errors at Almonte by pendulum pairs and grand means.

dulum values may be considered as true errors in the pendulum results.

Measurements were made alternately at Ottawa and Almonte. The first set consisting of four measurements at Ottawa and three at Almonte yielded six determinations of Δg. The errors for these measurements by pendulum pairs as well as for the mean of all pendulums are shown in Figure 18. From these results it was discovered that humidity at Almonte was causing a systematic error (Valliant, 1969a). As a result the pendulum storage chamber was desiccated and a second set of measurements yielding eight observations of Δg was made. The change in the characteristics of the error distribution for this second set of measurements may be attributed to the influence of the desiccant which was causing a monotonic change in the pendulum length due to continued drying. It was believed and later confirmed (Valliant, 1969a) that the rate of change in the pendulum's length due to drying was diminishing. As all pendulum measurements are conducted in some well defined pattern, such as ladder sequence, to reduce the effect of long term drift, the mean of an ingoing and outcoming measurement would normally be used to derive a final value for Δg. Thus the means of conjugate pairs for the second set of observations are also shown in Figure 18.

The average error in the mean of conjugate points is −0.06 mgal indicating that no serious systematic errors are present. The higher r.m.s. error of 0.20 mgal reflects the influence of desiccation as mentioned above. As these tests indicated that no systematic error larger than 0.06 milligal and no random errors larger than 0.2 milligal existed complete field trials of the apparatus was begun.

Complete field test. In order to evaluate the performance of the Canadian pendulum apparatus under actual field conditions, measurements were performed at selected sites along the North American Calibration Line (NACL). As the sites have been well connected with gravimeter measurements a comparison between gravimeter and pendulum derived values would indicate the magnitude of random

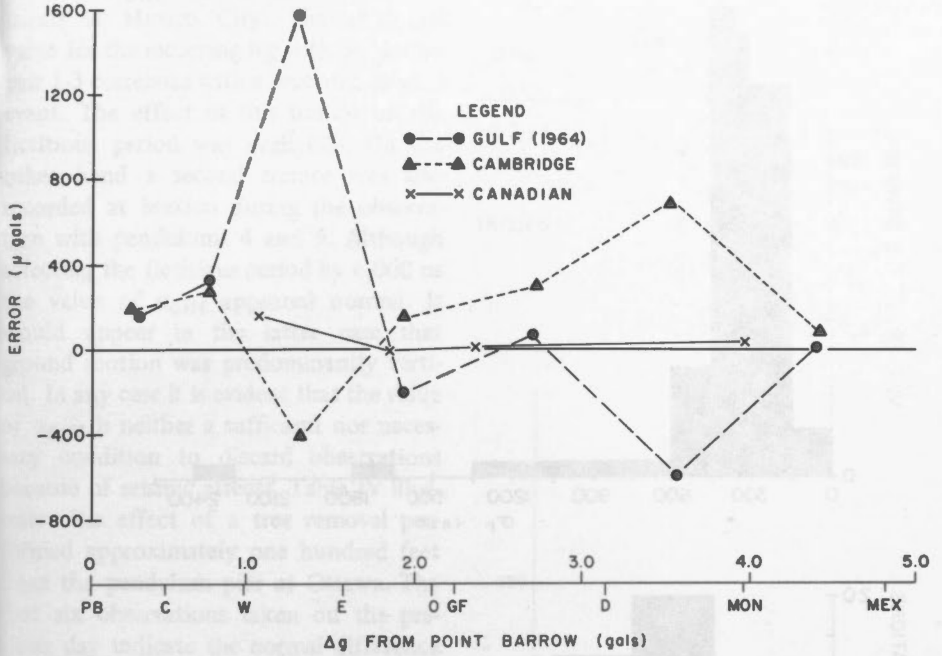

Figure 19. Observed errors for measurements on the North American calibration line compared with errors for the Gulf and Cambridge pendulums.

errors in the pendulum measurements. Also a comparison could be made with the results of measurements made with the Gulf and Cambridge apparatuses which have also been deployed along the NACL. It was further hoped that these measurements if successful, would make a contribution to the establishment of the final gravity values adopted for the NACL.

These results have been fully analyzed elsewhere (Valliant, 1969b). Figure 20 reproduces the comparison between the various pendulum measurements and gravimeter values. In summary it was found that the r.m.s. difference between the pendulum and gravimeter results was 0.08 milligal. As this parameter includes both pendulum and gravimeter errors it can be concluded that pendulum errors did not exceed this value. Indeed the standard deviation of the gravimeter adjustment (0.09 milligal) indicates that both sources contribute about equally to the error distribution and therefore the r.m.s. error of the pendulums alone amounted to about 0.06 milligal (0.08/$\sqrt{2}$). As was expected from the Almonte tests, the effects of continued drying of the pendulums was much diminished. The mean difference between gravimeter and pendulum values amounted to +0.04 milligal which is similar to that found in the Almonte tests. As the gravimeter analysis was based upon the European standard this is equivalent to a difference in scale of approximately 35 ppm between the Canadian pendulum and European standard.

Recently measurements on the NACL with the "Faller laser-interferometer free-fall apparatus" have been completed. Absolute measurements were made at the exact sites of the pendulum observations at College and Denver. The results tabulated below indicate agreement within the estimated error bounds:

***Δg from absolute measurements**
− 2637.24 ±0.07 mgal.

Δg from pendulum measurements
− 2637.38 ±0.10 mgal

Difference
0.14 ±0.12 mgal

***Recently completed measurments from College, Alaska to Ottawa, Ont. combined with gravimeter ties from Ottawa to Boston agree with the absolute Δg from College to Boston to within .04 mgal.**

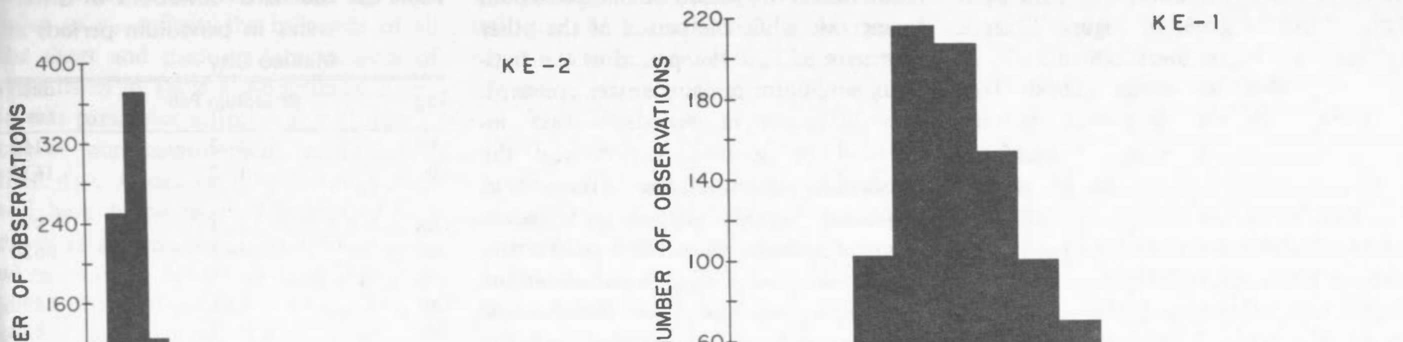

Figure 20. Distribution of σ_s' for knife-edges one and two for NACL results.

With the successful completion of these tests the Canadian pendulum apparatus is considered fully operational for gravity measurements in regions where gravity values are not closely controlled.

Normal test parameters. Several parameters are normally calculated in the complete analysis of pendulum data. Forty-nine measurements of the average period for each observation is obtained by taking all possible differences of the seven initial and final zero-crossing measurements. Their standard deviation σ_S' is a measure of the short term stability and is related to the standard deviation of individual periods by: $\sigma_S' = \sigma_{S/N}$ (where N is the number of oscillations of the pendulum, usually 900). This parameter indicates the proper functioning of the data acquisition system. It also serves as an indicator of sources of malfunction in the binary accumulator, transmission gates, or recorder as a fault associated with a particular binary level produces a well defined value of σ_S'. For example, an error in the 2^{+11} binary level yields a σ_S' in a small range centred around 16,000 ns; a σ_S' of 32,000 ns corresponds to an error in the 2^{+12} binary and $\sigma_S' = 8{,}000$ ns corresponds to an error in the 2^{+10} binary. If the 49 observed periods are listed in a suitable array, the offending binary counter can be quickly determined because it produces an error that affects all elements in either a row or a column. The distribution of σ_S' for both knife edges is given in Figure 20 as a reference for future measurements.

The individual mean periods (i.e. the mean of the 49 average periods for each pendulum) is also determined. This parameter yields information regarding the effect of ground motion and, when pendulums are swung together in sets, a study of individual periods can isolate the offending pendulum in the event of a tare (i.e. sudden unexplicable changes in the length of the pendulum). Note, however, that no significant tares have been observed with the Canadian pendulum apparatus during any of the laboratory tests or field trials.

In the bi-pendulum method horizontal ground motion in the plane of oscillation causes the period of one pendulum to increase while the period of the other decreases so that the period of the fictitious pendulum remains nearly constant. The difference in periods is then influenced by ground motion and the standard deviation of the differences in individual periods (σ_{diff}) is to some extent a measure of stability of the site. Hence the value of σ_{diff} is calculated for each day's observations (i.e. the standard deviation of the difference in periods for the twelve observations made on an observing day). The distribution of σ_{diff} for the NACL measurements is given in Figure 21 for reference purposes.

The influence on σ_{diff} of three isolated events are worth discussing in greater detail. Table III lists the observed

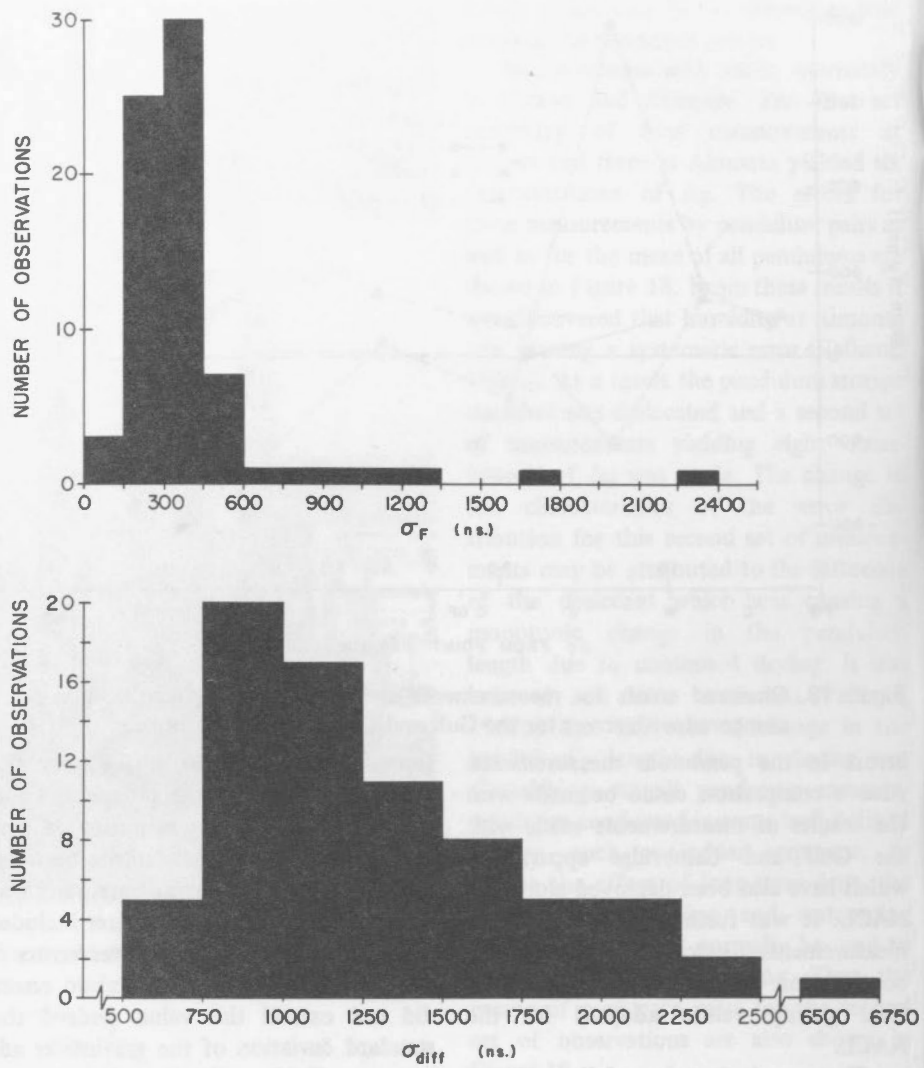

Figure 21. Distributions of σ_F and σ_{diff} for NACL results.

Table III. Standard deviations of differences in pendulum periods at Mexico City

Leg	Pendulum Pair	diff. (ns.)
Out	1–2	1110
In	1–2	1630
Out	1–3	990
In	1–3	6620
Out	2–3	1340
In	2–3	830
Out	4–5	990
In	4–5	980
Out	4–6	1350
In	4–6	1100
Out	5–6	1170
In	5–6	1250

value of σ_{diff} for the NACL measurements at Mexico City. The abnormal value for the incoming leg with pendulum pair 1-3 correlates with a recorded seismic event. The effect of this tremor on the fictitious period was negligible. On the other hand a second tremor was also recorded at Mexico during the observation with pendulums 4 and 5. Although affecting the fictitious period by 6,000 ns the value of σ_{diff} appeared normal. It would appear in the latter case that ground motion was predominantly vertical. In any case it is evident that the value of σ_{diff} is neither a sufficient nor necessary condition to discard observations because of seismic effects. Table IV illustrates the effect of a tree removal performed approximately one hundred feet from the pendulum pier at Ottawa. The first six observations taken on the previous day indicate the normal difference in periods for that particular pair of pendulums. The change in the difference in periods is striking on the second day. The main trunk of the tree was felled during observation 2 accounting for the exceptionally large value of ΔT for this observation. Note that although there was a significant increase in the value of σ_{diff} for all other observations the value of the fictitious period was not disturbed except for observation of 2.

The fictitious periods, the mean of the fictitious periods and their standard deviation (σ_F) is also prepared on a daily basis. The mean of the fictitious periods are used directly in determining Δg. The value of σ_F reflects the influence of all the short and medium term sources of error listed in Table I. Abnormal changes in this parameter reflect a disturbance in one or more measurements made during that day. Occasionally a measurement will have to be rejected because it produces an abnormally high σ_F. Frequently when seismic records are available a rejected observation can be correlated with seismic activity that has not affected σ_{diff} as in the example cited from the NACL observations. The distribution of σ_F for the NACL measurements is also given in Figure 21 for reference purpose.

The correlation between σ_F and σ_{diff} is given in Figure 22. A least squares linear fit is also plotted and a correlation coefficient of 0.16 was determined. Although large values of σ_{diff} indicate the influence of ground motion this parameter is a poor criterion for the rejection of data because of the influence of seismic activity.

Table IV. Effect of tree removal on differences in period

Date	Observation No.	Period Pend. 1 (sec)	Period Pend. 2 (sec)	Fictitious Period (sec)	ΔT (μs)
16/2/66	1	1.0008639	1.0008649	1.0008644	+ .13
	2	1.0008646	1.0008643	1.0008644	- 0.3
	3	1.0008635	1.0008659	1.0008647	+ 2.4
18/2/66	1	1.0008573	1.0008680	1.0008627	+ 10.7
	2	1.0003137	1.0008673	1.0005905	+553.6
	3	1.0008572	1.0008684	1.0008628	+ 11.2
	4	1.0008594	1.0008685	1.0008639	+ 9.1
	5	1.0008580	1.0008689	1.0008634	+ 0.9

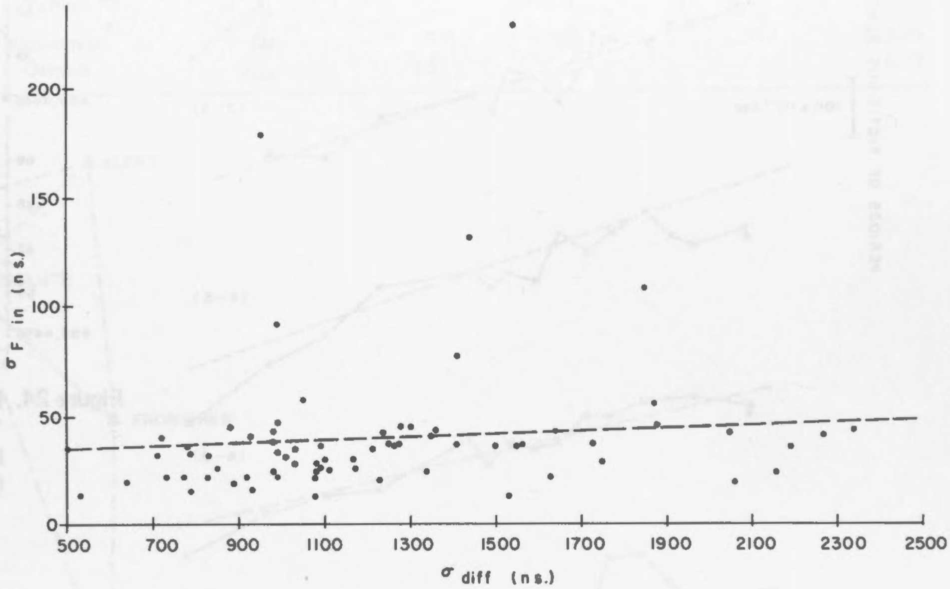

Figure 22. Correlation of σ_F with σ_{diff}. The dotted line represents a least-squares linear fit.

Long term trends in the pendulum periods. Factors affecting the long term characteristics of the distribution of fictitious periods are listed in Table I. All these appear negligible except for the long term drift in the effective length of the pendulums. Drift in the pendulum's apparent length may be due to creep effects, simple mechanical slippage at the joints of their component parts, or secular variation in gravity. Any or all of these factors may contribute to the distribution of fictitious periods shown in Figure 23.

A least squares linear fit to the time curves for each of the pendulum pairs is also shown. The statistics of the straight line fit is given in Table V. As can be seen there is a remarkable degree of linear correlation as well as a remarkable similarity in the slopes of all the curves. The average drift amounts to 3.3 ns per day which is equivalent to a monotonic shortening of the pendulums of about 2.4 ppm per year or in terms of gravity 2.4 mgal per year.

Hamilton (1961) published a similar graph (Figure 24) for pendulums 4, 5 and 6 for the years of 1957 to 1959 inclusive. Unfortunately the introduction of a new environmental control system introduces a singularity in the curves between 1959

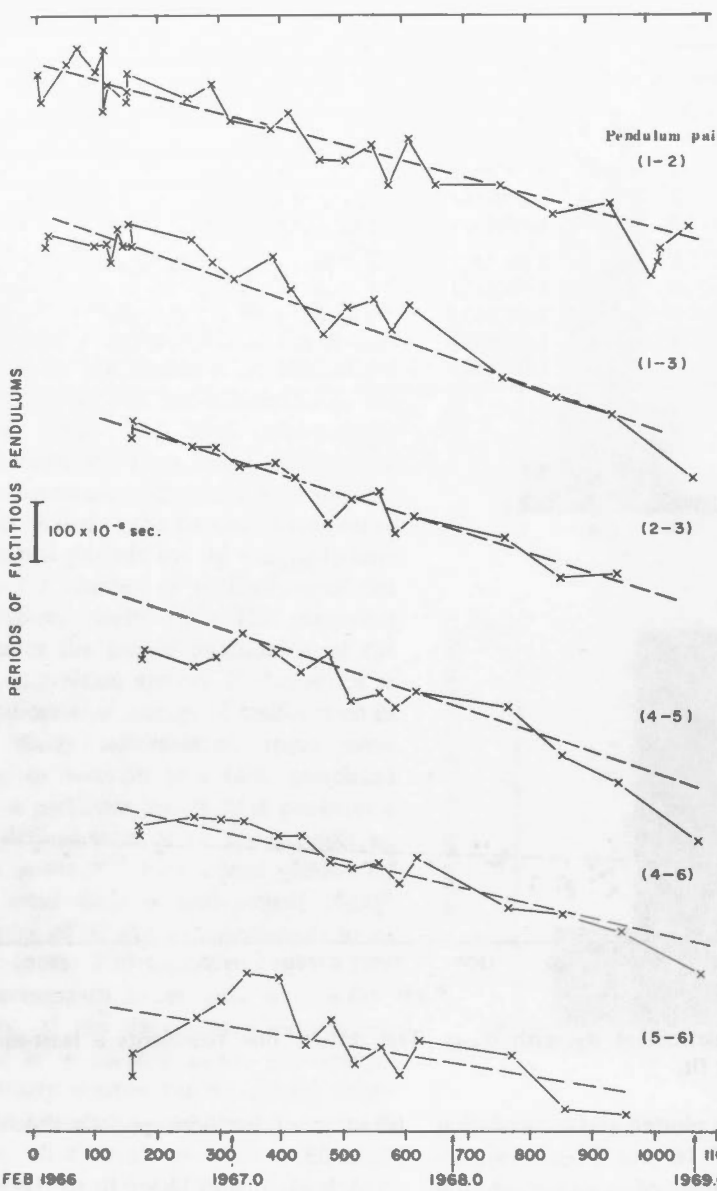

Figure 23. Long term trend in pendulum periods (effective length) from 1966 to 1969. The dotted lines represent a least-squares linear fit.

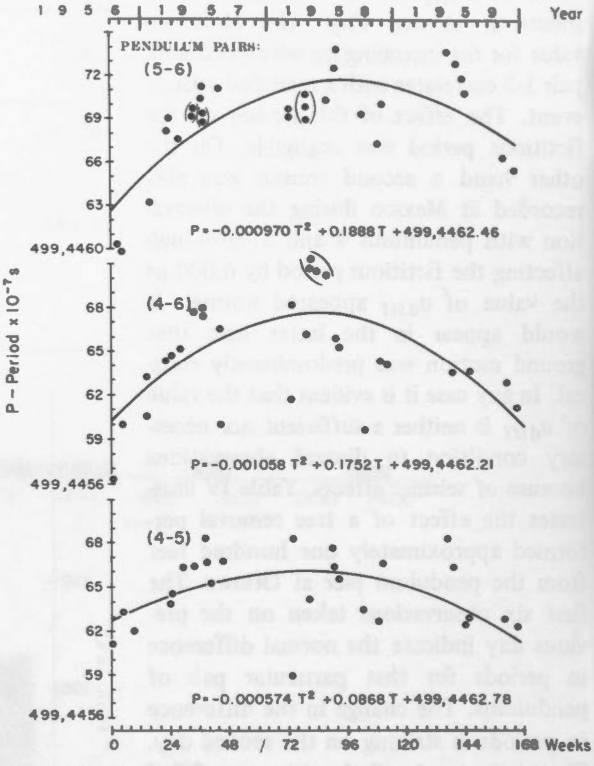

Figure 24. Long term trend in pendulum periods from 1956 to 1959. The dotted lines represent a least-squares second order polynominal fit. (After Hamilton, 1961.)

Table V. Statistics of long term drift analysis

Pendulum	Slope (ns/day)	Correlation Coefficient	Standard Deviation (ns)
1–2	−2.68	−0.92	328
1–3	−3.39	−0.95	303
2–3	−3.34	−0.96	236
4–5	−2.69	−0.91	306
4–6	−2.51	−0.95	208
5–6	−1.73	−0.60	573

and 1966 so that the two portions of the trend curves cannot be compared directly. The earlier data are also subject to large errors because of inadequate temperature control. It is further suggested by Saito (1963) that fitting a quadratic curve to these data is not statistically justified. Nevertheless comparison of the slopes of two sets of curves suggests that the present drift, having the same direction but greater magnitude than the slope in 1959, is a continuation of the negative trend which apparently began in 1958. It is interesting to postulate from these curves that the length of the pendulums (or gravity) is varying cyclically with a period in the order of 40 years and a half amplitude amounting to about 16 ppm in gravity. Notwithstanding the fact that the pendulums are all maintained in the same environment it is a remarkable coincidence that the behaviour of all six pendulums is nearly identical. There is no *a priori* reason to expect that two sets of pendulums which were constructed at different times, subject to widely different early histories, exhibit different magnetic properties, have a slightly different design and period, to exhibit identical long term drift characteristics.

Hysteresis in the thermal expansion curve is one possible explanation of the long term trend. In this case the pendulums do not attain their original length, but are slightly shorter after the temperature has been cycled from warm to cool and back to the original temperature. The pendulums are kept at constant temperature, however, even in their storage container, and the only temperature cycling occurs when they are transferred from storage to the vacuum chamber. The long term trend was also analyzed as a function of observation number (rather than time), and it was found that the fit to linear regression was poorer. This suggests that the drift is more properly considered a time dependent process which excludes thermal hysteresis as a possibility. Also the effect of thermal hysteresis is not in accord with Figure 24. As observations were carried out on a daily basis from 1957 to 1959 one would expect the slope to be greater for this period if hysteresis is the causative factor.

Table VI. Canadian network pendulum results

Interval	Parameter	Outgoing leg (mgal)	Incoming leg (mgal)	Difference (mgal)	Mean (mgal)
Resolute-Yellowknife	$\bar{\Delta g}$	839.54	839.72	0.18	839.63
	$\sigma_{\bar{\Delta g}}$	0.25	0.16		0.15
Yellowknife-Edmonton	$\bar{\Delta g}$	855.90	855.92	0.02	855.91
	$\sigma_{\bar{\Delta g}}$	0.84	0.27		0.44
Edmonton-Vancouver	$\bar{\Delta g}$	232.61	231.98	−0.63	232.29
	$\sigma_{\bar{\Delta g}}$	0.88	0.20		0.45
Winnipeg-Vancouver	$\bar{\Delta g}$	59.94	59.40	−0.54	59.67
	$\sigma_{\bar{\Delta g}}$	0.33	0.31		0.23
Vancouver-Ottawa	$\bar{\Delta g}$	372.89	373.22	0.33	373.05
	$\sigma_{\bar{\Delta g}}$	0.51	0.19		0.27

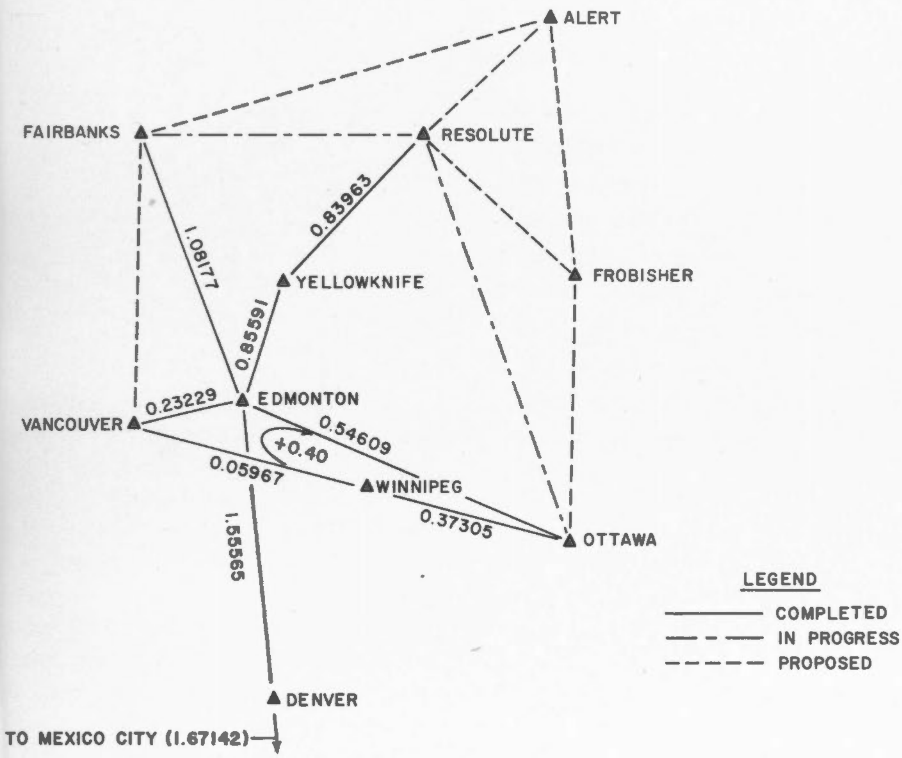

Figure 25. Network of pendulum stations in Canada.

Appendix I

Canadian Network of Pendulum Stations

Pendulum measurements at selected sites according to Figure 25 are being carried out. Measurements between Ottawa, Resolute Bay and Fairbanks were unsatisfactory because of an abnormally high apparent drift rate and are presently being repeated. The results of completed measurements are summarized in Table VI. A complete analysis of these results is expected to provide the subject of a future paper.

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