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PETR VANÍČEK, EDITOR

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**Canadian Geophysical Union
The University of Waterloo
Waterloo, Ontario, Canada
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Foreword

In 1974 the executive of the newly born Canadian Geophysical Union considered satellite application in geodesy and geodynamics significant enough to warrant the particular attention of the Canadian geophysical community. As a result, I was asked to convene a symposium on "Satellite Geodesy and Geodynamics" within the framework of the 2nd annual meeting of the Canadian Geophysical Union. The papers collected in this volume represent the contributions presented to this symposium. The questions and answers given here which pertain to the individual papers purport to have captured only the main points of the discussions that ensued during the meeting.

I should like to take this opportunity to thank the contributors for their cooperation and the promptness with which they complied with the deadlines imposed on them. My thanks are also due to the chairmen of the two subsessions, Mr. H.E. Jones of Surveys and Mapping Branch of the Department of Energy, Mines and Resources, and Dr. W. Cannon, Professor of Geophysics at York University, for their able and efficient conduct of the symposium. Assistance of the many officers of the Geological Association of Canada and the Canadian Geophysical Union who helped in preparing the meeting, and of the Gravity and Geodynamics Division, Earth Physics Branch of the Department of Energy, Mines and Resources, who made the printing possible, is here gratefully acknowledged.

P. Vanícêk
Department of Surveying Engineering
University of New Brunswick
Fredericton, N.B.

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Canadian cooperative satellite programs

A. Hittel*

Abstract. The geodesy community in Canada has successfully carried out major satellite projects by pooling resources from industry, government, university and the private sector. Programs conducted in this manner have recently been recognized for their high level of research and application content. This paper describes the results achieved and the nature of these large scale programs, made possible through sharing equipment and collectively utilizing specialist personnel from all the above sectors during such operations. The paper deals with the methods that were used to create a spirit of cooperation during these projects and outlines considerations that should be taken into account in order to achieve a more effective contribution to Canada's research application goals.

Résumé. Les géodésiens du Canada ont réalisé avec succès d'importants programmes de travaux par satellites avec la mise en commun des ressources de l'industrie, du gouvernement, des universités et du secteur privé. Des programmes réalisés de cette manière ont été récemment reconnus pour les résultats de haut niveau obtenus en recherche et en application. La présente étude décrit ces résultats, outre la nature de ces vastes programmes rendus possibles, grâce au partage de l'équipement et à l'utilisation collective au cours des opérations des spécialistes des secteurs mentionnés. L'auteur analyse les méthodes utilisées pour créer un esprit de coopération durant l'exécution de ces projets et souligne les considérations à prendre en compte pour apporter une contribution plus efficace aux objectifs canadiens en recherche appliquée.

Introduction

In the past 20 years two major scientific events have occurred that will continue to have an impact on the surveying community at large far beyond our present comprehension.

The introduction of highly precise Electronic Distance Measuring (E.D.M.) equipment during the 1950's and the launching of Sputnik II in 1957 has once more awakened the sleeping giant of geodesy.

More recent developments indicate that inertial guidance systems will play an even greater part in the surveying and mapping community.

The declassification in 1967 of the U.S. Navy Navigation Satellite System developed by the Applied Physics Laboratory of John Hopkins University, Baltimore, provided a new challenge not only for the geodetic community but for industry, and government agencies. Artificial orbiting objects in space opened up the possibilities of establishing highly precise coordinates anywhere on the earth's surface.

The Doppler Satellite System provided a clearly defined challenge to a country such as Canada. Our large landmass, together with a continental shelf extending several hundred miles offshore coupled with remote unpopulated areas, lends itself well to space geodesy. At the outset it was realized that both our resources and expertise were too limited, to take on any large scale programs. These limiting factors were successfully overcome by a spirit of cooperation between all interested agencies. This paper described some of the projects undertaken and the results achieved.

Early field Doppler receiver trials

It was during 1968 that Shell Canada Limited and the Bedford Institute of Oceanography acquired the first satellite Doppler system in Canada.

Our knowledge of all aspects of this equipment was indeed limited. The results achieved were often discouraging. The magic black box, which was to be the ultimate in surveying, was often referred to as the "black monster". The project seemed doomed during 1968 but clearer heads prevailed. It was during this era that an exchange of information in all aspects reached its highest level. No other event can draw a handful of people together with such determination than the threat of imminent failure. Early claims were made that the Doppler system would yield accuracies in the order of 1/10 of a nautical mile. Initially we could not achieve this precision. Collection of field data continued in Edmonton, Calgary and along the eastern coast line of Canada during 1968. The results began to improve late in 1968 and seemed to be directly proportional to the knowledge of the user.

It soon became apparent that we were able to improve the results by refining satellite computer programs, together with equipment modifications that could eventually lead us to fulfilling the rigorous objectives initially proposed. One such objective included the position determination within ± 100 metres of geophysical ships underway several hundred miles from shore at various intervals along designated seismic lines.

*Shell Canada Limited, Calgary, Alberta, Canada

Joint programs

By mid-1969 improvements in the Doppler satellite system began to appear on several fronts. Based on the knowledge available, to the end of 1965 from tracking satellites, the Applied Physics Laboratory adopted a reference ellipsoid with equatorial radius 6,378.166 kilometres and flattening 1/298.30. Refinements during 1967 reduced the equatorial radius by 22 metres to 6,378.144 kilometres, and the flattening was altered from 1/298.30 to 1/298.23.

Early in 1969 a new satellite computer program was acquired which would correct for out-of-plane component. This program simply takes into account the lateral movement or deviation from the satellite orbital plane through the poles. Prior to 1969 our satellite program neglected this correction. The deviation of the satellite from the orbital plane would fluctuate between ±0.25 kilometres. The above refinements together with modifications in equipment and computer programs began to produce improved data. The Atlantic Oceanographic Laboratory (Bedford Institute) and Shell Canada Limited worked in harmony sharing equipment, and personnel and comparing data during 1969 and 1970. Experiences with satellite navigation equipment were documented in several reports by D.E. Wells and A. Hittel in 1969. The passes processed in Edmonton were selected from data which show five consecutive Doppler counts centred on either side of closest satellite approach. The R.M.S. error obtained for these passes was of the order of 75 metres.

Major cooperative programs

During 1970, plans were made by the Canadian users, at that time, to conduct a large scale Doppler experimental program. The purpose of this program was to investigate fully the accuracies and applications that could be obtained by using satellite positioning in the orbital, translocation and simultaneous modes.

The navigation group of Bedford Institute, exploration and production of Shell Canada Limited, and the Department of Surveying Engineering of the University of New Brunswick, agreed to join manpower, resources and equipment and operated seven Doppler Satellite Receiver Systems simultaneously during the fall of 1970 (Hittel and Kouba, 1971).

This major undertaking by the above mentioned representatives of government, industry and university led us to new scientific concepts that have only recently been acknowledged.

At the outset, it was agreed that the combination of experience and effort would be of mutual benefit. There was an understanding among the participants that a satellite user country, such as Canada, should play the role of a modern day explorer and experiment with ideas and implement new technology for the benefit of the nation.

Technical achievements as a result of cooperative program

The program led to the development of a rigorous three dimensional satellite program during 1971. Although refinements have been implemented, the basic concept is still valid and in use today.

As a result of our data analysis, computer and receiver modifications were made to permit Doppler counts of 4.6 second intervals rather than the original two minute Doppler count interval.

The complete area of datum transformations was investigated and computer programs were developed that would permit transformation of coordinates readily from earth centre to the reference datum of the nation.

Improvements were made in equipment design leading to a highly portable J.M.R. satellite receiver system which was interfaced to a Wang calculator for on-site positions.

Offshore applications for geophysical purposes could now be accomplished with reliability of position, hundreds of miles offshore, because of interface and computer developments between radio positioning and the Doppler receiver systems.

Present state of art

During the summer of 1974, J.M.R. Instruments Incorporated finally gave birth to a Doppler satellite receiver system that incorporates the present state of the art. The land surveying community had finally been blessed with very nearly the little black box that included features like: world wide coverage, zero error, cost free, completely automatic, almost weightless and surveyor proof.

This new equipment together with computer capabilities was predicted to provide standard deviations of each component i.e. NAD 1927 latitude, longitude and elevation in the order of two metres using the precise ephemeris data.

Recent application of Doppler satellite control 1974 and 1975

Accuracies of the order of ± 1 metre had been successfully obtained on several experimental projects early in 1974. From these data it became apparent that the Doppler satellite system could be used effectively for first and second order surveys. The establishment of control for photogrammetric mapping by satellite now appeared a reality. Three major projects have been completed using Doppler satellite systems in the past twelve months.

1. The Geodetic Survey of Canada conducted an Arctic Island control program establishing some 40 stations by satellite. The preliminary results of this work have been reported by Kouba (1974).

2. The Alberta Government conducted a Doppler satellite control network for photogrammetric mapping establishing control at 24-mile intervals throughout the Tar Sands area.

3. Between November 1974 and May 1975 a major project was undertaken by Shell Canada to provide satellite control for photogrammetric purposes for Shell Gabon in Africa.

It is gratifying to note that accuracies better than two metres were generally obtained with the satellite doppler control network in Fort McMurray area of Alberta during 1974. From 75 metres in 1969 to two metres in 1975 was in our view a remarkable achievement.

The Gabon project was also conducted for the purposes of photogrammetric mapping, however, the accuracy standards were somewhat relaxed compared to (1) and (2). The objective was to place control stations at designated intervals which would produce an RMS no greater than five metres in the X, Y, Z earth-centred components. These precisions were achieved by occupying each station for approximately four days which would produce about 30 to 40 acceptable satellite passes. The geographic field operation was near the equator about 3.0 degrees north latitude.

Future systems on the horizon

The purpose of the foregoing material serves to demonstrate that the Doppler system has now been developed far beyond the initial expectations. Many of the experimental projects played an important role in the refinements of the present Doppler system. The systems that are now on the horizon will require even greater investigation research and development before they can be used operationally. The following three systems are worthy of receiving considerable attention by the surveying community in Canada.

GEOS-C satellite

The launching of this satellite in April 1975 will place us in a much better position to carry out applied research activities because of the instrumentation carried onboard. The laser retroflectors, stable frequency transmissions and radar altimeter instrumentation will provide greater data capabilities and accuracy in the determination of geoid undulations and gravity anomalies. The value of these data to oceanography and geophysical exploration appear enormous when we consider the sparse information presently available on our ocean surfaces.

Inertial guidance and control systems

Position and azimuth determining systems (PADS) developed by Litton Systems, Inc. are just now beginning to receive attention from the surveying community. It appears likely that guidance and control systems will play an important part of the future for a large country such as Canada.

The densification of both vertical and horizontal survey control is urgently needed in order to manage our resources effectively. Inertial systems may provide the means to perform a very important surveying and mapping task for Canada.

Data transmission by satellites

Artificial satellites with large core memories could act as storage banks for geophysical and geological field data being acquired. Collections of field data could be transmitted to such banks and retrieved once the particular satellite was in the vicinity of a computer centre. Such systems would of course greatly reduce the time required between the collection of raw data to the final processed results.

Our effective utilization of such systems depends largely on the degree of participation during the initial research and development for such systems. The cooperative approach is one means of sharing in the initial investigations.

Cooperative research programs

The programs earlier discussed accomplished many of the objectives initially considered. The success was largely because of some of the ingredients which the programs contained.

1. The programs contained clearly defined goals and objectives.
2. There existed a tremendous challenge for participants.
3. The prime investigators had an equal share of the failures and achievements.
4. The participants displayed great determination largely because of a common interest of the program objectives.
5. The programs undertaken were able to respond to a clearly defined need.

There are also some disadvantages that can be identified. These centre about the following areas:

1. New scientific concepts that are developed often are held within the participating representatives longer than necessary.
2. Because there is no Surveying Research Institute in Canada it becomes difficult to disseminate and circulate experimental activities.
3. The scientific geodesist seems to use a conservative approach when publishing experimental results in the area of surveying research in Canada. He often seeks to publish his work in other countries before they are known to the local community.

The need for a surveying research and development institute

A number of the research projects undertaken have made a clear contribution for the Canadian society. Within the programs now on the horizon we predict that future cooperative projects will be undertaken. The momentum already gained by the investigators should be continued although ways and means should be found soon to make these programs more effective for the general surveying community.

At the present time there is no institute in Canada to conduct research and development in the related disciplines of surveying and mapping for purpose of developing new base system concepts which could be commercially implemented by the private sector.

The public and private sectors deserve high praise for their individual in-house contributions to research and development. However, the role of these agencies is often such that they cannot respond to a research need solely for the nations benefit.

A more unified approach by the sectors is desirable on any future programs if the nation is to reap the full benefit. However, even more important is the formulation of a structure in which cooperative programs can best be implemented. The creation of a Geodetic Research and Development Institute would indeed have a far reaching impact on future excellence in Canada. On March 17, 1975 the University of Calgary was presented with a proposal to house a Surveying Research and Development Institute. The University of Calgary has agreed to offer their support in housing such an institute provided cooperation by the various sectors can be obtained.

The institute proposed would be structured on a partnership basis, with participation from the provinces of Canada, federal agencies, private sectors and universities. The enthusiasm and response at this stage is favourable. The institute even during its tender years could no doubt offer considerable coordination in the following areas:

1. The introduction of new information systems.
2. Merging of cadastre with geodetic control.
3. The eventual adoption of an earth-centred coordinate system together with a world reference ellipsoid.
4. Photogrammetry and cartography applications in environmental studies.
5. Data transmission by artificial satellites and remote sensing applications.
6. Determination of gravity anomalies over the oceans.
7. Inertial guidance and control systems for land surveys.
8. Calibration and testing of electronic equipment for the land surveyor.

There are of course many other areas that require attention including laser and electronic instrumentation, mathematical models and the study of ionization layers and their effects on various transmitted frequencies.

Through cooperative programs under the direction of an independent institute, work priorities can be chosen and efforts coordinated to achieve effective results. The institute would also prevent a certain degree of duplication in proposed experimental programs.

Conclusion

The nation's research in surveying and mapping depends largely on obtaining agreements between various agencies towards a common goal. It is firmly believed that these agreements will be made in the best interest of Canada's future. The researchers in Canada have earned a respectable reputation in the area of satellite space geodesy, on an international scale. Cooperate programs have worked successfully but may lack some direction from an overall research aspect for Canada. Methods of conducting large scale experimental research programs have been reviewed. The enthusiasm that now exists among the surveying community on application research in Canada should be encouraged. The level of monetary commitment from federal, provincial and private sectors needs to be increased in order to participate in new programs and sustain present research efforts in surveying and mapping.

References

Hittel, A. and J. Kouba. 1971. Accuracy tests of Doppler satellite positioning in orbital, translocation and simultaneous modes, *13th Congress FIG*, Wiesbaden

Kouba, J. 1974. Reduction of Doppler satellite data observed in Canada. *The Can. Surv. 28, 5*, 480–486.

Discussion

Q: When speaking about the root-mean square error, do you mean self consistency?

A: Yes.

Q: Why is the height component the worst determined?

A: Because of the uncertainty in the geoidal height.

Doppler satellite control in establishing geodetic control networks

J. Kouba*

Abstract. The U.S. Navy Navigation Satellite System was originally conceived for navigation with an accuracy of 50 to 100 metres. A decade later, after some spectacular improvements in satellite geodesy and instrumentation, accuracy exceeding classical geodetic triangulations are routinely achieved. And in the U.S.A. and Canada, Doppler Satellite positioning has been used over the past several years to provide the necessary high precision skeleton for geodetic control networks. Methods and reduction techniques are briefly discussed, as well as the approach(es) taken by the Geodetic Survey of Canada. Some results, and data analysis are also presented.

Résumé. Le système de navigation Doppler par satellites de la US Navy était conçu à l'origine pour la navigation, avec une précision de 50 à 100 mètres. Dix ans plus tard, après certains progrès spectaculaires en instrumentation et en levés géodésiques, une précision supérieure à celle de la triangulation géodésique classique était réalisée. Ces dernières années, aux États-Unis et au Canada, la navigation Doppler par satellites a été utilisée pour fournir le canevas de haute précision nécessaire à l'établissement du réseau de triangulation géodésique. L'auteur résume les techniques et méthodes de réduction et les approches choisies par les Levés géodésiques du Canada. Quelques résultats et analyses de données y sont aussi donnés.

1. Introduction

The U.S. Navy Navigation Satellite System (NNSS) consists of several satellites in near polar, circular orbits and a supporting ground station tracking network. The satellites, equipped with ultra-stable oscillators, transmit two coherent frequencies (approximately 150 to 400 MHz) which are phase modulated to allow transmission of orbital and timing information. A user on the ground is equipped with a portable receiver with another ultra-stable oscillator standard. Measurements consist of accumulated Doppler counts (over usually constant periods of time) of received satellite frequency, and some receiver models also collect broadcast orbital information. The user then can compute the position either by using the broadcast ephemeris, or alternatively using the more accurate historical "precise ephemeris" generated by the U.S. Naval Weapons Laboratory (NWL) [Sims, 1972].

For more details about the NNSS, the reader is referred to [Moffet, 1971].

Results reported in this paper are based on data collected by the Geodetic Survey of Canada during two seasons, 1973 and 1974, as shown in Figures 1 and 2, using three Canadian Marconi receivers models CMA 722A for 1973 Doppler survey (along with one ITT5001) and eight improved models CMA 722B in the 1974 Doppler operation.

The reduction program GEODOP used in this analysis was developed by the author during the period of 1970 to 1974 while with Shell Canada and later with the Geodetic Survey. For description, documentation and history of the development of the program, the reader is referred to [Kouba *et al.*, 1975]. Some earlier results of 1973 Doppler survey were reported earlier in [Peterson, 1974; Kouba *et al.* 1974].

*Gravity and Geodynamics Division, Earth Physics Branch, Department of Energy, Mines and Resources, Ottawa, Ontario, Canada.

2. Methods of satellite positioning

From available satellite orbits one can compute positions on the ground using the integrated Doppler measurements by purely geometrical consideration, analogous to conventional surface hyperbolic navigation systems.

When points are observed and reduced independently, the determinations are usually referred to as *point positioning*. Usually, more than 30 satellite passes are recommended for any one point determinations.

Because of slowly varying system biases in orbit generation and also in observation, point positionings are referred to a mean of the set of observed passes and atmospheric conditions during the relatively short observing time. Consequently, long-term repeatability is considerably greater than the internal noise within the observing period, usually a fraction of a metre for the latter. In order to increase the relative accuracy, more points, usually 2, are observed and processed simultaneously. Such procedure is called *translocation*. Often strict correspondence in satellite passes and Doppler data is required in the hope that both stations will be biased in the same way.

An alternative arises where points are observed at approximately the same time and processed independently as two point positionings. The relative position is then obtained by subtracting the two solutions. Such procedure is referred to as *simultaneous point positioning*. There is a considerable variety of reduction methods (error modelling) and data editing employed for all three methods by different authors, see e.g. DOD, 1972, Wells, 1974, Anderle, 1974(c).

This paper describes a logical combination of the advantages of the above methods, referred to as *multistation solution* which is a unique approach taken by the Geodetic Survey of Canada [Kouba *et al.*, 1974]. The stations are operated independently, similar to point positioning, with

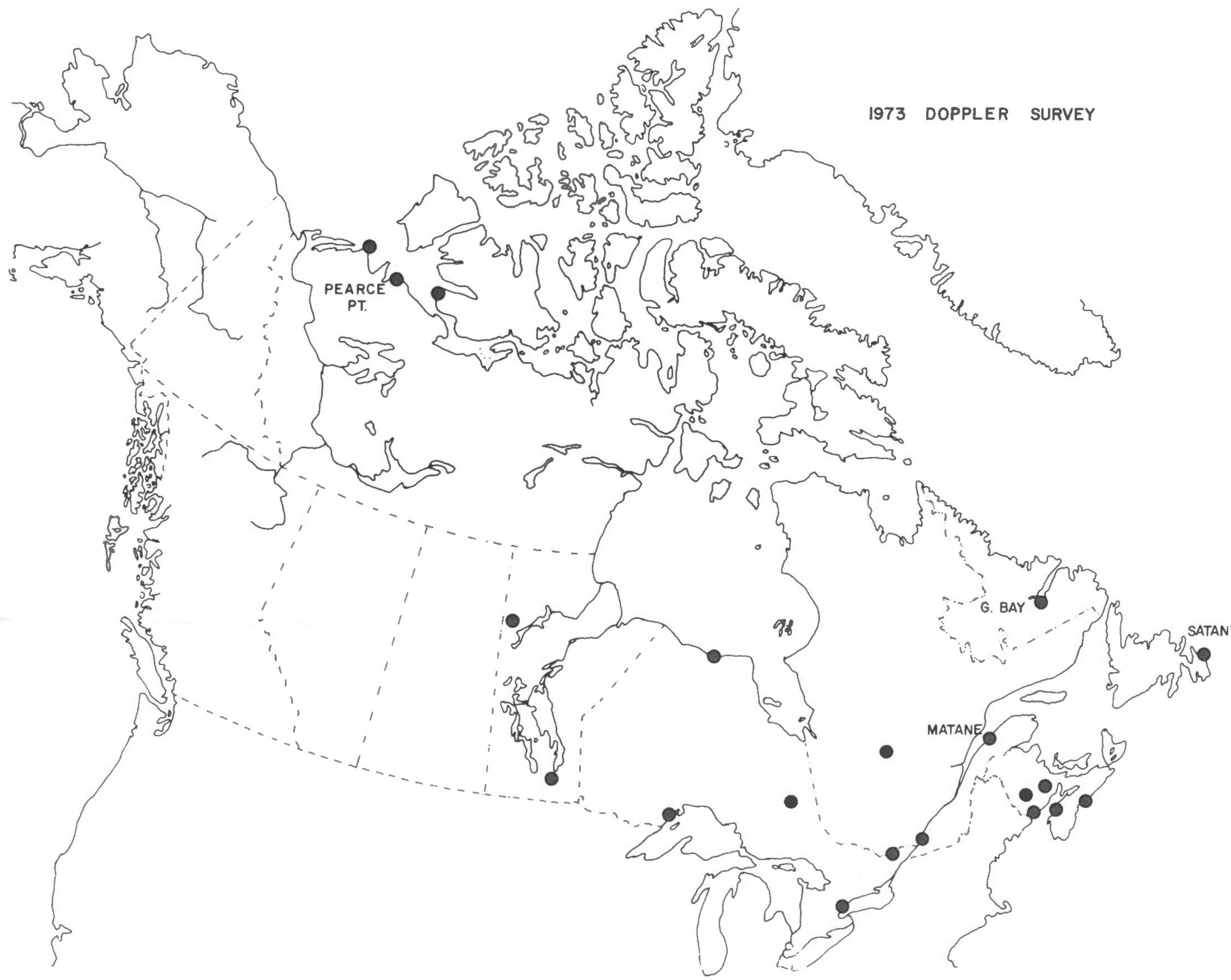


Figure 1

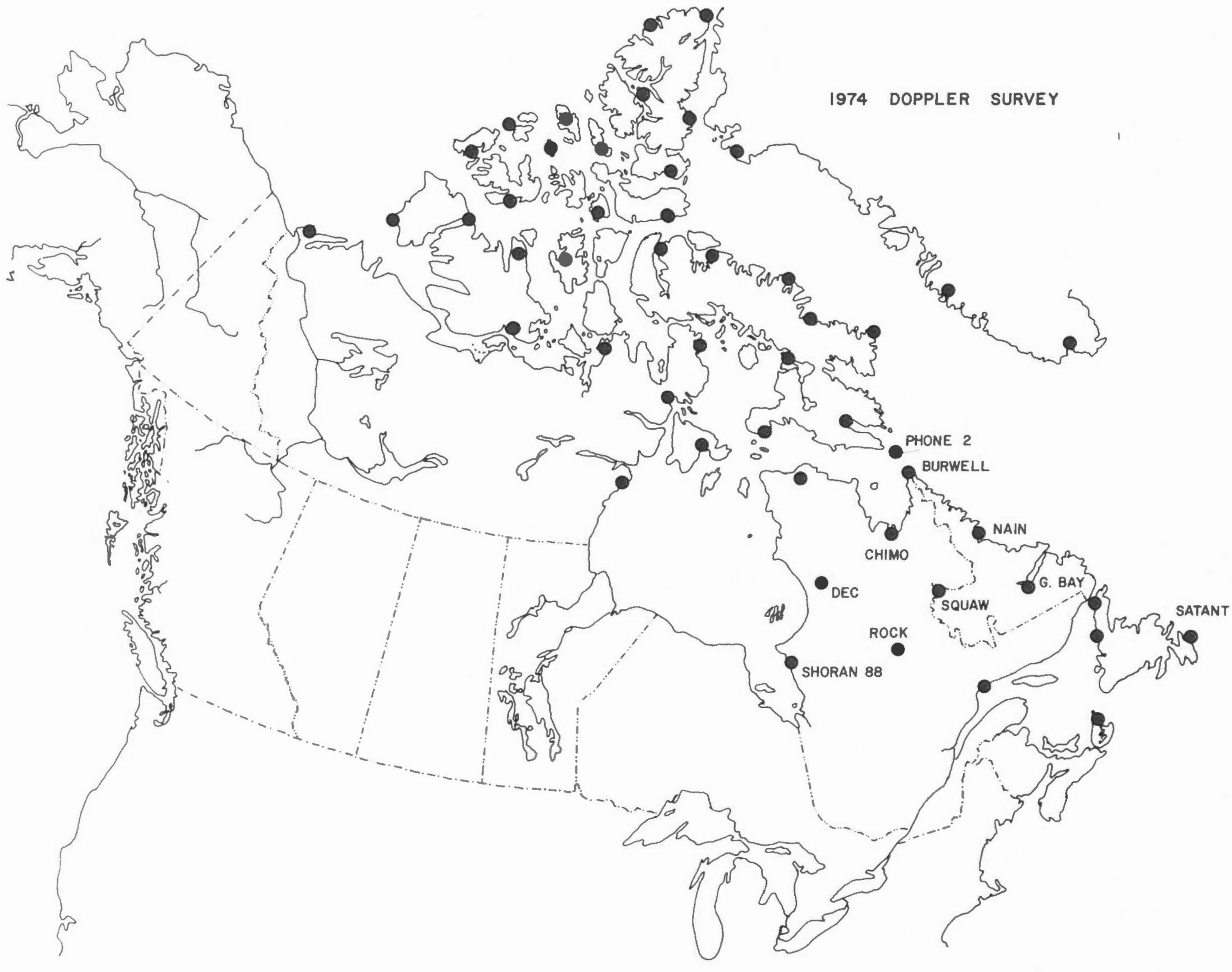

1974 DOPPLER SURVEY
PHONE 2
BURWELL
NAIN
CHIMO
G. BAY
DEC
SQUAW
SATANT
ROCK
SHORAN 88

Figure 2

some overlap, e.g. with one or two stations observing continuously to provide the "link" between the rest of the stations. The stations are then processed simultaneously with the three orbital biases (along, across-track, and out-of-plane) which are common to all stations observing a particular pass. Apart from the orbital biases, each station has three additional biases, frequency-offset, tropospheric refraction scaling, and timing (synchronization) bias. No trimming for the same passes or Doppler data is enforced to retain a sound data balance at each station as in point positioning. Therefore, all stations will only be biased consistently (i.e. small translations, rotations and scale). The relative accuracy will be significantly increased (more than for translocation or simultaneous point positioning) and proper correlation will be preserved.

This approach is somewhat analogous to the short-arc concept [Brown and Trotter, 1969], however, the requirement that all stations are observing simultaneously is not enforced and the satellite orbits which define scale and orientation are obtained from external sources. More likely the multistation solution can be considered an extension of simultaneous point positioning with proper station interaction and subsequent correlation.

The above methods are referred to as either the *broadcast* or *precise* positioning methods depending on the particular type of satellite ephemeris used.

3. Error analysis

The main sources contributing to Doppler position determination uncertainties can be grouped as follows:

(a) Instrument noise (including both receiver and antenna noise).

(b) Atmospheric refractions (ionospheric and tropospheric).

(c) Orbit determination uncertainties.

The *instrument noise* has been significantly reduced from 1-2 m (1σ) in Doppler counts for the navigational equipment to 0.10-0.30 m (1σ) on present geodetic receivers, which employ their own clock. This noise reduction resulted not only in stronger determinations, but also in much finer data editing. At present, position determination with a single satellite orbit is about 3 m (1σ) in each coordinate. As ground positions are based on more than 30 satellite passes, the error contribution is reduced well below the 1 m level. More serious are errors which are constant or slowly varying such as antenna phase centre uncertainty, ground reflection, receiver delay, etc. [Wells, 1972; Kouba, 1975]. These errors, if undetected, can cause errors of up to several metres.

Atmospheric refractions: Both ionospheric and tropospheric refraction irregularities shorter than the Doppler interval will be reflected in the estimated instrument noise. However, constant or slowly varying errors, which can reach up to 1 per cent for ionospheric and 5 per cent for the tropospheric refraction (of the nominal correction) can produce errors of 1-2 metres, mostly in height, for well balanced data.

Orbit uncertainty: Constant shifts (biases) in a satellite orbit are presently estimated at 20 to 30 m (1σ in each coordinate) for broadcast [Wells, 1974] and 1 to 3 m for precise ephemeris [Anderle, 1974a]. Such errors will influence all methods of satellite positioning as the expected value

$$E(r_o) \neq 0 \tag{1}$$

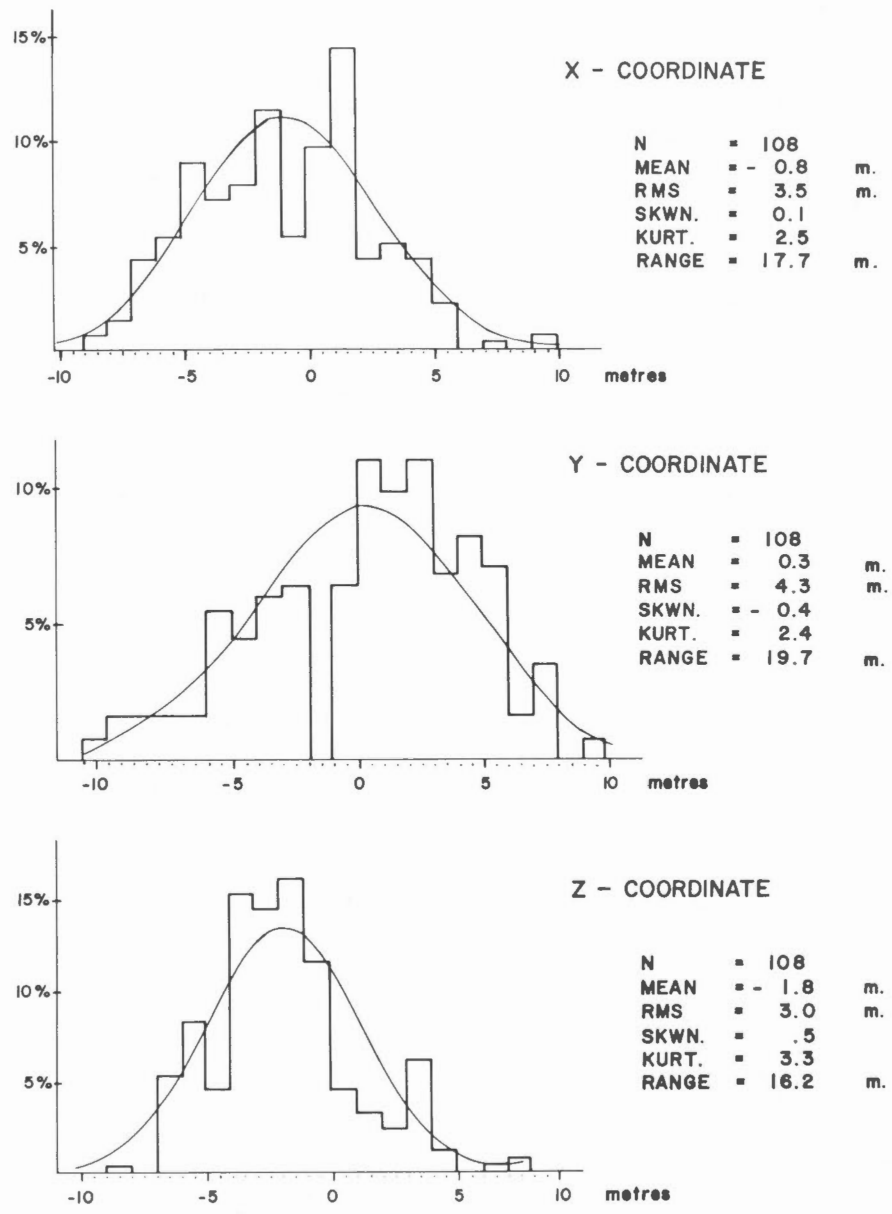
of a set of orbital biases r_o during the observational period will in general not be equal to zero. Thus the least squares solution for the parameters, i.e. station coordinates, will also be biased [Hamilton, 1964]. This is crucial especially for broadcast positioning, where the station(s) can be biased by as much as 10 metres.

The orbital biases are, however, changing to a certain extent during the pass as demonstrated by Anderle *et al.*, 1969 for precise and by Wells, 1974 for broadcast ephemeris. The most significant change is probably in the along-track direction, where it will appear as an error in the satellite speed or the scale of the orbit, which is equivalent. It was observed that the speed (the scale) variations are absorbed by the tropospheric refraction scaling bias, where 1 ppm (on the ground) corresponds to about 3 per cent of the scaling bias. This seems to be logical as tropospheric refraction influences the slant range to the satellite, and so does any scale error. From this point of view, it is advisable that a tropospheric scaling bias of at least 5 per cent (1σ) be included in the solution to account for changes of 1 to 2 ppm in the scale of individual satellite passes and other biases.

Both, tropospheric and orbital bias influences on relative positions are reduced in translocation, simultaneous point positioning and multi-station solutions. Well balanced data (such as for simultaneous point positioning and multi-station method) will decrease any error in latitude (ϕ) and longitude (λ) only.

External accuracy of satellite positioning

1974 Doppler points were processed in both precise and broadcast point positioning. Using a two-minute Doppler count, see Figure 3, both types of point positionings are compared to demonstrate how much broadcast point positioning can be biased. Each determination is from more than 30 passes, with internal standard deviation (1σ) typically less than 1.5 m in each coordinate. External accuracy of the precise point positioning is about 1 m (1σ). Despite these facts the RMS of the comparisons are significantly larger, varying from 3 m in Z to 4.3 m in the Y-coordinate. They can be attributed only to the solution biases discussed earlier. Similarly, for precise point positioning Anderle estimates external accuracies of 1.2 to 1.6 m (1σ in each coordinate) for data spanning several years. For shorter spans (two years and less), the accuracy for precise point positioning is somewhat improved with σ = .60 to 1.0 m. [Anderle, 1974b; Beuglass, 1974].



COMPARISONS OF PRECISE AND BROADCAST

POINT POSITIONINGS (DAYS 138 – 221, 1974)

Figure 3

Despite the σ's of the determination being only fractions of a metre (based on weighted satellite orbit or not) *the external accuracy,* in sense of repeatability over several years, *is not better than 3 metres for broadcast and 0.6 to 1 m* (1σ level in each coordinate) *for precise point positioning.* It is cautioned that the figures for broadcast point positioning may be significantly higher outside North America as they are based mainly on experience obtained from positioning in the Canadian Arctic (*see* Figure 2), relatively close to the four U.S. tracking stations used in the broadcast ephemeris system. A similar situation is discussed by Klosko *et al.*, 1974.

About the same external accuracy applies to translocation, simultaneous point positioning and multi-station solutions. Two or more stations observed during the same period are affected by a translation (with σ approximately the same as for point positioning) as well as small rotations and scale (significant for broadcast solutions only, corresponding to σ ≃ .1" and σ ≃ .6 ppm for rotations and scale, respectively).

Correlation for precise point positioning

Correlation between simultaneous precise point positionings based only on weighting of the satellite orbits (biases) is shown in Figure 4. The results are based on simulations, using data from two stations (about 270 km apart) observing simultaneously with about 75 per cent of common data. Distance, azimuth as well as observation time overlap were altered to produce the desired combinations, some of which are shown in Figure 4 and Table I. In all cases, the usual data deletion below 7.5° and satellite passes below 15° elevation angle were employed, causing a decrease of data for larger distances. Receiver noise of σ = 0.15 m, uncorrelated weighting of Doppler counts together with orbit σ's of 3 m, 1.2 m, and 3 m along-track, across-track and out-of-plane were assumed.

Apart from dependence on distance and azimuth, which is well known, the correlation is also a function of receiver noise (i.e. weights assigned either to Doppler counts or orbits) as well as on time (data) overlap. In Table I, correlation for different amounts of time overlap are listed for a distance of 1,000 km and the two critical directions, α = 0° and α = 90°. In the north-south direction there is about 70 per cent common data and for east-west only 50 per cent of common data for the same time periods. For smaller time overlaps, the percentage of common data is proportionately lower.

The correlation is only very slowly decreasing with reduced overlapping of the observing period, indicating that a small number of different passes, or Doppler data will hardly change the correlation, a fact utilized in the multi-station solution. Another interesting observation from Table I is that the correlation matrices are nearly symmetric, resembling in pattern the X, Y, Z station correlation. This suggests that the prevailing bias will be a nearly constant translation for both stations. Another interesting point is that while correlation coefficients up to 0.3 between X, Y, Z at a station can be observed, the φ,λ,h are virtually uncorrelated, with correlation less than 0.1, reflecting perhaps the fact that NNSS satellite are in polar orbits. Correlation is decreased when orbits are weighted according to σ's of 1.5 m, .6 m and 1.5 m for the three directions, from 0.7 to about 0.4 maximum. For broadcast point positioning (with σ's of 26, 5 and 10 m in the three orbital biases respectively), the correlation is much higher, namely from 0.80 to 0.97.

With satellite timing (BR) data, receiver σ≃ 1 to 2 m, the precise simultaneous point positioning is virtually uncorrelated (0.04) and simultaneous broadcast point positionings have correlation coefficients of not more than 0.7 in each coordinate.

Correlation of error model parameters

The preceding analysis does include likely the most significant source of correlation, namely constant orbital biases. And thus the correlation discussed is only true as far as the model, including the Doppler weighting, is representative of the actual situation, It is natural to expect that orbit biases are *not* independent (uncorrelated) between passes, although they have been assumed independent in the earlier data processing, since the orbits are results of least squares fits, subject to an adopted force model, over periods of up to 48 hours.

Correlation between passes is due to drag, pole positions and other parameters, as well as a truncated gravity field. None of these are considered to be significant with respect to the magnitude of the orbital biases, 2 to 3 m. [Anderle, 1975, pers. comm.]. To verify this, a simple analysis was done on orbital biases obtained from precise multi-station solutions. The correlation coefficients were calculated using only solutions for the biases from at least three observing stations.

TABLE I

Correlation Coefficients for Simultaneous Precise Point Positionings (σalong = 3; σacross = 1.2 and σout = 3.m; receiver σ = .15m.)

Per cent of time overlap Distance s and Azimuth α		100 per cent of time overlap			60 per cent time overlap			30 per cent time overlap		
		x	y	z	x	y	z	x	y	z
s = 1000 km.	x	.64	.11	−.09	.57	.10	−.08	.50	.09	−.07
α = 0° (180°)	y	.12	.69	.23	.11	.62	.22	.10	.57	.19
	z	−.15	.38	.72	−.13	.34	.65	−.12	.32	.58
s = 1000 km.	x	.39	.04	−.20	.34	.04	−.17	.28	.03	−.15
α = 90° (270°)	y	.10	.50	.40	.08	.44	.36	.07	.40	.32
	z	−.19	.35	.58	−.16	.32	.52	−.14	.28	.46

CORRELATION COEFFICIENTS

PRECISE POINT POSITIONING

(100% time overlap ; 75% common data at S = 0 km. ; ϕ = 45° λ = 297°, orbits weighted 3, 1.2 and 3 m. for along, across and out of plane)

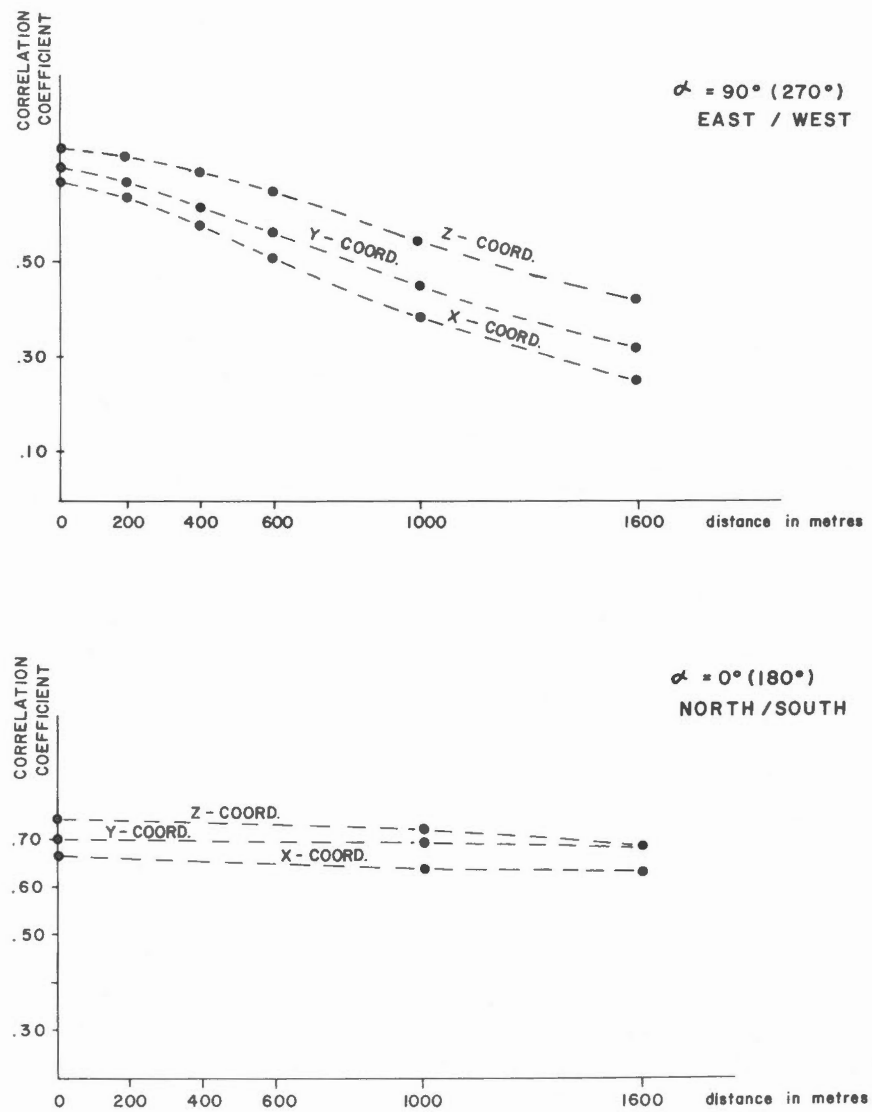


Figure 4

TABLE II

Correlation Coefficients for Orbital Biases With Time

Orbital bias	A priori σ (m)	n	mean (m)	σ (m)	Correlation coefficient for Δt 110 min	σ	220 min	σ	330 min	σ
Along-track	3.0	52	.27	1.11	.10	.19	.21	.21	−.39	.27
Across-track	1.5	65	.04	.35	−.40	.19	−.06	.23	−.06	.37
Out-of-plane	3.0	57	−.19	1.78	−.03	.20	.11	.21	−.09	.33

The correlation coefficients listed in Table II, were calculated as

$$\rho_{ij} = \frac{1}{n} \sum^{n} X_i X_j ;$$

where n pairs of (X_i, X_j) are either one, two or three orbital periods apart (i.e. 110, 220 or 330 min.).

Also shown are statistical limits σ of the determination obtained as

$$\sigma_\rho = \left(\frac{1-\rho^2}{n-2}\right)^{\frac{1}{2}}$$

When a correlation coefficient exceeds these limits, it can be considered statistically different from zero. Only two values $\rho_{110} = .4$ for across and $\rho_{330} = -.39$ for along-track, exceeded the above significance limits, though 0.4 in itself is usually not considered a significant correlation [Hamilton, 1964]. Also calculated in Table II are the means and standard deviations of the biases, the latter being approximately 1/2 of the a priori σ's.

Similarities of vertical profiles, both dry and wet, between stations up to several hundred km apart, as well as the fact that orbital systematic effects are absorbed into the scaling bias for tropospheric refraction, suggest that this bias parameter should also be considered correlated between stations (a function of distance). For similar reasons, the scaling bias will likely be correlated between successive passes (with time). A separate study is required to investigate the significance of this correlation on Doppler positioning.

4. Discussion of results

1973 Doppler survey

The 1973 Doppler survey was conducted with navigation-type receivers (Marconi BR data mode) in southern Canada as shown in Figure 1. Results of the extensive comparisons with freely adjusted Canadian triangulation were reported earlier in Peterson, 1974; Kouba *et al.*, 1974. In Figure 5, histogram of distance comparisons between the 1973 precise point positionings and the adjusted triangulation [Pinch, 1974] is shown in ppm for distances over 300 km. The Doppler points were scaled by −1.1 ppm as suggested by Anderle, 1974(a) prior to the comparisons. The agreement with RMS of 2.5 ppm is considered realistic when both Doppler and triangulation distance standard deviations are taken into account. The negative mean of −.6 ppm ± .3 ppm is likely because of the tellurometer scale control since that instrument tends to measure short in warm and humid regions [Jones, 1974]. The rotation around the Z-axis between the triangulation and the Doppler reported in Kouba *et al.*, 1974 of about 0.4 to 0.6" has not been explained yet but has given impetus to a detailed study into early astronomical azimuth determinations in Canada.

In order to verify that the results using satellite timing are compatible with results obtained from geodetic receivers, which employ their own clock (Marconi CBR data), and also to provide an overlap between seasons to assure accuracy in relative positions, four points were re-observed. The points are named in Figure 1. The results are tabulated in Table III, and are similar in magnitude to repeated positionings obtained at station Ottawa using both types of data [Kouba, 1974; Kouba *et al.*, 1974]. Thus the repeatability obtained using either the satellite timing (BR) data or receiver timing is approximately equal to geoceiver (receiver timing data) repeatability, namely 1.5 m with 90 per cent probability in each coordinate, as reported in DOD, 1972. However, for satellite timing (ITT5500) data the repeatability given in the preceding reference is doubled, to 3 m with 90 per cent probability level, which gives an indication of the older design of the ITT equipment.

TABLE III

Repeatability of Precise Point Positionings Using BR and CBR (2 min) Doppler Data

Station Name	No. of passes 1973	1974	Precise point positioning: 1974 value (CBR data) − 1973 value (BR data) Δφ(m)	Δλ(m)	Δh (m)
Pearce Pt.	93	62	− .62	−1.65	.24
Matane	270	54	.19	− .16	−1.49
G. Bay	167	144	+ .09	− .89	.74
Sat. Ant	166	84	+ .43	− .56	− .64
Mean			.02 m	+.82 m	− .28 m
RMS			.45 m	.63 m	.98 m

DISTANCE COMPARISONS (> 300 KM.)
TRIANGULATION — TWO MIN. PRECISE POINT
POSITIONING (1973 DOPPLER BR DATA)

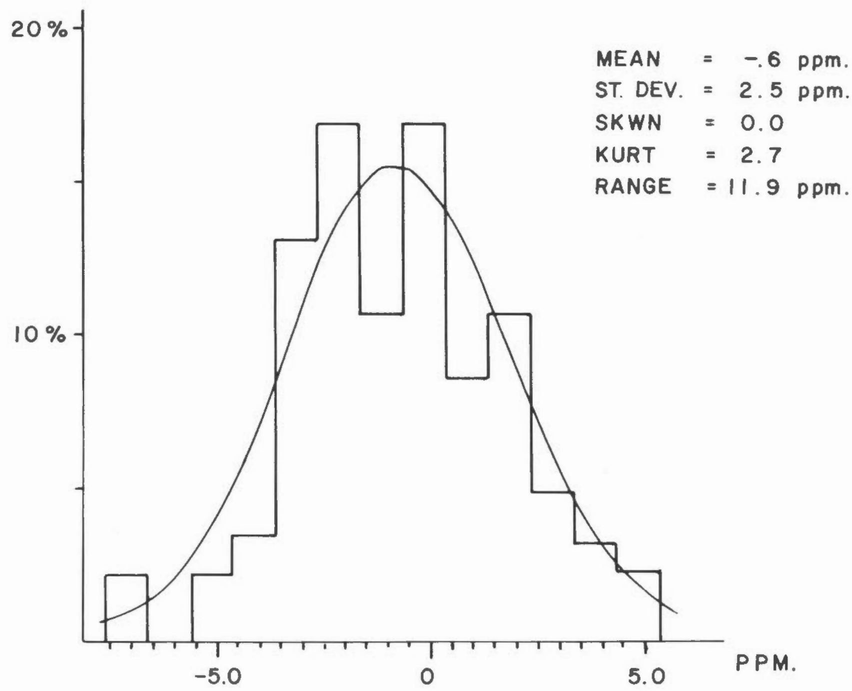


Figure 5

1974 Doppler survey

Altogether 52 points were observed using geodetic receivers (Marconi CMA 722B) in the receiver timing data collection mode (CBR). All the points were grouped into figures of 10 to 15 stations each for final processing to reflect the high degree of time overlap, with one or two stations being common to adjacent figures to provide links for increasing relative accuracy among all 1974 Doppler positions.

Initially, all the points were reduced independently in precise point positioning mode. Estimated receiver σ's (i.e. Doppler σ) from the point positioning processing for each receiver are shown in Figure 6, where each dot is assigned to the mean of the observing period at a particular station. One can see that with the exception of the two receivers, #110 and #103, which were later found to be in faulty operating condition, the receiver σ's are grouped within 0.10 and 0.30 m, with a distinctively different but constant σ for a particular receiver.

For the multi-station processing, the broadcast orbits were weighted according to σ's of 26, 5, 10 m for the along-track, across-track and out-of-plane directions. The precise orbits were weighted according to σ's of 1.5, .6 and 1.5 m in the three directions. The timing and the tropospheric refraction scaling biases were weighted with σ's of 100 μ sec and 10 per cent respectively for both broadcast and precise runs. A smoothing developed by Wells, 1974 was used for broadcast orbits.

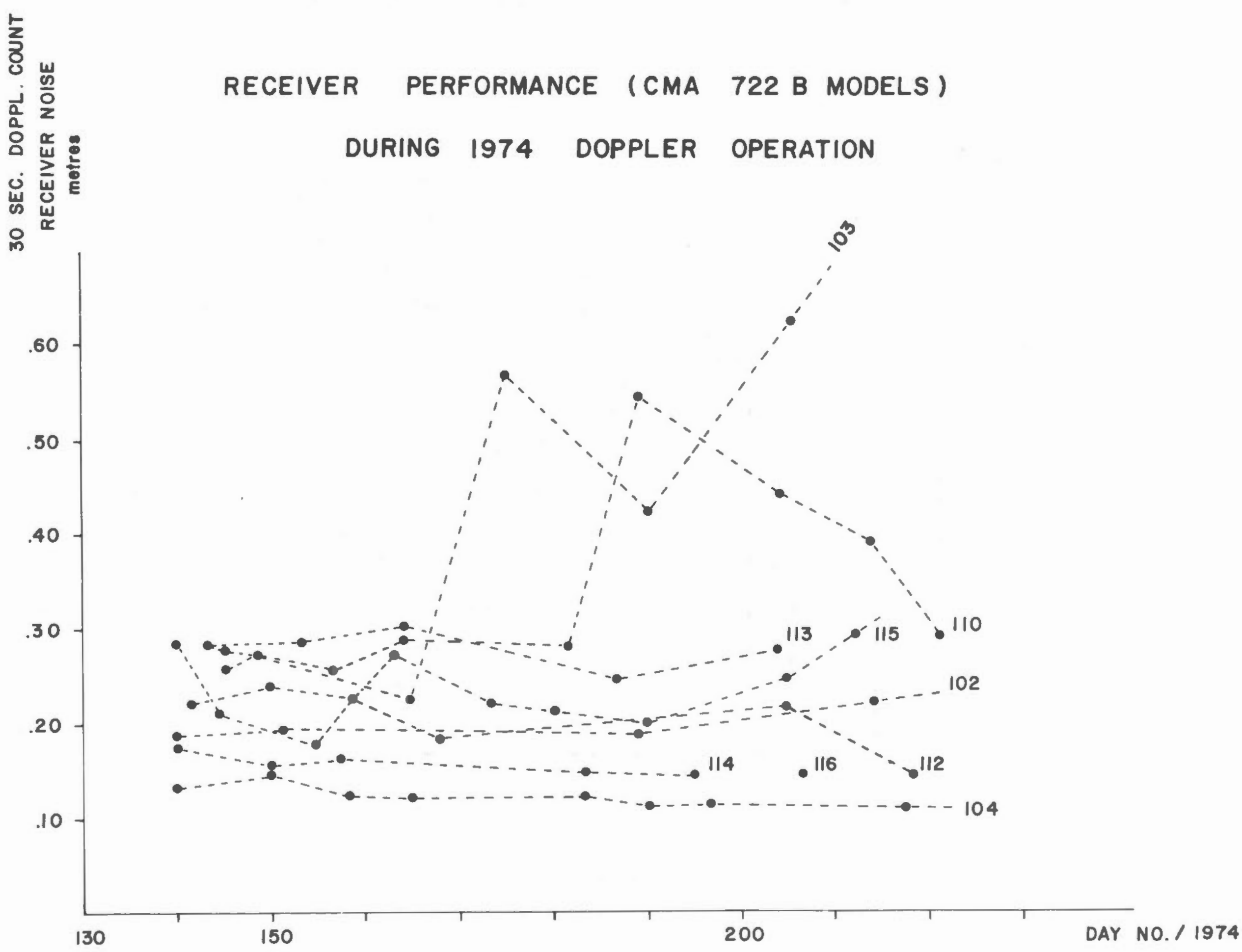


Figure 6

In Table IV, differences between 2-min. precise point positioning (APL Sat. 13, 19) and broadcast multi-station solutions broken into 5 two-day periods (using only Sat. 12, 14, 18, 20) are shown. Observing patterns (also used for precise point positionings) and data used in broadcast multi-station runs are shown in Figure 7. Short, 2-day periods used for broadcast processing were chosen to maximize relative accuracy and to minimize the computation time (number of actually observing stations), as program GEODOP [Kouba *et al.*, 1975] is designed to perform a least squares solution for only designated active stations. Minimum requirement set was that a station had to be observing 50 per cent of the time or more during any 2-day period to be considered active.

Both differences and RMS shown in Table IV include possible errors in precise point positioning. In Table V, the results are grouped according to stations to find the internal consistency among 2-day broadcast multi-station determination. The weighted mean RMS of a single 2-day determination (according to the number of comparisons and σ of a 30 sec Doppler count) is .55 m, .83 m, and 1.23 m in for φ,γ and h respectively, with weighted mean Doppler σ of .18 m. The RMS values of col. 4, 6 and 8 represent the precise point positioning result which is within the expected limits of 0.6 and 1 m [Beuglass, 1974; Anderle, 1974(b)].

In Figure 8, the mean differences in ϕ and λ from Table V, as well as circles with radius equal to the combined RMS of ϕ and λ of individual 2-day period determinations (Table V, col. 11) are shown. In Figure 9, the RMS values from Table V are compared with receiver noise. Very strong dependence is observed for ϕ, less for λ and h which may signify that those are also dependent on the distance.

To demonstrate a possible use of both broadcast and precise multi-station determinations, the different period or satellite solutions were adjusted separately according to variance-covariance matrices augmented in the following way:

$$\Sigma = \Sigma_g + \begin{bmatrix} \Sigma_b & \Sigma_b & \cdots & \Sigma_b \\ \Sigma_b & \Sigma_b & & \\ \vdots & & \ddots & \\ \Sigma_b & & & \Sigma_b \end{bmatrix}$$

to reflect a possible constant shift with variance Σ_b for all stations during the current period with the internal covariance matrix Σ_g estimated by program GEODOP, as discussed earlier. The Σ_b used was a diagonal (3 by 3) matrix with 1 m^2 along the main diagonal assigned to each satellite for precise and 16 m^2 for each 2-day broadcast multi-station solution. Strictly speaking, a 7-parameter transformation should have been used. The 3D adjustment program GDLSAT used is described in Peterson, 1974.

The estimated variance factors were 2.5 and 2.6 m^2 for precise or broadcast adjustment respectively, significantly larger than the expected value of 1.0. This may indicate that either the relative accuracy as estimated by GEODOP for each 2-day period, or satellite for precise run, may be optimistic. Or alternately, it may be the result of the two malfunctioning

TABLE IV

Two-min Precise Point Positioning (Sat. 13, 19) Minus (2 Day) Multi-station Broadcast Solution (Sat. 12, 14, 18, 20)

(Geodetic Survey of Canada Doppler 1974, Figure North Quebec)

Station	No. of Passes	Days 200–201 metres			No. of passes	202–203 metres			No. of passes	204–205 metres			No. of passes	206–207 metres			No. of passes	208–209 metres		
		$\Delta\phi$	$\Delta\lambda$	Δh		$\Delta\phi$	$\Delta\lambda$	Δh		$\Delta\phi$	$\Delta\lambda$	Δh		$\Delta\phi$	$\Delta\lambda$	Δh		$\Delta\phi$	$\Delta\lambda$	Δh
Precise - broadcast		.4	−1.0	−1.2		+2.6	2.2	−1.4		.4	−1.2	−4.9		− .7	.9	−5.6		4.4	1.7	−4.1
Burwell					17	.3	1.1	− .5	23	.1	1.4	.2	28	− .5	1.1	−1.3	28	.2	.5	−.4
Dec	24	.0	− .8	− .2	26	−1.1	− .4	−1.4	22	.2	−1.1	1.5	17	1.1	− .1	1.3				
Nain	24	−.1	+ .1	+ .7	24	− .8	− .3	2.2	19	1.4	−1.2	2.1	24	− .5	− .6	− .5				
G. Bay																	22	−1.1	.3	.2
Chimo 2	35	.6	− .1	− .9	31	.7	− .4	− .6	26	− .1	.2	.3								
Phone 2					16	− .7	+ .4	+1.3	21	− .4	− .9	−1.0	29	− .5	2.1	− .8	28	.0	.5	.4
Rock					29	+ .7	+ .9	− .4	20	−1.6	.7	−3.3	22	.5	− .5	1.1				
Satant																	21	.6	− .6	1.2
Shoran	30	−.3	.6	+1.5	18	−1.3	− .4	−1.2												
Squaw	16	−.3	.3	−1.1	24	.7	− .9	+ .4	23	.5	1.1	.1	20	− .2	− .2	.2	20	.2	− .6	−1.4
Mean		.1	.1	.0		− .1	0.0	− .2		− .1	.2	− .1		− .1	0.0	0.0		− .1	.1	.0
RMS		.4	.5	1.1		.9	.7	1.2		.9	1.1	1.8		.7	1.4	1.1		.4	.6	1.0

TABLE V
Summary of Two-min Precise Point Positioning (Sat. 13 and 19) Versus 2-day, Multi-station Broadcast Solution (Sat. 12, 14, 18, 20)

Station	Doppler σ * (metres)	No. of 2 day periods	φ(metres)		λ (metres)		h (metres)		2D RMS (φ, λ) (metres)	
			mean	σ	mean	σ	mean	σ	around 0	around mean
1	2	3	4	5	6	7	8	9	10	11
Precise-broadcast shift		5	+1.42	2.05	.52	1.55	-3.44	2.02	4.01	2.57
Burwell	.18	4	- .25	.36	1.02	.38	- .50	.62	1.32	.52
Dec	.23	4	.50	.90	- .60	.44	.30	1.36	1.34	1.00
Nain	.38	4	.00	.98	- .50	.55	1.12	1.28	1.26	1.12
G. Bay	.12	1							1.14	
Chimo 2	.16	3	.40	.43	- .10	.30	- .40	.62	.72	.52
Phone 2	.12	4	- .40	.29	.53	1.23	- .25	1.08	1.47	1.26
Rock	.52	3	-1.33	1.27	1.17	.98	- .86	2.24	2.70	1.60
Satant	.16	1							.84	
Shoran 88	.10	2	- .80	.71	.10	.71	.15	1.91	1.00	1.51
Squaw	.16	5	.18	.43	- .60	.79	- .36	.82	.89	.90
Mean			- .21		.11		- .10			
RMS	.18		.62	.55**	.61	.83**	.61	1.23**	1.27	1.00**

*30 second Dopplers.
**Weighted according to Doppler σ and number of comparisons, unweighted RMS = .67, .67, 1.24 metres for φ, λ, and h.

receivers (#110 and #103) causing one large discrepancy at station Nain (about 3 m) even for each satellite of precise determinations, which is rather unusual. However, when adjusted broadcast and precise solutions were combined in a final adjustment, the variance factor of 0.92 m^2 was obtained, easily passing a Chi-square statistical test.

The results are summarized for both broadcast and precise adjustments in Table VI, where station Squaw was held fixed to demonstrate relative accuracy within the figure. Even for single 2-day broadcast periods, the σ's in φ, λ and h compare well with precise values, and for several 2-day periods the σ's are equal to or better than corresponding precise σ's.

The differences in φ, λ and h between the two adjustments are small, in most cases less than standard deviations of either determination. A large height difference at Nain is probably due to the troublesome receiver at that station. The scale factor of -.86 parts in 10^6 (ppm) was obtained as a parameter of the final adjustment combining precise and broadcast which is in agreement with mean $\Delta h = -3.44$ m obtained in Table V. An improvement in estimated σ's by a factor of $\sqrt{2}$ was observed.

The last comparison, Table VII, is for distances originating from fixed station Squaw, to demonstrate that the broadcast multi-station solution is equivalent to the precise one. Agreement in distances is remarkably good, with respect to σ's of distances. (Note that the broadcast values were scaled down by -.86 ppm.) Clearly both are dependent on receiver noise and degree of overlap. And even for as few as 22 or 21 passes the σ's are 1.1 and 2.5 ppm. The longest distance, Satant – Shoran 88, of nearly 2,000 km compares very well with the precisely determined one, despite the fact that only 21 and 22 passes at each end were observed and at different times, as listed below: (refer to Figure 7)

Shoran 88 – Satant

broadcast	1 962 436.2 m ±1.6 m
precise	1 962 436.8 m ± .6 m
difference	-.6 m

Similar good agreement was observed for 12 other distances between 1,000 km and 1,800 km, where the largest difference was 1.2 m or 1.1 ppm.

The only comparison between a 1973 Doppler determination, triangulation, and a 1974 Doppler determination was possible for Goose Bay – Satant line. The individual values are listed below:

Goose Bay – Satant

	from:
1973 precise	837 567.1 m ± 1.3 m [Peterson, 1974]
Triangulation	837 565.8 m ± 1.7 m [Pinch, 1974]
1974 broadcast	837 566.7 m ± 1.1 m
1974 precise	837 566.9 m ± .5 m

where all Doppler values were scaled by -1.1 ppm (i.e. broadcast -1.96 ppm), as suggested by Anderle *et al.*, 1974(a). This is a quite close agreement, especially for all Doppler determinations. When both precise and broadcast adjustments

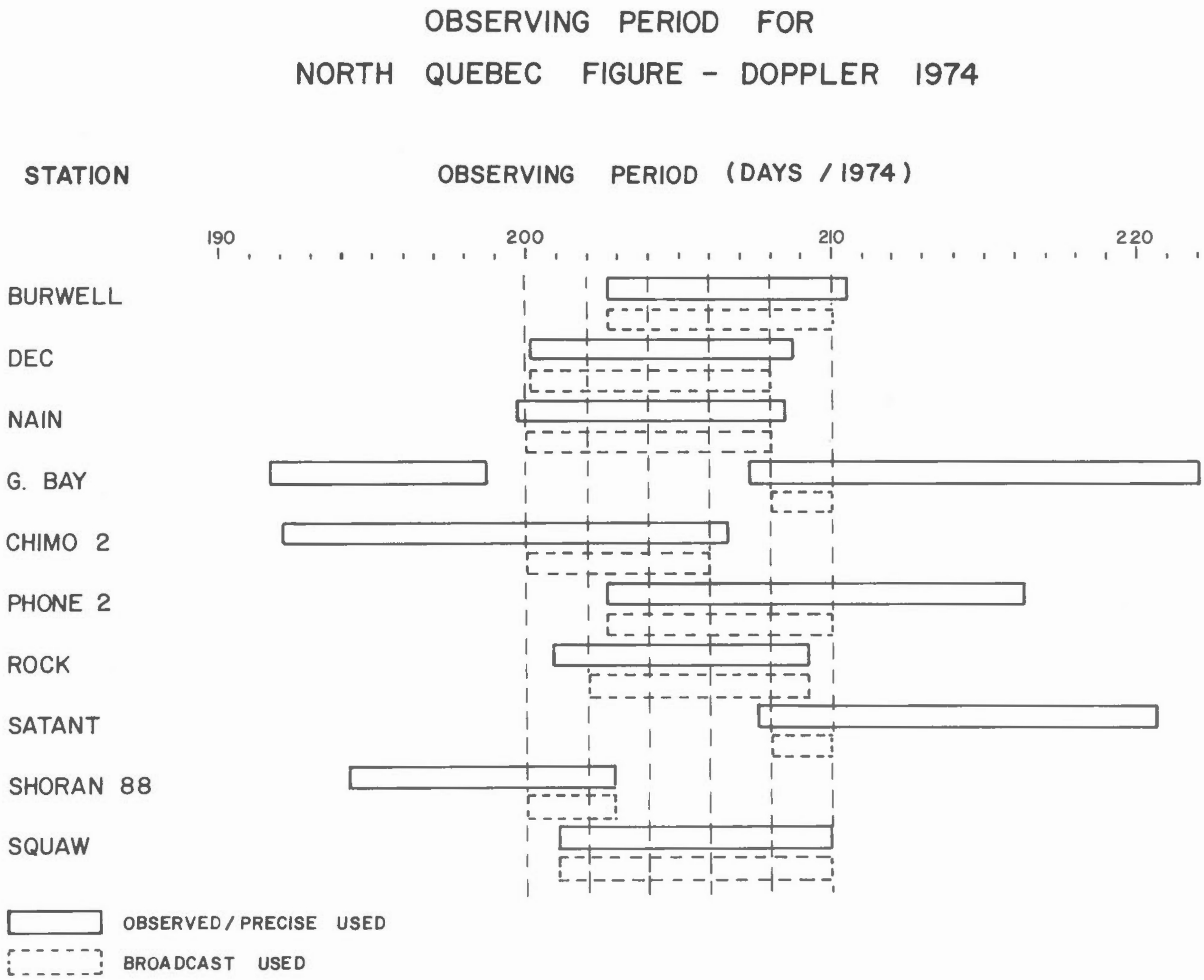

OBSERVING PERIOD FOR
NORTH QUEBEC FIGURE - DOPPLER 1974
STATION
OBSERVING PERIOD (DAYS / 1974)
190
200
210
220
BURWELL
DEC
NAIN
G. BAY
CHIMO 2
PHONE 2
ROCK
SATANT
SHORAN 88
SQUAW
OBSERVED / PRECISE USED
BROADCAST USED

Figure 7

TWO MIN. PRECISE POINT POSITIONING (SAT. 13 & 19)

VERSUS (2 DAY) MULTISTATION BROADCAST SOLUTION (SAT. 12, 14, 18, 20)

Figure 8

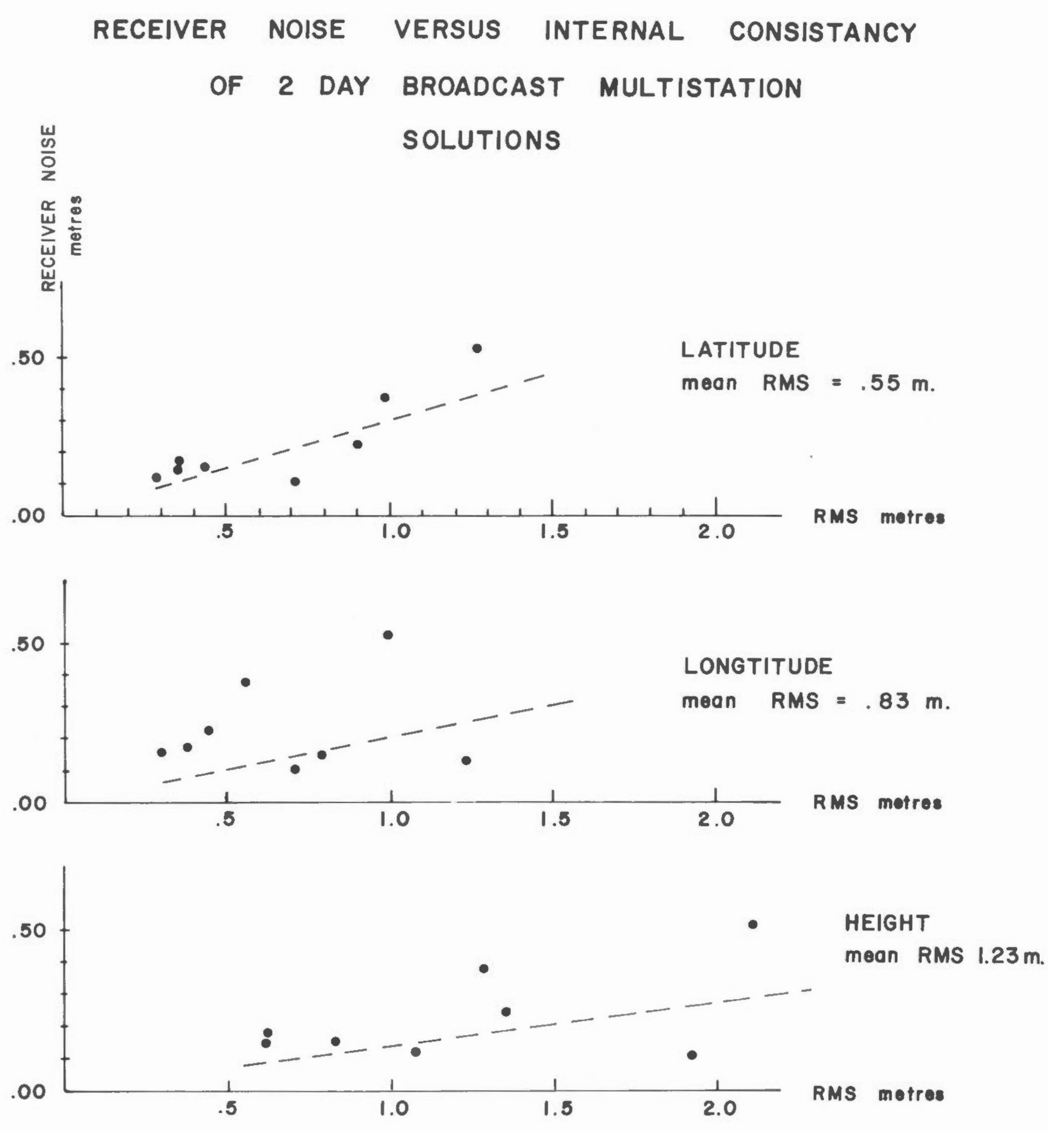

RECEIVER NOISE VERSUS INTERNAL CONSISTANCY
OF 2 DAY BROADCAST MULTISTATION
SOLUTIONS
RECEIVER NOISE
metres
.50
.00
.5
1.0
1.5
2.0
RMS metres
LATITUDE
mean RMS = .55 m.
LONGTITUDE
mean RMS = .83 m.
HEIGHT
mean RMS 1.23 m.

Figure 9

were combined, an improvement by a factor of about $\sqrt{2}$ was observed in σ's for φ,λ and h as well as for distances, indicating a substantially stronger determination (*see* Table VII).

Even though all the above comparisons, with the exception of the Goose Bay – Satant line are internal, using the same equipment and time periods, though using different data and orbits, it is expected that this represents the relative accuracy of each determination (broadcast and precise) in φ and λ for normally operating receivers and well-balanced data.

The high accuracy of precise point positioning had been established on several occasions previously with respect to independent external standard, see Anderle, 1974; Anderle *et al.*, 1974(a). The precise point positioning was thus the highest accuracy standard available for the comparisons. Height determination accuracy, on the other hand, cannot be considered representative, as the same antenna position and the same observation period does not reflect potentially significant errors in the height determination, which should be expected over large areas even for well-balanced data.

TABLE VI

Comparisons of 30 Second Multistation Precise (Sat. 13, 19) and Broadcast (Sat. 12, 14, 18, 20) Multi-station Solution North Quebec Figure, Doppler 1974 *

Station	Doppler σ metres)	No. of passes		Latitude (metres)			Longitude (metres)			Height (metres)		
		Prec.	Broad.	Δ Prec-BRD	σBRD	σPRC	Δ(m) PREC-BRD	σBRD	σPRC	Δ PREC-BRD	σBRD	σPRC
Squaw**	.16	59	103	.00	.00	.00	.00	.00	.00	.00	.00	.00
Burwell	.18	70	96	.36	.39	.44	1.55	.68	.69	.12	.43	.43
Dec	.23	52	89	.78	.43	.51	.00	.81	.86	.09	.60	.54
Nain	.38	59	92	.60	.54	.61	.50	1.04	1.07	1.93	.65	.64
G. Bay	.12	120	22	1.02	.64	.38	−1.37	1.10	.59	.32	.88	.36
Chimo 2	.16	130	92	.72	.37	.38	− .34	.63	.60	− .12	.41	.37
Phone 2	.12	125	94	.06	.36	.39	−1.72	.64	.57	− .20	.40	.35
Rock	.52	60	71	− .51	.80	.84	1.56	1.31	1.32	− .22	.93	.94
Satant	.16	64	21	.27	.70	.48	1.11	1.68	.74	− .50	1.42	.47
Shoran 88	.10	79	48	.09	.44	.40	.33	.90	.62	.80	.78	.38
Standardized:												
$\Delta/\sqrt{\sigma^2_{BRD} + \sigma^2_{PRC}}$	Mean			.59			.06			.29		
	RMS (around 0)			.89			1.08			.85		

*Scaled by −.86 PPM.
**Station Squaw held fixed.

TABLE VII

Distance Comparisons Multi-station Precise (Sat. 13, 19) and Broadcast (Sat. 12, 14, 18, 20)*

From Squaw to	Doppler σ (metres)	No. of passes** PREC	BRD	Chord Distance KM	Distance (PPM) Δ PREC-BRD*	σBRD	σPRC	Combined (PPM) σBRD + PRC
Burwell	.18	70	96	632.5079	+1.0	.6	.7	.5
Dec	.23	52	89	548.7506	+ .6	1.4	1.4	.9
Nain	.38	59	92	372.7517	− .3	2.5	2.6	1.7
G. Bay	.12	120	22	452.8604	+1.8	2.3	1.21	.9
Chimo 2	.16	130	92	377.8359	+1.5	1.0	1.0	.7
Phone 2	.12	125	94	762.6578	+ .4	.5	.5	.3
Rock	.52	60	71	349.1232	3.0	3.4	3.5	2.3
Satant	.16	64	21	1267.7223	+ .5	1.0	.5	.4
Shoran 88	.10	79	48	800.7435	− .4	1.1	.7	.6

*Broadcast solution scaled by −.86 PPM.
**The same number of passes does not necessarily imply simultaneous observations, see Figure 7.

5. Conclusions

Repeatability over several years is not better than 3 metres for broadcast and 0.6 m for precise point positioning from at least 30 satellite passes (1σ level of each coordinate). This is true no matter how small the internal σ of the determination may be.

Translocation, simultaneous point positioning and multi-station determinations are subject to approximately constant shifts and small rotations and scale biases with σ corresponding to the same values as for point positioning quoted above.

Two simultaneous precise point positionings can be highly correlated, as high as 0.7, depending on receiver (i.e. Doppler) noise, distance and azimuth between the two points. However, the correlation decreases only very slowly with a decrease in over-lapping time period (common data).

On the other hand, simultaneous precise point positioning using navigation-type receivers, with receiver σ of 1 to 2 m, are virtually uncorrelated.

The multi-station solution using broadcast ephemeris is equivalent to corresponding precise multi-station solution as far as relative accuracy is concerned. This approach compares favourably with geoceiver translocation with precise ephemeris for short distance [DOD, 1972], however much shorter period, greater observational and processing flexibility and almost no decrease in relative accuracy in any direction for distances up to 2,000 km are the expected advantages.

It is cautioned that results of comparisons are not based on totally independent determinations (same receiver, antenna, and observing period), though data and orbits were different. It is expected, however, that these comparisons represent the accuracy of each, broadcast and precise, determination (of relative position) in ϕ and λ for normally operating receivers and well-balanced data. The high accuracy of precise point positioning had been established on several occasions previously with respect to independent external standards, see Anderle, 1974(b); Anderle *et al.*, 1974(a). The precise point positioning was thus the highest accuracy standard available for the comparisons.

Height determination accuracy, on the other hand, cannot be considered representative, as the same antenna position and observation period do not reflect potentially significant errors in the determination, which should be expected even for well-balanced data.

Acknowledgment

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Discussion

Q: Is the repeatability of the positions determined from broadcast and precise ephemeris different?

A: Yes. The former are more scattered.

Q: When will the coordinates of all the Canadian Doppler points be available?

A: Hopefully in 1976. Most of the observations should have been made at the end of 1975.

Alternative solutions to the combination of terrestrial and satellite geodetic networks

D.B. Thomson and E.J. Krakiwsky*

Abstract. The parameters used to relate satellite and terrestrial geodetic network coordinate systems are outlined. Assumptions regarding these parameters determine, in part, what network combination procedures can be utilized. Several approaches to the combination of satellite and terrestrial geodetic networks are presented. Some experimental results are given. In the Atlantic Canada portion of the Canadian geodetic framework, the addition of satellite distances to the horizontal network estimation procedure caused a scale change of −2.3 ppm and a change in orientation of +0."1. Using data extracted from the literature, several three-dimensional models were tested. Test results for each of these models are presented.

Résumé. Les auteurs énumèrent les paramètres utilisés pour relier les systèmes de coordonnées des réseaux géodésiques levés par satellite et au sol. Des hypothèses concernant ces paramètres déterminent, en partie, les procédés à employer pour la combinaison des réseaux. Les auteurs proposent diverses approches pour cette combinaison des réseaux géodésiques levés par satellite et au sol, et donnent certains résultats expérimentaux. Dans la région atlantique du canevas de triangulation géodésique canadien, l'adjonction des mesures de distances faites par satellite au procédé d'estimation du réseau planimétrique a causé un changement d'échelle de $-2,3 . 10^{-6}$ et un changement d'orientation de +0,1". A l'aide de données d'études, plusieurs modèles tridimensionnels ont fait l'objet d'essais. Les auteurs donnent les résultats des tests de chaque modèle.

Introduction

The three dimensional coordinates of terrestrial points can be accurately determined through the use of observations of artificial earth satellites. Utilizing dynamic and semi-dynamic solutions, the standard deviation of coordinates are 3 m or less [Anderle, 1974; Meade, 1974; Kouba *et al.*, 1974; Wells, 1974], while geometric solutions yield values whose standard deviations are 10 m or less [Schmid, 1974]. The positional accuracies of satellite determined terrestrial coordinates are relatively independent of interstation spacing, and networks of such points are comparatively free of systematic errors. These features, and others, lead to the use of satellite determined coordinate data for the overall improvement of terrestrial geodetic networks.

There are two satellite networks in Canada. The North American Densification of the Worldwide Geometric Satellite Triangulation [Schmid, 1974] has recently been completed [Pope, 1975]. The establishment of a Doppler satellite network (using the United States Navy Navigation Satellite System) is presently under way in Canada [McLellan, 1974]. The task now is to determine the most effective and efficient manner by which these satellite networks can be combined with terrestrial geodetic networks. There are several alternative models that can be utilized. The choice of any one of them is dependent on the ultimate objectives of the user.

*Department of Surveying Engineering, University of New Brunswick, Fredericton, New Brunswick, Canada

General relationships

Before dealing with the alternative methods and associated mathematical models that can be used in the combination of terrestrial and satellite geodetic networks, it is necessary to outline the relationships that exist between a satellite network and its coordinate system, and a terrestrial network and its related coordinate systems.

A satellite network coordinate system may be defined by the three-dimensional satellite determined coordinates of terrain points [Vaníĉek, 1975]. In Figure 1, the coordinate system is designated as AT (Average Terrestrial), such as would be defined in a dynamic solution of satellite data [Anderle and Tannenbaum, 1974]. The terrain point coordinates are given by the components of the position vector $\bar{\rho}_i$. The Geodetic coordinate system (G) is the reference frame of the terrestrial network. The Z_G axis is coincident with the minor axis of the reference ellipsoid (horizontal datum), while the X_G Y_G plane is coincident with the equatorial plane of the reference ellipsoid. The Geodetic coordinate system is usually positioned and oriented via some specified geodetic parameters at the terrestrial network initial point k [Vaníĉek and Wells, 1974; Vaníĉek, 1975; Thomson and Krakiwsky, 1975]. The third coordinate system involved is the Local Geodetic (LG) (Figure 1). It is a left-handed system, located at the terrestrial initial point (k), with its X_{LG} axis oriented towards geodetic north, and the Z_{LG} axis coincident with the ellipsoid normal at k. The position vectors of terrestrial geodetic network points, $\bar{r}_{ki}$, are expressed in this system.

In the establishment of a terrestrial geodetic network, the aim is to have the axes of the Geodetic coordinate system parallel to those of the Average Terrestrial system. Further,

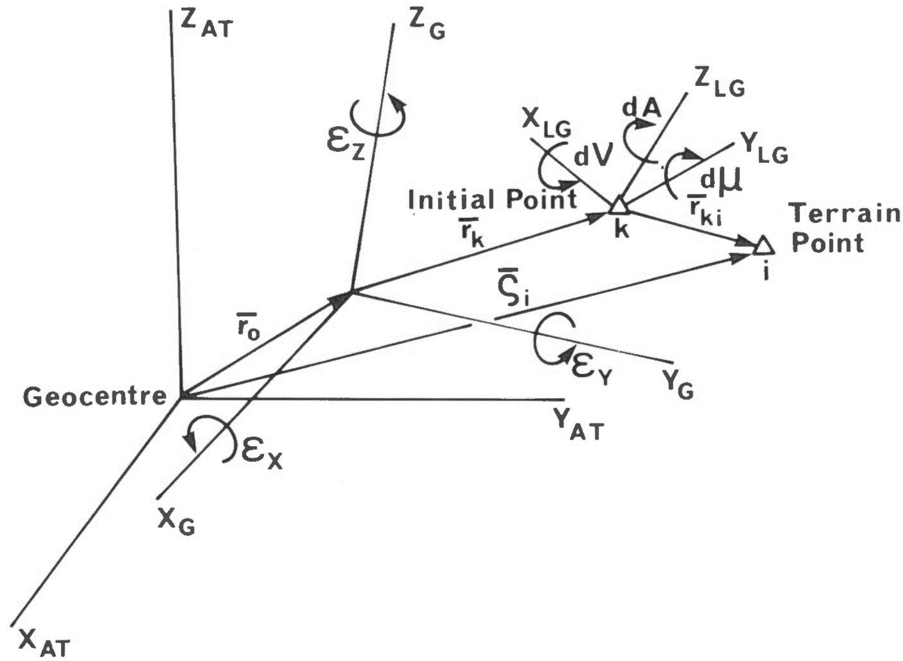

Figure 1. Geocentric Satellite (AT), Geodetic (G), and Terrestrial Network (LG), coordinate systems.

there should be no significant accumulation of systematic errors in the network. These are difficult objectives to attain using strictly terrestrial methods. Departures from the ideal situation present problems in terrestrial geodetic network computations and in the combination of terrestrial and satellite networks.

For several reasons, the origin of the Geodetic coordinate system is usually displaced from the geocentre. This displacement is given by the components (x_o, y_o, z_o) of the translation vector, r_o (Figure 1). A knowledge of this vector is essential, for some procedures, if full use of satellite coordinate data is to be made in terrestrial networks. The Geodetic and Average Terrestrial system axes may not be parallel. The non-parallelity is expressed by three rotations, ϵ_x, ϵ_y, and ϵ_z (Figure 1). The non-parallelity adversely affects terrestrial network computations, and some combination procedures.

Because of the accumulation of systematic errors in a terrestrial geodetic network, distortions in network orientation exists. This is expressed by rotations in the azimuth (dA), the meridian ($d\mu$), and the prime vertical ($d\nu$) directions about the Local Geodetic system axes at k (Figure 1).

Finally, there is the problem of scale difference (κ) Using the scale of the satellite system as a base, a scale difference can be determined between it and the terrestrial system. There are two ways in which this quantity can be introduced into a combination model, each of which has a different interpretation [Krakiwsky and Thomson, 1974; Vaníĉek, 1975; Thomson and Krakiwsky, 1975]. In one case, κ can be interpreted as a difference in the scales of Geodetic and Average Terrestrial coordinate systems (system scale difference). This difference may be expressed as either a change in size and shape of the Geodetic reference ellipsoid, or a change in ellipsoid height at the terrestrial initial point. Alternately, the scale difference can be interpreted as a terrestrial network scale distortion.

Assumptions regarding the aforementioned transformation parameters (x_o, y_o, z_o, ϵ_x, ϵ_y, ϵ_z, κ, dA, $d\mu$, $d\nu$) dictate, primarily, what methods and models are used in the combination of terrestrial and satellite geodetic networks.

Alternative combination procedures

The alternative procedures that can be utilized for the combination of satellite and terrestrial geodetic networks may be divided into two broad groups:

(i) Those in which the necessary transformation parameters (x_o, y_o, z_o, ϵ_x, ϵ_y, ϵ_z, κ, dA, $d\nu$, $d\mu$) are considered known;

(ii) those which treat the transformation parameters as unknowns to be solved for in the combination solution.

If the transformation parameters between the satellite and terrestrial geodetic coordinate systems and networks are known, the satellite determined coordinates may serve as a three-dimensional framework for the terrestrial geodetic network. Such a framework may be used in its inherent three-dimensional state, or, in order to be consistent with common geodetic practice, the satellite determined coordinates may be split into two parts. The geodetic latitude (φ) and longitude (λ) may be utilized in horizontal networks, and the ellipsoid height (h) for vertical networks or geoid determination. Alternately, the satellite coordinates can be used to derive distances, azimuths, and vertical angles that are used in terrestrial network computations.

The position of the origin of the Cartesian reference frame of satellite coordinates determined by geometric methods is arbitrary [Schmid, 1974], while coordinates determined in dynamic solutions refer to a geocentric coordinate system [Anderle and Tannenbaum, 1974]. The tertiary (Z) axis of each is referred to the CIO pole by the use of the BIH values for pole position [Anderle and Tannenbaum, 1974; Schmid, 1974]. The dynamically determined coordinates can be used, together with the terrestrial geodetic coordinates, in the recovery of the transformation parameters $x_o, y_o, z_o, \epsilon_x, \epsilon_y, \epsilon_z$.

Since there are minimal systematic errors in satellite networks, they can be used to model systematic scale (κ) and orientation (dA, dμ, dν) in a terrestrial network.

Combination models tested

Several investigators have dealt with the problem of combining terrestrial and satellite geodetic networks. Many recent studies have treated the problem by assuming an a priori knowledge of the coordinate system transformation parameters [Meade, 1974a; Meade, 1974b; Anderle, 1974]. The authors have concentrated on approaches that do not require a knowledge of these transformation parameters.

It is possible to introduce satellite distances to a terrestrial horizontal network when the cordinate system transformation parameters are unknown. This is accomplished by an "approximate" reduction of the spatial distances to geodesic distances, the latter of which are treated as distance observations in the network adjustment procedure. It is recognized that this approach is not completely rigorous. However, the effects on the distances are only second order in nature because of the unknown transformation parameters.

Three models for the combination of satellite and terrestrial geodetic networks have been tested in which the Cartesian coordinates of satellite network points are used directly.*

The Bursa model (Figure 5), in which the observables are the network position vectors, $(\bar{r}_i)_G$ and $(\bar{\rho}_i)_{AT}$, is given by [Bursa, 1962]

$$\bar{F}_i = (\bar{r}_o) + (1+\kappa)\, R_1(\epsilon_x)\, R_2(\epsilon_y)\, R_3(\epsilon_z)\, (\bar{r}_i)_G - (\bar{\rho}_i)_{AT} = 0$$

where R_1, R_2, and R_3 are three-dimensional rotation matrices. When solved using a combined case least squares estimation procedure [Thomson and Krakiwsky, 1975], one obtains a set of coordinate system transformation parameters, and a set of adjusted three-dimensional coordinates. The parameter κ is interpreted as a system scale difference, and the coordinates of the terrestrial initial point are redefined (scaled and rotated).

The Veis model (Figure 6), which is solved using the same estimation procedure as the Bursa model, is given by [Veis, 1960]

$$\bar{F}_i = (\bar{r}_o) + (\bar{r}_k)_G + (1+\kappa)\, R_V (\bar{r}_{ki})_G - (\bar{\rho}_i)_{AT} = 0 \quad ,$$

where

$$R_V = R_3(180-\lambda_k) R_2(90-\phi_k) P_2 R_1(d\nu) R_2(d\mu)$$
$$R_3(dA) P_2 R_2(\phi_k-90) R_3(\lambda_k-180)$$

in which ϕ_k, λ_k are the geodetic coordinates of the terrestrial initial point, and P_2 is a three-dimensional reflection matrix. The observables in this model are the satellite network position vectors, $(\bar{\rho}_i)_{AT}$, and the terrestrial network position vector differences, $(\bar{r}_{ki})_G$. The results are a set of coordinate system translation components (x_o, y_o, z_o), the orientation and scale difference parameters of the terrestrial geodetic network (dA, dμ, dν, κ) and a set of adjusted Cartesian coordinates. The terrestrial network initial point ($\bar{r}_k$) is not redefined.

Mathematical models have been developed in which there exists two sets of rotations (Figure 7) – coordinate system and terrestrial network [Hotine, 1969; Krakiwsky, Thomson, 1974].

The model proposed by the authors can be written [Krakiwsky and Thomson, 1974; Thomson and Krakiwsky, 1975],

$$\bar{F}_1 = (\bar{r}_o) + (1+\kappa) R_1(\epsilon_x) R_2(\epsilon_y) R_3(\epsilon_z)\, (\bar{r}_k)_G + R_V (\bar{r}_{ki})_G 0 - (\bar{\rho}_i)_{AT} = 0.$$

In this version of the proposed model, the scale difference is interpreted as a system scale difference. By rewriting the model as

$$\bar{F}_i = (\bar{r}_o) + R_1(\epsilon_x) R_2(\epsilon_y) R_3(\epsilon_z) \left[(\bar{r}_k)_G + (1+\kappa) R_V (\bar{r}_{ki})_G \right] - (\bar{\rho}_i)_{AT} = 0 \quad ,$$

the parameter κ is applied only to the terrestrial network coordinate difference vector, $(r_{ki})_G$. Because of the nature of the relationships among the unknowns being solved for (a maximum of ten), a phased, combined case least-square estimation technique is used in which the sets of common satellite and terrestrial Cartesian coordinates are split into two zones [Thomson and Krakiwsky, 1975]. The inner zone position vectors, which are close to the terrestrial initial point so as to minimize the effects of systematic terrestrial network errors, are used in the first phase to solve for a first approximation of the coordinate system transformation parameters. The results of phase one are then used in phase two, along with the outer zone position vectors, to determine the final values for the coordinate system transformation parameters, the terrestrial network orientation and scale parameters, and the final set of three-dimensional coordinates.

Experimental results

The effects of the addition of satellite determined distance data on a horizontal geodetic network have been examined. In Atlantic Canada, satellite determined coordinate data were available for eight Doppler and three Satellite Triangulation stations that are coincident with terrestrial geodetic network

*Since the satellite data used are the results of a dynamic solution, the satellite network coordinate system is denoted AT.

stations (Figure 2). Distances, and associated variances, were computed from the satellite coordinate data and reduced to the reference ellipsoid. A comparison of the above distances with those computed using previously adjusted terrestrial geodetic coordinates, indicated that the Doppler distances were *longer* by a mean of 5.7 ppm, the Satellite Triangulation distances were *longer* by a mean of 9.1 ppm (Table I). When compared with precise terrestrial surveys, Doppler network distances have been found to be longer by 1 ppm [Anderle and Tannenbaum, 1974; Meade, 1974a]. No correction for the 'known' scale bias was applied to the Doppler distances used in this test.

Distances among all Doppler stations (28) and all Satellite Triangulation stations (3) were employed in a parametric least squares adjustment of the terrestrial network. A brief summary of the horizontal network adjustment is given in Table II. As a result of the addition of the satellite network distances, there were changes in the horizontal geodetic coordinates. The maximum coordinate shifts detected (adjustment *without* satellite data minus adjustment *with* satellite data) were -0."133 (-4.00 m) in latitude and 0."078 (1.72 m) in longitude. A graphical illustration of the coordinate changes at 13 selected stations is given in Figure 3. The corresponding changes in network scale and orientation are shown in Figure 4 by means of the differences in 24 interstation geodetic distances and azimuths (sign convention equivalent to that used for coordinate differences). The scale changes vary from -5.0 ppm to +2.1 ppm, with a mean of -2.3 ppm, while azimuth changes range from -1."116 to +0."783, with a mean of 0."080.

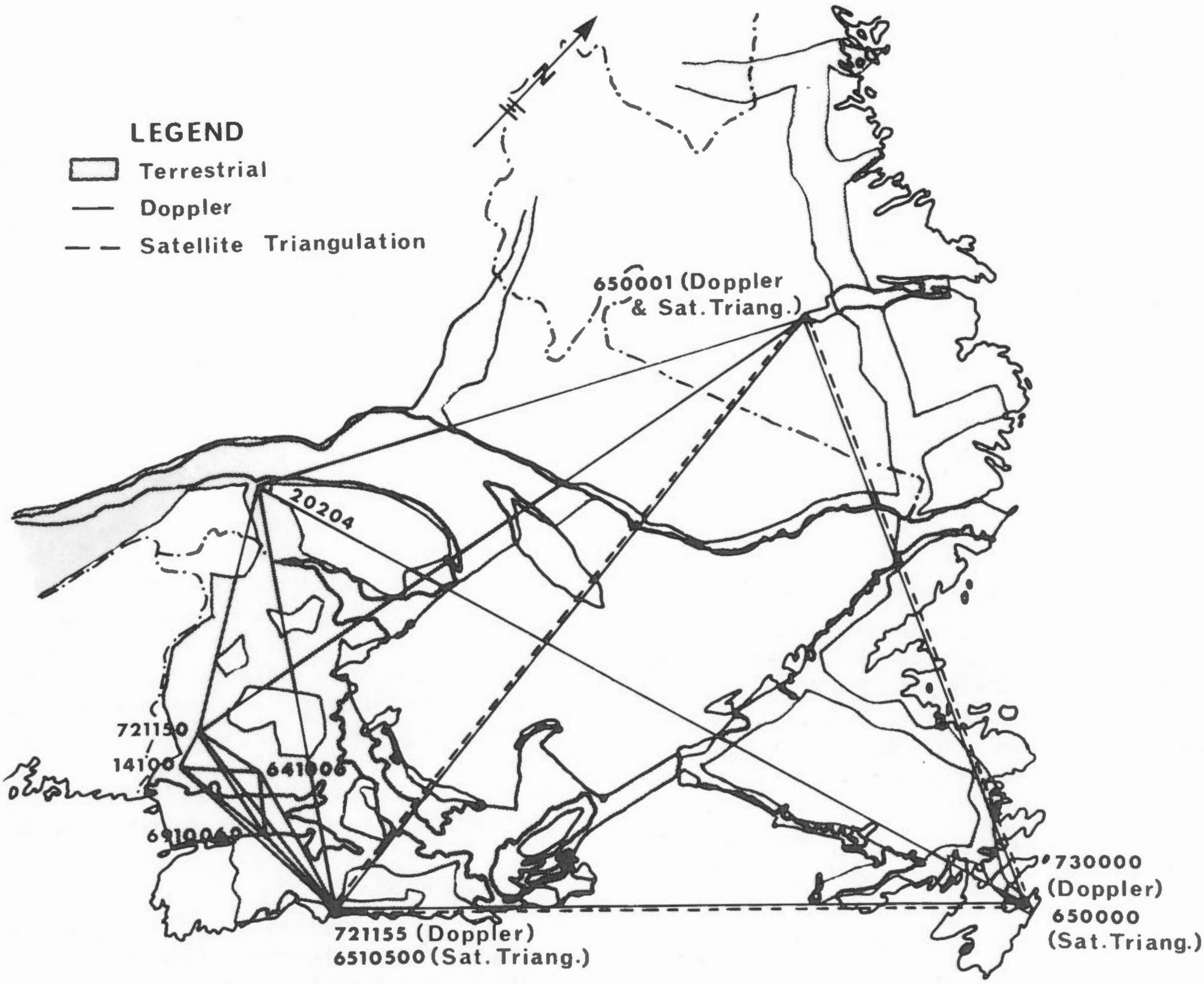

Figure 2. Atlantic Canada geodetic networks.

TABLE I
Comparison of Terrestrial and Satellite Distances in Atlantic Canada Test Network

Line	Terr. Dist. (m)	Satellite Dist. (m)	Δ = Terr.–Satellite Δ (m)	Δ (ppm)	σ(m) Sat. Dist.
721150-721155	276123.67	276122.00	1.67	6.0	1.13
721150-14100	59255.49	59257.56	−2.07	−35.0	1.07
721150-641006	93125.82	93124.86	0.96	10.3	2.87
721150-6910060	165349.97	165350.02	−0.05	−0.3	1.67
721150-650001	934815.77	934817.85	−2.08	−2.2	0.76
721150-20204	326498.14	326500.88	−2.74	−8.4	0.65
721150-730000	1079122.73	1079118.12	4.61	4.3	1.21
721155-14100	265440.16	265440.51	−0.35	−1.3	2.44
721155-641006	195550.83	195551.51	−0.68	−3.5	2.42
721155-6910060	115503.18	115502.49	0.68	5.9	2.23
721155-650001	987957.03	987962.02	−4.99	−5.0	0.75
721155-20204	549621.05	549626.02	−4.97	−9.0	1.15
721155-730000	901850.65	901849.44	1.21	1.3	1.54
14100-641006	117393.74	117396.86	−3.12	−26.6	3.34
14100-6910060	149939.11	149940.22	−1.11	−7.4	2.88
14100-650001	992515.68	992521.71	−6.03	−6.1	1.51
14100-20204	381057.36	381061.17	−3.81	−10.0	1.10
14100-730000	1107918.28	1107918.31	−0.03	0.0	2.47
641006-6910060	101802.76	101806.47	−3.71	−36.4	1.73
641006-650001	904210.34	904211.56	−1.22	−1.4	1.54
641006-20204	364181.07	364183.62	−2.55	−7.0	1.74
641006-730000	991760.90	991756.79	4.11	4.1	2.94
6910060-650001	981754.14	981760.98	−6.84	−6.9	1.16
6910060-20204	465706.40	765712.65	−6.25	−13.4	1.32
6910060-730000	988405.14	988404.72	0.42	0.4	2.18
650001-20204	708726.05	708731.46	−5.37	−7.6	1.34
650001-730000	838159.09	838160.18	−1.09	−1.3	1.11
20204-730000	1111555.74	1111558.98	−3.24	−2.9	1.73
*6510500-650001	991071.30	991081.37	−10.07	−10.2	7.01
*6510500-650000	894477.25	894481.72	−4.47	−5.0	9.41
*65001-650000	832760.79	832770.79	−10.00	−12.0	7.99

*Satellite distances derived from satellite triangulation coordinates. All other satellite distances derived from Doppler coordinates
Mean deviation (Δ) of Doppler distances: −5.7 ppm
Mean deviation (Δ) of satellite triangulation distances: −9.1 ppm
Mean deviation (Δ) of all satellite distances: −6.0 ppm

TABLE II
Results of the Horizontal Network Adjustment when using Satellite Network Distances

Unknown stations	795
Fixed stations	1 (721150)
Direction observations	4593
Azimuth observations	38
Terrestrial distance observations	456
Doppler distances	28
Satellite triangulation distances	3
A priori variance factor	1.00
Degrees of freedom	2583
Estimated variance factor	1.10

TABLE III

Results of Bursa and Veis Model Tests

	Bursa	Veis
Test #1	x_o: −23.9 m ±6.5	x_o: −42.2 m ±1.7
	y_o: 152.1 m ±6.0	y_o: 163.2 m ±1.6
	z_o: 194.8 m ±6.9	z_o: 182.0 m ±1.7
	ϵ_x: −0″.14 ±0.24	dA: −0″.45 ±0.19
	ϵ_y: 0″.07 ±0.22	$d\mu$: 0″.15 ±0.25
	ϵ_z: 0″.78 ±0.19	$d\nu$: −0″.63 ±0.21
	κ : −2.2 ppm ±0.9	κ : −2.3 ppm ±0.9
Test #2	x_o: −25.8 m ±4.2	x_o: −41.1 m ±1.9
	y_o: 149.5 m ±4.1	y_o: 163.0 m ±1.8
	z_o: 191.3 m ±3.8	z_o: 182.2 m ±1.8
	ϵ_z: 0″.76 ±0.18	dA: −0″.54 ±0.21
	κ : −2.2 ppm ±0.8	κ : −2.3 ppm ±1.0
Test #3	x_o: −42.0 m ±2.2	x_o: −40.3 m ±2.0
	y_o: 151.5 m ±5.2	y_o: 162.9 m ±2.0
	z_o: 191.7 m ±4.8	z_o: 182.3 m ±2.0
	κ : −2.3 ppm ±1.1	κ : −2.3 ppm ±1.1

The three dimensional models were tested using data taken from the literature [Badekas, 1969]. The distribution of the 11 Doppler tracking stations used is shown in Figure 8. Three solutions were computed for each of the Bursa and Veis models (Table III).

The indicator for the acceptance of results was the comparison of the relative magnitudes of the estimated parameters and their corresponding standard deviations. Test #1 was rejected for both models. Test #2, in which two rotations were eliminated from the models (ϵ_x, ϵ_y from Bursa; $d\mu$, $d\nu$ from Veis), also yielded unacceptable results. Only test #3, which yielded translation components and a scale difference, proved to be acceptable. A set of adjusted Cartesian coordinates were generated for each of the solutions for both models.

The same data were employed in a solution of the proposed model, although the limited amount of data inhibited extensive testing. The procedure used was to solve phase one in an iterative fashion, varying the number of inner zone coordinates and unknown coordinate system transformation parameters in each iteration. When, according to an analysis of the estimated variance factor, an acceptable solution was obtained, the results of phase one were employed in phase two. The latter phase was also solved iteratively.

Two sets of results are given in Table IV. Based on these results, it may be stated that the only significant transformation parameters between the two sets of data are the translation components (x_o, y_o, z_o), the longitudinal rotation (ϵ_z), and the scale difference (κ). Because of the limited amount of data involved, no conclusions may be drawn regarding the performance of the proposed model and the method of solution employed. Further testing, with more extensive data, is required.

TABLE IV

Results of Proposed Model Tests

Inner zone stations 5

Outer zone stations 6

	Test #1	Test #2
PHASE I	x_o: −14.6 m ±13.8	x_o: −14.6 m ±13.8
	y_o: 156.4 m ±3.8	y_o: 156.4 m ±3.8
	z_o: 180.7 m ±2.0	z_o: 180.7 m ±2.0
	ϵ_z : 1″.24 ±0.57	ϵ_z : 1″.24 ±0.57
PHASE II	x_o: −17.1 m ±10.2	x_o: −22.6 m ±3.8
	y_o: 158.1 m ±1.8	y_o: 158.8 m ±1.0
	z_o: 181.2 m ±0.8	z_o: 181.1 m ±0.7
	ϵ_z : 1″.11 ±0″.43	ϵ_z : 0″.90 ±0.16
	dA: 0″.26 ±0.30	dA: 0″.10 ±0″.52
	$d\mu$: 0″.20 ±0.15	
	$d\nu$: 0″.16 ±0.36	
	κ : −2.2 ppm ±0.6	κ : −1.6 ppm ±0.5

Summary

Several methods can be employed in the combination of terrestrial and satellite geodetic network. The amount of satellite data that can be used in classical terrestrial geodetic computations is dependent on whether or not one knows the appropriate coordinate system transformation parameters.

The introduction of satellite network distances in a horizontal terrestrial network has a significant effect on the station coordinates. This approach may be employed to assist in the detection of systematic scale and orientation errors in the terrestrial network.

The three-dimensional models permit a full use of satellite network data without a priori knowledge of the coordinate system transformation parameters. The flexibility of the models and particularly that of the proposed model, is advantageous. In addition, the scale and orientation unknowns in the terrestrial network can be determined, along with an adjusted set of three-dimensional coordinates.

Acknowledgement

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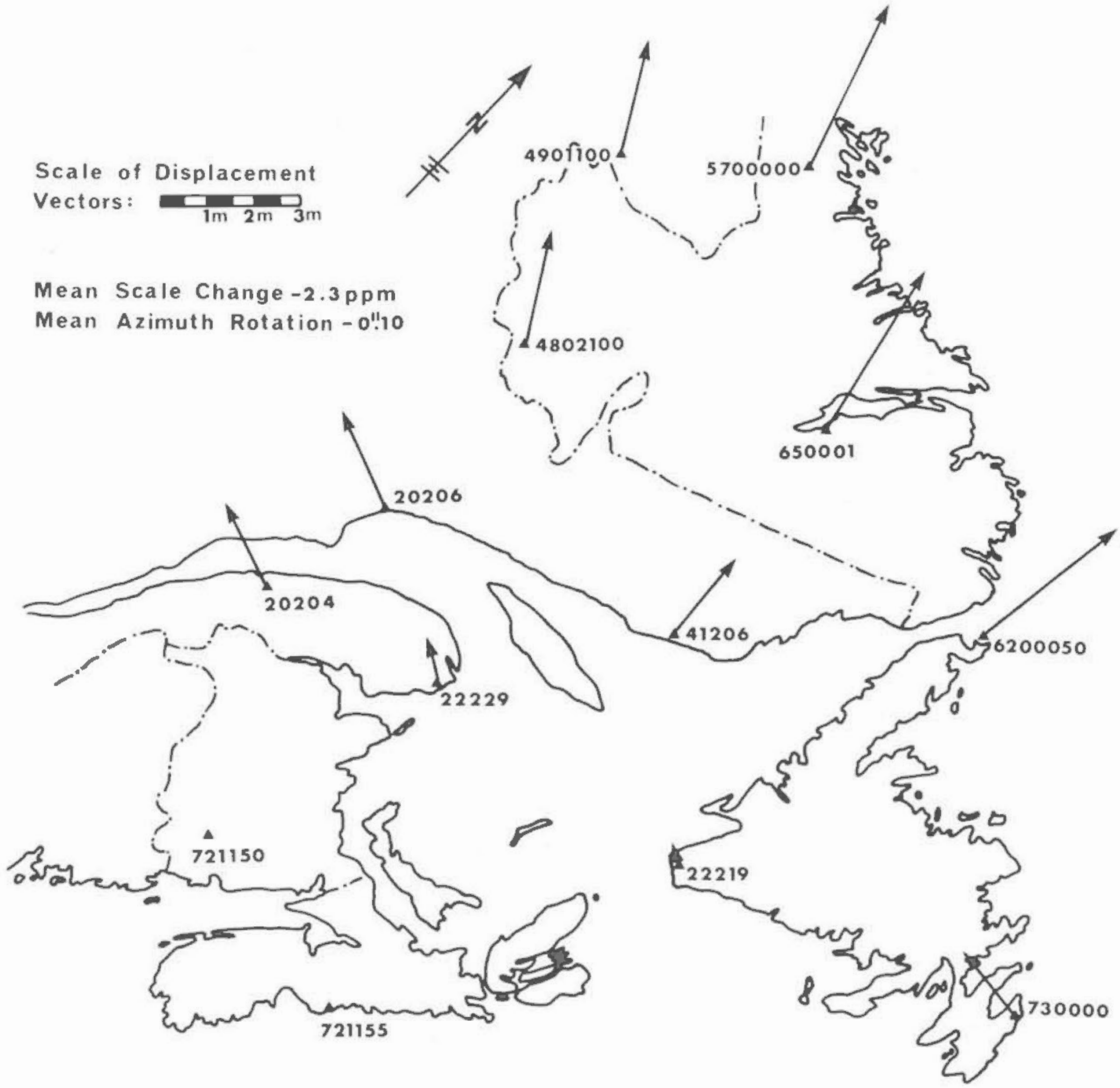


Figure 3. Shift in coordinates because of the addition of satellite distances.

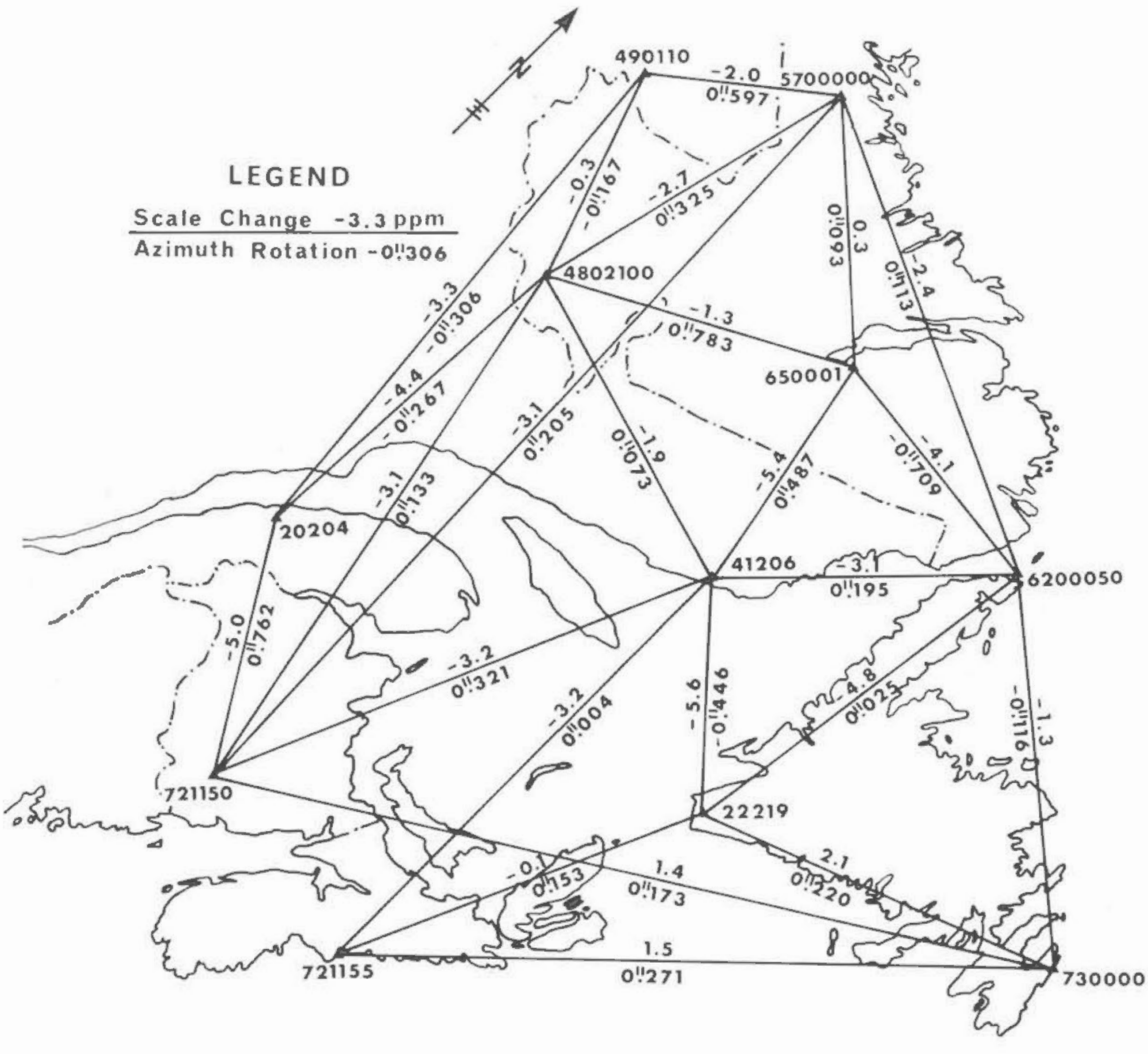
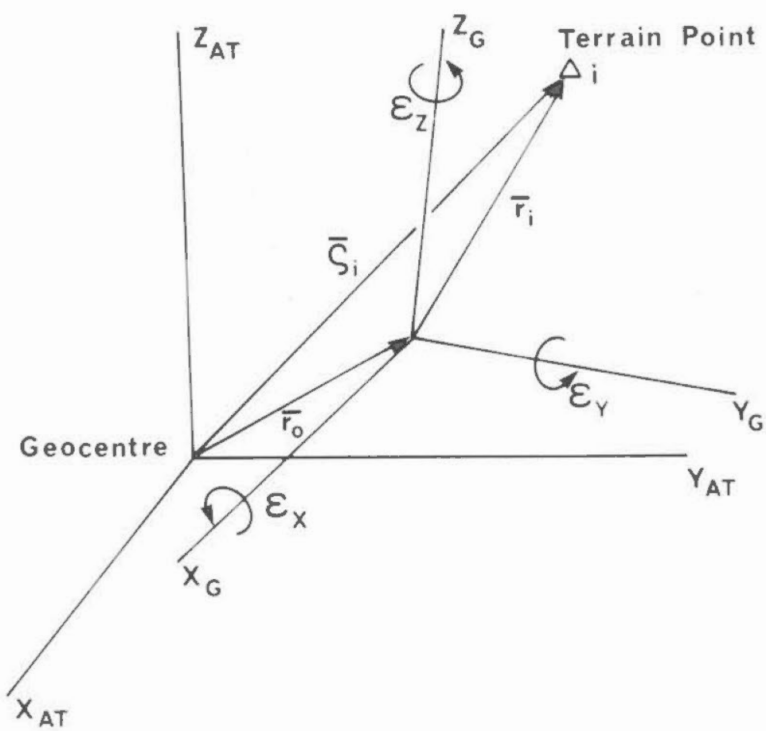
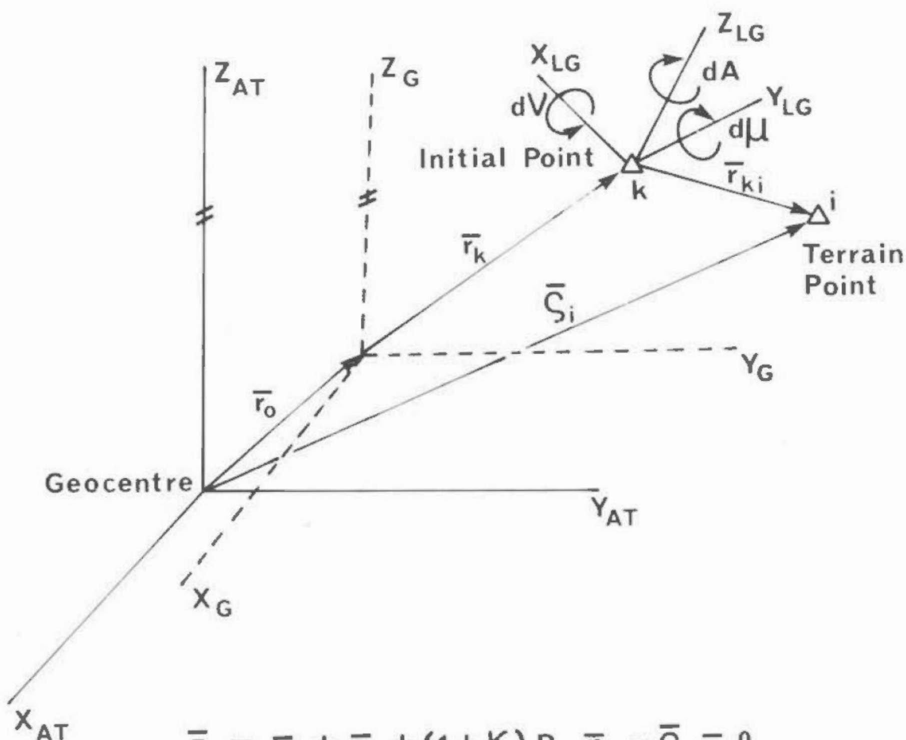


Figure 4. Changes in horizontal network scale and orientation because of the addition of satellite distances.



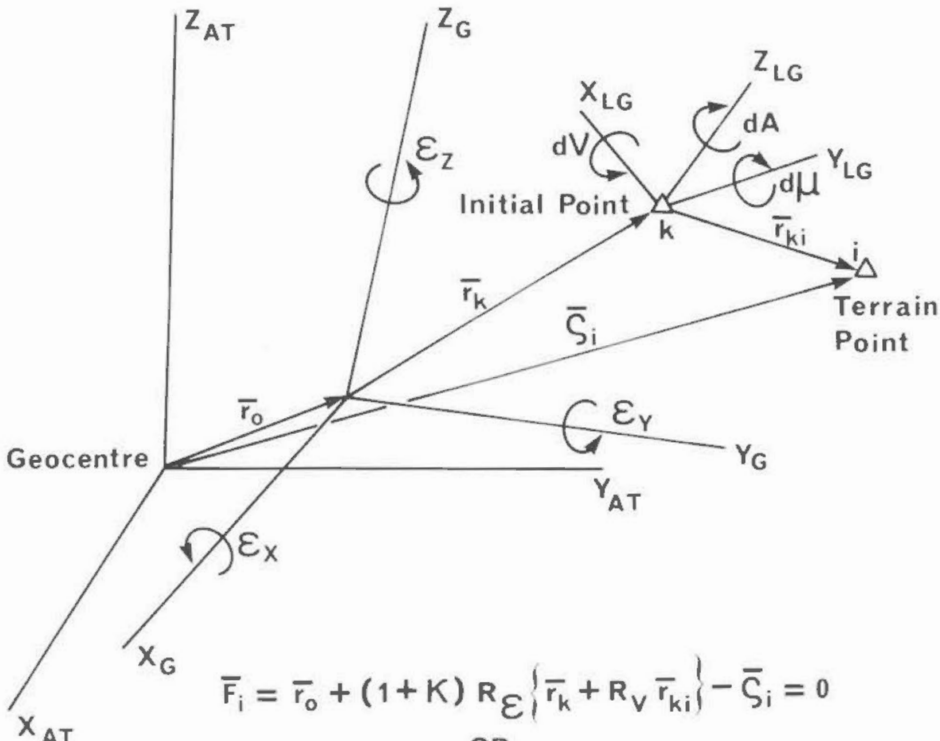
$$\bar{F}_i = \bar{r}_o + (1 + K)\, R_1(\varepsilon_X)\, R_2(\varepsilon_Y)\, R_3(\varepsilon_Z)\, \bar{r}_i - \bar{\varsigma}_i = 0$$

Figure 5. Bursa model.



$$\bar{F}_i = \bar{r}_o + \bar{r}_k + (1 + K)\, R_V\, \bar{r}_{ki} - \bar{\varsigma}_i = 0$$

Figure 6. Veis model.



$$\bar{F}_i = \bar{r}_o + (1+K)\,R_{\varepsilon}\left\{\bar{r}_k + R_V\,\bar{r}_{ki}\right\} - \bar{\varsigma}_i = 0$$

OR

$$\bar{F}_i = \bar{r}_o + R_{\varepsilon}\left\{\bar{r}_k + (1+K)\,R_V\,\bar{r}_{ki}\right\} - \bar{\varsigma}_i = 0$$

Figure 7. Proposed model.

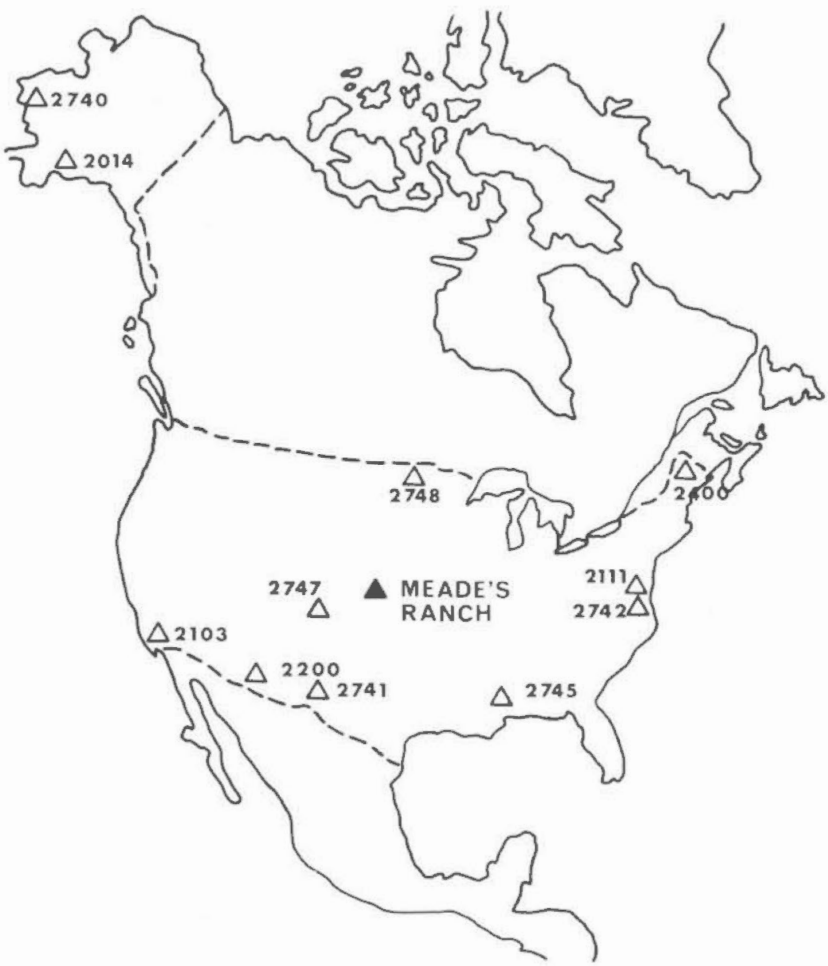


Figure 8. NWL Doppler tracking stations.

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Discussion

Q: Which coordinates for the Doppler points did you use? Based on broadcast or precise ephemeris?

A: Broadcast.

Semi-dynamical Doppler satellite positioning

D.E. Wells*

Abstract. In semi-dynamical satellite geodesy, the satellite orbit is determined beforehand by dynamical methods, and tracking station coordinates are determined geometrically from satellite observations and the accepted satellite coordinates. This paper analyzes the use of the Transit Doppler system in the semi-dynamical mode, presenting some test network numerical results. Four error sources are considered: errors in the satellite coordinates, refractive errors on the Doppler observations, errors contributed by the tracking receivers, and invalid assumptions in the observation equation. Compensatory techniques are discussed, such as relaxing the satellite orbit, ionospheric and tropospheric refraction reductions, tracking receiver time recovery, and translocation.

Résumé. En géodésie semi-dynamique par satellite, l'orbite du satellite est déterminée à l'avance par des méthodes dynamiques, et les coordonnées de la station de poursuite sont définies géométriquement à partir des observations du satellite et des coordonnées acceptées pour le satellite. La présente étude analyse l'utilisation du système Transit Doppler en semi-dynamique, et donne quelques résultats numériques d'un réseau d'essai. Quatre sources d'erreurs sont envisagées: erreurs dans les coordonnées du satellite; erreurs dues à la réfraction sur les observations Doppler; erreurs des récepteurs de poursuite; hypothèses nulles dans l'équation de l'observation. L'auteur étudie les techniques de correction compensatoires, telles que la relaxation de l'orbite du satellite, la réduction des réfractions ionosphérique et troposphérique, une correction de temps au récepteur de poursuite, et le déplacement.

Introduction

The observation techniques of satellite geodesy have included radio and laser ranging between earth and satellites, satellite triangulation, and the technique to be discussed in this paper, the measurement of the Doppler shift of satellite transmitted radio signals. In Doppler satellite geodesy, the satellite transmits one or more stable frequencies, f_s, in the 100 to 1,000 MHz region and ground receivers measure the amount by which these stable frequencies have been changed owing to the relative velocity between satellite and ground stations (Gill, 1965). There are different ways of measuring this Doppler shift. The most widely used method is to measure an integrated cycle count, D, of the beat frequency between the received satellite signal and a local oscillator, f_g:

$$D = (f_g - f_s)\ (T_2 - T_1) + (f_g/c)\ (S_2 - S_1) \tag{1}$$

This Doppler count is simply related to the change in slant range $(S_2 - S_1)$ during the integration interval $(T_2 - T_1)$. The slant ranges in turn are functions of the receiver and satellite coordinates. 'c' is the velocity of light.

Of the satellite geodesy techniques used so far, Doppler observations have been the most fruitful. Three inherent reasons for this are that Doppler tracking is passive, not requiring any interrogation or directionally sensitive antennae; the data obtained, Doppler counts, are in digital form; and the radio frequencies used permit all weather day and night tracking. Because of these inherent advantages, work has been done to improve the usefulness of Doppler tracking. Automatic portable equipment is available from several suppliers (including a Canadian firm, Canadian Marconi Company). Since 1967 there have been at least four operational satellites continuously available. Much effort has gone into identifying and eliminating the sources of error in Doppler work, with the result that the accuracies achieved have steadily improved (Brown, 1970; Anderle, 1974; Wells, 1974; Kouba, 1975).

The Doppler satellite system in general use in the *Transit* system (Guier and Weiffenbach, 1960; Dunnell, 1967; Newton, 1967; Stansell, 1971; Sims, 1972; Piscane *et al.*, 1973). Figure 1 shows the operating principles of *Transit.*

The two traditional techniques of analyzing satellite geodesy measurements are *geometrical,* in which satellite position is assumed unknown and independent of other positions, and *dynamical,* in which successive satellite positions are assumed to be related in a manner governed defined by the forces affecting satellite motion. Dynamic analyses of Doppler data from many stations and many satellite passes, for example, have provided from one solution the station and satellite coordinates, and coefficients of the force model (Anderle, 1974).

On a more modest level, if the satellite orbit has already been determined, using a dynamic analysis, then we need no longer be concerned with determining the satellite coordinates and force model coefficients, and can with greater ease determine the tracking station coordinates. This technique has been called *semi-dynamical Doppler satellite positioning* (Gaposhkin, 1973).

*Bedford Institute of Oceanography, Dartmouth, Nova Scotia, Canada.

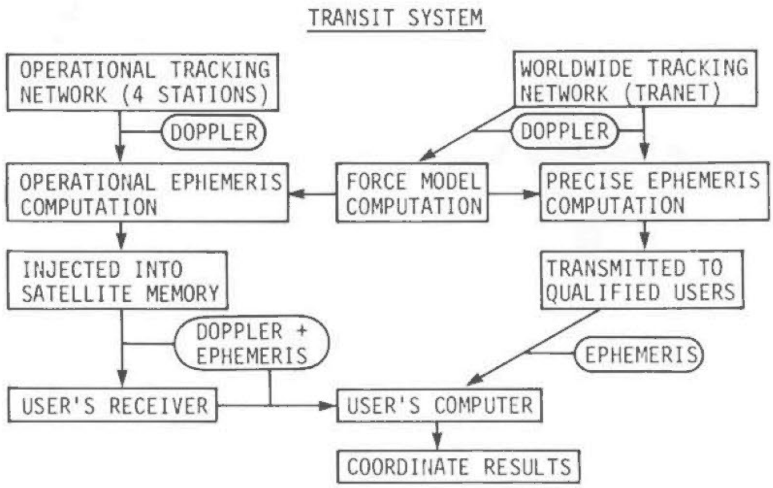

Figure 1. *Operating principles of the Transit Doppler satellite system.*

Every few years a new dynamic force model computation is performed, including the coordinates of permanent tracking stations. These results are then used in the day to day computation of orbit ephemerides. The *precise ephemeris,* since 1975 provided by the U.S. Defence Mapping Agency, is a set of Average Terrestrial positions and velocities at one minute intervals, computed by fitting 48 hour orbital arcs to Doppler data from the worldwide TRANET network (Sims, 1972). The *operational ephemeris,* injected into satellite memories twice per day and broadcast automatically to users, is computed by fitting 36 hour orbital arcs to Doppler data from the four U.S. Naval Astronautics Group stations situated in Maine, Minnesota, California and Hawaii, and extrapolating these arcs 16 hours beyond the time of the last data used (Piscane *et al.,* 1973). The top half of the figure represents the dynamic satellite geodesy portion of the Transit system, and the bottom half the semi-dynamical portion, in which the user obtains either the precise or operational ephemerides with which to define satellite trajectories.

Error sources

Transit error sources can be divided into four classes: errors in the satellite coordinates; refractive errors on the Doppler counts; errors contributed by the receiver; and invalid modelling assumptions. Since in semi-dynamic satellite geodesy we are accepting an orbit computed elsewhere, the first of these is the most pertinent, and we consider it in greatest detail.

The two processes involved in obtaining Doppler geodetic coordinates are the acquisition of observations, and the transformation from observation space to solution space. For each of these processes there are tools which are useful in investigating and reducing the effect of these error sources.

The use of two receivers in various configurations aids in identifying the effect of various error sources in the acquisition process. The four modular components of a Transit receiving system are antenna, local oscillator, the receiver itself, and a data recording or processing device. The effect of receiver characteristics is best investigated using *collocated* receivers (common antenna and common oscillator). The effect of oscillator characteristics is best investigated using *pseudo-collocation* (common antenna, separate oscillator). The effect of refraction (or residual refraction if some correction has been made) is best investigated using *translocation* (separate antenna, separate oscillator) for several station separations.

Expressing the solution vector in the coordinate system which is most natural to the geometry of Doppler satellite positioning is a useful tool in analyzing the effect of error sources in the transformation from observation to solution space. Selection of this most appropriate solution space is based on two principles. Since errors in the satellite trajectory are small, the actual and estimated trajectories are nearly parallel (Guier, 1965). Second, the three parameters which most fully represent the information contained in a set of satellite Doppler observations from a single pass are the time and frequency coordinates and slope at closest approach. The solution space formed by these three parameters consists of the frequency offset ($f_g - f_s$), and two geometrical parameters, closest approach time (along track) and range (cross track), which are coordinates in the two dimensional plane containing the receiver position and the satellite velocity vector at closest approach (Guier, 1965). This *Guier plane* is unique to each satellite pass. Refraction and oscillator drift effects show up in the cross track coordinate, timing errors in the along track coordinate, and satellite orbit biases in both. Thus to the extent that the above two principles are valid, the Doppler residuals resulting from the transformation into this solution space will be uncontaminated by the error sources.

A third useful tool in identifying systematic effects of the error sources is statistical analysis of the parameters of the solution vector, and the residuals. One characteristic of Doppler satellite positioning which facilitates the use of this tool is the fact that data are plentiful.

Orbit errors

The accuracy of the precise ephemeris is somewhat better than three metres (Anderle, 1974). In contrast, the operational ephemeris accuracy is much worse, since it is predicted rather than post-computed; is based on data from four tracking stations rather than a worldwide network of stations; and is computed using an earlier force model than the precise ephemeris (Piscane *et al.*, 1973). On the other hand the operational ephemeris is available in real time, for all satellites, while the precise ephemeris is available only after delays of weeks to months and only for one or two satellites. It is of interest to evaluate what accuracy penalty is involved in using the operational ephemeris.

The operational ephemeris consists of a set of fixed orbit parameters which describe a precessing, osculating orbital ellipse, and a set of variable orbit parameters which describe the departures of the predicted trajectory from this ellipse (Moffett, 1973). These departures are resolved into along track (ΔE), radial (Δa) and cross plane (η) components. The accuracy of the operational ephemeris is the accuracy with which these variable orbit parameters represent the departures of the actual trajectory from the mean ellipse. ΔE is broadcast with a resolution of 0.0001 degree (13 m), and Δa, η with a resolution of 10 m.

The accuracy of the operational ephemeris has been investigated in three ways (Wells, 1974). First for two sets of passes for which both precise and operational ephemerides were available, the operational ephemeris variable orbit parameters were compared to equivalent values computed from precise ephemerides. A typical pass involved the comparison of eight values for ΔE and Δa and four values for η. From these comparisons it was concluded that the operational and precise trajectories are closely parallel, that is for each pass the set of differences for each of the three available orbit parameters is well described by a mean value (the bias between the precise and operational ephemerides). The scattering about such mean values is small compared to the resolution of the operational ephemeris. The figures shown in Table I are the RMS values of these means or biases.

Operational ephemerides are computed and injected twice daily for each satellite. If a user happens to be tracking a satellite at the time such an injection occurs, he receives both the old, or stale, and the new, or fresh, operational ephemerides for that pass. In comparing these two sets of parameters we are essentially comparing two long arcs, each computed using the same force model, each fitted to 36 hours of tracking data from the same four stations. However only 24 hours of these data are common to both arcs, the fresh arc being fitted to 12 hours fresher data than the stale arc. Hence in this comparison we eliminate the differences in force model and tracking station configuration which were involved in the previous comparison. We should then obtain some measure of the extrapolation error, that is how well the drag and radiation pressure forces in particular can be predicted. Comparing the two trajectories as before, the RMS values of the single pass biases are shown in Table I. We can interpret the precise/operational and fresh/stale comparisons as indicating the contribution of surface force modelling errors to be 14 metres, geopotential modelling errors 17 metres, and rounding errors 6 metres (Wells, 1974).

In the third orbit error investigation, 2,877 pairs of Guier plane coordinates obtained from passes tracked at eight stations, using the operational ephemerides, were treated as statistical samples. The sample standard deviations of the along track coordinate was 39 metres, and for the slant range coordinate (which samples both the radial and out of plane biases) 12 metres. The corresponding best fitting normal distributions had standard deviations of 16 and 9 metres respectively. These results are more consistent with the fresh/stale comparison than the precise/operational comparison.

Based on these three studies values of 26, 5 and 10 metres respectively were adopted for the standard deviations of the orbit biases in the along track, radial and out of plane directions. These values were adopted because they correspond to 2, 0.5 and 1 broadcast units respectively.

Satellite positions can be computed from the operational ephemeris only for even minutes of Universal time. If Doppler integration intervals shorter than two minutes are to be used, positions in between the two-minute broadcast positions must be used. There are many ways of computing such positions.

TABLE I

Operational Ephemeris Accuracy

	Passes	ΔE	Δa	η	RMS
Comparison with precise ephemeris					
1970 data	99	31 m	4 m	11m	33m
1972 data	126	20	4	11	23
Comparison between fresh and stale orbits					
1973 data	94	13	2	6	14
Guier plane navigation					
Sample std dev	2877	39	12 (Δa, η combined)		
Best fit normal std dev		16	9 (Δa, η combined)		
Adopted std dev		26	5	10	

Here we treat this as a problem in least squares approximation. For each satellite we want to best fit curves, in the least squares sense, to each of the three sets of variable orbit parameters. In order to do this we must first choose appropriate base functions, that is base functions which closely represent the shape of the functions we wish to approximate, which we now consider.

Relating the operational ephemeris parameters to the parameters which describe a linearly perturbed Keplerian orbit (Kaula, 1966), we find that several of the perturbations contained in the variable orbit parameters have frequencies close to twice the orbit frequency, examples are nonlinear oblateness, direct and indirect lunar and solar, and air drag effects. Others such as solar radiation pressure (Brouwer, 1963), and second order tesseral harmonics (Anderle, personal communication) produce longer period perturbations which, over the duration of one pass (20 minutes) can be represented by a linear trend. Thus, including the orbit biases already discussed, we have four candidates for appropriate base functions Φ.

$$\Phi = [1, \cos 2\,nt, \sin 2\,nt, t] \qquad (2)$$

where n is the satellite mean motion and t is time.

To test the appropriateness of these base functions, the variable orbit parameters from a series of 125 consecutive passes, computed from precise ephemerides, were treated as three time series, and analyzed in two ways (Wells, 1974). In the first case least squares spectral analyses of the first few hundred values in the time series were performed, and in the second case the values from the entire time series were superposed as a function of time since perigee passage. The least squares spectrum of a time series is derived from the sequence of residual norms obtained from the best fit, for each spectral frequency ω of the base functions sin ωt and cos ωt, to the time series (Vaníček, 1971). The results of these two analyses are shown in Figure 2. They confirm the appropriateness of the trigonometric functions in Equation 2.

As a confirmation of the entire set, Φ was fitted to about 1,000 sets of operational ephemerides, and 200 sets of precise ephemerides. It was found that Φ can be fitted to both the precise and operational ephemerides to within the respective roundoff errors of 0.3 and 3 metres respectively. When Φ is fitted to precise ephemeris *coordinates*, $\dot{\Phi}$ then also fits precise ephemeris *velocities* to within ten times the roundoff error of 0.3 mm/s. The biases between the operational and precise ephemerides and the biases between the approximant to the operational ephemeris and the precise ephemeris are essentially unchanged.

If the operational ephemeris (or any other ephemeris) is assumed to perfectly model the actual satellite trajectory, then in the solution for receiver coordinates, any errors in the ephemeris will result in a spurious increase in the size of the observation residuals, and be absorbed into (bias) the solution. On the other hand it is possible to consider the orbit as only approximately modelled by the ephemeris. If we can assume that the differences between the trajectory and ephemeris have zero mean (that is over many passes we assume the ephemeris errors are balanced in sign), then the ephemeris errors can be represented by an appropriately chosen covariance matrix. Based on the results just presented, the variable orbit parameters can be replaced by the polynomial coefficients of the four adopted base functions, with no appreciable decrease in accuracy. The covariance matrix of the operational ephemeris then becomes the covariance matrix of these polynomial coefficients. We have also found that the differences between the precise and operational ephemerides is well represented by three biases, for which standard deviations have been given. Since one of the adopted base functions is a bias, we can then assign the adopted standard deviations to each of the three bias terms, and zero standard deviations to the other polynomial coefficients. Using this procedure the satellite orbit is allowed to "relax" parallel to itself during the transformation to solution space, consistent with the relative magnitude of the covariance matrices for the orbit and the a priori solution vector.

Refraction

In travelling from satellite to receiver, Transit signals pass through both the earth's ionosphere and troposphere, and are refracted by both, but in different ways. Ionospheric refraction is the more serious, so that Doppler measurements are made at two coherent frequencies, from which a first order correction can be made, since in the ionosphere, refraction is frequency dependent (Weiffenbach, 1967). Given a model for the vertical profile of tropospheric refractivity, a correction can be computed from either observed or average surface weather conditions; several such models exist, perhaps the most satisfactory being that of Hopfield (1969; 1971; 1972). For both ionospheric and tropospheric refraction, residual effects will remain even after these corrections have been made. Analogously to our treatment of the orbit biases, it is possible to account for these residual refractive effects by specifying an appropriate variance (Fell, 1975; Kouba, 1975).

When two receivers, separated by up to several hundred kilometres, observe the same satellite pass, we can expect that the effect of satellite orbit errors and uncorrected or residual refraction will be quite similar for both. This is the principle behind the technique of translocation; it states that provided identical Doppler integration periods are observed at both receivers, the relative position of one receiver with respect to the other can be determined more precisely than the absolute position of either (Westerfield and Worsley, 1966; Gloeckler, 1973). The results presented in Figure 3 indicate tropospheric and ionospheric refraction effects are strongly correlated for station separations of 100 km. Similar analyses, performed for station separations out to 1,100 km indicate that this correlation persists out to about 500 km.

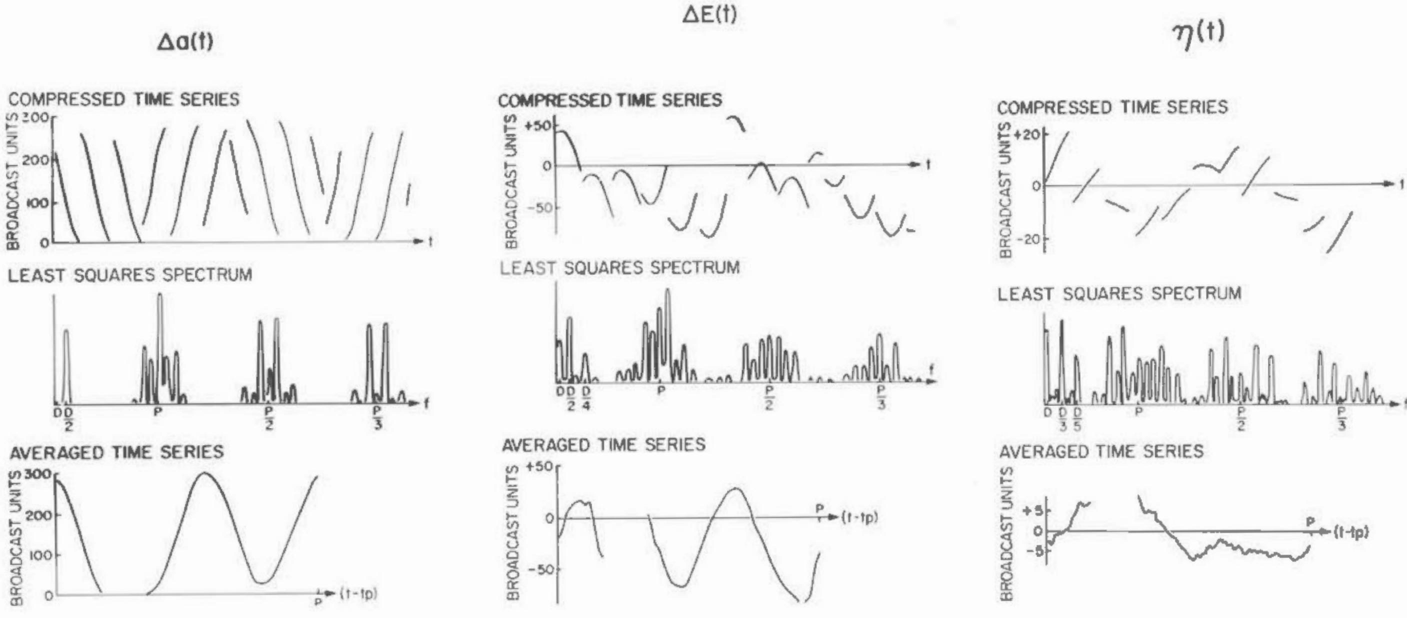

Figure 2. *a) Time series of the radial variable orbit parameter.*

At the top of the figure are the first few segments of the time series, omitting the gaps between segments (these gaps account for 94 per cent of the time series duration). In the centre of the figure is the least squares spectrum of that part of the time series shown above. This spectrum contains a barely discernible diurnal peak (D), a strong semi-diurnal peak (D/2), and families of seven peaks centred about P, P/2, P/3 etc., where P is the orbit period. The three pairs of side lobes in each family are the beat frequencies between the central peak and, moving outward, D, D/2 and D + D/2 respectively. The high frequency end of the spectrum is not shown, but continues this behavior. The principal periods are evidently D and P and harmonics thereof. At the bottom of the figure is the averaged time series obtained by superposing values from the entire time series of 125 segments as a function of time since perigee passage. It clearly indicates that the principal period is P/2.

b) Time series of the along track variable orbit parameter.

The least squares spectrum has a stronger D peak, and a D/4 peak as well, adding more side lobes to the P families. From the averaged time series, the principal period is again P/2.

c) Time series of the cross plane variable orbit parameter.

The least squares spectrum has a weak D/2 peak, strong D/3 and D/5 peaks, and even more side lobes in the P families. Evidently the cross plane parameter is not sufficiently correlated from pass to pass to reveal a principal period through averaging over many passes. However since we are interested in approximating its shape over just one pass at a time, and the least squares spectrum does not differ markedly from the previous two cases, it was assumed that its shape also had a principal period of P/2.

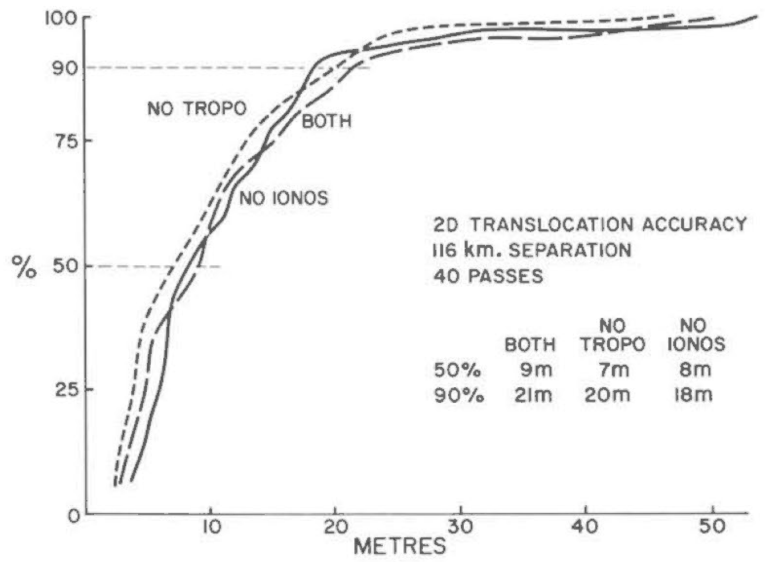

Figure 3. *Effect of refraction on two-dimensional translocation.*

From each of 40 sets of simultaneous single pass Doppler data a two-dimensional interstation vector was determined, and the median interstation vector subtracted from it. The lengths of these difference vectors were computed, ordered according to magnitude and the resulting cumulative percentage error curve plotted on this figure. This analysis was repeated three times; a) applying ionospheric and tropospheric refraction corrections to the Doppler data, b) omitting the tropospheric correction, and c) omitting the ionospheric correction. In comparison with these results, the cumulative percentage error curves for the *absolute* positions of each station had 50 per cent values of 20 m for (a), 30 m for (b) and 90 m for (c).

Receiver

The point for which we wish to determine coordinates is the phase centre of the receiving antenna. Ideally the Doppler integration epochs should occur when the satellite time signals arrive at this phase centre. However the propagation and processing of Doppler and timing signals through the receiver involves delays. The timing signal delay has a nominal value (which can be measured in the laboratory) of between 500 and 1,000 microseconds, depending on the model of receiver. However variations on this nominal value, called *timing jitter*, of as much as 100 microseconds can occur, again depending on the receiver used. After detection of this timing signal, the Doppler register is available to begin counting, however the integration epoch actually occurs at the next positive-going zero crossing of the Doppler beat frequency, up to 45 microseconds later.

There are a number of approaches which can be taken to account for these delays. One method called *constant bit rate* (used by Canadian Marconi receivers) is to establish fictitious satellite time marks corresponding to integration epochs of the measured Doppler counts, which are actually gated not by real satellite time marks, but by time marks generated by the local oscillator. Marconi reduces the zero crossing delay by multiplying the (f_g-f_s) beat frequency by 100.

A second method called *improved time recovery*, (used by receivers built for the Arctic Ice Dynamics Joint Experiment by Satellite Positioning Corporation), is to make additional Doppler and local clock timing measurements each integration interval to correct the measured Doppler counts, in this case gated by satellite time marks. Analysis of 19 passes tracked by two collocated Aidjex receivers, obtained cumulative 50 per cent errors of 2 m and 37 m respectively, when computed with and without improved time recovery.

Modelling assumptions

In the derivation of Equation 1 relating integrated Doppler counts to receiver coordinates, five generally invalid assumptions have been made: a) the integration epochs are coincident with arrival of time marks at the antenna phase centre; b) the ephemeris exactly defines the satellite position at the time of transmission of time marks; c) satellite signals propagate as if in a vacuum; d) there are no relativistic effects; and e) f_s and f_g are constant during a pass. We have just discussed methods of accounting for the invalidity of assumption (a). Assumption

(b) is invalid due to orbit biases, which we have seen can be taken into account. Also, lack of synchronization between ephemeris epochs and time signal epochs invalidates this assumption. However, such a lack of synchronization is indistinguishable from an along track orbit bias, the standard deviation of which we adopted as 26 metres, equivalent to 3,000 microseconds of satellite travel time. Any lack of synchronization between ephemeris and time signals is likely to be of the order of 50 microseconds (Anderle, 1974), only a few per cent of the along track orbit bias. The invalidity of assumption (c) can be corrected for as described above in the discussion on refraction. Assumption (d) ignores both special (time dilation) and general (gravitational red shift) relativistic effects. However their form is well known (Moller, 1952; Rindler, 1969) and they can be corrected for (Brown, 1970). It turns out that relativistic effects influence the determination of the local clock frequency, f_g, and not the receiver coordinates, and that this influence is about at the resolution with which f_g can be determined. The effect of assumption (e) is that a linear drift in either f_s or f_g of 10 parts in 10^{10} per day, over the 20 minute duration of a pass, will cause positioning errors of the order of one quarter of a metre. The long term drift of the satellite oscillators is of the order of 0.5 parts in 10^{10} per day. Local oscillator quality can vary widely, however high quality oscillators with stabilities of a few parts in 10^{10} per day can be obtained.

Summary

One of the basic aims of geodesy, if not the basic aim, is the determination of the geometry of the earth's physical surface, and the description of the earth's gravity field, in a common unambiguously defined coordinate system. In solving this problem, the limitations of classical geodesy have been overcome within the past decade or so, through the advent of geodetic satellites. The principle objective of satellite geodesy has been to establish such a unified geodetic reference system, in which are described station coordinates, the earth's gravity field, and the orbits of the geodetic satellites used. The original objective has now been more or less achieved (Henriksen and Mueller, 1974). Satellite and station positions can now be determined with an accuracy of about a metre and values for over 400 spherical harmonic coefficients of the earth's gravity field have been computed (Anderle, 1974). These spherical harmonics describe details of the gravity field with wavelengths as small as 2,000 km.

The current objectives of satellite geodesy are three refinements of the original objective; the determination of shorter wavelength gravity features; obtaining more accurate station positions (10 cm is an often quoted goal); and determining accurate station coordinates in "real time", which in this context means a few days.

Semi-dynamical Doppler satellite positioning now makes it possible to achieve this last objective. The result is to transfer the benefits of satellite geodesy to a much broader community of users.

Figures 4 and 5 describe two applications of semi-dynamical Doppler satellite positioning. A number of other applications are described elsewhere in these proceedings.

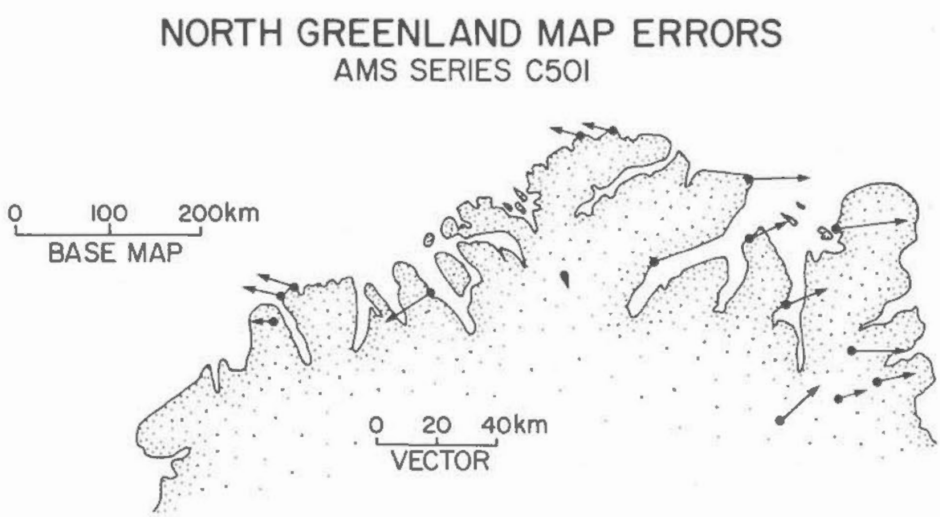

Figure 4. *North Greenland map errors.*

Results of an investigation of mapping errors caused by a lack of adequate ground control for the aerial photographs from which the maps had been compiled (Lillestrand *et al.*, 1968). Using a Transit receiver and other geodetic instruments, positioning errors of up to 20 km were found. The actual area of north Greenland is 8,000 km^2 greater than shown by the best maps then available. This application demonstrated the flexibility of this new geodetic tool, since with less than one week's notice, a shipboard receiver was taken from Halifax to northern Greenland and was producing useful geodetic data.

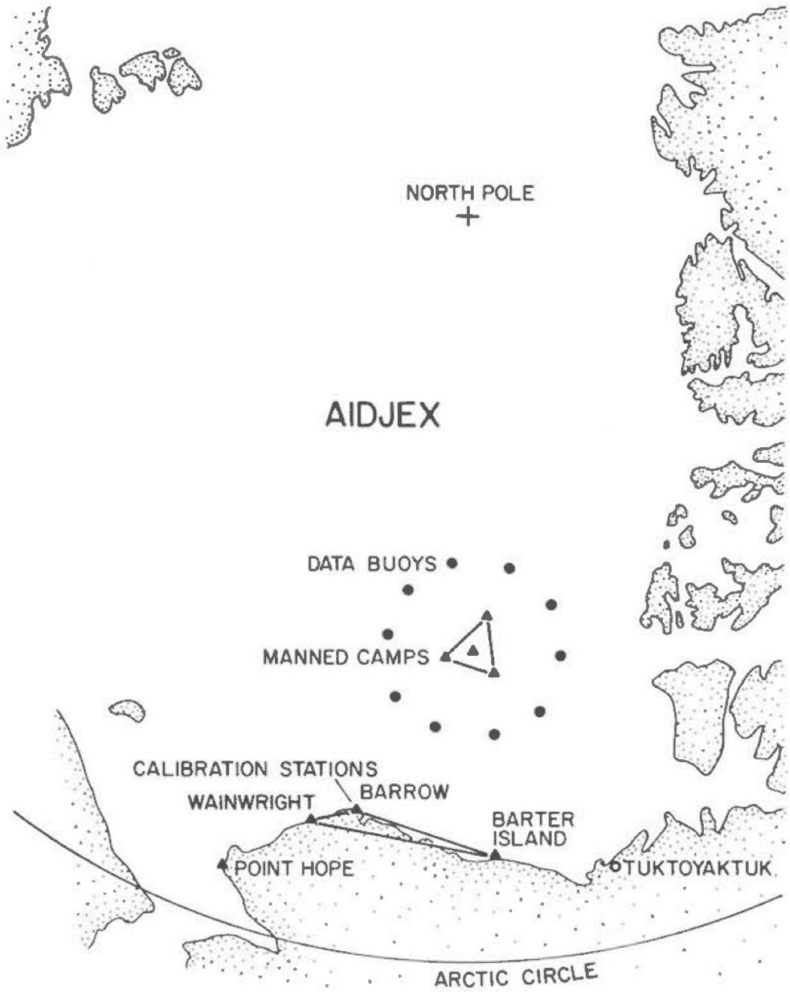

Figure 5. *The Arctic ice dynamics joint experiment.*

AIDJEX is a joint US/Canadian geophysical experiment on drifting sea ice in the Arctic Ocean which began in March 1975 and is scheduled to run for 14 months. The purpose of the experiment is to relate the large scale response of sea ice to its environment. An important component of this experiment is a semi-dynamical Doppler satellite positioning system consisting of receivers at each of four manned camps, and at each of ten unmanned data buoys (Thorndike, 1973; 1974). Environmental data from the data buoys and manned camps, plus motions of the data buoys derived from the Doppler data will be used to drive a numerical model relating the large scale sea ice stress and strain. Motions of the manned camps derived from Doppler data will be used to test this numerical model. All receivers are single channel, making no provision for ionospheric refraction correction, since the translocation technique will be used. The manned camp receivers have improved time recovery and a sophisticated computer-controlled satellite signal acquisition system. The manned camps are each equipped with two receivers for redundancy, and to provide azimuth measurements accurate to a few degrees using two antennas separated by 100 metres. Single pass relative positioning (translocation) accuracies of the order of 10 metres between camps are expected.

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Discussion

Q: What is the difference between the repeatability and the absolute error in positions determined from precise ephemeris?

A: Repeatability is about ±1 m and absolute error about ±1.5 m.

Polar motion program of the Earth Physics Branch Department of Energy, Mines and Resources

J. Popelar*

Abstract. The Polar Motion Group was established in 1973 as part of the newly organized Geodynamics Section within the Gravity Division to carry on studies of polar motion, earth's rotation and crustal plate movements. This long term program includes the operation of the two Canadian PZT observatories located near Ottawa and Calgary which have been accumulating astronomical data on latitude and longitude variations since 1951 and 1968 respectively. Both PZT which appear to be capable of detecting secular motions greater than 10 cm, contribute data regularly to the international time (BIH) and polar motion (IPMS) services.

In 1974 two Doppler satellite tracking stations were collocated with the PZT observatories to compare polar motion as determined by the two independent techniques. It is hoped that the Doppler satellite method will increase the accuracy and time solution of pole determinations to facilitate studies of short-term variations of polar motion. A mini-computer system is being used to automate the satellite data acquisition, processing and communication. As part of the world-wide TRANET satellite tracking network the Doppler stations contribute data to the Dahlgren Polar Monitoring Service.

Résumé. Le Groupe du mouvement des pôles, constitué en 1973 et élément de la nouvelle Section de géodynamique de la Division de la gravité, a la mission d'étudier le mouvement des pôles, la rotation de la Terre et les mouvements des plaques de la croûte. Ce programme à long terme comporte l'utilisation des deux observatoires PZT canadiens, situés près d'Ottawa et de Calgary, qui enregistrent des données astronomiques sur les variations de longitude et de latitude depuis 1951 et 1968 respectivement. Les deux PZT peuvent détecter des mouvements séculaires, au-delà de 10 cm, et relèvent régulièrement des données pour les services internationaux du temps (BIH) et du mouvement des pôles (IPMS).

En 1974, les données de deux stations de poursuite des satellites de navigation Doppler ont été confrontées aux données des observatoires PZT pour comparer les mouvements polaires déterminés par deux techniques différentes. On espère que la méthode de navigation Doppler par satellite améliorera la précision et le temps de résolution des déterminations de position des pôles pour faciliter l'étude des variations à court terme des mouvements des pôles. Un système de mini-ordinateur est utilisé pour automatiser l'acquisition des données de satellites, le traitement et la transmission. Rattachées au réseau de poursuite de satellites mondial TRANET, les stations Doppler fournissent des données au Service de surveillance des pôles de Dahlgren.

Introduction

The Polar Motion Group was formed in 1973 as part of the newly organized Geodynamics Section within the Gravity Division to carry on studies of polar motion, earth's rotation and crustal plate movements. The group was organized around the well established PZT operations in Ottawa and Calgary which were attached to the geophysical divisions when the Earth Physics Branch was formed in 1970 after the dissolution of the Dominion Observatory. The Polar Motion Group was to continue the PZT work which still contributes much excellent data on the global dynamics of the earth, and at the same time it is supposed to investigate and experiment with newer techniques which could eventually supersede the optical astronomical observations. Since the Polar Motion Group presently consists of only one reasearch scientist and four technical officers, two in Ottawa and two in Calgary, automation of routine observing procedures and data processing is of necessity an important aspect of the polar motion program.

*Gravity and Geodynamics Division, Earth Physics Branch, Department of Energy, Mines and Resources, Ottawa, Ontario, Canada.

PZT program

The PZT operation in Ottawa was set up in 1951 as part of the Positional Astronomy Division and has since been producing data nearly continuously (Thomson, 1955). During this period the PZT location was changed three times and the original instrument was replaced in 1968. Since 1970 the Ottawa PZT No. 2 has been located at Shirleys Bay approximately 16 km due west of its original site at the Dominion Observatory. With the exception of the first period between 1952 and 1955, when the scatter of observations was rather large due to changing operation procedures and the lack of an atomic standard for time comparisons, the Ottawa PZT observations of time and latitude provide excellent material for studies of polar motion and earth's rotation. Analyzing the PZT data accumulated between 1956 and 1970, Woolsey (1972) concluded that the observational noise in both coordinates is around 10 cm for averaging intervals of about 15 days. He clearly identified the Chandler term as the most prominent in the latitude observations with more than twice the amplitude of the annual term. For the time observations long-term variations prevail while the annual and semi-annual terms can be observed on the background of short-term variations.

Realizing the potential of precise PZT observations and in an attempt to eliminate biases between different PZT star catalogues which define the reference frame, the IAU strongly recommended in 1963 to locate new PZT sites on the latitudes of existing PZT observatories whenever possible. Following this recommendation a site for a new PZT was chosen at Priddis near Calgary on the same latitude as the Hertsmonceux PZT of the Royal Greenwich Observatory in England. The regular observations at Calgary commenced in 1968 using the PZT No. 1 from Ottawa and the Hertsmonceux star catalogue. The data obtained at Calgary from 1968 to 1972 contributed to a major revision of the star catalogue and the new catalogue was introduced at the beginning of 1975.

The nightly operations of both PZT are fully automated using an electro-mechanical system which has to be manually reset once a day. Although the measurement of PZT plates requires manual setting on program star images the data are automatically punched on cards for computer processing. The PZT results from both stations are transmitted weekly to the BIH in Paris and monthly to the IPMS in Mizusawa in a form of average time and latitude observations for star groups. Both stations also participated in the so called rapid service during critical phases of the Mariner series interplanetary flights when UT1 and pole position were evaluated daily from observations of only 14 BIH stations (Feissel, Guinot 1974).

The average night observations at Ottawa and Calgary together with reduced smoothed values of latitude and time are summarized in annual reports published in the Geodynamics Series Bulletins. The 1974 Ottawa PZT time results are shown in Figure 1 where the standard deviation of the average night UT2 from the smoothed curve is about 4 ms. Figure 2 shows the average night latitude variations at Ottawa in 1974; the standard deviation from the smoothed curve is about 0.″035. The scatter for the Calgary observations is about 40 per cent larger due to the shorter focal length of the telescope built to match the Hertsmonceux PZT and its higher latitude which affects the time observations (Figure 3). The smoothing is performed using the graduation method generalized by Vondrak (1969) for uneven data distribution with different weights. For this method different degrees of smoothing can be achieved by changing a single parameter, the so called coefficient of roughness, from one extreme when the smoothed curve is a least squares parabola, to the other when the smoothed curve passes through each observation while minimizing the sum of third differences. The coefficient of roughness can be varied to produce a smoothed curve with a certain standard deviation. Figure 4 shows the same time observations at Calgary with a standard deviation from the smoothed curve reduced by 1 ms (from 6 to 5 ms). How much of the variations are due to observing conditions (meteorological or instrumental) and what is due to short term variations in the earth's rotation and polar motion remains to be determined. However, it has been generally accepted that short term variations as well as major changes particularly in the rate of rotation do exist although their causes are not fully understood. A significant change in the earth's rate of rotation occurred at the end of 1973 as shown in Figure 5; the change in the slope of UT2 against UTC amounts to about 0.5 ms per day. A similar change but in the opposite direction took place between 1964 and 1966 (Woolsey, 1972).

Recent developments in polar motion studies

Greater interest particularly in irregular changes in polar motion was stimulated by the hypothesis put forward by Smylie and Mansinha (1967) relating such changes to major earthquakes. Disadvantages of the optical astronomical observations to detect positively abrupt changes in the polar motion are obvious: only night observations under favourable weather conditions are possible and data from a number of observatories are necessary to determine the pole position; the processing itself tends to suppress any abrupt fluctuations. In 1969, Anderle and Beuglass (1970) found that polar motion could be determined by analysis of Doppler satellite observations. A difference between the true and assumed pole position produces a systematic effect on along track residuals of satellites in polar orbit. The period of this systematic effect is 24 hours with maxima occurring at the times when the meridian plane containing the true and assumed poles coincides with the satellite orbital plane and nulls when these planes are perpendicular to each other. This effect can be evaluated from data of a single station providing the distribution of satellites is favourable. Smith *et al.* (1970) reported on a method using laser satellite tracking to determine polar motion effect on station latitude by measuring the apparent variations in the inclination of the satellite orbit. At the same time very promising results for polar motion and UT1 determinations were reported using ratio astronomical methods, VLBI in particular (Gold, 1967; Shapiro *et al.*, 1974).

Doppler satellite tracking program

These developments led in 1971 to the formation of a committee consisting of R.W. Tanner and R.I. Walcott to study possibilities of using some of the new methods for geodynamic studies to complement the existing PZT operations of the Earth Physics Branch. After extensive consultations the committee found Doppler satellite tracking as the best immediate prospect for a satellite geodynamics program. This was supported by the resolution of the IAU Symposium on Earth's Rotation in 1971 which strongly recommended collocation of Doppler satellite tracking equipment with existing observatories participating in the international time and polar motion services. Such collocations were negotiated for other sites, Misuzawa and Brussels, and Ottawa and Calgary were considered as important additions because of their locations and rather prominent position in the BIH and IPMS systems. More important support came from the NWL which was responsible for data processing and pole determinations for the world-wide TRANET satellite tracking network. The Doppler satellite tracking equipment, personnel training as well as close cooperation in data processing and evaluation have been offered in exchange for data. This was rather important considering the high costs of equipment and

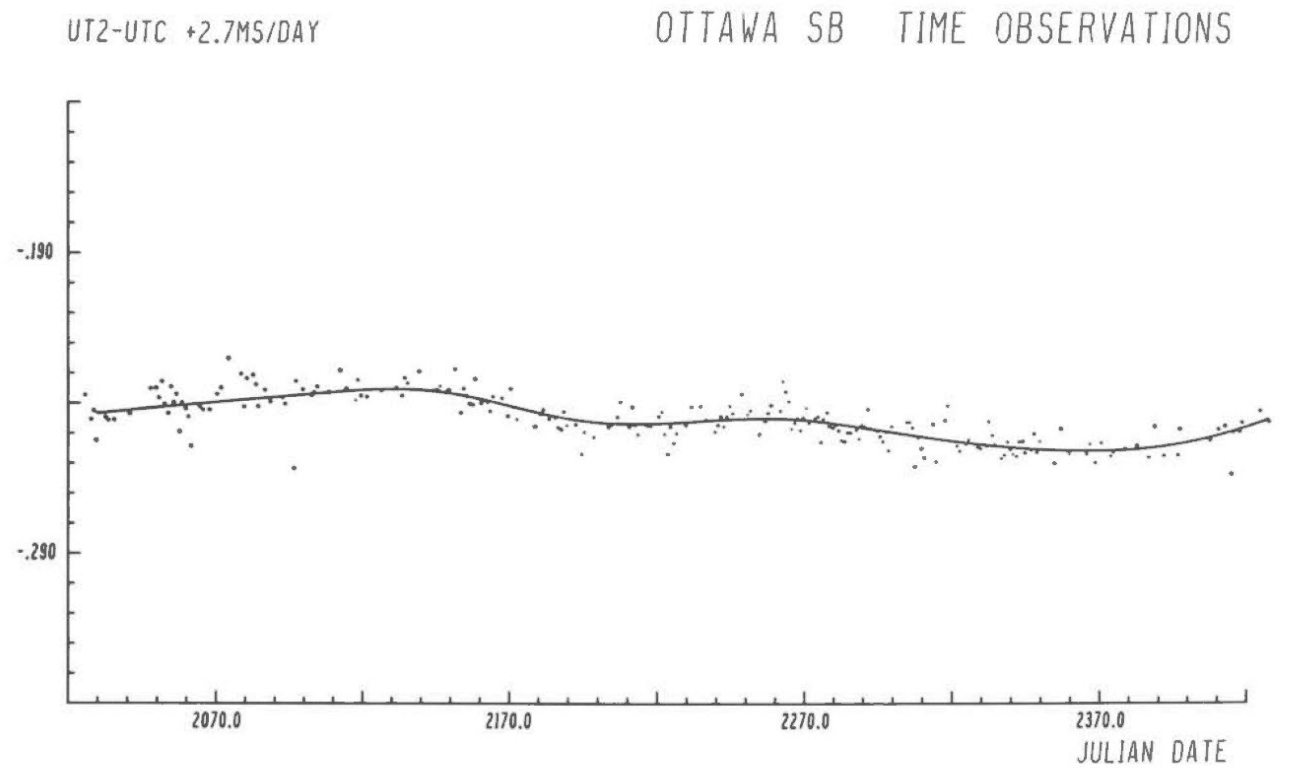

Figure 1. Individual night observations of time with the Ottawa PZT in 1974 and the smoothed curve for UT2 − UTC + 2.7 ms/day (STD 4.1 ms; UTC step adjustments removed).

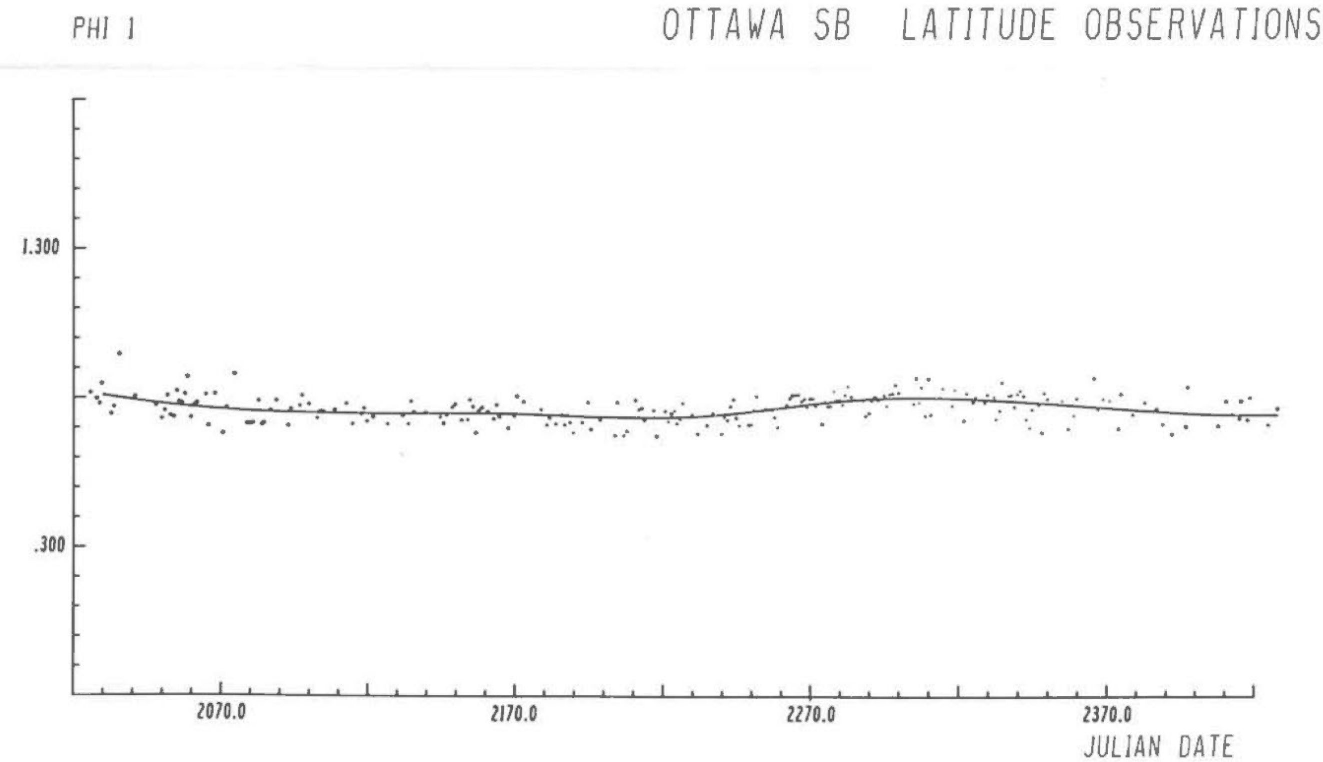

Figure 2. Individual night observations of latitude with the Ottawa PZT in 1974 and the smoothed curve for PHI 1 (latitude corrected for the 1974 BIH pole positions; STD 0."035).

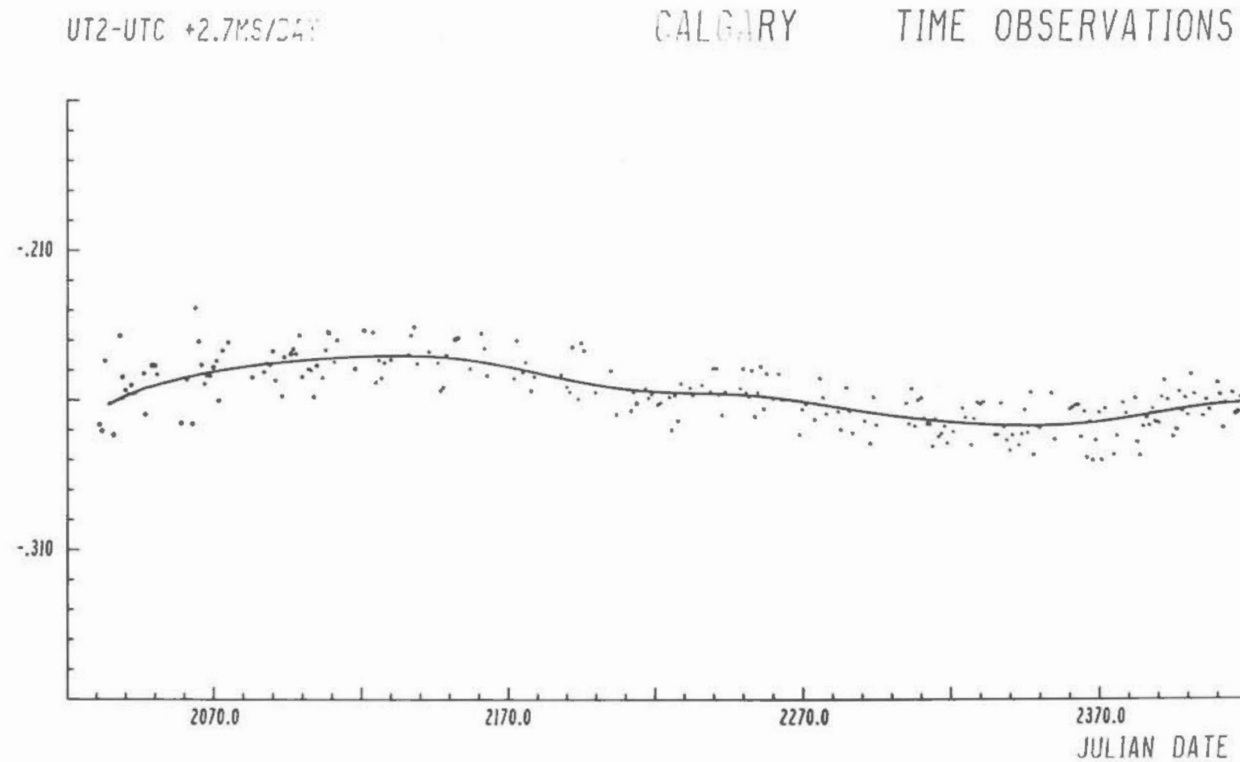

Figure 3. Individual night observations of time with the Calgary PZT in 1974 and the smoothed curve for UT2 – UTC + 2.7 ms/day (STD 6.1 ms; UTC step adjustments removed).

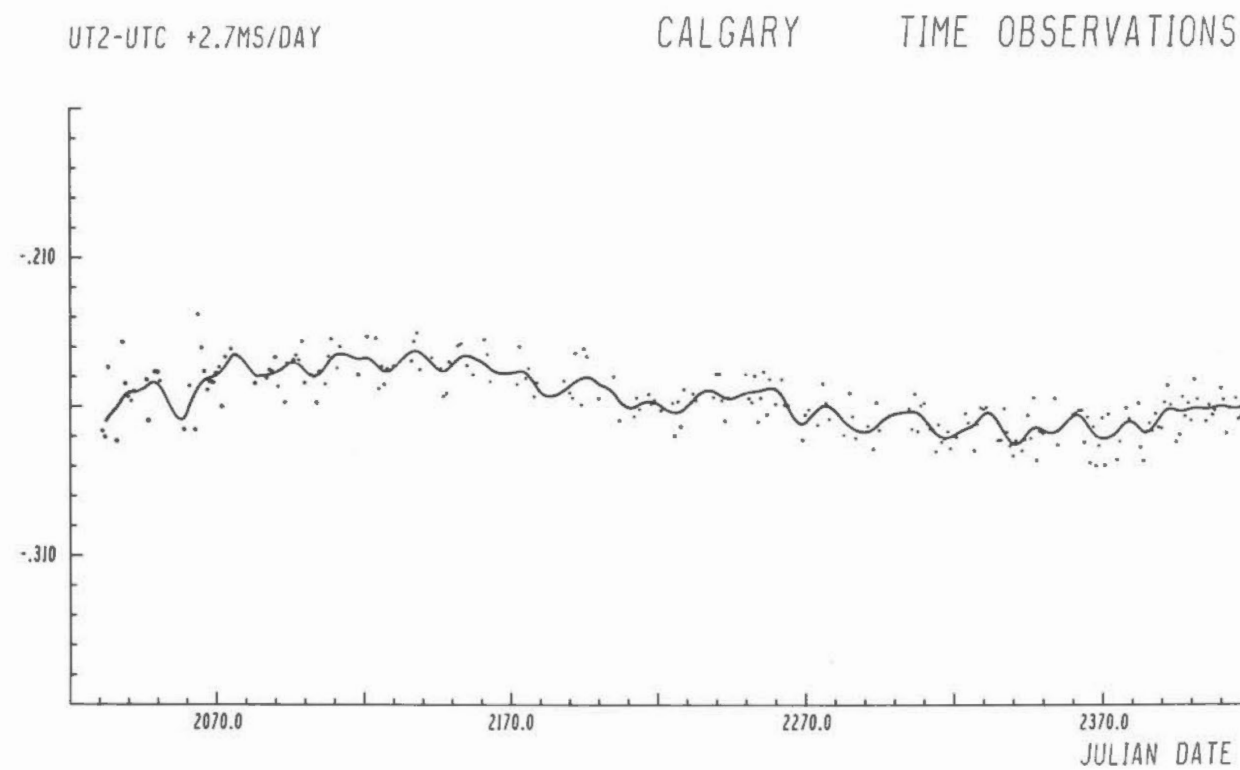

Figure 4. Individual night observations of time with the Calgary PZT in 1974 and the smoothed curve for UT2 – UTC + 2.7 ms/day using an increased coefficient of roughness (STD 5.1 ms; UTC step adjustments removed).

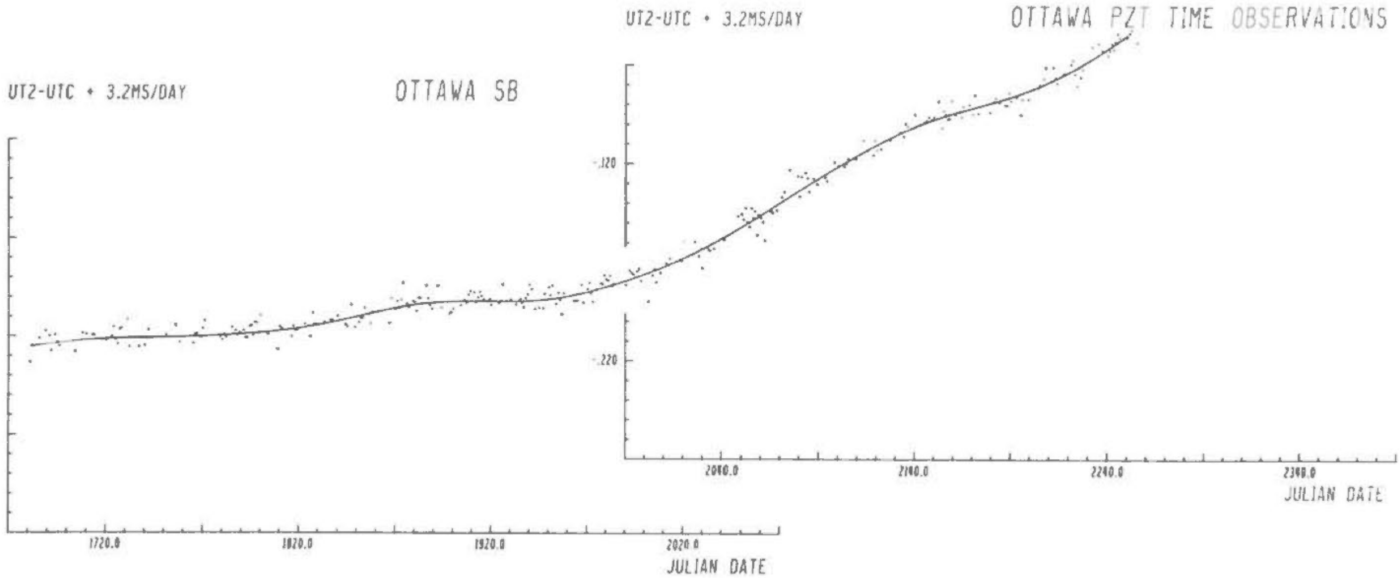

Figure 5. Individual night observations of time with the Ottawa PZT in 1973 and 1974.

operation. The agreement covering the loan, operation and maintenance of Doppler satellite tracking stations between the U.S. Defense Mapping Agency Topographic Center and the Earth Physics Branch was reached in January 1974. In April 1974 a geoceiver was set up at Calgary and a TRANET van was brought to Ottawa and installed in September.

The Doppler satellite tracking is a 24-hour all weather operation producing a large amount of raw data. It has been apparent from the very beginning that without proper automation of the Doppler stations one man can never acquire and manually process the satellite data to meet daily transmissions to the Satellite Control Center (SCC) at the John Hopkins University in Maryland. Moreover, any independent data processing and analysis would be impossible. The Doppler satellite operations were therefore carefully analyzed and after surveying the market of automatic measurement instruments, a real-time data acquisition processing and communication system was procured toward the end of 1974.

The system as presently constituted (Figure 6) is based on a Hewlett-Packard distributed mini-computer system with a central unit in Ottawa and a terminal in Calgary. The central unit consists of a 24K HP2100 mini-computer operating under multiprogramming real-time executive with file manager, paper tape reader, 5 magabyte cartridge disc drive, 30 cps console printer and card reader. The distributed system package provides on-line communication between the terminal and central computers giving the terminal full access to all the resources of the central system such as data storage and file manipulation on the central disc, program scheduling on the central computer and terminal program development and storage. The terminal system which consists of an 8K HP2100 mini-computer, paper tape reader, console printer and card reader, operates under terminal communication executive as a sophisticated input-output device with its own programming capability. Both mini-computers are equipped with interface cards to enable direct real-time data input from the satellite tracking stations and eventually basic computer control of the stations. The latter will probably be limited to a simple switching function for the geoceiver whereas the modular design of the TRANET station is more suitable for implementation of additional control functions. The mini-computers will eventually be directly interfaced with automatic weather stations so that meteorological parameters (temperature, pressure and humidity) can be automatically recorded during satellite passes.

Immediately after a satellite pass the data will be checked for internal consistency and identified by comparison with satellite predictions before being temporarily stored on the central disc. Another program will prepare a uniform observation file for permanent storage and select passes for transmission to the SCC as requested.

The HP Remote Data Transmission System in connection with a 2,000 bps data phone enables the central system to emulate an IBM 2780 terminal to any large computer installation supporting a dial-up port using the 2780 protocol. This system is operational and will be used to transfer the observation file onto high density magnetic tapes for permanent storage as well for batch job processing.

Since the SCC supports only teletype communication a buffered telex interface has to be used for the daily data transmissions of requested passes from the central computer disc to the SCC.

At the present time most of the hardware has been assembled with the exception of the weather stations. A concrete pier has been built for the geoceiver antennae on the PZT observatory grounds at Calgary and the TRANET antennae is presently mounted on the roof of the Geophysical Building at the old Dominion Observatory grounds in Ottawa.

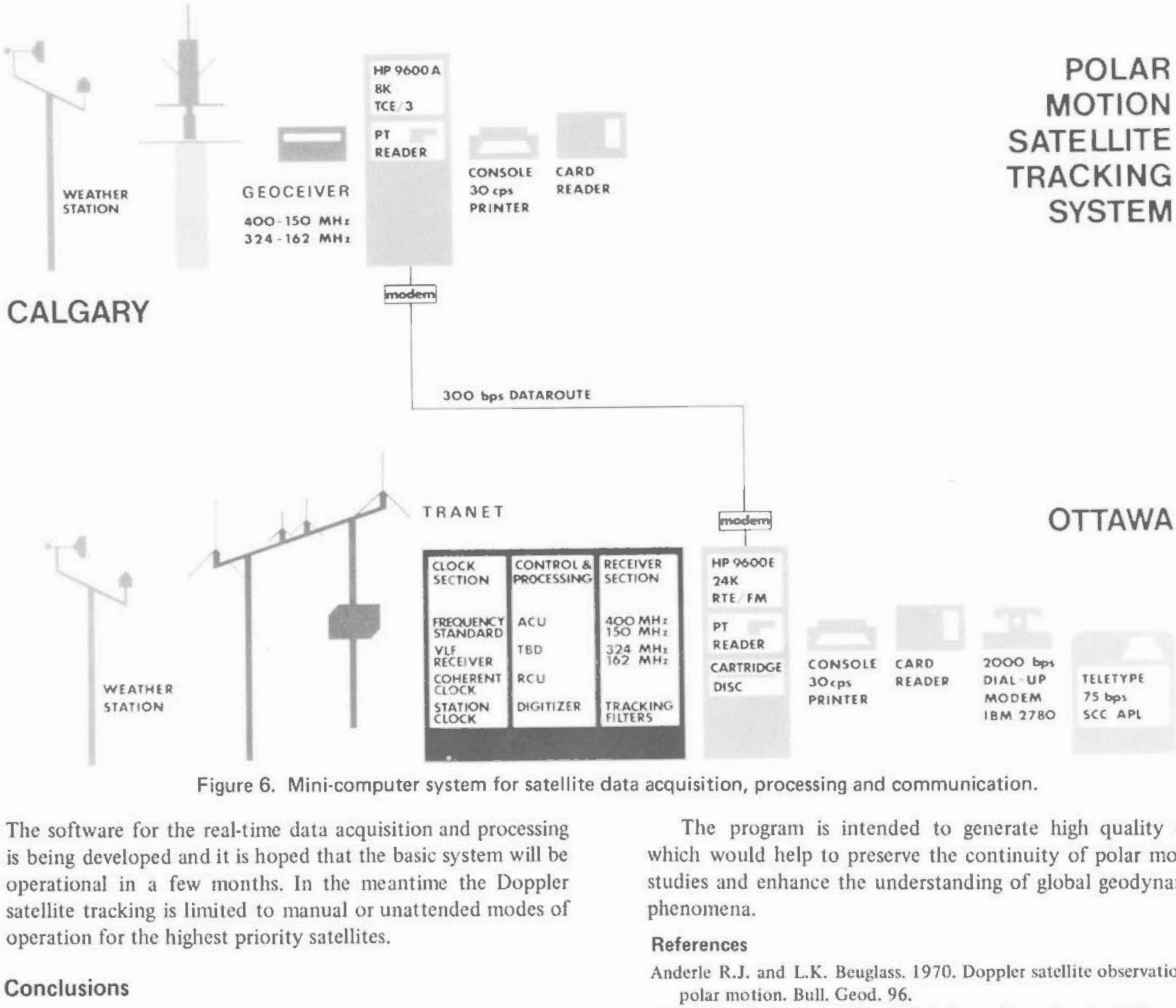

Figure 6. Mini-computer system for satellite data acquisition, processing and communication.

The software for the real-time data acquisition and processing is being developed and it is hoped that the basic system will be operational in a few months. In the meantime the Doppler satellite tracking is limited to manual or unattended modes of operation for the highest priority satellites.

Conclusions

The primary objectives of the Polar Motion Group of the Earth Physics Branch can be summarized as follows:

1. Continuous observation of variations of the station coordinates at Ottawa and Calgary using the astronomical and Doppler satellite methods to study polar motion and analyse differences between the two systems; since the error sources in PZT and Doppler results are largely independent any common changes in polar motion can be considered to be due to physical rather than computational sources.

2. To use the Doppler satellite method to increase the accuracy and time resolution of pole determinations in order to facilitate studies of short-term variations of polar motion.

3. To study variations in the rate of rotation of the earth.

4. To provide permanent reference points for satellite and astronomical observations and for long term studies of crustal plate movements.

The program is intended to generate high quality data which would help to preserve the continuity of polar motion studies and enhance the understanding of global geodynamics phenomena.

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Discussion

Q: How would you explain the large deviations of your PZT time and latitude observations from the average smoothed values for some nights?

A: They are most likely due to abnormal atmospheric conditions.

Q: What is the Canadian attitude towards replacing the classical methods by some modern ones?

A: We would give up the PZT if a more modern technique, which we could afford, would provide better data for the global geodynamic studies. The Doppler satellite method is very suitable for monitoring polar motion but presently it does not provide data on the earth's rate of rotation comparable to PZT. We also recognize the need for overlapping period of observations before abandoning the classical methods in order to preserve the continuity of the polar motion and earth's rotation studies.

Q: What is the accuracy of your satellite observations?

A: We have not done an analysis of the accuracy yet.

GEOS-C and measurement of the Earth tide

D.R. Bower*

Abstract. The Earth tide parameters of interest can be measured in principle through analysis of the orbital dynamics of a satellite or through analysis of the radial displacement of the Earth's surface as revealed by difference-ranging measurements to the satellite from two or more laser stations. The first method makes use of multi-pass orbital arcs and requires knowledge of the global ocean tide and other long term perturbing influences. The second method, because the effect of the ocean tide needs to be evaluated only at the laser sites, requires knowledge of mainly regional tides and shorter orbital arcs should be usable. Because there is little immediate prospect of the global ocean tide being determined to the accuracy required by the first method, a Canadian GEOS–C investigating team has chosen the second method to test the feasibility of determining the Love number h_2 to an accuracy of 1 per cent and the solid earth phase lag to 0.5°. Assuming an accuracy in difference-ranging of 1 cm from two sites in the GEOS–C calibration area, 650 independent measurements will be required if the M_2 constituent (period 17.2 days) is used and 1,500 measurements will be needed if the K_1 constituent (period 132 days) is used.

Résumé. Les paramètres d'intérêt de marée terrestre peuvent être en principe mesurés par l'analyse de la dynamique orbitale d'un satellite ou l'analyse du déplacement radial de la surface terrestre révélé par les mesures télémétriques de la distance du satellite, du point de deux stations laser ou plus. La première méthode est à partir d'arcs orbitaux de plusieurs passages et nécessite la connaissance de la marée océanique globale et autres influences perturbatrices à long terme. Dans la seconde, du fait que l'évaluation de l'effet de la marée océanique n'est nécessaire qu'à la station du laser, le calcul est à partir des données de marées surtout régionales et des arcs orbitaux plus courts peuvent être utilisables. Étant peut probable que la marée océanique globale puisse être déterminée avec la précision exigée dans la première méthode, une équipe canadienne GEOS–C a choisi la seconde pour tester la faisabilité d'une détermination du nombre de Love h_2, avec une précision de 1%, et le déphasage en arrière de la marée terrestre de $0{,}5^\circ$. Une différence de 1 cm dans la mesure télémétrique de la distance, à partir de deux points de la zone d'étalonnage de GEOS–C, entraînera la prise de 650 mesures différentes, avec l'emploi de la composante M_2 (période 17,2 jours) et 1 500 avec la composante K_1 (période 132 jours).

Introduction

The aim of earth tide research has been to confirm the theory of earth tides as expressed by the Love numbers h, k and ℓ, the load-deformation coefficients h', k' and the solid-earth tidal phase lag ϵ. Earth tide measurement techniques have reached the stage where the effect on the measurements of the ocean tides is the principal factor limiting the accuracy with which the Love numbers and the phase lag can be determined. This accuracy is below that needed to distinguish between the various earth models for which Love numbers (Alsop and Kuo, 1964) and load deformation coefficients (Farrell, 1972) have been calculated, or to reveal realistic Q values for the solid earth.

The ocean tide effect can be predicted if the tide is known accurately and there is some hope that this will be the case in a few years through progress in satellite altimetry over the oceans. However, another problem has recently become apparent. Both tidal-tilt measurements and tidal-gravity measurements are required before Love numbers can be calculated since separately they depend on the combinations $(\ell - h + k)$ and $(\ell + h - 3/2k)$, and it has recently been demonstrated that tilt measurements are susceptible to gross errors due to tilt-strain coupling effects. These effects are peculiar to the measuring site and depend on the configuration of the instrument cavity (King and Bilham, 1973) and the nature of the local topography (Harrison, 1974). It is not yet clear to what degree of accuracy the effects can be calculated or whether it is feasible to select sites where they would be negligible. Thus even when the ocean tide influence is accounted for surface-derived estimates of the Love numbers may not be representative of global conditions. These are the reasons why earth tide researchers are looking to satellites as the means of ultimately determining the Love numbers and the phase lag.

The Earth tide may be thought of as a component of the Earth's gravitational field which rotates with the attracting body rather than with the Earth. In principle therefore the amplitude and phase distribution of the tide can be determined through a study of its perturbing effect on the orbit of a satellite. This is the method which will be used by several investigating teams working with the newest NASA satellite, GEOS–C. A team including members from both the Earth Physics Branch and the University of New Brunswick will use a different approach. They will attempt to study the Earth tide through the geometric effect on the Earth as revealed by laser ranging measurements to the satellite.

GEOS–C is the forerunner of a new series of geodetic satellites intended to support experiments in solid earth physics and oceanography. The nominal orbit parameters are as follows: Mean altitude–843 km; Inclination–115°; Eccentricity–0.000; Period–101.8 min.

*Gravity and Geodynamics Division, Earth Physics Branch, Department of Energy, Mines and Resources, Ottawa, Ontario, Canada.

Theory

Kaula has developed the theory for the motion of a satellite in the gravitational field of the Earth (Kaula, 1966) and for the perturbations due to the attraction of the Moon and the tidally-displaced masses of the Earth (Kaula, 1964). Lambeck *et al.* (1974) have extended the theory to predict the perturbations due to a global ocean tide represented by a spherical harmonic expansion such as that of Hendershott (Hendershott and Munk, 1970) and he has analyzed the effect of both Earth and ocean tides on the orbit of GEOS–2 and other satellites.

Theory – Perturbation due to the non-central field

The six numbers (a, e, i, M, ω, Ω) (all symbols are listed at the end) describing the Keplerian ellipse followed by a satellite will be affected by the non-central gravitational potential of the Earth according to:

$$\dot{a} = \frac{2}{ma}\frac{\partial R}{\partial M} \tag{1}$$

$$\dot{e} = \frac{1-e^2}{ma^2e}\frac{\partial R}{\partial M} - \frac{(1-e^2)^{\frac{1}{2}}}{ma^2e}\frac{\partial R}{\partial \omega}$$

$$\dot{\omega} = \frac{-\cos i}{ma^2(1-e^2)^{\frac{1}{2}}\sin i}\frac{\partial R}{\partial i} + \frac{(1-e^2)^{\frac{1}{2}}}{ma^2e}\frac{\partial R}{\partial e}$$

$$\dot{i} = \frac{\cos i}{ma^2(1-e^2)^{\frac{1}{2}}\sin i}\frac{\partial R}{\partial \omega} - \frac{1}{ma^2(1-e^2)^{\frac{1}{2}}\sin i}\frac{\partial R}{\partial \Omega}$$

$$\dot{\Omega} = \frac{1}{ma^2(1-e^2)^{\frac{1}{2}}\sin i}\frac{\partial R}{\partial i}$$

$$\dot{M} = m - \frac{1-e^2}{ma^2e}\frac{\partial R}{\partial e} - \frac{2}{ma}\frac{\partial R}{\partial a}$$

where the disturbing potential R is a complete solution of Laplace's equation with a particular term given by:

$$R_{\ell m} = \frac{\mu a_e^{\ell}}{\upsilon^{\ell+1}} P_{\ell m}(\sin\phi)(C_{\ell m}\cos m\lambda + S_{\ell m}\sin m\lambda) \tag{2}$$

Expressing the coordinates (r, ϕ ,λ) of the evaluation point in terms of the Keplerian elements of the satellite leads to:

$$R_{\ell m} = \frac{\mu a_e^{\ell}}{a^{\ell+1}} \sum_{p=0}^{\ell} F_{\ell mp}(i) \sum_{q=-\infty}^{\infty} G_{\ell pq}(e) S_{\ell mpq}(\omega, M, \Omega, \Theta) \tag{3}$$

Where:

$$S_{\ell mpq} = \begin{bmatrix} C_{\ell m} \\ -S_{\ell m}\end{bmatrix}_{\ell-m\ \text{odd}}^{\ell-m\ \text{even}} \cos[(\ell-2p)\omega + (\ell-2p+q)M + m(\Omega-\Theta)] + \begin{bmatrix} S_{\ell m} \\ C_{\ell m}\end{bmatrix}_{\ell-m\ \text{odd}}^{\ell-m\ \text{even}} \sin[(\ell-2p)\omega + (\ell-2p+q)M + m(\Omega-\Theta)] \tag{4}$$

and $F_{\ell mp}(i)$ and $G_{\ell pq}(e)$ are functions only of the inclination and eccentricity respectively and are listed by Kaula (1966). Because the term C_{20} in the expansion for the noncentral field of the Earth is so much larger than any other term the effects of all other disturbances can be considered as perturbations of the principal rates determined by equation (1) with $R = R_{20}$. Further, of all the terms in the series (3) those for which the coefficient of M is zero will be the principal ones since the others will pass through a complete cycle every revolution of the satellite; thus only the term R_{2010} is needed to determine the principal rates. Substituting this term in (1) yields:

$$\dot{M} = m - \frac{3m C_{20} a_e^2}{4(1-e^2)^{\frac{3}{2}} a^2}(3\cos^2 i - 1) \tag{5}$$

$$\dot{\omega} = \frac{3m C_{20} a_e^2}{4(1-e^2)^2 a^2}(1-5\cos^2 i)$$

$$\dot{\Omega} = \frac{3m C_{20} a_e^2}{2(1-e^2)^2 a^2}\cos i$$

$$\dot{a} = \dot{e} = \dot{i} = 0$$

These rates can now be substituted in (4) and equations (1) solved for the perturbation due to other disturbing functions.

Theory – Perturbations due to the Earth tide

The potential due to a second body of mass m*, such as the Moon, evaluated at a point distant r from the centre of the Earth and angular distance S from the second body at a distance r* is given by:

$$R = \frac{\mu m^*}{m_e r^*}\sum_{\ell=2}^{\infty}\left(\frac{r}{r^*}\right)^{\ell} P_\ell(\cos S) \tag{6}$$

Transforming the coordinates of the 2nd body to Keplerian elements leads to:

$$R = \sum_{\ell mpq} R_{\ell mpq} \tag{7}$$

$$R_{\ell mpq} = B^*_{\ell m}\, C^*_{\ell mpq}\, r^{\ell} P_{\ell m}(\sin\phi) \begin{Bmatrix}\cos \\ \sin\end{Bmatrix}_{\ell-m\ \text{odd}}^{\ell-m\ \text{even}} \{\nu^*_{\ell mpq} - m(\lambda+\Theta)\} \tag{8}$$

$$B^*_{\ell m} = GM^* \frac{(\ell-m)!}{(\ell+m)!}(2-\delta om)$$

$$C^*_{\ell mpq} = \frac{1}{a_*^{\ell+1}} F_{\ell mp}(i^*) G_{\ell pq}(e^*)$$

$$\nu^*_{\ell mpq} = (\ell-2p)\omega^* + (\ell-2p+q)M^* + m\Omega^*$$

The evaluation is made at the point (r, ϕ, λ) and θ is the sidereal time of the reference meridian. This is the tide-raising potential. According to Love's theory all elastic deformations produced in a laterally-homogeneous Earth due to this potential can be expressed as the product of this potential and a simple combination of up to three numerical coefficients. These three coefficients are known as Love's numbers h, k and l. The various combinations required for radial displacements, for tidal gravity, strain, tilt and dilatation are given by Melchior (1966). The radial displacement is hR/g. The indirect potential due to the masses of the Earth which have been displaced by the tide-raising potential is equal to kR/g.

The phase of the solid-earth response to the tide-raising potential lags the potential by a small angle ϵ because of non-elastic effects within the Earth. This phase is related to the whole Earth Q by:

$$Q = \ell / \tan 2\epsilon$$

The disturbing potential acting on a satellite will be the sum of the tide-raising potential and the deformation potential evaluated at the satellite. After transforming to the Keplerian coordinates, a particular term of the part of the potential due to the deformation (the solid-earth tide) becomes:

$$T_{\ell mpq} = \kappa_\ell a_e^{2\ell+1} B^*_{\ell m} C^*_{\ell mpq} \sum_{h,j} C_{\ell mhj} \cos \left\{ \nu^*_{\ell mpq} - \epsilon_{\ell mpq} - \nu_{\ell mhj} \right\} \qquad (9)$$

where:

$$\nu_{\ell mhj} = (\ell - 2h)\omega + (\ell - 2h + j) M + m\Omega$$

Theory – Perturbation due to the Ocean tide

The global ocean tide can be represented by a variable surface load whose spherical harmonic expansion is (Hendershott and Munk, 1970):

$$\xi(\phi, 2) = \Sigma \xi_n^o (\phi, \lambda) \cos (2\pi m f t - \epsilon(\phi, \lambda)) \qquad (10)$$

The potential of such a gravitating layer is (Munk and MacDonald, 1960):

$$R_m = \frac{4\pi G a}{2m+1} \left(\frac{r}{a} \right)^m \xi_m \qquad (11)$$

This surface load will cause a deformation of the Earth and a new potential related, in a manner analogous to the derivation of Love's numbers, to the deforming potential R_n through a numerical coefficient which Munk and MacDonald (1960) have termed the load deformation coefficient k'_n, Lambeck *et al.* (1974) have expressed the sum of these two potentials in terms of the Keplerian elements of the satellite and has solved the equations of motion for the long-term perturbation of i and Ω for a number of satellites assuming several ocean tide models given by Hendershott (1972) and others.

The global ocean tide has not yet been accurately determined for any harmonic constituent owing to the lack of ocean tide observations, particularly at mid-ocean. The global solutions of Hendershott are theoretically the most satisfactory but various empirical charts are more accurate in particular areas.

The radial deformation of the Earth's surface, due to the load of the ocean tide and given by $h'_n R_n/g$, will be considered later in connection with its effect on the position of observing stations.

Predictions for GEOS–C

The periods of the principal tidal constituents as seen by GEOS–C and the corresponding long-term (longer than the orbital period) perturbations in the inclination of the satellite due to the solid-earth tide are presented in Table I. In the case of M_2 the estimated long-term perturbation due to the ocean tide as represented by the Hendershott tidal Model 1 is listed also. Also presented is the period of each constituent as seen by an Earth-fixed site, such as an Earth-tide recording station, and the radial displacement of the surface at a point in the calibration area assuming h = 0.638.

Lambeck *et al.* find in general that perturbations in both inclination and nodal rate due to this ocean tide model range between 10 per cent of the Earth-tide perturbation in the case of high inclination satellites such as GEOS–2, and 30 per cent in the case of low inclination satellites. They have found relative effects equal to about one half of this for the K_1 and 0_1 ocean tides but knowledge of these tides is rudimentary at this time. Their results for GEOS–2, which has an inclination of 105°, an eccentricity of 0.03 and perigee of 1,000 km, are similar to the estimates found here for GEOS–C.

It is interesting to note that the basic diurnal and semi-diurnal tidal frequencies as seen by an Earth-fixed station appear to the satellite to cover a range of periods from 10 days to over 1,500 days. This produces no advantage however with regard to resolving power, because the shift in frequency is the same for all neighbouring constituents and difference frequencies remain the same. The relative amplitude of the various constituents however may differ greatly from that seen by an Earth-fixed tidal gravity station and this may facilitate the separation of adjacent constituents.

Analysis of Earth tides by the gravimetric effect on satellites

The Love number k_2 and the solid-Earth phase lag have been estimated from satellite orbit perturbations in several studies but Lambeck *et al.* (1974) was the first to attempt to correct for the effects of the ocean tide. Table II is a summary, presented by Lambeck *et al.*, of the results obtained by various authors.

TABLE I

Predictions for GEOS–C

Name	Doodson Argument	Period (GEOS–C)	Period (earth-fixed)	Δi Solid Earth (GEOS–C)	Displace Tide at 36°N	Δi Ocean Tide (GEOS–C)
O_1	145555	15.3	1.076	.074	6.12	
π_1	162556	1529.7	1.006	.202	2.84	
P_1	163555	479.8	1.003		2.84	
S_1	164556	207.3	1.000		0.07	
K_1^m	165555	132.3	0.997	.608	13.49	
K_1^s	165555	132.3	0.997	.277		
ψ_1	166554	97.1	0.995		0.11	
φ_1	167555	76.1	0.992		0.19	
N_2	245655	10.6	0.527	.043	1.94	
M_2	255555	17.2	0.518	.363	10.15	.061
S_2	273555	103.6	0.500	1.000	4.72	
K_2^m	275555	66.1	0.499	.173	1.28	
K_2^s	275555	66.1	0.499			
		days	days	arc secs	cms	arc sec

It is difficult to summarize the accuracy of the various experiments referred to by Lambeck *et al.* because of the variety of measurement and analysis techniques used. Laser ranging is certainly required in the study of tidal perturbations and Smith *et al.* (1974) have estimated the accuracy of such systems since 1970 as between 30 and 40 cm although periodic trends with amplitudes of several metres are commonly seen in range residuals. The better results referred to by Lambeck *et al.* report residuals of this order and experimental errors in the determination of k_2 of about 10 per cent and a few degrees in phase. Smith has found that the large trends observed are largely dependent on the gravity field used in the analysis and are likely to be removed in the near future by improved gravity models.

Lambeck *et al.* (1974) have shown that the ocean tide, not considered in the other studies, may introduce errors of 8 to 30 per cent in the determination of k_2. This corresponds to errors in the estimate of phase lag of between 4 and 24 degrees. Since fraction-of-a-degree accuracy is required for geophysical significance this is the main problem in the foreseeable future.

TABLE II

Summary of Results Obtained for k_2 and ϵ_2 using Orbit Perturbations

	Satellite	Tide	Element Analyzed	k_2 Observed	ϵ_2 Observed, deg.
Kozai	5900101	$K_1^m + K_1^s$	i	0.22	−5.5
	6000902	$K_1^m + K_1^s$		0.31	1.3
	6206001	$K_1^m + K_1^s$		0.32	+0.7
Newton	Mean of	M_2	i	0.27	1.5
	four polar		Ω	0.29	1.7
	satellites	S_2	i	0.34	1.6
			Ω	0.33	1.2
Douglas *et al.*	6508901	$K_1 + S_2$	i	0.22	
	6800201	$K_1 + S_2\ P_1$	i	0.31	
Smith *et al.*	6502801	$K_1 + S_2\ P_1$	i	0.25	3.2
Lambeck *et al.*	6800201	M_2	$i{+}\Omega$	0.29	9.0

(From Lambeck, 1974)

Analysis of Earth tides by the geometric effect on laser sites

The accuracy of laser ranging systems is approaching a point where it may be feasible to measure the Earth tide directly from the height changes of laser sites rather than indirectly from the gravitational effects on satellite orbits. That is, to measure the Love number h rather than the number k. This approach has the advantage of being less influenced by the ocean tide. Also, differential height changes of neighbouring laser sites can be used and since these can be obtained from simultaneous ranging measurements fitted to very short orbital arcs the results will be less influenced by long-term orbital dynamics. In fact, Escobal *et al.* (1973) have shown that if six simultaneous laser measurements are made the location of the laser stations and the satellite can be obtained purely from geometric considerations. The accuracy in this case approaches the laser accuracy.

Smith *et al.* (1974) have determined the 'ultimate accuracy' demonstrated by simultaneous laser tracking measurements by Goddard and Lake Seneca lasers of the BE–C satellite in 1970. 'Ultimate accuracy' in this instance refers to the internal accuracy of the system in determining inter-site

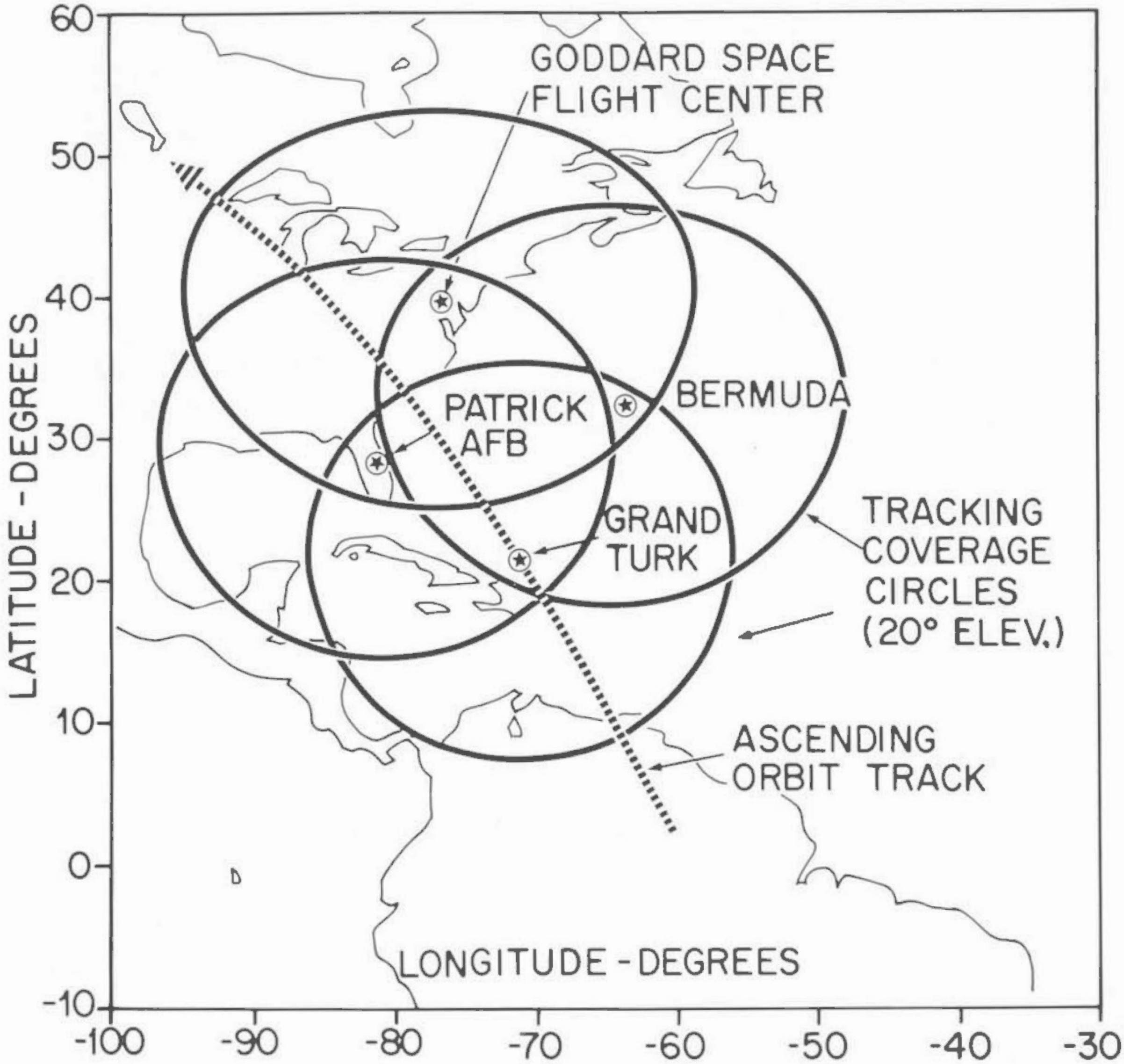

Figure 1. GEOS-C calibration area.

position and represents the ultimate capability of the technique if knowledge of everything affecting the orbit of the satellite were known perfectly. The ultimate accuracy in latitude was found to be 15 cm, in longitude about 8 cm, and 9 cm for height. They conclude that the technique has the capability of reaching the 1 cm level in all coordinates with the laser systems of 5–10 cm noise level in use today.

We can apply these estimates to predict the ultimate accuracy which might be attained in measuring h_2 by simultaneous ranging on GEOS–C from two laser stations situated in the GEOS–C calibration area (*see* Figure 1). The two sites which are separated by the greatest longitude are Bermuda and Patrick Air Force Base. The height difference between these two sites due to the M_2 Earth tide has an amplitude of 6.4 cm and a period of 17.2 days. The height difference due to the K_1 tide has an amplitude of 4.2 cm and a period of 132 days. Either tidal signal, if well separated from other constituents, can be determined with an accuracy of $\epsilon \sqrt{N/2}$ by least-squares fitting to N equi-spaced measurements of height differences with an rms error of ϵ. In order to determine h_2 to an accuracy of 1 per cent or the phase lag with an accuracy of 0.5 degree 650 independent measurements with an rms error of a cm would be required for the M2 tide and about 1,500 for the K_1 tide.

Measurements of h_2 at the K_1 frequency would be significant at the 5 per cent level because of its nearness to the core resonant frequency of Molodensky's Model I, (Molodensky, 1961). Figure 2 illustrates the behaviour of the Love number h_2 near the predicted frequency of core resonance. k_2 behaves similarly; the functional relations for Molodensky's Model I earth are

$$h = 0.6206 - 0.4711 \times 10^{-3}\, \beta$$
$$k = 0.3070 - 0.2384 \times 10^{-3}\, \beta$$

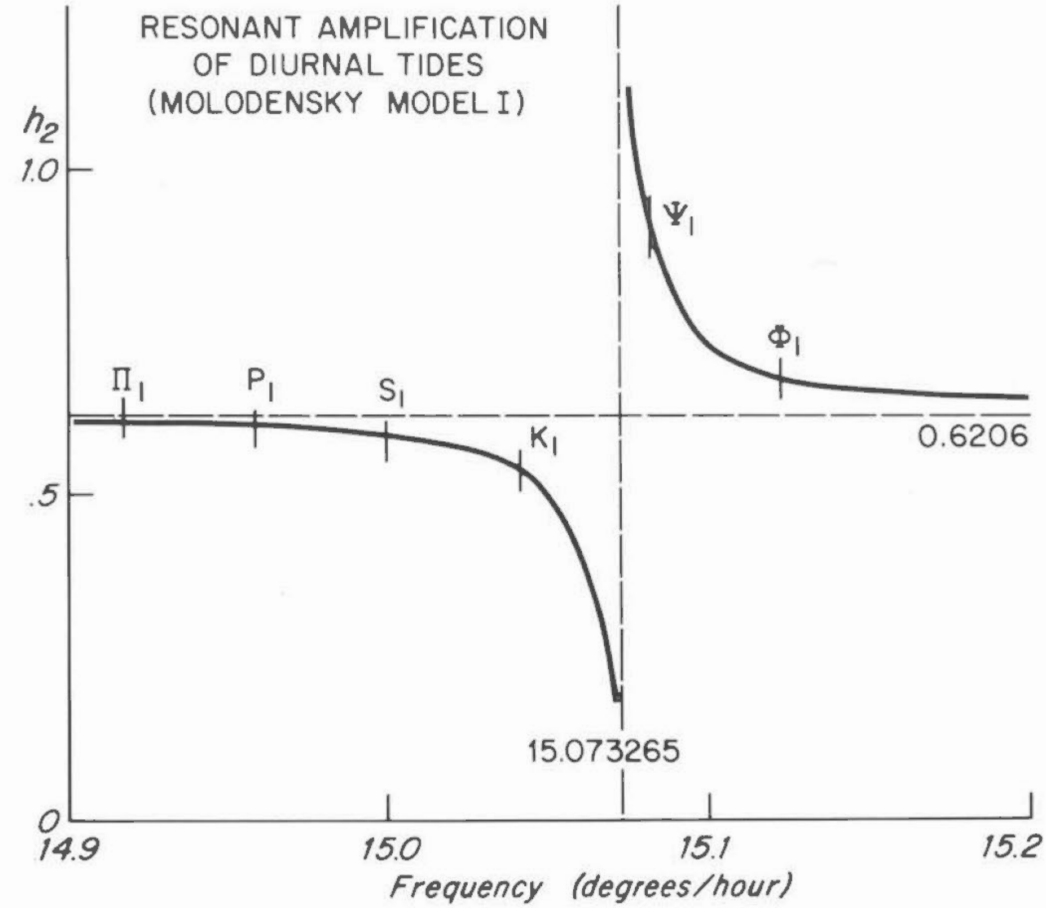

Figure 2

where;

$$\beta = \frac{41.87}{.2136 - 100\left(1 - \dfrac{\omega_o}{\sigma}\right)} + 1.9$$

$$\omega_o = 15.0410686\ ^\circ/\text{hr}$$

and σ is the forcing frequency. Measurement of h_2 at the frequency of K_1 with an accuracy of 5 per cent or better would be useful for evaluating core models.

Measuring height differences with the above precision requires careful consideration of the effect of the ocean tide but mainly with regard to its loading effects at the two laser sites, not for its gravitational effects on the satellite orbit. This is an important advantage since loading decreases rapidly with distance from the ocean, and moreover it can be predicted accurately in particular regions where the ocean tide is well described such as the British Isles (Bower, 1969; Baker and Lennon, 1973) and the east coast of Canada (Lambert, 1970, Beaumont and Lambert, 1972). The tide in the calibration area is likely to become sufficiently well known in the coming months as a result of GEOS–C investigations.

The loading effect has been calculated for the M_2 ocean tide at each of the laser sites in the calibration area by convolving existing tidal data with the Green's function for displacement published by Farrell (1972). The results, presented in Table III, indicate the variations to be expected for a range of sites from mid-continent to coastal. Ideally, for this type of differential measurement, the laser stations should be situated

TABLE III

Predicted Radial Deformation Due to Loading by Global Ocean Tides (M_2)

Site	Lat.	Long.	Deform. Ampli. (cm)	Deform. Phase (Greenwich)
Pt. Arena, Cal.	38.90	236.29	1.245	-60°
Reno, Nev.	39.54	240.19	1.201	-48°
Ephraim, Utah	39.36	248.40	0.683	-54°
Denver, Col.	39.74	255.01	0.545	-52°
Manhattan, Kans.	39.20	263.42	0.362	-49°
Urbana, Ill.	40.11	271.77	0.293	-36°
Oxford, Ohio	39.57	275.25	0.262	-32°
Carlisle, Penn.	40.20	282.00	0.335	-16°
New York, N.Y.	40.80	286.03	0.633	$+5^\circ$
Godlas, GSFC	39.02	283.17	0.456	-11°
Ramlas, AFETR.	28.00	278.00	0.566	$+51^\circ$
Wallops I.	37.80	284.49	0.513	$+1^\circ$
Grand Turk	21.00	288.00	1.014	$+96^\circ$
Bermuda	33.00	296.00	1.215	-57°

in mid-continent. At great distances from the oceans there remains a residual loading effect due to the global oceans but it tends to be small and fairly constant over large areas. To show this, the displacement has been calculated for each of the nine sites used by Kuo *et al.* in their trans-U.S.A. tidal-gravity profile (Kuo *et al.*, 1970). The data for this table were computed using a variety of world-wide empirical tidal data and programs GLOBAL and PRESS (Bower, 1971). New ocean-tide data which has recently become available for the GEOS–C calibration area is now being used in a detailed study of differential loading there.

Conclusions

The principal Earth-tide parameters of interest at this time are k_2 or h_2 near the predicted core-resonance frequency, and the phase lag. Both parameters can be determined in principle through analysis of orbital dynamics or through analysis of surface deformations as revealed by difference-ranging measurements from laser stations. The first technique requires knowledge of the ocean tide which is uniformly good throughout the world to yield a net accuracy of 10 per cent in the determination of its effect on the orbit and make possible the experimental determination of k_2 to an accuracy of 1 per cent and phase lag to 0.5 degree. This global knowledge of ocean tides is not likely to be achieved in the next few years. In the second technique, because the ocean tide effect is evaluated only at the laser sites, less weight is placed on distant tidal waters and 10 per cent accuracy for regional tides will be sufficient in most cases for the determination of h_2 and phase lag to the required accuracy. This knowledge of the tides is likely to be achieved in the GEOS–C calibration area. Using the ranging method, assuming 1-cm difference-ranging accuracy, 650 independent measurements will be required if the M_2 constituent tide is used and 1,500 measurements if the k_1 constituent tide is used.

Notation

a	semi-major axis of satellite orbit
a^*	semi-major axis of attracting body
a_ℓ	mean equatorial radius of earth
$C_{\ell m}$	cosine coefficient of spherical harmonic potential term
e	eccentricity of satellite orbit
$F_{\ell mp}$	inclination function
$G_{\ell pq}$	eccentricity function
G	gravitational constant
i	inclination
M	mean anomoly
m^*	mass of attracting body
m	mass of attracting Earth
n	mean motion
$P_{\ell m}$	legendre associatal polynomial
R	disturbing potential
$S_{\ell m}$	sine coefficient of spherical harmonic potential term
t	time
ϵ	solid earth phase lag
θ	Greenwich sidereal time
λ	longitude
μ	gravitional constant times Earth's mass
ϕ	latitude
Ω	longitude of node
ω	argument of perigee

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Discussion

Q: Which laser data do you intend to use in your analyses?

A: Data from the "calibration area".