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Investigation of Frozen Soils Using Time Domain Reflectometry

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Earth Physics Branch Open File Number 80-12

Ottawa, Canada, 1980

59p.

Price/Prix: $.18.50

Abstract

A new technique for determining the volumetric unfrozen water content of frozen soils is reported. Time Domain Reflectometry is used to measure the dielectric constant of the soil which is largely independent of soil and ice content but highly dependent on the proportion of unfrozen water. Results are compared with those obtained by dilatometry and other techniques.

Résumé

Une nouvelle méthode pour determiner la teneur volumetrique en eau non gelée des sols gelés est introduit dans ce rapport. La réflectometrie par intervalles de temps est utilisée pour mesurer l'équilibre diélectrique du sols qui est largement indépendent du sols et de la teneur en glaces, mais qui est aussi grandement dépendant de la proportion d'eau non gelée. Les résultats sont comparés à ceux obtenus par plusieurs methodes, en autre la dilatométrie.

FINAL REPORT

INVESTIGATION OF FROZEN SOILS USING TIME DOMAIN REFLECTOMETRY

for the

Department of Energy, Mines and Resources

Earth Physics Branch

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DSS File No. 05SU-23235-9-0482
Serial No. 0SU79-00047

June 1, 1980

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ABSTRACT

A new technique for determining the volumetric unfrozen water content of frozen soils is reported, which uses time domain reflectometry to measure the dielectric property. The technique has been developed previously by Davis, Topp and Annan (1977) for unfrozen soils; their results show that the dielectric property varies in a consistent way with volumetric water content, and, further, that the relationship is largely unaffected by soil type and bulk density (Topp, Davis and Annan, 1980).

Using precise temperature control, the technique has now been successfully applied to measurement of water contents of frozen soils. Curves of the dielectric property versus temperature show a close similarity to unfrozen water content curves, for a variety of soils. The results from ice/water experiments and combined TDR-dilatometry experiments suggest that the relationship obtained by Topp, Davis and Annan (1980) may be applicable to frozen as well as unfrozen soils. Using this relationship, dielectric values were converted to unfrozen water contents, and the results agreed very closely with published data for similar soils.

1. Introduction

A non-destructive and portable technique for the measurement of the unfrozen water content of frozen soils would have widespread and important applications. Of particular significance at the present time are investigations into the coupled heat and moisture regimes of freezing soils and frost heave. Applying an understanding of soil freezing phenomena to practical problems, such as the construction of refrigerated gas pipelines, requires basic knowledge of the thermal and hydraulic properties of freezing soils. As is the case for unfrozen soils, these properties, and hence the heat and moisture flows in the soil, depend upon the liquid (unfrozen) water content (θ_{uf}, by volume). the variation of θ_{uf} with negative temperature (i.e. the freezing characteristic curve) is, therefore, an important property of the soil. A knowledge of this can be used to calculate the volumetric heat capacity (e.g. Williams, 1967) and the thermal conductivity (e.g. Johansen, 1973, 1975). It may further prove useful in estimating the hydraulic conductivity of frozen ground.

The method described in this report for measuring θ_{uf} would have distinct advantages over previous methods:

i) It relies on a soil property (the dielectric constant) which is a function primarily of water content, and which can be measured in a direct and straightforward manner.

ii) It is non-destructive.

iii) It is suited to laboratory or field use, and could be used in situ or on undisturbed soil samples (e.g. see Davis, Topp and Annan, 1977).

iv) It could be used to give bulk (average) values or for vertical profiles (e.g. see Davis and Annan 1977).

Many different methods are in use for determining the unfrozen water content of frozen soils. Anderson and Morgenstern (1973) provide a review of the more usual methods; they include adiabatic calorimetry, dilatometry, the suction-moisture relationship and nuclear magnetic resonance. None of these techniques are applicable to measurements in situ or in general to undisturbed samples (with the possible exception of some nuclear methods). Further, they are subject to various other shortcomings. Calorimetry yields no information on the distribution of the unfrozen water; dilatometry cannot be used on natural soils, since samples must be slurried and completely de-aired. The suction-moisture technique, although straightforward, is time-consuming and the results are sensitive to sample treatment (see Hotzel, 1974). Further, it does not account for the influence of salts; at temperatures between 0° to $-0.5^{\circ}C$, the osmotic potential can influence θ_{uf} substantially (see El Khoraibi, 1975). Finally, nuclear magnetic resonance requires separate calibration for each sample, and the method is very sensitive to temperature.

2. Dielectric Properties of Soils

2.1 Electrical notation

The terms dielectric constant and relative permittivity are used interchangeably to describe the ability of a material to store electrical potential energy under the influence of an electrical field, relative to that of a vacuum. By definition, the dielectric constant, K', is the ratio of the permittivity of the material, ε_1 (F m^{-1}), to that of a vacuum, ε_o (8.854×10^{-12} F m^{-1}):

$$K' = \frac{\varepsilon_1}{\varepsilon_o} \quad (1)$$

The complex dielectric constant, K*, is defined as (Davis and Annan, 1977):

$$K^* = K' + j(K'' + \frac{\sigma_{dc}}{\omega\varepsilon_o}) \quad (2)$$

where K'' is dielectric loss and ($\sigma_{dc}/\omega\varepsilon_o$) represents conductive loss (σ_{dc} is the d.c. conductivity, ω is angular frequency (2 π f) and j is $\sqrt{-1}$). The losses can be defined in terms of the electrical loss tangent (Davis and Annan, 1977):

$$\tan\delta = \frac{K'' + \frac{\sigma_{dc}}{\omega\varepsilon_o}}{K'} \quad (3)$$

For low loss materials, $\tan\delta \ll 1$ (i.e. $(K'' + \sigma_{dc}/\omega\varepsilon_o) \ll K'$),

and therefore $K^* \simeq K'$.

Davis and Annan (1977) indicate that whilst K'' appears to vary a great deal with soil type, it is, however, considerably smaller than K' in the frequency range 1MHz to 1GHz. Conductive losses vary with, amongst other things, soil type and water, ice and salt content. Conductive losses increase with σ_{dc}, but decrease with the frequency. In general, clay soils are more "lossy" than sands, and lossiness increases with water content and soluble salts. However, at frequencies between 1MHz to 1GHz, such losses are small. Thus, in the range 1MHz to 1GHz, $K^* \simeq K'$. A more complete discussion of electrical losses can be found in Olhoeft (1975).

2.2 Dielectric properties of soil components

The dielectric property of a soil is affected by the same factors that influence its conductivity. The dielectric constant of most soil minerals lies in the range 2 to 5 (Thomas, 1966). K' for water is much higher, varying slightly with temperature (from 87.7 at 0°C to 80.1 at 20°C), but with minimal frequency-dependence from d.c. to about 3GHz (Von Hippel, 1954). Tan δ is also very small in this range (for water at 25°C, tan δ = 0.04 at 1MHz and 0.016 at 300 MHz; Von Hippel, 1954). Ice has a high d.c. dielectric constant (about 100), but it falls to a value of about 3.2 at frequencies above 30 KHz

(Hoekstra and Spanogle, 1972). Tan δ for ice decreases from 0.4 at 1MHz to about 0.0005 at 1GHz (Yoshino, 1967). Both K' and tan δ are minimally influenced by ice density and temperature (Evans, 1965; Yoshino, 1967).

2.3 The effect of water content on the dielectric property of soils

The dielectric constant for dry soils generally falls within the range 2.2 to 3.5, at frequencies above about 1MHz. K', though, is very sensitive to the volume of water present in the soil (see Chernyak, 1964; Thomas, 1966; Davis and Chudobiak, 1975; Davis, Topp and Annan, 1977), and soils containing water generally show a dielectric constant somewhere between 2.2 to 40, depending upon the water content. When water is present in the soil, the measured dielectric property can show a substantial frequency variation. Jumikis and Slusarchuk (1973) showed that K' is frequency-dependent in the range 100Hz to 100KHz. However, Von Hippel (1954), Ngoc Lan, Chaigne and Philippe (1970), Hoekstra and Delaney (1974) and Chudobiak, Syrett and Hafez (1979) present data for soils which indicate that K' is not strongly frequency-dependent in the range about 1MHz to 1GHz, and further that tan δ is generally very small (e.g. Figures 1 and 2). Measurement of K' in this frequency range should, therefore, be a sensitive indication of volumetric water content and the use of time domain reflectometry for such measure-

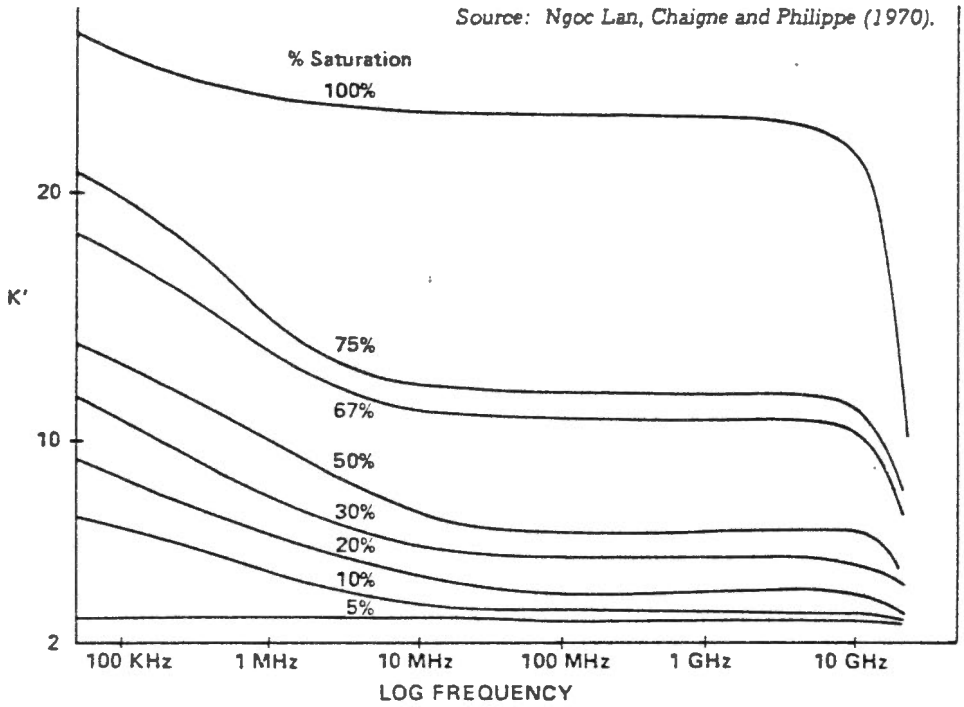

Figure 1 Variation of the Dielectric Constant of Pyla Sand with Frequency

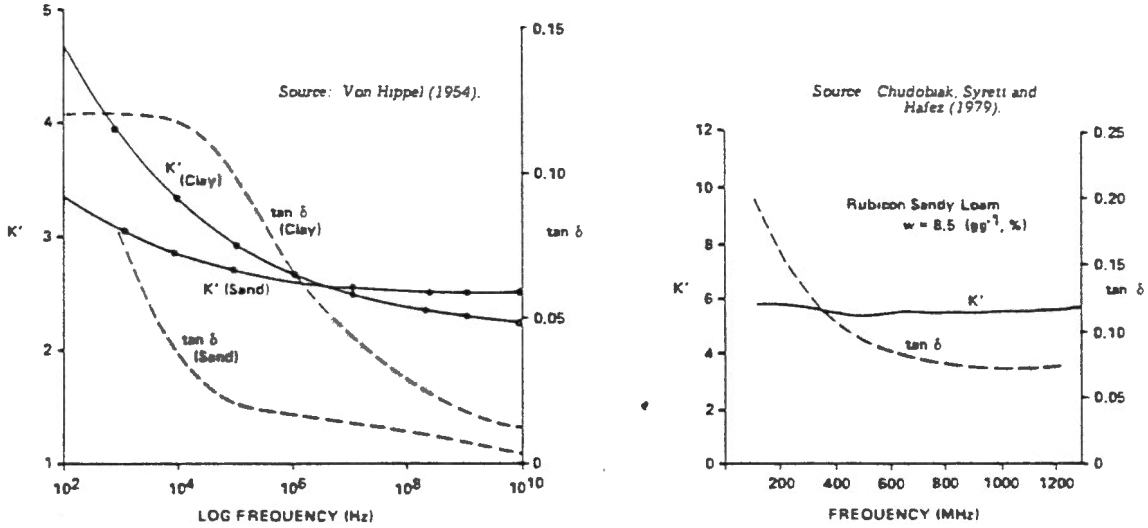

Figure 2 Dispersion Curves for Various Soils

ments in this range has been described by Davis and Annan (1977) and Davis, Topp and Annan (1977). The TDR technique registers the complex dielectric constant, K*; but in the range 1MHz to 1GHz, since tan δ is very small, $K^* \simeq K'$. However, since the effects of electrical loss (however small they might be) are present in such an estimate of K', Topp, Davis and Annan (1980) have proposed the term "apparent dielectric constant", K_a, to denote a measured value. The term K_a will be used throughout the rest of this report.

Topp, Davis and Annan (1980) obtained an empirical relationship between K_a and volumetric water content (θ_v) based on results for several mineral soils (Figure 3a), which they expressed as:

$$(4) \qquad K_a = 3.03 + 9.3\,\theta_v + 146.0\,\theta_v^2 - 76.7\,\theta_v^3$$

They found that the relationship was only weakly sensitive to soil type, dry density and soluble salt content, and that θ_v could be determined with a probable maximum error of ±2½%. This uncertainty is due to a combination of measurement error and variations in electrical properties of different soils. The error in θ_v from K_a measurements using the TDR technique is about ±1½% (Patterson, 1980). It is possible, of course, to obtain a separate relationship between K_a and θ for individual soils, if so desired. Figure 3b shows the results of such an experiment, carried

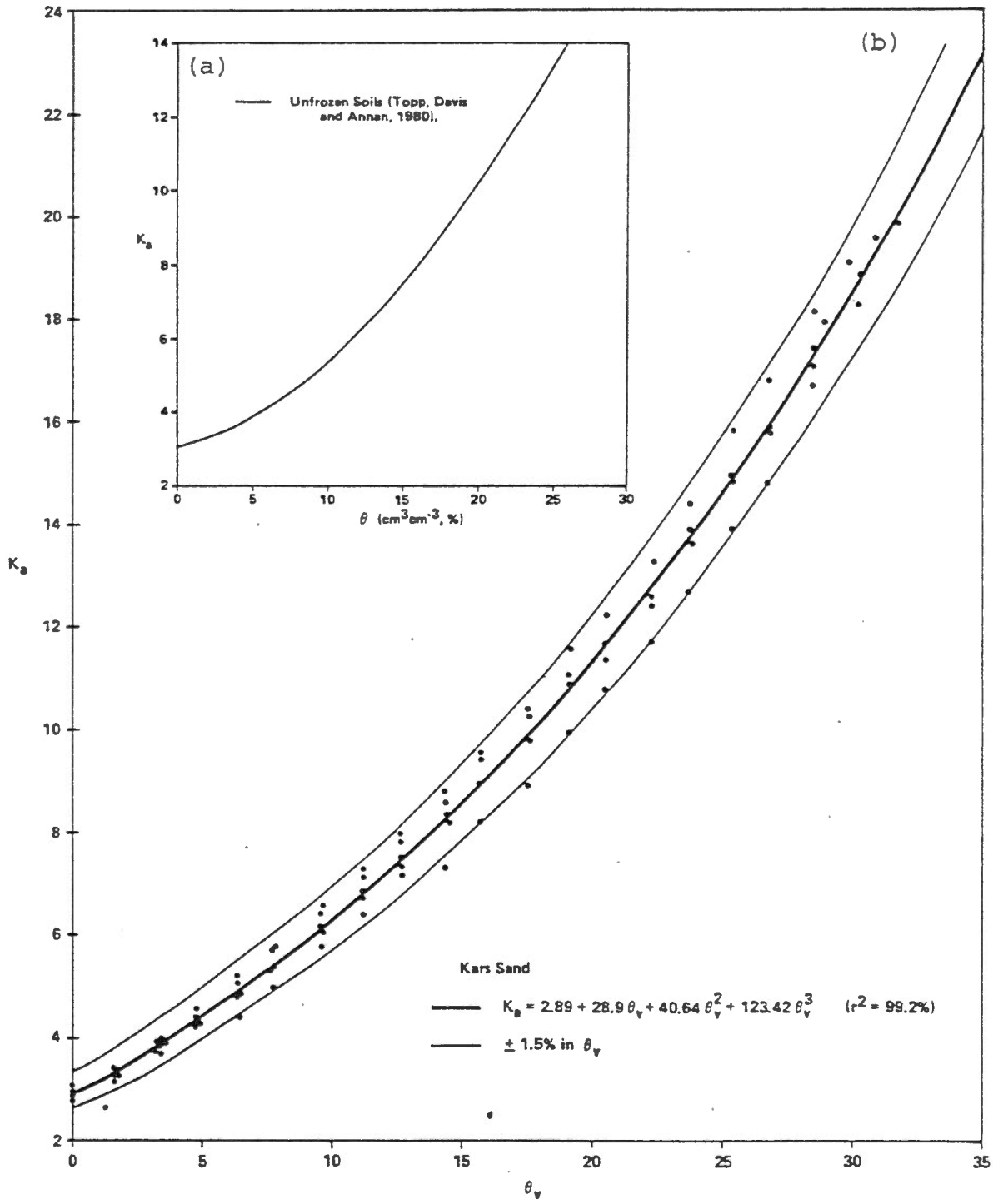

Figure 3 Variation of K_a with Volumetric Water Content, Unfrozen Soils

out by the authors; 100% of the data points fall within ±1½% in θ_v. For some applications it may be impossible or impractical to determine a separate relationship; use of (4), however, will generally limit errors to ±2½% in θ_v.

2.4 The unfrozen water content of frozen soils

Since ice and dry mineral soil have such similar dielectric constants (3.2 and 2.2 to 3.5 respectively), and K_a is very sensitive to the volumetric water content, it was felt that K_a measurements on frozen soils might yield a sensitive indication of the unfrozen water content (θ_{uf}, cm cm^{-3}). Davis and Annan (1977) present a graph which shows a strong temperature-dependence for K_a below 0^oC (Figure 4), although they make no reference to this nor discuss its significance. The main purpose of the research now being reported was to investigate the possibility of determining the unfrozen water content (θ_{uf}) of frozen soils from K_a measurements by TDR.

Before going on to describe our experiments and results, we will present an outline of the working principles of the TDR technique, which has been described by Davis and Annan (1977) and Topp, Davis and Annan (1980). A brief description is included in this report for the benefit of the present reader. TDR operation is outlined in Appendix I.

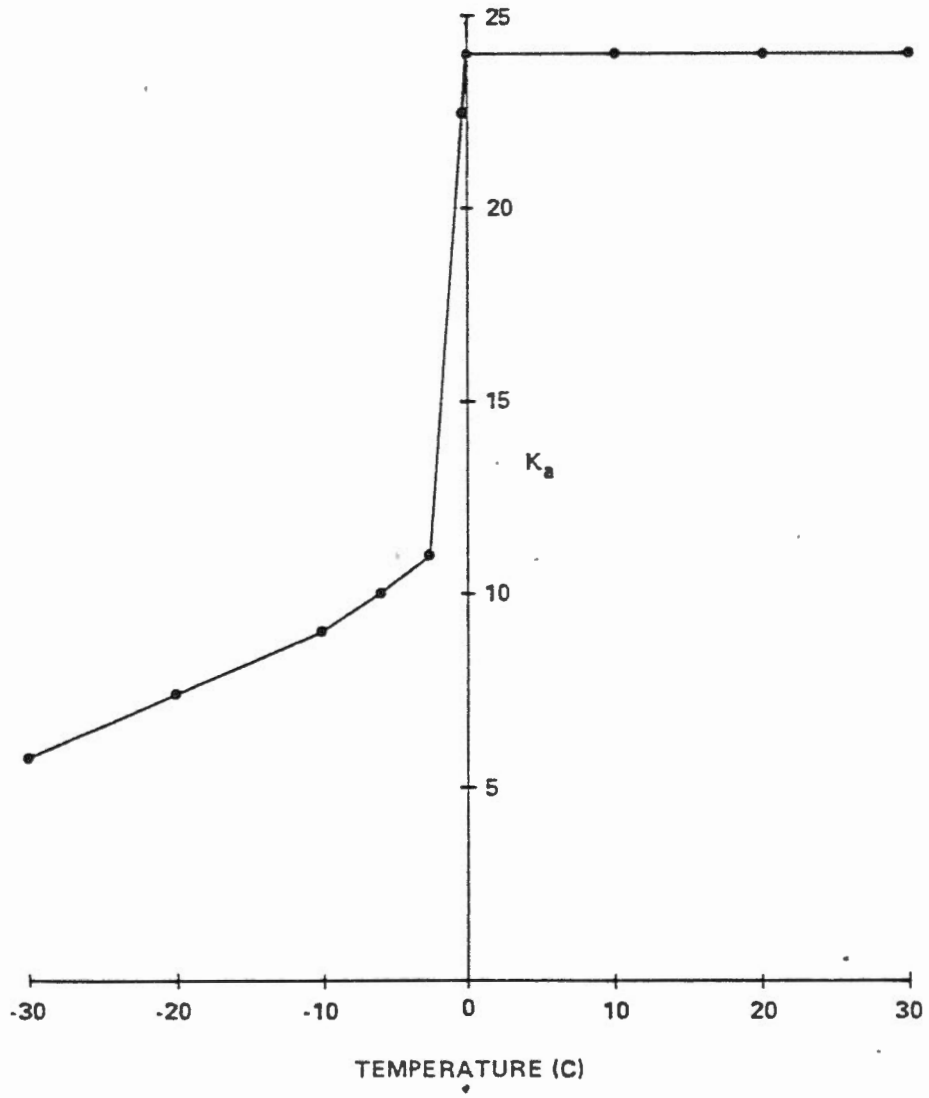

Figure 4 Variation of K_a with Temperature for Rideau Clay (from Davis and Annan, 1977)

3. Time Domain Reflectometry

3.1 K_a Measurement

Traditionally, the dielectric constant, K', was determined via capacitance measurements. However, it can also be determined from the velocity of propagation of electromagnetic waves through a medium, using time domain reflectometry (TDR). The measurement of dielectrics in the time domain was first described by Fellner-Feldegg (1969), and the use of TDR for measuring the dielectric constant of moist soils was first reported by Davis and Chudobiak (1975). The propagation velocity of an electromagnetic wave in free space (c) is 3×10^8 m s^{-1}. The propagation velocity (V) of a wave in a transmission line is the same as that in the medium surrounding, or contained within, the line. For a low loss medium ($\tan \delta << 1$), the propagation velocity is approximated by (Davis and Annan, 1977):

$$V \simeq \frac{c}{\sqrt{K_a}} \qquad (5)$$

If the propagation velocity is known, then the dielectric constant, K_a, of the medium can be determined.

The TDR unit[1] consists of a pulse generator which produces a fast rise-time step voltage, a sampler which transforms

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[1] The TDR unit is commercially-available. We have used the Tektronix 1502.

a high frequency signal into a lower frequency output, and an oscilloscope or other display or recording device. (Davis, Topp and Annan (1977) used Polaroid film as well as magnetic tape. We have used an auxiliary X-Y recorder; also an integral Y-t recorder is available on the Tektronix 1502.) TDR measurements are made by launching the step signal into a known length of transmission line and measuring the travel time along it.

Now, as the pulse travels the line, it will remain unaltered as long as the characteristics of the line stay the same. However, any discontinuity in the line (impedance mismatch) will give rise to partial reflection and partial transmission of the pulse. The incident and reflected pulses are sampled and compared and displayed on the TDR unit's crt. The vertical axis on the trace represents the reflection coefficient (from which impedance can be determined) and the horizontal axis represents electrical distance along the line (from which the travel time can be determined).[2]

When a parallel transmission line is embedded in a wet soil, for example, a trace such as is shown in Figure 5 is obtained. Point "A" denotes where the pulse just enters the line embedded in the soil; point "B" denotes where the pulse encounters the open circuit end of the line.[3] The

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[2] See Appendix I for TDR operation.

[3] For details, see Appendix I.

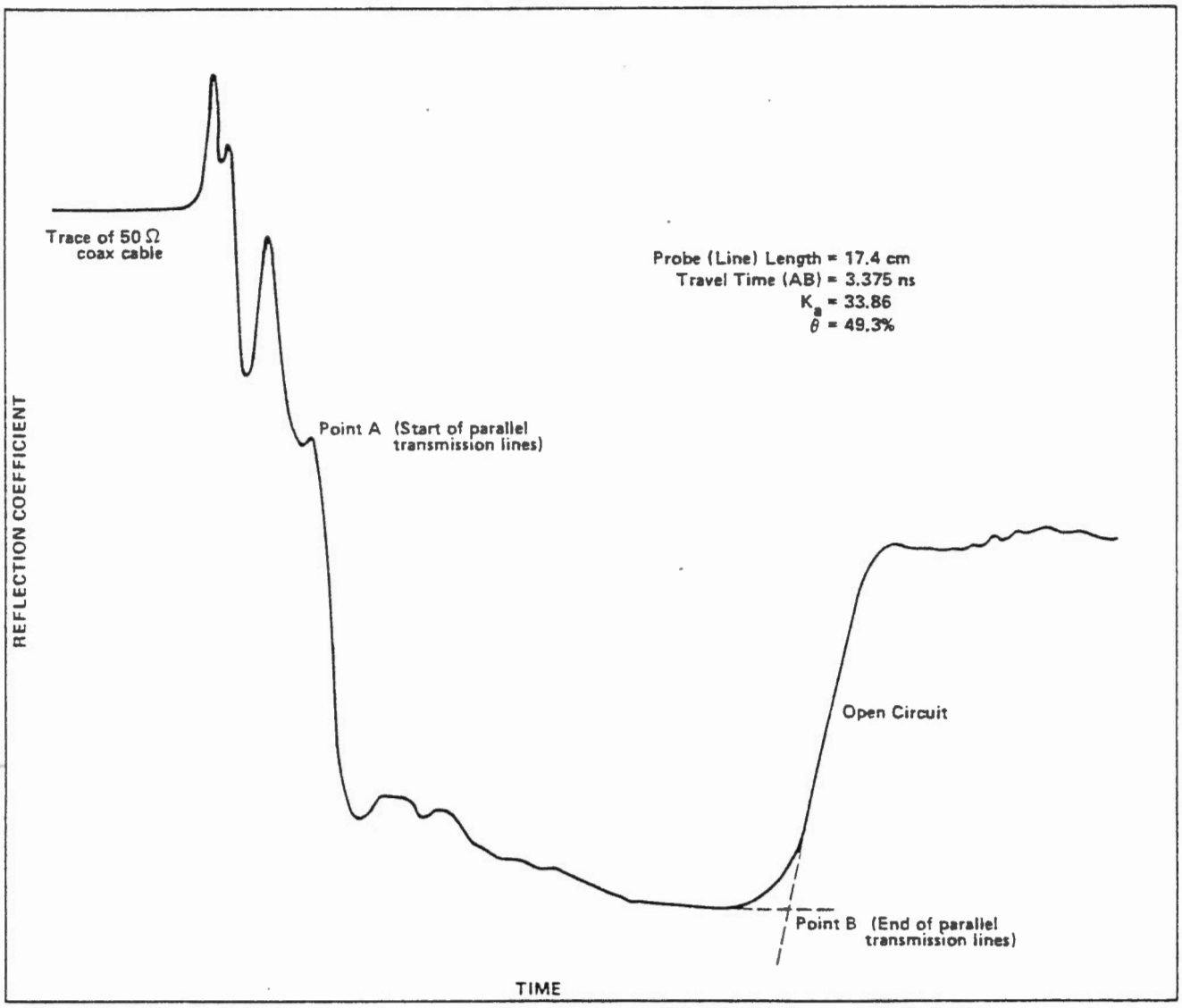

Figure 5 Typical TDR Trace in Wet Soil

A to B horizontal distance yields the trace length, from which the one-way travel time (tt) of the pulse in the line can be determined. The velocity of propagation (V) is then given by:

$$(6) \qquad V = \frac{L}{tt}$$

where L is the length of the transmission line. Since the propagation velocity in the line is the same as that in the medium surrounding the line, one can equate equations (5) and (6). Therefore,

$$\frac{c}{\sqrt{K_a}} = \frac{L}{tt}$$

from which K_a can be determined:

$$(7) \qquad K_a = \left[\frac{c \times tt}{L}\right]^2$$

(for low loss materials). As the soil water content increases, the travel time is seen to increase (i.e. as K_a increases). Measurement of K_a allows estimation of θ (by Figure 3, for example).

3.2 Probe characteristics

Two types of transmission lines have been used: a balanced parallel line which is inserted into the soil, and a coaxial line which is designed to contain a soil sample (Figure 6). Both designs have been used in the laboratory experiments; for in situ measurements, a parallel line probe would obviously be used.

With parallel lines there is a practical lower limit to line spacing, S, in that it should be large enough to ensure a representative volume of material is "sampled".

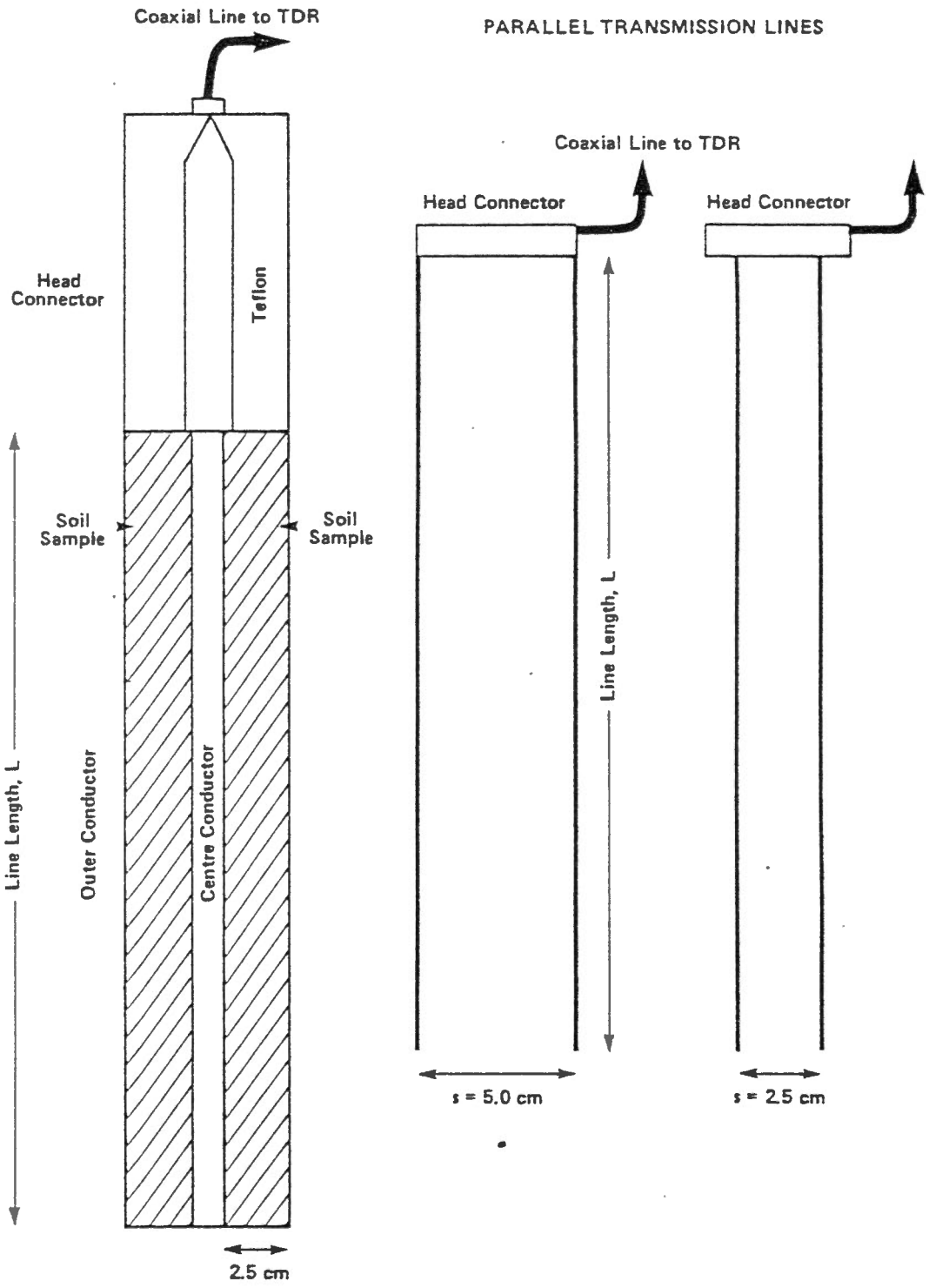

Figure 6 Probe Configurations

Davis and Chudobiak (1975) determined that the travel time of the signal was affected mainly by the material between the parallel lines. Material at a distance of about 2S from the lines has a negligible effect. There is also an upper limit to S, beyond which modes other than TEM occur (see Davis and Chudobiak, 1975). The rigidity of the probe is a function of rod diameter, d - for example, one may want it stiff enough to force or hammer into the soil. The authors have used parallel lines with S = 2, 2.5 and 5 cm and d = 0.3 and 1 cm. Lines with S = 5 cm and d = 1.3 cm were used by Davis and Chudobiak (1975). We have also used a coaxial tube with a diameter of 5 cm.

There are also practical limits to line length. The travel time of the TDR pulse through the soil is a function of K_a and line length, L. When the travel time is long, significant signal attenuation can occur. This loss manifests itself as a lengthening of the rise time of the reflected pulse, making it more difficult to fix point B. The lower limit is related to system resolution. As a guide, parallel lines over 100 cm have been used by Davis (1975) and we have used lines as short as 10 cm. Topp, Davis and Annan (1980) used a coaxial line of 100 cm; we have also used such lines as short as 25 cm. Further work is needed to determine the practical limits to line length for measuring K_a in earth materials.

When the parallel line configuration is used, a RF pulse transformer should be used to match the probe to the TDR unit and prevent any possible electrical interference. Nothing is required for the coaxial tube.

4. Experiments and Results

4.1 Introduction

To assess the potential for determining the volumetric unfrozen water content (θ_{uf}) of frozen soils from measurement of the dielectric constant (K_a), the following experiments were carried out.

4.1.1 Influence of ice content on K_a

As stated in Section 2.4, in order to apply the TDR technique to the determination of θ_{uf}, it is necessary to assume that ice content, *per se*, will have little or no effect on the values of K_a measured on frozen soils - i.e. variations in K_a will be due to variations in θ_{uf}, and will not be affected by variations in ice content. Thus an experiment was carried out to examine any variation in K_a with ice content, involving measurements on solidly frozen sand in coaxial lines (tubes).

4.1.2 Relationship between K_a and θ_{uf}

In order to "calibrate" the TDR technique to measurement of θ_{uf}, it was necessary to determine a relationship between K_a and θ_{uf}. This was pursued via the following:

i) Since the dielectric constant for ice is very similar to that for dry soil, it was assumed that an ice/water mixture should closely resemble a frozen soil containing unfrozen water, in a dielectric sense. Hence, measurements were made of K_a for ice/water mixtures in coaxial tubes, where the liquid water content (θ_{uf}) was known.

ii) A combined TDR/dilatometer experiment was devised which permitted simultaneous measurements of K_a and θ_{uf} for the same soil sample, at various freezing temperatures.

4.1.3 Freezing characteristic curves

Freezing characteristic curves (θ_{uf} vs. temperature) were determined by TDR, in the laboratory, for a variety of soils. The results were compared to published data, obtained by a variety of other techniques.

4.2 Experimental procedures

4.2.1 Influence of ice content on K_a

To examine the influence of ice content, per se, on K_a, a coarse Ottawa sand was used as the soil, since it would not contain any liquid water even at freezing temperatures close to $0^{\circ}C$. Dry sand was poured into coaxial lines (tubes) which were manually vibrated to obtain a fairly uniform packing. The tubes were 5 cm in diameter, and of

lengths 25 and 33 cm. K_a was measured for the dry state. A known volume of water was then added at one end of the tube, and permitted to distribute throughout the sample. The tube was then placed in a freezer overnight at a temperature of -18°C (i.e. so that all the water was frozen), and K_a determined. This procedure was repeated for various initial liquid water contents. The water content was kept below saturation to prevent rupture of the coaxial tube upon freezing.

4.2.2 K_a vs. θ_{uf} Relationship

4.2.2.1 Ice/water experiment

This experiment entailed adding known volumes of previously supercooled water to a coaxial tube, of known volume, which contained crushed ice. For each volume of water added, K_a was determined. The time required to obtain a reading after a volume of water was added was less than 30 seconds. The water was drained after each reading to check the volume.

4.2.2.2 Combined TDR/dilatometer experiment

The TDR/dilatometer is essentially a modified dilatometer which permits both measurement of sample volume expansion due to freezing (from which θ_{uf} can be calculated) and TDR determination of K_a. The apparatus consists of inner and outer chambers made of steel tubing (Figure 7). A de-aired soil sample is placed in a latex rubber membrane and sus-

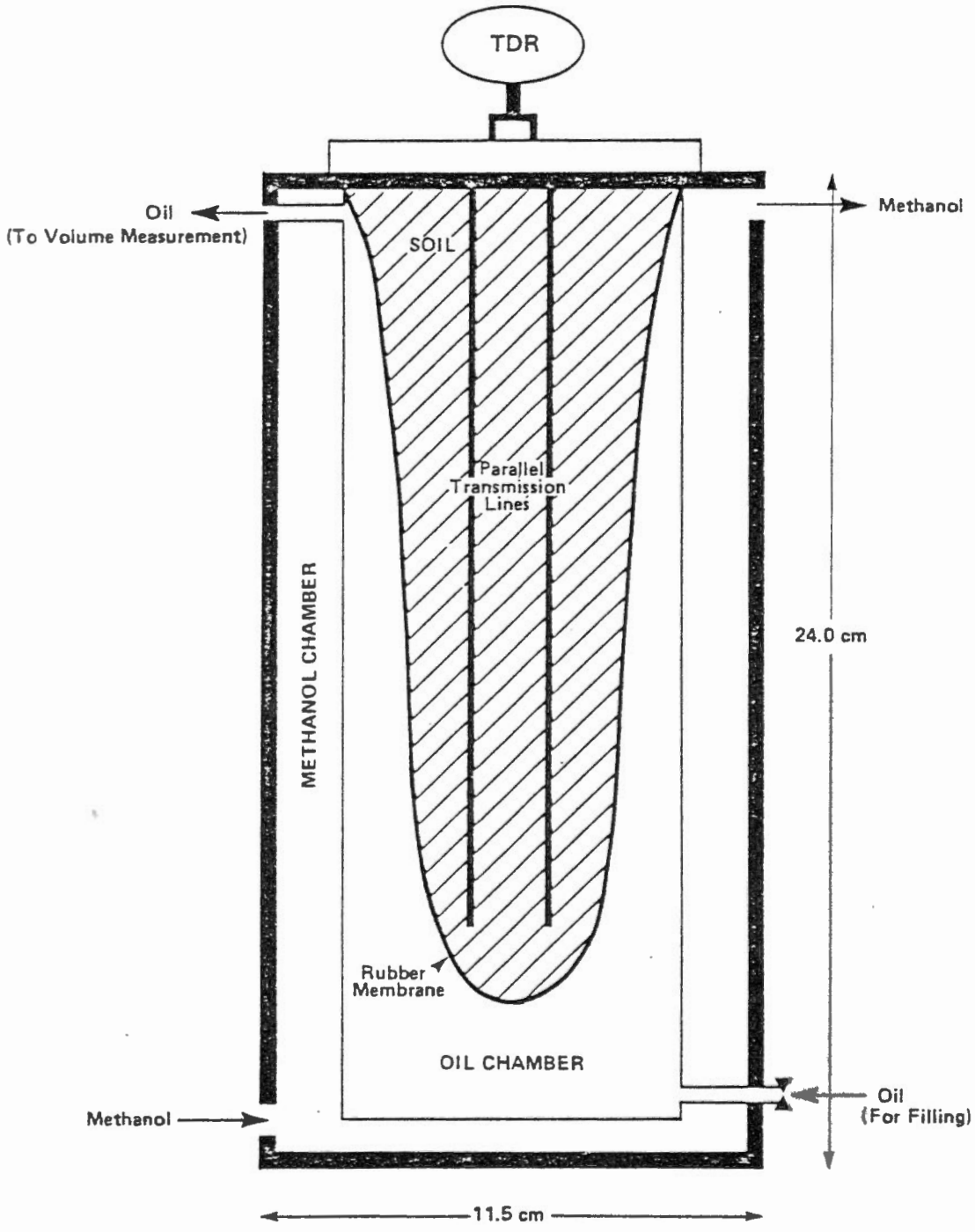

Figure 7 Combined TDR-Dilatometer Apparatus

pended in the inner chamber; the remaining volume in the chamber is filled with silicon oil which is connected to a graduated burette. A plexiglass top plate, which incorporates a balanced parallel transmission line, is mounted on the top of the inner chamber. Sample temperature is controlled by circulating methanol through the outer chamber from a regulated bath (±0.01°C). The whole apparatus is insulated and placed in a controlled-temperature chamber.

After maintaining the apparatus at 0°C for 72 hours, to check for leaks, sample freezing was initiated by reducing the methanol temperature to -4°C for about 30 minutes. Once nucleation occurred (as indicated by a volume expansion), the methanol temperature was set at the desired value, and the experiment left to equilibrate. At temperatures between 0° to -1°C, 80 to 90% of the final volume expansion occurred within 3 to 4 hours; however, the experiment was routinely left at each temperature for 24 hours. At the end of this time the volume expansion was recorded and K_a was measured.

Two soils were used in the experiment, Ellwood Clay Loam and Castor Silt Loam, since these should possess distinctly different freezing characteristic curves. Many difficulties were encountered, however, over sample treatment, and this has somewhat limited the amount of data obtained. The greatest difficulty was in completely de-airing the sample; thus, in many instances, the sample volume expansion was

greater than expected based on the volume of water in the sample. A new dilatometer design will permit better sample treatment. A more complete discussion of sample treatment and experimental procedures is given in Patterson (1980).

4.2.3 Freezing characteristic curves

In these experiments, soil samples were placed in a thin-walled PVC tube 20 cm long (4.5 cm I.D.) covered with a latex rubber membrane. This could be immersed in a circulating methanol bath which permits temperature control to $\pm 0.01^{o}C$ for extended periods of time. A balanced parallel line probe of 0.3 cm (1/8") stainless steel rod was embedded in the sample; a line length of 17.5 cm was used, with a line spacing of 2.5 cm. The samples were ramped through a temperature cycle and K_a was determined at each temperature. Nucleation was initiated, in the first instance, by subjecting the sample to a temperature of about $-2^{o}C$ for 30 minutes or so. K_a values were converted into unfrozen water contents by means of an empirical relationship (see Section 4.3.2).

A variety of soils were tested, including Leda Clay, Ellwood Clay Loam, Castor Silt Loam and Manchester Silt (formerly New Hampshire Silt). The soils selected were as similar as possible to those for which published data on θ_{uf} were already available. This provided a basis of comparison for evaluating the TDR technique. It is realised

that variations in soil properties exist between different soils, and even different samples of the "same" soil, and such differences must be considered when comparing data. In general, the properties which have the greatest influence on unfrozen water content, at a given temperature, are texture and structure. These determine the pore geometry and surface area. Other properties, such as mineralogy and soluble salt concentration, are also important, but data are rarely reported in the literature.[4] Since our experiments have been carried out on slurried samples, we have matched soils according to grain size characteristics. No real effort was made to obtain complete sample saturation.

4.3 Results and Discussion

4.3.1 Influence of ice content on K_a

The data for K_a vs. initial water content for solidly frozen Ottawa Sand are presented in Figure 8. This shows that the K_a value for a soil containing ice and air is little affected by ice content, and is similar to that of dry soils. K_a does increase slightly from a value of 2.5 with no ice present, to 3.5 for an initial volumetric water content of 35% before freezing. It is likely that this

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[4] For a discussion of the soil properties which influence θ_{uf}, see Anderson and Tice (1972) and Anderson and Morgenstern (1973).

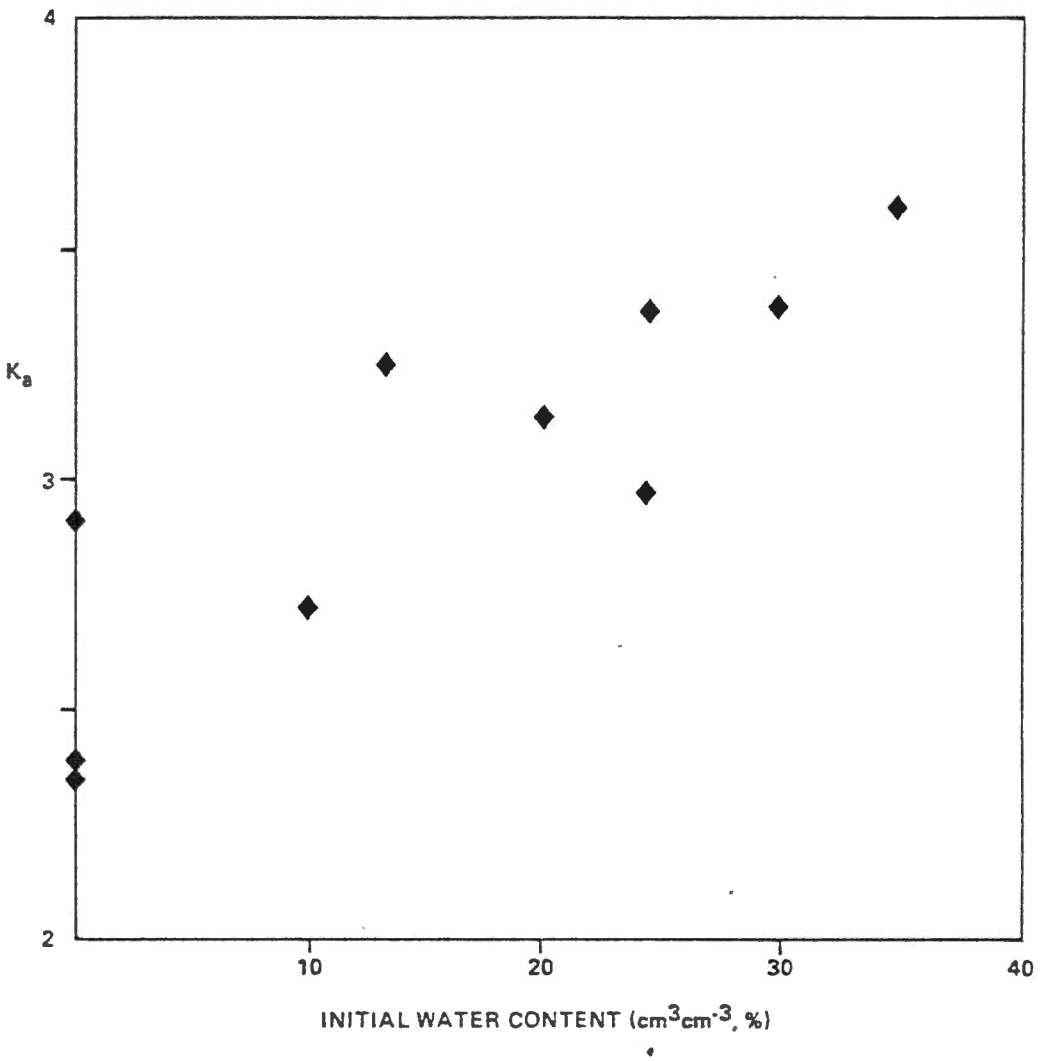

Figure 8 Variation of K_a with Ice Content, Ottawa Sand

is due to the replacement of air (K' = 1) by ice (K' = 3.2) in the soil, hence increasing K_a slightly. However, compared to the variation in K_a determined in subsequent experiments, when unfrozen water was present, the variation in Figure 8 is in-insignificant. For example, Figure 12 shows the variation in K_a with negative temperature for three soils (see also Figure 4). At different temperatures, and for the different soils, different amounts of unfrozen water are present; the total range in K_a values is considerable, compared to Figure 8.

These data suggest that variations in ice content will not have any appreciable effect on K_a values for frozen soils, since any liquid water present (K' = 87.7 at 0°C) clearly tends to dominate.

4.3.2 K_a vs θ_{uf} relationship

Simultaneous measurements of K_a and liquid water content were obtained for ice/water mixtures and from combined TDR/dilatometry for frozen soils. These results are presented in Figure 9, together with the best-fit plot of K_a vs. θ_v obtained for a variety of mineral soils in the unfrozen state by Topp, Davis and Annan (1980). Our data for frozen media fall generally within the 95% band of their curve. The K_a data for the Castor Silt Loam at θ_{uf} < 10% appear to be high; based on our experience with the dilatometer experiment, it is believed that the θ_{uf}

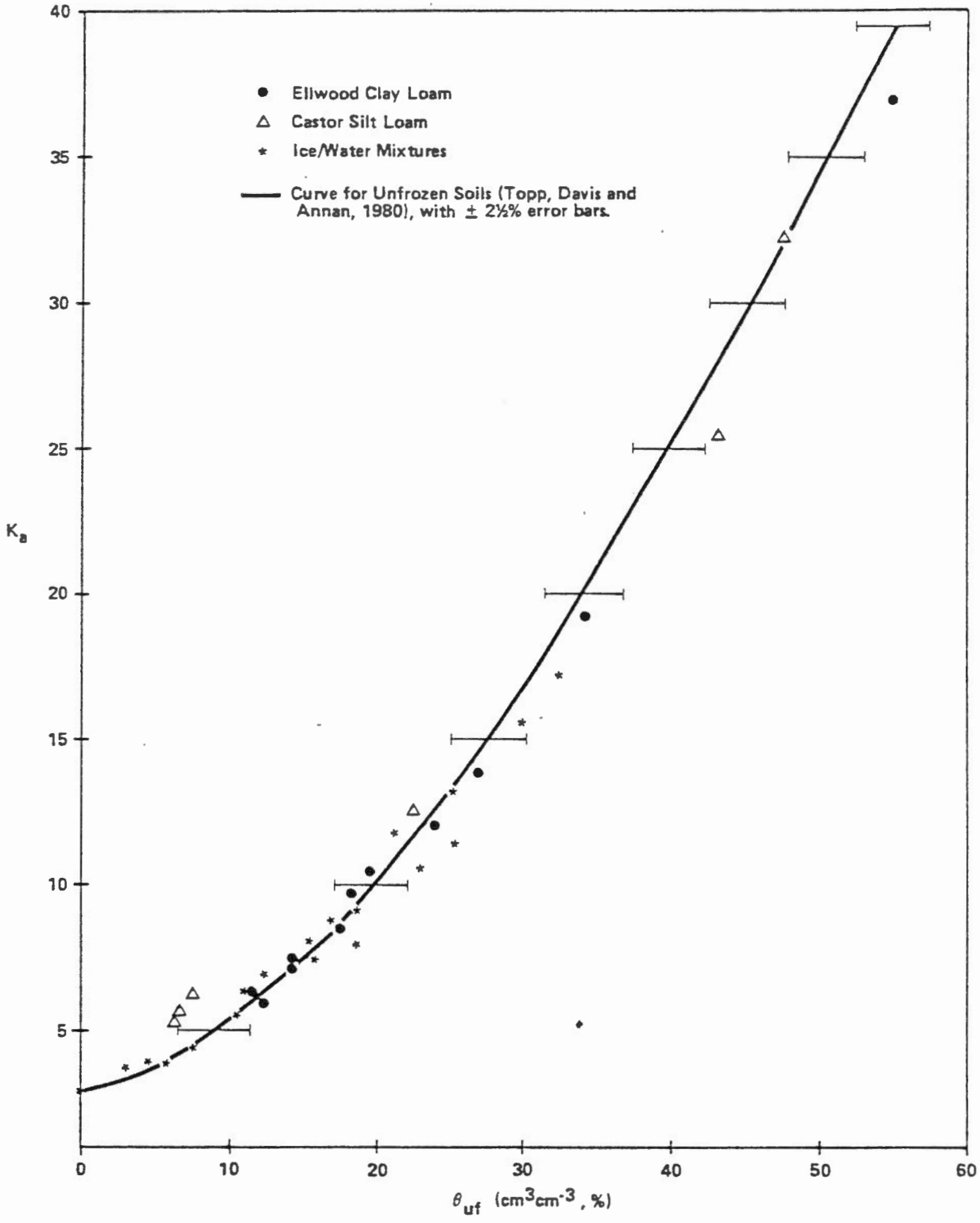

Figure 9 Variation of K_a with Unfrozen Water Content

values are in error, and are too low. Comparison of the freezing characteristic curve for the Castor to those for other similar soils (see Figure 15) further suggests that the dilatometer values are low in this case. However, the general agreement portrayed in Figure 9, together with the minimal influence of ice content on K_a, suggests that the relationship obtained by Topp, Davis and Annan (1980) could be applicable to frozen, as well as unfrozen, soils for determining θ from K_a. Their relationship between K_a and θ is tabulated in Appendix II. Further TDR/dilatometer experiments are desirable to examine the relationship between K_a and θ_{uf}.

Using the table in Appendix II, the K_a values from the TDR portion of the combined TDR/dilatometer experiment have been converted to θ_{uf} and are compared, in Figure 10, to the values determined independently from the dialtometer for Ellwood Clay Loam. The agreement between the two methods is remarkably good; the single largest difference is 3% in θ_{uf}, but agreement is generally within 1 to 1½%. A substantial difference in θ_{uf}, at about -0.75°C, was recorded between the first and second freeze cycles (about 8% in θ_{uf}). This difference was registered by both techniques and it is possible that it reflects physical changes in the soil (i.e. consolidation) during the first freezing cycle.

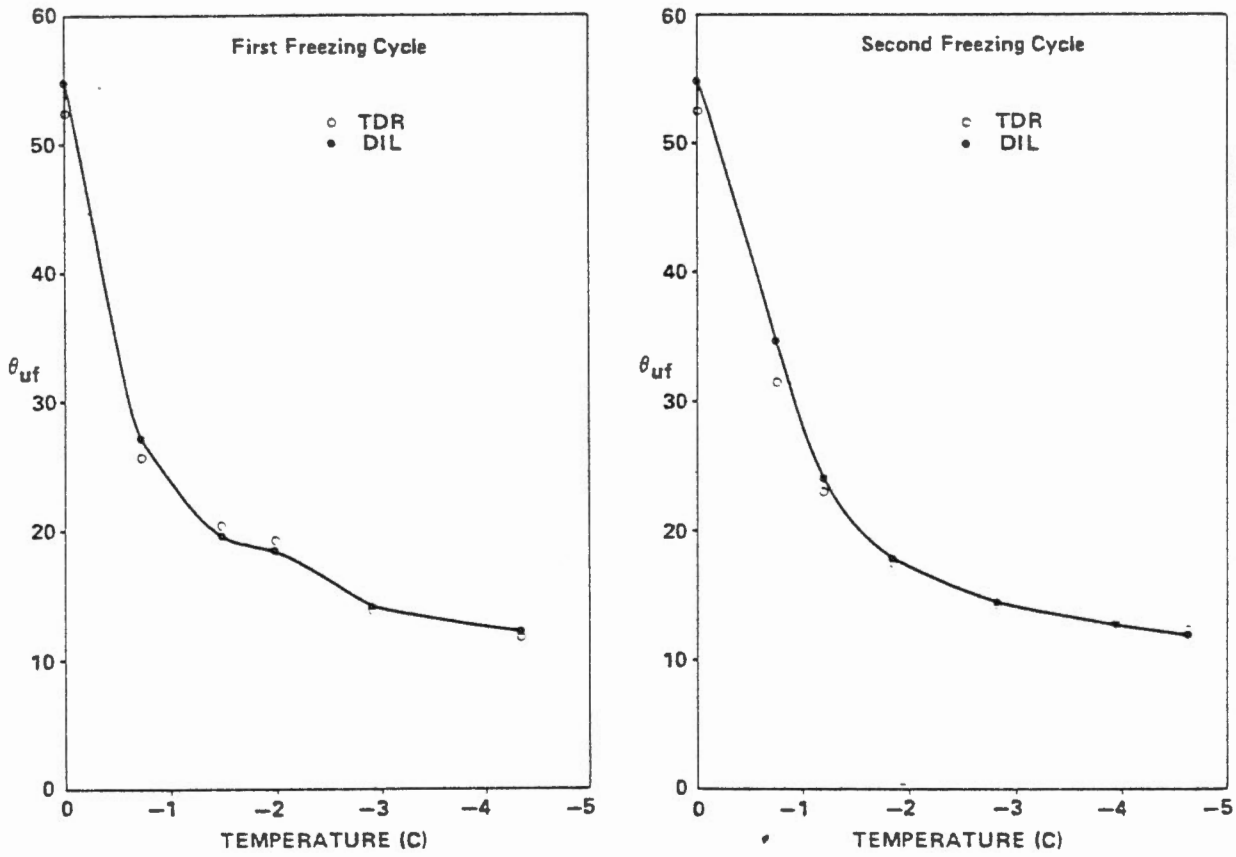

Figure 10 Comparison of Freezing Characteristic Curves Determined by TDR and Dilatometry, Ellwood Clay Loam

4.3.3 Freezing characteristic curves

When a soil freezes, as the temperature decreases, θ_{uf} decreases, and the travel time of an electromagnetic pulse in a transmission line also decreases (i.e. as K_a decreases). This is illustrated in Figure 11. When K_a values are plotted against temperature, relationships such as shown in Figure 12 are obtained. The curves show a rapid decrease in K_a over the temperature range 0° to $-1.0^{\circ}C$, with a decreasing rate of change at lower temperatures. Further:

i) the finer the soil texture, the greater the K_a value at a given negative temperature;

ii) K_a changes more rapidly with temperature, in the range 0° to $-1.^{\circ}C$, the coarser the texture.

Such characteristics are similar to those observed in freezing characteristic curves (θ_{uf} vs. temperature) and there is thus a very strong indication that the curves in Figure 12 represent the variation in unfrozen water content.

Since the TDR reading is largely affected by the soil between the two parallel lines, any radial moisture migration within the sample during freezing could be a source of error and confound the measurements. However, this can be discounted in our results, since inspection of the samples at the end of each experiment revealed no evidence of radial ice segregation; rather, the samples were randomly lensed (Figure 13). The possibility of radial ice segregation can be further discounted by considering the hydraulic

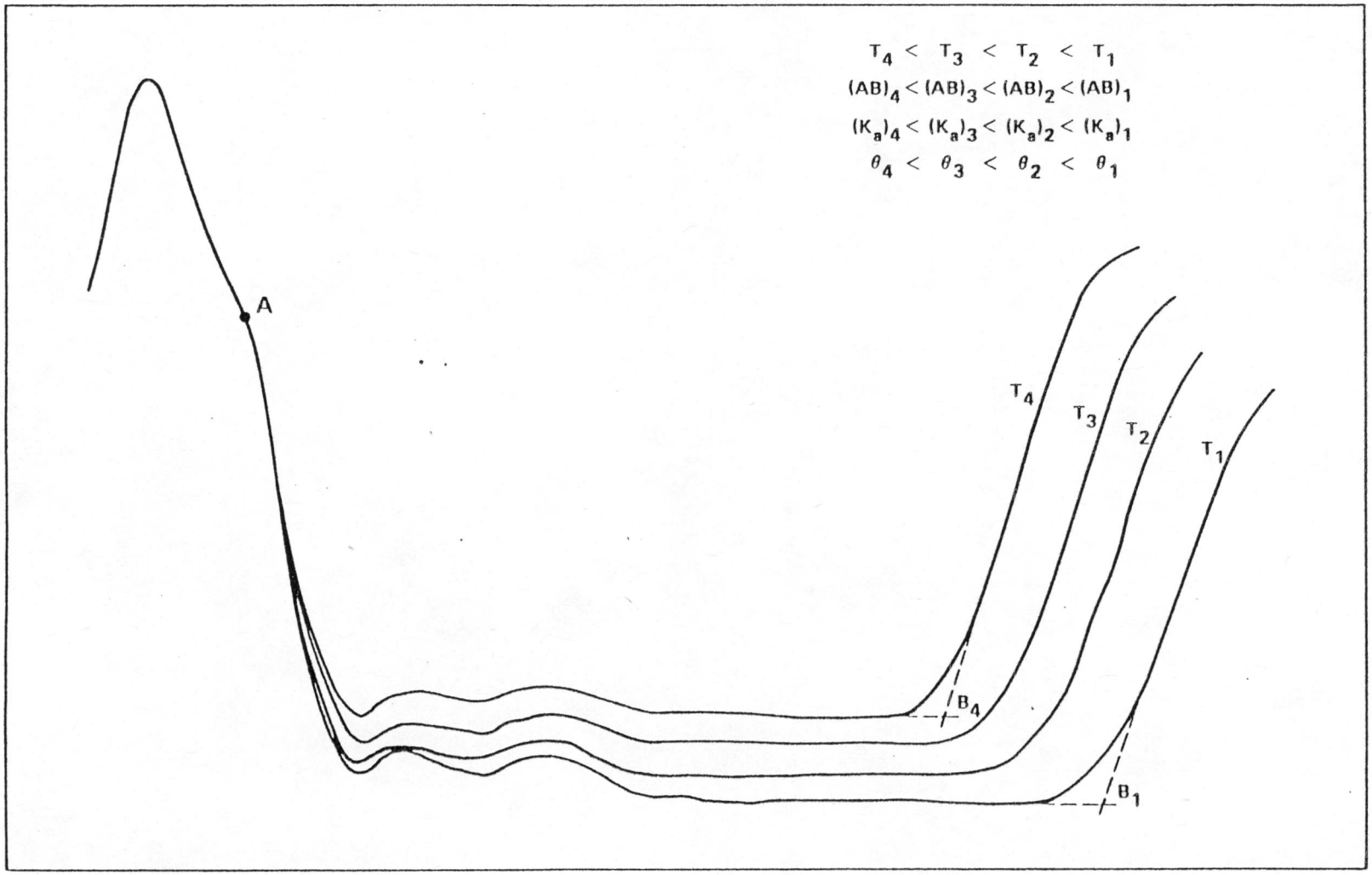

Figure 11 TDR Traces at Various Freezing Temperatures

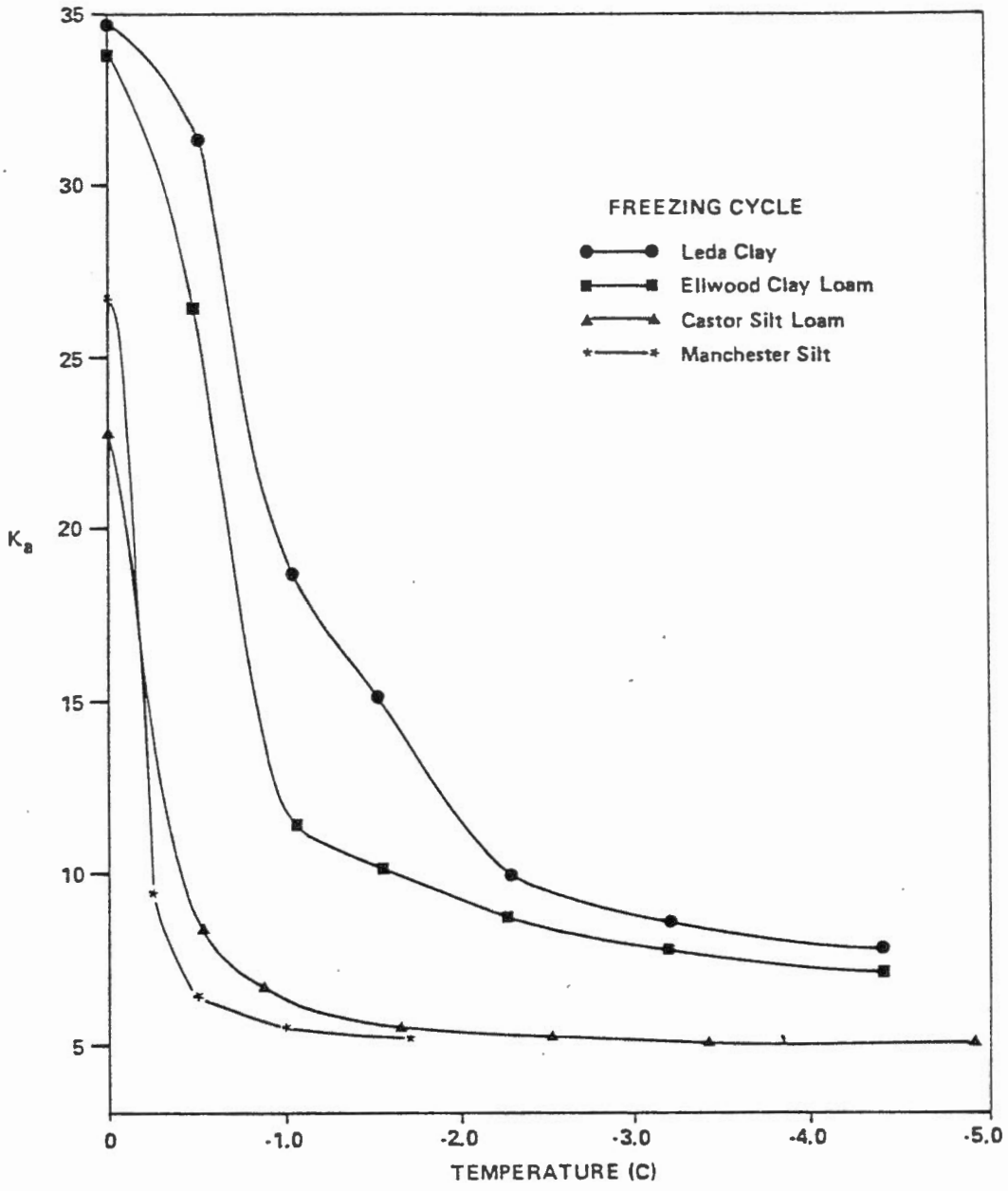

Figure 12 Variation of K_a with Freezing Temperature for Various Soils

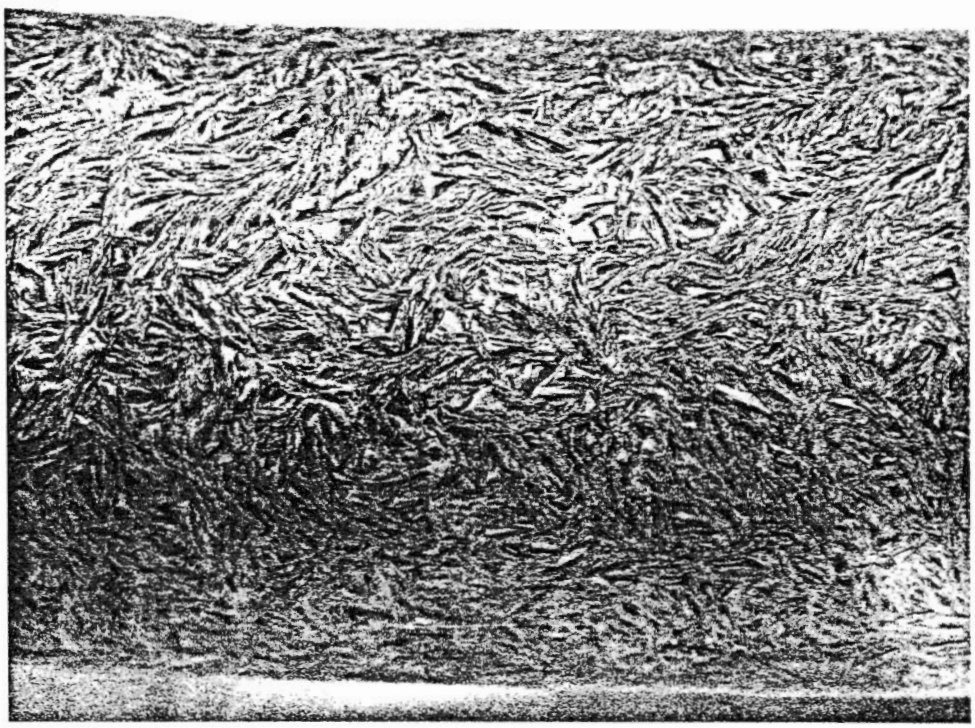

Figure 13 Random Ice Lensing in Sample of Castor Silt Loam

conductivities of the frozen sample and the lengths of time involved. Even at temperatures as warm as $-0.2^{\circ}C$, hydraulic conductivies are very low, 10^{-8} cm s^{-1} or so (Burt and Williams 1976). The use of a thin-walled sample container in our experiments meant that soils reached temperature equilibrium quickly; data from the dilatometer experiments showed that 80 to 90% of the phase change at temperatures in the range 0 to $-0.5^{\circ}C$ occurred within 3 or 4 hours. For a heavy clay, this was longer, but the hydraulic conductivity would probably be 10^{-9} cm s^{-1} or so.

To compile a curve such as shown in Figure 12 takes about 2 to 3 days for a silt loam, allowing 4 or 5 hours for each point to reach equilibrium. One can check for equilibrium by monitoring K_a a number of times at each temperature until a steady TDR trace is obtained. For a heavy clay at temperatures between 0° to $-1^{\circ}C$, it can take up to 24 hours to finally reach equilibrium in the sample. A complete curve can then take 4 days or so.

Curves of K_a vs. temperature determined for various soils were converted to freezing characteristic curves (Figure 14) by means of the relationship shown in Figure 9 and Appendix II. The curves for Castor Silt Loam, Leda Clay and Manchester Silt (formerly New Hampshire Silt) have been compared to published data for closely similar soils, and this is shown in Figures 15, 16 and 17 respectively. Grain size characteristics

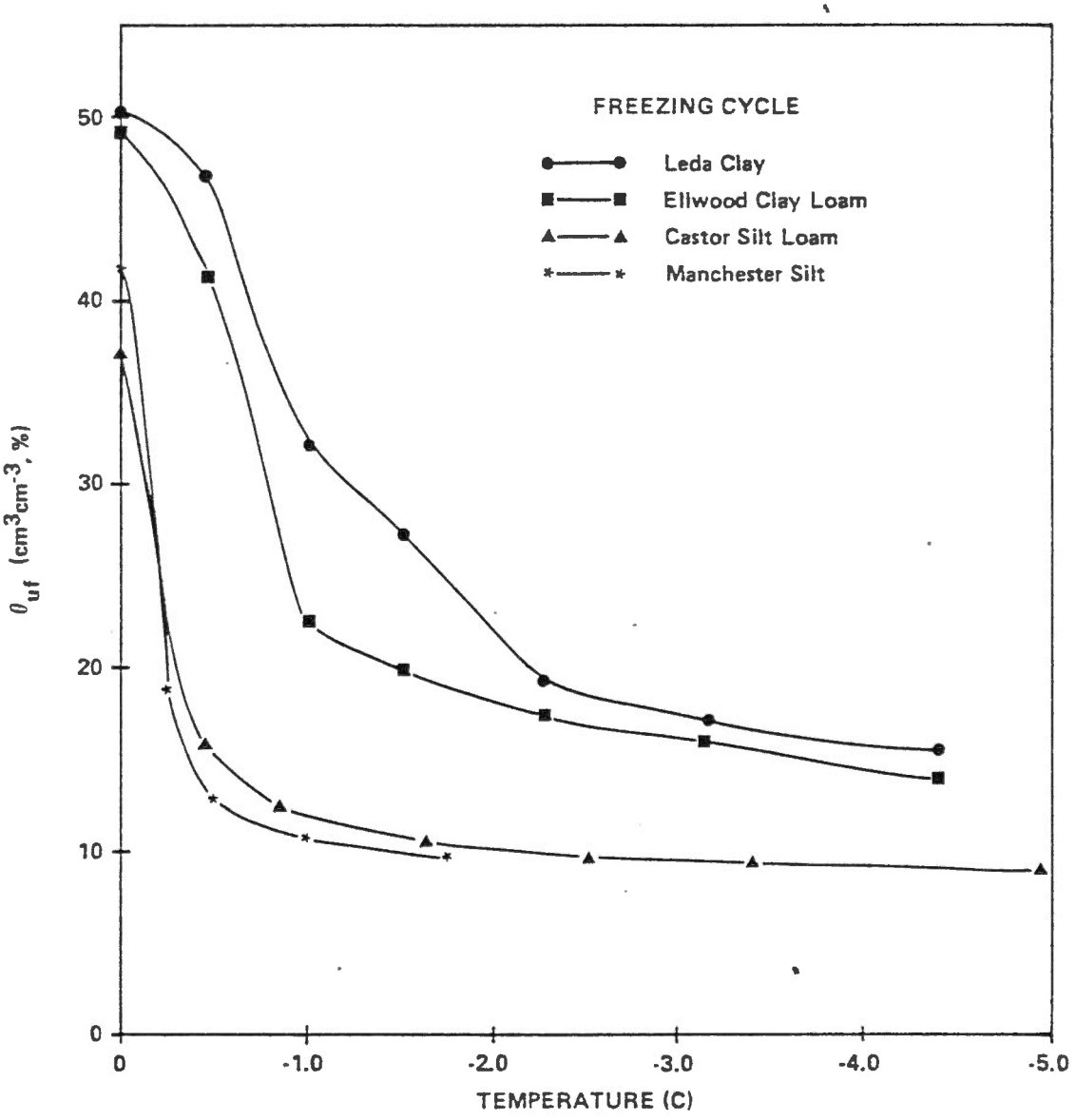

Figure 14 Freezing Characteristic Curves for Various Soils

are given for all the soils and determinations were for slurried samples in all cases.

The freezing characteristic curves for various silt loams are compared in Figure 15. As shown, the agreement is generally very good; the maximum difference in θ_{uf} is ±2½%, but the agreement is more typically within ±1½% between all curves. It is also satisfying that the TDR data fall between the other curves. The unfrozen water content data for Leda Clays also compare favourably, at all temperatures (Figure 16). In this case the maximum difference between the curves is ±5%, although the agreement is generally within ±3%. The variation shown in these Figures could be due to sample differences, experimental errors, or errors associated with converting the published gravimetric data to a volumetric basis (see Appendix III).

The existing data on unfrozen water contents for Manchester Silt are limited (see Figure 17); therefore, the TDR results were compared to those for Fairbanks Silt, an essentially identical soil (Dr. A. Tice, CRREL). θ_{uf} data have been determined previously by pulsed nuclear magnetic resonance, NMR (Tice, Burrous and Anderson, 1978) and calorimetry (Anderson and Tice, 1973). As shown in Figure 18, the NMR and TDR data compare favourably (within ±2% in w); however, both these sets of data are 3 to 4% higher than the calorimetric data. We can offer no obvious explanation for this

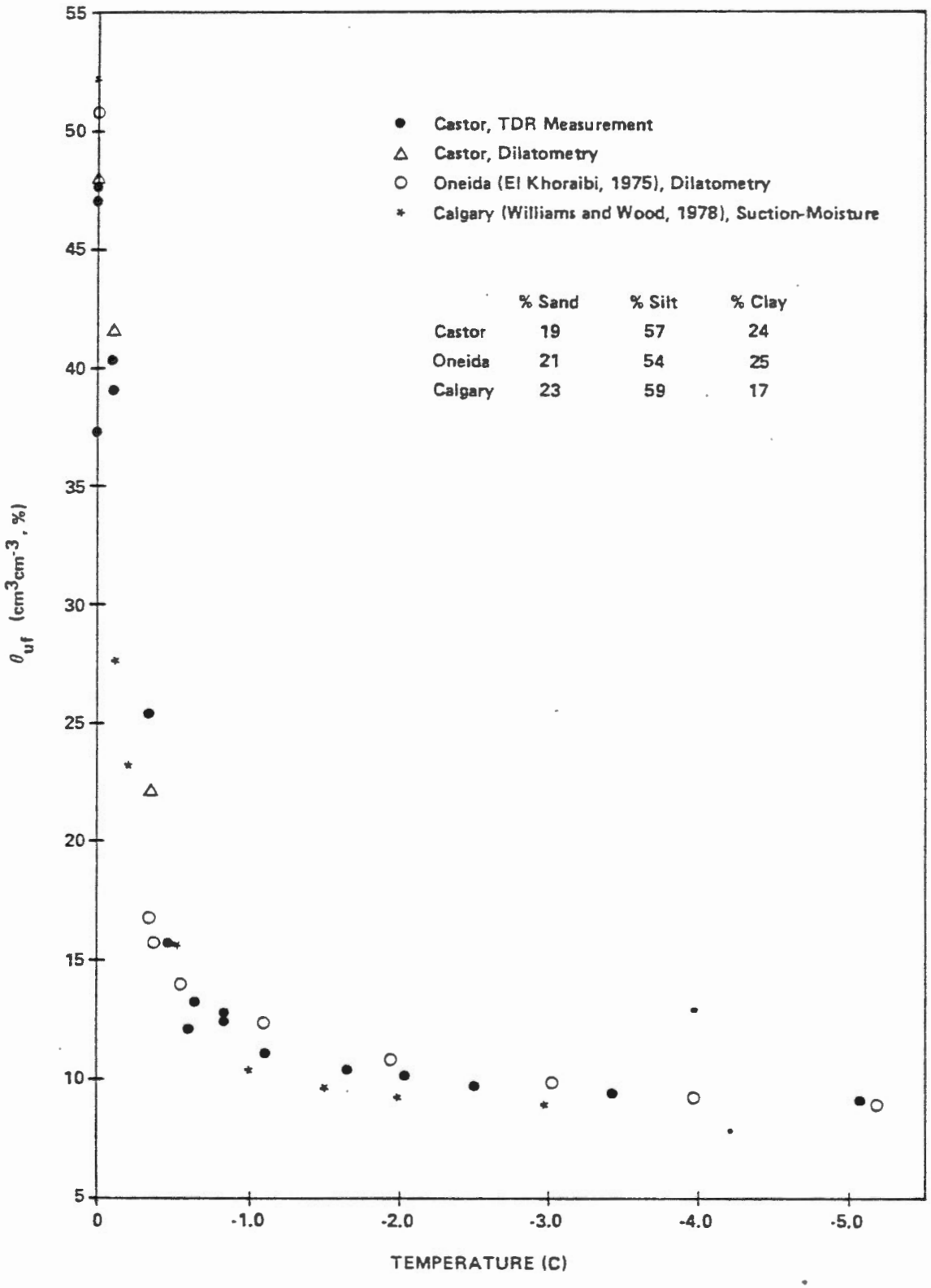

Figure 15 Comparison of Freezing Characteristic Data for Various Silt Loams

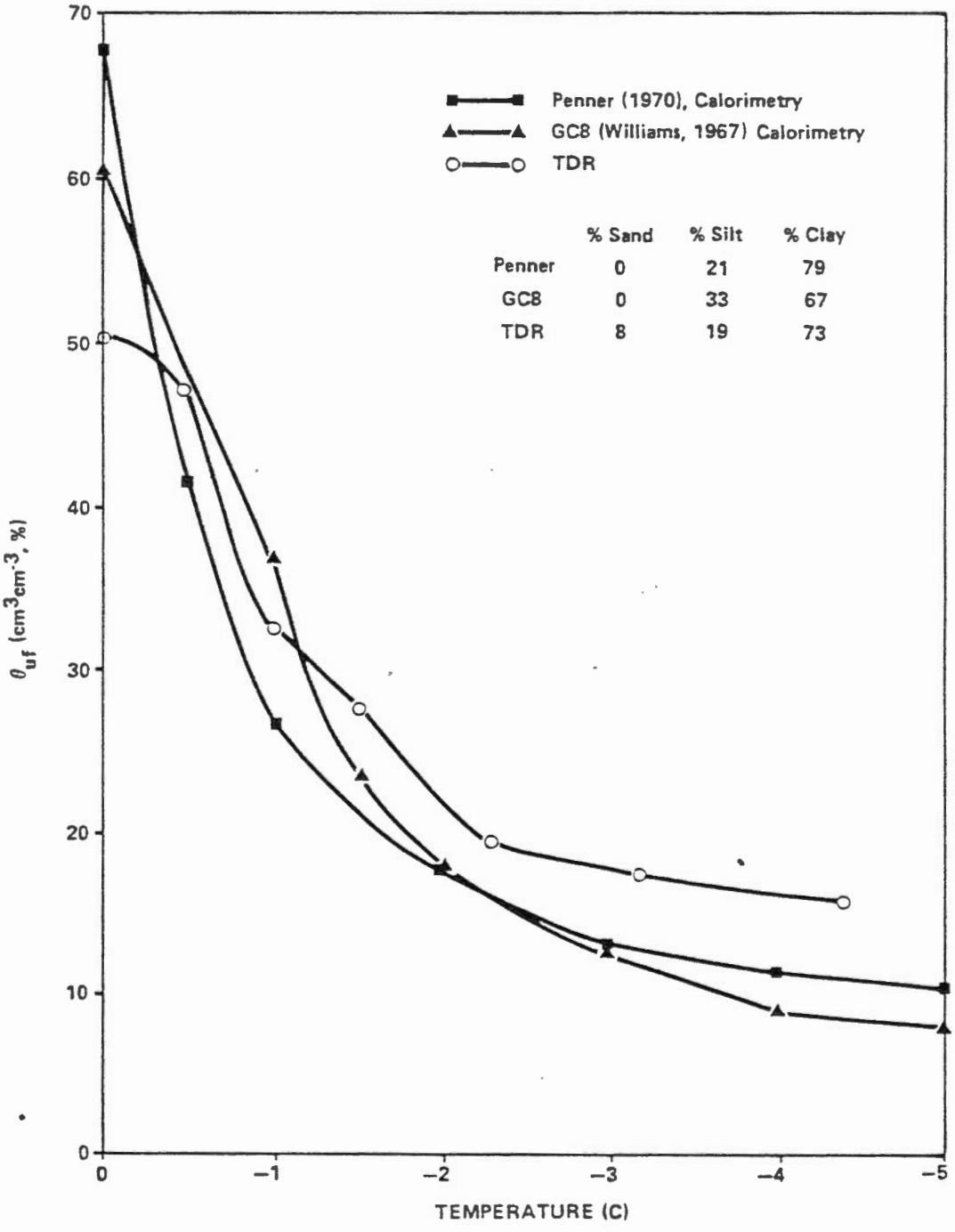

Figure 16 Comparison of Freezing Characteristic Data for Various Leda Clays

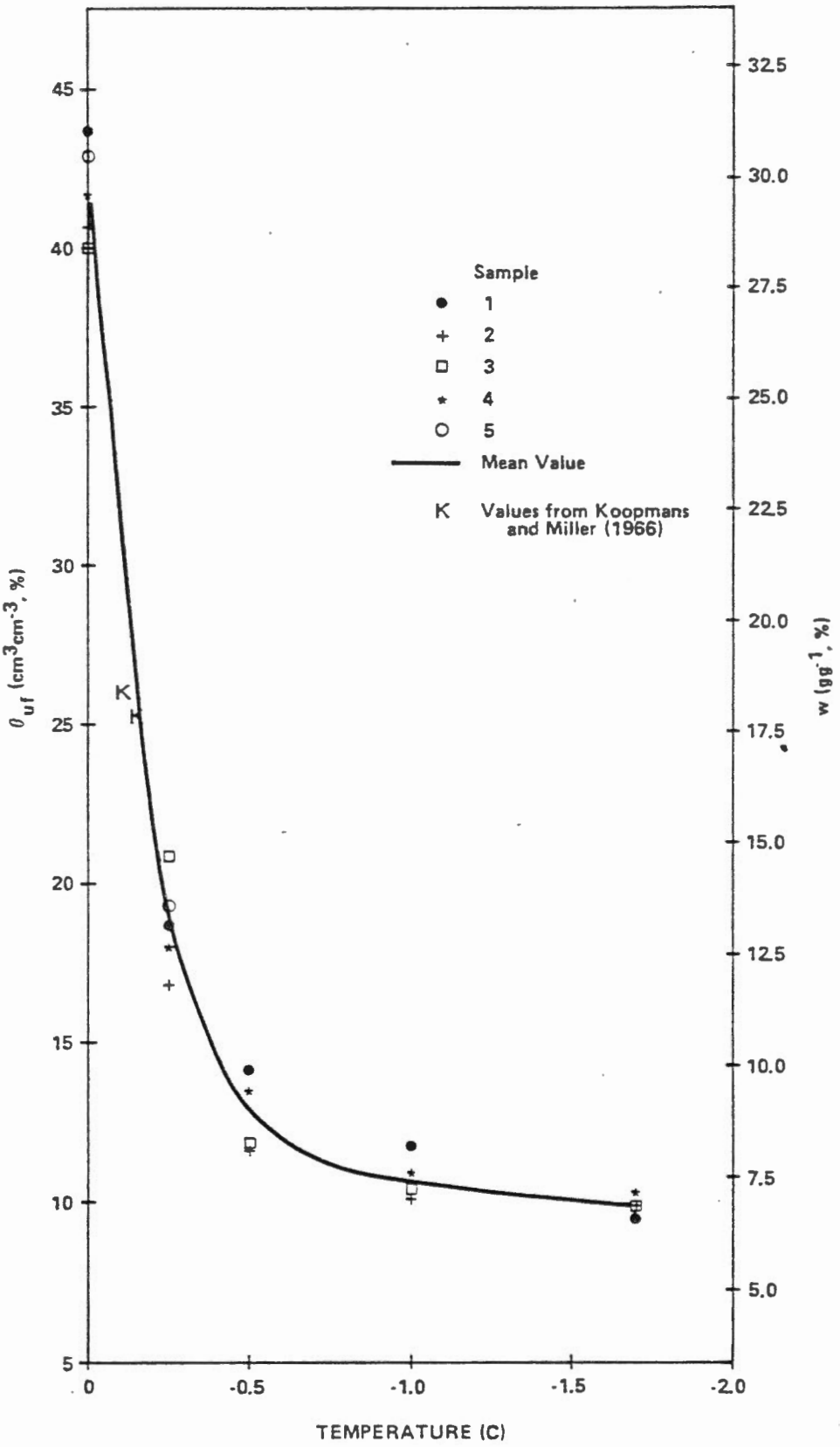

Figure 17 Freezing Characteristic Curve for Manchester Silt

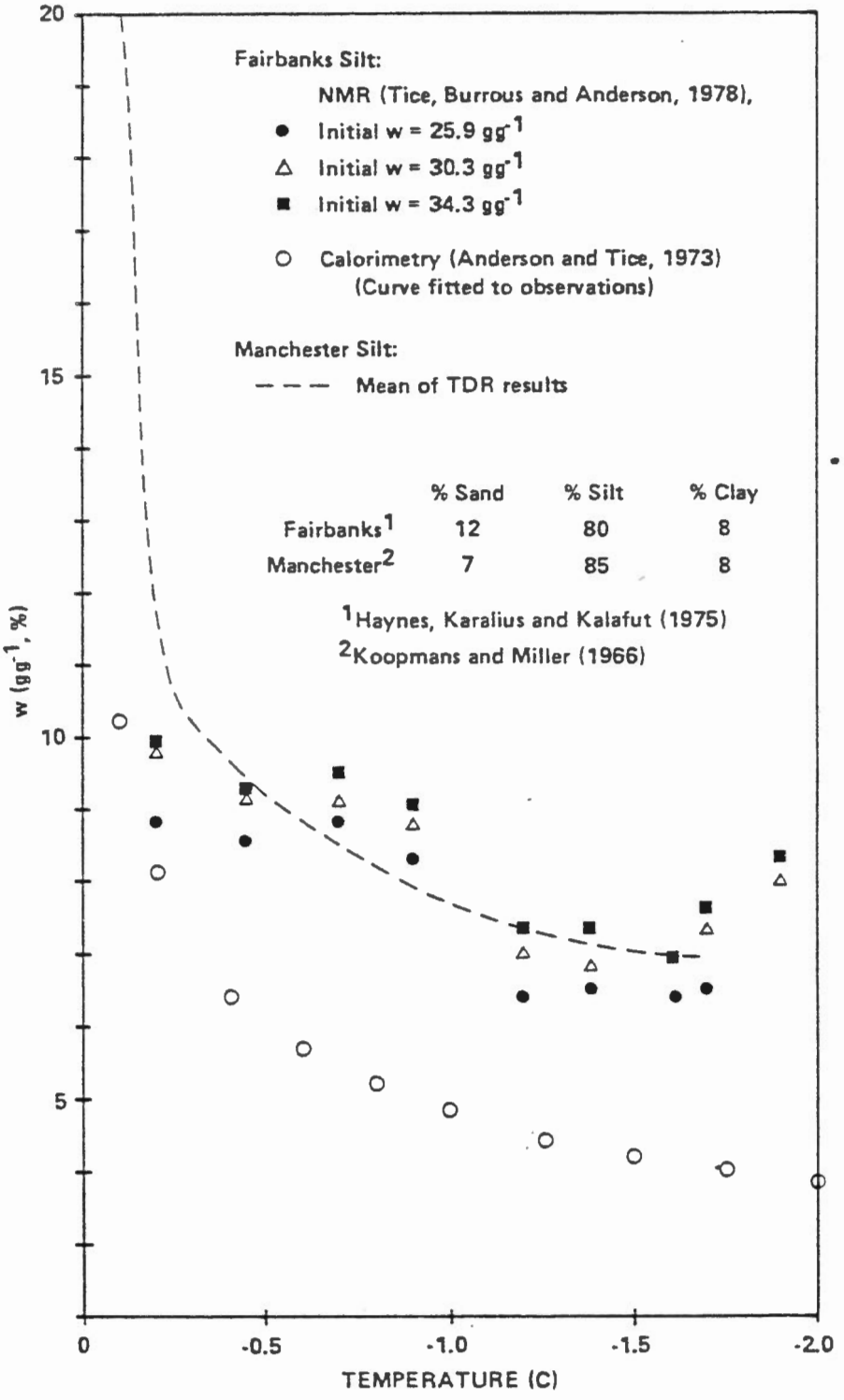

Figure 18 Comparison of Freezing Characteristic Data for Fairbanks and Manchester Silts

slight discrepancy.

The Leda Clay sample was subjected to a number of freezing-thawing cycles to see whether the TDR technique could detect any hysteresis in the θ_{uf} vs. temperature relationship, as described by previous authors (e.g. see Williams' 1963 calorimetry data). A difference in θ_{uf} between freezing and thawing cycles has been consistently measured (Figure 19); the differences are substantial over the first freeze-thaw cycle, but much reduced for the second cycle. Structural changes take place in the soil under freeze-thaw cycles, as evidenced by multiple ice lensing and consolidation in our samples, and this will affect the unfrozen water content relationship. This implies that laboratory measurements of frozen slurries will generally not be representative of the materials in situ. One advantage of the TDR technique is that it can be on undisturbed samples.

4.3.4 Reproducibility

A freezing characteristic curve for any soil will generally not be replicated exactly, particularly at temperatures close to $0^{\circ}C$, because of sample variability and experimental procedures. To examine reproducibility with the TDR technique, the θ_{uf} vs. temperature relationship was determined for several samples of the same soil, for three different soil types. The data for Manchester Silt (Figure 17) show a maximum difference of about 4% in θ_{uf}, at $-0.25^{\circ}C$, with the variation

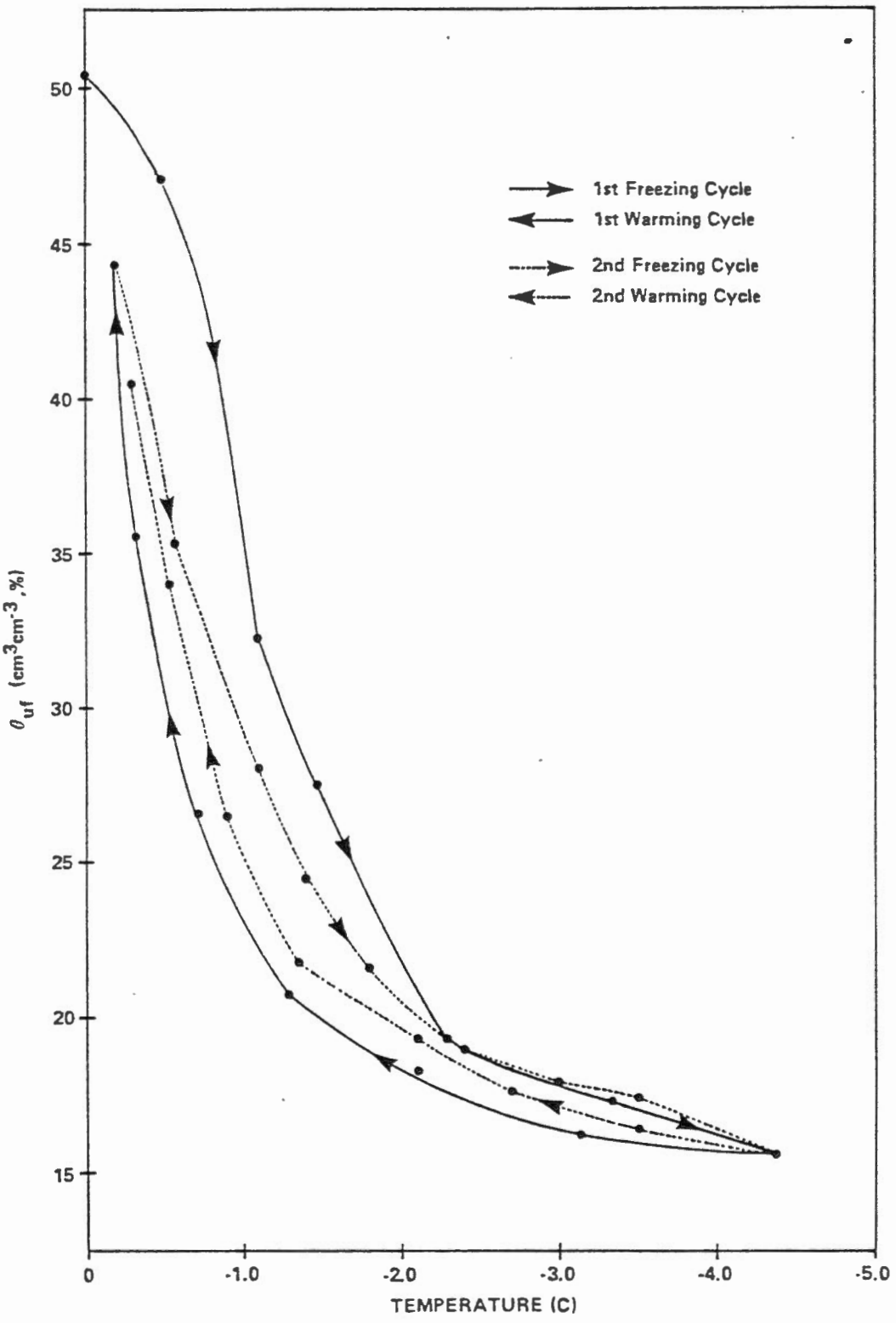

Figure 19 Freezing-Thawing Hysteresis in Leda Clay

decreasing to about 1% at -1.75°C. Data for Castor Silt Loam and Ellwood Clay Loam are summarised in Figure 20; values for θ_{uf} generally fall well within ±2½% at any given temperature, for each soil. It is felt that the variations shown in these Figures are likely largely due to sample differences (e.g. pore size distribution).

5. Conclusions

The TDR technique shows great promise for routine measurement of volumetric liquid water contents in frozen soils. The various experiments carried out have shown that:

i) K_a is not strongly sensitive to ice content in soil samples of coarse sand containing only ice and air;

ii) K_a measurements at various negative temperatures for a variety of finer textured soils indicate that unfrozen water is detected in the samples;

iii) data of K_a vs θ_{uf} for ice/water mixtures compare favourably (within ±2½%) to the empirical relationship for unfrozen soils obtained by Topp, Davis and Annan (1980);

iv) data of K_a vs. θ_{uf} for frozen soil samples, obtained from a combined TDR/dilatometer experiment, seem to further corroborate the applicability of this empirical relationship to frozen soils;

v) θ_{uf} values converted from K_a (using equation (4)) for

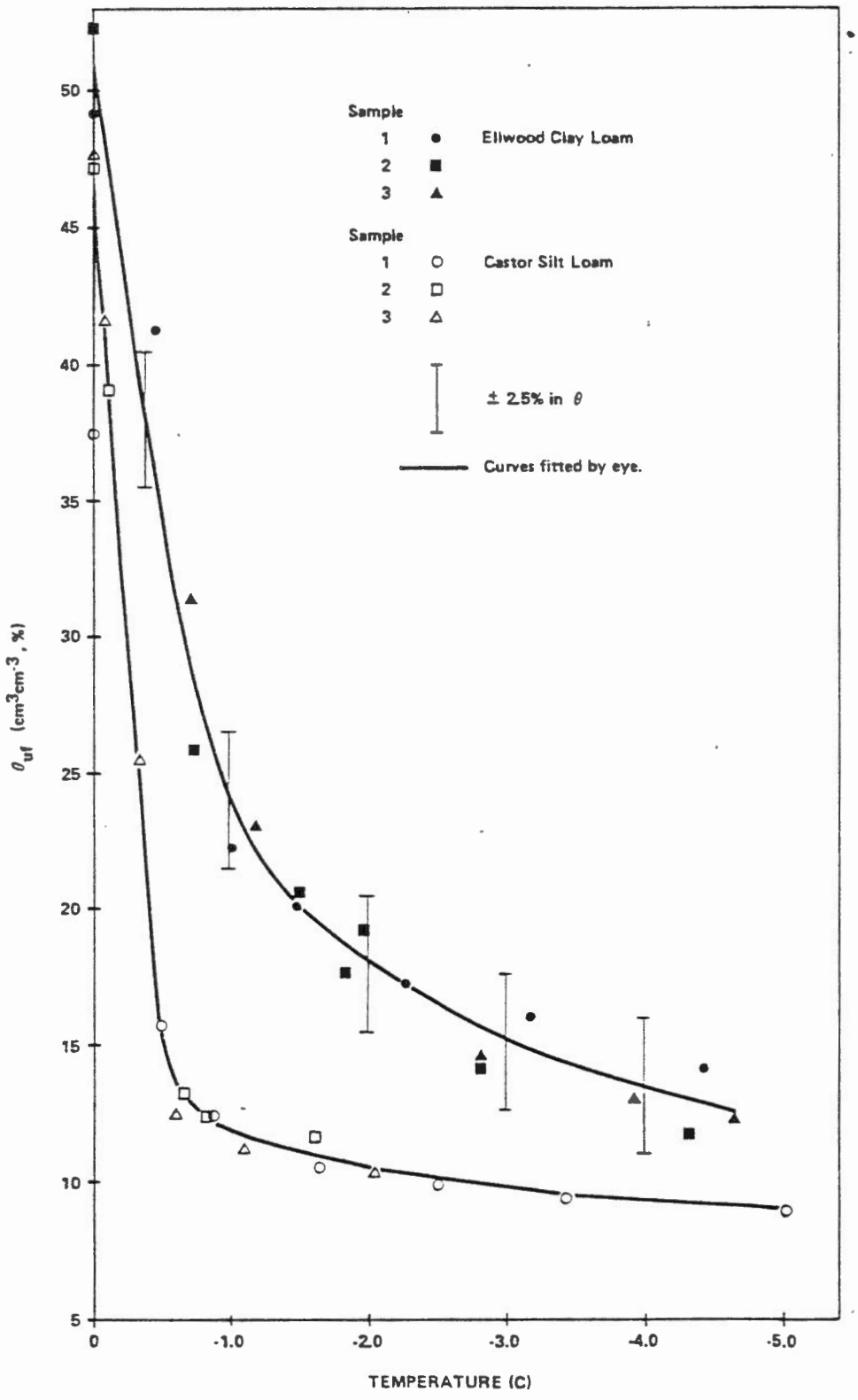

Figure 20 Reproducibility of Freezing Characteristic Curves by TDR

a number of soils compare favourably with published data for closely similar soils; agreement is typically within ±1½% in θ_{uf} for silts and ±3% for Leda clays.

Further data are desirable from the combined TDR/dilatometer experiment for a variety of soils to establish the $K_a - \theta_{uf}$ relationship. Also, determination of freezing characteristic curves for undisturbed core samples should be attempted, provided that the parallel probe can be inserted into the sample with a minimum of disturbance.

6. Acknowledgements

The success of this research is due, in some measure, to the invaluable help and cooperation of Dr. G.C. Topp (Agriculture Canada, Ottawa) and Dr. J.L. Davis (formerly from the Department of Energy, Mines and Resources, Ottawa). Their assistance is gratefully acknowledged. This research has also benefitted from the continued interest and support of Dr. A.S. Judge (Earth Physics Branch, Department of Energy, Mines and Resources). Thanks also to A. Pendlington and L. Boyle of the Geotechnical Science Laboratories, Carleton University, and to D. Halliwell and B. Rigby. Dr. A. Tice (CRREL, Hanover, New Hampshire) kindly provided the sample of Manchester Silt.

Appendix I

USING THE TEKTRONIX 1502 TDR UNIT

A.I.1 Introduction

Figure A.1 and Table A.1, in this appendix have been included to familiarize the reader with the Tektronix 1502 TDR control panel.

The Tektronix 1502 TDR has been used since it is reasonably priced and field portable. This TDR unit provides the CABLE connector with a 200 mV step-voltage which has an incident rise time of less than 0.1 ns. This rise time contains a wide bandwidth of frequencies up to a possible maximum of 3.5 GHz. The TDR's crt is calibrated in units of m ρ/DIV vertically and METRES/DIV horizontally. The vertical scale represents the ratio of the reflected to incident voltage (or voltage reflection coefficient, ρ). If a cable is open circuited (i.e. infinite impedance), $\rho = 1$; for a short circuited cable (zero impedance), $\rho = -1$. When a cable is terminated with its characteristic impedance there is no reflection and $\rho = 0$. Once can convert the reflected pulse amplitude to impedance since ρ is dependent upon the characteristic impedance, Z_o, of the line under test and the load on the cable, R_L, (or the impedance of a discontinuity within a cable):

$$(1) \qquad \rho = \frac{R_L - Z_o}{R_L + Z_o}$$

Therefore,

$$(2) \qquad R_L = Z_o \left(\frac{1 + \rho}{1 - \rho} \right)$$

Figure A.1

TDR MAINFRAME

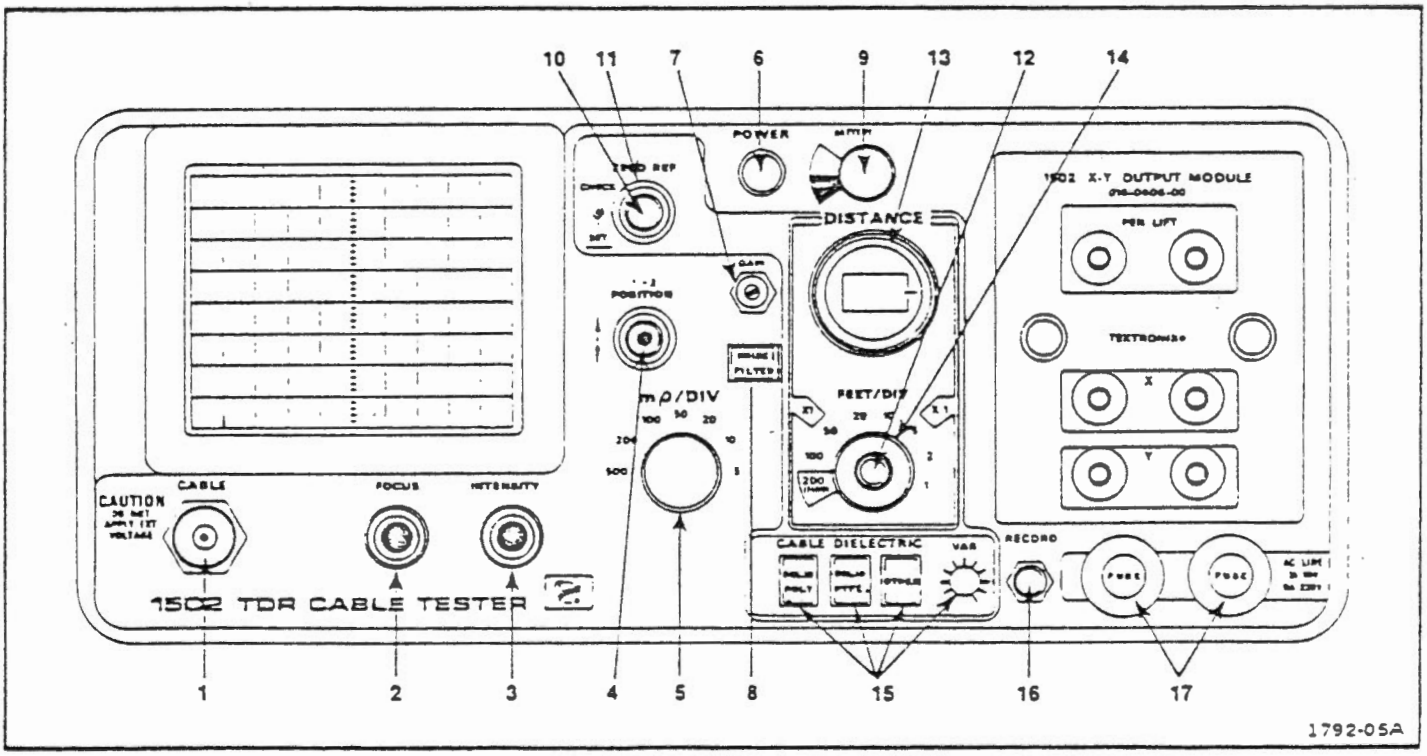

TABLE A.1

CONTROLS FOR TEKTRONIX 1502 TDR

1.	CABLE	BNC connector, delivers 110 ps rise pulse and receives reflected pulse
2.	FOCUS	visual controls for crt
3.	INTENSITY	visual controls for crt
4.	POSITION/FINE	vertical position control of the crt
5.	m ρ/DIV	vertical scaling control
6.	POWER	ON-OFF
7.	GAIN	for adjusting gain of the vertical amplifier
8.	NOISE FILTER	reduces noise and improves trace appearance
9.	BATTERY	battery level indicator
10.	ZERO REF CHECK	returns trace to reference setting
11.	ZERO REF SET	horizontal position control
12.	MULTIPLIER	horizontal scale setting factor: 0.1 or 1.0
13.	DISTANCE	moves crt display to any location in cable
14.	METRES/DIV	horizontal scale selector
15.	CABLE DIELECTRIC	permits selection v_p
16.	RECORD	activates x-y or y-t recorder
17.	AC LINE FUSES	

The horizontal axis is readily converted to a time axis from the following when CABLE DIELECTRIC is set for air (all buttons out):

$$\text{time/division} = \frac{\text{metres/division}}{0.3 \text{ m ns}^{-1}}$$

There are several other pieces of information which can be determined from the display on the TDR's crt. First, the impedance of a transmission line can be determined (or an impedance mismatch within a line) from (2) above, where Z_o is the impedance of a 'known' cable (or impedance mismatch in a line). If the vertical scale setting is known, the impedance at any point along the trace displayed on the crt can be determined. Also, the length of a cable can be determined directly off the crt display in the CABLE DIELECTRIC is set for the type of cable being tested. The dielectric constant of an 'unknown' cable can also be determined if the cable length is known, from equation (7) (p. 14).

A.I.2 TDR Operational Procedure

The first step in the procedure is to calibrate the device which records the information presented on the TDR's crt. Photographing the crt can be used in the field unless a Y-t module is used (the 1502 TDR has this option).

Operational Procedure

1. Set the TDR for air dielectric (VARIABLE control fully clockwise or all the CABLE DIELECTRIC buttons in the default position);
2. Pull the power switch on.

If remote X-Y recorder is used:

3. Connect a two-wire cable to the X-axis jacks on the TDR and the X-Y recorder;
4. Set the X and Y axis scales on the X-Y recorder to accommodate 1 V full scale.

5. Push up the RECORD switch on the TDR; this initiates the recorder of the trace displayed on the crt. The length of the trace will equal 10 divisions on the TDR's crt. Set the Y-axis on the S-Y recorder to the same setting as used for the X-axis.

If a Y-t recorder is used:

6. Since the horizontal axis is time, connect the X cables from the TDR to the Y connectors on the Y-t recorder. Set the chart speed of the Y-t recorder somewhere between 20 to 60 cm min^{-1}.
7. Set the Y-axis scale on the Y-t recorder to accept 1 V.
8. Push up the RECORD switch on the TDR. The slow ramp which will appear on the Y-t recorder represents the 100 mV/DIV that the TDR produces. The distance from the start to the end of the ramp equals 10 division on the TDR's crt.

Once the scaling factor is determined (i.e. trace length on the recorder used, divided by 10 divisions on the TDR), the starting point of the transmission line used (i.e. coaxial or parallel line) must be known. This is the A point introduced on p. 12 (also Figure 5).

9. Connect a 50Ωcable to the transmission line (for parallel lines, the cable is an integral part of the line).
10. Drain static charge from the line by connecting a 50Ω terminator and cable adapter (supplied with the TDR) to the end of the cable.
11. Connect the cable to the CABLE connector of the TDR.

Repeat the following steps 12 to 14 for the transmission line system with and without the actual lines of transmission connected to the probe head for various horizontal scale settings.

12. Adjust the DISTANCE dial and the POSITION control until the open circuit appears on the crt.
13. Set the mρ/DIV scale so that the trace appears completely on the screen vertically.

14. Record the crt display.

By superimposing both traces, the point of divergence (point A) can be determined (Figure A.2). Once determined, it remains a characteristic of the probe (line) on all traces. We determine point B as the intersect of two tangents to the trace where it encounters the open or short circuit (see also Figure 5).

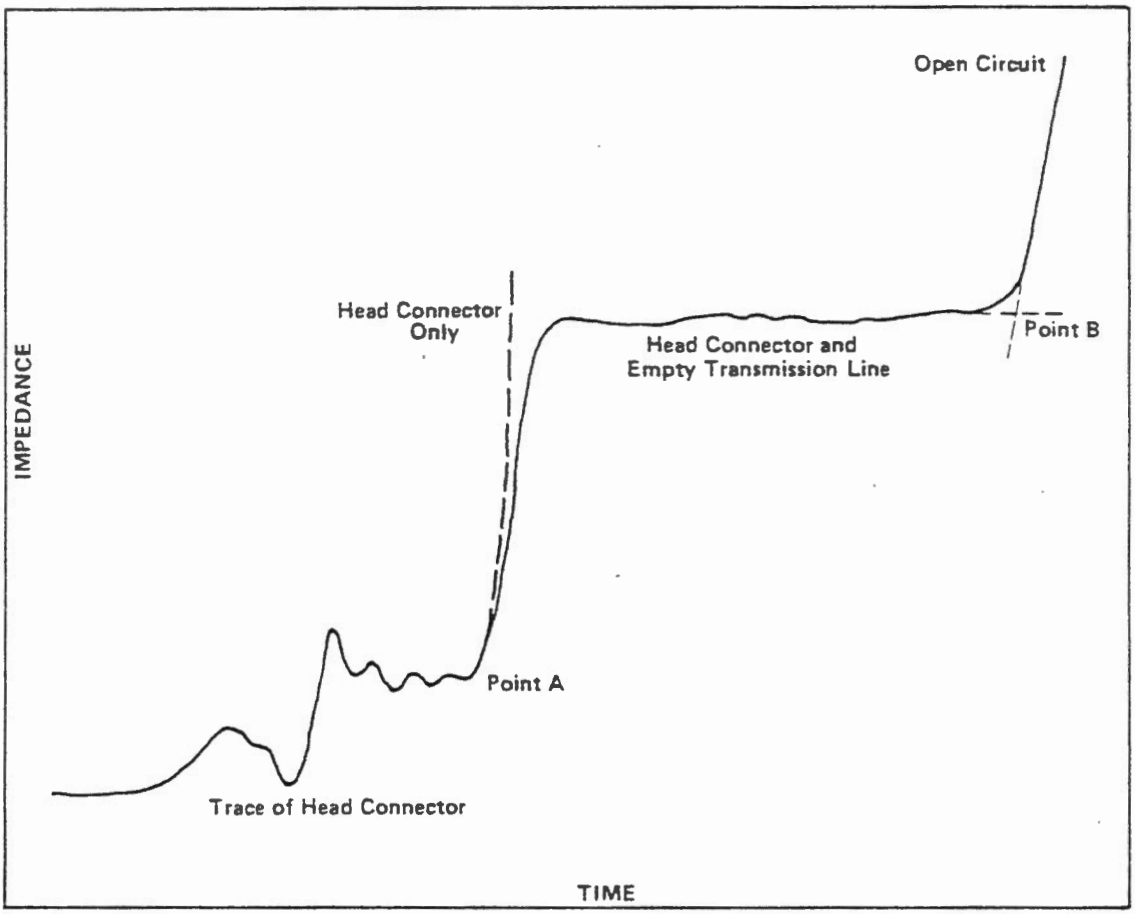

Figure A.2 Determining A and B points on TDR Trace

Appendix II

TABULATION OF THE EMPIRICAL RELATIONSHIP GIVEN BY TOPP, DAVIS AND ANNAN (1980)

K_a	Vol. Water Cont. (decimal fraction)	K_a	Vol. Water Cont. (decimal fraction)
3.03	0.000	11.64	0.225
3.08	0.005	11.96	0.230
3.14	0.010	12.28	0.235
3.20	0.015	12.61	0.240
3.27	0.020	12.94	0.245
3.35	0.025	13.28	0.250
3.44	0.030	13.62	0.255
3.53	0.035	13.97	0.260
3.63	0.040	14.32	0.265
3.74	0.045	14.67	0.270
3.85	0.050	15.03	0.275
3.97	0.055	15.40	0.280
4.10	0.060	15.76	0.285
4.23	0.065	16.13	0.290
4.37	0.070	16.51	0.295
4.52	0.075	16.89	0.300
4.67	0.080	17.27	0.305
4.83	0.085	17.66	0.310
4.99	0.090	18.05	0.315
5.17	0.095	18.44	0.320
5.34	0.100	18.84	0.325
5.53	0.105	19.24	0.330
5.72	0.110	19.65	0.335
5.91	0.115	20.05	0.340
6.12	0.120	20.47	0.345
6.32	0.125	20.88	0.350
6.54	0.130	21.30	0.355
6.76	0.135	21.72	0.360
6.98	0.140	22.15	0.365
7.21	0.145	22.57	0.370
7.45	0.150	23.00	0.375
7.69	0.155	23.44	0.380
7.94	0.160	23.87	0.385
8.19	0.165	24.31	0.390
8.45	0.170	24.76	0.395
8.72	0.175	25.20	0.400
8.99	0.180	25.65	0.405
9.26	0.185	26.10	0.410
9.54	0.190	26.55	0.415
9.83	0.195	27.01	0.420
10.12	0.200	27.47	0.425
10.41	0.205	27.93	0.430
10.71	0.210	28.39	0.435
11.02	0.215	28.85	0.440
11.33	0.220	29.32	0.445

Appendix II Continued

K_a	Vol. Water Cont. (decimal fraction)	K_a	Vol. Water Cont. (decimal fraction)
29.79	0.450	32.65	0.480
30.26	0.455	33.13	0.485
30.74	0.460	33.62	0.490
31.21	0.465	34.10	0.495
31.69	0.470	34.59	0.500
32.17	0.475	35.08	0.505

Appendix III

CALCULATING θ_{uf} FROM w

This Appendix lists the unfrozen water content data, taken from published sources, used for comparisons to the TDR-determined data. The method used for converting the published gravimetric water contents (w) to volumetric (θ_{uf}), for comparing to the TDR data, is also described. With the exception of El Khoraibi's (1975) data the following was assumed:

mass of the solids (M_s)	= 1 g
water content at 0°C (w, g g^{-1})	= saturation (i.e. no air)
particle density (ρd)	= 2.65 g cm^{-3}

Therefore,

volume of the solids (V_s)	= 0.377 cm^3 ($V_s = M_s/\rho_d$)
volume of water (V_w)	= M_w (by definition)
total volume (V_t) at 0°C	= $V_s + V_w$

As the soil freezes, the water that turns to ice expands by 9.05%, therefore, V_t must be corrected:

$$V_t \text{ (at } T^{\circ}C) = V_t \text{ (at } 0^{\circ}C) + (V_{w(0^{\circ}C)} - V_{w(T^{\circ}C)}) \times 0.0905$$

The unfrozen water content expressed volumetrically is:

$$\theta_{uf} = V_{w(T^{\circ}C)}/V_{t(T^{\circ}C)}$$

El Khoraibi (1975) provides sufficient data to calculate θ_{uf}, so his data and the method of conversion is presented separately.

Oneida Silt Loam (El Khoraibi, (1975) from dilatometric methods)

Temp. (^{o}C)	Observed Volume (cm^3)	Thermal Contr.^{n} (cm^3)	Volume Change (cm^3)	V_t (cm^3)	M_w (cm^3)	M_i (cm^3)	θ_{uf} ($cm^3 cm^{-3}$, %)
0.0	0.0	0.0	0.0	144.02	73.03	0.0	50.70
-0.328	4.361	0.0	4.361	148.38	24.82	48.18	16.73
-0.370	4.434	0.001	4.435	148.45	24.02	49.00	16.13
-0.586	4.703	0.005	4.708	148.73	21.00	52.02	14.12
-1.097	4.941	0.017	4.958	148.98	18.24	54.78	12.24
-1.943	5.109	0.035	5.144	149.16	16.18	56.84	10.84
-3.009	5.202	0.058	5.260	149.28	14.90	58.12	9.98
-3.994	5.251	0.079	5.330	149.35	14.12	58.90	9.45
-5.199	5.273	0.103	5.376	149.39	13.62	59.40	9.12

Thermal contraction = 0.0215 $cm^3\ {}^{o}C^{-1}$

M_i = volume change / 0.0905

$M_w = M_{w(0^{o}C)} - M_i$

$V_t = V_{t(0^{o}C)}$ + volume change

Calgary Silt Loam (Williams and Wood, (1978) from suction moisture data; the points represent those taken from a best fit curve)

Temp. (^{o}C)	w (gg^{-1})	θ_{uf} ($cm^3 cm^{-3}$)
0.0	0.4165	0.524
-0.1	0.22	0.277
-0.2	0.185	0.233
-0.5	0.125	0.157
-1.0	0.0825	0.104
-1.5	0.075	0.098
-2.0	0.075	0.094
-3.0	0.0725	0.091

Leda Clay (Penner) (Penner, 1970 from calorimetric methods for an undisturbed sample; the points represent those taken from a best fit curve)

Temp. (^{o}C)	w (gg^{-1})	θ_{uf} ($cm^{3}cm^{-3}$)
0.0	0.80	0.68
-0.5	0.50	0.415
-1.0	0.325	0.267
-2.0	0.22	0.179
-3.0	0.16	0.13
-4.0	0.14	0.113
-5.0	0.125	0.101

Leda Clay (GC8) (Williams, 1967 from calorimetric methods for a disturbed sample; the points represent those taken from a best fit curve)

Temp. (^{o}C)	w (gg^{-1})	θ_{uf} ($cm^{3}cm^{-3}$)
0.0	0.57	0.602
-1.0	0.355	0.367
-1.5	0.23	0.235
-2.0	0.175	0.178
-3.0	0.125	0.127
-4.0	0.085	0.086
-5.0	0.075	0.076

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