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INVESTIGATION OF PRESSURE CHANGES IN FROZEN SOILS DURING THERMALLY INDUCED MOISTURE MIGRATION

by

P.J. Williams and J.A. Wood

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FINAL REPORT

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to the

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ABSTRACT

The report examines the application of the Clapeyron equation in describing freezing-point depression attributing freezing point depression to solute presence and to interface pressures. Mechanisms are examined by which moisture migrates through frozen soil in both the liquid and solid phase. In general solid phase regelation mechanisms appear dominant in non-colloidal soils at marginally sub-zero temperatures whereas moisture transport dominates in colloidal soils. Preliminary pressure gradient measurements during thermally induced moisture migration suggest a reduction in effective soil stress with a reduction in temperature.

RESUME

Ce rapport examine l'application de l'équation de Clapeyron à la description de l'abaissement de la température de congélation, en attribuant cet abaissement à la présence d'ions dissouts et aux pressions d'interface. Les mécanismes de la migration d'eau, dans la phase liquide et la phase solide, à travers un sol gelé sont aussi étudiés. En général les mécanismes de regel de la phase solide semblent dominer dans les sols non-colloidaux pour des températures très peu sous zéro. Par contre le transfert de l'eau liquide domine dans les sols colloidaux. Des mesures préliminaires de gradients de pression établis lors de la migration d'eau induite par un gradient thermique suggèrent une diminution de la contrainte effective du sol et un abaissement de la température.

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NOMENCLATURE

P	pressure
T	temperature
s	entropy
v	specific volume
G	specific free energy
L	heat of fusion of water
σ	surface tension
r	radius of curvature
x	mole fraction
Π	osmotic pressure
ψ	electrical potential
n, c_s	ion concentration
ρ	density
θ_i	ice content
θ_w	water content
V_T	total flux
ν_i	flux of ice
ν_w	flux of water
∇	differential operator
ϕ	total hydraulic potential
k	hydraulic conductivity
R	universal gas constant
τ	film thickness

Introduction

This final report represents a summary of the theory behind, and findings from, recent experiments on frozen soils carried out between June 1980 and June 1981 under the terms of the contract OSU80-00040. The instrumentation developed to carry out the tests is reported, along with an analysis of results from a preliminary experiment. The types of experimental difficulties encountered as well as refinements and modifications to the apparatus are also discussed.

The work represents an extension of earlier research into the phenomenon of thermal osmosis or temperature induced moisture migration in frozen soils. It became apparent by the end of the previous phase of this investigation (Williams and Perfect, 1980) that a full analysis of the process requires a knowledge of the pressure state of the frozen soil within the sample holder. The present phase of the investigation involves construction of a special permeameter in which pressure gradients are measured within the frozen soil during thermal osmosis.

Owing to lengthy delays in the shipping and receiving of the pressure sensing instruments, construction of the experimental apparatus was only recently completed. However, good use was made of the available time and additional investigations have been conducted into the time-decay in the hydraulic conductivity of frozen soils as well as moisture migration induced by a gradient in osmotic potential. Results of both experiments are also summarized in this report (section 3) and have proven meaningful with regard to some of the recent theoretical work on mechanisms of moisture migration in frozen soils. The results have also answered some of the questions which had been raised by Perfect in his Master's thesis (1980) regarding the possibility of autonomous behaviour at opposite ends of a frozen soil sample.

The importance of a theoretical understanding of freezing soils became apparent from the earlier work, and a substantial theoretical analysis is included in a significant portion of the present report.

The report is divided into 4 sections. The first section is intended to introduce some of the basic theoretical concepts in thermodynamics and physical chemistry which are used in the study of frozen soils. The second section of the report deals largely with the theoretical aspects of moisture migration in frozen soils, particularly those arising from very recent work by ourselves and others. In the last two sections of the report experimental results are discussed. The results are examined with regard to current theoretical and applied studies of frozen ground.

In accordance with the terms of the contract, much of the research was carried out by J.A. Wood, a graduate student. Dr. Williams has maintained an overall supervising role as 'Principal Investigator'.

J.A. Wood P.J. Williams

SECTION I

Thermodynamics of Freezing Soil

1. Behaviour of Bulk Water

The presence of unfrozen films of water surrounding the particles in frozen soils was first postulated by Bouyoucos (1916), Taber (1930) and Beskow (1935) and later verified using a variety of techniques including dilatometry, adiabatic and isothermal calorimetry, nuclear magnetic resonance and x-ray diffraction (Anderson and Tice, 1973). The existence of these unfrozen films implies a freezing-point depression in a fraction of the water retained within the soil pores. The mechanism by which freezing-point depression is produced in soils, was and still remains, a subject of controversy in much of the literature.

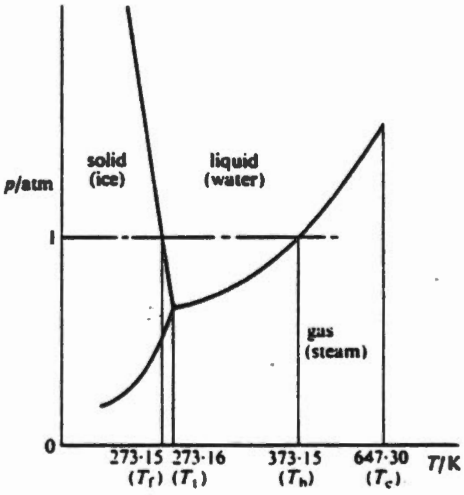
Figure 1.1 shows a phase diagram for bulk water. The diagram illustrates the pressure dependence of the freezing, boiling and sublimation temperatures for water. All three phases can coexist at only one temperature and pressure. This is called the triple point T_t. The boundaries between the solid, liquid and gaseous phases of water at all locations other than the triple point are defined by the Clapeyron equation:

$$\frac{dP}{dT} = \frac{s_2 - s_1}{v_2 - v_1} \qquad (1.1)$$

where P = the pressure,
T = the temperature,
s = the entropy,
v = the specific volume

and the subscripts 1 and 2 denote the appropriate phase.

Figure 1.1



The phase diagram for water.

Atkins (1978)

The Clapeyron equation indicates the pressure dependence of the temperature at which phase change occurs in any substance. It is an exact result and applies to any phase change, provided equilibrium is maintained between the two phases (Atkins, 1978). The quantity s_2-s_1 denotes the change in specific entropy of the system in going from phase 1 to phase 2.

Examining Figure 1.1, it is evident that the boiling-point of water increases very rapidly with pressure. In contrast, the freezing-point decreases very slowly and linearly as the pressure is increased.[1] This relates to the fact that as a solid or a liquid,water is practically incompressible whereas, as a gas its specific volume decreases rapidly with pressure.

A fundamental equation used in thermodynamics, is the Gibbs-Duhem equation:

$$dG = vdP-sdT \tag{1.2}$$

where G = the specific free energy (Gibbs free energy) of the system (ie. free energy per gram of substance).

This equation relates changes in the Gibbs free energy of a system to changes in its temperature and pressure. Figure 1.2 illustrates the temperature dependence of the Gibbs free energy for water at atmospheric pressure P. (T_f and T_b denote the freezing and boiling points). The discontinuity

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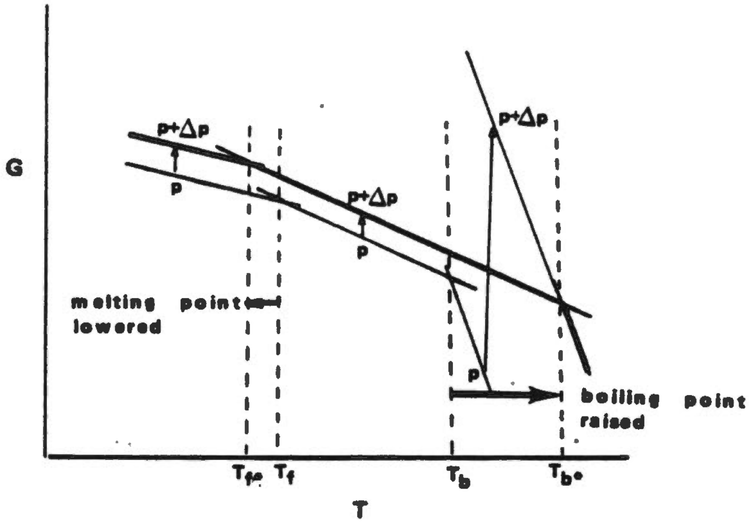
[1] Water is an unusual substance since its freezing-point decreases as the pressure increases. (With most substances, the freezing-point increases with pressure). The decrease in the freezing-point of water relates to the fact that the specific volume of ice (v_i = 1.090 $cm^3\ gm^{-1}$) is greater than that of water (v_w = 1.000 $cm^3\ gm^{-1}$). Substituting these values in the Clapeyron equation, gives a negative value for $\frac{dP}{dT}$.

Figure 1.2



The pressure and temperature dependence of the Gibbs free energy and its effect on the melting and boiling points of water. The phase with the lowest G is the stable phase at that temperature.

after Atkins (1978)

in the slope of G vs T at phase transition represents the change in entropy in passing from one phase to the next. If the pressure of the water is raised by an increment $P+\Delta P$ the specific free energy of the water increases in accordance with the equation (1.2) given above. Note, however, that the increase is very much less in the liquid or solid phase than it is in the gaseous phase. This relates to the fact that the specific volume of water as a liquid or a solid is constant whereas in the gaseous phase it is pressure sensitive. At any temperature and pressure the phase with the lowest free energy is the most stable phase under those conditions. Thus, during any change on a system of coexisting phases, the phase with the greater free energy will disappear.

1.2 Application of the Clapeyron Equation to a System of Water and Ice

One difficulty in applying the Clapeyron equation in the form shown in equation (1.1) to a real system, is that it relates the change in the freezing-point to the total change in pressure on the system. That is, it does not indicate how the freezing-point changes when the change in pressure on two coexisting phases are different. It is clear that in soils, for example, the ice and water are not, generally, at the same pressure. A generalized form of the Clapeyron equation was derived by Edlefsen and Anderson (1943) as:[2]

$$v_w\, dP_w - v_i dP_i = \frac{L dT}{T} \qquad (1.3)$$

where P_i, P_w = the pressure of the ice and water phases,

---

[2] The original derivation of this equation is attributed to Le Chåtalier (1892) (in Miller, 1971).

v_i, v_w = the specific volume of the ice and water,
dT = the freezing-point depression of the water,
T = the freezing temperature of pure free water (= 273.15K at 1 atm. pressure),
L = the latent heat of fusion of water.

Equation (1.3) indicates that for a system of water and ice in equilibrium with each other, freezing-point depression is produced by a rise in the pressure of the ice or a fall in the pressure of the unfrozen water or a combination of both changes. If P_i is increased by 1 atm. while P_w remains constant dT = -0.0899K. If, however, P_i remains constant while P_w is decreased by 1 atm., dT = -0.0824K. If P_i and P_w are both raised by 1 atm. then dT = -0.0075K. This value represents the difference between dT in the two cases given above. (Edlefsen and Anderson, 1943).

1.3 Interface Effects

The Clapeyron equation given above, indicates that in freezing soils,freezing-point depression is caused by a rise in the pressure of the pore ice or a fall in the pressure of the unfrozen pore water or a combination of both changes. However, it is not by itself, an explanation for the existence of unfrozen films of water in frozen soils.

Everett (1961) investigated the behaviour of small ice crystals immersed in a bath of their own melt. In order for a small crystal to exist in equilibrium with its own melt, the Gibbs free energy of the crystal must equal that of the water. However, Figure 1.2 indicates that the specific free energy of bulk ice is less than that of water at any temperature below freezing. Thus, in order for a small ice crystal to maintain equilibrium with its own melt, at hydrostatic pressure P_w, its specific free energy must exceed that of bulk ice by an amount given by:

$$G(P_i) = G(P_w) = v_i \, \sigma_{iw} \, \frac{dA}{dV} \qquad (1.4)$$

where $G(P_i)$ = the specific free energy of the crystal,

$G(P_w)$ = the specific free energy of bulk ice at the same temperature,

σ_{iw} = the free energy or surface tension of the interface (= 30.5×10^{-3} Nm^{-1}),[3]

A = the area of the interface,

and V = the volume of the crystal.

The increase in the specific free energy of the crystal above that of bulk ice is attributed, at least formally, to an increase in the pressure of the ice within the crystal above that of the melt, such that:

$$G(P_i) - G(P_w) = v_i \, (P_i - P_w) \qquad (1.5)$$

Rearranging terms in equation (1.4) and substituting for $G(P_i) - G(P_w)$, we obtain the Laplace equation:

$$P_i - P_w = \sigma_{iw} \, \frac{dA}{dV} \qquad (1.6)$$

Everett (1981) postulated that the freezing-point of water at a convex interface, such as a small crystal, is

---

[3] The free energy of an interface is defined by Edlefsen and Anderson (1943) as the energy necessary to maintain its surface area. The surface tension is numerically and dimensionally equal to the surface free energy and represents the work done by the interface ΔW, in changing its area by ΔA isothermally.

depressed below that of a planar interface. This is achieved by a rise in the pressure of the ice. The equilibrium between the crystal and its melt is maintained by adopting a shape for which $\frac{dA}{dV}$ satisfies $P_i - P_w$, the equilibrium condition being determined by P_i, P_w and the temperature T.

Woodruff (1963) describes the growth of a spherical ice nucleus in its own melt. The freezing temperature of the system is defined as the temperature at which the specific free energy of the solid and liquid phases is the same. According to Woodruff (1978), the total free energy of the crystal represents the sum of a contribution from the interface as well as one from its volume. For a spherical nucleus of radius r, the interfacial energy term is simply,

$$G_s = 4\pi r^2 \sigma_{iw} \tag{1.6}$$

where $4\pi r^2 = A =$ the area of a sphere

The volume contribution to the free energy of the nucleus relative to the liquid phase is:

$$G_v = \frac{-4\pi r^3}{3} \frac{L}{T} \Delta T \tag{1.7}$$

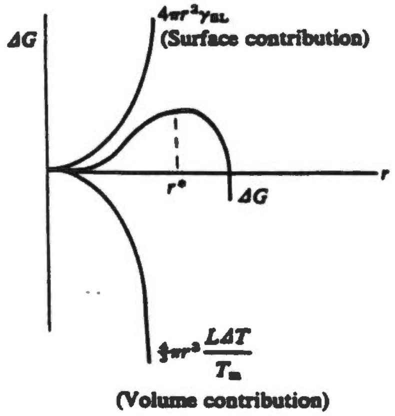
where $\frac{4\pi r^3}{3} = V =$ the volume of a sphere.

At any temperature ΔT below the freezing temperature of bulk water T, the excess free energy of the nucleus is given by:

$$\Delta G = G_{nuc} - G_{liq} = \frac{-4\pi r^3}{3} \frac{L}{T} \Delta T + 4\pi r^2 \sigma_{iw} \tag{1.8}$$

This relationship is illustrated in Figure 1.3. Note that the maximum value of ΔG corresponds with a certain critical radius r^*. This implies that there is a thermodynamic barrier to nucleation in creating a nucleus of radius r^* (Woodruff,

Figure 1.3



The surface and volume contributions and the total excess free energy ΔG of a spherical solid nucleus within a supercooled liquid as a function of the nucleus radius r.

Woodruff (1963)

1963). This barrier is caused by the interfacial free energy. "Once a nucleus of radius $r_{iw} > r^*$ has been created, further transformation results in a continued decrease in ΔG which proceeds spontaneously "(Loch, 1975). In other words, a slight perturbation to a larger radius of curvature results in spontaneous growth of the crystal until all of the melt solidifies or the latent heat brings the temperature of the system back to T.

The relationship between r^* and ΔT can be obtained by solving equation (1.8) for $\frac{d(\Delta G)}{dr} = 0$. (This corresponds with a maximum value of ΔG at r^*). This gives us the Kelvin equation:

$$r^* = \frac{2\,\sigma_{iw}\,T}{L\Delta T} \qquad (1.9)$$

This equation describes the "critical radius" of a spherical ice body that is in metastable equilibrium with its own melt at temperature ΔT below the freezing temperature of bulk ice. Note that r^* decreases with colder temperatures.

A condition of metastable equilibrium between the crystal and its melt exists when $r=r^*$ (ΔT = the freezing-point depression). Under these conditions, the specific free energy of the ice and the water is the same. Growth of the crystal occurs when the temperature is lowered so that $r>r^*$ (ΔT = the degree of supercooling). Under these conditions, the free energy of the crystal is less than its equilibrium value. The driving mechanism for the growth of the crystal is the fact that the free energy of the crystal is less than that of the melt at that temperature and so the crystal grows spontaneously until equilibrium is attained.

The theory of ice crystal growth in small capillaries and porous media was formally worked out and described by Everett (1961). The conditions for equilibrium and growth of an ice crystal in a small capillary are the same as those in a bath of its own melt except that r* is a stable rather than a metastable equilibrium radius. For a spherical interface, the Laplace equation is given as:

$$P_i - P_w = \frac{2\sigma_{iw}}{r_{iw}} \qquad (1.10)$$

Everett (1961) suggested two possible mechanisms by which the Laplace equation can be satisfied in frozen soils.

(1) By an increase in the ice pressure above that of bulk ice, the water pressure remaining constant and equal to the bulk ice pressure.

(2) A fall in the pore water pressure below that of bulk ice. In this case, the pressure of the ice within the capillaries remains constant and equal to the pressure of bulk ice.

In subsequent tests Williams (1967) and other investigators determined that in frozen soils, the Laplace equation and the Clapeyron equation are satisfied "at least in part by a fall in the pressure of the unfrozen pore water."[4]

Everett (1961) also demonstrated that the geometry of the capillaries imposes restrictions on the size of the ice

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[4] The degree to which the Laplace and the Clapeyron equations are satisfied by a rise in the ice pressure and a simultaneous fall in the pressure of the unfrozen pore water has not been answered satisfactorily in the literature. One is led to assume that the pore ice pressure is atmospheric at the surface of the soil and increases according to its hydrostatic value with depth.

The main limitation to the recording network was the lack of stations immediately to the west of the aftershock zone (Figure 3.2). Access to this area was impossible during the field survey. This lack of coverage was partially offset by the more distant stations LL and BB shown on Figure 3.3

The equipment deployed included both seismographs and accelerographs. The two types of recorders were often operated side-by-side because few suitable recording sites were available. The digital recorders had to be housed in heated shelters. These were generally summer cottages in which a portable kerosene heater was installed. The analogue equipment was often deployed out of doors without any shelter other than a covering of snow. All the equipment suffered from the low temperatures experienced, and many good data sets were lost because of cold-related equipment malfunctions.

In spite of these difficulties a wealth of good data was obtained including continuous monitoring of the aftershock sequence by at least three stations within 25 km for 15 days from January 10 to 24 and continuous more detailed monitoring by three stations within 10 km for the four days from January 19 to 22. The larger aftershocks in the 7-day period from January 16 to 22 included 14 events of magnitude up to 3.5 recorded on at least three digital stations and many other events recorded on one or two digital stations. The SMA network has recorded a magnitude 4.8 aftershock on March 31, 1982, at 17:02 AST and several smaller events (see Munro and Pomeroy).

PRELIMINARY SEISMICITY RESULTS

The deployment of seismographs in the epicentral area from January 19 to 22 (Figure 3.2) allowed the aftershocks to be located with the most accuracy, probably better than ± 1 km in position and ± 2 km in depth. A partial analysis of these data shows that the activity is concentrated in a volume of approximately 4 km NS by 6 km EW by 7 km deep (Figure 3.4) with some indication of a southwardly steeply-dipping trend in the focal depths. (See also Figure 4.3 of Pulli). These results must be considered provisional at this time (May 1982) however, and may be changed by a more thorough analysis of the data for this period.

The composite focal mechanism (local focal sphere) shown in Figure 3.4 is in particular very uncertain. It shows a tendency for a composite strike-slip mechanism for the aftershocks in this period, but the solution is not well defined by the data and there are many readings inconsistent with this solution.

(See also the SSA abstract by R.J. Wetmiller in Chapter 23.)

that the freezing temperature of a concave shaped ice-water interface (positive curvature relative to the ice) is lower than that of a convex shaped interface. For a pore which is at a uniform temperature, this explains the presence of unfrozen water in the wedge shaped regions between the soil particles while the remainder of the pore fills with ice. Since the ice-water interface in these regions is concave the freezing-point depression is greater than in the rest of the pore.

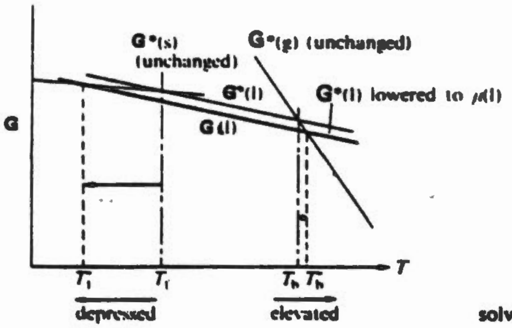
1.4 Osmotic Effects

In addition to interface (capillary) effects, freezing-point depression in soils is also attributed to the osmotic activity of the ions in the diffuse electric double layer surrounding the soil particles (Bolt and Miller, 1958). For most soils, the effect of the double layer tends to predominate particularlily in the lower temperature ranges of the soil.

It is a well known fact that the freezing temperature of a solution is depressed below that of pure solvent. The phenomenon is caused by an increase in the entropy of the solution which lowers its specific free energy below that of pure solvent. (see Figure 1.4). In other words, the enhanced random molecular behaviour of the solution increases the tendency of the solid phase to break up at lower temperatures than for pure solvent. For a dilute solution, the freezing-point depression ΔT expressed as a function of the mole fraction of solute present, is given by Blagden's Law:

$$\Delta T = \frac{RT^2}{L} X_{s_1} \qquad (1.13)$$

Figure 1.4



The Gibbs potential of a solvent in the presence of a solute.

Atkins (1978)

where T = the freezing temperature of pure solid,

R = the universal gas constant,

L = the latent heat of fusion of the solvent,

X_{s_1} = the mole fraction of solute ($X_{s_1} + X_{s_2} = 1$, and X_{s_2} = the mole fraction of solvent).

When an ice crystal grows in a brine solution, the solute is excluded from the crystal. As the temperature is lowered, ice continues to form which increases the concentration of the remaining solution. This effect is illustrated in the binary phase diagram for a single sodium chloride solution in Figure 1.5.

If a solvent such as pure water is separated from a solution by a membrane impermeable to the solvent, a process known as osmosis causes the solvent to pass through the membrane into the solution. (See Figure 1.6), Equilibrium is attained when the two become equal. This is achieved by an increase in the pressure of the solution ($P+\Pi$) above that of pure solvent (P) thereby raising its specific free energy. The pressure required to maintain equilibrium is termed the osmotic pressure Π. This is related to the concentration of the solution by the Van't Hoff equation:

$$\Pi = \frac{RTX_{s_1}}{v_m} \qquad (1.14)$$

where T = the temperature of the solution

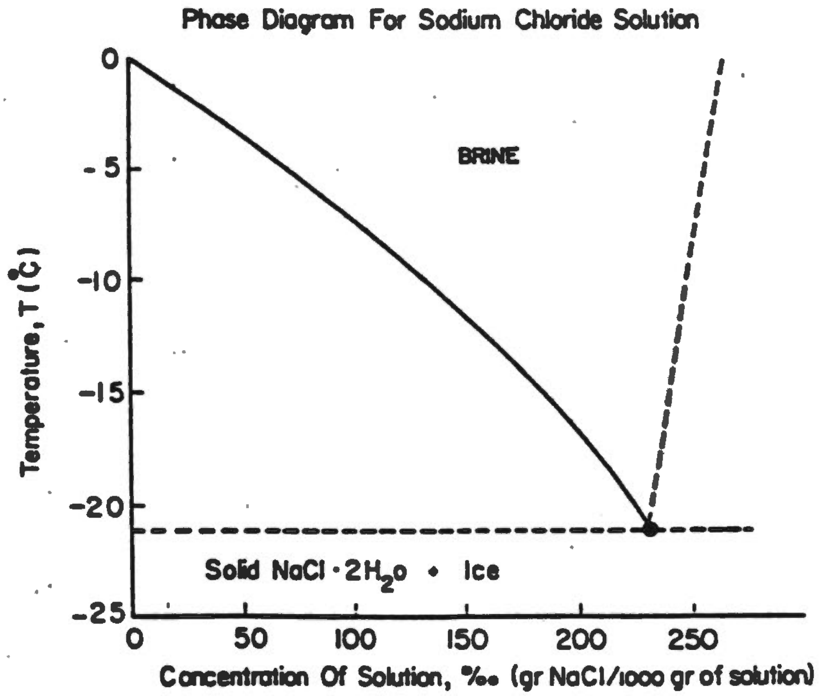
and v_m = the partial molar volume of the solvent.

Hoekstra and Miller (1967) give the relationship between freezing-point depression and osmotic pressure for a simple dilute solution as:

$$\Pi = -\left(\frac{L}{T\Delta v}\right) \Delta T \qquad (1.15)$$

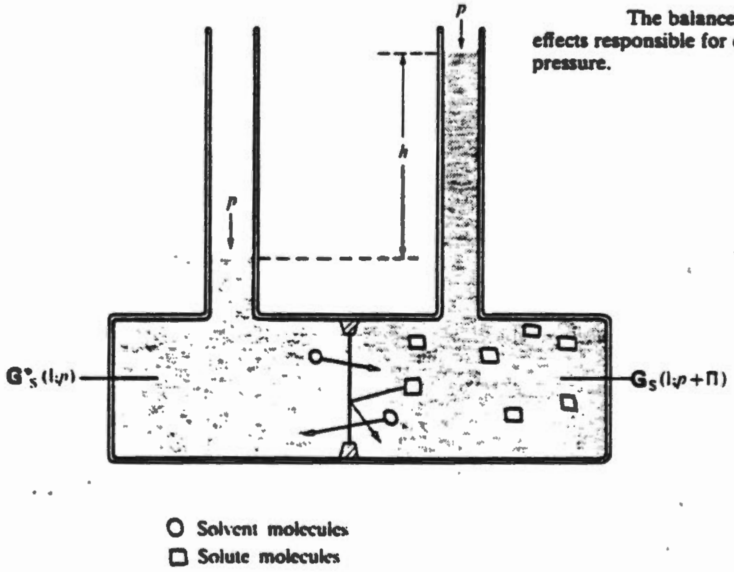
where Δv = the difference in the partial molar volume of ice and water.

Figure 1.5



Sheeran and Yong (1975)

Figure 1.6



The balance of effects responsible for osmotic pressure.

Atkins (1978)

This equation indicates that the freezing-point depression increases with osmotic pressure.[5] The reader should note that both the osmotic pressure and the freezing-point depression are colligative properties (Atkins, 1978). That is, they depend on the amount rather than the nature of the solute present.

In soils, the osmotic pressure associated with the electric double layer is caused by the electrical restraints imposed upon the movement of ions by the coulombic forces emanating from the surface of the particles. According to the Gouy-Chapman model of the double layer, the decrease in the electrical potential ψ, with distance x from the charged surface of an infinite plane,

$$\psi = \psi_0 \exp(-Kx) \qquad (1.16)$$

where $\psi_0 = \frac{4kT}{Z\varepsilon} =$ the electrical potential at the surface of the plane,
the plane,
$k =$ the Boltzmann constant,
$K =$ decay constant ($\frac{1}{K} =$ the thickness of the double layer),
$Z =$ the valence of the ion,
and $\varepsilon =$ the charge of an electron.

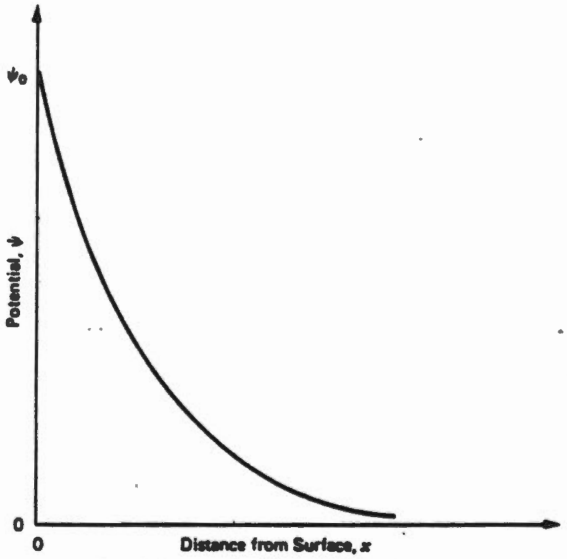
This follows from Poissons equation which relates the divergence of the gradient in the electrical potential at a given point within the double layer, to the charge density at that point (Adamson, 1976).

Equation (1.16) indicates an exponential decrease in the electrical potential with distance from the surface of a charged plane. (See example Figure 1.7). This results in a similar distribution in the number of ions associated with the charged surface. The distribution of cations and anions between parallel

---

[5] It is not evident how Hoekstra and Miller (1967) obtained equation (1.15). Expressing equation (1.15) and (1.14) in terms of x_s and equating, does not give the result indicated in equation (1.15).

Figure 1.7



Variation of electrical potential with distance from a charged surface. Except in very unusual cases ψ in soils is negative.

Mitchell (1976)

charged plates is given by the Boltzmann equation

$$n^+ = n_o \exp\left(\frac{Z\varepsilon\psi}{kT}\right) \quad \text{and} \quad n^- = n_o \exp\left(-\frac{Z\varepsilon\psi}{kT}\right) \qquad (1.17)$$

where n^+, n^- = cation and anion concentration at distance x from the charged surface,

n_o = the concentration at an infinite distance away from the surface,

and T = the temperature.

Since the net concentration of ions increases towards the charged surface (as shown in Figure 1.8 for a monovalent ion system), this results in an osmotic pressure which also increases towards the surface. For a monovalent electrolyte, the gradient in osmotic pressure is given by Cass and Miller (1959) as:

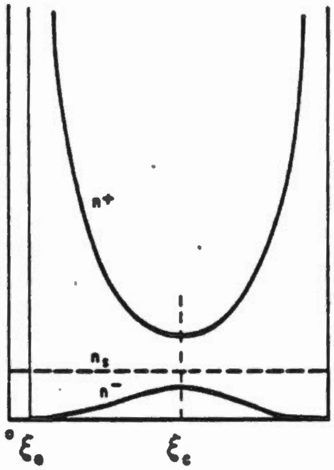
$$\frac{d\Pi}{dx} = 2n_s k\, T \frac{d \cosh y}{dx} \qquad (1.18)$$

where $y = -\frac{Z\varepsilon\psi}{kT}$

and $n_s = n^+ = n^-$ = the total number of ions present

Comparing this result with equation (1.15), it is evident that freezing-point depression within the double layer increases exponentially with distance towards the charged surface of the soil particles. This accounts for the presence of significant amounts of unfrozen water retained in many soils even at very low temperatures (-1.5 to -10°C). (Examples are shown in Figure 1.9). It also explains the attenuation in slope of the soil freezing characteristic curve at lower temperatures since large changes in temperature are required to produce any significant freezing of the double layer.

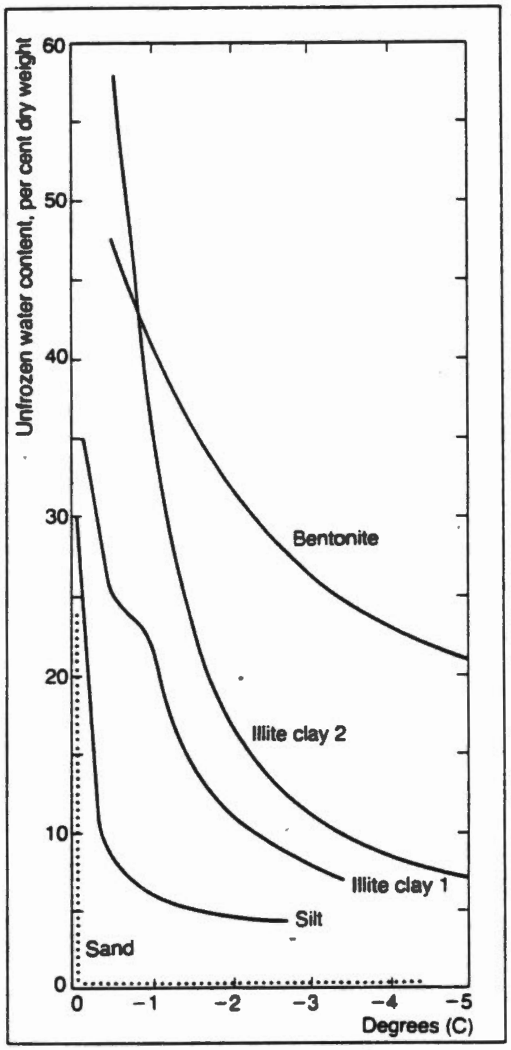
Figure 1.8



Sketch of distribution of cations and anions between parallel charged plates. Ordinate is at plane of infinite charge density with surface of clay at ξ_0.

Cass and Miller (1959)

Figure 1.9



Water contents of frozen soils. Water remains unfrozen as a result of capillary, osmotic and surface adsorption effects, but is progressively replaced by ice at lower temperatures. Thus the water contents depends on temperature and type of soil.

Williams (1979)

It should be pointed out that double layers form spontaneously on particles of all sizes, non-colloidal as well colloidal (Miller, 1980). This explains the presence of significant amounts of unfrozen water in silts as well as clays at low temperatures. However, since most silts are composed largely of quartz which has a low charge density in comparison with aluminium silicates (clay minerals), the thickness of the double layers and their respective ion concentrations will be considerably lower than in most clay materials. Consequently, in the lower temperature ranges, clays usually contain greater amounts of unfrozen water than silts at the same temperature.

With most soils, the freezing characteristic curve shows an initially sharp decline in the unfrozen moisture content (θ_{uf}) followed by an abrupt change in the slope of the curve, θ_{uf} thereafter decreasing more slowly with temperature. In general, interface (capillary) effects tend to be the dominant cause of freezing-point depression during the initial rapid decline of θ_{uf} while double layer effects predominate following the inflection in slope.[6]

1.5 Osmotic Effects and the Clapyeron Equation

According to Dalton's law of partial pressures, the total pressure of a solution $P_{\ell} = P_w + \Pi$

where P_w = the partial pressure of the water

and Π = the partial pressure of the ions

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[6] Usually the more clay present in a soil, the less abrupt the inflection in the slope of the soil freezing characteristic curve.

(osmotic pressure generated by the solutes).

In a frozen soil, the osmotic pressure generated by the double layer can be regarded as the pressure which must be exerted on the film in order to maintain equilibrium between the water in the double layer and that in the bulk solution of the soil pores (Miller, Baker and Kolainan, 1960).

Substituting P_ℓ for the liquid phase in the Laplace equation (1.10) gives:

$$P_\ell = P_i - \frac{2\sigma_{iw}}{r_{iw}} \qquad (1.20)$$

A general expression for freezing-point depression in soils may be obtained by combining equations (1.19) and (1.10) and substituting for P_w in the Clapyeron equation (Loch, 1975).

$$v_w\left(P_i - \frac{2\sigma_{iw}}{r_{iw}} - \Pi\right) - v_i P_i = \frac{L\Delta T}{T} \qquad (1.21)$$

This includes terms for the freezing-point depression associated with the effects of a curved interface as well as the presence of solutes within the double layer. The reader should note that equation (1.21) does not include a term for freezing-point depression caused by the presence of solutes in the bulk solution of the soil pores.

SECTION 2

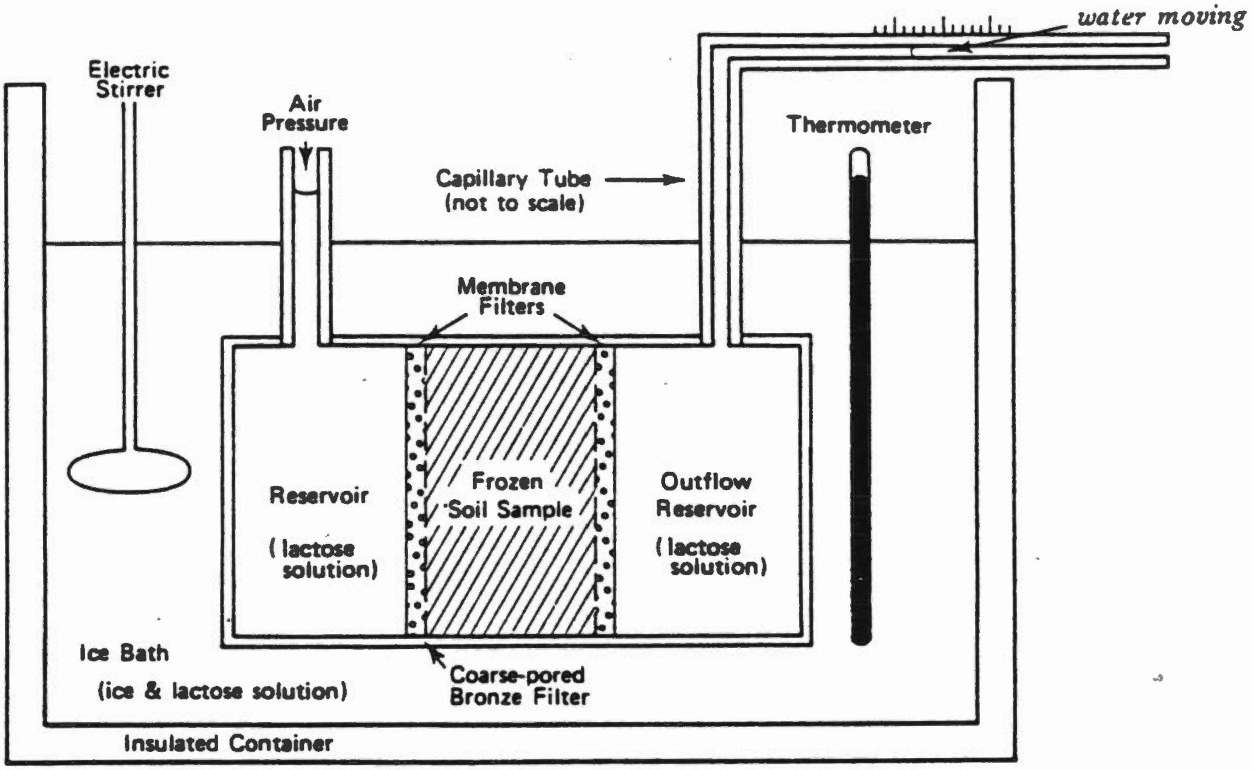
Mobility of Water in Frozen Soils

1.1 Measurement of Permeability of Frozen Soils

In recent years a number of investigators have demonstrated that substantial movements of water can occur in frozen soils. Williams and Perfect (1979) in an earlier phase of this contract and Burt and Williams (1976) used a simple permeameter to measure the hydraulic conductivity of frozen soils under isothermal conditions. The apparatus used in the experiments consisted of a frozen soil sample confined to a perspex ring and sandwiched between two reservoirs containing lactose solution. (See Figure 2.1). The soil sample was separated from the solution by membranes whose pores are permeable to water but impermeable to lactose. The lactose maintains a freezing-point depression in the water retained within the reservoirs, the concentration being adjusted so that the specific free energy of the water in the reservoirs equals that of the unfrozen water in the soil. The flow of moisture across the sample was produced by the application of a hydrostatic pressure to one reservoir. The whole assembly was clamped together and immersed in a bath of crushed ice and lactose solution which ideally maintains stable temperatures as low as -0.4°C

Examples of the results obtained by Burt and Williams (1976) are shown in Figure 2.2. The diagram indicates that the hydraulic conductivity of frozen soils declines sharply

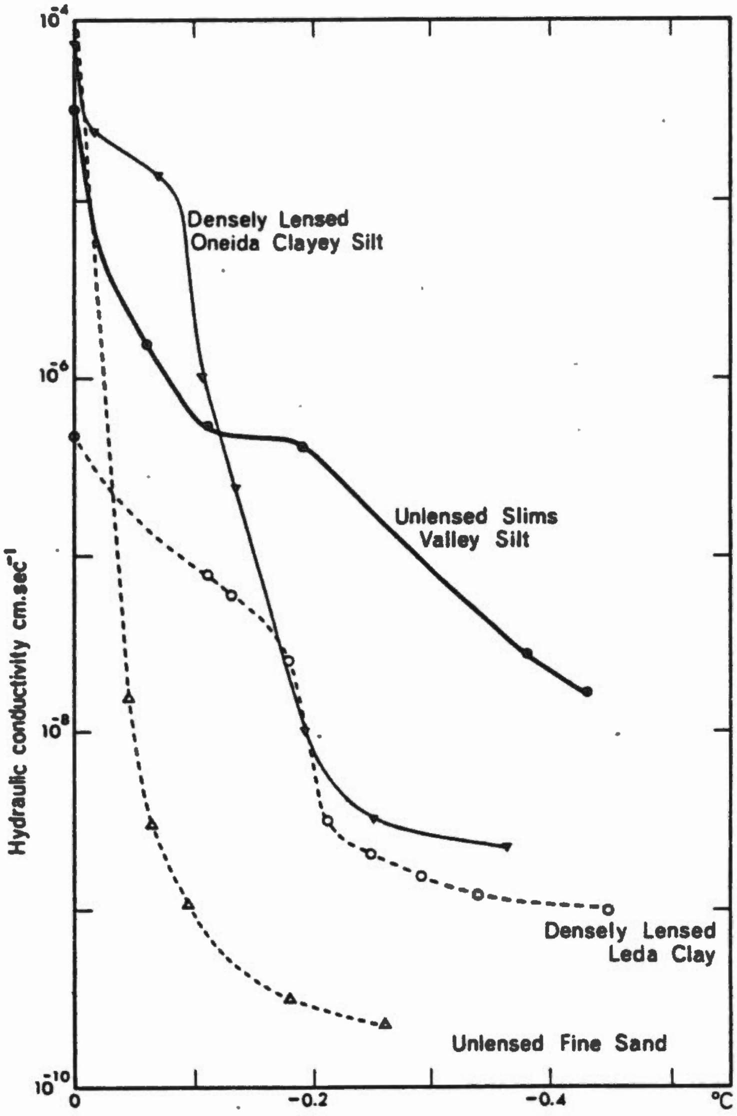
Figure 2.1



A schematic diagram of the permeameter used by Burt (1974) to measure the hydraulic conductivity of frozen soils.

Williams and Perfect (1979)

Figure 2.2



The influence of soil type on the hydraulic conductivity of frozen soils

Burt and Williams (1976)

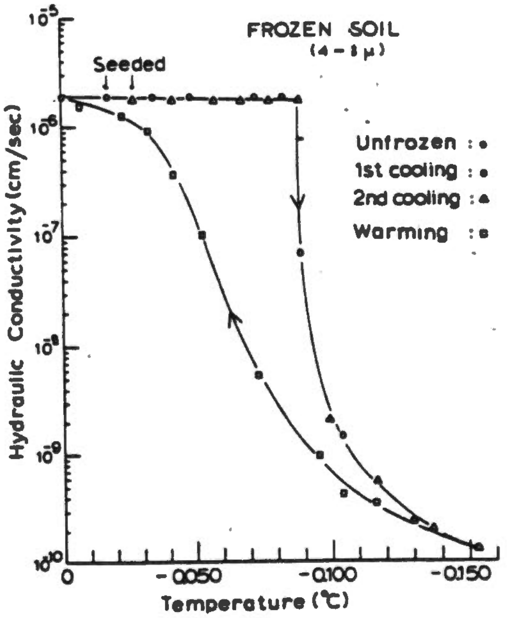
at temperatures within a few tenths of a degree below 0°C. This was attributed to the rapid attenuation in the amount of unfrozen water retained within the soil in this temperature range. Commonly observed values varied between 10^{-5} and 10^{-10} $cmsec^{-1}$ in the temperature range 0°C to -0.4°C.

One difficulty with the apparatus used by Burt and Williams (1976) was that the membranes were not entirely impermeable to lactose. This resulted in a gradual thawing of the sample over a period of several days. Horiguchi and Miller (1980) state that "the presence of significant concentrations of lactose would compromise interpretation of the results to some unknown degree." To avoid this problem, they constructed an apparatus which was similar in design to that of Burt and Williams (1976) but contained supercooled water in the end reservoir instead of lactose solution. However, results of tests on a 4-8 μm silt appear at least superficially to be very similar to those obtained by Burt and Williams (1976) using a similar soil (see Figures 2.3a and b). This suggests that the penetration of lactose into the frozen soil has little effect on the results given by Burt and Williams (1976).

2.2 Thermal Osmosis in Frozen Soils

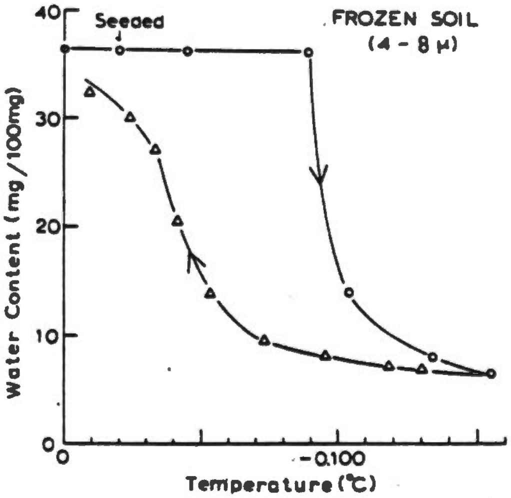
Experiments by Dirksen and Miller (1966), Hoekstra (1966) and Torrance and Williams (1976) demonstrated that water also moves through frozen soils under temperature gradients. More recently Williams and Perfect in the second phase of this

Figure 2.3a



Apparent hydraulic conductivity as a function of temperature, 4-8 μm silt fraction.

Figure 2.3b



Unfrozen water content, 4-8 μm silt, as determined with permeameter in the dilatometer mode.

Horiguchi and Miller (1980)

contract (see Final Report submitted to E.M.R. 1980), constructed an apparatus which allows direct measurement of thermally induced moisture migration in frozen soils under steady state conditions. The apparatus was essentially similar in design to that used by Burt and Williams (1970) except that flow was induced by a temperature gradient rather than a hydraulic gradient across the sample. (See Figure 2.4). The perspex sample container was sandwiched between two aluminium end plates containing Peltier modules.[1] Temperature control for the system was provided by a thermoelectric cooling control unit which maintains temperatures constant to within ± 0.02°C. The aluminium plates contained small end reservoirs which were connected to capillary tubes from which inflow and outflow even recorded. The sample was separated from the reservoirs by semi-permeable membranes which were supported by porous brass or aluminium end plates mounted within the reservoirs. The temperature of the system was monitored with 5 thermistors placed at different locations within the sample each calibrated to an accuracy of ± 0.01°C. The apparatus was wrapped in a jacket of insulation 3-4 cm thick and placed in a 'Precision' low temperature incubator which was maintained at a temperature approximately midway between that of the two end plates. The air temperature within the incubator remained constant to within ± 0.1°C of the mean temperature of the soil sample.

As in the case with Burt and Williams' (1976) experiment,

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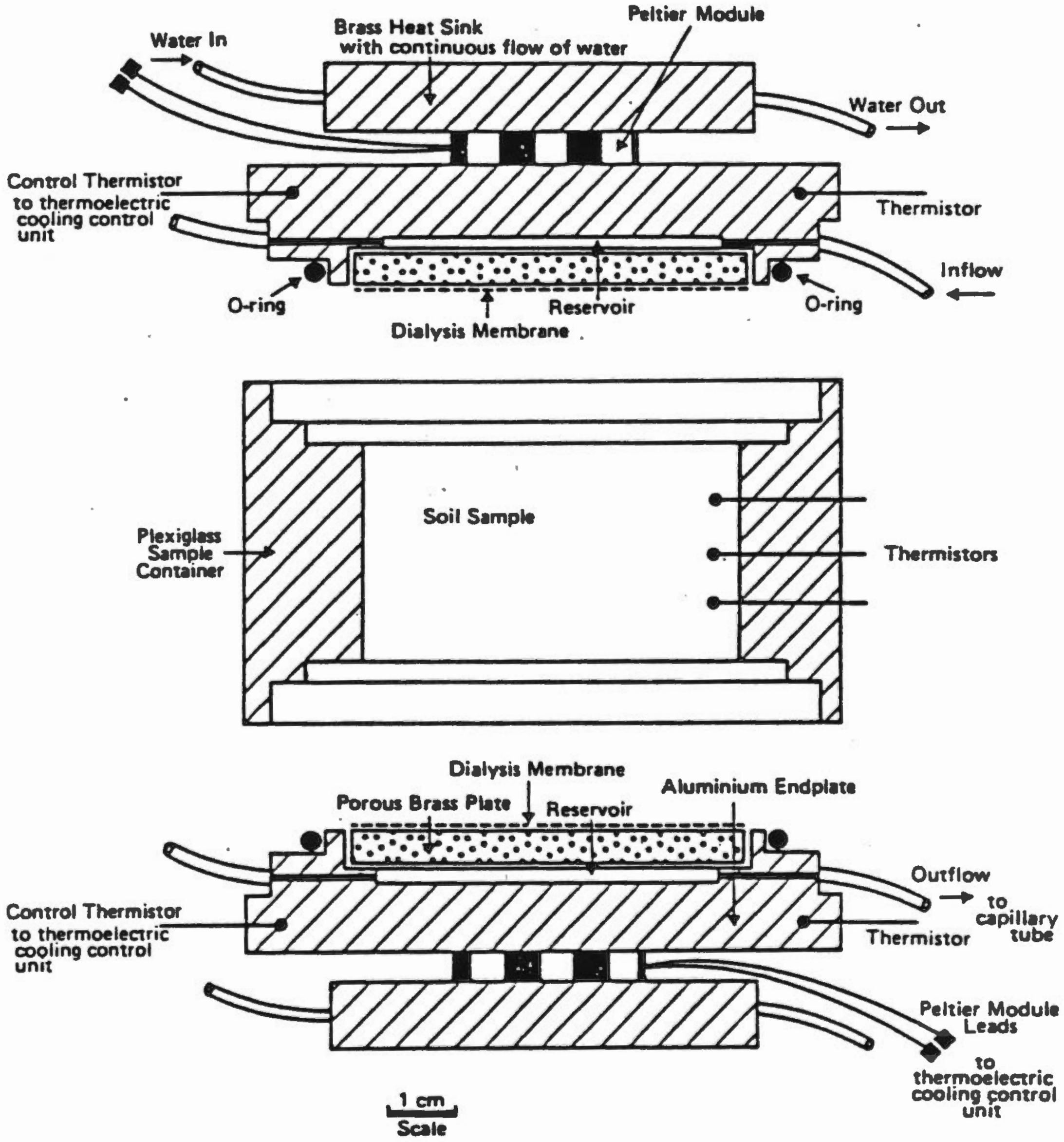
[1] A Peltier element consists of a group of p and n-type semi-conductors connected in series by metal junctions. Passage of a current through the circuit results in heat loss from one junction and the absorption of heat at the other (Gray and Issacs, 1975).

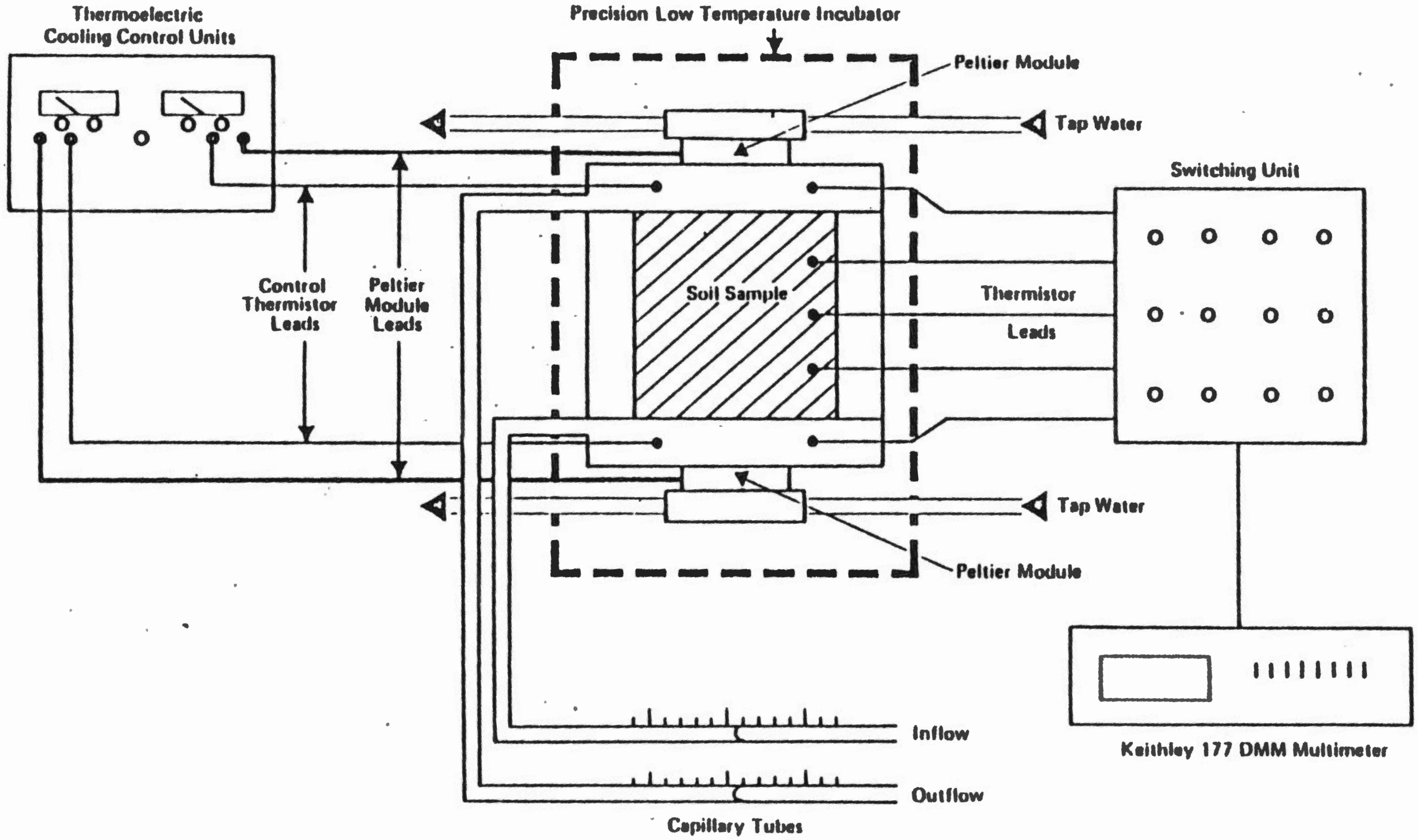
Figure 2.4a



CROSS-SECTION OF THE EXPERIMENTAL APPARATUS.

Williams and Perfect (1980)

Figure 2.4b



The jacket of fibreglass insulation that the cell is wrapped in is not shown.

SCHEMATIC DIAGRAM OF THE COMPLETE EXPERIMENTAL APPARATUS

Williams and Perfect (1980)

lactose solution was used in the end reservoirs. Unfortunately, however, the same lactose concentration was used in both reservoirs, counteracting in part,the effects of the thermal gradient at the 'warm' end of the sample.[2] The lactose concentration should have been adjusted so that the Gibbs free energy of the water in each reservoir was equal to that of the unfrozen water in the adjacent soil. At any temperature below 0°C, the free energy of the water in the reservoirs and in the soil is less than that of bulk water by an amount given by:

$$dG = \frac{L}{v} \frac{dT}{T} \qquad (2.1)$$

where dT = the freezing-point depression

and T = the freezing temperature of pure free water (= 273.15K or 0°C at 1 atm. pressure).

This result is obtained from the Gibbs-Duhem equation (1.2) and is subject to the condition that the ice phase is pure and at atmospheric pressure (Williams, 1976). To demonstrate that lactose does, in fact, produce equilibrium between the water in the reservoirs and that in the adjacent soil, an experiment was performed in which the sample was maintained at a constant and uniform temperature. Once a uniform temperature was attained, inflow and outflow were found to be negligible indicating that an equilibrium condition had been achieved.

Harlan (1971, 1973), Miller (1978) and Miller and Koslow (1980) describe equations for mass transport in frozen soils. These are based on the equation of continuity for partly saturated soils in the unfrozen state. For saturated

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[2] The concentration used in the experiments corresponded with the appropriate freezing-point depression at the 'cold' end of the system.

frozen soils, the equation of continuity may be written as:

$$\frac{\partial\,(\rho_i\theta_i)}{\partial t} + \frac{\partial\,(\rho w \theta w)}{\partial t} = -\nabla\cdot\underline{V}_T \qquad (2.2)$$

where $\rho_i\theta_w$= the mass of ice per unit volume of soil

$\rho w\theta w$ = the mass of water per unit volume of soil (θ_i and θ_w = the volumetric ice and water contents)

$\underline{V}_T$ = the total flux of the ice and water on a mass basis, in a unit volume of soil.

The operator ∇ is defined as:[3]

$$\nabla = (\underline{i}\frac{\partial}{\partial x} + \underline{j}\frac{\partial}{\partial y} + k\frac{\partial}{\partial Z}) \qquad (2.3)$$

Note that ∇ operates on the flux V_T, a vector quantity. The term $\nabla\cdot\underline{V}_T$ is usually referred to as the divergence of flux (div V_T).

Equation (2.2) states that if the total flux into a volume element of soil exceeds the total flux out of the soil, then the soil must be storing the excess as ice or water or both phases and its total moisture content will increase with time. The term V_T represents the sum of the mass transfer in the ice and water phases. That is:

$$V_T = \rho_i\nu_i(T) + \rho_w\nu_w(T) \qquad (2.4)$$

where $\nu_i(T)$ and $\nu_w(T)$ = the volumetric flux of the ice and water phases expressed as a function of temperature.

The flux of the pore ice can be expressed in relation to the pore ice velocity by:

$$\nu_i(T) = \theta_i(T)\ V_I \qquad (2.5)$$

where V_I = the velocity of the pore ice

Assuming that the fluid phase is continuing throughout

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[3] The terms i, j and k are the unit vectors in the x,y and z directions of the Cartesian co-ordinate system.

the frozen soil and that it obeys Darcy's law, the liquid flux is described by:

$$\nu_w(T) = -k(T)\nabla\phi \tag{2.6}$$

where $k(T)$ = the hydraulic conductivity of the soil expressed as a function of temperature,
and $\nabla\phi$ = the gradient in total hydraulic potential across the soil.[4]

Note that:

$$\phi = \psi + \varepsilon + \Pi \tag{2.7}$$

where ψ = the pressure or suction head,
ε = the elevation or datum head,
and Π = the osmotic head

Applying equation (2.6) to Williams and Perfect's (1980) experiment, it is evident that, once a linear temperature gradient has been established across the soil, the gradient in potential across the system will also be linear. Since the permeability of the soil decreases rapidly with temperature, the flux of water across the system will decrease by a proportional amount. Given this condition, the equation of continuity (2.2) described earlier indicates that the total moisture content of the soil should increase throughout the duration of the experiment, ice lenses forming where the divergence of flux across the system is maximum (Loch and Kay, 1978). (That is, where the rate of change of flux with temperature is maximum). This corresponds with the point where the decrease in the hydraulic conductivity of the soil with temperature is maximum.

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[4] $\nabla\phi$ is a scalar quantity and is usually referred to as grad ϕ

Contrary to expectations, Williams and Perfect (1980) observed a net decline in the moisture content of the sample during all of the experiments, the mean rate of outflow exceeding the inflow by approximately an order of magnitude. This may be a result of the fact that the wrong lactose concentration was used in the reservoir at the 'warm' end of the sample. Perfect and Williams (1980) offer the following explanation.

> "Moisture migration within the frozen soil may result in ice lensing at the 'cold' end of the sample. This would give rise to an increase in the frost heave pressures within the confined sample. Consequently some melting may take place towards the warm end; expulsion of water may then counteract intake and reduce the observed rates of inflow. Secondly, the concentration of the lactose solution used was such that the potentials of the water in the 'cold' end reservoir and the adjacent unfrozen soil are in equilibrium. Since the same concentration of lactose solution was used in the 'warm' reservoir, the resultant local osmotic potential gradient coupled with the presence of the dialysis membrane could be sufficient to partly counteract the temperature induced potential gradient at the warm end. A decline in inflow rates may again result on this account."

During the approach to steady state heat flow conditions moisture was expelled from both the 'warm' and 'cold' ends of the sample. Once a steady state was attained (indicated by a linear temperature gradient) a constant flux of water was observed in the direction of decreasing temperature. Equation (2.6) indicates that the rates of flow across the system depend upon the temperature of the soil which affects its permeability as well as the temperature gradient which acts as the driving force for moisture flow across the system. Since the hydraulic conductivity of the soil is continuously changing with temperature the overall permeability of the system K represents the sum of the hydraulic conductivity of a series of infinitesimal layers integrated over the thickness of the soil. That is:

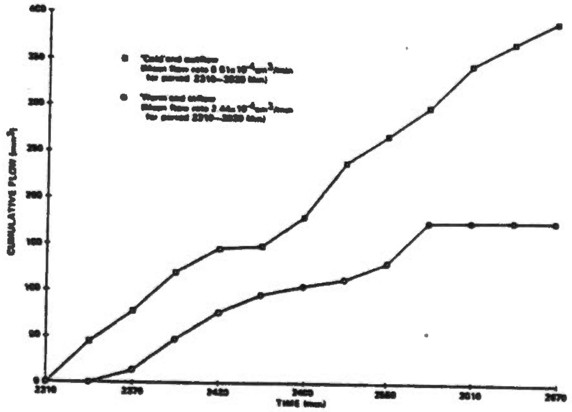
$$K = \frac{\ell}{\int_0^{\ell} \frac{1}{k(x)}\, dx} \tag{2.8}$$

where ℓ = the thickness of the soil,

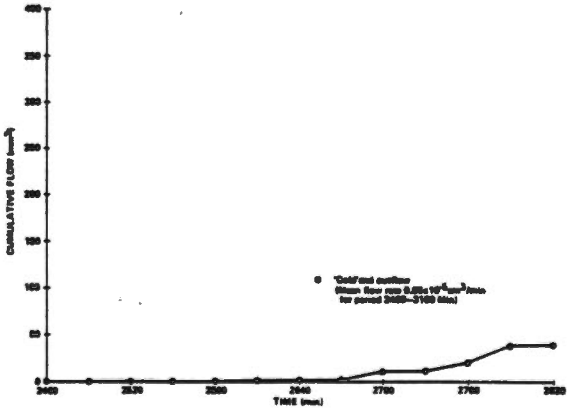
$k(x)$ = the hydraulic conductivity of the soil as a function of distance (see explanation for this equation Appendix A).

Williams and Perfect's (1980) results indicate the dominant influence that the overall permeability has on the rates of flow through a frozen soil. Substantially larger amounts of inflow and outflow were observed at relatively warm temperatures, even under small temperature gradients, than at colder temperatures. (Compare Figures 2.5a and b). This can be attributed to the fact that near 0°C the permeability of the soil increases by several orders of magnitude with only a small increase in temperature. (See Figure 2.5c). When the soil was maintained at relatively warm mean temperatures the magnitude of the temperature gradient appeared to have a much greater effect on the rates of flow. (Compare results of experiment 7B and 8B). In contrast, at colder temperatures despite a large change in the temperature gradient, little change was observed in the amount of inflow or outflow. The

Figure 2.5a



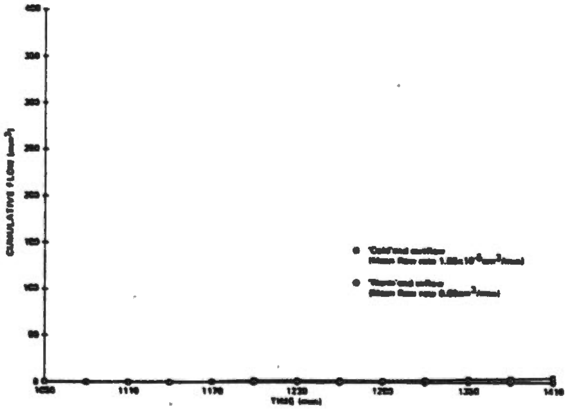
Flow induced by a large temperature gradient at relatively warm temperatures. (Experiment No. 7B).



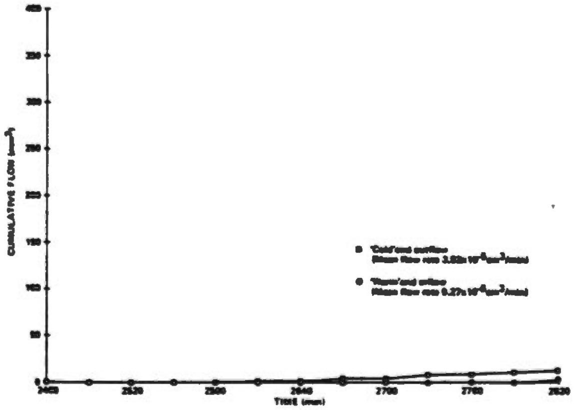
Flow induced by a small temperature gradient at 'warm' temperatures. (Experiment No. 8B).

Perfect and Williams (1980)

Figure 2.5b



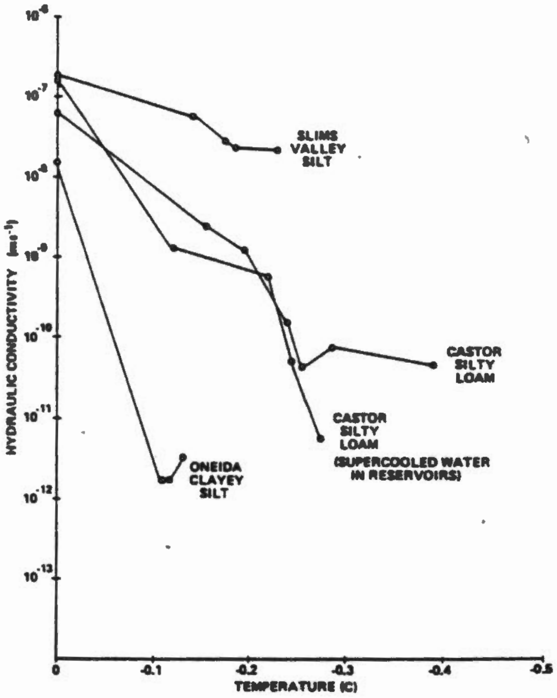
Flow induced by a large temperature gradient at 'cold' temperatures. (Experiment No. 10B).



Flow induced by a small temperature gradient at relatively cold temperatures. (Experiment No. 9A).

Perfect and Williams (1980)

Figure 2.5c



Hydraulic conductivities of frozen soils as a function of temperature. (Experimental determinations).

Perfect and Williams (1980)

explanation for this is that at colder temperatures, any change in the temperature of the soil has comparatively little effect on its permeability.

2.3 Series-Parallel Transport Mechanism

Following Philip and DeVries (1957) example for liquid and vapour transport in unfrozen unsaturated soils, Miller, Loch and Bresler (1975) proposed that in frozen non-colloidal soils,moisture migration occurs by a series-parallel transport mechanism,in which water traverses the pores in both the solid and fluid phases. The parallel component of movement involves transport within the fluid films surrounding the soil particles as well as plug movement and plastico-viscous flow of pore ice. The series component refers to movement by regelation.[5]

(1) Parallel Transport

Freden (in Loch, 1975) attributes the mobility of water molecules in the unfrozen films to two possible causes:

(i) The influence of surface forces generated by the soil minerals,which reduces the free energy of the water in the double layer thereby depressing its freezing point.

(ii) The presence of unbalanced bonds along the edges of the ice crystals within the pores,which gives the molecules sufficiently high mobility that they in effect behave as a liquid.

Miller, Loch and Bresler (1975) point out that it is questionable whether Darcy's law can be used to describe fluid transport in frozen soils since the drag of a moving ice body will affect the transport of film water. Thus "the driving force

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[5] A rigorous analysis of the flux in the solid and liquid phases during parallel transport in a system of parallel cylindrical pores is described in Miller, Loch and Bresler (1975).

for film flow is not the liquid pressure gradient alone; it must include a term for momentum exchange with the ice phase."

Experiments by Glen (1958) established that the deformation of ice under stress is plastico-viscous (non-Newtonian) in character. (See Figure 2.6). With single crystals of ice, the deformation is attributed to shearing along the basal plane of the crystal as well as the movement of dislocations within the crystals. Dislocations are "irregularities in the crystal structure which allow planes of atoms to move over one another more easily than would occur in a perfect crystal" Paterson (1969). With polycrystalline ice the mechanism of movement is somewhat more complex. In addition to deformation along the dislocations and basal planes, ice also moves by a process of recrystallization under stress as well as the migration of crystal boundaries (Glen, 1958). Glen (1958) proposed that the deformation can be described by an exponential function of the form:

$$\dot{\gamma} = k\tau^n \qquad (2.9)$$

where $\dot{\gamma}$ = the shear strain rate
and τ = the shear stress.

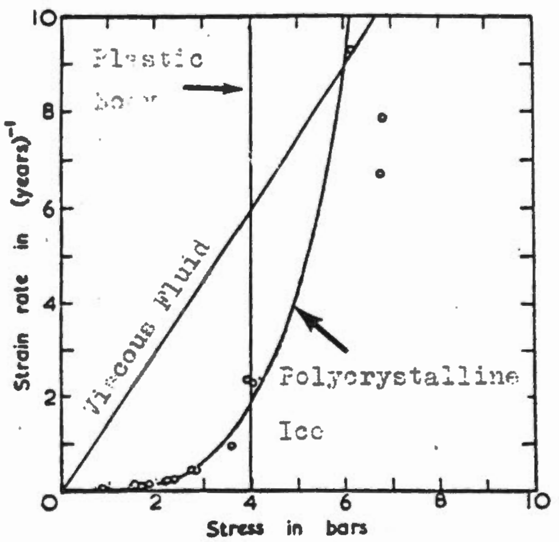
k and n are constants,

where $n \simeq 3$ for stresses between 1 and 10 atmospheres

and $k = \exp(-Q/RT)$
Q = an activation constant,
R = universal gas constant,
T = the temperature.

The term 'plug flow' refers to a condition which occurs when the applied stress is less than the yield stress and so the

Figure 2.6



The variation of the steady state creep rate with stress.

Glen (1952)

ice moves as a rigid body. Under these conditions all of the shear stress occurs along a single plane within the ice body (Nye, 1957).

(2) Series Transport

The process of regelation is adequately described in a simple experiment performed by Horiguchi and Miller (1980). They enclosed a layer of ice between two reservoirs containing supercooled water with porous phase barriers separating the ice from the water. Application of a pressure to one reservoir produced a slow discharge out of the other reservoir; the explanation for the movement being that water freezes at the inflow side, traversing the 'ice sandwich' in the solid phase and melts at the outflow side. Miller, Loch and Bresler (1975) offer the following explanation.

> "The Clapeyron equation (1.3) indicates that, when $dT < 0^{o}C$, P_i exceeds P_w. When on one side P_w is increased somewhat, the equilibrium temperature rises above ambient favouring formation of ice at that side and the ice pressure rises. As the increase in ice pressure is felt at the other side where P_w has not been changed, equilibrium temperature falls below ambient favouring melting. The result is translational movement of ice in its chamber and pure series transport. Phase transition induced by differential liquid pressures produce a temperature peak at the high pressure (freezing) side and a trough at the low pressure (melting) side as heats of transition are liberated and absorbed."[6]

Results of Horiguchi and Miller's (1980) experiments on regelation transport are summarized in Figures 2.7a and b. In general, their results indicate that the rate of transport during regelation is practically independent of temperature.

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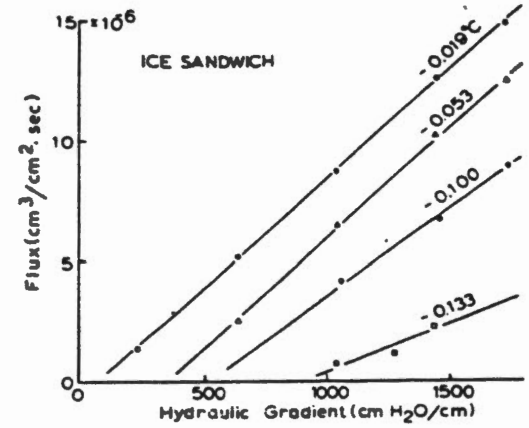
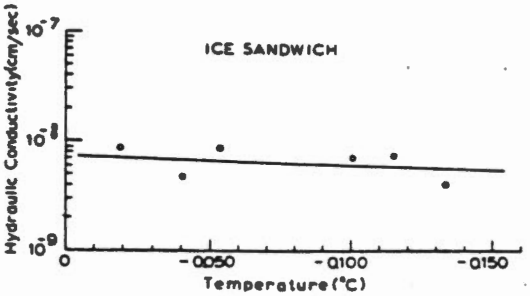
[6] Horiguchi and Miller (1980) were actually able to measure this temperature difference with thermocouples mounted on either side of the 'ice sandwich'. (See Figure 2.7c).

Figure 2.7a



Transport data at various temperatures and hydraulic gradients in ice sandwich mode.

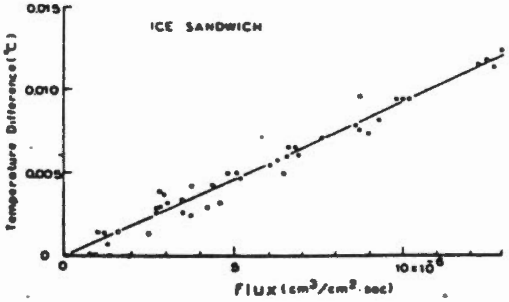
Figure 2.7b



"Apparent hydraulic conductivity" of ice in ice sandwich as a function of temperature.

Horiguchi and Miller (1980)

Figure 2.7c



Temperature differentials observed by thermocouples outside phase barriers in ice sandwich mode as a function of observed fluxes at various temperatures and with various hydraulic gradients.

Horiguchi and Miller (1980)

Instead, it is governed primarily by the pressure gradient across the ice as well as the rate of exchange of heat between phase barriers at the 'warm' and 'cold' ends of the system and between the phase barriers and the surrounding bath. The most interesting feature of the graphs is that regelation appears to obey Darcy's law. This is indicated by the fact that the relationship between the hydraulic gradient and the resultant flux is linear (See Figure 2.7a).

In a frozen soil,regelation can be viewed as a continuous process involving freezing on the upstream sides of the pore ice and melting on the downstream sides. Motion within the pore ice proceeds at a uniform translational velocity and is sustained by internal exchanges of latent heat between the ice-water interfaces on the upstream and downstream sides of the ice body. The regelation component of transport is added to whatever flow might be taking place in the films if the ice were not moving. Ice movement also induces the transport of sensible heat in the opposite direction and a fraction of this heat is recirculated by conduction between heat sources and sinks within the soil (Miller, Loch and Bresler, 1975).

Miller, Loch and Bresler (1975) derived an expression ice and water transport in a simplified soil system. The soil was assumed to consist of a plate of mineral matter containing uniform cylindrical pores of radius R, each being filled with ice except for an adsorbed film of water with thickness τ. The total flux J within the soil was given as:

$$J = - (\theta_i / 6\eta) \left[\rho_i \ \tau^2 \nabla P_\ell \ + \rho_\ell \ R\tau \ \nabla P_i \right] \qquad (2.10)$$

where ∇P_ℓ and ∇P_i = the gradients in the liquid and ice pressure,

ρ_ℓ and ρ_i = the densities of water and ice,

θ_i = the volumetric ice content of the soil,

and η = the viscosity of the fluid phase.

(The unfrozen film was assumed to behave as a Newtonian fluid).

The first term on the right side of equation (2.10) represents the transport entirely within the fluid phase while the second term describes the transport within the pore ice.

If the ice phase were assumed to be immobile, the total flux within the soil becomes:

$$J = -(\rho_\ell \theta_i / 6\eta)(\tau^3/R)\ \nabla P_\ell \tag{2.11}$$

Comparing these two equations, it is clearly evident that regelation transport increases the total mass transfer within the soil by a very large factor (Miller, Loch and Bresler, 1975).

Miller, Loch and Bresler (1975) also describe equations for the transfer of mass J and sensible heat j in a permeameter containing a frozen soil. These were given as:

$$J = -\Lambda_m (1/_{\rho_\ell})(\Delta P/_{\ell_f} - \Lambda_{mh}\ (1/T)\ \Delta t/_{\ell_f} \tag{2.12}$$

and $$j = -\Lambda_{hm}\ (1/_{\rho_\ell})\ \Delta P/_{\ell_f} - \Lambda_h\ (1/T)\ \Delta t/_{\ell_f} \tag{2.13}$$

where $\Delta P/_{\ell_f}$ = the hydraulic gradient between the input and and output ports of the permeameter,

$\Delta t/_{\ell_f}$ = the temperature gradient between the outer most walls of the permeameter,

and $\Lambda_m, \Lambda_{mh}, \Lambda_{hm}, \Lambda_h$ = phenomenological direct and cross-coefficients for heat (h) and mass (m) transfer.

The flux of mass J is measured as the total input or output of water from the permeameter while the quantity j represents the sensible heat flux as would be measured by a flux plate placed in the outer wall of the permeameter (Miller, Loch and Bresler 1975). (See Figure 2.8). Unfortunately, analytical expressions for the direct and cross-coefficients (Λ_m etc.) are not yet available and so equations (2.12) and (2.13) cannot be applied to Burt's experiment.

2.4 Role of Osmotic Forces in Water Migration in Frozen Soils

In frozen soil subjected to a temperature gradient, the movement of water within the soil can be attributed at least formally to the gradient in the specific free energy or chemical potential of water between the 'warm' and 'cold' ends of the soil.[7] However, the reader should note that a thermodynamic treatment of the subject is, by itself, merely a superficial explanation for the mechanism by which the temperature gradient induces moisture migration.

Römkens and Miller (1973) suggest two possible mechanisms to account for thermally induced moisture migration in frozen soils. Both models attribute the movement to temperature induced gradients in the osmotic activity of the ions in the fluid films surrounding the soil particles.

Figure 2.9 shows an annular cross-section of a particle embedded in ice, subjected to a temperature gradient. Note that the film thickness decreases towards the 'cold' end of the particle in

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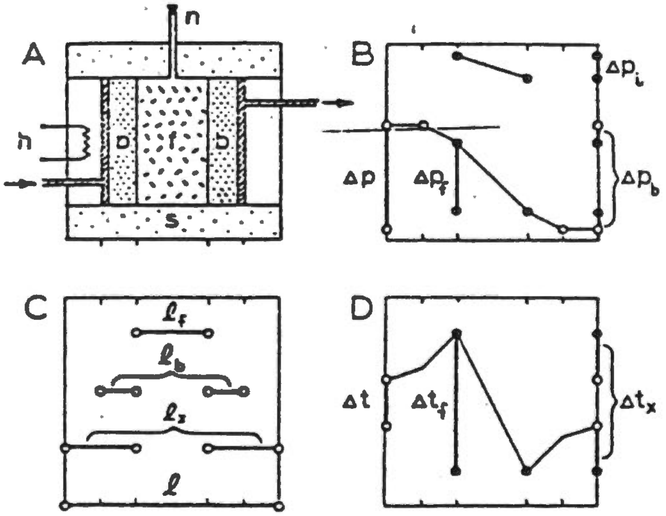
[7] The chemical potential is the free energy per mole of substance. The specific free energy is the free energy per gram of substance.

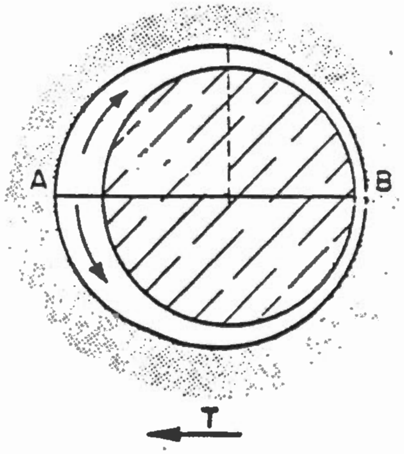
Figure 2.8



Schematic frozen soil permeameter in insulating sleeve (*s*), with rigid porous phase barriers (*b*), separating supercooled water (*hatched*) from frozen soil (*f*). A heat flux sensor (*h*) is required. Freezing of a supercooled specimen is nucleated by means of a side arm (*n*). (*B*) Schematic pressure profile for the permeameter. Upper segment represents ice pressure; lower transect represents liquid pressure. (*C*) Lengths identified in derivations. (*D*) Schematic temperature profile for the permeameter.

Miller, Loch and Bresler (1975)

Figure 2.9



Particle embedded in ice. A transition layer separates the ice and the particle.

Hoekstra and Miller (1967)

accordance with the relationships discussed in section 1.4. Römkens and Miller (1973) assumed that the freezing-point depression produced by the solutes within the unfrozen film is equal to the temperature at the interface. Under this condition, the solute concentration increases toward the 'cold' end of the particle according to Blagden's law. (see equation 1.13). This concentration differential tends to be equalized by ionic and/or water transport in the unfrozen film. The gradient, however, is maintained by continued local freezing of water on the 'cold' side of the particle and melting on the 'warm' side (Römkens and Miller, 1973).

(1) Diffusion Model

With the first model which is referred to by Römkens and Miller (1973) as the 'diffusion model', the particles are assumed to be located in a pocket of brine in which the solutes are concentrated during freezing. The most important feature of the model is that the surface charge of the particle is regarded as being insufficient to arrest the movement of the ions within the film, yet sufficient to prevent separation of the brine and the particle. In other words, the coulombic forces emanating from the particle's surface are not sufficient to restrict the mobility of the ions in the unfrozen film. Thus, the temperature induced gradient in concentration results in diffusion of the solutes from the 'cold' end of the film towards the 'warm' end. Diffusion proceeds according to

Fick's law:

$$J_s = D\nabla c_s = vc_s \tag{2.14}$$

where J_s = the flux of ions per unit area and time,

∇c_s = the concentration gradient,

D = the diffusion coefficient (diffusivity) of salt in water,

and v = the velocity of movement of the salt.

The relationship between the concentration gradient ∇c_s and the temperature gradient ∇T was given by Römkens and Miller (1973) as:

$$\nabla c_s = \alpha \nabla T \tag{2.15}$$

where α = the slope of the freezing-point depression curve at the prevailing temperature of the particle.[8]

Salt transport is matched by an equal volume of water transport in the opposite direction. (That is, from the 'warm' side of the particle to the 'cold' side).[9] In the frozen soil, the concentration gradient is maintained during diffusion, by the continuous addition of salts to each film, from the preceding film located at a slightly warmer temperature.

(2) Double Layer Model

The second model, which is referred to as the 'double layer model' assumes that the solutes in the unfrozen films are part of the diffuse electric double layer. Figure 2.10 shows a cross-section of a particle taken in the plane normal to the temperature gradient. The distribution of cations and anions in

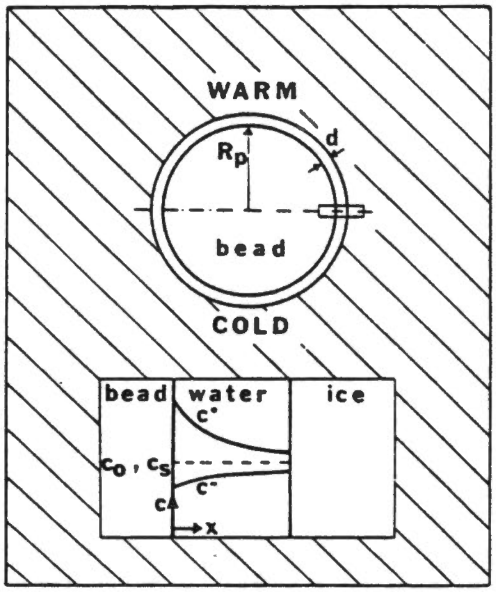
---

[8] α can be interpreted from equation (1.13) to be:

$$\alpha = \frac{RT^2}{L}$$

[9] Migration of the water occurs in response to a gradient in the specific free energy G of the solvent. G is lower at the colder (more concentrated) end of the system due to the hydration of additional ions.

Figure 2.10



A schematic presentation of a glass bead embedded in ice and an enlargement of the film region in the annular.

Römkens and Miller (1973)

the unfrozen film, according to the double layer model is shown in the inset. An important distinction between this model and the diffusion model is that the lateral movement of ions within the double layer is restrained by the electric charges emanating from the particle's surface.[10] Under these conditions, the gradient in ion concentration between the 'warm' and 'cold' ends of the particle generates an osmotic pressure which increases with colder temperatures. Since the total pressure of the solution represents the sum of the partial pressures of the ions and water (see equation 1.19), the osmotic gradient within the film is countered by an equal but opposite gradient in the partial pressure of the water. Expressed as a function of temperature gradient, the gradient in the partial pressure of the water was given by Römkens and Miller (1973) as:

$$\nabla P_w \simeq \alpha RT \ \nabla T \qquad (2.16)$$

where P_w = the partial pressure of the water,

R = the universal gas constant,

and α = the slope of the freezing-point depression curve at the prevailing temperature T of the particle.

Römkens and Miller (1973) assumed laminar-viscous flow in the unfrozen films. Under this condition the flux of water J_v (volumetric basis), between the 'warm' and 'cold' ends of the particle is described by:

$$J_v = L_\ell \nabla P_w \qquad (2.17)$$

where $L_\ell = \dfrac{-\tau^2}{12\eta_w}$ = the coefficient of laminar-viscous flow through the film,

---

[10] Miller (1980) indicates that double layers develop spontaneously on particles of all sizes. Thus, the double layer model is applicable to any type of soil.

τ = the film thickness,

η_w = the viscosity of the water at the prevailing temperature.

Hoekstra (1969) (in Loch, 1975) argues that the flow of water in the double layer is diffusional rather than laminar. For diffusional flow:

$$L_d = \frac{D_s M_s}{RT} \qquad (2.18)$$

where D_s = the diffusivity of salt in water

and M_s = the molecular weight of the soil.

Stein (1967) (in Loch, 1975), however, points out that it makes no difference which mechanism is assumed since:

$$L_\ell = L_d \qquad (2.19)$$

Thus equation (2.13) is equally applicable in describing diffusional as well as laminar flow. Equation (2.13) does, however, ignore the impediment to flow caused by the presence of quasi-stationary ions in the double layer. According to Römkens and Miller (1973) this impediment can be dealt with by "computing the induced streaming potentials and subtracting the electro-osmotic counterflow from the flow predicted by the Navier-Stokes equation."[11]

Römkens and Miller (1973) also tested the above mentioned theories of moisture transport in frozen soils. In general, their results indicate the prevalence of the diffusion model as the mode of transport at temperatures near 0°C.

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[11] For discussion on electro-osmosis and streaming potentials see Kruyt (1963) p. 198-207.

The most likely explanation for this result is that at warmer temperatures, the film thickness exceeds the thickness of the double layer and so the mechanisms governing the rate of flow are no longer determined by the properties of the double layer.

It is fairly clear from the discussion above, that the macroscopic behaviour of frozen soils is governed largely by what goes on at the microscopic level. One difficulty with many of the theories discussed above is that they are extremely difficult to prove. It is not yet within our technical means to measure directly the ice pressure in a pore or the gradient in salt concentration in a fluid film surrounding a soil particle. However, sometimes it is possible to infer what is going on at the microscopic level from the behaviour of the soil at the macroscopic level. Several examples of this are given in the next section.

SECTION 3

Investigation of Moisture Movements Through Frozen Soils Induced by Hydrostatic Pressure and Osmotic Potentials

3.1 Physical Properties of Soils Used in the Experiments

Unfortunately the supply of soil used by Perfect in his experiments was exhausted. Instead, two locally obtained soils, Castor sandy loam and Allendale silty clay (U.S.D.A. Classification) have been used. The latter is essentially a colloidal material containing slightly less than 50% clay sized particles. In structure, it resembles a well flocculated garden soil with spheroidal aggregates about 2 mm. in diameter. Rapid freezing of the soil results in a dense pattern of fairly thick ice lenses which are randomly oriented throughout the soil. Analysis of x-ray diffraction peaks indicates that it consists largely of the clay minerals illite and chlorite with smaller amounts of quartz, plagioclase and orthoclase feldspar also present. Traces of amphiboles are also apparent.

The Castor soil was chosen mainly because it exhibits the opposite properties to that of the Allendale. It is a non-colloidal soil having less than 3% clay present. In structure, it is a granular soil, the particles having no cohesive strength, and resembles a coarse silt. This makes it ideal for present purposes, since no ice lenses are formed within it when it is rapidly frozen. (That is, all of the ice formed within the soil is pore ice). Analysis of x-ray diffraction peaks reveals that it is composed chiefly of quartz with smaller quantities of orthoclase and plagioclase feldspar. Small amounts of chlorites, amphiboles and hydrous micas are also present.

Properties of the two soils including saturation moisture content and bulk density are listed in Table 3.1 Soil freezing characteristic curves were determined by D. Patterson (Geotechnical Science Laboratories) using time domain reflectometry. (See Figure 3.1).

The hydraulic conductivity of both soils was determined under isothermal conditions at various temperatures below freezing. This was done for two reasons:

(1) To obtain some idea of the flow rates to be expected at the 'warm' and 'cold' ends of the sample in the thermal osmosis experiment (that is, with a temperature gradient across the sample).

(2) To aid in understanding which type of membrane should be used as a phase barrier at any particular temperature setting in the experiment. Ideally, the hydraulic conductivity of the soil should be less than that of the dialysis membrane,otherwise the membrane acts as an impediment to the flow of moisture through the soil. In the latter situation, a more porous bacterial filter is used instead of the dialysis membrane, the hydraulic conductivity of the filter being several orders of magnitude greater than that of the frozen soil.

TABLE 3.1

Grain Size Characteristics of Soils Used in Study*

	% Sand (>50μm)	% Silt (50-2μm)	% Clay (<2μm)
Castor sandy loam	44.2	52.8	2.9
Allendale silty clay	17.7	33.9	48.5

Physical Properties

	Castor sandy loam	Allendale silty clay
ρ_B*	1.55	1.25
ρ_s*	2.73	2.68
ω_{sat}	27.0	57.0
θ_{sat}	42.0	72.0
K_{uf}	$1.2X10^{-2}$	$8.2X10^{-3}$

ρ_B = bulk density, g cm^{-3}

ρ_s = particle density, g cm^{-3}

ω_{sat} = saturated moisture content (% dry weight)

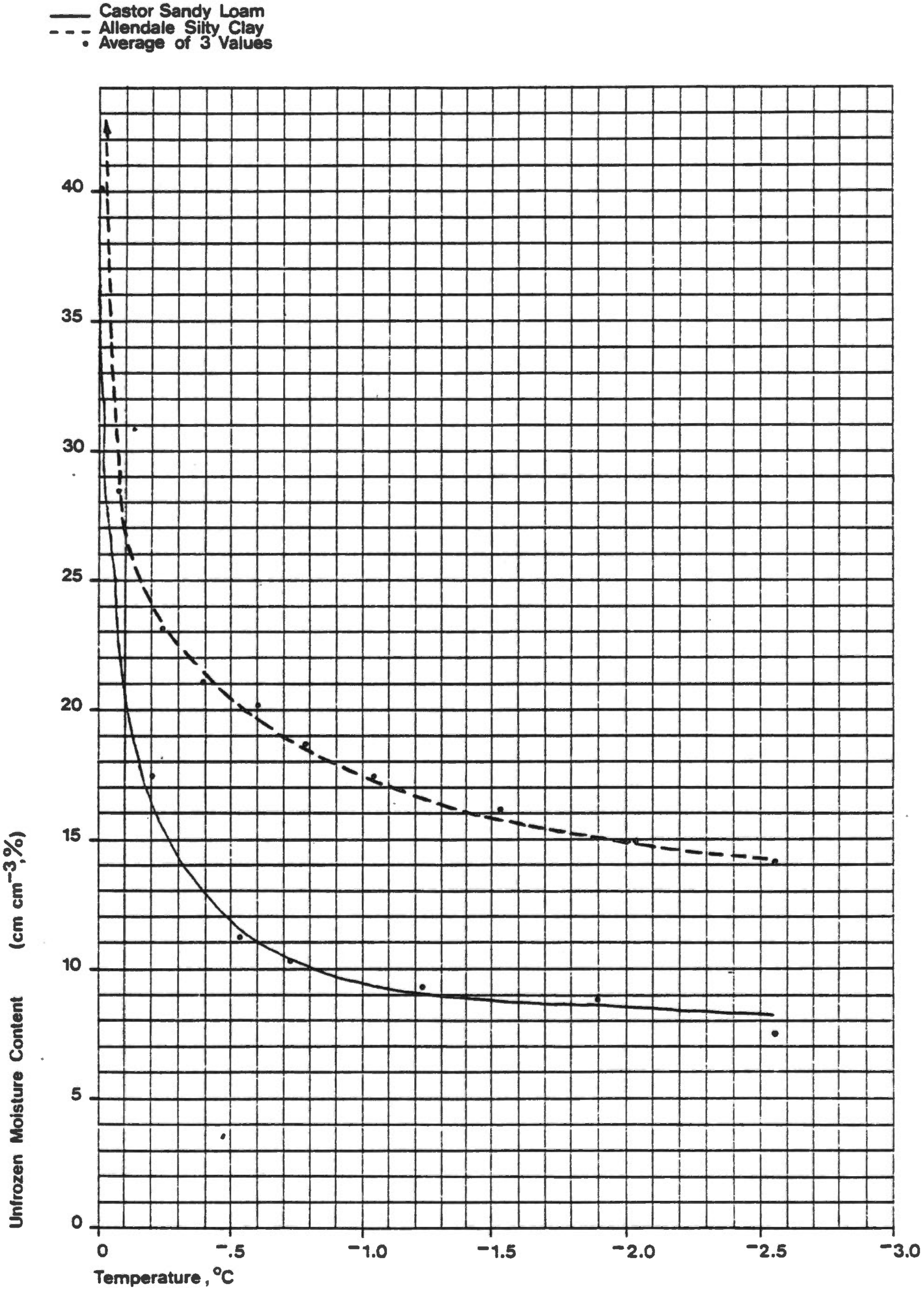
θ_{sat} = saturated moisture content, volumetric (% total volume $\theta=\omega\rho_B$)

K_{uf} = unfrozen hydraulic conductivity, saturated state, cm s^{-1}

* Data courtesy of Mr. D. Patterson

Figure 3.1

Freezing Characteristics of Soils Used In Study



3.2 Modifications to Burt's Experiment

Several attempts were made to repeat Burt's experiment using the apparatus shown in Figure 2.1. One difficulty with this arrangement is that the ice-lactose bath used to cool the system often warmed up too quickly for the ice and water within the soil to reach a state of equilibrium. In addition to this, it is doubtful whether stable or uniform temperatures occurred within the bath, since over a period of a day or so the ice tended to fuse together to form a single lump floating in the lactose solution.[1]

This problem was overcome by substituting a 'Hotpack' refrigerated bath circulator for the ice-lactose bath used by Burt and Perfect. The bath is filled with methanol and provides constant temperature control to within ± 0.01°C for the system. This arrangement not only permits measurement of the hydraulic conductivity under stable temperature conditions, it also allows investigation of the phenomenon of declining flow rates observed by Burt (1974) and Williams and Perfect (1979).[2] Burt (1974) noted the need for such an

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[1] Thermistors placed at different locations within the bath indicated a marked variation in temperature with position and with time.

[2] Williams and Perfect (1979) considered using a refrigerated bath circulator instead of the ice-lactose bath but rejected the idea because "variations in the electric current supplied caused significant temperature fluctuations in the methanol bath." It has now been found that changes in the line voltage had no significant effect on the bath temperature. However, they appear to affect the output from the multimeter (connected to a thermistor) which Perfect used to measure the temperature of the bath.

arrangement:

> "Further research is required on this point but since the present apparatus cannot allow temperatures to be maintained for long periods, especially at lower temperatures, the short term situation is all that can be investigated. Single tests lasting for 8 hours are complicated by the fact that the temperature may have increased by 0.1°C or more by the end of the test."

Williams and Perfect (1979) indicate that "if the soil does have a lower long term permeability then this is the permeability which should be investigated, since it most closely resembles the natural field situation .... it is uncertain whether the conductivity will stabilize at some lower long term value or continue to decrease until the sample becomes impermeable."

3.3 Experimental Procedure

All samples were prepared as slurries,using actively boiling deionized water and gently spooned into the perspex sample holder. The sample holder was placed in a vacuum dessicator for approximately 15 minutes to remove any excess air trapped within the soil. The sample was then frozen rapidly in a freezer to a temperature of about -18°C and left there for 24 hours. Prior to assembly the ends of the sample were scraped down until flush with the perspex sample holder. Once assembled,to prevent gradual seepage of methanol into the reservoirs,any possible openings in the apparatus were sealed with a silicon caulking compound and waterproof tape.

Both Burt's original apparatus (cross-sectional area 11.34 cm^2, length 3.02 cm) and the larger permeameter designed by Perfect (cross-sectional area 22.88 cm^2 length 3.29 cm) were used in the experiments. In all experiments a constant head of 1500 cm of water (=1.45 atmospheres) was used. Pressure was generated by a water filled screw piston connected to a manometer (see Plate 3.2). A rigid network of steel tubing and swagelock valves enabled pressure to be maintained in both permeameters simultaneously. Measurements of the decline in the hydraulic conductivity for both soil samples were made at temperatures between -0.6°C and -0.1°C in 0.1°C increments. At each increment in temperature, the reservoirs were flushed with the appropriate concentration of lactose solution and left to equilibriate for approximately 24 hours before taking flow readings.

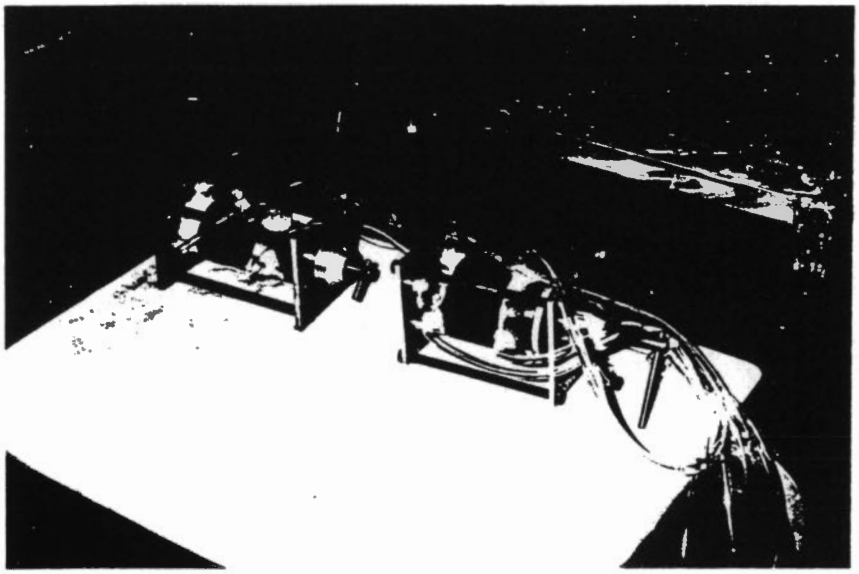
Outflow from each permeameter was measured with two small capillaries (internal diameter 0.047 cm) mounted on a finely divided scale and read with a magnifying lens. At each temperature increment, readings of approximately 4-5 minutes duration were taken continuously for a period of 4 or 5 hours.

Dialysis membranes were used in all of the experiments. Darcy's law was used to complete the hydraulic conductivity coefficients (See computer program Appendix B). The values were corrected for the presence of a thawed layer at either end of the soil sample,caused by the gradual diffusion of lactose molecules through the membranes (See Appendix C).

3.4 Decline in Apparent Permeability of Soil with Time

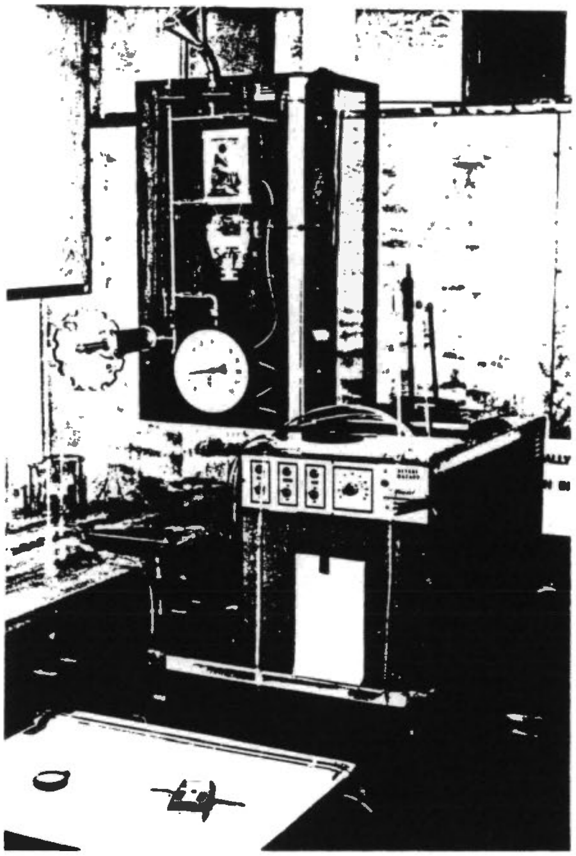
Initial and final moisture contents for both soils

Plate 3.1



View of the two permeameters used to measure the hydraulic conductivity of frozen soils.

Plate 3.2



General view of the hydraulic conductivity experiment: Methanol bath on the right. Screw piston apparatus and manometer on the left. Capillary tubes used to measure flow in the foreground.

TABLE 3.2

Initial and Final Values of Moisture Content For Soils Used in the Experiment

Experiment No.	H1	H2	I1	I2
Initial Moisture Content, % dry wt.	31.8	33.0	54.6	57.7
Final Moisture Content, % dry wt.	27.0	27.7	53.7	58.1

are shown in Table 3.2. With the Allendale soil there was no significant change in the moisture content throughout the duration of the experiment. In contrast, the Castor soil appeared to experience a moderate decrease in moisture content from about 32 - 33% to about 27 - 28%. This may have been due to the fact that the initial moisture content of the Castor was somewhat greater than saturation(=28%) and so application of pressure to the sample expelled the excess moisture until an equilibrium condition was attained (ie. at saturation).

In most instances, the flow was too rapid in the first minute or two of each experiment to be recorded with the small capillaries. This may have been due to sliding of the sample within its perspex container ,following the application of pressure to the system.

Results of the tests performed on the Castor sandy loam (Experiment Nos. H1 and H2) and the Allendale silty clay (Experiment Nos. I1 and I2) are plotted in Figures 3.2 - 3.5. The dashed lines represent visually interpolated best fit curves for the experimental data. In general, the results indicate a rapid decline in the 'apparent' hydraulic conductivity of both soils during the first hour or so of the experiment, approaching assymptotically a constant value some 2 to 6 hours following the initial application of pressure to the system.[3] In all of

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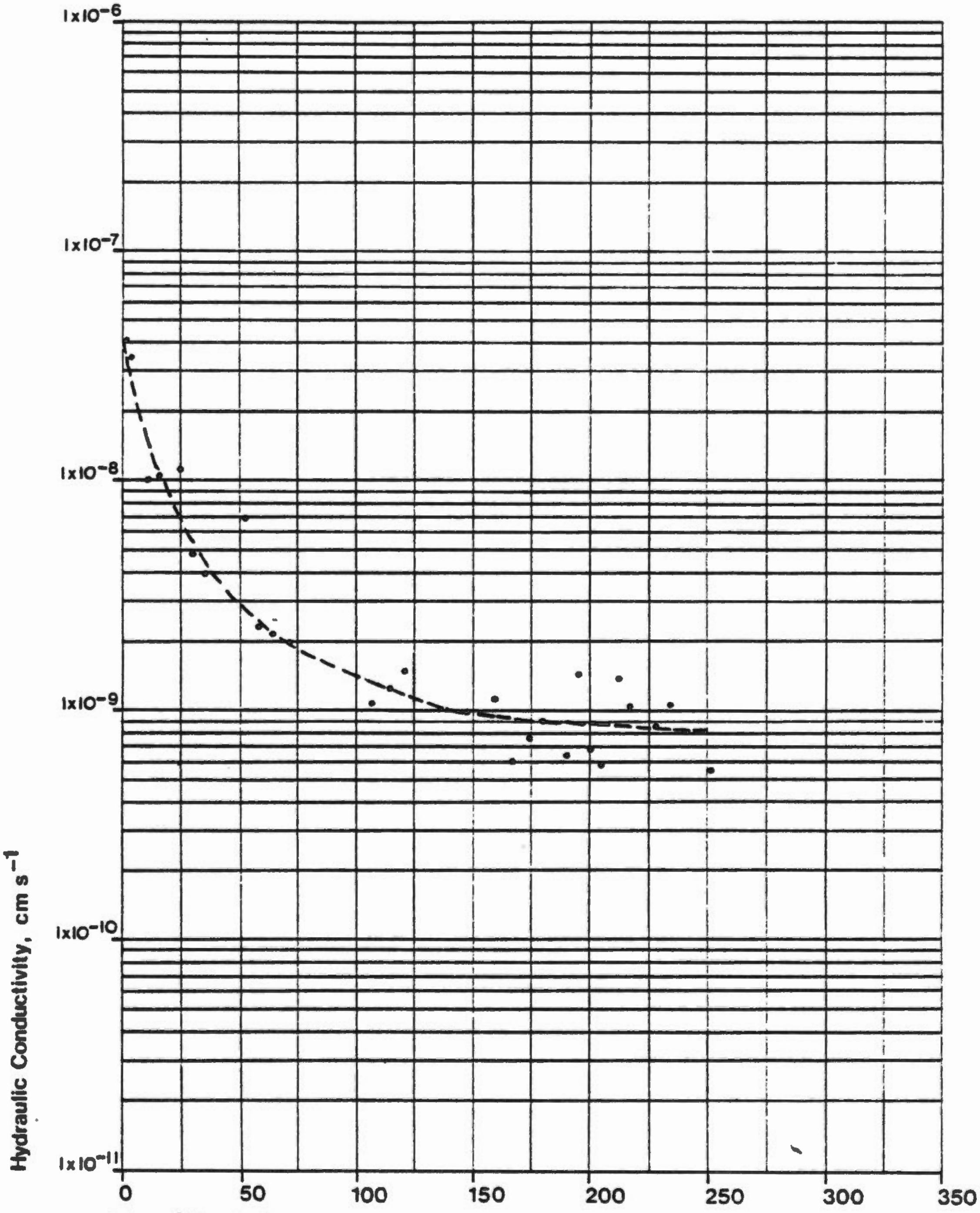
[3] Miller (1976) recommends the use of the term 'apparent' hydraulic conductivity since, in addition to movement entirely within the fluid phase, undoubtedly a large component of transport is by regulation. Also, as mentioned in section 2.3, it is doubtful whether transport within the fluid films obeys Darcy's Law.

Figure 3.2a
Experiment Number H1

Decline In Apparent Hydraulic Conductivity
Of Castor Sandy Loam With Time

Temperature of Soil - 0.613 °C

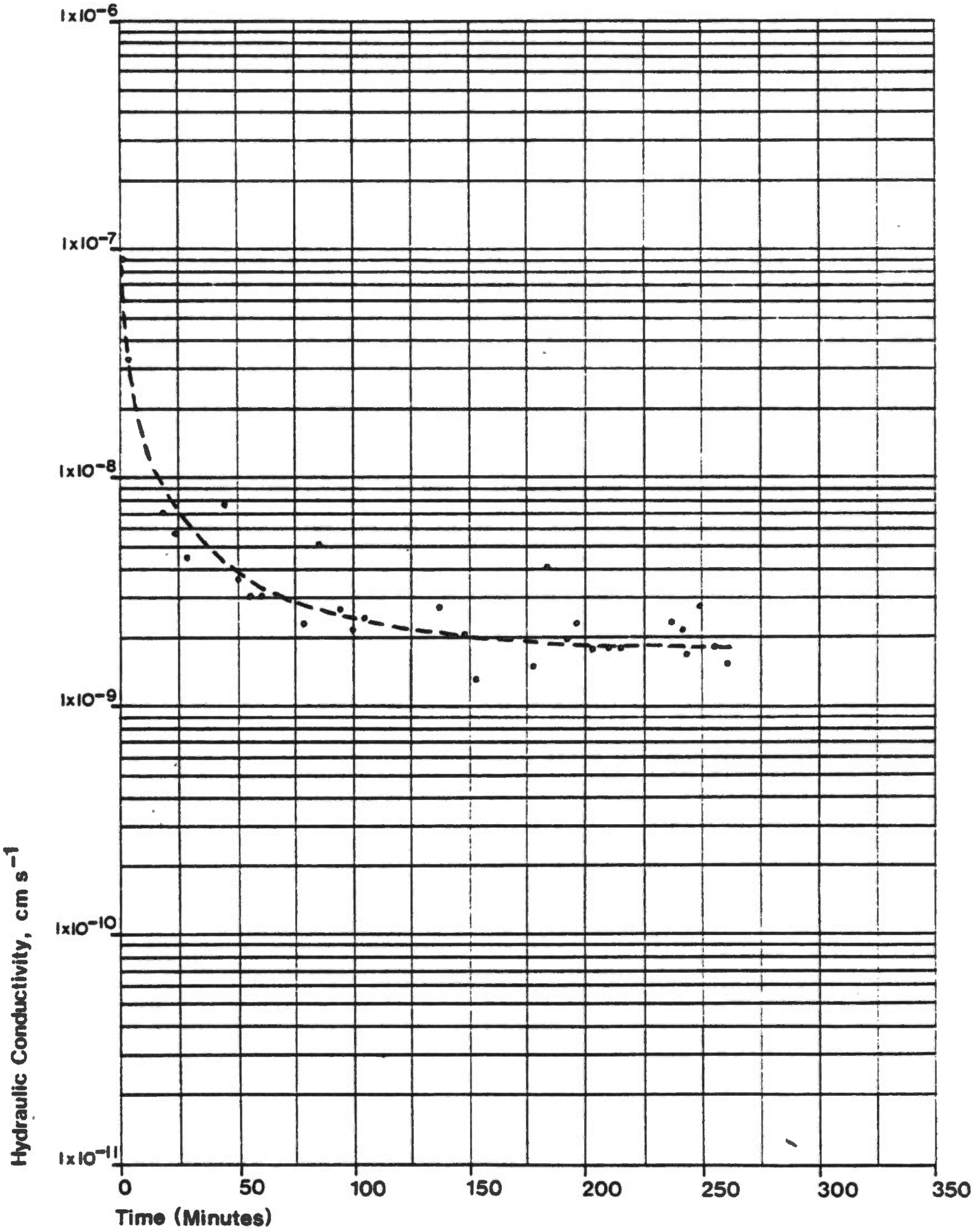


• experimentally determined values

Figure 3.2b
Experiment Number H1

Decline In Apparent Hydraulic Conductivity Of Castor Sandy Loam With Time

Temperature of Soil – 0.503 °C



• experimentally determined values

Figure 3.2c
Experiment Number H1

Decline In Apparent Hydraulic Conductivity
Of Castor Sandy Loam With Time

Temperature of Soil - 0.397 °C

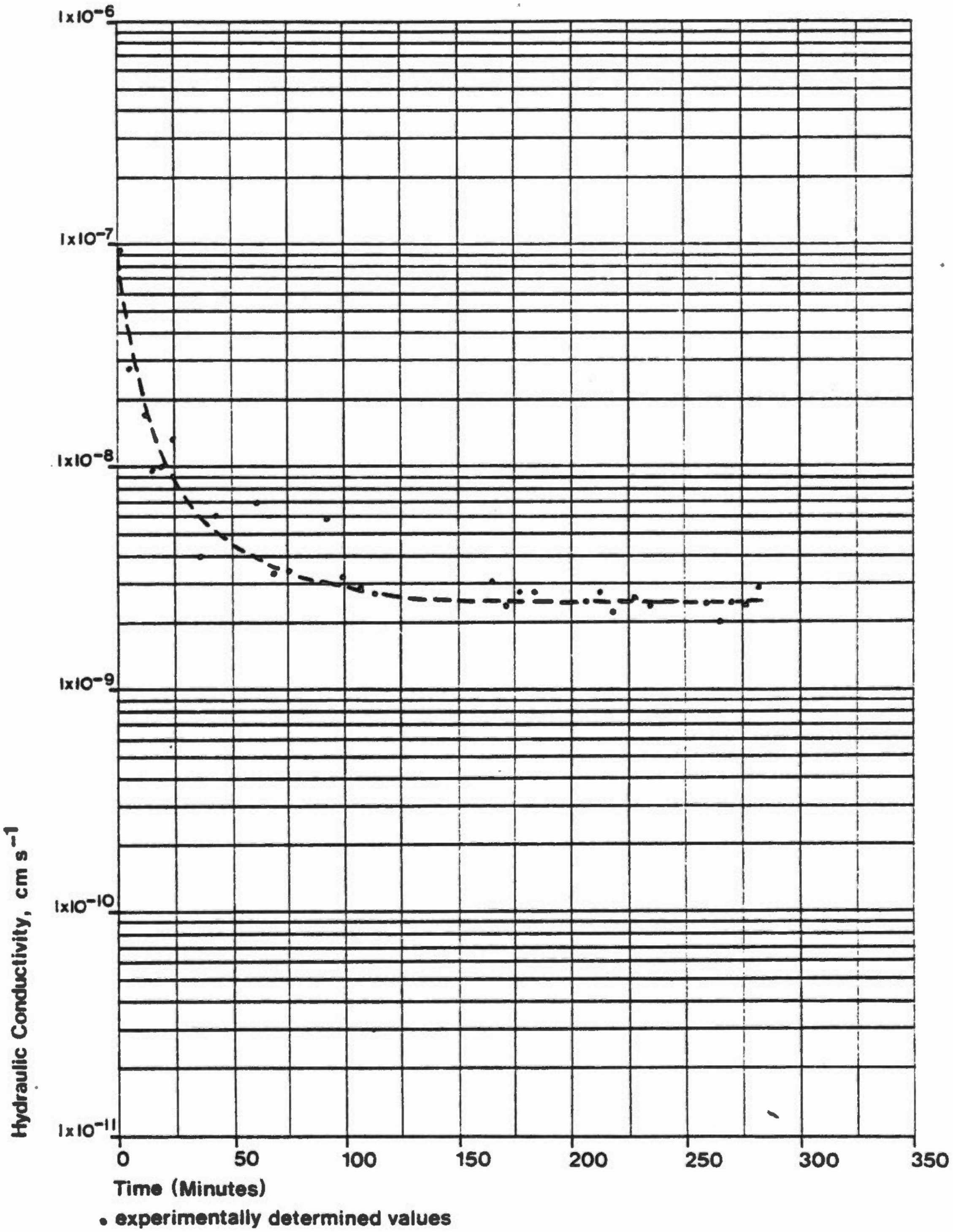
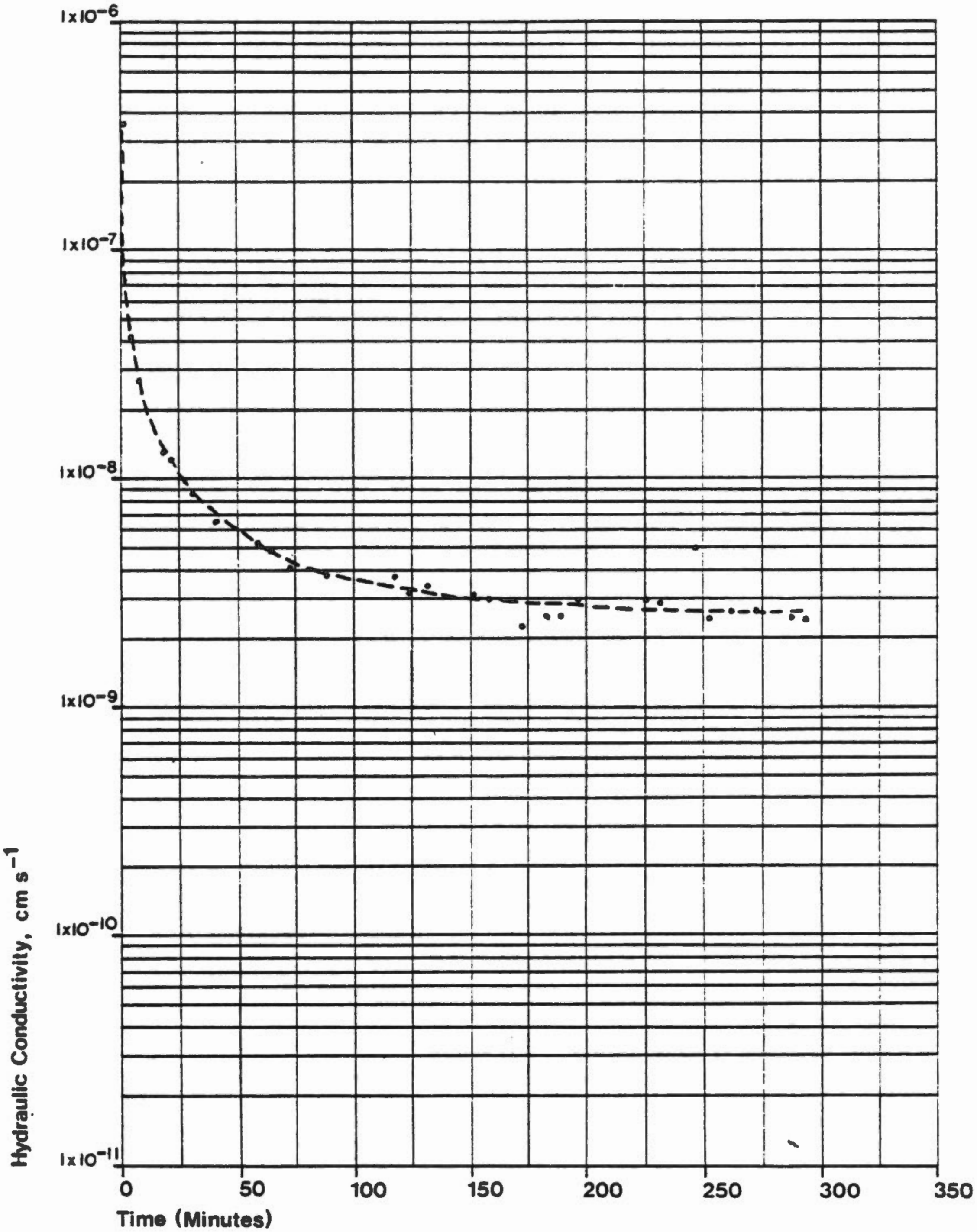


Figure 3.2d
Experiment Number H1

Decline In Apparent Hydraulic Conductivity Of Castor Sandy Loam With Time

Temperature of Soil - 0.291 °C

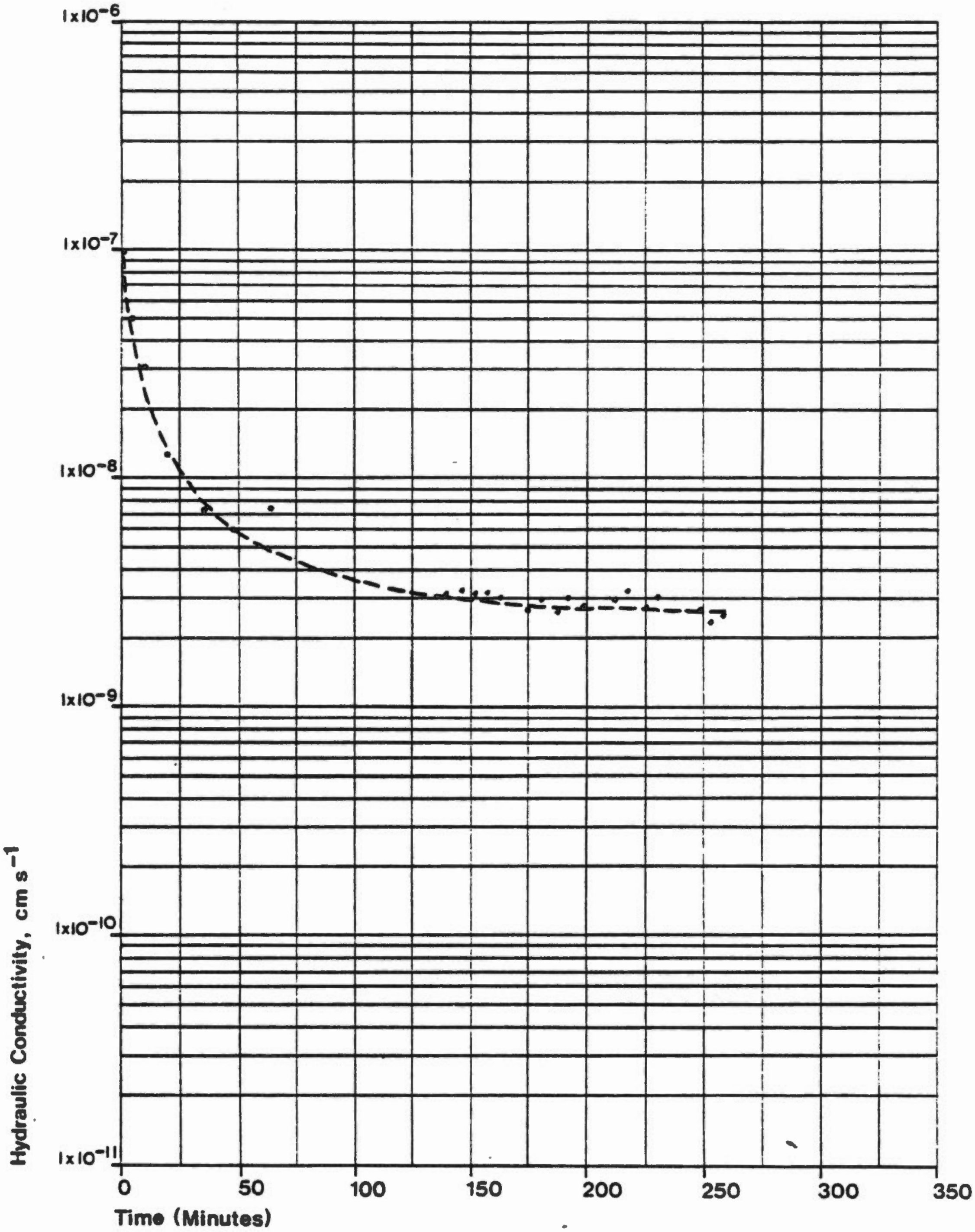


• experimentally determined values

Figure 3.2e
Experiment Number H1

Decline In Apparent Hydraulic Conductivity Of Castor Sandy Loam With Time

Temperature of Soil - 0.201 °C

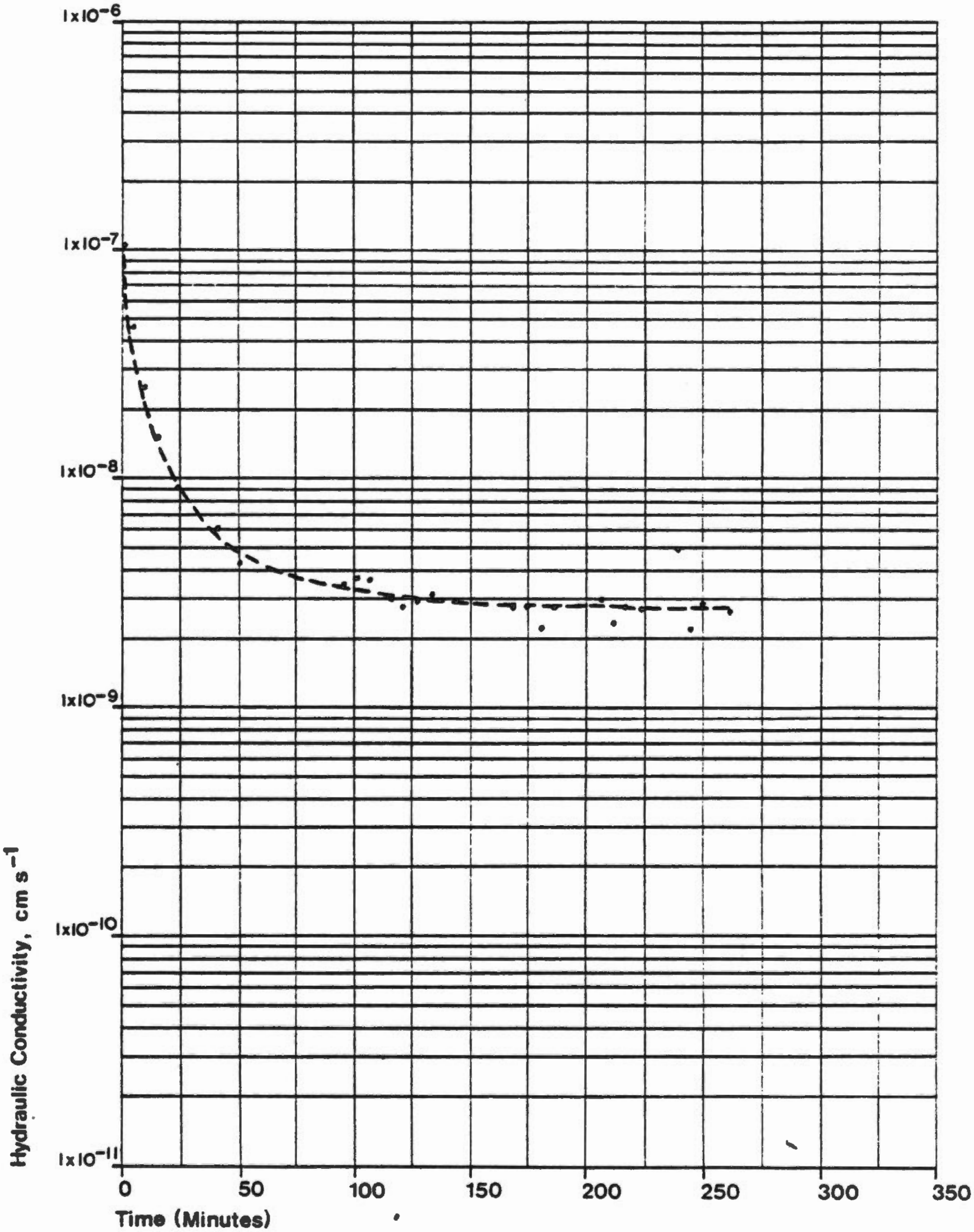


• experimentally determined values

Figure 3.2f
Experiment Number H1

Decline In Apparent Hydraulic Conductivity Of Castor Sandy Loam With Time

Temperature of Soil - 0.100 °C

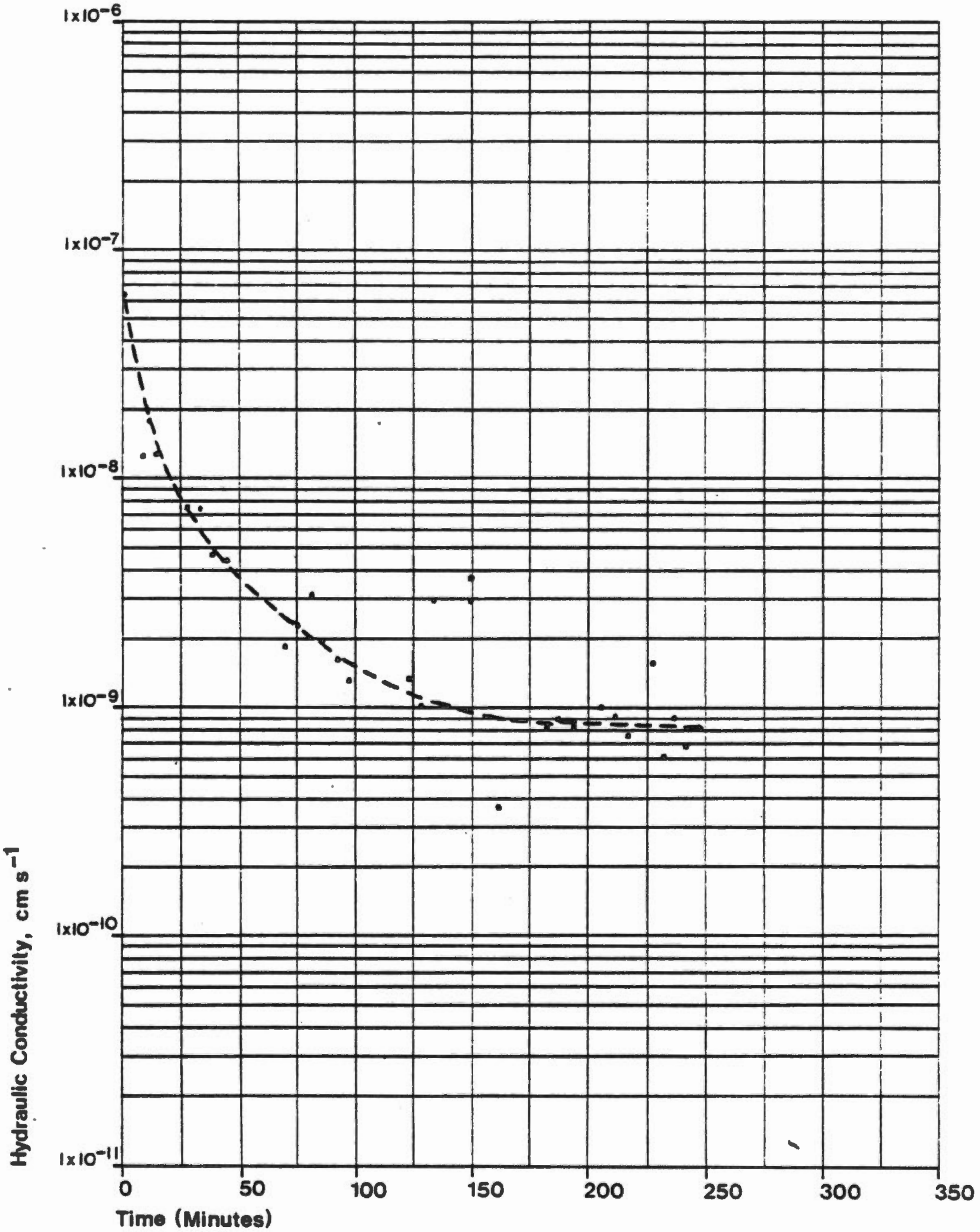


• experimentally determined values

Figure 3.3a
Experiment Number H2

Decline In Apparent Hydraulic Conductivity Of Castor Sandy Loam With Time

Temperature of Soil – 0.608 °C

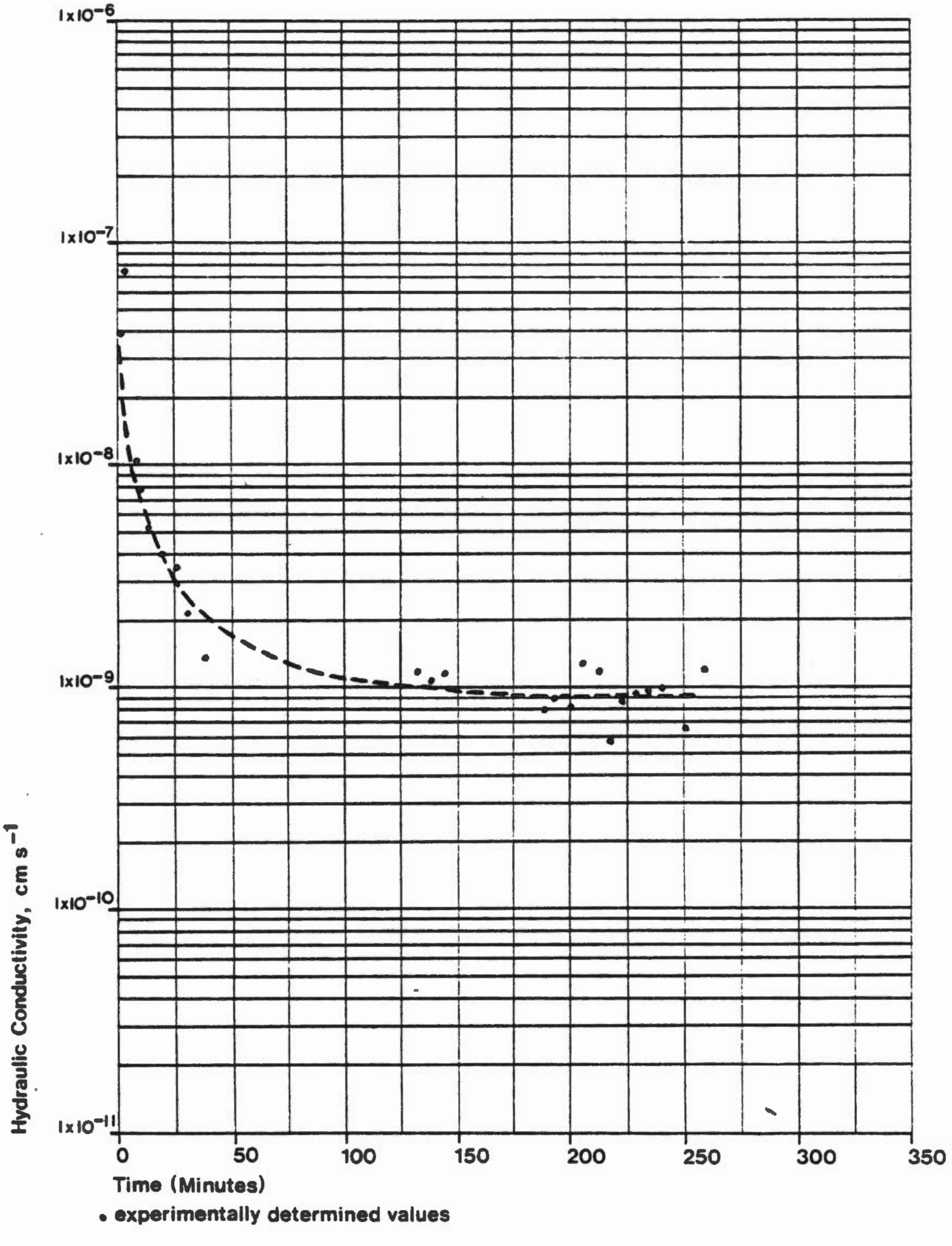


• experimentally determined values

Figure 3.3b
Experiment Number H2

Decline In Apparent Hydraulic Conductivity Of Castor Sandy Loam With Time

Temperature of Soil - 0.503 °C



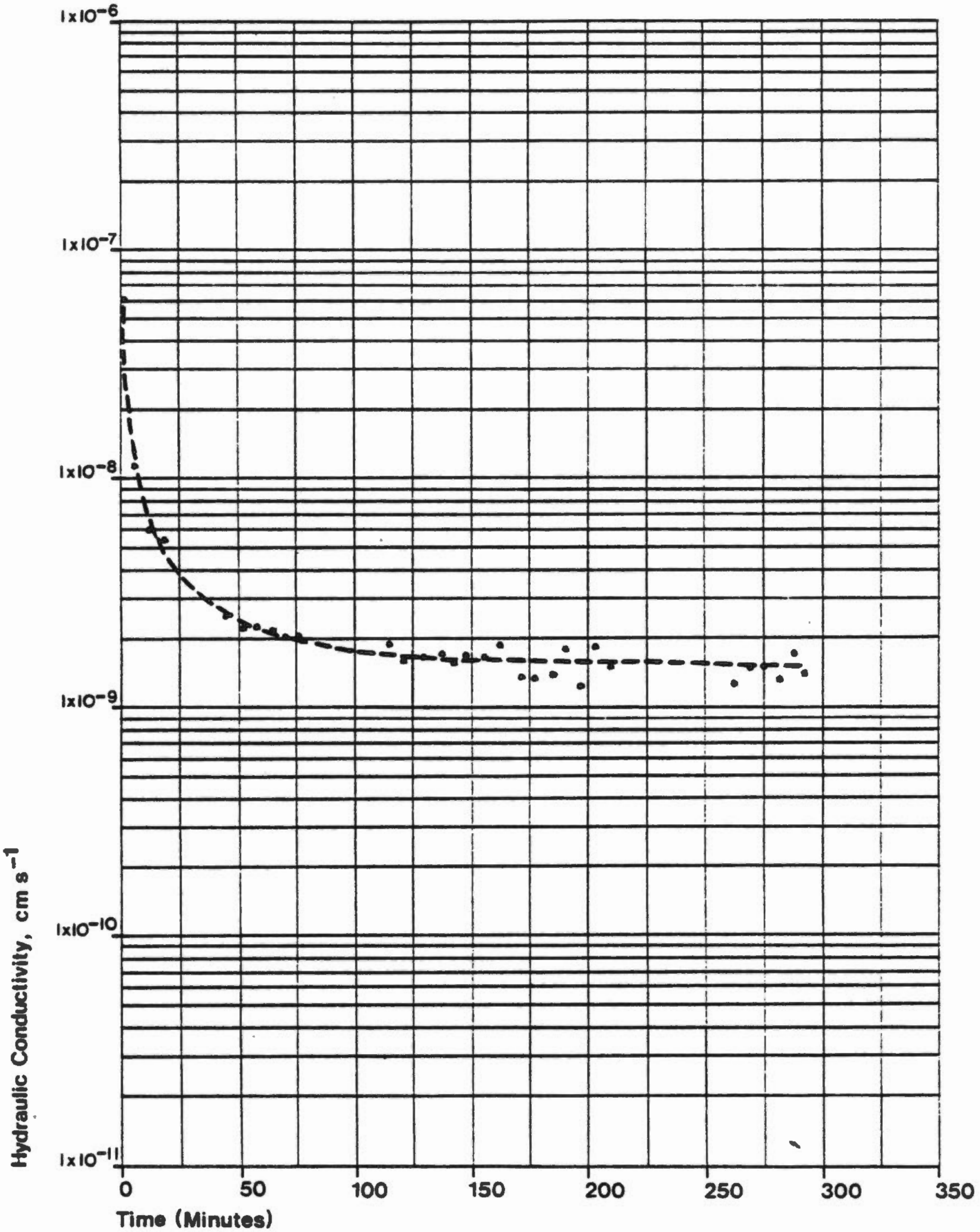
Time (Minutes)

• experimentally determined values

Figure 3.3c
Experiment Number H2

Decline In Apparent Hydraulic Conductivity Of Castor Sandy Loam With Time

Temperature of Soil – 0.397 °C

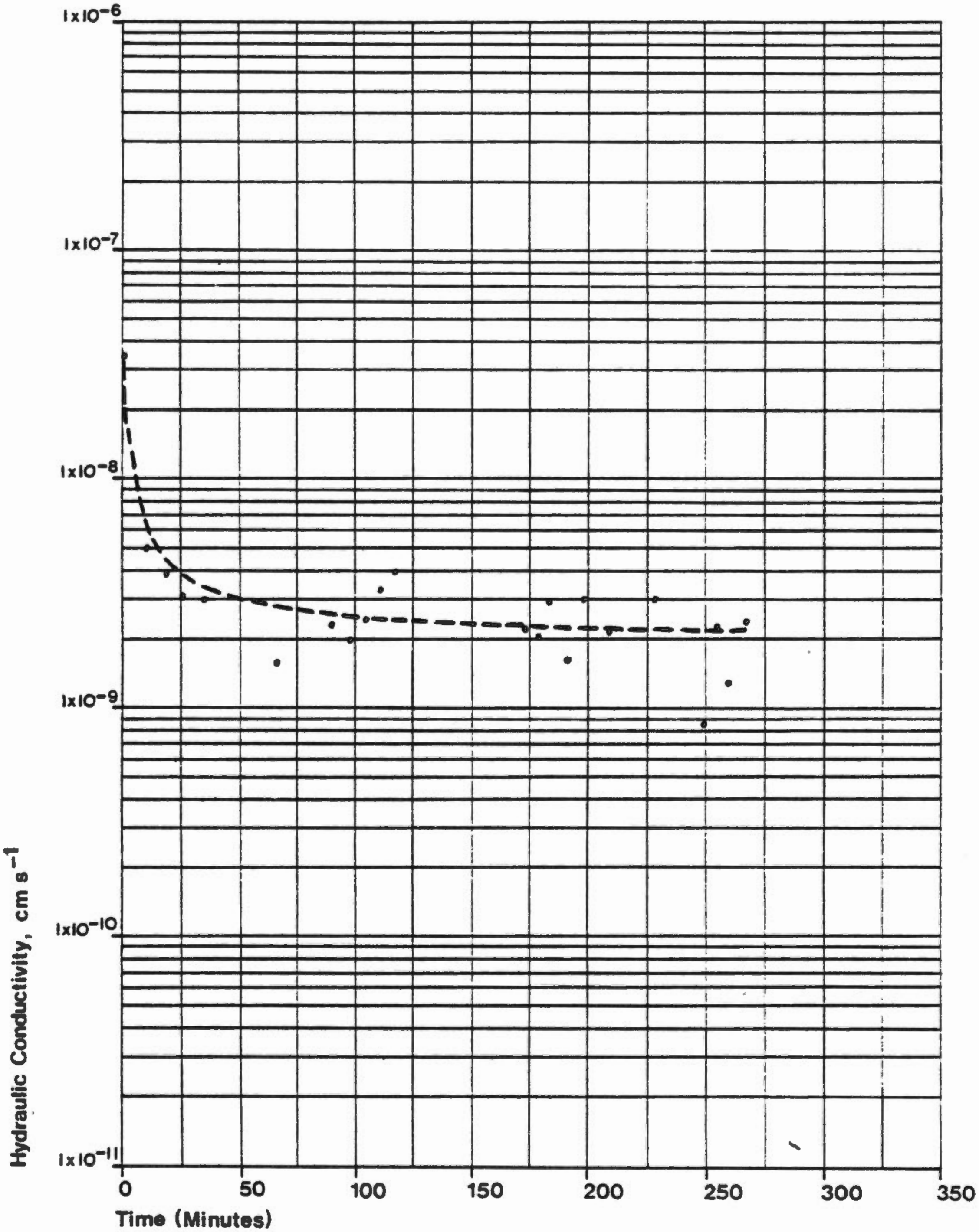


• experimentally determined values

Figure 3.3d
Experiment Number H2

Decline In Apparent Hydraulic Conductivity Of Castor Sandy Loam With Time

Temperature of Soil - 0.291 °C



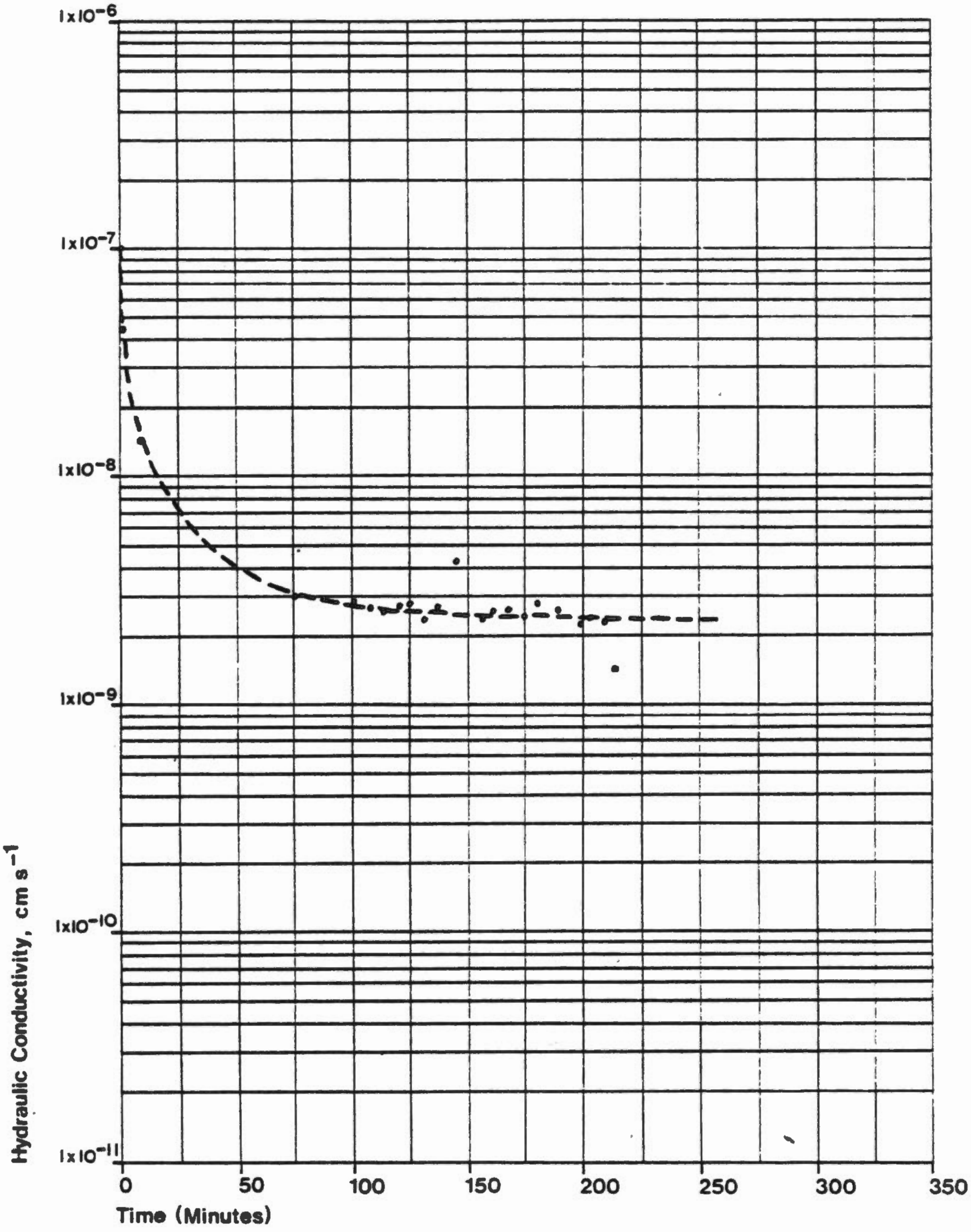
Time (Minutes)

• experimentally determined values

Figure 3.3e
Experiment Number H2

Decline In Apparent Hydraulic Conductivity
Of Castor Sandy Loam With Time

Temperature of Soil - 0.201 °C

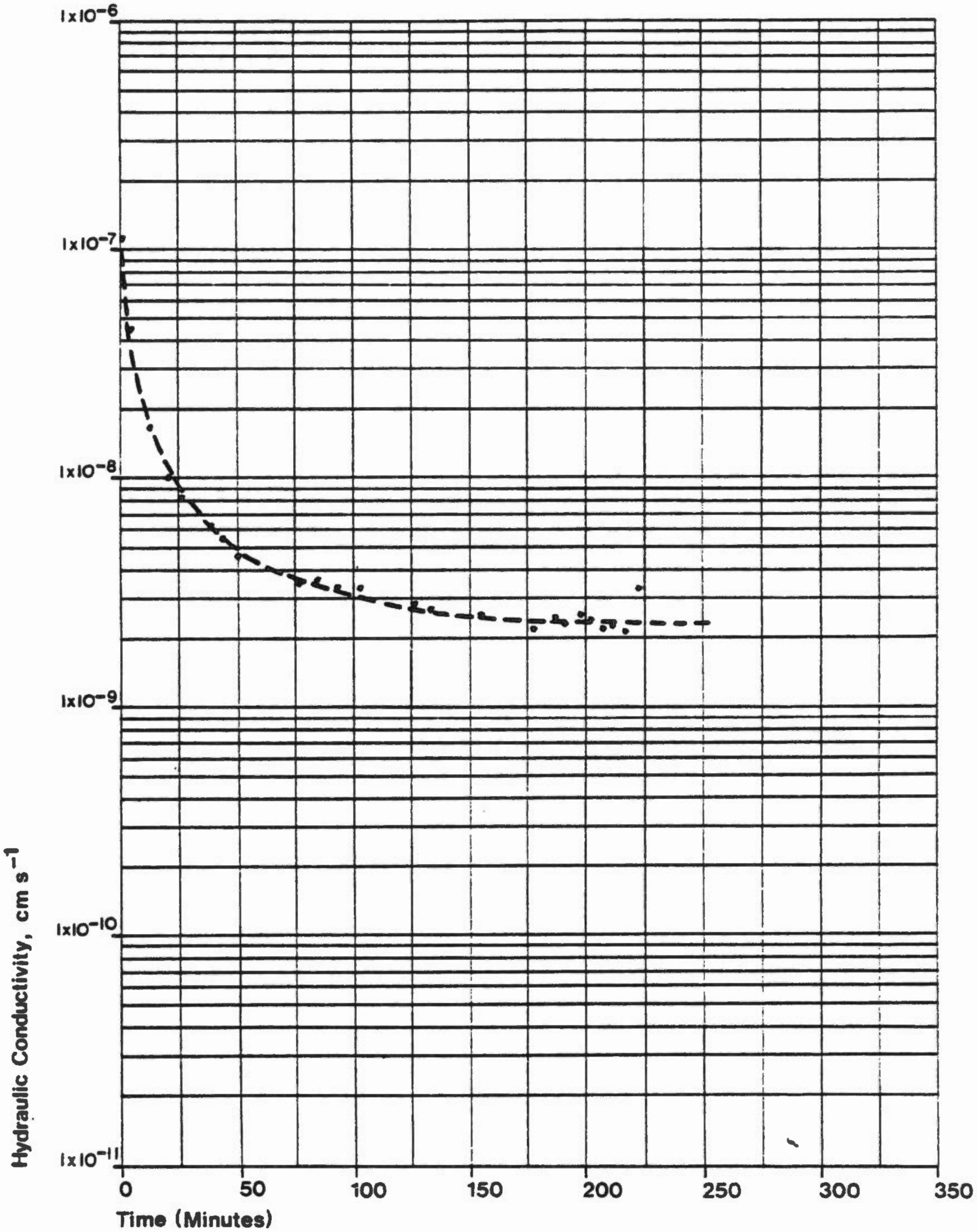


• experimentally determined values

Figure 3.3f
Experiment Number H2

Decline In Apparent Hydraulic Conductivity
Of Castor Sandy Loam With Time

Temperature of Soil – 0.100 °C

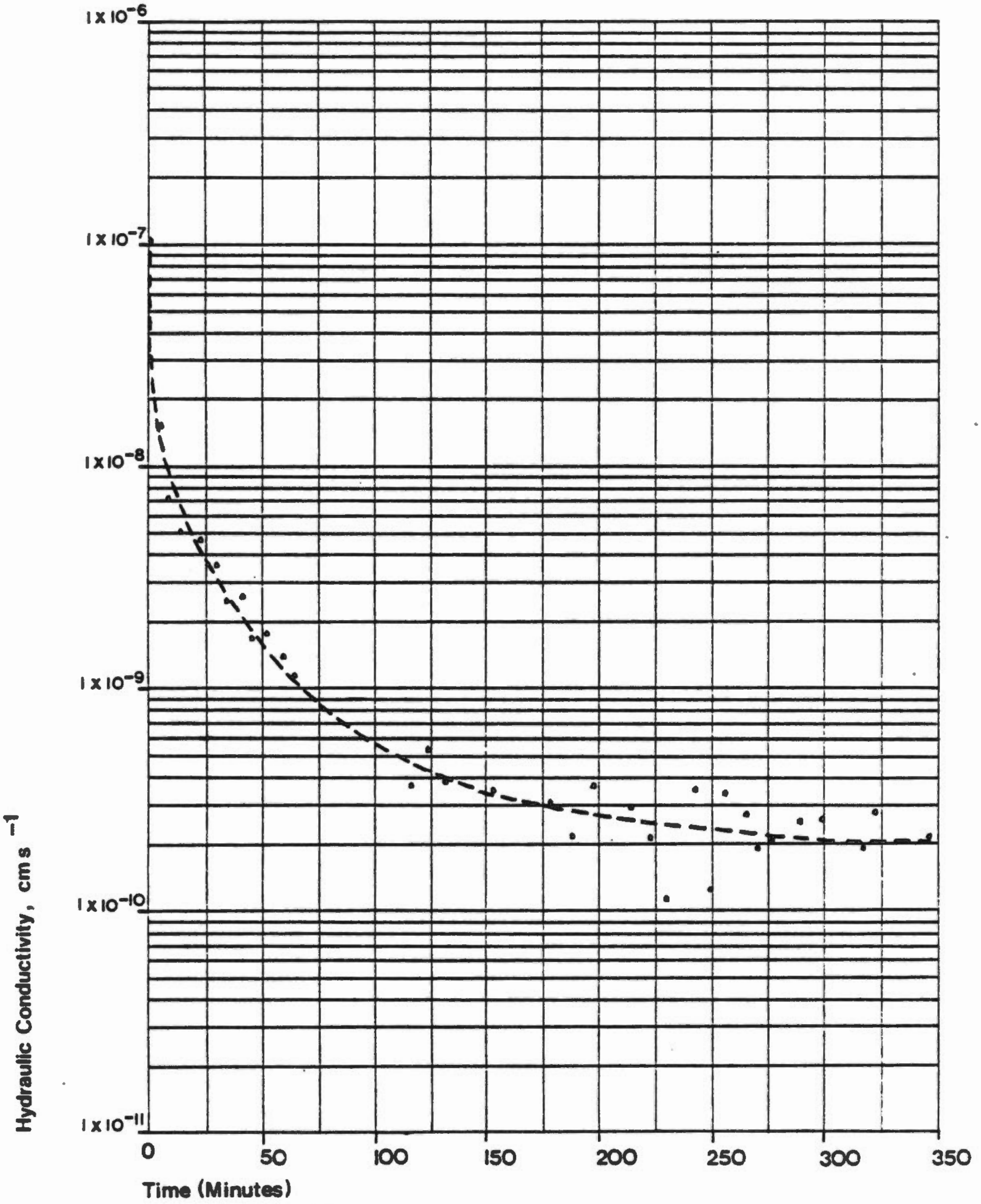


• experimentally determined values

Figure 3.4a
Experiment Number 11

Decline In Apparent Hydraulic Conductivity
Of Allendale Silty Clay With Time

Temperature of Soil - 0.598°C



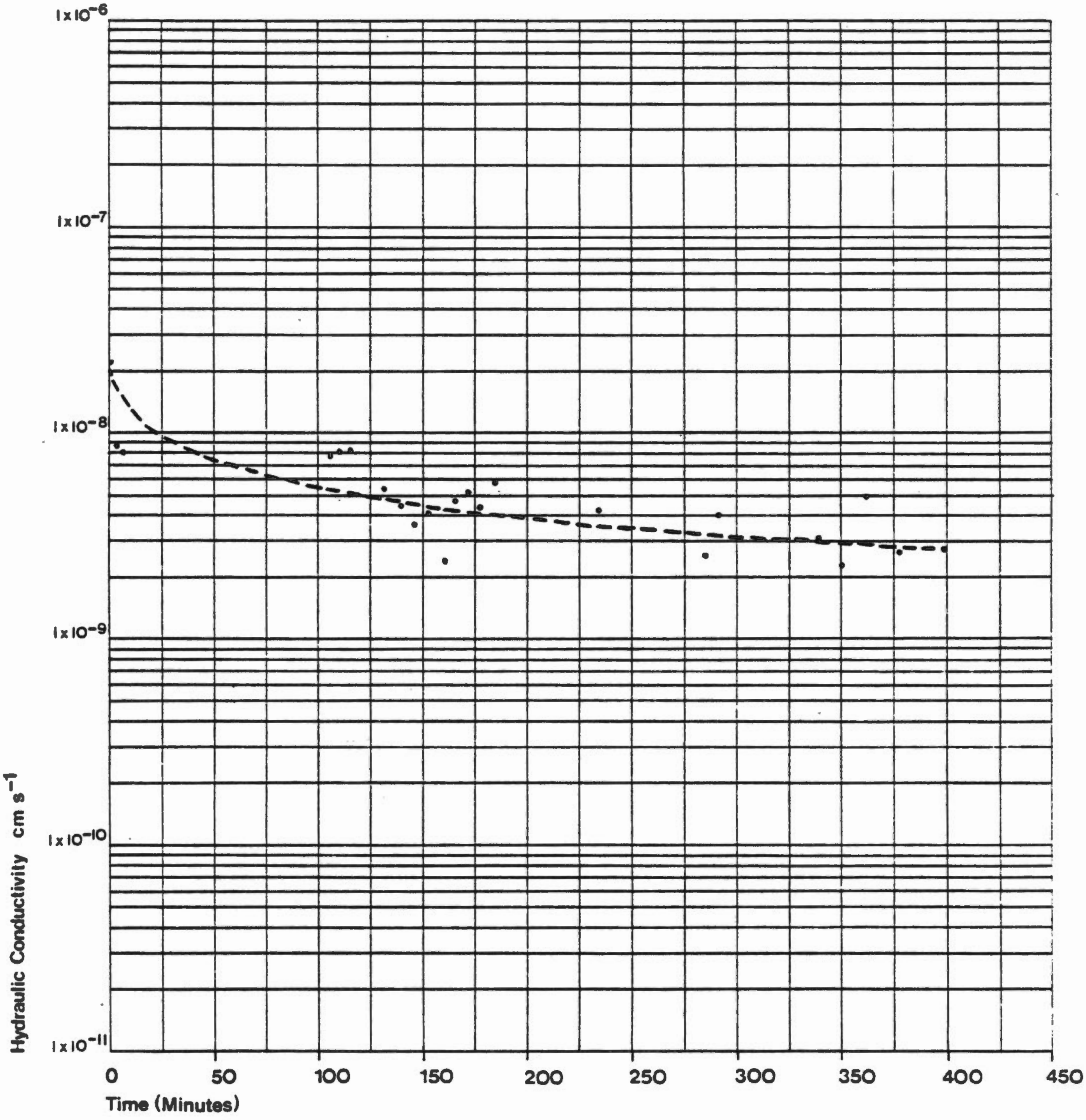
Time (Minutes)

• experimentally determined values

Figure 3.4b
Experiment Number 11

Decline In Apparent Hydraulic Conductivity Of Allendale Silty Clay With Time

Temperature of Soil -0.497 °C

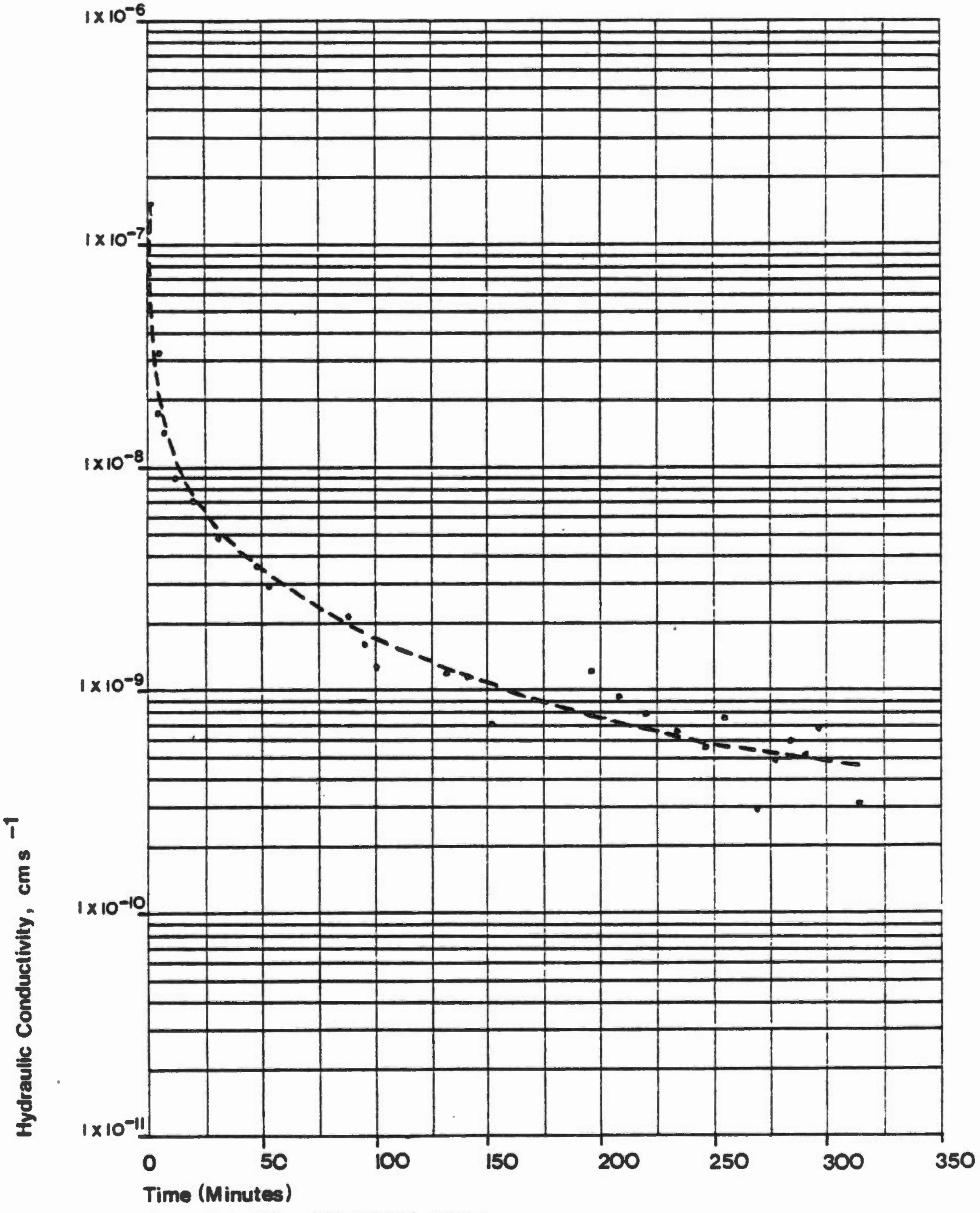


• experimentally determined values

Figure 3.4c
Experiment Number 11

Decline In Apparent Hydraulic Conductivity
Of Allendale Silty Clay With Time

Temperature of Soil - 0.397°C

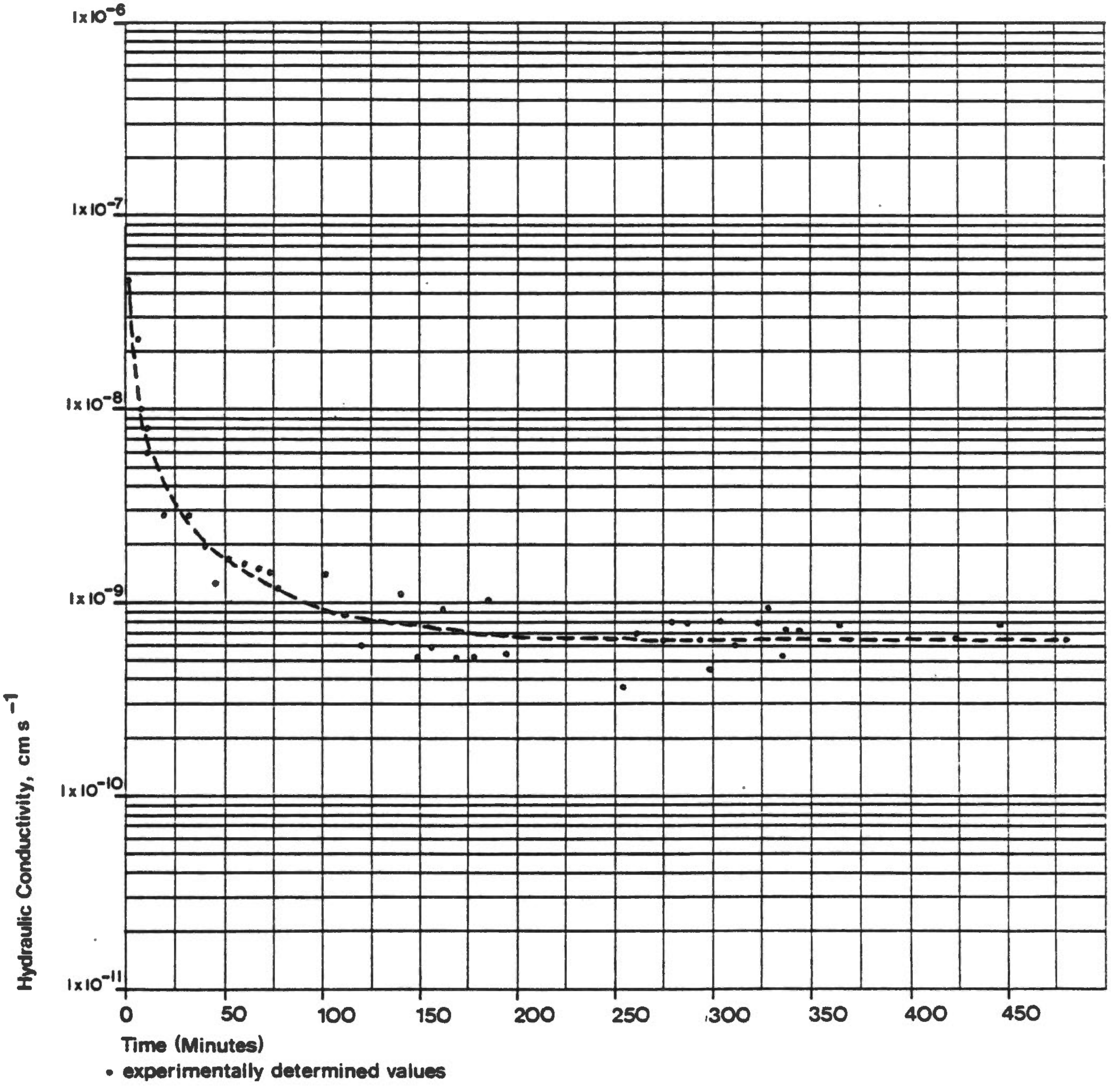


• experimentally determined values

Figure 3.4d
Experiment Number 11

Decline In Apparent Hydraulic Conductivity Of Allendale Silty Clay With Time

Temperature of Soil − 0.291 °C

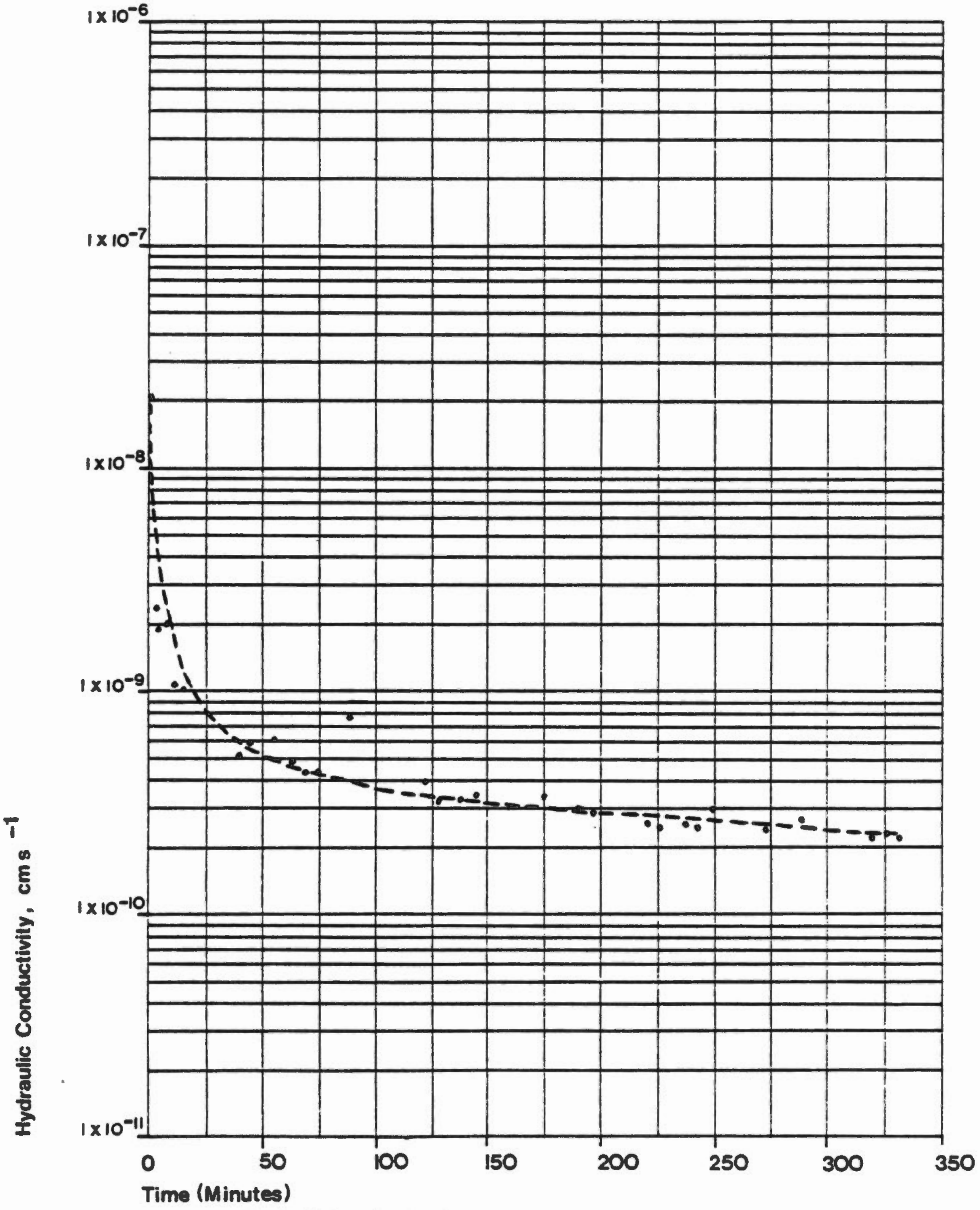


• experimentally determined values

Figure 3.4e
Experiment Number 11

Decline In Apparent Hydraulic Conductivity
Of Allendale Silty Clay With Time

Temperature of Soil - 0.216°C

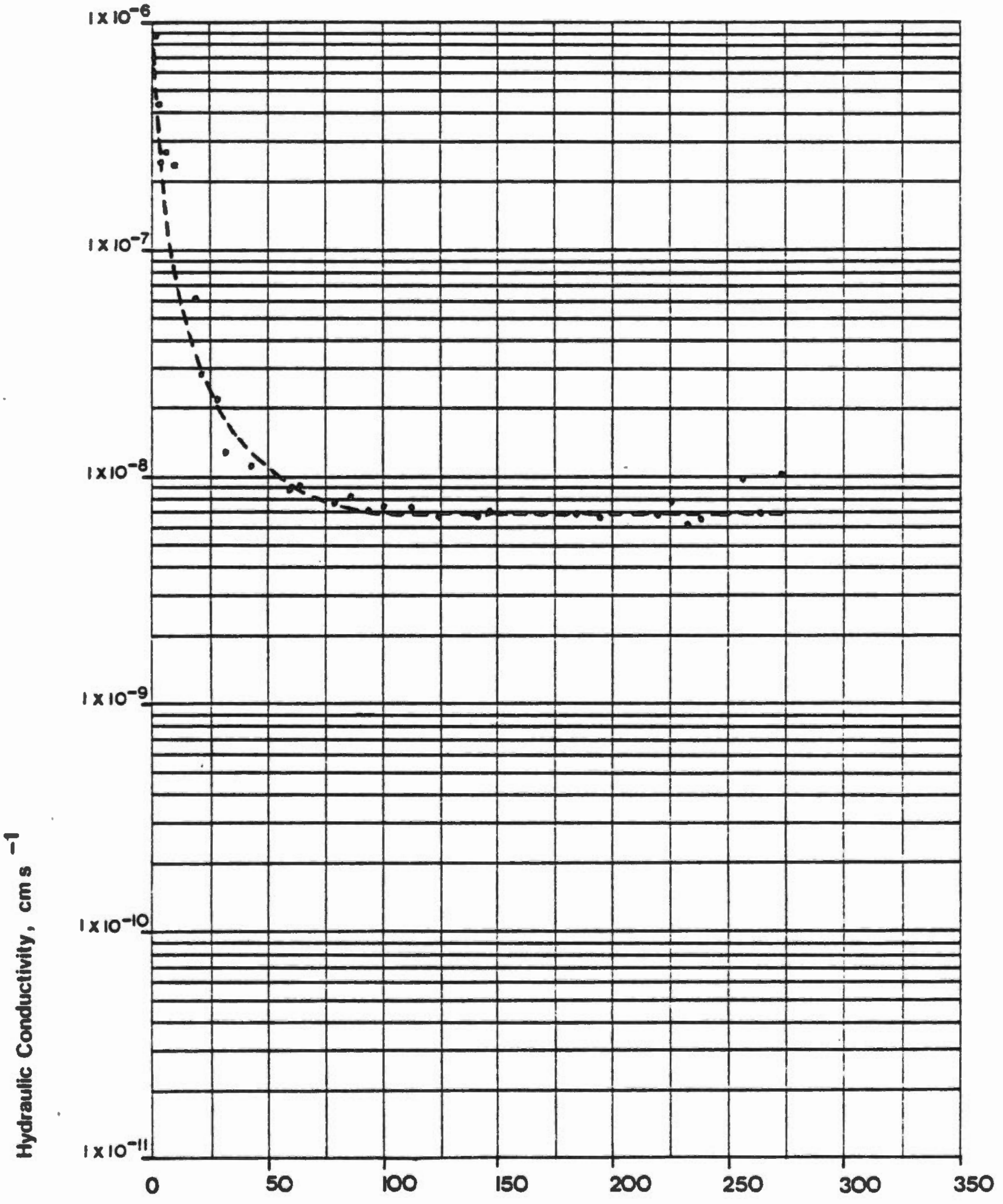


• experimentally determined values

Figure 3.4f
Experiment Number 11

Decline In Apparent Hydraulic Conductivity
Of Allendale Silty Clay With Time

Temperature of Soil - 0.095°C



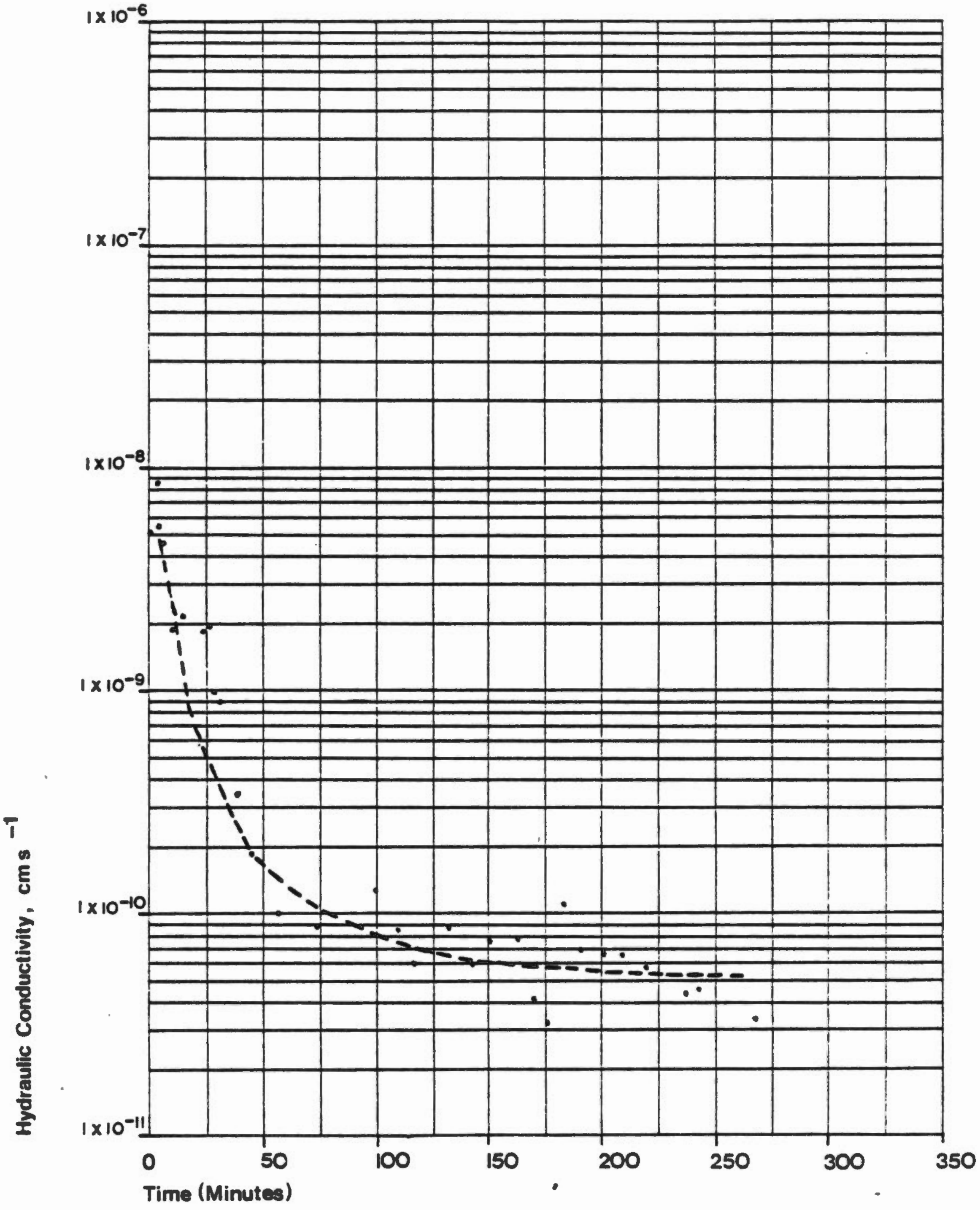
Time (Minutes)

• experimentally determined values

Figure 3.5a
Experiment Number 12

Decline In Apparent Hydraulic Conductivity Of Allendale Silty Clay With Time

Temperature of Soil - 0.598°C

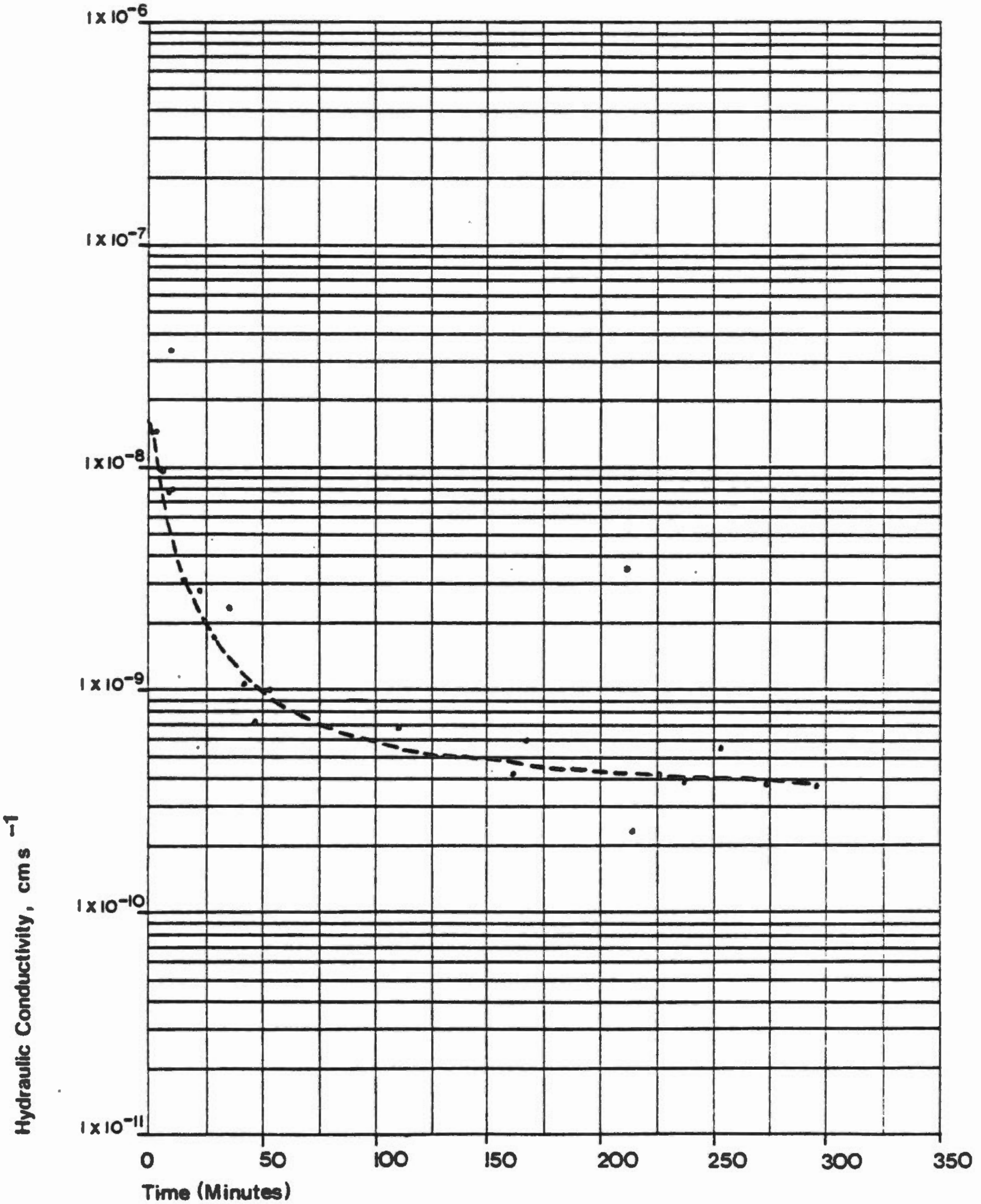


• experimentally determined values

Figure 3.5b
Experiment Number 12

Decline In Apparent Hydraulic Conductivity Of Allendale Silty Clay With Time

Temperature of Soil - 0.497°C

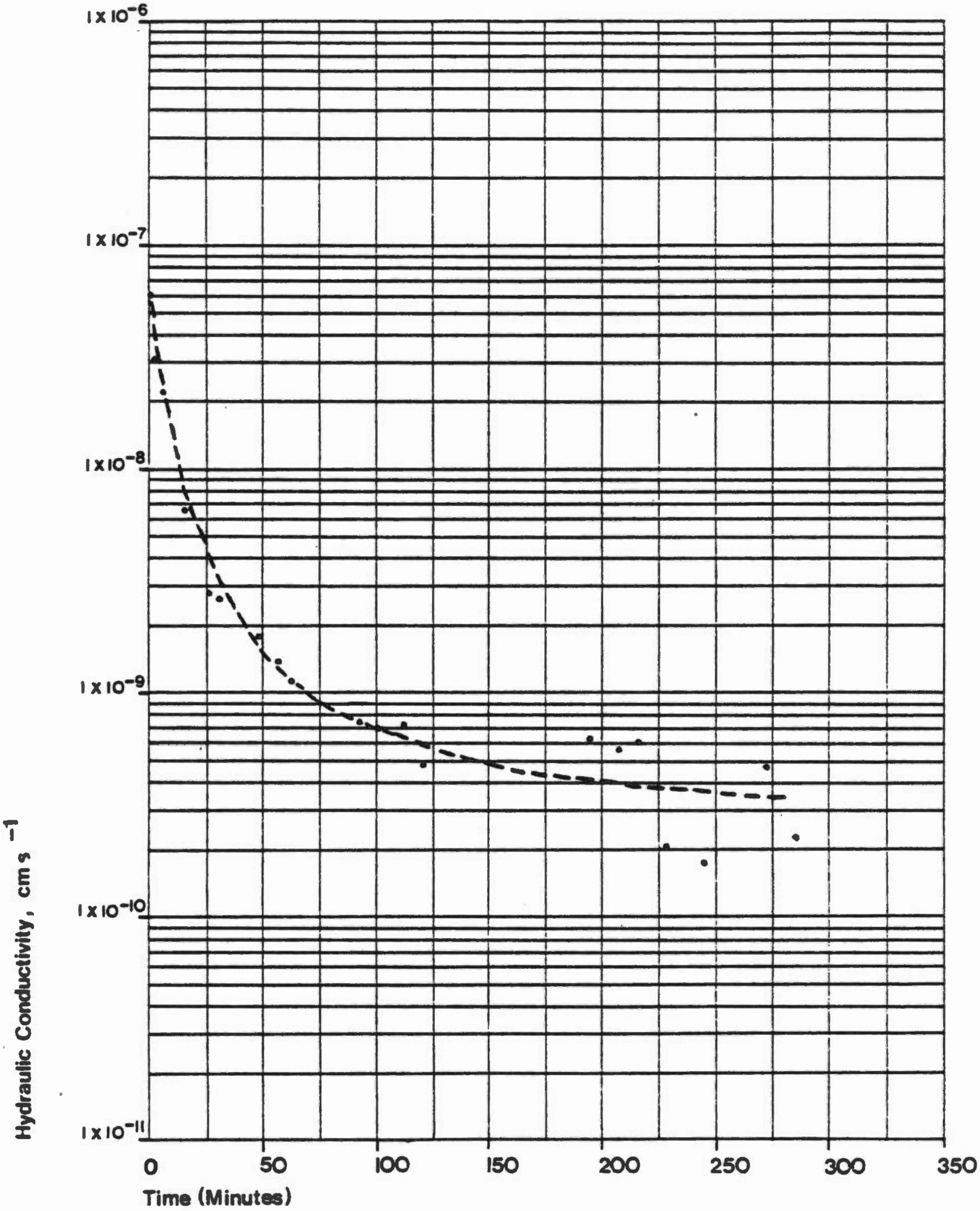


• experimentally determined values

Figure 3.5c
Experiment Number 12

Decline In Apparent Hydraulic Conductivity
Of Allendale Silty Clay With Time

Temperature of Soil - 0.397°C

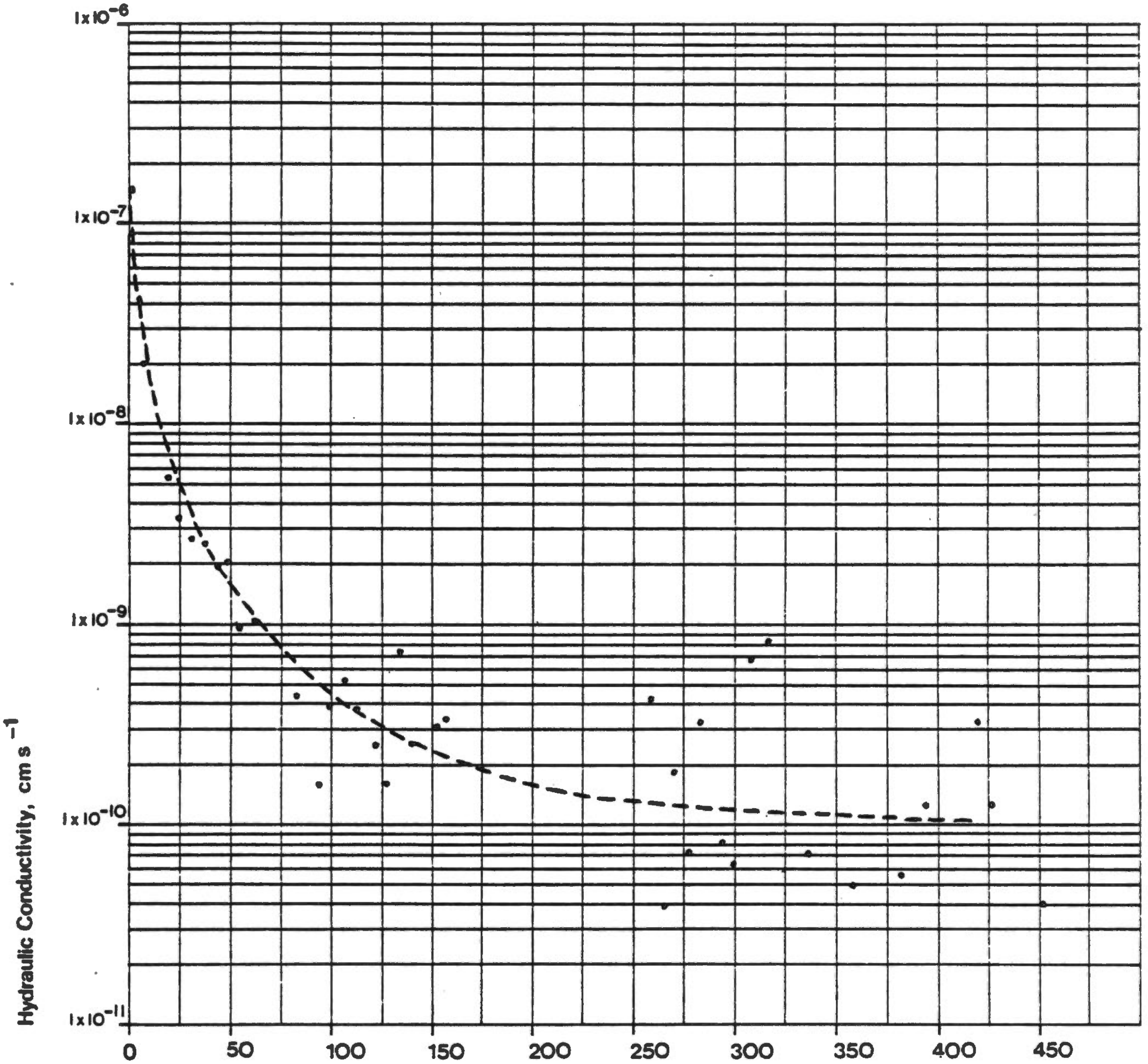


• experimentally determined values

Figure 3.5d
Experiment Number 12

Decline In Apparent Hydraulic Conductivity
Of Allendale Silty Clay With Time

Temperature of Soil −0.291°C



Time (Minutes)

• experimentally determined values

Figure 3.5e
Experiment Number 12

Decline In Apparent Hydraulic Conductivity Of Allendale Silty Clay With Time

Temperature of Soil - 0.216 °C

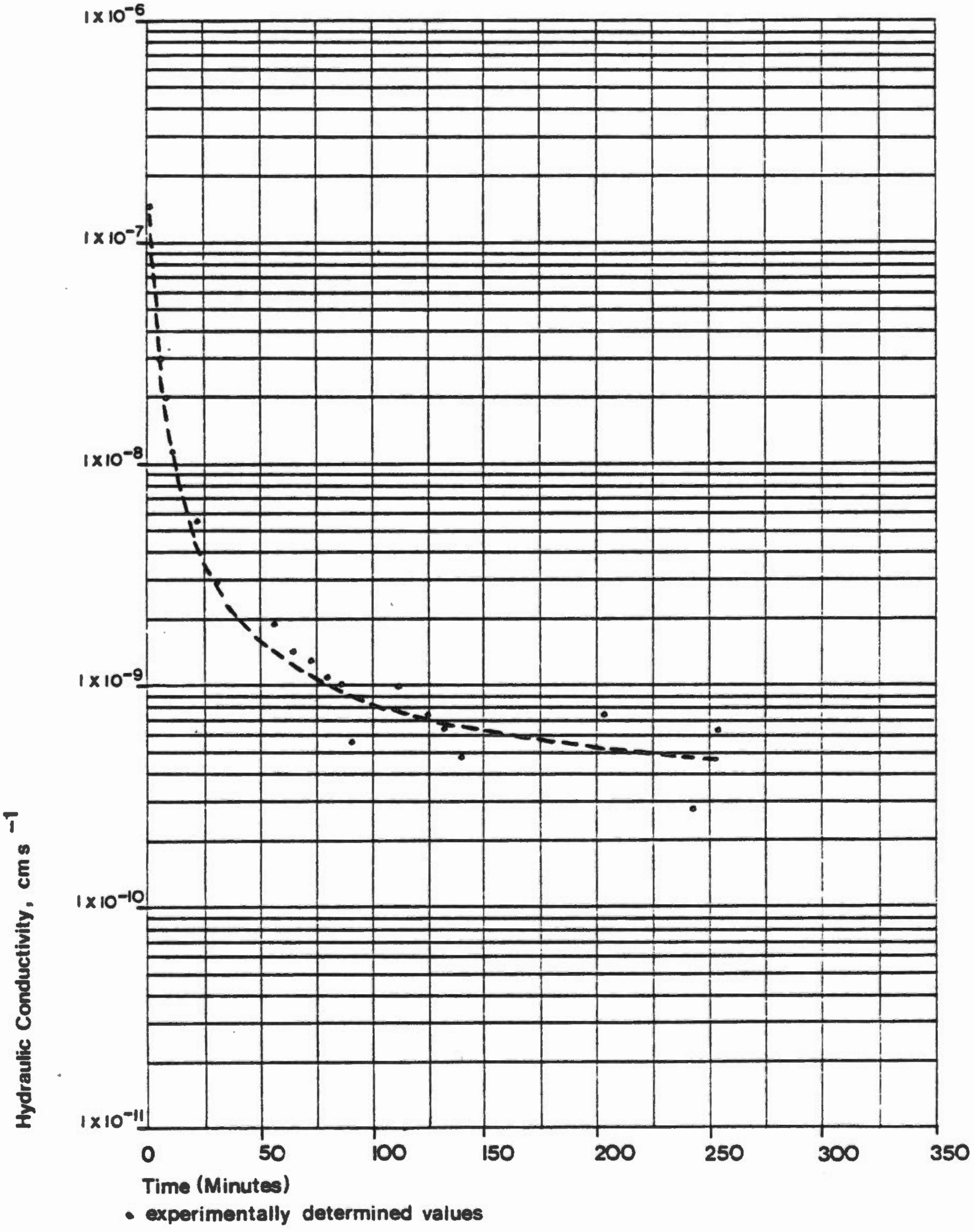
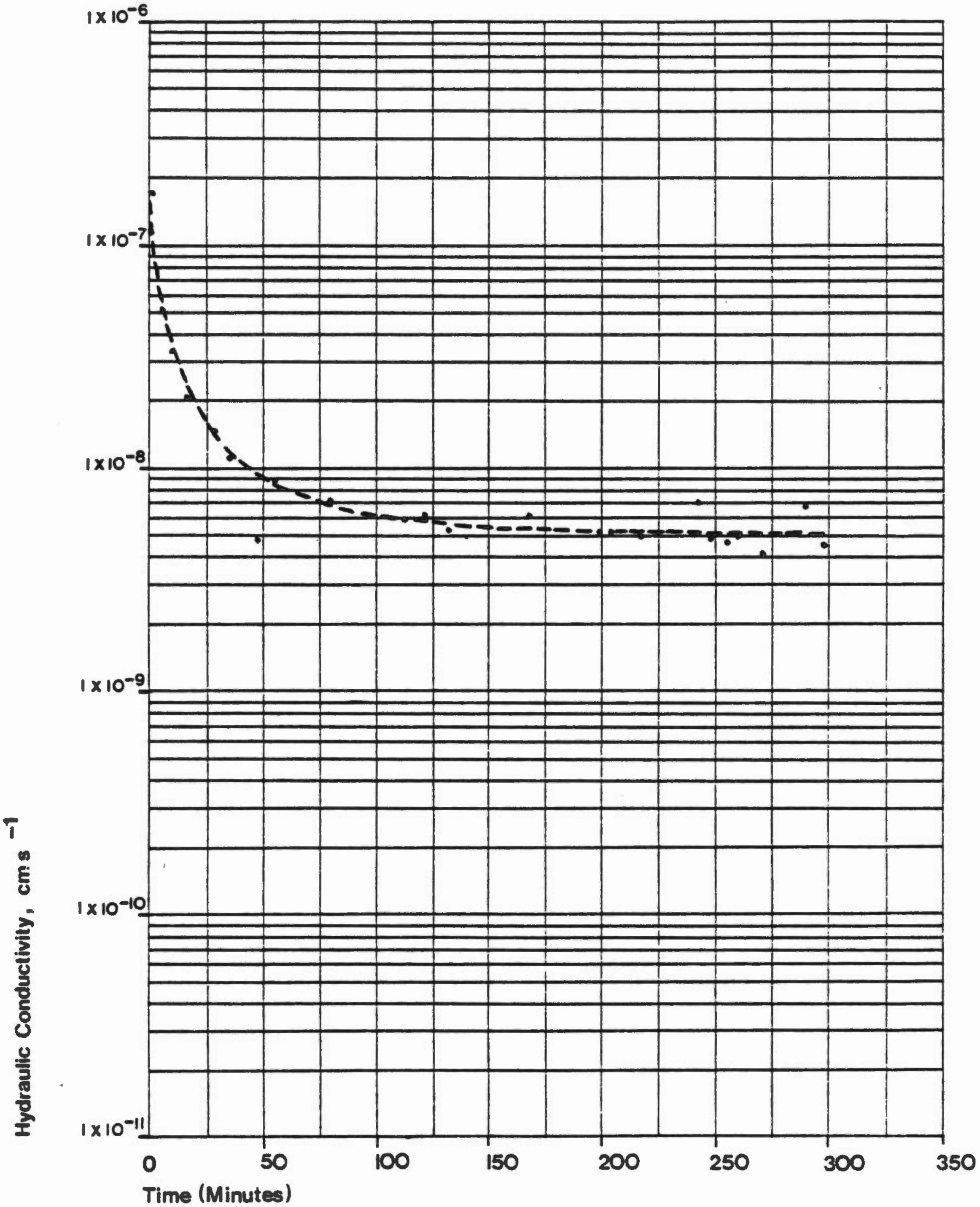


Figure 3.5f
Experiment Number 12

Decline In Apparent Hydraulic Conductivity Of Allendale Silty Clay With Time

Temperature of Soil - 0.115 °C



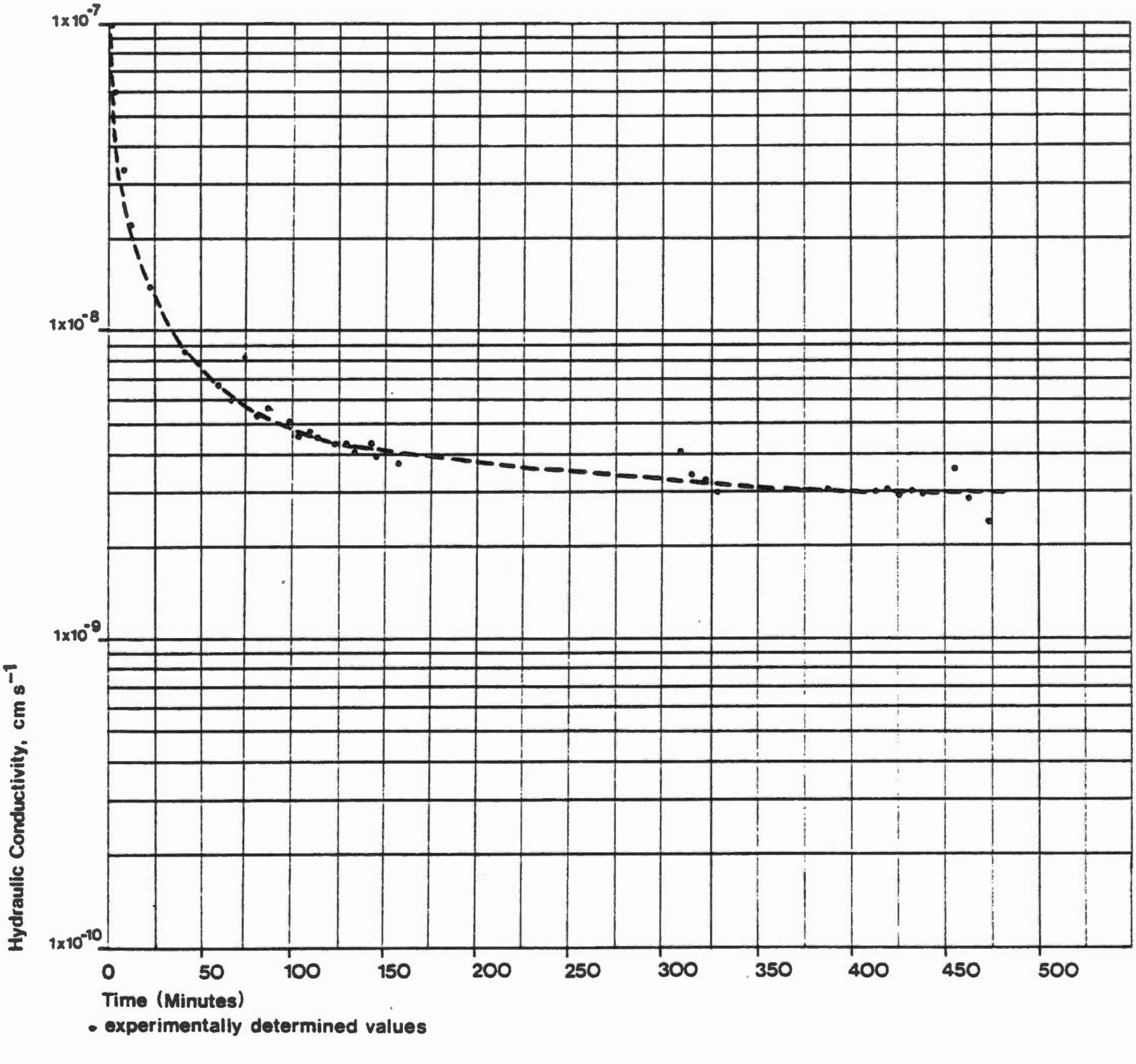
Time (Minutes)

• experimentally determined values

Figure 3.6
Experiment Number F1

Decline In Apparent Hydraulic Conductivity Of Castor Sandy Loam With Time

Temperature of Soil −0.206 °C



• experimentally determined values

the tests, the decline in the permeability of the soil was in the order of 2 to 3 orders of magnitude.

Two of the tests were run for 8 hours to determine whether the flux of water through the soil achieved steady state conditions over an extended period of time. (See Figures 3.4d and 3.6). In both cases nearly constant rates of flow were observed after 5 or 6 hours.

Statistical curve fitting techniques performed on the data indicate that the decline in the apparent permeability of the soil with time can be approximated by a function of the form:

$$k = at^{n} \qquad (3.1)$$

where k = the hydraulic conductivity coefficient, cm s^{-1},

t = the time, min.,

n = the slope of the function,

and a = the intercept

Taking the log of both sides, the equation above becomes:

$$\log k = \log a + n \log t \qquad (3.2)$$

This plots as a straight line on logarithmic graph paper. Examples of this for 3 of the tests performed at -0.2°C are shown in Figures 3.7-3.9. (R denotes correlation coefficients).

Williams and Perfect (1979) suggest a number of possible mechanisms to account for the decline in the flow rates through the soil with time. These include:

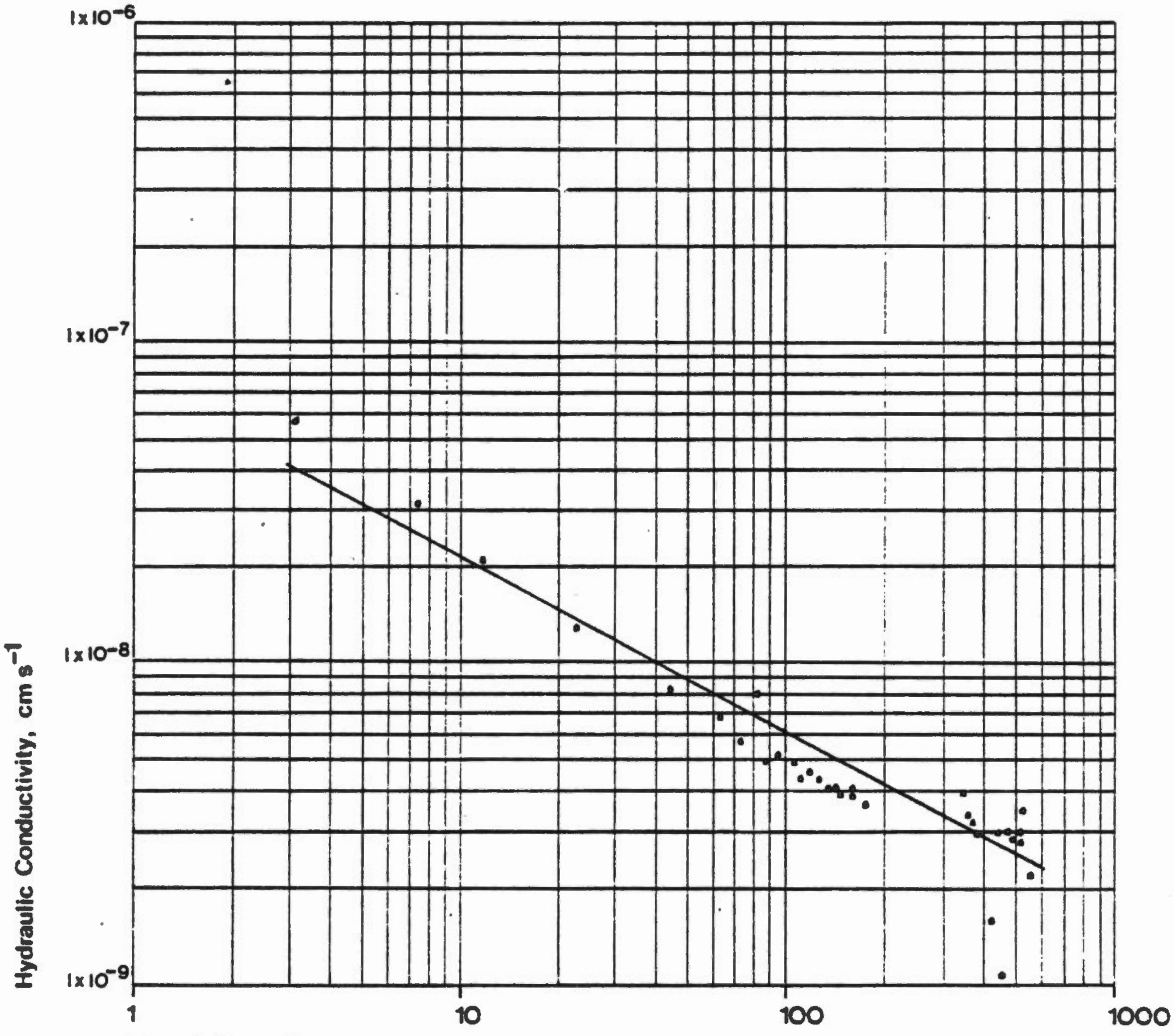
(1) Redistribution of ice within the sample, increasing the thickness of the existing ice lenses.

(2) Consolidation of the soil samples with time.

Figure 3.7
Experiment Number F1

Log-Log Plot Of Hydraulic Conductivity Of Castor Sandy Loam vs. Time
Temperature of Soil - 0.206 °C

$n = -0.56$
$a = 7.70 \times 10^{-8}$
$R = -0.94$



Time (Minutes)

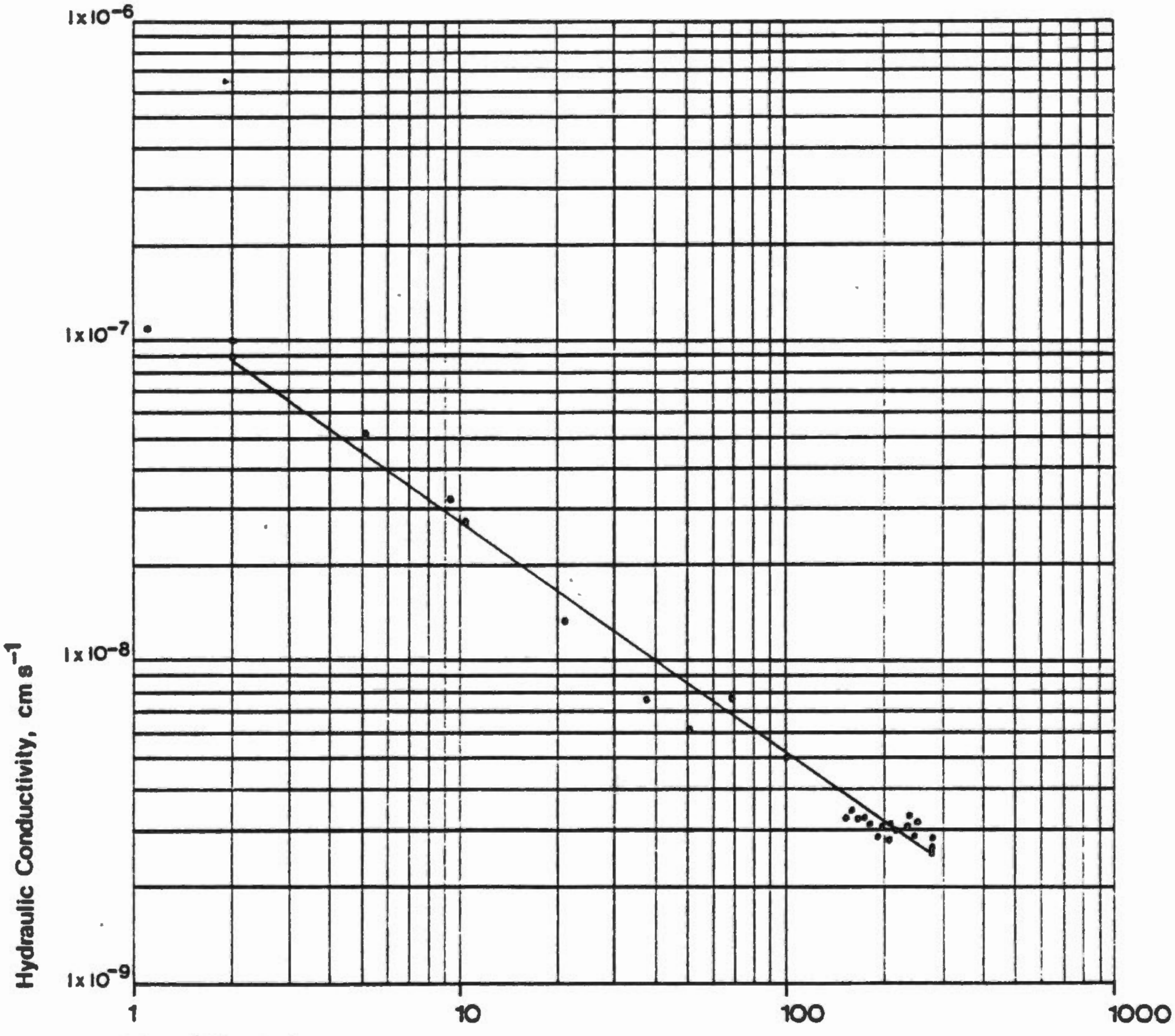
- experimently determined values

Figure 3.8
Experiment Number H1

Log-Log Plot Of Hydraulic Conductivity Of Castor Sandy Loam vs. Time

Temperature of Soil - 0.201 °C

n= -0.73
a= 1.45×10^{-7}
R= -0.99



Time (Minutes)

• experimently determined values

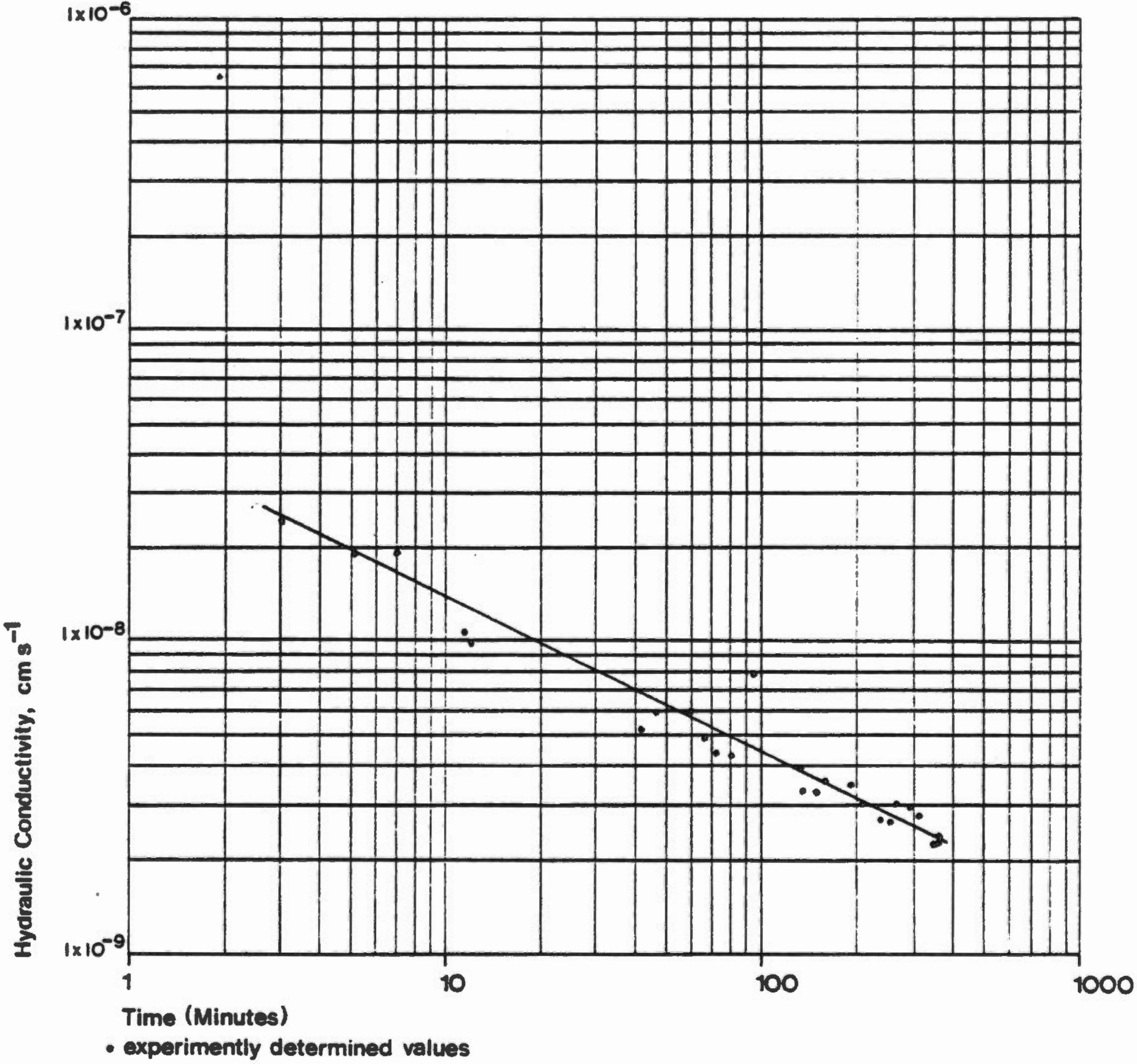


Figure 3.9
Experiment Number I1

Log-Log Plot Of Hydraulic Conductivity
Of Allendale Silty Clay vs. Time

Temperature of Soil - 0.216 °C

n= -0.59
a= 4.50 x 10^{-8}
R= -0.96

(3) Development of a counter osmotic potential gradient caused by concentration of lactose in the inflow reservoir and dilution in the outflow reservoir. This, however, seems to be an unlikely explanation since Burt (1974) found that the reverse osmotic potential represents only 3% of the total hydraulic gradient. This would have an insignificant effect so long as rapid diffusion of the lactose molecules occurred in the end reservoirs.[4]

(4) Development of a pressure induced temperature gradient across the soil sample. As mentioned earlier, this causes freezing on the high pressure (inflow) side of the soil sample and melting on the low pressure (outflow) side. The induced temperature gradient would be in such a direction as to diminish transport through the permeameter. Once the temperature gradient has been established, it takes time for the pore water and pore ice to reach a state of equilibrium since latent heat produced by the thawing or melting of pore ice must be transferred into or out of the soil. Thus, the decline in the permeability of the soil would be governed by the rate of heat transfer to the soil from the surrounding bath of methanol.

3.5 Effect of Temperature on Apparent Permeability of Soils

The effect of temperature on the 'apparent' hydraulic conductivity of the Castor and the Allendale soils is plotted in Figures 3.10 and 3.11 at t=10 minutes and t=250 minutes after the initial application of pressure to the sample. (The dots

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[4] Miller (1970) in his experiments with an 'ice sandwich' "regularily observed a time-decay in the rate of transport when dilute solutions of NaCl and $CaCl_2$ (freezing-point depression = -0.03°C were present in the input chamber at pressures exceeding the osmotic pressure." The initial transport rate was rapidly restored when the input chamber was flushed with fresh solution; a constant rate prevailed if flushing was continuous. He attributes the phenomenon to a "local accumulation of solutes at the input face which tended to be dispersed by mixing and replacement in flushing" (Miller, 1976).

Figure 3.10
Experiment Number H1, H2

Variation In Apparent Hydraulic Conductivity Of Castor Sandy Loam With Temperature At 10 Minutes And 250 Minutes After The Initial Application Of Pressure To The Soil

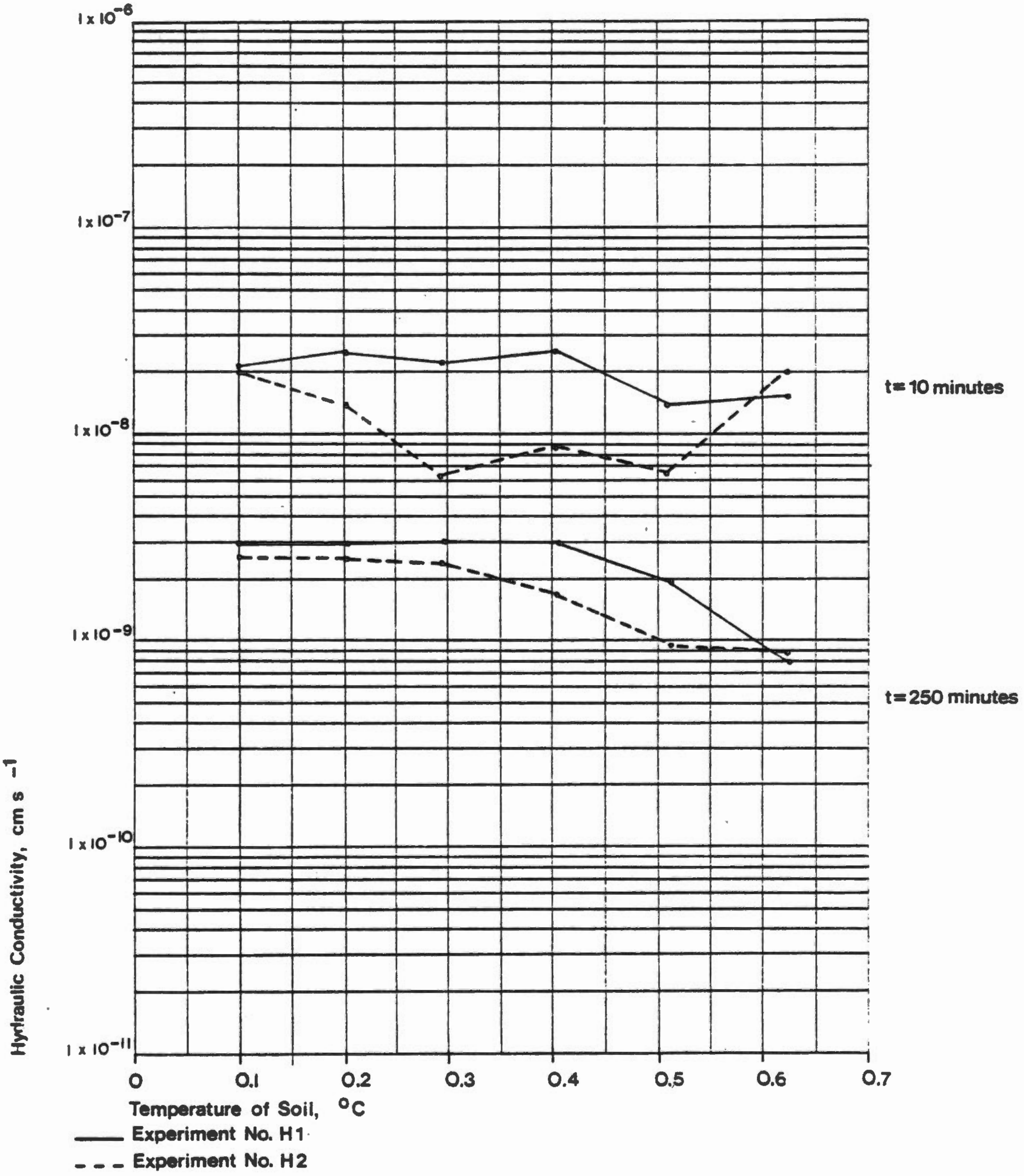
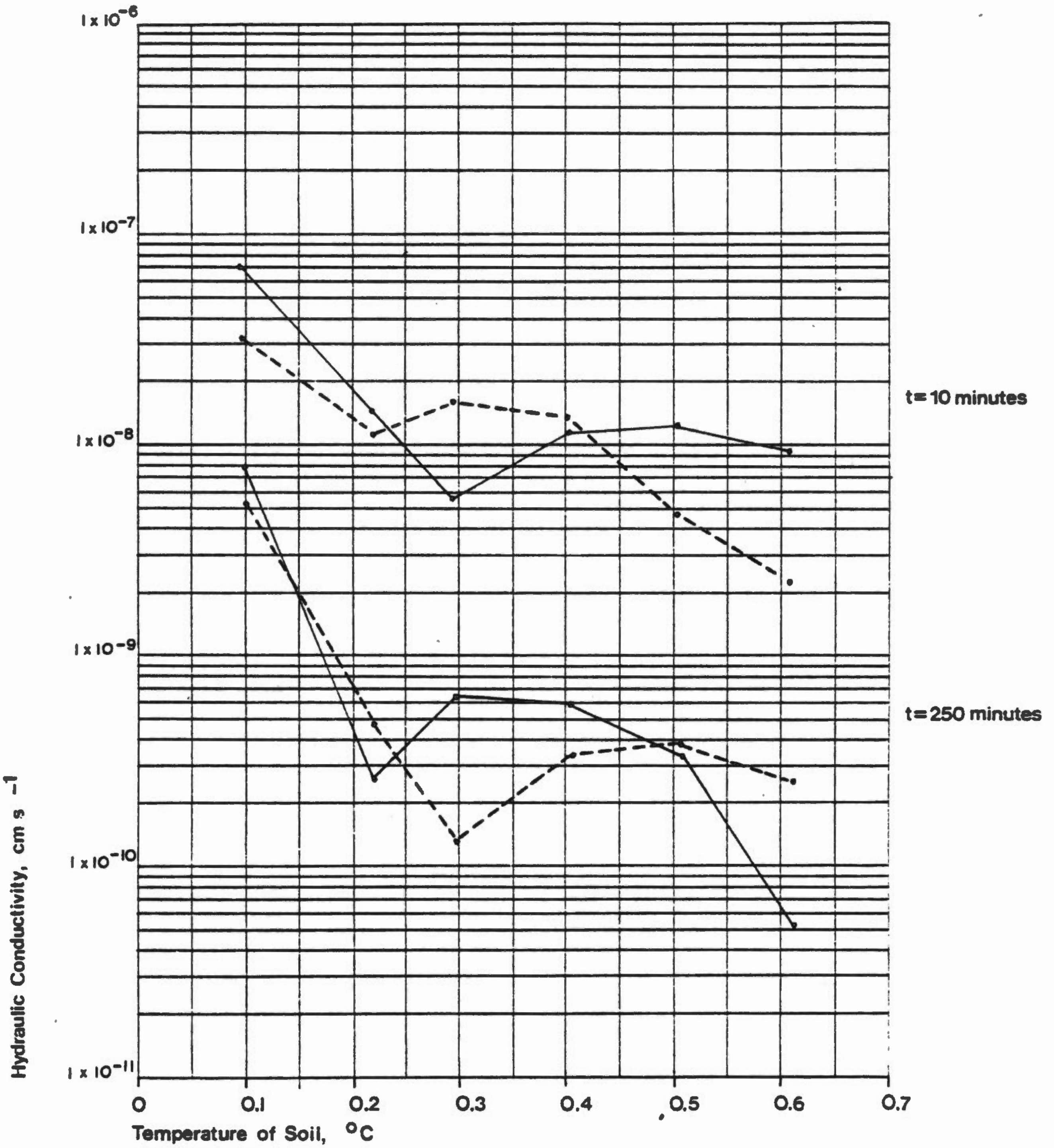


Figure 3.11
Experiment Number 11, 12

Variation In Apparent Hydraulic Conductivity Of Allendale Silty Clay With Temperature At 10 Minutes And 250 Minutes After The Initial Application Of Pressure To The Soil



Temperature of Soil, °C

—— Experiment No. 11

- - - Experiment No. 12

represent values taken from the best fit curves in Figures 3.2-3.5). These two time periods were chosen to determine whether there was any significant change in the temperature dependence of the permeability,as the moisture flux through the soil approached a steady state condition. Comparing curves at t=10 minutes and t=250 minutes, it is evident that there is a slightly greater decrease in the permeability of both soils as steady state conditions are approached (i.e. at t=250 minutes). The explanation for this, however, is unclear at this time.

In general, there appears to be a fair degree of consistency between results obtained with the two samples used in each experiment, individual values at each temperature differing by less than one-half an order of magnitude. The overall decrease in the permeability of the Castor over the indicated time period, was somewhat less than that of the Allendale soil,being about 1 order of magnitude as opposed to 1-2 orders of magnitude for the Allendale.

The most interesting feature of the graphs is that the hydraulic conductivity of the Castor soil shows only a slight decrease with temperature. In contrast, the hydraulic conductivity of the Allendale decreases by about 1-2 orders of magnitude over the indicated temperature range. This suggests that the primary mode of transport in the Castor soil is regelation, which according to Horiguchi and Miller's (1980) findings, is only slightly temperature dependent. With the Allendale soil, although regelation probably represents an important component of movement, the flux of water in the fluid films surrounding the soil particles also appears to form a significantly large component of the total moisture transport within the soil. Thus, the greater temperature dependence of the permeability of this particular soil. These conclusions are further evidenced by the fact that the unfrozen moisture content of the Allendale is roughly twice that of the Castor in the indicated temperature range . Thus, one would expect a much greater component of fluid transport.

3.6 Moisture Flux in Frozen Soils Induced by a Gradient in Osmotic Potential

In addition to using a hydraulic gradient to induce moisture flux through the soil, experiments were also performed in which Burt's apparatus was used as an osmometer. That is, the lactose concentration in the end reservoirs was adjusted so as to produce a gradient in thermodynamic potential between the unfrozen water at the ends of the soil sample and the water in the adjacent reservoirs.[5] The chief advantage with this type of experiment is that it avoids some of the difficulties that are inherent in Burt's experiment, such as pressure induced temperature gradients established within the soil sample.

The samples were prepared in the same manner outlined in Section 3.3. Soil temperatures were kept constant throughout the duration of the experiments by immersion of the permeameters in the methanol bath.

Three different lactose concentrations were used in the end reservoirs, 17.9 gl^{-1}, 94.4 gl^{-1} and 52.9 gl^{-1}. These concentrations produced a difference in potential (or osmotic head) between the end reservoirs and the adjacent soil equivalent to -2440 cm of water, +2470 cm of water and -202 cm of water respectively. Negative values indicate that the potential of the water in the reservoir was greater than that in the soil thereby causing water to flow into the soil. The method of determining the difference in potential between the reservoirs and the adjacent soil is outlined in Appendix D.

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[5] The free energy is often referred to as the thermodynamic potential.

To prevent rapid diffusion of lactose molecules into the soil, dialysis membranes were used in both experiments. The reservoirs were flushed with fresh lactose solution in each experiment and left to equilibriate for 24 hours prior to taking flow readings. Readings were taken at 20 minute intervals for a period of 4 or 5 hours every day. The results are summarized in tabular form in Appendix D.

Figure 3.12 shows the cumulative inflow and outflow caused by large osmotic heads at both ends of the soil sample. As expected, outflow occurred at the end of the sample with the more concentrated lactose solution while inflow occurred at the end with the lower concentration. Both the inflow and the outflow showed a nearly linear increase with time which indicates a constant rate of moisture flow through the soil.[6] The outflow, however, exceeded the inflow by about one-half an order of magnitude. This phenomenon was also observed by Perfect in his thermal osmosis experiment. In this case, however, the discrepancy may have been due to partial freezing of the porous plate in the inflow reservoir since the concentration of lactose in this reservoir (17.9 gl^{-1}) was much less than the amount required to prevent freezing at $-0.30^{o}C$.[7] (55.9 gl^{-1} of hydrated lactose are required to produce a freezing-point depression of $-0.30^{o}C$).

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[6] It should be pointed out that, this result disproves Miller's (1976) theory that, the development of an counter osmotic potential gradient caused by the concentration of lactose in the inflow reservoir and dilution in the outflow reservoir, acts to reduce the rate of flow over time.

[7] It should be mentioned that, in some experiments (Expt. 2D Jan 7 and Expt. 3B Jan 7), the discrepancy between the inflow and outflow was much less than that indicated in Figure 3.14. No ice was observed to form in the inflow reservoirs during these experiments.

Figure 3.12
Experiment No. 2A

Cumulative Inflow–Outflow Induced By An Osmotic Gradient At Both Ends Of Soil Sample

Temperature of Soil – 0.30°C

Allendale Silty Clay
Date: Jan. 3, 1981

	Res.1	Res.2
Lactose Concentration (g/l)	17.9	94.4
Osmotic Head (cm. H_2O)	-2444	+2470

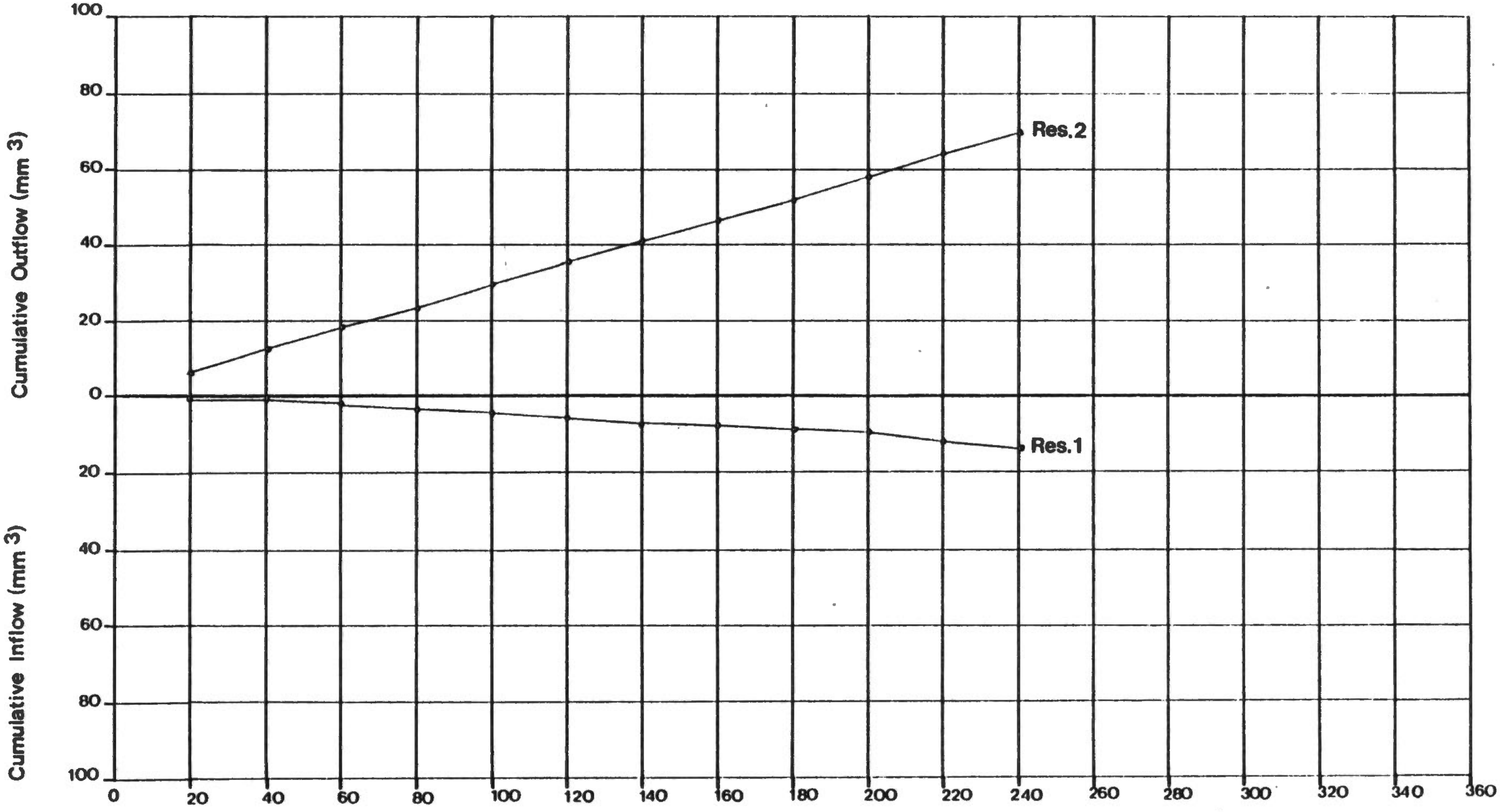

Figure 3.13
Experiment No. 2C

Cumulative Inflow-Outflow Induced By An Osmotic Gradient At One End Of Soil Sample

Temperature of Soil - 0.30°C

Allendale Silty Clay
Date: Jan. 5, 1981

	Res.1	Res.2
Lactose Concentration (g/l)	52.9	94.4
Osmotic Head (cm. H_2O)	-202	+2470

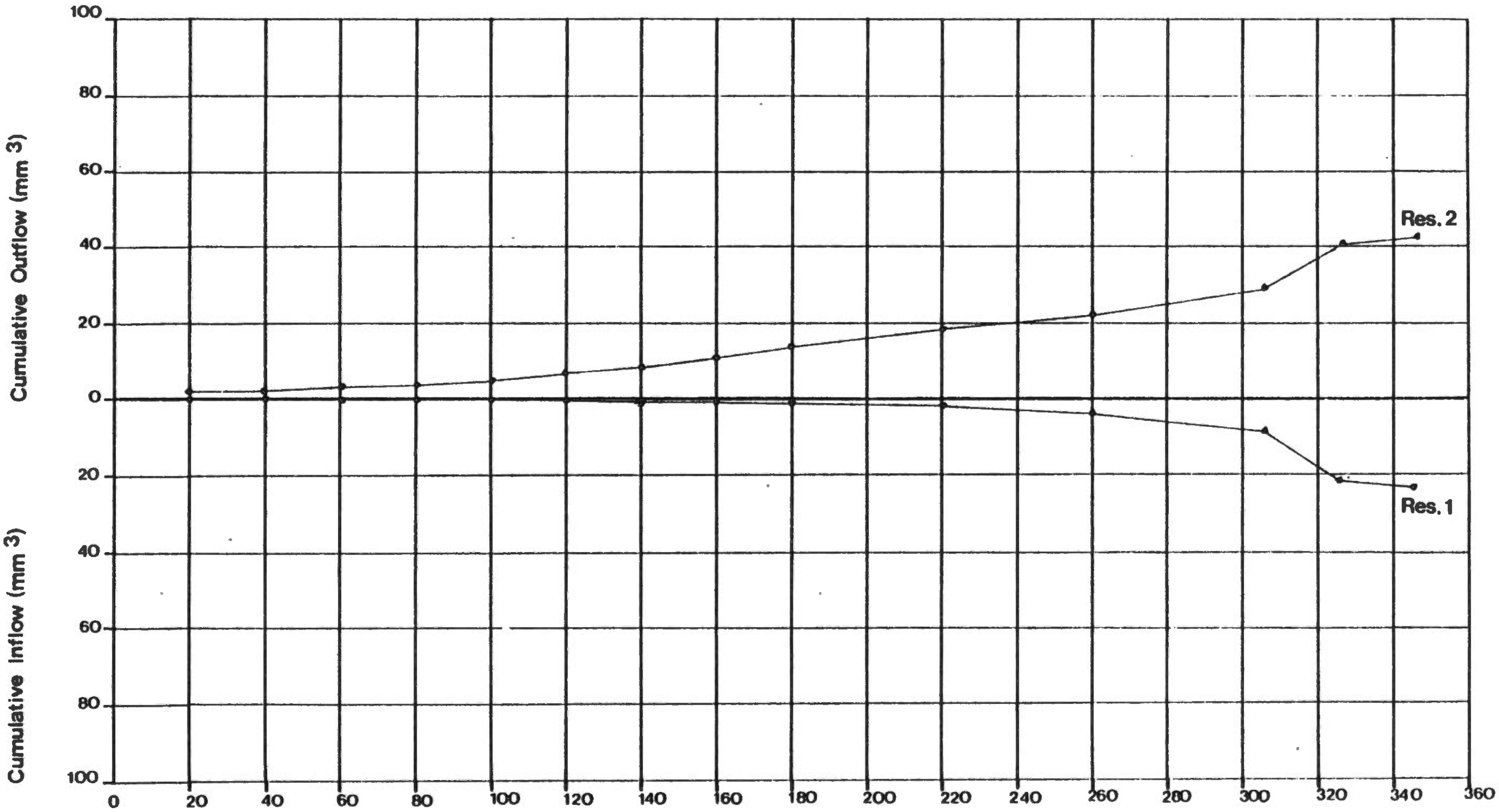

Figure 3.14
Experiment No. 3B

Cumulative Inflow–Outflow Induced By An Osmotic Gradient At One End Of Soil Sample

Temperature of Soil – 0.30°C

Allendale Silty Clay
Date: Jan. 5, 1981

	Res.1	Res.2
Lactose Concentration (g/l)	94.4	52.9
Osmotic Head (cm. H_2O)	+2670	-202

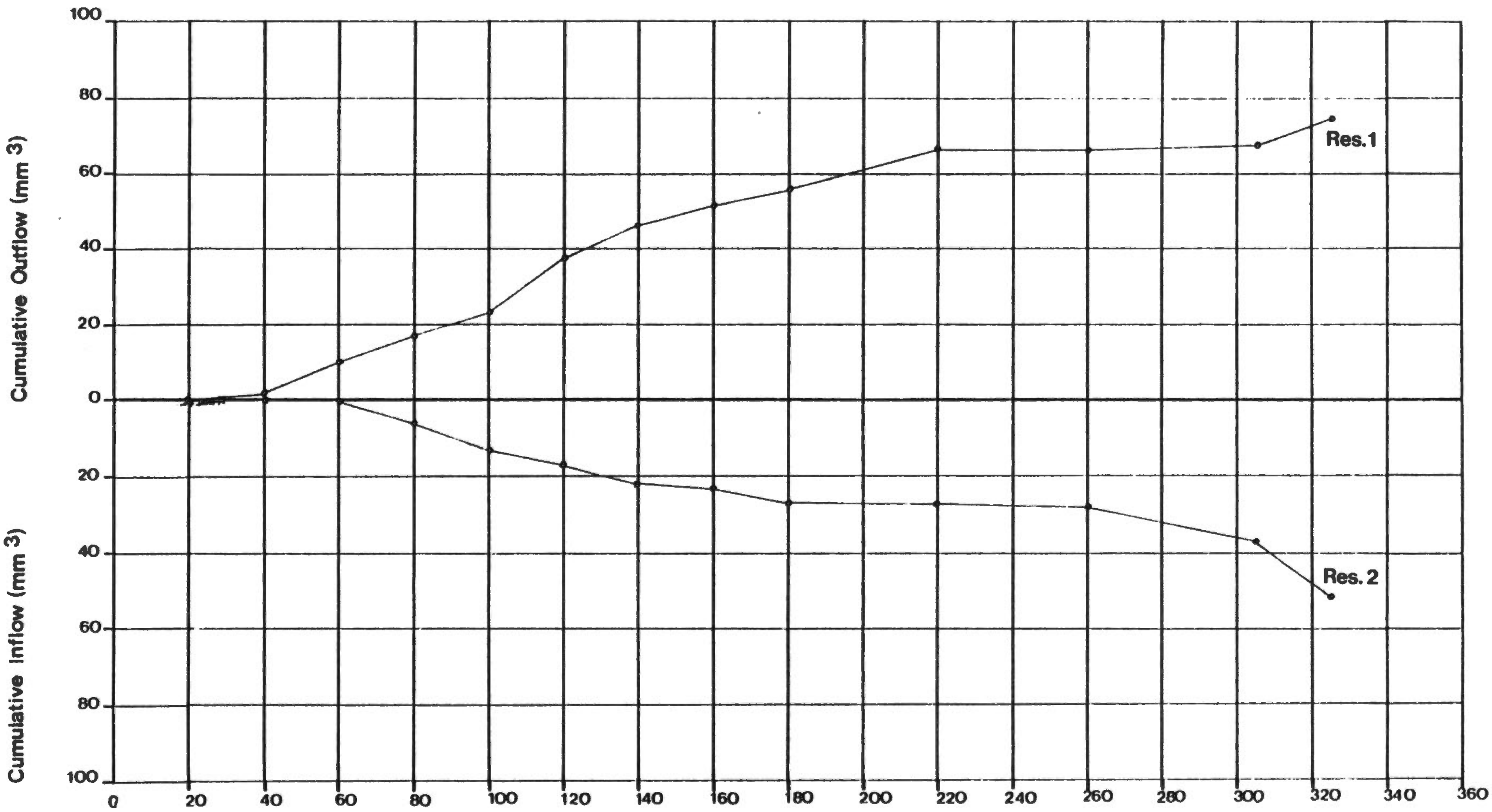

Figure 3.15
Experiment No. 3A

Cumulative Inflow – Outflow With Equilibrium Lactose Concentration At Start Of Experiment

Temperature of Soil – 0.30°C

Castor Sandy Loam
Date: Jan. 4, 1981

	Res.1	Res.2
Lactose Concentration (g/l)	52.9	52.9
Osmotic Head (cm. H_2O)	−202	−202

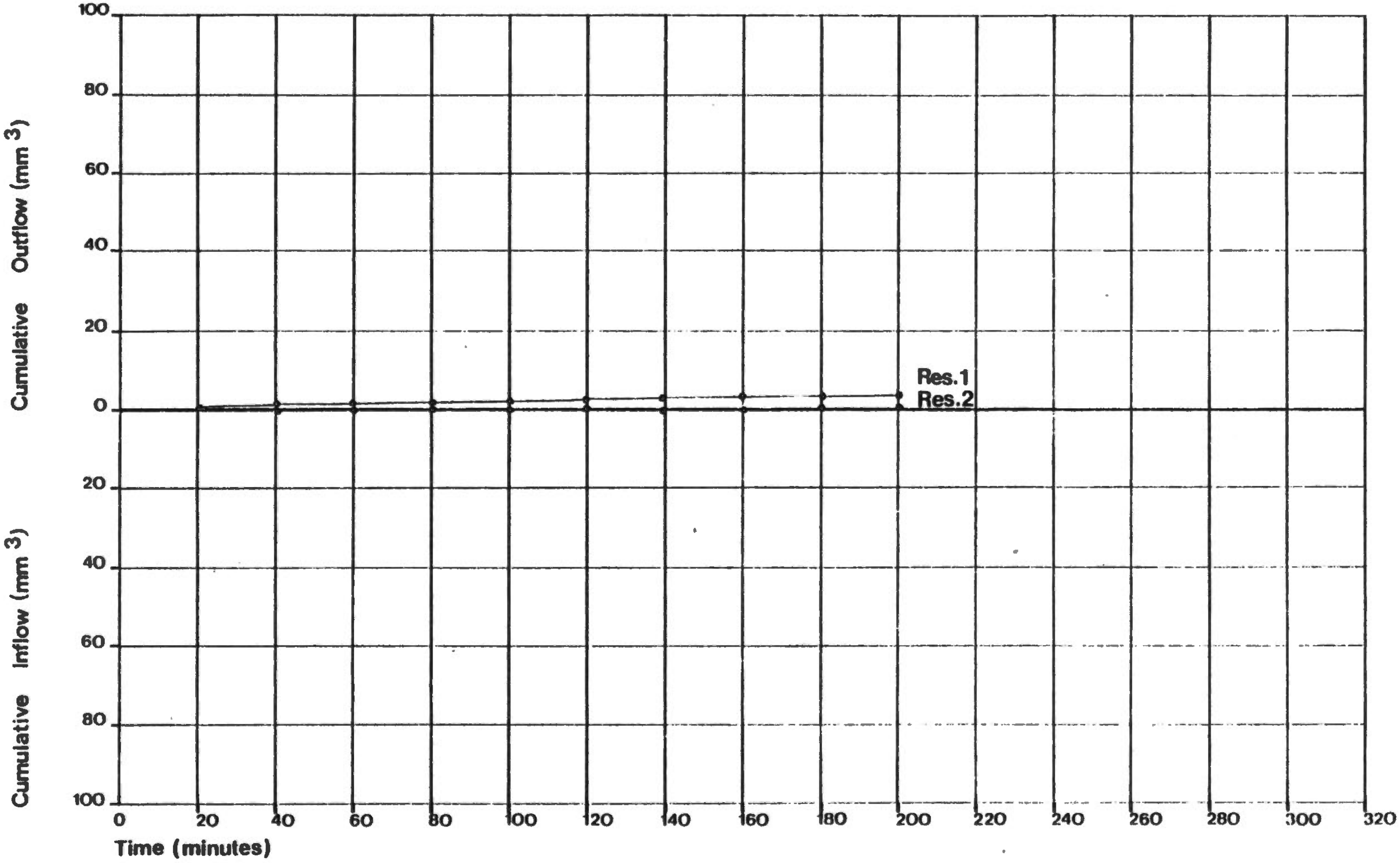

Figure 3.16
Experiment No. 3C

Cumulative Inflow – Outflow With Equilibrium Lactose Concentration 4 Days After Start Of Experiment

Temperature of Soil −0.30°C

Castor Sandy Loam
Date: Jan. 8, 1981

	Res.1	Res.2
Lactose Concentration (g/l)	52.9	52.9
Osmotic Head (cm. H_2O)	−202	−202

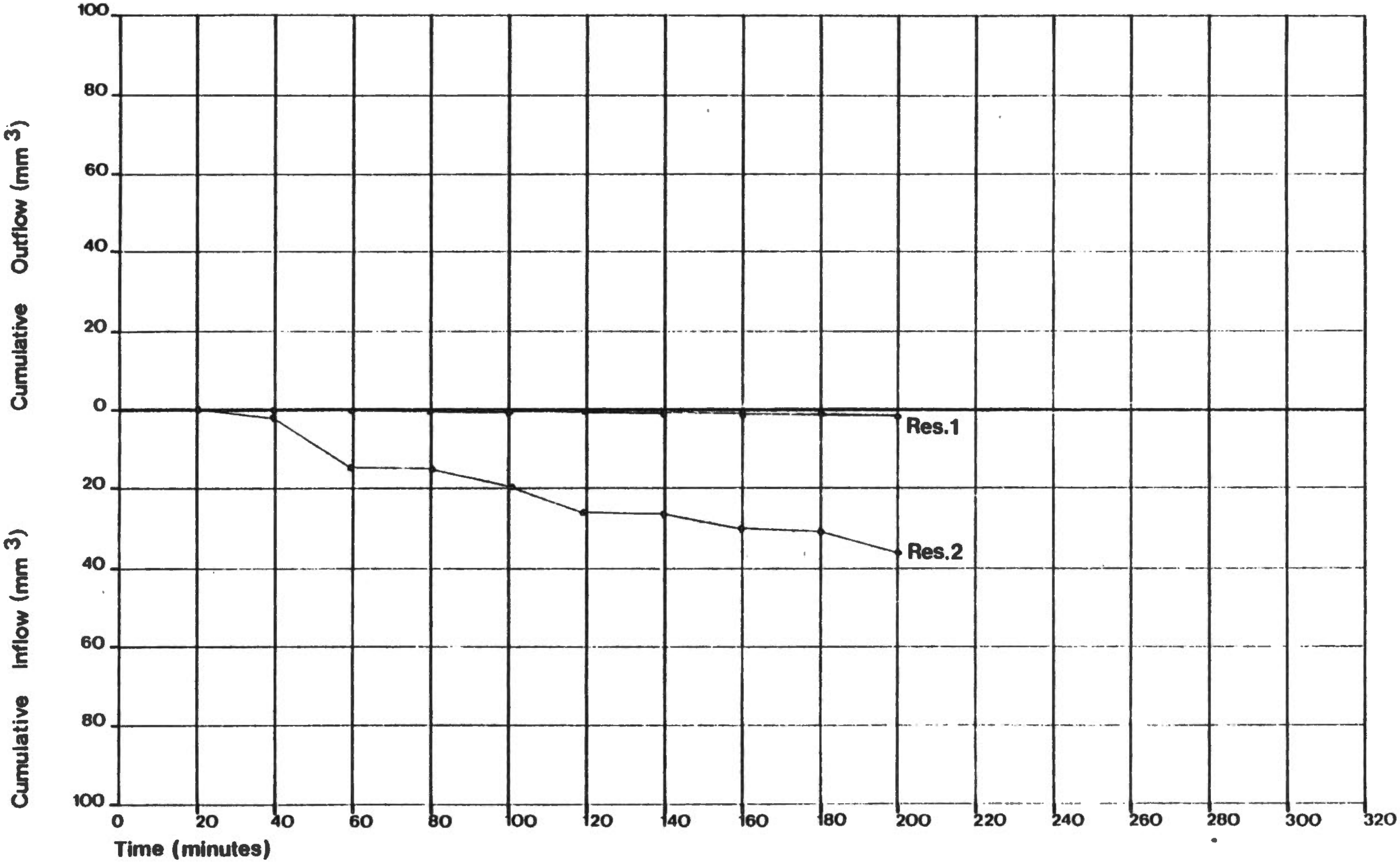

Figure 3.13 shows a plot of the flow through the Allendale soil when the lactose concentration in one reservoir (Res. 1) was adjusted so as to produce equilibrium with the adjacent soil (52.5 gl^{-1}). That is, both the inflow and the outflow even produced almost entirely by the osmotic head generated at the reservoir with the more concentrated lactose solution (94.4 gl^{-1}). The importance of this result is that it demonstrates that events occurring at one end of the system do, in fact, influence events occurring at the other end. (This question had been raised by Perfect in his Masters thesis).

During the first three hours of the experiment, there was a net loss of moisture from the soil, no inflow occurring while the outflow steadily increased. Thereafter, the inflow increased rapidly. Explanation for the inflow is quite simple.

Dessication of the sample resulted in a decrease in the thermodynamic potential of the water retained within the soil.[8] (This is expressed as an increasing suction). This disturbed the equilibrium between the soil and the first reservoir (Res. 1),resulting in inflow.

The most interesting feature of the graph is that changes in the rate of inflow correspond almost exactly with changes in the rate of outflow. (This effect is also illustrated in Figure 3.14). This indicates that changes in the potential at one end of the system are communicated over the length of the sample almost immediately.

Unfortunately the experiment was not run long enough to determine whether the inflow eventually equalled or exceeded the outflow. However, barring the possibility of freezing in the reservoir, it is likely that

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[8] The free energy or thermodynamic potential of the water retained within the soil is zero at saturation and negative when less than saturation. Thus, a decrease in the free energy of the water in the soil implies that it is becoming more negative as the saturation ratio decreases.

this does in fact occur otherwise large suctions would develop within the soil.

One experiment was performed in which a lactose concentration of 52.9 gl^{-1} was used in both reservoirs. (See Figure 3.15). This should, in effect, produce equilibrium between the soil and the adjacent reservoirs.[9] On the first day of the experiment (Jan. 4),the observed rates of flow were found to be almost negligible indicating that the potential of the water in the reservoirs and in the soil is more or less identical. However, 4 days later, the equilibrium between the soil and reservoir 2 appears to be very much disturbed. (See Figure 3.16). This is caused by the diffusion of lactose molecules through the dialysis membranes, thawing a thin layer of soil at this end of the sample. The inflow probably arises from the fact that the potential of the water in the frozen section of the sample is lower than that in the thawed section and so water is drawn into the frozen section. The lack of inflow in reservoir 1 appears to be a result of an ice lens approximately .5 mm thick which formed at that end of the sample. As demonstrated by Miller (1970) in his ice sandwich experiment, an ice lens behaves as a very effective barrier for solutes and so no lactose would be able to penetrate the soil at this end of the sample.

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[9] Ideally a concentration of 52.9 gl^{-1} in the end reservoirs would produce equilibrium with the adjacent soil, if annhydrous lactose was used (molecular wt. 342.3). However, owing to the fact that the lactose was in hydrated form (molecular wt. 360.3) a small gradient in potential equivalent to -202 cm of water was generated between the reservoirs and the soil. The concentration of lactose required to produce equilibrium should have been 55.6 gl^{-1}.

SECTION 4

Investigation of Pressure Changes in Frozen Soils During Temperature Induced Moisture Migration

4.1 Apparatus Design

This section of the report deals with the experimental determination of pressure changes in frozen soils during thermal osmosis. The apparatus used in the experiments is similar to that used by Perfect except that a number of modifications have been made to the original design. Modifications to the perspex sample holder are described below.

(1) The addition of two Kulite VQS-250 series pressure transducers (rated pressure 50 psi and 100 psi spaced 1.20 cm apart. This arrangement not only enables the measurement of pressure changes within the frozen soil but also pressure gradients as well. This is an important consideration when determining the magnitude and direction of moisture movements in the soil. (Test reports for both instruments are shown in Appendix H).

(2) Changes in thermistor housings: The measurement of temperatures and temperature gradients within the sample is an important aspect of the experiment ,since moisture flow is coupled with heat flow through the soil. However, in order for pressure to be maintained within the soil sample, it is necessary that the thermistors be separated from the sample holder by a pressure tight seal. This has proven to be a more difficult problem than was originally anticipated. The difficulty lies in the fact that the seals must not conduct excess heat into or out of the system during the experiment, since this will disturb the temperature gradient across the system. A number

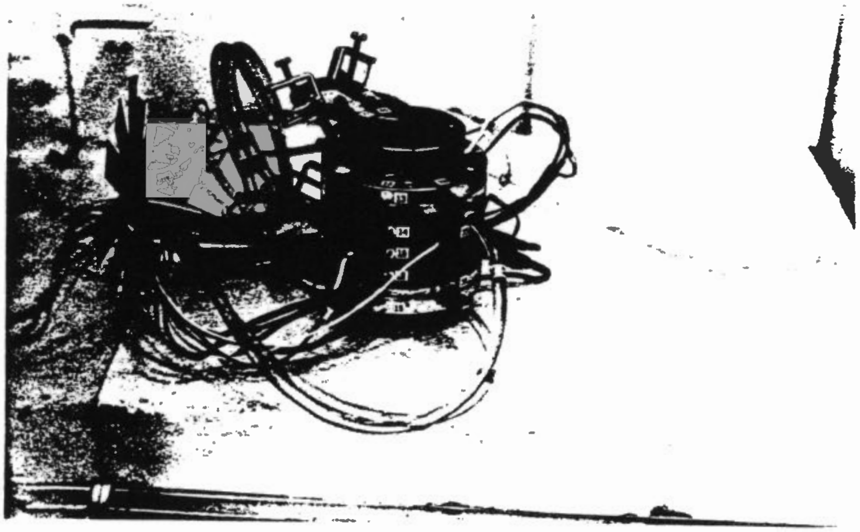
of different designs have been considered. However, most of these have been rejected owing to excessive heat conduction. At present a design is being tested in which the thermistors are inserted into thin brass tubes mounted along the inner wall of the sample holder and extending several millimeters into the soil. If this design proves successful the apparatus will be further modified to accommodate these changes.

Perfect's experiment was repeated a number of times in order to gain familiarity with the apparatus used to measure thermally induced moisture migration in frozen soils (see results Appendix E). Running the experiment revealed a number of difficulties with the apparatus and experimental procedure used by Perfect. Because of this, a number of additional modifications have been made to the apparatus presently being used. These are described below.

(1) To inhibit nucleation when super cooled water (or lactose solution) is used in the experiments, a teflon coating has been applied to the reservoirs in the aluminium end plates. Porous teflon plates are also being used to hold the membranes in place, instead of the porous brass or aluminium plates used by Perfect. Since teflon (a flurocarbon) is a water repellant substance, it is believed that this precaution will reduce the possibility of water freezing to the walls of the reservoirs during the experiments.

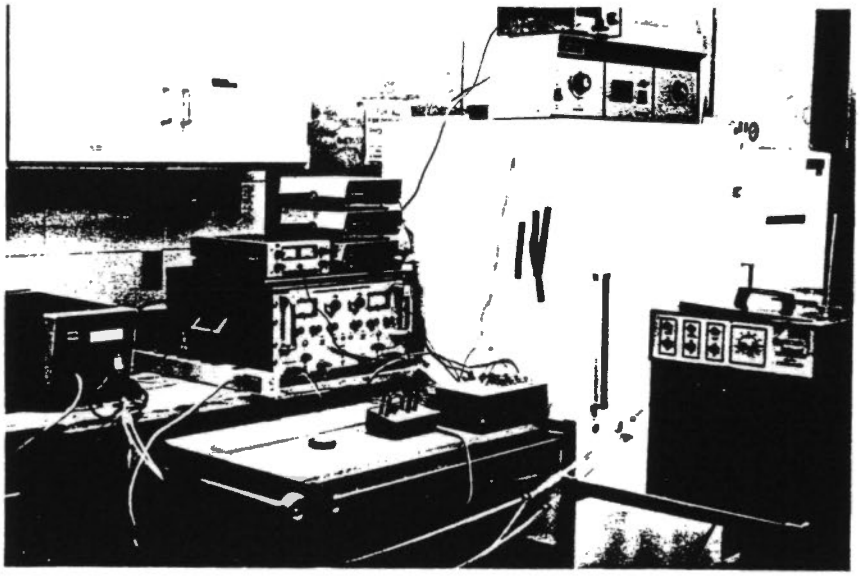
(2) A second modification involves minimizing the exposure to room temperatures of the tygon tubing connecting the capillaries used to measure inflow and outflow to the reservoirs. The reason for this precaution is that during tests, large numbers of bubbles were observed forming in the section of tubing extending outside of the

Plate 4.1



Apparatus used to measure pressure changes in frozen soils during thermal osmosis. (Insulating jacket not shown). Pressure transducers with reference tubes and thermistors are inserted in the perspex wall of the sample holder. Tubing in the foreground leads from the reservoirs in the end plates to capillaries used to measure inflow and outflow. Tubing in the background leads from the brass heat sinks to the bath circulator.

Plate 4.2



General view of the experiment: Incubator containing the apparatus in the background. Thermoelectric cooling unit, multimeters and power supply for pressure transducers next to incubator. Bath circulator on the extreme right and voltage regulator on the extreme left. Capillary tubes and switching units for thermistors and pressure transducers on the table in the foreground.

incubator. The bubbles are believed to come from air dissolved in the water during the preparation of the lactose solution. Expansion of the bubbles could cause significant errors in the flow readings, increasing the outflow and decreasing the inflow. This problem has been largely eliminated by shortening and insulating the tubing extending outside of the incubator. In addition to this, lactose solution is now prepared from an actively boiling (deaired) reservoir of water.

(3) Additional precautions have also been taken to ensure that the sample remains frozen during the assembly of the apparatus. The assembly is done in two stages and usually takes about one-half hour to complete. In some instances, this was sufficient time for melting to have occurred within the soil. Melting of the soil hastens the diffusion of lactose molecules through the dialysis membranes in the early part of the experiment and disturbs the equilibrium between the sample and the end reservoirs. To avoid this problem, the apparatus is placed back in the freezer for about one-half hour after the first stage in the assembly is completed.

(4) The long term stability of temperature control for the system has also been improved by substituting a 'Hotpack' constant temperature bath circulator, for the tap water used to cool the brass heat sinks in thermal contact with

the 'Hot' side of the Peltier molecules. Normally an experiment runs for 2 or 3 days, during which time, the temperature of the tap water may change by several degrees. It was discovered that even relatively small changes in the temperature of the water were sufficient to cause the Peltier modules to seek a new equilibrium temperature. Overnight changes in the tap water temperature frequently resulted in a gradual drift in the temperature gradient across the system from the desired setting. This disturbed the equilibrium of the system, producing local osmotic gradients between the reservoirs and the adjacent soil. The use of a constant temperature bath circulator enhances the stability of the system and helps to maintain steady-state heat flow conditions across the soil for long periods of time.

(5) Fluctuations in the line voltage of the building during an experiment can cause significant changes in the output from the multimeters used to measure pressure and temperature. To minimize this problem all pressure and temperature sensing instruments are now connected to a 120 volt, 8.3 amp Sola Basic voltage regulator.

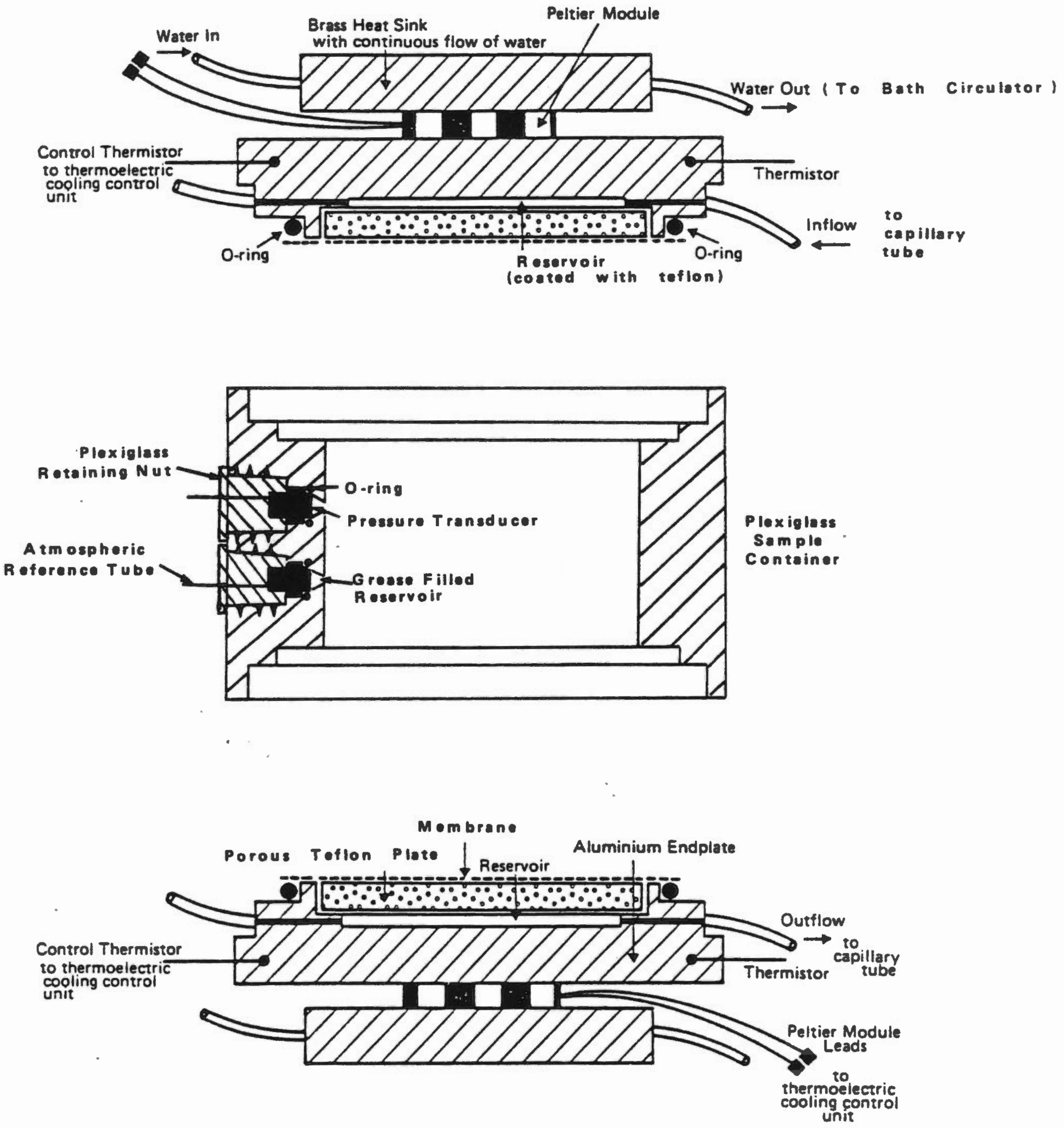
4.2 Operation of Pressure Transducers

The pressure transducers are mounted in the perspex wall of the sample holder and are separated from the soil by small reservoirs filled with silicon grease. The reservoirs are approximately 0.5 cm wide at the base of the transducers and taper to a hole 0.1 cm in diameter at the inner wall of the sample holder. (See Figure 4.1) Each transducer is held in place by a perspex retaining nut which screws into the wall of the sample holder. A small O-ring maintains a pressure tight seal around the pressure transducer. To accommodate these changes, the sample holder had to be extended slightly in length from 3.28 cm in the original design, to 3.50 cm in the present apparatus. The transducers were also offset from each other in the sample holder, since placing them in an aligned position would have required further extension of the test cell. This would not only reduce the rate of moisture flow through the soil, it would also reduce the magnitude of the temperature gradients which could be established across the system.

The transducer circuit itself is fairly simple, consisting of a wheatstone bridge and a zero shift temperature compensating module mounted on a thin silicon diaphram which acts as the pressure sensitive area. Operation of the transducers requires external excitation from a constant voltage source. (10 V d.c. is used). This is provided by a Hewett Packard Model 6205B d.c. power supply. Output from the transducers is given in millivolts, which can be readily converted into units of pressure by a simple conversion factor. The output leads are connected via a switching unit to a Keithley 1191 multimeter.

The pressure transducers are referenced to the atmosphere by small tubes extending out the back of the steel

Figure 4.1



CROSS-SECTION OF THE EXPERIMENTAL APPARATUS.

casing which protects the circuit. (See Plate 4.1). Absolute values of pressure within the soil are obtained by adding the value indicated by the transducer to the atmospheric pressure in the laboratory. The atmospheric pressure reading is obtained from a mercury barometer mounted on the wall of the laboratory.

One difficulty with the operation of the pressure transducers which had not been anticipated, is that the output failed to stabilize even over quite long periods of time. It is suspected that this may be due to heating effects caused by excitation of the transducer circuit. The heat Q in Joules, developed by an electric circuit, with current I flowing through a resistor R for t seconds is given by:

$$Q = RI^2t \qquad (3.3)$$

For a period of 5 seconds the heat produced by a transducer with an input impedance = 890 Ω and I = 0.34 amps, is 545 Joules which is sufficient to melt about 2 grams of water. This equation, however, assumes that all of the electrical energy in the circuit is turned into heat energy, which is not the case. Usually, a temperature is reached, at which point the rate of emission of heat from the surface of the resistor is equal to its rate of generation. Also, not all of the heat produced by the circuit will be transferred to the soil. Thus, one would expect Q to be somewhat lower than the value indicated above.

One method of testing to see if heating effects are responsible for the drift in output from the pressure transducers is to insert a small thermistor in the grease filled reservoirs and observe any changes in temperature that occur when power is supplied to the transducer circuit. However, for the time being, to minimize any possible heating of the soil during the experiments, power is applied to the pressure transducers for only a brief period (about 5 seconds) when the readings are taken. A switching unit enables the power supply to be left on while the pressure transducers are switched off.

4.3 Results and Discussion

Unfortunately, owing to lengthy delays in the shipping and receiving of the pressure transducers, the apparatus was not completed until May of this year.[1] Once the apparatus had finally been constructed a number of difficulties were encountered which are fairly typical in experimental work of this nature. For example, in one trial the thermoelectric cooling unit failed. In another test,a teflon screw broke off during assembly of the experiment.

Results of a preliminary experiment using the new

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[1] The pressure transducers were received in the fall of 1980. Following this, it was discovered that they would have to be returned to the manufacturer to have a special protective coating applied to the diaphragm, otherwise damage would result to the circuit from contact with the silicon grease in the reservoirs. Much of the delay was caused by customs, who are required to inspect all electronic components entering the country and are apparently understaffed for the job.

apparatus are shown in Appendix F. The sample (Castor sandy loam) was prepared in the manner discussed in section 3.3. On assembly, the 'warm' plate was brought to a temperature of -0.10°C while the 'cold' plate was maintained at -0.20°C. Temperature of the plates remained constant to within ± -0.05°C of these values. Lactose concentrations of 17.9 $gl^{-1}$and 35.8 gl^{-1} were used in the 'warm' and the 'cold' reservoirs.

Owing to the relatively warm temperatures at which the experiment was run Sartorius nitrocellulose bacterial filters (pore size 0.05 μm) were used in both reservoirs, rather than dialysis membranes. The filters serve as an effective phase barrier,since at 0.05 μm pore size, the temperature would have to be lowered to -0.999°C for ice to enter the pores. The filters also have a hydraulic conductivity of 2.5×10^{-2} cm s^{-1} which is considerably greater than that of the soil and so do not impose a serious impediment to moisture flow through the system. Initially outflow occurred from both reservoirs, the volume being 4 times greater in the 'warm' reservoir than in the 'cold' reservoir. Later the flow direction reversed in the 'cold' reservoir and a small amount of inflow occurred.

At the start of the experiment,the pressure of the soil sample was very high,being about 2 atmospheres (absolute pressure). Thereafter,the pressure decreased over a period of about 3 hours to a fairly constant value near 1 atmosphere (absolute pressure). This may be due to the fact that the soil was very cold at the start of the experiment, since the sample had only recently been removed from a freezer (at -18°C). As mentioned earlier, although temperature gradients are rapidly established within the soil, it takes time for the pore water and the pore ice to reach a state of equilibrium since latent heat produced by the melting of the pore ice must be exchanged with the surroundings. This is a fairly slow process, owing to the fact that the sample is well insulated and subjected to only a small temperature gradient.

One interesting finding is that, throughout the duration of the experiment, pressures were observed to be greater at the 'warm' end of the sample than at the 'cold' end (about 10 kPa difference). (Note that at the 'cold' end of the system,the pressures were often slightly less than atmospheric). This result appears to be in keeping with a recent hypothesis by Prof. R.D. Miller at Cornell University. Miller (1978) indicates that the pressure generated by a frozen soil represents a combined reaction of the pore ice and the pore water operating on the soil particles. The effect is described by a term called the effective stress. (A discussion of the effective stress is provided in our Research Proposal 'Investigation of Pressure Changes in Frozen Soils During Secondary Frost Heaving' submitted April of this year). Miller (1978) also proposed that,in a soil subjected to a temperature gradient, the effective stress will be greater at the 'warm' end of the soil than at the 'cold' end. However, whether the pressure transducers in the present experiment are actually measuring the effective stress is difficult to determine at this time.

During the second day of the experiment, inflow was observed at the 'warm' end of the soil and outflow at the 'cold' end. However, despite the fact that there was a net loss of moisture from the soil, the outflow being somewhat greater than the inflow, the pressure of the sample remained fairly constant changing by only 3kPa over a 6 hour period.

Although preliminary results are encouraging, further refinements to the experiment are needed before any definitive conclusions can be made. Work is presently underway on modifications to the apparatus, to allow the measurement of temperature within the soil sample as well as pressure. It may also be necessary to redesign the apparatus to eliminate the effects of heat generated by the pressure transducers, if this proves to be a problem. Tests will also be done to compare the use of osmotic potentials, to establish intial thermodynamic equilibrium between the water in the reservoirs and in the frozen soil sample, with the use of supercooled water. A further modification to the apparatus will allow the application of back pressures to the outflow reservoir sufficient to stop the flow. This will provide an indication, in mechanical terms, of the temperature induced driving forces across the system. The experiments are expected to continue throughout the summer and into the fall. It is hoped that the results will enable testing of the validity of some of the recent theoretical work by the research group at Cornell University.

Summary of the Conclusions

1. Unlike many substances the freezing temperature of water decreases when the pressure increases. This relates to the fact that the specific volume of water is greater than that of ice. A fundamental equation in thermodynamics is the Clapyeron equation. This indicates the pressure dependence of the temperature at which phase change occurs in any substance. For a system of ice and water in thermodynamic equilibrium, the Clapyeron equation predicts that the freezing-point depression arises from an increase in the pressure of the ice or a decrease in the pressure of the water or a combination of both changes. The application of the Clapyeron equation in describing freezing-point depression in soils is discussed in Section I.

2. The physical explanation for freezing-point depression in soils can be attributed to two main causes:
   - (i) A decrease in the specific free energy of the pore water below that of bulk water. This is associated with the development of a concave ice-water interface. (i.e. negative curvature with respect to the water).
   - (ii) The presence of solutes in the diffuse electric double layer surrounding the soil particles as well as in the bulk solution of the soil pores. The solutes lower the specific free energy of the water in the double layer below that of pure water, which depresses its freezing-point.

In general,interface (capillary) effects tend to be the principle cause of freezing-point depression in soils in the warmer temperature ranges while double layer effects predominate at lower temperatures.

3. The mechanism of moisture movement through frozen soils is still not fully understood and remains a subject of controversy. A U.S. research group has proposed a series-parallel transport mechanism in which water traverses the soil pores in both the solid and fluid phases. The parallel component of transport involves movement within the fluid films surrounding the soil particles as well as plug movement and plastico-viscous flow of pore ice. The series component refers to movement by regelation.

It is doubtful whether Darcy's law can be used to describe the fluid component of transport in a frozen soil since the drag of a moving ice body will affect the transport of film water. Experiments by Horiguchi and Miller (1980) indicate that regelation does obey Darcy's law. Their results also indicate that the process is only slightly dependent upon the temperature.

4. Two models have been proposed to account for the mechanism by which a temperature gradient induces moisture migration in a frozen soil. Both models attribute the movement to temperature induced gradients in the osmotic activity of the ions in the unfrozen films of water

surrounding the soil particles. With the 'diffusion' model, ions are assumed to be fully mobile, water migration occurring in response to the diffusion of ions along the temperature gradient. With the 'double layer' model, ions are assumed to be 'quasi-stationary', water migration occurring as a result of a gradient in the osmotic pressure between the 'warm' and 'cold' ends of the system.. Tests of both models, suggest the diffusion model as the predominant mode of transport at temperatures near 0°C.

5. Investigation of the time-decay in the permeability of frozen soils using Burt's permeameter, indicate that the decline levels off to a constant value some 2 to 4 hours, following the application of pressure to the soil. The total decline in the permeability of the soil over this time period was about 2-3 orders of magnitude. Two soils were used in the study, one exhibiting colloidal properties and the other non-colloidal properties. Results of the experiments were examined in relation to current theoretical and applied studies on moisture transport in frozen soils.

With the non-colloidal soil, regelation appears to be the primary mode of moisture transport in the temperature ranges between -0.1 and -0.6°C. This is indicated by the fact that the hydraulic conductivity of the soil is only slightly temperature dependent. With the colloidal soil although regelation probably represents an important component of movement, the flux of water in the fluid films surrounding the soil particles also appears to form a significantly large component of the total moisture transport. Thus, the greater sensitivity of the hydraulic conductivity of this soil to changes in temperature. These conclusions are also supported by the fact that, the unfrozen moisture content was much greater in the colloidal soil than the non-colloidal soil over the indicated temperature range.

6. Burt's apparatus was also used to investigate moisture movements in frozen soils induced by a gradient in osmotic potential. Outflow was observed to occur from the soil towards the reservoir concentrated lactose solution while inflow occurred at the end with more dilute solution. In general, the results indicate that the two ends of the system are not autonomous. That is, changes in the potential at one end of the system are communicated over the entire length of the sample almost immediately.

   It was also demonstrated that the lactose concentration can be adjusted so as to equalize potentials between the water in the reservoirs and in the soil. However, the equilibrium condition is disturbed considerably by the gradual passage of lactose molecules from the reservoirs into the soil over a period of several days.

7. A preliminary experiment has been conducted in which pressure gradients are measured in a frozen soil during

thermally induced moisture migration. The apparatus used in the experiments is similar although somewhat more sophisticated than that used by Williams and Perfect (1980) in the previous phase of this investigation. Results of the experiment are encouraging. Inflow was generally observed at the 'warm' end of the system and outflow at the 'cold' end. Results of the experiment also seem to confirm a recent hypothesis by Dr. R.D. Miller at Cornell University that, in a frozen soil subjected to a thermal gradient, the effective stress diminishes with colder temperatures. However, further modifications,to eliminate some of the technical difficulties with the apparatus,will be required before any definitive conclusions can be made. The experiments are expected to continue throughout the summer. Results will be examined with regard to some of the recent hypotheses put forward by the research group at Cornell University.

REFERENCES

Adamson A.W., 1976, "Physical Chemistry of Surfaces", 3rd ed. John Wiley and Sons, New York, p. 196-208.

Anderson D.M. and Tice A.R., 1973, "The Unfrozen Interfacial Phase in Frozen Soil Water Systems "In: Physical Aspects of Soil Water and Salt in Ecosystems, Springer Verlag, Berlin, p. 107-124.

Atkins P.W., 1978, "Physical Chemistry", Oxford University Press, Great Britain, p. 151-247.

Beskow G., 1935, "Soil Freezing and Frost Heaving With Special Reference to Roads and Railroads", Swedish Geological Society, Series C, No. 375, 26th Yearbook No. 3, Transl. by J.O. Osterberg, Technological Institute, Northwestern University, Evanston Ill.

Bouyoucos G.J., 1916, "The Freezing Point Method as a New Means of Measuring the Concentration of the Soil Solution Directly in the Soil, Mich. Agri. College Exp. Sta. Tech. Bull. 36,48 pp.

Burt, T.P., 1974, "A Study of the Hydraulic Conductivity in Frozen Soils" Unpubl. M.A. Thesis, Carleton University, Ottawa, 78 p.

Burt, T.P. and Williams, P.J., 1976, "Hydraulic Conductivity in Frozen Soils" Earth Surface Processes, V. 1, p. 349-360.

Cass, L.A. and Miller, R.D., 1959, "Role of the Electric Double Layer in the Mechanism of Frost Heaving" CRREL Res. Rep. No. 49, 15 p.

Edlefsen, N.E. and Anderson, A.B.C., 1943, "Thermodynamics of Soil Moisture", Hilgardia, V. 15, No. 2, 298 p.

Everett, D.H., 1961, "Thermodynamics of Frost Damage to Porous Solids", Trans. Für. Soc., V.57, p. 1541-1551.

Everett, D.M. and Haines J.M., 1965, "Capillary Properties of Some Model Pore Systems With Special Reference to Frost Damage", Bull. RILEM, No. 27, p. 31-36.

Glen, J.W., 1952, "The Creep of Polycrystalline Ice", Proc. Royal Soc.,V. A 228, p. 519-538.

Glen, J.W., 1958, "The Mechanical Properties of Ice And Plastic Properties of Ice", Advances in Physics V.7, p. 254-265.

Gray, H.J. and Issacs, A. (eds.),1975,"A New Dictionary of Physics", Longman Group, London.

Harlan, R.L., 1971, "Water Transport in Frozen and Partly Frozen Porous Media", Can. Hydro. Symp. Proc. 8th V. 1, p. 109-129.

Harlan, R.L., 1973, "Analysis of Coupled Heat-Fluid Transport in Partially Frozen Soil", Water Resources Res. V. 9, p. 1314-1322.

Hoekstra, P. and Miller, R.D., 1967, "On the Mobility of Water Molecules in the Transition Layer Between Ice and a Solid Surface", Journ. Colloid Interface Sci., V. 25, p. 166-173.

Horiguchi K. and Miller, R.D., 1980, "Experimental Studies with Frozen Soil in an 'Ice Sandwich' Permeameter", Cold Reg. Sci and Tech. V. 3, p. 177-183.

Kruyt, H.R. (ed.), 1963, "Colloid Science", V. 1, Elsevier Publ. Co., London, p. 127-146 and p. 203-207.

Loch, J.P.G., 1975, "Secondary Heaving: Experiments and Analysis of Frost Heaving Pressure in Soils", Ph.D. Thesis, Cornell University, 102 p.

Loch, J.P.G. and Kay, B.D., 1978, "Water Distribution in Partially Frozen Saturated Silt Under Several Temperature Gradients and Overburden Loads", Soil Sci. Soc. Am. Journ. V. 42, p. 400-406.

Miller, R.D., 1970, "Ice Sandwich: Functional Semipermeable Membrane", Science, V. 169, p. 584-585.

Miller, R.D., 1973, "The Porous Phase Barrier and Crystallization", Sep. Sci. V. 8, p. 521-535.

Miller, R.D., 1976, "Reviewers Comments: The Variation of Hydraulic Conductivity in Frozen Soils", T.P. Burt and P.J. Williams, personal communication.

Miller, R.D., 1978, "Frost Heaving in Non-Colloidal Soils", Proc. 3rd Int. Conf. on Permafrost, V. 1, p. 708-713.

Miller, R.D., 1980, "The Adsorbed Film Controversy", Agronomy Paper No. 1327, Cornell University, 9 p.

Miller, R.D. and Koslow, E.E., 1980, "Computation of Rate of Heave vs. Load Under Quasi-Steady State", Cold Reg. Sci. Tech., V. 3, p. 243-251.

Miller, R.D., Loch, J.P.G. and Bresler, E., 1975, "Transport of Water and Heat in a Frozen Permeameter", Soil Sci. Soc. Am. Proc. V. 39, p. 1029-1036.

Mitchell, J.R., 1976, 'Fundamentals of Soil Behaviour' John Wiley and Sons, U.S.A., p. 187.

Perfect, E., 1980, "Temperature Induced Water Migration in Saturated Frozen Soils", Unpub. M.A. Thesis Carleton Univ. Ottawa, 81 p.

Perfect E. and Williams, P.J., 1980, "Thermally Induced Water Migration in Frozen Soils", Cold Reg. Sci. and Tech., V. 3, p. 101-109:

Philip, J.R. and De Vries, D.A., 1957, "Moisture Movement in Porous Materials Under Temperature Gradients", Trans. Am. Geophys. Un., V. 38, p. 222-231.

Römkens, M.J.M. and Miller, R.D., 1973, "Migration of Mineral Particles in Ice with a Temperature Gradient", J. Colloid Interface Sci., V. 42, p. 103-111.

Sheeran, D.E. and Yong, R.N., 1975, Water and Salt Redistribution in Freezing Soils", Proc. Conf. on Soil-Water Problems in Cold Regions, Calgary, Alta., p. 61-69.

Taber, S., 1930, "The Mechanics of Frost Heaving", J. Geol. V. 38, p. 303-317.

Williams, P.J., 1967, "Properties and Behaviour of Freezing Soils", Norwegian Geotechnical Institute, Publ. N. 72, 119 p.

Williams, P.J., 1979, "Pipelines and Permafrost", Geography and Development in the Circumpolar North" Longmans Inc., N.Y., p. 33.

Williams, P.J. and Perfect, E., 1979, "Investigation of Rates of Water Movement Through Frozen Soils", Final Report for Dept. Energy, Mines and Resources, Ottawa, DSS File No. 02SU-KL229-7-1562, 36 p.

Williams, P.J. and Perfect, E., 1980, "Investigation of Thermally Actuated Water Migration in Frozen Soils", Final Report for Dept. Energy, Mines and Resources, Ottawa, DSS File No. 05SU-23235-9-0484, 57 p.

Woodruff, D.P., 1963, "The Solid-Liquid Interface, Cambridge University Press, Great Britain, p. 21-25.

Nye, J.F., 1957, "The Distribution of Stress in Glaciers and Ice Sheets" Proc. Royal Society, V.A 239, p. 113-133.

Paterson, W.S.B., 1969, The Physics of Glaciers, Pergamon Press. Oxford, 247 pp.

APPENDIX A

Permeability of a Layered Soil

For flow through a soil consisting of a group of layers arranged in series, the overall permeability of the soil is given by:

$$K = \frac{\ell_T}{\frac{\ell_1}{k_1} + \frac{\ell_2}{k_2} \ldots . \frac{\ell_n}{k_n}}$$

where ℓ_n = the thickness of layer n

k_n = the hydraulic conductivity of the nth layer

and $\ell_T = \sum_{1}^{n} \ell_n$ = the total thickness of the soil.

For a medium in which the conductivity is continuously changing, the expression above reduces to:

$$K = \frac{\ell_T}{\frac{1}{k(x)} dx}$$

APPENDIX B

Computer Program in BASIC Used to Calculate Hydraulic Conductivity Coefficients

```
LIST

10 PRINT "      TEMP."
20 PRINT "       (C)"
30 PRINT
40 DISP "ENTER L,A,R,(CM),TEMP.";
50 INPUT L,A,R,T1
60 WRITE (15,520)T1
70 DISP "ENTER TOTAL NO. OF READING SESSIONS ";
80 INPUT X
90 DISP "ENTER PRESENT READING SESSION";
100 INPUT Y
110 DISP "ENTER LENGTH OF FROZEN SOIL AFTER EXP. DISASSEMBLED";
120 INPUT L1
130 M=Y/X
140 L2=(L-L1)*M
150 L5=L-L2
160 DISP "ENTER UNFROZEN CONDUCTIVITY (CM/SEC)";
170 INPUT K2
180 DISP "ENTER NO. OF READINGS";
190 INPUT I
200 PRINT
210 PRINT "      TIME    DISTANCE   HEAD     HYDRAULIC     CORRECTED"
220 PRINT "                                CONDUCTIVITY      VALUE"
230 PRINT "       SEC       CM       CM      CM/SEC          CM/SEC"
240 PRINT
250 J=0
260 K=0
270 K1=0
280 K3=0
290 K4=0
300 DISP "ENTER T(SEC),H1,H2(CM),P1,P2(CM)";
310 INPUT T,H1,H2,P1,P2
320 D=P2-P1
330 Q=(R*R*D*PI)/T
340 H=(H1+H2)/2
350 K=(Q*L)/(H*A)
360 K3=L5/(((A*H)/Q)-(L2*K2))
370 K1=K1+K
380 K4=K4+K3
390 WRITE (15,510)T,D,H,K,K3
400 J=J+1
410 IF J=I THEN 430
420 GOTO 300
430 K1=K1/J
440 K4=K4/J
450 PRINT
460 PRINT "                                  AVERAGE       CORRECTED"
470 PRINT "                                CONDUCTIVITY      VALUE"
480 PRINT "                                  CM/SEC          CM/SEC"
490 PRINT
500 WRITE (15,530)K1,K4
510 FORMAT F10.1,F10.2,F9.0,E13.2,E14.2
520 FORMAT F11.3
530 FORMAT E42.2,E14.2
540 STOP
```

APPENDIX C

Correction in the Hydraulic Conductivity of a Frozen Soil for the Presence of a Thawed Layer

A correction for the presence of a thawed layer in a frozen soil is based on a procedure used to correct for membrane filters (Williams, 1964). The corrected hydraulic conductivity of the frozen soil is given by:

$$k_f = \frac{\ell_f}{\frac{Ah}{Q} - \frac{\ell_{uf}}{k_{uf}}}$$

k_f = hydraulic conductivity of frozen soil,

ℓ_f = thickness of frozen soil,

k_{uf} = hydraulic conductivity of unfrozen soil, (see values Table 3.1)

ℓ_{uf} = thickness of unfrozen soil,

A = cross-sectional area of sample,

Q = volume of flow discharge,

h = hydraulic head difference across sample.

APPENDIX D

Moisture Movement Through Frozen Soils Induced By an Osmotic Head: A Record of the Results Obtained

The difference in potential between the water in the reservoirs and the soil was calculated in the following manner.

The difference in potential ΔG, between the unfrozen water in the soil and free water is given by equation (2.1):

$$\Delta G = \frac{L}{v_w} \frac{\Delta T}{T}$$

where ΔG is expressed in cm of hydraulic head,

$L = 3.33549 \times 10^5 \ \mathrm{JKg}^{-1}$,

$v_w = 1 \times 10^{-3} \ \mathrm{m}^3 \ \mathrm{Kg}^{-1}$,

ΔT = the freezing point depression (degrees Kelvin),

$T = 273.15$ K.

The osmotic head ΔH_o in the end reservoirs is given by:

$$\Delta H_o = RT\Delta c$$

where ΔH_o is expressed in cm of hydraulic head

R = universal gas constant (=84.84 cm of water),
T = the temperature of the solution (degrees Kelvin),
Δc = lactose concentration in the end reservoir g-mol ℓ^{-1}.

Taking the difference $\Delta H_o - \Delta G$, gives us the hydraulic head between the end reservoirs and the soil. For example at a concentration of 17.94 gl^{-1}, $\Delta C = \frac{17.94}{360.3} = 0.0498$ g-mol ℓ^{-1}

If the soil temperature is -0.3°C (= 272.85 K) $\Delta H_o = 1153$ cm of water and $\Delta G = 3597$ cm of water. Thus $\Delta H_o - \Delta G = -2444$ cm of water

Experiment No.: 2
Sample: Allendale Silty Clay
Initial Moisture Content of Soil = 55.9%
Temperature of Soil = -0.30°C
Experiment Assembled: Jan. 3, 1981
Experiment Disassembled: Jan. 9, 1981
Total Length of Sample = 3.02 cm
Length of Frozen Section Remaining when Disassembled = 2.60 cm
Internal Diameter of Capillary Tubes = 0.047 cm

Note: Positive values indicate outflow (that is flow from the soil towards the reservoir) and negative values indicate inflow.

Experiment No. 2A

Date: January 3, 1981

Time Interval (Min)	Cumulative Flow (mm^3) Res. 1	Res. 2
0-20	- .61	+ 5.64
20-40	- .69	+11.79
40-60	- 2.51	+17.35
60-80	- 3.99	+22.29
80-100	- 5.38	+28.45
100-120	- 6.42	+34.00
120-140	- 7.81	+39.73
140-160	- 8.41	+45.02
160-180	- 9.28	+50.31
180-200	-10.06	+56.42
200-220	-12.58	+62.20
220-240	-13.79	+67.40

Lactose Concentration, $g\ell^{-1}$ Res. 1	Res. 2	Osmotic Head, cm. H_2O Res. 1	Res. 2
17.9	94.4	-2444	+2470

Experiment No. 2B

Date: January 4, 1981

Time Interval (Min)	Cumulative Flow (mm^3) Res. 1	Res. 2
0-20	- .34	+ .43
20-40	- .61	+ 1.47
40-60	- .69	+ 7.55
60-80	- .87	+10.93
80-100	-1.30	+16.30
100-120	-1.47	+21.08
120-140	-2.00	+25.60
140-163.5	-2.25	+29.75
163.5-180	-2.69	+33.57
180-200	-3.21	+38.52
200-220	-3.64	+43.98
220-240	-3.82	+48.58

Lactose Concentration, gℓ^{-1} Res. 1	Res. 2	Osmotic Head, cm. H_2O Res. 1	Res. 2
17.9	94.4	-2444	+2470

Experiment No. 2C

Date: January 5, 1981

Time Interval (Min)	Cumulative Flow (mm^3) Res. 1	Res. 2
0-20	0	+ 1.65
20-40	0	+ 2.60
40-60	0	+2.78
60-80	- .08	+ 3.12
80-100	- .17	+ 4.68
100-120	- .17	+ 6.68
120-140	- .35	+ 8.50
140-160	- .52	+10.07
160-180	- .95	+14.05
180-220	- 1.90	+18.39
220-260	- 2.95	+22.03
260-305	- 7.98	+28.97
305-325	-20.82	+40.33
325-345	-22.46	+42.42

Lactose Concentration, $g\ell^{-1}$ Res. 1	Res. 2	Osmotic Head, cm. H_2O Res. 1	Res. 2
52.9	94.4	-202	+2470

Experiment No. 2D

Date: January 6, 1981

Time Interval (Min)	Cumulative Flow (mm^3)	
	Res. 1	Res. 2
0-20	0	+ 10.15
20-40	+ .35	+ 27.32
40-60	+ .69	+ 38.6
60-80	+1.04	+ 49.88
80-100	+1.39	+ 63.59
100-120	+1.74	+ 70.69
120-140	+1.91	+ 80.59
140-160	+2.43	--
160-180	+2.52	+ 80.94
180-200	+2.78	+ 89.96
200-220	+2.95	+ 99.06
220-240	+3.12	+104.70

Lactose Concentration, $g\ell^{-1}$		Osmotic Head, cm. H_2O	
Res. 1	Res. 2	Res. 1	Res. 2
17.9	94.4	-2444	+2470

Experiment No. 2E

Date: January 7, 1981

Time Interval (Min)	Cumulative Flow (mm^3)	
	Res. 1	Res. 2
0-30	+14.49	-14.66
30-60	+15.00	-32.36
69-90	+25.07	-41.55
90-120	+34.09	-55.26
120-150	+46.06	-68.44
150-180	+57.77	-80.59
180-210	+69.57	-91.55
210-240	+81.19	--

Lactose Concentration, $g\ell^{-1}$		Osmotic Head, cm. H_2O	
Res. 1	Res. 2	Res. 1	Res. 2
94.4	17.9	+2470	-2444

Experiment No. 2F

Date: January 8, 1981

Time Interval (Min)	Cumulative Flow (mm^3)	
	Res. 1	Res. 2
0-20	- 5.11	- 5.89
20-40	- 6.67	- 9.19
40-60	- 6.94	-11.10
60-80	- 7.63	-12.66
80-100	- 8.76	-14.31
100-120	- 9.63	-15.52
120-140	-10.24	-17.61
140-160	-10.32	-18.91
160-180	-10.32	-20.30
180-200	-10.24	-23.42

Lactose Concentration, $g\ell^{-1}$		Osmotic Head, cm. H_2O	
Res. 1	Res. 2	Res. 1	Res. 2
17.9	94.4	-2444	+2470

Experiment No. 2G

Date: January 9, 1981

Time Interval (Min)	Cumulative Flow (mm^3) Res. 1	Res. 2
0-20	0	- 1.13
20-40	0	- 2.52
40-60	0	- 3.99
60-80	0	- 5.46
80-100	0	- 6.59
100-120	0	- 8.78
120-140	0	-10.15
140-160	0	-11.19
160-180	0	-12.66
180-200	0	-14.05
200-220	0	-15.09
220-240	0	-16.13

Lactose Concentration, $g\ell^{-1}$ Res. 1	Res. 2	Osmotic Head, cm. H_2O Res. 1	Res. 2
52.9	52.9	-202	-202

Observations: Reservoir 1 appeared frozen when the apparatus was disassembled.

Experiment No.: 3
Sample: Castor Sandy Loam
Initial Moisture Content of Soil = 28.0%
Temperature of Soil = -0.30°C
Experiment Assembled: Jan. 4, 1981
Experiment Disassembled: Jan. 9, 1981
Total Length of Sample = 3.285 cm
Length of Frozen Section Remaining when Disassembled = 3.10 cm
Internal Diameter of Capillary Tubes = 0.047 cm

Note: Positive values indicate outflow (that is flow from the soil towards the reservoir) and negative values indicate inflow.

Experiment No. 3A

Date: January 4, 1981

Time Interval (Min)	Cumulative Flow (mm^3) Res. 1	Cumulative Flow (mm^3) Res. 2
0-20	0	+0.35
20-40	+0.09	+0.87
40-60	+0.09	+1.21
60-80	+0.17	+1.56
80-100	+0.17	+1.99
100-120	+0.26	+2.69
120-140	+0.26	+3.04
140-160	+0.35	+3.29
160-180	+0.35	+3.38
180-200	+0.35	+3.47

Lactose Concentration, $g\ell^{-1}$ Res. 1	Lactose Concentration, $g\ell^{-1}$ Res. 2	Osmotic Head, cm. H_2O Res. 1	Osmotic Head, cm. H_2O Res. 2
52.9	52.9	-202	-202

Experiment No. 3B

Date: January 5, 1981

Time Interval (Min)	Cumulative Flow (mm^3)	
	Res. 1	Res. 2
0-20	+ .26	+ 0.09
20-40	+ 2.42	0
40-60	+10.58	- 0.35
60-80	+17.18	- 6.33
80-100	+23.42	-12.93
100-120	+39.12	-16.83
120-140	+45.72	-21.42
140-160	+51.01	-22.55
160-180	+55.26	-25.68
180-220	+65.32	-26.63
220-260	+64.88	-27.76
260-305	+66.45	-36.69
305-325	+73.65	-50.49

Lactose Concentration, $g\ell^{-1}$		Osmotic Head, cm. H_2O	
Res. 1	Res. 2	Res. 1	Res. 2
94.4	52.9	+2470	-202

Experiment No. 3B Cont'd.

Date: January 6, 1981

Time Interval (Min)	Cumulative Flow (mm^3) Res. 1	Res. 2
0-20	+ 8.67	0
20-40	+15.67	+0.09
40-60	+23.42	+0.09
60-80	--	+0.09
80-100	--	0
100-120	+36.17	0
120-140	+38.43	0
140-160	+44.15	0
160-180	+48.75	0
180-200	+52.92	+0.09
200-220	+56.30	+0.09
220-240	+59.77	+0.09

Date: January 7, 1981

0-30	-27.70	+19.34
30-60	-45.28	+72.09
60-90	-70.87	+82.58
90-120	-88.22	+125.00
120-150	-111.47	+142.44
150-180	-134.89	+152.24
180-210	-152.2	+170.46
210-240	-165.1	+195.01

Lactose Concentration, g Res. 1	Res. 2	Osmotic Head, cm. H 0 Res. 1	Res. 2
17.9	94.4	-2444	+2470

Observations: Reservoir 2 appeared frozen on January 6.

Experiment No. 3C

Date: January 8, 1981

Time Interval (Min)	Cumulative Flow (mm^3) Res. 1	Cumulative Flow (mm^3) Res. 2
0-20	0	- 0.43
20-40	- .17	- 1.21
40-60	- .26	-14.22
60-80	- .34	-14.57
80-100	- .43	-18.91
100-120	- .52	-25.50
120-140	- .69	-25.50
140-160	-1.04	-29.32
160-180	-1.21	-30.01
180-200	-1.91	-36.09

Date: January 9, 1981

Time Interval (Min)	Res. 1	Res. 2
0-20	+0.09	- 4.77
20-40	+0.09	- 9.19
40-60	+0.09	-14.23
60-80	+0.17	-21.86
80-100	+0.17	-27.24
100-120	+0.17	-32.62
120-140	+0.17	-56.38
140-160	+0.17	-61.76
160-180	+0.26	-69.31
180-200	+0.26	-72.78
200-220	+0.34	-78.77
220-240	+0.43	-85.45

Lactose Concentration, gℓ^{-1} Res. 1	Lactose Concentration, gℓ^{-1} Res. 2	Osmotic Head, cm. H_2O Res. 1	Osmotic Head, cm. H_2O Res. 2
52.9	52.9	-202	-202

Observations: An ice lens 1 mm. thick was observed at the end of the sample adjacent to Res. 1.

APPENDIX E

Thermally Induced Moisture Migration in Frozen Soils: A Record of the Results Obtained

Note: The experiments are numbered 1,2,3, etc. according to the sample that was used in the experiment. In some cases, the temperature gradient was changed part way through the experiment. This is indicated by a change in the letter following the experiment number. Results of experiments nos. 1,3 and 5 are not shown because the flow of moisture through the soil was arrested by freezing in the end reservoirs.

Prefrozen samples were used in all of the experiments. Moisture contents are expressed as percent dry weight. Temperatures were measured with themistors calibrated to an accuracy of ± 0.01°C.

Thermistor Code

T1 - Temperature of end plate ('warm')
T2 - Temperature of sample 2.25 cm from T1
T3 - Temperature of sample 3.10 cm from T1
T4 - Temperature of sample 3.95 cm from T1
T5 - Temperature of end plate ('cold'), 6.25 cm from T1

The time interval in column 1 represents the total cumulative time since the experiment was started. The starting point is considered to be the time when the thermoelectric cooling unit was switched on.

Flow readings were measured with small capillary tubes (internal diameter 0.047 cm) mounted on a finely divided scale. Outflow (that is, flow from the soil towards the reservoir) is denoted by a positive (+) sign and inflow by a negative (-) sign.

Note that with most of the results shown here outflow occurred from both ends of the sample. This may have been due to the fact that the moisture content of the soil was much greater than saturation and so the soil expelled the excess moisture until a saturation value was achieved.

Experiment No. 2B

Castor Sandy Loam

Date: June 20, 1980

						Meniscus Movement mm		Cumulative Flow mm^3	
Time Interval Min.	T1	T2	T3	T4	T5	'Warm' End	'Cold' End	'Warm' End	'Cold' End
2615-2645	-0.075	--	-0.115	-0.160	-0.295	+ 1.5	+ 0.5	+0.26	+0.09
2645-2675	-0.06	--	-0.115	-0.160	-0.295	+ 3.0	0	+0.78	+0.09
2675-2705	-0.055	--	-0.110	-0.160	-0.295	+14.0	+12.0	+3.21	+2.24
2705-2735	-0.065	--	-0.110	-0.160	-0.290	+ 1.0	+ 0.5	+3.38	+2.25
2735-2765	-0.06	--	-0.110	-0.155	-0.290	+ 1.0	0	+3.56	+2.25
2765-2795	-0.08	--	-0.115	-0.165	-0.290	0	0	+3.56	+2.25
2795-2825	-0.07	--	-0.110	-0.155	-0.285	0	+10.0	+3.56	+3.99
2825-2855	-0.065	--	-0.110	-0.155	-0.290	0	-10.0	+3.56	+2.25
2855-2885	-0.065	--	-0.115	-0.165	-0.290	0	0	+3.56	+2.25
2885-2915	-0.065	--	-0.115	-0.155	-0.290	0	0	+3.56	+2.25

Date Experiment Assembled: June 18, 1980
Initial Water Content = 39.7%
Temperature Gradient = 0.23°C
Lactose Concentration 'Warm' Plate Reservoir = 20.1 g ℓ^{-1}
Lactose Concentration 'Cold' Plate Reservoir = 56.1 g ℓ^{-1}

Experiment No. 2C

Castor Sandy Loam

Date: June 21, 1980

Time Interval Min.	T1	T2	T3	T4	T5	Meniscus Movement mm 'Warm' End	Meniscus Movement mm 'Cold' End	Cumulative Flow mm^3 'Warm' End	Cumulative Flow mm^3 'Cold' End
4223-4253	-0.110	-0.110	-0.175	-0.270	-0.470	- 40.0	+39.5	- 6.93	+ 6.85
4253-4283	-0.12	-0.100	-0.195	-0.275	-0.455	+ 22.5	+ 5.5	- 3.04	+ 7.81
4283-4337	-0.08	--	-0.015	-0.130	-0.465	+128.0	+68.5	+19.17	+19.69
4337-4350	-0.08	--	-0.020	-0.140	-0.460	+115.0	+50.5	+39.12	+28.89
4350-4380	-0.110	-0.100	-0.155	-0.240	-0.470	+196.5	+43.0	+73.2	+36.35
4380-4410	-0.155	-0.165	-0.215	-0.285	-0.465	+ 1.0	0	+73.2	+36.35
4410-4440	-0.120	-0.095	-0.165	-0.265	-0.445	0	0	+73.2	+36.35

Date Experiment Assembled: June 18, 1980
Initial Water Content = 39.7%
Temperature Gradient = 0.36°C
Lactose Concentration 'Warm' Plate Reservoir = 20.1 g ℓ^{-1}
Lactose Concentration 'Cold' Plate Reservoir = 82.5 g ℓ^{-1}

Comments: Experiment disassembled June 23, 1981.
Sample was observed to be completely thawed.

Experiment No. 4A

Castor Sandy Loam

Date: July 17, 1980

Time Interval Min.	T1	T2	T3	T4	T5	Meniscus Movement mm 'Warm' End	Meniscus Movement mm 'Cold' End	Cumulative Flow mm^3 'Warm' End	Cumulative Flow mm^3 'Cold' End
140-165	-0.175	-0.235	-0.270	-0.360	-0.495	+57.0	+22.0	+ 9.89	+ 3.82
165-177	-0.165	-0.235	-0.265	-0.355	-0.475	+17.0	-28.0	+12.83	- 1.04
177-207	-0.175	-0.220	-0.250	-0.335	-0.485	+26.0	-62.0	+17.34	-11.79
207-237	-0.160	-0.215	-0.240	-0.330	-0.475	+ 9.0	+12.5	+18.91	- 9.63
237-264	-0.255	-0.135	-0.150	-0.260	-0.470	--	+ 2.0	--	- 9.28
264-278	-0.165	-0.140	-0.125	-0.085	-0.510	+ 8.0	- 2.5	+20.29	- 1.18

Date: June 18, 1980

Time Interval Min.	T1	T2	T3	T4	T5	Meniscus Movement mm 'Warm' End	Meniscus Movement mm 'Cold' End	Cumulative Flow mm^3 'Warm' End	Cumulative Flow mm^3 'Cold' End
1085-1115	-0.170	-0.130	-0.113	-0.245	-0.500	+165.0	+14.0	+28.63	+ 2.43
1115-1145	-0.175	-0.155	-0.160	-0.280	-0.485	+ 43.0	+87.0	+36.09	+17.52
1145-1175	-0.165	-0.155	-0.185	-0.295	-0.495	+ 18.0	+30.5	+39.21	+22.81
1175-1205	-0.295	-0.155	-0.175	-0.295	-0.550	+ 30.0	--	+44.41	--
1205-1215	-0.165	-0.155	-0.115	-0.295	-0.475	--	--	--	--
1215-1235	-0.165	-0.175	-0.190	-0.305	-0.485	- 17.0	+10.0	+41.47	+24.55
1235-1265	-0.165	-0.195	-0.220	-0.320	-0.485	0	+55.5	+41.47	+34.18
1265-1295	-0.165	-0.195	-0.215	-0.320	-0.495	+ 6.0	+49.5	+42.51	+42.77
1295-1325	-0.165	-0.195	-0.215	-0.320	-0.485	+ 14.5	-21.0	+45.02	+39.12
1325-1355	-0.165	-0.195	-0.215	-0.320	-0.490	+ 19.0	+19.0	+48.32	+42.42
1355-1385	-0.165	-0.195	-0.215	-0.320	-0.500	+ 13.5	+ 2.0	+50.66	+42.77
1385-1415	-0.165	-0.190	-0.215	-0.320	-0.505	+ 12.0	+29.5	+52.74	+47.88
1415-1445	-0.150	-0.195	-0.215	-0.300	-0.490	+ 19.0	+42.5	+56.04	+55.26
1445-1475	-0.180	-0.195	-0.215	-0.320	-0.530	--	--	--	--

Date Experiment Assembled: July 17, 1980
Initial Water Content = 46.7%
Temperature Gradient = 0.320 °C
Lactose Concentration 'Warm' Plate Reservoir = 30.3 g ℓ^{-1}
Lactose Concentration 'Cold' Plate Reservoir = 82.5 g ℓ^{-1}

Observations: Experiment disassembled June 19, 1981.
Total length of sample = 3.28 cm.
Length of frozen section remaining when disassembled = 3.0 cm.

APPENDIX F

Results of a Preliminary Experiment Measuring Pressure Changes in a Frozen Soil During Temperature Induced Moisture Migration

The sample was prepared with actively boiling deionized water and prefrozen prior to the experiment. The length of the sample holder was 3.50 cm, cross-sectional area 22.90 cm^2. Temperatures of the end plates were measured with thermistors calibrated to an accuracy of ± 0.01°C.

Thermistor Code:

T1 - Temperature of 'warm' plate (plate 1)
T2 - Temperature of 'cold' plate (plate 2)

Pressures were measured with Kulite VQS-250 series pressure transducers:

Pressure Transducer Code:

P1 - Pressure at 'warm' side of sample, 1.15 cm from end of sample
P2 - Pressure at 'cold' side of sample, 1.20 cm from P1

Specifications:

	Rated Pressure	Sensitivity
P1	100psi = 689.5kPa	0.126 mV/kPa
P2	50psi = 344.8kPa	0.215 mV/kPa

Note: To convert pressures from kPa to cm of hydraulic head use the following conversion factor ($cm\ H_2O = kPa \times 10.198$)

Atmospheric pressure was recorded at the start of each experiment from a mercury barometer mounted on the wall of the laboratory.
Lactose solution was used in both reservoirs.

Reservoir code:
R1 - reservoir in 'warm' plate
R2 - reservoir in 'cold' plate
Outflow (that is, flow from the soil towards the reservoir) is indicated by a (+) sign and inflow by a (-) sign.

Experiment No.: 1
Sample: Castor Sandy Loam
Initial Moisture Content = 25.4%
Final Moisture Content = 23.2%
Bulk Density of Sample = 1.66 g cm^{-1}
Temperature Gradient = 0.10°C
Lactose Concentration Res. 1 = 17.9 gl^{-1}
Lactose Concentration Res. 2 = 35.8 gl^{-1}
Experiment Assembled: June 11, 1981
Experiment Disassembled: June 12, 1981
Internal Diameter of Capillary Tubes = 0.047 cm

Date: June 11, 1981

Atmospheric Pressure = 100.4 KPa

Time Interval Minutes					Meniscus Movement mm		Total Cumulative Flow mm^3	
	T1	T2	P1	P2	R1	R2	R1	R2
100-130	-0.10	-0.20	197.3	141.4	+ 26.0	0	+ 4.51	0
130-165	-0.10	-0.21	148.4	126.3	0	+64.5	+ 4.51	+11.19
165-185	-0.095	-0.20	124.0	112.1	+ 26.0	+22.0	+ 4.51	+15.01
185-224	-0.10	-0.205	118.4	108.0	+ 32.0	0	+10.06	+15.01
224-255	-0.10	-0.20	113.7	105.3	+229.0	0	+49.79	+15.01
255-285	-0.095	-0.20	110.1	99.8	+ 64.5	0	+60.98	+15.01
285-315	-0.10	-0.205	115.3	99.5	+ 19.5	0	+64.36	+15.01
315-345	-0.095	-0.20	113.5	99.7	+ 4.0	0	+65.06	+15.01
345-375	-0.095	-0.20	107.1	99.4	0	-11.55	+65.06	-5.03
375-405	-0.115	-0.235	107.1	99.4	0	-11.55	+65.06	-5.03
405-435	-0.09	-0.19	118.6	98.9	0	-11.55	+65.06	-5.03
435-465	-0.10	-0.20	108.7	98.7	0	-11.55	+65.06	-5.03

Date: June 12, 1981

Atmospheric Pressure = 100.8 KPa

Time Interval Minutes					Meniscus Movement mm		Total Cumulative Flow mm^3	
	T1	T2	P1	P2	R1	R2	R1	R2
1315-1345	-0.10	-0.20	114.2	100.9	0	+ 4.0	0	+0.69
1345-1375	-0.11	-0.24	114.7	101.0	+1.5	+136.0	+0.26	+24.28
1375-1380	-0.095	-0.22	114.6	100.1	0	+ 75.5	+0.26	+37.38
1380-1430	-0.10	-0.205	113.7	99.9	0	+ 1.5	+0.26	+37.64
1430-1460	-0.10	-0.205	113.3	99.5	0	+ 1.2	+0.26	+37.82
1460-1520	-0.09	-0.205	112.2	98.9	0	0	+0.26	+37.82
1520-1550	-0.09	-0.20	108.9	98.4	0	+328.0	+0.26	+94.73
1550-1580	-0.09	-0.185	114.7	98.0	0	- 57.2	+0.26	+84.84
1580-1610	-0.095	-0.185	110.6	97.6	-18.0	- 10.0	-2.86	+83.10
1610-1640	-0.10	-0.21	111.0	97.4	-112.0	- 3.0	-22.29	+82.58
1640-1670	-0.10	-0.20	111.9	98.3	-175.0	- 1.0	-52.65	+82.41

APPENDIX G

Freezing-Point Depression, $\Delta T^{o}C$ as a Function of Lactose Concentration

Reference: Handbook of Chemistry and Physics, 58th ed.
Page D232-233

LACTOSE, $C_{12}H_{22}O_{11} \cdot 1H_2O$

Molecular weight = 342.30
Formula weight = 360.31

Anhydrous Solute Concentration g/l	Molar Concentration g-mol/l	Freezing Point Depression Δ°C
0.50	5.0	0.027
1.00	10.0	0.055
1.50	15.1	0.083
2.00	20.1	0.112
2.50	25.2	0.140
3.00	30.3	0.169
3.50	35.4	0.198
4.00	40.6	0.228
4.50	45.7	0.258
5.00	50.9	0.288
5.50	56.1	0.319
6.00	61.4	0.351
6.50	66.6	0.385
7.00	71.9	0.420
7.50	77.2	0.456
8.00	82.5	0.495
8.50	87.8	No value given
9.00	93.1	" " "
9.50	98.5	" " "
10.00	103.9	" " "

APPENDIX H

Pressure Transducer Test Reports

PRESSURE TRANSDUCER TEST REPORT

MODEL NO. VQS-250-50

SERIAL NO. 3876-2-61

CUSTOMER CARLETON UNIVERSITY

CUSTOMER P.O. No. REPAIR

STANDARD ELECTRICAL CONNECTIONS: (Per ISA 37.1)

Red	+ Input	Green	+ Output
Black	– Input	White	– Output

SPECIAL CONNECTIONS:

TEST CONDITIONS:

Rated Pressure 50 psi G

Maximum Pressure 100 psi G

Maximum Reference Pressure 50 psi G

X Gage

____ Sealed Gage

____ Absolute

____ Differential

Tested at 10 VDC Excitation

Maximum Excitation: 15 VDC

SPECIFICATIONS:

Sensitivity: 1.48 mV/psi

Zero Pressure Output: < ±3% F.S.

Thermal Effect on Zero: < ±1 % FS/100°F

____ Not Compensated

Thermal Effect on Sensitivity: < ±2 %/100°F

____ Not Compensated*

Compensated Temperature Range: 80°F to 180°F

Output Impedance 358 ohms

Input Impedance 890 ohms

*See Bulletin ____ for external compensation method.

REMARKS:

"O" RING SUPPLIED.

NEW TEST REPORT BEING ISSUED AFTER DIAPHRAGM WAS COATED WITH SILASTIC.

QUALITY ASSURANCE:

Tested by R. D.

Date 4-10-81

Inspected by A2

Date APR 14 1981

PRESSURE TRANSDUCER TEST REPORT

MODEL NO. VQS-250-100

SERIAL NO. 4417-2-19

CUSTOMER CARLETON UNIVERSITY

CUSTOMER P.O. No. REPAIR

STANDARD ELECTRICAL CONNECTIONS: (Per ISA 37.1)

Red	-	+ Input	Green	-	+ Output
Black	-	− Input	White	-	− Output

SPECIAL CONNECTIONS:

TEST CONDITIONS:

Rated Pressure 100 psi G

Maximum Pressure 200 psi G

Maximum Reference Pressure 50 psi G

X Gage ____ Sealed Gage

____ Absolute ____ Differential

Tested at 10 VDC Excitation

Maximum Excitation: 15 VDC

SPECIFICATIONS:

Sensitivity: .870 mV/psi

Zero Pressure Output: < ±3% F.S.

Thermal Effect on Zero: < ±1 % FS/100°F ____ Not Compensated

Thermal Effect on Sensitivity: < ±2 %/100°F ____ Not Compensated*

Compensated Temperature Range: 80°F to 180°F

Output Impedance 316 ohms Input Impedance 422 ohms

*See Bulletin ____ for external compensation method.

REMARKS:

"O" RING SUPPLIED.

NEW TEST REPORT BEING ISSUED AFTER DIAPHRAGM WAS COATED WITH SILASTIC.

QUALITY ASSURANCE:

Tested by R. D.

Date 4-10-81

Inspected by A27

Date APR 14 1981

KULITE SEMICONDUCTOR PRODUCTS INC. • [illegible]

APPENDIX I

N.R.C. Correction for Glass Thermometer Used In Methanol Bath

National Research Council
Canada

Conseil national de recherches
Canada

REPORT

Liquid-in-Glass Thermometer

Serial No.:	112213
Manufacturer:	Brooklyn Thermo Co.
Submitted By:	Carleton University Ottawa, Ontario
Description:	Mercury in Glass, 4 in. immersion
Range:	-6° to 0°C
Graduation:	0.01°C

THE ABOVE DESCRIBED THERMOMETER HAS BEEN COMPARED WITH THE STANDARDS OF THE NATIONAL RESEARCH COUNCIL AND FOUND, AT THIS DATE, TO HAVE THE CORRECTIONS LISTED BELOW.

Thermometer Reading	Correction
-6°C	-0.086°C
-3°	-0.072°
0°	-0.055°

TO USE THE CORRECTIONS PROPERLY REFERENCE SHOULD BE MADE TO THE FOLLOWING NOTES ON THE REVERSE SIDE OF THIS SHEET: A & D

REMARKS:

The estimated limit of accuracy was ± 0.01°C.

for the Director, Division of Physics, Ottawa.

REPORT No. APH 2173

DATE 16 October 1978