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INERTIAL INSTRUMENTATION AT THE GEODETIC SURVEY OF CANADA

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INTRODUCTION

1. Since the middle sixties a great amount of money and manpower have been made available for space research work in the U.S.A. Up to date, this has resulted in an immense amount of sophisticated hardware and specific knowledge which made the success in space possible.

2. The spin-offs of these special projects, for this specific technology, are now being utilised in other scientific fields; so far the science of electronics has been the main beneficiary of the space research.

3. One of the oldest sciences, geodesy, is characterised by others as being very conservative, in that geodesy in the 1950's was not much different than it was in the 1850's. Today, it seems that the geodesists have created a communication gap between themselves and the other sciences, and they are certainly slow in taking advantage of the immense technological advances made during this century, especially in the second half of it. However, in the past few years, efforts have been made by space firms to utilise their wealth in other fields as well and, in this manner, we are becoming recipients and beneficiaries of their work.

4. The impact of satellites on geodesy is well known today, but another aspect of the space experiments in other fields is bringing even more profound changes which will not only affect geodesy, but surveying of less than geodetic order as well.

5. Although inertial navigation has been used on airplanes for two decades, the necessity of greatly increased accuracy for high velocity rockets and satellites resulted in increasingly more sophisticated hardware and software. This objective resulted in a precision that appears useful even for geodesy, for it has a high economic importance.

6. We, the Geodetic Survey of Canada of the Surveys and Mapping Branch, entered into inertial geodesy at its lowest level, and involved ourselves in azimuth determinations by suspended gyroscopes. This situation brought us in contact with the aerospace industry, where we were able to become involved with some more sophisticated developments.

7. In this paper I will describe the goals that we have reached in this field.

8. All gyroscopes are drifting. The drift is an unwanted torque on the gimballing, whatever it may be. In the case of the suspended gyroscope, the gimballing is the suspension band (an extremely fine gimballing) and the bearing of the spin-axis.

9. The drift manifests itself as a slow change in the position of the equilibrium which defines the direction of North. Whenever the reasons causing this slow change are discovered, they can be eliminated or minimised and thus define the position of the North more accurately. Although the reasons for the drift cannot be detected, the effect can be measured and a correction applied.

10. Some of the existing hardware was modified and some procedures were initiated to eliminate the drift effect. This method was described in the report presented at the F.I.G. Congress held in Washington in 1974 (Reference #1). I will only present some of the results here:

- Using M.O.M. Gi-B2 gyroscopes in clamped mode (more-or-less like a Gi-B1) on medium latitudes, the interior accuracy of the mean from 6 to 8 sets of determinations became ±1".0 to 2".5. The exterior accuracy (in comparison to known azimuths) was found to be ±1" to 3".
- In 1972, the gyroscope was used to determine ground azimuths on all stations in an aerodist net, where the stations were 40 to 70 miles apart and not intervisible (Table 1). Because it was not possible to make a direct comparison to known values, the instrument was compared before, during, and after the job on an established calibration line. A change in the E-factor would have fallen outside the usual accuracy; no such phenomenon was noted.
- Since the aero-space industry, for some unknown reason, discarded the principle of suspension and therefore was unable to achieve such accuracy, a great deal of doubt arose regarding the validity of these results. Finally, one of the top aero-space firms, TRW of California, was awarded a contract to investigate the validity of the Canadian claims. For five months, a team of engineers and scientists dug into our system and came up with the conclusion that the claims were fully backed with proof. These investigations produced the following results which were obtained under strict supervision:

  On Medium Latitude 45° 19'

1st day	σ_{mean} of 6 sets ±1".2,	abs. error -2".0
2nd day	σ_{mean} of 6 sets ±1".4,	abs. error +1".3
3rd day	σ_{mean} of 6 sets ±2".0,	abs. error +1".4
	Total mean	+0".2

On High Latitude 69° 08'

1st day	σ_{mean} of 6 sets	±1″.7,	abs. error −3″.2
2nd day	σ_{mean} of 6 sets	±1″.6,	abs. error −1″.2
3rd day	σ_{mean} of 6 sets	±1″.7,	abs. error −1″.1
	Total mean		−1″.8

On Medium Latitude 45° 19', Intense Cold −7° to −18°C.

1st day	σ_{mean} of 5 sets	±1″.3,	abs. error −2″.2
2nd day	σ_{mean} of 4 sets	±2″.1,	abs. error −2″.5
3rd day	σ_{mean} of 3 sets	±3″.2,	abs. error −3″.4
4th day	σ_{mean} of 7 sets	±1″.7,	abs. error +4″.3
	Total mean		−1″.0

- In 1974, a gyroscope was used to tie in the new Doppler stations in the Arctic Archipelago, and the north perimeter of the continent to existing surveys in these areas.
  This situation involved the use of the gyroscope in the most northern azimuth determinations ever made for ten stations between 71° and 80° latitudes (Table 2). The surprising results have shown that the accuracy of gyroscopic determinations does not decrease strictly by the cosine of the latitude as anticipated. The reason is simple; in that the error consists of two components, one of which (i.e.,: reading on the R.O., setting the H.C., band torque errors, etc.) is the same on all latitudes, while the other (i.e.,: directive moment of the Earth's centripetal force of rotation) is definitely a function of cos ϕ.
  The fourth column in Table 2 contains values which may be expected if it is assumed that the accuracy deteriorates as a function of cos ϕ. The fifth column is the expectation obtained by solving the equation by the above mentioned functions. The experience obtained on a latitude of 49°19' from more than 100 sets of determinations was used as the norm.
  When the Arctic determinations were compared to the four astronomic determinations, which were simultaneously measured, there was a strong hint that the effect of the deviation of the vertical on the gyroscopic azimuth is quite different from that on the astronomic azimuth. An investigation will be undertaken in April and, hopefully, the results will be available in time for the Congress.
  Our gyroscopic system is now employed by the maritime Provinces, Manitoba and Saskatchewan, and also in use at the Defense Mapping Agency's Topographic Center in the United States.

GROUND ELEVATION METER (GEM)

11. This equipment, which accidentally landed in our hands, is a so-called "strapdown" inertial device with a damped and torqued pendulum to measure the slope angle, and a fifth wheel to measure distances with which the slope angle can be utilised to give elevations. As far as inertial instrumentation goes, it was probably already outdated when it came out of the factory.

12. We discovered that by using proper electronic and mechanical calibration procedures, and with the careful designing and execution of the measurements it is a useful tool for measuring elevations for mapping purposes.

13. It can be stated that where roads are available, a 20 to 25 km. line can be completed, even in a considerable relief area, in about 2 to 2½ hours between two points of known elevation. This procedure ensures that the error in elevations for any intermediate point will be no more than 1 m. or less. For mapping purposes this is very useful, especially if one considers the speed with which such a high relief level line can be run using standard levelling procedures. See Reference #3. The applicability of this procedure is illustrated in the following example: the course is 2 x 26 km. long and has elevation differences of 1000 feet. When run by two men (driver and recorder) in the GEM, it requires 2½ hours of time and costs $30.00 but when run by standard levelling procedures, it takes 1 observer, 1 notekeeper, 2 rodmen, and 1 driver five days at a cost of $750.00.

14. The system has peculiarities and requires all the ability and training of the operator to produce a better than 1 m. accuracy. Another factor to remember is that it is tied to roads and that the accuracy is a function of the quality of the road surface, which presents a considerable disadvantage.

15. Neither the concept nor the technology of this system are modern by todays standards, for eight years is a long time on the scale of our present technology. However, it is more economical for running vertical control for mapping purposes than spirit levelling. It is also more accurate than barometric elevations.

POSITION AND AZIMUTH DETERMINATION SYSTEM (PADS)
INERTIAL POSITIONING SYSTEM (IPS)
AUTOSURVEYOR

A) General Development

16. This trio of names describes an inertial navigation system with an extremely high accuracy.

17. The PADS was originally designed for certain military applications, and may be considered to be the "grandfather" of the real inertial equipment of the present day and that of the future. This system has proven, in many tests, that it is able to deliver an accuracy of about ±10 m. in latitude, longitude and elevation in open traverses up to 200 km. long. The system can be operated in both four-wheel drive vehicles and helicopters.

18. The IPS is a highly refined version of the original PADS with significant changes in hardware, software and the method of application. In preliminary tests, it gave latitude,

longitude and elevation, on a course over 40 km. long, with mixed terrain (flatland and mountainous) and road conditions (hardtop, gravel and dirt) to an accuracy of better than 1 m. in both coordinates in closed traversing. Gravity anomaly and deflection of the vertical could also be obtained with an accuracy of ±1 to 3 mg. or 2 seconds respectively. The Autosurveyor is the commercial version of the IPS and its output only contains latitude, longitude and elevation. Otherwise, both the hardware and software are identical (see Reference #2).

19. The IPS-Autosurveyor is the most exciting development which, in future applications, will supplement satellite geodesy without overshadowing it and it might make many parts of our present day surveying technology redundant, except in special circumstances.

20. This statement considers the usual way of development from prototype through the first generation into a future second, third, etc. (IPS-Autosurveyor is a first generation).

21. This inertial equipment is the first serious step towards the fulfilment of the geodesists and surveyors dreams, the "magic black box", which instantaneously gives us all the necessary geodetic data wherever we put it down. In its present form it is highly automatic, the operator only has to do some initialisation and maintenance. It is grossly independent of outside influences, no line of sight is required, the need for towers and cutlines is eliminated and it can be operated in cold or hot climates, day or night, rain or shine. All the measurements are executed at the high velocity of a motor vehicle or helicopter.

22. The Geodetic Survey of Canada had an opportunity to participate in this extraordinary development partly in active and partly in passive roles. We participated as observers in some parts as well as performing our own tests with the prototype. We own the first production model, with both the IPS and Autosurveyor programs.

23. At the beginning of February, we embarked on an ambitious test program for the purpose of some additional refinements of methodology. We also intend to find out if there are any aspects of the system which have not yet been experimented with, at least not to the extent that our special geographical location would require. The following are the phases of the project that have either already been completed or will be in the near future:

a) we tested the equipment in Los Angeles on the D.M.A. testline (see Appendix A);
b) we are now testing the equipment in Ottawa in cold weather (see Appendix B);
c) we will test the equipment in the high relief and gravity anomaly areas in the Rocky Mountains and

test the connecting of points 150 km. and further apart (this test will be completed in time for the Congress and a report on it will be handed out as Appendix C during the Congress);

d) The helicopter mounting and high latitude tests will be executed after August 15 so their results will be reported in separate papers.

B) Description of the Inertial System

24. With respect to the importance of this new development in survey technology, I will give a short description of the system and its operation, avoiding the aerospace science and industry terminology, in order to keep the explanation simple and easier to understand.

25. Basically, the system is a precise inertial platform with gyroscopes keeping its three othoganal axes oriented in space. Each one of the three axes contains a gyroscopic control. The orientation system is the local vertical and local North, thus the three axes are oriented N-E-Down.

26. Each axis also contains a very sensitive accelerometer. These are the sensor-torquer types which, via a quantizer system, define the acceleration in that particular orientation direction. A computer keeps track of the accelerations in the horizontal channels which are integrated by time to give velocity, and integrated again by time to give the distances travelled in that particular direction.

27. The vertical (z) accelerometer is biased by the normal gravity which is also acceleration. The biased output is also double integrated to give relative elevation. Whenever the vehicle is stopped (where both the velocity and acceleration are zero), the z accelerometer measures the local gravity and calculates the normal gravity, elevation, anomaly and deflection of the vertical.

28. The travelling system executes acceleration and time measurements every 15 milliseconds, and the on-board computer continuously executes the double integration to keep track of the distance travelled on the Earth's surface in the N-E and Down directions. The computer calculates the amount of torque necessary to keep the platform travelling perpendicularly to the changing direction of the gravity. Thus, it commands the torquer system to follow the curvature of the reference spheroid exactly in the direction to be travelled.

C) The Hardware

29. The hardware consists of the following items:
- the Inertial Measuring Unit (IMU);
- the on-board computer;

- the power supply;
- the cassette recorder;
- the display and command unit.

The following is a short description of each of these essentials:

1. Inertial Measuring Unit

Its "stable element" consists of three quantizer cards, four gimbals, three accelerometers (N, E, Z), and two two-degree of freedom gyroscopes.
The gyroscopes are the floated, self-generating, air-bearing type. The two horizontal accelerometers have a sensitivity of 8 to 10 µg., and the Z accelerometer 1 to 2 µg. The gimbals are the torqued type. The quantizer cards "scale" the accelerometer's electronic impulses into dynamic units.
All gyroscopes and accelerometers drift, the drift is unwanted torque on the gimbals.
Fortunately, the gyroscope drift is usually very regular and quite linear in functions of time. The drift of the accelerometer is non-linear, partly due to its normal physical nature. The double integration results in distance errors being functions of t^2.

2. On-board Computer

This is also typical space hardware. Its dimensions are 15 x 15 x 40 cm. and it is capable of storing 336,000 digits. The six plates are triple-layered, and were manufactured under a microscope. It is a formidable electronic contraption characterised by the fact that the flow chart of the operations it executes fills a 186-page book. The computer has a double function: it calculates the mission and, at the same time, controls the system. It is quite a miniature "Hal" the quasi-human computer of "Space Odyssey 2000". (Its operation will be described later in Part D, The Software).

3. Power Supply

This unit supplies a 24-volt current to the system which consumes 65 amps/hour in the alignment phase and about 35 amps/hour in the running phase. The current is derived from an independent circuit from the vehicle's motor. In the case of engine stoppage, the system automatically switches over to battery operation for about one hour at the maximum.

4. Cassette Recorder

This unit also has a double function: it records the measurement data calculated by the computer and records all the actual biases, outputs, and changes occurring in the entire system, enhancing the operational maintenance; it monitors all the functions and is therefore able to describe which parts of the system have gone wrong and how.

5. Display and Command Unit

The display and control panel contains the following features:

- a continuous display of the state of the system;
- a visual display of the measurement data as required;
- it contains switches to command the system;
- it enables the operator to interrogate the system on the state of any of its functions;
- it also enables the computer to reverse communication to the operator.

30. All the hardware units can be placed anywhere in the vehicle, except the display and control panel which must be placed close to the operator.

D) The Software

31. As already stated, the software is immensely complicated.

32. Its basis is the Kalman filtering, which is something beyond the usual geodetic least-squares approach. However, the successes in space in the last decade would have been impossible without it. The following is only a brief description of its operation.

33. The Kalman filtering is an optimal algorithm formulated by using à-priori knowledge of the statistical nature of the errors which the navigation measurements are likely to contain. It gives the best estimate of the state of the system. Inherent in the Kalman filter is the information on how accurate its own estimate is. Thus, the Kalman filter enables us to evaluate the present performance of the systems by comparing the present propagation of errors to à-priori data. When the filter notices systematic changes developing in any of the error parameters, it automatically updates the à-priori knowledge and immediately develops a new error budget which now describes the new contribution of each error source to the total navigation error.

34. This changed error budget is the most useful tool for exposing a critical error source.

35. In simplified mathematical terms it could be said that the Kalman filter enables us to bring out some likely solutions for single equations with many unknowns by à-priori correlation. Because it can also forecast regularity, it can judge any new data for their reliability. It can also update the à-priori correlation and distribute the aggregate error differently if the systematic changes warrant it.

36. Beyond this, the computer is also mechanised to execute the adjustment of its own calculated data when outside information, for example: closing on a point of known coordinates or azimuth, has proven that the estimates were not exactly correct. It re-examines its own data in light of the

residual errors and uncorrected effects of noise sources, and it develops a new error budget to fit the outside information. All these computer activities occur in three-dimensional correlation.

37. The computer program also automatically calibrates the system when it is turned on and aligns the platform with the accelerometers in the N-E-D direction.

38. The system has a great number of error sources to consider, for example: a gyroscope alone has about 90 different possible sources of error in theory, only a few of which are of any significance and could show up in a mission of relatively short (few hours) duration. To consider and monitor all those error sources independently in the system would require a computer with a memory consisting of millions of bits which could not be on-board. Therefore, the computer has its Kalman filter mechanised to only consider seventeen of the most significant error sources.

E) The Method of Operation

39. The IMU or, more commonly, the platform is aligned into the local horizon and local north at the initial point of the survey. This is an hour-long automatic operation where the computer calculates and checks all the biases, levels the platform into the local horizon and aligns the north axis into the meridian.

40. Once the alignment hour is over, the vehicle is ready to begin measuring. The velocity of the vehicle is restricted to a maximum of 230 m./sec. change in acceleration. On a paved road one may travel at 60 Mph. or so, but on gravel or dirt roads lower speeds will aid in avoiding excess bumps which would be greater than the shock and vibration absorbing system could handle.

41. During the run, the computer calculates the exact distance travelled E-W and N-S every fifteen milliseconds, it calculates the necessary curvature of the NAD spheroid and commands the platform gimbal torques to torque the platform so that the horizontal plane remains tangent to the geoid, and the N. gyroscope axis remains pointed towards the North.

42. After a four minute run the vehicle must be stopped for twenty seconds. When the vehicle is not moving the accelerometers and the velocities should read zero; the Kalman filter looks at both values and considers the accelerometer errors to be caused by drift and from the velocity errors it estimates how this drift developed in time. The data are adjusted accordingly. This updating of accelerometers is called Zero Velocity Updating (ZUPT). Preferably, this ZUPT must occur in a four-minute time interval. The drift of the accelerometers is non-linear and after four minutes of time it drifts away so rapidly that precise integrations are no

longer possible. Once this adjustment has been executed the platform is automatically relevelled perpendicular to the new local gravitation direction. The amount of torque used for this relevelling describes the change in the direction of the deviation of the vertical, plus the uncompensated gyroscope drifts in the horizon.

43. When the point where the coordinates are to be determined is reached, a ZUPT is executed and the results in ϕ, λ, h, $\Delta\xi$(SUME), and $\Delta\eta$(SUMN), are registered in the memory. The computer can store the observed data for thirty stations.

44. Once the closing control point has been measured, the known coordinates can be entered and the smoothing ordered.

45. All the measurement data and the results of the smoothing are registered on a tape recorder and this information can be printed out in hard copy by a Wang computer.

46. Missions should preferably be run in straight lines, which is not easy to do even in a helicopter, since the vehicle must come to a stop every four minutes and although it is possible to do the ZUPT while hovering, this question has not yet been solved to our complete satisfaction. When rapid changes in direction must occur (for example: on a winding mountain road) a very small deterioration in accuracy may be expected.

47. The best results can be achieved in a traverse between two control points placed at extreme distances apart. Closing a traverse on the initial point itself may leave a scale error in the traverse.

48. At this time it appears that a traverse should be no longer than 30 to 50 km. Longer traverses must be broken down into segments, the junction points of which may be adjusted in an off-board computer.

49. Please note that the Geodetic Survey of Canada is not involved in measurement of gravity and therefore operates the system in the Autosurveyor mode. Readers having a special interest in gravity measurements should refer to Reference #2.

APPENDICES

50. The final experiments are in progress and should be completed in time for the Congress, therefore, it is necessary to describe the experiments and their results in the form of Appendices.

Appendix A

1) Tests on the D.M.A. testline in Los Angeles

1. The United States Defense Mapping Agency's testline is 42 km. long (Figure A-1). About half the line runs along city streets on land that is quite flat; both ends run up into considerable relief on tortuous twisting gravel and dirt roads.

2. At the time of the test, the portion from Station 6 to Station 1 was impassable due to heavy rains, washouts and landslides.

3. Tests were run between the following stations:
   - Station 6 to Station 11 - 4 times;
   - Station 6 to Station 12 - 2 times.

4. An E-W line was also established (see the dotted line on Figure A-1) and was run twice. The result of these tests is shown in Table A-1.

2) Conclusions

- A line 20 to 30 km. long can be established between two known points so that the mean error of any intermediate point is less than $\frac{1}{2}$ meter in all three coordinates.

- In this accuracy bracket the plumbing of the vehicle above the point is very critical. The EMR-designed plumbing device worked very well and resulted in an accuracy improvement of about fifty percent.

- It appears that the four minute ZUPT period can be increased with no great harm to the accuracy. This question should be thoroughly investigated.

- Lines that are only closed at the initial point are not of much use. A scale error will remain in the results.

- Since the gyroscope's drift is continuous, idling times (ZUPT, Mark, and Update) should be kept to a minimum.

- It seems that a mission should not be much longer than 30 to 40 km. Longer lines must be broken into sections and a final adjustment for the junction points will be needed.

- The smoothing process appears to be in need of some revision.

Appendix B

Experiments in the Ottawa area
(Incomplete when this paper was written)

1. The experiments in the Ottawa area were undertaken for two main reasons:

a) To obtain cold weather experience with the system; and
b) to discover any peculiarities in the system in a very regular and flat gravity field.

2. For these purposes, a line was established in October 1974 and was marked so that points could be located even when covered by many feet of snow. Figure B-1 shows the line which was established between four first-order triangulation points. The intermediate second-order points were adjusted into the first-order system.

3. The test series began with great difficulties due to the extended strikes with the airlines, airports and mail services. There were some errors in the manufacturing of the IMU and as well, the office computing-printing equipment malfunctions slowed down the operation.

4. Canada has also had one of the coldest springs on record and at the time that this paper was written, the temperature was still below freezing and the ground was covered by one to two feet of snow.

5. Table B-1 contains a description of the conditions and results of the 19 runs made on the testline before April 15.

6. Table B-2 contains detailed results of a sample run made on April 5, which was one of the most consistent runs.

7. Table B-3 contains the deflection of the vertical analysis of the last 12 runs.

Conclusions on the test as of April 15

- The driving method is very important. Excessive speed may result in accidentally hitting potholes (free-fall = 1000 μg) and the A-1000 accelerometer (the z-accelerometer) finds this difficult to handle.

- Gravel surfaces, especially when the vehicle is equipped with balloon tires, requires attention, but even "washboard" does no damage as it is only potholes that must be avoided.

- Gyroscopes are continually drifting, therefore it is advisable to run the lines at a sufficient speed. A driving speed that is too slow increases the drift-time.

- The three or four minutes of ZUPT apparently gives good results. Up to six minutes, the result deteriorates only slightly, but after that, the accelerometers drift too rapidly and irregularly.

Note: A six minute ZUPT limits the gyroscope drift by faster overall speed, but it causes more accelerometer drift. A three minute ZUPT needs more stops which increases the running time. We found that the four minute ZUPT is a good compromise.

- The vehicle may rock too much in strong winds causing incorrect ZUPT's.
- Snow and rain may increase road hazards but they do not influence the measurements.
- Cold temperatures (down to -14°C) definitely do not cause any disturbance.
- When a 26 km. long line is updated at both ends, it appears that any intermediate points are defined by a mean error (regardless of sign) of about 15 to 40 cm. in all three coordinates.
- Errors on longer lines appear to increase as $\sqrt{D}$ when updating is done at both ends.
- Closing a circuit will eliminate drift errors but will not correct orientation or scale errors. The same is valid for the so-called "dangling" lines which end in unknown points.
  This is the most important problem which must be solved. Under Canadian conditions, and with the scarcity of available control points, the most expected frequent condition will be the line with an uncontrolled end.
  We must find an acceptable solution for this problem during the following phases of the experiment.

General Conclusions (at the end of April 1975)

a.) The inertial system seems to be a highly accurate surveying tool, by which ϕ, λ, H and the deviation of the vertical can be determined with acceptable accuracy (might be second-order). We will not comment on gravity as Dr. Mancini is investigating this matter.

b.) It is especially suitable for breaking down first-order systems.

c.) The velocity of operation is tremendous. Large areas can be covered in a matter of a few days especially when using a helicopter. No towers, cutlines or line of sight is required and weather and refraction have no effect.

d.) The high price of the system is a deficiency at the present time, but this will decrease either when the development costs have been recovered or some competition appears on the market.

e.) Up to date, geodesy was used to extrapolate and

interpolate data for unreasonable distances because good data acquisition was a painfully slow and expensive operation.
We may now be able to replace mathematical dreams with the cold reality of measurements.

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2) Dr. A. Mancini; Dr. J. Huddle: "Gravimetric and Position Determinations Using Land Based Inertial Systems" (Paper presented at the A.C.S.M. Congress of 1975, in Washington).

3) S. Vamosi: "Test of the Ground Elevation Meter for Rapid Vertical Control" (Interim report, Geodetic Survey of Canada, 1974).

TABLE 1

Gyroscope Azimuths Derived From Field Observations
During The Summer of 1972 With A MOM GiB-2 Gyro-Theodolite
Using The Modified Observing Technique
And The Improved Mathematical Model For Computing

Chrono-logical Order	Station	Mean Gyro. Azimuth	σ_{mean}	σ_{mean}	No. of Det.	Gyro.Az. Minus Stellar Azimuth
1	East Pier*	356°-17'-32".4	1".0	2".4	6	0".0
2	Carden	209°-20'-32".4	1".8	4".0	10	
3	Herveux	66°-51'-41".7	1".0	2".4	6	
4	Mono	202°-10'-37".8	0".9	2".4	7	
5	Reservoir**	42°-11'-03".3	2".4	5".4	5	+0".9
6	East Pier*	356°-17'-33".3	0".9	2".2	6	+0".9
7	Lowther*	292°-06'-54".1	1".6	3".5	8	+1".5
8	Ganong	359°-12'-25".2	1".4	2".4	4	
9	Mace	18°-44'-47".8	1".0	1".0	2	
10	Smoothrock	331°-15'-45".6	3".1	3".1	2	
11	Moosonee	165°-00'-14".5	1".2	2".1	4	
12	Nagami	267°-06'-18".2	2".8	2".8	2	
13	Pled	67°-29'-53".0	0".2	0".2	3	
14	Hearst	51°-18'-15".8	1".8	3".1	4	
15	Longlac	211°-12'-06".0	0".0	0".0	2	
16	Lowther*	292°-06'-51".2	1".8	4".8	8	-1".3
17	Ogoki	106°-29'-48".3	1".5	1".5	2	
18	Pebble	250°-56'-59".3	1".0	1".3	3	
19	Quanta	61°-30'-07".8	0".9	1".6	5	
20	Rapid	271°-19'-51".5	1".4	2".7	7	
21	"64"	7°-23'-40".4	1".4	2".9	5	
22	Lost	260°-08'-31".4	1".6	2".8	5	
23	Lush	300°-40'-00".8	1".5	2".6	6	
24	Equan	126°-32'-45".6	0".9	1".3	3	
25	Attawapiskat	7°-47'-30".4	1".3	2".7	5	
26	"695"	277°-37'-33".6	1".3	2".3	5	
27	Winisk	124°-31'-12".2	1".1	1".8	5	
28	Island	106°-22'-24".2	1".7	3".0	4	
29	"6518"	201°-44'-23".5	1".9	2".8	5	
30	"6510"	24°-05'-16".0	1".4	2".1	4	
31	Goose	206°-44'-37".8	1".8	3".2	4	
32	Lowther*	292°-06'-53".7	0".5	1".3	7	+1".2
33	East Pier*	356°-17'-30".5	0".2	0".6	4	-1".9
		Mean	1".3	2".3		
		Sum			152	

**Grazing Line

Note: *Station used for control of the change in E-factor.

TABLE 2

1	2	3	4	5		
STATION NAME OR NUMBER	ϕ	AZIMUTH $\pm\sigma$	EXPECTATION BY COS ϕ (1)	EXPECTATION MODIFIED (2)	Δ' (σ-1)	Δ'' (σ-2)
749167	79° 59' 20"	246° 29' 44".4	±12".2	±6".0	-6".6	-0".4
749171	78° 52' 15"	42° 26' 01".4 ±4".9	±11".0	±5".6	-6".1	-1".2
749165	77° 45' 19"	202° 57' 30".9 ±4".9	±10".0	±5".3	-5".1	-0".2
Lougheed Shoran	77° 23' 43"	134° 13' 55".5 ±4".0	± 9".7	±5".2	-5".7	-1".2
749172	76° 25' 00"	300° 02' 09".2 ±5".6	± 9".0	±5".0	-3".4	+0".6
749177	76° 15' 00"	2° 25' 48".0 ±7".3	± 8".9	±4".9	-1".6	+2".4
749162	74° 49' 06"	308° 51' 40".9 ±4".0	± 8".1	±4".6	-4".1	-0".6
Resolute	74° 43' 12"	75° 24' 52".9 ±5".2	± 7".9	±4".6	-2".7	+0".6
Pond Inlet	72° 41' 43"	191° 56' 33".0 ±1".6	± 6".9	±4".3	-5".0	-2".7
Home	71° 33' 30"	42° 47' 22".8 ±6".6	± 6".7	±4".2	-0".1	+2".4
				Σ	-40".4	-0".3

Note: σ = standard error of a single determination.

TABLE A-1

Errors of intermediate points in cm.

Date	Nature of Test	Description of line	ϕ RMS	ϕ σ	ϕ MAX	λ RMS	λ σ	λ MAX	H RMS	H σ	H MAX
Feb. 4	POS.UD	2 x 21 km. (6-11-6)	40	44	46	35	50	70	28	34	43
Feb. 5	POS.+AZ.UD	2 x 27 km. (6-12-6)	32	39	64	18	22	-18	41	50	94
Feb. 7	POS.AZ.UD with corrected smoothing	2 x 21 km. (6-11-6)	29	33	60	11	12	22	34	37	51
Feb. 8	POS.UD	E-W line 2 x 29 km.	6	6	6	42	52	54	19	19	-20
Feb. 10	POS.UD	E-W line 2 x 29 km. quantizer modified	24	27	28	9	12	14	14	17	18
Feb. 10	Static test for new plumbing system		8			13			5		
Feb. 10	Measuring the same point in four different directions		4			3			4		
Feb. 11	POS.UD	2 x 21 km. (6-11-6) Using IPS program	39	42	50	35	40	45	28	32	60
Feb. 12	POS.UD A-21 program	2 x 21 km. (6-11-6) Using new Plumbing syst.	17	17	20	31	38	54	21	26	40

TABLE B-1

Results of the Ottawa Experiments

No.	Date	Run		σ	MAX.	Δξ	Δη	Remarks
1	3-12	1	φ λ H	.49 .21 .26	.65 .31 .47			Trial run, observer with some training.
2	3-13	1	φ λ H	.67 1.00 .47	.96 -1.64 .64			New operator, first time on console.
3	3-13	1	φ λ H	.63 1.33 .54	1.14 -2.07 - .60			New operator, first time on console.
4	3-14	1	φ λ H	.60 1.35 .32	.74 -2.05 - .38			Excessive gyroscope drift. Inverse airflow discovered.
	3-15							Attempts made to define biases for gyros and accelerom. after changing airflow.
5	3-17	1	φ λ H	1.12 .39 .08	1.48 - .59 .10			East gyro alignment error. Recalculating and setting new heading biases. Time: 90 minutes.
6	3-21	1	φ λ H	.55 .14 .36	.88 .22 .60			Heading bias improved, gyro alignment still not satisfactory.
	3-26							Recalculating and re-setting acc. bias E.
7	3-27	1	φ λ H	.08 .26 .30	.18 .54 .47	-1.2	+0.8	First satisfactory performance. Time: 134 minutes.
8	4-1	1	φ λ H	.10 .07 .14	- .17 .10 - .22	-1.4	+0.6	Calm weather, dry roads. Time: 120 minutes.
9	4-2	1	φ λ H	.66 .59 .74	.77 .83 -1.24	-2.4	+1.9	Wheel bumped in potholes. Snowstorm began.
10	4-4	1	φ λ H	.32 .16 .28	.46 .26 - .49	-1.3	+0.5	Roads not properly cleaned after snowstorm. Snowing and high winds.
11	4-5	1	φ λ H	.14 .09 .14	.40 .37 .48	-1.4	+2.4	Very careful driving on deteriorated road conditions. Time: 135 minutes.
12	4-5	2	φ λ H	.24 .28 .43	.43 .49 .92	-0.8 -0.3	+0.4 -0.9	Updating also in center at Manotick. Road conditions still very bad.

No.	Date	Run		σ	MAX.	Δξ	Δη	Remarks
13	4-7	3	φ	.59	.86			No updating - only run straight through and back.
			λ	.79	.98			
			H	.28	- .41	-1.9	+0.9	
14	4-7	4	φ	.44	.67			Updating only at end. Some bumping in potholes.
			λ	.34	.61	-0.3	+1.9	
			H	.20	- .38	+0.1	-0.3	
15	4-8	1	φ	.23	.52			6 minute ZUPT.
			λ	.39	.78			
			H	.18	.30	-3.3	+0.2	
16	4-8	1	φ	.27	.46			3 minute ZUPT.
			λ	.17	.26			
			H	.16	.26	-1.8	+0.8	
17	4-9	1	φ	1.18	-1.82			9 - 10 minute ZUPT.
			λ	1.93	-4.03			
			H	1.37	2.83			
18	4-10	1	φ	.19	.30			4 minute ZUPT.
			λ	.14	.16			
			H	.28	.38	-0.4	+1.1	
19	4-15	1	φ	.55	.89			Very badly broken up hard-top and gravel roads in spring melting.
			λ	.43	.78			
			H	.09	.36			

Note:

a) Types of runs (length and nature):

1. TIE-MAN-TIE, update both ends, 2 x 26.3 km., 8 intermediates.
2. TIE-MAN-JOHNSON-MAN-TIE, update on ends and center, 2 x 45 km., 12 intermediates.
3. TIE-MAN-TIE, 52.6 km., update only at TIE, 7 intermediates.
4. TIE-JOHNSON-TIE, update on both ends, 2 x 45 km., update on both ends.

b) All errors are given in meters.

c) Deflection of vertical differences between end points of the lines are given in seconds of arc. Where two figures are given, data are calculated for Manotick (upper figure) and Johnson (lower figure).

TABLE B-2

The First Run On April 5

		ϕ			λ			Elevation		
		Known	Mean	Error	Known	Meas.	Error	Known	Meas.	Error
Direct	TIE	45°19'43".850	43.850	.000	75°52'03".293	03.293	.000	115.55	115.55	
	39	17'30".885	30.892	.007	52'13".679	13.696	.017	108.97	109.05	.08
	DUBROY	14'32".325	32.338	.013	50'36".754	36.764	.010	96.36	96.48	.12
	642	15'35".131	35.132	.001	49'32".736	32.744	.000	94.64	94.81	.17
	71	16'09".170	09.168	-.002	45'05".058	05.063	.005	99.74	100.22	.48
	MANOTICK	14'51".680	51.680	.000	42'48".093	48.093	.000	106.04	106.04	.00
Reverse	71	16'09".170	09.179	.009	45'05".058	05.044	-.014	99.74	99.42	-.32
	642	15'35".131	35.138	.007	49'32".736	32.729	-.007	94.64	94.24	-.40
	DUBROY	14'32".325	32.323	-.002	50'36".754	36.738	-.016	96.36	96.50	.14
	39	17'32".325	30.887	.002	52'13".679	13.670	-.009	108.97	109.28	.31
	TIE	19'43".850	43.850	.000	52'03".293	03.293	.000	115.55	115.55	.00
$\sigma_{sec.}$				±.00671			±.01116			
$\sigma_{cm.}$				20.7 cm.			24.3 cm.			±28.7 cm.

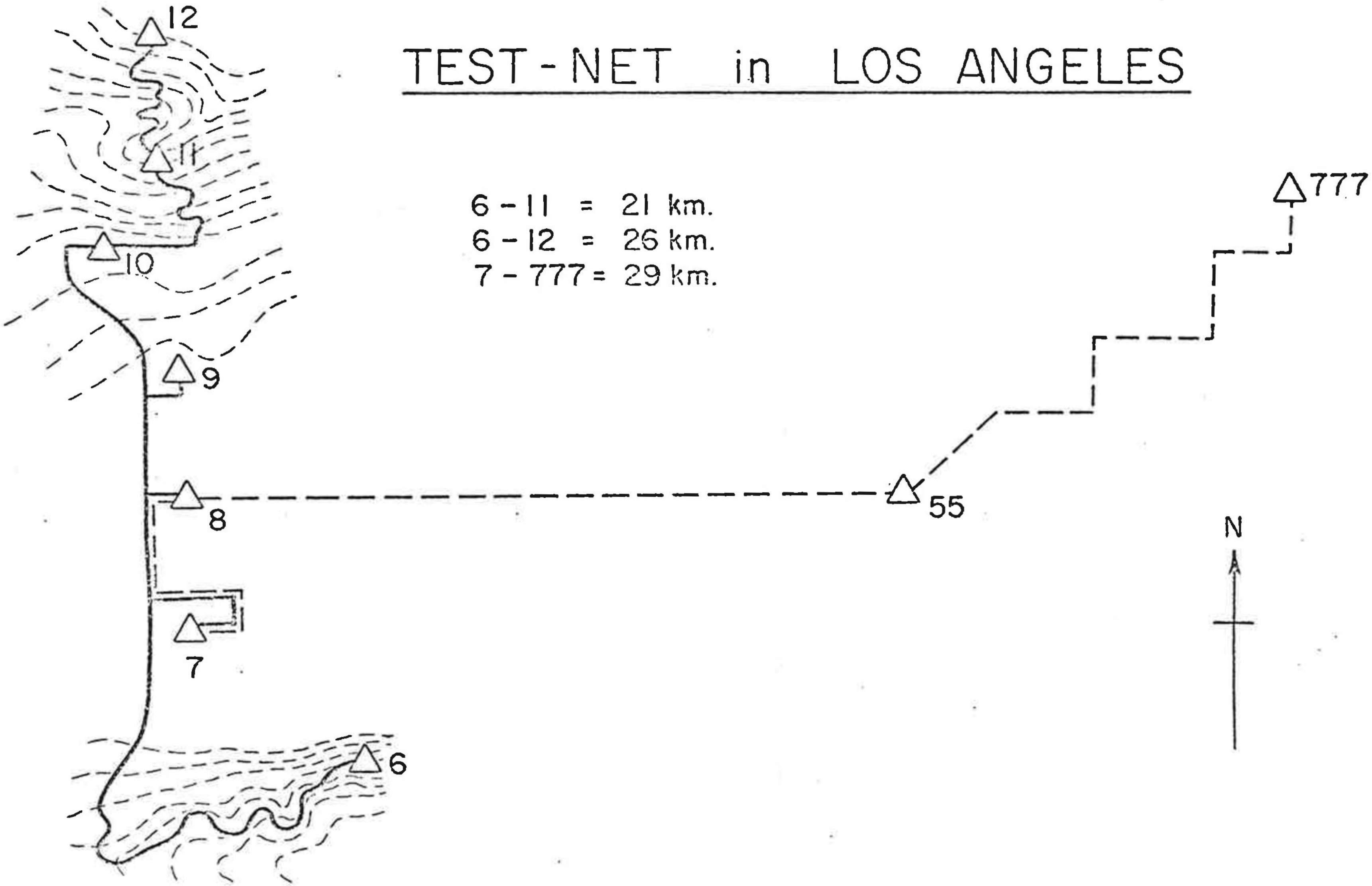
	ϕ Mean	ϕ Error	λ Mean	λ Error	Elevation Mean	Elevation Error
TIE	43.850		03.293		115.55	.195
39	30.8895	.0045	13.683	.004	109.165	.13
DUBROY	32.3305	.0055	36.751	-.003	96.49	-.095
642	35.135	.004	32.741	.005	94.525	.14
71	09.1735	.0035	05.0535	-.0045	99.88	
MAN	51.680		48.093		106.04	
$\sigma_{sec.}$		.00443		.00419		
$\sigma_{cm.}$		13.7 cm.		09.1 cm.		14.4 cm.

TABLE B-3

Relative Deviation of The Vertical From Station Tie to Station Manotick

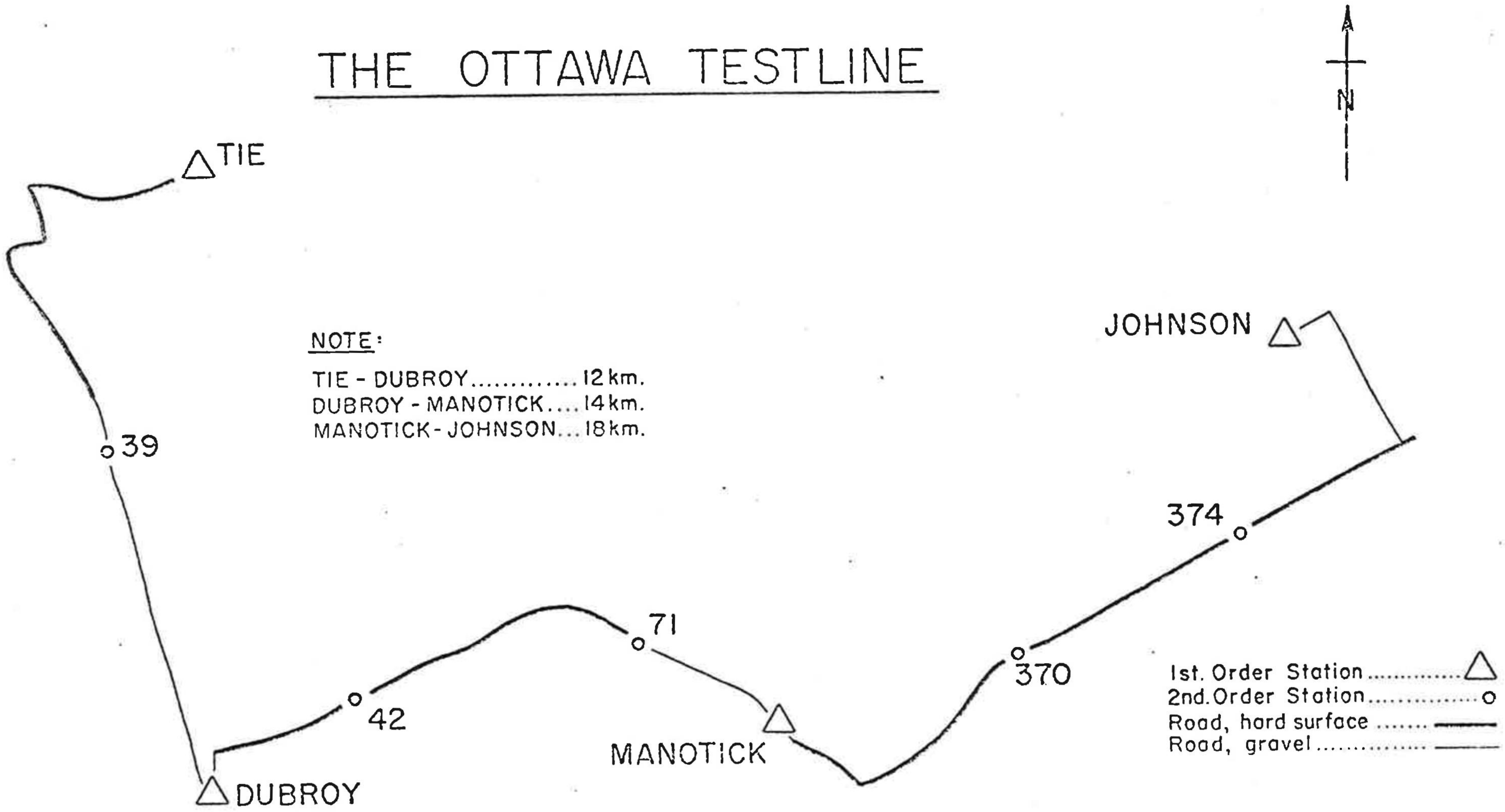
Set	$\Delta\xi$	$\Delta\eta$
8	-1.2	0.8
9	-1.4	0.6
10	-2.4	1.9
11	-1.3	0.5
12	-1.4	2.4
13	-0.8	0.4
14	-1.9	0.9
15	-0.3	1.9
16	-3.3	0.2
17	-1.8	0.8
18	-0.4	1.1
19	-1.3	-0.1
Mean	-1.45 ±0.79	+0.95 ±0.72
Known	-1.3	+1.0

Figure A-1



TEST-NET in LOS ANGELES
6 - 11 = 21 km.
6 - 12 = 26 km.
7 - 777 = 29 km.
12
11
10
9
8
7
6
55
777
N

Figure B-1



THE OTTAWA TESTLINE
N
TIE
JOHNSON
NOTE:
TIE - DUBROY............12 km.
DUBROY - MANOTICK....14 km.
MANOTICK - JOHNSON...18 km.
39
374
71
370
42
MANOTICK
DUBROY
1st. Order Station
2nd. Order Station
Road, hard surface
Road, gravel