SOME ASPECTS OF MATHEMATICAL GEOGRAPHY

J. E. R. ROSS, D.T.S., Dominion Geodesist

Down through the ages we find the fundamental conceptions of geography are mathematical and deal with the relations of space and form. The figure and dimensions of the Earth are the first of these, and are ascertained by a combination of actual measurement of the highest precision on the surface and by angular observations of the positions of the stars. The science of geodesy as now established is a part of mathematical geography, of which the arts of surveying and cartography are well known applications. In what follows an effort will be made to embrace in broad outline some of the factors which increase geographic knowledge.

EARLY CONCEPTIONS

To primitive man the Earth was a flat disk with its surface diversified by mountains, rivers and seas. In many theories of creation this disk was encircled by water, unmeasurable by man and extending to a junction with the sky; and the disk stood as an island rising up through the waters from the floor of the universe, or was borne as an immovable ship on the surface. Such was the belief of the Babylonians and Hebrews; Homer states the same idea and Hesiod regarded the Earth as a disk midway between the sky and the infernal regions.

When it was realized that the same sun and stars reappeared in the east after their setting in the west, a modification of the downward extent of the Earth had to be made, and man slowly realized that the Earth was isolated in space, floating freely as a balloon, and much speculation was associated about that which supported the Earth. Tunnels in the foundations to permit the passage of the sun and stars were suggested; the Greeks considered twelve columns to support the heaven, and the god Atlas appears condemned to support the columns; while the Egyptians had the Earth supported by four elephants, which themselves stood on a tortoise swimming on a sea. Earthquakes were regarded as due to a movement of these foundations; in Japan this was considered to be due to the motion of a great spider, an animal subsequently replaced by a catfish; in Mongolia it is a hog; in India, a mole; and among some of the North American Indians, a giant tortoise.

GREEK INFLUENCE

The first forms of scientific curiosity regarding the world in which primitive man moved were satisfied by such mythologies. The flat form of the Earth appears to have been in general acceptance until the days of Pythagoras (b. 582—d. 506?), although it was not until Aristotle (384-322), founder of scientific geography, that the doctrine of the spherical form was advanced with vigour. His arguments are those which we employ today:—the ship gradually disappearing from hull to mast as it recedes from the harbour to the horizon; the circular shadow cast by the Earth on the moon during an eclipse, and the alteration in the appearance of the heavens as one passes from point to point on the Earth's surface. He records attempts made to determine the circumference; but the first scientific investigation in this direction was made 150 years later by Eratosthenes (276-194). The spherical form, however, only became generally accepted after the Earth's circumnavigation by Magellan (1480-1521).

Eratosthenes, Greek scientific writer and librarian at Alexandria, was one of the most learned men of antiquity. He is said to have died of voluntary starvation, being threatened with total blindness. His great achievement was his measurement of the Earth. Being informed that at Syene (Assuan), on the day of the summer solstice at noon, a well was lit up through all its depth, so that Syene lay on the tropic, he measured, at the same hour, the zenith distance (vertical angle) of the sun at Alexandria and obtained the

value 7° 12′. He thus found the distance between Syene and Alexandria (known to be 5000 stadia) to correspond to 1/50th of a great circle, and so arrived at 250,000 stadia as the circumference of the Earth.

We are not now interested in the degree of accuracy attained by Eratosthenes and others using somewhat similar methods; suffice it to say that his principle of measuring an arc to determine the curvature and dimensions of the Earth is in effect today.

The knowledge of the Greeks was all lost as their civilization declined, and for more than a thousand years Europe, sunk in intellectual darkness, made no inquiry concerning the size or shape of the Earth. Only in Arabia were the sciences at all cultivated during this period. In the fifteenth century, the first gleams of light broke in upon the darkness of the middle ages, and men began to think again of the Earth's shape and size. Navigators began to doubt that its surface was a level plane, and here and there one, like Columbus, asserted it to be globular. In the sixteenth century, the doctrine of the spherical form was again generally accepted, and one of the ships of Magellan, after a three years' voyage, accomplished the Earth's circumnavigation.

THE APPROACH OF SCIENCE

With the acceptance of this idea arose also the question as to the size of the globe. In 1617 the Dutch astronomer, Snellius, conceived the idea of triangulating from a known base line, and thus, near Leyden, he measured a meridian arc. To him belongs the credit of introducing the principle of triangulation extensively used in modern surveys, the angles of his triangles having been measured with a quadrant and semicircles with sights attached. The application of the telescope to angular instruments was the next important step and Picard in 1669 measured an arc near Paris with a quadrant of ten feet radius, furnished with a telescope in which he had inserted cross hairs. Compared with the open sight method of previous geodesists, this innovation of cross hairs added greatly to the accuracy of his results. He found the length of one degree to be 57,060 toises (1 toise=6.3946 English feet), and this was the result that Newton used when making his famous calculation which proved that the moon gravitated toward the Earth.

Hitherto, geodetic observations had been confined to the determination of the magnitude of the Earth as a sphere, but a discovery made by Jean Richer (d. 1696) turned the attention of mathematicians to its deviation from a spherical form. In book III of Newton's "Principia" (first edition 1687) are discussed the observations of Richer, who having been sent to Cayenne, in equatorial South America, on an astronomical expedition, noted that his clock which kept accurate time in Paris, there continually lost two seconds daily, and could only be corrected by shortening the pendulum by about 1/12th of an inch. Newton showed, after making allowance for the effect of centrifugal force, that the force of gravity at Cayenne, compared with that at Paris, was too small for the hypothesis of a spherical globe; in short, that Cayenne was farther from the centre of the globe than Paris, or that the Earth was an oblate spheriod flattened at the poles.

Newton's philosophy did not gain ready acceptance in France and this investigation in particular called forth much argument. Between 1684 and 1718, J. and D. Cassini, starting from Picard's base, carried a triangulation northwards from Paris to Dunkirk and southwards from Paris to Collioure. For the northern portion the length of a degree was found to be 56,960 and for the southern portion, 57,097 toises. The immediate inference from this was that, the degree diminishing with increasing latitude, the Earth must be a prolate spheroid. This conclusion was totally opposed to the theoretical investigations of Newton and Huygens, and the Academy of Sciences of Paris determined to apply a decisive test by the measurement of arcs at a great distance from each other—one near the equator, the other in a high latitude.

Accordingly, two parties set out in 1735, one for Lapland, the other for Peru. The Lapland expedition measured its base upon the frozen surface of the Tornea river, executed its triangulation and latitude observations, and returned in two years. The Peruvian expedition measured two bases, executed its triangulation and latitude work, and returned in seven years. From these the following results could be written:

Arc	Mean Latitude	Toises in One Degree
Lapland	N. 66° 20′	57,438
France	N. 49 22	57,060
Peru	S. 1 34	56,728

The length of one degree of latitude was thus shown to increase with latitude and proved beyond question the correctness of Newton's theory.

The era of modern geodesy may be said to have begun at the close of the eighteenth century, with the proposals in 1783 to connect by triangulation the observatories of Paris and Greenwich; and in 1791 to determine the Earth's meridian quadrant from the measure of an arc of about 9° 40′, extending south from the extreme northern end of France; one ten-millionth part of this meridian quadrant was to be used as a standard unit of length to be called a metre.

General interest in the subject awakened, geodetic surveys began to extend over Europe, and the degree of accuracy attained in these, in some respects at least, compares not unfavourably with that of the present time. For instance, in that part assigned to the English, large triangles were easily closed with the 36-inch Ramsden theodolite to within 3 seconds of their theoretical value. Modern practice in first-order triangulation has accepted this as a maximum limit and specifies an average closure of one second observation error.

The accumulated data have been made available to international bureaus from time to time with the result that in the present state of knowledge the Earth's dimensions are as follows:

SPHEROIDS OF REVOLUTION

	Equatorial Semi-Axis	Polar Semi-Axis
Clarke 1866	6378206.4 metres	6356583.8 metres.
Hayford 1909	6378388.4 "	6356908.8 "

The first spheroid is in practical use for all North American surveys and the second has been adopted by the International Union of Geodesy and Geophysics for scientific purposes.

The need for geodetic surveys employing methods of the highest precision and working from the whole to the part in order to minimize accumulation of error has been recognized by all highly developed nations. Forty cultural nations are now contributors of data to the International Union of Geodesy and Geophysics, the only organization which brings together the various earth sciences and treats them from the viewpoint of the world as a whole.

The Geodetic Survey of Canada was established in 1907 under the direction of the then Chief Astronomer, Dr. W. F. King, and authorized to conduct field operations relating to Triangulation, Precise Levelling, Precise Astronomy and related sciences, which would furnish the necessary control for surveys of detailed mapping, engineering work, in the delimitation of boundaries, and other operations of a national character in which precision is a foremost requirement.

TRIANGULATION

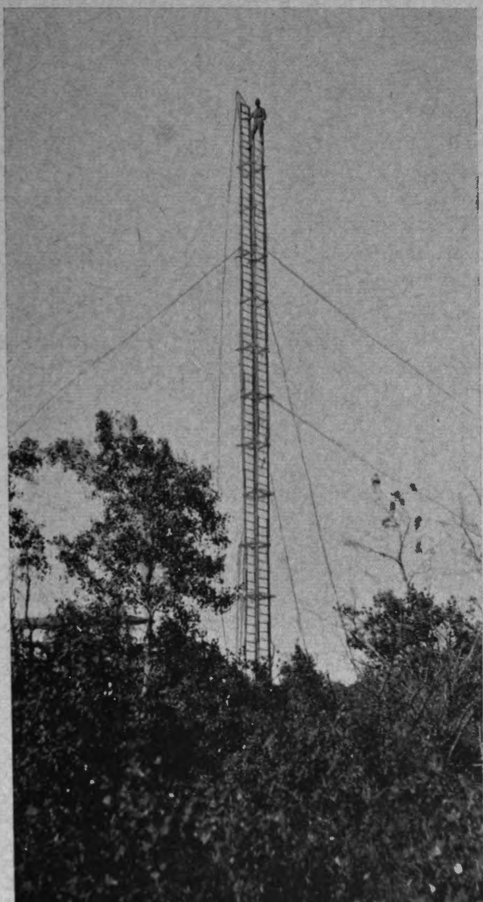
When Snellius in 1617 introduced his method of determining position on the Earth's surface by the measurement of a base line from which he could expand to other points simply through the measurement of the angles of continuous triangles, he used only what today is common knowledge, i.e., that given one length and two angles of a triangle, the remaining parts may be deduced. When, however, the third angle is also measured difficulties arise in that the sum of the angles of each spherical triangle now must precisely be equal to a quantity in excess of 180° dependent on its area, due to the earth's shape. Snellius arbitrarily distributed his misclosures, in some cases amounting to 3 minutes of arc, an amount 180 times greater than present day standards. By this we gain some conception of the perfection attained since then in instrumental equipment.

Looking north from station Lucky (elevation about 4,000 feet) of the Vancouver Island secondary triangulation net. The mountain in the foreground is a sister peak half a mile from the main station. This picture shows how rough the mountain tops are and the difficulty of access for triangulation in this area.

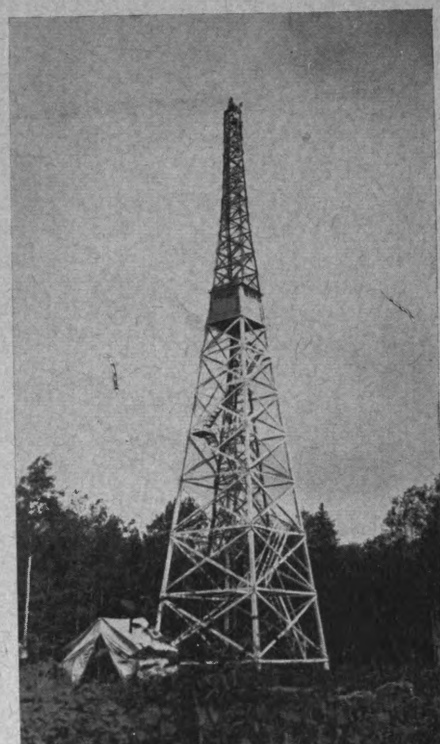
Precise triangulation today involves the Snellius principle. A selection of the stations to be occupied is made by a reconnaissance engineer, who first of all gathers what knowledge is available of the high relief of an area so that he may enter the field for a more accurate examination to determine the amount of clearing to be done, or the height of tower necessary to insure intervisibility between stations. He is faced with a problem in design. His observations are necessarily rough and pertain to general location, and are often from high up in the branches of a swaying spruce or pine tree. The geometrical figures must be well shaped to insure mathematical strength, be economical for occupation, be placed in the area of immediate use not losing sight of continuity with projected or proposed schemes. In all cases his figures must allow independent routes of calculation, such as a quadrilateral with two diagonals. An analogy in strength is a rectangular gate frame with one diagonal brace; if the other brace is attached, the gate becomes more rigid, hence stronger.

Following the selection of stations, each site must be prepared for use of the instrument and involves the cutting of trails and lines, building a tripod to support the instrument, and permanent marking by bolts, tablets, or concrete monument. For tower sites the inner structure, the instrument tripod, must be free of the outer, as movements of the observer would otherwise adversely affect the measurement of angles.

Electric signal lights operated by clocks are directed from the surrounding stations to the observer, affording targets for the angular observations, made at night as accuracy is improved, and moreover the lengths of lines usually prevent daytime work. Lines 93 and 99 miles long have been observed over water on the Atlantic and Pacific coasts. In some cases lines have been used where the terminal elevations are not quite sufficient to clear the water surface under usual atmospheric conditions, and all must then be in readiness to observe during the periods of abnormal refraction that occur following air conditions which produce inverted temperature gradients over the area. The light rays thus received at the instrument do not always give a single image and as many as eight overlapping images, indicating as many paths, have been seen. The observations subsequently show the rays were refracted vertically only, and hence without error to the horizontal angles. The horizontal angles are measured not less than 16 times.



Reconnaissance Tower, 80 feet high.



Ashton Tower, New Brunswick. Height of tripod 81 feet, height of lamp-stand 132 feet.

When the component geometrical figures in a triangulation net attain a length of some 300 miles, it becomes necessary to step down the size of figures and connect the main framework to a base-line of 4 to 10 miles' length, easily measured with 50-meter invar tapes to an accuracy of 1 inch in 10 miles. This affords a length control and the triangulation, when adjusted, must reproduce this length. The lengths of the tapes are found, both before and after a field operation, through comparison with the one-metre bar, a reproduction of the International metre.

The Geodimeter designed by Dr. Bergstrand to measure lines up to 20 miles in length by means of high frequency polarized light promises to replace the older method of measuring base-lines by invar tapes.

At a few stations it is necessary to observe astronomically for the latitude and longitude, and the direction from north, or the azimuth, of a line The astronomic values in this case are not used to determine the station's position—that comes from the triangulation—but these values are used in combination with the triangulation values to secure an azimuth for orientation of the net, free from the deflection of the vertical.

A net of triangulation may be regarded as a continuous chain of overlapping triangles extending from one base-line to the next and including azimuth controls for certain triangle sides. The angles must be subjected to an extensive mathematical process so that each triangle closes exactly, and that any length calculated through different routes is in exact agreement. The calculation of all geographic positions and azimuths is then made on a particular mathematical surface which fits very closely the sea-level surface extended through the land surface.

The accuracy of modern surveys may be judged by an example. In co-operation with the United States Coast and Geodetic Survey, the triangulation of the two geodetic surveys in the area easterly from Toronto to New Brunswick in Canada and through New York and Maine in the United States has been adjusted as a unit to eliminate all the misclosures formed by 6 loops, of which the outside perimeter is 2,200 miles and the misclosure 34.5 feet, or a ratio of less than 1 inch in 5 miles. The discrepancies have been removed and all the work placed on the 1927 North American datum, which simply means that geodetic surveys on this continent are international in character and reduced on a uniform basis.

TRILATERATION

The immediate necessities of control for mapping in our northern regions have resulted in the use of shoran electronic equipment for the measurement of the lines joining stations separated on the average by 210 miles. In this system the geometrical figures are more strongly interlaced than in triangulation where angles are measured, a necessity because of the fewer geometrical conditions which exist between the measured lengths. The system is called trilateration.

Line-crossing operations were commenced in 1949 and continued in 1950 and 1951. With the improvement in operating efficiency gained through experience, a marked increase in the amount of work accomplished is noted in that the number of lines measured in the three years has been 28, 56, and 65 respectively. To date the longest line measured is 330 miles. The resulting geographic positions of the stations by this method are not yet comparable in accuracy to that obtained by first-order triangulation but are definitely superior to astronomic positioning, the only other feasible and economical method at present available due to the remoteness of the area. Shoran-controlled photography in which the position of the plane is obtained at the time of each camera exposure is the next forward step and is now under adoption by the mapping bureaus.

PRECISE LEVELLING

Precise levelling is the art of determining the relative heights as referred to a surface which cuts the direction of gravity everywhere at right angles. When a line of instrumental levels is begun at sea-level, a series of heights is determined corresponding to what would be found by perpendicular measurements upwards from the surface of water communicating freely with the ocean in underground channels. The oceans, undisturbed by the action of wind and tide, may thus be considered to be canalized through the continents.

In relation to the Earth we then have a continuous water surface—or shape—called the Geoid, which is a closer approximation than the mathematical spheroid, but at present its form can only be determined for local areas.

Mean sea-level is the natural plane of reference and is determined by the mean of observations on automatic registering gauges taken over a period of years. The daily rise and fall of tide is evident, but the mean position from day to day is not constant, and observations must cover a long period to provide the necessary datum of zero elevation.

In executing continent-wide levelling, the operations of about nine instrument set-ups in a mile, the readings on forward and backward graduated staffs properly standardized, the balancing of length of sights, and other factors require constant care to avoid systematic error.

Fundamental Bench Mark Pier in Memorial Park, Assiniboia, Sask. The bench mark is a bronze tablet, three inches in diameter set in the top of the monument. On a bronze plate on one face of the monument is inscribed the elevation of the bench mark above mean sea level.

The Canadian levelling system is based on six tidal stations defining mean sea-level; Halifax, Yarmouth, Father Point on the Atlantic and Churchill; Vancouver and Prince Rupert on the Pacific. The elevation obtained at Vancouver from Halifax across the continent differed by only 0.6 feet from the tidal value, and this was considered to arise solely in the operations and not to be due to any difference in sea-level on the two coasts.

Where both triangulation and levelling operations are available in areas subject to earthquakes, the amount of displacement of the Earth's crust after a severe quake, both vertically and horizontally, may be obtained through remeasurement of the angles and relevelling. This has been utilized in one area of Quebec and quantitatively confirmed the deductions arrived at by the geologist and the seismologist.

With the close of the 1951 field season the system comprised 50,274 miles of levelling. Elevations are available for permanent marks set in engineering structures at intervals of three to four miles, along the railway systems, and in public and other buildings in the towns through which levelling has been done. A fundamental bench-mark monument, whenever possible, at intervals of about 40 miles, has been constructed in a park or public square, the selection of the site being carried out in collaboration with the municipal authorities in order that, as far as can be foreseen, the location shall be free from destruction.

PRECISE ASTRONOMY

In precise astronomy we obtain the latitude and longitude of a point and the azimuth of a triangulation line by observations on the stars. The instruments are of the precision type equipped with long level vials and sufficiently heavy to remain stable under changing temperatures: in fact they constitute a portable observatory. A wireless set is also required to receive broadcasts of Greenwich time by national observatories.

As mentioned earlier, the astronomic position is not used directly in the triangulation because the instrument when levelled is in a position perpendicular to the direction of gravity and not perpendicular to the spheroid upon which the calculations are made. It is possible, however, to utilize both the astronomic and geodetic positions of points in the study of the geoid.

Custom has dictated the legal definition of many boundaries by astronomic position because of two factors, i.e. the location of boundaries far removed from geodetic control required to be monumented for political purposes, and the invariability of a value based upon the stars. Due to the uneven distribution of matter in the Earth's crust and irregularities of topography, the plumb line is deflected; the deflections in Canada on the average amount to 3 seconds of arc, equivalent to 300 feet. Thus for the 49th parallel of latitude the observations indicate a series of points through which the chords defining the arc must pass, and actually the boundary is sometimes north and sometimes south of geodetic latitude 49°. Similarly, a meridian projected astronomically cannot be a continuous straight line when the distance between terminals is of considerable size. The largest deflection so far observed in Canada is at Cap Madeleine L.H., Gaspé, where the astronomic latitude is displaced 2053 feet to the north of the geodetic latitude, indicating the strong attraction of the Shickshock mountains lying to the south. There is very little change in the east-west direction.

While the astronomic method is used extensively in boundary surveys and in the control of aerial surveys for small-scale mapping, its use as a control for general survey purposes becomes deficient when two astronomic stations are relatively close to one another as the distance calculated therefrom does not agree with the correct measurement between them on the ground. In boundary demarcation the astronomic position defines where the boundary line must pass through, but no stipulation can be made that the intervening distance, as measured on the ground, is to be consistent with that which can be deducted mathematically from the data. In this sense its use then is directional rather than one of length control. The advent of easy transportation in the remote areas by aeroplane dictates the use of astronomic positions of secondary precision as an economic factor in the production of maps of small scale of our vast hinterland now urgently required in its development.

MAP PROJECTION AND LONG LINES

An important contribution of the science of geodesy is the development of the laws by which the curved surface of the earth may be represented on a plane, as it is through these laws that the details of mapping are framed in their proper place. Map projections are of many kinds and are designed for special purposes. Instances are for the mariner, who needs those upon which he can lay courses of either constant bearing or shortest course; and the air navigator, who needs one in which the courses to be flown are angle true. Neither of these is suitable for flight in the polar regions, for which other types are designed.

The schoolroom globe provides a ready means of visualizing the shortest distance and course between two points, but upon the spheroidal earth there are some differences which are not generally realized. Thus for a plane, the shortest distance between two points is a straight line; for a sphere, an arc of a great circle; and for the spheroid, a curve somewhat like a very flat letter "S". We may clarify this by an example. Let us suppose that we wish to know the shortest course over the Earth's surface for two points on the equator diametrically opposite. The meridian passes through the pole, latitude 90°, and its course is the least and is 21 miles shorter than along the equator, but if we move our far point along the equator 18 miles closer our shortest path is still northward but now only reaches latitude 60°. On some world maps the straight line joining two widely separated points cannot give the shortest course except for the meridian; in other cases this course lies on the side of the straight line away from the equator.

THE GEOID

We have noted that the differences in astronomic and geodetic positions are due to a deflection of the plumb line as caused by the uneven distribution of matter in the Earth's crust. Given sufficient data for an area, these deflections may be used to construct the form of the geoid—or a continuation of the undisturbed sea surface through the land surfaces—with reference to our mathematical spheroid.

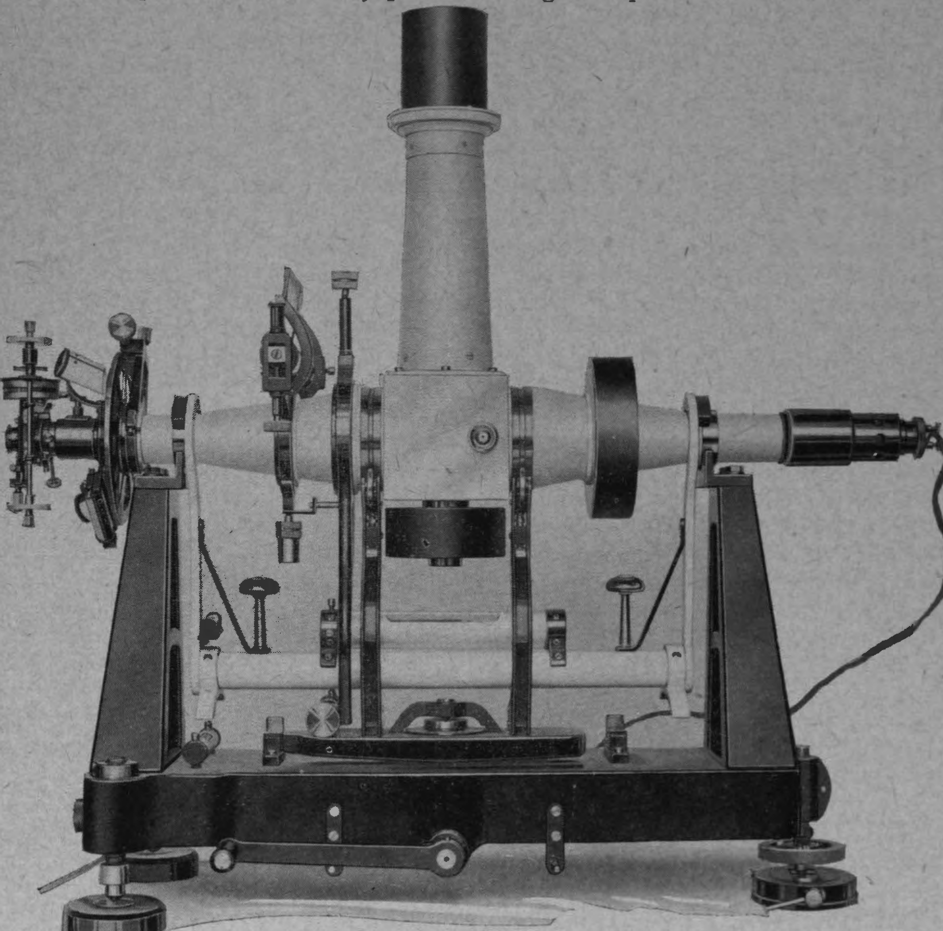
The method is similar to that of drawing contours. At each station we are given the direction of slope and its amount; and its effect, for lack of intermediate data, is assumed to extend half way to the adjoining stations. If a value of the separation from the spheroid for one station be assumed, it is possible to deduce relative elevations for each of the stations, and then from the lines of equal value, visualize the form of the geoid. We cannot know by how much the geoid lies above or below the spheroid until connected surveys have been made throughout the world.

The gulf of St. Lawrence area bordering on Gaspé, New Brunswick, and Nova Scotia, has been given a critical examination, and a predominant rise of 5 metres occurs from Calais, Me., eastward to the Magdalen Is. The fall from the Magdalens to the St. Lawrence River is 9 metres and extends through the Shickshocks of Gaspé with only slight modification. In general a rise in the geoid indicates either excessive density within the area, or a deficiency of density in the surrounding area.

The St. Lawrence River valley forms the separation of two distinct geological features, as to the north the Canadian Shield is exposed, and being composed of older and deeper rocks, its density should be greater than the area to the south; a geoidal rise may then be expected northward of the valley when data are available. The tentative conclusion is that the Gulf area is one of excess density, which receives some support in that the Atlantic coast to the south also indicates a geoidal rise towards the continental shelf. This view is also supported by United States and Canadian data in that the fall continues westward to the Great Lakes with the least geoidal height in the Lake Superior area.

With sufficient data we could sav that the contours so obtained represent the true form of the geoid in the Gulf area. Recently under co-operation with the then Commission of Government for Newfoundland, the entire Gulf area has been enclosed by triangulation, but unfortunately with rather limited astronomic data. The geoidal rise compiled from data to the west of the Gulf is fully supported by this further data in that the rise continues eastward through Newfoundland.

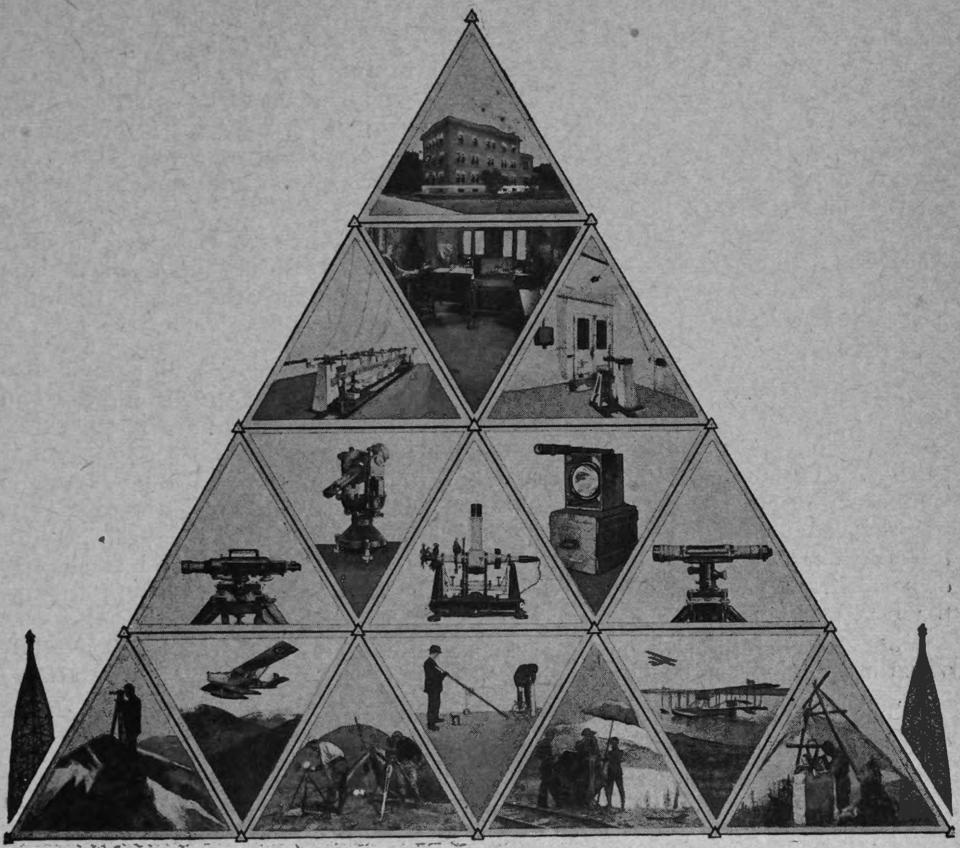
The outer portion of the Earth rests on the material of the interior in a condition of equilibrium which, presumably, is only disturbed by the shifting of loads in geological time over its surface. If we consider two unit areas, say of 100 square miles, one in the Rockies where elevations exceed 6000 feet, and another adjacent in the Prairies of 3000 feet elevation, it is easy to realize that it would be impossible for each of these volumes to exert equal pressure on bases at 2000 feet elevation directly underneath. At what depth, then, is the pressure exerted by prisms resting on equal areas uniform?



Transit of the broken telescope type used in astronomical determination of Latitude and Longitude.

The crust, or outer shell, contains materials of various densities at different places; and while supported by its own rigidity and strength, it is believed that a slow process of readjustment is constantly taking place through which excessive loads in an area are being compensated by plastic flow of the material underneath.

According to the principle of Isostasy (equal pressure), if the Earth's crust were cut into prisms of the same cross section by imaginary vertical planes, the prisms would have the same mass if the isostatic condition were perfect. Hence to allow for the topographic relief, we must say that for the higher ground the density of the material below is less; and the lower



Top—Geodetic Survey Building at Ottawa.

Second row, left to right—

North end of Standards building, showing fine-meter bar apparatus.
Office, Precise Level Division.
Fiducial point at south end of 50-metre comparator, in Standards building.

Third row, left to right—

Precise Level, U.S.C. & G.S. Pattern.
Modern Primary Triangulation Theodolite.
Astronomical Transit.
Electric Signal Lamp for Primary Triangulation.
Precise Level, Zeiss Model.

Bottom row ,left to right—

Observing on Secondary Triangulation.
Photographic and Transport Hydroplane.
Sending instructions to light keepers by heliograph.
Setting rear end of tape in Baseline measurement.
Observing Precise Levels in the Yukon Territory.
A Transport Hydroplane.
Observing on Primary Triangulation.

On flanks—

Triangulation Tower used near Chatham, Ont., with Lamp-stand extended 37 feet. Height of Lamp-stand, 147 feet.

the solid surface of the Earth, the greater is the density of the crustal material underneath.

The Gaspé region has been repeatedly subjected to elevation and depression as revealed by its geology. Confirmation of this is given by the charts of the Gulf of St. Lawrence which show the river canyon to be from 1000 feet to 1500 feet below sea level, indicating a subsidence from former times. Movements of elevation are indicated by the raised beaches around the coast and terraces at the mouths of streams. The valleys and summits exhibit all the characteristics of a drowned topography, which has not been exposed to erosion for any great geological time. With the spread of the Labrador ice sheet, a tremendous load was imposed to which the crust must have yielded. With its retreat, uplift must have begun, and the extent to which it has now been completed was the object of investigation.

As before, we make use of the observed deflections and assume that isostasy prevails for the area. The effect of the topography on the deflections may be calculated from the best maps available for the region and is subtracted from the observed values. What remains is accounted for by variations in density. Through the assumption that isostasy exists, various depths of compensation are tried, and that solution which most nearly harmonizes with the reduced deflections is by hypothesis and mathematics the best. While too much stress should not be laid on the numerical result for a small area deduced from a general principle for large areas, it is interesting to note that the probable depth of compensation in Gaspé is placed at 100 miles. In contrast, the depth has been found to be 70 miles for a more extensive study of data applying to the United States.

The deflections observed in Gaspé have a persistent easterly trend with one exception, and thus point out a strong attraction in the Gulf of St. Lawrence or farther eastward. It follows also that the trend might be caused by a deficiency of material locally of about 400 feet in thickness. For this latter viewpoint the isostatic equilibrium would be restored when the general level of the region becomes by geological process some 400 feet higher.

CONCLUSION

In the above we have outlined some of the problems and results of geodetic surveys in relation to geography with the view of stimulating an interest beyond the routine of practice. If, at times, Mother Earth be disappointed at the slow progress made by her children in their efforts to clothe her in well-fitting raiment and to search out her secrets, we can only say that we are doing the best we can with our present knowledge.