Version 2 of the National Transformation Between NAD27 and NAD83 and Its Importance for GPS Positioning in Canada

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Introduction

The North American Datum of 1983 (NAD83) was adopted by the Canadian federal government in 1990, superseding the North American Datum of 1927 (NAD27) (SMRSS, 1990). Shifts resulting from the move from NAD27 to NAD83, ranged up to 100 metres in geodetic coordinates (Pinch, 1990). NAD83 is compatible with GPS (Global Positioning System) positioning due to its geocentric reference system and the removal of major distortions which were present in NAD27. To support users in the move to NAD83, the Geodetic Survey Division, along with its provincial partners, developed a transformation between NAD27 and NAD83. Version 1 of this standard utility has been available since 1990 (Junkins, 1990), and is widely used by private and government organizations with holdings of geospatial data. Updates and improvements to this transformation made in response to user feedback, were incorporated in the National Transformation Version 2 (NTv2) and published on CD-ROM in November 1995.

GPS measurements and NAD27 data are incompatible due to the consistency of the GPS measurements as compared to the distortions inherent in NAD27 data. Consequently, it is recommended that data be transformed to NAD83 using NTv2 and that NAD83 be used for all data collection and data management. In situations where conversion to NAD83 is still forthcoming interim options to combine GPS measurements with NAD27 data are possible. However in applying these interim options, users face a dilemma: to preserve the integrity of the GPS distance measurements or to preserve the integrity of the relationships with the existing base information. The suitability of each option is dependent on a user's situation.

The purpose of this paper is to provide the foundation for making informed decisions on how to combine NAD27 and GPS data. A basic understanding of the two datums and the working principals of NTv2 underlie the ability to make the best decisions dealing with these issues, and hence are described in the first portion of this paper. Options and trade-offs for combining GPS observations with existing NAD27 data are discussed. An appendix contains details of methods for assessing accuracy and distortion as reference material. This paper also explains how NTv2 can be easily adapted to a user's environment to simplify and expedite working in NAD83. Finally, recommendations on making the best of the combined use of NAD27, GPS data and NTv2 in Canada are presented.

Characteristics of NAD27 and NAD83

Horizontal datums have traditionally been realized as a large number of survey markers (i.e. monuments in the ground with accurate latitude and longitude) which have been precisely surveyed and adjusted together to form a uniform network upon which all other measurements may be based. Such is the case with NAD27, and with NAD83 as adopted by the federal government in 1990. NTv2 performs the transformation between these realizations of NAD27 and NAD83.

Two fundamental improvements were made in the move from NAD27 to NAD83 as summarized in Figure 1. The change from the Clarke 1866 ellipsoid, which was a "best fit" over North America, to the geocentric Geodetic Reference System 1980 (GRS80) ellipsoid facilitated compatibility with satellite positioning techniques such as GPS (Moritz, 1980). The complete readjustment of observations in NAD83 resulted in an elimination of most of the distortions which plagued NAD27. Random and systematic errors had accumulated in NAD27 as a result of geometrical weaknesses in the network, lack of an accurate geoid model and the application of non-rigorous adjustment methods (Pinch, 1990).

NAD27				**NAD83**
distortions	→	*readjustment*	→	removal of major distortions
Clarke 1866, best fitting ellipsoid for North America	→	*change of reference ellipsoid & its origin*	→	GRS80, geocentric ellipsoid

Figure 1. Improvements in Moving from NAD27 to NAD83

Since both NAD27 and NAD83 (as adopted in 1990) are realized by coordinates for monumented control points, the differences in coordinates at each of these points defines the relationship between them. The NAD83 coordinates adopted federally in 1990 consist of the results of the 1986 Continental Adjustment and the subsequent secondary integration adjustments, namely the 1990 ESIHBA (Eastern Secondary Integration Helmert Block Adjustment) for Eastern Canada and the NMIP93 (Network Maintenance Integration Project 1993) for Western Canada. In practical applications, the relationship between NAD27 and NAD83 is typically sought at points other than control monuments. As this relationship is not explicitly known, it becomes necessary to model it for any possible point in Canada. Such a model was developed for NTv2.

The distinction between the realization of datums through coordinates, and their underlying reference system is very significant. The NAD83 reference system is described in Schwarz(1989). For the NAD27 - NAD83 relationship, if only the change in datum (the reference ellipsoid and its origin and orientation) is considered, shifts of 20 m or more may still remain after coordinate conversion. This is due to omitting the influence of the readjustment, and its effect in removing major distortions in the coordinate realization of NAD83.

Recent advances in precise satellite positioning have both prompted a need for and enabled a refinement of the realization of NAD83 referred to as NAD83(CSRS). Here CSRS stands for the Canadian Spatial Reference System (Duval et al., 1996). While minor distortions may still exist in NAD83 as adopted in 1990 due to limitations of the observations available for the readjustment, NAD83(CSRS) may be considered essentially distortion-free. Currently NAD83(CSRS) is in its early stages of development. The refinements in NAD83(CSRS) as compared to NAD83 as adopted in 1990, are expected to be less than a few decimetres in most cases, and in rare cases may exceed a metre. In contrast, shifts of up to 100 m exist when transforming from NAD27 to NAD83 as adopted in 1990.

The nature of the changes is similar to normal network maintenance which is often required whenever new surveys are conducted. With no change in reference system, or other gross effects, a revised version of the National Transformation is not generally considered necessary. NTv2 is the tool to satisfy users who must deal with both NAD27 and NAD83 data in Canada.

NAD83 terminology distinctions are summarized in Table 1. In practice, the term "NAD83" is often used without clarification of its specific intent, making it important to pay attention to the context in which it is used.

Table 1. NAD83 Terminology

NAD83 reference system	defined by origin, orientation, scale and ellipsoidal parameters (See NOAA, 1989.)
NAD83 as adopted federally in 1990	realization through horizontal coordinates assigned to survey control markers based on results of network readjustments
NAD83 (CSRS)	refinement in the realization of NAD83 to eliminate minor distortions, consistent with the NAD83 reference system at the sub-decimetre level for Canada's most accurate control points

Combining GPS Measurements with NAD27 Data

The improvements made in the move to NAD83 support GPS positioning in two main ways: the removal of major distortions provide NAD83 coordinates consistent with satellite positioning techniques; and the geocentric reference ellipsoid chosen is compatible with GPS. Consequently, working with GPS data in NAD83 is straight forward. It is recommended that existing NAD27 data be transformed to NAD83 and that NAD83 be used for all data collection and data management. The ease with which NTv2 can be adapted to perform this transformation in the user's data and software environment is described in a following section. This section describes considerations and options when data holdings are in NAD27 and there is a desire to use GPS measurements.

The characteristics and source of a user's NAD27 data holdings and the key outcomes sought from combining NAD27 and GPS data are at the root of selecting the most suitable option. The source of a user's NAD27 data may vary widely.

In most cases user's NAD27 data is well tied with NAD27 coordinates for surrounding control points and therefore has distortions which are consistent with those modelled in NTv2. As an example, a GIS system based on NAD27 map products would fit well with control points as the underlying maps are usually based on these.

In some cases "local datums" have been developed based on the measurements from one point (or variations of this which have the same effect). Starting with the NAD27 coordinates at a selected point, measurements would have been used to propagate coordinates for all the other points of the user's data set. Often these approaches have been taken when the accuracy and integrity afforded by precise measurement techniques have taken priority over integrating with other local control. Engineering surveys are an example of where a "local datum" may have been established. Another example is where all of a user's data is based on measurements taken with respect to a single GPS base station.

In cases where the user's NAD27 data holdings are well connected to control points, it is recommended that NTv2 be used to bring the NAD27 data holdings to NAD83 for combination with the GPS derived coordinates. In cases where NAD27 data holdings are based on a local datum, it is recommended that a local datum shift between NAD27 and NAD83 be determined and applied to the data, moving the NAD27 data holdings to NAD83 for combination with the GPS derived coordinates. These two situations and the corresponding recommendations are summarized in Table 2.

In both cases, converting the existing data holdings to NAD83 eliminates any further concerns regarding the compatibility with new GPS observations, and establishes a consistent environment for future data integration. Where a local datum extends over a broad area overlapping with data holdings from other sources, use of a local datum shift to move to NAD83 would have the added benefit of making this data consistent with these overlapping but previously incompatible data holdings.

Often, the source of the NAD27 coordinates for data holdings is unknown, yet a choice between one of the two recommendations presented in Table 2 must be taken. Unless the data is specifically known to be based on a local datum, or that that it has been treated to avoid NAD27 distortions, adherence to the NTv2 standard is preferred.

Table 2.
Recommendations for Combining
NAD27 Data and GPS Derived Coordinates

Characteristics of User's NAD27 Data Holdings	Recommended Procedure for Combining NAD27 and GPS Derived Coordinates
well tied to control points	use NTv2
based on local datum (i.e. measurements made with respect to a single control point)	use local datum shift

For the opposite situation, in which the existing data must remain in NAD27 for some other reason, converting the GPS observations to NAD27 can pose a dilemma. Use of NTv2 has the benefit of preserving the relationships with the NAD27 base data, but potentially degrades the integrity of the distance measurements. Use of a local datum shift has the benefit of preserving the integrity of distance measurements but potentially jeopardizes the relationship with the existing base data. The pros and cons or using NTv2 or a local datum shift are summarized in Table 3.

Table 3.
Pros and Cons of Using NTv2 or a Local Datum Shift to Work in NAD27

Procedure for Using GPS Derived Coordinates in NAD27	Pros	Cons
NTv2	preserves local relationship between GPS points and base data	may degrade integrity of GPS measurements
local datum shift	preserves integrity of GPS measurements	may degrade local relationship between GPS points and base data

Some considerations and evaluations may be made when choosing between these two approaches. The distortion in the area may be evaluated through use of NTv2 features, as described in the appendix. If the distortion in the area is very small, a local datum shift may be applied to the GPS observations, preserving the integrity of distance measurements, while still having a good relationship with the existing base data. If the distortion in an area is quite large, yet it is very important to preserve the relationship with the base data, the user may decide to forsake the integrity of the GPS measurements and use NTv2 rather than a local datum shift.

From this discussion, it is evident that there are no hard set rules for choosing how to combine NAD27 data holdings and GPS derived coordinates. However, information about the NAD27 data holdings, known priorities, and NTv2 utilities can be used to guide one towards the most suitable route. If NTv2 is to be used, software and accompanying users manuals provide guidance in its application. If a local datum shift is to be used, the following procedures may be followed.

Determining a Local Datum Shift

To compute a local datum shift, a control survey point with known coordinates available in both NAD83 and NAD27 needs to be selected, and for which the NAD27 coordinates correspond to the user's data. For both the NAD83 and NAD27 coordinates, the latitude, longitude and height should be converted to Cartesian X, Y and Z coordinates. The same height may be used in both computations, as will be explained later. The local datum shift is the difference between the NAD83 and NAD27 Cartesian coordinates, i.e. ΔX, ΔY and ΔZ. Since only one point is being used, it is not possible to compute any rotation or scale parameters. The complete local datum transformation is the combination of the shifts and the change in reference ellipsoid, which may then easily be applied to coordinates determined through differential GPS.

In effect, this process makes the surfaces of the two reference ellipsoids coincident at the datum point, as illustrated in Figure 2. As also can be seen by this diagram, the geoid separation is a component of the height of the point above the ellipsoid, but the geoid models refer only to geocentric systems, and their relationships to the NAD27 reference system are not known. (This fact, together with the variable distortion of the NAD27 coordinates, are the basic reasons why no precise definitive relationship exists between NAD27 and NAD83). So no matter which height is used for the local datum shift computation - orthometric from spirit levelling, ellipsoidal from GPS, or no height at all - the results will be identical.

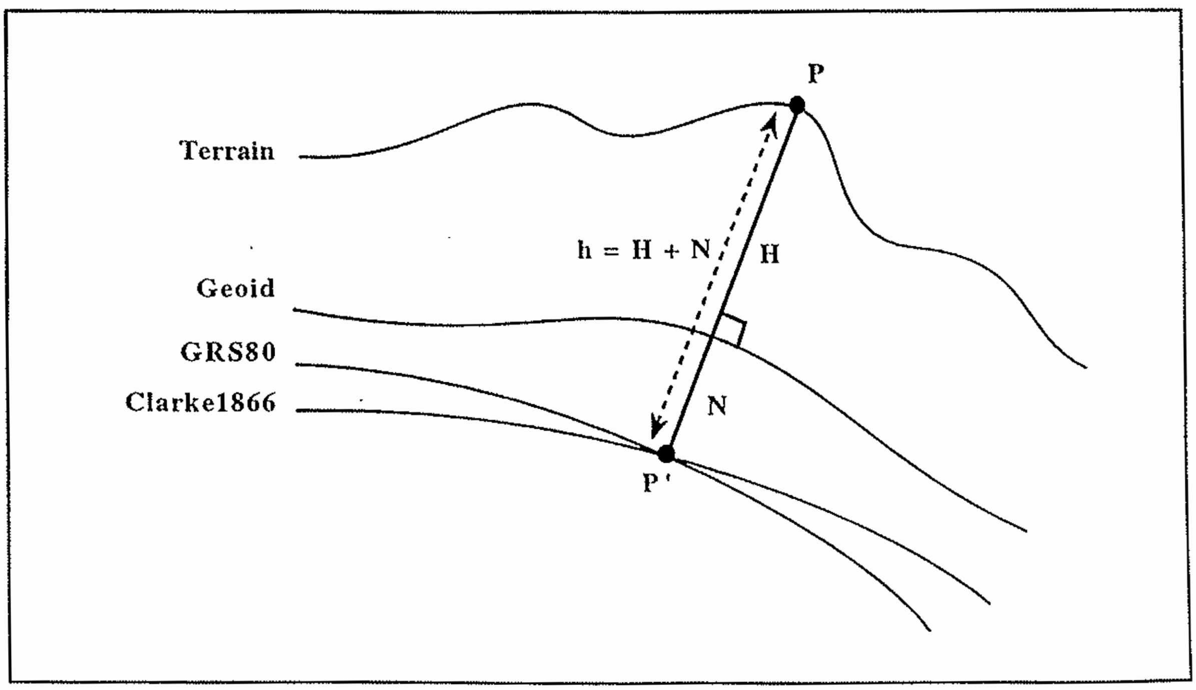

Figure 2. Local Datum Shift Between NAD27 and NAD83

This technique is identical to the commonly used relationships built into most GPS receivers and processing software for converting between datums. With this description, the user can determine an exact NAD27 to NAD83 local datum relationship rather than using approximate values from another source. The local datum shifts can be input by the user into most GPS receivers or software.

Adapting NTv2 to the User's Environment

NTv2 is designed to enable simple implementation on a broad range of user platforms and provide for easy incorporation into a wide range of GIS, GPS or other software packages. Three qualities of NTv2 support its adaptation to the user's environment.

The first is the portability of the software to various computer platforms. Although the NTv2 CD-ROM is strictly in a PC compatible format, source code for the software is included that can be recompiled on the user's preferred system. The programs are written in FORTRAN following the ANSI77 standard. Most changes that are required for other computer systems are already embedded in the code. One of the software utilities facilitates the conversion of the binary grid shift file to ASCII for transfer between systems, and then converts the file back to binary on the destination platform.

The second is the ease of modifying input and output data formats to match user's needs. The user's manual details the fixed format and delimited ASCII format that are accepted by the software. By having the source code available, these formats can be altered to read and write data in the formats familiar to the user, rather than reformatting the data to an externally imposed specification. All of the data input and output is found in four subroutines, which correspond to the input and output of geographic and UTM coordinates.

The third is the ease with which NTv2 can be incorporated into existing software systems. At the heart of the transformation program there are three subroutines that provide the core functions for accessing and applying the information from the grid shift file. The rest of the program involves peripheral operations such as the user interface, data input and output, and the conversion between geographic and transverse Mercator coordinates. Using the core subroutines, a developer can implement a feature that provides NTv2 functionality into an existing software system. This approach has the advantage of acting directly on the data within its own environment, without having to export it for processing and then re-import it. This can result in great time and cost savings for users.

To support the integration of NTv2 within the user's environment, a Developer's Guide (Junkins and Farley, 1995) details the grid shift file specifications and the concepts underlying the three procedures that access it. The Developer's Guide, together with the source code for the subroutines and a minimal main program to demonstrate their usage, form a developer's kit that enables any end user to create custom implementations for their own use. The full potential is realized when such a feature is implemented in commercial application software, such as GIS, mapping, and coordinate computation packages.

The advantages of incorporating NTv2 into commercial software are multiple: users do not have to do any development work to access the NTv2 in their own data environment; more users become aware of and use the standard simply by discovering it in their application ensuring data compatibility amongst users and consistency with GPS; developers can extend the standard to more complex features rather than just to discrete data points; and users can convert their data to work in their preferred reference system without having to mount a separate conversion campaign.

The Geodetic Survey Division licenses all implementations of the NTv2 in commercial software products. The grid shift file structure is necessarily complex to accommodate the variable grid density that makes the NTv2 such a precise tool in municipal regions, while covering Canada's vast territories at a reasonable density. Licensing ensures the full quality of NTv2 is retained in its implementation. It prevents use of simplifications to reduce the amount of data or maintain compatibility with existing algorithms such as Version 1 of the National Transformation or NADCON, (the US counterpart to the National Transformation). Licensing ensures the standard is followed and provides users of reassurance through the endorsement process. Incentive for vendors of commercial products is created by demand from their user community.

Recommendations

Based on the foregoing discussions, the following recommendations are presented for users of GPS positioning who are faced with combining their new observations with existing NAD27 based data:

- Evaluate the significance of the distortion in the area of concern.

- Determine whether the existing NAD27 data is homogeneous with the distorted NAD27 control survey network, or based on a local datum point.

- Using the appropriate methodology (NTv2 or local datum shift), move the existing data forward into NAD83 if possible, eliminating further analysis and concerns.

- If it is necessary to convert GPS observations into NAD27 where distortions are significant, determine whether it is more important to preserve the GPS measurement accuracy (apply a local datum shift) or to overlay the GPS positions on the existing base data (apply NTv2 shifts).

- Seek out licensed NTv2 features in commercial software products to facilitate its use in the native environment of the existing data.

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Appendix

NTv2 Shifts, Accuracies and Distortions

Modelling NAD27-NAD83 Shifts and Accuracies

Using the latitude and longitude shifts between NAD27 and NAD83 for 136,188 federal, provincial and municipal control points in Canada, a model enabling prediction of shifts for anywhere in Canada was developed using a weighted mean approach. After approximate modelling the systematic trends of datum difference and broad regional distortion, the shifts at a point of estimation *P* are based on a weighted mean of the residual coordinate shifts at the surrounding survey control points. This can be written for any latitude or longitude shift *y* as

$$y = \frac{\sum w_i x_i}{\sum w_i}, \tag{1}$$

where x_i is the shift at control point *i*, and w_i is the assigned weight given to the shift at control point *i*.

The weights assigned for each control point *i* are a function of the distance d_i between the control point and the point of estimation *P*, and the network density r_P at the point of estimation. The weights are computed as

$$w_i = e^{-(d_i / r_P)^2}. \tag{2}$$

The relationships between control points, the point of estimation, the distances and the network density are illustrated in Figure A1. The network density is a key factor which allows the model to adapt to the vast range of interstation spacing, which varies from as little as a hundred metres in a municipal environment to as much as a hundred kilometres in hinterland regions. The network density used for computation at the point of estimation is the distance from it to the nearest control point, as shown in Figure A1.

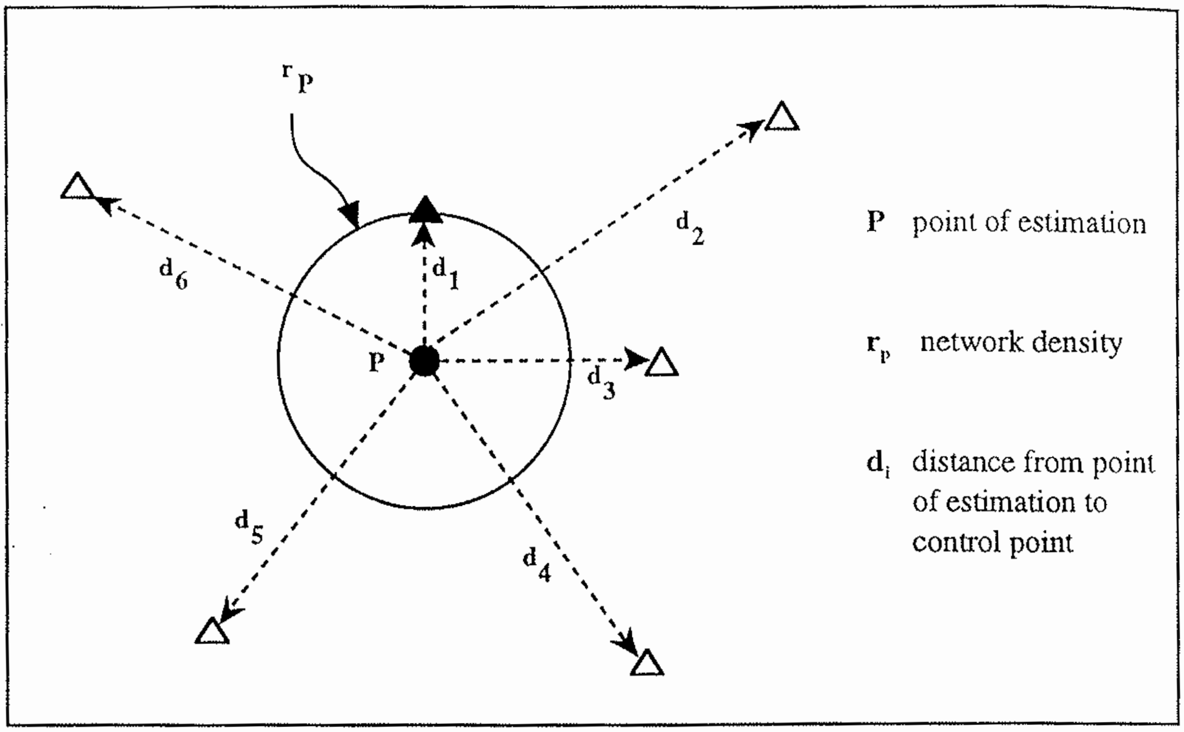


Figure A1. Parameters for Estimating Weighted Mean Shift at Point of Estimation

If the point of estimation falls within the exclusive neighbourhood of the nearest control point, the network density of the control point is used. In such cases the control point network density is defined as the distance to its nearest neighbouring control point (nearby eccentric points are not considered), and an area defined by a circle around the control point with a radius equal to half of its network density represents the exclusive neighbourhood of that control point. The exclusive neighbourhood of a control point does not overlap with the exclusive neighbourhood of any other control point. This ensures a smooth transition in the weighting function (equation 2), and prevents the network density from approaching zero as the point of estimation approaches a control point (Junkins, 1993).

When computing the weighted mean shift at a point of estimation, if a control point is coincident with the point of estimation it receives a weight of 1.0, the maximum weight possible. If a control point is at a distance equal to the network density determined for the point of estimation, it receives a weight of 0.37. The weights diminish to virtually zero at a distance of three times the network density, as shown in Figure A2. With this modelling technique, the displacement fields (i.e. the estimated latitude and longitude shifts for Canada) and their first derivatives are continuous, meaning the intervals between survey control points are smooth. In practical applications, this is very important as it avoids any jumps or breaks in transformed data sets.

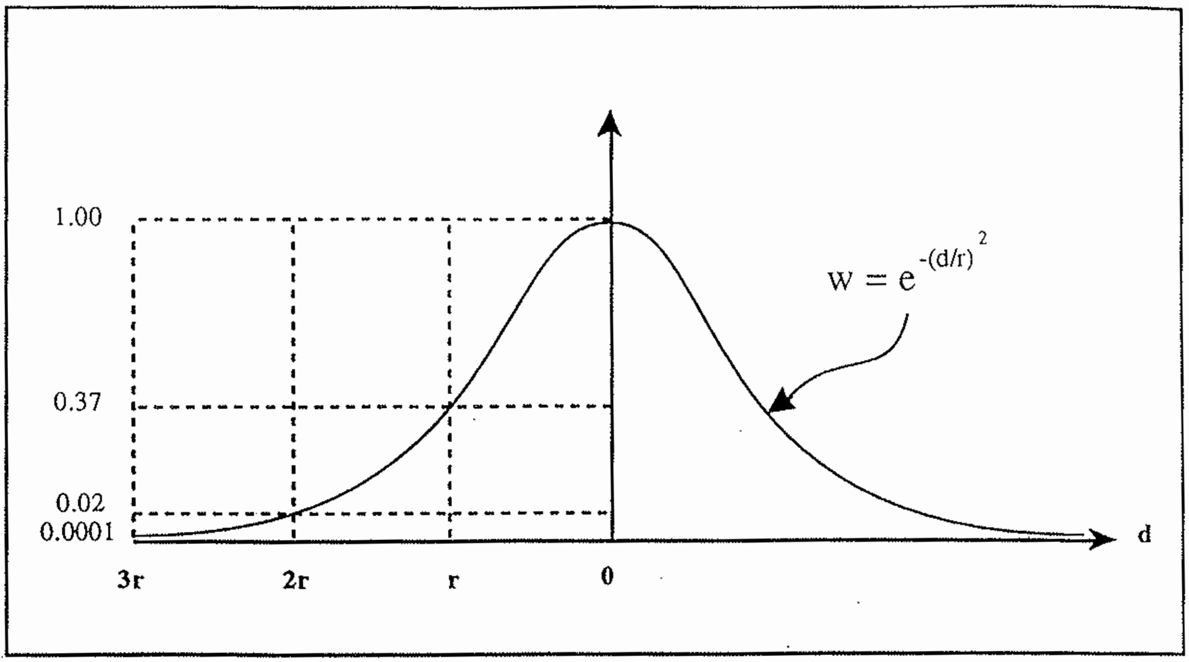


Figure A2. Weighting Function for Computing Weighted Mean Shifts

Since the modelling of shifts is based on a weighted mean, rather than fixing the shift at each control point to the actual difference between NAD27 and NAD83 coordinates, there are residuals, similar to those computed in regression analysis or any other least-squares process. Thus, unless the shifts at all the neighbouring points are identical, the weighted mean shift at a control point will differ from the actual shift at that control point.

The accuracy of the shifts, s_y is estimated as

$$s_y = \sqrt{\frac{\sum w_i^2}{\left(\sum w_i\right)^2} \cdot \frac{\sum\left(x_i - \bar{x}\right)}{(n-1)}} \qquad (3)$$

where x_i is the shift at neighbouring control points and $\bar{x}$ is the weighted mean of the shifts at all the neighbouring control points and n is the number of control points being used to estimate the shift.

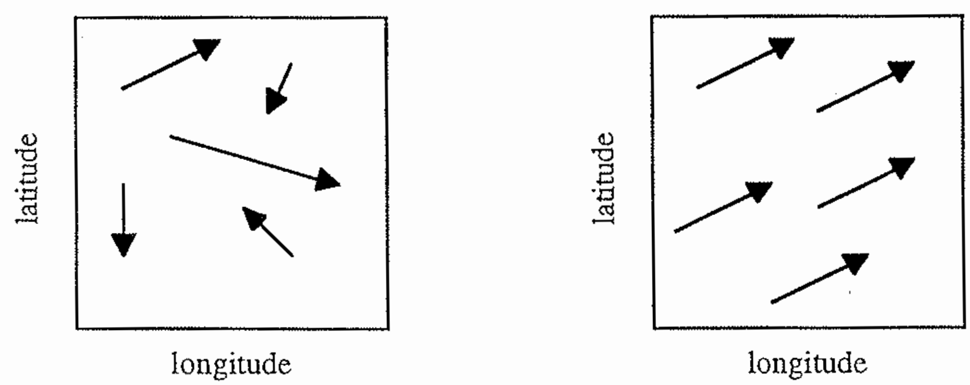
The modelled surfaces of latitude and longitude shifts and associated accuracies were used to create a table of "grid shifts and accuracies" within NTv2. Such a table allows the user to easily interpolate the NAD27 - NAD83 shift for any point in Canada they wish. In Version 2 a base grid interval of 5 minutes is used (approximately 9 km of latitude, less for longitude), while in areas of dense municipal survey control the grid spacing interval is reduced, typically to 30 seconds (just under 1 km). The grid interpolation software provided with NTv2 automatically selects the correct grid for each point to be transformed.

With an understanding of how NAD27-NAD83 coordinate shifts and accuracies have been derived and made accessible in NTv2, it is now possible to review the significance of the information they convey regarding accuracy and distortion.

Assessing Accuracy and Distortion

Accuracy and distortion have a large influence on how to best combine NAD27 and NAD83 data in various situations. Consequently it is important to be aware of how they may be assessed using NTv2.

NTv2 accuracy estimates give a measure of the confidence in the predicted shifts between NAD27 and NAD83 at a given point. If neighbouring control points have significant discrepancies amongst their shifts, then the accuracy will be low and the estimated shift can be considered poorly known. If all of the survey control points in a neighbourhood have almost exactly the same shifts, then the accuracy will be high and the estimated shift can be considered well determined. Figure A3 illustrates this concept.



Low Accuracy

- neighbouring control points have discrepancies amongst shifts
- NAD27-NAD83 poorly defined

High Accuracy

- neighbouring control points have common shifts
- NAD27-NAD83 well defined

Figure A3. Interpretation of NTv2 Shift Accuracies

Distortion means twisted or bent out of shape. In the case of NAD27 it is the angles and distances between control points, usually on a regional level, which are distorted. Distortions are often significant over long distances, but may not be particularly noticeable in the local context. For example, a primary network meeting First Order standards has an accuracy of better than 20 ppm in relative positions at the 95% confidence level (Geodetic Survey, 1978). This allows for up to 20 metres of distortion over a distance of 1000 km without exceeding the specification for allowable error. Similarly, Second Order specifications would allow for up to 50 metres of distortion.

It is quite possible to have large distortions even though the accuracy of the shifts is very good. Conversely, it is possible for points with poor accuracy due to local network problems to have very little distortion between them.

Distortion may be evaluated in NTv2 using contour plots of shifts, tabulations of shifts, cell gradients or direct computation of shift differences. Using each of these four techniques it is necessary to distinguish the true distortion from the effect of the change in reference ellipsoid. The magnitude and direction of distortion is highly variable across the country. In contrast, the change in the reference ellipsoid causes geographic coordinates to change systematically.

Distortion can be envisioned from contour plots of shifts by understanding what their pattern would be without distortions. If only the change in reference ellipsoids were considered, all of the contours would be regularly spaced with a uniform trend. In the presence of distortions, the contours are more variably spaced, either widely separated or closely bunched.

Distortion may also be assessed using the cell gradient option of the NTv2 program INTGRID. It determines numerically the maximum change in shifts within the grid cell containing the point submitted for transformation as described on page 35 of the NTv2 User's Guide (Junkins and Farley, 1995). This is analogous to the local slope of the surface represented by the contour plots.

As well, distortions may be evaluated by scanning tabulations of shifts for a given area. Such tables can be generated using the INTTAB software included with NTv2. Again, the trend must first be discerned, and the variations from the trend represent the distortions.

The fourth technique is one which enables numerical evaluation of distortion by comparing the difference between the shifts at two points. As mentioned previously, comparisons must be carried out on the same reference surface to avoid misconstruing the effect of changing reference ellipsoid as distortion. Thus, at each point the difference between the shifts from NTv2 and the shifts derived by using only a datum conversion (i.e. ignoring distortions) must be used. The datum conversion need only be approximate for this exercise, which essentially shows changes that would have resulted if the control networks had been readjusted without changing the reference system.

Distortion evaluated by this fourth method is represented by the change in line AB in Figure A4. Points A' and B' represent the undistorted line AB after an approximate datum transformation, while points A'' and B'' represent the distorted line AB after applying the NTv2 shifts. Both are on the same reference ellipsoid. The vector difference ΔB - ΔA represents the distortion between the two points. In taking this difference, any error in approximating the datum shift for A' and B' is cancelled.

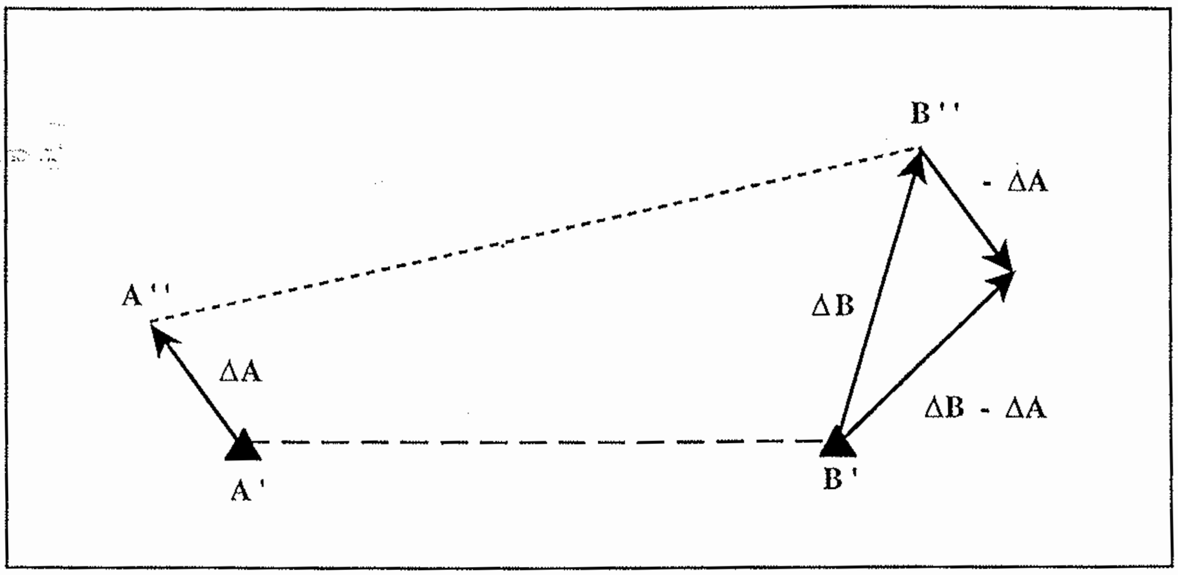


Figure A4. Distortion Between Two Points

The accuracy of the distortion between any given points A and B, s_{AB} can be computed by propagating the accuracies of the individual shifts into the difference. This is given by the relationship

$$s_{AB}=\sqrt{s_A^2+s_B^2}\,, \tag{4}$$

where s_A is the shift accuracy at point A and s_B is the shift accuracy at point B.

Through reviewing the characteristics of NAD27 and NAD83, describing the modelling techniques to predict the shifts and corresponding accuracies between them for any place in Canada, and presenting means for evaluating and interpreting accuracy and distortion, the background has been set for addressing how to combine GPS measurements with NAD27 data.

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