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PILLAR AND STOPE STABILITY ASSESSMENT OF THE NIOBEC MINE USING THE THREE-DIMENSIONAL FINITE ELEMENT TECHNIQUES

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Abstract

This paper describes a cooperative ground control research program with the Niobec Mine, Quebec, from 1985 to 1988.

Following a brief technical background on the mine and the scope of the study, the paper briefly summarizes the results obtained from using the three dimensional finite element technique to simulate large, open, underground stopes. In addition to the finite element technique, elastic theory and "voussoir" arch analyses were also used to examine the stability of the surface crown pillars.

Stress distributions were determined around the large underground open stopes using finite element modelling. Stope stability analyses was carried out based on an empirical and the Drucker/Prager failure criteria. Results of their application are presented and discussed. Using an empirical strength reduction relationship, the in situ pillar strength is estimated. C-102-23, the largest stope, with a planned stope span of 73.5m is currently being mined and no instability has been observed to date.

Introduction

An extensive co-operative ground control research project has been carried out with Niobec mines, Chicoutimi, Quebec. The project, initiated in 1985, involves the participation of the Niobec Mine, Centre de Recherches Minérales, Quebec, and the Mining Research Laboratories of CANMET. The program was structured to meet both the ground control information requirements of the mine and the broader ground control interests of its two partners. The objectives of the program are indicated below:

(a) to contribute to the development of a geomechanical data-base on the physical and mechanical properties of mine rocks; (b) to determine the field stresses at the mine, and to design and test instruments to monitor stress changes as mining progresses; (c) to assess the regional and local stability of the mine structure by means of numerical simulation; and (d) to provide design guidelines for mine design and ground control for use by mine personnel.

Numerical modelling constituted an integral part of this project. This paper primarily deals with items (c) and (d) involving stope and pillar stability evaluation of two working levels (upper and lower) of the mine, and an evaluation of in-situ pillar strength. Stress redistributions at several stages of mining the stopes were examined. The three-dimensional finite element technique was used in these studies. In addition, the stabilities of the surface crown pillars, composed of massive unjointed limestone spanning underground openings and located in zones of niobium enrichment of a carbonatite intrusive, were analysed using elastic theory and "Voussoir" arch approach. Critical stable opening spans were calculated.

Following mining of the largest stope, C-102-23, the mine plans to extract additional stopes below the sill pillar separating the upper and lower levels and, in time, the sill pillar (Figure 1). Recovery of all pillars on the upper level is also planned. The present study was directed at evaluating the stability of stopes on the upper level, and the impact of mining lower stopes C-102-17 and C-102-19 (2nd level) on the workings located on the upper level. The stability of stopes C-102-23, C-102-17, and C-102-19 with partial removal of sill pillar S-L-102-17 was also examined. Ultimately, the stability of the huge stope which would result from recovery of all support pillars on the upper level was studied.

Niobec Mine

Mining activity takes places in a carbonatite pluton located in the Pre-Cambrian Shield. The carbonatite and host rocks are capped by flat lying limestone. The carbonatite, which is rich in carbonate minerals, contains small vertical zones of niobium enrichment. Mining follows irregularly distributed ore concentrations, and a large blasthole stoping method is used. The overlying surface crown pillars are generally 60m thick. Each mining level is 90m high. The upper level begins with the surface crown pillar. The second level starts below a 30m sill pillar. The lower stopes follow the same ore zone as the upper stopes. Primary stoping in the upper level is almost completed and the mine will shortly begin to mine the support pillars. No backfilling is carried out, for economic reasons. Only limited primary stoping has been carried on the lower level.

C-102-23 stope at the upper level is currently being mined with a planned stope span of 73.5m and width of 30m. No instability has been observed. When all the upper level support pillars are extracted the stope span will be increased to approximately 274m.

3-D Niobec Mine Model

To simplify mine geometry for modelling purposes, the mine model was constructed with openings symmetric around the centre plane of pillar L-102-17. As a result, only one quarter of the total structure had to be modelled. However, this did result in including in the model a mirror image of stope C-102-23 not shown in Fig. 1. Its inclusion, however, was justified on the basis that the resulting model would result in analytic studies providing a conservative estimate of opening stabilities. An isometric view of the finite element model used in the study is shown in Fig. 2.

The mining block, which was modelled, extends 600m vertically, 800m along the strike and 210m from hangingwall to footwall. The size of the model consists of 6500-7650 8-node brick isoparmetric elements which yielded about 21000-25000 equations. The element size around the areas of interest is about 6 x 6 x 7 m. It is realized that with the element size this resolution is not sufficient to determine accurately stress concentrations around stope corners. However, the stresses resulting from various mining stages are sufficient to assess the stability of the mine structures.

The computer code, SAP3D, used in this study is based on a three-dimensional, linear, elastic finite element formulation. An isoparmetric eight-node brick element with three degrees of freedom at each node is used in this model. At an earlier stage in the project the model was run on a Cyber 840 computer using VE/NOS operating system. It was soon realized that the size of the 3-D model had reached its practical limit in terms of both computer capacity and costs. A recent benchmark test of Cray 1S, in 1987, conducted by Energy, Mines and Resources Canada and Cray Canada Inc. indicates that the code used in this study can be run more cost effectively on the Cray 1S.

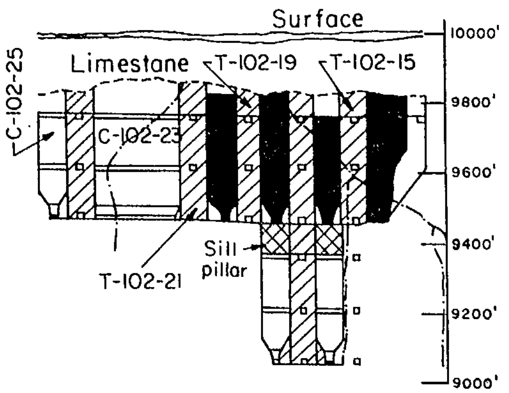

Fig.1 A typical section showing stope C-102-23 and surrounding mine workings (mined out stopes in black)

The model was used to simulate the mine at the following stages of development:

<u>Stage 1</u> - All stopes on the upper level including stope C-102-23 mined; <u>Stage 2</u> - Stopes C-102-17 and C-102-19 below the sill pillar mined; <u>Stage 3</u> - Sill pillar S-L-102-17 between stopes C-102-17 and C-102-19 extracted; and <u>Stage 4</u> - All support pillars on the upper level removed while the sill pillar remains intact.

The modulus of deformation, uniaxial compressive, and tensile strengths of both carbonatite and limestone formations were determined in laboratory tests [Bétournay et al. 1986, Labrie, 1986]. A summary of the mechanical properties for the formations used in the study is given in Table 1.

In situ stress determinations were carried out in Niobec Mine at sites located 270m and 320m in depth [Arjang,1986]. In the Canadian Shield, it is known that

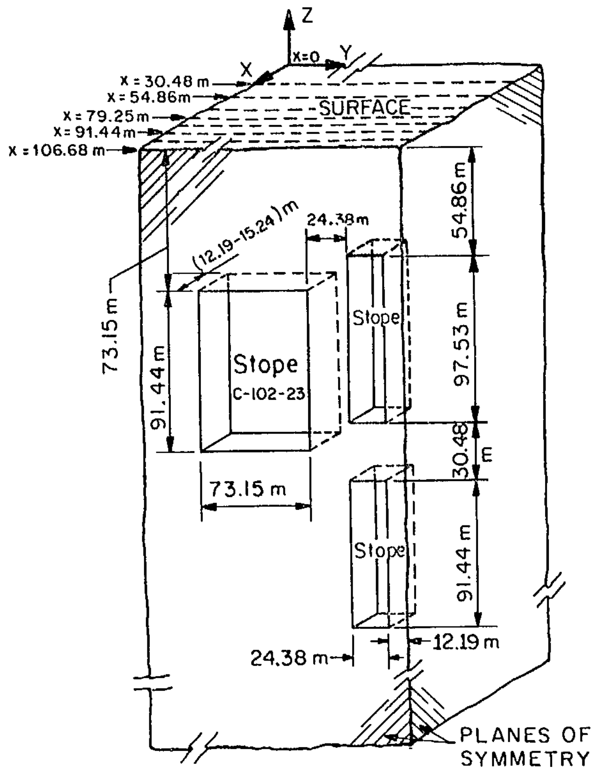

Fig. 2 Isometric view of the mine model showing quarter of the structure

the horizontal stress is greater than vertical stress. Based on the limited stress data developed for the Niobec Mine, the following far field stress conditions were established.

$$\sigma_z = \gamma Z$$

$$\sigma_x = 1.5\sigma_z$$

$$\sigma_y = 2.0\sigma_z$$

where σ_z,σ_x,σ_y are the vertical stress at depth Z, and the horizontal stress in the EW and NS directions respectively. γ is the unit weight of rock material.

Results and Discussions

The analytic study indicated that C-102-23 stope with a span of 73.5m in the strike direction, following primary stopping at the upper level, is stable under estimated field stress conditions. The effect of mining C-102-17 and C-102-19 stopes (lower level) on ground stability conditions around C-102-23 stope will be

Table 1 - Material Properties of the Geological Formations used in Modelling Study

Item	Limestone		Carbonatite (altered)		Carbonatite (intact)	
	Laboratory	Estimated *	Laboratory	Estimated *	Laboratory	Estimated *
Young's modulus - MPa	36,000	30,000	56,400	40,000	64,000	40,000
Poisson's ratio	0.21	0.25	0.25	0.25	0.29	0.25
Compressive stresgth - MPa	92	45	86	45	128	45
Tensile strength - MPa	5.8	3.0	10.3	3.0	16.8	3.0
Cohesion - MPA	10	2 - 4	17	2 - 4	25	2 - 4
Friction angle - degrees	36	40	50	40	51	40
Empirical constant m	9.30	3.30	4.8	3.30	4.8	3.30
Empirical constant s	0.04	0.1111	0.014	0.1111	0.014	0.1111
RQD - %	90		90		95	
* Used for modelling study						

nominal. The largest compressive stresses occur near stope corners with a magnitude of about 22 MPa. They are not sufficiently high to cause stability problems. The C-102-23 stope is currently being mined and no instability has been observed to date.

Tensile stress developed along the walls of C-102-23 stope (T-102-21 pillar) are relatively small. The stresses are less than 1.0 MPa. However, when sill pillar S-L-102-17 between stopes C-102-17 and C-102-19 is extracted (mining stage 3), tensile stresses will develop along most of the length of L-102-17 pillar. The maximum tensile stress is approximately 1.9 MPa, and occurs at about one-eighth of the pillar height from the pillar toe. Even though the tensile strength of carbonatite is estimated at 5 - 10 MPa, if joints are unfavorably orientated, pillar stability could be a problem. Since the pillar is only 24m wide, consideration should be given to ensuring its integrity. In other words, the sill pillar seems to be providing important structural support and should be maintained until mining on the upper level is completed.

At the fourth stage of development, a large stope spanning about 350m along the strike direction will be created. The actual stope span excluding the mirror image of C-102-23 is approximately 274m. The major and minor principal stresses for the central section of the model for the fourth stage of development are shown in Figs. 3a and 3b.

Potential Failure Areas: It is difficult to find a realistic failure criterion for a rock mass surrounding an underground excavation, since the stability of the rock in the immediate vicinity of underground openings is related to pre-existing discontinuities and fractures induced in the rock by blasting, and drilling processes, etc. Back analysis seems to be a rational approach which will be possible in the future and should be considered. At the moment the mining is at the end of stage 1. Visual observations indicate no tensile failure in the rock mass surrounding stope C-102-23.

No one criterion is capable of predicting potential ground failure for all types of rock. The Drucker/Prager yield criterion takes into account triaxial stress conditions but fails to consider any tensile failure. In recent years, Hoek and Brown's empirical failure criterion appears to be gaining popularity in rock mechanics applications. Although it does not take into consideration intermediate principal stress, it does take into account rock mass quality and both shear and tensile modes of failure.

Drucker/Prager Yield Criterion: As an indicator for potential failure around stopes, the strength/stress ratio or the local factors of safety (LFS) in terms of the Drucker/Prager yield function have been applied.

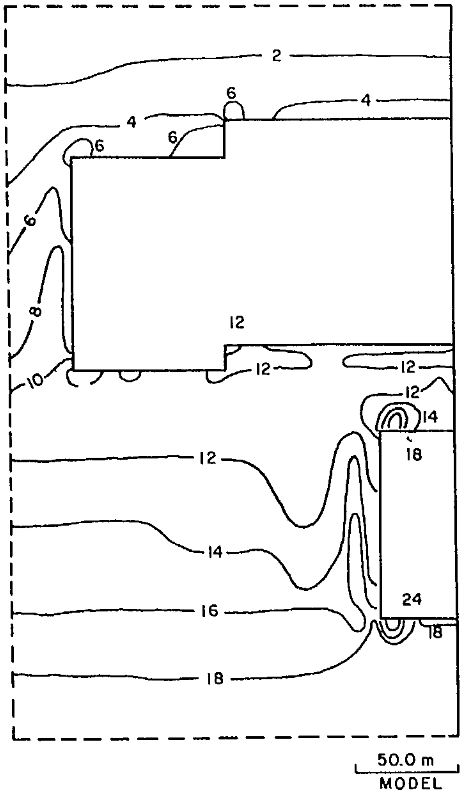

Fig. 3a σ_1 Stress Contours

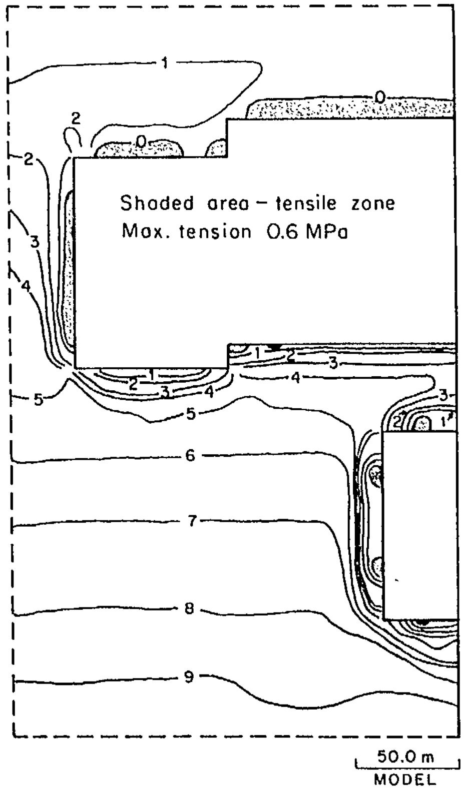

Fig. 3b σ_3 Stress Contours

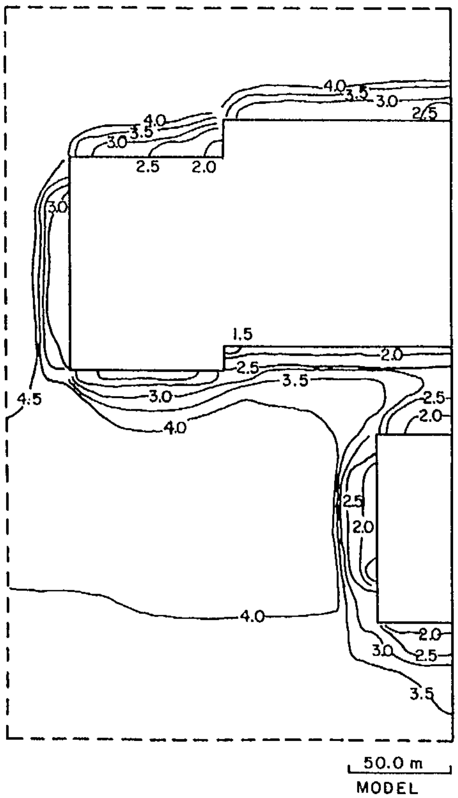

Fig. 4a LFS Contours - Drucker/Prager

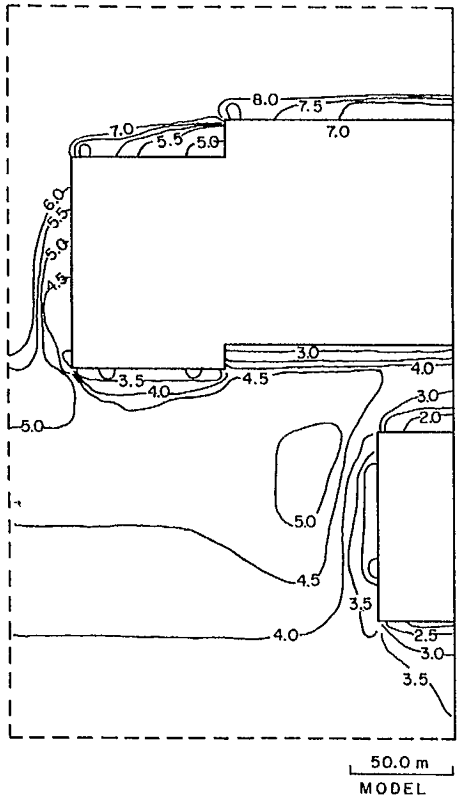

Fig. 4b LFS Contours - Empirical

The material constants used in a first trial analysis were $C = 2MPa$ and $\phi = 40°$, where C is the cohesive strength and ϕ the angle of internal friction. Only one mine area was identified with a LFS ≤ 1.0. This area occurred in the lower portion of L-102-17 pillar. However, the values of C and ϕ used in this first trial were conservative. A second study was carried out using $C = 4MPa$, with $\phi = 40°$. The minimum LFS increased from 0.95 to 1.30. The LFS contour maps for the central section are shown in Fig. 4a.

The laboratory determined compressive strengths for the intact and altered carbonatite rocks are 130 MPa and 90 MPa, respectively [Labrie,1986]. No standard procedures presently exist to estimate in situ rock mass strength based from laboratory tested specimens. A worst case scenario would be a concentration of altered carbonatite rock in the lower level of the Niobec Mine. If we apply a reduction factor of 2 to the strength of the intact samples, then, the in situ compressive strength of the altered carbonatite would be 45 MPa. Assuming the material obeys the Coulomb criterion of brittle failure, then the compressive strength, σ_c, can be related to the rock cohesive strength and angle of internal friction by the equation: $\sigma_c = 2C(1 + \sin\phi)/(1 - \sin\phi)$. For altered carbonatite $\sigma_c = 45MPa$, and $\phi = 40°$ and the cohesive strength of carbonatite is estimated as 8.7 MPa. Therefore, a value of C = 4 MPa used in shear failure analysis seems to be reasonable and would provide conservative estimate of structural performance. On this basis, the local factors of safety calculated for the pillars and stope walls appear to be more than adequate. Therefore, no imminent shear failure is anticipated. If failure should occur, it would be localized tensile failure rather than shear failure.

Empirical Failure Criterion: Another indicator for failure is based on Hoek and Brown's empirical approach. The equation relating the principal stresses at failure is given by [Hoek et al. 1980]:

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2} \tag{1}$$

σ_1, σ_3 and σ_c are the major and minor principal stresses at failure, and the uniaxial compressive strength of the intact rock material, respectively; m and s are rock property constants whose values on the the extent to which the rock has been broken before being subjected to stresses σ_1 and σ_3.

The uniaxial compressive strength, σ_c, and the uniaxial tensile strength, σ_t, of the rock specimens can be related to the uniaxial compressive strength of the intact rock material using the following equations [Hoek and Brown, 1980]:

$$\sigma_c = \sqrt{s\sigma_c^2} \tag{2}$$

$$\sigma_t = \frac{1}{2}\sigma_c\left(m - \sqrt{m^2 + 4s}\right) \tag{3}$$

Eq. 3 can be rewritten as follows:

$$|m| = \frac{\sigma_c}{\sigma_t}\left\{s - (\frac{\sigma_t}{\sigma_c})^2\right\} \tag{4}$$

Hoek and Brown's empirical failure criterion allows for tensile and shear failure [Eissa, 1985]:

Tensile failure occurs when:

$$\sigma_3 < \sigma_t = \frac{1}{2}\sigma_c\left(m - \sqrt{m^2 + 4s}\right).$$

The strength/stress ratio is calculated as:

$$\frac{\sigma_t}{\sigma_3} = \frac{\sigma_c\left(m - \sqrt{m^2 + 4s}\right)}{2\sigma_3}.$$

The strength/stress ratio for a shear failure is given by:

$$\frac{\sigma_{1f}}{\sigma_1} = \frac{\sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2}}{\sigma_1}.$$

where σ_{1f} is the principal stress at failure and calculated from Eq.(1).

Little information was available on the m and s values for the Niobec mine rocks. However, the $RQD's$ for the limestone, the intact and altered carbonatite are estimated as 90%, 95% and 90%, respectively. They are all representative of very good quality rock mass. It is not unreasonable to assume that the altered carbonatite has a compressive strength of 45 MPa, thus, the tensile strength can be assumed to be one tenth of its compressive strength, i.e., 4.5 MPa. Based on these values and using Eq.2 and Eq.4, m and s were estimated as 4.95 and 0.25 respectively. The same values were assumed for the carbonatite, and the limestone.

Analytic studies were carried out using m = 4.95 and s = 0.25 for the altered and intact carbonatite, and limestone. The strength/stress ratio for various sections of the model were calculated. No potential failure zones around the stopes were indicated.

A second series of analytic studies, using m = 3.30 and s = 0.1111 for altered carbonatite, were conducted. These lower m and s values were arrived at by assuming the compressive strength of altered carbonatite to be 30 MPa and the tensile strength to be 3 MPa. The strength/stress ratio maps, for the central section are shown in Fig. 4b. Similarly, no failure areas were observed. The smallest strength/stress ratio, 1.24, is

located in the lower portion of pillar L-102-17.

The strength/stress ratio or LFS calculated for the pillars or stope walls based on the above assumptions, should be adequate against shear failure.

<u>In-Situ Pillar Strength:</u> It is a recognized fact that for most rocks a strength reduction occurs with increasing specimen or block size. An approximate relationship between uniaxial compressive strength and specimen size for a number of rock types has been developed [Hoek and Brown,1980]:

$$\sigma_c = \sigma_{c50}\left(\frac{50}{d}\right)^{0.18}$$

where σ_{c50} is the uniaxial compressive strength of a specimen of 50mm diameter and d is the diameter of the specimen under consideration. Although this relationship is derived from laboratory tests on unjointed intact rock, it may be used as a crude indication of pillar strength.

Pillar L-102-17 has a dimension of 24.39m by 24.39m in plan. The equivalent diameter of a cylindrical pillar would be approximately 13.76m (13,760mm). Thus, the pillar strength, ignoring the effect of height, is estimated as:

$$90.0 \times (\frac{50.0}{13,760.})^{0.18} = 90.0 \times 0.3638 = 32.74\ MPa$$

This seems to be a reasonable estimate. However, the actual strength of the pillar should be higher because the two sides of the pillar in the E-W direction are confined.

<u>Surface Crown Pillars :</u> The stability of the limestone surface crown pillars has been evaluated using elastic analyses, "voussoir" solutions and numerical modelling [Bétournay et al, 1987]. The methods used together permit an integrated approach for assessment: consideration of various failure mechanisms and calculation of margins of stability.

Voussoir solutions were used to evaluate localized detachment of stratas and stability in a voussoir mode ($F_s = 1$) as shown in Fig. 5. The curve provides stability values for various stope lengths and breadths versus arching layer thicknesses (Fig. 6a). Stability of the existing and future pillars is shown in Table 2.

Elastic analysis, though rarely representative of conditions and failure mechanisms (e.g., a rock mass has no tensive strength), is applicable to this massive unjointed rock mass. Large size beam tests which were conducted to obtain the tensile resistance of the rock, yielded a value of 5.8$\pm$2.6 MPa; compressive strength tests yielded a value of σ_c = 91.5 $\pm$ 22.5 MPa with

Table 2 - Elastic and Voussoir Anaysis of Pillar Stability

stope	limestone dimension (m) h	b	l	two-sided support lab strength/induced tension	four-sided support lab strength/induced tension	required voussoir layer thickness (m)
Largest Existing Opening						
102-23	37.5	24.4	73.2	5.8/1.91 = 3.0	5.8/0.35 = 16.8	3.3
102-17	48.4	26.5	48.8	5.8/0.64 = 9.2	5.8/0.28 = 21.0	2.0
203-27	50.0	32.0	34.1	5.8/0.31 = 19.0	5.8/0.19 = 30.3	0.7
203-23, 25	50.0	25.9	54.9	5.8/0.80 = 7.3	5.8/0.21 = 26.8	2.8
206, 202-14	40.8	27.4	73.2	5.8/2.11 = 2.8	5.8/0.28 = 20.9	3.5
203-13, 15	64.0	15.2	91.4	5.8/1.74 = 3.4	5.8/0.05 = 106.7	4.7
Largest Future Openings						
102-25→ 102-15	37.5	24.4	274.3	5.8/26.8 = 0.2	5.8/1.00 = 5.8	20.6
T203-29→ T203-13	50.0	22.9	160.0	5.8/ 7.1 = 0.8	5.8/0.16 = 37.6	10.4
202-16→ 202-14	43.4	21.3	91.4	5.8 /2.7 = 2.2	5.8/0.11 = 53.4	4.76
201-13→ 203-09	67.2	60.0	100.0	5.8 /2.0 = 2.9	5.8/0.54 = 10.9	5.4

$E = 35.8 \pm 9.9$ GPa [Bétournay et al, 1986]. Elastic beam and plate theory provided, respectively, the following fomulae for fully supported cases:

$$\sigma_{ind} = \frac{ql^2}{2h^2} \tag{5}$$

$$\sigma_{ind} = \frac{\beta q b^2}{h^2} \tag{6}$$

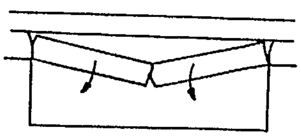

Fig. 5 Voussoir Action

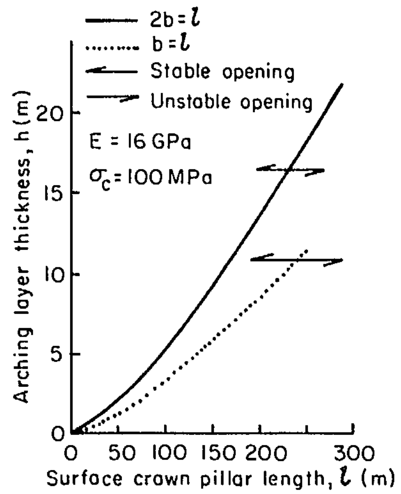

Fig. 6b Design Curves for Surface Crown Pillars - Voussoir Solution

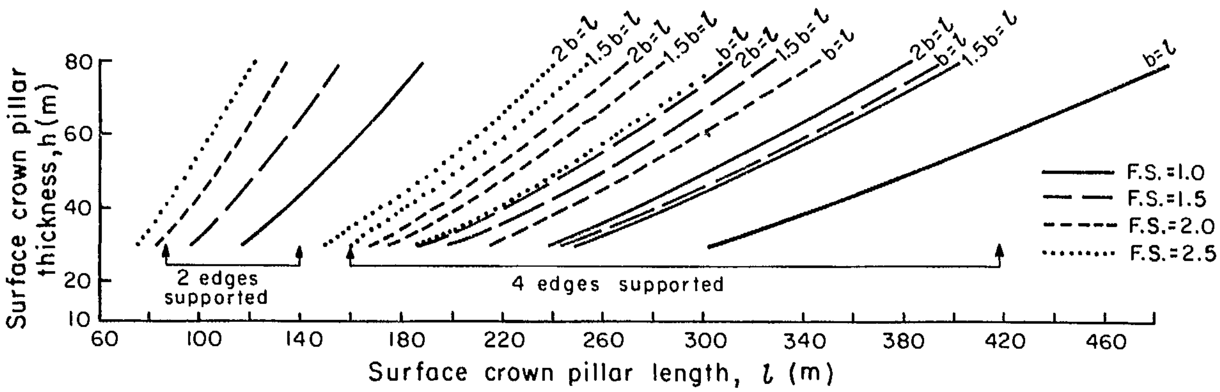

Fig. 6a Design Curves for Surface Crown Pillars - Elastic Analysis (b = breadth, l = length)

where σ_{ind} is the induced tensile stress; l, h, b, and q are the stope span, beam thickness, breadth, and distributed load respectively; β is the shape factor [Roark and Young, 1975].

When the induced tensile stresses reach the laboratory determined value, i.e., $F_s = 1$, the structure is expected to fail in tension followed by a complete and generalized degradation. Figure 6a portrays design curves for critical spans for $F_s = 1$ to 2.5. The support contribution of rock bolts is not considered. The overlying soil and rock mass is dry and thus the effect of water can be neglected in the analyses. Table 2 presents stability estimates for existing and future surface crown pillar using this analysis. Existing pillars fall above a $F_s > 2.5$ using beam and plate analysis. Only two future pillars will meet the minimum F_s of 1 when applying beam analysis (Fig. 6a). F_s conditions for the future stopes are more acceptable when plate analysis is applied.

Previous described numerical modelling for stage 1 mining, was run under both gravity and estimated field stress conditions. The advantage of this approach lied in establishing factor of safety for all areas of the uppermost openings, including the surface crown pillars.

Field stress conditions provided better localized stress conditions for ground stability. The large compressive stresses associated with field stress conditions kept localized tensile stress low in the mine model. As a result the surface crown pillars and vertical supporting pillars were estimated to have an F_s value greater than 2.

It is believed that the massive nature of the limestone and relatively intact carbonatite would preclude tensile failures. Voussoir failures are not expected because of the limited parting in the limestone beds. The massive limestone can reach strength levels comparable to that determined from large size lab tests. When support pillars are removed, localized failure in the carbonatite and limestone may result. Application of cable bolting may prove to be necessary.

Conclusions

Following are the main conclusions of this study:

(a) Analytic study and visual observations indicate that C-102-23 stope with a span of 73m in the strike direction following primary stopping at the upper level is stable under actual field stress conditions.

(b) As expected, the mining of C-102-17 and C-102-19 stopes will induce higher compressive stresses in the lower portion of the stopes. However, the effect of mining these stopes (lower level) on ground stability conditions around C-102-23 stope would be nominal. Compressive stress levels of about 22 MPa will develop near the stope floor; however, they are not sufficiently high to cause a ground stability problem.

(c) This study also indicates that after the partial removal of the sill pillar (S-L-102-17, stage 3 mining), and with the height of L-102-17 pillar exceeding 235m localized tensile failure may occur at about one-eighth of the pillar height measured from the pillar toe. If failure develops, it may jeopardize the integrity of L-102-17 pillar and possibly the stability of stopes in the upper level (300 level) as well. Therefore, it is not recommended to remove the sill pillar at an early stage.

(d) Based on the Drucker/Prager yield criterion, as well as Hoek & Brown's empirical failure criterion, a series of stability analysis studies was carried out. Results indicated no potential failure zones are developing around the stopes. The material properties assumed in both analyses were considered conservative. If failure should occur, it would be localized tension failure rather than shear failure.

(e) Tensile stresses occur in stope walls, and pillars. Although these stresses are small (0 - 2 MPa) when compared with the tensile strength of carbonatite (5 - 10 MPa), localized tensile failure may develop if joints are oriented unfavorably with respect to tensile stresses. In other words, the structural stability of the pillars will be greatly influenced by the presence of vertical or sub-vertical joints. The area for potential tensile failure is located in the lower portion of L-102-17 pillar.

(f) Only localized failures are expected in the surface crown pillars when the uppermost openings are increased to very large spans.

(g) Finite element analysis is, by now, well established for simulation of structural behaviour. Larger size model with 7700 elements, 9200 nodes and 25000 equations or more can be handled very cost-effectively on a super computer such as Cray 1S. The three dimensional finite element technique can be used efficiently and economically in practical mining applications.

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