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PLUME DISPERSION COMPUTATIONS FOR INCO AND FALCONBRIDGE STACK EMISSIONS UNDER LIMITED-MIXING CONDITIONS

by

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ABSTRACT

This report summarizes selected dispersion computations, carried out on behalf of the Department of Justice, to determine if the severity of SO_2 fumigations measured at Lake Penage during the growing seasons of 1961 to 1971 could be related to SO_2 stack emissions from four smelters in the Sudbury area. By utilizing factual rather than postulated dispersion data for limited-mixing conditions, it was shown that the trends for the computed and measured ground-level SO_2 concentrations are in good agreement and that high ground-level SO_2 concentrations measured at Lake Penage are a valid indicator of fumigation conditions over the nearby Whitefish Lake Indian Reserve No. 6. The computations indicate that the three smelters operated by the International Nickel Co. Ltd. and the one smelter operated by Falconbridge Nickel Mines Ltd. would have contributed 92.4 and 7.6 per cent respectively of the ground-level SO_2 to which the Whitefish Lake Indian Reserve No. 6 was exposed during fumigation conditions. Statistical tests, outlined in the appendix, established that the computational analysis can be applied with a reasonable degree of confidence.

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1. INTRODUCTION

The Director of Civil Litigation of the Department of Justice in a letter dated 12 April 1973, requested the assistance of the Mines Branch of the Department of Energy, Mines and Resources in conducting computations related to the dispersion of sulphur dioxide (SO_2) stack emissions over the Sudbury area. The purpose of these computations was to determine if the daily quantities of SO_2 emitted from four smelters - three operated by the International Nickel Co. Ltd. (INCO) and one operated by Falconbridge Nickel Mines Ltd. - could be correlated with the frequency and severity of the fumigation incidents recorded at the Whitefish Lake Indian Reserve No. 6 (IR No. 6) during the growing season of each year from 1961 to 1971.

Insofar as possible, the input data for these computations were to be based on factual information with the fumigations of interest as selected by the Department of Justice, being indicated by observed instances of tree damage in IR No. 6 and by SO_2 records from a ground-level monitor at Lake Penage southwest of IR No. 6. In a few instances, the required information was unavailable making it necessary to statistically interpolate relevant data records to obtain the necessary input values.

The following report (a) lists the sources of input data, (b) outlines the methods of data selection, (c) describes the dispersion computations, (d) compares the computed and measured ground-level SO_2 concentrations under fumigation and selected neutral conditions at Lake Penage, and finally (e) presents computations of ground-level SO_2 concentrations for fumigation and selected neutral conditions at a central point in IR No. 6. In addition, the statistical tests used to validate the dispersion computations are given in the Appendix.

2. INPUT DATA

With the exception of the plume dispersion parameters, all of the input data used in this report were provided by the Department of Justice. As tabulated below the input data were abstracted from reports and records compiled by a number of organizations.

2.1 SO_2 Emission Rates

(a) SO_2 emission rates were abstracted from INCO records for the Copper Cliff and Coniston ore smelting complexes and the Iron Ore Recovery Plant southwest of Copper Cliff. These abstracted data, which covered the period April 1st to October 31st for the years 1960 to 1972 inclusive, summarized the monthly SO_2 emissions from each of the three ore processing operations.

(b) SO_2 emission rates were abstracted from records compiled by Falconbridge Nickel Mines Ltd for the Falconbridge smelter complex. These abstracted data, which covered the period April 1st to October 31st for the years 1961 to 1971 inclusive, summarized the monthly SO_2 emissions from the Falconbridge complex.

2.2 Plume Dispersion Parameters

Quantitative dispersion data for limited-mixing (fumigation) conditions over the Sudbury area were obtained from a Mines Branch Investigation Report submitted previously to the Department of Justice[(1)]. This report describes the behaviour of the three INCO plumes and one Falconbridge plume during a limited-mixing situation as measured on September 12, 1970.

2.3 Meteorological Data

(a) Details of surface wind speed and direction, relative humidity and temperature of the ambient air, and the direction of the plumes from both the Copper Cliff and Coniston stacks were abstracted from INCO records. These abstracted data, which covered the period May 1st to October 31st for the years 1962 to 1968 inclusive, summarized the above meteorological parameters on an hourly basis, except for periods when no observations were recorded.

(b) Summaries of maximum and minimum daily temperatures, daily mean wind speed and direction, maximum hourly wind speed and direction, relative humidity and daily weather observations were obtained from weather records for the period 1961 to 1972 inclusive filed by the Ministry of Transport at Sudbury airport. These data, being daily values, were less precise than the hourly data recorded by INCO; thus the daily data were only used to obtain missing data by interpolation as mentioned earlier.

2.4 Ground-Level SO_2 Measurements

(a) The ground-level SO_2 concentrations at Lake Penage, based on integrated 30-minute measurements of SO_2 on a Thomas Autometer were obtained from records filed by the Ontario Ministry of the Environment. These records covered the period May 1st to October 31st for the years 1962 to 1971 inclusive, except for periods when the Autometer was inoperative.

3. DATA SELECTION AND ANALYSIS

Persistent ground-level impingement of SO_2 bearing plumes generally occur during limited-mixing conditions. These conditions, which are characterized by a ground-based neutral layer that is capped by an inversion aloft, effectively inhibit upward plume dispersion resulting in high concentrations of SO_2 at ground-level.

Accordingly, the data selected for analysis were restricted to those days on which northeasterly winds coincided with (a) tree damage recorded on IR No. 6 and (b) high SO_2 concentrations at Lake Penage. A few days on which good or neutral dispersion conditions occurred with northeasterly winds were also analyzed to verify the correlation of the computations with measured correlation between the computed and measured values; it was assumed that low SO_2 concentrations at Lake Penage were consistent with neutral conditions or a reduction of emissions during a limited-mixing situation such as would occur when the plants are shut down by a labour strike.

3.1 SO_2 Emission Rates

The SO_2 emission-rate data provided for each of the three INCO smelters and the Falconbridge smelter were monthly totals. Therefore, the emission rates on the days of interest were determined by dividing the monthly emission rates by the appropriate number of days in each month. It is important to note that this figure is an average daily emission rate for the entire month and that actual emission rates for specific days may have been higher or lower than average and could have varied by as much ± 5% under normal operating conditions. The Iron Ore Recovery Plant is likely to have a much greater variation than ± 5% due to fluctuating operational characteristics.

3.2 Plume Dispersion Parameters

The location of the plume axis and the values of the plume-spread parameters were derived by using the method of finite differences to numerically analyze the spatial SO_2 measurements taken within the INCO and Falconbridge plumes on September 12, 1970 On that day, limited-mixing conditions prevailed over the rural area around Sudbury and as shown in Figures 1 and 2, respectively, the vertical spread of both the INCO and Falconbridge plumes over the rural area was limited to about 800 metres by an elevated inversion. The underlying neutral layer resulted in good downward dispersion and ground-level plume impingement over a large area(1).

3.3 Meteorological

3.3.1 Wind Speed and Direction

Wind speed and direction vary considerably during the day. For this reason only daytime average values, based on the hourly readings between 0800 and 1600 hrs from INCO records were used. This is the period when limited-mixing conditions would prevail due to progressive break-up of a nocturnal ground-based inversion. When data were unavailable from the INCO records the best statistical estimate was made from the meteorological data recorded at Sudbury airport. It is known that the wind speeds aloft can be appreciably higher than those measured at ground level. In fact, it was found that the mean wind speed over the vertical zone of plume dispersion was 25 per cent higher than the ground during the limited-mixing conditions on September 12, 1970. For this reason the average daytime ground-surface wind speeds calculated from the INCO hourly records were increased by 25 per cent before being incorporated into any dispersion computations. The ground surface wind data are summarized in Table 1.

3.3.2 Plume Direction

Analysis of the axial direction and variation of the Copper Cliff and Coniston plumes showed that convergence of these two plumes occurred over IR No. 6 on the majority of the selected days during which fumigations were recorded. Hence, the theory postulated previously that the Copper Cliff, Coniston and Falconbridge plumes can merge over IR No. 6 is reinforced(1). This being the case, it was decided that the axial SO_2 concentrations were unnecessary.

3.4 Ground-Level SO_2 Concentrations

The Thomas Autometer records of SO_2 concentration at Lake Penage were used to abstract the maximum average 30-minute concentration of SO_2. These average 30-minute concentrations of SO_2 were then compared with the ground-level SO_2 values computed from the dispersion equations given in Section 4.

4. DISPERSION COMPUTATIONS

4.1 Neutral Conditions

The standard "Gaussian" dispersion equation for computing ground-level SO_2 concentrations resulting from an elevated point source is given by:

$$C_o = \frac{10^6 \quad Q}{\pi U \ \sigma_y \sigma_z} \left[\exp - 1/2 \left(\frac{He}{\sigma_z} \right)^2 \right] \qquad \text{Equation I}$$

where C_o = ground-level SO_2 concentration beneath the plume axis, ppm by volume

Q = SO_2 emission rate, std m^3/s

U = mean wind speed over the height of the dispersion zone m/s

He = height of the plume axis above ground elevation, m

σ_y, σ_z = horizontal and vertical standard deviations of plume spread respectively, m

The equation is applicable to all cases where there is no atmospheric layering caused by changes in the vertical temperature gradient. The equation using values of σ_y and σ_z for neutral conditions is usually employed by pollution control agencies to estimate potential ground-level SO_2 concentrations. Under neutral conditions relatively good dispersion occurs and it follows that ground-level SO_2 computations based on Equation I will give lower values than those computed for limited-mixing conditions. As mentioned previously, a few dispersion computations under neutral conditions were necessary to establish an accurate correlation between the computed and measured ground-level SO_2 concentrations for fumigation conditions. The neutral days selected for inclusion in the correlation are given in Table 2 together with their corresponding maximum average 30-minute ground-level SO_2 concentrations recorded at Lake Penage.

4.2 Limited-Mixing Conditions

To compute the ground-level SO_2 concentrations under limited-mixing conditions, Equation I must be modified to account for restricted upward dispersion of the plume. The dispersion equation for limited-mixing conditions, as modified by Hewson[2], is given by:

$$C_o = \frac{10^6 Q}{\sqrt{2\pi} U \sigma_y H} \qquad \text{Equation II}$$

where Q, U and σ_y are as defined in Equation I and

H = the mixing height of the dispersion zone, m; this value was assumed to be 800m (as found during the aerial probing on September 12, 1970) in all computations.

The days on which fumigations occurred, together with their corresponding computed and maximum average 30-minute SO_2 concentrations recorded at Lake Penage are given in Table 2.

5. DISCUSSION OF COMPUTATIONAL DATA

The computed ground-level SO_2 concentrations at the Lake Penage recorder station and a central point in IR No. 6 (i.e. the confluence of Blackwater Creek and Round Lake), due to emissions from the three INCO smelters and the one Falconbridge smelter are given in Tables 2 and 3 respectively. Also given in Table 3 is a breakdown of the percentage contribution of each smelter plume to the computed ground-level SO_2 concentration in IR No. 6.

Figure 3 (a) shows the excellent correlation between the computed and measured ground-level SO_2 concentrations at Lake Penage. This figure indicates that the values of the dispersion parameters used in Equation I and II closely approximate those that prevailed during actual ground-level plume impingement. Figure 3 (b) compares the computed and measured ground-level SO_2 concentration at Lake Penage for both the neutral days required for correlation of Figure 3 (a) and the fumigation days selected by the Department of Justice for the years 1961 to 1971 inclusive. It can be seen that the two SO_2 curves agree reasonably well in both magnitude and time. Statistical tests noted in the appendix established confidence in the computations.

Figure 3 (b) shows that the trend of the SO_2 concentrations computed for the central point in IR No. 6, although higher in magnitude, is not significantly different than the trend of the SO_2 concentrations recorded at Lake Penage. Thus, the recording station at Lake Penage is a good indicator of fumigation conditions over IR No. 6 and it can be assumed with confidence that the SO_2 concentrations at Lake Penage will generally be lower than those prevailing over IR No. 6.

The computational method was validated using the statistical tests given in the Appendix. These tests established that the computed and measured ground-level SO_2 concentrations at Lake Penage are in good agreement for both the neutral and the limited-mixing conditions of interest.

Statistical treatment of the computed ground-level SO_2 data given in Table 3, when averaged over the years 1961 to 1971 inclusive, revealed that the three INCO operations and the single Falconbridge operation would contribute 92.4% and 7.6% respectively of the ground-level SO_2 experienced at the central point in IR No. 6 during the selected limited-mixing conditions. These SO_2 percentages are within 1% of those reported previously in Mines Branch Investigation Report IR 71-37(1).

6. CONCLUSIONS

6.1 An excellent correlation was obtained when the computed and measured ground-level SO_2 concentrations at Lake Penage were compared with the theoretical curve.

6.2 The trend of the computed maximum ground-level SO_2 concentration under fumigation conditions at the confluence of Blackwater Creek and Round Lake, although higher in magnitude, agreed reasonably well with the trend of the SO_2 concentration measured farther downwind at Lake Penage, during the growing seasons of 1961 to 1971 inclusive.

6.3 Considering only the limited-mixing days selected by the Department of Justice, statistical analysis indicated that the INCO and Falconbridge operations contributed 91.4% and 7.6% respectively of the ground-level SO_2 concentrations that prevailed at the confluence of Blackwater Creek and Round Lake.

6.4 The computations in this report further confirm the assumptions and less rigorous calculations reported previously in Mines Branch Investigation Report IR 71-37. In that report (i) it was assumed that the axes of the plumes from Copper Cliff, Coniston and Falconbridge could simultaneously pass over the confluence of Blackwater Creek and Round Lake, and (ii) it was estimated that the INCO and Falconbridge operations would contribute 91.5% and 8.5% respectively of the ground-level SO_2 concentrations that would prevail at the point mentioned in (i) above.

7. REFERENCES

7.1 Whaley, H. and Lee, G. K., "The Dispersion of Smelter Plumes in the Sudbury Area", Mines Branch IR 71-37, May 1971.

7.2 Hewson, E. W., and Gill, G C., "Meteorological Investigation in the Columbia River Valley, Trail, B.C.", U.S. Bureau of Mines Bulletin 4.53, U.S. Govt Patent Office, Washington, D.C., 1944.

7.3 Spiegel, M.R. "Theory and Problems of Statistics" Schaum Publishing Co., New York, N.Y. 1961.

TABLE 1

Wind Data

Date	Meteorological Condition	Daily Wind Data Sudbury Airport Speed, m/s	Daily Wind Data Sudbury Airport Direction	Daytime Wind Data INCO Speed, m/s	Plume Direction, deg Copper Cliff	Plume Direction, deg Coniston
28-6-61	Limited-Mixing	5.0	N	10.3*	-	-
6-6-61		4.5	N	7.6*	-	-
8-6-61		4.7	NE	8.0*	-	-
19-10-61		10.4	NE	12.1*	-	-
5- 8-62		3.5	NE	6.3	226	209
4- 6-63		4.6	NE	6.3	209	211
9- 6-63		7.1	NE	6.3	240	219
2- 6-65		7.5	N	10.5	207	221
13- 8-65		2.0	N	6.5	170	186
6- 9-65		3.6	N	7.2	198	226
16-10-65		-	-	6.9	216	218
27- 8-66		3.6	NE	8.3	200	196
30- 8-66		4.5	NE	9.2	184	220
28-10-66		-	-	10.1	206	237
19- 8-67		3.2	NE	6.3	-	241
27- 8-67		5.8	NE	6.8	219	220
10- 9-67		4.7	NE	7.6	222	249
25- 5-68		3.1	N	5.7	191	233
7- 6-68		4.9	N	7.5	237	232
9- 6-68		5.0	N	7.1	250	260
25- 6-68		6.9	NE	10.1	208	223
10- 7-68		4.5	NE	7.0	204	248
7- 8-68		5.0	NE	8.9*	-	226
6- 7-69		3.5	NNE	6.3*	-	-
6- 8-69		3.1	NE	5.4*	-	-
11- 5-70		6.1	NE	9.8*	-	-
12- 6-70		6.2	NE	8.9*	-	-
13- 8-71		3.6	N	8.5*	-	-
11- 5-62	Neutral	3.8	NE	3.7	254	253
24- 8-63		6.1	NE	5.2	210	214
9- 9-64		4.2	NE	6.1	226	246
10- 7-67		3.2	NE	6.8	240	280
31- 7-67		4.2	NE	6.8	226	260
22- 9-68		3.0	NE	5.0*	177	167
			Mean Plume Direction, deg		214	228

*INCO Records Not Available
Extrapolated From Airport Data

TABLE 2

Comparison of Computed and Measured Ground-Level

SO_2 Concentrations at Lake Penage

Date	Meteorological Conditions	Ground-Level SO_2 Concentrations, ppm Computed	Measured*
28-5 -61	Limited-Mixing	0.54	0.52
6-6 -61		0.73	0.52
8-6 -61		0.69	1.09
19-10-61		0.46	0.53
5-8 -62		0.70	0.49
4-6 -63		0.64	0.89
9-6 -63		0.64	0.61
2-6 -65		0.54	0.88
13-8 -65		0.86	1.17
6-9 -65		0.76	0.76
16-10-65		0.79	1.04
27-8 -66		0.48	0.78
30-8 -66		0.43	1.01
28-10-66		0.49	0.73
19-8 -67		0.80	0.53
27-8 -67		0.74	0.93
10-9 -67		0.68	0.58
25-5 -68		0.90	1.01
7-6 -68		0.67	1.15
9-6 -68		0.71	0.87
25-6 -68		0.50	0.55
10-7 -68		0.71	0.61
7-8 -68		0.59	0.30
6-7 -69		0.54	0.32
6-8 -69		0.09	0.09
11-5 -70		0.52	0.83
12-6 -70		0.55	0.74
13-8 -71		0.54	0.91
11-5 -62	Neutral	0.07	0.15
24-8 -63		0.13	0.19
9-9 -64		0.15	0.37
10-7 -67		0.19	0.05
31-7 -67		0.19	0.21
22-9 -68		0.14	0.10

*Maximum 30-minute Average
**INCO on Strike

TABLE 3

Percentage Contribution of Each Smelter to the Computed Ground-Level SO_2 Concentration at a Central Point in Whitefish Lake Indian Reserve No. 6

Date	Computed Ground-Level SO_2, ppm	SO_2 Contribution from Each Smelter, %			
		Copper Cliff	Coniston	Iron Ore Recovery Plant	Falconbridge
28-5 -61	0.747	76.0	8.2	9.2	6.6
6-6 -61	1.007	74.4	8.2	10.5	6.9
8-6 -61	0.957	74.5	8.1	10.6	6.8
19-10-61	0.637	76.3	7.4	8.5	7.8
5-8 -62	0.965	71.8	9.1	11.0	8.1
4-6 -63	0.932	52.6	3.2	35.7	8.5
9-6 -63	0.932	52.6	3.2	35.7	8.5
2-6 -65	0.779	57.1	4.3	33.1	5.5
13-8 -65	1.235	59.3	4.7	29.9	6.1
6-9 -65	1.092	56.8	4.9	31.9	6.4
16-10-65	1.145	56.4	4.9	33.2	5.5
27-8 -66	0.679	62.3	8.5	21.8	7.4
30-8 -66	0.609	62.4	8.4	21.8	7.4
28-10-66	0.711	58.1	4.8	31.3	5.8
19-8 -67	1.149	56.7	7.0	29.1	7.2
27-8 -67	1.066	56.6	7.0	29.1	7.3
10-9 -67	0.971	58.1	6.3	28.6	7.0
25-5 -68	1.265	69.0	6.6	16.8	7.6
7-6 -68	0.940	66.3	7.0	17.9	8.8
9-6 -68	0.992	66.3	7.1	17.9	8.7
25-6 -68	0.698	66.3	7.0	17.9	8.8
10-7 -68	1.014	62.1	6.2	24.3	7.4
7-8 -68	0.843	61.7	6.4	23.8	8.1
6-7- 69	0.760	58.9	8.4	20.9	11.8
6-8- 69**	0.106	nil	nil	nil	100.0
11-5 -70	0.712	73.4	7.6	9.7	9.3
12-6 -70	0.756	72.0	8.4	11.1	8.5
13-8 -71	0.761	64.5	7.1	20.4	8.0
Mean Value for 1961-1971,%		63.8	6.6	22.0	7.6
Standard Deviation, %		7.4	1.7	8.9	1.4

*For limited-mixing conditions at the confluence of Round Lake and Blackwater Creek.
**Data excluded from averaging - INCO on strike.

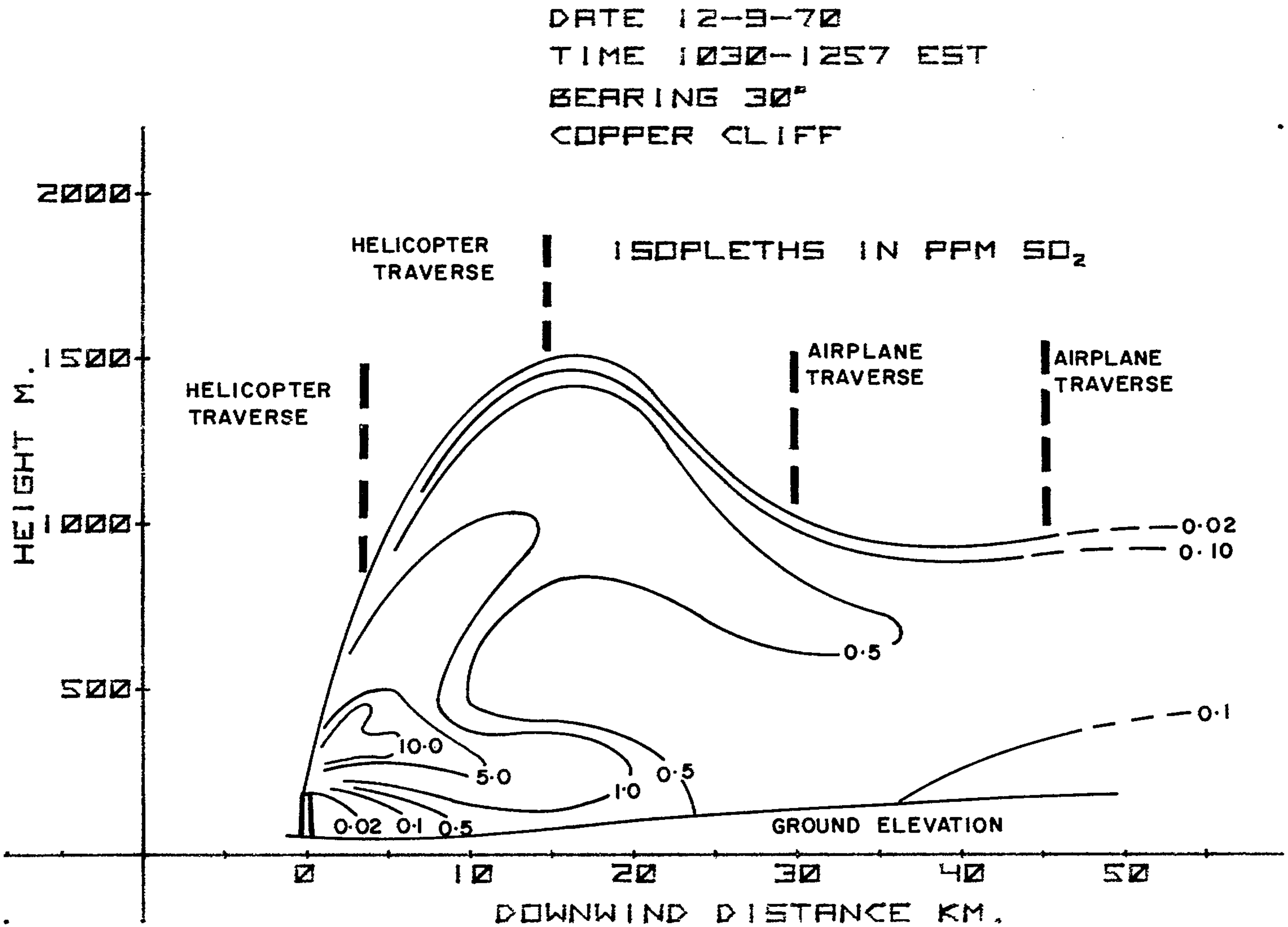

Figure 1. Side view of Copper Cliff plume during limited-mixing conditions.

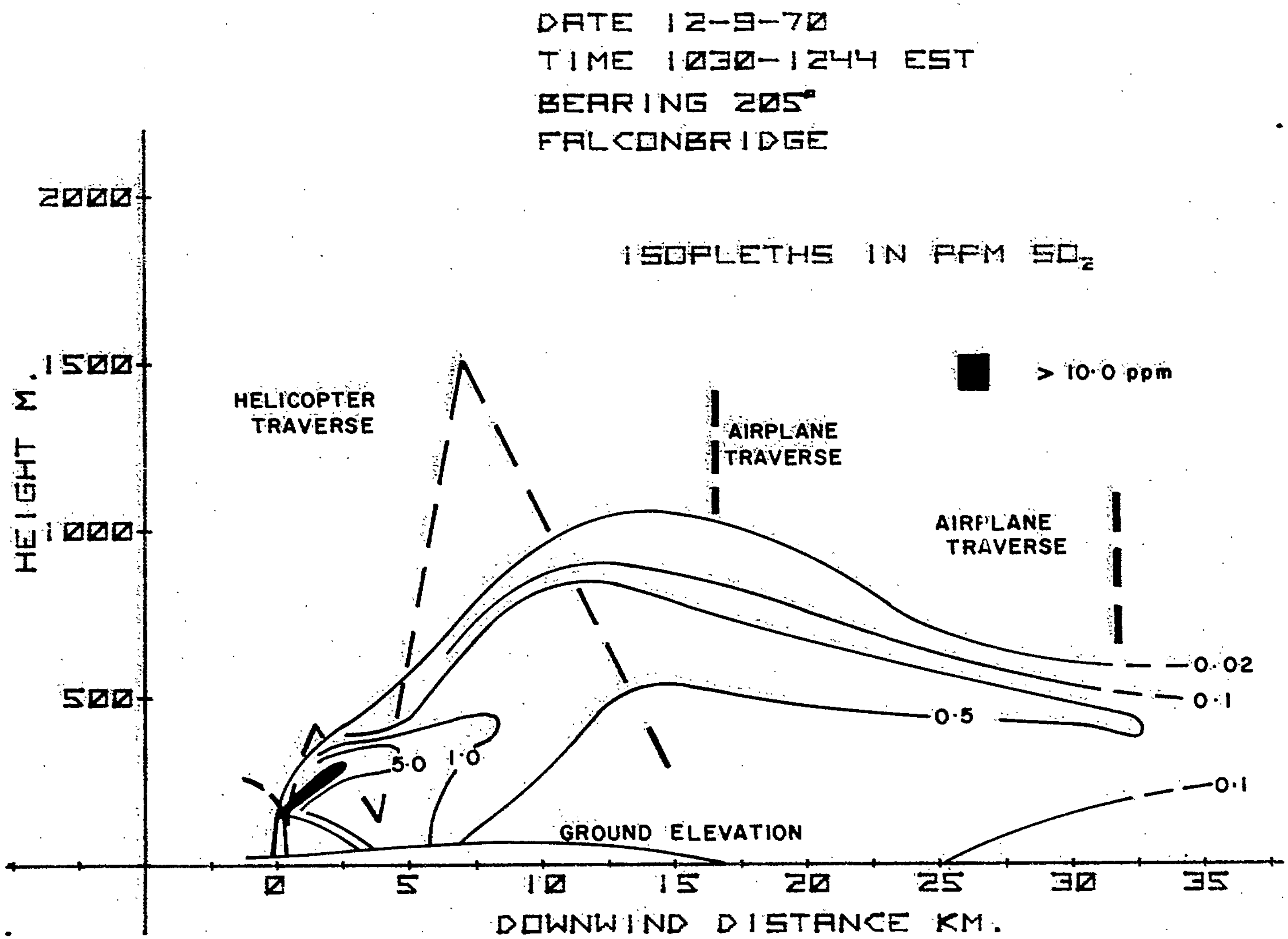

Figure 2. Side view of Falconbridge plume during limited-mixing conditions.

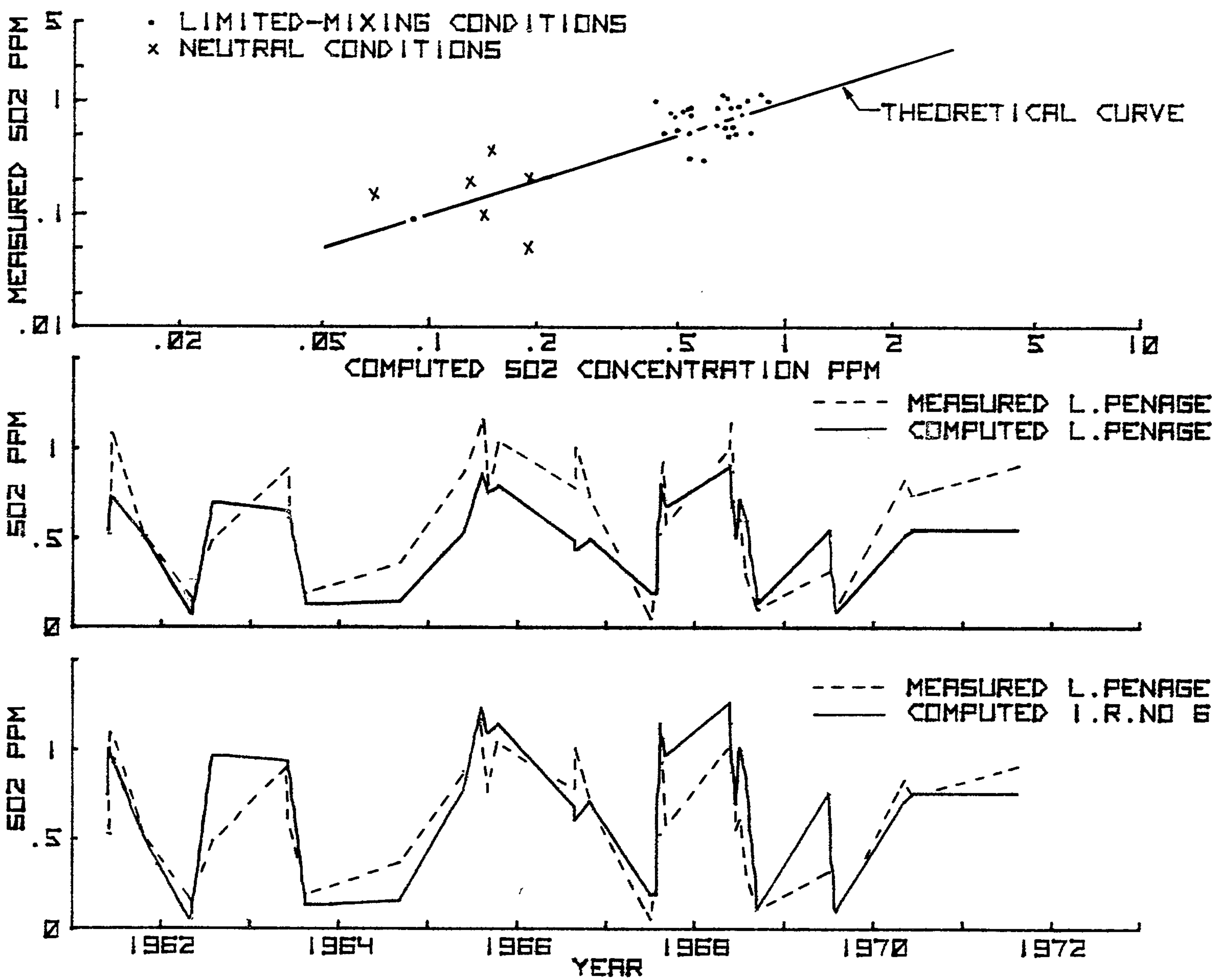

Figure 3. Comparison of computed and measured ground-level SO_2 values:
(a) Computed SO_2 versus measured SO_2,
(b) Computed and measured SO_2 at Lake Penage versus time,
(c) Computed (IR No. 6) and measured (Lake Penage) SO_2 versus time.

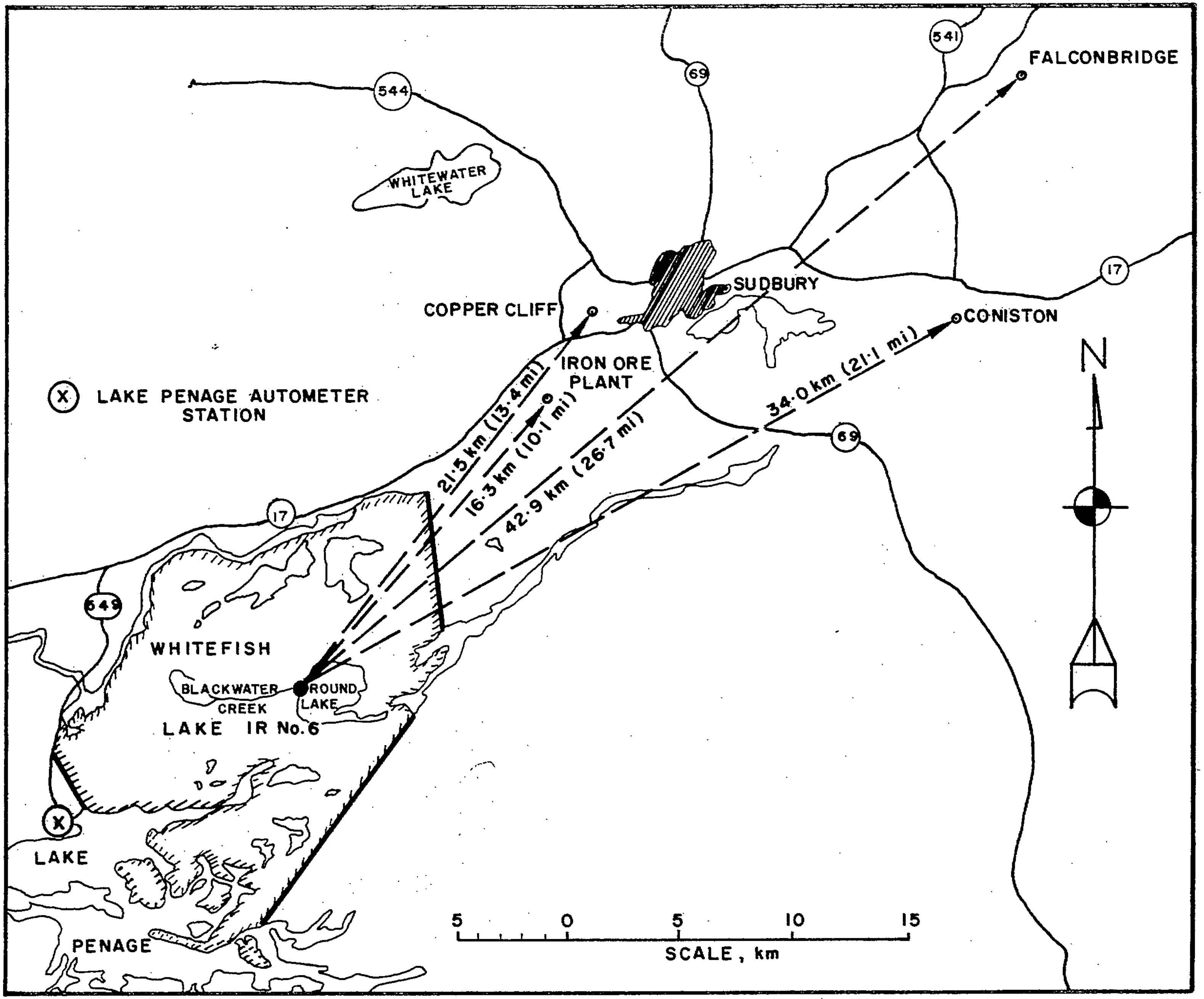

Figure 4. Map of Whitefish Lake Indian Reserve No. 6 showing distances to the smelting operations in the Sudbury area.

APPENDIX

STATISTICAL TESTS OF COMPUTED SO_2 VALUES

Statistics is concerned with scientific methods for collecting, organizing, summarizing, presenting and analysing data, as well as drawing valid conclusions and making reasonable decisions on the basis of such analysis[3]. It is in the latter context that the following statistical tests were applied to the computed and measured values of SO_2 at Lake Penage to establish confidence in the computational method used in this report.

(a) Regression Analysis

Regression is a means of determining the best fit line through data points by the method of least squares. In Figure 3 (a), which shows a plot of the computed versus the measured SO_2 concentrations at Lake Penage, the superimposed theoretical line represents the condition for complete data agreement. For comparative purposes, the theoretical line has a slope of 45° and a constant of 1.00 in the equation $y = ax^b$ whereas a line based on a logarthimic regression of the data points would have a slope of 43.3° and a constant of 1.07. This comparison indicates good agreement between the theoretical and the regression lines.

The correlation coefficient is a measure of the departure of the data points from the regression line. A correlation coefficient of 1 means that all of the points are on the regression line whereas a correlation coefficient of 0 means that a random scattering of points exists with neither a correlation nor a meaningful regression line possible. In the case of Figure 3 (a), the correlation coefficient is 0.82 indicating that the data points are grouped fairly close to the regression line and that scatter is not excessive.

(b) Students t Test

The t statistic is used to establish if two sets of data, in this case the measured and computed values of SO_2 at Lake Penage, are statistically consistent.

The basic equation for the student's t statistic is as follows:

$$t = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{1}{n_x} + \frac{1}{n_y}} \sqrt{\sum(X_i - \bar{X})^2 + \sum(Y_i - \bar{Y})^2}}$$

where:

t	=	the t statistic
$\bar{X}$	=	mean value of computed SO_2 values, ppm
$\bar{Y}$	=	mean value measured SO_2 values, ppm
n_x, n_y	=	total number of measured and computed SO_2 values respectively (in this case $n_x = n_y$)
X_i	=	the "i" th measured SO_2 value, ppm
Y_i	=	the "i" th computed SO_2 value, ppm
	=	mathematical sign representing summation of the above defined terms for all values of i.

To illustrate the significance of this test complete agreement between the means of the computed and measured SO_2 values would give $t = 0$. This would correspond to a probability of 100% that the two sets of data were consistent. Therefore, it follows that any value of t greater than zero represents a deviation from complete agreement and the probability of agreement is reduced. In order to enable the scientist to make a judgement, confidence limits are selected, and if the t value falls inside these limits, then the data are considered consistent. As an example, the 95% confidence limits selected for these dispersion computations, would correspond to t values between ±2.04. For the two sets of SO_2 data $\bar{X}$ = .533 ppm, $\bar{Y}$ = .633 ppm and $n_x = n_y = 34$ giving $t = -1.4$; this t value falls within the 95% confidence limits and therefore we can be reasonably confident that the measured and computed SO_2 values are consistent.

(c) <u>Chi-square Test</u>

The chi-square (χ^2) test is used to verify how well theoretical models predict actual conditions. For this test, a comparison is made between the measured and computed SO_2 concentration by use of a contingency table. As shown in Table 1A, each of the SO_2 concentrations is categorized into intervals of 0.1 ppm and those points which are paired together (i.e. fall into the same interval) are given a weighting factor of 2 or 3 respectively depending on whether they are more or less than half an interval apart. Thus, a contingency table can be drawn up subject to the constraint that there must be equal numbers of measured and computed points after weighting. The values of χ^2 is then estimated as follows:

$$\chi^2 = \sum \frac{(|o_i - e_i| - 0.5)^2}{e_i}$$

where o_i = "i" th observed frequency of SO_2 occurring in a given class (Table 1A)

e_i = "i" th expected frequency of SO_2 occurring in a given interval (Table 1A)

The numerical value of 0.5 in the equation, known as Yate's correction for continuity, is necessary because the SO_2 data have been categorized in a discontinous manner.

If the χ^2 value of this equation is 2.60 there is a 99.5 per cent probability that the computational method is correct whereas if X^2 exceeds 19.7 there is less than a 5 per cent probability of it being correct. A probability of less than 5 per cent for natural phenomenom implies a significant variance between the computed and its corresponding measured SO_2 interval; hence the computational method would be rejected.

Using the SO_2 data in Table 1A, $\chi^2 = 5.927$ for eleven degrees of freedom. This result confirms the validity of the computational method since the probability of it being correct is 88 per cent.

TABLE 1A

Observed and Expected Frequencies* of Computed and Measured SO_2 Concentrations at Lake Penage

SO_2 Interval ppm	Observed Number of SO_2 Data Points										Expected Number of SO_2 Data Points Average of Computed and Measured Weighted Totals
	Computed					Measured					
	Unweighted Total	Weighting Factor 1	Weighting Factor 2	Weighting Factor 3	Weighted Total	Unweighted Total	Weighting Factor 1	Weighting Factor 2	Weighting Factor 3	Weighted Total	
<0.1	2	1	0	1	4	2	1	0	1	4	4.0
0.100 to 0.199	5	3	1	1	8	3	1	1	1	6	7.0
0.200 to 0.299	0	0	0	0	0	1	1	0	0	1	0.5
0.300 to 0.399	0	0	0	0	0	3	3	0	0	3	1.5
0.400 to 0.499	4	4	0	0	4	1	1	0	0	1	2.5
0.500 to 0.599	8	6	1	1	11	6	4	1	1	9	10
0.600 to 0.699	5	4	0	1	7	2	1	0	1	4	5.5
0.700 to 0.799	7	6	0	1	9	4	3	0	1	6	7.5
0.800 to 0.899	2	2	0	0	2	4	4	0	0	4	3.0
0.900 to 0.999	1	1	0	0	1	2	2	0	0	2	1.5
1.000 to 1.099	0	0	0	0	0	4	4	0	0	4	2.0
> 1.1	0	0	0	0	0	2	2	0	0	2	1.0
Total Points	34	27	2	5	46	34	27	2	5	46	46.0

*Frequencies of SO_2 data points derived from Table 2.