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CANADA

DEPARTMENT OF ENERGY, MINES AND RESOURCES

OTTAWA

MINES BRANCH INVESTIGATION REPORT IR 72-7

**APPLICATIONS OF A 2^N FACTORIAL DESIGN
IN A MINERALS TESTING AND PROCESSING
LABORATORY**

by

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MINERAL SCIENCES DIVISION

COPY NO.

09

FEBRUARY 1, 1972

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by
Sutarno* and W.S. Bowman**

SUMMARY

The two-level factorial experimental design is one of the most useful designs for use in those types of experiment that involve a general survey of the effects and interactions of many operational parameters. This report describes the possible uses of this design in a minerals testing and processing laboratory. A computer program for this purpose is provided. It will set up the design matrix and will evaluate the experimental results.

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INTRODUCTION

In both laboratory- and in production-scale operations, one very often encounters a problem of determining the effects on a certain process of various parameters of which their theoretical relationships are often not fully understood. In many cases, the experimenter is guided by his "intuition" to consider that a certain variable, A, will have an effect on the process, whereas another variable, B, which, by itself, will have no effect but will, in the presence of A, have a significant effect on the product of the process.

In general, the experimenter encounters the following questions:

- (1) Do variables A, B, C, D, etc., either by themselves or in the presence of the operation of the others, have a significant effect on the process?
- (2) If so, to what extent?

In the absence of a theoretical answer, an empirical relation must be developed to answer the above questions.

One of the most economical experimental designs, to perform a survey to provide the answer to Question (1), and, to a certain extent, to Question (2), is the two-level factorial design (2^N design). In this design, all variables (factors) are investigated at two levels. This design will involve 2^N experimental runs, where N is the number of factors. Thus 2^N will be the number of all possible combinations of these factors.

POSSIBLE APPLICATIONS OF A 2^N FACTORIAL DESIGN IN A MINERALS TESTING AND PROCESSING LABORATORY

In a laboratory for testing and/or processing minerals, the 2^N factorial design might be used in analytical chemistry, complex syntheses and in empirical modelling. These applications are discussed hereunder.

(1) Analytical Chemistry

(a) The Development of an Analytical Procedure

Many analytical procedures are developed for a specific purpose. This is particularly true in the field of ore analysis. The development of the procedures is commonly conducted by a highly-qualified scientist. However, once these procedures have been established, they will be more valuable if the routine analyses can then be performed by a reasonably competent technician. For this reason, the developer of such procedures should report the variables in his recommended procedure that require very strict control and those that require only a moderate degree of control in order to achieve a result with the desired level of precision. Thus, the user of these methods will then be able to apply them with minimum cost to provide an adequate control and to produce an acceptable precision in the final results.

In this case, a 2^N factorial experiment will provide a thorough survey of the effects of the appropriate variables.

(b) Determination of the Effect of the Environment on the Accuracy of an Analytical Procedure

Let us suppose that one wanted to analyse a leaching solution by a direct physical method. The solution will probably contain elements besides the one or ones of interest. In this case, one would like to know the effects of these other elements on the instrumental readings for the main elements so that appropriate corrections could be made, if necessary. If the problem can be solved, a 2^N experiment would be a very suitable means of finding the solution. In this case, the various factors can be accurately controlled because they are in solution form. This type of experiment can be extended to a solid system. However, in this case, one must be careful in using the results because some of the physical variables, such as particle size distribution, particle orientation, etc. cannot be either fully controlled or specified.

(c) Calibration of Electronic or Other Physical Testing Equipment Used in a Laboratory

There are numerous types of electronic equipment used in a modern analytical laboratory. To operate these items of equipment, the operator is sometimes required to determine various suitable settings of the electronic and/or mechanical dials, depending on the nature of the sample to be analysed. For a given sample, the readings (responses) may change from one setting to another. The 2^N experimental design is suited to determining the sensitivities of these various dial settings in controlling the response of the equipment.

(2) Complex Syntheses

One example of this type of problem is the investigation of the effect of process variables on the magnetic and ceramic properties of ferrites. There are numerous variables that may effect the final properties of the ferrite ceramic; therefore, a 2^N factorial experiment should be done because it is the most economical method of investigating all these variables simultaneously.

(3) A Tool for Empirical Modelling

Should the model be either linear or very close to linear at a given range of variable levels, a 2^N factorial experiment would be very suitable. If the model is found to be non-linear with respect to some of the variables in the ranges of interest, the design can be extended into a more complicated design based on $Z^k 2^{N-k}$ etc., instead of on 2^N , where $Z > 2$, depending on the requirements of the information to be obtained.

DESIGN PROCEDURE

The term "factor" in a 2^N factorial design refers to the controllable process variables. For example, in chemical analyses, it could be pH, concentration, etc. These factors can be either qualitative or quantitative in nature. It is obvious that, for the qualitative factors, all that can be determined is whether they have any effect on the response

or not. The term "response" refers to any measurable property of the process product(s) that is of interest to the experimenter. In chemical analysis, it means, of course, the final analytical result achieved.

In this discussion, it is convenient to assume that all factors are quantitative because the qualitative factors can always be "artificially" quantified by expressing them as a "dummy" number.

These quantitative factors are then given coded values of -1 and +1 for the lowest and the highest levels, respectively, of the factor in which the experimenter is interested; i.e., if the experimenter is interested in seeing the effect of a pH change from 7 to 8, then he would assign pH = 7 as (-1) and pH = 8 as (+1). In this case, therefore, one coded unit of this particular factor is equal to 0.5 pH unit. Having assigned the low and high values to all the factors, the experimenter then combines all these factors for his experimental run. There will be a total of 2^N different combinations for the N factors. This schematic plan is called the "design matrix". The following is an example of a design matrix:

Run No.	Factors					
	X_1	X_2	X_3	X_4	X_N
1	-1	-1	-1	-1	-1
2	+1	-1	-1	-1	-1
3	-1	+1	-1	-1	-1
4	+1	+1	-1	-1	-1
---	---	---	---	---	---
---	---	---	---	---	---
2^N	+1	+1	+1	+1	+1

To eliminate any possible time-dependence of the response (the experimental result), it is desirable to conduct the actual experiments in a random sequence with respect to the run numbers.

EVALUATION OF THE RESULTS

One of the advantages of a two-level factorial design is the ability to evaluate the effects of all factors, including their interactions, using only relatively simple arithmetic. This simplicity is a consequence of the orthogonal properties of the design matrix. The interaction columns in the matrix can be considered as the product of the interacting factors(1).

The mathematical model of the response can be expressed by the following polynomial:

$$y_i = a_1 + a_2 x_2 + a_3 x_3 + a_4 x_2 x_3 + \dots + \mathcal{E}_i \quad \dots [\text{Eq. 1}],$$

where

- y_i = response in run number i ;
- a_j = constants (i. e., the coefficients); and
- \mathcal{E}_i = error in run number i .

The coefficients, a_j 's, can be computed by the least-squares method(2).

Equation 1 can be written in a matrix form, thus:

$$Y = X A + \mathcal{E}$$

where

- Y = a column vector of response y_i ;
- A = a column vector of coefficient a_j ;
- X = a $(2^N \times 2^N)$ matrix of x_{ij} ; and
- \mathcal{E} = a column vector of error \mathcal{E}_i .

If one can assume that the error, \mathcal{E}_i , is normally distributed with an expected value of 0 and a variance of σ^2 , then the fitted regression line can be computed as follows:

From Equation 1, we get

$$\mathcal{E} = Y - X A \quad \dots [\text{Eq. 2}]$$

The sum of the squares of the error, \mathcal{E}_i , for all values of i 's can be expressed in the following matrix form:

$$\begin{aligned} \mathcal{E}^T \mathcal{E} &= [Y - X A]^T [Y - X A]^* \\ &= [Y^T - A^T X^T] [Y - X A] \end{aligned}$$

whence

$$\mathcal{E}^T \mathcal{E} = Y^T Y - A^T X^T Y - Y^T X A + X^T A^T X A \quad \dots [\text{Eq. 3}]$$

In order for the sum of the squares of the errors to be a minimum,

$$\frac{\partial (\mathcal{E}^T \mathcal{E})}{\partial A} = 0 \quad \dots [\text{Eq. 4}]$$

and consequently,

$$X^T X A = X^T Y \quad \dots [\text{Eq. 5}]$$

Therefore,

$$A = (X^T X)^{-1} X^T Y \quad \dots [\text{Eq. 6}]$$

Equation 5 is commonly known as the normal equation.

If the design is such that the X-matrix is orthogonal, the resulting pre-multiplication by its own transpose will be a diagonal matrix. In the case of 2^N factorial design, this diagonal matrix will have diagonal elements of 2^N . The right-hand side of Equation 5 is a column vector with elements of

$$\sum_{i=1}^{i=2^N} x_{ij} y_i \cdot$$

* X^T is a transpose of matrix X.

The left-hand side of Equation 6 is a column vector with elements of a_j 's. Therefore, Equation 6 can be rewritten as:

$$a_j = \frac{\sum_{i=1}^{2^N} x_{ij} y_i}{2^N} \dots [\text{Eq. 7}]$$

where:

$$j = 1, 2, \dots, 2^N;$$

and

$$x_{i1} = 1.$$

A computer program has been written in FORTRAN language for the 2^N factorial design and evaluation. Depending on the option selected by the user, the program will write the design matrix only, or will compute the coefficients, or will compute the coefficients and write the X-matrix. An example of a program listing and of a test run is illustrated in the last section of this report.

INTERPRETATION OF THE RESULTS

In interpreting the results of a two-level factorial experiment, one must note that the coefficients, a_j , were computed by drawing a straight line between two points or between two groups of points, if repeat runs were performed. Therefore, Equation 1 is only a crude linear approximation of the model. However, the magnitude of the a_j 's will provide a good estimate of the extent of the effect of a particular factor. In many literature references, a_0 is commonly called the effect total and the a_j - values are half of the effect of factor j , including the interaction.

The variance of the a_j - values, $V [a_j]$, can be computed from the variance - covariance matrix $(X^T X)^{-1} \sigma^2$. In this case, because $(X^T X)$ is a diagonal matrix with elements of 2^N ,

$$V[a_j] = \frac{\sigma^2}{2^N} \quad \dots [\text{Eq. 8}]$$

where σ^2 is the variance of each observation. In practice, σ^2 is not known; therefore, it must be estimated, either by repeating or by using previous knowledge of σ^2 values. Another method of estimating the value of σ^2 is by observing the sum of squares associated with the coefficients. Using "intuition", the smaller sums of squares can be pooled and used to estimate the value of σ^2 .

The confidence interval of the coefficients, a , can then be constructed, based on the degrees of freedom of their variance.

The variance of the expected response, \hat{Y}_k , at $X = X_k$, $V[\hat{Y}_k]$, can be computed by the following formula:

$$V[\hat{Y}_k] = X_k^T X^T X X_k \sigma^2 \quad \dots [\text{Eq. 9}]$$

where X_k is a vector with elements of $(1, x_{k1}, \dots, x_{kN})$,

Having computed the statistical significance of each of the coefficients [i. e., of each factor], one can then evaluate the importance of each of these factors. This stage of the evaluation cannot be generalized, because it depends on various factors such as the units used, economic factors, etc.

FRACTIONAL FACTORIAL DESIGN

On the assumptions that the observations are uncorrelated and have equal variance, the 2^N factorial design provides independent minimum-variance estimates of the grand average and of the $(2^N - 1)$ effects which consist of:

- (i) k main effects;
- (ii) $\frac{k(k-1)}{2}$ two-factor interaction effects;
- (iii) $\frac{k(k-1)(k-2)}{2 \cdot 3}$ three-factor interaction effects;

(iv) $\frac{k(k-1)(k-2) \dots (k-h+1)}{h!}$ h-factor interaction effects; and

(v) a single k - factor interaction effect (1).

Hence, this design is the most economical one to undertake in a complete survey of all possible variables.

In many cases, however, the number of possible factors is large and a complete 2^N factorial design becomes prohibitive in cost. Furthermore, it is often the case that the high-order interactions are either negligible or difficult to explain in terms of their physical meaning. In these cases, unnecessary to perform a complete survey of the effects of these variables. In these situations, a fractional two-level factorial design may be more economical. This design is commonly referred to as a 2^{N-P} factorial design. It must be noted, however, that, by decreasing the number of experiments, some information will be lost.

Depending on the information that the experimenter wishes to obtain from his experiments, there are various types of fractional factorial design. There are no definite rules that will be applicable to all situations. However, numerous papers have been written on this subject. Among these, papers by Box and Hunter (1, 3) gave an excellent description of this subject and an N. B. S. publication (4) gives a list of plans for various fractional replications of a factorial design up to 16 factors.

EXAMPLE OF THE APPLICATION OF A 2^N FACTORIAL DESIGN IN THE CALIBRATION OF AN ANALYTICAL EQUIPMENT

In industry, one very often encounters the problem of performing the quick analysis of a solution containing several elements that interfere with one another in their determinations. In a laboratory, this problem is commonly solved by separating the interfering elements before a particular physical or chemical method is applied for the desired element. This procedure, however, can be tedious and, in some cases, impossible to perform. For example, if one has to analyse a leaching solution on-line, most probably the solution will contain several other elements that might

interfere with the determination of the desired element. Therefore, it is necessary to calibrate the instrument with a solution containing approximately the same set of elements. For this problem, a 2^N factorial experiment is economically acceptable. Having done the calibration, a correction procedure may then be developed that can be programmed directly into the instrument.

Suppose the leaching solution contains six elements: A, B, C, D, E, and F and that the typical operating concentration of these elements are A_0 , B_0 , C_0 , D_0 , E_0 , and F_0 , respectively. The most probable limiting concentrations are $A(-)$, to $A(+)$, $B(-)$ to $B(+)$, etc.; therefore, one can assign $A(-)$ as (-1) , $A(+)$, as $(+1)$, etc., in coded units. A unit transformation can be found from the requirement that $[A(+)-A_0] = [A_0 - A(-)] = 1$ in coded units.

The next step will be to set up the design matrix or the experimental plan. For this purpose, 2^N bottles of the solutions, containing all the elements concerned with concentration combinations according to the design matrix, are prepared. N readings (for the N elements) are then taken on each bottle. The total number of instrumental readings will thus be $N \cdot 2^N$. From these readings, N sets of coefficients, a_j , can be computed. There will be a total of $N \cdot 2^N$ such coefficients for the N elements. For the purposes of further discussion, these coefficients will be called the calibration coefficients.

The development of the corrections will involve the solution of the following set of equations:

$$y_v = a_{vi} f(C_v) + a_{vj} f[C_1, C_2] + \dots + a_{vk} f(C_1 \dots C_N) \quad \dots [\text{Eq. 10}]$$

where

y_v = the reading for element v ;

a_{vi} = coefficients of the main effect for element v ;

a_{vj} and a_{vk} = coefficients of the interaction effects.

C_v = concentration of element v^* .

The above Equation 10 gives rise to a set of non-linear equations. However, the solution that will be of interest will be a set of real and positive values of C_v . The complexity of the solution of Equation 10, of course, depends on the nature of the calibration coefficients, a , which may be positive or negative.

In many practical cases, the extent of the interferences is not too severe. It probably will fall into either of the following two categories:

(i) The interference is small:

In this case, as a first approximation, one can simplify Equation 10 into a set of independent equations:

$$y_v = a_{v0} + a_{v1} C_v \quad \dots [\text{Eq. 11}]$$

The value of C_v^1 from this first approximation can be substituted into Equation 10 to compute a closer approximation until the value of $|C_v^n - C_v^{n+1}|$ will be smaller than a certain arbitrary quantity. A statistical parameter, such as the standard deviation or a multiple thereof, can be used as this arbitrary quantity for this purpose.

(ii) The interference is not small but is concentrated in the linear portion of Equation (10).

In this case, as a first approximation, one can use the root of a set of N linear equations of the type:

$$y_v = a_{v0} + a_{v1} C_1 + a_{v2} C_2 \dots a_{vN} C_N \quad \dots [\text{Eq. 12}]$$

above. Further procedures will be the same as in (i).

* f represents the functional dependence of the response on this quantity.

COMPUTER PROGRAM FOR A 2^N FACTORIAL EXPERIMENT

a. Purpose: To set the design matrix and to evaluate the experimental results of a 2^N factorial experiment for N having values up to 7.

The output of this program is in the form of regression coefficients for all factors and their interactions. To compute the factor effect, these coefficients, except a_0 (which is constant), must be multiplied by a factor of 2 to allow for the complete range of the factor values. The constant, a_0 , is the effect total.

b. Language: The program was written in FORTRAN language. The user should also provide a suitable set of control cards for his particular system.

c. General Description: The flow-chart of this program is illustrated in Figure 1.

(i) Data Cards:

Card 1. Title card. Up to 80 columns of alphanumeric information.

Card 2. Number of factors. Integer input in Column 2.

Options: Integer input in Column 5. There are three options that will control the output:

(a) 1; design matrix only will be printed;

(b) 0 or blank; the experimental results and the regressions, and the sum of squares will be printed; and

(c) 2; the same as (b), plus the X-matrix will be printed.

Cards 3 to (3 + N). Lower level of factor i in Columns 1-10. Upper level of factor i in Columns 11-20. Both are floating-point inputs with decimal point. Name of factor i in Columns 21-26. Unit of factor i in Column 31-36. Both are alphanumeric informations.

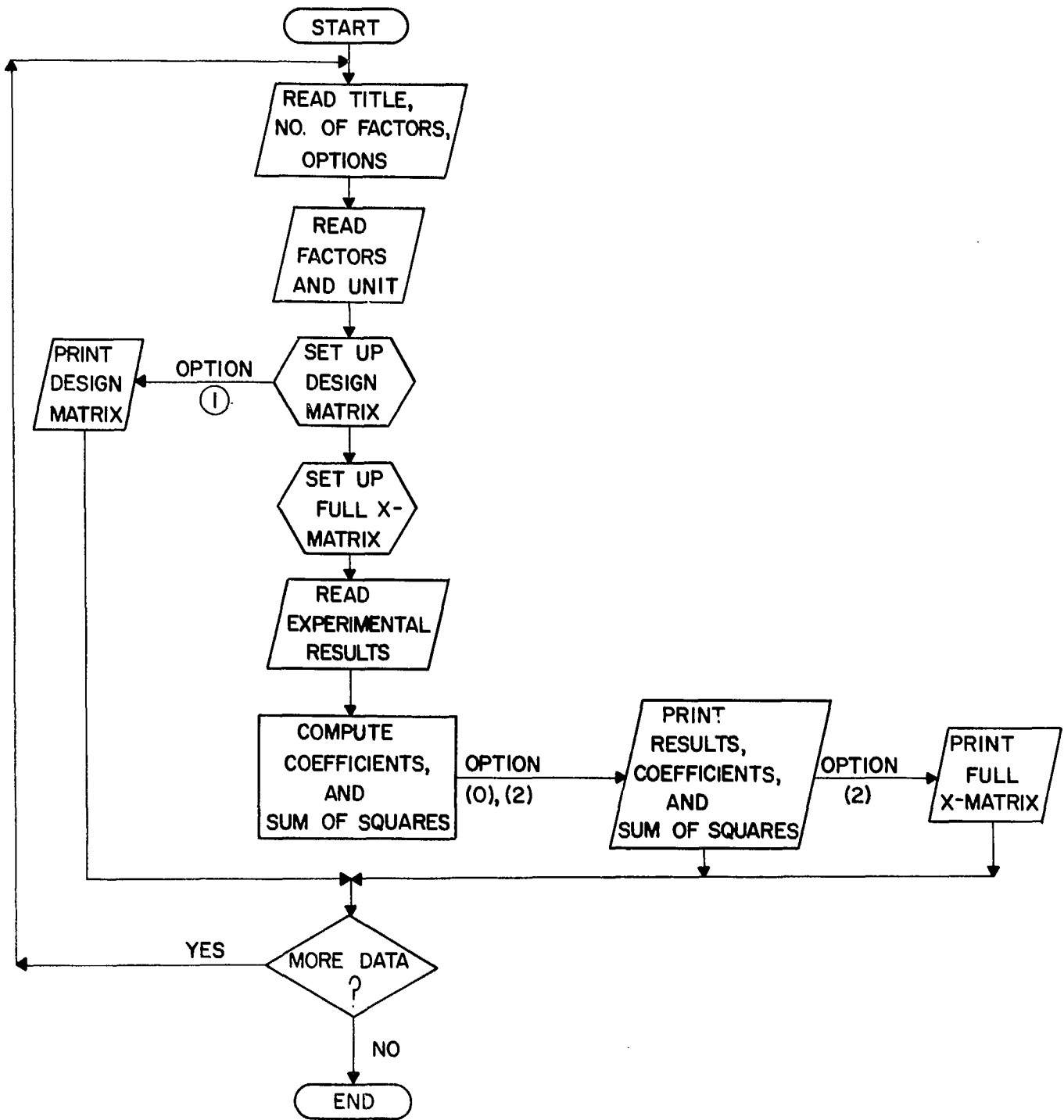


Figure 1. Flow-chart of the Computer Program for a 2^N Factorial Experiment.

Cards $(4 + N)$ to $(3 + N + 2^N)$. Observed response of run v in Columns 1-10. Floating-point inputs with decimal point. Run sequence, sample number or other identification numbers in Columns 77-80. Integer input.

(These cards are only required if the option is other than 1).

NOTE: Since the program was written in a standard sequence of runs, the responses must be entered in the same sequence as with the design matrix (from Run No. 1 to Run No. 2^N).

(ii) Test runs:

The program listing and the test run (Example run) for Options 1 and 2 are given immediately following the References in this report (see pages 17 to 27). This run was performed using a CDC 6400 computer. The data used in this run were the results of a measured weight-loss on the Mn-Zn ferrite that occurred during sintering. Six factors were investigated in this experiment. They were calcination temperature, milling time, forming pressure, sintering temperature, soaking time and sintering atmosphere. The specimens were in the form of toroids with 1.5 in. O.D. and 0.75 in. I.D. The weight losses were expressed in per cent of the original weight of the specimens.

The result of the analysis of these data, in the form of regression coefficients and sums of squares of errors associated with the corresponding factors, are listed on pages 29 and 31 . Both the design matrix and the complete X-matrix are listed for this test run on pages 25 and 33 to 43. Any one or both of these matrices can, of course, be omitted at the option of the operator.

REFERENCES

1. G.E.P. Box and J.S. Hunter, "The 2^{k-p} Fractional Factorial Design, Part I", Technometrics, 3, [3] 311-351 (1961).
2. N.R. Draper and H. Smith, "Applied Regression Analysis", John Wiley and Sons, Inc., New York, (1966).
3. G.E.P. Box and J.S. Hunter, "The 2^{k-p} Fractional Factorial Design, Part II", Technometrics 3, [4] 449-458 (1961).
4. Statistical Engineering Laboratory, "Fractional Factorial Experiment Designs for Factors of Two Levels", National Bureau of Standards Applied Mathematics Series 48, April 15, 1957.

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APPENDIX

- 17 -

PROGRAM

MAIN

TRACE

CDC 6400 FTN V3.0-P265 OPT=0

PROGRAM MAIN(INPUT,OUTPUT,TAPE1=INPUT,TAPE3=OUTPUT)

C
C
C
C-----
PROGRAMME FOR TWO LEVEL FACTORIAL DESIGN FORTRAN V

5

REAL LABEL

DIMENSION LABEL(128),XM(128,7),X(128,128),Y(128),A(128),SS(128)
DIMENSION FACT(7),UNIT(7),XFACT(7,2),RUN(128),TITLE(16),NX(128)
DIMENSION HDG(7)

10

DATA ICR/1/,LPT/3/

DATA LABEL/6HCONST.,6HA

	,6HBC	,6HABC	,6HCD	,6HAD	,6HBD	,6HABD	,6HCD	,
	26HACD	,6HBCD	,6HABCD	,6HE	,6HAE	,6HBE	,6HABE	,
	36HCF	,6HACE	,6HBCE	,6HABCE	,6HDE	,6HADE	,6HBDE	,
15	46HARDE	,6HCDE	,6HACDE	,6HBCDE	,6HABCDE	,6HF	,6HAF	,
	56HBF	,6HARF	,6HCF	,6HACF	,6HBCF	,6HABCF	,6HDF	,
	66HADF	,6HRDF	,6HARDF	,6HCDF	,6HACDF	,6HBCDF	,6HABCDF	,
	76HEF	,6HAEF	,6HBEF	,6HABEF	,6HCEF	,6HACEF	,6HBCEF	,
	86HARCEF	,6HDEF	,6HADEF	,6HRDEF	,6HABDEF	,6HCDEF	,6HACDEF	,
20	96HBCDEF	,6HARBCDEF	,6HG	,6HAG	,6HRG	,6HABG	,6HCG	,
	16HACG	,6HBCG	,6HARCG	,6HDG	,6HADG	,6HBDG	,6HABDG	,
	26HCDG	,6HACDG	,6HBCDG	,6HARCDG	,6HEG	,6HAEG	,6HBEG	,
	36HAREG	,6HCEG	,6HACFG	,6HRCEG	,6HARCEG	,6HDEG	,6HADEG	,
25	46HRDEG	,6HARDEG	,6HCDEG	,6HACDEG	,6HBCDEG	,6HABCDEG	,6HFG	,
	56HAFG	,6HRFG	,6HARFG	,6HCFG	,6HACFG	,6HBCFG	,6HABCFG	,
	66HDFG	,6HADFG	,6HRDFG	,6HARDFG	,6HCDFG	,6HACDFG	,6HBCDFG	,
	76HARCDFG	,6HEFG	,6HAEFG	,6HBEFG	,6HABEFG	,6HCEFG	,6HACEFG	,
	86HARCEFG	,6HARCEFG	,6HDEFG	,6HADEFG	,6HRDEFG	,6HARDEFG	,6HCDEFG	,
	96HACDEFG	,6HRCDEFG	,6HSEVEN /					

30

1 READ (ICR,1000) TITLE

1000 FORMAT (16A5)

READ (ICR,1001) NFACT,NOPT

1001 FORMAT (I2,I3)

N = 2**NFACT

35

AN= N/2

DO 2 I=1,N

DO 2 J=1,NFACT

AI=I

AJ=J

40

L = AI/(2.0**(AJ-1.0)) + 1.0 - 1.0/AN

2 XM(I,J) = (-1)**L

READ (ICR,1010) (XFACT(I,1),XFACT(I,2),FACT(I),UNIT(I),I=1,NFACT)

1010 FORMAT (2F10.2,A6,4X,A6)

WRITE (LPT,1050)

45

1050 FORMAT (1H1,T20,"FACTORIAL EXPERIMENTS"//)

WRITE (LPT,1051) TITLE

1051 FORMAT (1H0,16A5//)

WRITE (LPT,1052) NFACT,N

1052 FORMAT (T10,"NUMBER OF FACTORS

:",I6//T10,"NUMBER OF RUN

50

IS :",I6//)

WRITE (LPT,1053)

1053 FORMAT (T4,"NO",T10,"FACTOR",T24,"LOW (-1)",T44,"HIGH (+1)",T71,"O

NE CODED UNIT EQ. TO"//)

DO 12 I=1,NFACT

55

T = (XFACT(I,2)-XFACT(I,1))/2.0

PROGRAM

MAIN

TRACE

CDC 6400 FTN V3.0-P265 OPT=0

```

      J = 2**(I-1)+1
      12 WRITE (LPT,1054) LABEL(J),FACT(I),XFACT(I,1),XFACT(I,2),UNIT(I),T,
        1UNIT(I)
60 1054 FORMAT (3X,A2,T10,A6,T20,F10.2,T38,F10.2,T52,A6,T70,F10.2,T84,A6/)
      IF (NOPT-1) 17,13,17
      13 WRITE (LPT,1055)
1055 FORMAT (1H1,T20,"THE DESIGN MATRIX"//)
      DO 14 K=1,NFACT
      J=2**(K-1)+1
65 14 HDG(K)=LABEL(J)
      WRITE (LPT,1056) (HDG(K),K=1,NFACT)
1056 FORMAT (T4,"RUN NO",T20,7(4X,A6)/)
      DO 16 I=1,N
      DO 15 K=1,NFACT
70 15 NX(K)=XM(I,K)+XM(I,K)/N
      16 WRITE (LPT,1057) I,LABEL(I),(NX(K),K=1,NFACT)
1057 FORMAT (T4,I4,T14,A6,T22,I3,6I10)
      GO TO 1
      17 DO 4 I=1,N
75 X(I,1)=1.0
      DO 4 J=1,NFACT
      N1 = 2**(J-1)
      DO 4 J1=1,N1
      J2 = N1 + J1
80 4 X(I,J2) = X(I,J1) * XM(I,J)
      READ (ICR,1002) (Y(I),RUN(I),I=1,N)
1002 FORMAT (F10.3,66X,I4)
      WRITE (LPT,1063)
85 1063 FORMAT (1H1,T20,"EXPERIMENTAL RESULTS"//)
      DO 34 K=1,NFACT
      J = 2**(K-1)+1
      34 HDG(K)= LABEL(J)
      WRITE (LPT,1064) (HDG(K),K=1,NFACT)
90 1064 FORMAT (T4,"RUN NO",T22,"EXPT.NO",T34,"OBSERVED",T50,7(8X,A2)/)
      WRITE (LPT,1070)
1070 FORMAT (1H0)
      DO 36 I=1,N
      DO 35 K=1,NFACT
      J=2**(K-1)+1
95 35 NX(K)=X(I,J)+X(I,J)/N
      36 WRITE (LPT,1065) I,LABEL(I),RUN(I),Y(I),(NX(K),K=1,NFACT)
1065 FORMAT (T4,I4,T14,A6,T22,I4,T32,F12.4,T50,7I10)
      DO 8 J=1,N
      A(J)=0
100 DO 7 I=1,N
      7 A(J) = A(J) + X(I,J)*Y(I)
      A(J) = A(J)/N
      8 SS(J) = A(J)*A(J)*N
      WRITE (LPT,1061)
105 1061 FORMAT (1H1//,T36,"REGRESSION COEFFICIENTS AND THEIR SUM OF SQUARE
        1S"//,T4,"NUMBER",T13,"VARIABLE",T25,"COEFFICIENT",T40,"SUM OF SQUA
        2RES",T70,"NUMBER",T80,"VARIABLE",T92,"COEFFICIENT",T107,"SUM OF SQ
        3UIARES"//)
      N1 = N/2
110 DO 30 I=1,N1

```


PROGRAM

MAIN

TRACE

CDC 6400 FTN V3.0-P265 OPT=0

```

      N2 = N1 + 1
      30 WRITE (LPT,1062) I,LABEL(I),A(I),SS(I),N2,LABEL(N2),A(N2),SS(N2)
1062 FORMAT ("0",T4,I4,T14,A6,F16.6,T37,F17.8,T70,I4,T81,A6,F16.6,F17.8
      1)
115      IF (NOPT -1) 1,1,21
      21 L = 1
      18 K1 = 12*(L-1) + 1
      K2 = 12*L
      19 WRITE (LPT,1058) L
120 1058 FORMAT (1H1//,T50,"X-MATRIX",I6//)
      WRITE (LPT,1059) (LABEL(J),J=K1,K2)
1059 FORMAT (T4,"RUN NO",T26,12(2X,A6)/)
      DO 22 I=1,N
      DO 20 J=K1,K2
125 20 NX(J)=X(I,J)+X(I,J)/N
      22 WRITE (LPT,1060) I,LABEL(I),(NX(J),J=K1,K2)
1060 FORMAT (I7,T12,A6,T21,12I8)
      L = L+1
      IF (N-K2) 1,1,24
130 24 K3 =N-K2
      IF (12-K3) 18,18,26
      26 K1=K2+1
      K2=K3+K2
      GO TO 19
135      END
```


FACTORTIAL EXPERIMENTS

TEST RUN NO. 1 (OPTION 1)

TWO LEVEL FACTORTIAL EXPERIMENTAL PLAN FOR FIVE FACTORS

NUMBER OF FACTORS : 5

NUMBER OF RUNS : 32

FACTOR	LOW (-1)	HIGH (+1)		ONE CODED UNIT EQ. TO	
CALC.	900.00	1000.00	DEG.C	50.00	DEG.C
MILL	24.00	48.00	HOUR	12.00	HOUR
PRESS	5000.00	10000.00	PSI	2500.00	PSI
TEMP.	1250.00	1300.00	DEG.C	25.00	DEG.C
SOAK	120.00	240.00	MINUTE	60.00	MINUTE

THE DESIGN MATRIX

RUN NO		A	B	C	D	E
1	CONST.	-1	-1	-1	-1	-1
2	A	1	-1	-1	-1	-1
3	B	-1	1	-1	-1	-1
4	AB	1	1	-1	-1	-1
5	C	-1	-1	1	-1	-1
6	AC	1	-1	1	-1	-1
7	RC	-1	1	1	-1	-1
8	ABC	1	1	1	-1	-1
9	D	-1	-1	-1	1	-1
10	AD	1	-1	-1	1	-1
11	RD	-1	1	-1	1	-1
12	ABD	1	1	-1	1	-1
13	CD	-1	-1	1	1	-1
14	ACD	1	-1	1	1	-1
15	RCD	-1	1	1	1	-1
16	ABCD	1	1	1	1	-1
17	F	-1	-1	-1	-1	1
18	AF	1	-1	-1	-1	1
19	RE	-1	1	-1	-1	1
20	ARE	1	1	-1	-1	1
21	CE	-1	-1	1	-1	1
22	ACE	1	-1	1	-1	1
23	RCE	-1	1	1	-1	1
24	ABCF	1	1	1	-1	1
25	DE	-1	-1	-1	1	1
26	ADE	1	-1	-1	1	1
27	RDE	-1	1	-1	1	1
28	ARDE	1	1	-1	1	1
29	CDE	-1	-1	1	1	1
30	ACDE	1	-1	1	1	1
31	RCDE	-1	1	1	1	1
32	ABCDE	1	1	1	1	1

FACTORIAL EXPERIMENTS

TEST RUN NO. 2 (OPTION 2)

MANGANESE ZINC FERRITE

MEASURED RESPONSE IS WEIGHT LOSS DURING SINTERING

NUMBER OF FACTORS ; 6

NUMBER OF RUNS ; 64

NO	FACTOR	LOW (-1)	HIGH (+1)		ONE CODED UNIT EQ. TO	
A	CALC.	900.00	1000.00	DEG.C	50.00	DEG.C
B	MILL	24.00	48.00	HOUR	12.00	HOUR
C	PRESS	5000.00	10000.00	PSI	2500.00	PSI
D	TEMP.	1250.00	1300.00	DEG.C	25.00	DEG.C
E	SOAK	120.00	240.00	MINUTE	60.00	MINUTE
F	OXYGEN	.50	4.00	PERC.	1.75	PERC.

EXPERIMENTAL RESULTS

RUN NO		EXPT. NO	OBSERVED	A	H	C	D	F	F
1	CONST.	33	1.1900	-1	-1	-1	-1	-1	-1
2	A	37	.0300	-1	-1	-1	-1	-1	-1
3	R	35	1.9400	-1	1	-1	-1	-1	-1
4	AR	39	1.2000	1	1	-1	-1	-1	-1
5	C	34	1.5600	-1	-1	1	-1	-1	-1
6	AC	34	.6400	1	-1	1	-1	-1	-1
7	RC	36	1.9500	-1	1	1	-1	-1	-1
8	ARC	40	1.1800	1	1	1	-1	-1	-1
9	D	1	1.5600	-1	-1	-1	1	-1	-1
10	AD	5	.7500	1	-1	-1	1	-1	-1
11	RD	3	1.9800	-1	1	-1	1	-1	-1
12	ARD	7	1.1800	1	1	-1	1	-1	-1
13	CD	2	1.5700	-1	-1	1	1	-1	-1
14	ACD	6	1.1700	1	-1	1	1	-1	-1
15	RCO	4	1.9400	-1	1	1	1	-1	-1
16	ARCO	8	1.2200	1	1	1	1	-1	-1
17	F	41	1.4300	-1	-1	-1	-1	1	-1
18	AF	45	.4700	1	-1	-1	-1	1	-1
19	RF	43	1.9400	-1	1	-1	-1	1	-1
20	ARF	47	1.1900	1	1	-1	-1	1	-1
21	CF	42	1.5200	-1	-1	1	-1	1	-1
22	ACF	46	1.0500	1	-1	1	-1	1	-1
23	RCF	44	1.4100	-1	1	1	-1	1	-1
24	ARCF	48	1.2300	1	1	1	-1	1	-1
25	DF	25	1.5400	-1	-1	-1	1	1	-1
26	ADF	29	1.2500	1	-1	-1	1	1	-1
27	RDF	27	1.3500	-1	1	-1	1	1	-1
28	ARDF	31	1.2700	1	1	-1	1	1	-1
29	CDF	26	1.6000	-1	-1	1	1	1	-1
30	ACDF	30	1.2300	1	-1	1	1	1	-1
31	RCDF	28	1.9600	-1	1	1	1	1	-1
32	ARCDF	32	1.2200	1	1	1	1	1	-1
33	F	49	1.1700	-1	-1	-1	-1	1	-1
34	AF	53	.2700	1	-1	-1	-1	1	-1
35	RF	51	1.9300	-1	1	-1	-1	1	-1
36	ARF	55	1.0200	1	1	-1	-1	1	-1
37	CF	50	1.5300	-1	-1	1	-1	1	-1
38	ACF	54	.3700	1	-1	1	-1	1	-1
39	RCF	52	1.9100	-1	1	1	-1	1	-1
40	ARCF	56	1.0700	1	1	1	-1	1	-1
41	DF	4	1.5200	-1	-1	-1	1	-1	1
42	ADF	13	.9200	1	-1	-1	1	-1	1
43	RDF	11	1.9600	-1	1	-1	1	-1	1
44	ARDF	15	1.2300	1	1	-1	1	-1	1
45	CDF	10	1.5300	-1	-1	1	1	-1	1
46	ACDF	14	1.2400	1	-1	1	1	-1	1
47	RCDF	12	1.9400	-1	1	1	1	-1	1
48	ARCDF	16	1.2700	1	1	1	1	-1	1
49	FF	57	1.3700	-1	-1	-1	-1	1	1
50	AEF	61	.1900	1	-1	-1	-1	1	1
51	RF	59	1.9100	-1	1	-1	-1	1	1
52	ARFF	63	1.1600	1	1	-1	-1	1	1
53	CEF	58	1.4500	-1	-1	1	-1	1	1
54	ACFF	62	.6000	1	-1	1	-1	1	1
55	RCFF	60	1.8900	-1	1	1	-1	1	1
56	ARCF	64	1.1100	1	1	1	-1	1	1
57	DEF	17	1.6200	-1	-1	-1	1	1	1
58	ADFF	21	1.2200	1	-1	-1	1	1	1
59	RDF	19	1.9600	-1	1	-1	1	1	1
60	ARDFF	23	1.2500	1	1	-1	1	1	1
61	CDFF	18	1.5400	-1	-1	1	1	1	1
62	ACDFF	22	1.2100	1	-1	1	1	1	1
63	RCDF	20	1.9400	-1	1	1	1	1	1
64	ARCDFF	24	1.1900	1	1	1	1	1	1

REGRESSION COEFFICIENTS AND THEIR SUM OF SQUARES

NUMBER	VARIABLE	COEFFICIENT	SUM OF SQUARES	NUMBER	VARIABLE	COEFFICIENT	SUM OF SQUARES
1	CONST.	1.350625	116.74802500	33	F	-.021562	.02975625
2	A	-.362812	8.42450625	34	AF	-.008750	.00490000
3	B	.213125	2.90702500	35	BF	.004063	.00105625
4	AB	-.014063	.01265625	36	ABF	.001875	.00022500
5	C	.049063	.15405625	37	CF	-.013750	.01210000
6	AC	.026250	.04410000	38	ACF	-.011562	.00855625
7	BC	-.054687	.19140625	39	BCF	.013125	.01102500
8	ABC	-.021250	.02890000	40	ABCF	.010312	.00680625
9	D	.119688	.91680625	41	DF	.025000	.04000000
10	AD	.068750	.30250000	42	ADF	.020313	.02640625
11	BD	-.089687	.51480625	43	BDF	-.008750	.00490000
12	ABD	-.056875	.20702500	44	ABDF	-.005938	.00225625
13	CD	-.031250	.06250000	45	CDF	.009687	.00600625
14	ACD	-.001562	.00015625	46	ACDF	.009375	.00562500
15	BCD	.028125	.05062500	47	BCDF	-.007812	.00390625
16	ABCD	.001562	.00015625	48	ABCDF	-.010625	.00722500
17	E	.040000	.10240000	49	EF	-.015937	.01625625
18	AE	.025312	.04100625	50	AEF	-.015625	.01562500
19	BE	-.033750	.07290000	51	BEF	.014687	.01380625
20	ABE	-.015937	.01625625	52	ABEF	.016250	.01690000
21	CE	-.020937	.02805625	53	CEF	.003750	.00090000
22	ACF	-.001250	.00010000	54	ACEF	.004688	.00140625
23	BCE	.012812	.01050625	55	BCEF	-.008125	.00422500
24	ABCE	-.005000	.00160000	56	ABCEF	-.010937	.00765625
25	DE	-.005937	.00225625	57	DEF	.004375	.00122500
26	ADE	-.005625	.00202500	58	ADEF	-.000312	.00000625
27	BDE	.003437	.00075625	59	BDEF	-.014375	.01322500
28	ABDE	-.003750	.00090000	60	ABDEF	-.007812	.00390625
29	CDE	-.010000	.00640000	61	CDEF	-.002812	.00050625
30	ACDF	-.027812	.04950625	62	ACDEF	.000625	.00002500
31	BCDF	.006875	.00302500	63	BCDEF	.005937	.00225625
32	ABCDE	.021562	.02975625	64	ABCDE F	.005625	.00202500

X-MATRIX 3

RUN NO		DE	ADE	RDF	ARDE	COF	ACOE	RCOE	ABCDE	F	AF	RF	ABF
1	CONST.	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1
2	A	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
3	R	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
4	AR	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
5	C	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1
6	AC	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1
7	RC	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1
8	ARC	1	1	1	1	1	1	1	1	-1	-1	-1	-1
9	D	-1	1	1	-1	1	-1	1	1	-1	1	1	-1
10	AD	-1	-1	1	1	1	-1	-1	1	-1	-1	1	1
11	RD	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1
12	ARD	-1	-1	-1	-1	1	1	-1	-1	-1	-1	-1	-1
13	CD	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1
14	ACD	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1
15	BCD	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
16	ARCD	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
17	E	-1	1	1	-1	1	-1	1	1	-1	1	1	-1
18	AE	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1
19	RE	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1
20	ARE	-1	-1	-1	-1	1	-1	-1	-1	-1	-1	-1	-1
21	CE	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1
22	ACE	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1
23	REE	-1	1	-1	1	-1	-1	-1	1	-1	1	-1	1
24	ARCE	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
25	DE	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1
26	ADE	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
27	RDE	1	-1	1	1	-1	1	-1	1	-1	1	-1	1
28	ARDE	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
29	COE	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1
30	ACOE	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1
31	RCOE	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1
32	ARCOE	1	1	1	1	1	1	1	1	-1	-1	-1	-1
33	F	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1
34	AF	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1
35	RF	1	-1	1	-1	-1	-1	-1	1	1	-1	1	-1
36	ARF	1	1	1	1	-1	-1	-1	-1	1	1	1	1
37	CF	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
38	ACF	1	1	-1	-1	1	-1	-1	-1	1	-1	-1	-1
39	RCF	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
40	ARCF	1	1	1	1	1	1	1	1	1	-1	-1	-1
41	DF	-1	1	1	-1	1	-1	-1	1	1	-1	-1	1
42	ADE	-1	-1	1	1	1	1	-1	1	1	-1	-1	-1
43	RDE	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1
44	ARDE	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1
45	COE	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1
46	ACOE	-1	-1	1	-1	-1	-1	1	1	1	-1	-1	-1
47	RCOE	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1
48	ARCOE	-1	-1	-1	-1	-1	-1	-1	-1	1	-1	-1	-1
49	EF	-1	1	1	-1	1	-1	-1	1	1	-1	-1	1
50	ADE	-1	-1	1	1	1	-1	-1	1	1	-1	-1	-1
51	RE	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1
52	ARE	-1	-1	-1	-1	1	1	1	1	1	1	1	1
53	CEF	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1
54	ACFE	-1	-1	1	-1	-1	-1	1	1	1	-1	-1	-1
55	RCFE	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1
56	ARCFE	-1	-1	-1	-1	-1	-1	-1	-1	1	-1	-1	-1
57	DEF	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1
58	ADEF	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1
59	RDEF	1	-1	1	-1	-1	-1	-1	1	1	-1	1	-1
60	ARDEF	1	1	1	1	-1	-1	-1	-1	1	1	1	1
61	COFE	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
62	ACDEF	1	1	-1	-1	1	1	-1	-1	1	-1	-1	-1
63	RCDEF	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
64	ARCFE	1	1	1	1	1	1	1	1	1	-1	-1	-1

X-MATRIX 6

RUN NO		C0FF	AC0FF	RC0FF	ARC0FF
1	CONST.	1	-1	-1	1
2	A	1	1	-1	-1
3	B	1	-1	1	-1
4	AR	1	1	1	1
5	C	-1	1	1	-1
6	AC	-1	-1	1	1
7	RC	-1	1	-1	1
8	ARC	-1	-1	-1	-1
9	D	-1	1	1	-1
10	AD	-1	-1	1	1
11	RD	-1	1	-1	1
12	ARD	-1	-1	-1	-1
13	CD	1	-1	-1	1
14	ACD	1	1	-1	-1
15	BCD	1	-1	1	-1
16	ARCD	1	1	1	1
17	E	-1	1	1	-1
18	AE	-1	-1	1	1
19	RE	-1	1	-1	1
20	ARE	-1	-1	-1	-1
21	CE	1	-1	-1	1
22	ACE	1	1	-1	-1
23	CE	1	-1	1	-1
24	ARCE	1	1	1	1
25	DE	1	-1	-1	1
26	ADF	1	1	-1	-1
27	RDF	1	-1	1	-1
28	ARDE	1	1	1	1
29	CDF	-1	1	1	-1
30	ACDE	-1	-1	1	1
31	RCDE	-1	1	-1	1
32	ARCDE	-1	-1	-1	-1
33	F	-1	1	1	-1
34	AF	-1	-1	1	1
35	RF	-1	1	-1	1
36	ARF	-1	-1	-1	-1
37	CF	1	-1	-1	1
38	ACF	1	1	-1	-1
39	RCF	1	-1	1	-1
40	ARCF	1	1	1	1
41	DF	1	-1	-1	1
42	ADF	1	1	-1	-1
43	RDF	1	-1	1	-1
44	ARDF	1	1	1	1
45	CDF	-1	1	1	-1
46	ACDF	-1	-1	1	1
47	RCDF	-1	1	-1	1
48	ARCDF	-1	-1	-1	-1
49	EF	1	-1	-1	1
50	AFF	1	1	-1	-1
51	RF	1	-1	1	-1
52	ARFF	1	1	1	1
53	CEF	-1	1	1	-1
54	ACFF	-1	-1	1	1
55	RCFF	-1	1	-1	1
56	ARCF	-1	-1	-1	-1
57	DEF	-1	1	1	-1
58	ADFF	-1	-1	1	1
59	RDFF	-1	1	-1	1
60	ARDEF	-1	-1	-1	-1
61	C0FF	1	-1	-1	1
62	AC0FF	1	1	-1	-1
63	RC0FF	1	-1	1	-1
64	ARC0FF	1	1	1	1