O T T A W A April 21st, 1944.

R E P O R T

of the

ORE DRESSING AND METALLURGICAL LABORATORIES.

Investigation No. 1630.

The Determination of Residual Stresses in 17-Pdr. Armour-Piercing Shot.

(Copy No. 17.)

Bureau of Mines
Division of Metallic
Minerals.
-
Ore Dressing
and Metallurgical
Laboratories

CANADA

DEPARTMENT
OF
MINES AND RESOURCES

Mines and Geology Branch

OTTAWA April 21st, 1944.

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Abstract

This report describes an effort made to determine the approximate residual stress distribution in a 17-pdr. armour-piercing shot.

Five ½-inch-thick discs were cut from the shot. In one of them the distribution of tangential and radial residual stresses was determined by using the Sachs method.

The longitudinal stresses in the main were relieved by the cutting operation. These stresses were assumed to be equal to and distributed in the same way as longitudinal stresses.

Two ways of calculating the residual stresses on an outside surface were applied. In both cases results obtained are comparable. It was found that stresses are in tension at the outside surface.

=========

Description of Material Used:

The armour-piercing shot used in this survey of distribution of the residual stresses remaining after heat treatment, was a standard 17-pdr. A.P.C. shot taken from a production lot.

According to the producer, the shot had been

(Description of Material Used, cont'd) -

quenched in oil from 1500° F. The temperature of the oil was 130° F. during quenching; the shot was held in oil for 13 minutes and afterwards was stress-relieved at 250° F. for three hours. The base was drawn by induction heating (the maximum temperature reached by the shot at the base end was 1200° F.) and then quenched in water.

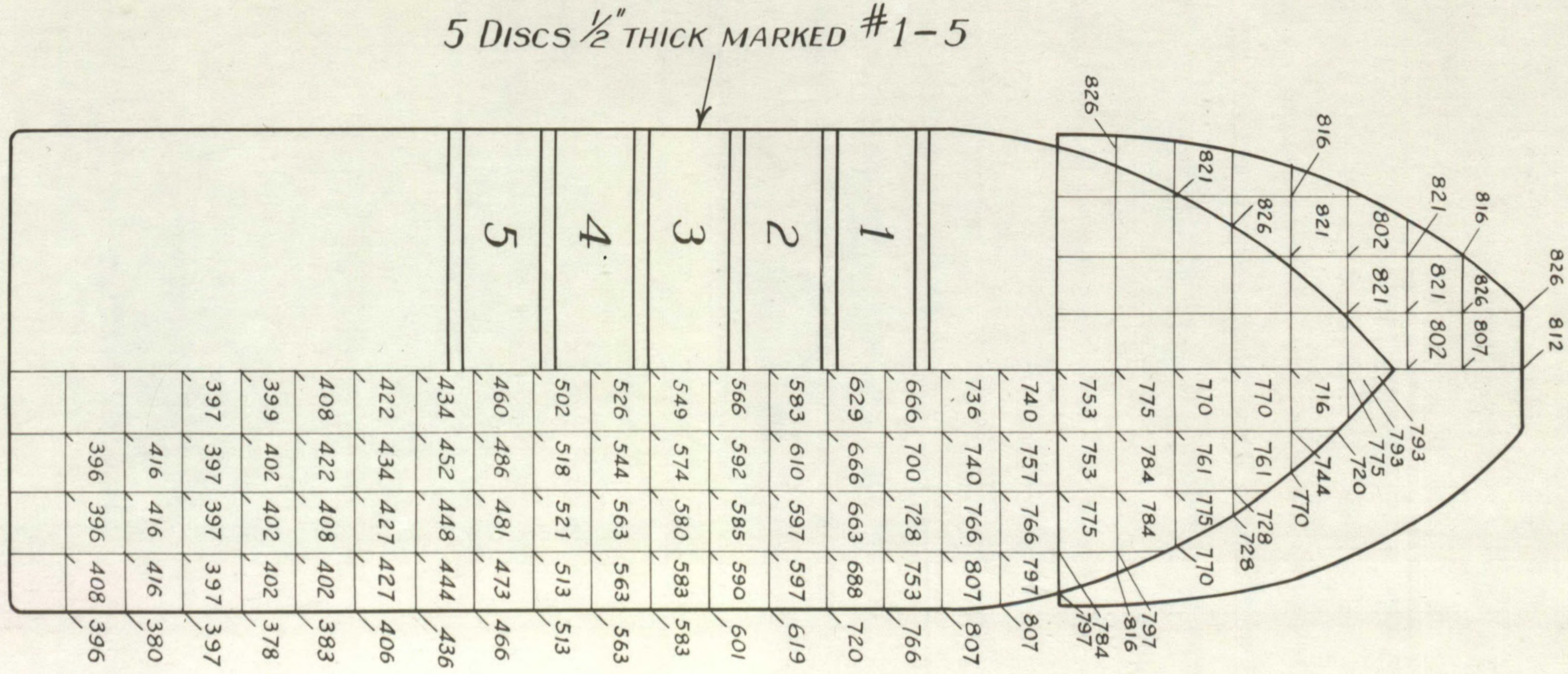
The chemical analysis of this shot, as determined at these Laboratories, is as follows:

		Per cent
Carbon	-	0.71
Manganese	-	0.84
Phosphorus	-	0.005
Sulphur	-	0.021
Silicon	-	0.31
Chromium	-	0.72
Molybdenum	-	0.22
Nickel	-	0.75
Vanadium	-	0.05

In Figure 1 are shown results of hardness tests performed on an average shot on an axial section (longitudinal) from which 1/16th of an inch was removed by wet grinding. (This hardness survey is taken from our P.M. Lab. Report No. 6418, dated July 22, 1943.)

(Figure 1 comprises Page 3.)
(Text continues on Page 4.)

FIG 1



VICKERS HARDNESS SURVEY OF 17-PDR.
A.P.C. SHOT USING A 50KG. LOAD

W.A.E.

STRESS SURVEY PROCEDURE:

In order to determine the residual stress distribution, the shot was divided as shown in Figure 1. The five discs, ½ inch thick, marked Nos. 1 to 5, were cut from the cylindrical part of the shot for the following purposes:

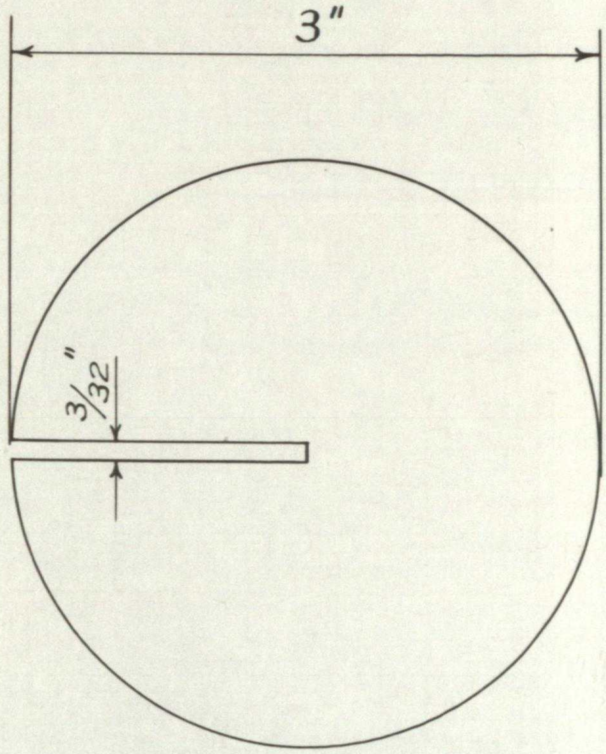
Disc No. 4, for complete survey of residual stress distribution. (Figure 3).

Discs Nos. 1, 2 and 3, for splitting from outside surface to centre in order to obtain, in a very approximate way, some comparable figures of the magnitude of the distribution of tangential stresses in these discs. (Figure 2).

Disc No. 5, for hardness test. This disc was taken 1/8 inch away from the driving groove. (Figure 4).

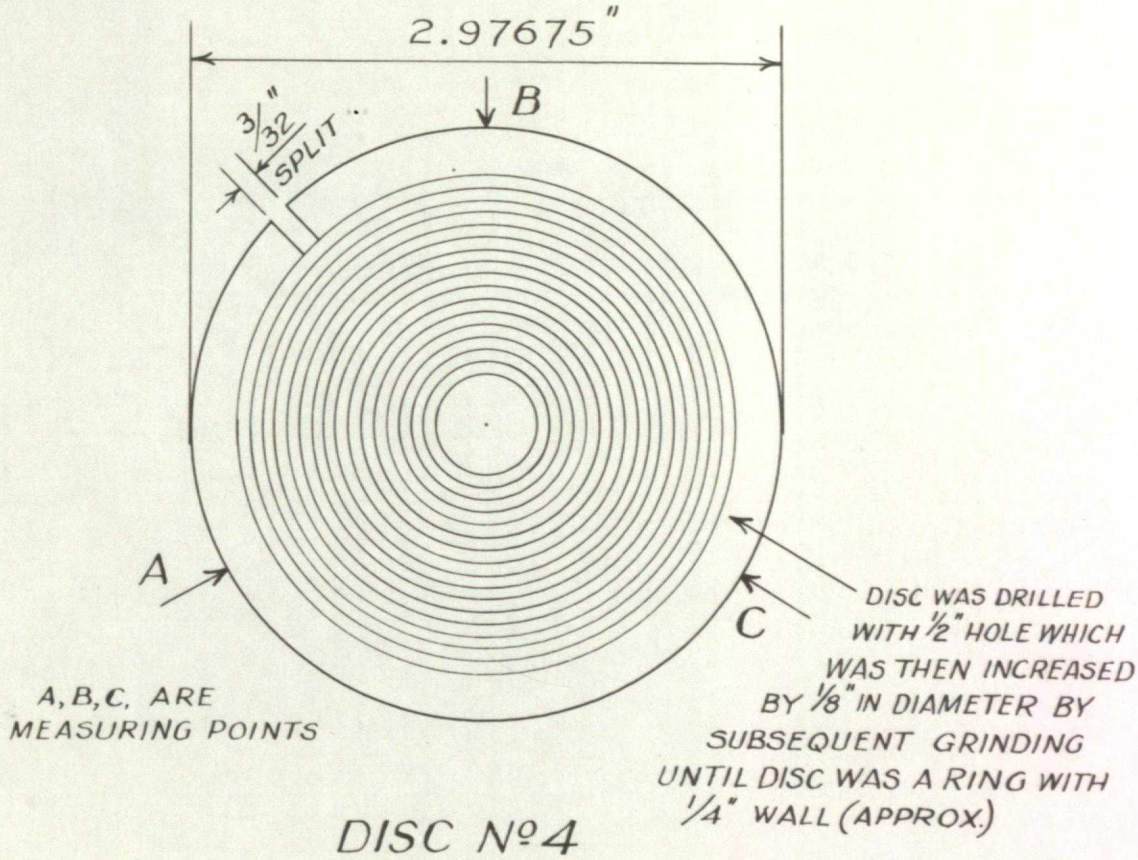
All machining operations, that is, grinding, drilling or cutting, were done very carefully, using coolant profusely and keeping local heating as low as possible.

(Figures 2, 3 and 4 follow,)
(on Pages 5 and 6.)
(Text continues on Page 7.)



DISCS Nos 1, 2, 3

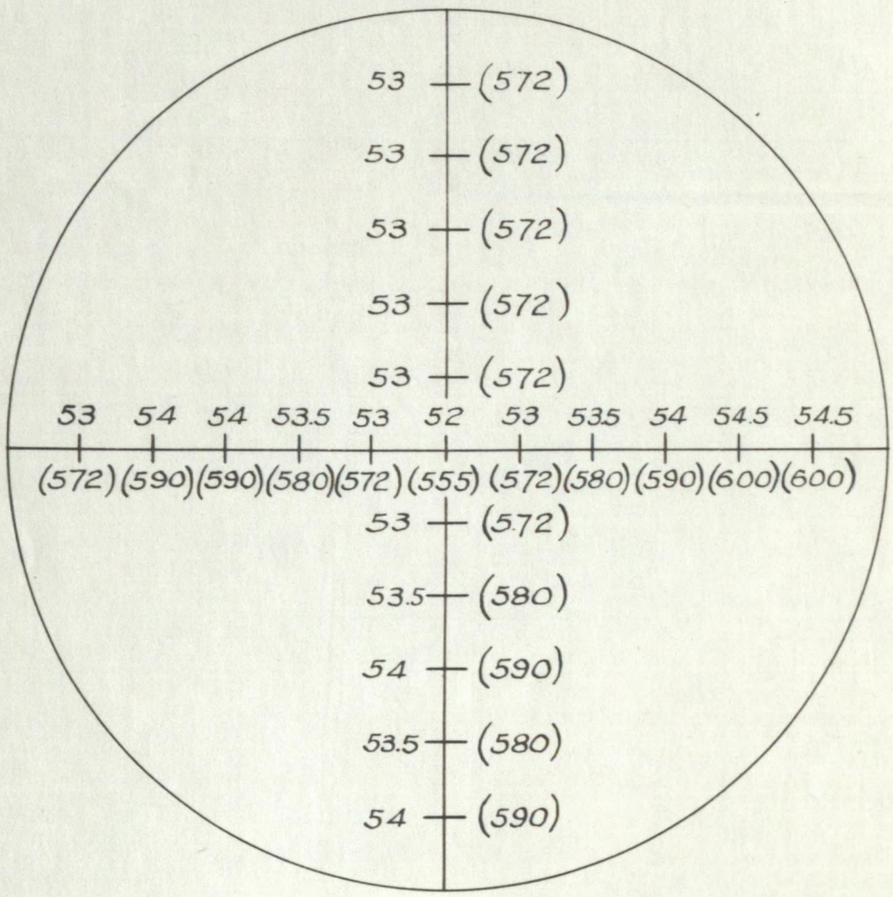
FIG. 2



DISC No 4

FIG. 3

W.A.E.



HARDNESS IN C SCALE 150 KG. ROCKWELL TAKEN ON DISC № 5 ON SURFACE BETWEEN DISCS №s 4 AND 5. THE FIGURES IN BRACKETS ARE VICKERS HARDNESS 50 KG. ALL READINGS WERE TAKEN AT 1/4" INTERVALS.

FIG. 4

W.A.E

Performing Measurements:

Disc No. 4 (Figure 3), about 3 inches in diameter and $\frac{1}{2}$ inch thick, was cut on a cutting machine with a wheel about 3/32 inch thick. The following further operations were performed:

1. Grinding of both faces of the disc parallel to each other and perpendicular to the axis of the disc.

2. Grinding and polishing the periphery of the disc.

3. Measuring outside diameters in three directions, 60° apart, ~~in places marked A, B, and C.~~ These measurements were taken, as an average from several readings in the centre of the disc thickness, with a Sheffield comparator (Dayton, Ohio) capable of reading to 25 millionths of an inch.

4. Drilling the $\frac{1}{2}$ in. diameter hole in centre of the disc.

5. Taking the outside diameters as given in (3 above) exactly in the same locations A, B and C in the centre of the disc; and the inside diameter. This last measurement is performed with an internal micrometer compared with gauge blocks.

6. Grinding of 1/16 in. thick layers of metal from the inside of the disc, i.e., enlarging the $\frac{1}{2}$-in. hole made in (4) to 5/8 in.

7. Taking the outside and inside diameter measurements.

8. Grinding as in (6) and measuring as in (5) until the thickness of the ring was about $\frac{1}{4}$ inch.

Throughout all these operations coolant was used generously, to eliminate any local excessive heating which would be a cause of cracks.

All diameter measurements were made at the same temperature (68° F.) and after a sufficiently long time had elapsed after grinding. Measurement results of both outside and inside diameters are given in Table I (Page 21).

During successive grindings of the internal hole, the outside diameter has increased after grinding 5/8-inch hole (position 2) and after grinding $1\frac{1}{2}$-inch hole (position 9). By all other grindings (positions 1 to 17) the outside diameter

(Performing Measurements, cont'd) -

was decreasing gradually. This is shown in Table II, (paragraphs 6 and 7), on Page 22.

(Note: Tables I to VI will be found)
(on Pages 21 to 27, inclusive.)

Calculation of Residual Stresses in Disc No. 4:

The calculation of the magnitude of the residual stresses was performed by the Sachs method.* This method involves drilling out the inside part of a disc and measuring the variations in diameter after each drilling, as previously described.

The residual stresses in each drilled-out layer may be calculated from the following formulae:

Tangential Stress --

$$t = E \left[(F_b - F) \times \frac{\Delta\beta}{\Delta F} - \frac{F_b + F}{2F} (\beta) \right]$$

Residual Stress --

$$r = E \left[\frac{(F_b - F)}{2F} (\beta) \right]$$

where

E = modulus of elasticity taken, 30,000,000 p.s.i.

F_b = complete cross-section area of disc, 6.9594 sq. in.

F = ground-out cross-section area (Table II, column 3) in sq. in.

ΔF = cross-section of one ground-out layer (Table II, column 5) in sq. in.

β = unit diameter change (Table II, column 7).

$\Delta\beta$ = unit diameter change after grinding one layer (Table II, column 8).

The calculation and values of stresses t and r are

---

* G. Sachs and G. Espey: "The Measurement of Residual Stresses in Metal." IRON AGE, September 18, 1941.

(Calculation of Residual Stresses in Disc No. 4, cont'd) -

given in Table III.

Unfortunately, it was not possible to take first measurements of position 1 in Table I, of the outside diameter of the disc, because by error the hole was drilled before the first measurements were taken. All other measurements from positions Nos. 2 to 17 were taken properly, i.e., before and after grinding the inside part of the disc in steps of 1/8 inch in diameter. For that reason the stresses were calculated, not as we would for a disc, but only as for a ring with a ½-inch hole.

The values of stresses t and r, given in Table III, are calculated directly by using $\frac{\Delta\beta}{\Delta F}$ (see Table II, column 9) as they were measured separately for each point from positions 1 to 17. The scattering of values $\frac{\Delta\beta}{\Delta F}$ is quite high, and the stress diagram would be badly distorted by using the finite differences $\frac{\Delta\beta}{\Delta F}$ between two successive borings. Therefore, the quantity $\frac{\Delta\beta}{\Delta F}$ should be determined as the slope of a smooth curve of β vs. F. This curve is shown in Figure 7 on Page 19. Diagrammatically calculated, corrected values of $\frac{\Delta\beta}{\Delta F}$ are given directly on Figure 7. The part of this curve covering the points 2, 3 and 4 is drawn in two alternatives,

(a) $\frac{\Delta\beta}{\Delta F} = -20$, and

(b) $\frac{\Delta\beta}{\Delta F} = +106$,

since the measuring point No. 2 is distinctly out of approximate line linking the points 3 and 4. Obtained from Figure 7, corrected values of β and $\frac{\Delta\beta}{\Delta F}$ with whole calculation of stresses $t^{(1)}$ (column 7) and $r^{(1)}$ (column 8) are given in Table IV.

Measuring point 18 on diameter 2¾ inches, marked in diagrams, Figure 7 (on Page 19) and Figure 8 (on Page 20), represents a point chosen freely for checking the calculation shown

(Calculation of Residual Stresses in Disc No. 4, cont'd) -

in Table IV. The stress values obtained for that point are in accordance with the remaining part of that diagram.

A more complete and sufficiently accurate method of residual stress measurement is the triaxial determination of stress distributions in circular rods, cylinders, and tubes. A section of tube, cylinder or rod, of a length of at least two or three diameters, is bored out in steps, and the outside diameter and length are measured after each boring.

The determination of stresses by mechanical methods depends on the principle that the cutting of a layer of a stressed part causes the deformation of the remaining part. By measurement of those dimension changes and using the known relations between strain and stresses, we are in a position to determine the approximate values of residual stress locked in material and relieved by the above-mentioned maching operations.

In the present investigation it was not possible to investigate the distribution of stresses in all three directions since the distribution of stresses was not uniform in the longitudinal direction because the shot was drawn from the base with diminishing temperature in the direction of the nose (hardness gradient from about 400 to 800 Vickers). See Figure 1.

For this reason it was advisable to cut a narrow disc ($\frac{1}{2}$ inch thick) and assume that the difference in magnitude of stresses in such disc in the direction of shot axis is small.

In order to save both time and expense the complete determination of tangential and radial stresses was performed only on Disc No. 4 (Figure 1).

Since the approximate determination of stresses in a disc permits only the calculation of tangential and radial stresses remaining in the disc after cutting it from the shot, without taking into account by this calculation the longitudinal

(Calculation of Residual Stresses in Disc No. 4, cont'd) -

stresses, it would be very interesting to strive to make up this calculation assuming an approximation of the relation between tangential and longitudinal stresses. The simplest assumption, but one which is also a little too far from the reality, would be that the tangential and longitudinal strains are equal.

From two equations given below, determinations of triaxial stresses distribution may be attempted:

<u>Tangential Stress</u> -

$$t^{(2)} = \frac{E}{1-V^2}\left[(F_b - F) \times \frac{\Delta\beta + v\Delta\alpha}{\Delta F} - \frac{F_b + F}{2F}(\beta + v\alpha)\right]$$

<u>Radial Stress</u> -

$$r^{(2)} = \frac{E}{1-V^2} \quad \frac{(F_b - F)}{2F} \quad (\beta + v\alpha)$$

where,

E = modulus of elasticity taken, 30,000,000 p.s.i.

F = cross-section of disc in sq. in.

V = Poisson's ratio, 0.3.

F = Ground-out cross-section of disc.

ΔF = Cross-section of one ground-out layer.

α = unit length change.

β = unit diameter change.

$\Delta\alpha$ = unit length change after grinding one layer.

$\Delta\beta$ = unit diameter change after grinding one layer.

By taking into account the above assumption, we find that the previously calculated values $t^{(1)}$ and $r^{(1)}$ given in Table IV, columns 10 and 11, should be multiplied by

$$\frac{1}{1-V^2} \times (1 + V) = \frac{1 + 0.3}{1 - 0.3^2} = 1.43$$

The changed values of the $t^{(1)}$ and $r^{(1)}$ are given in Table IV, columns 9 and 10 (stresses $t^{(2)}$ and $r^{(2)}$). The residual

(Calculation of Residual Stresses in Disc No. 4, cont'd) -

stresses calculated in Table IV are shown in the diagram, Figure 8.

Checking of Calculations:

The ring remaining after the 17th grinding operation from the inside has been performed has a thickness of about ¼ inch. In this thickness we may expect that some residual stresses may be locked in the ring. To check this, the ring was split in two as shown in Figure 3.

We may assume that a major part of the stresses were relieved by bending the ring in the direction of the surface which was under tension. The maximum stresses in a split ring may be calculated from the equation,

$$t^{(3)}max. = E \quad x \quad \frac{d}{2} \quad x \quad \frac{R_1 - R_0}{R_0 \; x \; R_1}$$

where,

d = thickness of ring.

R_0 = radius before splitting.

R_1 = " after "

Calculated in this way, $t^{(3)}max.$ is of a magnitude of $t^{(3)} = \pm$ 4338.9 p.s.i., which is in tension on the outside surface of the ring and in compression on the inside surface of the ring.

Assuming that the values of $t^{(1)}$ stress calculated previously by grinding-out method (see Table IV, column 7, position 17) on the inside surface fibre of the ring is +3941 p.s.i. without correction in regard to the longitudinal stresses, the above calculated stress should differ from stress on the outside surface of the remaining ring by double the numerical value of $t^{(3)}max.$; it is

$$2 \; x \; 4338.9 = 8677.8 \text{ pounds per sq. in.}$$

It is also a way to compare the magnitude of residual stress in the outside fibre by taking into account the whole

(Checking of Calculations, cont'd) -

change of the outside diameter before the grinding from the inside was started, and after grinding the last layer, No. 17.

D_o, diameter before grinding from inside (see Table I, column 7, position 1) = 2.976750 in.

D_{17}, diameter after last grinding from the inside (see Table I, column 7, position 17) = 2.976025 in.

The approximate magnitude of that stress may be calculated from the equation:

$$t^{(4)} = -E.\delta = +7,320 \text{ p.s.i.}$$

where,

$$\delta = \frac{D_{17} - D_o}{D_o}$$

The above figure of $t^{(4)}$ does not include the stresses relieved by splitting the ring (see Table V) i.e., $t^{(3)}max.$ = ± 4,338.9 p.s.i.

The resultant of these stresses, i.e., $t^{(1)}$ and $t^{(4)}$, will give the stress $T^{(1)}$ in the outside fibre.

$$T^1 = +7,320 + 4,338.9 = \underline{+11,658 \text{ p.s.i.}}$$

This value is comparable with stress $t^{(1)}_{pos. 17}$ received in Table IV, column 7, position 17, i.e., $t^{(1)}_{(17)}$ = 3,941 p.s.i. plus 2 x $t^{(3)}max.$ = 8,677.8 = $\underline{+12,619 \text{ p.s.i.}}$

On diagram, Figure 8, are marked the values $T^{(1)}$ and $T^{(2)}$ corresponding to cross-section bore (19).

$T^{(1)}$ = +11,658 p.s.i. and

$T^{(2)}$ = +11,658 x 1.43 p.s.i. = +16,670 p.s.i.,

that is, taking into account the influence of longitudinal stresses.

Points $T^{(1)}$ and $T^{(2)}$ obtained by the above described calculation are sufficiently covered by the prolongated dotted curves.

Since the hardness of the investigated shot is

(Checking of Calculations, cont'd) -

gradually changing along the axis as shown in Figure 1, of the given hardness pattern, we may expect that the magnitude of stresses will increase in the direction of the nose. For that reason three additional discs were cut from the shot (see Figure 1, Discs Nos. 1, 2 and 3). These rings were split from the outside surface to the centre, as shown in Figure 2, and the diameter measurements were taken, as shown in Table V.

The Changes of Magnitude of Stresses Along the Axes of the Shot:

The increase in diameter changes $D_1 - D_o$ give a picture of approximate relation of maximum stresses in top part of the shot. In Table VI, column $D_1 - D_o$, the increases in diameter for Discs Nos. 1, 2 and 3 are:

No. 1	-	5/10,000 of an inch.
No. 2	-	4/10,000 "
No. 3	-	2/10,000 "

This relation gives a little more information about the ratio of stresses in places where Discs Nos. 1, 2 and 3 were taken. With an approximation we can assume for an orientation that the ratio of residual stresses in Discs Nos. 1, 2, 3 and 4 is like 5:4:2:2. This means that to obtain an approximate picture of stress magnitude in places where Discs Nos. 1, 2 and 3 were taken it is necessary to multiply the obtained values for Disc No. 4 by 1, 2 or 2½, but for obtaining a much more accurate picture of residual stress distribution it is also desirable to perform a full examination of stress on every disc, as was done on Disc No, 4.

To throw a little more light on the relations between tangential and longitudinal stresses distribution in A/P shot, it would be very interesting to perform, also, a triaxial measurement of residual stresses on a shot which was equally

(The Changes of Magnitude of Stresses Along the Axes of the Shot, cont'd) -

heat-treated, i.e., the whole shot was quenched and drawn in the same way. From a shot heat treated in this way, it will be possible to take a 4-inch cylinder (see Figure 5) and determine the stress distribution by grinding out in a similar way as was done on the disc, but also with measurements of length changes after each grinding.

Comparing the diagrams of complete stress distributions, we will have more to say about the relation between longitudinal and tangential stresses in this case, and we may strive to fix the approximate relation between these stresses, and use it in a simplified survey performed on the disc. In Figure 6 (as for an example) the longitudinal and tangential stresses distribution in a rod shows that the differences are quite large, but the similarity may be seen.

In other cases, where it may be sufficient to know only the magnitude of the surface stresses, both in longitudinal and tangential direction, we may apply SR-4 electrical strain gauge. A simple way of using this gauge is shown on Figure 5 which describes the "cut-off segment" method employed in these Laboratories for simplified residual stress examinations upon the surface of elements.

(Figures 5 and 6, which follow,)
(comprise Pages 16 and 17.)
(Text is resumed on Page 18.)

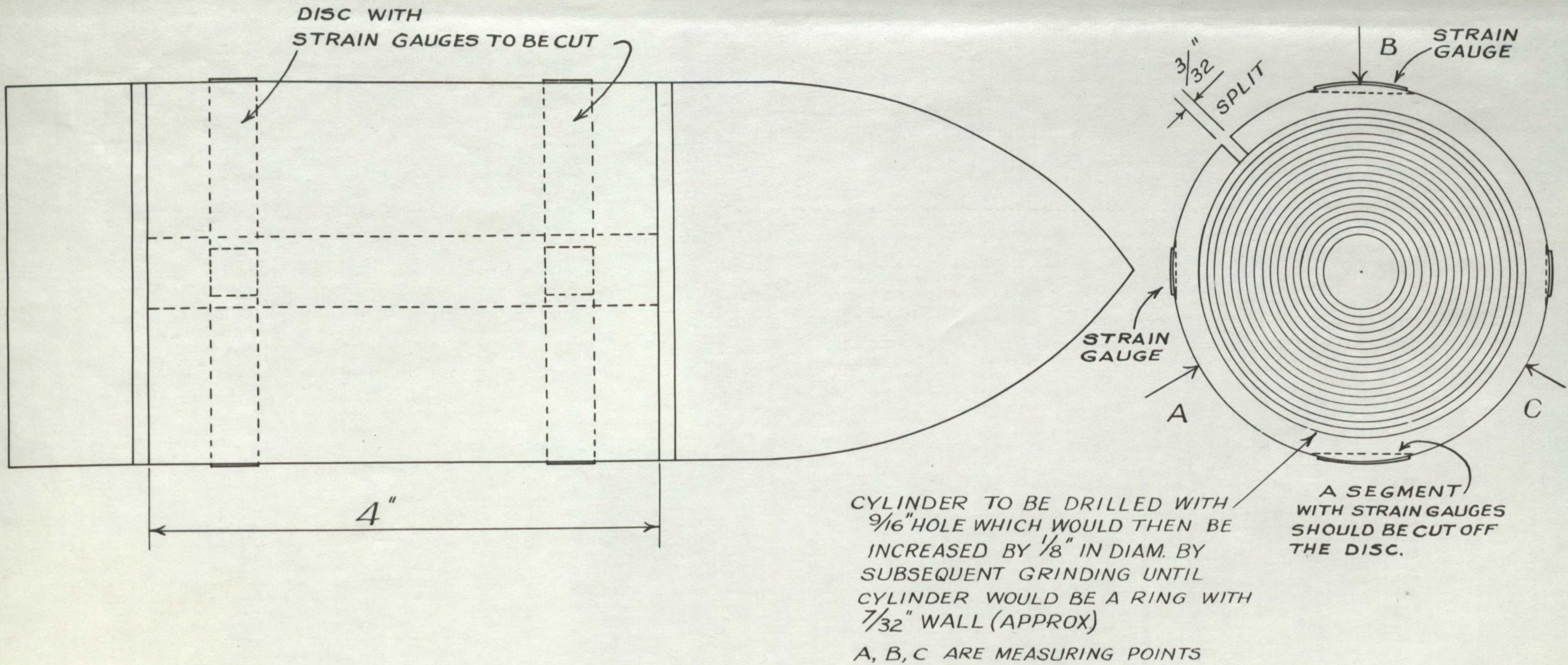
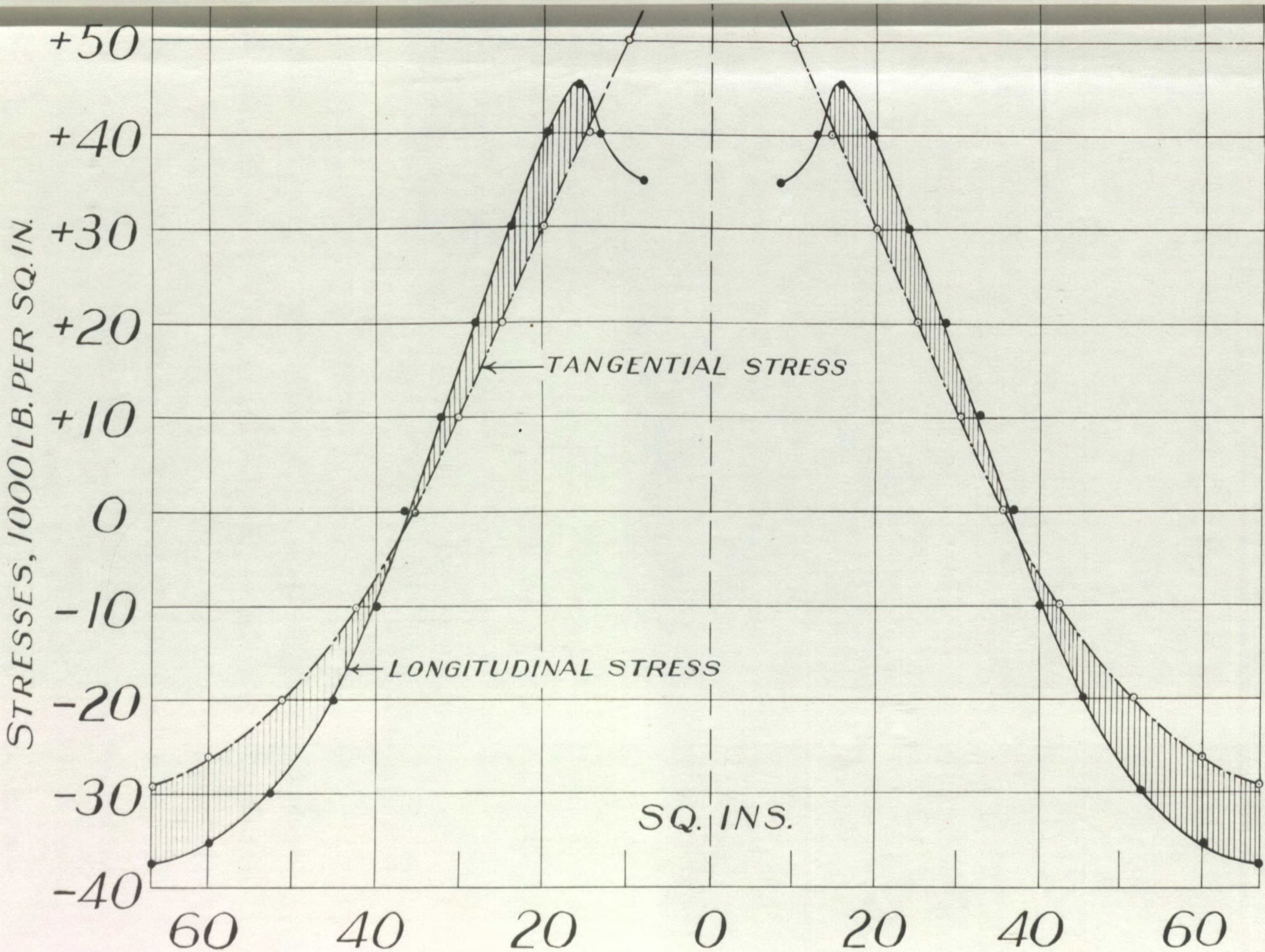

DISC WITH
STRAIN GAUGES TO BE CUT
4"
3/32"
SPLIT
B
STRAIN
GAUGE
STRAIN
GAUGE
A
C
A SEGMENT
WITH STRAIN GAUGES
SHOULD BE CUT OFF
THE DISC.
CYLINDER TO BE DRILLED WITH
9/16" HOLE WHICH WOULD THEN BE
INCREASED BY 1/8" IN DIAM. BY
SUBSEQUENT GRINDING UNTIL
CYLINDER WOULD BE A RING WITH
7/32" WALL (APPROX)
A, B, C ARE MEASURING POINTS

FIG. 5

W.A.E.



FROM Dr G. SACKS, IRON AGE SEPT. 18. 1941

W.A.E.

FIG. 6

Summary:

Determined in the above described way, the residual stresses ~~will~~ allowed the following preliminary conclusions to be drawn.

The applied heat treatments ~~have~~ produced approximate maximum residual tensional stresses as given below:

	(a) On outside surface without correction for longitudinal stresses	(b) On outside surface with correction for longitudinal stresses
Disc No. 4 -	$T^{(1)}$ + 12,620 p.s.i.	$T^{(2)}$ + 16,670 p.s.i.
Disc No. 3 (approx.) -	$T^{(1)}$ x 1 = + 12,620 p.s.i.	$T^{(2)}$ x 1 = +16,670 p.
Disc No. 2 (approx.) -	$T^{(1)}$ x 2 = + 25,240 p.s.i.	$T^{(2)}$ x 2 = +33,340 p.
Disc No. 1 (approx.) -	$T^{(1)}$ x 2.5 = +31,550 p.s.i.	$T^{(2)}$ x 2.5 = +41,675

The magnitude of stresses in the curve of A/P shot itself was not determined, as explained before.

The maximum tensile stresses which might be expecte in the front part of the shot (Disc No. 1) would be of a magn of about 40,000 pounds per square inch.

In order to obtain a complete proving of residual stresses distribution it would be advisable to perform comple examinations in the way adopted by these Laboratories and sho in Figure 5.

The complete knowledge of residual stresses distribution in each type of A/P shot should be of great importance in determining optimum condition for shot heat treatment and, through this, optimum shot performance.

TWW:HLL:GHB.

(Figures 7 and 8 follow, on Pages 19 and 20.)
(Tables I to VI comprise Pages 21 to 27.)

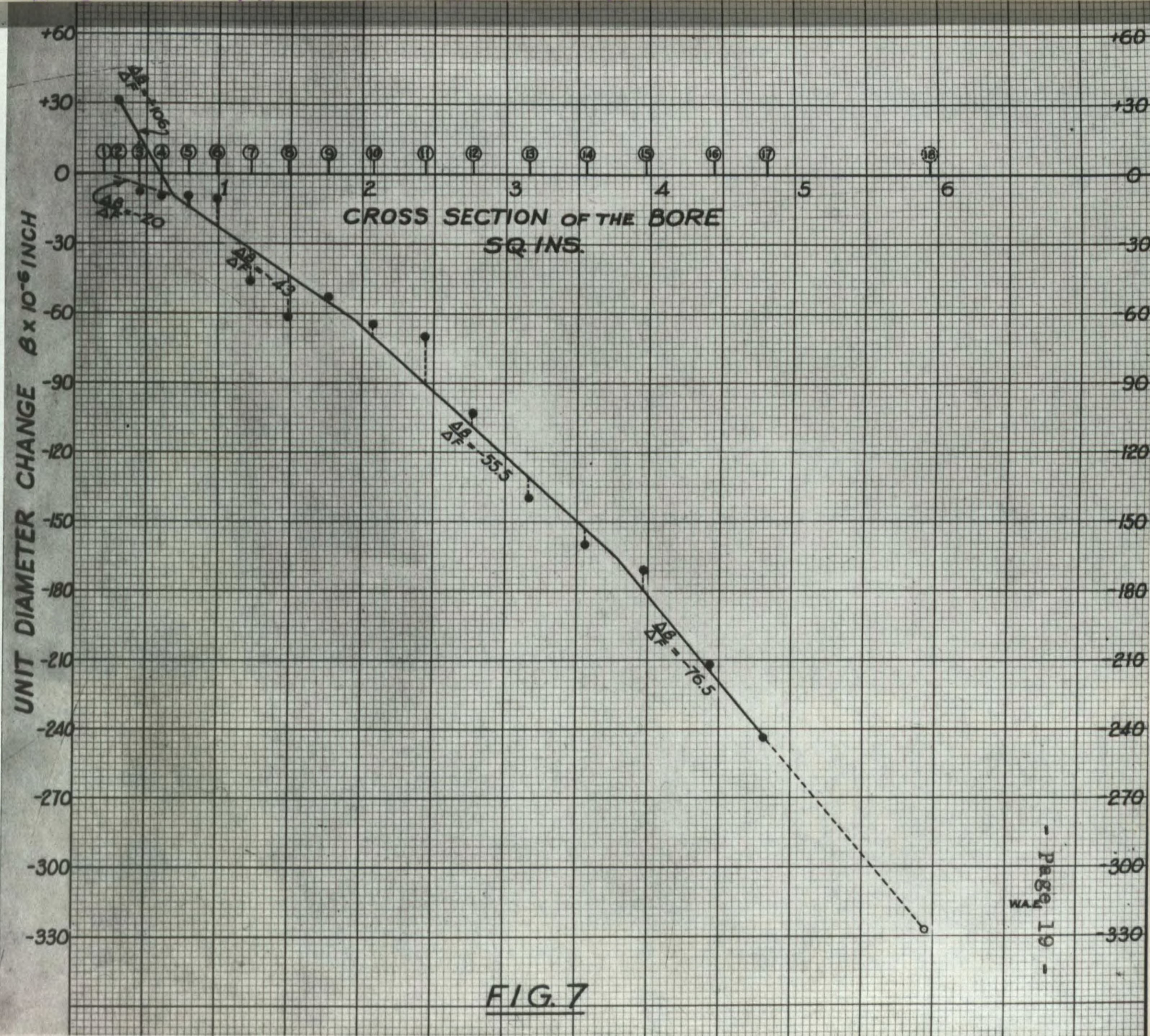

UNIT DIAMETER CHANGE β x 10⁻⁶ INCH
CROSS SECTION OF THE BORE
SQ. INS.
Δβ/ΔF = +106
Δβ/ΔF = -20
Δβ/ΔF = -43
Δβ/ΔF = -55.5
Δβ/ΔF = -76.5
+60
+30
0
-30
-60
-90
-120
-150
-180
-210
-240
-270
-300
-330
1
2
3
4
5
6
W.A.E.

FIG. 7

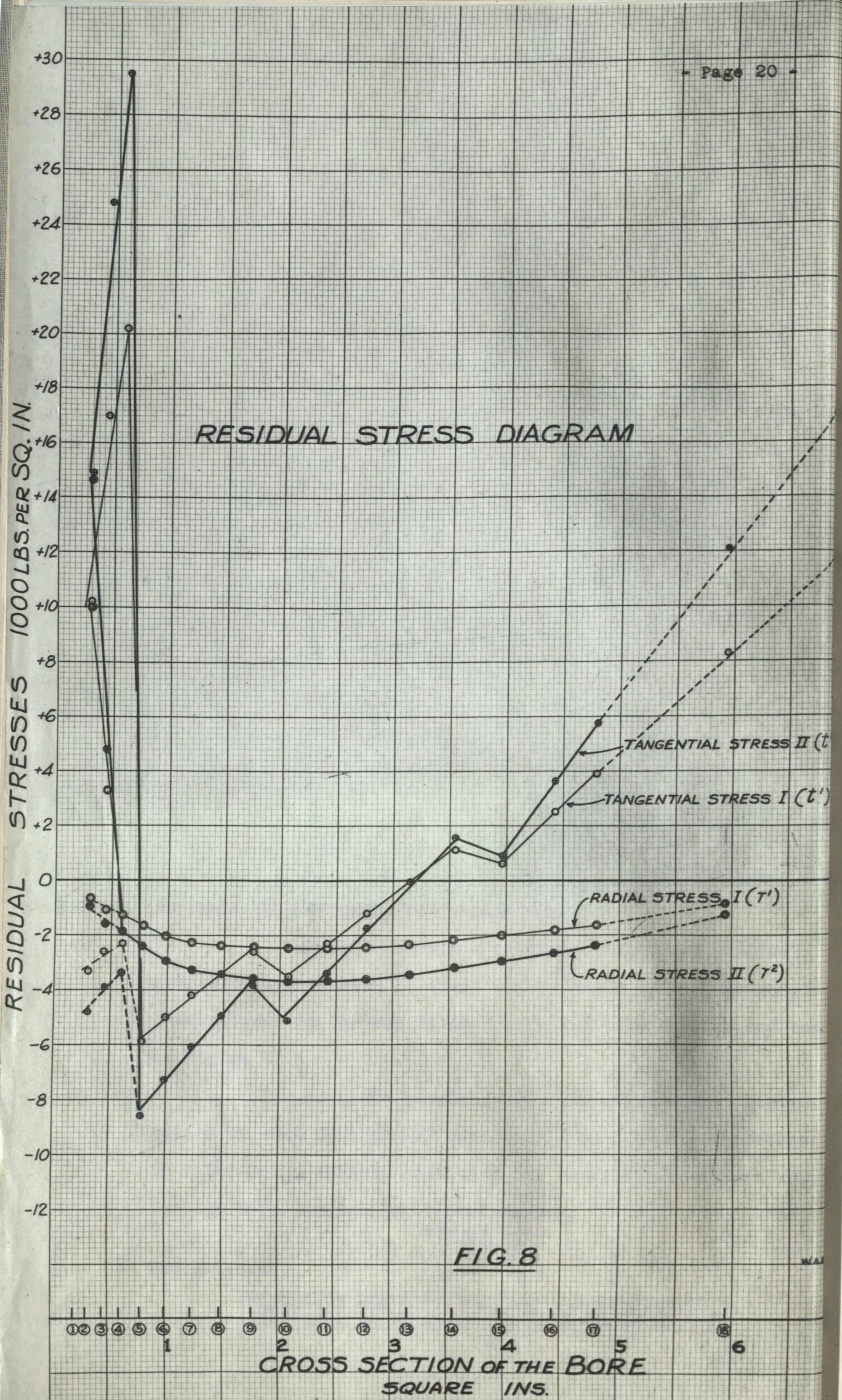

RESIDUAL STRESS DIAGRAM
RESIDUAL STRESSES 1000 LBS. PER SQ. IN.
+30
+28
+26
+24
+22
+20
+18
+16
+14
+12
+10
+8
+6
+4
+2
0
-2
-4
-6
-8
-10
-12
TANGENTIAL STRESS II (t
TANGENTIAL STRESS I (t')
RADIAL STRESS I (r')
RADIAL STRESS II (r²)
1
2
3
4
5
6
CROSS SECTION OF THE BORE
SQUARE INS.

FIG. 8

TABLE I.

1) Position No.	INSIDE DIAMETER (in inches) 2) Nominal	3) True	OUTSIDE DIAMETER (in inches) 4) At Point A	5) At Point B	6) At Point C	7) Average
1	1/2	0.50000	2.976750	2.976750	2.976750	2.976750
2	5/8	0.62580	2.976825	2.976850	2.976850	2.976842
3	3/4	0.7538	2.296725	2.976725	2.976725	2.976725
4	7/8	0.8761	2.9767125	2.976725	2.976725	2.976721
5	1	1.001	2.9767125	2.976725	2.976725	2.976721
6	1 1/8	1.1251	2.976700	2.976725	2.976725	2.976717
7	1 1/4	1.2501	2.976600	2.976640	2.976600	2.976613
8	1 3/8	1.3750	2.976550	2.976575	2.976575	2.976567
9	1 1/2	1.500	2.976575	2.976600	2.976600	2.976592
10	1 5/8	1.625	2.976550	2.976575	2.976550	2.976558
11	1 3/4	1.760	2.976525	2.976550	2.976550	2.976542
12	1 7/8	1.880	2.976425	2.976450	2.976450	2.976442
13	2	2.005	2.976325	2.976350	2.976325	2.976333
14	2 1/8	2.1270	2.976250	2.976300	2.976275	2.976275
15	2 1/4	2.250	2.976225	2.976250	2.976250	2.976242
16	2 3/8	2.377	2.976125	2.976100	2.976125	2.976117
17	2 1/2	2.473	2.9760	2.97605	2.976025	2.976025
(18)	2 3/4	2.750	--	--	--	--

TABLE II.

Position No. 1)	Nominal inside diameter, in inches 2)	F Ground-out cross-section area, in sq. in. 3)	2 x F., in sq. in. 4)	ΔF Cross-section of one ground-out layer, in sq. in. 5)	$D_n - D_{n-1}$ Change in outside diameter, in inch x 10^{-6} 6)	$D_n - D_0/D_0 = \beta$ Unit diameter change x 10^{-6} 7)	$\Delta\beta$ Unit diameter change after grinding one layer, in inch x 10^{-6} 8)	$\frac{\Delta\beta}{\Delta F}$ in inch^{-1} x 10^{-6} 9)
1	1/2	0.1963	--	--	--	--	--	--
2	5/8	0.3076	0.6152	0.1113	+92	+30.9	+30.9	+277.63
3	3/4	0.4462	0.8924	0.1386	-117	- 8.4	-39.3	-283.5
4	7/8	0.6028	1.2056	0.1566	-4	- 9.74	- 1.34	- 8.5
5	1	0.7869	1.5738	0.1841	0	- 9.74	0	0
6	1 1/8	0.9942	1.9884	0.2073	-4	-11.1	- 1.36	- 6.56
7	1 1/4	1.2274	2.4548	0.2332	-104	-46.0	-34.9	-149.72
8	1 3/8	1.4849	2.9698	0.2575	-46	-61.5	-15.5	- 60.19
9	1 1/2	1.7671	3.5342	0.2823	+25	-53.1	+ 8.4	+ 29.75
10	1 5/8	2.0739	4.1478	0.3068	-34	-64.5	-11.4	- 37.16
11	1 3/4	2.4328	4.8656	0.3589	-16	-69.9	- 5.4	- 15.05
12	1 7/8	2.7758	5.5518	0.3431	-100	-103.5	-33.6	- 97.93
13	2	3.1573	6.3146	0.3814	-109	-140.1	-36.6	- 95.96
14	2 1/8	3.5532	7.1064	0.3959	-58	-159.6	-19.5	- 49.25
15	2 1/4	3.9761	7.9522	0.4229	-33	-170.6	-11.0	- 26.01
16	2 3/8	4.4376	8.8752	0.4615	-125	-212.6	-42.0	- 91.00
17	2 1/2	4.8032	9.6064	0.3656	-92	-243.6	-31.0	- 84.79
(18)	2 3/4	5.9395	11.8790	1.1363	--	--	--	--

TABLE III.

Position No. 1)	F_b-F in sq. in. 2)	$\frac{F_b+F}{2F}$ 3)	$\frac{F_b-F}{2F}$ 4)	$\frac{\Delta\beta}{\Delta F}$ in inch^{-1} x10^{-6} 5)	$\beta \times 10^{-6}$ 6)	$(F_b-F)\frac{\Delta\beta}{\Delta F}$ x 10^{-6} 7)	$\frac{F_b+F}{2F}\beta$ x 10^{-6} 8)	$\frac{F_b-F}{2F}\beta$ x 10^{-6} 9)	t Tangential stresses, p.s.i. 10)	r Radial stresses, p.s.i. 11)
1	6.7631	--	--	--	--	--	--	--	--	--
2	6.6518	11.8124	10.8124	+277.63	+30.9	+1846.74	+365.00	+334.10	+44,452.20	+10,023.00
3	6.5132	8.2985	7.2985	-283.5	-8.4	-1846.49	-69.70	-61.30	-53,303.70	- 1,839.00
4	6.3566	6.2726	5.2726	-8.5	-9.74	-54.03	-61.09	-51.35	+ 211.80	- 1,540.50
5	6.1725	4.9220	3.9220	0	-9.74	0	-47.94	-38.20	--	- 1,146.00
6	5.9652	4.0080	3.0000	-6.56	-11.1	-39.13	-44.49	-33.3	+ 160.80	- 999.00
7	5.7320	3.3350	2.3350	-149.72	-46.0	-858.19	-153.41	-107.41	-21,143.40	- 3,222.30
8	5.4745	2.8434	1.8434	-60.19	-61.5	-329.51	-174.89	-113.37	- 4,638.60	- 3,401.10
9	5.1923	2.4692	1.4692	+29.75	+53.1	+154.47	+131.15	+78.01	+ 699.60	+ 2,340.30
10	4.8855	2.1779	1.1779	-37.16	-64.5	-181.54	-140.47	-75.97	- 1,232.10	- 2,279.10
11	4.5266	1.9303	0.9303	-15.05	-69.9	-68.25	-134.93	-65.03	+ 2,004.30	- 1,950.90
12	4.1835	1.7535	0.7535	-97.93	-103.5	-409.69	-181.49	-77.99	- 6,846.00	- 2,339.70
13	3.8021	1.6021	0.6021	-95.96	-140.1	-364.85	-224.45	-84.35	- 4,212.00	- 2,530.50
14	3.4062	1.4793	0.4793	-49.25	-159.6	-167.75	-236.09	-76.49	+ 2,050.20	- 2,294.70
15	2.9833	1.3752	0.3752	-26.01	-170.6	- 77.60	-234.60	-64.01	+ 4,710.00	- 1,220.30
16	2.5218	1.2841	0.2841	-91.00	-212.6	-229.48	-272.99	-60.39	+ 1,305.30	- 1,811.70
17	2.1562	1.2245	0.2245	-84.79	-243.6	-191.45	-298.28	-54.69	+ 3,204.90	- 1,640.70
(18)	1.0199	1.0858	0.0858	--	--	--	--	--	--	--

Remark: F_b = constant = 6.9594 sq. in.

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TABLE IV.

Position No. 1)	2) $\frac{\Delta\beta}{\Delta F}$ in inch^{-1} x 10^{-6} a	2) b	3) β x 10^{-6} a	3) b	4) $(F_b-F)\frac{\Delta\beta}{\Delta F}$ x 10^{-6} a	4) b	5) $\frac{F_b+F}{2F}\beta$ x 10^{-6} a	5) b	6) $\frac{F_b-F}{2F}\beta$ x 10^{-6} a	6) b
1	--	--	--	--	--	--	--	--	--	--
2	-20	+106	-2	+30.9	-133.0	+705.1	-23.6	+365.0	-21.6	+334.1
3	-20	+106	-5	+15.0	-130.3	+690.4	-41.5	+124.5	-36.5	+109.5
4	-20	+106	-8	+	-127.1	+673.8	-50.2	+	-42.2	+
5	-43.0		-14.0		-265.4		-68.9		-54.9	
6	-43.0		-22.5		-256.5		-90.2		-67.5	
7	-43.0		-32.0		-246.5		-106.7		-74.7	
8	-43.0		-43.0		-235.4		-122.3		-79.3	
9	-43.0		-55.0		-223.3		-135.8		-80.8	
10	-55.5		-70.5		-271.1		-153.5		-83.0	
11	-55.5		-90.1		-251.2		-173.9		-83.8	
12	-55.5		-109.5		-232.2		-192.0		-82.5	
13	-55.5		-131.0		-211.0		-209.9		-78.9	
14	-55.5		-152.0		-189.0		-224.9		-72.9	
15	-76.5		-180.5		-228.2		-248.2		-67.7	
16	-76.5		-215.0		-192.9		-276.1		-61.1	
17	-76.5		-242.0		-164.9		-296.3		-54.3	
(18)	-76.5		-327.0		- 78.0		-355.1		-28.1	

Remark: F_b+F = constant = 6.7631 sq. in.

7) $t^{(1)}$ Tangential stresses, in p.s.i.		8) $r^{(1)}$ Radial stresses, in p.s.i.		9) $t^{(2)} = t^{(1)} \times 1.43$ in p.s.i.		10) $r^{(2)} = r^{(1)} \times 1.43$ in p.s.i.	
a	b	a	b	a	b	a	b
-3282.3	+10202.6	- 648.7	+10023.1	-4693.9	+14589.7	- 927.6	+14333.0
-2663.1	+16977.7	-1094.8	+ 3284.3	-3808.2	+24278.1	-1565.6	+ 4696.5
-2308.5	+20214.0	-1265.4	0	-3301.1	+28906.0	-1809.5	0
-5895.3		-1647.2		-8430.3		-2355.5	
-4989.7		-2025.0		-7135.3		-2895.7	
-4192.7		-2241.6		-5995.6		-3205.5	
-3394.1		-2378.0		-4853.6		-3400.5	
-2623.9		-2424.2		-3752.2		-3466.6	
-3528.1		-2491.3		-5045.2		-3313.4	
-2319.2		-2514.6		-3316.4		-3595.9	
-1205.3		-2475.2		-1723.6		-3539.5	
- 34.2		-2366.3		- 48.9		-3383.9	
+1074.3		-2185.6		+1536.3		-3125.4	
+ 600.0		-2031.7		+ 858.0		-2905.3	
+2494.9		-1832.4		+3567.7		-2620.3	
+3941.4		-1629.9		+5636.2		-2330.7	
+8311.0		- 841.7		+11884.7		-1203.6	

TABLE V. - THE DETERMINATION OF STRESSES IN THE REMAINING RING BY SPLITTING.

$$t^3 \text{ max.} = \pm E \cdot \frac{d}{2} \cdot \frac{R_1 - R_0}{R_0 \cdot R_1}$$

Key: d = the thickness of the ring;
R_0 = mean radius before splitting; and
R_1 = " " after splitting.

BEFORE SPLITTING

Reading taken at	Outside diameter, inches	Inside diameter, inches
A	2.9759	2.4745
B	2.97595	2.4748
C	2.97595	2.4755
	Average O.D. - 2.97593	Average I.D. - 2.47493

AFTER SPLITTING

Reading taken at	Outside diameter, inches	Inside diameter, inches
A	2.9801250 2.9801375	2.47860
B	2.9802125 2.9802125	2.479150
C	2.9801750 2.9801625	2.48010
	Average O.D. - 2.98017	Average I.D. - 2.47928

Results:

$$t^3 \text{ max.} = \underline{30,000,000} \cdot \frac{0.2505}{2} \cdot \frac{0.0021475}{1.8598}$$

R_0 = 1.3627 inch, R_1 = 1.3648 inch

$R_1 - R_0$ = 0.0021475 inch

d = 0.2505 inch

t^3 max. = 4,338.9 lb./sq. inch.

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TABLE VI. - THE COMPARISON OF STRESSES IN FULL DISCS NOS. 1, 2, AND 3 AFTER SPLITTING TO THE CENTRE.

DIAMETER OF DISCS BEFORE SPLITTING

(D_0 measured in inches)

Disc No.		A	B	C	Average
1	I	2.9828	2.9829	2.9830)	
	II	2.9830	2.9830	2.9832)	2.9830
	III	2.9830	2.9829	2.9833)	
2	I	2.9830	2.9831	2.9828)	
	II	2.9832	2.9834	2.9830)	2.9831
	III	2.9832	2.9834	2.9831)	
3	I	2.9840	2.9841	2.9843)	
	II	2.9840	2.9841	2.9843)	2.9841
	III	2.9840	2.9841	2.9843)	

DIAMETER OF DISCS AFTER SPLITTING

(D_1 measured in inches)

Disc No.		A	B	C	Average	D_1-D_0
1	I	2.9835	2.9833	2.9836)	2.9835	0.0005
	II	2.9836	2.9833	2.9836)		
2	I	2.9832	2.9834	2.9836)	2.9835	0.0004
	II	2.9834	2.9836	2.9837)		
3	I	2.9843	2.9843	2.9844)		
	II	2.9844	2.9843	2.9843)	2.9843	0.0002

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April, 1944
Ottawa, Canada.
TWW:HLL:GHB.