

O T T A W A

February 12th, 1942.

FILE COPY

R E P O R T

of the

ORE DRESSING AND METALLURGICAL LABORATORIES.

Investigation No. 1157.

Armour Plate Quality and Its Relation
to Physical and Chemical Tests.

(This is Report No. 2 of the Canadian Bureau
of Mines 1942 Armour Plate Statistics Series.)

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PART ONE.

Origin of Request:

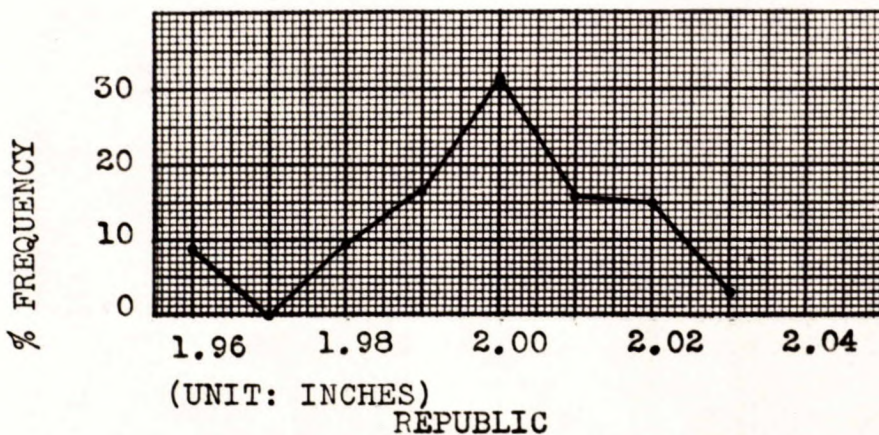
At the request of the British Central Scientific Office, Washington, D. C., statistical analysis of homogeneous armour plate data has been undertaken. Investigation No. 1144 (January, 1942) described the statistical methods proposed and showed their application to one source. Further investigation, on four sources which are designated as follows,

GENERAL STEEL (General Steel Castings Corporation),
UNION STEEL (Union Steel Casting Division, Blaw-Knox Co.),
AMERICAN STEEL (American Steel & Wire Co.), and
REPUBLIC STEEL (Republic Steel Corporation),

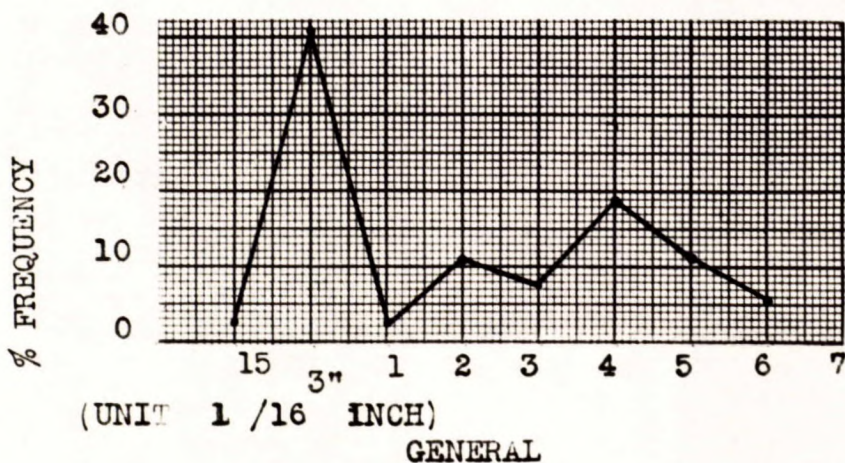
was undertaken. The results of this work are included in the present report.

THICKNESS OF ARMOUR EXAMINED

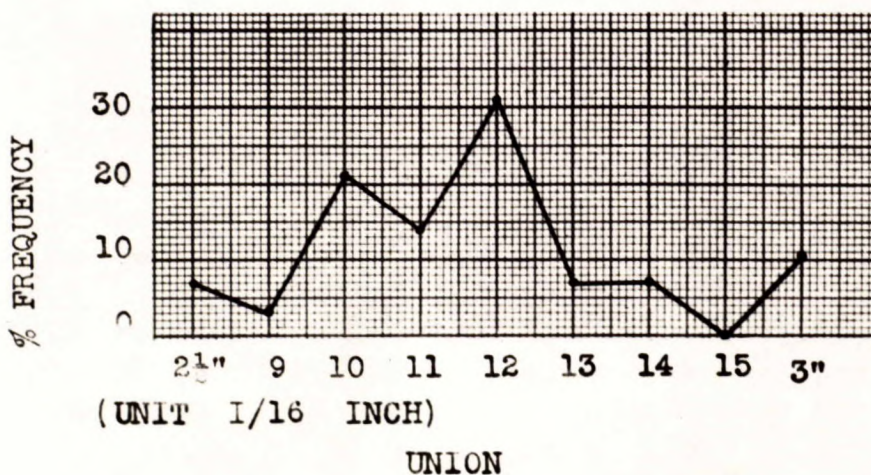
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1.96
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2 15/16 - 3 3/8



2 1/2 - 3"

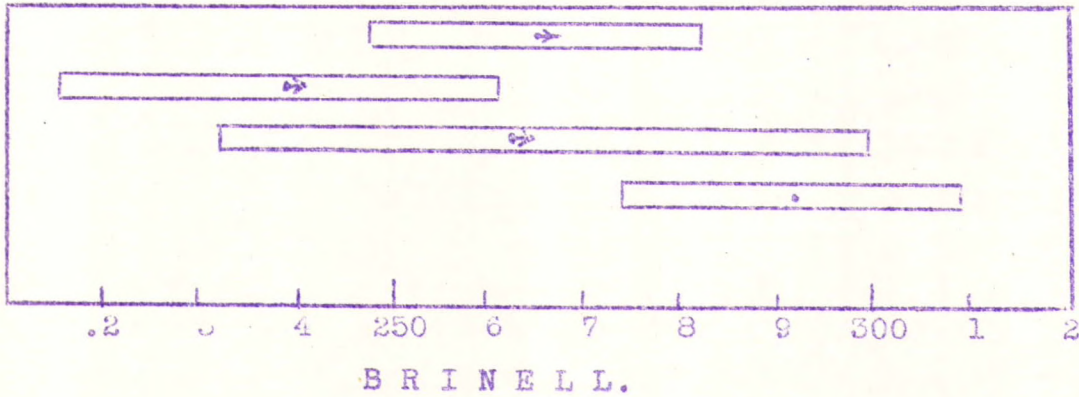
(ALL REPORTED AS 3 3/16 " THICK)

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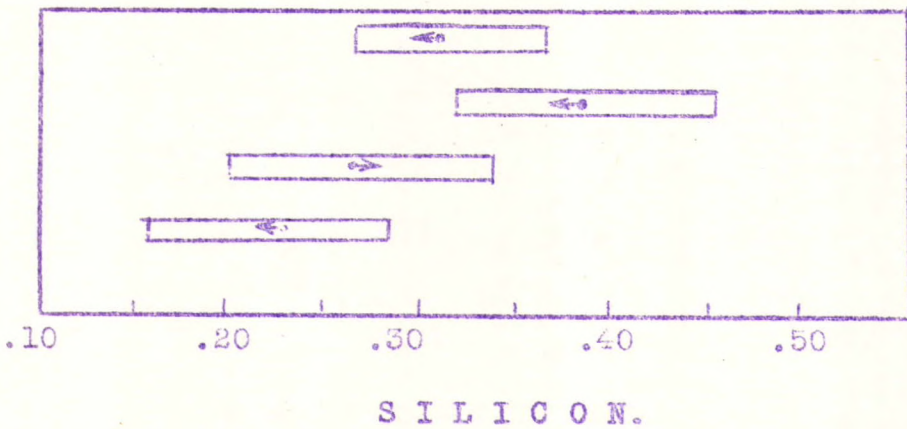
AMERICAN

GRAPHIC SUMMARY OF TRENDS EVIDENT IN DATA EXAMINED:

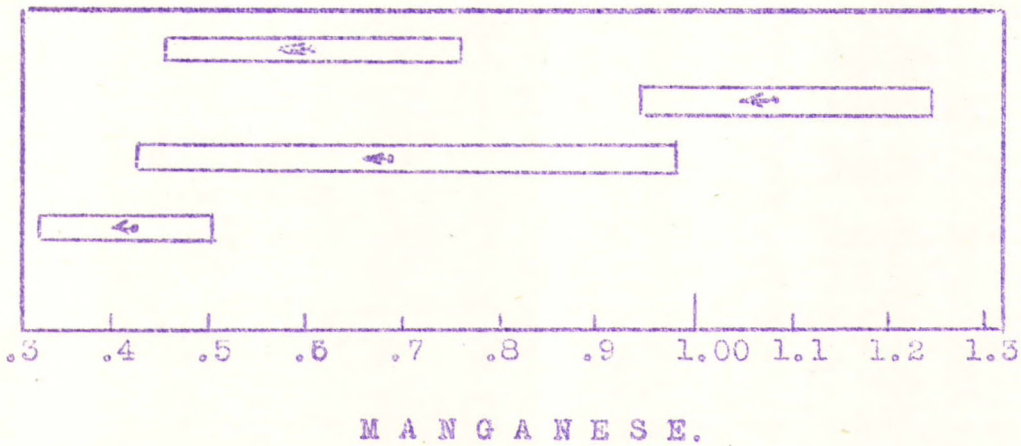
(KEY TO GRAPHS:)
(• Average of test values..)
([] 3 Sigma Limits (Spread))
(→ Direction in which improvement)
(is indicated.)



GENERAL
AMERICAN
UNION
REPUBLIC



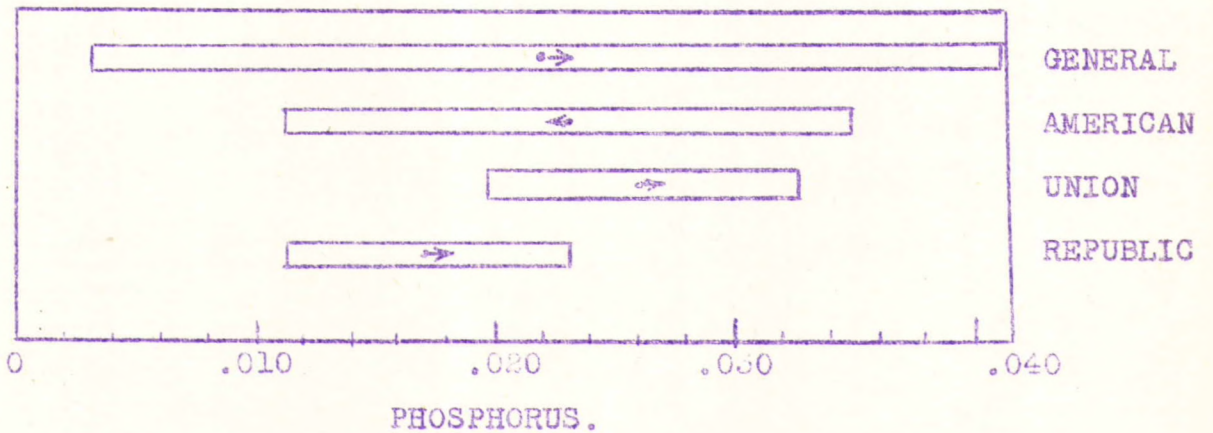
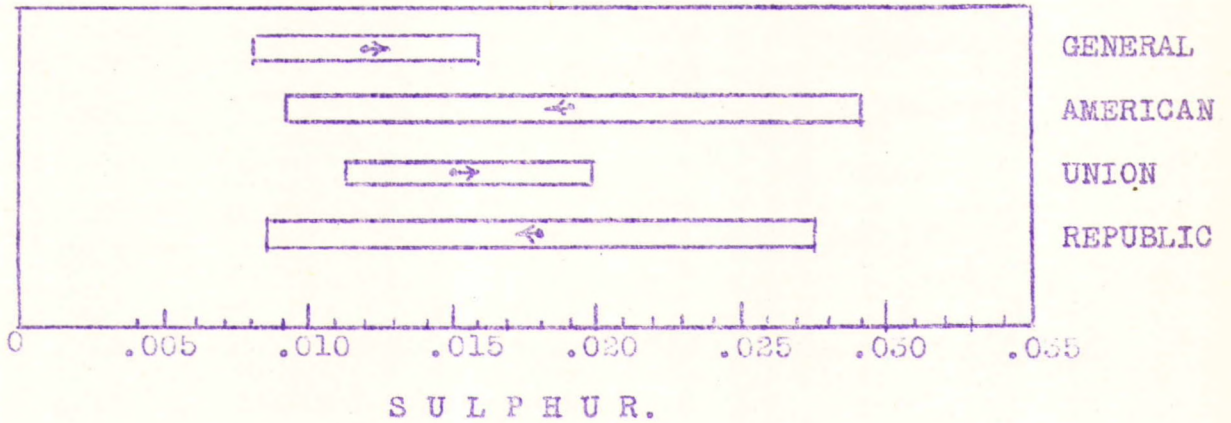
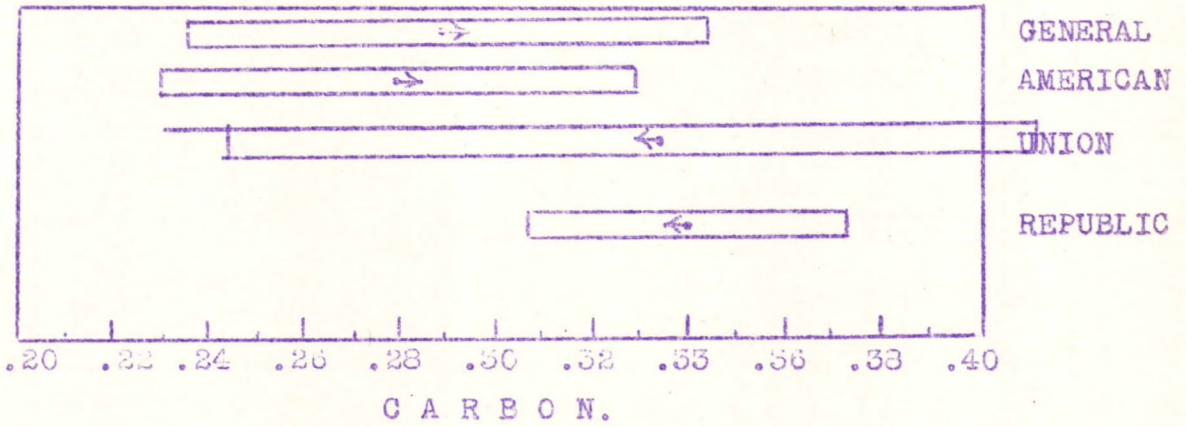
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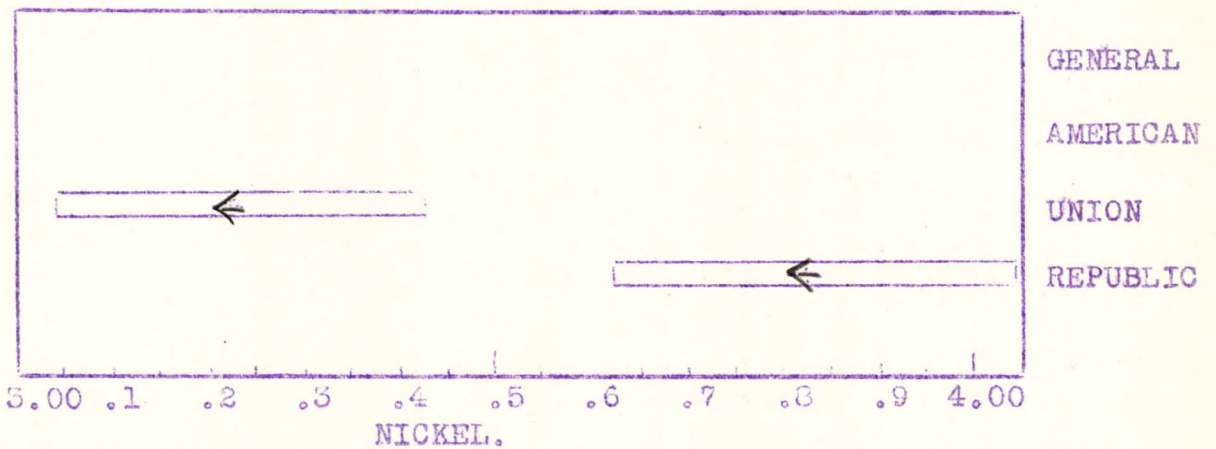
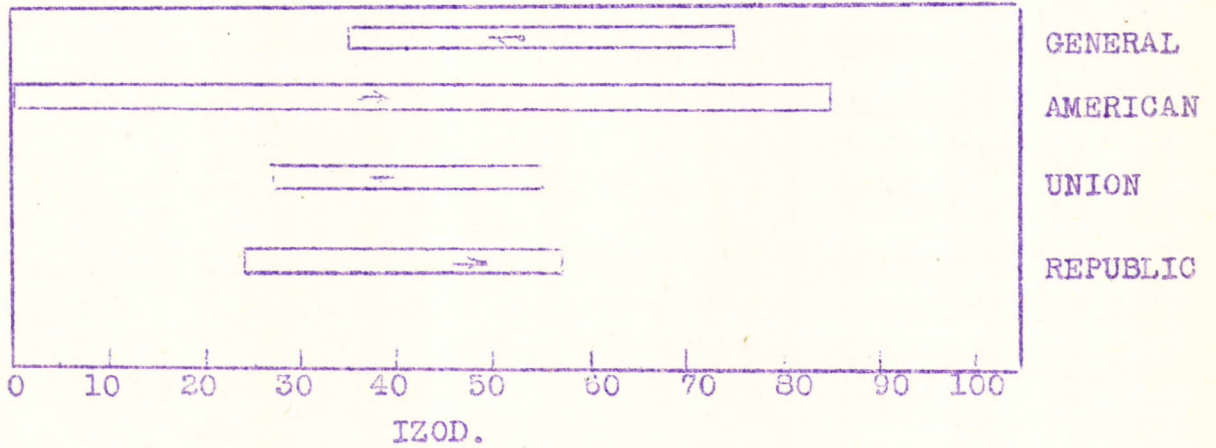
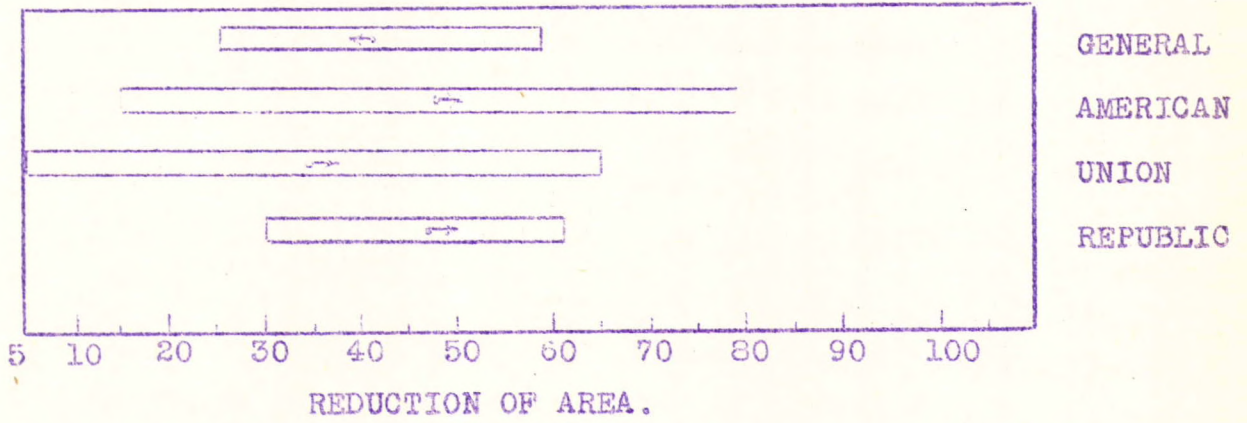
(Graphic Summary of Trends Evident in Data Examined, cont'd) -

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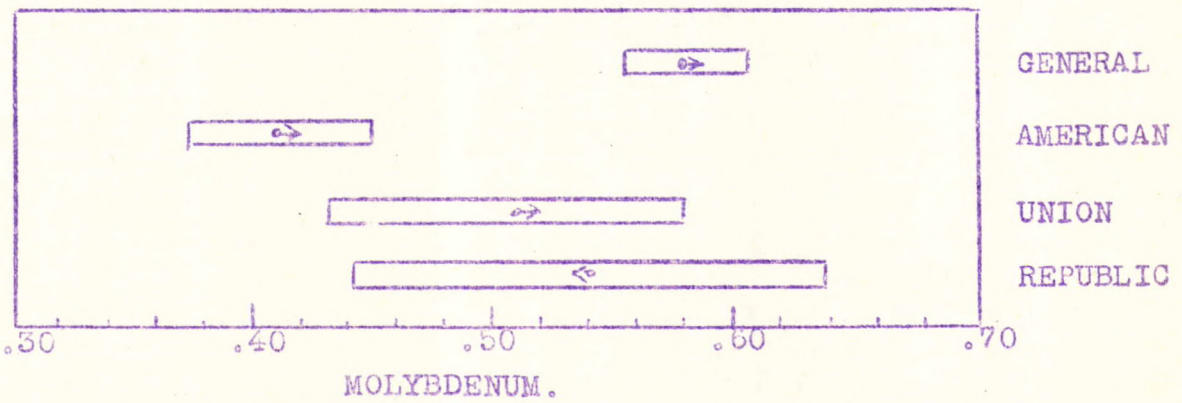
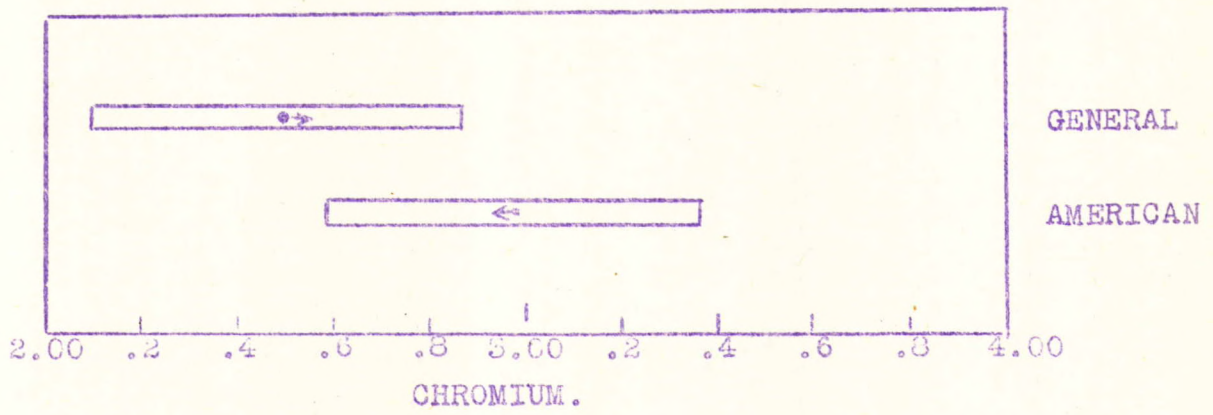
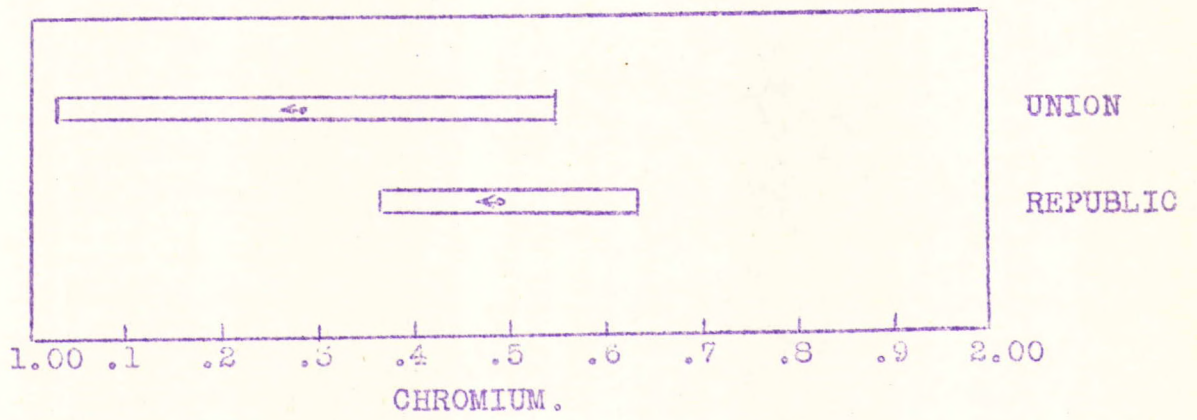
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(KEY TO GRAPHS:)
(• Average of test values.)
(▭ 3 Sigma Limits (Spread))
(→ Direction in which improvement)
(is indicated.)



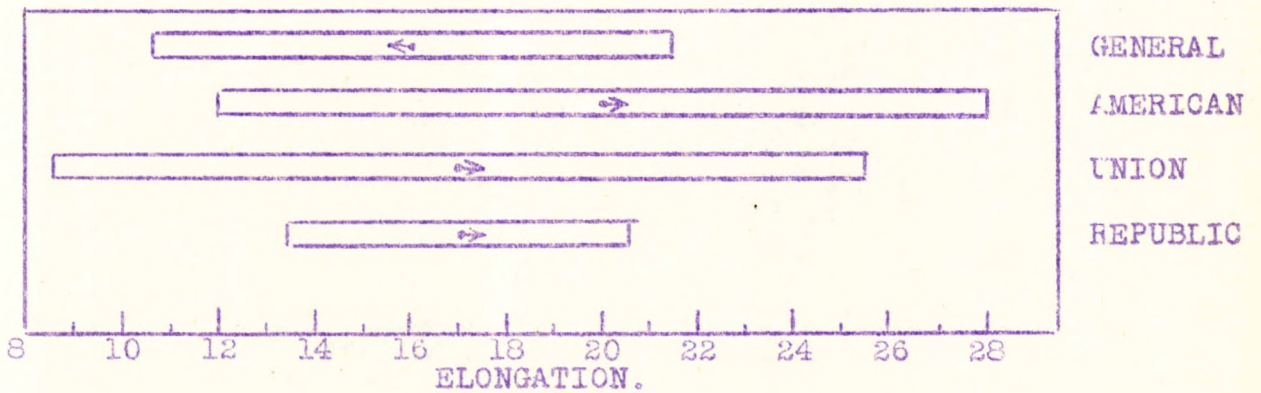
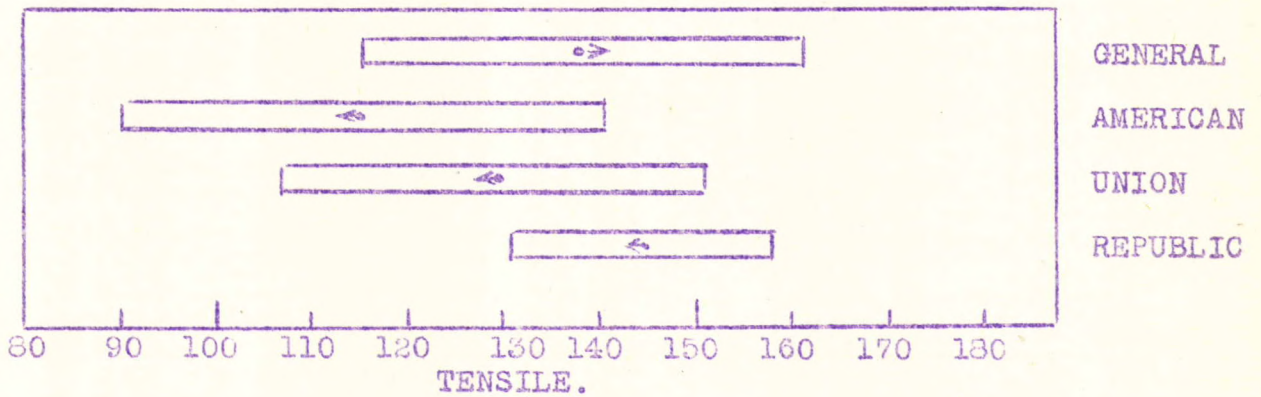
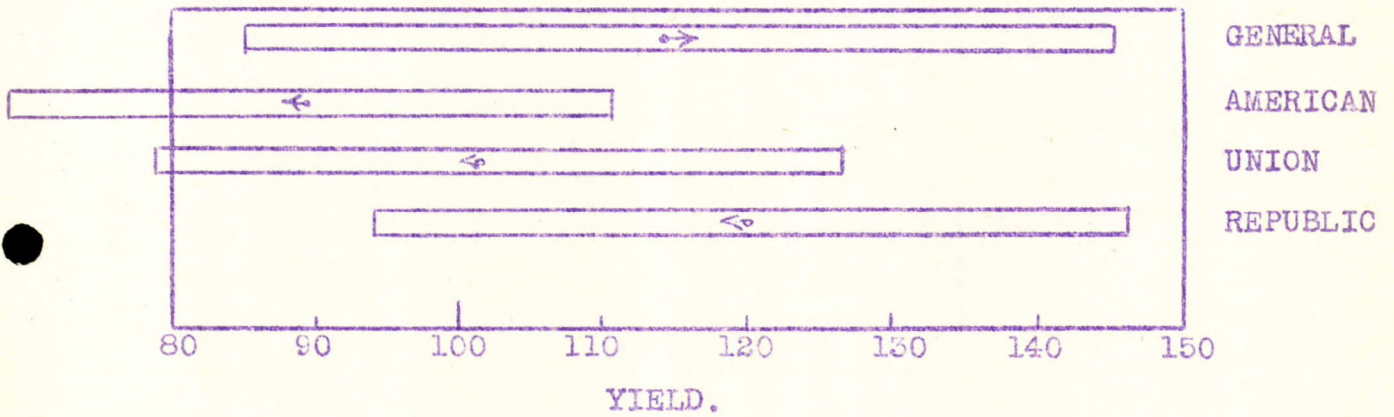
(Graphic Summary of Trends Evident in Data Examined, cont'd) -

(KEY TO GRAPHS:)
(. Average of test values...)
([] 3 Sigma Limits (Spread))
(→ Direction in which improvement)
(is indicated.)



(Graphic Summary of Trends Evident in Data Examined, cont'd) -

(KEY TO GRAPHS:)
(• Average of test values...)
([] 3 Sigma Limits (Spread))
(→ Direction in which improvement)
(might result.)



Conclusions:

The accuracy and amount of bias in the results charted are not known. The ability of a test in any source to serve as an indicator of armour quality can be determined very accurately by correlation methods.

Data accumulated from successive periods of analysis, when available, will give incontrovertible evidence about the fundamental laws underlying armour manufacture.

Since the physical and chemical tests are correlated with armour quality, it is almost certain that there are other variables in manufacture which are closely related to armour quality. Suggested for investigation are:

Pouring temperature,
Melt down time,
Time of boil,
Quench temperature,
Cooling rate on quenching,
and many others.⁶

Since steel properties vary widely, it is obvious that there are variations in manufacturing processes.

The relationship between a value and the ballistic limit of armour plate is so dependent upon manufacturing conditions that it can have meaning only for the source from which it is derived. For this reason a test value (Brinell hardness, for example) can not be specified with any degree of certainty that satisfactory armour will result. Sensible specifications for a source can only originate from analysis of data taken from the source.

The most practical solution to the problem of improving armour plate lies in the establishment of metallurgical statistical departments in the armour plate manufacturing concerns. Metallurgical observers should collect

⁶ See Investigation No. 1144, January, 1942.

(Conclusions, cont'd) -

data on all phases of the steel making and testing processes. These data properly recorded and correlated will give information which will enable improvements to be made.

Characteristics of different processes can be accurately compared by the frequency distribution method.

PART TWO.

NATURE OF VARIABLES RECORDED:

Ballistic Limit is assumed to mean the maximum speed, in feet per second, of a 37 mm. shot which will not defeat the plate. These data were correlated to ballistic limit results using only 37 mm. shot.^① Heavier projectiles or lighter projectiles would require different kinds of plate to defeat them most successfully.

Independent Variables - Alloys.
(Carbon, Chromium, Molybdenum, Nickel, and Manganese)

The effect of variations in these elements on steel properties is universally known and therefore there should be some degree of similarity between their optimum values in each plant. To a certain extent they are independent of the steel process used.

Carbon "caught coming down", however, might have a different relationship to ballistic limit than when carbon was boiled down below the control range and then the bath recarburized up to the ideal point (the usual method).

Since chromium oxidizes so readily, it is quite

^① See "Armour for Fighting Vehicles", a British Ordnance publication.

(Nature of Variables Recorded, cont'd) -

possible that early chrome additions would result in a different ideal chrome content than would late chrome additions.

Since manganese may exist in the iron as oxide, sulphide, and in solution, each process may have a different ideal manganese range.

Dependent Variables - Melting.
(Sulphur, Phosphorus, Silicon)

The ideal or optimum range for these variables is dependent upon the melting methods.

Sulphur and Phosphorus -

Temperature of bath, slag characteristics, sulphur in charge, time allowed for slag reaction, and other factors vary from source to source. Assuming a practice is fairly constant, correlation of sulphur and phosphorus to ballistic limit may point out how slight changes may improve armour. 0.03 per cent sulphur may be desirable in one plant and 0.01 per cent sulphur in another. Non-metallic inclusions are probably closely related to sulphur and phosphorus.

Silicon -

The amount of silicon in armour may with reservations be considered as a measure of the extent to which the bath was oxidized before ferrosilicon was added. Ideal silicon content, therefore, will vary according to the manner in which silicon is added and the extent of the boil-down.

Dependent Variables - Heat Treatment.
(Brinell, Tensile, Yield, Elongation, Reduction of Area, Izod)

Brinell hardness is dependent upon hardenability, tempering temperature, and amount of decarburization. Optimum

(Nature of Variables Recorded, cont'd) -

Brinell hardness in each source will vary according to the amount of notch sensitivity. Non-metallics, oxides, carbides, and structural inhomogeneity affect notch sensitivity. This test is notoriously the most subject to the effects of surface preparation and individual interpretation.

Tensile strength is related to Brinell hardness, although imperfection in the steel may lower tensile strength without affecting Brinell.

Yield point may vary considerably for the same hardness. This is due to variations in the cooling rate through the transformation period. In general, the higher transformation temperatures produce lower yield points. Lower yield points (for approximately the same strengths) mean that before rupture a greater amount of energy is absorbed.

Elongation and Reduction of Area -

Elongation indicates the amount of plastic flow the armour is capable of before rupture. Homogenizing affects the elongation and, of course, the Brinell hardness has an effect on elongation.

Reduction of Area is affected by the homogenizing treatment. Temperature of transformation also affects reduction of area. Austempering heat treatment, by preventing a low temperature transformation, results in -

low yield,
high elongation,
high reduction of area, and
high impact strength.

Therefore it would seem very important to concentrate on

(Nature of Variables Recorded, cont'd) -

getting the armour to transform on quenching within a definite temperature range. The temperature of the quench medium will have an effect on this.

Yield, elongation, and reduction of area will serve as guides in arriving at the ideal quenching method.

Izod impact strength is affected by inclusions (carbides, ferrite, oxides, non-metallics, etc.). The significance of Izod as an indicator, therefore, is that it indicates the degree of structural inhomogeneity.

CHANGES SHOWN TO BE DESIRABLE:

TEST	SOURCE	C H A N G E			Direction	ODDS THAT THIS CHANGE WOULD IMPROVE ARMOUR
		From average	To average			
Carbon	Republic	.335	.325	-	:80 out of 100	
	Union	.335	NO CHANGE	0	:	
	American	.28	.315	+	:40 out of 100	
	General	.295	.325	+	:99 out of 100	
Silicon	Republic	.215	.18	-	:60 out of 100	
	Union	.255	.305	+	:20 out of 100	
	American	.395	.365	-	:60 out of 100	
	General	.305	.295	-	:94 out of 100	
Manganese	Republic	.41	.37	-	:40 out of 100	
	Union	.685	.605	-	:50 out of 100	
	American	1.07	1.00	-	:94 out of 100	
	General	.595	.545	-	:84 out of 100	
Sulphur	Republic	.016	.014	-	:85 out of 100	
	Union	.015	NO CHANGE	0	:	
	American	.018	.016	-	:98 out of 100	
	General	.011	.013	+	:80 out of 100	

(Continued on next page)

(Changes shown to be Desirable, cont'd) -

TEST	SOURCE	C H A N G E			Direction	ODDS THAT THIS CHANGE WOULD IMPROVE ARMOUR
		From average	To average			
Brinell	Republic	292	NO CHANGE	0		
	Union	265	278	+	25 out of 100	
	American	238	249	+	55 out of 100	
	General	267	273½	+	99.85 out of 100	
Tensile	Republic	143	138½	-	10 out of 100	
	Union	128	122	-	85 out of 100	
	American	113	100	-	40 out of 100	
	General	142	145	+	95 out of 100	
Elongation	Republic	16	18½	+	80 out of 100	
	Union	17	20	+	55 out of 100	
	American	19	22	+	95 out of 100	
	General	15	12	-	92 out of 100	
Yield	Republic	119	111½	-	95 out of 100	
	Union	99	93½	-	20 out of 100	
	American	86	73	-	80 out of 100	
	General	119	126	+	96.5 out of 100	
Reduction in Area	Republic	46	54	+	40 out of 100	
	Union	32	42	+	93 out of 100	
	American	48	58	+	95 out of 100	
	General	40	32	-	87 out of 100	
Phosphorus	Republic	.016	.021	+	60 out of 100	
	General	.024	.030	+	65 out of 100	
	Union	.026	.0235	-	40 out of 100	
	American	.022	.029	+	30 out of 100	
Nickel	Republic	3.82	3.73	-	70 out of 100	
	Union	3.23	3.18	-	85 out of 100	
Molybdenum	Republic	.53	.49	-	80 out of 100	
	Union	.50	.53	+	40 out of 100	
	American	.40	.445	+	60 out of 100	
	General	.58	.595	+	85 out of 100	
Chromium	Republic	1.49	1.45	-	20 out of 100	
	Union	1.27	NO CHANGE	0		
	American	2.94	2.73	-	30 out of 100	
	General	2.49	2.59	+	60 out of 100	
Izod	Republic	44	59	+	85 out of 100	
	American	35	57	+	97 out of 100	
	General	50	45	-	85 out of 100	

(THESE FIGURES ARE MERELY TARGETS TO AIM AT. IT IS)
 (EXPECTED THAT NORMAL VARIATION WILL OCCUR ABOVE AND)
 (BELOW THE TARGET.)

RELATIVE SIGNIFICANCE OF VARIABLES AS FOUND:

It was immediately obvious that a variable can be closely related to ballistic limit in one source and have little relationship to ballistic limit in another source.

TABLE I. - Relative Significance of Metallurgical Variables As Indicators of Armour Quality.

Relative Significance	REPUBLIC	GENERAL	UNION	AMERICAN
1	: Yield	: Brinell	: Red. area	: Sulphur
2	: Sulphur, Izod	: Carbon	: Ult. strength	: Izod
3	:	: Yield	: Nickel	: Red. area
4	: Elong., Carbon, Molybdenum	: Ult. strength	: Elongation	: Elongation
5	:	: Silicon	: Manganese	: Manganese
6	:	: Elongation	: Phosphorus	: Yield
7	: Nickel	: Red. area	: Molybdenum	: Silicon, Molybdenum
8	: Phosphorus	: Izod	: Brinell	:
9	: Silicon	: Molybdenum	: Silicon, yield	: Brinell
10	: Manganese, Red. area	: Manganese	:	: Ult. strength
11	:	: Sulphur	: Carbon, Chrome, Sulphur	:
12	: Chromium	: Phosphorus	:	: Chromium
13	: Ult. strength	: Chromium	:	: Phosphorus
14	: Brinell	:	:	:

(Continued on next page)

(Relative Significance of Variables As Found, cont'd) -

Table I gives a clue to the nature of armour and what affects its properties. Note that in one source Brinell hardness was more closely related to ballistic limit than any other variable. In another source Brinell hardness was the least significant of the physical and chemical variables.

This fact shows that too much faith should not be put in any one metallurgical test performed on armour plate.

When Brinell is closely associated with ballistic limit, it may be inferred that a change in the Brinell hardness would improve the plate. It might also be inferred that the Brinell test is performed in such a way as to be a sensitive indicator of armour quality.

When the Brinell hardness is not closely related to ballistic limit, it may be inferred either

- (1) that the hardness of the material examined remained in the ideal range,

or

- (2) that the Brinell test was performed in a way that made it a poor indicator of armour quality. This latter may well be the case, since instances of decarburized castings are well known,

or

- (3) that change in that part of the process affecting Brinell hardness obscured the relationship.

It will be noted that in one source sulphur proved to have the highest significance as an indicator of armour quality. This is a clue which the experienced metallurgist should not fail to interpret. It means that

(Relative Significance of Variables As Found, cont'd) -

there are variations in the melting practice which have a great effect on the quality of the armour and incidentally upon the sulphur. By a careful study of actual melting conditions and their relationship to sulphur content, improvement of armour from this source can be obtained.

In another source sulphur content is shown to have little relationship to ballistic limit. Note, however, that sulphur and izod impact strength tend to occupy about the same position on the table of relative degree of correlation. From this it would seem likely that impact strength and non-metallic inclusions were very closely related.

Chromium does not appear to be of very great significance[Ⓢ] in controlling armour qualities. We might assume then that if alloy content is above a certain minimum limit variations are of little significance.

Nickel occupies a higher position in degree of correlation with ballistic limit. This might be due to the fact that nickel prolongs the transformation time and hence variations in nickel affect the homogeneity of micro-structure. Chromium and molybdenum steels transform in such a short time that transformation may be assumed to be complete in all cases.

The significance of any test, therefore, can be evaluated. Should this significance change drastically from one period to another, it is almost certain that some drastic variation in either testing procedure or manufacturing methods has taken place. The correlation technique, however, guards against giving too much weight to any test value.

[Ⓢ] Statistically speaking.

DISTRIBUTION CHARACTERISTICS:

After observing the results in Table I it was decided to study the characteristics of each group of test data. APPENDIX B shows in detail how distribution characteristics are calculated. Each process, method of operation, and method of testing, results in a type of distribution which is as individual as a fingerprint. By comparing distributions of tests reported by different manufacturers, an idea can be obtained as to the relative efficiency of each department in the steel mill. Considering phosphorus, for example, we find that one source is able to control phosphorus to within ± 0.001 per cent, while another plant can control phosphorus only within ± 0.0063 per cent. Now, while the desirability of maintaining a narrow range has not been proven, still it is obvious that in one plant a different method of operation is being used. Further investigation may reveal which practice is most desirable. In such a case information will be available so that a manufacturer may, if he wishes, change his method of operating so as to produce more satisfactory results.

The kurtosis factor[Ⓢ] serves as a quantitative measure of the frequency of extreme results. Extreme results are undesirable because their effect on the armour cannot be predicted. Therefore, some thought should be given to the correction of operating conditions which produce these extreme results. For example, one source has a kurtosis factor of 1612 whereas another source has a kurtosis factor of 206565 (unit is 0.01 per cent chromium). It should be possible by a comparison of conditions in these

[Ⓢ] See Appendix B. - Relative Kurtosis and Skewness.

(Distribution Characteristics, cont'd) -

two steel plants to find out the reason for the extreme variations in the one case.

In general, the greatest variation appears to be not in the ballistic limit but rather in the tensile and yield strength. It would seem from the limited amount of data on hand that the greatest improvement to armour would come, first, through a closer control over heat-treatment operations and, secondly, from closer control over melting operations. At present the variations due to heat-treatment and melting operations obscure the effects of variations in alloy content (chromium, molybdenum, nickel, etc.). Appendix C shows distribution characteristics of all variables recorded for the four sources.

A notation was made on each distribution characteristic result indicating whether or not a variable was under statistical control. The fact that only a small percentage of metallurgical variables are under statistical control is not startling. Variations in raw materials and in furnace conditions are to a certain extent uncontrollable and, therefore, metallurgical operations cannot always be under perfect control. At present it is sufficient if progress towards statistical control can be made. What is aimed at is not merely a control of the variables such as sulphur or Brinell hardness within a narrow limit, but a uniform, consistent method of manufacture. It would seem that the method of manufacture is the thing of greatest importance and the purpose of metallurgical tests is to guide the manufacturing process.

ACTUAL CONTROL RANGES:

After examining data from the four sources the narrowest control ranges encountered were tabulated as follows:

Ballistic Limit	-	± 1.65 ft./sec.
Brinell	-	± 18
Tensile	-	$\pm 15,000$ p.s.i.
Yield	-	$\pm 21,000$ "
Elongation	-	± 3.5 per cent
Reduction in Area	-	± 15 "
Izod	-	± 12 ft./lbs.
Carbon	-	± 0.03 per cent
Silicon	-	± 0.05 "
Manganese	-	± 0.09 "
Sulphur	-	± 0.005 "
Phosphorus	-	± 0.007 "
Nickel	-	± 0.19 "
Chromium	-	± 0.15 "
Molybdenum	-	± 0.03 "

It should not be inferred that these are the most desirable limits within which the variables should be expected to remain. Much further work will be necessary before control limits can be accurately outlined.

QUALITY CONTROL DEPARTMENT:

On the basis of the work done to date it would appear desirable that each manufacturer of armour plate collect and analyse his own operating data. It has been proven fairly conclusively both by statistical analysis and by practical experience that physical properties and chemical analysis cannot be transplanted from one steel plant to

(Quality Control Department, cont'd) -

another. Therefore, there are only two alternatives,

(a) By a series of costly trial-and-error experiments determine how to make armour; or

(b) By a statistical analysis of operating data determine optimum conditions of each factor in the environment which produces armour plate.

The program which should be followed by each manufacturer is briefly outlined as follows:

1. Metallurgical observers should measure and record test data and conditions surrounding the manufacture of the product under the direction of a competent metallurgical engineer.

2. This data should be classified, analysed, and correlated by an engineer familiar with elementary statistical procedure.

3. The first results should indicate in which direction lies the optimum range of each variable.

4. The relative degree of significance of each variable should then be determined. This would indicate which part of the process was in the most immediate need of corrective steps.

5. It should then be decided what changes in the process are necessary to shift test data results into a more desirable range.

6. After this program has been set in motion periodic changes should be made to determine the relative significance of each variable and to calculate the frequency distributions so that any change in relationship or in the process can be detected.

(Continued on next page)

(Quality Control Department, cont'd) -

7. Finally, a statistical quality control program should be instituted. This phase of industrial management has been described in detail in

"Engineer's Manual of Statistical Methods",
by L. E. Simon. (John Wiley Pub. Co., New York)

"Applied General Statistics", by Croxton and Cowden.
(Prentice-Hall Pub. Co., London, England)

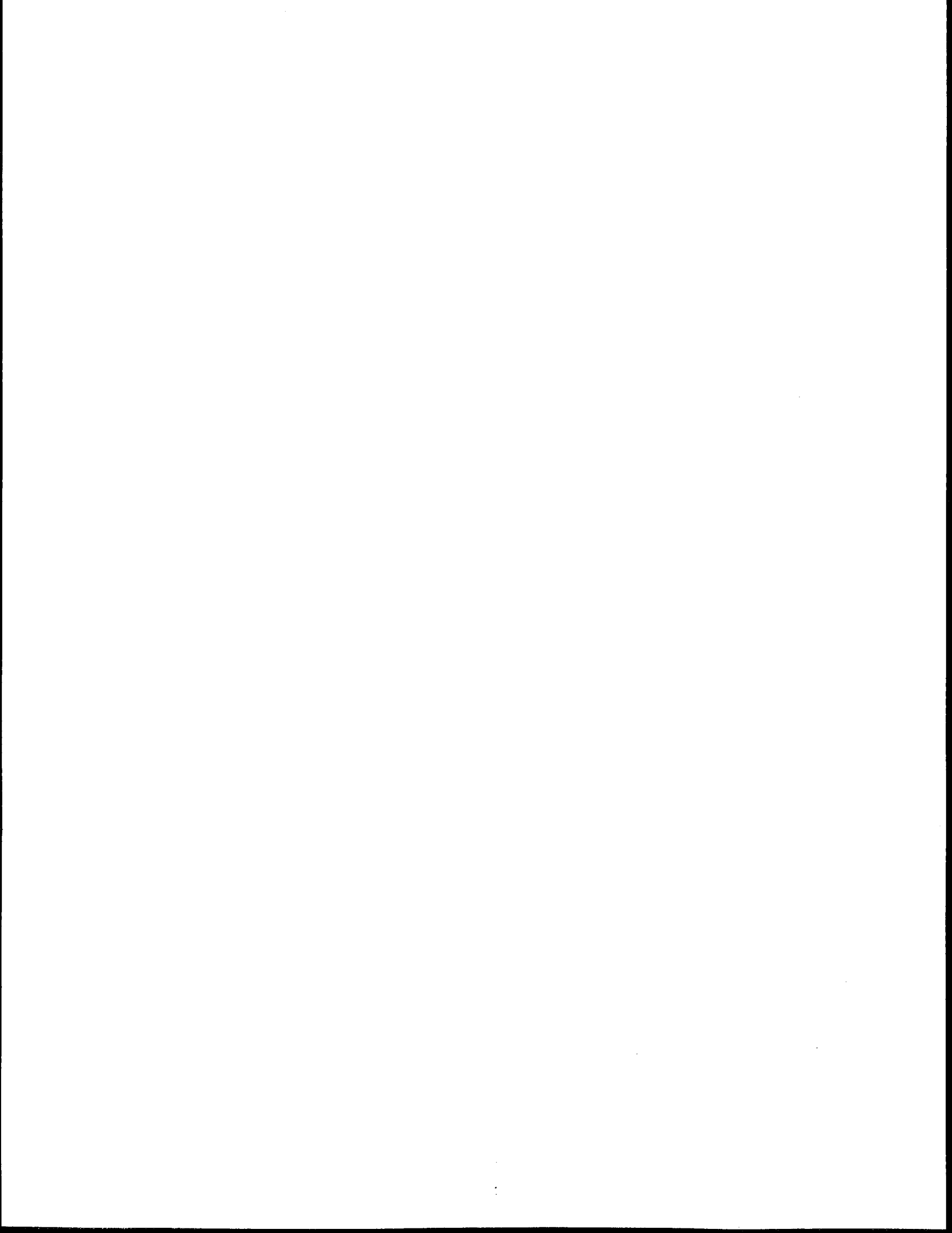
"A. S. T. M. Manual on the Presentation of Data."

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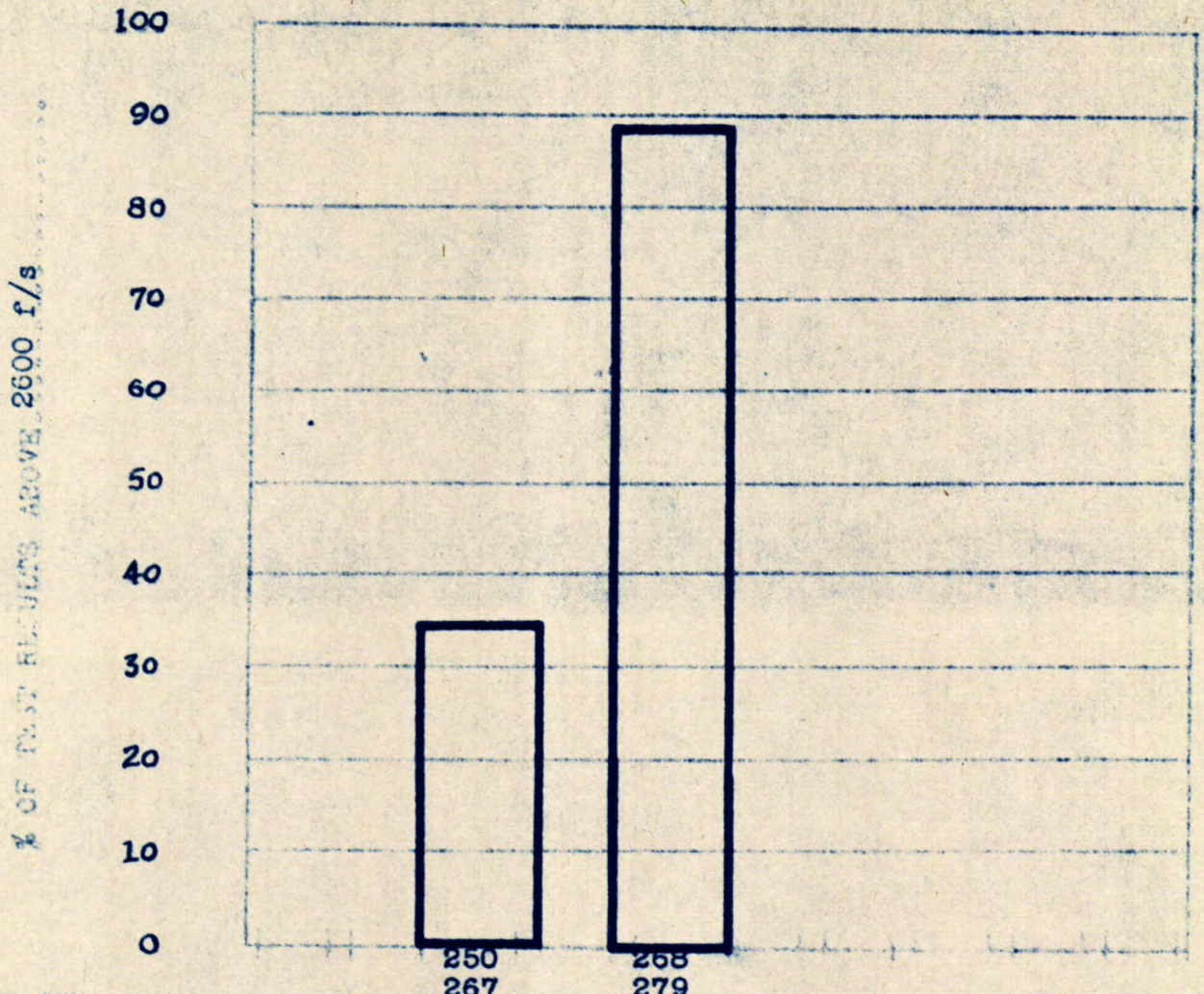
-- APPENDIX A --
CONFIDENTIAL - SECURITY INFORMATION

Correlation Between Metallurgical Tests
and Ballistic Limits.



CORRELATION BETWEEN Brinell & Ballistic Limit ..

IN. 3rd Armour FROM General



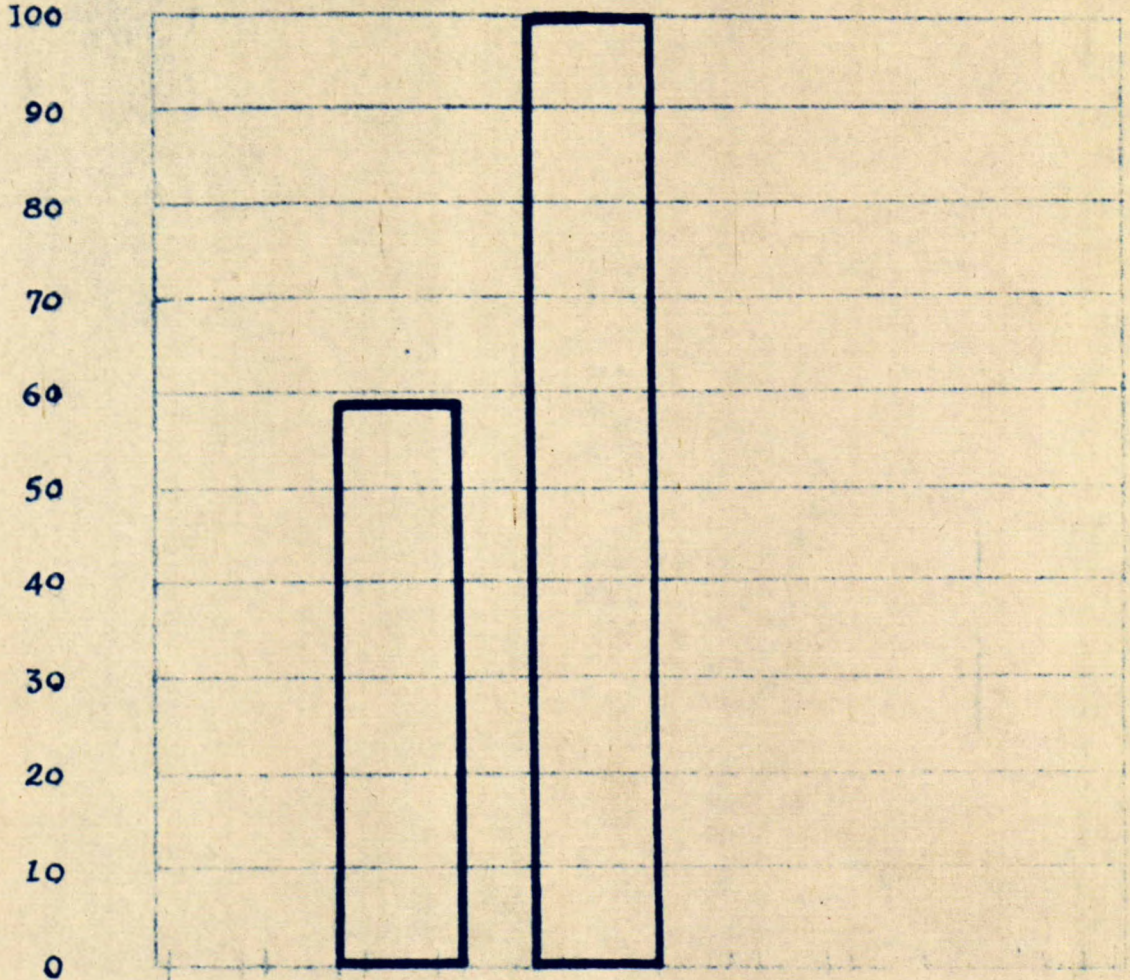
RANGE		
No. above 2600 f/s:	7	16
Total number of test results:	20	18
Percentage above:	35	89
Ratio difference:	54	
Number effect:	.05	+ 0.0555 = .1055
Significance:	.99:85 %.

(NOTE: This means that (100-99.85 equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Carbon & Ballistic Limit

IN. 3" Armour FROM General

% OF TEST RESULTS ABOVE 2600 f/s



.26 .30
.29 .35

No. above 2600 f/s 13 20

Total number of test results 22 20

Percentage above59 100

Ratio difference:41

Number effect:095

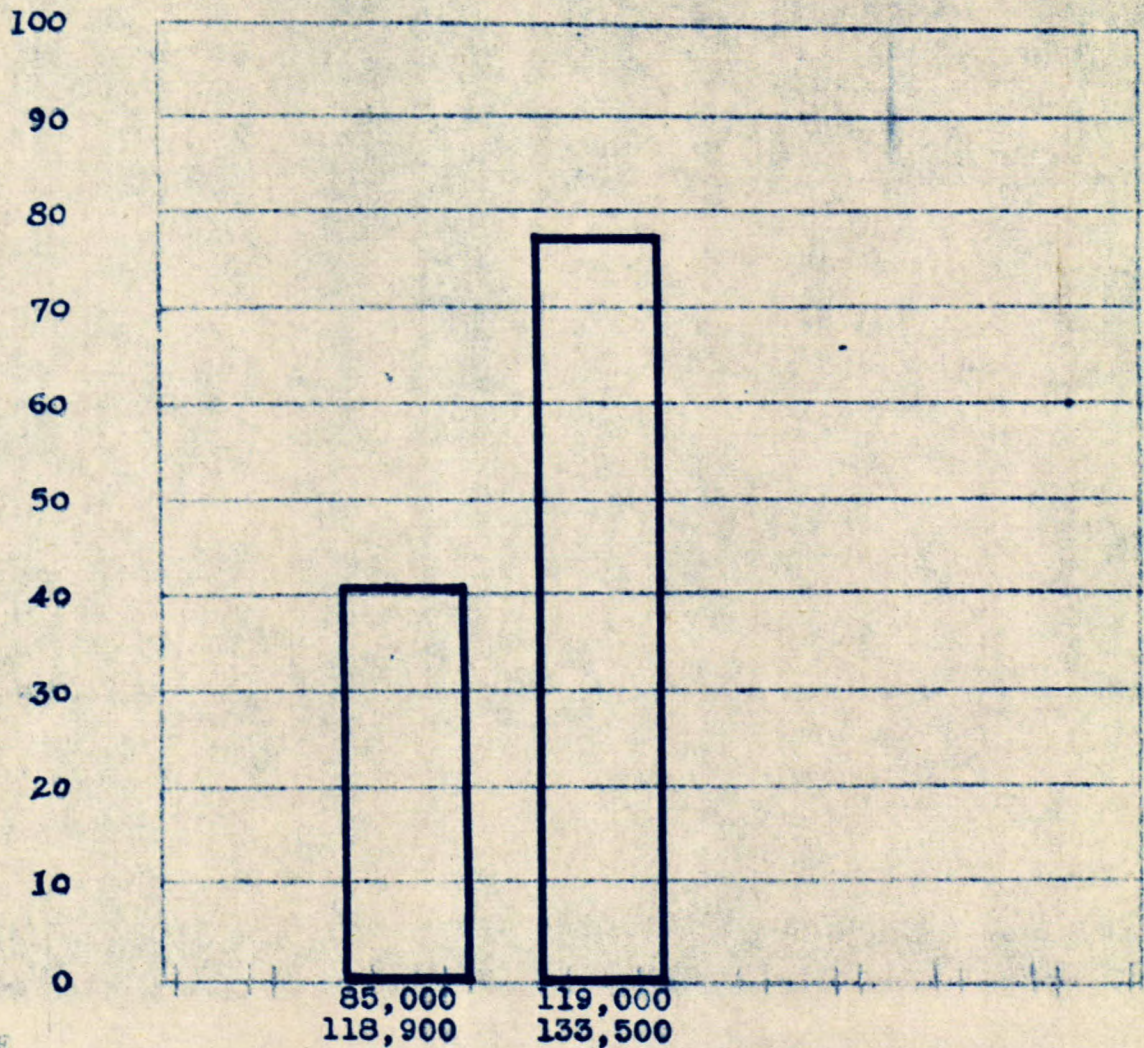
Significance: * 99 %

(NOTE: This means that (100-99%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN **yield** & **Ballistic Limit**

IN **3" Armour** FROM **General**

% OF TEST RESULTS ABOVE **2600 f/s**



No. above 2600 f/s:

7 17

Total number of test results:

17 22

Percentage above:

.411772

Ratio difference:

.361

Number effect:

.0588 ± .0454 = .1042

Significance: *

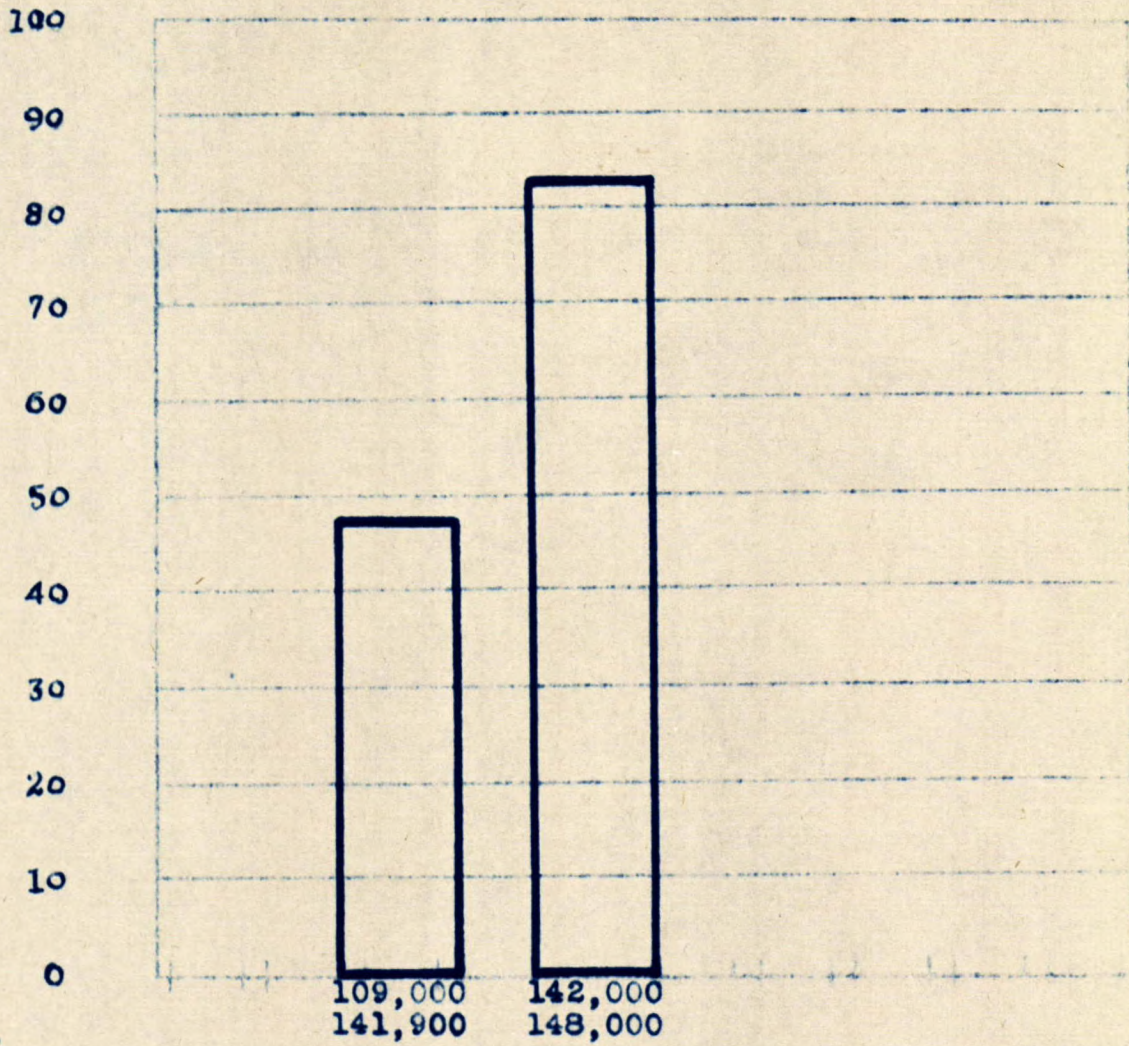
..... 96.5%

(NOTE: This means that (100-96.5%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Tensile Strength & Ballistic Limit

IN. 3" Cast Armour FROM. General

% OF TEST RESULTS ABOVE 2600 f/s.



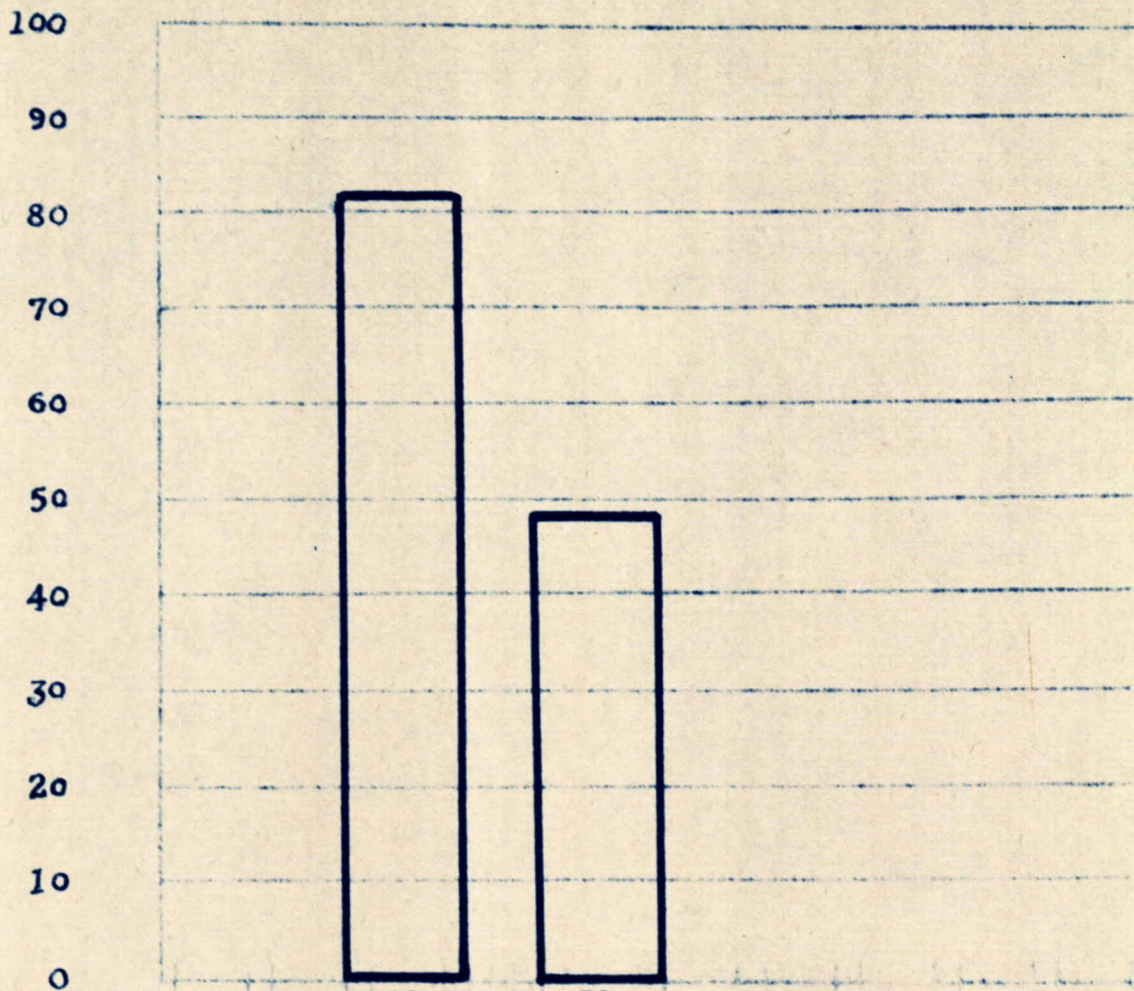
RANGE	109,000 141,900	142,000 148,000
No. above 2600 f/s:	10	14
Total number of test results:	21	17
Percentage above:	.476	.823
Ratio difference:	.347	
Number effect:	.0476 plus .0589 = .1065	
Significance:	95 %	

(NOTE: This means that (100-95...%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Silicon & .. Ballistic Limit

IN. 3" Armour FROM, General

% OF TEST RESULTS ABOVE 2600 f/s



RANGE

No. above

2600 f/s:

.29
.30 .31
.35

..... 13 11

Total number of
test results:

16 23

Percentage
above:

.81 .48

Ratio difference:

.81 - .48 = .33

Number effect:

.0625 plus .0434 = .1059

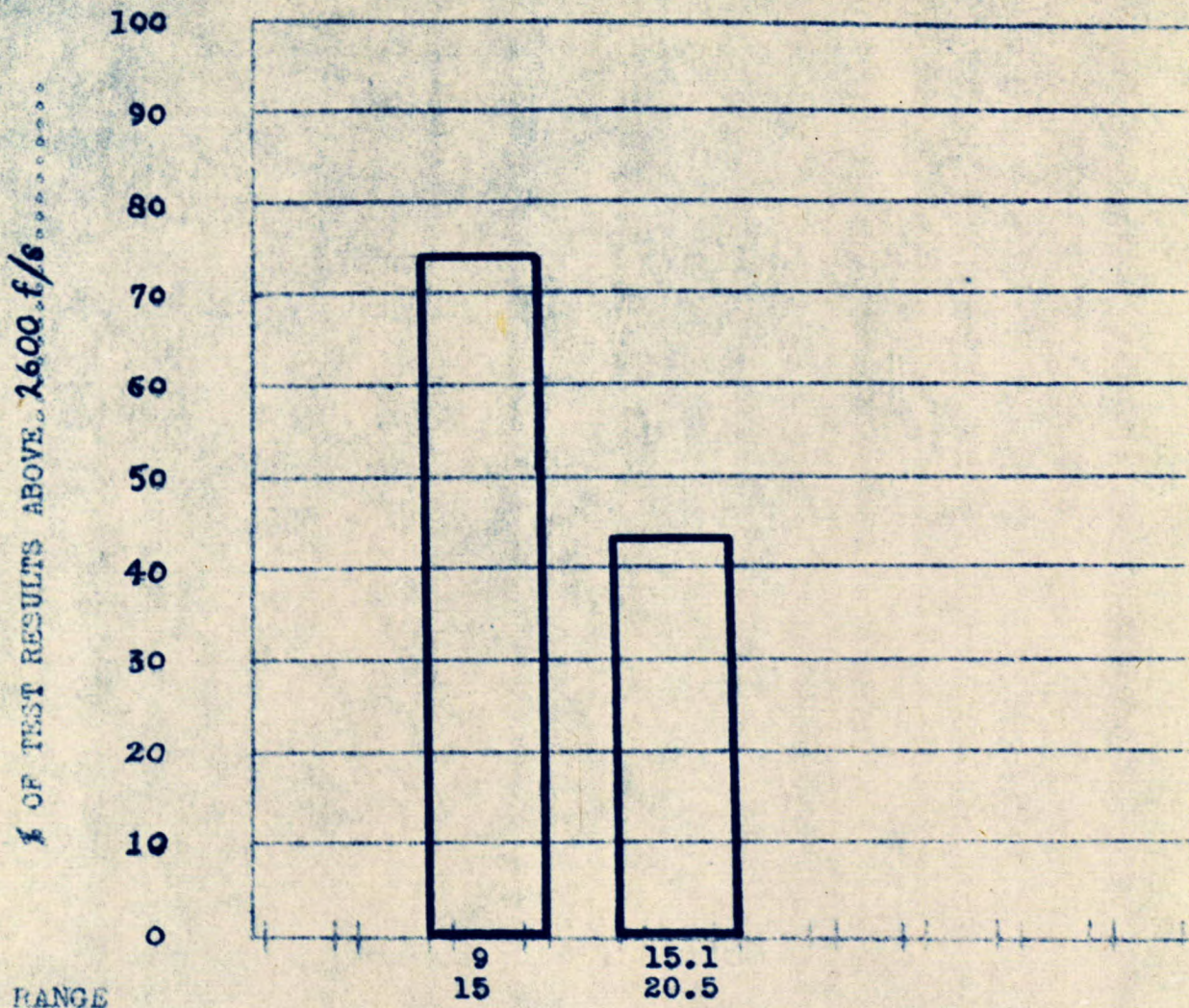
Significance:*

94. %

(NOTE: This means that (100-...94%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Elongation.....&..Ballistic Limit..

IN. 3" Armour..... FROM. General.....



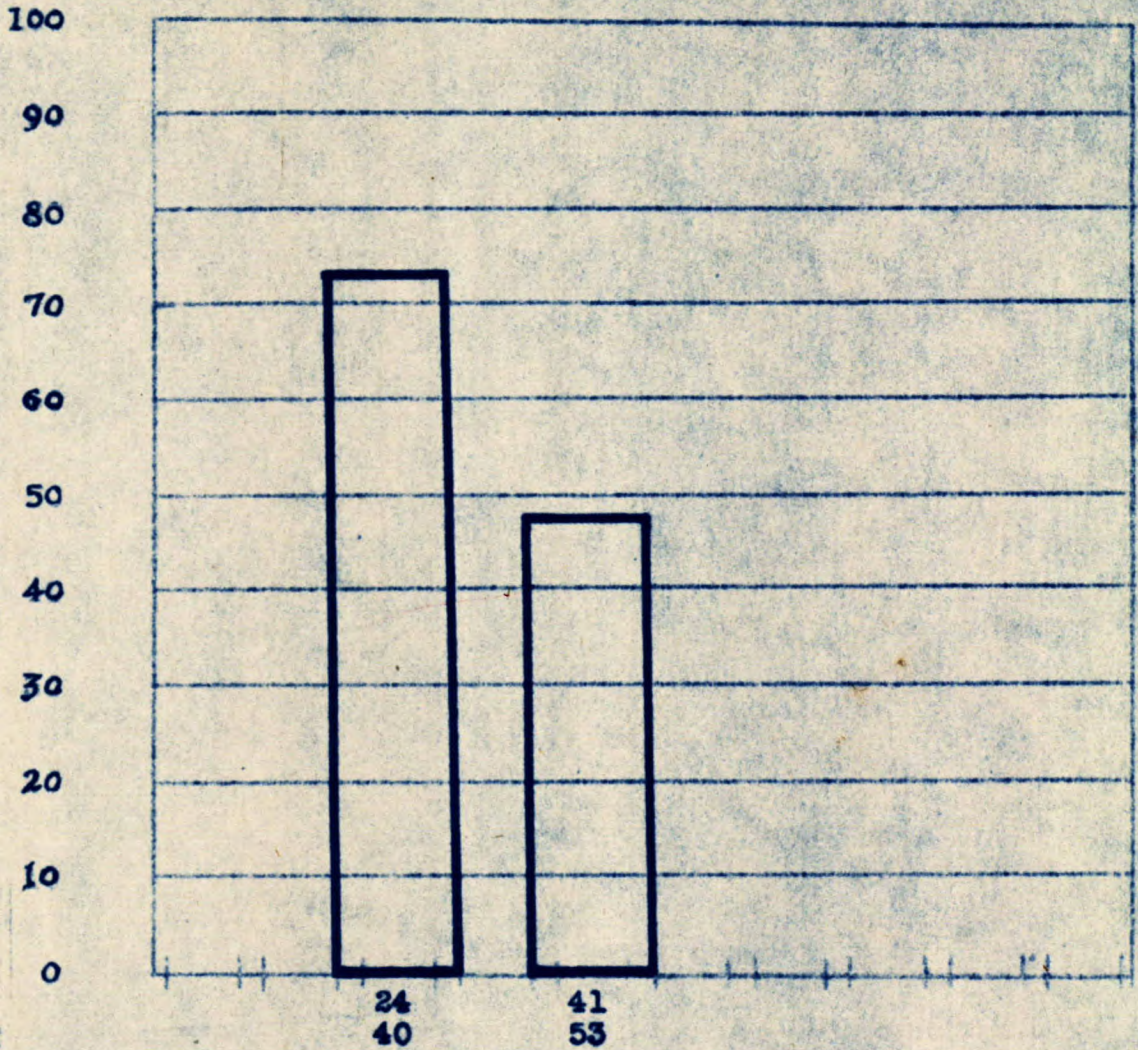
RANGE	9 15	15.1 20.5
No. above 2600 f/s:	17	7
Total number of test results:	23	16
Percentage above:	.74	.437
Ratio difference:	.303	
Number effect:	.0425 plus .0625 = .106	
Significance:	92 %	

(NOTE: This means that (100-92%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Red. of Area...&...Ballistic Limit..

IN. 3" Armour..... FROM.General.....

% OF TEST RESULTS ABOVE 2600 f/s.

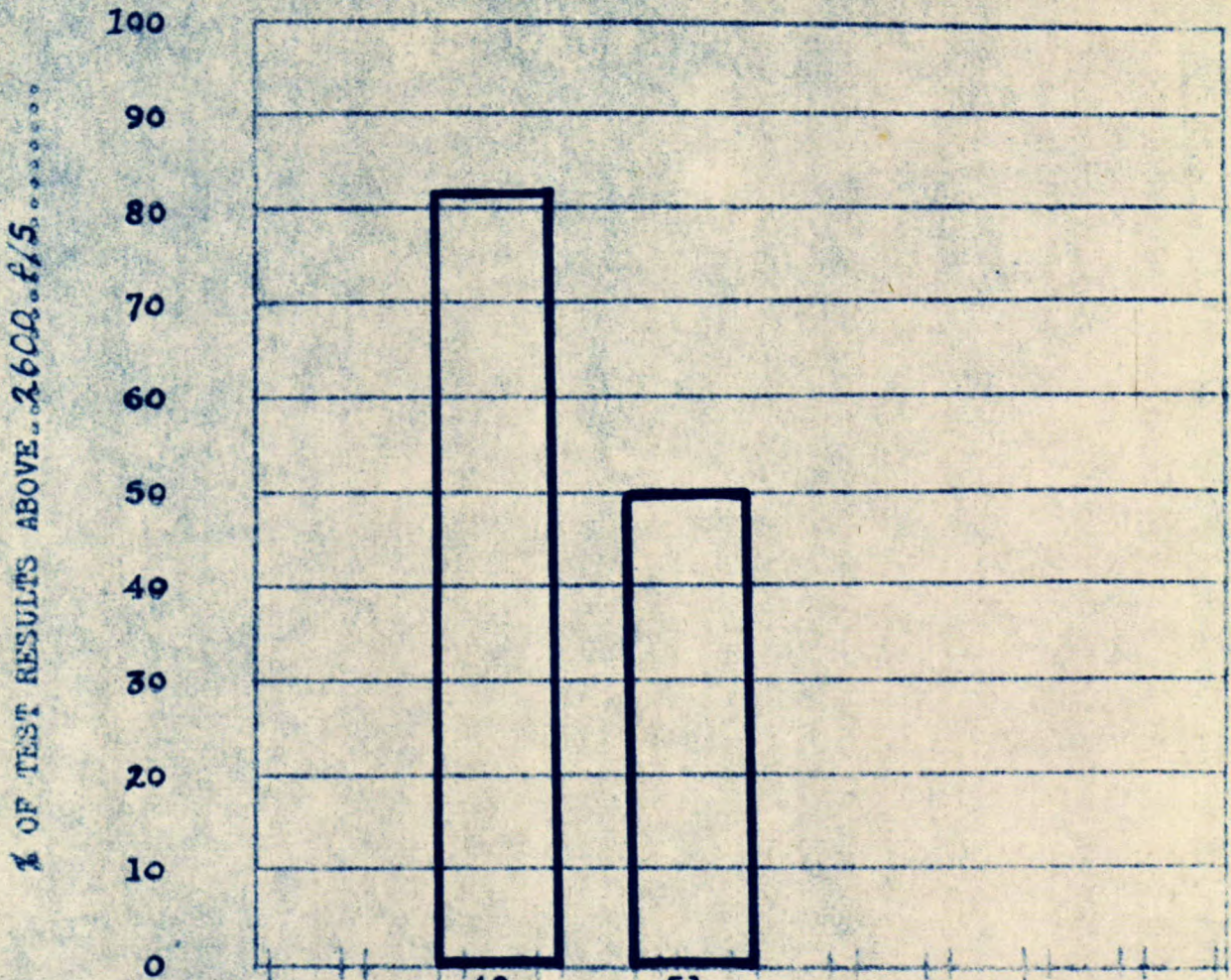


RANGE	24	41
	40	53
No. above f/s:	14	9
Total number of test results:	19	19
Percentage above:	73.8	47.4
Ratio difference:	.264	
Number effect:	.1055	
Significance:	87 %	

(NOTE: This means that (100-87%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN IZOD & Ballistic Limit

IN. 3" Armour FROM. General



RANGE

No. above 2600 f/s:

40 50 51 67

9 11

Total number of test results:

11 22

Percentage above:

.82 .50

Ratio difference:

.32

Number effect:

.091 plus .0454 = .1364

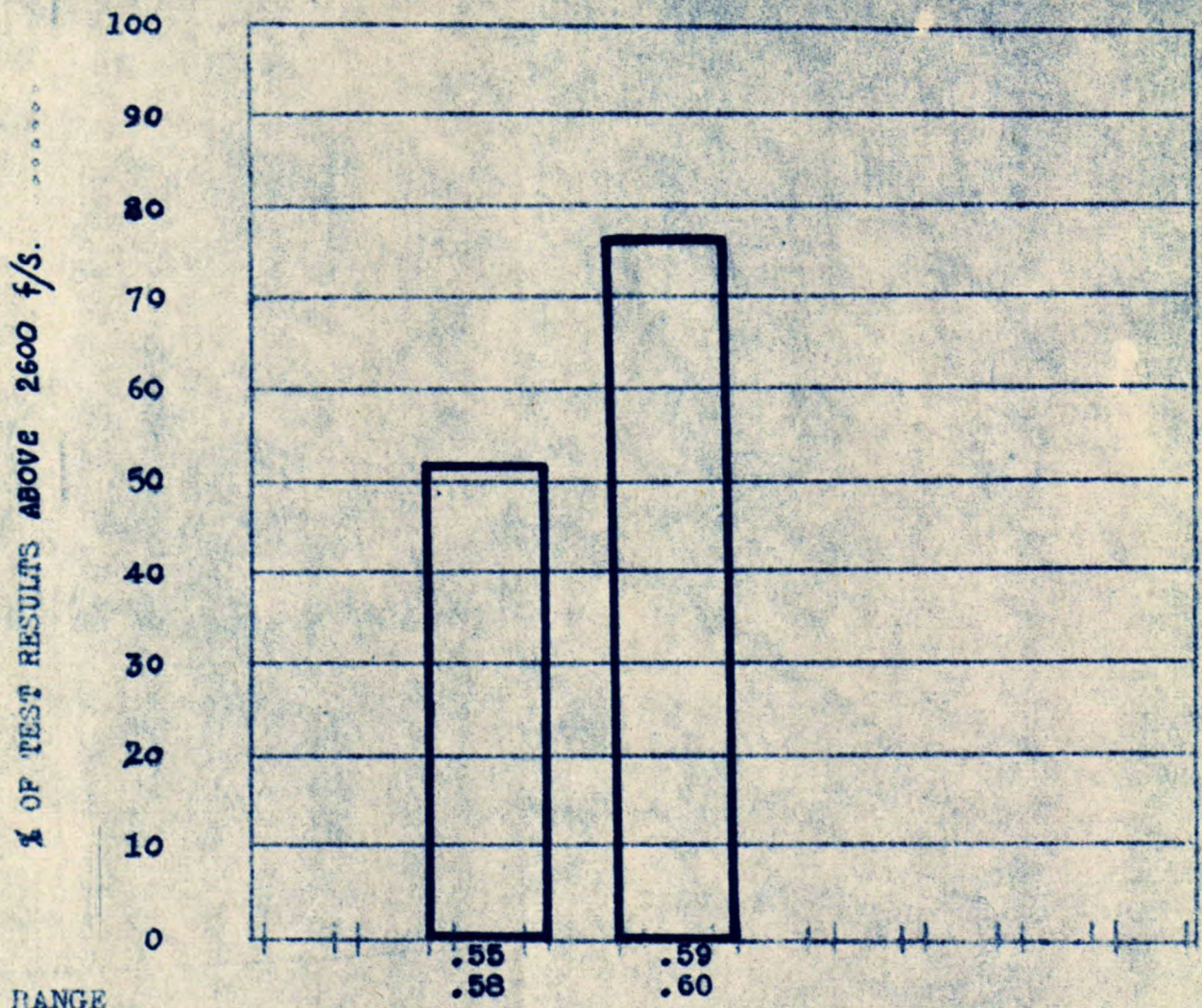
Significance:

85 %

(NOTE: This means that (100-85...%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Molybdenum....&..Ballistic Limit..

IN. 3" Armour..... FROM...General.....



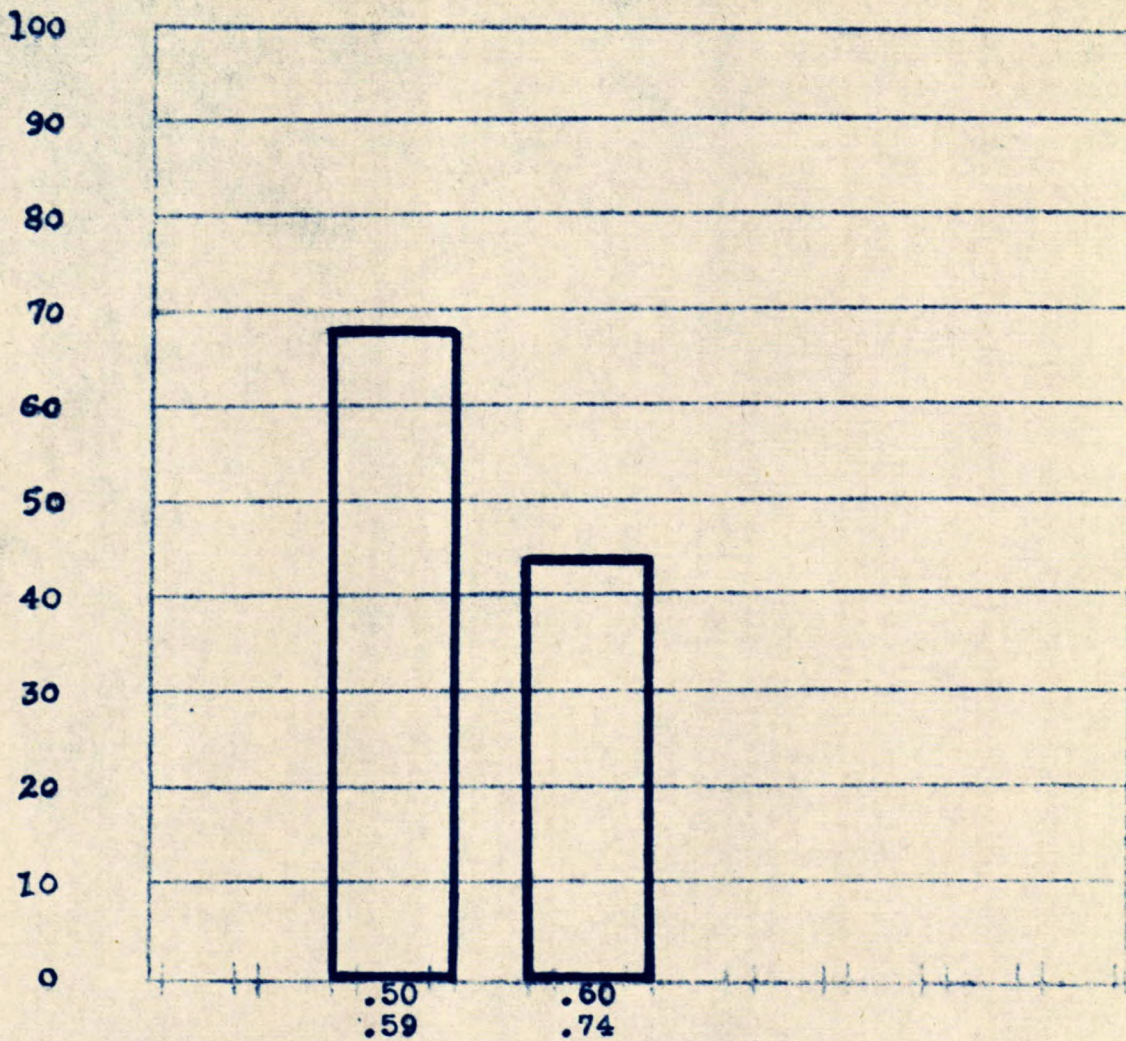
RANGE		.55	.59
		.58	.60
No. above 2600 f/s:	13	13
Total number of test results:	25	17
Percentage above:52	.76
Ratio difference:24
Number effect:04 plus .06 = .1	
Significance:	85 %	

(NOTE: This means that (100-.85%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Manganese..... & Ballistic Limit.....

IN. 3". Armour..... FROM. General.....

% OF TEST RESULTS ABOVE 2600 f/s.....



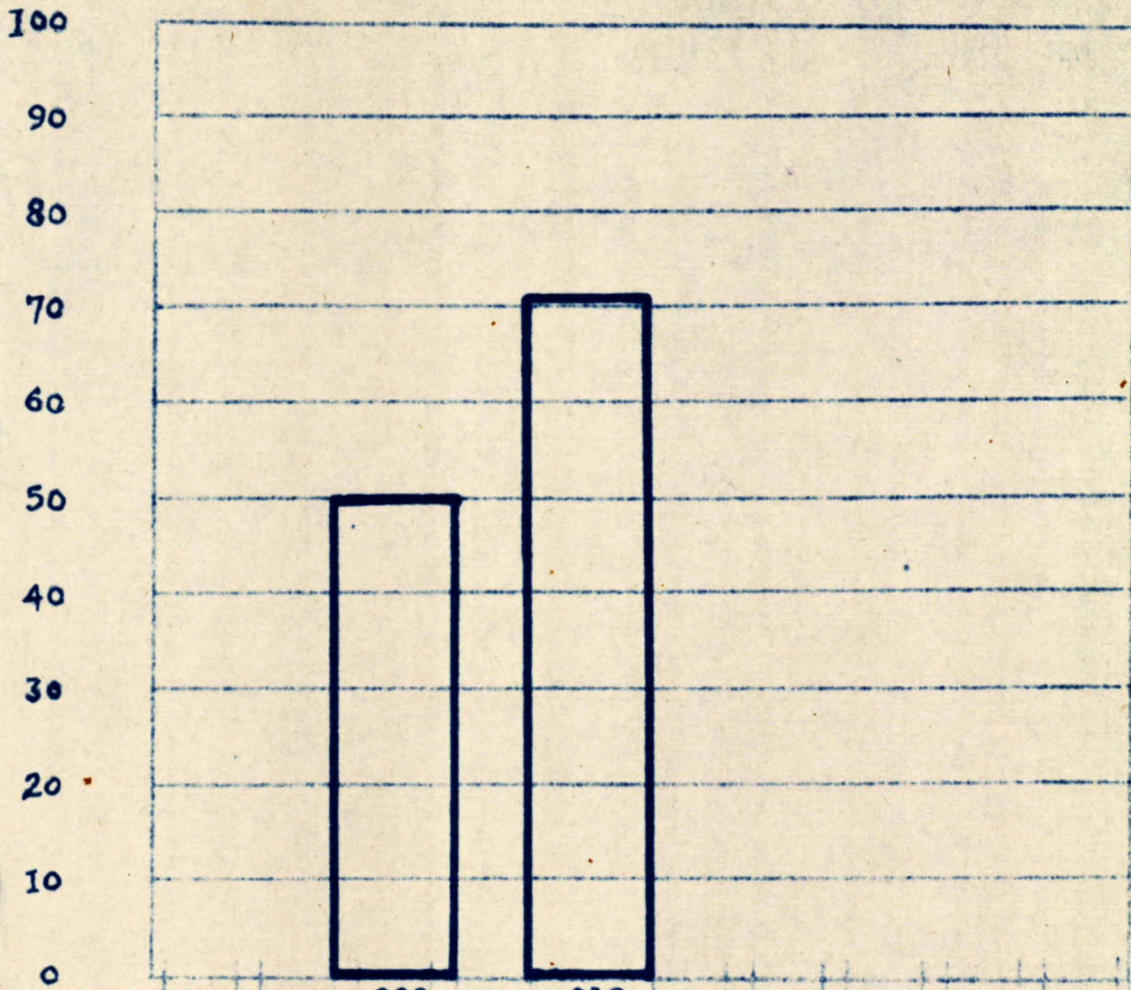
RANGE		
No. above 2600 f/s: 17 7
Total number of test results: 25 16
Percentage above:6844
Ratio difference:24	
Number effect:04 plus .062 = .102	
Significance: 84 %	

(NOTE: This means that (100-84%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Sulphur & .. Ballistic Limit ..

IN. 3" Armour FROM. General

% OF TEST RESULTS ABOVE 2600 f/s



RANGE

No. above 2600 f/s: 10 15
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Total number of test results: 20 21
-------------------------------	----------------	----------------

Percentage above:5071
-------------------	-----------------	-----------------

Ratio difference:21
-------------------	-----------------

Number effect:05 plus .0476 = .0976
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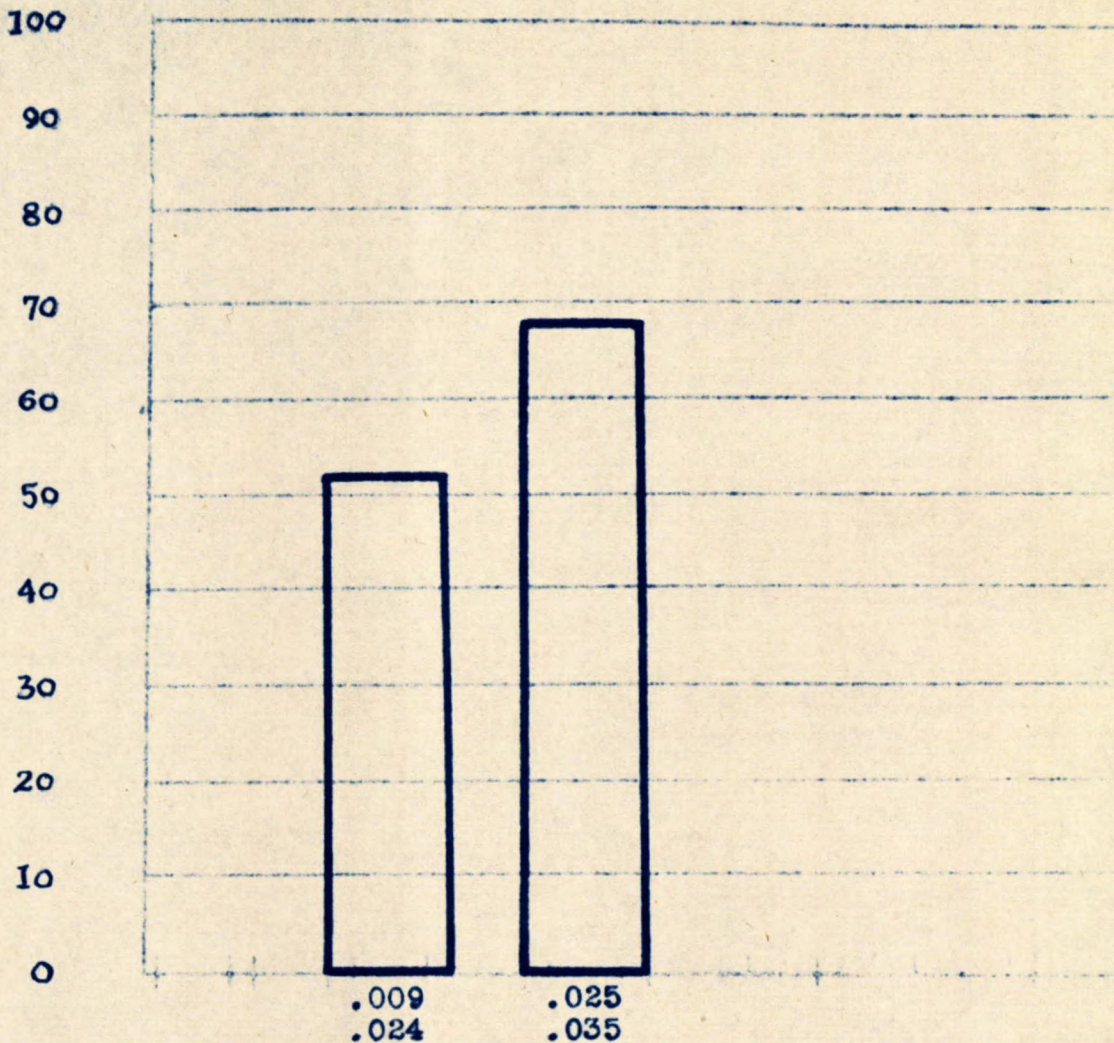
Significance: 80 %
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(NOTE: This means that (100-80%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Phosphorus & Ballistic Limit

IN. 3" Armour FROM. General

% OF TEST RESULTS ABOVE 2600 f/s



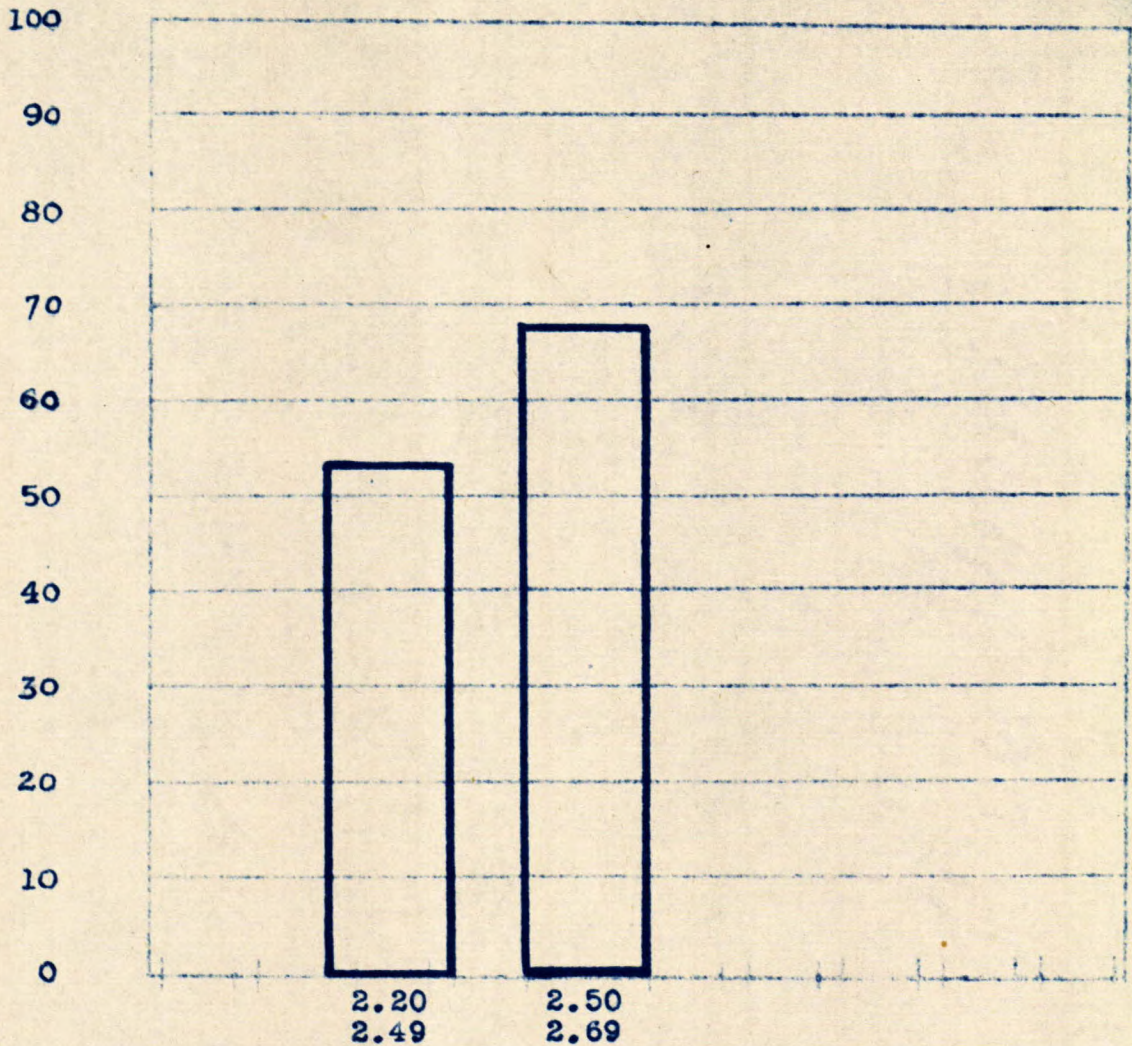
RANGE	.009 .024	.025 .035
No. above 2600 f/s	12	13
Total number of test results;	23	19
Percentage above;	.52	.68
Ratio difference:	.16	
Number effect:	.043 ÷ .052 = .095	
Significance:*	65 %	

(NOTE: This means that (100-.65%) equals the
 (percentage of the time the above ratio
 (difference would occur due to chance.)

CORRELATION BETWEEN Chromium & Ballistic Limit

IN. 3" Armour FROM General

% OF BEST RESULTS ABOVE 2600 F/S.

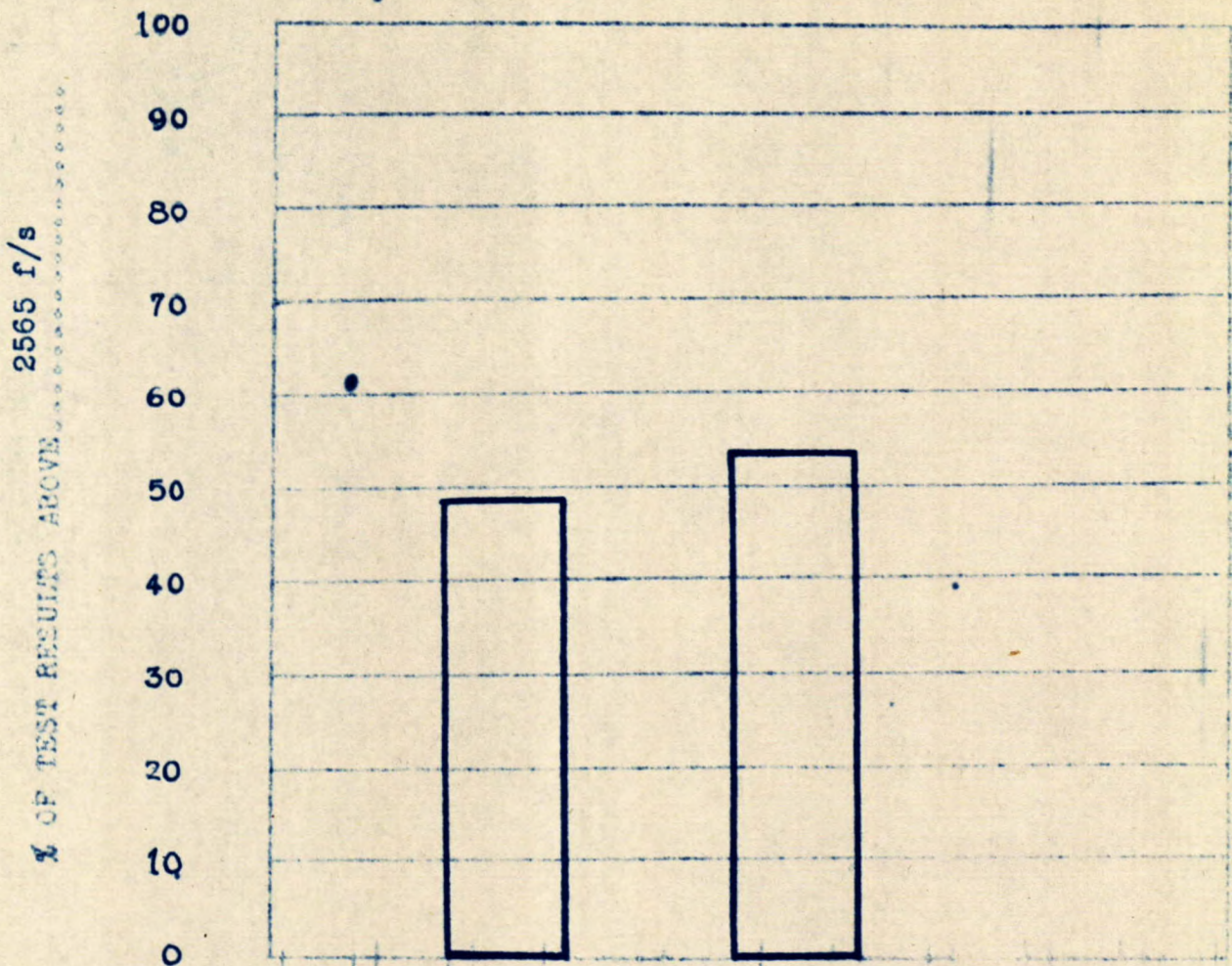


RANGE	2.20 2.49	2.50 2.69
No. above 2600 F/S:	9	15
Total number of test results:	17	22
Percentage above:	.53	.68
Ratio difference:	.15	
Number effect:	$.059 \pm .045 = .104$	
Significance:*	.60 %	

(NOTE: This means that (100-.60%) equals the
 (percentage of the time the above ratio
 (difference would occur due to chance.)

CORRELATION BETWEEN CARBON & BALLISTIC LIMIT

IN 3" ARMOUR FROM AMERICAN



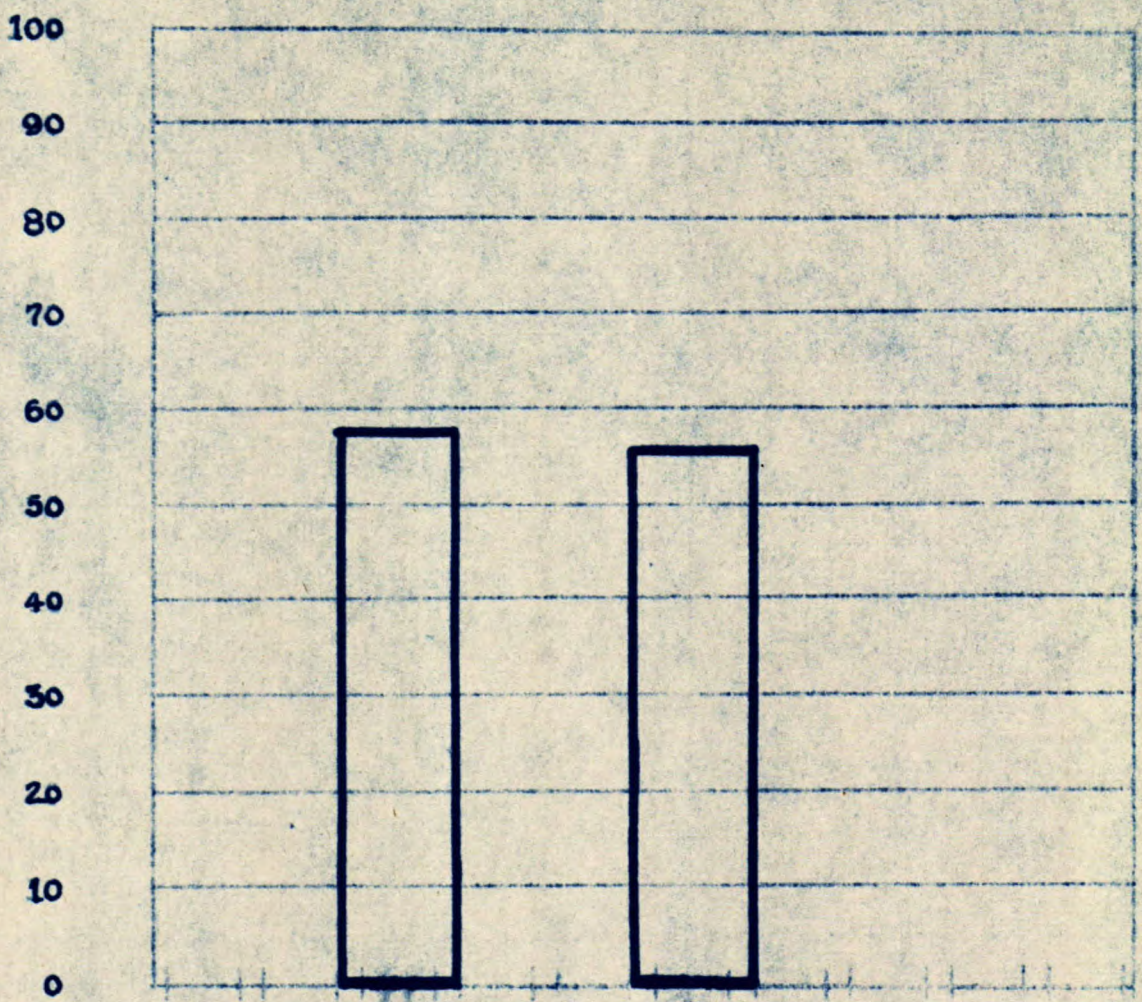
RANGE	.262	.282
No. above 2565 f/s	.281 22	.348 25
Total number of test results	46	47
Percentage above	48	53
Ratio difference	.05	
Number effect	$\frac{1}{46} + \frac{1}{47} = .0217 + .0213 = .0430$	
Significance	40 %	

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Carbon & Ballistic Limit

IN. 2 3/4" Armour FROM Union Steel

% OF TEST RESULTS ABOVE 2254 F/S

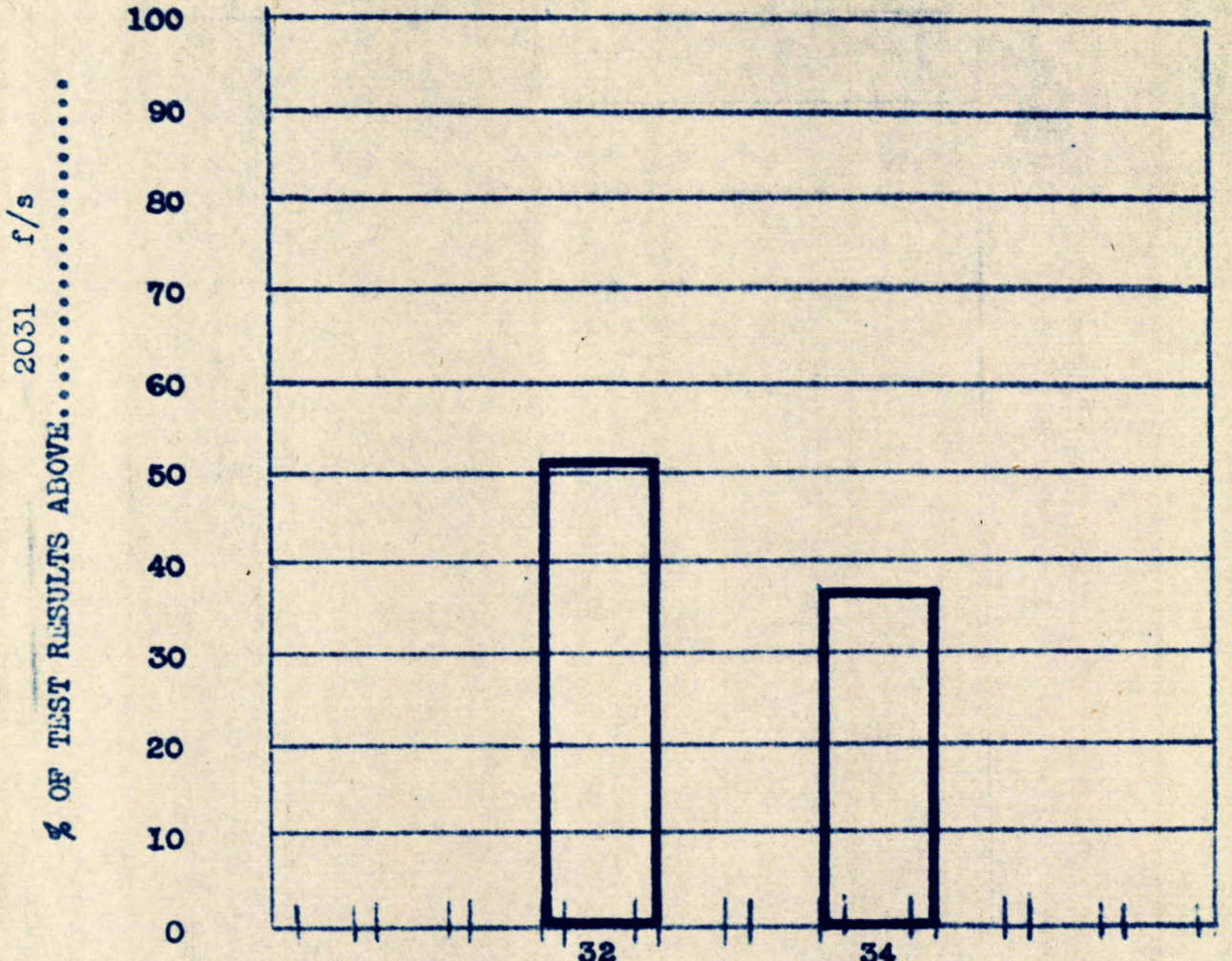


RANGE	28	34
	33	46
No. above 2254 f/s:	7	9
Total number of test results:	12	16
Percentage above:	58	56
Ratio difference:	.02	
Number effect:	$1/12 + 1/16 = .0835 + .0625 = .1460$	
Significance:	0 %	

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Carbon.....&..Ballistic..Limit.

IN..2"..Armour..... FROM...Republic.....

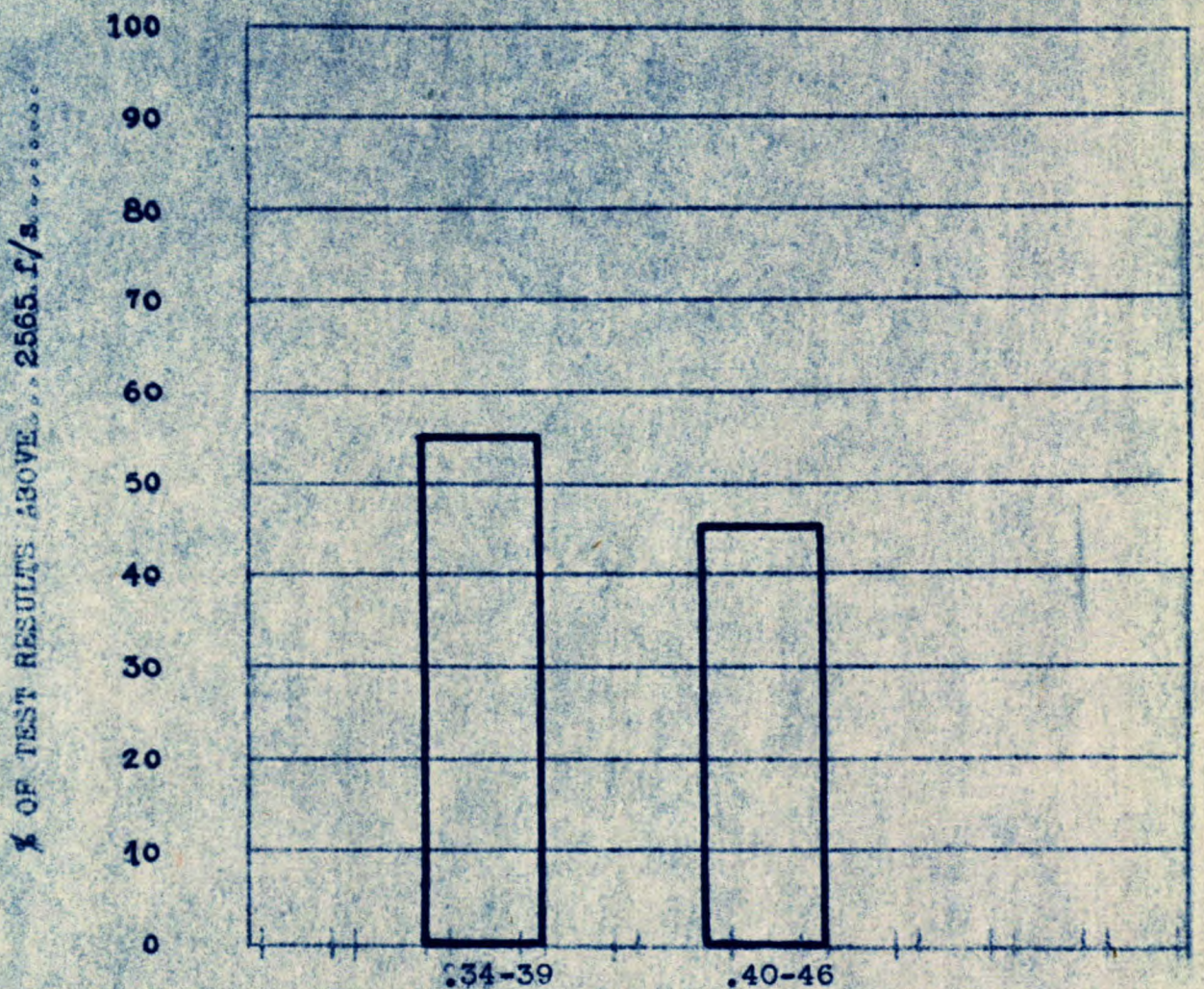


RANGE	32	34
No. above:	33	36
2550 f/s:	17	11
Total number of test results:	33	30
Percentage above:	.52	.37
Ratio difference:	.15	
Number effect:	.029 plus .034 = .063	
Significance:	80 %	

(NOTE: This means that (100-80%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN . . . SILICON . . . & . . . BALLISTIC LIMIT . . .

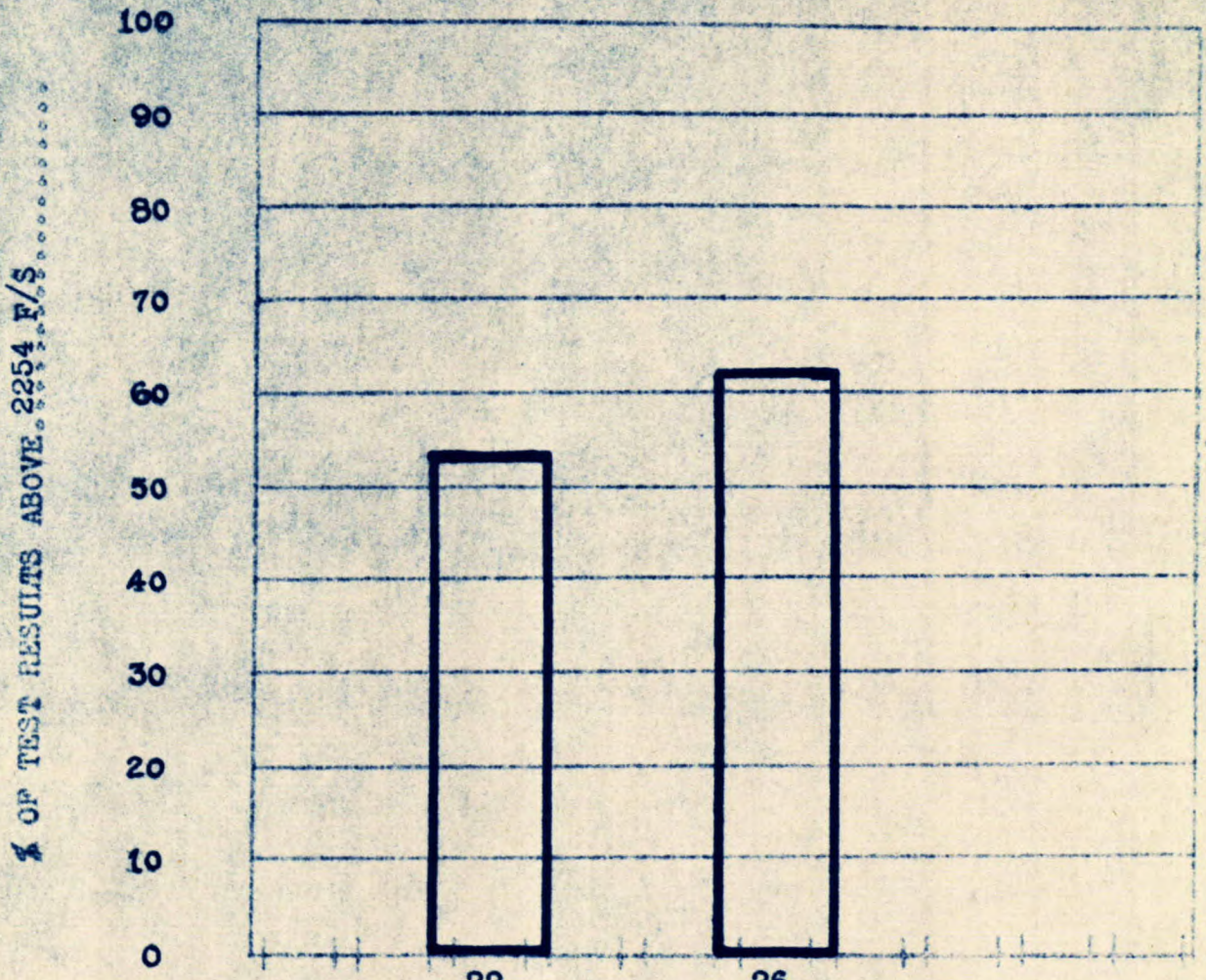
IN . 3" . ARMOUR FROM . . AMERICAN



RANGE		
No. above 2565 f/s:	27	20
Total number of test results:	49	44
Percentage above:	55	45.4
Ratio difference:	.096	
Number effect:	$\frac{1}{49} \pm \frac{1}{44} = .02040 \pm .0227 = .0431$	
Significance: 60 %	

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

IN. 2 3/4" Armour ... FROM ... Union Steel

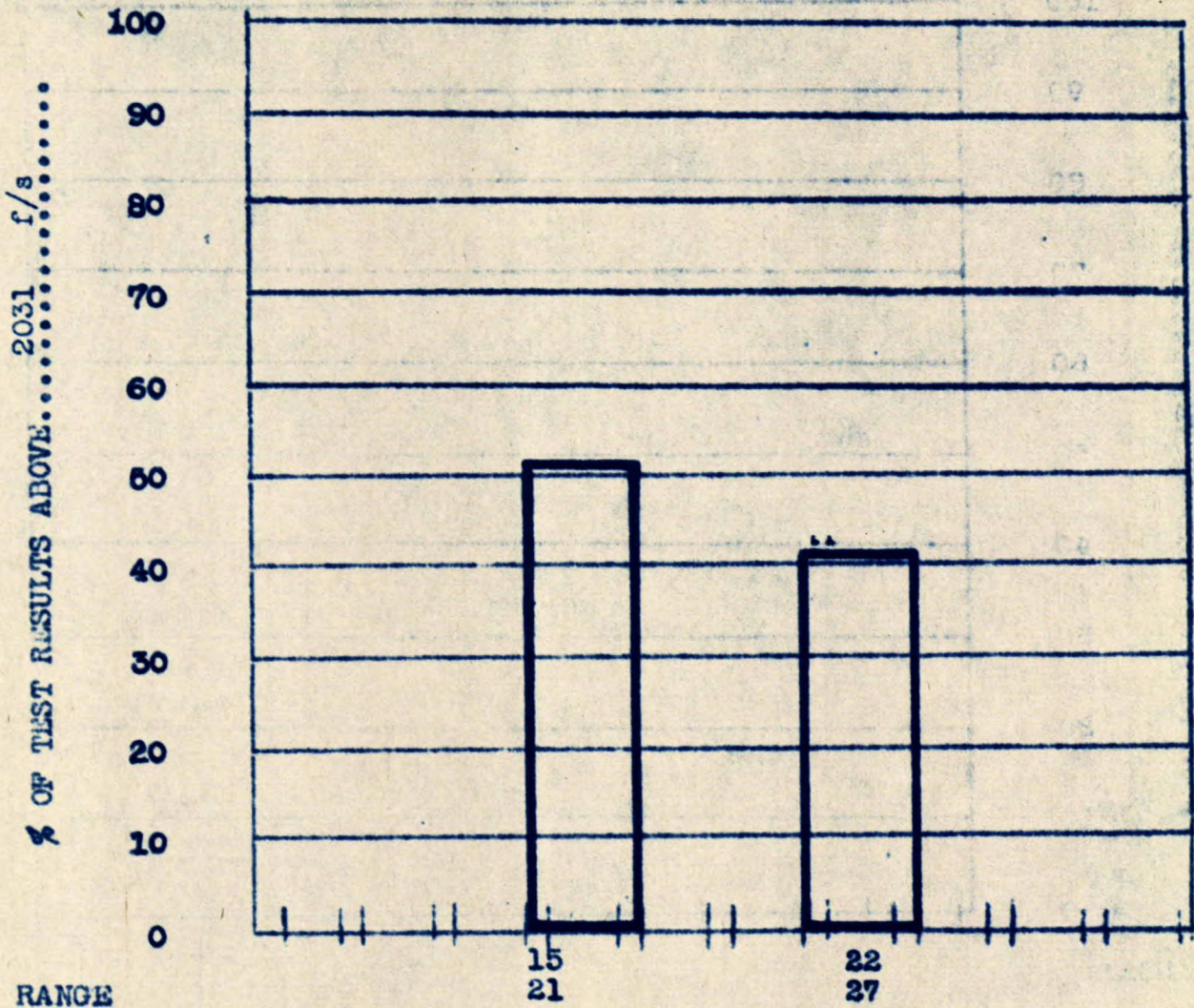


RANGE	22 25	26 35
No. above 2254. f/s:	8	8
Total number of test results:	15	13
Percentage above:	53	62
Ratio difference:	.09	
Number effect:	$1/15 + 1/13 = .067 + .077 = .144$	
Significance:	20 %	

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN ..Silicon.....&..Ballistic...Limit

IN. 2."...Armour..... FROM... Republic.....



No. above: 2550 f/s: 14 15

Total number of test results: 27 36

Percentage above:5242

Ratio difference:10

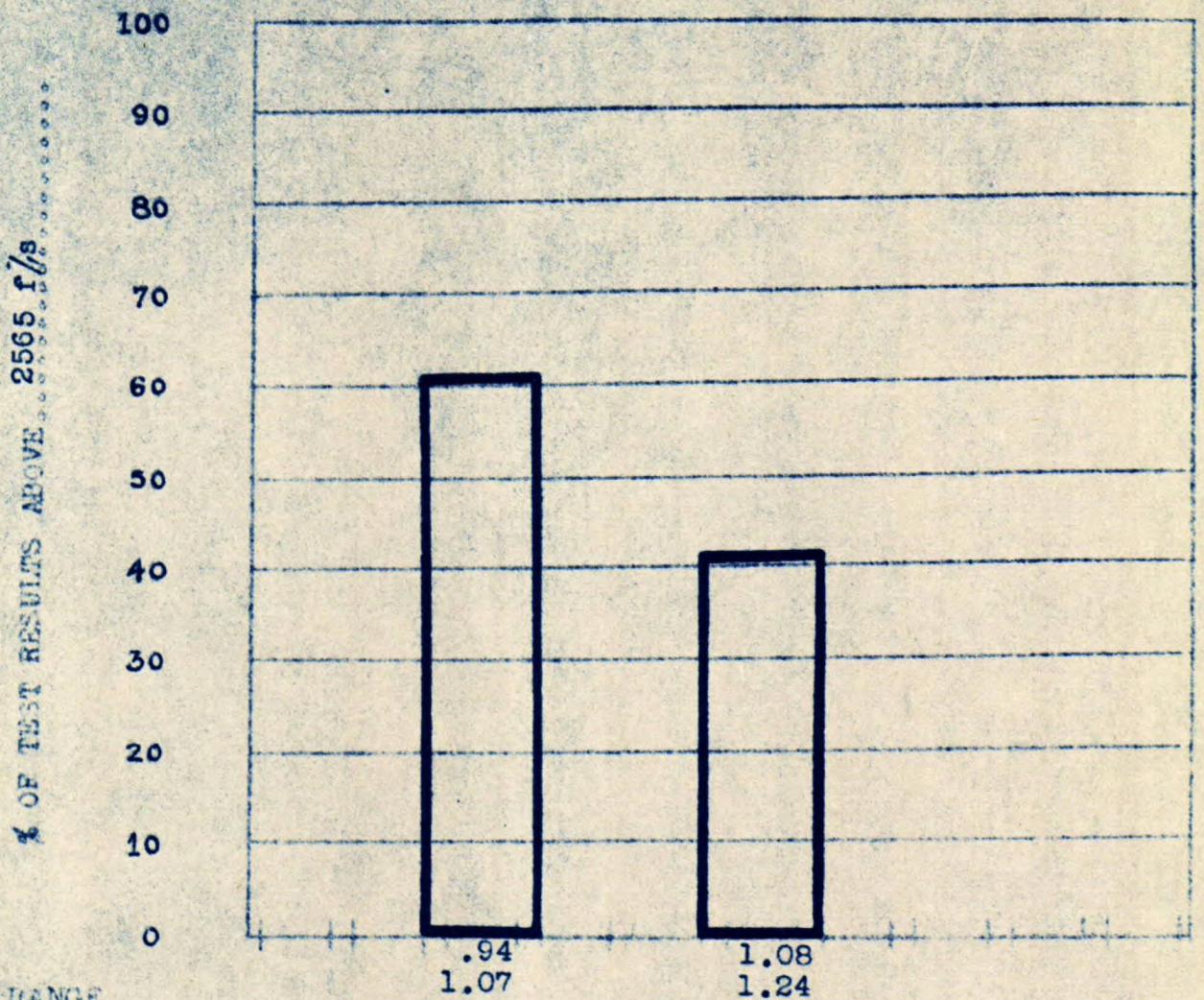
Number effect:037 plus .028 = .065

Significance:° 60..... %.

(NOTE: This means that (100-60%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN MANGANESE & BALLISTIC LIMIT

IN 3" ARMOUR FROM AMERICAN



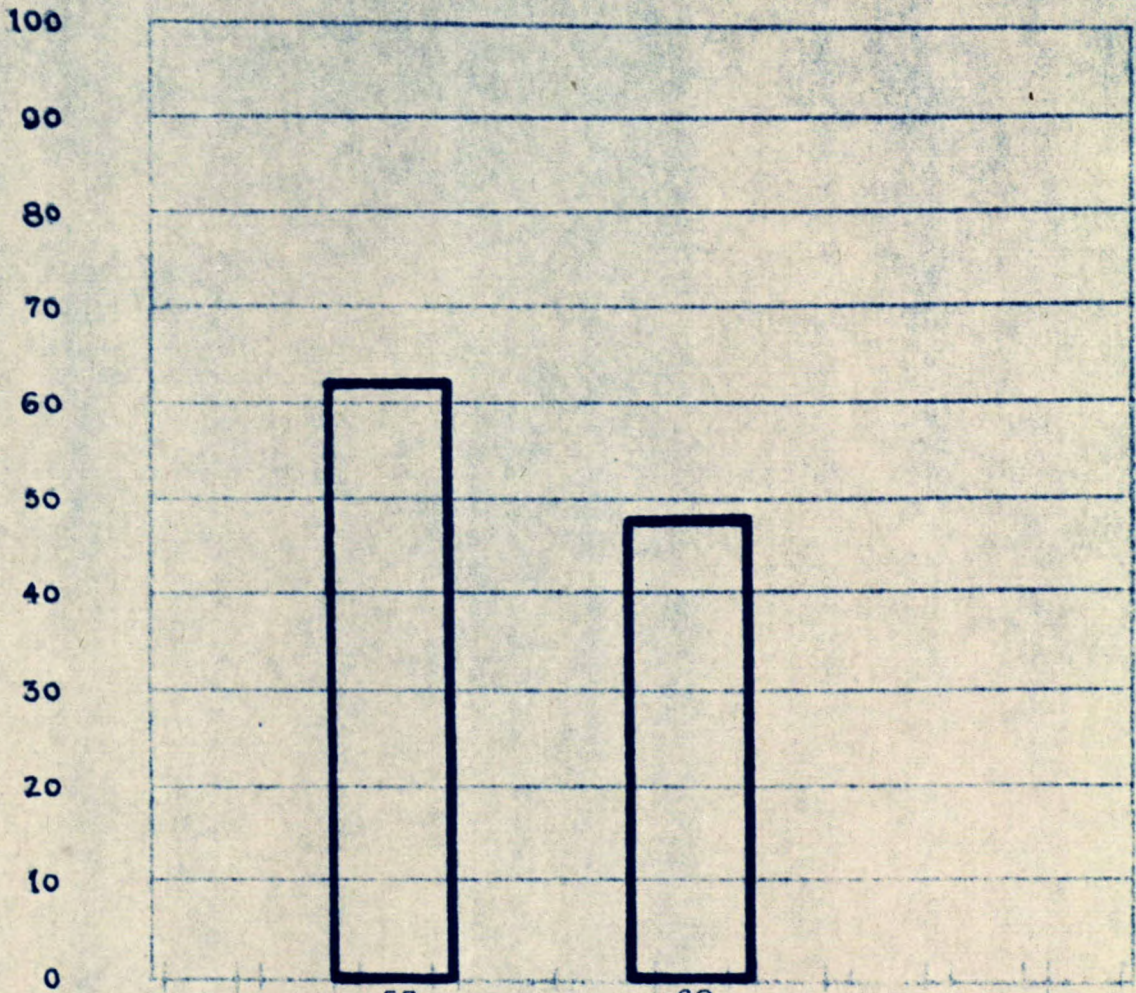
RANGE
 No. above 2565 f/s 26 21
 Total number of test results: 43 50
 Percentage above: 61 42
 Ratio difference: 19
 Number effect: $\frac{1}{61} + \frac{1}{42} = .0164 + .0238 = .0402$
 Significance: 94 %

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Manganese & B₁ L₁

IN. 2 3/4" Armour FROM Union Steel

% OF TEST RESULTS ABOVE 2254 f/s



RANGE

53
68

69
88

No. above

2254 f/s

8

7

Total number of test results

13

15

Percentage above

62

47

Ratio difference:

15

Number effect:

$1/13 + 1/15 = .077 + .067 = .144$

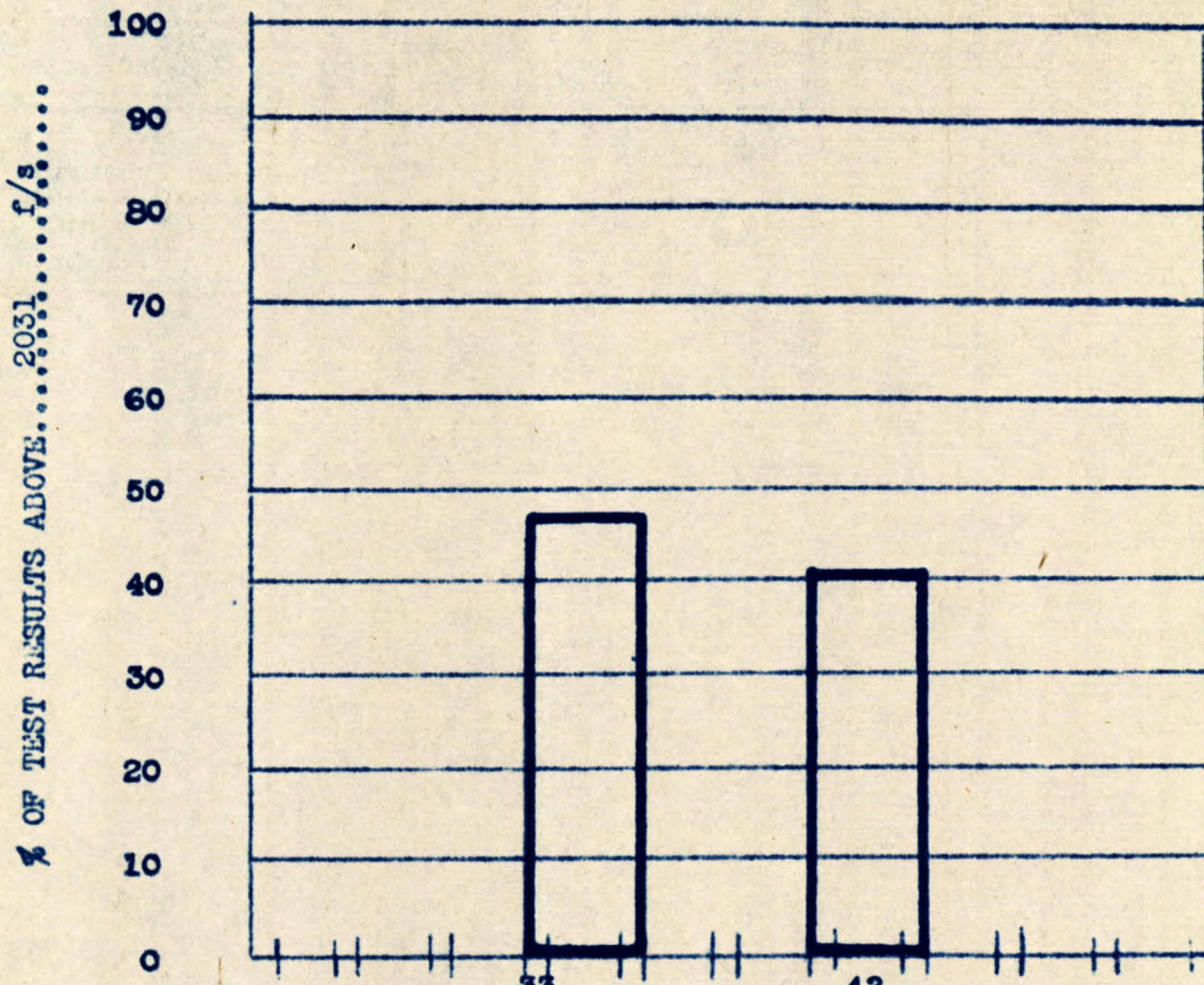
Significance:

...50..... %

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN **Manganese**.....&...**Ballistic Limit**.

IN **2"** Armour..... FROM **Republic**.....



RANGE

No. above:
2550 f/s:

33
41

42
46

14

14

Total number of
test results:

30

34

Percentage
above:

47

41

Ratio difference:

.06

Number effect:

.334 plus .0294 = .0628

Significance:•

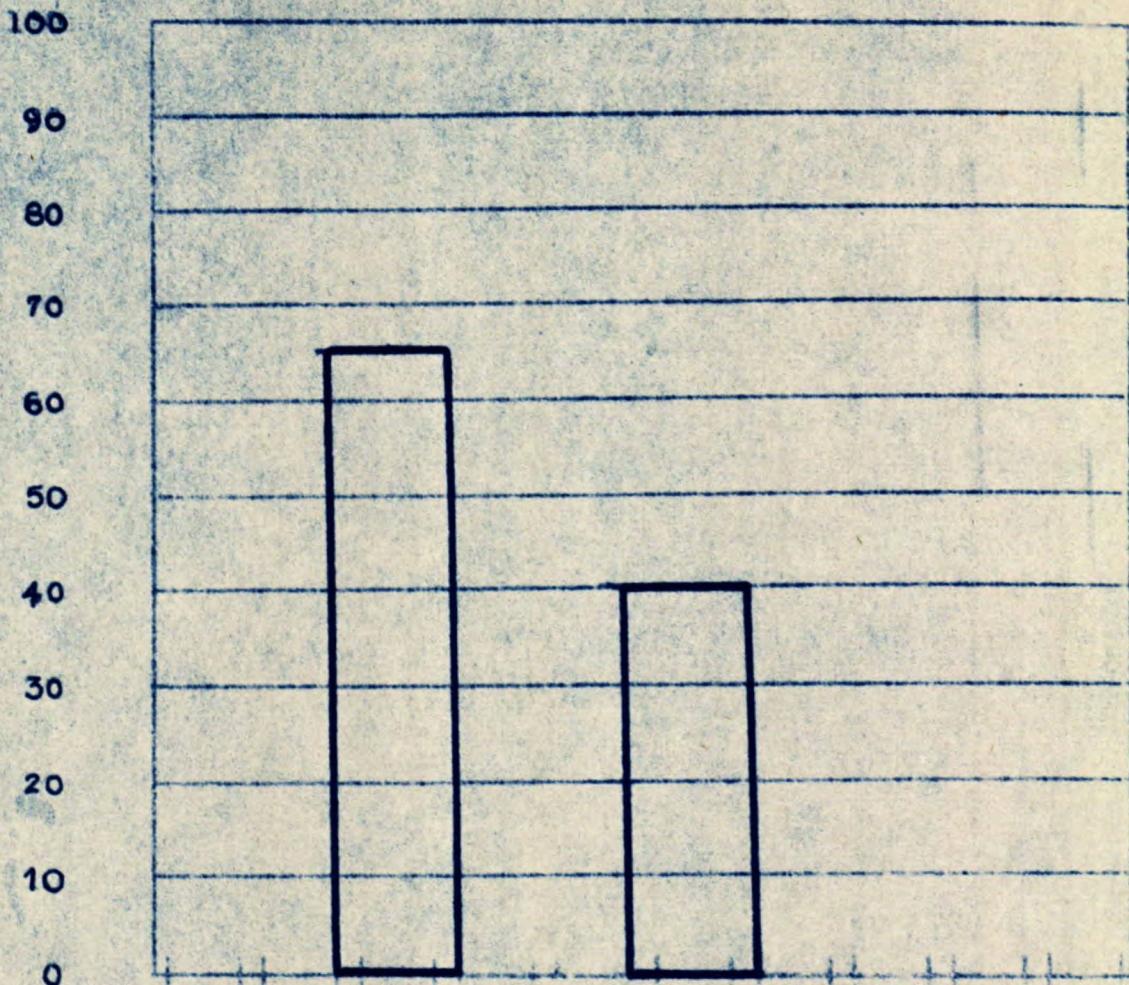
40 %.

(NOTE: This means that (100-40 %) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN .. SULPHUR .. & .. BALLISTIC LIMIT.

3rd ARMOUR IN. FROM. AMERICAN

% OF TEST RESULTS ABOVE 2565 f/s

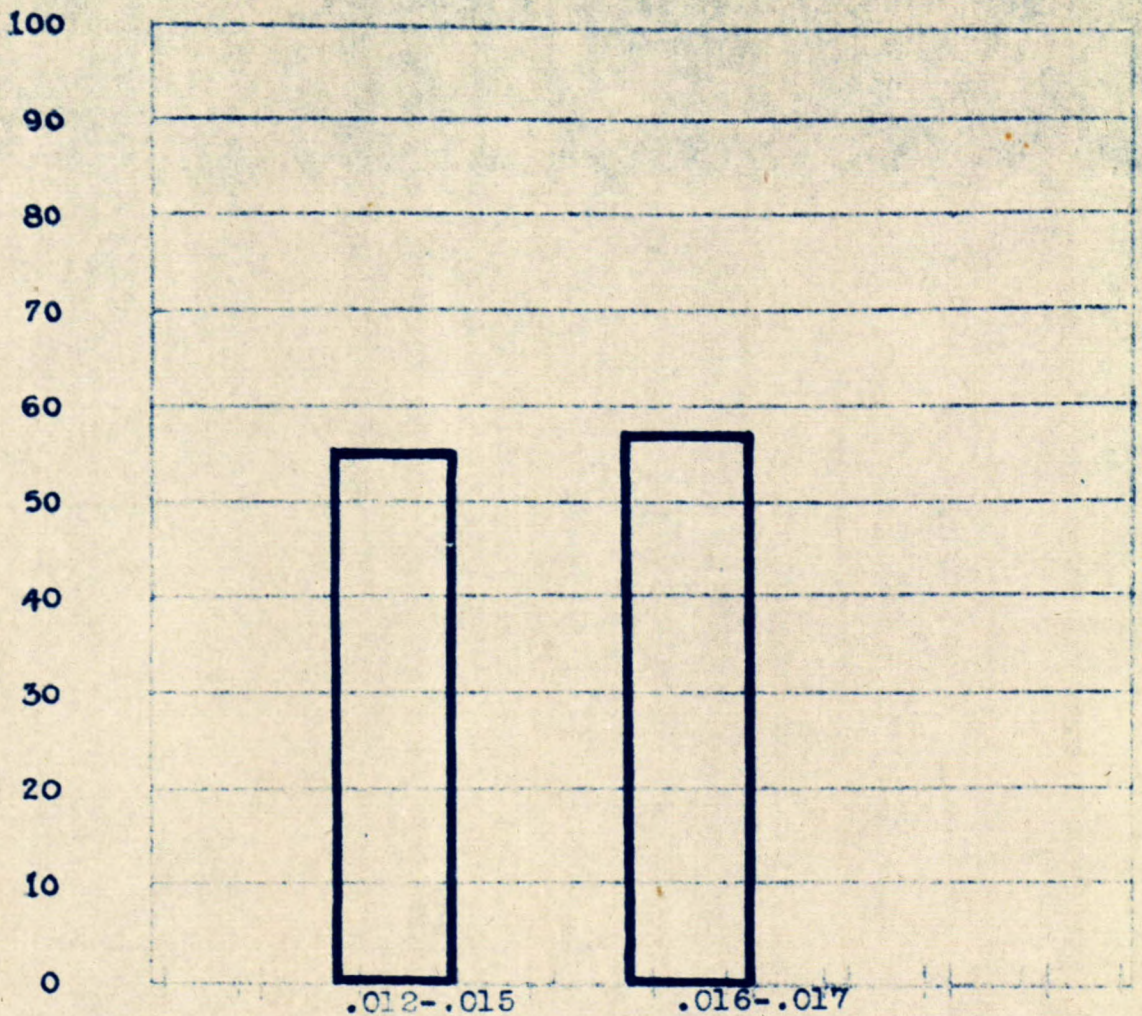


RANGE	.014-18	.019-30
No. above 2565. f/s:	30	19
Total number of test results:	46	47
Percentage above:	65.	40.
Ratio difference:	.25	
Number effect:	$\frac{1}{46} + \frac{1}{47} = .0217 + .0213 = .0430$	
Significance:	98 %.	

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

IN. 2 3/4" Armour FROM. Union Steel

% OF TEST RESULTS ABOVE 2254 F/S



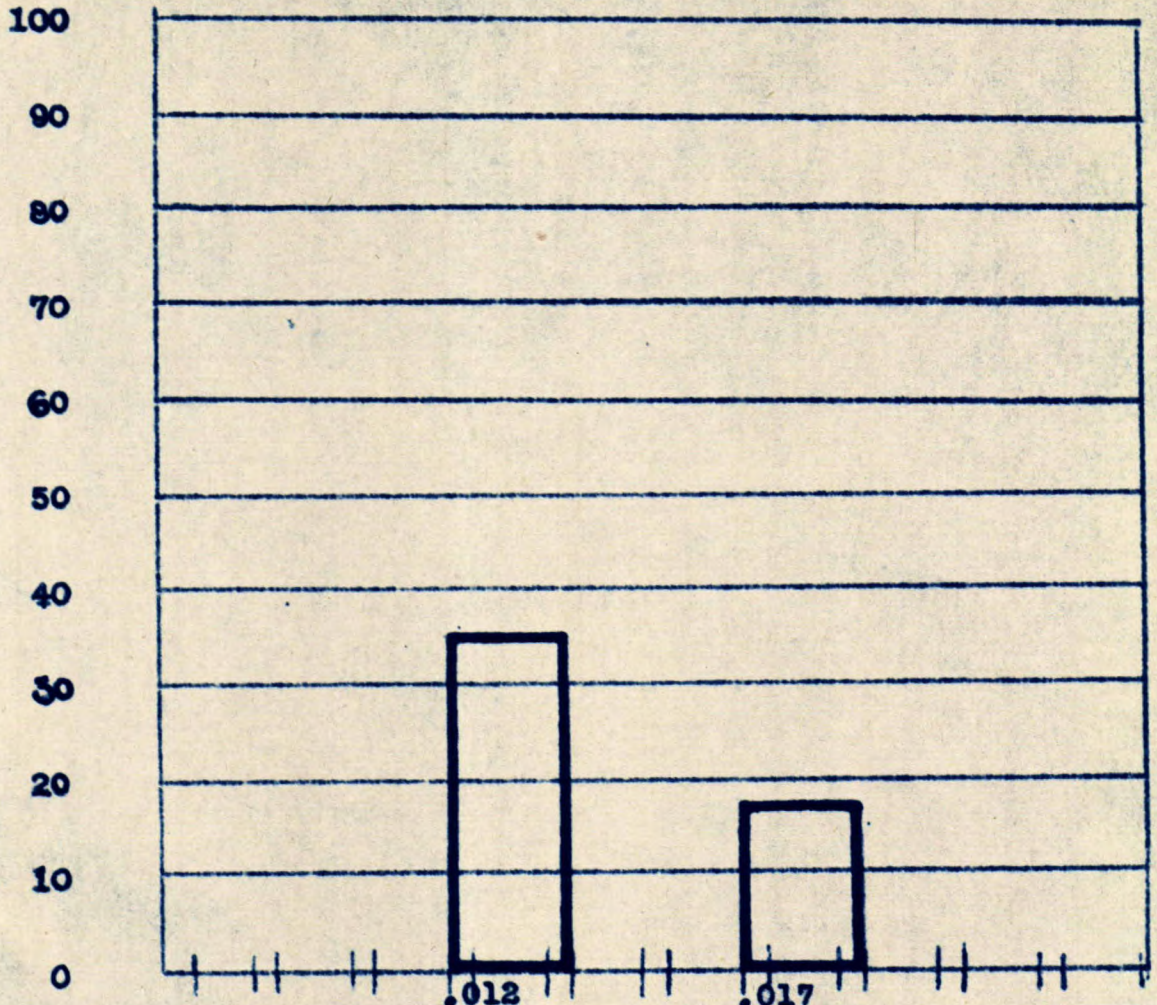
RANGE		
No. above 2254 f/s:	9	7
Total number of test results:	16	12
Percentage above:	56	58
Ratio difference:	.02	
Number effect:	$\frac{1}{16} + \frac{1}{12} = .0625 + .0835 = .1460$	
Significance:	0 %	

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN ..Sulphur.....&..Ballistic..Limit.

IN...2" Armour..... FROM..Republic.....

% OF TEST RESULTS ABOVE.....2031.....f/s.....



RANGE

No. above:
2550 f/s:

Total number of
test results: 26 38

Percentage
above:3518

Ratio difference:17

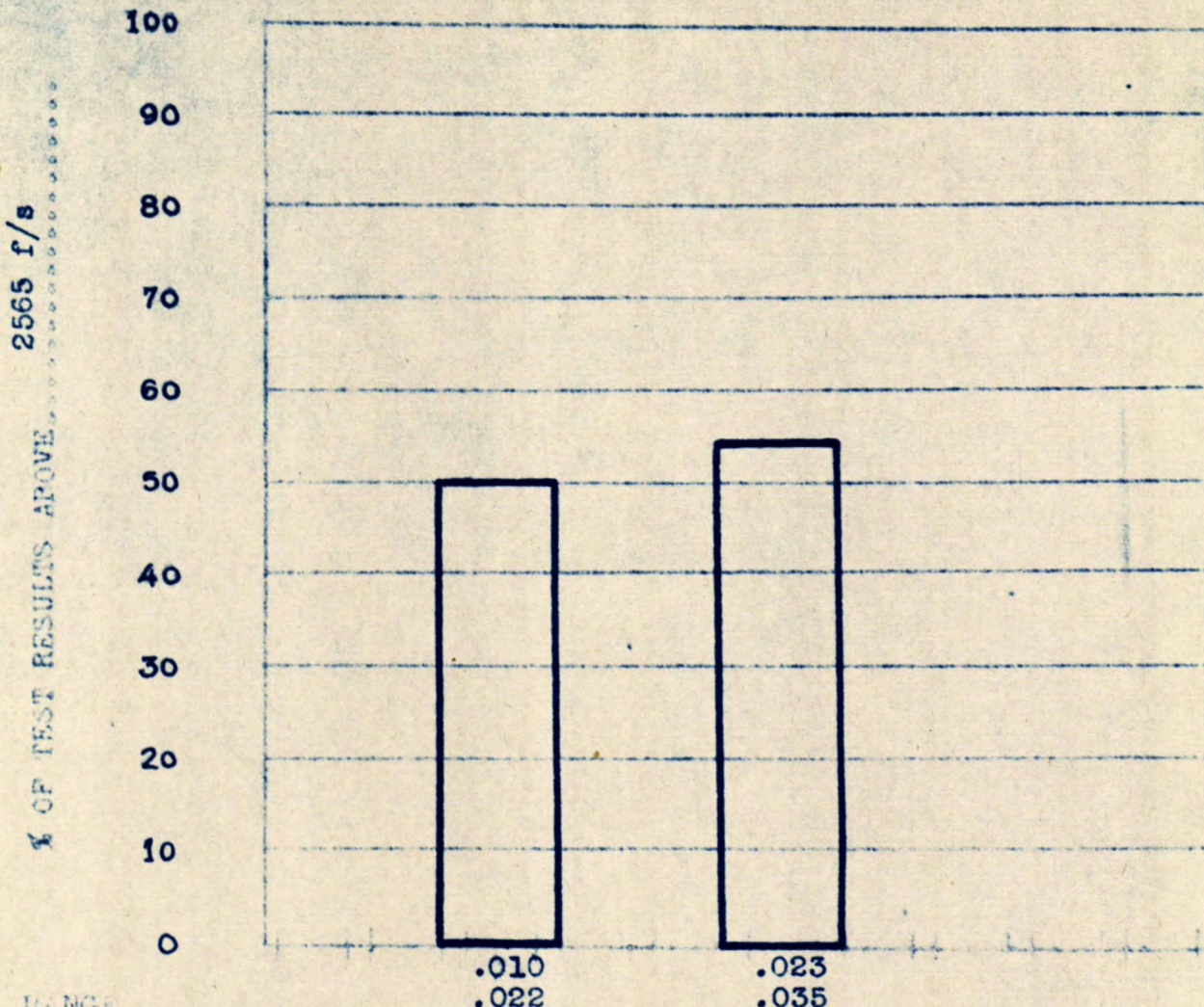
Number effect:0385 plus0263 =0648

Significance: 85 %.

(NOTE: This means that (100-85 %) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN PHOSPHORUS BALLISTIC LIMIT

IN. 3" ARMOUR FROM AMERICAN



RANGE

No. above 2565 f/s

Total number of test results:

Percentage above:

Ratio difference: .04

Number effect: $\frac{1}{50} + \frac{1}{54} = .020 + .0185 = .0385$

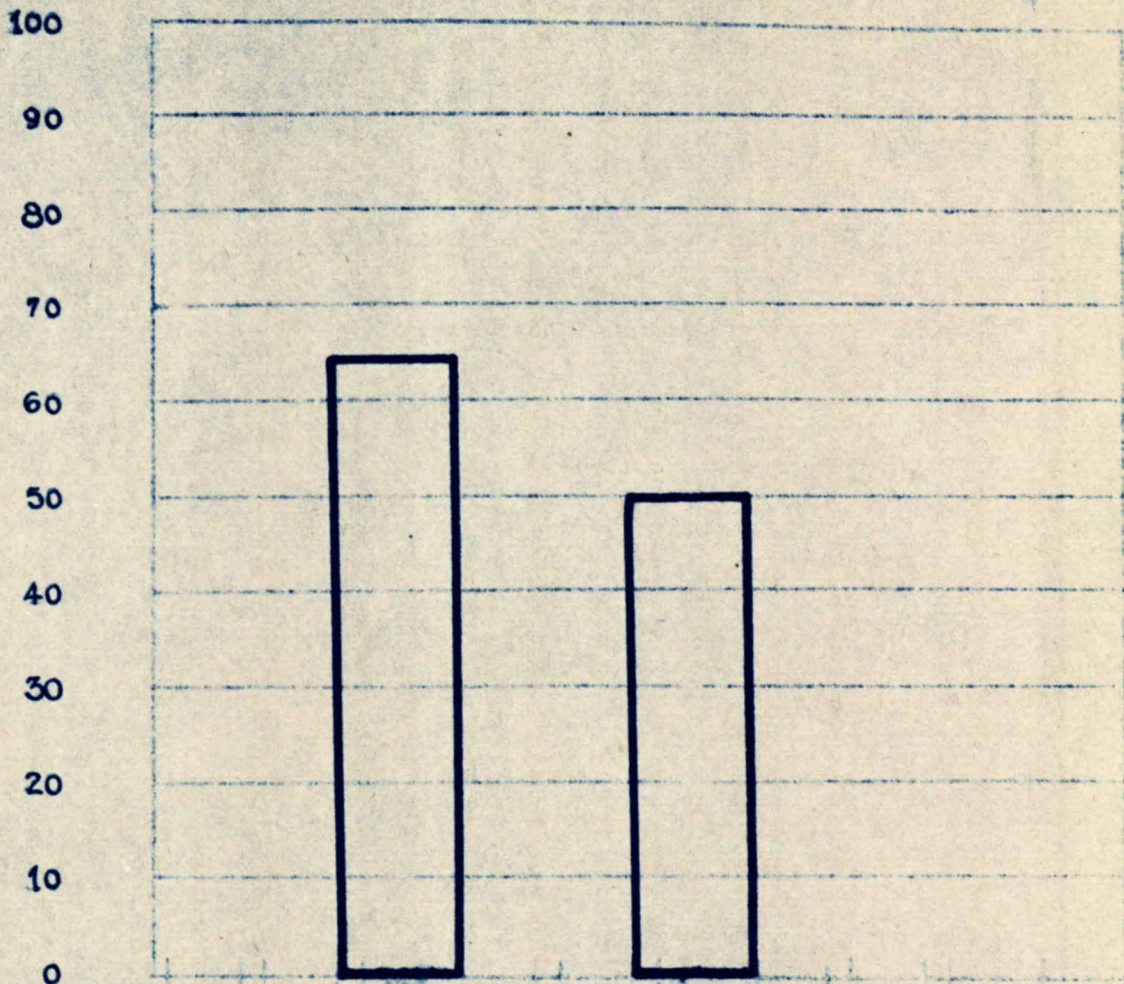
Significance: 30%

(NOTE: This means that (100-30%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Phosphorus B. L.

IN 2 3/4" Armour ... FROM ... Union Steel.

% OF TEST RESULTS ABOVE 2254 F/S

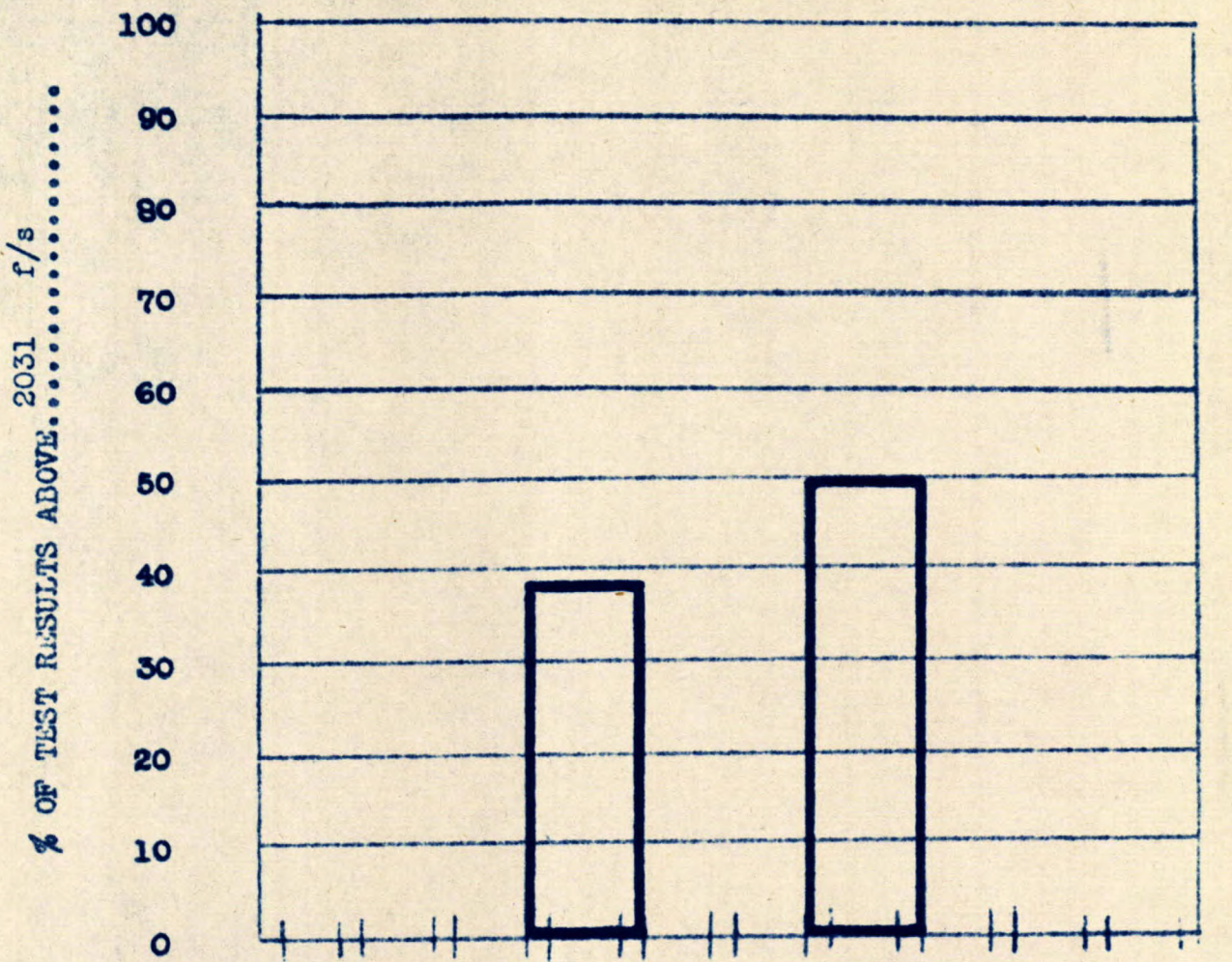


RANGE	.021 .026	.027 .029
No. above 2254 f/s:	9	7
Total number of test results:	14	14
Percentage above:	.64	50
Ratio difference:	.14	
Number effect:	.1/14 + 1/14 = .143	
Significance:*	40 %	

(NOTE: This means that (100-.....%) equals the
 (percentage of the time the above ratio)
 (difference would occur due to chance.)

CORRELATION BETWEEN Phosphorus & Ballistic Limit

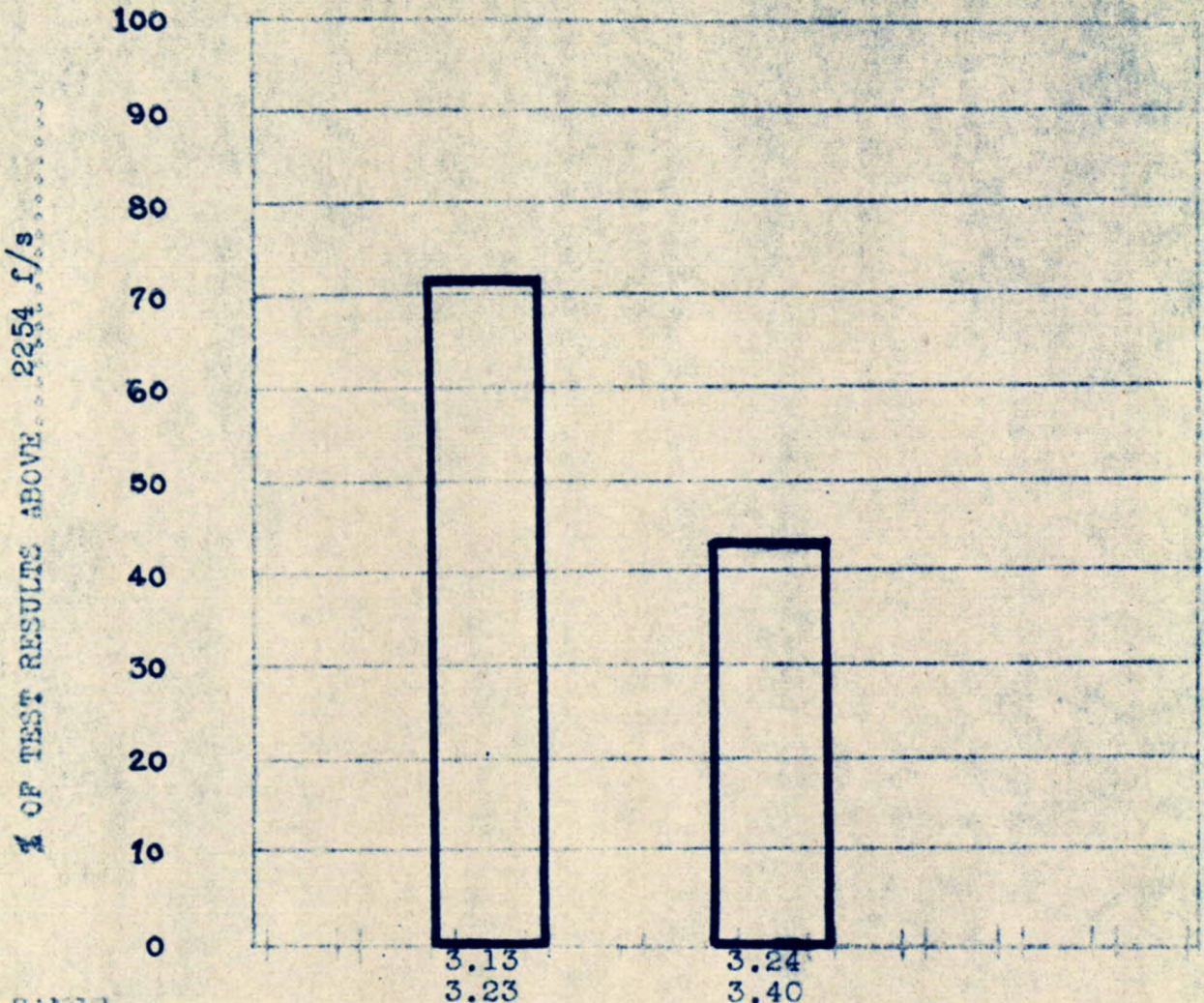
IN . . 2 . ? . Armour FROM . Republic



RANGE	.014 .016	.017 .025
No. above: 2550 f/s:	14	14
Total number of test results:	36	28
Percentage above:	39	50
Ratio difference:	.11	
Number effect:	.028	plus .036 = .064
Significance:*	60	%

(NOTE: This means that (100 ~~50~~ %) equals the percentage of the time the above ratio difference would occur due to chance.)

IN. 2 3/4" Armour FROM Union Steel

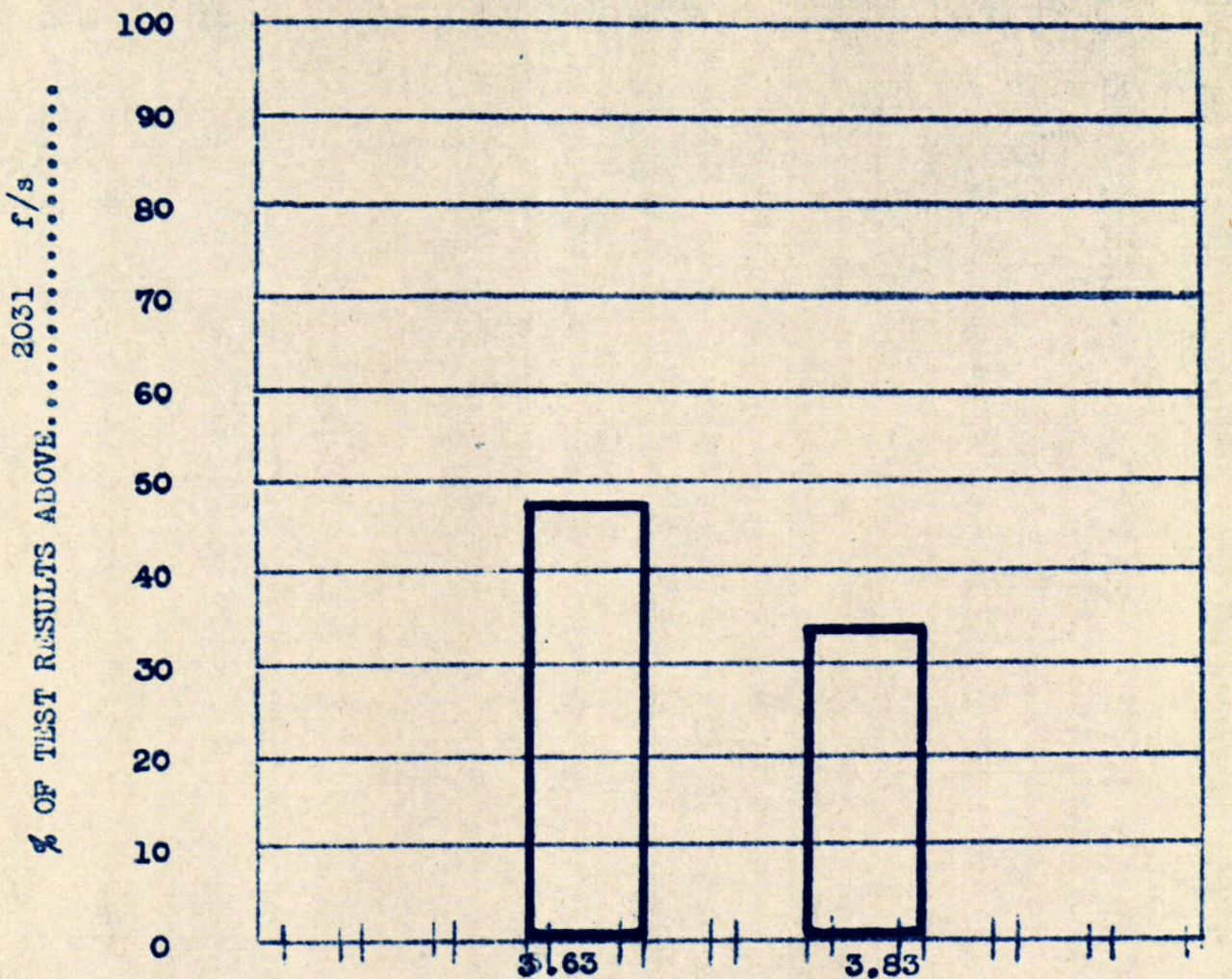


RANGE	3.13 3.23	3.24 3.40
No. above 2254 f/s:	10	6
Total number of test results:	14	14
Percentage above:	72	43
Ratio difference:	29	
Number effect:	143	
Significance:	85	%.

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Nickel.....&..Ballistic..Limit.

IN...2"....Armour.. FROM..Republic.....



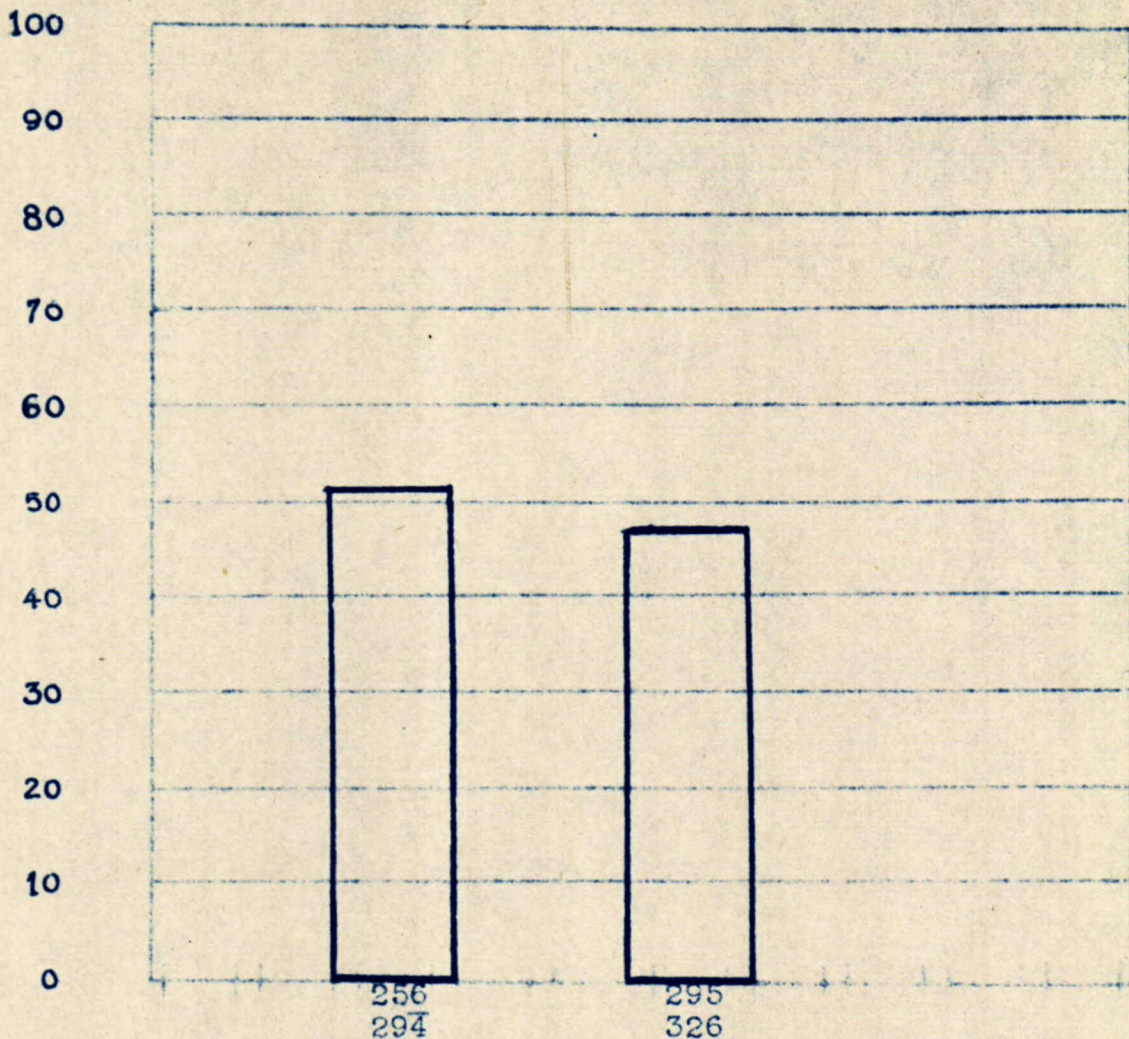
RANGE	3.82	4.00
No. above; 2550 f/s:	14	15
Total number of test results:	29	43
Percentage above:	.48	.35
Ratio difference:	.13	
Number effect:	.0345	plus .0233 z .0578
Significance:*	70	%.

(NOTE: This means that (100-70 %) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN CHROMIUM & BALLISTIC LIMIT

IN 3" ARMOUR FROM AMERICAN

% OF TEST RESULTS ABOVE 2565 f/s



RANGE

No. above 2565 f/s: 19 26

Total number of test results: 37 55

Percentage above: 51 47

Ratio difference:04

Number effect: $\frac{1}{37} + \frac{1}{55} = .027 + .0184 = .0454$

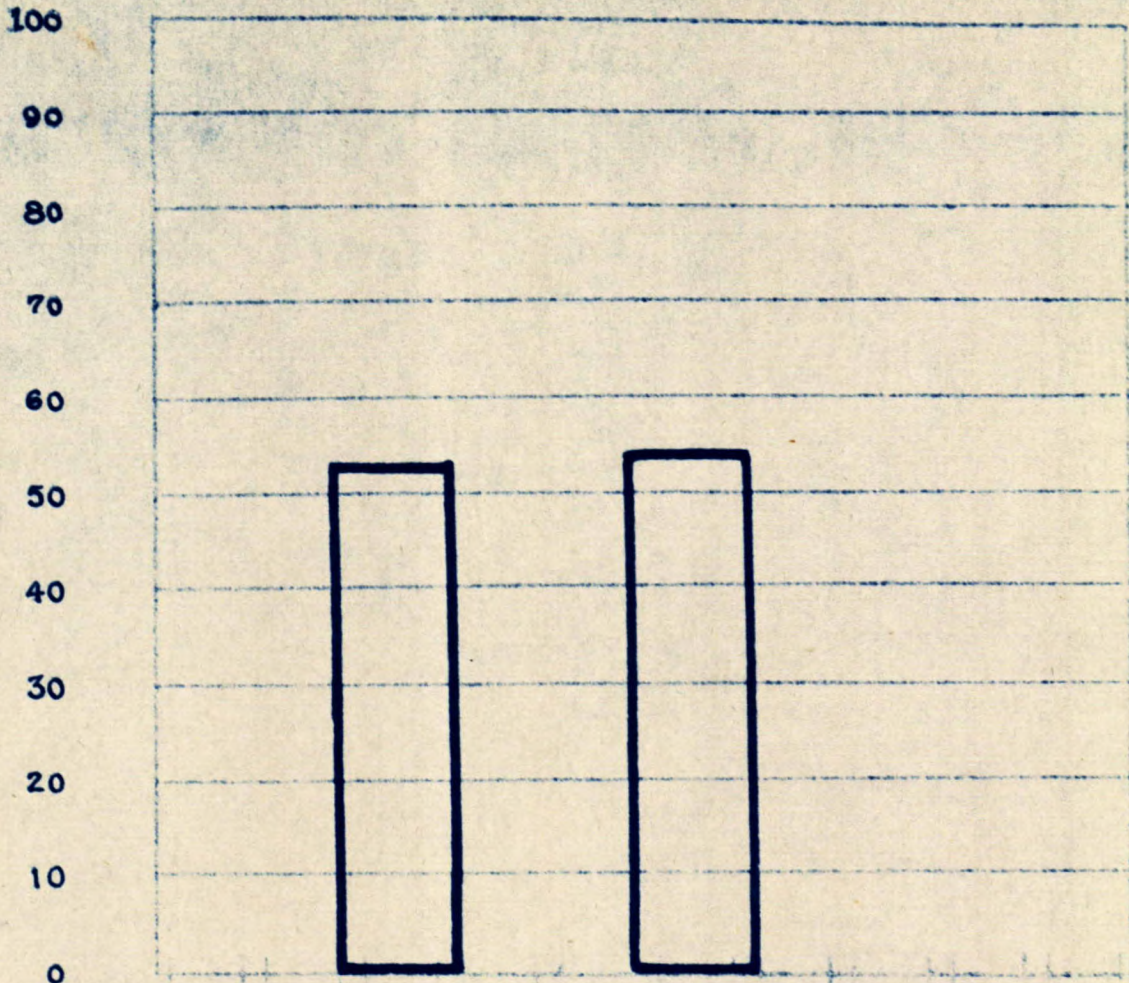
Significance: 30 %

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Chromium & Ballistic Limit

IN. 2 3/4" Armour FROM. Union Steel

2254 F/S
% OF TEST RESULTS ABOVE



RANGE

1.15
1.27

1.28
1.62

No. above
2254 F/S:

8

7

Total number of
test results:

15

13

Percentage
above:

53

54

Ratio difference:

.01

Number effect:

$$1/15 + 1/13 = .067 + .077 = .144$$

Significance:

0

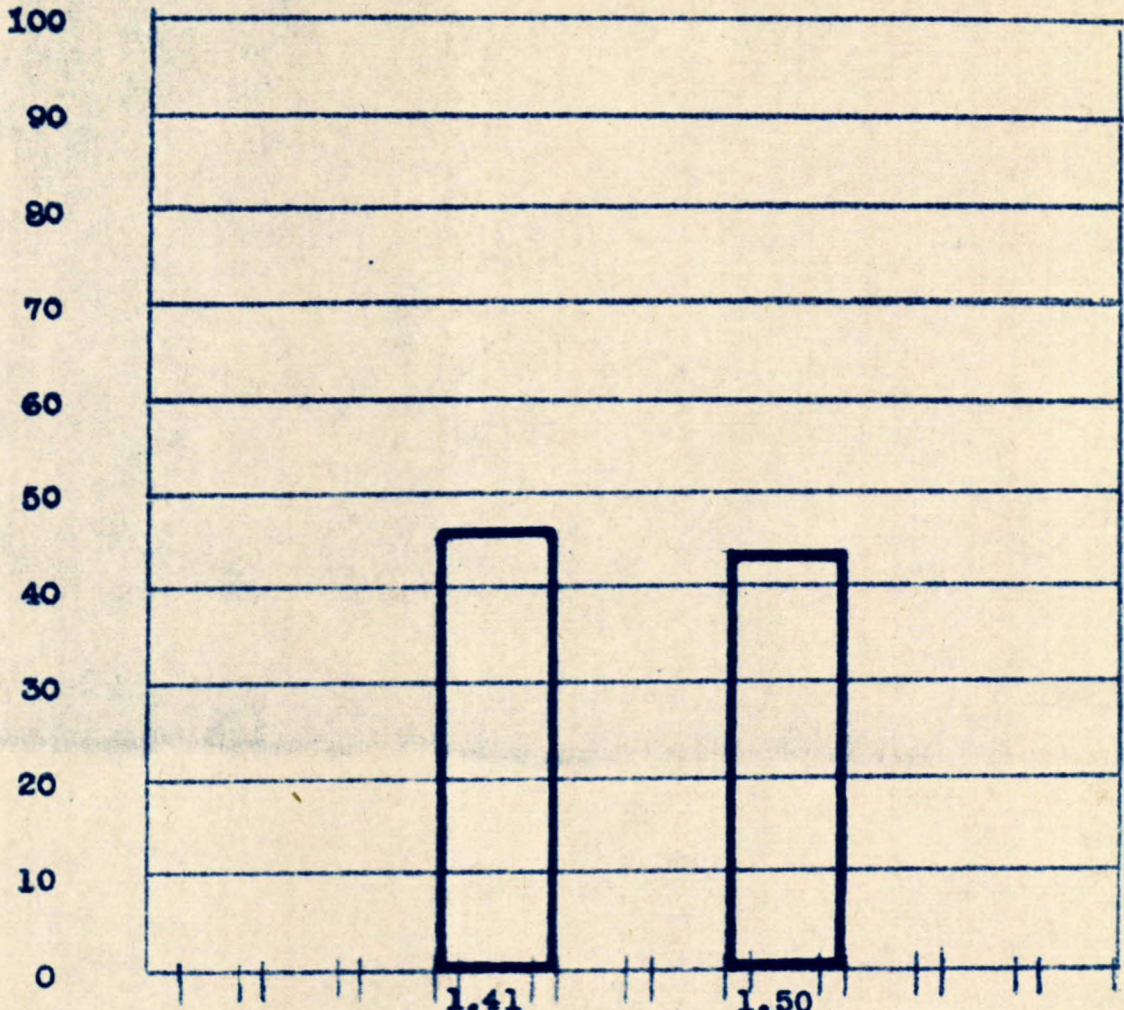
%.

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN .Chromium.....&...Ballistic..Limit

IN..27..Armour..... FROM..Republic.....

% OF TEST RESULTS ABOVE.....2031...f/s.....



RANGE

No. above:
2550 f/s:

1.41

1.50

1.49

1.64

17

12

Total number of
test results:

36

27

Percentage
above:

.47

.44

Ratio difference:

.03

Number effect:

.028 plus .037 ± .065

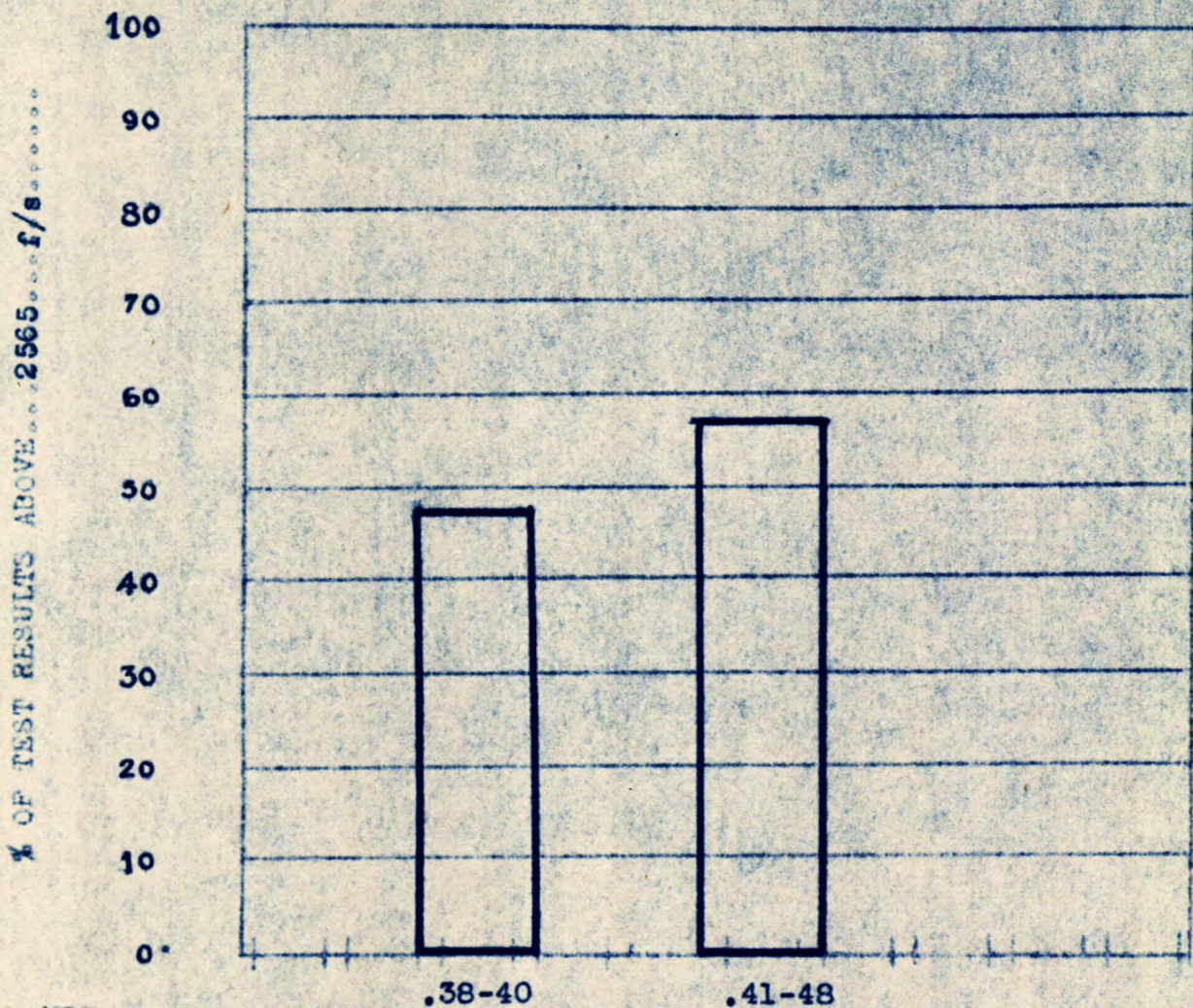
Significance:°

20 %

(NOTE: This means that (100-20 %) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN . MOLYBDENUM . . . & . BALLISTIC LIMIT . . .

IN . 3" . ARMOUR FROM . . AMERICAN



RANGE

No. above
2565 f/s:

.38-40

.41-48

31

16

Total number of
test results:

65

28

Percentage
above:

48

57

Ratio difference:

.09

Number effect:

$$\frac{1}{65} + \frac{1}{28} = .0154 + .0357 = .0511$$

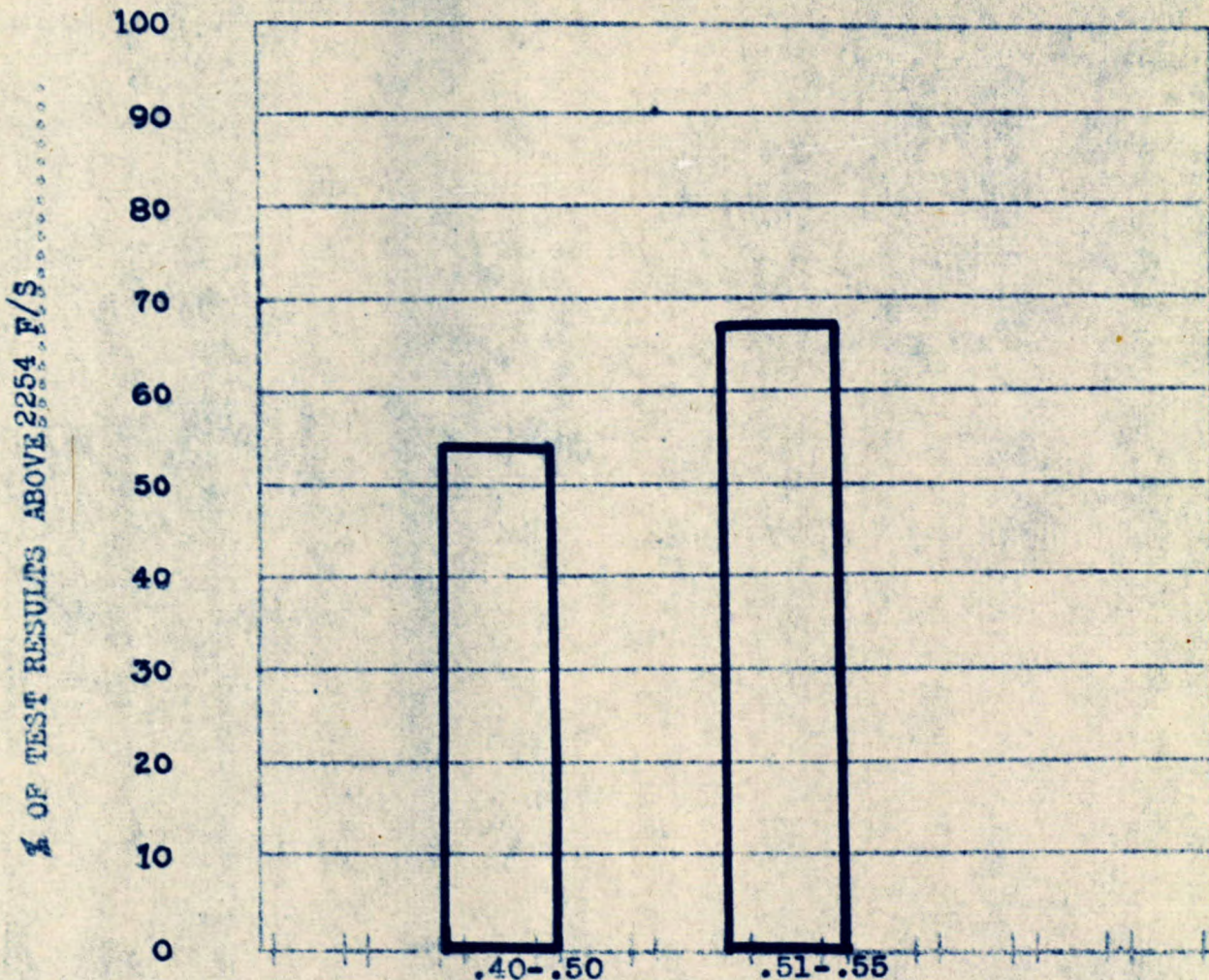
Significance:

60 %

(NOTE: This means that (100-60%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Molybdenum . . . & . . . B. I.

IN. 2 3/4" Armour . FROM. . Union Steel . .



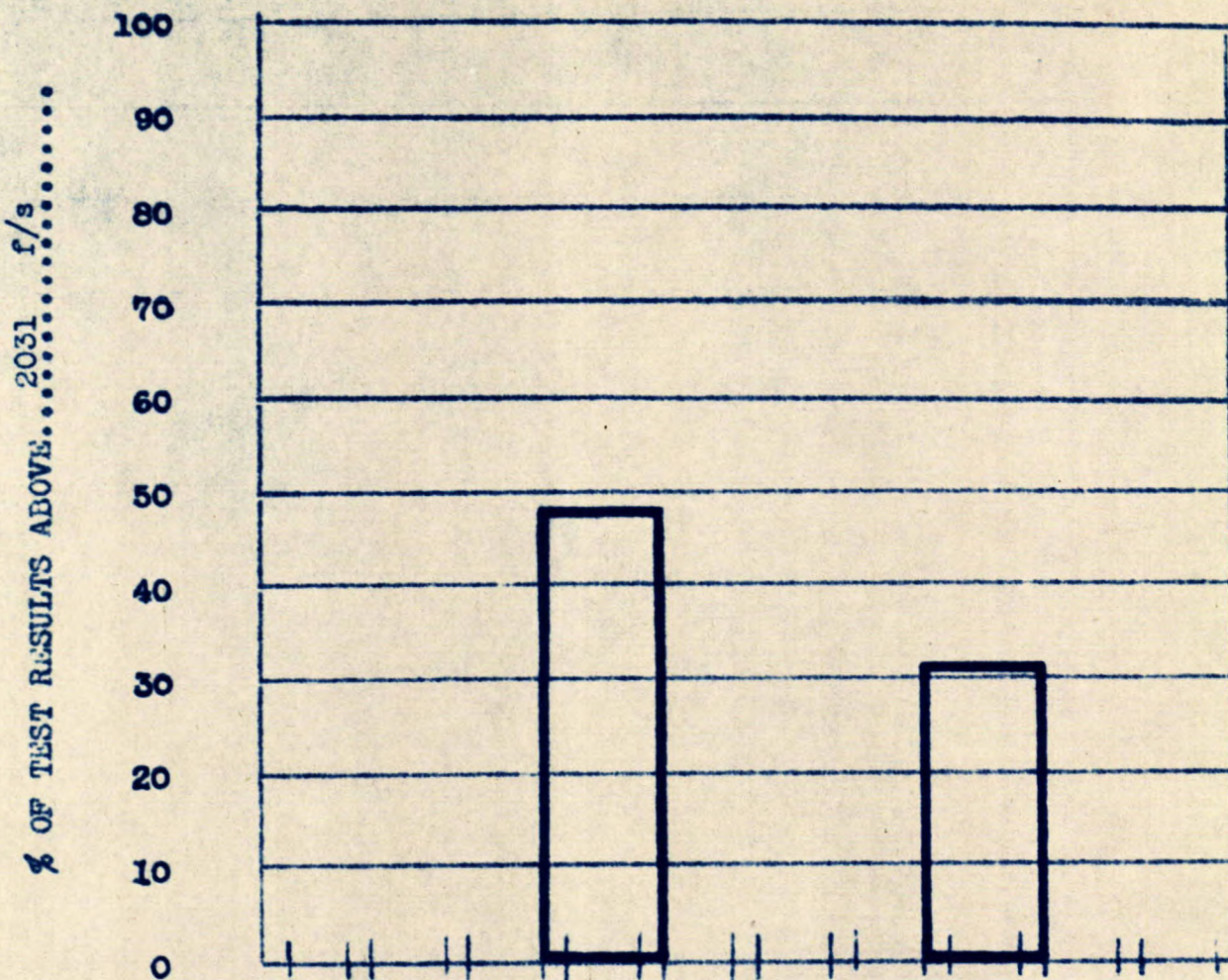
RANGE

No. above 2254 f/s:	7	10
Total number of test results:	13	15
Percentage above:	54	67
Ratio difference:	13	
Number effect:	$1/13 + 1/15 = .077 + 06.7 = .144$	
Significance:	40 %	

(NOTE: This means that (100-40%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN . Molybdenum ... & ... Ballistic Limit.

IN ... 2" Armour ... FROM ... Republic ...



RANGE

.45
.53

.54
.62

No. above:
2550 f/s:

14

11

Total number of
test results:

29

34

Percentage
above:

.48

.32

Ratio difference:

.16

Number effect:

.0345 plus .0294 = .0639

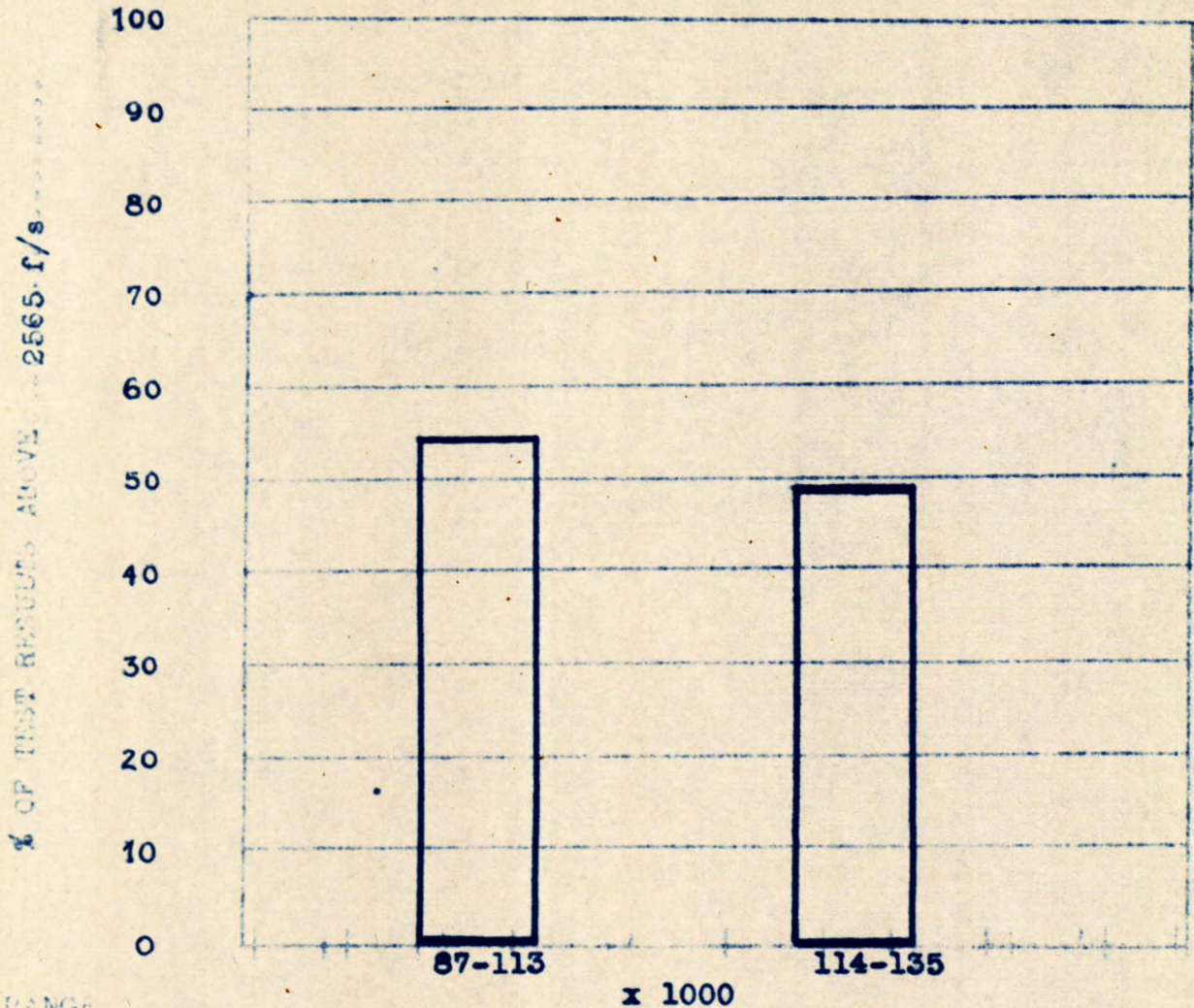
Significance:°

80 %

(NOTE: This means that (100-80 %) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN TENSILE & BALLISTIC LIMIT

IN 3" ARMOUR FROM AMERICAN

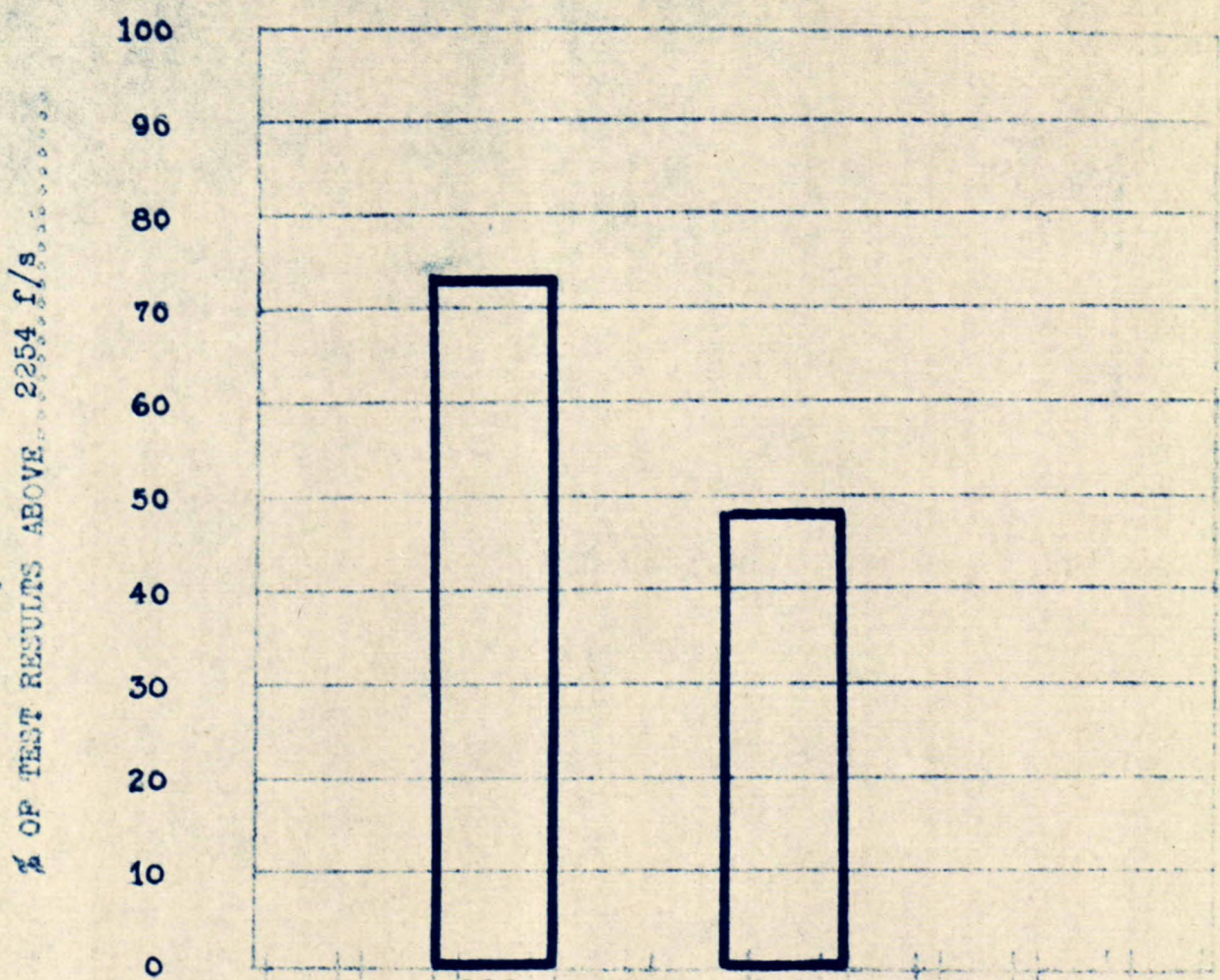


PAGE 2

No. above 2565 f/s	21	26
Total number of test results	39	53
Percentage above	54	49
Ratio difference:	.05	
Number effect:	$\frac{1}{39} + \frac{1}{53} = .0256 + .0189 = .0445$	
Significance:	40 %	

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

IN. 2³/₄" Armour. FROM Union Steel



RANGE: 116 128 x1000 129 142
 No. above 2254 f/s: 11 6

Total number of test results: 15 14

Percentage above: 73.5 43

Ratio difference:305

Number effect: $1/15 + 1/14 @ .067 + .0715 = .1385$

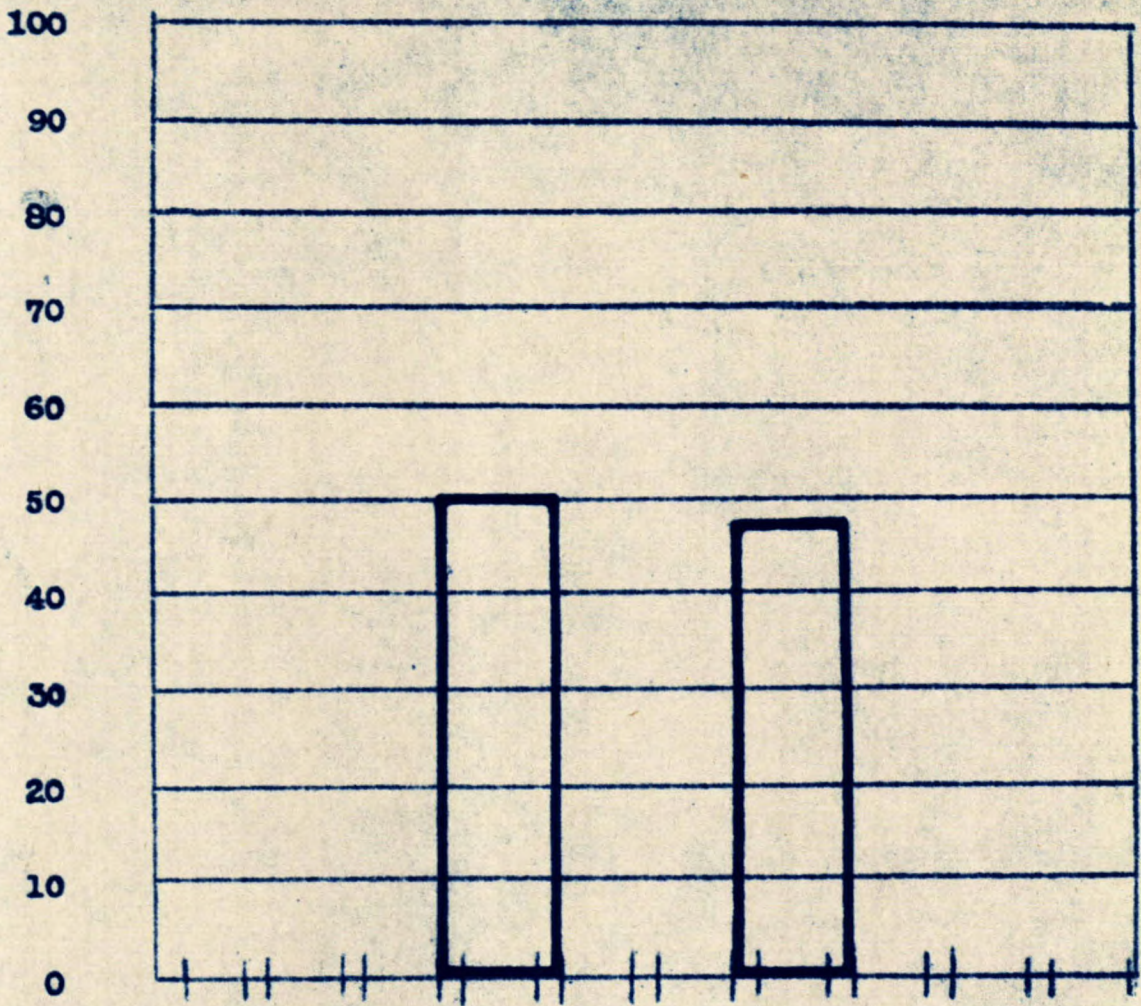
Significance: 85 %.

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Ultimate Strength & Ballistic Limit

IN... 2" Armour FROM Republic

% OF TEST RESULTS ABOVE... 2031 f/s.....



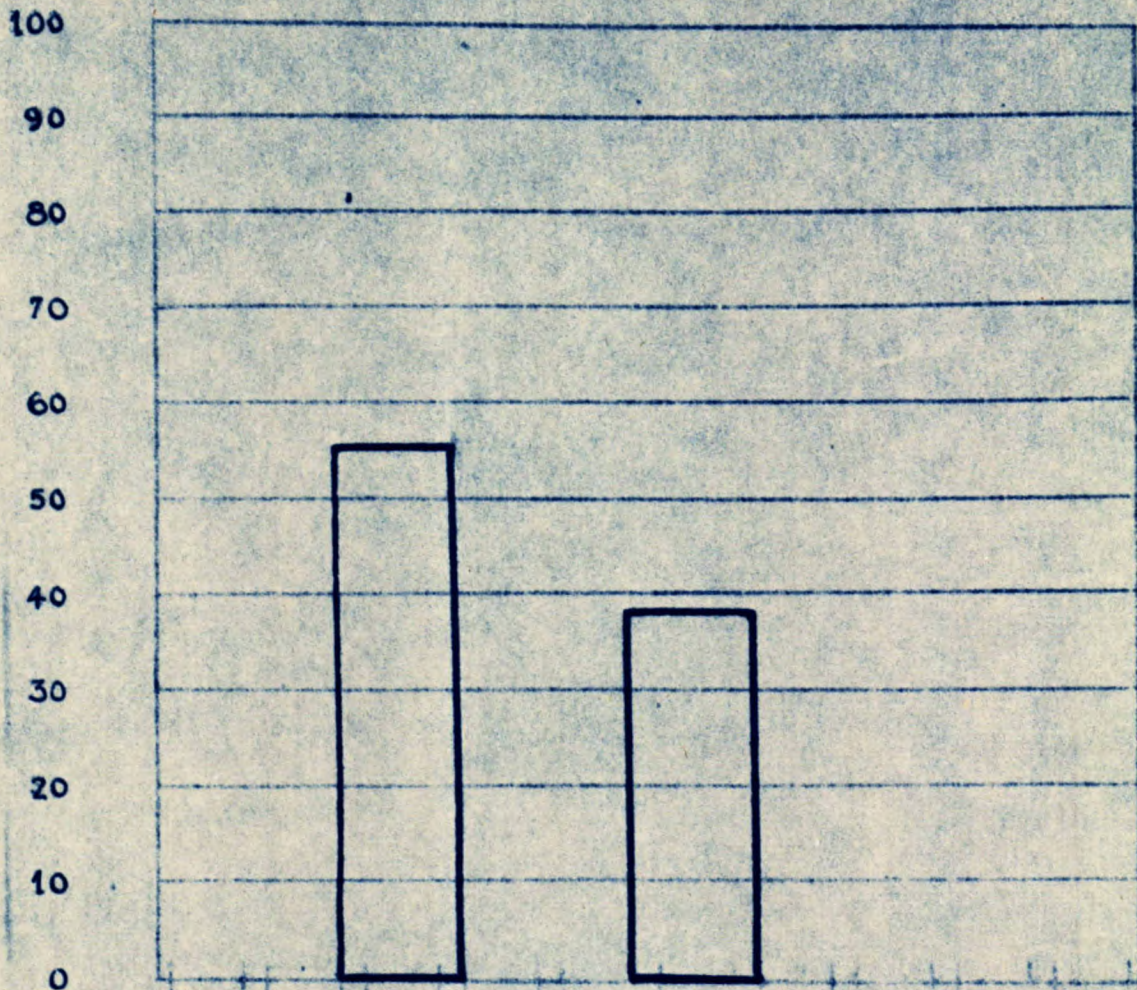
RANGE	134	144
No. above:	143	152
2550 f/s:	16	15
Total number of test results:	32	31
Percentage above:	.50	.48
Ratio difference:	.02	
Number effect:	.0312 plus .0323 = .0635	
Significance:	10 %	

(NOTE: This means that (100-10%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN YIELD..... & .. BALLISTIC LIMIT

IN.. 3" ARMOUR..... FROM..... AMERICAN.....

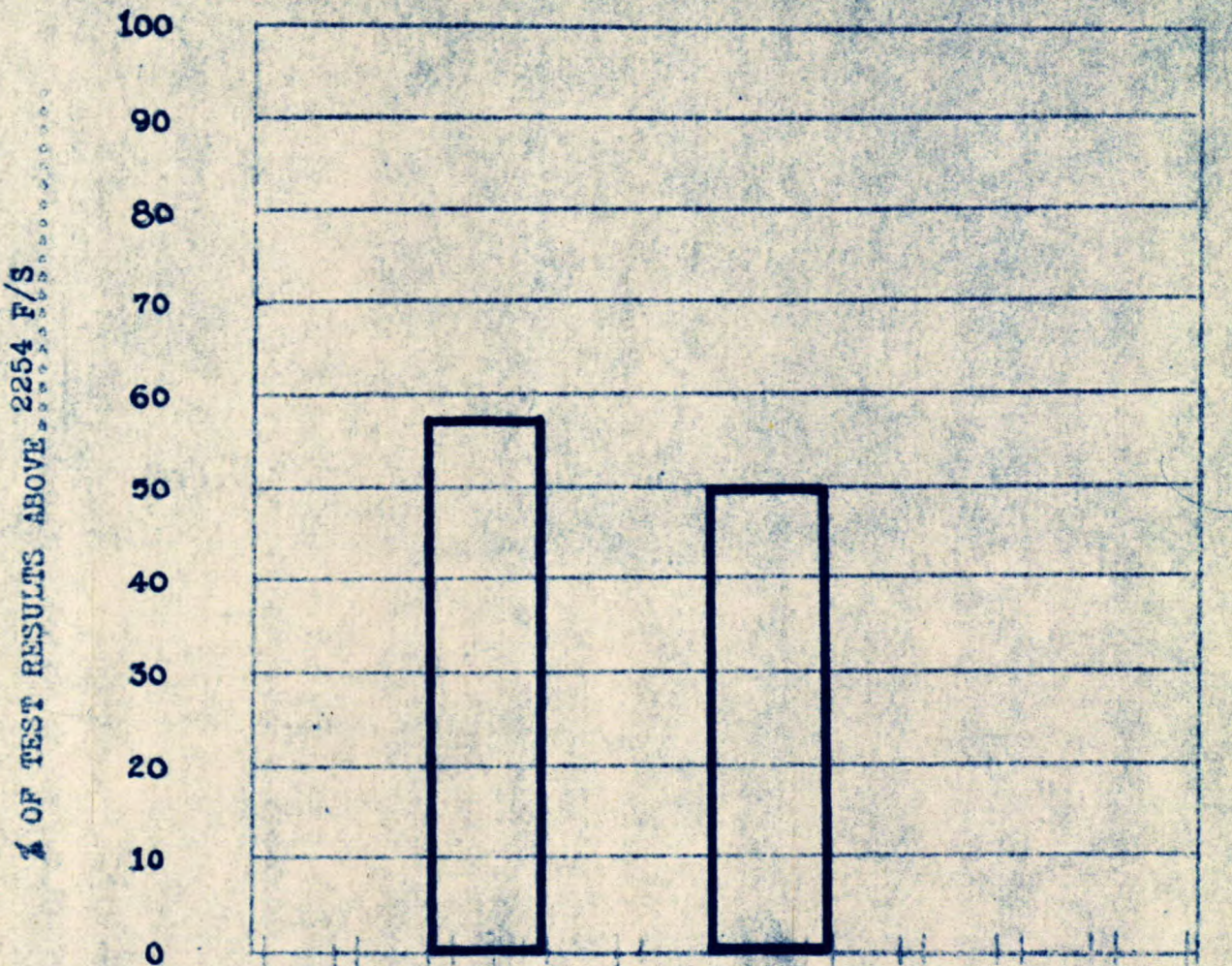
% OF TEST RESULTS ABOVE 2565 f/s



RANGE	59-86	87-103
No. above 2565 f/s:	16	15
Total number of test results:	29	39
Percentage above:	55	38.5
Ratio difference:	.165	
Number effect:	$\frac{1}{29} + \frac{1}{39} = .0345 + .0256 = .0601$	
Significance:80.... %.	

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

IN. 2 3/4" Armour FROM. Union Steel



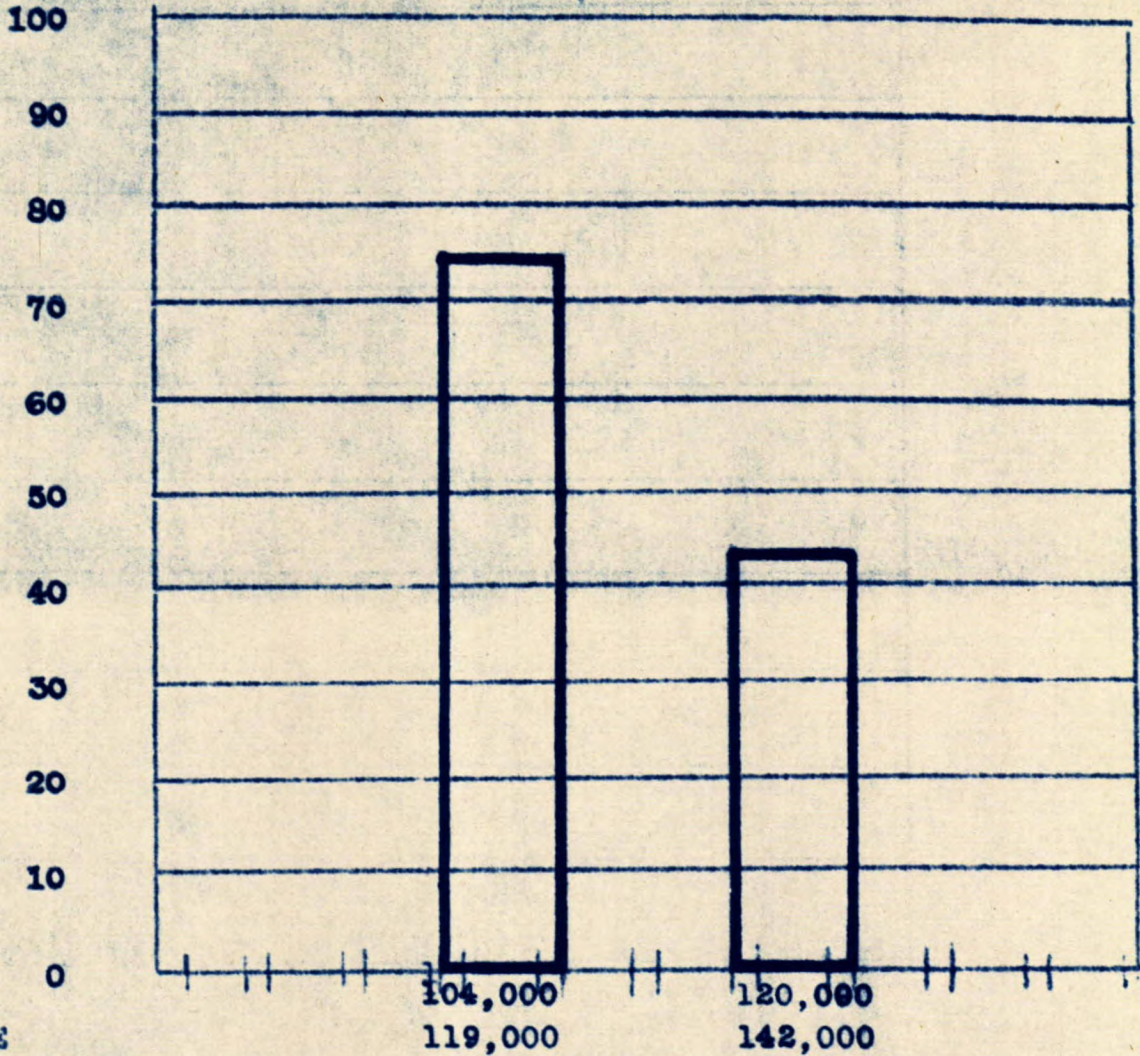
RANGE	88	100
	99 x 1,000	116
No. above 2254 f/s:	8	7
Total number of test results:	14	14
Percentage above:	57	50
Ratio difference:	.07	
Number effect:	...0715 + .07115 = .14 30	
Significance: 20 %	

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEENYield.....&...Ballistic.....

IN..2"....Armour..... FROM.Republ19.....

% OF TEST RESULTS ABOVE.....2031.....f/s.....



no. above:
2550 f/s:

12

16

Total number of
test results:

16

36

Percentage
above:

.75

.44

Ratio difference:

.31

Number effect:

.0278

.0623

.0901

Significance:*

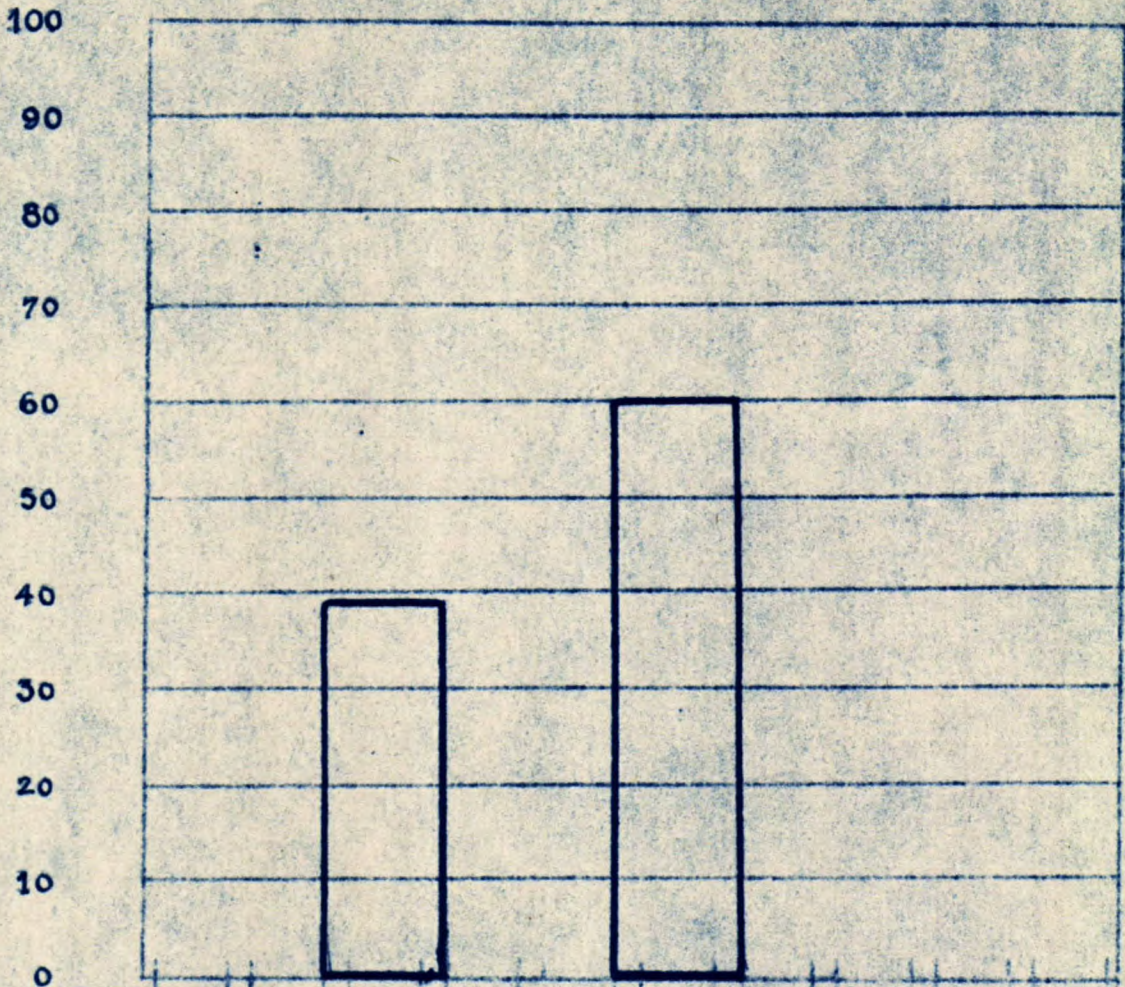
...95..... %.

(NOTE: This means that (100-95 %) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN ELONGATION... & BALLISTIC LIMIT...

IN ARMOUR..... FROM... AMERICAN.....

% OF TEST RESULTS ABOVE 2565 f/s.....



RANGE	10-19	20-24
No. above 2565 f/s:	19	27

Total number of test results:	49	45
-------------------------------	----	----

Percentage above:	39	60
-------------------	----	----

Ratio difference:21.....

Number effect: $\frac{1}{49} + \frac{1}{45} = .0240 + .0222 = .0462$

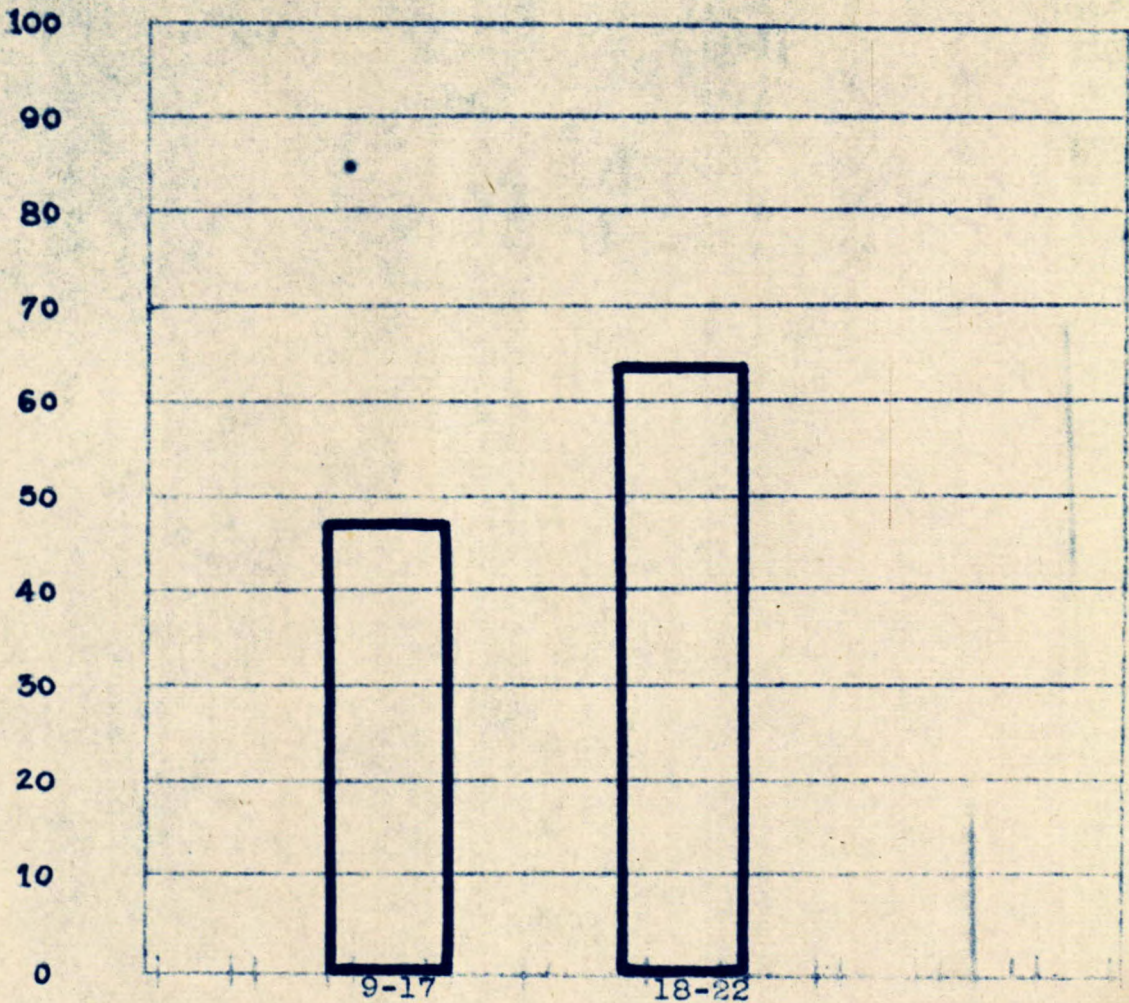
Significance: •95..... %.

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Elongation & R. L.

IN. 2 1/2" Armour FROM Union

% OF TEST RESULTS ABOVE 2254 f/s



RANGE

No. above 2254 f/s: 7 9

Total number of test results: 15 14

Percentage above: 47 64

Ratio difference: 17

Number effect: $1/15 + 1/14 = .067 + .0715 = .1385$

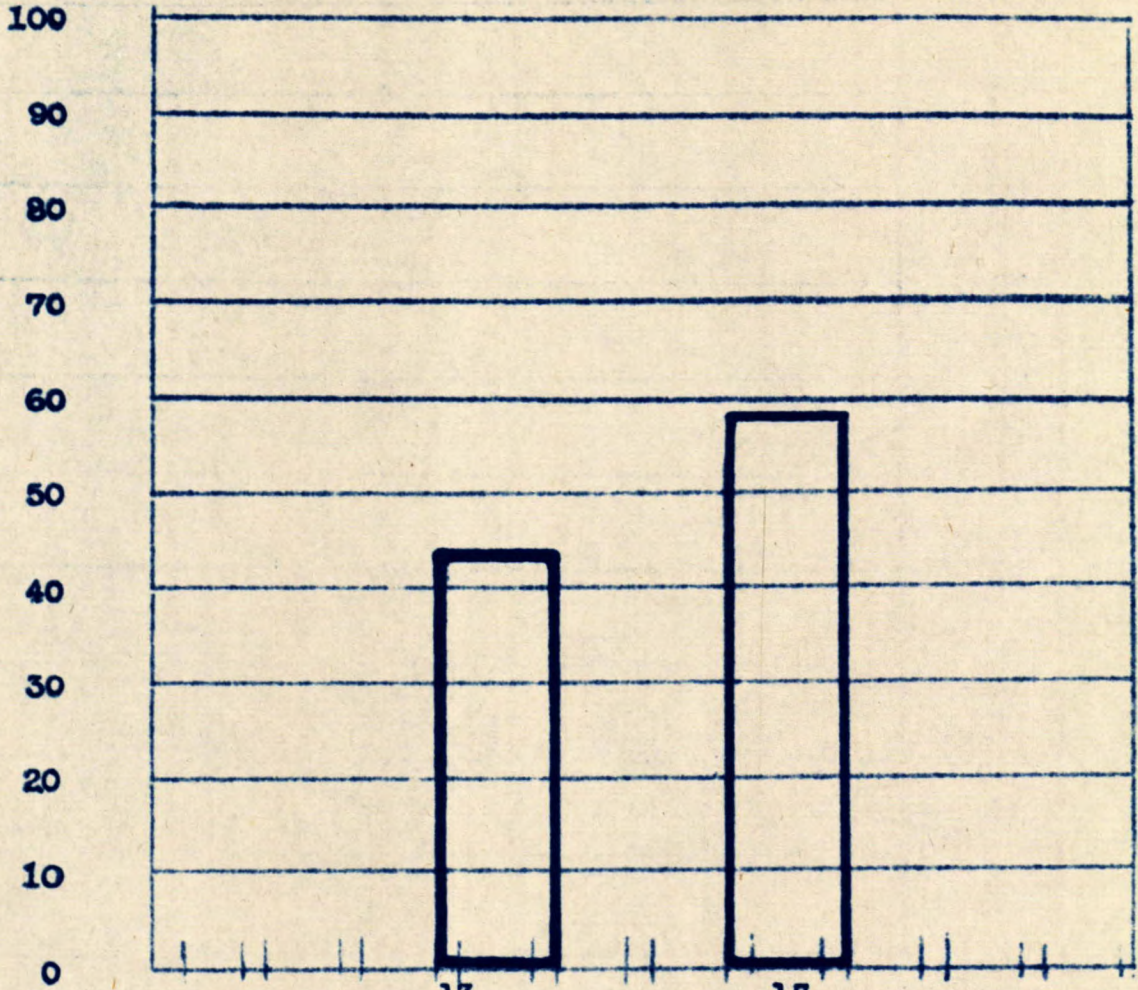
Significance: 55 %.

(NOTE: This means that (100-55%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Elongation . . . & . . . Ballistic Limit . . .

IN . . . 2 . . . Armour . . . FROM . . . Republic . . .

2031 f/s
 % OF TEST RESULTS ABOVE . . .

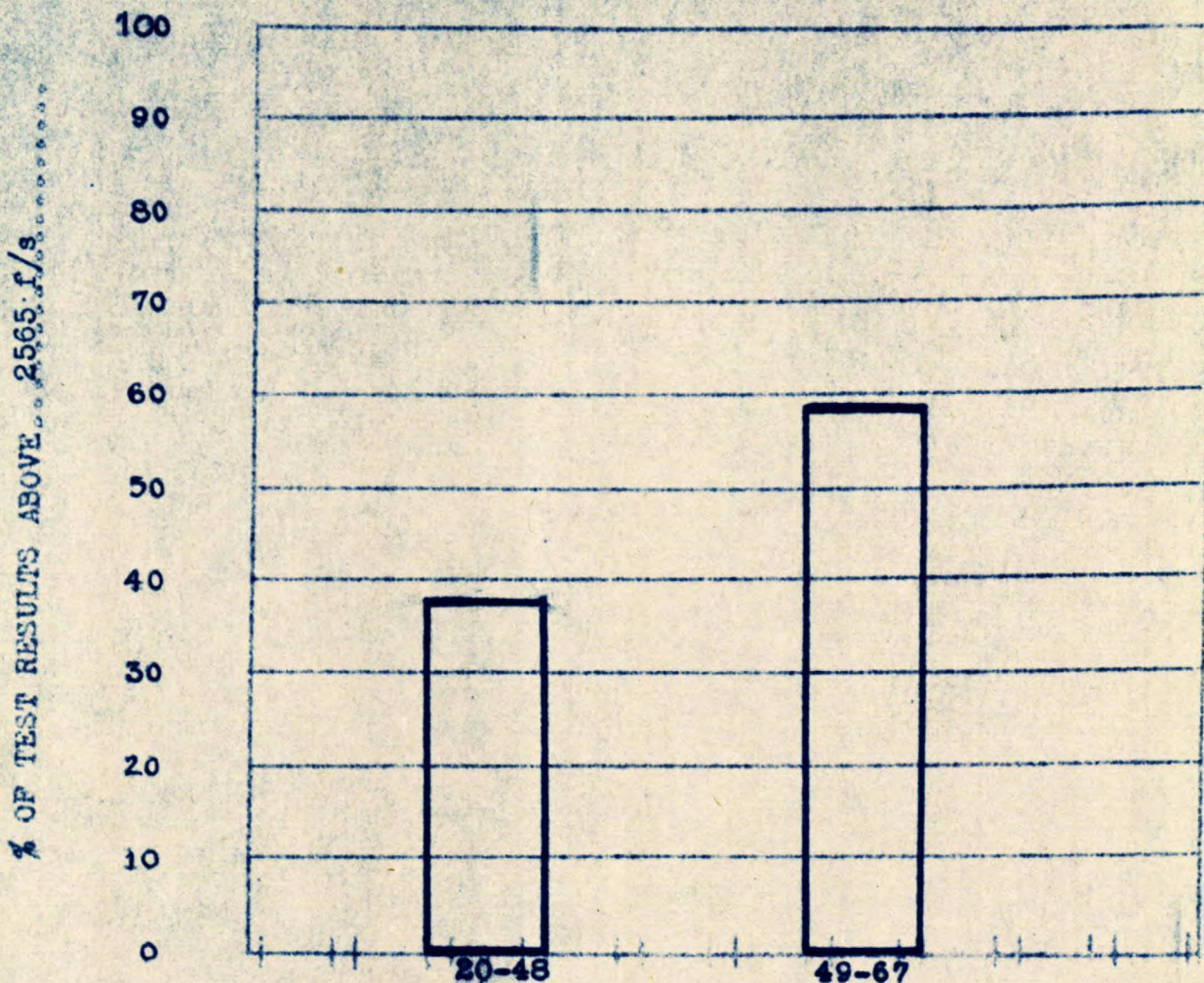


RANGE	13 16	17 20
No. above: 2550 f/s;	14	19
Total number of test results:	32	32
Percentage above:	.44	.59
Ratio difference:	.15	
Number effect:	.031 plus .031 = .062	
Significance:	80 %	

(NOTE: This means that (100-80 %) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN RED. AREA & BALLISTIC LIMIT

IN 3" ARMOUR FROM AMERICAN



RANGE

No. above 2565 f/s: 17 26

Total number of test results: 46 45

Percentage above: 37 58

Ratio difference: 21

Number effect: $\frac{1}{46} \pm \frac{1}{45} = .02175 + .0222 = .0439$

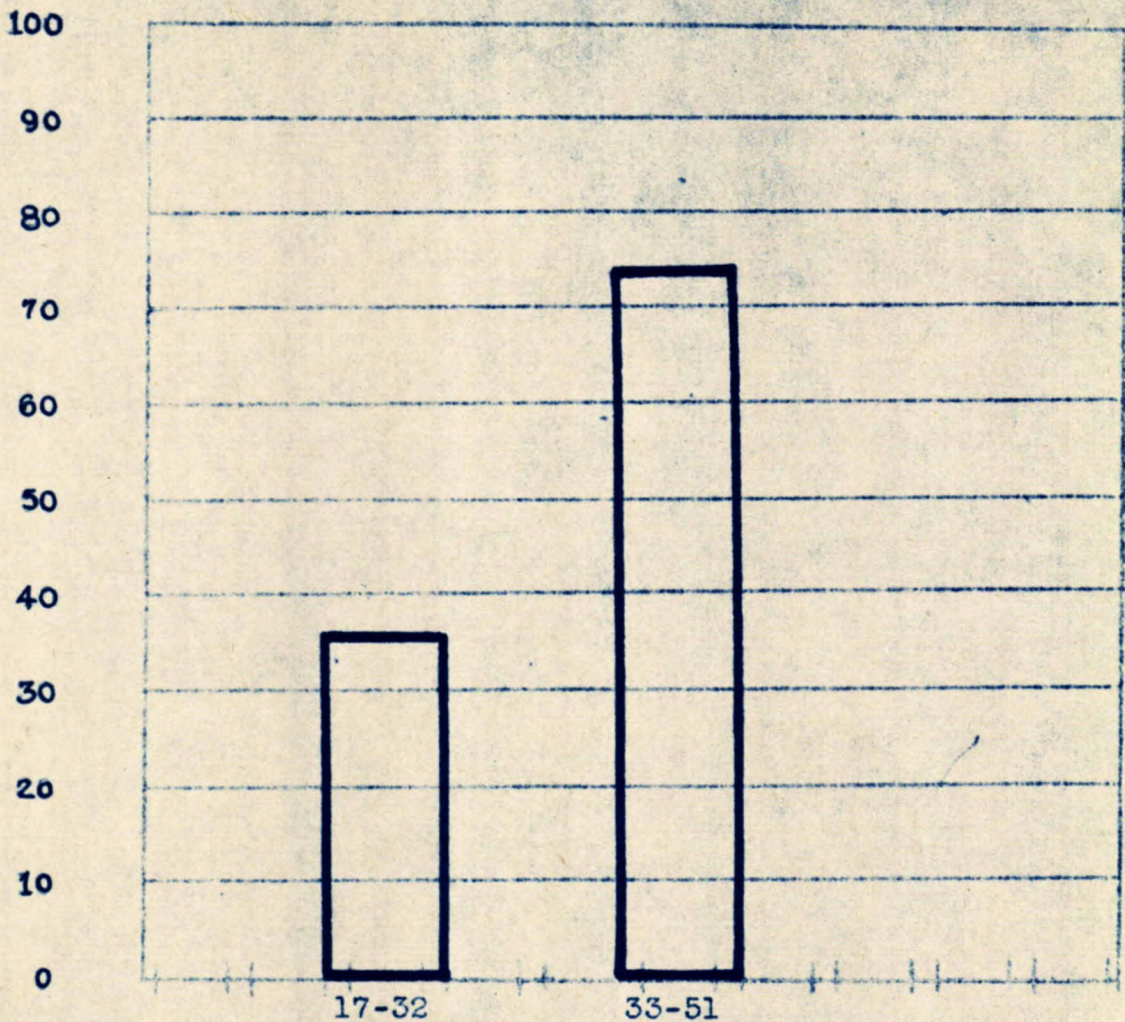
Significance: 95 %.

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Red. in Area & B. L.

IN. 2^{1/2}" Armour. FROM. Union.

% OF TEST RESULTS ABOVE 2254 f/s



RANGE

No. above 2254 f/s: 5 11

Total number of test results: 14 15

Percentage above: 36 73

Ratio difference: .37

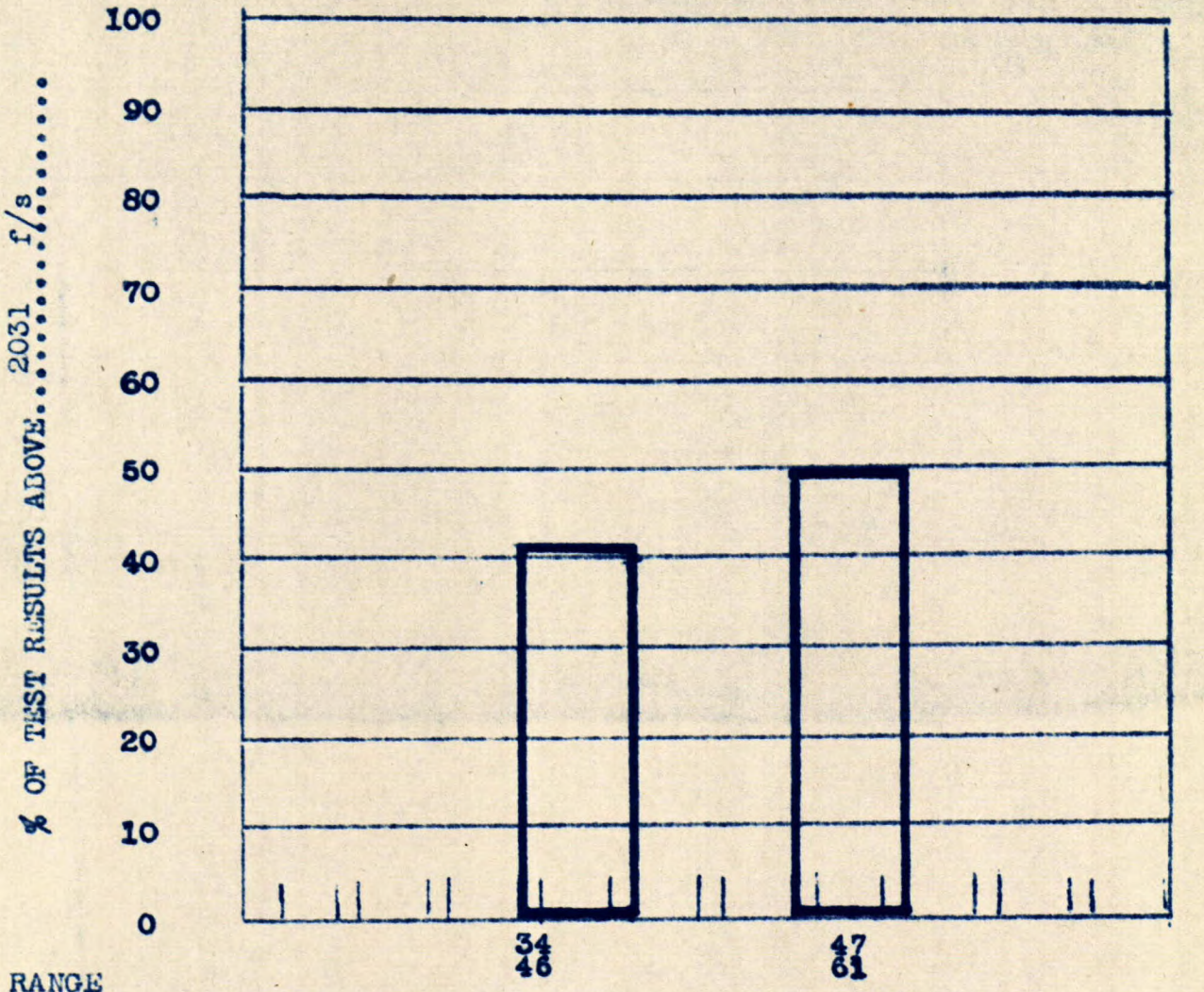
Number effect: .067 + .0715 = .1385

Significance: 93 %.

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

Reduction
CORRELATION BETWEEN of **Area**.....& **Ballistic**..Limit.

IN **2.74** **Armour**..... FROM **Republic**.....



No. above:
2550 f/s: 18 10

Total number of test results: 43 20

Percentage above: 42 50

Ratio difference: 08

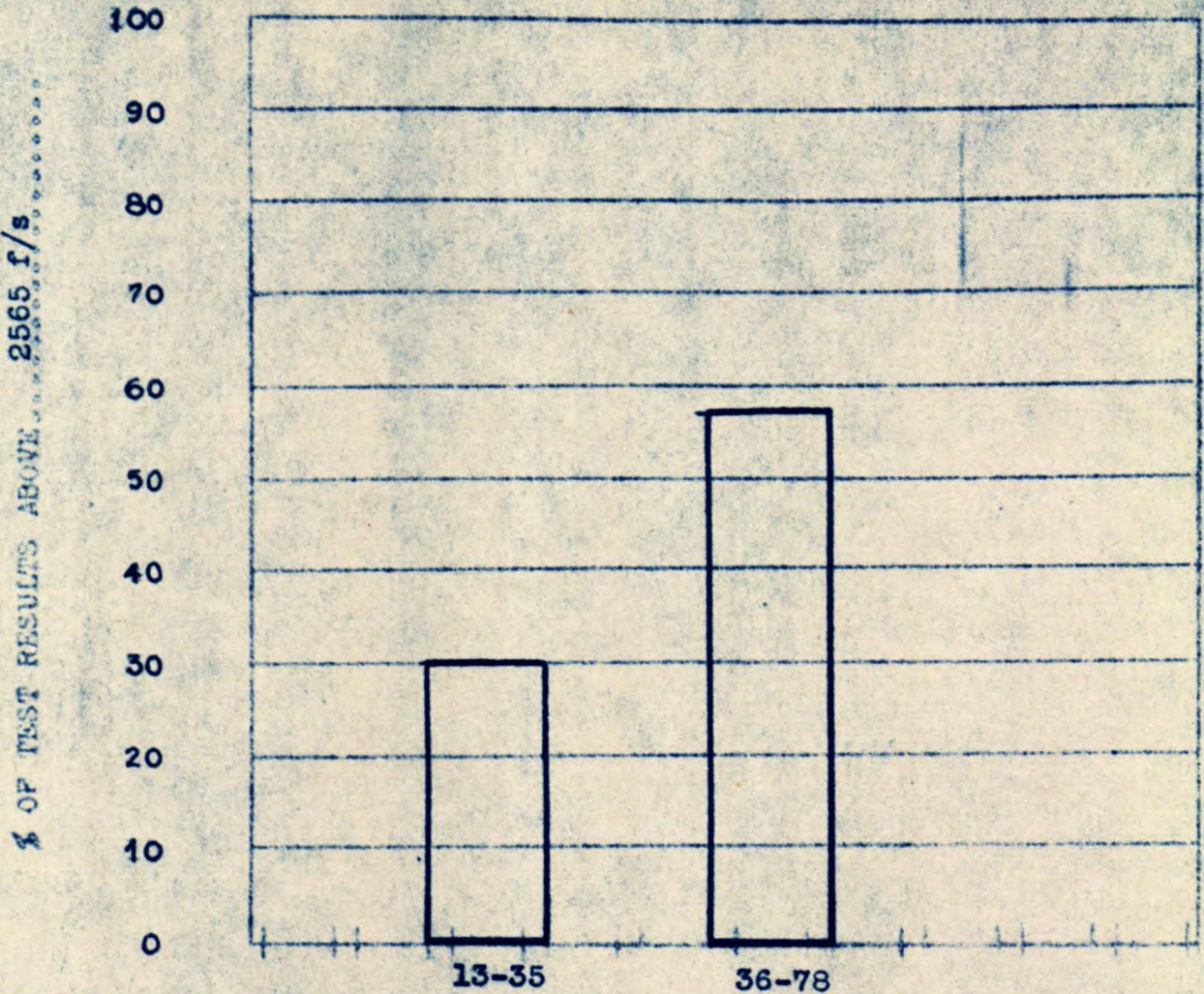
Number effect: 0233 Plus 05 0733

Significance: • ... 40 %.

(NOTE: This means that (100-40%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN IZOD & BALLISTIC LIMIT

3" ARMOUR FROM AMERICAN



RANGE

No. above
2565 f/s:

12

16

Total number of
test results:

40

28

Percentage
above:

30

57

Ratio difference: 27

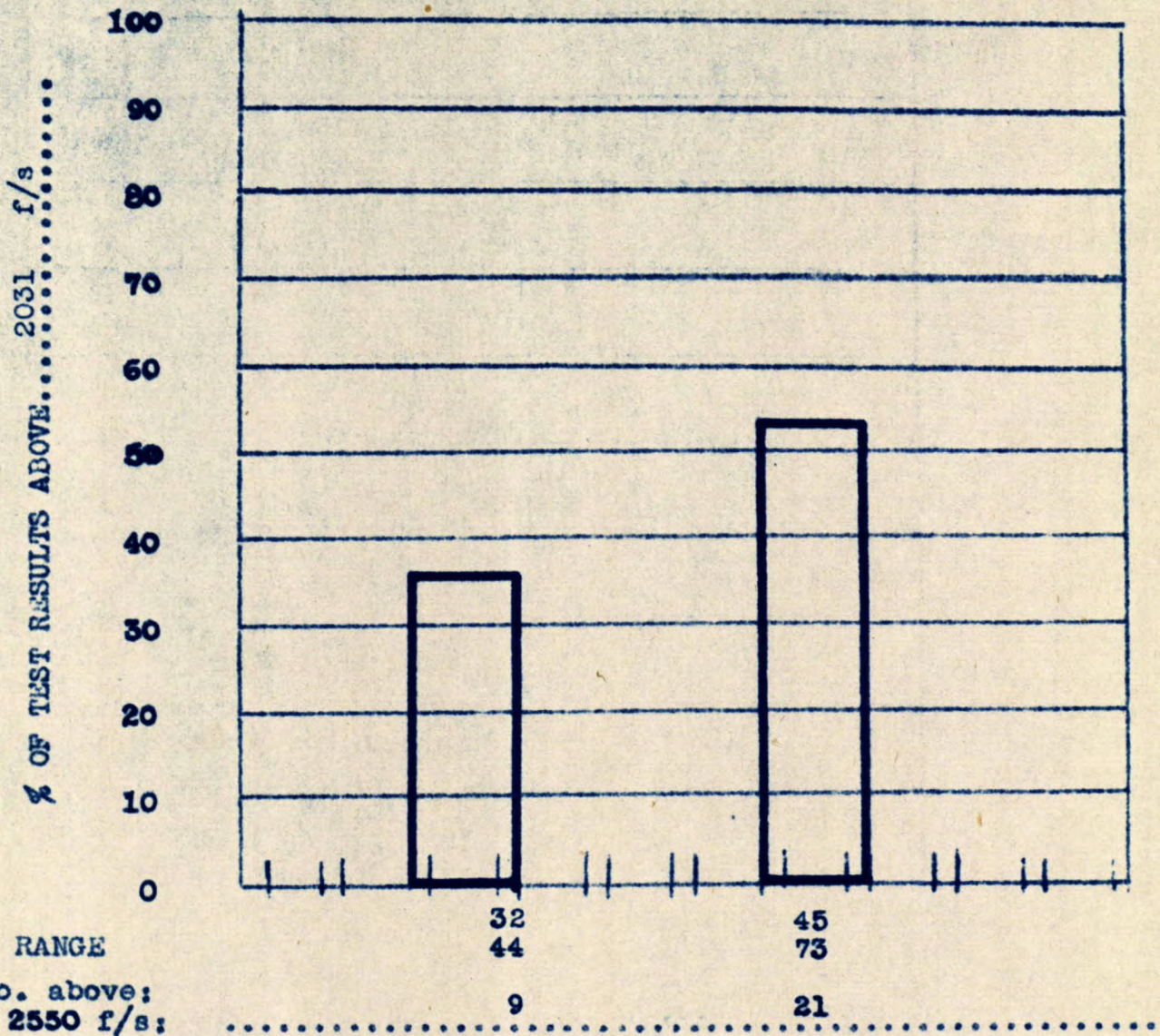
Number effect: $\frac{1}{40} + \frac{1}{28} = .0250 + .0357 = .0607$

Significance: 97%.

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN ...Ized.....&...Ballistic...Limit

IN..2"....Armour..... FROM....Republic.....



Total number of test results:25.....39.....

Percentage above:36.....54.....

Ratio difference:18.....

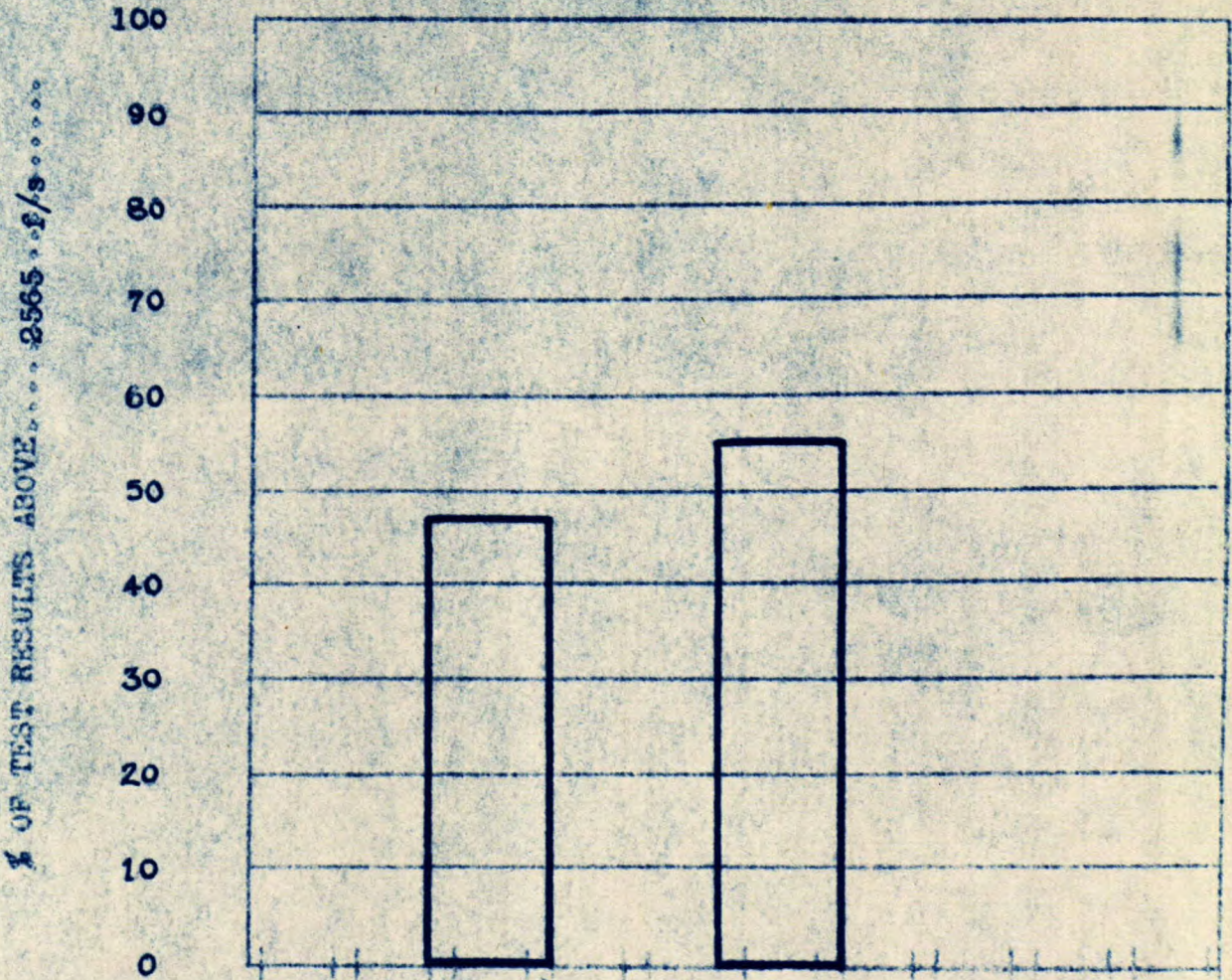
Number effect:040 plus .026 = .066

Significance:85... %.

(NOTE: This means that (100-85 %) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN BRINELL & BALLISTIC LIMIT

IN.3" ARMOUR FROM AMERICAN



RANGE

226
238

239
259

No. above
2565 f/s:

27

28

Total number of
test results:

57

51

Percentage
above:

47.5

55

Ratio difference: .075

Number effect: $\frac{1}{57} - \frac{1}{51} = .0371$

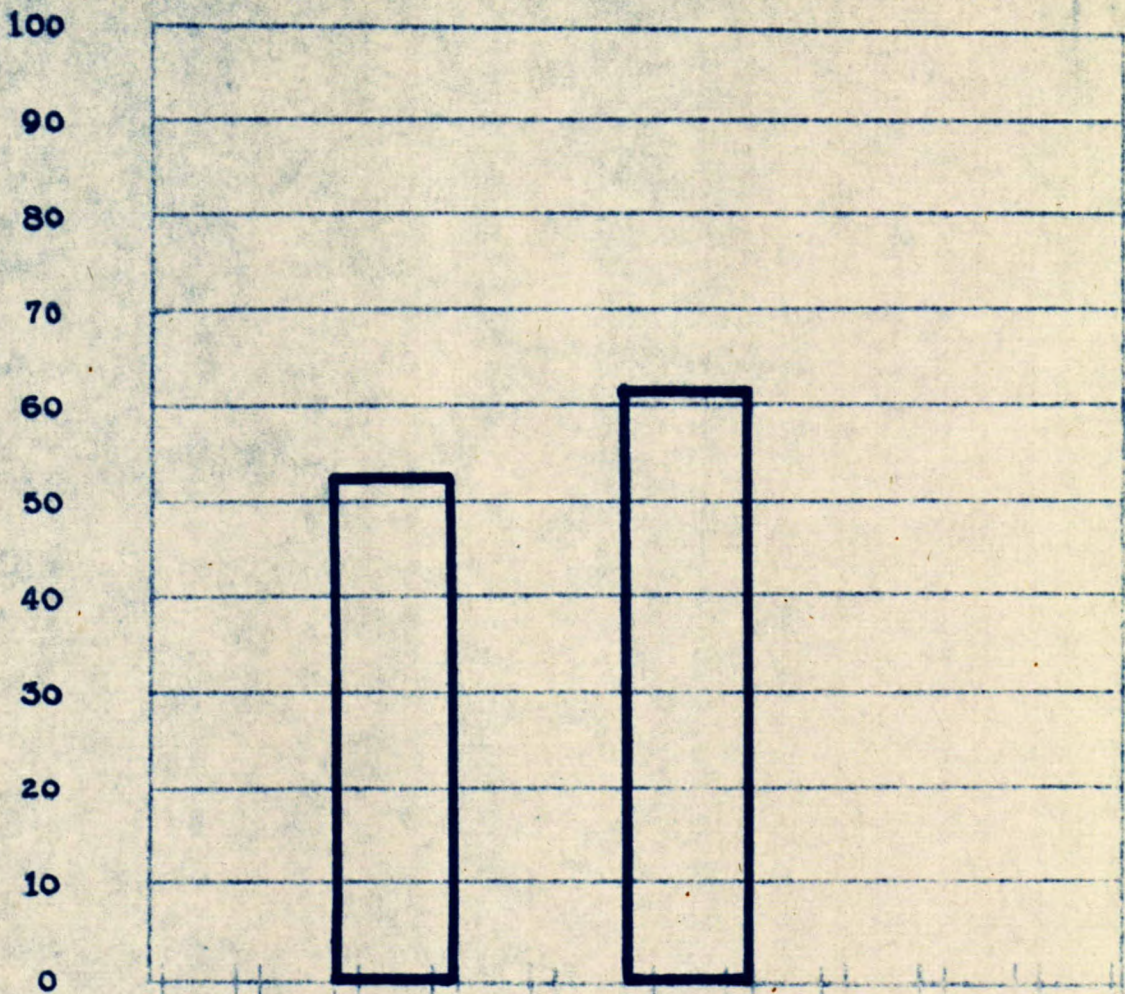
Significance: .55 %

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN **Brinell** & **Ballistic Limit**

IN. **2 3/4"** Armour FROM. **Union**

% OF TEST RESULTS ABOVE **2254** f/s

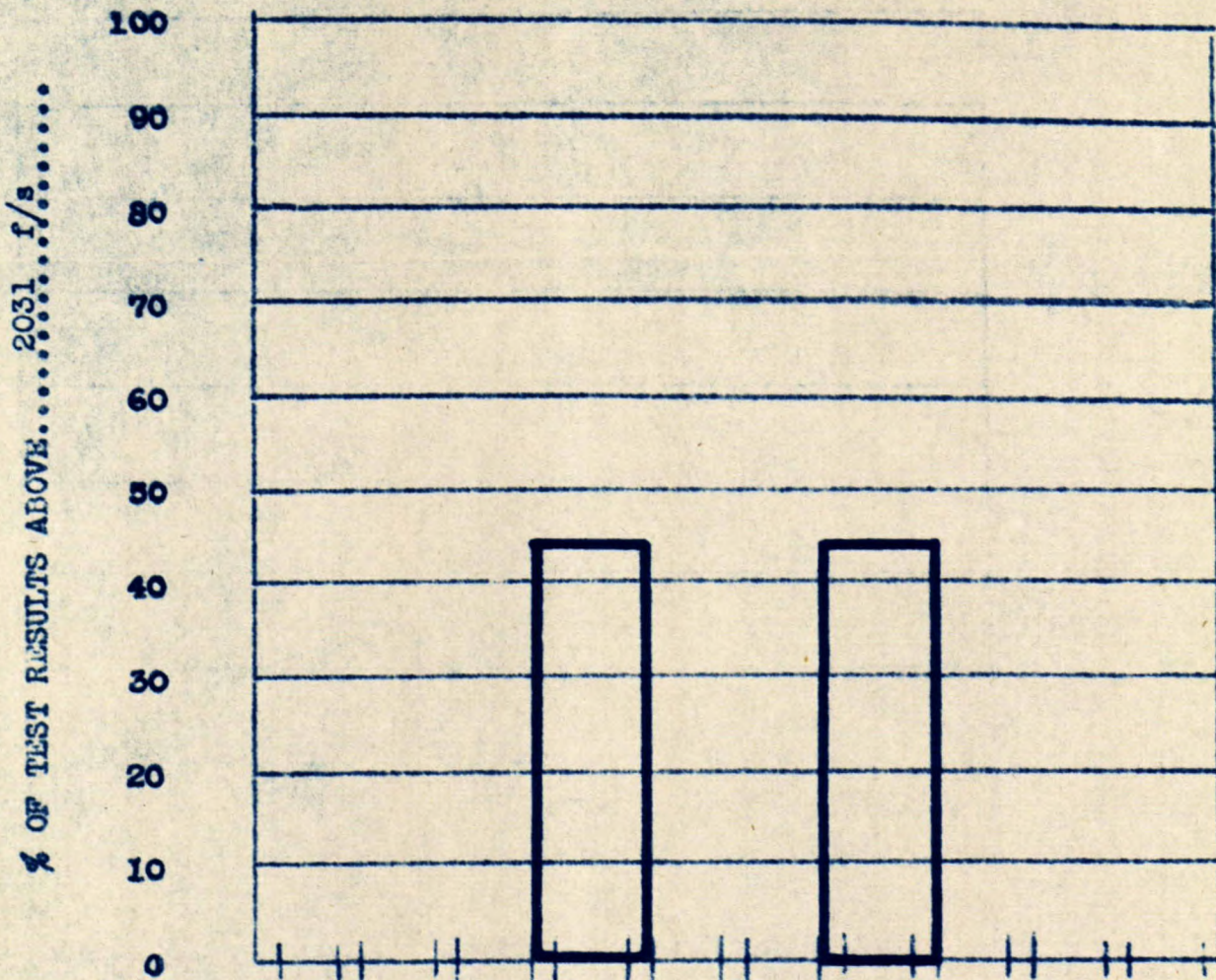


RANGE	247	266
	265	289
No. above 2254 f/s:	8	8
Total number of test results:	15	13
Percentage above:	53	62
Ratio difference:	.09	
Number effect:	$1/15 + 1/13 = .067 + .077 = .144$	
Significance: 25	

(NOTE: This means that (100-.....%) equals the percentage of the time the above ratio difference would occur due to chance.)

CORRELATION BETWEEN Brinell.....&...Ballistic Limit

IN...27.. Armour..... FROM..... Republic.....



RANGE

No. above:
2550 f/s:

277
292

292
302

12

16

Total number of
test results:

27

36

Percentage
above:

.44

.44

Ratio difference:

.00

Number effect:

.037 plus .0278 = .0648

Significance:°

...0..... %.

(NOTE: This means that (100-0 %) equals the percentage of the time the above ratio difference would occur due to chance.)

- APPENDIX B -

Distribution Characteristics.

In the following four examples, test data are plotted graphically in order to show the meaning of

STANDARD DEVIATION,

SKIWNNESS (Relative), and

KURTOSIS (Relative).

Compare 3 and 4 and note the great difference in the way sulphur results are distributed. It is obvious that the source of the data used for Example 3 must have a different method of operating than the source for Example 4. Note how the Standard Deviation, Skewness, and Kurtosis measure quantitatively the characteristics of a distribution, as follows:

Standard Deviation indicates the spread within which the majority of results are included.

Skewness gives a measure of the degree of departure from symmetry. Example 1 is the most symmetrical and Example 4 is the least symmetrical.

Kurtosis: By using the fourth power of the difference from the average, the extreme results are measured.

The power of this method to detect changes from month to month can be readily seen.

Sources also can be compared.

EXAMPLE I

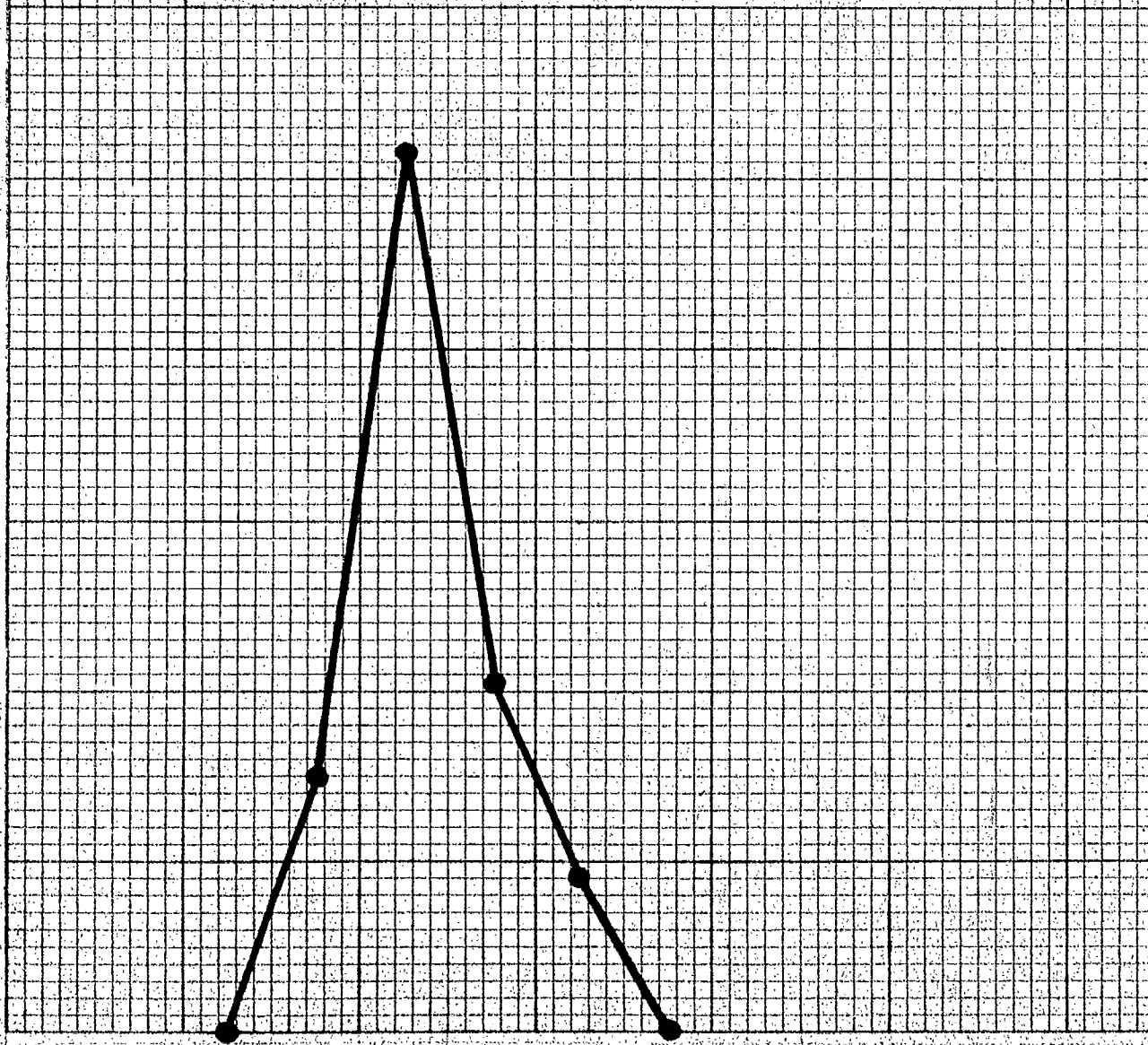
AVERAGE	58
STANDARD DEVIATION	0.87
SKEWNESS	0.39
KURTOSIS	1.85

FREQUENCY OF RESULTS (PER CENT)

60
50
40
30
20
10
0

56 57 58 59 60 61 62

MOLYBDENUM (UNIT : 0.01%)



CALCULATION SHEET FOR THE
 CHARACTERISTICS OF A DISTRIBUTION
 PROPERTY **Molybdenum**..... IN **General**.....
 (Unit: 0.01 %)

X	$X - \bar{X}$	$(X - \bar{X})^2$	$(X - \bar{X})^3$	$(X - \bar{X})^4$
58	0	0		
58	0	0		
57	-1	1	-1	1
60	2	4	8	16
60	2	4	8	16
59	1	1	1	1
58	0	0		
58	0	0		
57	-1	1	-1	1
59	1	1	1	1
57	-1	1	-1	1
59	1	1	1	1
58	0	0		
58	0	0		
59	1	1	1	1
59	1	1	1	1
59	1	1	1	1
57	-1	1	-1	1
59	1	1	1	1
60	-2	4	-8	16
58	0	0		
58	0	0		
58	0	0		
58	0	0		

X	X - \bar{X}	(X - \bar{X}) ²	(X - \bar{X}) ³	(X - \bar{X}) ⁴
59	1	1	1	1
58	0			
58	0			
58	0			
58	0			
58	0			
58	0			
58	0			
57	-1	1	-1	1
1923		25	13	61

N = number of results = 33.

$\bar{X} = \frac{\sum X}{N} = \dots \frac{1923}{33} = 58.$

Standard deviation = $\sigma = \sqrt{\frac{\sum (X - \bar{X})^2}{N}} = \sqrt{\frac{25}{33}} = \sqrt{0.76} = 0.87$

Skewness = $\pi_3 = \frac{\sum (X - \bar{X})^3}{N} = \frac{13}{33} = 0.39$

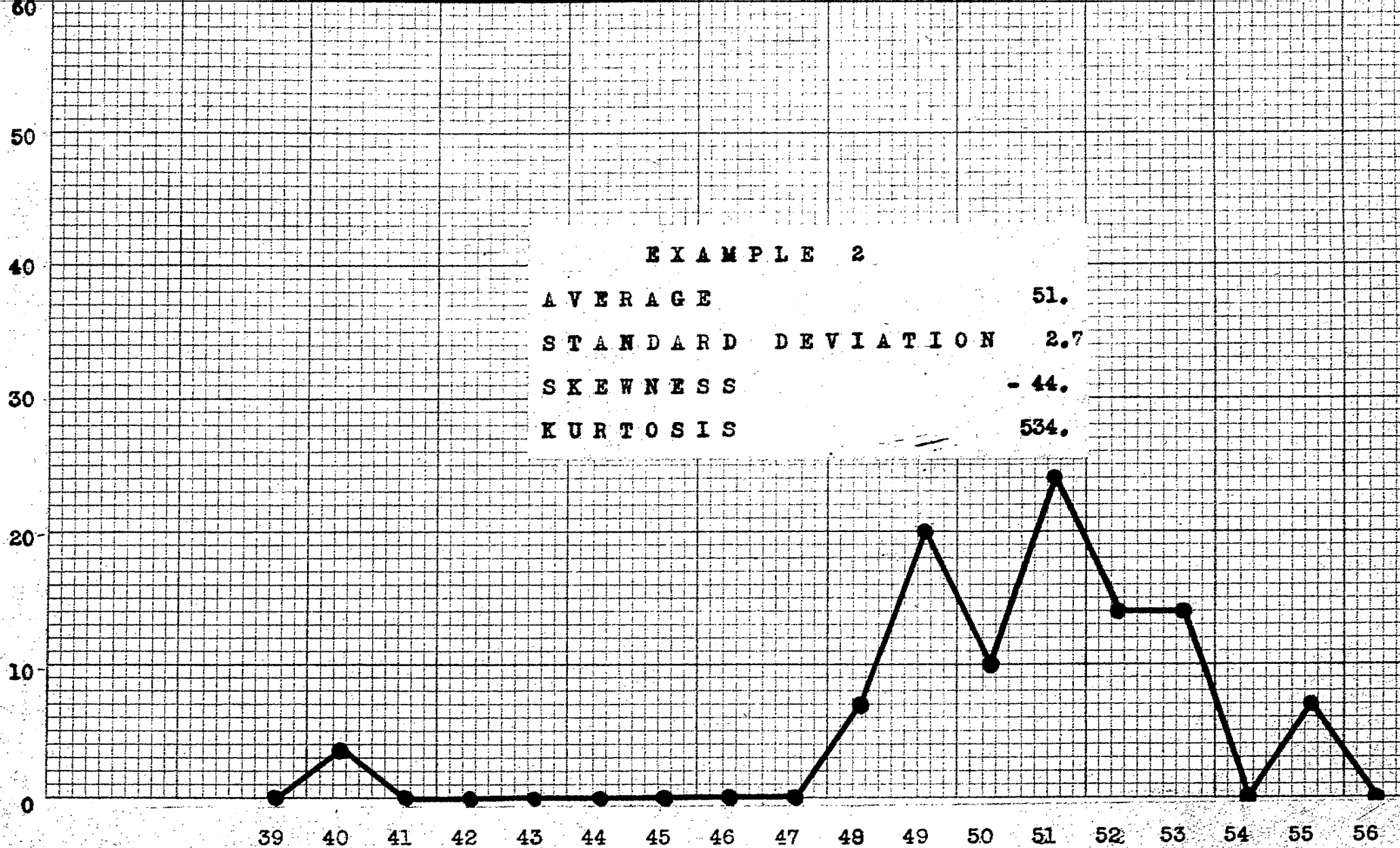
Kurtosis = $\pi_4 = \frac{\sum (X - \bar{X})^4}{N} = \frac{61}{33} = 1.85$

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FREQUENCY OF RESULTS
(PER CENT)

EXAMPLE 2

AVERAGE 51.
STANDARD DEVIATION 2.7
SKEWNESS - 44.
KURTOSIS 534.



MOLYBDENUM (UNIT : 0.01 %)

CALCULATION SHEET FOR THE
CHARACTERISTICS OF A DISTRIBUTION

ELEMENTARY **Molybdenum** IN **Union**
(Unit : 0.01 %)

X	\bar{X}	$(X - \bar{X})^2$	$(X - \bar{X})^3$	$(X - \bar{X})^4$
49	-2	4	-8	16
48	-3	9	-27	81
49	-2	4	-8	16
48	-3	9	-27	81
50	-1	1	-1	1
51	0	0	0	0
51	0	0	0	0
49	-2	4	-8	16
53	+2	4	8	16
49	-2	4	-8	16
51	0	0	0	0
52	1	1	1	1
51	0	0	0	0
50	-1	1	-1	1
53	2	4	8	16
40	-11	121	-1331	14,641
53	2	4	8	16
52	1	1	1	1
55	4	16	64	256
53	2	4	8	16
50	-1	1	1	1
55	4	16	64	256
51	0	0	0	0
51	0	0	0	0

5'					
X	X - \bar{X}	(X - \bar{X}) ²	(X - \bar{X}) ³	(X - \bar{X}) ⁴	
52	1	1	1	1	
52	1	1	1	1	
51	0	0	0	0	
49	-2	4	-8	16	
49	-2	4	-8	16	
<hr/>					
1467		218	- 1271	15,482	

N = number of results = 29.

$$\bar{X} = \frac{\sum X}{N} = \dots\dots\dots \frac{1467}{29} = 50.6$$

$$\text{Standard deviation} = \delta = \sqrt{\frac{\sum (X - \bar{X})^2}{N}} = \sqrt{\frac{218}{29}} = \sqrt{7.52} = 2.7$$

$$\text{Skewness} = \pi_3 = \frac{\sum (X - \bar{X})^3}{N} = \frac{-1271}{29} = -43.8$$

$$\text{Kurtosis} = \pi_4 = \frac{\sum (X - \bar{X})^4}{N} = \frac{15,482}{29} = 534.$$

=====

EXAMPLE 3

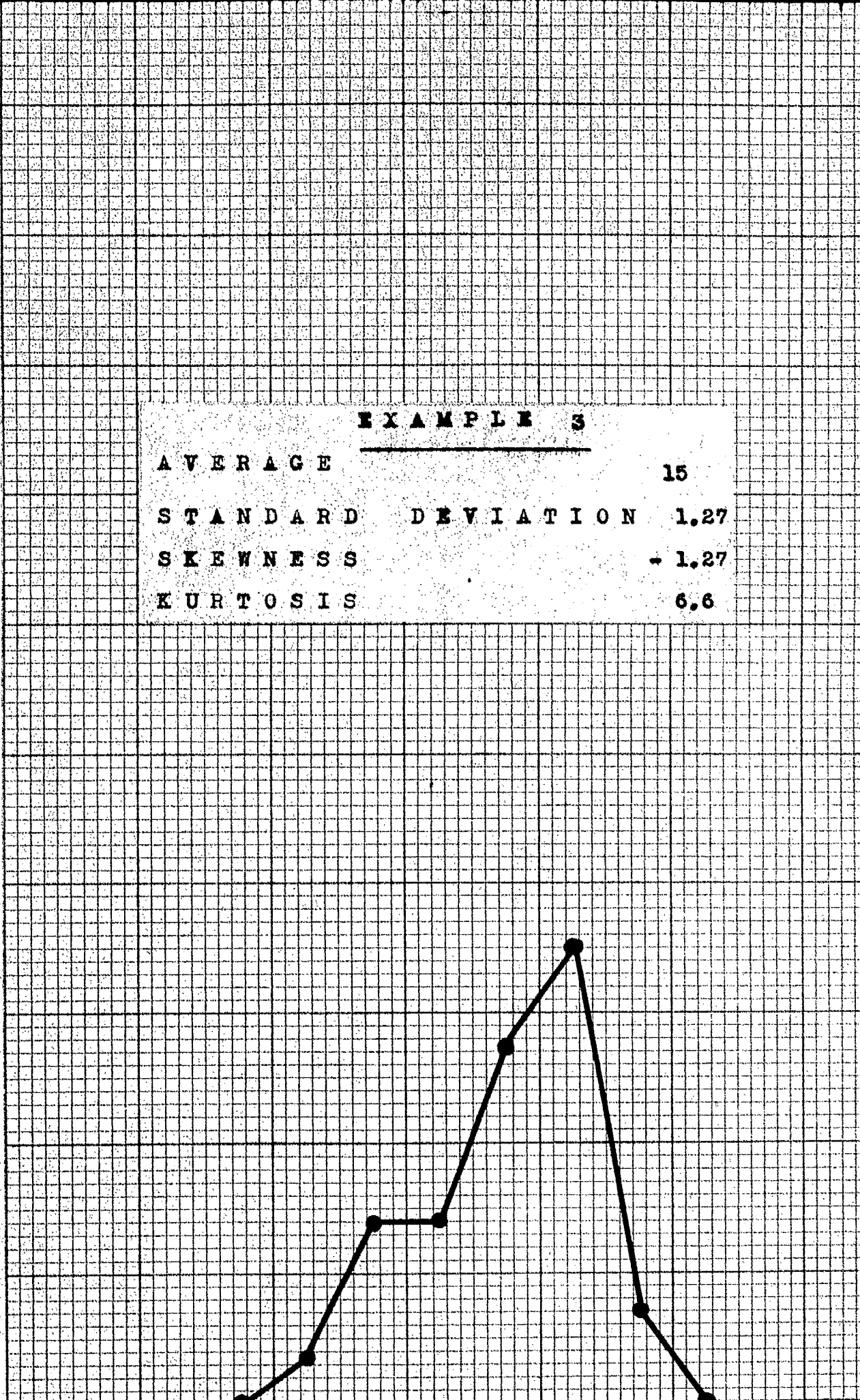
AVERAGE	15
STANDARD DEVIATION	1.27
SKENNESS	+ 1.27
KURTOSIS	6.6

FREQUENCY OF RESULTS (PER CENT)

60
50
40
30
20
10
0

11 12 13 14 15 16 17 18

SULPHUR (UNIT : 0.01 %)



CALCULATION SHEET FOR THE
 CHARACTERISTICS OF A DISTRIBUTION
 PROPERTY .. Sulphur..... IN Union.....
 (Unit : 0.001 %)

x	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^3$	$(x - \bar{x})^4$
15	0	0	0	0
15	0	0	0	0
14	-1	1	-1	1
14	-1	1	-1	1
16	1	1	1	1
15	0	0	0	0
15	0	0	0	0
16	1	1	1	1
16	1	1	1	1
15	0	0	0	0
16	1	1	1	1
16	1	1	1	1
16	1	1	1	1
15	0	0	0	0
15	0	0	0	0
15	0	0	0	0
13	-2	4	-8	16
14	-1	1	-1	1
16	1	1	1	1
17	2	4	8	16
16	1	1	1	1
12	-3	9	-27	81
16	1	1	1	1

X	X - \bar{X}	(X - \bar{X}) ²	(X - \bar{X}) ³	(X - \bar{X}) ⁴
13	-2	4	-8	16
14	-1	1	-1	1
17	2	4	8	16
16	1	1	1	1
13	-2	4	-8	16
13	-2	4	-8	16
434		47	- 37	191

N = number of results = 29.

$\bar{X} = \frac{\sum X}{N} = \dots\dots\dots \frac{434}{29} = 15.$

Standard deviation = $\sigma = \sqrt{\frac{\sum (X - \bar{X})^2}{N}} = \sqrt{\frac{47}{29}} = \sqrt{1.62} = 1.27$

Skewness = $\pi_3 = \frac{\sum (X - \bar{X})^3}{N} = \dots\dots \frac{-37}{29} = - 1.27$

Kurtosis = $\pi_4 = \frac{\sum (X - \bar{X})^4}{N} = \frac{191}{29} = 6.58$



FREQUENCY OF RESULTS
(PER CENT)

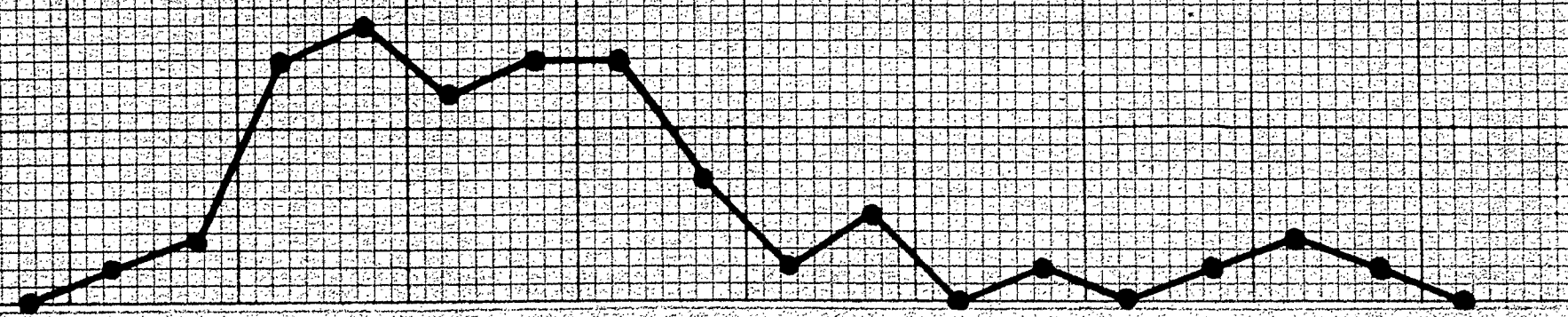
50
40
30
20
10
0

13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

SULPHUR (UNIT : 0.001 %)

EXAMPLE 4

AVERAGE	19.
STANDARD DEVIATION	3.3
SKENNESS	517.
KURTOSIS	2,435.



CALCULATION SHEET FOR THE
 CHARACTERISTICS OF A DISTRIBUTION
 PROPERTY Sulphur..... IN American.....
 (Unit : 0.001 %)

X	X - \bar{X}	(X - \bar{X}) ²	(X - \bar{X}) ³	(X - \bar{X}) ⁴
27	8	64	512	4,096
22	3	9	27	81
16	-3	9	-27	81
14	-5	25	-125	625
16	-3	9	-27	81
16	-3	9	-27	81
15	-4	16	-64	256
17	-2	4	-8	16
15	-4	16	-64	256
19	0	0	0	0
16	-3	9	-27	81
17	-2	4	-8	16
17	-2	4	-8	16
23	4	16	64	256
17	-2	4	-8	16
16	-3	9	-27	81
16	-3	9	-27	81
19	0	0	0	0
19	0	0	0	0
18	-1	1	-1	1
19	0	0	0	0
20	1	1	1	1
16	-3	9	-27	81
21	2	4	8	16
19	0	0	0	0
17	-2	4	-8	16
16	-3	9	-27	81

CALCULATION SHEET FOR THE
CHARACTERISTICS OF A DISTRIBUTION

PROPERTY IN

19

X	$X - \bar{X}$	$(X - \bar{X})^2$	$(X - \bar{X})^3$	$(X - \bar{X})^4$
19	0	0	0	0
19	0	0	0	0
21	2	4	8	16
17	-2	4	-8	16
20	1	1	1	1
18	-1	1	-1	1
20	1	1	1	1
18	-1	1	-1	1
19	0	0	0	0
21	2	4	8	16
21	2	4	8	16
17	-2	4	-8	16
20	1	1	1	1
20	1	1	1	1
17	-2	4	-8	16
20	1	1	1	1
18	-1	1	-1	1
18	-1	1	-1	1
17	-2	4	-8	16
18	-1	1	-1	1
20	1	1	1	1
23	4	16	64	256
23	4	16	64	256
20	1	1	1	1
28	9	81	729	59,049
29	10	100	1,000	10,000
28	9	81	729	59,049

X	X - \bar{X}	(X - \bar{X}) ²	(X - \bar{X}) ³	(X - \bar{X}) ⁴
25	6	36	216	1,296
18	1	1	1	1
1085		615	2899	136,347

N = number of results = 56.

$$\bar{X} = \frac{\sum X}{N} = \dots\dots\dots \frac{1085}{56} = 19.$$

$$\text{Standard deviation} = \sigma = \sqrt{\frac{\sum (X - \bar{X})^2}{N}} = \sqrt{\frac{615}{56}} = \sqrt{11} = 3.3$$

$$\text{Skewness} = \pi_3 = \frac{\sum (X - \bar{X})^3}{N} = \dots\dots\dots \frac{2899}{56} = 51.7$$

$$\text{Kurtosis} = \pi_4 = \frac{\sum (X - \bar{X})^4}{N} = \frac{136,347}{56} = 2435.$$



- APPENDIX C -

Distribution Characteristics of Test Data.

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DISTRIBUTION CHARACTERISTICS FOR CARBON, SILICON, MANGANESE IN 2"-3" ARMOUR PLATE

SOURCE	PROPERTY	No. of results:		AVERAGE:		STANDARD DEVIATION:		SKEWNESS FACTOR	KURTOSIS FACTOR	Remarks
		As found:	Probable error:	As found:	Probable error:					
REPUBLIC GENERAL UNION AMERICAN	CARBON	60	34		1.1		-0.9	3.9	(UNIT: 0.01 %)	(a)
	"	33	29		1.8		+7.9	66.6		(b)
	"	29	33.5		3.0		9.8	172.		(a)
	"	59	28.0		1.7		12.	87.		(b)
REPUBLIC GENERAL UNION AMERICAN	SILICON	61	22		2.1		-12.9	81.	UNIT: 0.01 %	(b)
	"	33	31		1.7		+5.2	25.		(a)
	"	28	25.5		2.5		+1.34	83.		(a)
	"	57	39.		2.3		+9.	99.		(a)
REPUBLIC GENERAL UNION AMERICAN	MANGANESE	61	41.		2.9		-13.	236.	UNIT: 0.01 %	(a)
	"	34	60		5.1		+99.7	2,476.		(a)
	"	29	69		9.4		88.6	15,760.		(a)
	"	60	109		5.1		+35.	3,126.		(a)

(a) IN CONTROL

(b) OUT OF CONTROL

DISTRIBUTION CHARACTERISTICS FOR SULPHUR, PHOSPHORUS, NICKEL IN 2"-3" ARMOUR PLATE

SOURCE	PROPERTY	: No. of : A V E R A G E : STANDARD DEVIATION:		: results: As. : Probable: As : Probable :		SKEWNESS FACTOR	KURTOSIS FACTOR	Remarks	
		: :	: :	: :	: :				: :
REPUBLIC 2"	Sulfur	61	18		3.2		9.5	271	UNIT: 0.001 % (a)
GENERAL 3"	"	35	12		1.3		-1.97	6.9	
UNION 3"	"	29	15		1.27		-1.27	6.6	
AMERICAN 2 1/4"	"	56	19		3.3		51.7	2,435.	
REPUBLIC	Phosphorus	60	17		1.9		11.6	90.4	UNIT: 0.001 % (b)
GENERAL	"	33	22		6.3		-83.	4,214.	
UNION	"	28	26		2.2		8.1	59.4	
AMERICAN	"	55	23		3.9		12.2	889.	
REPUBLIC	Nickel	60	3.83		7.		1.3	7,464.	UNIT: 0.01% (a)
GENERAL	"	29	3.23		6.4		-70.4	9,794.	
UNION									

(a) IN CONTROL
(b) OUT OF CONTROL

DISTRIBUTION CHARACTERISTICS FOR CHROMIUM, MOLYBDENUM, IN 2"-3" ARMOUR PLATE.

SOURCE	PROPERTY	: No. of :		: A V E R A G E :		: STANDARD DEVIATION :		SKELNESS FACTOR	KURTOSIS FACTOR	Remarks
		: results :	: found :	: As :	: Probable :	: As :	: Probable :			
REPUBLIC GENERAL UNION AMERICAN	CHROMIUM	61	1.49		4.6			43.7	1,612	(b) UNIT : 0.01 %
		34	2.48		13.1			302.	206,505	(b)
		29	1.28		8.7			+1,368.	50,015	(b)
		56	297.		13.			38.	11,647	(a)
REPUBLIC GENERAL UNION AMERICAN	MOLYBDENUM	61	54.		3.3			-16	438	(a) UNIT : 0.01 %
		33	58		0.87			+0.39	1.85	(a)
		29	51		2.7			-44	534.	(b)
		56	41		1.58			+11	79.5	(b)

(a) IN CONTROL
(b) OUT OF CONTROL

DISTRIBUTION CHARACTERISTICS FOR BRINELL, YIELD, TENSILE STRENGTH, IN 2"-3" armour plate

SOURCE	PROPERTY	: No. of : A V E R A G E : STANDARD DEVIATION:		: results: As : Probable: As : Probable :		SKEWNESS FACTOR	KURTOSIS FACTOR	Remarks	
		: :	: :	: :	: :				: :
REPUBLIC	BRINELL	65	292		5.9		-200	4,471	(a) 10 UNIT ± 1
GENERAL	"	30	264		6.0		-100	2,855	(a)
UNION	"	28	266		11.8		-3.3	66,400	(a)
AMERICAN	"	78	239		7.9		+1,346.	63,371	(b)
REPUBLIC	YIELD	64	120		6.8		+1,932.	8,522	(b) UNIT: 1,000 p.s.i.
GENERAL	"	32	114		10.25		-1,689.	50,610	(a)
UNION	"	29	102		8.2		+485.	11,700	(b)
AMERICAN	"	69	89		7.3		-2.6	6,945	(a)
REPUBLIC	TENSILE	61	144		4.5		+60.	1,461	(b) UNIT: 1,000 p.s.i.
GENERAL	"	33	138		7.6		-742.	24,175	(b)
UNION	"	29	129		7.4		+22.	5,417	(a)
AMERICAN	"	89	115		8.4		-369.	19,904	(b)

(a) IN CONTROL
(b) OUT OF CONTROL

DISTRIBUTION CHARACTERISTICS FOR ELONGATION, REDUCTION OF AREA, IZOD IN 2nd-3rd ARMOUR PLATE

SOURCE	PROPERTY	No. of results:		AVERAGE:		STANDARD DEVIATION:		SKEWNESS FACTOR	KURTOSIS FACTOR	Remarks
		As found:	Probable error:	As found:	Probable error:					
REPUBLIC GENERAL UNION AMERICAN	ELONGATION	66	17		1.2			-1.5	16.4	UNIT : 1 % (b)
	"	33	16		1.8			+4.9	46.0	(a)
	"	27	17		2.8			-13.5	214.	(a)
	"	83	20		2.7			-30.	794.	(b)
UNIT : 1 %										
REPUBLIC GENERAL UNION AMERICAN	REDUCTION OF AREA	65	46		4.9			48.	2,439.	(a)
	"	33	41		5.9			-73.7	3,340.	(a)
	"	29	34		9.8			215.	17,848.	(a)
	"	88	47		10.7			-1,616.	62,114.	(b)
REPUBLIC GENERAL UNION AMERICAN	IZOD	64	46		7.2			+436.	14,999.	UNIT : 1 FT.LB. (b)
	"	28	53		6.4			+127.	4,787.	(a)
	"	16	40		3.9			-2,332.	93,116.	(b)
	"	66	36		16.2			+1,107.	153,600.	(a)

(a) IN CONTROL
(b) OUT OF CONTROL

DISTRIBUTION CHARACTERISTICS FOR BALLISTIC LIMIT TESTS ON CAST AND ROLLED 2"-3" ARMOR PLATE

PROPERTY SOURCE	: No. of		: A V E R A G E :		: STANDARD DEVIATION:		SKEWNESS FACTOR	KURTOSIS FACTOR	Remarks
	: results:		: As : Probable:		: As : Probable :				
	:		: found: error :		: found : error :				
UNION	29	2258		54.9		-93.5×10^3	43.7×10^6	(b)	
AMERICAN	49	2565		69.6		-82.8×10^3	94.6×10^6	(b)	
GENERAL	30	2569		110.8		-1509×10^3	629×10^6	(b)	
REPUBLIC	68	2031		54.4		$+354.7 \times 10^3$	55.8×10^6	(a)*	

(a) IN CONTROL
(b) OUT OF CONTROL