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THE DIGITAL VELOCITY-GENERATING COMPUTER FOR THE DOMINION OBSERVATORY MIRROR TRANSIT

G. A. Brealey

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ERRATA

PAGE 61, right coln., line 9, *for* milliseconds, *read* microseconds.

PAGE 63, left coln., line 30, ("Whence $C' C'' = \ldots$") *for* $(\alpha a + \alpha s)$ *read* $(\alpha a - \alpha s)$.

The Digital Velocity-generating Computer for the Dominion Observatory Mirror Transit

G. A. Brealey

Abstract: The original velocity-generating computer of the Ottawa mirror transit has been found to lack sufficient accuracy for the tracking of stars during meridian passage. This shortcoming is shown to be caused by a combination of observing techniques, atmospheric refraction, thermal expansion effects, and mechanical alignment errors. The first two of these effects are referred to briefly (with references to a previous paper) and the last two are examined in detail and shown to exceed tolerable amounts.

The new computer uses electro-magnetic clutches and mechanical differentials to supply a mechanical analogue of binary arithmetic. The differentials are used to add angular velocities which in turn are selected by the energizing of appropriate clutches. The selection of velocity is done through a four-column decimal keyboard in which the observer enters the digits of cosine declination directly. A diode matrix converts each decimal number into its binary equivalent; and this in turn energizes the required clutches.

The new velocity generator is shown to be an extremely accurate device. The output velocity error does not exceed plus or minus 2 parts in 10,000 whereas the original generator had errors of about ten times this value.

Résumé:—On s'est rendu compte à l'observatoire d'Ottawa que la première calculatrice génératrice de vitesse reliée à l'instrument de passage à réflexion n'était pas suffisamment précise pour suivre les étoiles à leur passage au méridien. Un ensemble d'éléments comprenant les techniques d'observation, la réfraction atmosphérique, des effets d'expansion thermique et des erreurs d'alignement mécanique sont responsables de cette faiblesse. L'auteur analyse succinctement les deux premiers éléments et renvoie le lecteur à une étude précédente, mais il examine les deux derniers en détail et démontre qu'ils causent des erreurs excédant la tolérance permise.

La nouvelle calculatrice emploie des embrayages électro-magnétiques et des différentiels mécaniques qui alimentent une calculatrice analogique mécanique binaire. Les différentiels servent à augmenter les vitesses angulaires qui à leur tour sont sélectionnées par la mise en mouvement des embrayages appropriés. Le choix de la vitesse se fait à l'aide d'un clavier à décimales en quatre colonnes où l'observateur inscrit directement les chiffres du cosinus de la déclinaison. Une matrice à diode convertit chaque chiffre décimal en son équivalent binaire qui, à son tour, fait mouvoir les embrayages répondants.

Cette nouvelle génératrice de vitesse est un instrument très précis. Les erreurs de vitesse ne dépassent pas ± 2 parties en 10,000 tandis que le premier instrument pouvait accuser des erreurs dix fois supérieures.

Introduction

In a previous paper (*Pub. Dom. Obs.* XXV, No. 3, 1963), the writer described in detail the mechanical and optical equipment used for the photographic registration of transits on the Ottawa mirror transit. In the summation, it was mentioned that the device used to drive the right ascension slide (at a velocity proportional to cos δ) failed to give a sufficiently accurate velocity for observations of stars within 30° or so of the pole; which in turn required the observer to record other observing parameters involving the time-relationship between the shutter operation and the incidence of flashes.

It was further mentioned that these difficulties led to the design of a more accurate device for deriving plate carriage velocities, in which the accuracy would be independent of δ, and would thus reduce the number of necessary parameters and simplify the reduction of observations.

1. Observing Techniques

A brief description of the original velocity generator will obviate reference to the previous paper. The mirror transit uses a flat mirror held in a cell, which is pivoted on an east-west axis, to reflect the selected star (at meridian passage) into one of two rigid horizontal telescopes that face one another on a north-south baseline. In general, the telescope on the same side of the zenith as the star is the one used, although near the zenith either telescope can be used. The mirror angle is set by means of a servo-mechanical transmitter with an accuracy of $\pm \frac{1}{2}'$ of arc, which ensures that the star's image will transit in the field of view of the telescope eyepiece.

As the star crosses the meridian its image moves horizontally at the focus of the telescope, where a fixed glass reticle is placed, having etched on it a pattern of horizontal and vertical lines. The reticle is flashed at precisely known times, by a high-intensity discharge tube giving a flash of some 10 milliseconds duration.

The right ascension slide is ideally made to move at exactly the speed of the star image, so that the latter will appear as a small circular dot on the film, which is carried on the slide. The successive flashes of the reticle (which is of course in motion relative to the film) appear on the film, the vertical lines being equi-spaced horizontally and the horizontal lines merely extending themselves with each flash.

The reduction of observations, which consists essentially of measurements of distances, on the film, from star image to horizontal and vertical wire images, is greatly complicated if the tracking velocity is incorrect and thus causes the star images to be elongated. One does not know whether the centre of gravity of the image corresponds to the middle of the exposure (since various atmospheric conditions, e.g., scattered clouds, could conceivably cause the last half of the exposure, for example, to contribute most of the light from the star). Furthermore the time-phase between the opening and closing of the shutter and the sequence of flashes is not predictable; and calculations show that this must be measured if the tracking velocity is in error by more than .001. These are "nuisance" measurements. If they are omitted or erroneously recorded the observation cannot be used; or if the erroneous data is unwittingly used, the observation, accepted in good faith, will be not indicative of the true position of the star.

2. The Original Computer and Associated Errors

The original velocity generator was designed to use the transmitted servo-mechanism signals that serve primarily to position the mirror axis. These signals are sent on transmission lines to the eyepieces of both telescopes, where servo-receivers cause a shaft to adopt a position corresponding to the declination of the star. Since the velocity of a star image is proportional to cos δ rather than δ, it is necessary to have the linear (with δ) rotational motion of this shaft generate a cos δ-functional velocity.

The method used will become apparent with reference to Figure 1A. A very accurately ground limaçon cam is fixed to the shaft, so that its associated cam follower performs a simple harmonic motion with cam rotation, hence a cos δ-functional displacement has been obtained. The variable velocity is obtained from a ball-and-disc integrator, for which it can be readily seen that the output-shaft velocity is linearly proportional to the ball-cage displacement from centre, provided that the input velocity is constant. In the mirror transit application, the integrator input and output are geared in such a way that about .95 of the maximum possible velocity corresponds to the equatorial stars. This value was selected at a time when the exact focal length, and hence maximum plate velocity, was unknown; so that it was necessary to keep slightly below the maximum setting of the integrator in calculating subsequent gear ratios in the mechanism. To ensure the required displacement of the ball cage, and to allow for future adjustments, (e.g., due to focal length changes with temperature) the cage push rod and the cam follower are linked with a lever with adjustable fulcrum as indicated. The reproducibility of velocity in a good ball-and-disc integrator is better than .1 of 1%. The first limitation to the accuracy of the system became serious with the realization that the star positions in declination could not be entered exactly on the servo-mechanical transmitters, but had to be rounded off to the nearest 3′ of arc so that certain other unavoidable conditions could be satisfied. So long as cos δ changes slowly with δ the velocity error is negligible and can be ignored, but close to the pole cos δ is changing quite rapidly and the errors become appreciable. The second limitation is the more serious of the two and, in a perverse way, did not come to mind at any time during the design stage. The atmospheric refraction for low stars makes them appear higher than they really are, so the servo-mechanically transmitted positions for the mirror axis have to be adjusted correspondingly. This also changes the limaçon cam angle and hence output velocity—which introduces an error as the refraction does not affect the horizontal velocity. Hence for sub-polar stars the errors of velocity due to cos δ and refraction combine sometimes in very serious ways. A detailed analysis of the contribution of these errors is given in the previous paper.

In addition to the above difficulties the computer gives rise to other errors in velocity which were at first incomprehensible, until with the passage of time it was observed that these errors increased with decreasing ambient temperature. This phenomenon seemed to indicate that thermal contraction of the various linkages could not be neglected, and led to the following analysis of potential sources of error.

Figure 1B condenses the components used in the analogue computer (Figure 1A) into a schematic system

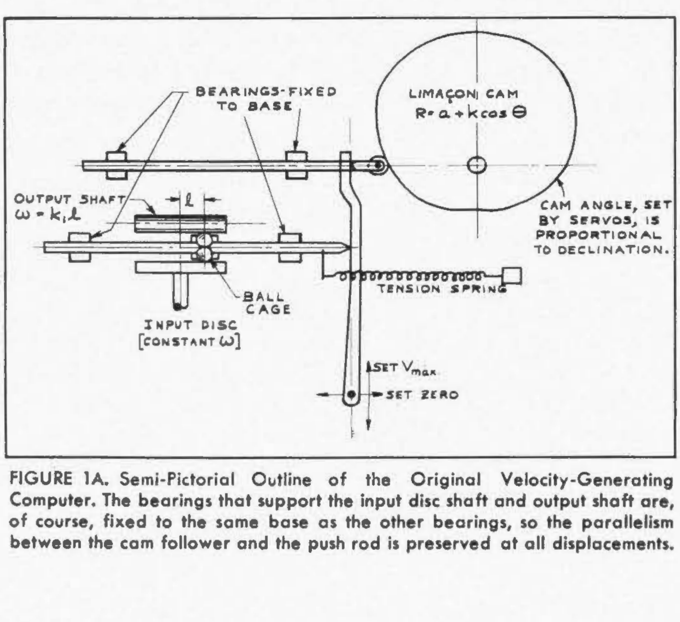

FIGURE 1A. Semi-Pictorial Outline of the Original Velocity-Generating Computer. The bearings that support the input disc shaft and output shaft are, of course, fixed to the same base as the other bearings, so the parallelism between the cam follower and the push rod is preserved at all displacements.

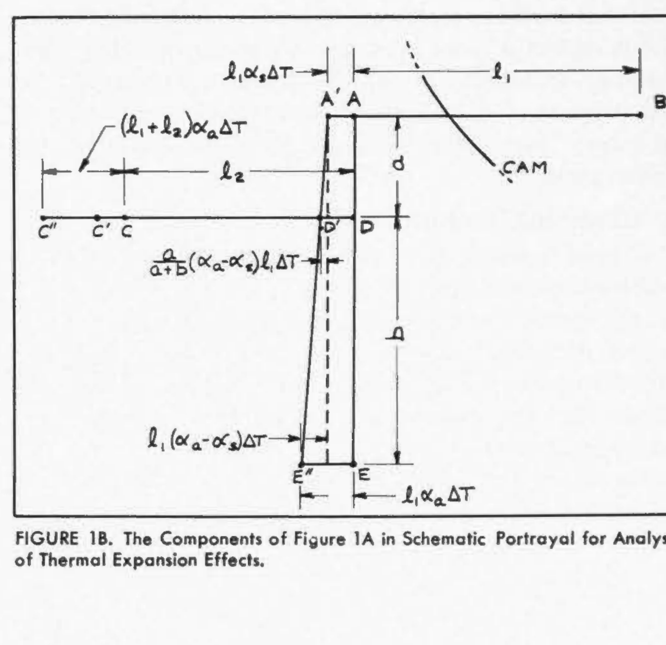

FIGURE 1B. The Components of Figure 1A in Schematic Portrayal for Analysis of Thermal Expansion Effects.

more adaptable to analysis. Examine first the thermal expansion effects. Suppose that at some temperature T the cam is set to correspond to $\delta = 90°$, so that the integrator ball cage is in the centre of the disc and hence gives the required zero output velocity. B represents the cam axis, ADE the lever, CD the integrator push rod, and C the axis of the input disc. AB, the cam radius vector plus a portion of the follower, can be treated as a unit since cam and follower are made of identical materials. It will also be in accordance with the facts if we assume that C, E, and B are on a common aluminum base having coefficient of expansion α_a, and that AB, ADE, and CD are made of identical steels having coefficient α_s; $\alpha_s < \alpha_a$.

Now suppose temperature increases to a new value T°. We take B as the origin. It can be shown (*see* Appendix 1) that the horizontal component of expansions only need be considered, therefore E moves to E″ and C moves to C″ as the base expands. A moves to A′ (hence D to D′) and C (as it indicates the ball cage) to C′ as the steels expand. The lever ratio, $\frac{b}{a+b}$ remains constant. We wish to find the order of magnitude of C′C″, which will effectively move the integrator push rod off its zero velocity position.

The displacement CC″ is given by

$$CC'' = (l_1 + l_2)\alpha_a \Delta T$$

and displacement CC′ by

$$CC' = (l_1 + l_2)\alpha_s \Delta T + \frac{al_1(\alpha_a - \alpha_s)\Delta T}{a+b}$$

$$\text{Whence } C'C'' = \left[(l_1 + l_2)(\alpha_a + \alpha_s) - \frac{al_1}{(a+b)}(\alpha_a - \alpha_s)\right]\Delta T$$

$$= (l_1 + l_2 - \frac{al_1}{a+b})(\alpha_a - \alpha_s)\Delta T$$

$$= (\frac{l_1 b}{a+b} + l_2)(\alpha_a - \alpha_s)\Delta T$$

assigning values $\alpha_a = 12.8 \times 10^{-6°F}$

$$\alpha_s = 9.2 \times 10^{-6°F}$$

$$\Delta T = 50°F$$

$$l_2 = l_1 = 3''$$

$$\frac{a}{a+b} = 1/6$$

$$C'C'' = (6 - .5)(3.6 \text{ x } 10^{-6})\, 50 = .99 \times 10^{-3}$$

or approximately .001″.

Since the total throw of the integrator push rod is .75″, the above shift introduces an error of about .13%. This is barely tolerable and then only if no other contributing errors act in the same direction. For the latitude of Ottawa, ΔT can easily be 70° with a proportionately greater error as the result. The effect of a temperature increase is, then, to move the ball cage farther out along the disc radius and increase speed. If the ball cage is on the "opposite" radius (as it is for lower culmination stars requiring reverse drive) then the effect is to decrease the speed.

We now investigate the nature of the errors resulting if the line of action of the cam follower does not pass through the origin of the cam, and refer to Figure 2. In this figure the ideal line of action is CO where O is the cam origin. Actually the line of action is C′DE, not directed at the origin.

We write the equation of the cam as

$$R = a + k \cos \delta \text{ so that}$$

$$\delta \text{ in the equation} = \delta \text{ the declination angle.}$$

The slope of the cam at any point is then $-\sin\delta$. Referring to Figure 2, and noting that the angles and displacements are small, we have

$$\Delta = (R-S)\varphi$$

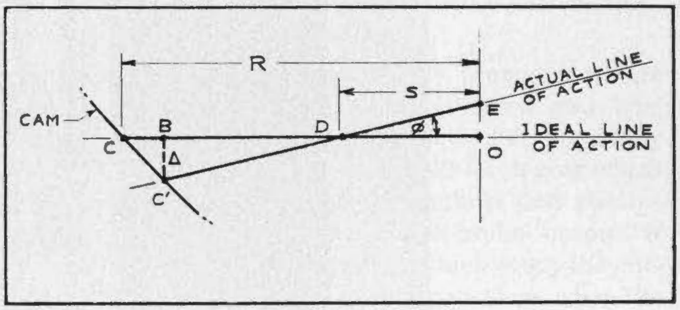

FIGURE 2. Geometry of Ideal and Actual Relationships of Cam Follower to Cam.

To a first order approximation C′O falls short of CO by CB, which is then the error in displacement of the cam follower.

We have

$$CB = \Delta\sin\delta = (R-S)\varphi \sin\delta$$

The error will not be identically zero unless $\varphi = 0$. In the general case it will be zero when $\sin \delta = 0$, i.e., when the cam is set for equatorial stars; and at the point where R = S. If S = O this corresponds to the case where the index is improperly set.

When $\sin\delta = 1 \qquad CB = (R-S)\varphi$

and if $R-S = 1.5$ inches

then φ must be less than 3′ for the displacement error to be less than .001

The errors introduced by differential thermal expansion could be eliminated by using a steel base instead of the present aluminum one, which would necessitate complete rebuilding of the velocity generator; or by temperature control of the whole velocity generator. Since refrigeration would not be feasible, it would be necessary to select some temperature, higher than the ambient due to normal power consumption, and use thermostatted heaters to maintain this value. This proposal leads in turn to a further problem, viz, the enclosure must be extremely well insulated so that heat dissipation into the atmosphere does not affect the astronomical seeing.

Errors due to misalignment of the cam follower are more elusive and hence more difficult to eliminate. It has been shown that the angle φ between actual and ideal lines of action of the cam follower must be less than 3′, and under the existing mechanical arrangements one cannot guarantee this order of accuracy in setting-up. Indeed the very act of removing the velocity generator from the test bench to the face of the mounting plate (on the instrument pier) may introduce errors of this order of magnitude. *See* Figure 3.

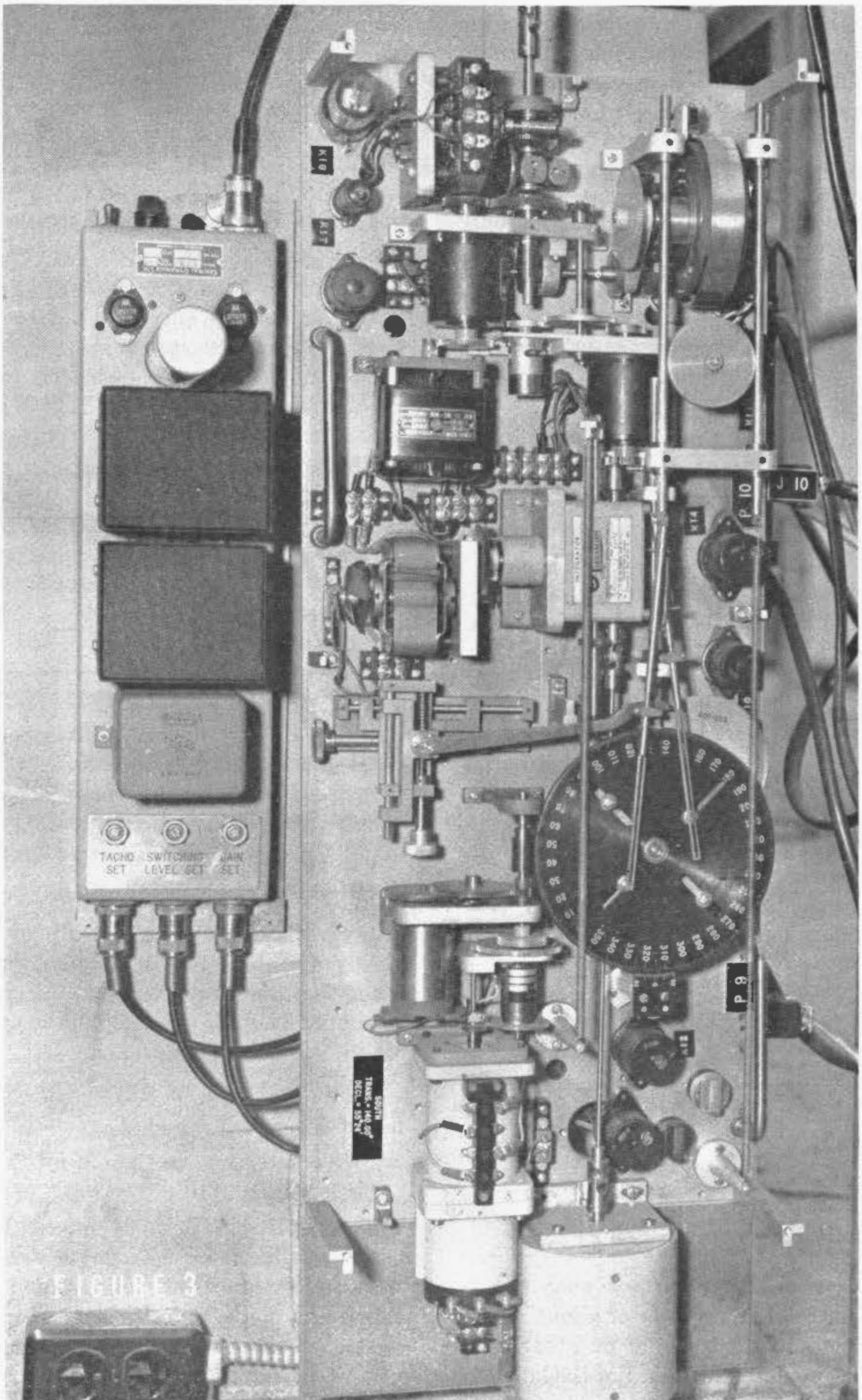

FIGURE 3. Original Velocity Computer on North Collimator Pier. It will be noticed that accuracy of alignment between cam follower and cam axis cannot be continually monitored because of the system of connecting rods on top of the graduated gear.

3. Principle of the New Computer

Consideration of the foregoing factors culminated in the decision to approach the whole problem of precise velocity generation from another viewpoint. First of all, it was clearly apparent that the velocity would have to be set independently of other parameters of observation. Furthermore, digital input of velocity would be a desideratum if the digits could be related one-to-one with $\cos \delta$—a factor always available from the various ephemerides. While there are probably several sophisticated ways of deriving a variable velocity accurate to .1% or better, (particularly if one has unlimited funds and time at their disposal) it is logical to give careful consideration to ease of maintenance and trouble-shooting in whatever method is selected. It would be of little profit to have such a complex system that specialized help would be required in the event of breakdown.

The possibility of using a synchronous motor and an electronic variable frequency oscillator-amplifier was considered and rejected. There are a few oscillators on the market with digital frequency selection but the advertised accuracy is never as good as .1%. To make such a system feasible the output frequency must be measured by a second precise unit. The difference between actual and desired frequency must then be used as an error signal to correct the oscillator.

The straightforward simplicity of a completely mechanical system always held great appeal. At first the problem of making a 1,000-speed gear box seemed insoluble within reasonable space limitations, until the mechanical analogues of binary arithmetic were considered. It is well known that the decimal digits from zero to nine can be made up from various combinations of four binary digits in the so-called 4-2-2-1 code. In computer techniques, this means that a "yes" in the least significant binary stage means a decimal "1", in the next stage a decimal "2", in the next again a "2", and in the most significant a "4". So that a decimal 8, for example, is made up of "yes" signals from 4, 2, and 2 of the binary stages.

If four velocities in the 4-2-2-1 ratio each-to-each could be extracted, as desired, from a single input velocity, and furthermore be added in various combinations, then a ten-speed (zero being considered as a speed) gear box would result. This would take care of one decimal digit in $\cos \delta$. If then there are three such gear boxes made, and number 2 is run at an input speed 1/10 of number one and number 3 at input speed 1/100 of number one, and if furthermore their outputs are added together mechanically, then a 1,000-speed gear box would result, in which the speeds will form an arithmetic series from 000 to 999.

Two rotating shafts can be combined into a single rotation by a mechanical differential, in which the output velocity is one-half the arithmetic sum of the two input velocities. This resultant can be further combined with other velocities (or indeed the resultant from another differential) through a second differential; and so on in a cascade of differentials. The only drawback is that net velocity is lost on each addition because one gets only the average, not the sum, of the two input velocities; and in some applications this might preclude extended use of differentials. It is however, an advantage in the possible mirror transit application, since the lead screw of the right ascension drive moves at only about 30 r.p.m. whereas the synchronous motors run at 1,800. Thus the differentials can occupy the low torque, high-speed stages of the drive system, and can be of small physical size. Furthermore the various gears will rotate many times during one excursion of the right ascension slide. No gear is completely perfect, and has small errors of concentricity and ellipticity, hence any two gears will have their errors combined in some way so that, depending on the phase of the mesh, the driving ratio and back-lash will differ slightly. The differences are minute for precision class gears; and for many rotations will average into negligibility insofar as the final velocity of the right ascension lead screw is concerned.

The first experimental test of the proposed system called for the construction of one 10-speed gear box, shown in Figure 4. The input shaft, on the left, nominally rotates at 1,800 r.p.m. and carries a 40-tooth gear which drives four 80-tooth gears on the inputs of four electro-magnetic clutch-brakes. The clutch-brake output is braked unless current is applied, in which event the output is clutched to the input. The outputs then, when the respective clutch-brakes are energized, rotate at 900 r.p.m. In Figure 4A, pinion (1) drives gear (2) through a 1:4 reduction and pinion (3) drives gear (4) through a 1:2 reduction. Gears (2) and (4) are the end gears of a differential (partly concealed by the post) for which the output appears at gear (5). If pinions (1) and (3) are driven, the velocity at (5) will be $\frac{(900}{4} + \frac{900)/2}{2} = 337.5$ r.p.m. Gear (5) is further coupled 1:1 (by an idler pinion which is concealed by the bracket B) into the gear 6 which is one of the end gears of the final output differential.

In Figure 4B, which is a view of the gear box from the opposite side to 4A, pinion (7) drives gear (8) 1:2, and pinion (9) drives gear (10) 1:1. Gears (8) and (10) are the end gears of a differential for which the output appears at gear (11). If (7) and (9) are driven, the output at 11 will be $\frac{(900}{1} + \frac{900)/2}{2} = 675$ r.p.m. Gear (11) is further coupled 1:1 by the idler pinion (14) into gear (12) which is the other end gear of the final output differential. This differential averages the two previous summation velocities, giving 506.25 r.p.m.

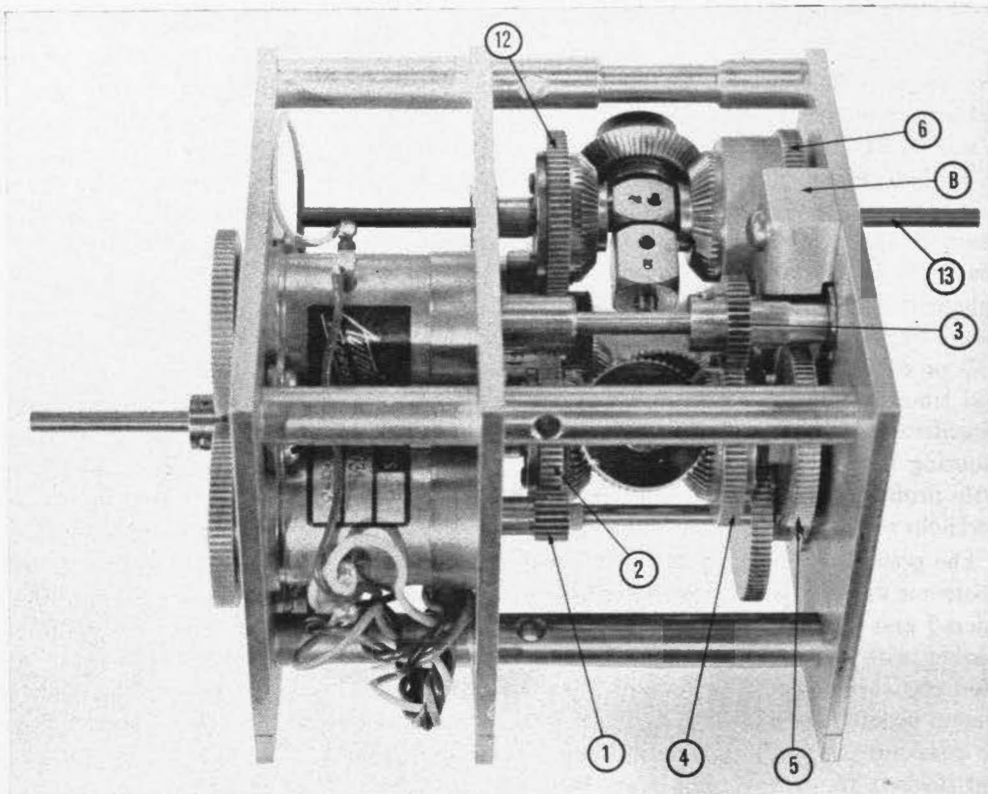

FIGURE 4

Two Views of the Experimental 10-speed Gear Box. The pinion at (7) is rather difficult to see, being mostly obscured by gear (8). The final mixing differential end gear (6) had to be of special extended design so that it would meet its idler pinion from gear (5).

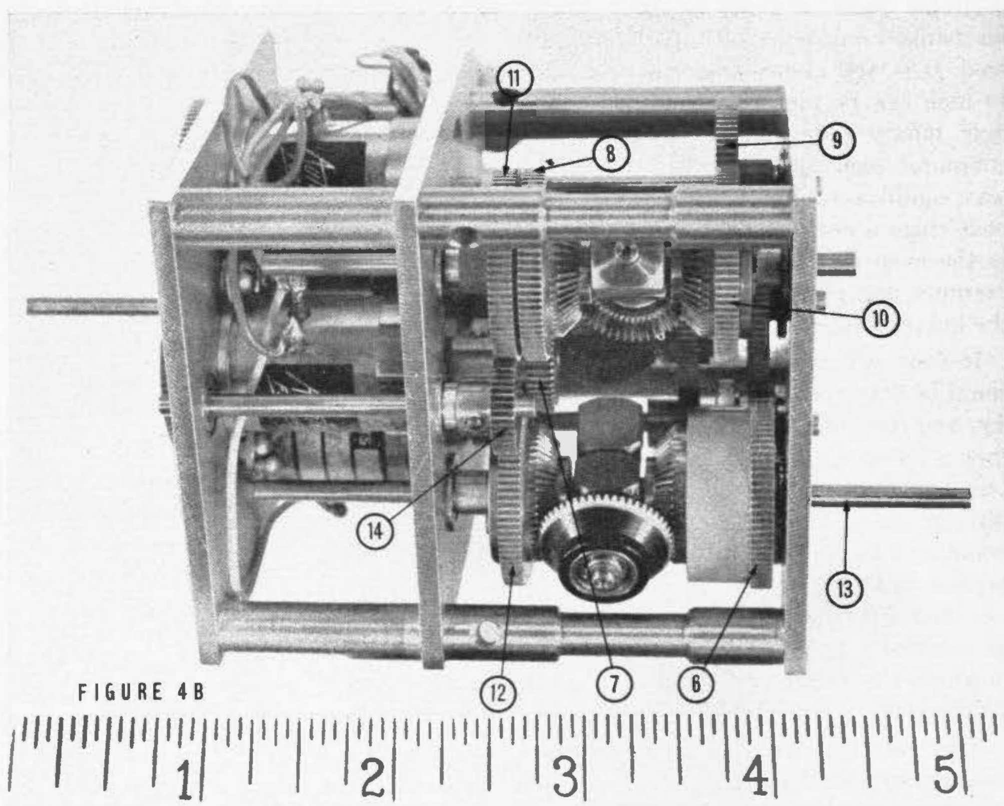

FIGURE 4B

(when all clutches are energized) at shaft (13). If the various clutches are energized in accordance with the following table, 10 decimally-related speeds are available. "O" indicates off, "X" indicates on.

TABLE 1. Decimal-to-binary conversion code.

Decimal Speed	Stage Reduction			
	1:1	1:2	1:2	1:4
0	O	O	O	O
1	O	O	O	X
2	O	O	X	O
3	O	O	X	X
4	O	X	X	O
5	X	O	O	X
6	X	X	O	O
7	X	X	O	X
8	X	X	X	O
9	X	X	X	X

For testing purposes the gear-box output was directly coupled to a photoelectric tachometer and the input driven at 1,800 r.p.m. from a synchronous motor. The pulses from the tachometer were electronically counted, the counter "gate open" time being set so that theoretically the counter would read 9,000 if all clutches were energized. On the first test the average counter reading was 8997 ± 0005. This was a rather alarming scatter and did not auger very well for the project, it being supposed that gear errors would prove too large to be tolerated after all. The test was repeated somewhat later, giving readings averaging 9002 ± 0004. The suggestion was then made that the synchronous motor be driven from a precise 60-cycle crystal-controlled source at the observatory time room. When this was done the readings became 9000 ± 0001, thus showing that the variations in domestic power-line frequency were the main cause of the previous scatter. The .0002 scatter is quite tolerable—actually the counter readings themselves contribute half of this because the phase between start and stop gates and incoming pulses changes for each sampling. For every decimal speed selected the nominal counter readings agreed with prediction and with the same scatter magnitude.*

4. Extension of Experimental Model into Complete Computer

The next problem was the combining of three such 10-speed gear boxes into a 1,000-speed gear box. Since the mechanical differential is able to combine only two velocities at once it would be necessary to do the combination of the three outputs in two stages. This implied that, for very little extra effort, a 10,000 speed box could be made by using four 10-speed boxes, using them in pairs through differentials to make two 100-speed gear boxes. The two resultants could then be combined into the final output. It is of course, still true that the output velocities of any gear box, digit for digit, must be one-tenth that of the gear box for the next-higher order of decimal digit, this being a net ratio, i.e., it does not matter when the ratio is achieved in the gear train so long as the final adding differential receives velocities having the "times 10" correspondence.

The array of the 10-speed gear boxes will depend on many engineering factors, not the least of which is the shape and size of the space available. On the mirror transit velocity-generating computer, the space available was nearly 12 inches square and about 4 inches high, and the way this space was utilized is shown in Figure 5.

The two 100-speed gear boxes are identical. Each is made by making two 10-speed gear boxes side by side, but with one being a mirror-image of the other so that the two output shafts will be relatively close together. In Figure 5 the unit below provides the two highest-order digits of cos δ. The other unit, which is inverted for over-all design reasons, provides the two lowest-order digits. In the lower unit, shafts (1) and (2) are the outputs of the two 10-speed assemblies. The right-hand assembly is driven directly from the 1,800-r.p.m. motor, the other at 2/11 of this speed through an idler and gear (3). The mixing differential is at (4). Its outer end is driven 1:1 by the fast assembly; and its inner end through a gear reduction of 11:40 from the slow assembly. The product of 2/11 and 11/40 reduction gives the required over-all 1:10 ratio between the inputs to differential (4), digit-for-digit, and hence gives 100 decimally-related possible output speeds. The output of differential (4) is geared, 1:1, onto one end of differential (5).

In exactly the same way the right-hand 100-speed gear box is driven by an 1,800-r.p.m. motor so that the output appears at shaft (6). Since this 100-speed gear box provides only the two lowest order digits of cos δ, its output, relative to the output of the other 100-speed gear box, must have a 100:1 reduction where it goes into the final mixing differential at (5). This is done by using a worm-and-worm gear input which gives the desired ratio directly, so that the shaft at (7) represents the output of a gear box having 10,000 speeds. For a star very close to the celestial equator, for which cos δ = 1.0000, it is necessary to compromise by programming 9999. This is sufficiently accurate for all purposes and removes the necessity for the highest order 10-speed gear box to be modified to provide 11 possible output speeds 0 to 10 inclusive.

It is a straightforward mathematical exercise to show that, for 1,800-r.p.m. inputs, the output velocity at shaft (7) is 140.611 r.p.m. when the 9999 selection is made. The lead screw of the mirror transit has to be rotated at 29.476 r.p.m. for a star having cos δ = .9999. There is another shaft rotating at twice this velocity into

*The 4-2-2-1 code selected is not the only possible one. For a discussion of other possible codes, see appendix II.

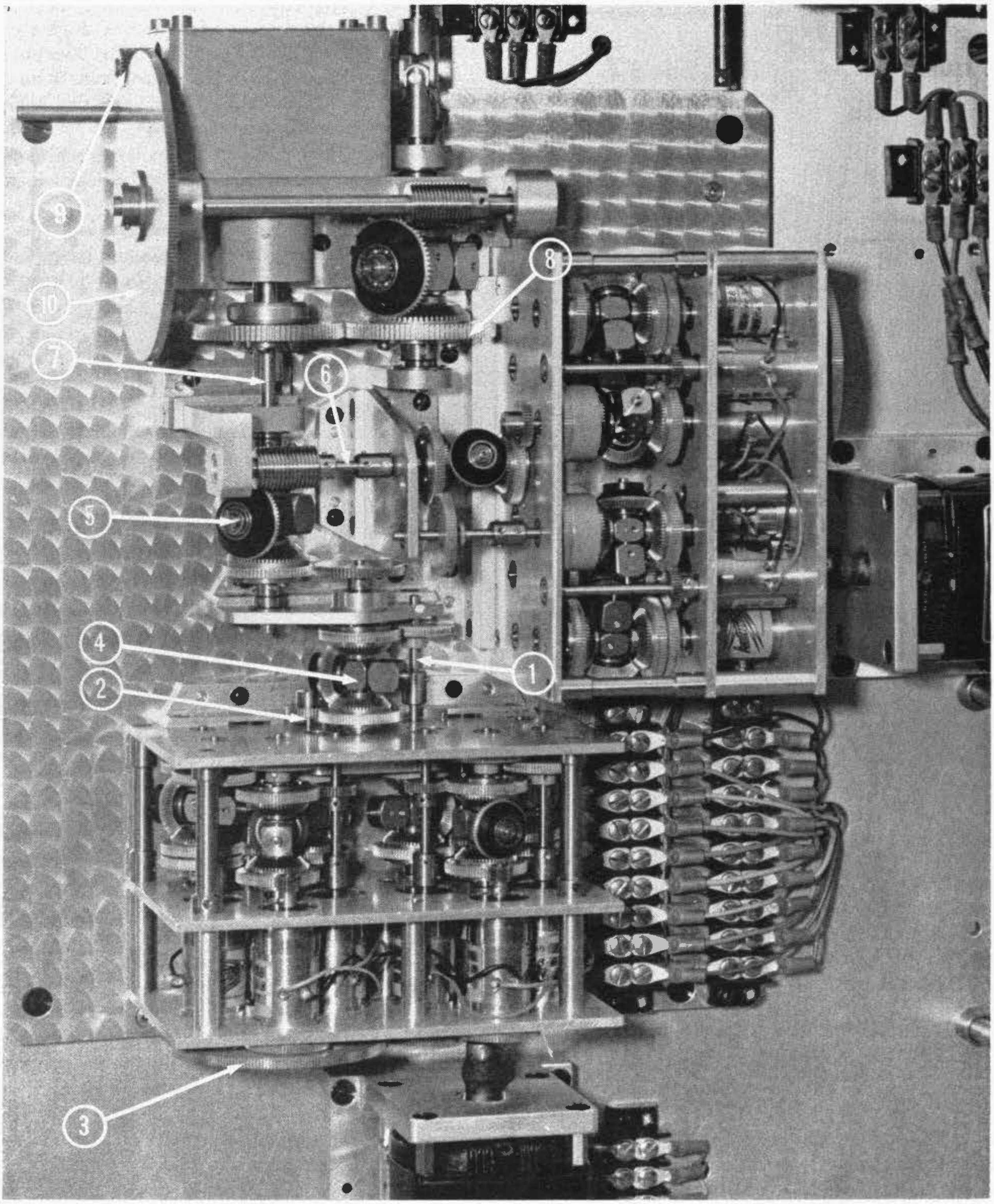

FIGURE 5. The Complete 10,000-speed Gear Box. The lower unit provides the two highest-order digits of cos δ, the right-hand unit the two lowest-order digits. Following the two 100-speed gear boxes, the differentials and gear-tooth sizes are increased in steps in order to handle the increasing torques as speed drops.

which the 10,000-speed gear box must be coupled; and in order to find the required gear ratio, one must divide 140.611 by 58.952, and hope the quotient will be expressible in terms of a proper fraction of reasonable size, where the numerator and denominator will then represent the numbers of teeth of the driver and driven gears. Actually the foregoing is a simplification. The value of 29.476 r.p.m. for lead screw velocity is an average figure for the two mirror transit collimators, whose mean focal lenghts, and hence lead screw velocities, differ by about a factor of .001. Furthermore, if the focal length of either collimator changes with the gross temperature changes from summer to winter, then the lead screw velocity will also change and this must be incorporated into the digital velocity-generator.

This is done by extracting from shaft (7) a small portion of its angular motion and adding it to, or subtracting it from, the original motion. In order to make this a continuously variable increment, use is made of the ball-and-disc integrator. Varying the position of the push rod will change the magnitude and sign of the small incremental motion, which can be fed into the original motion via a differential and associated gears. At first thought it might be supposed that the nominal position of the push rod would correspond to the zero output of the integrator; but this is not the case as the integrator should not be run with the ball cage at dead centre for any length of time. A little reflection will show that the disc centre and the contacting ball abrade one another in this position. Hence a better way is to start with the push rod half way out from the zero position in either direction and calculate subsequent gear ratios using this as the nominal zero. The resulting increment can be either additive or subtractive as one wishes. Suppose for purposes of argument it is additive (as in the present application) then if the push rod is moved towards zero, the added amount will be decreased, and conversely if the push rod is moved out, the added amount will be increased. The ratio between the direct input to one end of the mixing differential and the ball-and disc-input to the other end will determine to what degree the added increment can be increased or decreased.

The large gear on shaft (7) is geared, 1:1, with the differential end gear (8), and in the absence of any input at the other end of the differential the latters's output velocity will be one-half of input. Shaft (7) also feeds directly into a ball-and-disc integrator, whose output at pinion (9) drives gear (10), which in turn drives the worm gear at the other end of the differential. The gear ratios are so arranged that, with the integrator push rod halfway out from the zero position, the corrective input is .0015 of the main input at the differential, and additive, so that the final output velocity becomes

$$140.611 \frac{(1 + .0015)}{2} = 70.411 \text{ r.p.m.}$$

The quotient of 70.411 and 58.952 is 1.19447 and fortuitously it happens that gears of 86 and 72 teeth have a ratio of 1.19444 which could not be more ideal. The ball-and-disc integrator adjustment makes possible velocities from 70.305 to 70.516 r.p.m. at the output, i.e., $\pm$ .15% of nominal. The entire velocity computer can be seen in Figure 6.

The switching array for the drive system is shown in Figure 7, where the four columns of pushbuttons on the right, each numbered 0-9, represent cos δ. The rest of the panel is devoted to other controls used in the mirror transit operation. The pushbuttons and associated switch modules are interlocked within decades so that pressing one releases the previous number. The conversion of the decimally selected signals to binary signals for clutch activation is done through a diode matrix, shown schematically in Figure 8.

The system has been in use for several observing periods and has given no difficulty. At the present time the synchronous motors are driven from the domestic 60-cycle power line, as present experience is that the frequency is fairly exact and stable during the observing periods, when domestic and industrial loads are light and constant. If evidence accumulates that controlled 60-cycle power must be used, it will be necessary only on the motor that provides the two highest-order digits.

The operating cycle is arranged so that the programmed gear train begins to run when the observer initiates the rapid moving of the lead screw to its start position; hence the backlash of all gears, up to and including the ball-and-disc integrator, is taken up prior to the initiation of the exposure.

After this point in the gear train, the only two meshes left are from the tracking clutch output to the "visual observation differential" referred to in Figure 8; and from the vertical drive shaft to the horizontal collimator lead screw. The shutter operation which opens the optical path is started simultaneously with the tracking drive. Since the shutter takes some 800 milliseconds to open, these two final backlashes have been taken up and the plate carriage is in uniform motion by the time the light of the star begins to fall on the film.

The writer wishes to acknowledge the cooperation of E. Sanders and his staff at the Observatory machine shop, where the velocity-generating computer was made. The execution of the work was almost solely the responsibility of J. C. Reynolds: and the fact that no jig-boring machine was available to him makes the fine workmanship and close-tolerance achievements doubly commendable.

Reference

Brealey, G. A. and Tanner, R. W. Photographic registration of transits and reduction of observation on the Ottawa mirror transit telescope. *Dom. Obs. Pub.* v. XXV, no. 3, 1963.

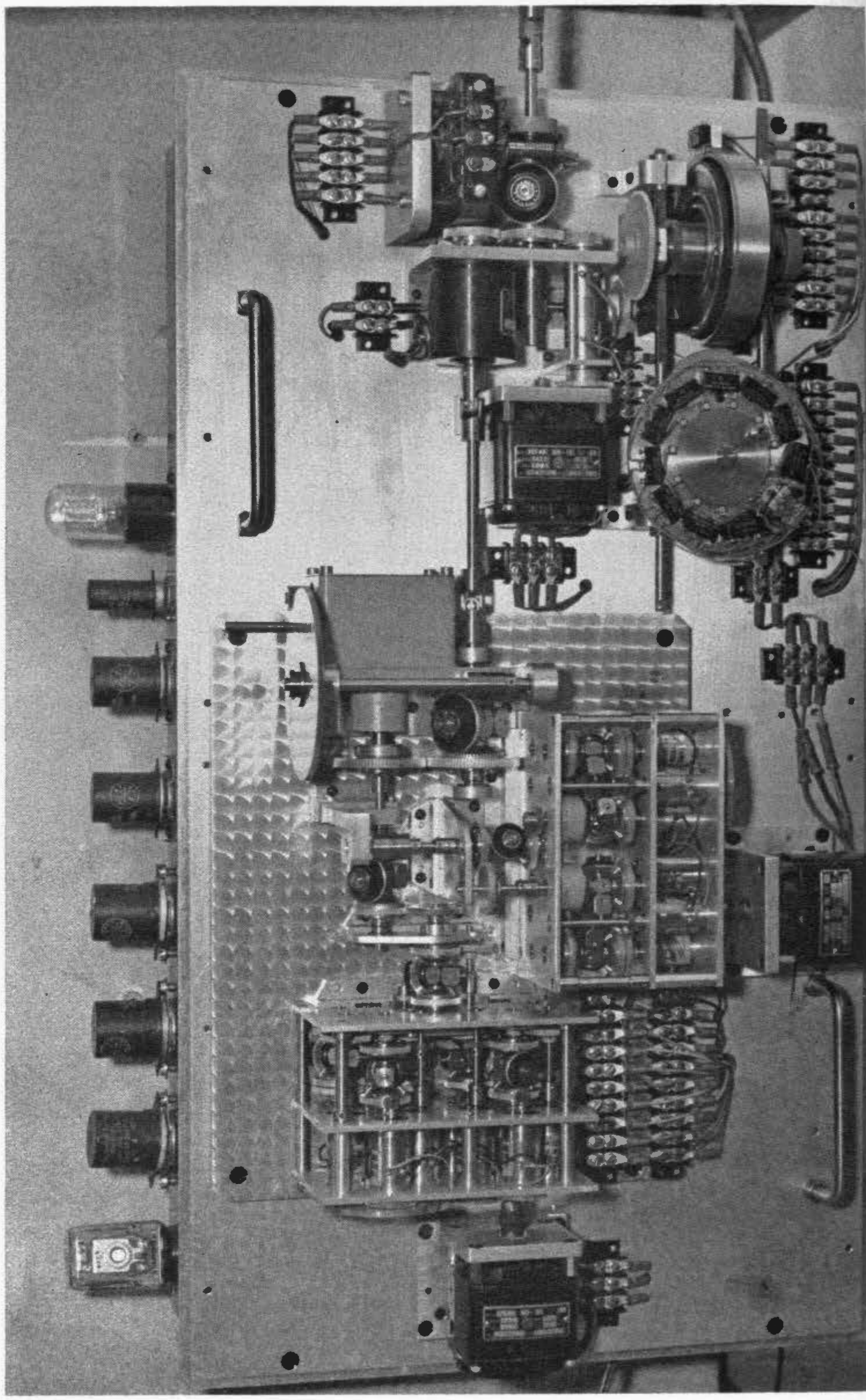

FIGURE 6

The Complete Velocity Computer on the North Collimator Pier. The circular array of microswitches uses the first decimal digit of cos δ to drive the two parallel racks below, which in turn rotate limit switches that shorten the travel of the right ascension slide proportionately as stars closer to the pole are observed, thus keeping the exposure time constant. The motor and clutch near the microswitches are used for high-speed motion of the lead screw. The motor and differential at the very top are used for adding to or subtracting from the lead screw a velocity increment for visual observations.

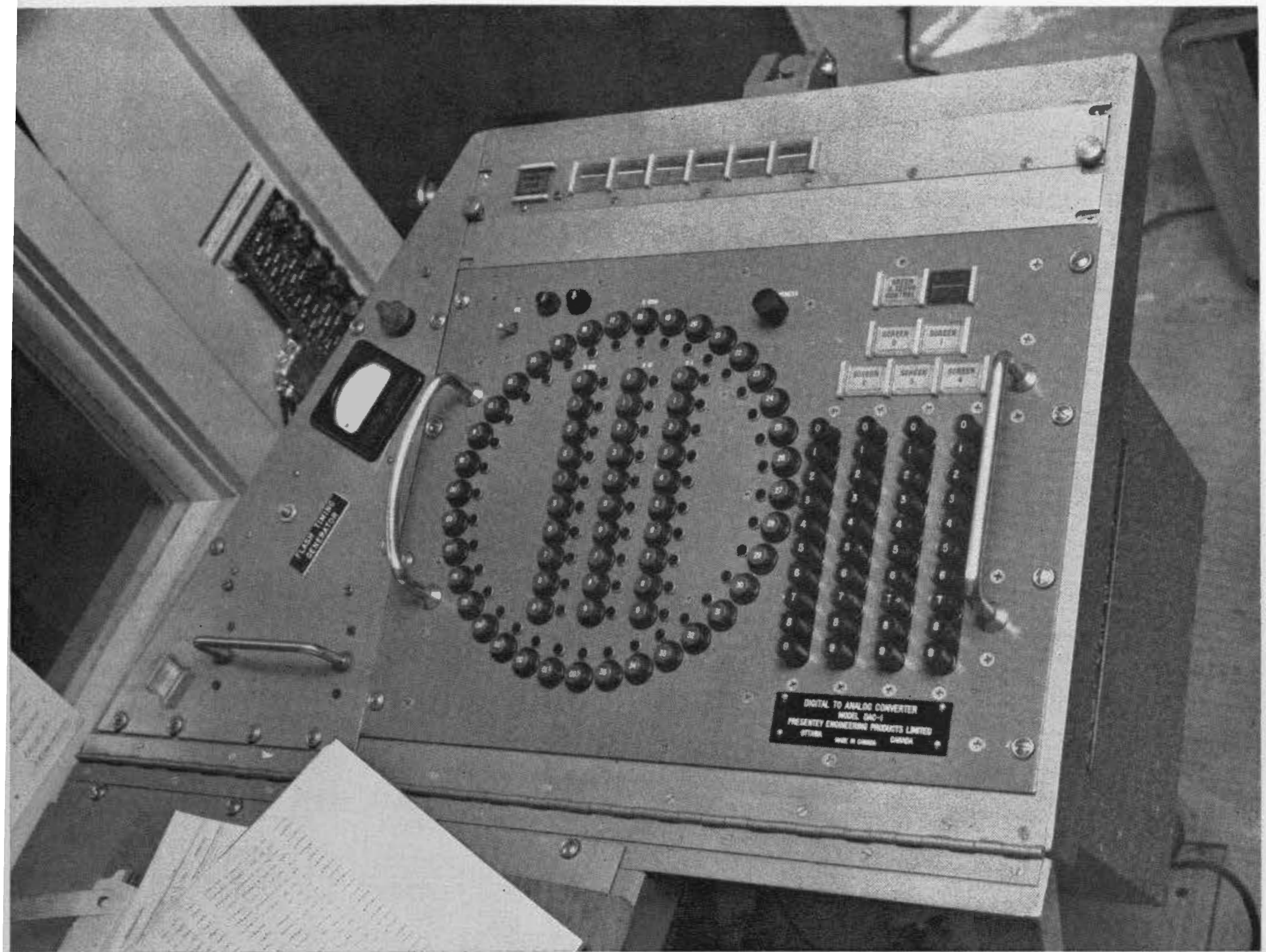

FIGURE 7. The Control Panel for Observing Parameters. The left side is devoted to the digital servo-transmitter, the lower right to velocity selection, and the upper right to the screen selector (the various screens dim the bright stars so they will not overexpose). These are the only data that have to be entered. The initiation of exposures is controlled by a programmable digital clock, not seen in the figure.

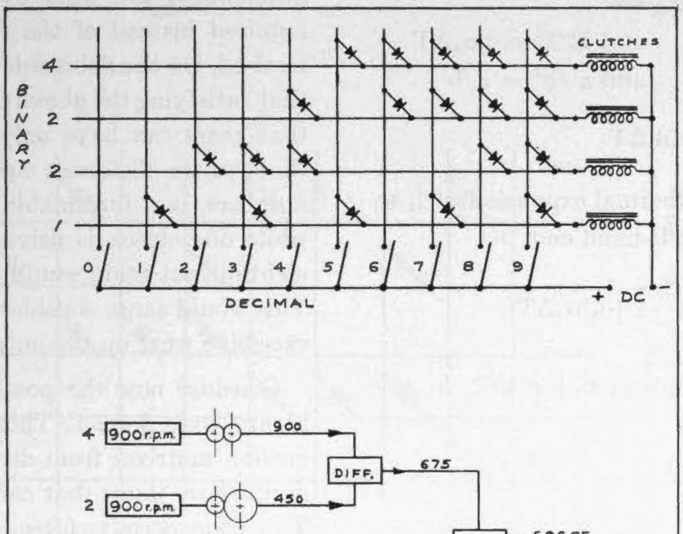

FIGURE 8. The Decimal-to-Binary Conversion Matrix (above) and a Block Diagram of a 10-speed Gear Box. The switches numbered from 0 to 9 are actually relay contacts associated with the decimal keyboard. The velocities shown for the gear box are those for the highest-order digit of cos δ.

Appendix I

The differential thermal expansion in both vertical and horizontal coordinates of Figure 1B will be considered, and the result will be shown to be equivalent to that obtained by considering the horizontal component only. Once again the origin is taken at B, (Figure 9), and with

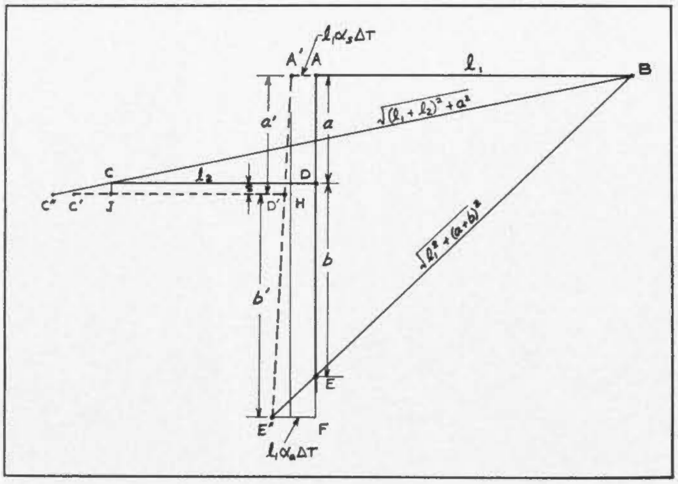

FIGURE 9. Schematic of original velocity-generating computer, showing relative motions with temperature change when both vertical and horizontal components are considered.

an increase in temperature E moves to E″ and C to C″ as the base expands; while A moves to A′ (and hence D to D′) and C to C′ as the steels expand.

$$EE'' = (\sqrt{l_1 + (a+b)^2})\alpha_a\Delta T$$

whence $EF = (a+b)\alpha_a\Delta T$ and $E''F = l_1\alpha_a\Delta T$

furthermore $S = a\alpha_s\Delta T$ and $a'/b' = a/b$

so that $D'H = \dfrac{a}{a+b}(\alpha_a - \alpha_s)l_1\Delta T$

To find C′J, we must add the thermal expansion of l_2 to the *total* displacement of its right-hand end.

$$\therefore\ C'J = \frac{a}{a+b}(\alpha_a - \alpha_s)l_1\Delta T + (l_1+l_2)\alpha_s\Delta T$$

This is identical to the expression for CC′ in the approximation. Now for C″

$$C''B = (\sqrt{(l_1+l_2)^2 + a^2})\alpha_a\Delta T$$

whence $C''J = (l_1+l_2)\alpha_a\Delta T$

which likewise is identical to the expression for CC″ in the approximation.

So that C″C′, the relative displacement of ball cage from push rod, is correct as given by the formula in the approximation.

Appendix II

The 4-2-2-1 binary code was selected as being the most suitable for the computer mechanism. However, there are other codes that are discussed briefly with the reasons for their rejection in favour of the 4-2-2-1 code.

One might first of all ask why not avoid binary equivalents altogether and use strictly decimal concepts throughout each decade. Suppose an input shaft carries a pinion that drives nine gears, each of which is the input of a clutch, which in turn has the property that when de-energized its output is free and when energized its output is connected to its input. At the output ends of the clutches place other gears that all engage with a single gear on an output shaft. Call the input gear ratios for each clutch $I_1, I_2 \ldots I_9$ and the output ratios $O_1, O_2 \ldots O_9$, then to obtain nine decimally related output speeds by energizing one of nine clutches the gear ratios must satisfy the following conditions, where K is any constant.

$$I_1O_1 = K$$
$$I_2O_2 = 2K$$
$$I_3O_3 = 3K$$
$$\vdots$$
$$I_9O_9 = 9K$$

Obviously the advantage of this proposal is that no differentials are used even though nine clutches are required instead of the four in the 4-2-2-1 conversion method. On the debit side it is submitted, without proof, that satisfying the above equations, within the limitation that gears can have only whole numbers of teeth and that centre distances are rigorously subject to tooth numbers, is a formidable design problem. Furthermore while one clutch is driving the output shaft the other eight output gears would be driven by the output shaft. This would cause considerable loading of the output and excessive wear on the output shaft gear.

Consider now the possibility of using the "straight" binary code 8-4-2-1. This has the advantage that conversion matrixes from decimal to binary and binary to decimal are items that can be purchased "off the shelf". This code permits fifteen possible speeds of which the highest six are of no value in the present application; however the main reason for its rejection is the wide divergence in the gear ratios between the highest and lowest order binary digits. A spur gear ratio of 1:8 is a rather cumbersome thing. If for example a pinion is

3/8 inch diameter, then the driven gear has a diameter of 3 inches and takes up nearly as much space as all the other gears put together.

A third possible code was seriously considered. To the writer's knowledge, it does not have any formal name and is a hybrid of products and sums. The decimal digits from zero to nine are obtained as follows:

$$0 = 0$$
$$1 = 1$$
$$2 = 3-1$$
$$3 = 3$$
$$4 = 3+1$$
$$5 = 3\times 2-1$$
$$6 = 3\times 2$$
$$7 = 3\times 2+1$$
$$8 = 3\times 3-1$$
$$9 = 3\times 3$$

The mechanical anologue of these expressions is difficult to describe in words but is clearly apparent in Figure 10.

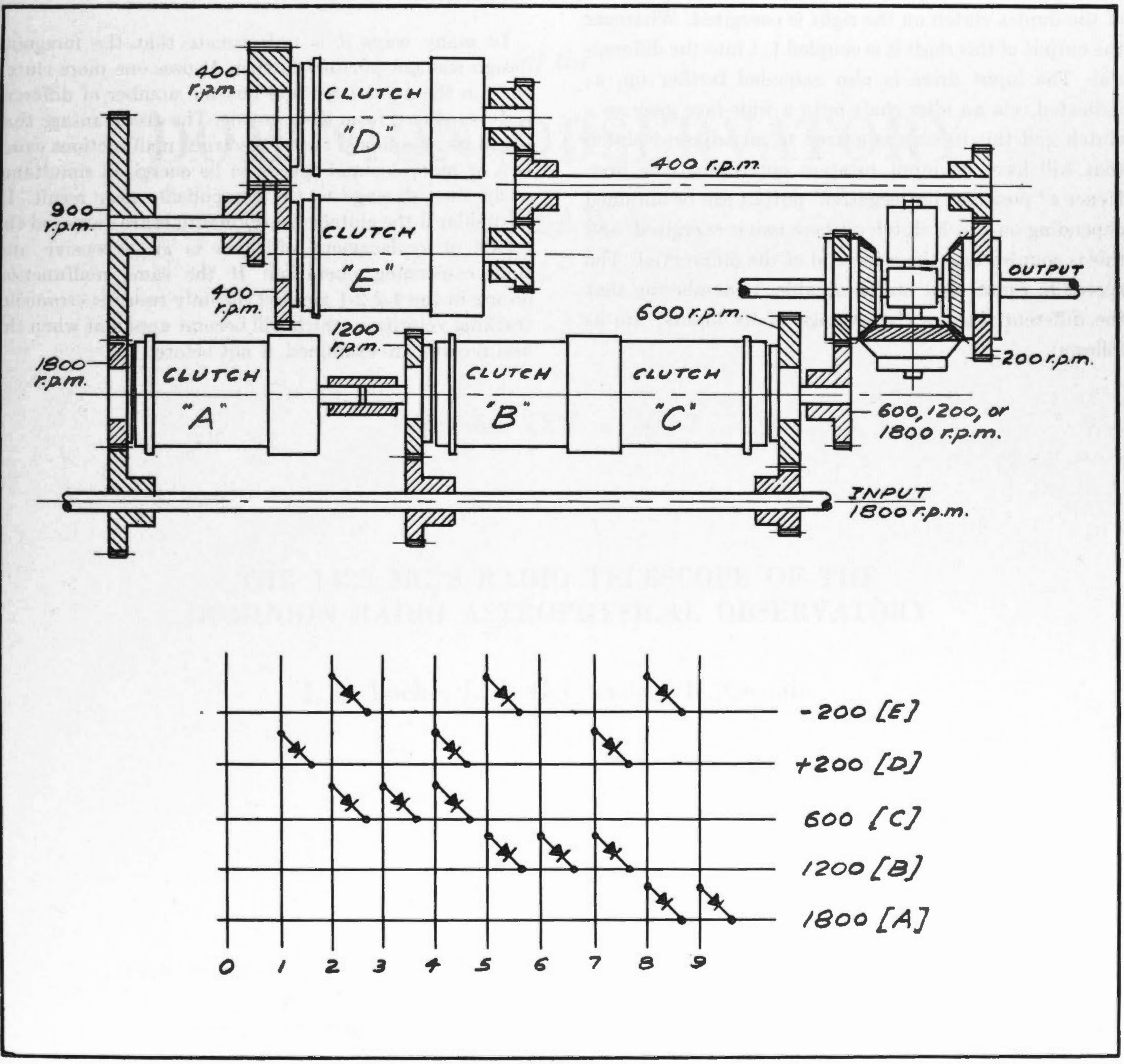

FIGURE 10. Assembly drawing for 10-speed gear box described in Appendix II, and diode matrix for conversion of decimal to 1-2-3 code. It is understood that clutch input gears are bolted to clutch input discs and output gears are set-screwed to output shafts. No support bearings for components or shafts are shown.

It is emphasized that for simplicity of examination the mechanism has been arranged all in one plane whereas in a final design a certain amount of "folding" would be desirable and could result in a very compact package. In the array shown, the clutches and the differential are drawn to scale and the gears are supposed to be of 80 diametral pitch. The lower shaft is the input, driven at 1,800 r.p.m. (so that the equivalence to the above table will later be seen). The next shaft up can have speeds of 1,800, 1,200, or 600 r.p.m. depending on whether the clutch on the left is energized or alternatively which half of the duplex clutch on the right is energized. Whatever the output of this shaft it is coupled 1:1 into the differential. The input drive is also extended farther up, as indicated, via an idler shaft onto a wide-face gear on a clutch and this in turn is geared to an adjacent clutch that will have its input rotation opposite to the first. Hence a "positive" or "negative" output can be obtained depending on which clutch of these two is energized; and this is coupled into the other end of the differential. The speeds in r.p.m. that are obtainable (remembering that the differential gives the average of its inputs) are as follows:

r.p.m. out	obtained by	*clutch(s) engaged (see Figure 10)*
0	0	none
100	200/2	D
200	(600−200)/2	C and E
300	600/2	C
400	(600+200)/2	C and D
500	(1200−200)/2	B and E
600	1200/2	B
700	(1200+200)/2	B and D
800	(1800−200)/2	A and E
900	1800/2	A
1,000*	(1800+200)/2	A and D

*if required for any purpose

In many ways it is unfortunate that the foregoing design was not pursued further. It uses one more clutch than in the 4-2-2-1 method but the number of differentials is reduced from three to one. The disadvantage that led to its rejection is that if electrical malfunctions cause two or more decimal speeds to be energized simultaneously then damage to the components may result. In particular if the clutches or differentials are damaged the repair or replacement of them is an expensive and time consuming operation. If the same malfunction occurs in the 4-2-2-1 method the only result is erroneous tracking velocities, which will become apparent when the film records are examined, if not before.