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TABLE OF CONTENTS

	PAGE
No. 1 A Temperature Control System for the Canadian Pendulum Apparatus, by H.D. Valliant, I.R. Grant and J.W. Geuer	1 1968 ✓
No. 2 An Electronic System for Measuring Pendulum Periods, by Herbert D. Valliant	11 1968 ✓
No. 3 Record of Observations at Victoria Magnetic Observatory, 1966, by D.R. Auld and P.H. Andersen	21 1968 ✓
No. 4 Polynomial Estimation of Certain Geomagnetic Quantities, Applied to a Survey of Scandinavia, by G.V. Haines	75 1968 ✓
No. 5 A Three-Component Aeromagnetic Survey of the Nordic Countries and the Greenland Sea, by W. Hannaford and G.V. Haines	113 1968 ✓
No. 6 The Effect of the Solar Cycle on Magnetic Activity at High Latitudes, by E.I. Loomer and G. Jansen van Beek	165 1968 ✓
No. 7 A Symposium on Processes in the Focal Region, by Keichi Kasahara and Anne E. Stevens, <i>Editors</i>	181 1968 ✓
No. 8 Record of Observations at Fort Churchill Magnetic Variometer Station, 1964-1965, by G. Jansen van Beek	237 1968 ✓
No. 9 Record of Observations at Great Whale Magnetic Observatory, 1967, by E.I. Loomer	335 1969
No. 10 Record of Observations at Agincourt Magnetic Observatory, 1967, by W.R. Darker and D.L. McKeown	411 1969 ✓

CONTENTS

	PAGE
Abstract	79
Introduction	79
Rotated Grid System	80
Maxwell's Equations	80
Taylor Expansion of U , V and Z	81
Estimation by Least Squares	82
Estimation of Geomagnetic Quantities	82
The Covariances between U , V and Z	83
The Polynomial Hypothesis	83
Results for Scandinavian Survey of 1965	83
Acknowledgments	105
References	105
Appendix	106
Tables	107

POLYNOMIAL ESTIMATION OF CERTAIN GEOMAGNETIC QUANTITIES, APPLIED TO A SURVEY OF SCANDINAVIA

G.V. HAINES

ABSTRACT: It is mathematically convenient to describe the geomagnetic field over a small portion of the earth's surface in terms of polynomials in a polar stereographic coordinate system. The method is applied to data from an airborne magnetic survey of Norway, Sweden, Finland and Denmark, conducted in 1965 by scientific institutes of the four countries and the Dominion Observatory of Canada. Polynomials of the third degree were fitted by the method of least squares to the observed values of two orthogonal horizontal components and the vertical component, in three independent analyses. From the three polynomials, values of the north (X), east (Y) and vertical (Z) components, together with their standard errors, were computed at grid points, and are shown as contour charts. Charts of the vertical gradients ($\partial X/\partial z$, $\partial Y/\partial z$, $\partial Z/\partial z$) and the vertical component of curl H , calculated from the polynomials, are also given. By statistical tests, the vertical component of curl H differs significantly from zero in two small regions. Geomagnetic components and vertical gradients calculated from the polynomials are compared with values computed from Cain's spherical harmonic model GSFC (12/66). Finally, charts of the residuals, or differences between the observed field and the polynomial representation of the field, are plotted, both as profiles along the flight paths and as vector diagrams in horizontal and vertical planes.

RÉSUMÉ: Mathématiquement parlant, il est commode d'exprimer le champ géomagnétique au-dessus d'une petite partie de la surface de la terre en fonction de polynômes dans un système de coordonnées stéréographiques polaires. La méthode s'applique aux données recueillies au cours d'une étude aéromagnétique couvrant les territoires de la Norvège, de la Suède, de la Finlande et du Danemark, étude qu'ont effectuée en 1965 des instituts scientifiques de ces quatre pays et la Direction des observatoires fédéraux du Canada. La méthode des moindres carrés a servi à déterminer les polynômes du troisième degré, équivalents aux valeurs de deux composantes horizontales orthogonales et de la composante verticale relevées dans trois analyses indépendantes. Les valeurs des composantes nord (X), est (Y) et verticale (Z), et leurs erreurs standards ont été calculées aux points du graticule, en fonction de ces trois polynômes, et sont présentées sous forme de cartes d'isogammes. Il en est de même pour les gradients verticaux ($\partial X/\partial z$, $\partial Y/\partial z$, $\partial Z/\partial z$) et la composante verticale de H . Il découle d'épreuves statistiques que la composante verticale de rotation H diffère considérablement de zéro dans deux petites régions. Les composantes géomagnétiques et les gradients verticaux calculés à partir des polynômes ont été comparés aux valeurs calculées à partir du modèle GSFC (12/66) des harmoniques sphériques de Cain.

Enfin, les résiduelles, c'est-à-dire les différences entre le champ mesuré et sa valeur algébrique calculée, sont représentées sous forme de profils le long des lignes de vol, et sous forme de diagrammes vectoriels dans les plans horizontal et vertical.

Introduction

When estimating a field component from a geomagnetic chart, it is desirable to know the accuracy of the estimate. This accuracy will depend, in general, on random and systematic errors in the observed data, the geographic distribution and density of the observations, and the appropriateness of the chosen method of data representation. When the data are represented by a Taylor series whose coefficients have been determined by the method of least squares, the variance of an estimate is readily derivable and gives a very good, and well-understood, figure of accuracy.

The variance of an estimate at a given position is a measure of the errors which occur at random. It is also a function of the observed positions, and is larger at the edge of the data area than at the center. This variance, however, is only as meaningful as the appropriateness of the mathematical model; that is, it is derived under the hypothesis that the data can be represented in the given manner. The appropriateness of a given model may be tested statistically and various models

tried until a suitable one is found. Systematic errors in the observations, though, can be detected only by comparisons with independently observed data.

It is also desirable for many purposes to know the vertical gradients of the geomagnetic field components. Estimates of these gradients can be obtained from the least-squares Taylor series by making use of Maxwell's equations. Errors in the estimates can be obtained statistically as in the case of the field components.

Chapman (1942) suggested that charts of any two orthogonal horizontal components of the magnetic field be made 'mutually consistent' by requiring the curl of the horizontal component to be zero. In a purely mathematical derivation of these horizontal components, however, it may be of some use not to enforce this *a priori* condition but rather use the curl as an indication of the consistency under the given derivation. Whether or not the curl is significantly different from zero can be tested statistically by knowing the standard error of the curl estimate.

Rotated Grid System

The Greenwich Grid System and the use of the polar stereographic transformation have been described by Hutchison (1949) and Haines (1967). When the flight lines of an aeromagnetic survey do not point grid east or west, in the Greenwich system, it is convenient to rotate the Greenwich system until they do. The convenience arises in plotting residuals or other information along the flight track. This will be shown later, in connection with Figures 5 to 9.

If the Greenwich system is rotated counterclockwise through an angle λ_0 , the rotated polar stereographic transformation is

$$u = -\tan\left(\frac{\theta}{2}\right) \cos(\lambda - \lambda_0) \quad (1)$$

$$v = \tan\left(\frac{\theta}{2}\right) \sin(\lambda - \lambda_0) \quad (2)$$

where θ is the geographic colatitude and λ is the geographic east longitude. In this coordinate system, grid north is the direction of true north at any point whose east longitude is λ_0 . The inverse transformation is

$$\begin{aligned} \theta &= 2 \arctan \sqrt{u^2 + v^2} \\ \lambda &= \lambda_0 + \pi - \arctan\left(\frac{v}{u}\right) \end{aligned}$$

where the arctangents are taken in the appropriate quadrants.

The magnetic field components U and V , in the direction of u and v , respectively, are defined by

$$U = H \cos(D - \lambda + \lambda_0) \quad (3)$$

$$V = H \sin(D - \lambda + \lambda_0) \quad (4)$$

where H is the horizontal field component and D is its declination relative to true north. In terms of the geographic north component X and the geographic east component Y , the grid components U and V become

$$U = X \cos(\lambda - \lambda_0) + Y \sin(\lambda - \lambda_0)$$

$$V = -X \sin(\lambda - \lambda_0) + Y \cos(\lambda - \lambda_0)$$

The expressions for X and Y , in terms of U and V , are derived easily:

$$X = U \cos(\lambda - \lambda_0) - V \sin(\lambda - \lambda_0) \quad (5)$$

$$Y = U \sin(\lambda - \lambda_0) + V \cos(\lambda - \lambda_0) \quad (6)$$

The rotation angle λ_0 is chosen so that the v -axis of the above coordinate system lies approximately in the direction of the average flight line.

Maxwell's Equations

Two equations of Maxwell pertaining to the magnetic field are

$$\operatorname{div} F = 0 \quad (7)$$

$$\operatorname{curl} F = J + \frac{\partial D}{\partial t} \quad (8)$$

where F is the total magnetic field vector, J is the electric current density vector, and D is the electric displacement vector.

In terms of the true north (X), true east (Y), and vertical downward (Z) components, the divergence and curl can be resolved into components in the spherical polar coordinate system as follows:

$$\operatorname{div} F = \frac{\partial Z}{\partial z} - \frac{1}{a} \left[2Z + X \cot \theta + \frac{\partial X}{\partial \theta} - \frac{\partial Y}{\partial \lambda} \cosec \theta \right] \quad (9)$$

$$\operatorname{curl}_\theta F = \frac{\partial Y}{\partial z} - \frac{1}{a} \left[Y + \frac{\partial Z}{\partial \lambda} \cosec \theta \right] \quad (10)$$

$$\operatorname{curl}_\lambda F = \frac{\partial X}{\partial z} - \frac{1}{a} \left[X - \frac{\partial Z}{\partial \theta} \right] \quad (11)$$

$$\operatorname{curl}_z F = -\frac{1}{a} \left[Y \cot \theta + \frac{\partial X}{\partial \lambda} \cosec \theta + \frac{\partial Y}{\partial \theta} \right] \quad (12)$$

where θ is the geographic colatitude, λ is the east longitude, z is the vertical downward direction, and a is the earth's radius.

Combining Equations (7) and (9) gives an expression for the vertical gradient of Z in terms of the measurable horizontal gradients of X and Y . Also, if it is assumed that the horizontal component of $(J + \partial D / \partial t)$ is zero, Equation (8) gives $\operatorname{curl}_\theta F = 0$ and $\operatorname{curl}_\lambda F = 0$. These, together with Equations (10) to (12), yield expressions for the vertical gradients of X and Y in terms of the measurable horizontal gradients of Z .

The assumption that the horizontal component of $(J + \partial D / \partial t)$ equals zero is justified. The current densities measured near the earth are only of the order of 10^{-3} ma/km² (see Chalmers, 1957), and since these are primarily vertical, the horizontal component would be even smaller. The effect on the vertical gradients of X and Y is less than 10^{-6} γ/km.

Referring to Equation (12), it is seen that the expression for $\operatorname{curl}_z F$ contains only horizontal components, and so is equivalent to $\operatorname{curl}_z H$, where H is the horizontal magnetic field vector. This expression is of interest since all terms on the right are measurable, and $\operatorname{curl}_z F$ should be less than 10^{-3} ma/km² as mentioned in the previous paragraph.

The equations, then, are as follows:

$$\frac{\partial X}{\partial z} = \frac{1}{a} \left[X - \frac{\partial Z}{\partial \theta} \right]$$

$$\frac{\partial Y}{\partial z} = \frac{1}{a} \left[Y + \frac{\partial Z}{\partial \lambda} \cosec \theta \right]$$

$$\frac{\partial Z}{\partial z} = \frac{1}{a} \left[2Z + X \cot \theta + \frac{\partial X}{\partial \theta} - \frac{\partial Y}{\partial \lambda} \operatorname{cosec} \theta \right]$$

$$\operatorname{curl}_z H = -\frac{1}{a} \left[Y \cot \theta + \frac{\partial X}{\partial \lambda} \operatorname{cosec} \theta + \frac{\partial Y}{\partial \theta} \right]$$

When the derivatives of X , Y and Z with respect to θ and λ are expressed in terms of the derivatives of U , V and Z with respect to u and v , using Equations (1) and (2), (5) and (6), the following expressions are obtained for the vertical gradients and $\operatorname{curl}_z H$:

$$\frac{\partial X}{\partial z} \equiv \frac{1}{a} \left[X + \frac{1}{2} \sec^2 \left(\frac{\theta}{2} \right) \left\{ \cos(\lambda - \lambda_0) \frac{\partial Z}{\partial u} - \sin(\lambda - \lambda_0) \frac{\partial Z}{\partial v} \right\} \right] \quad (13)$$

$$\frac{\partial Y}{\partial z} = \frac{1}{a} \left[Y + \frac{1}{2} \sec^2 \left(\frac{\theta}{2} \right) \left\{ \sin(\lambda - \lambda_0) \frac{\partial Z}{\partial u} + \cos(\lambda - \lambda_0) \frac{\partial Z}{\partial v} \right\} \right] \quad (14)$$

$$\frac{\partial Z}{\partial z} = \frac{1}{a} \left[2Z - X \tan \left(\frac{\theta}{2} \right) - \frac{1}{2} \sec^2 \left(\frac{\theta}{2} \right) \left\{ \frac{\partial U}{\partial u} + \frac{\partial V}{\partial v} \right\} \right] \quad (15)$$

$$\operatorname{curl}_z H = \frac{1}{a} \left[Y \tan \left(\frac{\theta}{2} \right) - \frac{1}{2} \sec^2 \left(\frac{\theta}{2} \right) \left\{ \frac{\partial U}{\partial v} - \frac{\partial V}{\partial u} \right\} \right] \quad (16)$$

Expressing X and Y in terms of U and V , and θ and λ in terms of u and v , the formulae for the vertical gradients of X , Y and Z in terms of measurements in the (u, v) system are as follows:

$$\frac{\partial X}{\partial z} = -\frac{1}{a \sqrt{u^2 + v^2}} \left[uU + vV + \frac{1+u^2+v^2}{2} \left\{ u \frac{\partial Z}{\partial u} + v \frac{\partial Z}{\partial v} \right\} \right] \quad (17)$$

$$\frac{\partial Y}{\partial z} = -\frac{1}{a \sqrt{u^2 + v^2}} \left[uV - vU + \frac{1+u^2+v^2}{2} \left\{ u \frac{\partial Z}{\partial v} - v \frac{\partial Z}{\partial u} \right\} \right] \quad (18)$$

$$\frac{\partial Z}{\partial z} = \frac{1}{a} \left[2Z + uU + vV - \frac{1+u^2+v^2}{2} \left\{ \frac{\partial U}{\partial u} + \frac{\partial V}{\partial v} \right\} \right] \quad (19)$$

$$\operatorname{curl}_z H = -\frac{1}{a} \left[uV - vU + \frac{1+u^2+v^2}{2} \left\{ \frac{\partial U}{\partial v} - \frac{\partial V}{\partial u} \right\} \right] \quad (20)$$

Taylor Expansion of U , V and Z

The components of U , V and Z can be expanded about a point (u_0, v_0) by Taylor's theorem as follows:

$$U = U_1 + U_2 a + U_3 \beta + \frac{1}{2} U_4 a^2 + U_5 a \beta + \frac{1}{2} U_6 \beta^2 + \frac{1}{6} U_7 a^3 + \frac{1}{2} U_8 a^2 \beta + \frac{1}{2} U_9 a \beta^2 + \frac{1}{6} U_{10} \beta^3 \quad (21)$$

$$V = V_1 + V_2 a + V_3 \beta + \frac{1}{2} V_4 a^2 + V_5 a \beta + \frac{1}{2} V_6 \beta^2 + \frac{1}{6} V_7 a^3 + \frac{1}{2} V_8 a^2 \beta + \frac{1}{2} V_9 a \beta^2 + \frac{1}{6} V_{10} \beta^3 \quad (22)$$

$$Z = Z_1 + Z_2 a + Z_3 \beta + \frac{1}{2} Z_4 a^2 + Z_5 a \beta + \frac{1}{2} Z_6 \beta^2 + \frac{1}{6} Z_7 a^3 + \frac{1}{2} Z_8 a^2 \beta + \frac{1}{2} Z_9 a \beta^2 + \frac{1}{6} Z_{10} \beta^3 \quad (23)$$

where $a = u - u_0$ and $\beta = v - v_0$. The coefficients U_1 , U_2 , etc., are actually partial derivatives evaluated at $u = u_0$ and $v = v_0$. The fraction appearing in the $a^i \beta^j$ term is $\frac{1}{i! j!}$. The equations have been truncated after the third order terms since ten coefficients are normally adequate for representation of U , V and Z , over areas of several million square kilometres.

In the analysis of Haines (1967) the $\operatorname{curl}_z H = 0$ condition was imposed on U and V , leading to certain relationships between their coefficients. From Equation (20) the relationship between V_2 and U_3 is

$$V_2 = U_3 + \frac{2(uV - vU)}{1 + u^2 + v^2} \quad (24)$$

Taking higher derivatives yields relations between the higher order coefficients. When uV approximately equals vU the second term can be ignored and $V_2 = U_3$. Other approximations lead to simple relations among the higher order coefficients, and the components U and V may then be solved simultaneously by the method of least squares.

When these approximations are not valid, however, the dependent variables U and V cannot be separated from the independent variables u and v , and the least squares solution cannot be obtained by the method of normal equations. The residual squares could be minimized by iteration, but this method would be very long and difficult from a computing standpoint.

It may, in fact, be preferable not to impose the curl condition even when the above approximations are valid. Chapman (1942) has suggested that the condition be applied to make magnetic charts 'mutually consistent'. That is, in areas where data are available for one horizontal component only, the other horizontal component should be estimated by requiring the vertical component of $\operatorname{curl} H$ to be zero. However, when the field is derived mathematically it is not necessary to use the curl relationship since all components (together with their standard errors) can be estimated at every point of the area under analysis. It is then easy to obtain a value of the curl and test it for statistical significance by knowing its standard error.

Estimation by Least-squares

If the mean μ_y of the distribution of a variable y , when other variables x_1, x_2, \dots, x_p are given, depends linearly on these other variables, the relationship

$$\mu_y = \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$$

is called the true regression equation. If, in addition, the variance σ_y^2 of this distribution is constant for all sets of x values, the method of least squares may be used to estimate μ_y and σ_y^2 . The estimate of μ_y is

$$\bar{y}' = b_1 x_1 + b_2 x_2 + \dots + b_p x_p \quad (25)$$

where the b_i are the least squares estimates of the β_i . The estimate of σ_y^2 is

$$s_y^2 = \frac{\sum_{i=1}^n (y - \bar{y}')^2}{n-p} \quad (26)$$

where n is the number of observations used in the analysis.

The $p \times p$ matrix whose $(i, j)^{\text{th}}$ element is $\sum_1^n x_i x_j$ is known as the matrix of 'sums of squares and products of independent variables'. This matrix will be denoted by A^{-1} . The summation subscript in $\sum_1^n x_i x_j$ has been suppressed since the meaning is obvious. The $(i, j)^{\text{th}}$ element of the inverse matrix A multiplied by s_y^2 is an estimate of σ_{ij} , the covariance between the least squares coefficients b_i and b_j . This estimate is denoted by s_{ij} . The element s_{ii} , of course, is an estimate of the variance σ_{ii} of b_i . These are always positive and so may be denoted by s_i^2 and σ_i^2 , respectively. The positive square root σ_i of σ_i^2 , is called the standard error of b_i , and the positive square root s_i , of s_i^2 , is the estimate of this standard error.

The statistic \bar{y}' , being a known function of the variables x_i , also has a distribution. Its mean when the x_i are held constant is μ_y (hence \bar{y}' is an unbiased estimator of μ_y) and its variance is

$$\sigma_{\bar{y}'}^2 = \sum_{i=1}^p \sum_{j=1}^p x_i x_j s_{ij}$$

which, of course, is estimated by

$$s_{\bar{y}'}^2 = \sum_{i=1}^p \sum_{j=1}^p x_i x_j s_{ij} \quad (27)$$

It should be emphasized that \bar{y}' is the estimated mean of the y' -distribution, and $\sigma_{\bar{y}'}^2$, is the variance of that estimated mean, not the variance of an estimated individual value. The variance of an individual value estimated (or predicted) by the least squares equation would be the variance of the estimated mean plus the variance of y' . Denoting the variance of the estimate of an individual value y' by $\sigma_{y'}^2$, the relationship is

$$\sigma_{y'}^2 = \sigma_{\bar{y}'}^2 + \sigma_y^2$$

which may be estimated by

$$s_{y'}^2 = s_{\bar{y}'}^2 + s_y^2 \quad (28)$$

The square root of a variance is always called a standard error. Thus $\sigma_{\bar{y}'}$, $\sigma_{\bar{y}'}$ and σ_y are standard errors of y' , \bar{y}' and y , respectively, and s_y , $s_{\bar{y}'}$ and s_y are their estimates.

Confidence intervals may be determined for the various statistics when the variable y is distributed normally, at fixed values of x_1, x_2, \dots, x_p . It can be shown, in fact, that $\frac{b_i - \beta_i}{s_i}$ (for $i = 1, 2, \dots, p$) and $\frac{y' - \mu_y}{s_y}$ are distributed with Student's t distribution, and $\frac{s^2}{\sigma_y^2}$ is distributed as Chi-Square/ df ,

where df is its degrees of freedom. Of course, the last two expressions are true when y is replaced by y' or \bar{y}' , remembering that $\mu_{y'} = \mu_{\bar{y}'} = \mu_y$.

Estimation of Geomagnetic Quantities

The coefficients U_1, U_2, U_3 , etc, in Equations (21), (22) and (23) may be determined by least squares to give estimates of U, V and Z . These estimates correspond to the \bar{y}' of Equation (25). Estimates of the standard errors (square root of the variance) can be calculated from Equation (27).

The geographic components X and Y can be estimated from U and V by using Equations (5) and (6). Their variances can be calculated knowing the variances of U and V :

$$\begin{aligned} \sigma_X^2 &= \sigma_U^2 \cos^2(\lambda - \lambda_0) + \sigma_V^2 \sin^2(\lambda - \lambda_0) \\ \sigma_Y^2 &= \sigma_U^2 \sin^2(\lambda - \lambda_0) + \sigma_V^2 \cos^2(\lambda - \lambda_0). \end{aligned}$$

The covariance between U and V is ignored in these equations; this will be discussed in a later section.

The vertical gradients of X, Y and Z , and the vertical component of curl H can be estimated by using Equations (17) to (20). (For computation purposes, Equations (13) to (16) are more useful if latitude and longitude are given rather than u and v .) The variances of these quantities can be estimated by expressing the quantities as functions of the least-square coefficients, and then using Equation (27) as before.

Confidence intervals may be obtained for U, V, Z , their gradients, and $\text{curl}_z H$ since they all are distributed normally when n , the number of degrees of freedom, is large. This is so because $\frac{b_i - \beta_i}{s_i}$ (for $i = 1, 2, \dots, p$) is distributed as Student's t , which tends to the Normal distribution as n becomes large. The convergence is quite rapid, and there is very little difference between the two distributions at $n = 100$. Of course, linear functions (such as U, V, Z , etc.) of normally distributed variables (such as β_i) are themselves distributed normally.

The Covariances between U , V and Z

The variance of any function of the U -coefficients can be calculated by knowing the variances of all these coefficients and the covariances between them. These come directly from the inverse of the matrix of 'sums of squares and products of independent variables', obtained in the least-squares solution. Similarly, the variance of any function of the V -coefficients can be calculated. But to obtain the variance of a function of both the U - and V -coefficients it is necessary to know the covariance between each U -coefficient and each V -coefficient, as well as their respective variances; and, in general, to obtain the variance of a function of the U , V and Z -coefficients, the covariance between every pair of coefficients must be available, in addition to all the variances.

This problem is discussed by Williams (1959) for the case when the sums of squares and products matrix is the same for each dependent variable. This occurs when U , V and Z are all obtained at every position point. In general, however, this is not the case. The element Z , for example, usually is obtained more frequently than U and V . The matrices for U and V , though, are the same, since U and V are both derived from the measurements D and H (see Equations (3) and (4)).

In practice, either matrix may be used since the number of Z -only observations is a small percentage of the total number of observations. This can be checked easily by running the computer program with the two matrices interchanged.

Once this is established, the covariances between the components U , V and Z can be calculated. The effect of these covariances can then be included in the calculation of any function of U , V and Z . For example, the variances of the components X and Y now become

$$\sigma_X^2 = \sigma_U^2 \cos^2(\lambda - \lambda_0) + \sigma_V^2 \sin^2(\lambda - \lambda_0) - 2 \sin(\lambda - \lambda_0) \cos(\lambda - \lambda_0) \sigma_{UV}$$

$$\sigma_Y^2 = \sigma_U^2 \sin^2(\lambda - \lambda_0) + \sigma_V^2 \cos^2(\lambda - \lambda_0) + 2 \sin(\lambda - \lambda_0) \cos(\lambda - \lambda_0) \sigma_{UV}$$

where σ_{UV} stands for the covariance between U and V .

In a similar way, these covariances can be included in the calculation of the variances of the vertical gradients and $\text{curl}_z H$.

The Polynomial Hypothesis

It is important to realize that any conclusions drawn from a 3rd degree polynomial analysis are based on the hypothesis that the field can be closely represented by a polynomial of this degree. Estimates can be calculated, and confidence limits constructed, for any number of parameters, but how meaningful these estimates and confidence limits are depends on how well the above hypothesis is satisfied.

It may not be very informative, for example, to compare statistically a 2nd degree estimate with a 3rd degree estimate even though the standard error of each is available. If the comparison takes place at a point where the field is represented well by the 3rd degree polynomial, but poorly by the 2nd,

then neither the mean nor the standard error of the 2nd degree estimate will have much significance.

This is especially important to remember when making estimates at points outside the data area. The estimate and its standard error are available, but these are only as good as the hypothesis that the field at that point can be represented by a polynomial of the given degree. It would be possible to have an extremely good fit to a plane, for example, but this would in no way imply that this plane extends far outside the area analyzed.

Normally it is not known beforehand how many terms should be included in a polynomial, and so a statistical method is used to truncate the series. The coefficients of any given order may be tested statistically, and when there is no significant contribution to the regression beyond a certain degree the terms beyond this point may be neglected. In neglecting terms in this manner, there is of course always the danger of making a 'type 2' error (that is, accepting a false hypothesis). The effect of this error on estimation within the area analyzed is small, however, and it is mainly outside this area where the problem becomes important.

This, in fact, is the reason a stepwise regression of a polynomial has little statistical justification. The hypothesis that a coefficient is zero may be rejected with a 5 per cent chance of making an incorrect decision, but the error in accepting the hypothesis when it is wrong is actually unknown, and may be quite large. The problem in this type of analysis, then, is not which terms to include, but how many. All well-behaved fields can be represented by a polynomial according to Taylor's theorem, and there is no special reason why any particular coefficient should be zero. But since the series converges, there is a reason for truncating the series as soon as the contribution from the truncated part becomes small enough. Also, because of limitations in the number and accuracy of observations, the coefficients of high-order terms simply cannot be determined with any degree of assurance.

Results for Scandinavian Survey of 1965

In the fall of 1965, a three-component aeromagnetic survey of Norway, Sweden, Finland and Denmark (see Figure 1) was conducted by the Dominion Observatory of Canada, and governmental agencies of the four countries concerned. The results of this survey will be reported by Hannaford and Haines (in press). The magnetometer and direction reference system were described by Hannaford, *et al.* (1967).

The area covered was 1.5 million square kilometres, and about 1400 measurements were made of the declination D , the horizontal intensity H and the vertical intensity Z . These measurements were made at an altitude of about 3 km, and represent averages over 5 minutes of time (or approximately 30 km of flight track). After being corrected for aircraft fields they were reduced to sea level by means of the inverse cube relationship, the correction to a component P being $4.6 \times 10^{-4} \times h \times P$, where h is the altitude in kilometres. These

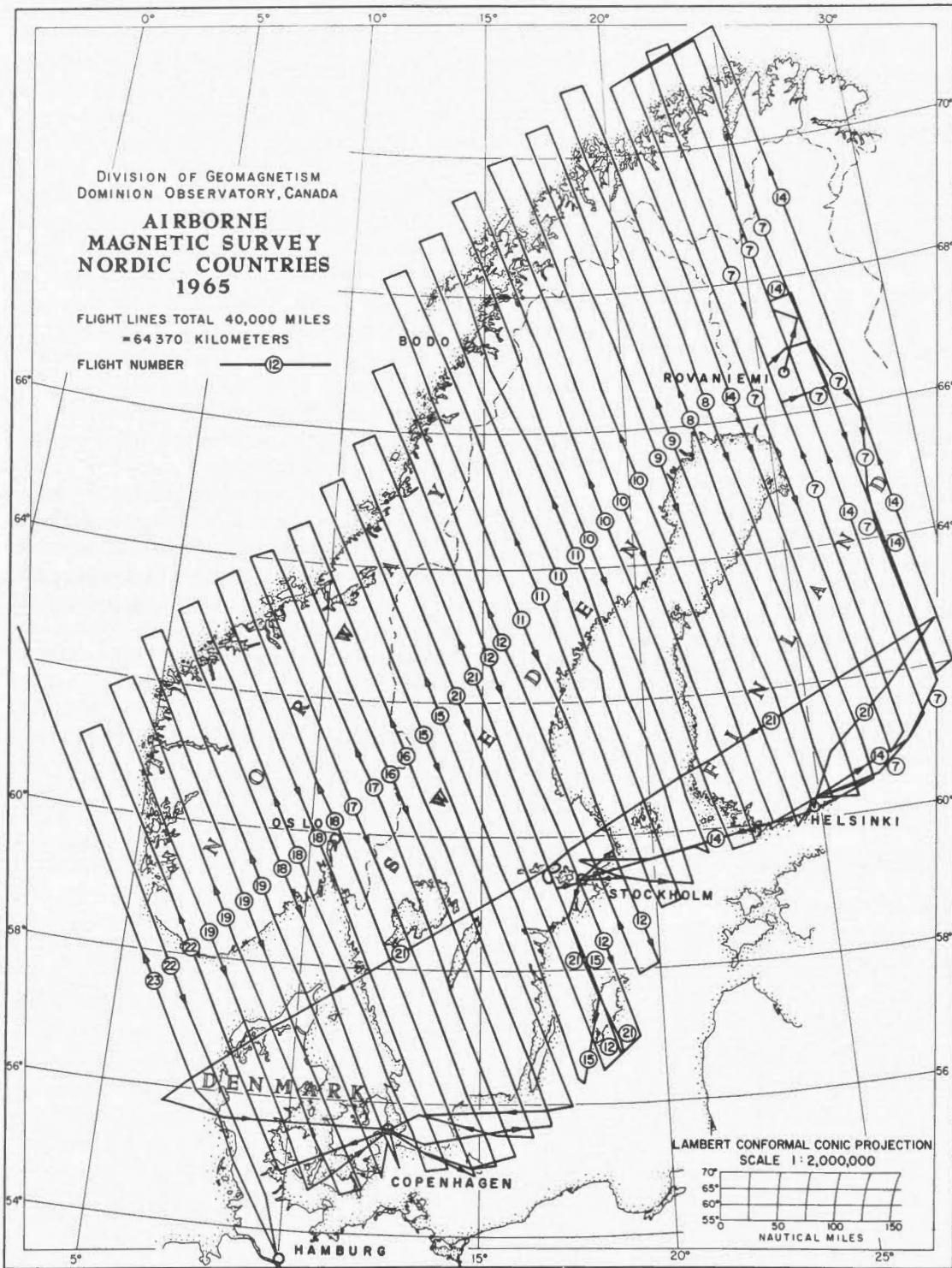


FIGURE 1.
Aeromagnetic survey of Scandinavia, 1965. Flight numbers and directions are shown.

Table 1

Coefficients of U , V and Z , and their standard Errors, Obtained by Method of Least Squares.
Value of t is Coefficient Divided by Standard Error.

		U			V			Z		
		Coeff. (γ)	St. Error (γ)	t	Coeff. (γ)	St. Error (γ)	t	Coeff. (γ)	St. Error (γ)	t
1st Degree	1	4.4529+3	.0042+3	1049.7	-13.1510+3	.0034+3	-3850.7	48.6890+3	.0044+3	10975.6
	2	-2.6916+4	.0139+4	-193.7	.5977+4	.0112+4	53.4	2.3316+4	.0145+4	160.6
	3	0.5671+4	.0142+4	40.0	-4.1965+4	.0114+4	-367.5	-3.1023+4	.0148+4	-209.0
2nd Degree	1	4.4562+3	.0068+3	655.1	-13.0850+3	.0054+3	-2411.7	48.7030+3	.0077+3	6343.6
	2	-2.7201+4	.0131+4	-207.1	.5383+4	.0105+4	51.4	2.3335+4	.0148+4	157.7
	3	.5196+4	.0133+4	38.9	-4.2443+4	.0106+4	-398.6	-3.1145+4	.0151+4	-206.9
	4	-0.0953+5	.0776+5	-1.2	-1.0154+5	.0619+5	-16.4	.3626+5	.0877+5	4.1
	5	-7.7965+5	.0452+5	-17.6	-4.4863+5	.0361+5	-13.5	.0960+5	.0510+5	1.9
	6	-6.5666+5	.0770+5	-8.5	-5.967+5	.0614+5	-9.7	-5.429+5	.0871+5	-6.2
3rd Degree	1	4.4511+3	.0072+3	621.0	-13.0920+3	.0057+3	-2312.6	48.7060+3	.0081+3	6030.4
	2	-2.6762+4	.0301+4	-88.9	.4890+4	.0238+4	20.6	2.3311+4	.0340+4	68.6
	3	.4235+4	.0297+4	14.3	-4.2102+4	.0235+4	-179.5	-3.1436+4	.0336+4	-93.7
	4	-0.0631+5	.0894+5	-0.7	-9.004+5	.0706+5	-12.8	.3346+5	.1007+5	3.3
	5	-6.835+5	.0642+5	-10.6	-3.265+5	.0507+5	-6.4	.0138+5	.0718+5	0.2
	6	-4.4957+5	.0970+5	-5.1	-4.001+5	.0766+5	-5.2	-6.245+5	.1089+5	-5.7
	7	-1.1005+6	.7494+6	-1.5	.1511+6	.5919+6	0.3	.0291+6	.8480+6	0.0
	8	.8724+6	.2810+6	3.1	.4115+6	.2220+6	1.9	-9.655+6	.3165+6	-3.1
	9	.3296+6	.2988+6	1.1	1.3980+6	.2360+6	5.9	-3.364+6	.3358+6	-1.1
	10	1.8587+6	.6932+6	2.7	.0331+6	.5475+6	0.1	1.5939+6	.7838+6	2.0

Note: Coefficients and their standard errors are in floating-point notation, a decimal fraction followed by a power of ten.
For example, $4.4529+3 = 4.4529 \times 10^{+3}$.

corrected and reduced measurements, which are referred to as 'observations', were used to obtain polynomial estimates as previously discussed. The computations were done on a CDC 3100 computer.

The $\text{curl}_z H = 0$ condition was not applied, as the approximations described in the section 'Taylor Expansion of U , V and Z ' are not valid. Hence, U , V and Z were solved independently, by the method of least squares. A rotation angle of $\lambda_0 = -55^\circ$ was chosen, so that the flight lines lie approximately parallel to the v -axis. The expansion was done about the point 63°N and 16°E , which is approximately at the center of the data area. For this point $u = -0.0782$ and $v = .2270$.

The least-squares coefficients for the 1st, 2nd and 3rd degree polynomials in U , V and Z are given in Table 1, together with their standard errors and t values. The t -value is the coefficient divided by its standard error, and gives an indication of the statistical significance of the coefficient. This has been explained in a previous paper (Haines, 1967).

The inverse of the matrix of 'sums of squares and products of independent variables' for the case of U and V is given in Table 2A, and for the case of Z in Table 2B. The elements of this 'sums of squares and products' matrix are $\sum_1^n x_i x_j$, as described previously. The independent variables x_i are as follows:

i	x_i
1	1
2	a
3	β
4	a^2
5	$a\beta$
6	β^2
7	a^3
8	$a^2\beta$
9	$a\beta^2$
10	β^3

When the inverse matrix is multiplied by s_y^2 , the estimate of σ_y^2 (see Equation (26)), the resulting matrix gives an estimate of the covariance between any two regression coefficients. For example, the estimate of the covariance between U_2 and U_6 , obtained from the (2,6)th term of the inverse matrix in Table 2A, is $19675 \times -1.4644 \times 10^1 = -2.8812 \times 10^5 \gamma^2$. An estimate of the variance of Z_8 is obtained from the (8,8)th term in Table 2B and is $25314 \times 3.9570 \times 10^6 = 1.0017 \times 10^{11} \gamma^2$. The standard error, or square root of the variance, of Z_8 , is $3.1649 \times 10^5 \gamma$, agreeing with that found in Table 1.

Table 2A
Inverse of 'Sums of Squares and Products' Matrix for U and V .

2.611289E-03 A (1, 1)	7.422348E-03 A (1, 2)	1.264439E-02 A (1, 3)	-2.192143E 00 A (1, 4)	-8.556475E-01 A (1, 5)
-2.126795E 00 A (1, 6)	-3.085387E 01 A (1, 7)	-2.532459E 01 A (1, 8)	-3.154100E 01 A (1, 9)	-5.133806E 01 A (1, 10)
7.422348E-03 A (2, 1)	4.601081E 00 A (2, 2)	1.013325E 00 A (2, 3)	-3.924691E 01 A (2, 4)	-1.921547E 01 A (2, 5)
-1.464402E 01 A (2, 6)	-9.684486E 03 A (2, 7)	-1.004828E 03 A (2, 8)	-1.399633E 03 A (2, 9)	-1.031612E 03 A (2, 10)
1.264439E-02 A (3, 1)	1.013325E 00 A (3, 2)	4.480464E 00 A (3, 3)	-2.616126E 01 A (3, 4)	-2.949616E 01 A (3, 5)
-3.692750E 01 A (3, 6)	-1.678010E 03 A (3, 7)	-2.002849E 03 A (3, 8)	-1.231156E 03 A (3, 9)	-8.047466E 03 A (3, 10)
-2.192143E 00 A (4, 1)	-3.924691E 01 A (4, 2)	-2.616126E 01 A (4, 3)	4.062932E 03 A (4, 4)	1.375145E 03 A (4, 5)
1.466941E 03 A (4, 6)	1.231606E 05 A (4, 7)	3.954836E 04 A (4, 8)	4.654455E 04 A (4, 9)	7.581533E 04 A (4, 10)
-8.556475E-01 A (5, 1)	-1.921547E 01 A (5, 2)	-2.949616E 01 A (5, 3)	1.375145E 03 A (5, 4)	2.095863E 03 A (5, 5)
2.290546E 03 A (5, 6)	3.256350E 04 A (5, 7)	5.003927E 04 A (5, 8)	6.261124E 04 A (5, 9)	9.539435E 04 A (5, 10)
-2.126795E 00 A (6, 1)	-1.464402E 01 A (6, 2)	-3.692750E 01 A (6, 3)	1.466941E 03 A (6, 4)	2.290546E 03 A (6, 5)
4.781409E 03 A (6, 6)	1.822811E 04 A (6, 7)	5.985830E 04 A (6, 8)	8.216248E 04 A (6, 9)	1.421456E 05 A (6, 10)
-3.085387E 01 A (7, 1)	-9.684486E 03 A (7, 2)	-1.678010E 03 A (7, 3)	1.231606E 05 A (7, 4)	3.256350E 04 A (7, 5)
1.822811E 04 A (7, 6)	2.854615E 07 A (7, 7)	2.932680E 06 A (7, 8)	3.668373E 05 A (7, 9)	-1.111472E 06 A (7, 10)
-2.532459E 01 A (8, 1)	-1.004828E 03 A (8, 2)	-2.002849E 03 A (8, 3)	3.954836E 04 A (8, 4)	5.003927E 04 A (8, 5)
5.985830E 04 A (8, 6)	2.932680E 06 A (8, 7)	4.014740E 06 A (8, 8)	1.899643E 06 A (8, 9)	1.991860E 06 A (8, 10)
-3.154100E 01 A (9, 1)	-1.399633E 03 A (9, 2)	-1.231156E 03 A (9, 3)	4.654455E 04 A (9, 4)	6.261124E 04 A (9, 5)
8.216248E 04 A (9, 6)	3.668373E 05 A (9, 7)	1.899643E 06 A (9, 8)	4.536794E 06 A (9, 9)	5.537757E 06 A (9, 10)
-5.133806E 01 A (10, 1)	-1.031612E 03 A (10, 2)	-8.047466E 03 A (10, 3)	7.581533E 04 A (10, 4)	9.539435E 04 A (10, 5)
1.421456E 05 A (10, 6)	-1.111472E 06 A (10, 7)	1.991860E 06 A (10, 8)	5.537757E 06 A (10, 9)	2.442353E 07 A (10, 10)

Table 2B
Inverse of 'Sums of Squares and Products' Matrix for Z

2.576942E-03 A (1, 1)	6.768310E-03 A (1, 2)	1.212066E-02 A (1, 3)	-2.155178E 00 A (1, 4)	-8.284378E-01 A (1, 5)
-2.082796E 00 A (1, 6)	-2.941856E 01 A (1, 7)	-2.436122E 01 A (1, 8)	-3.035632E 01 A (1, 9)	-4.976280E 01 A (1, 10)
6.768310E-03 A (2, 1)	4.559644E 00 A (2, 2)	9.810290E-01 A (2, 3)	-3.855341E 01 A (2, 4)	-1.887838E 01 A (2, 5)
-1.393020E 01 A (2, 6)	-9.614126E 03 A (2, 7)	-9.943831E 02 A (2, 8)	-1.378796E 03 A (2, 9)	-9.693962E 02 A (2, 10)
1.212066E-02 A (3, 1)	9.810290E-01 A (3, 2)	4.449464E 00 A (3, 3)	-2.534696E 01 A (3, 4)	-2.902662E 01 A (3, 5)
-3.635349E 01 A (3, 6)	-1.614939E 03 A (3, 7)	-1.986962E 03 A (3, 8)	-1.212301E 03 A (3, 9)	-7.989012E 03 A (3, 10)
-2.155178E 00 A (4, 1)	-3.855341E 01 A (4, 2)	-2.534696E 01 A (4, 3)	4.004086E 03 A (4, 4)	1.327866E 03 A (4, 5)
1.405025E 03 A (4, 6)	1.212322E 05 A (4, 7)	3.810996E 04 A (4, 8)	4.472681E 04 A (4, 9)	7.308263E 04 A (4, 10)
-8.284378E-01 A (5, 1)	-1.887838E 01 A (5, 2)	-2.902662E 01 A (5, 3)	1.327866E 03 A (5, 4)	2.036412E 03 A (5, 5)
2.218984E 03 A (5, 6)	3.182946E 04 A (5, 7)	4.824599E 04 A (5, 8)	6.042595E 04 A (5, 9)	9.302963E 04 A (5, 10)
-2.082796E 00 A (6, 1)	-1.393020E 01 A (6, 2)	-3.635349E 01 A (6, 3)	1.405025E 03 A (6, 4)	2.218984E 03 A (6, 5)
4.688518E 03 A (6, 6)	1.673626E 04 A (6, 7)	5.763909E 04 A (6, 8)	7.942495E 04 A (6, 9)	1.392833E 05 A (6, 10)
-2.941856E 01 A (7, 1)	-9.614126E 03 A (7, 2)	-1.614939E 03 A (7, 3)	1.212832E 05 A (7, 4)	3.182946E 04 A (7, 5)
1.673626E 04 A (7, 6)	2.840390E 07 A (7, 7)	2.910699E 06 A (7, 8)	3.247412E 05 A (7, 9)	-1.244108E 06 A (7, 10)
-2.436122E 01 A (8, 1)	-9.943831E 02 A (8, 2)	-1.986962E 03 A (8, 3)	3.810996E 04 A (8, 4)	4.824599E 04 A (8, 5)
5.763909E 04 A (8, 6)	2.910699E 06 A (8, 7)	3.957018E 06 A (8, 8)	1.832912E 06 A (8, 9)	1.921123E 06 A (8, 10)
-3.035632E 01 A (9, 1)	-1.378796E 03 A (9, 2)	-1.212301E 03 A (9, 3)	4.472681E 04 A (9, 4)	6.042595E 04 A (9, 5)
7.942495E 04 A (9, 6)	3.247412E 05 A (9, 7)	1.832912E 06 A (9, 8)	4.454349E 06 A (9, 9)	5.448800E 06 A (9, 10)
-4.976280E 01 A (10, 1)	-9.693962E 02 A (10, 2)	-7.989012E 03 A (10, 3)	7.308263E 04 A (10, 4)	9.302963E 04 A (10, 5)
1.392833E 05 A (10, 6)	-1.244108E 06 A (10, 7)	1.921123E 06 A (10, 8)	5.448800E 06 A (10, 9)	2.426606E 07 A (10, 10)

Note: Elements of matrices are in floating-point notation. For example, $2.611289E-03 = 2.611289 \times 10^{-3}$.

Table 3
Residual Errors

	<i>U</i>	<i>V</i>	<i>Z</i>	<i>D</i>	<i>H</i>	<i>X</i>	<i>Y</i>
Observations	1372	1372	1382	1372	1378	1372	1372
1st Degree	157 γ	126 γ	165 γ	0.7°	122 γ	121 γ	161 γ
2nd Degree	141	113	160	0.6	113	113	141
3rd Degree	140	111	159	0.6	112	112	140

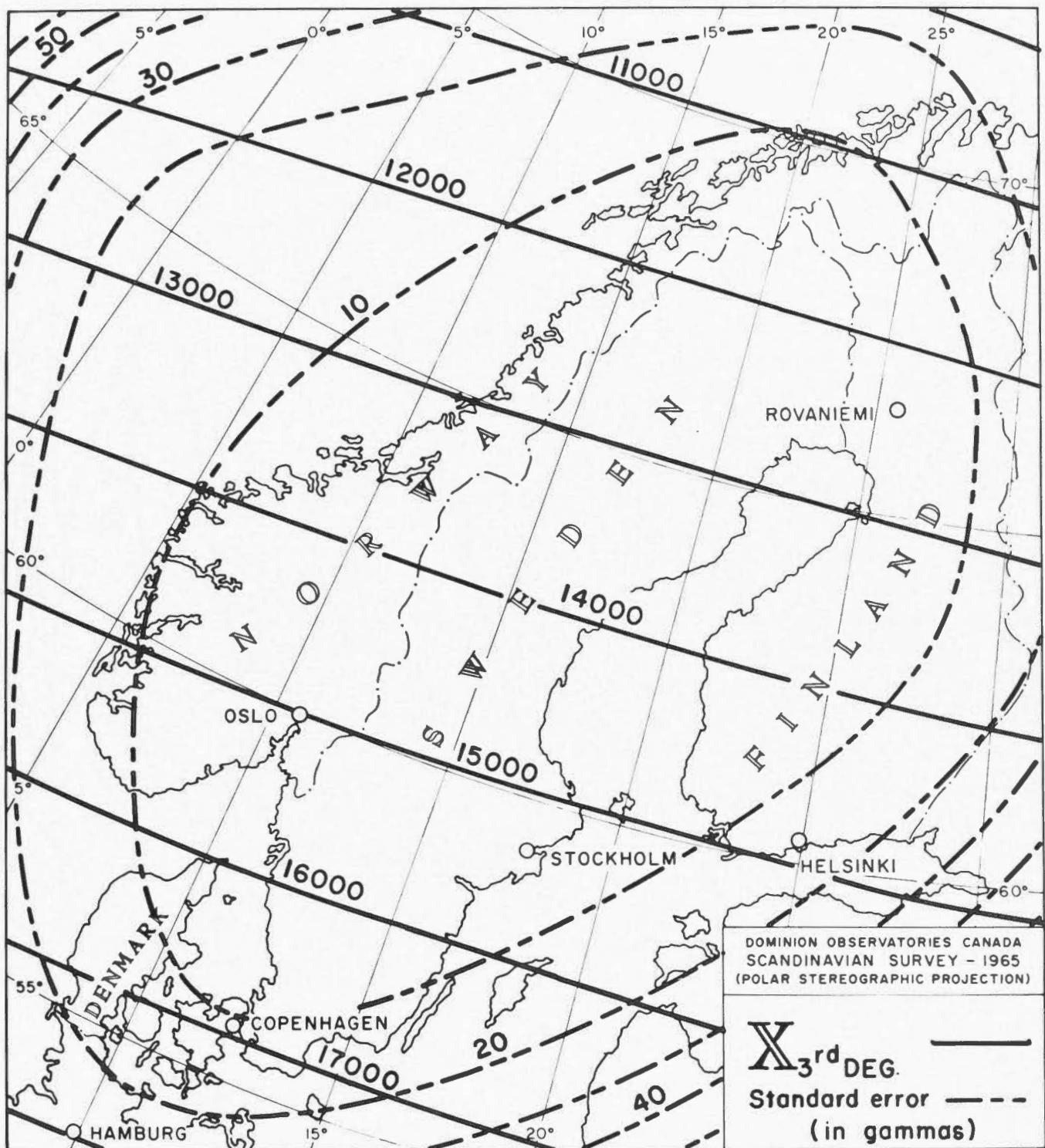


FIGURE 2. North component of magnetic field, and its standard error. Calculated from 3rd degree polynomial whose coefficients were determined by method of least squares.

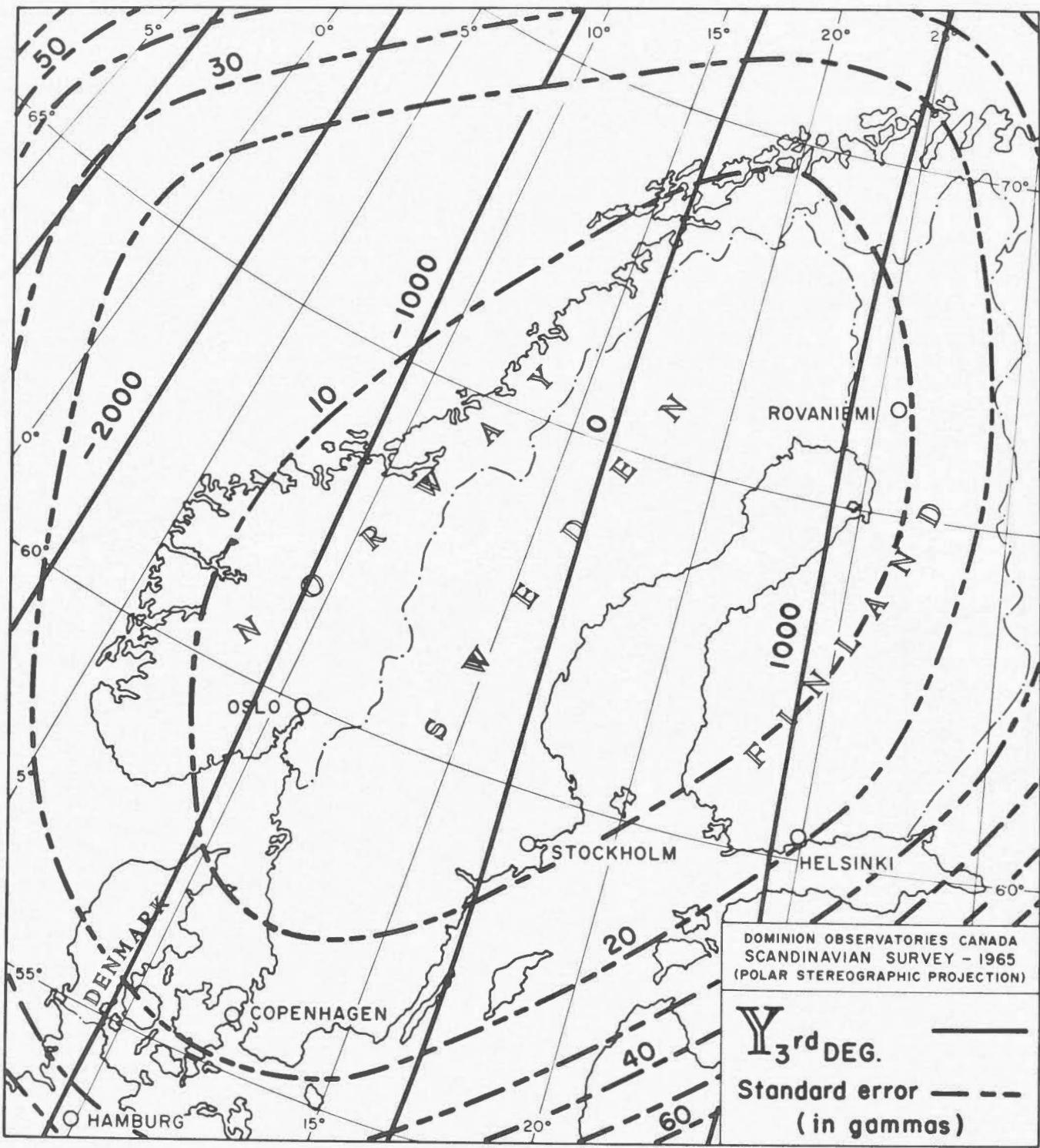


FIGURE 3. East component of magnetic field, and its standard error.

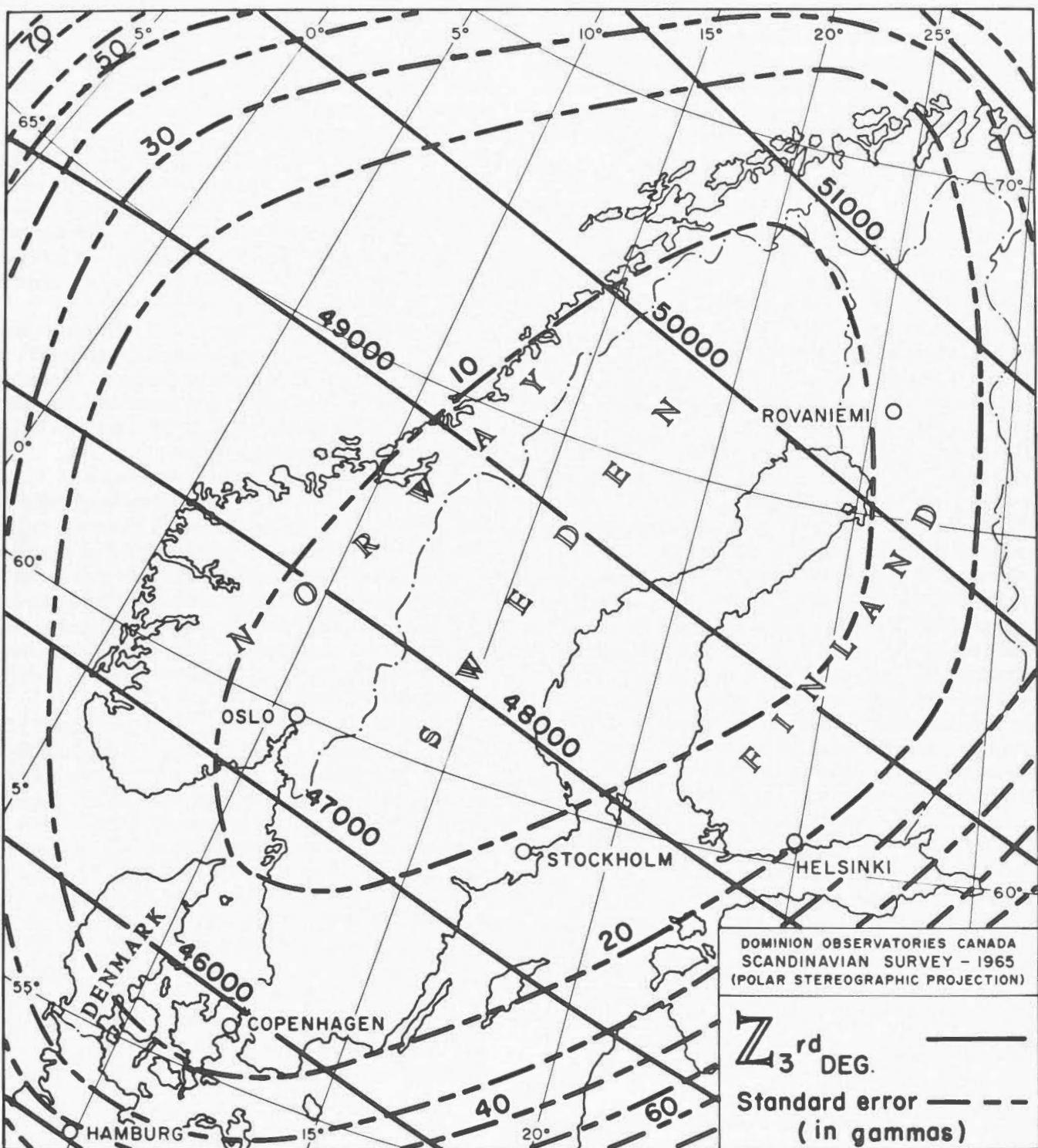


FIGURE 4. Vertical downward component of magnetic field, and its standard error.

Table 3 lists the residual errors of the various components for the three degrees. It will be noticed that there is practically no decrease in these errors in going from the 2nd to 3rd degree. This means the 2nd degree polynomial fits the observed data almost as well as the 3rd degree, and there would probably be no further improvement in going to the 4th degree.

Contours of X , Y and Z , for the 3rd degree, are given in Figures 2 to 4, with contours of their standard errors. Note that these standard errors correspond to the s_y of the section 'Estimation by Least Squares'; that is, they refer to the mean of the X , Y and Z distributions at any given position. The standard errors of the X , Y and Z observations (about these means) are those given in Table 2. They correspond to the s_y in the section just referred to, and are independent of position. The standard error of an individual X , Y or Z estimate could be obtained by using Equation (28). An example will help make this clear.

Consider, in Figure 4, the point 59°N , 25°E , approximately, where $Z' = 48,000 \gamma$ and $s_{z'} = 30 \gamma$. Table 2 gives $s_z = 159 \gamma$. Then Equation (28) gives $s_{z'}^2 = (30)^2 + (159)^2 = 26181 \gamma^2$, and $s_{z'} = 162 \gamma$. Thus the least-squares estimate of the mean of the Z distribution at 59°N , 25°E is $48,000 \gamma$, and the standard error of this estimate is 30γ . The standard error of a Z -value about its mean is 159γ ; being independent of position it has the same value at any point on the chart. (In fact, one of the least-squares assumptions is that σ_Z^2 is constant over the data range.) The standard error of an individual Z -estimate, however, is 162γ since it includes the error in the mean-estimate as well as the error in the Z -value itself.

Contours of the standard errors of estimates of individual values could be added easily to Figures 2 to 4 if these were required. For example, the contour of $s_{z'} = 20 \gamma$ serves also as the contour for $s_z = 160 \gamma$. That of $s_{z'} = 60 \gamma$ is equivalent to $s_z = 170 \gamma$.

It is immediately obvious how the accuracy of estimation decreases with increasing distance from the data area. Most of the observed data, as a matter of fact, lie inside the 20γ contours; outside the area of observed data the accuracy decreases rapidly, standard errors reaching over 100γ in the southeast corner where there were no observed data whatsoever. For example, at 56°N , 30°E , the standard errors of X , Y and Z are 103γ , 129γ and 145γ , respectively. The accuracy decreases more slowly in the northwest corner since some data

from the Greenland-Norwegian Sea survey were used in the analysis.

Figures 5 to 7 show the 'residuals' of X , Y and Z . The residual is the observed value of the component minus the 3rd degree estimate. It is plotted to the east of the flight track when positive and to the west when negative. Figure 8 gives the 'vector residuals' in the horizontal plane, and Figure 9 the 'vector residuals' in a vertical plane passing through the flight track. In the latter case, when a vector is pointing in the direction of increasing Z (i.e., downward) it has been plotted to the east of the flight track, in conformity with the directions in Figure 7. Figures 5 to 9 display the observed data in a most convenient way for those interested in large-scale features.

The residual charts of Figures 5 to 7 demonstrate one of the advantages of rotating the grid axis by $\lambda_0 = -55^\circ$. It is quite a simple matter to plot, by computer, the residuals perpendicular to the flight track, since this direction is simply that of the position vector u which can then be made the direction of the y -axis on a plotter.

It was found that the covariances between U , V and Z could be neglected. Including these covariances makes only slight differences in the standard errors, amounting to 1 or 2 percent at the center of the data area and 3 or 4 percent at the edges. The 'residual mean products' are given in Table 4; of course, those of UU , VV and ZZ are just the square of the U , V and Z residual errors in Table 2, listed here for comparison with the mean products UV , UZ and VZ . Note that they are mean products, i.e., the residual products divided by their degrees of freedom.

It may not be clear how to obtain an estimate of the covariance between a U -coefficient and a V -coefficient. In Table 4 the residual mean product of UV , for the 3rd degree case, is given as $895 \gamma^2$. This figure, multiplied by the inverse matrix listed in Table 2A, gives the required estimates. For example, the covariance between U_2 and V_6 , obtained from the (2,6)th term of the inverse matrix, is $895 \times -1.4644 \times 10^1 = -1.3106 \times 10^4 \gamma^2$. Of course, to obtain an estimate of the covariance between a U -coefficient and a Z -coefficient, the matrices in Tables 2A and 2B would have to be the same.

Many are interested in how a particular polynomial model compares with a spherical harmonic model. In Figures 10 to 12 the 3rd degree polynomials have been compared to Cain's 10th degree spherical harmonic model GSFC (12/66) Set 1 Rounded, reduced to 1965.8. The agreement over the main

(continued on page 105)

Table 4

Residual Mean Products

	UU	VV	ZZ	UV	UZ	VZ
Observations	1372	1372	1382	1372	1370	1370
1st Degree	$24,670 \gamma^2$	$15,991 \gamma^2$	$27,182 \gamma^2$	$2,913 \gamma^2$	$-1,713 \gamma^2$	$-216 \gamma^2$
2nd Degree	19,931	12,679	25,596	827	-1,308	387
3rd Degree	19,675	12,274	25,314	895	-1,230	587

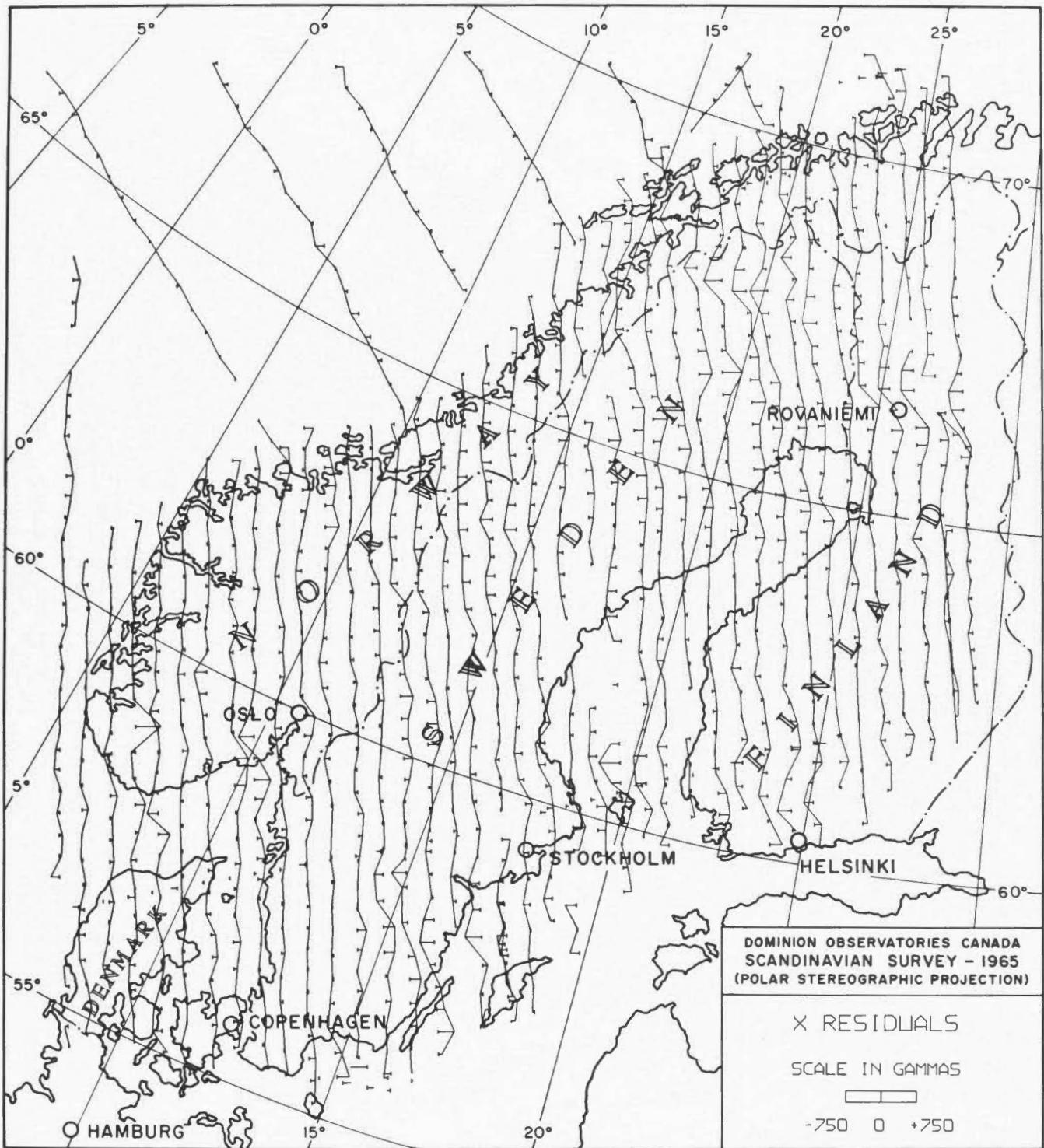


FIGURE 5. Residuals of north component of magnetic field. The residual of a component is defined as the observed value of component minus the value obtained from the 3rd degree polynomial. Plotted to the right of the flight track when positive, to the left when negative.

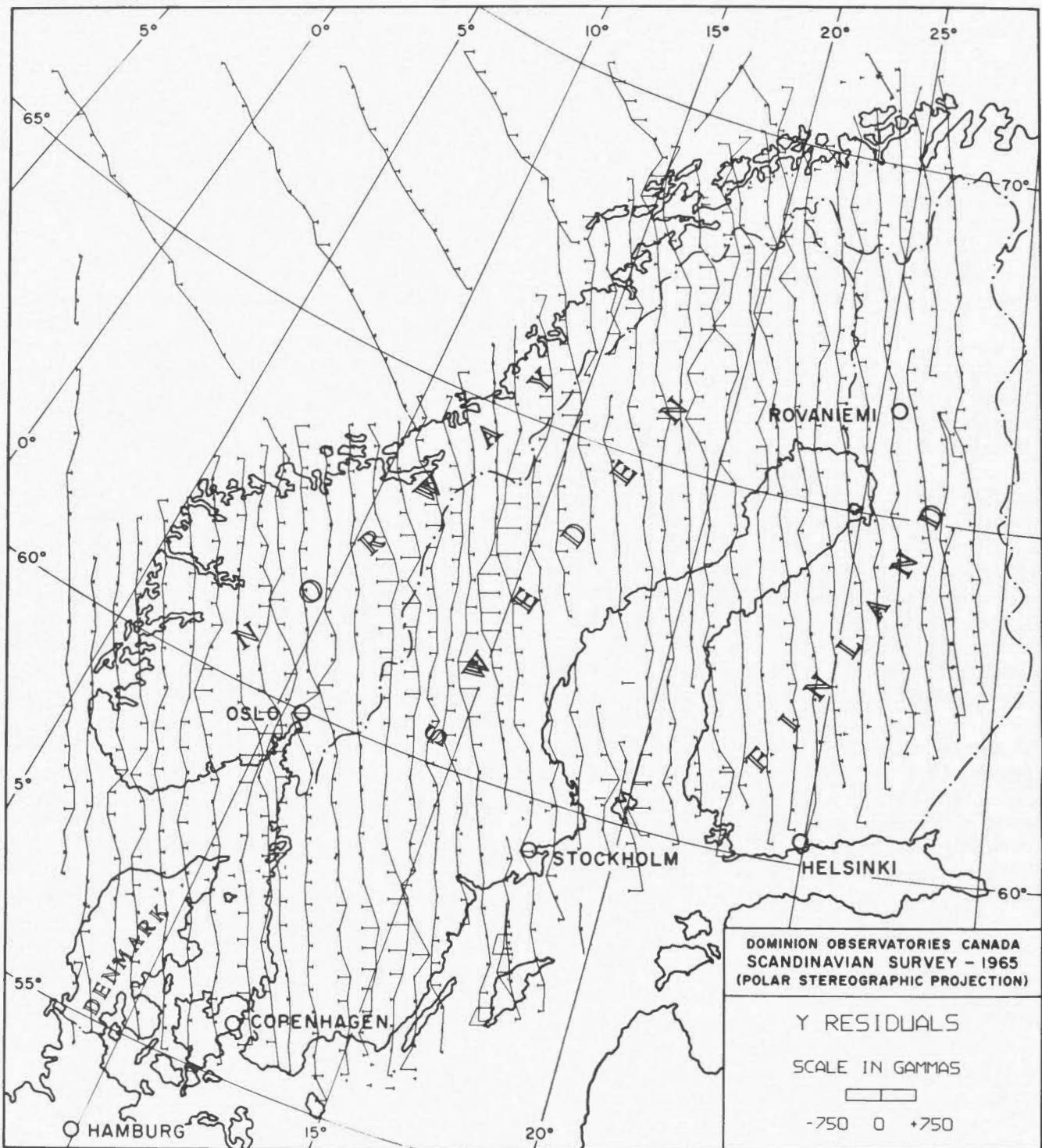


FIGURE 6. Residuals of east component of magnetic field.

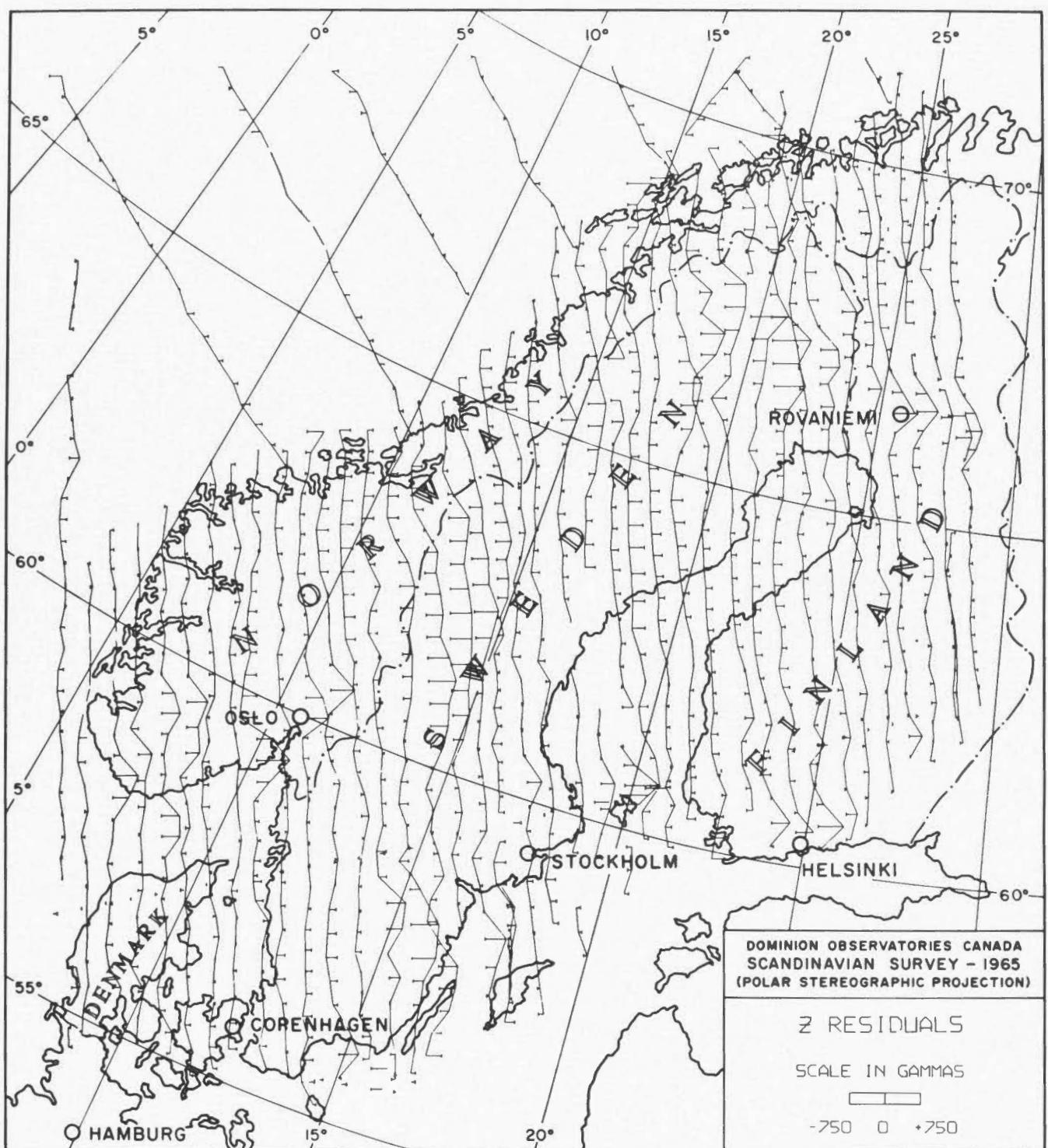


FIGURE 7. Residuals of vertical downward component of magnetic field.

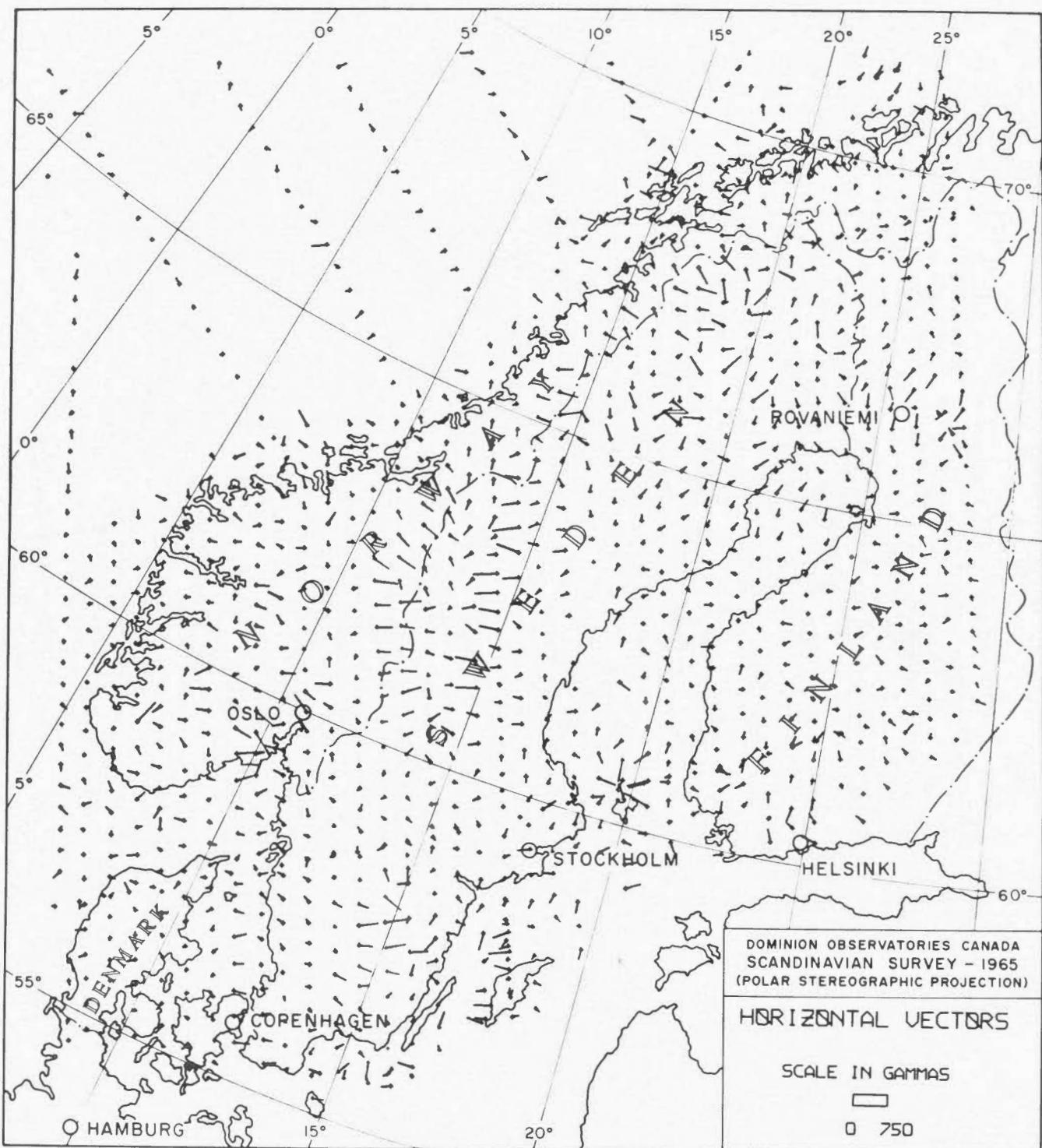


FIGURE 8. Projection of total residual vector onto horizontal plane.

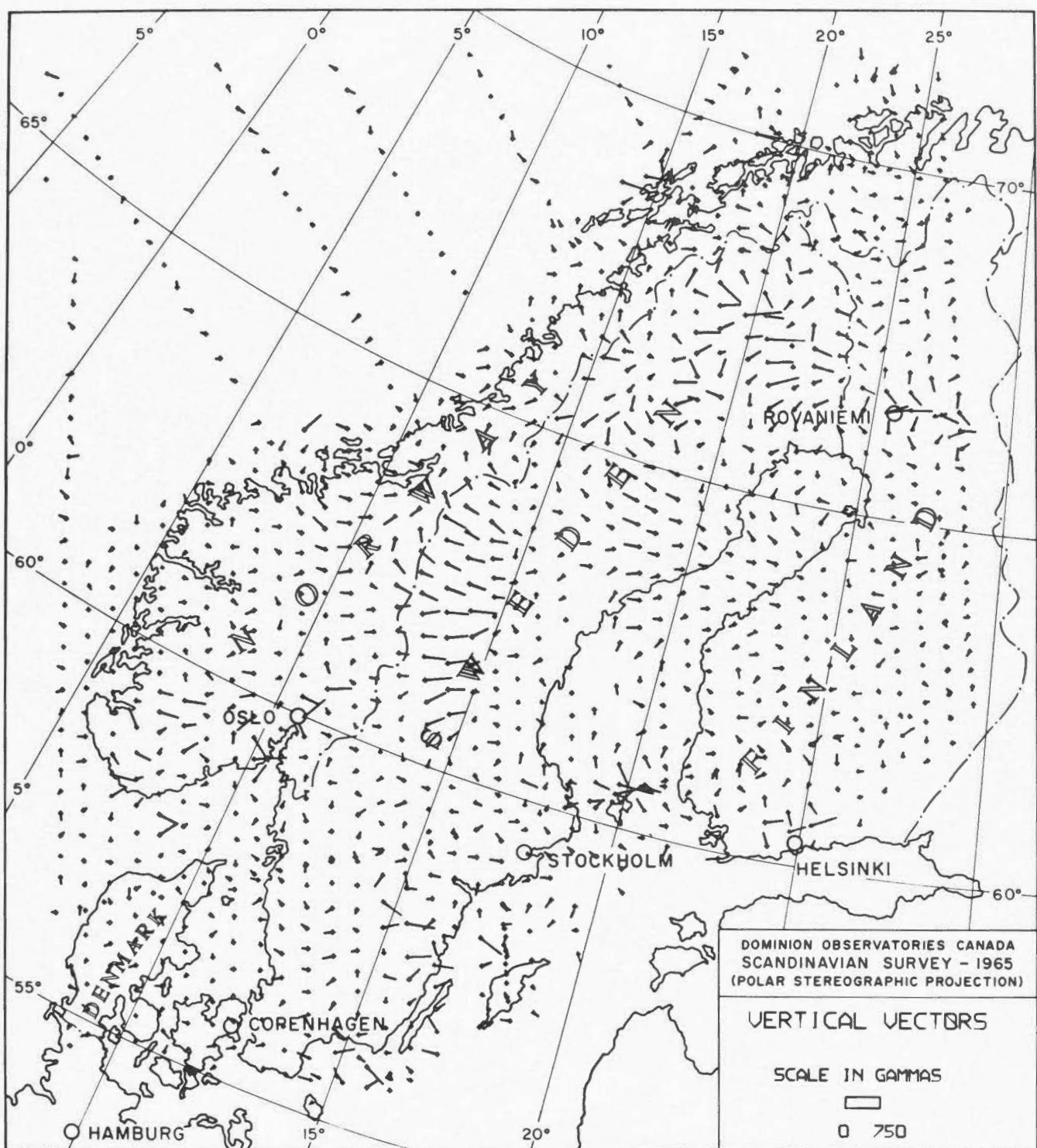


FIGURE 9. Projection of total residual vector onto vertical plane passing through the flight track.

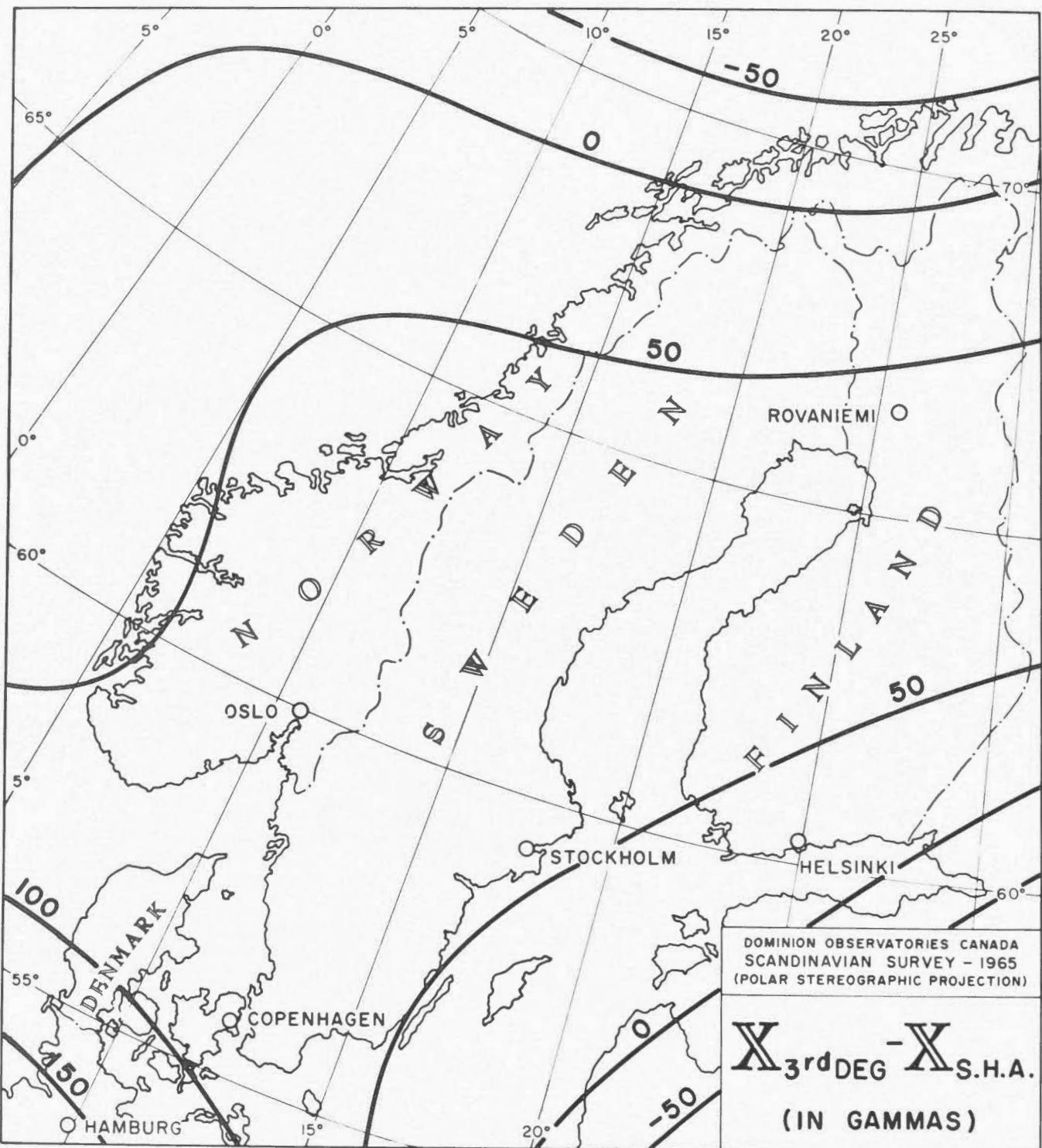


FIGURE 10. North component of magnetic field, obtained from 3rd degree polynomial, compared with that obtained from Cain's spherical harmonic analysis GSFC(12/66).

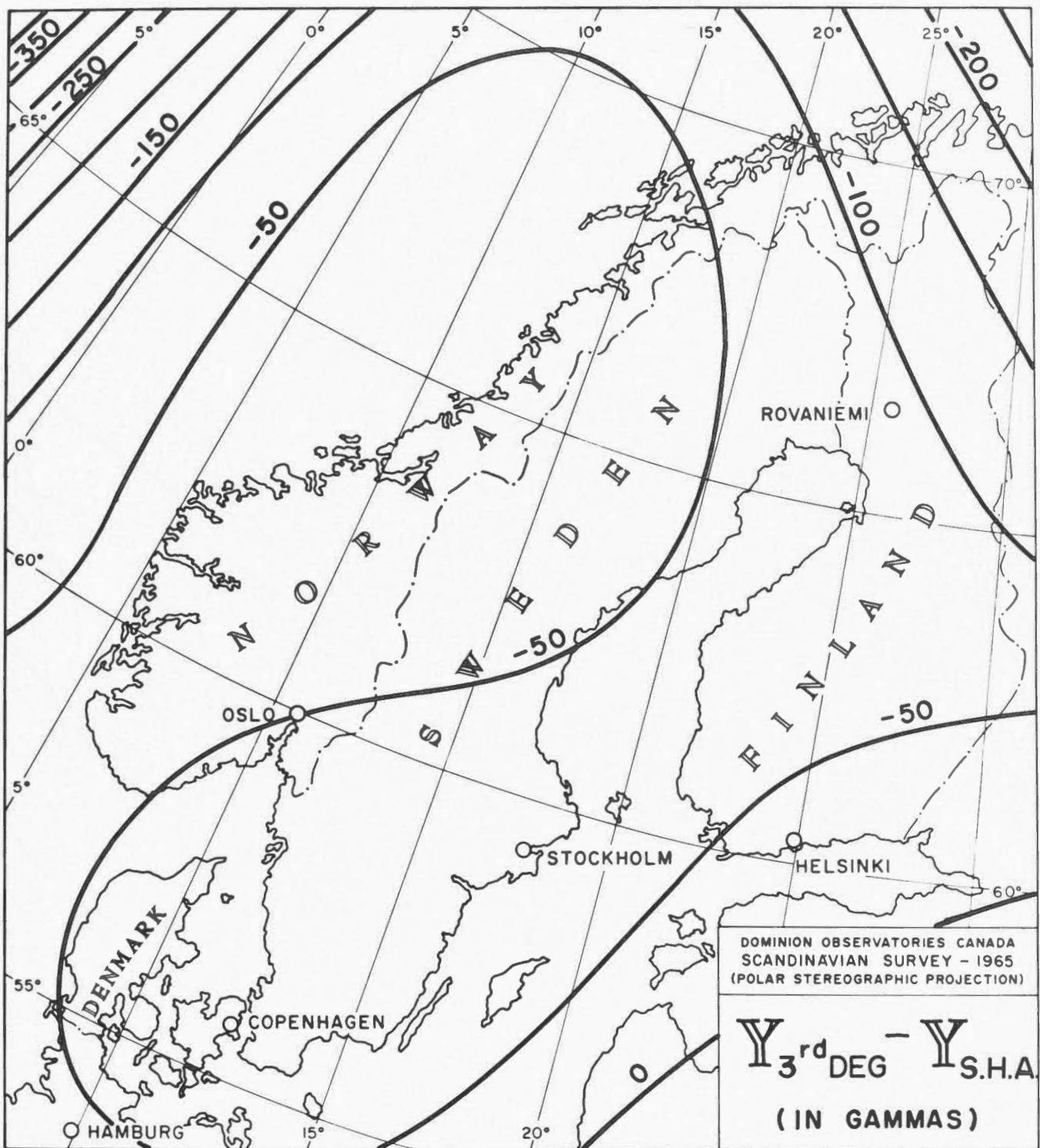


FIGURE 11. East component obtained from 3rd degree polynomial, compared with that from Cain's spherical harmonic analysis.

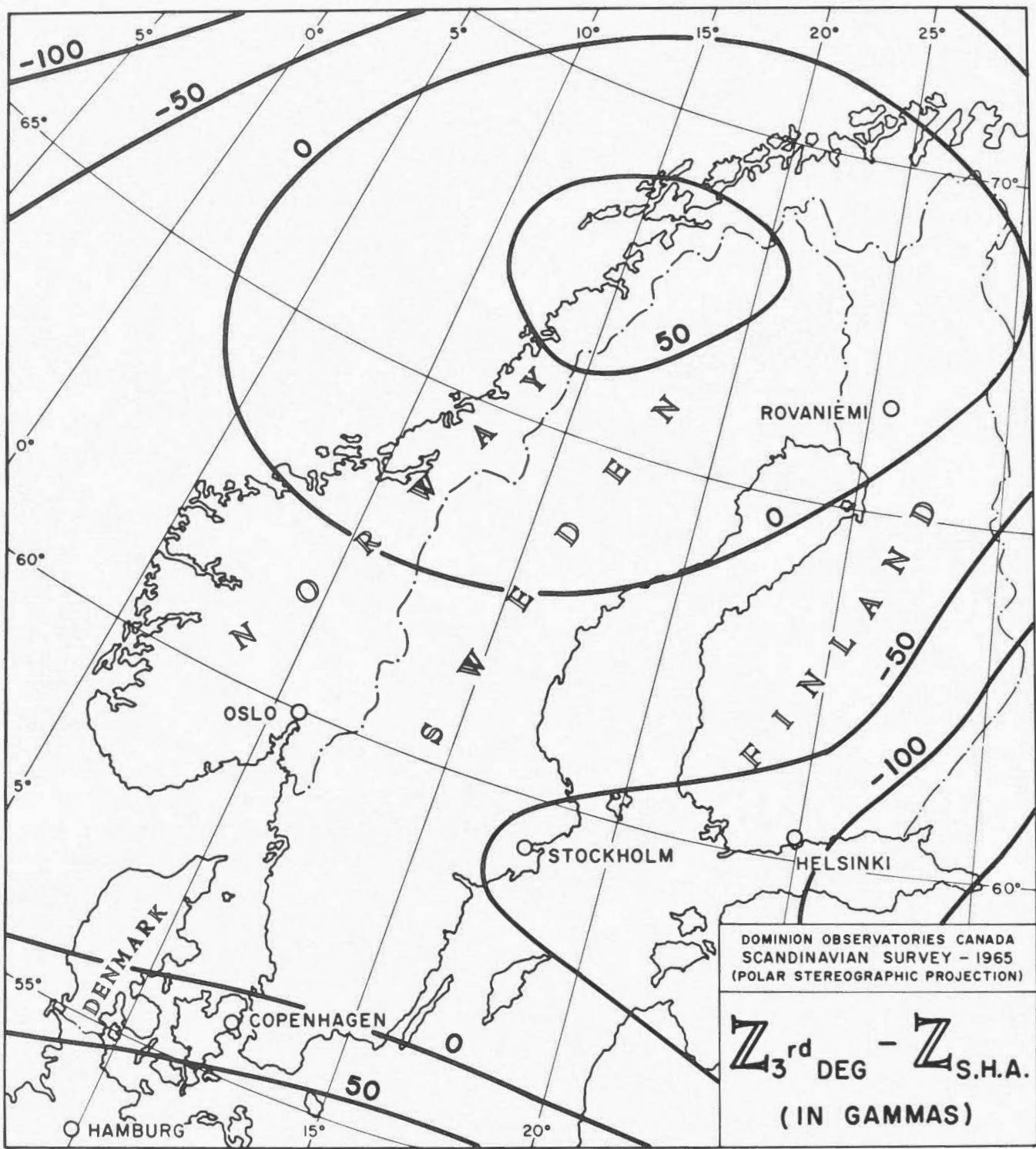


FIGURE 12. Vertical downward component obtained from 3rd degree polynomial, compared with that from Cain's spherical harmonic analysis.

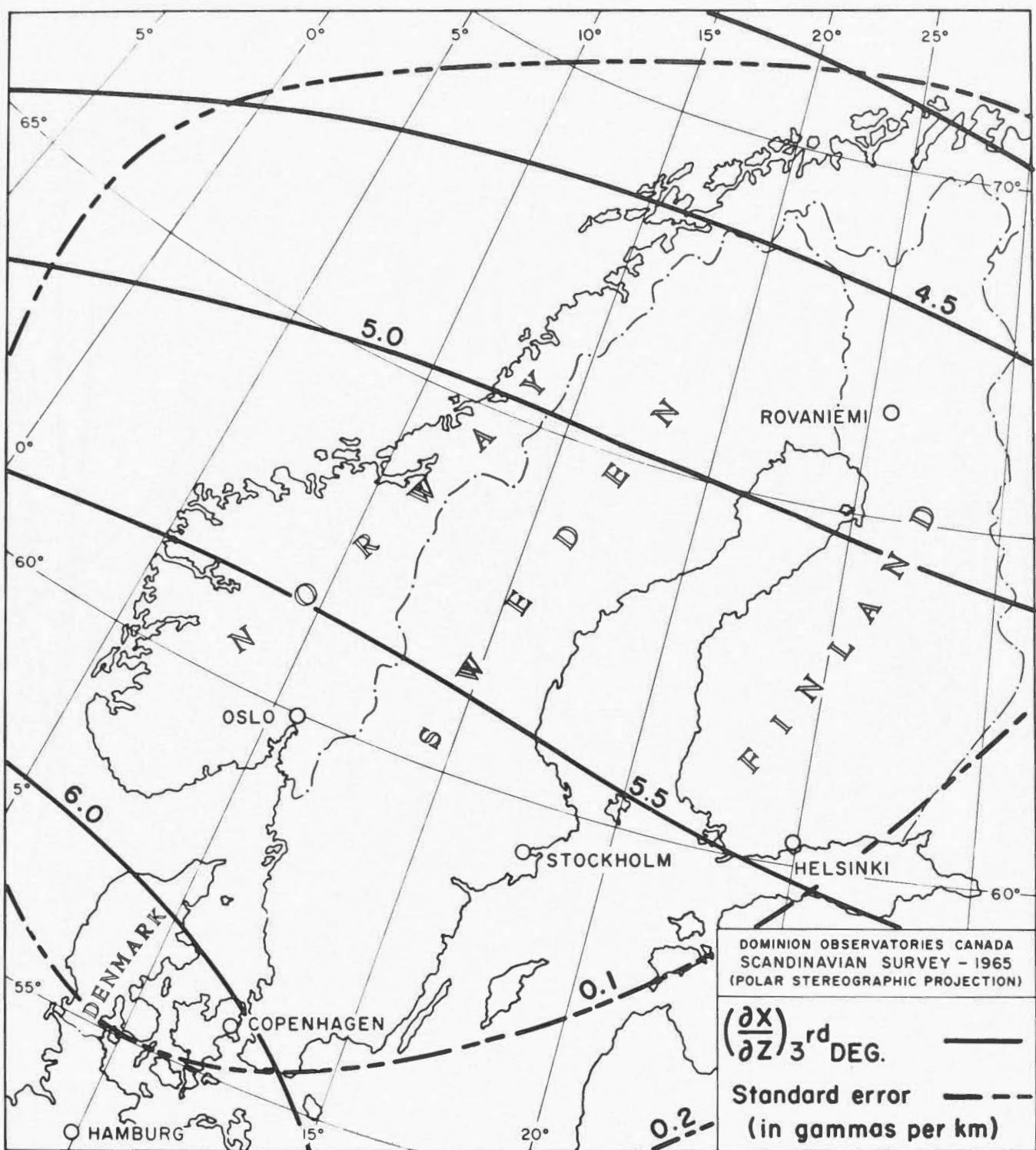


FIGURE 13. Vertical downward gradient of north component, and its standard error. Calculated from 3rd degree polynomial.

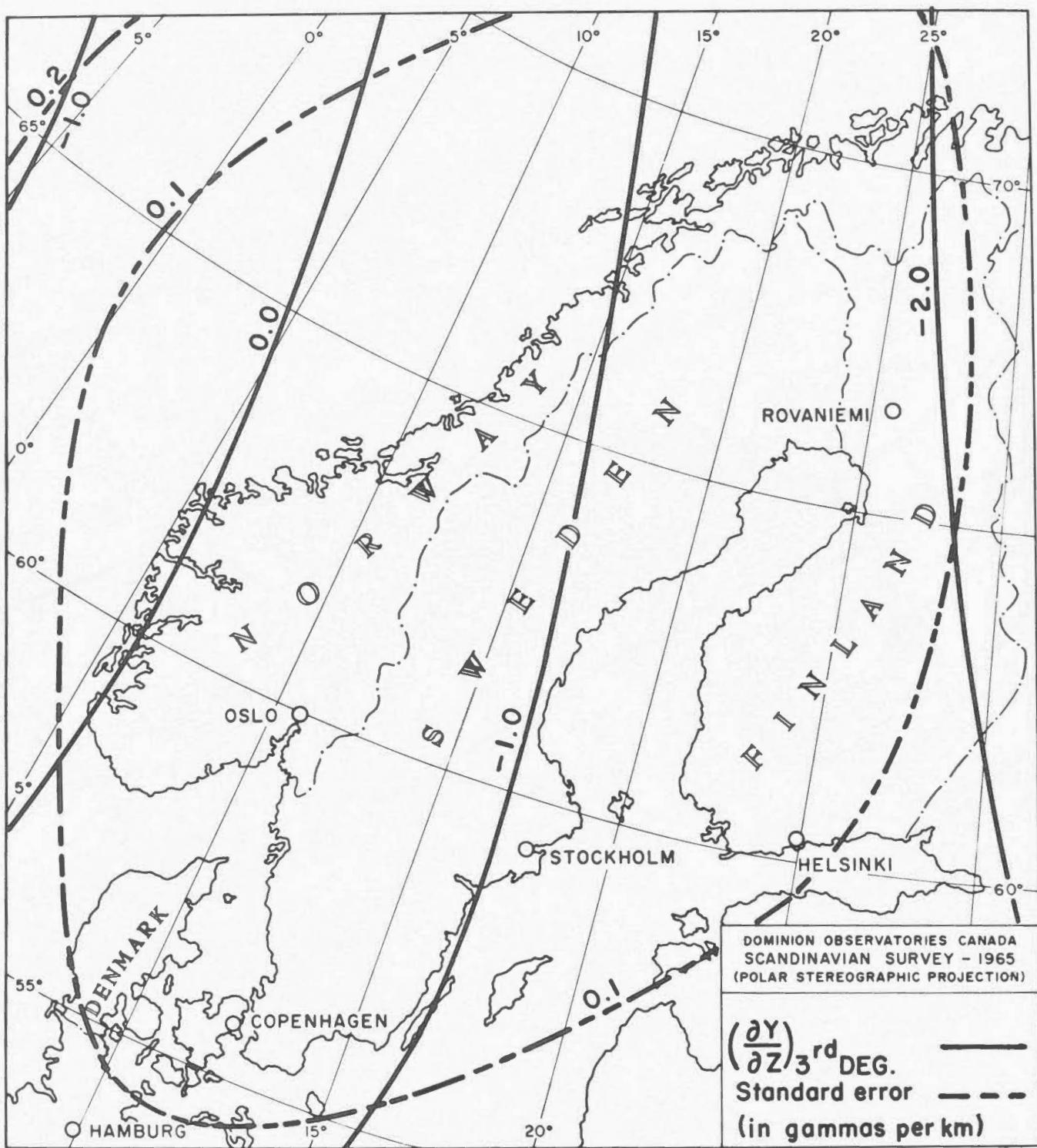


FIGURE 14. Vertical downward gradient of east component, and its standard error.

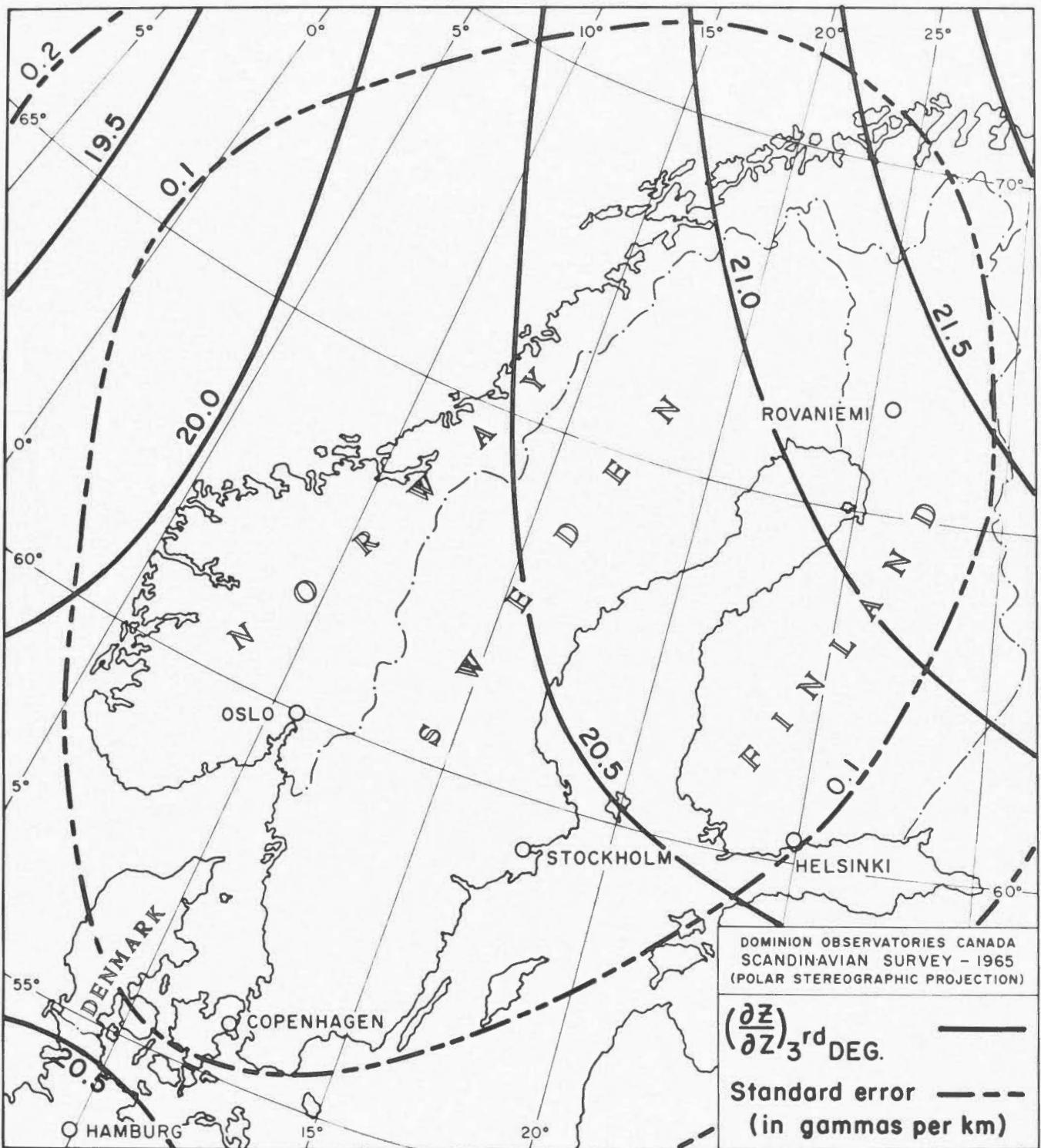


FIGURE 15. Vertical downward gradient of vertical component, and its standard error.

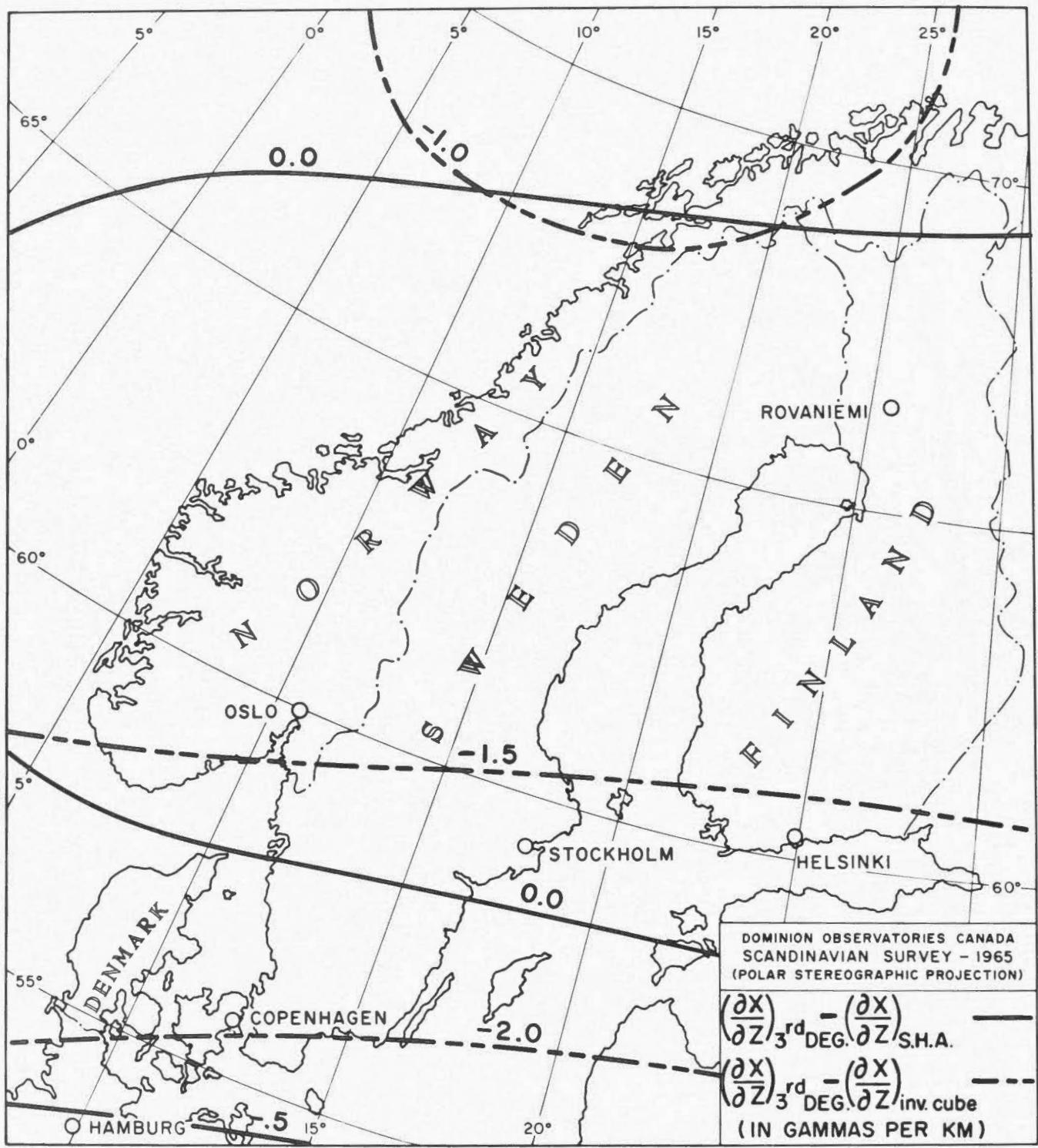


FIGURE 16. Vertical gradient of north component obtained from 3rd degree polynomial, compared with that obtained from (i) Cain's spherical harmonic analysis GSFC(12/66), and (ii) the 'inverse cube relationship' for a dipole field.

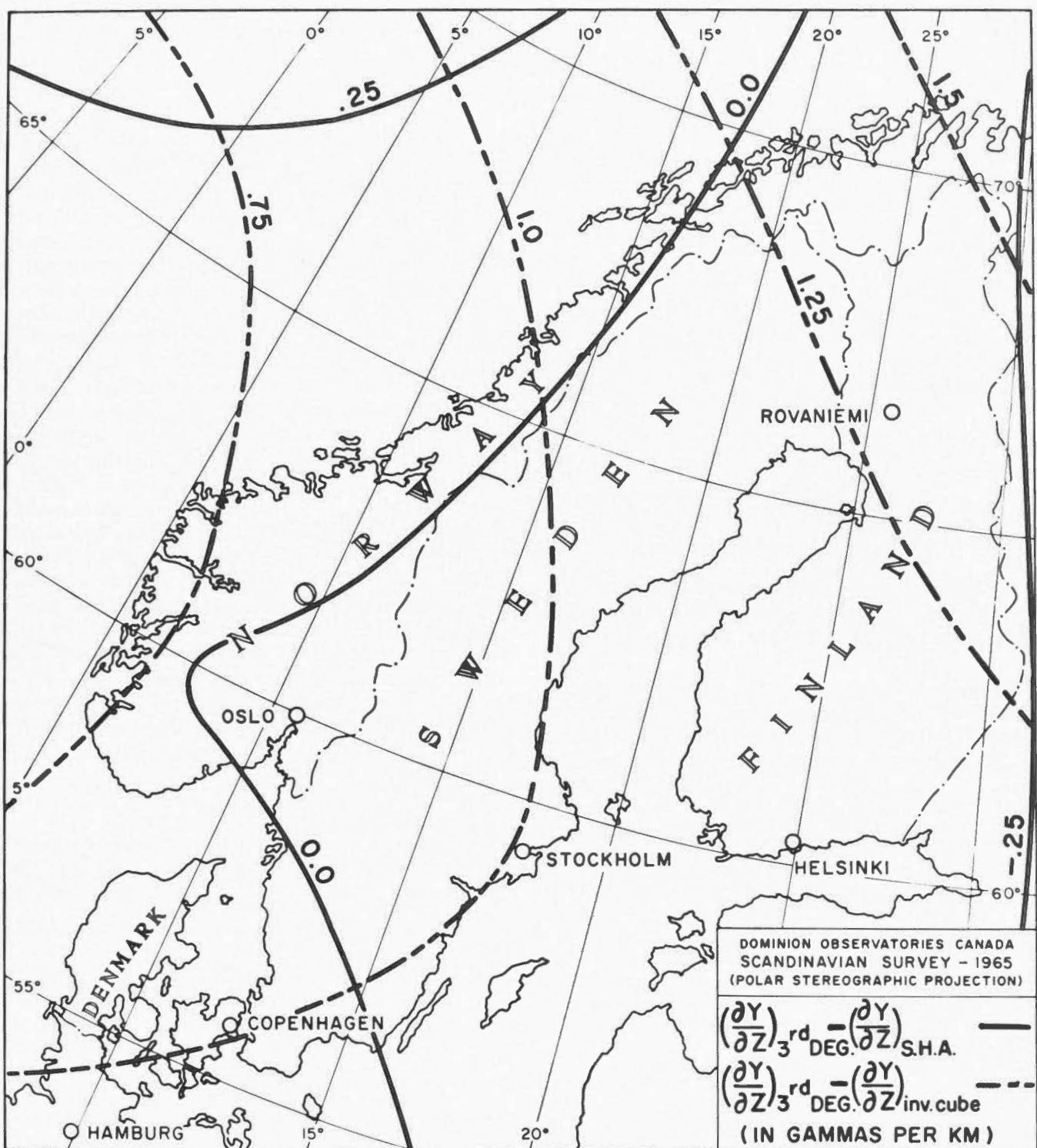


FIGURE 17. Vertical gradient of east component obtained from 3rd degree polynomial, compared with that obtained from (i) Cain's spherical harmonic analysis, and (ii) the 'inverse cube relationship'.

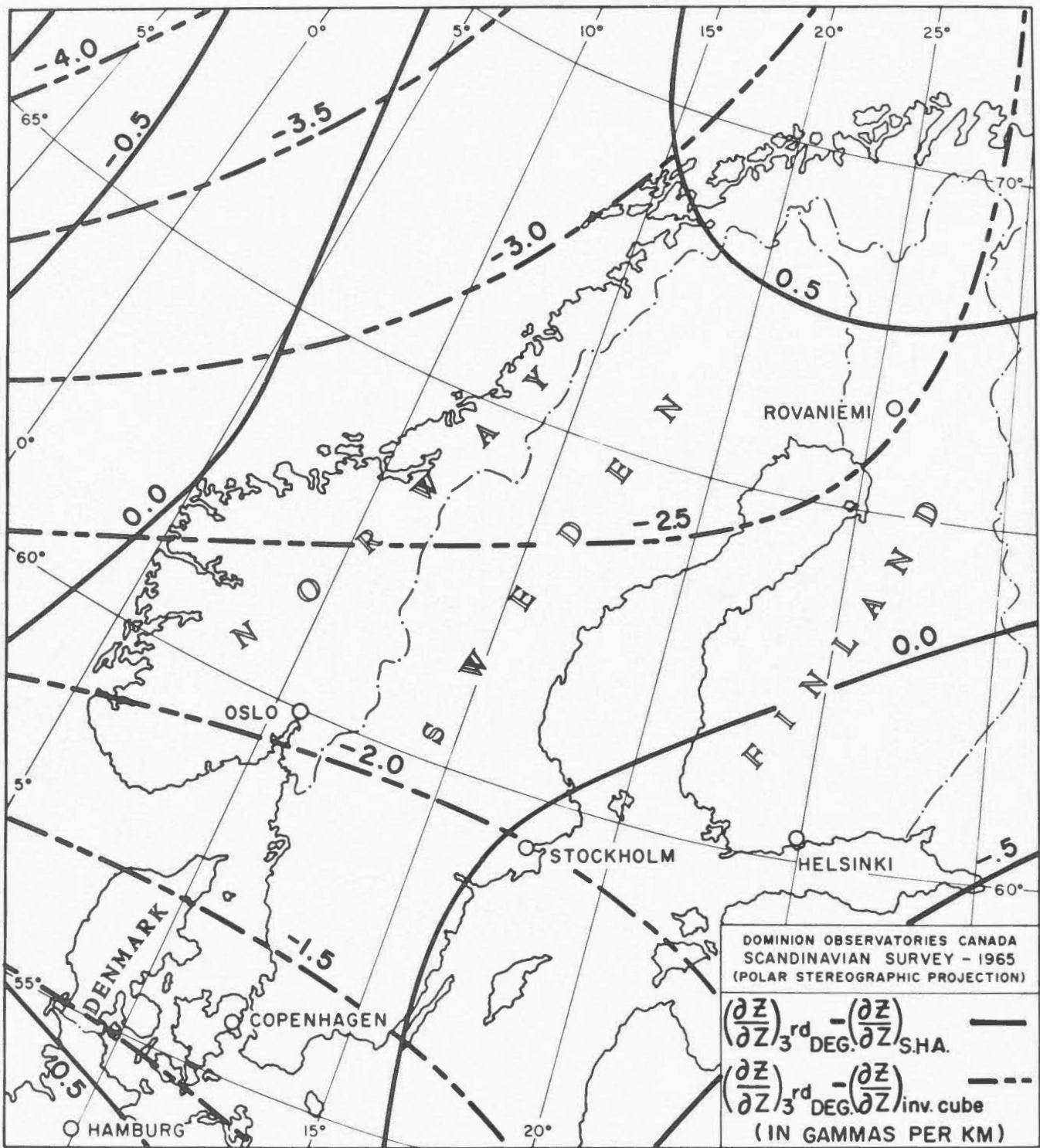


FIGURE 18. Vertical gradient of vertical component obtained from 3rd degree polynomial, compared with that obtained from (i) Cain's spherical harmonic analysis, and (ii) the 'inverse cube relationship'.

data area is always within about 75γ , but is naturally very poor outside. The disagreement in Y , in the upper left-hand corner of Figure 11, is rather puzzling since some data from this area were used in the least-squares analysis. These came from several flights across the Greenland and Norwegian Seas, and can be seen in Figures 5 to 9. (In fact, a polynomial expansion over the Greenland-Norwegian Sea area agrees quite well with the spherical harmonic model.) With this one exception, though, the agreements are generally in conformity with the standard errors in Figures 2 to 4.

It is of interest to know how the degree of a polynomial expansion compares with that of a spherical harmonic expansion. Bullard (1967) suggests that the number of constants per unit area is a reasonable basis of comparison. By this method, a 3rd degree polynomial fitted to the Scandinavian data area of 2 million square kilometres (see Figure 5) is 'equivalent' to a spherical harmonic expansion of degree 49. Bullard also shows that the shortest wavelength which can be represented in an n^{th} degree spherical harmonic expansion is c/n , where c is the circumference of the earth. A harmonic expansion of degree 49, then, would have a cutoff wavelength of 800 km, that of degree 10 a cutoff of 4000 km.

The vertical gradients of X, Y and Z , from the 3rd degree polynomials and Equation (17), with their standard errors, are contoured in Figures 13 to 15. Comparisons with the gradients calculated from the aforementioned spherical harmonic model of Cain are given in Figures 16 to 18, as well as comparisons with the gradients calculated from the inverse cube relationship. The latter comparisons are of interest since the data were reduced to sea level by using this relationship. Since most of the data were observed at about 3 km, the error introduced by using the inverse cube formula is less than 10γ .

TABLE 5

Vertical component of $\text{curl}_z H$, in Units of ma/km^2 , and
(in brackets) the Modulus of its t value.

Latitude	Longitude ($^{\circ}$ E)						
	0	5	10	15	20	25	30
($^{\circ}$ N) 70	-171 (1.1)	-140 (1.3)	-123 (1.5)	-122 (1.9)	-137 (2.3)	-167 (2.5)	-214 (2.3)
68	-125 (1.1)	-87 (1.2)	-66 (1.4)	-62 (1.7)	-75 (2.3)	-106 (2.4)	-155 (2.0)
66	-92 (1.1)	-47 (1.1)	-20 (0.7)	-12 (0.5)	-23 (1.1)	-54 (1.5)	-106 (1.4)
64	-72 (1.0)	-19 (0.6)	14 (0.6)	27 (1.0)	18 (0.8)	-13 (0.3)	-67 (0.8)
62	-65 (0.8)	-4 (0.1)	36 (1.3)	53 (2.2)	47 (1.9)	18 (0.3)	-37 (0.4)
60	-72 (0.7)	-2 (0.0)	45 (1.7)	68 (3.0)	66 (1.6)	38 (0.5)	-18 (0.1)
58	-94 (0.7)	-13 (0.2)	42 (1.2)	71 (1.7)	73 (0.9)	47 (0.4)	-9 (0.1)
56	-130 (0.7)	-40 (0.4)	24 (0.4)	61 (0.8)	68 (0.5)	45 (0.2)	-11 (0.0)

In Table 5 are listed the computed values of the vertical component of $\text{curl } H$ (see Equation (20)), with the modulus of its t value in brackets. The t value is the value of the $\text{curl}_z H$ estimate divided by its standard error. An estimate is 'statistically significant' at the 5 percent level whenever $|t|$ is larger than 2.0. It will be noted that only in two small areas can the curl be considered significantly non-zero. Of course, the test that $\text{curl}_z H = 0$ is not a very powerful one, in a statistical sense, because of the type 2 error mentioned in the previous section. That is, the hypothesis that $\text{curl}_z H = 0$ may be rejected with only a 5 percent chance of being wrong, but cannot be accepted with the same degree of confidence.

An appendix is included giving 3rd degree estimates of the geomagnetic elements U, V, Z, D, H, X, Y, I and F , the vertical gradients of X, Y, H and Z , and the vertical component of $\text{curl } H$. The angle I is the inclination of the total field vector F relative to the horizontal, and the vertical gradient of H is calculated from

$$\frac{\partial H}{\partial z} = \frac{1}{H} \left[X \frac{\partial X}{\partial z} + Y \frac{\partial Y}{\partial z} \right]$$

The other components have been defined previously.

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Appendix

Estimates of Several Geomagnetic Quantities, Calculated from 3rd Degree Polynomial

Vertical gradients (DX , DY , DH , DZ) of X , Y , H and Z are in gammas per kilometre, and vertical component of curl H (CURL) is in millamps per square kilometre.

LAT	LONG	U	V	Z	D	H	X	Y	I	F	DX	DY	DH	DZ	CURL
54.00	2.00	7879	-15900	44908	-6.6	17745	17626	-2052	68.4	48287	6.5	-0.3	6.5	20.5	-138.7
54.00	4.00	7591	-16056	44930	-5.7	17760	17673	-1762	68.4	48313	6.5	-0.0	6.4	20.5	-99.0
54.00	6.00	7290	-16207	44976	-4.8	17771	17710	-1481	68.4	48340	6.4	.2	6.4	20.5	-63.9
54.00	8.00	6979	-16350	45045	-3.9	17778	17737	-1204	68.5	48426	6.3	.4	6.3	20.5	-33.3
54.00	10.00	6661	-16484	45133	-3.0	17779	17754	-930	68.5	48508	6.2	.6	6.2	20.5	-7.3
54.00	12.00	6337	-16605	45238	-2.1	17773	17761	-655	68.6	48604	6.2	.8	6.1	20.5	14.1
54.00	14.00	6010	-16712	45357	-1.2	17760	17756	-379	68.6	48710	6.1	.9	6.1	20.5	30.7
54.00	16.00	5681	-16804	45489	-0.3	17739	17738	-100	68.7	48825	6.0	1.0	6.0	20.4	42.4
54.00	18.00	5352	-16881	45632	.6	17709	17708	182	68.8	48947	5.9	1.2	5.9	20.3	49.2
54.00	20.00	5023	-16940	45783	1.5	17669	17663	467	68.9	49074	5.9	1.3	5.9	20.3	50.8
54.00	22.00	4695	-16983	45941	2.5	17620	17604	755	69.0	49204	5.8	1.4	5.9	20.2	47.2
54.00	24.00	4369	-17009	46105	3.4	17561	17530	1043	69.1	49336	5.8	1.4	5.8	20.1	38.2
54.00	26.00	4044	-17018	46273	4.4	17492	17442	1332	69.3	49469	5.7	1.5	5.8	20.0	23.7
54.00	28.00	3720	-17012	46447	5.3	17414	17339	1619	69.4	49604	5.7	1.6	5.8	19.9	3.4
54.00	30.00	3397	-16992	46624	6.3	17328	17223	1903	69.6	49740	5.7	1.7	5.9	19.8	-22.7
54.00	32.00	3072	-16958	46806	7.3	17234	17095	2181	69.8	49878	5.7	1.8	5.9	19.8	-54.7
55.00	2.00	7578	-15478	45322	-6.9	17234	17108	-2074	69.2	48488	6.4	-0.3	6.4	20.3	-112.8
55.00	4.00	7311	-15625	45337	-5.9	17251	17159	-1780	69.2	48508	6.3	-0.1	6.3	20.4	-75.4
55.00	6.00	7031	-15768	45376	-5.0	17265	17200	-1495	69.2	48549	6.3	.2	6.3	20.4	-42.4
55.00	8.00	6740	-15906	45436	-4.0	17275	17232	-1216	69.2	48609	6.2	.4	6.2	20.4	-13.8
55.00	10.00	6440	-16035	45515	-3.1	17280	17254	-940	69.2	48685	6.2	.5	6.1	20.5	10.3
55.00	12.00	6135	-16153	45611	-2.2	17279	17266	-665	69.3	48774	6.1	.7	6.1	20.4	29.9
55.00	14.00	5825	-16260	45722	-1.3	17272	17267	-389	69.3	48876	6.0	.9	6.0	20.4	44.8
55.00	16.00	5512	-16354	45847	-0.4	17252	17257	-113	69.4	48987	6.0	1.0	5.9	20.4	55.1
55.00	18.00	5197	-16433	45982	.6	17235	17235	166	69.5	49106	5.9	1.1	5.9	20.3	60.5
55.00	20.00	4882	-16498	46127	1.5	17205	17199	446	69.5	49231	5.8	1.2	5.9	20.3	60.9
55.00	22.00	4567	-16548	46280	2.4	17166	17151	728	69.6	49361	5.8	1.3	5.8	20.2	56.3
55.00	24.00	4252	-16583	46441	3.4	17119	17089	1010	69.8	49495	5.7	1.4	5.8	20.2	46.5
55.00	26.00	3938	-16603	46608	4.3	17063	17014	1292	69.9	49633	5.7	1.5	5.8	20.1	31.4
55.00	28.00	3623	-16608	46781	5.3	16999	16926	1572	70.0	49773	5.7	1.6	5.8	20.0	10.8
55.00	30.00	3308	-16601	46959	6.3	16927	16826	1848	70.2	49917	5.7	1.7	5.8	20.0	-15.4
55.00	32.00	2991	-16581	47144	7.2	16849	16715	2119	70.3	50064	5.7	1.8	5.9	19.9	-47.3
56.00	2.00	7294	-15069	45729	-7.2	16742	16611	-2091	69.9	48697	6.2	-0.3	6.2	20.2	-90.8
56.00	4.00	7047	-15208	45739	-6.1	16761	16665	-1793	69.9	48713	6.2	-0.1	6.2	20.3	-55.6
56.00	6.00	6786	-15344	45771	-5.1	16778	16710	-1504	69.9	48749	6.2	.1	6.1	20.3	-24.7
56.00	8.00	6514	-15475	45824	-4.2	16790	16746	-1222	69.9	48804	6.1	.3	6.1	20.4	2.0
56.00	10.00	6233	-15600	45896	-3.2	16799	16772	-944	69.9	48874	6.1	.5	6.0	20.4	24.3
56.00	12.00	5945	-15715	45985	-2.3	16802	16789	-668	69.9	48959	6.0	.7	6.0	20.4	42.2
56.00	14.00	5652	-15821	46089	-1.3	16800	16795	-393	70.0	49056	6.0	.8	5.9	20.4	55.6
56.00	16.00	5355	-15915	46207	-0.4	16792	16791	-119	70.0	49163	5.9	1.0	5.9	20.4	64.4
56.00	18.00	5055	-15997	46336	.5	16776	16776	157	70.1	49279	5.8	1.1	5.9	20.3	68.6
56.00	20.00	4753	-16065	46476	1.5	16754	16748	433	70.2	49403	5.8	1.2	5.8	20.3	67.9
56.00	22.00	4451	-16121	46625	2.4	16724	16709	710	70.3	49533	5.7	1.3	5.8	20.3	62.4
56.00	24.00	4147	-16163	46782	3.4	16686	16657	987	70.4	49669	5.7	1.5	5.8	20.2	51.9
56.00	26.00	3843	-16192	46947	4.4	16641	16593	1263	70.5	49809	5.7	1.6	5.8	20.2	36.3
56.00	28.00	3538	-16207	47119	5.3	16589	16518	1536	70.6	49954	5.7	1.7	5.8	20.2	15.4
56.00	30.00	3231	-16211	47299	6.3	16530	16431	1806	70.7	50104	5.7	1.8	5.8	20.1	-10.8
56.00	32.00	2922	-16203	47486	7.2	16465	16334	2070	70.9	50259	5.7	1.9	5.9	20.1	-42.4
57.00	2.00	7023	-14674	46128	-7.4	16268	16132	-2102	70.6	48912	6.1	-0.4	6.1	20.1	-72.8
57.00	4.00	6795	-14804	46134	-6.3	16289	16189	-1800	70.6	48926	6.1	-0.1	6.1	20.2	-39.7
57.00	6.00	6553	-14933	46162	-5.3	16308	16238	-1508	70.5	48958	6.1	.1	6.0	20.3	-10.7
57.00	8.00	6300	-15058	46210	-4.3	16323	16277	-1223	70.5	49008	6.0	.3	6.0	20.3	-14.1
57.00	10.00	6037	-15178	46276	-3.3	16334	16307	-943	70.6	49074	6.0	.5	5.9	20.3	34.7
57.00	12.00	5766	-15290	46359	-2.3	16341	16327	-667	70.6	49155	5.9	.6	5.9	20.4	51.0

LAT	LONG	U	V	Z	D	H	X	Y	I	F	DX	DY	DH	DZ	CURL
57.00	14.00	5489	-15394	46457	-1.4	16343	16338	-392	70.6	49248	5.9	.8	5.9	20.4	63.0
57.00	16.00	5207	-15487	46568	-0.4	16339	16339	-119	70.7	49352	5.8	1.0	5.8	20.4	70.5
57.00	18.00	4922	-15570	46692	.5	16330	16329	154	70.7	49465	5.8	1.1	5.8	20.4	73.5
57.00	20.00	4634	-15642	46827	1.5	16314	16308	427	70.8	49588	5.7	1.2	5.8	20.3	71.9
57.00	22.00	4344	-15702	46972	2.5	16291	16276	700	70.9	49717	5.7	1.4	5.8	20.3	65.6
57.00	24.00	4051	-15749	47127	3.4	16262	16233	972	71.0	49854	5.7	1.5	5.8	20.3	54.5
57.00	26.00	3758	-15785	47290	4.4	16226	16178	1242	71.1	49996	5.7	1.6	5.8	20.3	38.5
57.00	28.00	3462	-15809	47462	5.4	16184	16113	1510	71.2	50145	5.6	1.7	5.8	20.3	17.4
57.00	30.00	3164	-15822	47642	6.3	16135	16037	1773	71.3	50300	5.6	1.8	5.8	20.3	-8.7
57.00	32.00	2864	-15824	47830	7.3	16081	15952	2032	71.4	50461	5.6	1.9	5.8	20.2	-40.0
58.00	2.00	6765	-14289	46520	-7.7	15810	15669	-2109	71.2	49133	6.0	-0.4	6.0	20.0	-58.6
58.00	4.00	6556	-14412	46523	-6.5	15833	15730	-1803	71.2	49144	6.0	-0.2	5.9	20.1	-27.5
58.00	6.00	6332	-14534	46548	-5.5	15853	15781	-1508	71.2	49174	5.9	.1	5.9	20.2	-0.4
58.00	8.00	6096	-14653	46592	-4.4	15870	15823	-1220	71.2	49220	5.9	.3	5.9	20.3	22.6
58.00	10.00	5850	-14767	46653	-3.4	15884	15856	-939	71.2	49283	5.9	.4	5.8	20.3	41.6
58.00	12.00	5596	-14876	46731	-2.4	15894	15880	-661	71.2	49360	5.8	.6	5.8	20.3	56.4
58.00	14.00	5335	-14977	46824	-1.4	15899	15894	-387	71.2	49449	5.8	.8	5.8	20.4	67.1
58.00	16.00	5068	-15070	46930	-0.4	15899	15899	-114	71.3	49550	5.8	.9	5.8	20.4	73.4
58.00	18.00	4797	-15153	47049	.6	15894	15893	157	71.3	49662	5.7	1.1	5.7	20.4	75.4
58.00	20.00	4522	-15226	47180	1.5	15884	15878	427	71.4	49782	5.7	1.2	5.7	20.4	72.9
58.00	22.00	4245	-15289	47322	2.5	15867	15852	697	71.5	49912	5.7	1.4	5.7	20.4	65.9
58.00	24.00	3964	-15341	47474	3.5	15845	15815	964	71.5	50048	5.6	1.5	5.7	20.4	54.3
58.00	26.00	3681	-15382	47636	4.5	15816	15768	1229	71.6	50193	5.6	1.6	5.7	20.4	37.9
58.00	28.00	3395	-15412	47806	5.4	15782	15711	1492	71.7	50344	5.6	1.7	5.7	20.4	16.8
58.00	30.00	3107	-15433	47987	6.4	15742	15645	1750	71.8	50503	5.6	1.8	5.7	20.4	-9.2
58.00	32.00	2815	-15443	48176	7.3	15697	15569	2003	72.0	50669	5.6	2.0	5.8	20.4	-40.2
59.00	2.00	6519	-13915	46903	-7.9	15367	15221	-2111	71.9	49356	5.8	-0.4	5.8	20.0	-48.1
59.00	4.00	6327	-14030	46906	-6.7	15391	15285	-1803	71.8	49366	5.8	-0.2	5.8	20.1	-18.9
59.00	6.00	6120	-14145	46928	-5.6	15412	15339	-1505	71.8	49394	5.8	.0	5.8	20.1	6.3
59.00	8.00	5902	-14258	46969	-4.5	15431	15383	-1215	71.8	49438	5.8	.2	5.8	20.2	27.7
59.00	10.00	5672	-14367	47026	-3.5	15446	15418	-931	71.8	49498	5.8	.4	5.7	20.3	45.2
59.00	12.00	5434	-14472	47100	-2.4	15458	15444	-652	71.8	49572	5.7	.6	5.7	20.3	58.6
59.00	14.00	5188	-14570	47189	-1.4	15466	15461	-378	71.9	49659	5.7	.8	5.7	20.4	67.9
59.00	16.00	4936	-14661	47291	-0.4	15469	15469	-106	71.9	49757	5.7	.9	5.7	20.4	73.2
59.00	18.00	4679	-14743	47407	.6	15468	15467	164	71.9	49866	5.6	1.1	5.7	20.4	74.2
59.00	20.00	4418	-14817	47534	1.6	15462	15456	432	72.0	49986	5.6	1.2	5.6	20.4	70.9
59.00	22.00	4152	-14882	47673	2.6	15450	15434	698	72.0	50114	5.6	1.4	5.6	20.5	63.3
59.00	24.00	3883	-14937	47822	3.6	15433	15403	962	72.1	50251	5.6	1.5	5.6	20.5	51.2
59.00	26.00	3611	-14982	47982	4.6	15411	15362	1223	72.2	50396	5.5	1.6	5.7	20.5	34.6
59.00	28.00	3335	-15017	48152	5.5	15383	15312	1480	72.3	50549	5.5	1.8	5.7	20.5	13.5
59.00	30.00	3056	-15043	48332	6.5	15350	15252	1733	72.4	50711	5.5	1.9	5.7	20.5	-12.3
59.00	32.00	2773	-15060	48521	7.4	15313	15184	1981	72.5	50880	5.5	2.0	5.7	20.6	-42.9
60.00	2.00	6283	-13550	47278	-8.1	14935	14785	-2111	72.5	49581	5.7	-0.4	5.7	19.9	-41.2
60.00	4.00	6107	-13657	47281	-6.9	14960	14852	-1799	72.4	49591	5.7	-0.2	5.7	20.0	-13.9
60.00	6.00	5917	-13765	47302	-5.7	14983	14908	-1498	72.4	49618	5.7	.0	5.7	20.1	9.6
60.00	8.00	5714	-13872	47340	-4.6	15003	14954	-1206	72.4	49661	5.7	.2	5.6	20.2	29.4
60.00	10.00	5501	-13976	47395	-3.5	15020	14992	-921	72.4	49718	5.7	.4	5.6	20.3	45.4
60.00	12.00	5278	-14076	47466	-2.4	15033	15020	-641	72.4	49790	5.6	.6	5.6	20.3	57.5
60.00	14.00	5048	-14171	47551	-1.4	15043	15039	-366	72.4	49874	5.6	.8	5.6	20.4	65.6
60.00	16.00	4810	-14259	47650	-0.4	15049	15049	-94	72.5	49970	5.6	.9	5.6	20.4	69.8
60.00	18.00	4567	-14341	47763	.7	15050	15049	174	72.5	50078	5.6	1.1	5.6	20.5	70.0
60.00	20.00	4318	-14415	47887	1.7	15047	15041	440	72.6	50195	5.5	1.2	5.6	20.5	66.0
60.00	22.00	4065	-14480	48023	2.7	15040	15023	703	72.6	50323	5.5	1.4	5.6	20.5	57.8
60.00	24.00	3807	-14537	48170	3.7	15027	14996	964	72.7	50460	5.5	1.5	5.6	20.6	45.4

LAT	LONG	U	V	Z	D	H	X	Y	I	F	DX	DY	DH	DZ	CURL
60.00	26.00	3546	-14584	48328	4.7	15009	14960	1221	72.7	50605	5.5	1.7	5.6	20.6	28.6
60.00	28.00	3281	-14623	48497	5.6	14987	14914	1474	72.8	50759	5.4	1.8	5.6	20.6	7.5
60.00	30.00	3011	-14653	48676	6.6	14960	14860	1723	72.9	50923	5.4	1.9	5.6	20.7	-18.0
60.00	32.00	2737	-14675	48865	7.6	14928	14798	1966	73.0	51095	5.4	2.1	5.6	20.7	-48.1
61.00	2.00	6054	-13192	47645	-8.3	14515	14361	-2107	73.1	49806	5.5	-0.4	5.5	19.9	-37.8
61.00	4.00	5894	-13292	47648	-7.1	14561	14430	-1793	73.0	49817	5.5	-0.2	5.5	20.0	-12.4
61.00	6.00	5720	-13394	47668	-5.9	14564	14488	-1490	73.0	49844	5.6	.0	5.5	20.1	9.5
61.00	8.00	5533	-13494	47706	-4.7	14585	14536	-1196	73.0	49885	5.6	.2	5.5	20.2	27.7
61.00	10.00	5335	-13593	47759	-3.6	14603	14574	-909	73.0	49942	5.5	.4	5.5	20.2	42.3
61.00	12.00	5128	-13688	47827	-2.5	14617	14604	-628	73.0	50011	5.5	.6	5.5	20.3	53.2
61.00	14.00	4912	-13779	47910	-1.4	14629	14624	-353	73.0	50094	5.5	.8	5.5	20.4	60.2
61.00	16.00	4688	-13865	48007	-0.3	14636	14636	-.81	73.0	50188	5.5	.9	5.5	20.4	63.5
61.00	18.00	4458	-13944	48116	.7	14640	14638	186	73.1	50294	5.5	1.1	5.5	20.5	62.8
61.00	20.00	4222	-14017	48238	1.8	14639	14632	450	73.1	50410	5.5	1.3	5.5	20.6	58.2
61.00	22.00	3981	-14082	48371	2.8	14634	14617	711	73.2	50536	5.4	1.4	5.5	20.6	49.5
61.00	24.00	3735	-14139	48516	3.8	14625	14592	969	73.2	50672	5.4	1.6	5.5	20.7	36.8
61.00	26.00	3485	-14189	48672	4.8	14610	14559	1222	73.3	50818	5.4	1.7	5.5	20.7	20.0
61.00	28.00	3230	-14230	48839	5.8	14592	14518	1471	73.4	50973	5.4	1.8	5.5	20.8	-1.1
61.00	30.00	2970	-14263	49018	6.8	14569	14468	1716	73.4	51137	5.3	2.0	5.5	20.8	-26.3
61.00	32.00	2706	-14288	49207	7.7	14542	14410	1954	73.5	51310	5.3	2.1	5.6	20.9	-55.9
62.00	2.00	5833	-12840	48002	-8.6	14103	13945	-2101	73.6	50031	5.4	-0.4	5.4	19.9	-38.0
62.00	4.00	5688	-12934	48006	-7.3	14129	14016	-1786	73.6	50043	5.4	-0.2	5.4	20.0	-14.3
62.00	6.00	5529	-13029	48028	-6.0	14153	14076	-1481	73.6	50070	5.4	.0	5.4	20.1	6.0
62.00	8.00	5357	-13123	48065	-4.8	14175	14125	-1185	73.6	50111	5.4	.2	5.4	20.2	22.8
62.00	10.00	5174	-13217	48117	-3.6	14193	14165	-896	73.6	50166	5.4	.4	5.4	20.3	36.0
62.00	12.00	4981	-13307	48184	-2.5	14209	14195	-615	73.6	50235	5.4	.6	5.4	20.3	45.7
62.00	14.00	4779	-13394	48265	-1.4	14221	14217	-339	73.6	50316	5.4	.8	5.4	20.4	51.7
62.00	16.00	4569	-13476	48359	-0.3	14230	14229	-67	73.6	50409	5.4	1.0	5.4	20.5	54.1
62.00	18.00	4352	-13553	48466	.8	14234	14233	199	73.6	50513	5.4	1.1	5.4	20.6	52.7
62.00	20.00	4129	-13624	48585	1.9	14235	14228	462	73.7	50628	5.4	1.3	5.4	20.6	47.5
62.00	22.00	3900	-13688	48717	2.9	14232	14214	721	73.7	50753	5.3	1.4	5.4	20.7	38.4
62.00	24.00	3665	-13745	48860	3.9	14225	14192	975	73.8	50888	5.3	1.6	5.4	20.7	25.5
62.00	26.00	3426	-13795	49014	4.9	14214	14161	1226	73.8	51033	5.3	1.7	5.4	20.8	8.6
62.00	28.00	3181	-13837	49179	5.9	14198	14121	1471	73.9	51188	5.3	1.9	5.4	20.9	-12.3
62.00	30.00	2932	-13871	49356	6.9	14178	14074	1712	74.0	51352	5.2	2.0	5.4	21.0	-37.2
62.00	32.00	2677	-13898	49544	7.9	14154	14019	1946	74.1	51526	5.2	2.2	5.5	21.0	-66.2
63.00	2.00	5618	-12493	48349	-8.8	13698	13537	-2093	74.2	50252	5.2	-0.3	5.2	19.8	-41.5
63.00	4.00	5487	-12581	48356	-7.4	13725	13610	-1776	74.2	50266	5.3	-0.1	5.2	20.0	-19.5
63.00	6.00	5342	-12669	48379	-6.1	13750	13671	-1470	74.1	50295	5.3	.1	5.2	20.1	-0.8
63.00	8.00	5184	-12758	48416	-4.9	13771	13721	-1173	74.1	50336	5.3	.3	5.3	20.2	14.6
63.00	10.00	5015	-12846	48468	-3.7	13790	13762	-883	74.1	50392	5.3	.4	5.3	20.3	26.6
63.00	12.00	4836	-12931	48534	-2.5	13806	13793	-601	74.1	50459	5.3	.6	5.3	20.4	35.1
63.00	14.00	4648	-13014	48613	-1.3	13819	13815	-325	74.1	50539	5.3	.8	5.3	20.4	40.2
63.00	16.00	4451	-13092	48706	-0.2	13828	13828	-54	74.2	50631	5.3	1.0	5.3	20.5	41.7
63.00	18.00	4247	-13166	48811	.9	13834	13832	212	74.2	50734	5.3	1.1	5.3	20.6	39.7
63.00	20.00	4036	-13234	48929	2.0	13836	13828	473	74.2	50847	5.2	1.3	5.3	20.7	34.0
63.00	22.00	3819	-13296	49058	3.0	13834	13814	730	74.3	50971	5.2	1.5	5.3	20.8	24.6
63.00	24.00	3596	-13352	49199	4.1	13828	13793	982	74.3	51105	5.2	1.6	5.3	20.8	11.5
63.00	26.00	3368	-13401	49351	5.1	13818	13763	1230	74.4	51249	5.2	1.8	5.3	20.9	-5.4
63.00	28.00	3134	-13444	49515	6.1	13804	13725	1472	74.4	51403	5.2	1.9	5.3	21.0	-26.1
63.00	30.00	2894	-13479	49690	7.1	13786	13680	1708	74.5	51566	5.1	2.1	5.4	21.1	-50.6
63.00	32.00	2649	-13506	49876	8.1	13764	13627	1939	74.6	51740	5.1	2.2	5.4	21.2	-79.0
64.00	2.00	5406	-12151	48687	-9.0	13299	13135	-2084	74.7	50471	5.1	-0.3	5.1	19.8	-48.4

LAT	LONG	U	V	Z	D	H	X	Y	I	F	DX	DY	DH	DZ	CURL
64.00	4.00	5289	-12232	48697	-7.6	13327	13209	-1766	74.7	50488	5.1	-0.1	5.1	20.0	-28.0
64.00	6.00	5158	-12315	48721	-6.3	13351	13271	-1459	74.7	50517	5.1	.1	5.1	20.1	-10.8
64.00	8.00	5014	-12397	48760	-5.0	13373	13322	-1161	74.7	50560	5.2	.3	5.1	20.2	3.3
64.00	10.00	4858	-12479	48812	-3.7	13392	13364	-871	74.7	50615	5.2	.5	5.1	20.3	14.1
64.00	12.00	4693	-12560	48877	-2.5	13408	13395	-588	74.7	50683	5.2	.6	5.1	20.4	21.6
64.00	14.00	4517	-12637	48956	-1.3	13421	13417	-311	74.7	50762	5.2	.8	5.1	20.5	25.7
64.00	16.00	4334	-12712	49047	-0.2	13430	13430	-41	74.7	50853	5.2	1.0	5.2	20.6	26.5
64.00	18.00	4142	-12782	49151	1.0	13436	13434	224	74.7	50954	5.1	1.2	5.2	20.7	23.8
64.00	20.00	3943	-12847	49266	2.1	13438	13430	484	74.7	51066	5.1	1.3	5.2	20.8	17.7
64.00	22.00	3738	-12907	49394	3.2	13437	13417	739	74.8	51189	5.1	1.5	5.2	20.9	8.0
64.00	24.00	3526	-12961	49533	4.2	13432	13395	989	74.8	51322	5.1	1.6	5.2	20.9	-5.2
64.00	26.00	3309	-13008	49683	5.3	13423	13366	1233	74.9	51464	5.1	1.8	5.2	21.0	-22.0
64.00	28.00	3086	-13050	49844	6.3	13409	13328	1472	74.9	51617	5.0	2.0	5.2	21.1	-42.4
64.00	30.00	2856	-13084	50017	7.3	13392	13283	1705	75.0	51779	5.0	2.1	5.2	21.2	-66.5
64.00	32.00	2622	-13112	50201	8.3	13371	13231	1932	75.1	51952	5.0	2.3	5.3	21.3	-94.3
65.00	2.00	5198	-11811	49015	-9.2	12905	12737	-2073	75.2	50685	4.9	-0.3	4.9	19.9	-58.5
65.00	4.00	5093	-11887	49028	-7.8	12932	12812	-1756	75.2	50705	5.0	-0.1	4.9	20.0	-39.7
65.00	6.00	4975	-11963	49054	-6.4	12956	12875	-1449	75.2	50737	5.0	.1	4.9	20.1	-23.9
65.00	8.00	4844	-12040	49094	-5.1	12978	12927	-1150	75.2	50781	5.0	.3	5.0	20.2	-11.2
65.00	10.00	4702	-12117	49147	-3.8	12997	12968	-859	75.2	50837	5.0	.5	5.0	20.3	-1.5
65.00	12.00	4549	-12192	49213	-2.5	13013	13000	-576	75.2	50904	5.0	.7	5.0	20.4	5.0
65.00	14.00	4386	-12264	49291	-1.3	13025	13022	-300	75.2	50981	5.0	.9	5.0	20.5	8.3
65.00	16.00	4215	-12334	49382	-0.1	13034	13034	-30	75.2	51073	5.0	1.0	5.0	20.6	8.4
65.00	18.00	4036	-12400	49484	1.0	13040	13038	234	75.2	51173	5.0	1.2	5.0	20.7	5.1
65.00	20.00	3849	-12462	49598	2.2	13042	13033	493	75.3	51284	5.0	1.4	5.1	20.8	-1.4
65.00	22.00	3655	-12518	49723	3.3	13041	13020	746	75.3	51405	5.0	1.5	5.1	20.9	-11.3
65.00	24.00	3455	-12570	49860	4.4	13036	12998	993	75.3	51536	5.0	1.7	5.1	21.0	-24.5
65.00	26.00	3248	-12615	50008	5.4	13027	12968	1235	75.4	51677	4.9	1.8	5.1	21.2	-41.2
65.00	28.00	3036	-12655	50168	6.5	13014	12930	1471	75.5	51828	4.9	2.0	5.1	21.3	-61.4
65.00	30.00	2817	-12688	50338	7.5	12997	12885	1700	75.5	51989	4.9	2.1	5.1	21.4	-85.0
65.00	32.00	2592	-12714	50519	8.5	12976	12833	1923	75.6	52159	4.8	2.3	5.1	21.5	-112.1
66.00	2.00	4992	-11474	49332	-9.5	12512	12341	-2063	75.8	50894	4.8	-0.2	4.7	19.9	-71.9
66.00	4.00	4899	-11543	49349	-8.0	12540	12418	-1746	75.7	50917	4.8	-0.0	4.8	20.0	-54.6
66.00	6.00	4793	-11614	49378	-6.6	12564	12481	-1439	75.7	50952	4.8	.2	4.8	20.1	-40.2
66.00	8.00	4674	-11685	49420	-5.2	12586	12534	-1140	75.7	50997	4.9	.4	4.8	20.3	-28.6
66.00	10.00	4544	-11756	49474	-3.9	12604	12575	-850	75.7	51054	4.9	.5	4.8	20.4	-20.1
66.00	12.00	4404	-11826	49540	-2.6	12619	12606	-567	75.7	51122	4.9	.7	4.9	20.5	-14.5
66.00	14.00	4254	-11893	49618	-1.3	12631	12628	-291	75.7	51201	4.9	.9	4.9	20.6	-12.0
66.00	16.00	4095	-11958	49708	-0.1	12640	12640	-22	75.7	51290	4.9	1.1	4.9	20.7	-12.6
66.00	18.00	3927	-12020	49809	1.1	12645	12643	241	75.8	51389	4.9	1.2	4.9	20.8	-16.3
66.00	20.00	3752	-12077	49922	2.3	12647	12637	498	75.8	51499	4.9	1.4	4.9	20.9	-23.2
66.00	22.00	3570	-12131	50046	3.4	12645	12623	749	75.8	51618	4.9	1.6	4.9	21.0	-33.2
66.00	24.00	3381	-12179	50180	4.5	12639	12600	995	75.9	51748	4.8	1.7	4.9	21.1	-46.5
66.00	26.00	3185	-12221	50326	5.6	12630	12569	1234	75.9	51887	4.8	1.9	5.0	21.3	-63.0
66.00	28.00	2983	-12258	50483	6.7	12616	12531	1467	76.0	52035	4.8	2.0	5.0	21.4	-82.8
66.00	30.00	2774	-12289	50650	7.7	12599	12484	1693	76.0	52194	4.7	2.2	5.0	21.5	-105.9
66.00	32.00	2560	-12314	50829	8.7	12577	12431	1912	76.1	52362	4.7	2.3	5.0	21.6	-132.3
67.00	2.00	4786	-11137	49638	-9.7	12121	11947	-2052	76.3	51097	4.6	-0.2	4.6	19.9	-88.5
67.00	4.00	4704	-11201	49659	-8.2	12149	12024	-1737	76.3	51124	4.6	.0	4.6	20.0	-72.6
67.00	6.00	4610	-11266	49692	-6.7	12173	12088	-1430	76.2	51161	4.7	.2	4.6	20.2	-59.5
67.00	8.00	4503	-11332	49736	-5.3	12194	12141	-1133	76.2	51209	4.7	.4	4.7	20.3	-49.1
67.00	10.00	4385	-11397	49791	-4.0	12211	12182	-843	76.2	51267	4.7	.6	4.7	20.4	-41.6
67.00	12.00	4256	-11461	49858	-2.6	12226	12213	-561	76.2	51335	4.7	.7	4.7	20.5	-36.9
67.00	14.00	4118	-11524	49936	-1.3	12237	12234	-285	76.2	51414	4.7	.9	4.7	20.7	-35.2

LAT	LONG	U	V	Z	D	H	X	Y	I	F	DX	DY	DH	DZ	CURL
67.00	16.00	3970	-11584	50026	-0.1	12245	12245	-17	76.2	51503	4.7	1.1	4.7	20.8	-36.3
67.00	18.00	3815	-11641	50126	1.1	12250	12247	245	76.3	51601	4.7	1.3	4.8	20.9	-40.5
67.00	20.00	3651	-11694	50237	2.3	12250	12240	500	76.3	51709	4.7	1.4	4.8	21.0	-47.6
67.00	22.00	3480	-11743	50359	3.5	12248	12225	749	76.3	51827	4.7	1.6	4.8	21.1	-57.8
67.00	24.00	3302	-11787	50492	4.6	12241	12201	992	76.4	51954	4.7	1.7	4.8	21.3	-71.1
67.00	26.00	3117	-11826	50635	5.8	12230	12168	1228	76.4	52091	4.7	1.9	4.8	21.4	-87.4
67.00	28.00	2925	-11860	50789	6.9	12216	12129	1458	76.5	52237	4.6	2.1	4.8	21.5	-106.8
67.00	30.00	2727	-11889	50953	7.9	12197	12081	1681	76.5	52393	4.6	2.2	4.8	21.6	-129.3
67.00	32.00	2523	-11911	51128	9.0	12175	12026	1896	76.6	52558	4.5	2.4	4.9	21.8	-155.0
68.00	2.00	4579	-10800	49934	-10.0	11730	11551	-2041	76.8	51293	4.4	-0.1	4.4	20.0	-108.2
68.00	4.00	4508	-10858	49959	-8.5	11757	11629	-1728	76.8	51323	4.5	.1	4.4	20.1	-93.7
68.00	6.00	4424	-10918	49994	-6.9	11781	11694	-1424	76.7	51364	4.5	.3	4.5	20.2	-81.8
68.00	8.00	4329	-10979	50041	-5.5	11801	11747	-1127	76.7	51414	4.6	.4	4.5	20.4	-72.6
68.00	10.00	4222	-11039	50098	-4.1	11818	11789	-839	76.7	51473	4.6	.6	4.5	20.5	-66.0
68.00	12.00	4105	-11097	50166	-2.7	11832	11819	-558	76.7	51543	4.6	.8	4.5	20.6	-62.2
68.00	14.00	3978	-11154	50245	-1.4	11842	11839	-284	76.7	51621	4.6	1.0	4.6	20.7	-61.1
68.00	16.00	3842	-11209	50334	-0.1	11849	11849	-17	76.8	51710	4.6	1.1	4.6	20.9	-62.8
68.00	18.00	3697	-11261	50433	1.2	11853	11850	243	76.8	51807	4.6	1.3	4.6	21.0	-67.4
68.00	20.00	3545	-11310	50543	2.4	11852	11842	497	76.8	51914	4.6	1.5	4.6	21.1	-74.8
68.00	22.00	3385	-11354	50663	3.6	11848	11825	744	76.8	52030	4.6	1.6	4.6	21.2	-85.0
68.00	24.00	3217	-11395	50794	4.8	11840	11799	984	76.9	52155	4.5	1.8	4.7	21.4	-98.2
68.00	26.00	3043	-11430	50934	5.9	11828	11765	1218	76.9	52290	4.5	1.9	4.7	21.5	-114.3
68.00	28.00	2862	-11460	51085	7.0	11812	11724	1444	77.0	52433	4.5	2.1	4.7	21.6	-133.3
68.00	30.00	2675	-11485	51246	8.1	11792	11674	1664	77.0	52585	4.4	2.3	4.7	21.8	-155.2
68.00	32.00	2481	-11504	51417	9.2	11768	11618	1875	77.1	52747	4.4	2.4	4.7	21.9	-180.1
69.00	2.00	4371	-10461	50217	-10.3	11338	11154	-2032	77.3	51481	4.3	-0.0	4.2	20.0	-130.9
69.00	4.00	4310	-10515	50247	-8.7	11364	11233	-1721	77.3	51516	4.3	.2	4.2	20.2	-117.8
69.00	6.00	4236	-10570	50286	-7.2	11387	11298	-1419	77.2	51559	4.4	.3	4.3	20.3	-107.1
69.00	8.00	4151	-10625	50335	-5.7	11407	11351	-1125	77.2	51612	4.4	.5	4.3	20.4	-99.0
69.00	10.00	4055	-10679	50394	-4.2	11423	11392	-838	77.2	51673	4.4	.7	4.3	20.6	-93.4
69.00	12.00	3948	-10733	50463	-2.8	11436	11422	-559	77.2	51743	4.4	.8	4.4	20.7	-90.3
69.00	14.00	3832	-10785	50542	-1.4	11445	11442	-287	77.2	51822	4.4	1.0	4.4	20.8	-89.9
69.00	16.00	3707	-10834	50631	-0.1	11451	11451	-22	77.3	51910	4.4	1.2	4.4	20.9	-92.1
69.00	18.00	3574	-10881	50730	1.2	11453	11450	236	77.3	52007	4.4	1.3	4.5	21.1	-96.9
69.00	20.00	3432	-10925	50839	2.4	11451	11441	488	77.3	52112	4.4	1.5	4.5	21.2	-104.5
69.00	22.00	3283	-10964	50957	3.7	11445	11422	732	77.3	52226	4.4	1.7	4.5	21.3	-114.8
69.00	24.00	3126	-11000	51085	4.9	11436	11395	970	77.4	52349	4.4	1.8	4.5	21.5	-127.8
69.00	26.00	2963	-11031	51223	6.0	11422	11359	1200	77.4	52481	4.3	2.0	4.5	21.6	-143.6
69.00	28.00	2792	-11058	51370	7.2	11405	11315	1424	77.5	52621	4.3	2.1	4.5	21.7	-162.2
69.00	30.00	2615	-11078	51527	8.3	11383	11264	1640	77.5	52769	4.3	2.3	4.5	21.9	-183.5
69.00	32.00	2432	-11094	51694	9.4	11357	11206	1848	77.6	52927	4.2	2.4	4.5	22.0	-207.6
70.00	2.00	4160	-10120	50489	-10.7	10942	10753	-2023	77.8	51662	4.1	.1	4.0	20.1	-156.7
70.00	4.00	4108	-10170	50523	-9.0	10968	10833	-1717	77.8	51700	4.1	.2	4.1	20.2	-144.9
70.00	6.00	4044	-10219	50566	-7.4	10990	10898	-1418	77.7	51747	4.2	.4	4.1	20.4	-135.4
70.00	8.00	3968	-10269	50618	-5.9	11009	10952	-1126	77.7	51801	4.2	.6	4.1	20.5	-128.2
70.00	10.00	3882	-10319	50679	-4.4	11025	10992	-842	77.7	51864	4.2	.7	4.2	20.6	-123.5
70.00	12.00	3786	-10367	50749	-2.9	11036	11022	-565	77.7	51935	4.3	.9	4.2	20.8	-121.2
70.00	14.00	3681	-10413	50829	-1.5	11044	11041	-296	77.7	52015	4.3	1.1	4.2	20.9	-121.3
70.00	16.00	3566	-10457	50917	-0.2	11049	11049	-33	77.8	52102	4.3	1.2	4.3	21.0	-124.0
70.00	18.00	3443	-10499	51015	1.2	11049	11047	223	77.8	52198	4.3	1.4	4.3	21.2	-129.1
70.00	20.00	3312	-10538	51122	2.4	11046	11036	472	77.8	52302	4.2	1.5	4.3	21.3	-136.9
70.00	22.00	3173	-10573	51239	3.7	11039	11016	714	77.8	52414	4.2	1.7	4.3	21.4	-147.1
70.00	24.00	3027	-10604	51364	4.9	11027	10986	948	77.9	52535	4.2	1.9	4.3	21.6	-160.0
70.00	26.00	2874	-10630	51499	6.1	11012	10949	1176	77.9	52663	4.2	2.0	4.3	21.7	-175.5

LAT	LONG	U	V	Z	D	H	X	Y	I	F	DX	DY	DH	DZ	CURL
70.00	28.00	2714	-10652	51643	7.3	10992	10903	1396	78.0	52800	4.1	2.2	4.4	21.8	-193.5
70.00	30.00	2547	-10669	51796	8.4	10969	10850	1608	78.0	52944	4.1	2.3	4.4	22.0	-214.2
70.00	32.00	2374	-10680	51958	9.5	10941	10789	1812	78.1	53097	4.0	2.5	4.4	22.1	-237.5
71.00	2.00	3944	-9776	50749	-11.0	10542	10347	-2017	78.3	51833	3.9	.2	3.8	20.2	-185.6
71.00	4.00	3901	-9821	50787	-9.3	10568	10427	-1715	78.2	51875	4.0	.3	3.9	20.3	-175.0
71.00	6.00	3846	-9866	50834	-7.7	10589	10494	-1420	78.2	51925	4.0	.5	3.9	20.4	-166.5
71.00	8.00	3779	-9911	50888	-6.1	10607	10547	-1132	78.2	51982	4.0	.6	3.9	20.6	-160.3
71.00	10.00	3703	-9955	50951	-4.6	10622	10588	-851	78.2	52047	4.1	.8	4.0	20.7	-156.4
71.00	12.00	3617	-9998	51023	-3.1	10632	10617	-577	78.2	52119	4.1	1.0	4.0	20.8	-154.8
71.00	14.00	3521	-10039	51103	-1.7	10639	10634	-310	78.2	52198	4.1	1.1	4.1	21.0	-155.5
71.00	16.00	3417	-10078	51191	-0.3	10642	10642	-51	78.3	52286	4.1	1.3	4.1	21.1	-158.5
71.00	18.00	3304	-10115	51288	1.1	10641	10639	202	78.3	52380	4.1	1.4	4.1	21.2	-163.9
71.00	20.00	3183	-10148	51394	2.4	10636	10626	448	78.3	52483	4.1	1.6	4.1	21.4	-171.7
71.00	22.00	3055	-10178	51508	3.7	10627	10605	687	78.3	52593	4.0	1.7	4.1	21.5	-182.0
71.00	24.00	2919	-10204	51631	5.0	10614	10574	918	78.4	52710	4.0	1.9	4.2	21.7	-194.6
71.00	26.00	2776	-10226	51762	6.2	10596	10535	1142	78.4	52836	4.0	2.1	4.2	21.8	-209.7
71.00	28.00	2627	-10243	51902	7.4	10575	10487	1359	78.5	52968	3.9	2.2	4.2	22.0	-227.3
71.00	30.00	2470	-10255	52051	8.5	10549	10432	1567	78.5	53109	3.9	2.4	4.2	22.1	-247.3
71.00	32.00	2307	-10262	52207	9.7	10518	10369	1767	78.6	53257	3.8	2.5	4.2	22.3	-269.7
72.00	2.00	3724	-9428	50997	-11.4	10137	9935	-2012	78.8	51995	3.7	.2	3.6	20.2	-217.3
72.00	4.00	3688	-9469	51039	-9.7	10162	10016	-1716	78.7	52041	3.8	.4	3.7	20.4	-207.9
72.00	6.00	3641	-9509	51089	-8.0	10183	10082	-1426	78.7	52094	3.8	.6	3.7	20.5	-200.5
72.00	8.00	3584	-9549	51146	-6.4	10200	10136	-1142	78.7	52153	3.9	.7	3.8	20.7	-195.3
72.00	10.00	3516	-9589	51211	-4.9	10213	10176	-865	78.7	52219	3.9	.9	3.8	20.8	-192.1
72.00	12.00	3439	-9626	51283	-3.3	10222	10205	-595	78.7	52292	3.9	1.0	3.8	20.9	-191.1
72.00	14.00	3353	-9662	51364	-1.9	10228	10222	-332	78.7	52372	3.9	1.2	3.9	21.1	-192.3
72.00	16.00	3259	-9696	51452	-0.4	10229	10229	-76	78.8	52459	3.9	1.3	3.9	21.2	-195.7
72.00	18.00	3156	-9728	51548	1.0	10227	10225	174	78.8	52553	3.9	1.5	3.9	21.3	-201.3
72.00	20.00	3045	-9756	51652	2.3	10220	10211	416	78.8	52653	3.9	1.6	3.9	21.5	-209.2
72.00	22.00	2926	-9781	51764	3.7	10209	10188	651	78.8	52761	3.8	1.8	4.0	21.6	-219.3
72.00	24.00	2801	-9802	51884	4.9	10194	10156	879	78.9	52876	3.8	1.9	4.0	21.8	-231.7
72.00	26.00	2668	-9819	52011	6.2	10175	10115	1099	78.9	52997	3.8	2.1	4.0	21.9	-246.4
72.00	28.00	2529	-9831	52147	7.4	10151	10066	1312	79.0	53126	3.7	2.2	4.0	22.1	-263.4
72.00	30.00	2382	-9838	52291	8.6	10123	10009	1516	79.0	53262	3.7	2.4	4.0	22.2	-282.7
72.00	32.00	2230	-9841	52442	9.8	10090	9944	1712	79.1	53404	3.6	2.5	4.0	22.4	-304.3