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An Electronic System for Measuring Pendulum Periods

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***Abstract*—An electronic system for measuring the period of freely swinging pendulums has been developed for use with the Canadian Pendulum Apparatus. Extensive testing with an electrically generated test signal suggests that the error in determining the mean period of a one-second pendulum, averaged over 1200 seconds, does not exceed 100 and may be as low as 13 ns. Further testing with actual pendulums indicates that the measured periods, for a group of 3000-second observations, have a standard deviation of 600 ns. It is believed that this increased error is due to some factor associated with the pendulums, such as the effects of microseisms, knife-edge effects, or the dimensional stability of the pendulums themselves.**

Introduction

THE Canadian Pendulum Apparatus (CPA) for relative gravity measurements has undergone extensive redevelopment since 1960. The main objective of this program, which was to replace the photographic method for measuring pendulum periods with an electronic system, has now been completed. Other ancillary problems such as temperature and pressure control have also been under study. The basis of this apparatus is a group of six bronze pendulums which are allowed to swing, two at a time, on agate knife-edges. The knife-edges are mounted on a common support so that the two pendulums oscillate in the same plane. While in use, the pendulums are housed inside a vacuum chamber, where pressure and temperature can be maintained at a constant value.

In operation the pendulums are first deflected by a mechanical device protruding through the wall of the chamber and suddenly released so that they oscillate freely in opposing phase. Antiphase operation essentially decouples the pendulums from their support in the manner described by Vening-Meinesz.[4] The mean period of two nearly identical pendulums swinging in antiphase with equal amplitudes on a common support may be thought of as the period of a fictitious pendulum swinging on a stationary support.

The period of the fictitious pendulum, which is approximately 1 second, must be determined to within 10^{-7} seconds in order to achieve a 0.2 mGal accuracy. This precision was achieved in the photographic method by determining the average period for 3000 oscillations of the pendulum. The procedure was to record the time of six successive null crossings of each pendulum at the beginning and end of each observation. An electronic system which provides a tenfold improvement in the accuracy of period measurements is outlined in the following sections.

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General Description of the Timing System

A block diagram of the timing system is shown in Fig. 1. The primary time source is a James Knight frequency standard having a drift rate of 5 parts in 10^{10} per day. A "clock" pulse, 10 μs wide, is formed by the control and clock pulse generator every 20 μs. The clock pulses are stored in a binary accumulator where they provide a measure of elapsed time and the resolution of a time interval measurement is plus or minus one count or 20 μs. A series of control pulses, each spaced exactly between two clock pulses, is also provided by the pulse generator. The control pulses regulate the operation of the print-out circuit so that the binary accumulator is interrogated only when it is dormant.

A timing pulse, originating from the photodetector, opens the selector gate so that a control pulse is applied to the time gate which consists of a series of transmission gates. As this control pulse also closes the selector gate only a single pulse is chosen. The transmission gates are arranged to provide a nondestructive read-out of the instantaneous state of the binary accumulator. All gates connected to binaries in the ONE state are open, while those connected to binaries in the ZERO state are closed. The selected control pulse is then transmitted through the open gates to a specially designed recorder[3] where it produces a pattern of dots. A dot indicates that the associated binary is in the ONE state; absence of a dot indicates that the associated binary is in the ZERO state.

The print-out control circuit automatically provides for the recording of seven successive periods after the print-out sequence has been initiated; a second series is recorded after a predetermined number of oscillations of the pendulum. If the total elapsed time is known the average period for any number of swings may be obtained. In addition to its instantaneous states, the number of recycles of the binary accumulator must be found to determine the elapsed time. An approximate period may be obtained from the six individual periods recorded at the start and finish of each sequence with sufficient accuracy to compute the number of recycles for any observation time up to approximately 3500 seconds. For longer averaging times it is necessary to make a preliminary observation of about 100 seconds duration to obtain a sufficiently accurate estimate of the period.

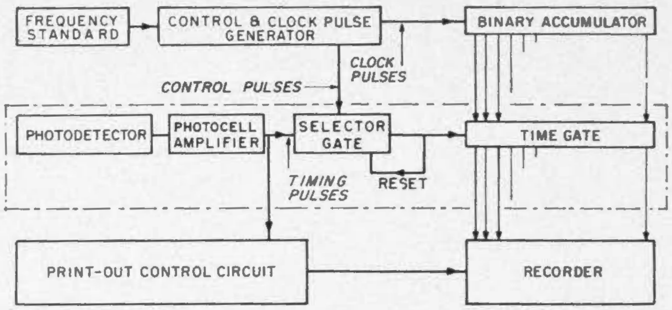

Fig. 1. Block diagram of timing system.

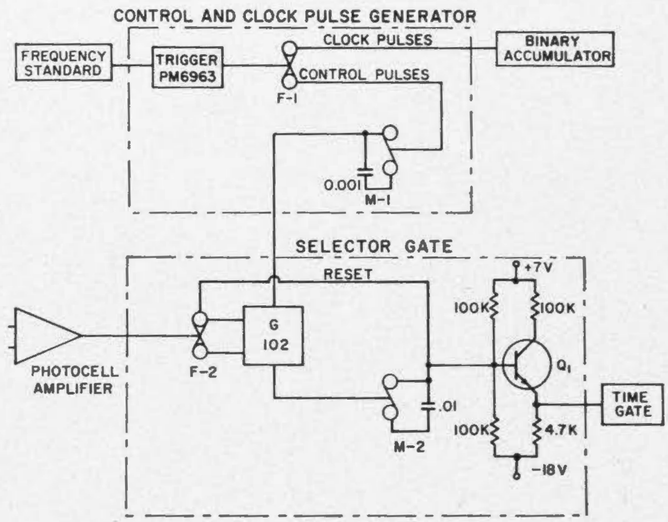

Fig. 2. Schematics: top half—control and clock pulse generator; bottom half—selector gate.

The Photodetector System

The purpose of the photodetector is to provide an exact electrical analog of the pendulum's motion. A beam of light which is reflected from a mirror on the pendulum is focused onto a differential photocell. As the pendulum oscillates, the beam plies back and forth across the photocell so that the difference between the outputs from its two halves is a sinusoidal voltage which is synchronized with the pendulum's motion and whose amplitude is proportional to the pendulum's amplitude. The light beam is adjusted to illuminate both halves of the photocells equally when the pendulum is vertical so that the zero crossings of the output waveform coincide with the null position of the pendulum.

In selecting a suitable photocell, both photo-voltaic and photo-conductive cells were evaluated under actual operating conditions; the latter type proved to be unsuitable because of its extremely high sensitivity to temperature. Also, the use of photo-voltaic cells simplifies the electronics because no bias current is required for their operation. Hence, a matched pair of selenium photocells, measuring approximately ¾ by ½ inch, has been chosen for each photodetector.

Photo-voltaic cells may be operated in one of two modes, either as a current or a voltage source. Operating the cell into a low impedance gives an improved temperature stability and a linear response, but requires an additional impedance matching stage which can give rise to temperature drifts equivalent to those experienced when the photocells are operated into a high impedance. As the differential photocell arrangement tends to reduce the effect of nonlinearities in the photocell response, other factors such as nonconformity of the light beam introduce greater departures from linear response than do the photocells. As a strictly linear response is not required and as the photocells are located inside the pendulum case, which is temperature controlled, the voltage mode of operation was chosen. A commercial operational amplifier connected to each photodetector provides a gain of 150 with an input impedance of 10 000 ohms. A Schmitt trigger then converts the amplified signal to a rectangular wave whose leading edge coincides with the positive-going zero crossing of the pendulum waveform.

The Gating Circuits

The control and clock pulse generator (Fig. 2) transforms the sinusoidal output from the frequency standard (100 000 Hz) into two 50 000-Hz rectangular waves

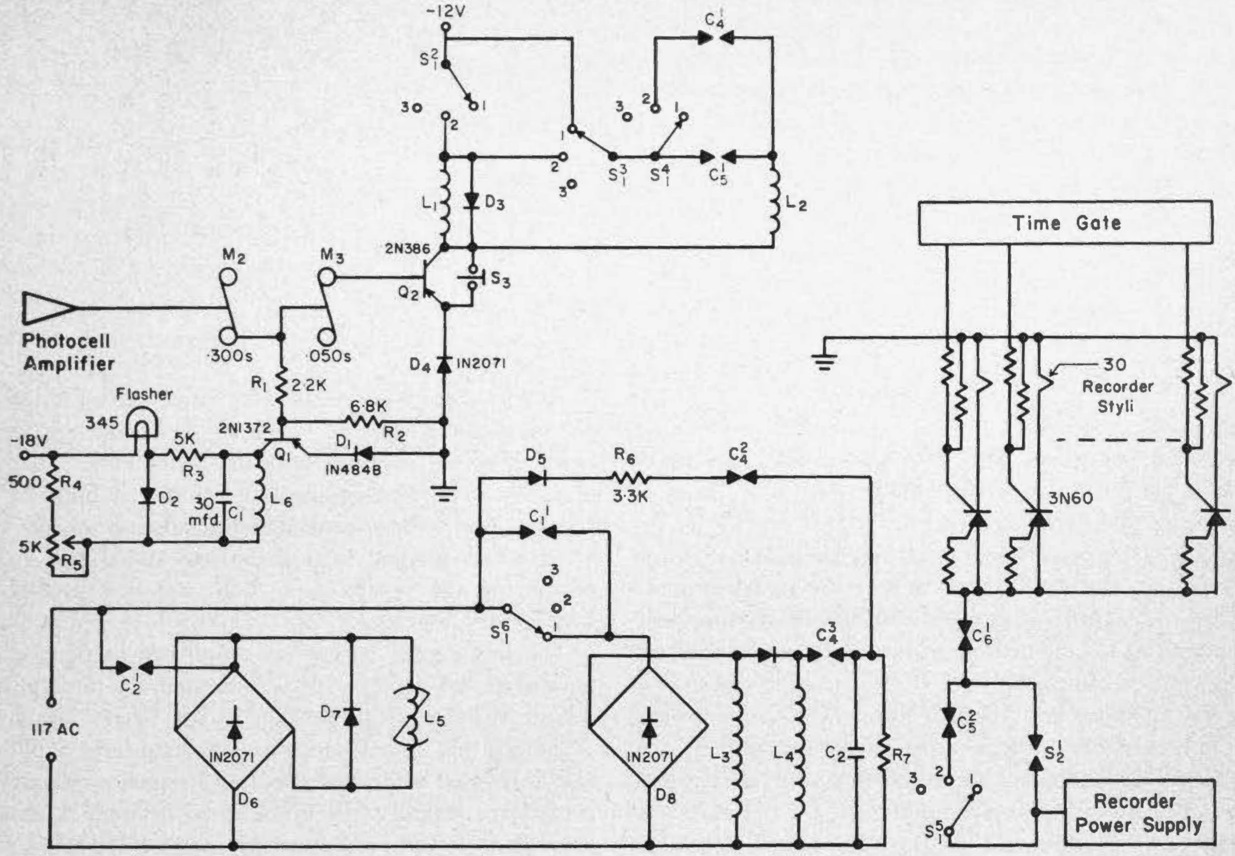

Fig. 3. Print-out control schematic. For simplicity the coils and contacts of the relays and switches have been separated into their individual components. C_4^2 refers to the second contact associated with relay L_4; S^3 refers to the third contact associated with switch S_1.

which are exactly out of phase. The input signal is squared with a trigger module and then divided by two with a flip-flop module (*F*-1) to provide the clock pulses. The second output from the flip-flop, which is out of phase with the first, is used to trigger a monostable multivibrator (*M*-1). The multivibrator, which is adjusted for a 10-second pulse width, generates the control pulses.

The selector gate (Fig. 2) consists of a standard 6-diode transmission gate[1] (designated *G*-102) along with other logic modules. The photodetector signal activates a flip-flop circuit (*F*-2) which controls the transmission gate. When the gate is open a control pulse is transmitted to the monostable multivibrator (*M*-2) which generates a 100 μs pulse. This pulse is returned to the reset line of the flip-flop and closes the gate. Thus, a single control pulse is selected for each positive-going zero crossing of the photodetector signal. An emitter follower (Q_1) provides the necessary power to drive the time gate which consists of 15 type *G*-102 transmission gates connected in parallel; each gate is associated with a particular binary in the accumulator.

Print-Out Control Circuit

The purpose of the print-out control circuit (Fig. 3) is to permit the automatic recording of seven successive pendulum periods at preselected intervals during a gravity measurement. A preset impulse counter (L_1) equipped with electrical reset initiates the print-out sequence each time a predetermined number of pendulum oscillations has elapsed. The counter, which may be set at any number between 0 and 10 000, counts backwards from this number (say 3000) until 0 is reached. The pulse, which advances the counter from 1 to 0, closes a contact which resets the counter to 3000 and starts the recorder. The counter is driven from the photocell amplifier which is associated with knife-edge 1 and, since the pendulum periods are nearly identical, it totalizes the number of swings for both pendulums.

Two 8-position wafer switches (C_5^1 and C_5^2) driven by the recorder motor (L_5), control the application of power to the styli and chart drive mechanism. These switches are closed in positions 2 to 8 inclusive and provide power for the recorder motor and styli circuits. In position 1 (standby) the switches are open, disabling the recorder until they are advanced to position 2 by some external method.

The output from the photocell amplifier triggers two monostable multivibrators (M_2 and M_3) connected in cascade. In addition to delaying the output of M_3 by 300 ms, M_2 also controls the operation of the indicator lamp and the print-out contrast circuit associated with Q_1. The negative output from M_2 turns on Q_1, allowing the indicator lamp to flash once for each pendulum period. C_1 is charged through R_4 and R_5 while Q_1 is conducting, and the potential across C_1 increases until relay L_6 pulls in and disconnects the stylus power source. The delay time and, hence, stylus current pulse width may be varied from 0 to 150 ms by R_5 which provides a convenient contrast control for the recorder print-out. The indicator lamp and R_3 form a voltage divider which in conjunction with the clamping diode (D_2) limits the potential across C_1 to a maximum of 12 volts in order to protect the relay coil from over-voltage. The output from M_3, a 50-ms negative pulse which is generated 300 ms after a pendulum null crossing, is applied to the impulse counter and recorder motor by means of Q_2. The 300-ms delay provides sufficient time for both pendulums to generate a print-out before the recording paper advances.

The first requirement in preparing for a gravity measurement is to reset the impulse counter, the binary accumulator, and the recorder motor to their zero or starting positions. When S_1 is switched from *stand-by* to *reset*, power is applied to C_6^1 through S_1^3 and, if the recorder motor is in any of the positions 2 to 8, it will automatically step around to position zero when the pendulums are swinging. S_3 permits manual operation of the recorder motor if required. When the recorder reaches position zero, C_5^1 opens to prevent any further movement. At the same time L_3 (counter reset coil) and L_4 (delay relay) are activated by S_1^6 while S_1^1 (not shown) resets the binary accumulator to zero. A capacitor (C_2), which is connected across L_4, delays the opening of C_4^1 for 1.5 seconds. The closing of C_4^1 permits the recorder motor to be advanced from position zero and the time delay assures that this contact remains closed for at least one pendulum period.

After zero reset has been completed a gravity measurement is started by allowing S_1 to return to the *operate* position. In order to prevent a miscount during the start procedure, S_1 must be released when the print-out control circuit is inactive. By synchronizing his movements with the flashing light, the operator may release S_1 between two successive pendulum pulses. Thus power is supplied through S_1^2 to the impulse counter which begins counting pendulum pulses and to C_4^1 which is closed for the first pendulum pulse by virtue of the 1.5-second time delay. The first pendulum pulse activates L_2 which advances the recorder to position 2 and closes contacts C_5^1 and C_5^2. C_5^1 allows the recorder motor to continue operating after C_4^1 opens while C_5^2 applies power to the styli circuits. The instantaneous time of the second zero crossing is printed out and the recorder is advanced to its third position. This sequence continues until seven print-outs have been obtained and the recorder motor has returned to zero where it disables the recorder circuits.

As the pendulums swing, the preset counter continues counting until it reaches zero and contact C_1^1 closes to apply power to L_3 and L_4. C_4^1 closes for 1.5 seconds so that L_2 may once again be activated and the recorder motor advanced. Thus a print-out sequence is automatically repeated each time the preset counter reaches zero.

Accuracy of the Timing System

Two series of tests have been conducted to evaluate the performance of the timing system. The first series used a simulated signal to measure the stability of the electronics (excluding photocells). The second series was intended to test the complete system including the pendulums and photocells.

Series I Tests

The first series of tests were intended to simulate the output of the photocells with a signal of much greater stability than that obtained with the pendulums. The generation of a 1-Hz test signal with a phase stability of about 10 μs proved to be exceedingly difficult, and was not in fact achieved. A method to utilize a less stable test signal by comparing the results from both channels proved to be satisfactory, however.

The test signal, having an amplitude of 65 mV and a period of 1.3 seconds was generated by integrating the output from the last flip-flop in the binary accumulator. Although this signal had excellent long-term stability because it was controlled by the frequency standard, its short-term stability left much to be desired. A sample of individual periods was measured with a Berkeley (Model 7161U) EPUT meter and found to have a standard deviation of 210 μs.

The test signal was applied to both amplifiers connected in parallel and four 1200-period observations were made in exactly the same manner as for the pendulums. The instantaneous times of seven successive zero crossings were recorded for each channel at the start and end of each interval. As well as the mean period, averaged over 1200 oscillations, 12 individual periods could be determined per channel for each observation. Each channel thus provided an independent observation of the duration of individual periods. The results of one of the four tests are shown in Table I and the strong correlation between the two channels indicates that variations in individual periods are being measured. Similar results from a typical pendulum measurement also recorded in Table I show no such correlation because the inputs to each amplifier are derived from independent sources. Since each channel yields an independent measurement of the same period, the differences in the measured periods are due to the combined triggering errors of both channels. The rms error computed from the combined results of the four tests was found to be 106 μs. Since this error is divided by the number of oscillations (N) used in determining the mean period, and since the mean of seven such observations is used in the determination of the final result, the error in the measured period of the fictitious pendulum, due to triggering effects, is given by: $\sigma_s = 106 \times 10^{-6}/N\sqrt{7}$. If N equals 3000, σ_s equal 13 ns, which is nearly one order of magnitude more precise than required.

The mean periods for each of the four tests are recorded

TABLE I
MEASUREMENT OF INDIVIDUAL PERIODS

Test Signal			Pendulum-Generated Signals		
Amp-1 (seconds)	Amp-2 (seconds)	[Diff] (seconds)	Pend-1 Amp-1 (seconds)	Pend-3 Amp-2 (seconds)	[Diff] (seconds)
1.31040	1.31032	0.00008	1.00074	1.00098	0.00024
1.31110	1.31112	0.00002	1.00126	1.00080	0.00046
1.31066	1.31064	0.00002	1.00024	1.00108	0.00084
1.31046	1.31048	0.00002	1.00088	1.00096	0.00008
1.31102	1.31104	0.00002	1.00158	1.00132	0.00026
1.31020	1.31008	0.00012	1.00104	1.00098	0.00006
1.31134	1.31136	0.00002	0.99996	1.00100	0.00104
1.31022	1.31024	0.00002	1.00188	1.00042	0.00146
1.31104	1.31104	0.00000	1.00010	1.00132	0.00122
1.31052	1.31048	0.00004	1.00046	1.00066	0.00020
1.31080	1.31084	0.00004	1.00036	1.00078	0.00042
1.31078	1.31078	0.00000	1.00048	1.00116	0.00068

$$\text{rms error} = \sqrt{\frac{\sum_{m=1}^{48} (\text{diff})^2}{48^*}} = 106\ \mu\text{s}$$

* For combined results of four tests.

TABLE II
SUMMARY OF TEST I RESULTS

Observation No.	Amp 1		Amp 2	
	T (seconds)	σ_s (ns)	T (seconds)	σ_s (ns)
1	1.31072012	250	1.31072013	150
2	1.31071998	320	1.31071995	350
3	1.31072023	250	1.31072008	300
4	1.31072008	280	1.31072001	300
Mean σ_M	1.3107201 seconds 98 ns			

in Table II. The short-term stability (σ_s) which is the standard deviation of the seven observed periods as well as the standard deviation of the mean (σ_M) reflects the "jitter" in the test signal waveform. From the standard deviation of the individual periods (210 µs) the values of

$$\sigma_s\left[\sigma_s = \frac{210 \times 10^{-6}\sqrt{2}}{1200}\right] \text{ and } \sigma_M\left[\sigma_M = \sigma_s/\sqrt{7}\right]$$

may be estimated at approximately 260 and 100 ns, respectively. Since the measured values of σ_M and σ_s are in close agreement with those estimated from the stability of the input waveform, it may be concluded that no sources of error, larger than approximately 10^{-7} seconds, contribute to the overall error level.

Series II Tests

In the second series, one pair of pendulums, which was maintained under carefully controlled environmental conditions, was used to generate test signals. Thirty 3000-second observations were carried out during a three-month interval. The fictitious periods, to which small corrections for variations in amplitude and temperature had been applied, were found to be normally distributed about the mean with a standard deviation of 600 ns.

Photocell Errors

It will be noted that Series I tests excluded photodetectors while Series II included them. The increased noise level in Series II tests may be due to either the pendulums, the photocells, or a combination of both. Although no clear-cut method has been found to distinguish between photocell- and pendulum-derived errors, there is some evidence to indicate that the photodetectors are not contributing significantly to the noise level.

1) The use of different varieties of detectors, including photo-voltaic and photo-conductive cells as well as photopots,† [1] does not alter the noise level.

2) Increasing the illumination by a factor of two, thereby increasing the S/N ratio of the photodetector, has no effect.

3) An analysis of pendulum results by Saito,[2] indicates that the statistical distribution of pendulum periods is determined by random deformations of the pendulums themselves.

CONCLUSIONS

An electronic system has been developed for measuring the fictitious period of two freely oscillating pendulums, averaged over 1200 periods, with an accuracy ($\pm 2\ \sigma$) of ±26 ns. A series of observations with actual pendulums indicates that the period of the fictitious pendulum for a single observation with one pair of pendulums may be ascribed a standard deviation of 600 ns, for a 3000-second observation. This increased error level is believed to be due to either the instability of the pendulums themselves or to external causes such as microseisms or knife-edge effects, rather than to errors in the electronic timing system. As a complete gravity determination normally involves 12 observations with each of six pendulum pairs the standard deviation of the final result will amount to approximately 0.15 mGal.

† Gianni Corporation trademark.

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