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Estimates of Crack Density Parameters in Near-Surface Rocks from Laboratory Studies of Core Samples and In-Situ Seismic Velocity Measurements.

by

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Abstract

A comprehensive theory of the elastic behaviour of cracked and porous solids has been derived by O'Connell and Budiansky. A summary has been given of those portions of the theory that are useful in the interpretation of seismic velocities, measured both in the laboratory and in the field, in terms of crack or fracture densities and fluid content. Numerical estimates of these crack parameters and the associated errors have been derived from both field and laboratory data. The results emphasize the importance of being able to measure reliable S wave velocities. The relevance of the theory to the study of potential radioactive waste disposal sites has been discussed, and a strategy for future studies has been described.

Introduction

The near-surface crystalline rocks of the Earth's crust contain fractures that provide openings through which water may flow. Radioactive material deposited permanently in a rock body might therefore be transported through fractures. An understanding of the distribution of microcracks, cracks, joints and faults is of key importance in determining whether or not a particular rock body may be useful as a potential disposal site. Seismic wave velocities in near-surface rocks are controlled primarily by the concentration and geometry of cracks, joints and other pore space. As the lithostatic pressure increases with depth, more and more of the cracks become closed, so that deep in the Earth's crust the elastic wave velocities depend largely on the mineralogical constitution of the rocks. It is therefore logical that an attempt should be made to use seismological techniques to infer something about the pore space and crack and fluid content of rocks both in situ and in the laboratory.

Studies of microcracks using seismological techniques for a number of rocks samples from the Chalk River site have already been reported by Simmons, Batzle and Cooper (1978). Seismological experiments in the Chalk River area have also been undertaken by both the Earth Physics Branch and the Geological Survey of Canada; these experiments yield P and S wave velocities at frequencies between 20 and 250 Hz within different regions of the rock body. In recent years, both experimental and theoretical studies of the elastic properties of cracked solids have made important advances (O'Connell and Budiansky, 1974, 1977; Kuster and

Toksöz, 1974 a, b; Mavko and Nur, 1978, 1979; Winckler, Nur and Gladwin, 1979, Toksöz, Johnston and Timur, 1979; Johnston, Toksöz and Timur, 1979), and consequently, the effect of cracks and pore fluids on the elastic parameters of crustal rocks can now be studied quantitatively. Further, since attenuation at seismic and laboratory frequencies is now believed to be primarily due to fluid flow between cracks, rough estimates of the permeability of a rock body using seismological techniques may eventually be possible.

The first objective of this work is to compare theoretical calculations of seismic velocities in cracked solids with those measured both in the laboratory on rock cores and in situ; the resulting crack density parameters can then be compared with porosity measurements and with the concentrations of fractures observed in thin sections of the samples collected from a borehole. The second and most important objective is to use the theory to determine the accuracy with which seismic velocity measurements would have to be made in order to derive useful crack parameters. Finally, the relationships between elastic constants and other parameters associated with cracked media, such as porosity, permeability and degree of saturation have been investigated so that firm recommendations for future studies can be presented. We have used the theory of O'Connell and Budiansky (1974, 1977) in this report, since it gives the most comprehensive treatment over the range of physical conditions that are likely to be encountered in the evaluation of potential waste disposal sites.

2. Models of Cracked Solids

The estimation of crack and saturation parameters requires four measurable quantities: the intrinsic or 'crack free' P and S wave velocities and in situ or laboratory P and S wave velocities. Wright and Kamineni (1979) have argued that the intrinsic P wave velocity should be calculated from the modal analyses of core samples. The intrinsic S wave velocity, however, may only be reliably determined from laboratory measurements on bore hole cores.

(i) Elastic Moduli and Seismic Velocities

O'Connell and Budiansky (1974) used a self-consistent approximation in using the energy of a single crack to estimate the elastic constants of a solid permeated with flat cracks. They considered four cases: dry cracks, saturated cracks and cracks saturated with soft fluid. For water-filled cracks, the difference between the simple case of saturated cracks and the more rigorous treatment in terms of soft fluid saturation is negligible, so that the soft fluid case is of no interest to us here. The physical conditions under which each of these cases is applicable was further elucidated by O'Connell and Budiansky (1977). We have summarized below the formulae from both papers that we have found useful in this investigation.

The crack density parameter ε is defined as follows:

$$\varepsilon = \frac{2N}{\pi}\left\langle\frac{A^2}{P}\right\rangle \qquad 1,$$

where A and P are the crack area and perimeter respectively, and N is the number of cracks per unit volume. With this definition of ε, O'Connell and Budiansky (1974) showed that the difference between the results for circular cracks and highly elliptical cracks is insignificant, so that we are justified in restricting our discussion to the case of circular cracks. For circular cracks,

$$\varepsilon = N \langle a^3 \rangle \qquad 2,$$

where 'a' is the crack radius. If the aspect ratio and radius of the jth crack are α_j and a_j respectively, the crack porosity is given by

$$\Delta V = \sum_{j=1}^{N} \frac{4\pi}{3} \alpha_j a_j^3 \qquad 3.$$

If we assume that all the α_j's are the same,

$$\Delta V = \frac{4\pi}{3} \alpha N \langle a^3 \rangle = \frac{4\pi}{3} \alpha \varepsilon \qquad 4.$$

Thus, if ε is calculated from the seismic velocities, we can estimate the aspect ratio required to give the measured crack porosity.

Let $\bar{K}$, $\bar{G}$ and $\bar{\nu}$ be the bulk modulus, rigidity and Poisson's ratio of the cracked material; K, G and ν are the corresponding parameters for the uncracked matrix. Then for dry cracks or fluid-filled cracks in which there is bulk flow of fluid out of or into cracks resulting from changes in fluid pressure in the cracks,

$$\frac{\bar{K}}{K} = 1 - \frac{16}{9}\left(\frac{1-\bar{\nu}^2}{1-2\bar{\nu}}\right)\varepsilon$$

$$\frac{\bar{G}}{G} = 1 - \frac{32}{45}\,\frac{(1-\bar{\nu})(5-\bar{\nu})}{(2-\bar{\nu})}\,\varepsilon \qquad 5,$$

$$\bar{\nu} = \nu - \frac{16}{45}\,\frac{(1-\bar{\nu}^2)(10\nu - 3\nu\bar{\nu} - \bar{\nu})}{(2-\bar{\nu})}\,\varepsilon \qquad 6.$$

For saturated cracks in which the shear stresses in the fluid relax completely, but no fluid can flow out of the cracks,

$$\frac{\bar{K}}{K} = 1 \quad , \qquad \frac{\bar{G}}{G} = 1 - \frac{32}{15}\left(\frac{1-\bar{\upsilon}}{2-\bar{\upsilon}}\right)\varepsilon \qquad 7,$$

$$\bar{\upsilon} = \upsilon + \frac{32}{45}\,\frac{(1-\bar{\upsilon}^2)(1-2\upsilon)}{(2-\bar{\upsilon})}\,\varepsilon \qquad 8.$$

This case is known as 'saturated isolated', and the changes in fluid pressure may not be the same in every crack. For the case of partial saturation, O'Connell and Budiansky (1974) gave expressions that correspond to a weighted average of the formulae for the 'dry' and 'saturated isolated' cases. Thus,

$$\varepsilon = \frac{45}{16}\cdot\frac{(\upsilon-\bar{\upsilon})}{1-\bar{\upsilon}^2}\cdot\frac{(2-\bar{\upsilon})}{\left[(1-\xi)(1+3\upsilon)(2-\bar{\upsilon})-2(1-2\upsilon)\right]} \qquad 9,$$

where ξ is the proportion of saturated cracks.

$$\frac{\bar{K}}{K} = 1 - \frac{16}{9}\cdot\frac{(1-\bar{\upsilon}^2)}{(1-2\bar{\upsilon})}\cdot(1-\xi)\varepsilon$$

$$\frac{\bar{G}}{G} = 1 - \frac{32}{45}\cdot(1-\bar{\upsilon})\left[1-\xi+\frac{3}{2-\bar{\upsilon}}\right]\varepsilon \qquad 10.$$

The seismic velocities within the cracked material are then given by

$$\bar{V}_S = V_S\left(\frac{\bar{G}}{G}\right)^{\frac{1}{2}} \qquad 11,$$

and

$$\bar{V}_P = V_P\left[\frac{(1-\bar{\upsilon})}{(1+\bar{\upsilon})}\cdot\frac{(1+\upsilon)}{(1-\upsilon)}\cdot\frac{\bar{K}}{K}\right]^{\frac{1}{2}} \qquad 12.$$

To make use of the theory, we require a series of tables of elastic moduli and seismic velocities as a function of ε. Each separate table corresponds to a different value of ξ and Poisson's ratio ν of the uncracked matrix. In preparing a table, the

initial step is to determine ε as a function of $\bar{\nu}$ keeping ν and ξ constant. The cracked moduli and velocities can then be calculated using equations 10 - 12.

There is, however, an additional case of saturation that is intermediate between 'saturated isolated' and 'dry'. If all the cracks are in communication, but no bulk flow of fluid out of the material occurs, the bulk modulus is the same as for the 'saturated isolated' case , but the material responds in shear as though the cracks were dry. This case is known as 'saturated isobaric'(O'Connell and Budiansky, 1977), and the elastic parameters are given by

$$\frac{\bar{K}}{K} = 1 \quad , \quad \frac{\bar{G}}{G} = 1 - \frac{32}{45}\,\frac{(1-\nu')(5-\nu')}{(2-\nu')}\,\epsilon \qquad 13,$$

where

$$\nu' = \nu - \frac{16}{45}\,\frac{(1-\nu'^2)(100-300\nu'-\nu')}{(2-\nu')}\,\epsilon \qquad 6a.$$

ν has been changed to ν' because it is no longer the effective Poisson's ratio of the bulk sample. The effective Poisson's ratio $\bar{\nu}$ is estimated from standard relations among elastic moduli and is

$$\bar{\nu} = \frac{(1+\nu)\,\bar{K}/K - (1-2\nu)\,\bar{G}/G}{2(1+\nu)\,\bar{K}/K + (1-2\nu)\,\bar{G}/G} \qquad 14.$$

$\bar{K}/K$ and $\bar{G}/G$ are given by 13.

The formulae for partial saturation if the saturated cracks conform to the 'saturated isobaric' case are simpler than equations 9 and 10. The relationship between ν' and ν is given by 6a; $\bar{G}/G$ is then given by 13. $\bar{K}/K$, however, is still calculated from 10 with ν' replacing $\bar{\nu}$. Elastic wave velocities measured on laboratory specimens of crystalline rocks correspond closely to this type of partial saturation, since measured S wave velocities are almost independent of the degree of saturation (Nur and Simmons, 1969).

(ii) Attenuation and Viscoelastic Behaviour

The transitions between the main cases discussed in the previous section involve the relaxation of shear stresses in a viscous medium and the flow of a viscous fluid in a permeable medium. O'Connell and Budiansky (1977) showed how these effects can be described in terms of linear viscoelasticity, and derived frequency-dependent elastic moduli. The imaginary parts of these complex elastic parameters express the dissipation of sinusoidal variations of stress and strain. For fluid saturated cracks equations 13 and 6a are modified to

$$\frac{\bar{K}(\omega)}{K} = 1 \quad , \quad \frac{\bar{G}(\omega)}{G} = 1 - \frac{32}{45}\left(\frac{1-\nu'}{2-\nu'}\right)\left[(2-\nu')D + 3C\right]\varepsilon \qquad 15,$$

where ν' is the root of

$$\nu' = \nu + \frac{16}{45}\left(\frac{1-\nu'^2}{2-\nu'}\right)\left[2(1-2\nu)C - (2-\nu')(1+3\nu)D\right]\varepsilon \qquad 16.$$

Here $D = f(\omega, \omega_1)$ and $C = g(\omega, \omega_2)$, where ω_1 and ω_2 are characteristic frequencies corresponding to the transition from 'saturated isolated' to 'saturated isobaric', and from 'saturated isolated' to the 'glued' case in which the cracks have no effect on the moduli. Analytical forms of the complex functions f and g are given by O'Connell and Budiansky (1977). For closely-spaced cracks having many intersections,

$$\omega_1 \simeq \left(\frac{K}{\eta}\right)\alpha^3, \qquad \omega_2 \simeq \left(\frac{G}{\eta}\right)\alpha \qquad 17,$$

where α is the aspect ratio and η is the fluid viscosity. A real crystalline rock will have a spectrum of characteristic frequencies owing to its distribution of aspect ratios that probably lie mostly within the range $10^{-2} - 10^{-4}$. Viscous relaxation in water-saturated cracks consequently becomes important at frequencies greater than about 10^7 Hz, if $\eta \sim 10^{-3}$ N sm^{-2}. Thus, for water-saturated thin cracks, field and laboratory elastic wave velocities correspond to a state ranging between saturated isobaric at the low frequency end and saturated isolated at high frequencies; fluid flow between cracks is therefore the dominant dissipative mechanism. Because the theory predicts a marked velocity dispersion between 1 and 10^6 Hz, ultrasonically measured velocities may be significantly higher than those at seismic frequencies. Since the quality factor Q is dependent on the spectrum of characteristic frequencies ω_1, and therefore on the spectrum of aspect ratios, measurements of seismic attenuation both in the field and in the laboratory may yield a rough estimate of the distribution of aspect ratios.

O'Connell and Budiansky (1977) also suggest the following relationship to replace ω_1 in 17 for moderate crack densities:

$$\omega_1 = \frac{4.4\,K k}{\eta a^2 (1 - 1.14\,\varepsilon^{1/5})} \qquad 18,$$

where a is the radius of a circular crack and k is the permeability. We thus have a direct mathematical relationship between the elastic constants and permeability. The k in 18 is for flow between adjacent cracks and will be highly variable over a rock body. Further, it may be rather different from the bulk permeability of a rock body over a long period of time. Nevertheless, the existence of this relationship suggests the intriguing possibility that a crude estimate of the permeability of a rock body might be made by purely seismological means.

3. The Estimation of Crack and Saturation Parameters from Seismic Velocity Measurements.

To illustrate the application of the theory, we have used some relevant velocity estimates from both field and laboratory data given by Wright and Kamineni (1979), and Lam and Wright (1979). The real difficulty is that we have no measured S wave velocities at 2 kb or other approximations to the intrinsic S wave velocities. The calculations were undertaken for the 'saturated isobaric' case of section 2. The measured and intrinsic S wave velocities therefore completely determine the crack density parameter; the measured and intrinsic P wave velocities then define the saturation parameter.

We have made plausible guesses for the intrinsic S wave velocities using the laboratory data published by Feves, Simmons and Siegfried (1977). The average P and S velocities measured at 2.0 kb on dry samples of four granites and one quartz monzonite were 6.33 and 3.77 km/s respectively, yielding an intrinsic V_P/V_S value of 1.68. The measured saturated and dry P velocities and the S velocities at 1 bar from Feves et al. are similar to those reported by Simmons et al. (1978) for Chalk River samples, though the differences between the measured dry and saturated P wave velocities are rather smaller for the Chalk River cores. We have used 'average layer' velocities in our calculations, as defined by Wright and Kamineni (1979), which represent the average laboratory velocities for samples taken from a layer that is petrologically reasonably homogeneous. To make realistic estimates of the errors in crack and saturation parameters, we have used the average P and S wave velocities of profile 3 of Lam and Wright (1979). The primary data and the results of our calculations are listed in tables 1 and 2.

In table 1, values of ξ have been determined for the average saturated and dry P velocities measured on cores from the quartz monzonite zone of borehole CR1 at Chalk River. The P velocity measured on dry samples at 2.5 kb was taken as a reasonable estimate of the intrinsic P velocity. ε was determined first from the average measured shear wave velocity at 1 bar, assuming $V_{PI}/V_{SI} = 1.63$. The calculations were then repeated using the systematically higher values of V_{PI} estimated from the modal analyses of core samples and mineral velocities obtained from the literature; V_{PI}/V_{SI} was kept at 1.63.

The first four rows of table 1 show the results. On changing from the measured to the calculated V_{PI}, the crack density parameter increases from 0.17 to 0.24, whilst the saturation parameters for both saturated and dry specimens drop.

Wright and Kamineni (1979) have suggested that the calculated values of V_{PI} are a better approximation to the true value because all of the pore space in a rock is unlikely to be closed at a pressure of 2.5 kb. On the other hand, the calculated values of V_{PI} may be slightly overestimated, because alteration of some minerals, especially feldspars, may be significant. We will return to this problem in the next section in which the interpretation of crack densities and porosity measurements in terms of crack aspect ratios is discussed.

Because V_{SI} is not accurately known, the values of ξ in table 1 may be too low. However, the significant feature of the results is that the difference in the degree of saturation between dry and saturated specimens is only about 0.3, suggesting that the process of saturating or drying rock samples affects only a small proportion of the cracks. This conclusion is supported by calculations on other samples from Chalk River and for data on other crystalline rocks discussed in the literature (O'Connell and Budiansky, 1974; Scholz, 1978). As a further illustration, we have used the velocity data on a sample of granite studied by Feves et al. (1977). Here V_{SI} was taken to be the measured shear velocity at 2.0 kb. The results are given in the last two rows of table 1; $\varepsilon \approx 0.13$, and ξ is 0.92 and 0.53 for saturated and dry specimens respectively.

Table 1 also shows the saturation parameters calculated using the average Vp from the borehole log between depths of 90 and 170 m (rows 5 and 6). The calculated V_{PI} yields $\xi = 0.51$. Since we would expect the cracks in situ to be close to saturation, the result suggests that V_{SI} has been underestimated. If V_{SI} is increased by 0.2 km/s, ε and ξ increase by 0.03 and 0.07 respectively.

The data of Lam and Wright (1979) yield a random error in P wave velocities of ± 0.09 km/s. Any further experiment designed specifically to measure seismic velocities should be capable of attaining at least that precision. Assuming a similar error in S velocities and an error of ± 0.10 km/s in V_{PI} and V_{SI}, the entire range of possible values of ξ and ε has been calculated. The selected measured velocities, V_P and V_S, correspond to profile 3 of Lam and Wright (1979), whilst the intrinsic velocity V_{PI} is a calculated average over the whole suite of borehole samples; V_{SI} is a plausible guess.

The numerical results are listed in Table 2. First the effect of varying one parameter at a time is discussed. An error of ± 0.09 km/s in V_S results in errors in ε and ξ of ± 0.02 and ± 0.06 - 0.09 respectively; the higher error in ξ occurs for lower input values of V_p or dryer rocks. An error of ± 0.10 km/s in V_{SI}, however, causes smaller errors of ± 0.015 and ±0.03 - 0.05 in ε and ξ respectively. A change in V_p of ± 0.09 km/s causes a change in ξ of ± 0.06 - 0.08; again the larger change occurs for dryer rocks. Similarly an error in V_{PI} of ± 0.10 km/s causes an error in ξ in the range 0.04 - 0.08 depending on the degree of saturation. Thus, the three parameters V_{PI}, V_P and V_S need to be known with similar accuracy,

because they have approximately equal effects on ξ . V_{SI}, however, has less influence on ε or ξ than V_s, so that errors in V_{SI} that are about twice those in the other velocities can be tolerated. These results concerning the errors are of importance in the practical application of the theory, because V_{SI} is the most difficult parameter to determine accurately. Allowing for simultaneous errors in all four velocities, the ranges of ε and ξ are from 0.20 - 0.27 and from 0.15 - 0.58 respectively. Our calculations consequently emphasize the need for very accurate determinations of the seismic velocities (~ 1-2%).

ε and ξ have also been determined for the saturated isolated case of section 2, using the same input parameters as set (a) of table 2; the results comprise set (e). The saturated isolated case yields much higher values of both ε and ξ . Note that because the shear wave velocity is no longer independent of saturation the value of V_p also influences ε. We shall discuss later why we consider the saturated isolated case to be less appropriate for both laboratory and field measurements.

4. Discussion

(i) Aspect Ratios

We have already indicated that the calculated P velocities for rocks at 2.5 kb are probably better approximations to the intrinsic values than measured velocities, since all the pore space in a rock is unlikely to be closed at 2.5 kb. We can show that this explanation is realistic by means of a crude but simple calculation using the crack density parameters and the

measured porosites. The closure pressure for dry cracks is given roughly by $E\alpha$, where E is Young's modulus and α is the aspect ratio. Taking $E \simeq$ 200 kb, at 2kb only those cracks with $\alpha < 10^{-2}$ will be closed.

Now consider the data of table 3 compiled for the quartz monzonites of borehole CR1 at depths between 90 and 170 m. From Simmons et al. (1978), the average crack porosity is 9.4×10^{-4}, which, from equation 4 and the crack densities of table 1, yields an average aspect ratio for thin cracks of 1.3×10^{-3}. Now the calculated intrinsic P velocity used in table 1 yields a higher crack density parameter, suggesting that cracks not closed at 2.5 kb contribute about 0.07 to the overall crack density parameter. We call this the residual crack density parameter. The average measured porosity is, moreover, about an order of magnitude larger than the estimated crack porosity. Thus, pore space that is not closed at 2.5kb accounts for most of the porosity, which we call the residual porosity. From the residual porosity and residual crack density parameter, it is possible to calculate a residual aspect ratio for those pores that remain open at 2.5 kb. This residual aspect ratio is 2.1×10^{-2}, which is consistent with the earlier estimate that pores with $\alpha > 10^{-2}$ will remain open at about 2 kb.

(ii) Magnitude of ε

The results of tables 1 and 2 indicate that the crack density parameters for real crystalline rocks lie in the range 0.1 - 0.3. This is a significant result, since such moderate values of ε correspond to the region in which the theory is fairly accurate. It is also the range

of ε for which O'Connell and Budiansky (1977) derive equation 18, in which the characteristic frequency ω_1 depends on the inter-crack permeability. Now a crack density of 0.25, corresponds to one circular crack with a diameter of 1.2 mm for each mm^3 of the rock. Observations of real rocks suggest that values much greater than this are unrealistic. If the 'saturated isolated' case is valid rather than the 'saturated isobaric' case, the estimates of ε would be much larger than 0.3. Thus, the 'saturated isolated' case gives results that are apparently discordant with observation.

(iii) Further Comments on the Use of the Saturated Isobaric Case.

If the saturated isolated case were correct, we would expect the shear wave velocity to increase as the degree of saturation increases, provided the crack density remains constant. The observation by Nur and Simmons (1969) that shear wave velocities are almost independent of saturation conditions would consequently appear to provide ample justification for the use of the saturated isobaric base. However, if the process of saturation introduces new cracks, the velocity would tend to decrease. Hence, an increase in velocity due to increased saturation could be approximately balanced by a decrease in velocity due to the creation of new cracks. It is therefore theoretically possible to maintain a constant shear wave velocity under saturated isolated conditions. The point that we are emphasizing is that the near independence of S wave velocities upon saturation is consistent with saturated isobaric conditions, but under unusual circumstances can also be consistent with saturated isolated conditions.

Some kind of fluid flow mechanism appears important in explaining elastic wave attenuation both in the laboratory and in the Earth's crust (Winckler and Nur, 1979), thus hinting at the relevance of the saturated isobaric case.

Mavko and Nur (1979) have presented a model of seismic attenuation in cracked rocks in which the cracks themselves are partially saturated; the attenuation results from fluid flow within each crack. Such a model of partial saturation may be more realistic than the models of O'Connell and Budiansky that consist of a mixture of totally dry and totally saturated cracks.

(iv) Crack Shapes

The theoretical results given in section 2 were derived for circular cracks. Calculations by Budiansky and O'Connell (1976) suggest that other crack shapes would yield results that differ from those given here by only a few percent provided the definition of ε remains the same. The equations for circular cracks therefore provide a satisfactory interpretation of seismic velocities measured in crystalline rocks.

Recommendations for Future Work

An attempt has been made to demonstrate the relevance of the mathematical theories of cracked solids to the 'RADWASTE' program, and to explain how an integrated program of seismological field and laboratory experiments combined with mathematical modelling can yield valuable information on the crack and saturation parameters of a rock body. Our recommendations for future work consequently fall into three categories: field studies, laboratory experiments and mathematical modelling.

(i) The first important step in field studies is to obtain reliable estimates of shear wave velocities in a rock body over distances of up to

a kilometre. Initially, the testing of a modified mechanical hammer as a source of shear waves should be undertaken, as suggested by Lam and Wright (1979).

(ii) Assuming that shear wave velocities can be accurately measured, the next step is to conduct a field experiment to measure velocities and, if possible, attenuation for both P and S waves using recording both in a borehole and on the surface.

(iii) Laboratory measurements of P and S wave velocities at pressures up to 2.0 kb should be made on rocks from the area in which a seismic experiment is performed; they are essential for the calculation of crack density and saturation parameters.

(iv) Laboratory measurements of attenuation of P and S waves in dry and saturated crystalline rocks should be made in order to better understand the attenuation mechanisms.

(v) Further work on the development and application of the mathematical theories of the elastic properties of cracked solids is necessary in order to devise a model that approximates better both attenuation mechanisms and crack shapes, and also predicts accurately any observed frequency dependence of seismic velocities and attenuation.

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Table 1 : Seismic Velocities and Estimates of the Crack Density and Partial Saturation Parameter

Intrinsic Velocities km/s				Measured P and S Velocities km/s						
				Laboratory Values						
Calculated V_{PI}	Measured V_{PI}	Assumed V_{SI}	V_{PI}/V_{SI}	V_{PS}	V_{PD}	V_S	Borehole V_P	Crack Density ε	Saturation Parameter ξ	Remarks
	6.28	3.86	1.63	5.58		3.28		0.17	0.53	Calculated and measured velocities are the average for layer No.4 of Wright and Kamineni (1979) (Quartz monzonite: depths 90-170m).
	6.28	3.86	1.63		5.28	3.28		0.17	0.21	
6.92		4.23	1.63	5.58		3.28		0.24	0.41	
6.92		4.23	1.63		5.28	3.28		0.24	0.19	
6.92		4.23	1.63			3.28	5.72	0.24	0.51	
6.92		4.43	1.56			3.28	5.72	0.27	0.58	
	6.50	3.82*	1.70	6.26		3.39		0.13	0.92	Sample 1336 (granite) of Feves et al. (1977)
	6.50	3.82*	1.70		5.91	3.39		0.13	0.53	

* Here the intrinsic S wave velocity is assumed equal to the measured value at 2.0 kb

Note: V_{PS} and V_{PD} are the velocities at 1 bar measured on saturated and dry samples, respectively.

V_S is the shear velocity measured on samples at 1 bar.

V_P is the average P velocity in the layer as determined from the borehole log.

Table 2 : Range of Possible Crack Density and Partial Saturation Parameters for a Specified Range of Input Velocities.

Intrinsic Velocities km/s V_{PI}	V_{SI}	Measured Velocity km/s V_P	V_S	Crack Density ε	Saturation Parameter ξ	Comments
6.50	3.98	5.49	3.09	0.242	0.58	(a) Saturated Isobaric
"	"	5.31	"	0.242	0.45	
"	"	5.49	3.27	0.201	0.47	V_P minimum
"	"	5.31	"	0.201	0.31	V_S minimum
6.50	4.18	5.49	3.09	0.268	0.65	(b) Saturated Isobaric
"	"	5.31	"	0.268	0.52	
"	"	5.49	3.27	0.230	0.52	V_P minimum
"	"	5.31	"	0.230	0.36	V_S maximum
6.70	3.98	5.49	3.09	0.242	0.50	(c) Saturated Isobaric
"	"	5.31	"	0.242	0.38	
"	"	5.49	3.27	0.201	0.31	V_P maximum
"	"	5.31	"	0.201	0.15	V_S minimum
6.70	4.18	5.49	3.09	0.268	0.55	(d) Saturated Isobaric
"	"	5.31	"	0.268	0.43	
"	"	5.49	3.27	0.230	0.41	V_P maximum
"	"	5.31	"	0.230	0.25	V_S maximum
6.50	3.98	5.49	3.09	0.413	0.88	(e) Saturated Isolated
"	"	5.31	"	0.381	0.76	
"	"	5.49	3.27	0.308	0.74	V_P minimum
"	"	5.31	"	0.255	0.52	V_S minimum

V_{PI} = 6.60 ± 0.10 km/s; V_{SI} = 4.08 ± 0.10 km/s; V_P = 5.40 ± 0.09 km/s; V_S = 3.18 ± 0.09 km/s.

Table 3 : Aspect Ratios Measured from Crack Density Parameters and Porosities for Quartz-Monzonite.

Parameter	Numerical Value
Depth Range.	90 - 170 m
Average Crack Porosity from Data of Simmons et al. (1978).	9.4×10^{-4}
Average Crack Density Parameter from Data of Simmons et al. (1978).	0.17
Average Aspect Ratio of Thin Cracks.	1.3×10^{-3}
Measured Porosity.	7.2×10^{-3}
Average Crack Density Parameter from Calculated Intrinsic P Velocity.	0.24
Residual Porosity.	6.3×10^{-3}
Residual Crack Density Parameter.	0.07
Residual Aspect Ratio.	2.1×10^{-2}