Internal Report No. 89-4
Geophysics Division,
Geological Survey of Canada, Ottawa.

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February, 1989

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INTRODUCTION

Specifications for a useful airborne gravity measuring system have been determined by *Bower and Halpenny* (1987) (*B&H*) as follows:

Aircraft Speed - 300 km/hr
Accuracy - 2 mGal
Resolution - 5 km (Altitude ~ 600m)

B&H describe a filter-and-average scheme satisfying these specifications which consists of low-pass filtering, with a high frequency cut off of .0083 Hz, followed by averaging over 60-second intervals. This scheme was shown to yield the specified accuracy and resolution when applied to simulated traverses over two 'typical' surface gravity fields, assuming perfect navigation and perfect correction for aircraft motion.

In practice, vertical accelerations of the aircraft at frequencies within the gravity passband ($0 < f < .0083$ Hz) must be determined independently and removed from the gravimeter signal. This is the principle problem of airborne gravity. and consequently the statistical and spectral characteristics of the anticipated acceleration 'noise' in the gravity passband is of central interest.

Residual undetermined vertical accelerations of the aircraft are traceable to uncorrected errors in measuring the altitude of the aircraft since the acceleration corrections to be applied to the gravimeter are determined by twice differentiating these measured altitudes. Note that this means that the power spectral density (PSD) of acceleration corrections is equal to the PSD of altitude times the fourth power of the angular frequency (ω^4). Thus errors in gravity measurement will be dominated by the higher frequency part of the altitude error spectrum.

In order to estimate how accurately the vertical accelerations of the aircraft can be determined B&H considered three possibilities for the behaviour of the altitude error as a function of frequency (see Figure 1): (1) The altitude noise is due to a single sinusoid located just below the high frequency boundary of the gravity passband; (2) The altitude noise spectrum has the same frequency dependency within the gravity passband as the PSD of actual aircraft motion); (3) It has a 'white' spectrum (ie. has the same power per unit frequency interval through out the passband). For each case they calculated the rms altitude error which yielded a 2 mGal error in computed acceleration, given a grid width of 5 km and a range of aircraft speeds. Note that the important parameter here is actually the high frequency boundary of the gravity passband (f_h), which was determined here by the grid width (w) and the aircraft speed (S) by:

$$f_h(\mathrm{Hz}) = S/2w$$

These results are illustrated in their Figure 21 and reproduced here in Figure 1 (curves 1 to 3 respectively).

For the same rms altitude meaurement error B&H's first case (curve 1) leads to the greatest error in acceleration and consequently in measured gravity. They find that for an aircraft speed of 300 km/hr the rms amplitude of this sinusoid must be less than 0.7 cm. The white-spectrum noise (curve 3) is the least demanding of the three cases and requires altitude determination to about 1.5 cm. Because the spectrum of typical aircraft vertical motions rises with frequency throughout the passband, the altitude error spectrum of the second case (curve 2) implies a greater rms acceleration error than a white altitude error spectrum but less than if all the altitude error power were concentrated at a single higher frequency, as in the periodic case.

Because information at the time indicated that the error spectrum of GPS phase measurements was white *B&H* felt that these curves represented the range of anticipated conditions.

DISCUSSION

B&H discuss various methods of measuring the altitude variations of the aircraft but the most viable method over land, independent of ground-based navigation aids, is probably utilizing GPS carrier-phase measurements. Using recent data *Georgiadou and Kleusberg* (1987) (*G&K*) and *Kleusberg* (1988) show that the main error sources in such measurements can be characterized in some detail. In the light of their results we have reviewed conditions in *B&H*'s gravity passband with a view to refining their estimates of overall system performance.

If altitude measurements are based solely on GPS carrier phase measurements it is likely that the spectrum of altitude errors will be characteristic of the GPS environment rather than of aircraft motion. The most serious source of error in altitude variation measurements based on GPS carrier-phase observations is probably that due to multipath interference, which remains after errors due to ionospheric delays have been eliminated through the reception of two GPS frequencies.

G&K report on carrier phase observations of two satellites using adjacent GPS receivers on successive days. They infer multipath effects from differential 'ionospheric delays' computed from two-frequency carrier phase measurements, a computation which should eliminate real ionospheric delays for closely spaced receivers. They demonstrate conclusively that these apparent ionospheric effects are in fact multipath effects by showing almost perfect repeatability on the following day.

Their results are reproduced in Figure 2. The PSD determined for both satellites is clearly 'white' above about .05 Hz but below this frequency a f^{-4} response is shown for PRN-11. The response for PRN-6 in this frequency range also suggests a basic f^{-4} power spectrum superimposed on a spectral peak near .01 Hz. The much lower power 'white' spectrum evident at higher frequencies is thought to represent receiver noise. If we assume that this receiver noise persists with the same PSD (10 mm^2/Hz) throughout the gravity passband it would contribute only 0.3 mm rms to the altitude error (we have assumed the passband to be $0 < f < .01$Hz). With regard to the PSD attributable to multipath effect, if we assume it to be white with a value of $10^4 mm^2/Hz$ (the maximum value) it's contribution to the altitude error would be 10.0 mm rms. It would in fact contribute less than this to the rms altitude error and very much less to the computed acceleration error because, as we have pointed out, the PSD is not white but falls off rapidly with increase in frequency.

Results similar to those of *G&K* are presented by *Kleusberg* (1988) and are reproduced here in Figures 3 and 4. Fitting a f^{-n} line to their PSD's in the gravity passband yields (approximate) values for n of 4, 3, 3 and 4 for satellites PRN-6, 8, 11 and 12 respectively.

On the basis of these data we have computed the permissible rms error in altitude for a 2 mGal error in acceleration measurement when the altitude error spectrum is 'red', specifically when it varies as f^{-2}, f^{-3} and as f^{-4}. These results have been added to those of B&H in Figure 1 (curves 4, 5 and 6 respectively).

It must be pointed out here that these latest rms values have been determined from the corresponding 'red' PSD's by integrating from .001 Hz to the top of the gravity passband, rather than from the minimum available spectrum frequency (equal to .00016 Hz, the reciprocal of the particular data sample length used). This was done because that part of the altitude error spectrum below .001 Hz, although large in power, contributed relatively little to the acceleration errors. Including the power below .001 Hz would yield a much larger permissible rms altitude error but most of the error power would not contribute to the acceleration errors and the result would be misleading.

The present results in Figure 1 suggest almost an order of magnitude increase in permissible error in altitude measurement when compared with the previous estimates. Clearly it will be important to characterize altitude errors in this frequency range.

SUMMARY AND CONCLUSIONS

Recent observations suggest that errors in GPS carrier phase observations at frequencies within the gravity passband of airborne gravity systems are due mainly to multipath interference. Further, the spectrum of these errors at frequencies within the anticipated gravity passband may generally be red, falling as rapidly with frequency as f^{-3} or even f^{-4}, in contrast to the white spectrum of receiver noise.

This result has a significant impact on the conclusions of *B&H* in regard to the allowable error in altitude reference. Given a continuous f^{-4} distribution of spectral power a rms altitude error as large as 12 cm would allow the computation of acceleration correction with an accuracy of 2 mGal. In comparison, the allowable rms altitude error given a 'white' distribution of spectral power is 1.5 cm.

These estimates do not take into account the degradation introduced by the procedures necessary for correcting for ionospheric refraction (factor of 3) and clock errors (factor of $2/\sqrt{2}$). When these degradation factors are taken into account the 12-cm altitude error referred to above requires that the rms ranging error be less than about 3 cm. A further degradation of 3 or 4 will be introduced by the unfavourable geometry of the current constellation of satellites (vertical dilution of precision) but this is expected to be reduced to 1 when the full constellation is in place in 1992.

Note that the allowable errors estimated here assume a continuous altitude error spectrum, as suggested by the observations, and that the presence of prominent spectral lines near the upper end of the gravity passband, representing strong reflections from discrete surfaces, could easily dominate the error spectrum and lead to results similar to *B&H*'s periodic case.

Finally, mention should be made that the effects of tropospheric refraction, although not discussed here, may be significant in humid air conditions and should be considered.

Acknowledgements

We am grateful to Alan Goodacre for initiating the GPS measurement tests and for valuable discussions, and to Alfred Kleusberg for reading the manuscript and for useful suggestions.

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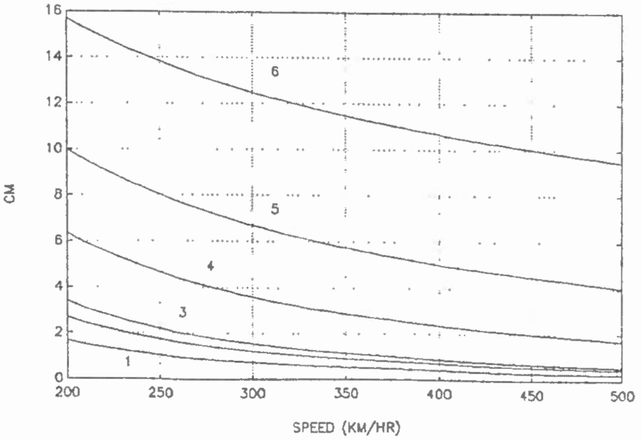

Figure 1. Maximum permissible RMS error in altitude determination at frequencies within the gravity passband for a grid width of 5 km and a gravity error of 2 mGal, as a function of aircraft speed and type of altitude error spectrum. Curves 1-3 are from Bower and Halpenny (1987), curves 4-6 assume an altitude error spectrum proportional to f^{-2}, f^{-3} and f^{-4} respectively.

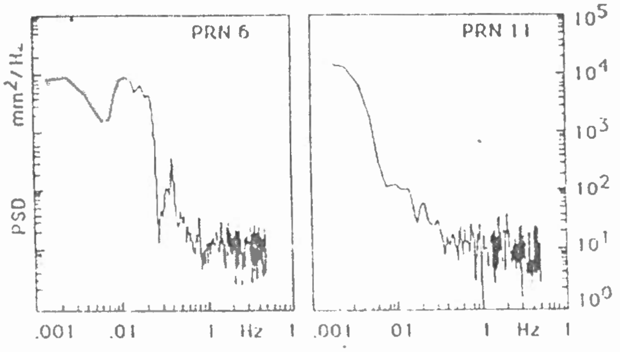

Figure 2. PSD of differential 'ionospheric delay'. (Georgiadou and Kleusberg, 1987)

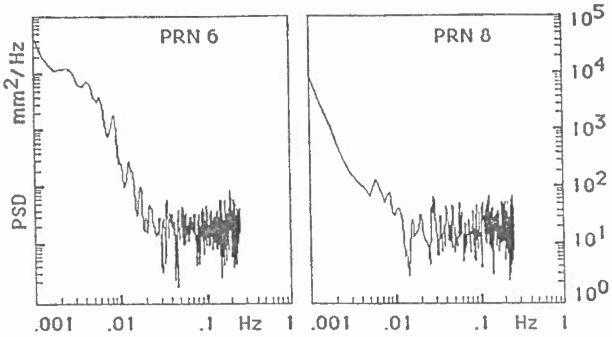

Figure 3. PSD of differential 'ionospheric delay'. (Kleusberg, 1988)

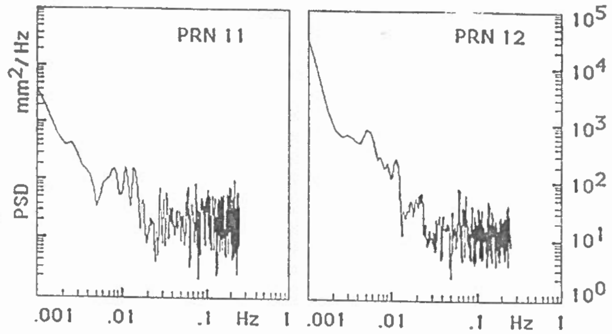

Figure 4. PSD of differential 'ionospheric delay'. (Kleusberg, 1988)