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1.0 Introduction

This Division has played a leading role in the development and evaluation of techniques and instrumentation for shipborne gravity surveys since 1960. Much of this activity is directly applicable to airborne measurements but the last active participation of the Division in airborne gravity consisted of an aerial test of the inertial platform used with the LaCoste and Romberg (L&R) Air/Sea Gravimeter (Valliant, 1976). This test demonstrated that the platform was sufficiently stable (given frequent, independent position data) to yield velocity data adequate for determining Eotvos corrections for a gravimeter. However further work was discouraged by the difficult problem of correcting for the vertical motion of the aircraft and the project was essentially moth-balled in 1975. Activities related to shipborne gravity continued however and significant work was done by this Division in testing the new L&R straight-line gravimeter (Valliant, 1983), which eliminated inherent cross-coupling effects, and in developing a new digital controller for the gravimeter (Valliant et al, 1985).

In 1983 interest in airborne gravity was renewed by the anticipated launching in 1986-87 (subsequently postponed until 1989-91) of a network of satellites to form the Navigation Satellite Timing and Ranging Global Positioning System (NAVSTAR GPS). (For an extensive list of documents available on GPS contact: NOAA, National Geodetic Information Center, N/CG17X2, Rockville, Maryland 20852 USA). Early experience with a network of six GPS satellites indicated that sub-decimetre accuracy in the positioning of moving platforms was possible. This suggested a solution to the vertical motion problem in airborne gravity, particularly for slower moving aircraft since the permissible uncorrected vertical motion is inversely proportional to the square of the aircraft speed.

In 1986 consideration was given to a proposal for the transfer of in-house expertise and methodology to the private sector for the purpose of developing a fixed-wing airborne gravimetry system. Before this could take place however the head of the Division's dynamic gravimetry group, H. D. Valliant, resigned to take a position with L&R. With the main source of expertise now missing the proposed development project was

reexamined and a decision made to cancel the Division's participation and support. At the same time, because the Division's view with regard to fixed-wing airborne gravimetry was pessimistic (including the view of Valliant himself who favoured using a slower moving dirigible or airship), a decision was made to initiate a study of the prospects for a longer term airborne gravity program. It was felt that such a study would also allow new personnel to become familiar with the project and be in a position to evaluate new developments. The present report is the first part of this study.

2.0 Considerations in the Design of Airborne Gravity Systems

2.1 Required Accuracy and Resolution

Conventional land-surface gravity surveys can be characterized as belonging to one of two accuracy and resolution classes. A grid spacing of 100 m to 1 km and an accuracy of 0.1 mGal are typical of detailed or target-specific surveys. Reconnaissance gravity surveys on land typically have a station spacing of 5 to 15 km and an anomaly accuracy of 1 to 2 mGal, most of the error being due to uncertainty in the measurement of the station height. In areas of high relief uncertainties in the determination of terrain correction can increase the anomaly uncertainty to 5 mGal or more. At sea, track spacing is typically 5 to 20 km with measurements reduced at 1 to 2 km spacing along track. Anomaly accuracy requirements at sea are generally 1 to 3 mGal.

2.2 Potential Applications for Airborne Gravity

The areas listed below were suggested by M^CConnell (private communication) and are candidates either because ice-surface or surface ship measurements are not possible or would take so long that it is likely that airborne gravity measurements would be competitive. For estimating costs it is reasonable to assume a ship speed of 12 knots and an aircraft speed of 350 km/hr. Totally dedicated ship time is

currently estimated to cost \$25,000 per day while costs for an aircraft of the type suitable for airborne gravity surveys are approximately \$1000 per flying hour. Thus the costs per kilometre of survey line for ship and aircraft are approximately \$50 and \$3 respectively. The candidate areas are:

Canada: Foxe Basin, because it is too shallow for safe operation of large surface ships and it does not freeze over completely. Approximate linear distance to be traversed, assuming a line spacing of 5 km, is 3.2×10^4 km.

Canada: Hudson Straits, Cumberland Sound and Ungava Bay because the currents here are so strong that solid ice cover does not form. Approximate linear distance is 2.4×10^4 km.

Canada: Northern part of Davis Strait mainly because coverage by surface ship would take so long. Approximate linear distance is 1.1×10^5 km.

Canada: North East Pacific. Coverage of this area by surface ship is currently under consideration as part of the NOREP project. Approximate linear distance is 3.0×10^4 km.

Canada: Parts of Northern British Columbia and the Yukon. This is the only land area in Canada that is a potential candidate but the difficulties are much greater than for over-water areas. Approximate linear distance is 1.2×10^5 km.

Continental shelf areas of the world. These are candidates because of both economic and military pressures and the enormous area involved which makes ship coverage in the near future unlikely. Estimated linear distance is 5.5×10^6 km.

2.3 Design Specifications

Based on the above, an accuracy of 2 mGal and a resolution of 5 km were

adopted in the present study as design specifications for a useful airborne gravity measuring system. The study will focus on over-water systems since this is the major need. Over-land airborne gravity measurements involve basically the same problems as over-water measurements but there are additional critical parameters. It will be shown later that height control by GPS or local land-based laser transponders is necessary for over-land measurements but may not be essential in certain cases over water. The over-land case is discussed in Section 6.

The required resolution and the vehicle speed determine the highest frequency necessary in the gravimeter-system passband. Although a low vehicle speed is desirable, and for this reason the use of airships has been proposed in the past, a fixed-wing aircraft with a speed of 300 km/hr is assumed here. This speed is in the low cruising range of available aircraft such as the NAE's Convair 580. Airships are not considered here because: (1) very little performance data are available; (2) the availability of airships is uncertain; (3) techniques developed for fixed-wing aircraft would be transferable to airships if they should become available. See Balaam (1982) for a report on angular motion and vibration measurements on the Skyship 500 dirigible.

2.4 Data Filtering for an Airborne Gravity System

A filter/averaging scheme was employed to evaluate the performance of a model airborne system under particular conditions of vertical motion etc. In this scheme, data from the gravimeter system are passed through an 8-pole, low-pass filter with a cutoff frequency of .0083 hz (equivalent to a period of 120 sec) and then a simple average is taken at intervals of 60 seconds. At an assumed vehicle speed of 300 km/hr this yields the average gravity over the intended grid spacing of 5 km. Trials were made to ensure that this scheme did not excessively attenuate expected gravity field variations. These results are illustrated in Figures 1 to 7.

Figure 1 shows the desired and achieved frequency responses of the filter. The filter response to a unit pulse of 60-sec length, corresponding to one grid width in the gravity field model, is

reproduced in Figure 2. The filter output has been corrected for a delay of 66 seconds introduced by the filter. Figure 3 illustrates the response of the gravity filter to a unit step change in gravity. Contoured gravity over a region located in northern Ontario (Nagy, 1986) is shown in Figure 4. Profiles A and B were selected from this figure and used as test input for the gravity filter model. Figure 5 is a plot of the gravity field along profile A overlain with the same gravity field passed through the gravity filter and corrected for the 66 second delay. This result demonstrates that the filter does not significantly distort the gravity field.

The same result is expressed in terms of the average gravity over 5-km grid widths in Figure 6. The solid line is that for the gravity field while the dashed line is the difference between the gravity field and the filter response. Note that the difference is well below 2 mGal throughout. Figures 7 and 8 show equivalent results for profile B, which exhibits larger gravity changes.

It is assumed that the airborne survey is taken at a low altitude (500 m or less) so that attenuation due to height is not a significant factor. Current experience at the Naval Research Laboratory (NRL) of the U.S. Navy (Brozena et al. 1987) suggests that 500 m may be the optimum altitude for airborne gravity surveys. (In this connection note that the Division has the software required to quickly upward-continue a gravity field described on a surface grid of dimensions 200x200 points).

We conclude that the filter/averaging scheme we have chosen for this study, while not necessarily optimal, is representative of a realistic system. Given statistical data describing the expected uncorrected vertical motion of the vehicle we can apply this filter/averaging scheme to determine the resulting error statistics in the gravity data.

3.0 Errors due to Vehicle Acceleration.

3.1 Horizontal Accelerations and Platform Off-Level Errors

Typical-magnitude periodic horizontal accelerations are shown by Brozena et al. (1986) not to cause significant platform errors. Non-

periodic horizontal accelerations are caused by course adjustments and these will induce platform errors. Techniques to correct (in real time or in postprocessing) for these errors are described in Brozena et al. (1986). The problem can be avoided also by switching the tuning of the platform dynamics to short period (eg. 4 min) during and for a few minutes after turns. This problem is probably not serious but if it does require a closer look we would need to know: (1) the impulse response of the autopilot to course corrections and (2) the statistics of course deviations caused by wind gusts. Brozena et al. (1986) describe the impulse response for the system used by them but course deviation statistics are probably not available.

3.2 Periodic Vertical Accelerations:

Implications for Gravimeter Dynamic Range

The vertical accelerations of the vehicle must be independently determined and subtracted from the gravimeter response. In determining this correction it is assumed that the gravimeter response is linear over the range of vertical accelerations experienced. This is the case so long as: (1) the gravimeter beam does not contact a stop and (2) the spring tension (expressed in gravity units) is within 1000 mGal of that required for beam equilibrium. To drive the heavily damped gravimeter beam from its centre position to one stop requires an integrated acceleration-time of 450,000 mGal-seconds. Thus, for example, the maximum tolerable rms amplitude of a 50-second vertical acceleration is 20,000 mGal, provided that the spring tension is within 1000 mGal of that required for equilibrium in the absence of the periodic acceleration. This is the case for the Geophysics Division's gravimeter system using the LSI-11 controller. The NRL system uses a different controller and requires the spring tension to be within 100 mGal of the equilibrium value for undistorted gravimeter response under the same conditions.

These specifications should not present a problem for normal airborne work although the large difference in Eotvos correction between east and west data tracks (up to 4000 mGal according to Brozena et al., (1986)) must be anticipated and the system adjusted accordingly at the beginning

of a new track.

3.3 Effect of Periodic Vertical Acceleration

This is a frequently used measure of system performance although vertical motion really has a continuous spectrum and probably can be best characterized as broad-band gaussian noise. Nevertheless the response to sinusoidal motion is a useful parameter for comparing strategies and suggesting constraints. A broad-band interpretation of vertical motion will be used in later sections.

Figure 9 illustrates the rapid increase in acceleration with increasing frequency for constant amplitude (1-cm) vertical motion. Frequency is an implicit function of vehicle speed and grid spacing in Figure 10. For these curves the system filter is assumed to have been adjusted so that the high frequency cutoff is just high enough to resolve spatial changes in gravity to the indicated grid spacing at the indicated vehicle speed. The error shown in Figure 10 is a 'worst-case' condition of a function which varies with the initial phase of the sinusoid. It can be seen that for a 300-km/hr vehicle speed and a 5-km grid spacing that an uncorrected sinusoidal height error of 10.0 cm, at a frequency within the passband of the system filter, will cause an error in gravity estimation of more than 8 mGal. For the same grid spacing a vehicle speed of 100-km/hr, with the appropriate change in system filtering, would reduce this error to less than 1 mGal.

We consider next the effect of more realistic, continuous-spectra vertical motion.

3.4 Predicted Vertical-Motion Effects based on Observed Continuous Spectra

Brozena et al. (1986) find that the piezometric-controlled automatic pilot flies a much better level flight than does a human pilot, generally keeping a given height within 3-5 m. This is confirmed by Leach (personal communication) of the National Aeronautical Establishment (NAE). There are three main sources of error in height keeping. Since conventionally the auto pilot attempts to follow the local isobaric surface the most obvious error (but not the largest) is

that due to the difference between this surface and that of constant height. A second major reason for height deviation is due to the transient response of the auto pilot to wind gusts. This effect interacts with the third major height keeping problem which involves the natural oscillations of the airframe, referred to in the literature as phugoid-mode oscillations. Brozena (Telecon 87) rates phugoid motion as the single most severe problem in airborne gravity. The mechanism for this motion is the inherent stability of the aircraft in pitch which leads to damped oscillations which then couple into roll and yaw motion.

Leach (personal communication) refers to an equation giving the approximate frequency of phugoid oscillations as: $f = g/(\sqrt{2}mS)$ where S is the aircraft speed. For 250 knots (463 km/hr) this is .0172 hz or T = 58.3 sec. Assuming the autopilot keeps this motion to within about 3 m the corresponding vertical acceleration at this frequency would be about 3500 mGal. We will see later that this is very close to observed values.

We will not consider the reasons for vertical motion further since for the purpose of predicting the performance of a given gravity-measuring system we need only characterize the overall height-keeping behaviour of a typical fixed-wing survey aircraft. For this purpose we will adopt the power spectral density distribution (PSD) of altitude variations reported by Brozena et al. (1986) for a P-3A aircraft under turbulent conditions. A smoothed version of this spectrum is reproduced in Figure 11.

There are two considerations with respect to excessive vertical motion. The first is that the vertical acceleration of the vehicle must be within the linear dynamic range of the gravimeter system and the second is that that part of the vertical acceleration spectrum within the passband of the overall gravity measuring system, taken to be from 0 to .0083 hz here, must be determined by independent means and removed from the gravimeter output. The latter is the main problem in measuring gravity from an aircraft and available techniques for doing this will be discussed later. It will be shown next that the gravimeter dynamic range is not normally a problem.

3.5 Dynamic Range Considerations

The amplitude spectrum shown in Figure 11 can be converted to a power spectrum (i.e. in units of length squared per unit frequency) by squaring (Figure 12) and to an acceleration power spectrum by further multiplying by $(2\pi f)^4$ (Figure 13). Brozena et al. (1986) estimate the acceleration experienced by the gravimeter beam by integrating the acceleration power spectrum in the range $0.01 < f < .03$ hz, taking the square root to obtain acceleration (.7 gal) and then reducing this to allow for the internal filtering of the gravimeter. This yields 200 mGal which they describe as close to the maximum tolerable acceleration for linear operation (ie. within the stops). We find their estimate of acceleration (.7 gal) to be inconsistent with their quoted rms altitude variation in this band (6.5m) and in fact we calculate an acceleration of 12.4 gals using the PSD function in Figure 13. Our higher acceleration value however is approximately consistent with their net figure of 200 mGal on the assumption of the internal filtering described in the gravimeter manual (ie. 3 stages of RC filtering with a cutoff frequency of .05 hz).

Accepting their estimate of 200 mGal for the net beam acceleration however we are puzzled by their conclusion that this is near the maximum tolerable acceleration for the gravimeter beam. According to the manufacturer's specifications (ie. the current beam calibration constant for SL-1) the gravimeter will tolerate a constant acceleration of 4500 mGal for 100 seconds before hitting a stop. This result requires clarification (a technical manual is not supplied by L&R for the SL-type gravimeters) but it appears that the turbulent conditions represented by Figure 11 are well within the dynamic capabilities of the SL-1 gravimeter.

3.6 Prediction of Effect on Measuring Gravity

That part of the vertical acceleration power spectrum (Figure 13) within the gravimeter passband of .0083 hz will be interpreted as a gravity signal unless corrected by an independent measurement of vertical acceleration. Without correction the spectrum shown in Figure 13 for turbulent conditions will contribute an rms error in gravity determination of approximately 500 mGal. The rms error for this and

other cutoff frequencies can be read from the accumulative rms vertical acceleration as a function of frequency shown in Figure 14. The corresponding rms altitude deviation can be read from the accumulative rms altitude shown in Figure 15. From these results it can be seen that in order to reduce the gravimeter error to 2 mGal we must reduce the rms altitude deviations to 1.2 cm ($290 \times 2 / 500$) by independent measurement.

The above result assumes that the reduced altitude spectrum, ie. the altitude spectrum after removing altitude deviations determined by some independent means, has the same spectrum shape as the original altitude spectrum. It can be shown too that if the reduced spectrum is white, ie. distributed over all frequencies at constant density, then the rms altitude deviation is 1.5 cm for the same error in gravity measurement.

This daunting requirement in altitude determination is reduced quickly with decreasing cutoff frequency. For example for a cutoff frequency of 1/2 that assumed above the allowable rms altitude deviation is 5.3 cm ($160 \times 2 / 60$). For the same gravity field resolution however this lower cutoff frequency would require 1/2 the vehicle speed, which would not be feasible using a fixed-wing aircraft.

4.0 Measurement of Vertical Motion

4.1 Differential GPS Navigation: Introduction

The determination of vehicle vertical acceleration due to motion is the crucial problem in the measurement of gravity from the air and GPS navigation offers, at least potentially, the only generally-applicable solution. This section will summarize GPS characteristics and current levels of performance.

Each satellite of the GPS system (Janiczek, 1978) transmits unique data continuously on two frequencies. These data include satellite orbit and timing information for satellite to receiver transit-time ranging and the synchronization of code receivers to GPS time. This permits instantaneous relative positioning (ie. relative to another receiver) to 10-30 m and several-hour relative positioning to the 1-m level. This accuracy can be improved greatly by recording carrier

phase differences. With appropriate geometry (quasi-simultaneous reception of 4 satellites is required) and software, relative positioning to the centimetre-level has been demonstrated (Remondi, 1985) using carrier phase mode.

4.2 Estimated Positioning Accuracy.

The following is a summary of current positioning accuracy prepared by Kouba (unpublished note) based on carrier phase solutions:

1. Ideal Conditions (ie. including instrumentation errors only)

Parameter:	Exp. Error:
Position (3 coord. GDOP=5)	1.5 cm (Note 1)
Velocity (instantaneous)	0.3 cm/sec (avg over 2 sec)
Velocity (avg over 10 sec)	0.1 cm/sec
Acceleration (instantaneous)	4.0 mm/sec ² (400 mGal)
Acceleration (avg. over 10 sec)	0.2 mm/sec ² (20 mGal) (Note 2)

2. Achieved Under Field Conditions (Mader, 1986) (Note 3)

Parameters:	Exp. Error:
Position	5.0 cm (1.2 sec)
Velocities (instantaneous)	2.0 cm/sec (avg over 2 sec)
Velocity (avg. over 10 sec)	0.6 cm/sec
Acceleration (instantaneous)	3.0 cm/sec ² (3000 mGal)
Acceleration (avg. over 10 sec)	1.0 cm/sec ² (100 mGal)

Notes:

1. GDOP and VDOP are measures of geometric dilution. They are weighting factors to be applied to error estimates to account for amplification due to geometry of the 4-satellite solution. They are equal to the mean square root of the appropriate covariance matrix. VDOP is the measure for the vertical component while GDOP is the average of the three components.

2. The error is less than that it would be just using the end

points. Acceleration is assumed constant throughout the averaging period and is determined by the best fit of a straight line through the samples obtained during the averaging period.

3. Possible error sources here are multipath, the atmosphere, the troposphere and the ionosphere. Troposphere error is likely to remain constant for up to several hours under normal conditions. Ionosphere error is of the order of $1 \text{ to } 3 \times 10^{-6} \times L$ where L is the distance between the two receivers. Since the process of combining the two signals to obtain an estimate of the ionospheric error has the effect of multiplying other errors by 3.0 this correction is not usually implemented when the two receivers are within about 50 km of one another.

4.3 Some Results in the Fixed-Point Monitoring of GPS Carriers

In order to obtain first hand experience with the potential precision of GPS phase measurements the carriers of two satellites were monitored for periods of approximately 20 minutes. The results of this and other tests are described in detail by Beach and Goodacre (1987). In addition, a subset of these data was treated using standard spectral analysis software (MATLAB) and these results are summarized below.

As an indication of the error we might expect in the determination of altitude variations through differential GPS phase measurements we concentrated on the short period variations in phase. The raw phase measurement of one carrier after removal of a 4th-order polynomial representing orbital motion (ie. the orbital motion was not explicitly determined) and after conversion to equivalent range in cm, is shown in Figure 16. The power spectral density function for Figure 16 is shown in Figure 17. Using the PSD function we calculated the accumulative rms range deviation (Figure 18) and then the equivalent rms acceleration in mGal (Figure 19). This result suggests that altitude variations within the gravimeter passband of .0083 hz can be determined through GPS differential phase measurements to an accuracy equivalent to 2 mGal in acceleration.

In reality there are many degrading factors involved in the final determination of position by differential GPS phase measurements which are not considered here. Nevertheless the above result impressed and encouraged some of us. Kouba (private communication) believes that much of the spectral power in Figure 17, particularly that at frequencies less than .06 hz, represents timing errors due to the receiver clock and that the rms range deviation should be sub-centimetre. The evidence of Beach and Goodacre (1987) is consistent with this theory and they suggest further experiments using more stable oscillators and antennas with moderate gain to increase the signal to noise ratio. Further in this connection, Georgiadou (private communication) has found after further analysis that at least part of the spectral power shown in Figure 17 is due to multipath reception.

4.4 Accuracies Achieved in Airborne Use of GPS

The most relevant data available on this subject appear to be that determined by Mader et al. (1986) during an airborne test of GPS carrier phase positioning and simultaneous laser altimetry over water. The report presents results obtained during taxiing, takeoff and climb, and during level flight. The following is a summary of significant points:

- 1) A TI 4100 GPS receiver was used with GESAR software and a bandwidth of 16 hz. Data were recorded at 3 second intervals. Level flight was at an altitude of 150m at a speed of 370 km/hr.
- 2) Solutions generated while the aircraft was stationary or moving slowly (up to 30 km/hr) showed a noise of 2 cm horizontally and 5 cm vertically. Vertical solutions while taxiing agreed with geodetic surveying within 10 cm. The receivers maintained phase lock during maximum takeoff accelerations and aircraft manoeuvres.
- 3) Once airborne and flying smoothly at a constant speed of 320 km/hr results are indistinguishable from the case when the aircraft is moving at 30 km/hr on the ground, so far as the

tracking loop is concerned.

4) Unrecoverable cycle slips (1 cycle = 19 cm) were experienced during four of five flights over the calibration area. Cycle slips appear unrelated to aircraft dynamics.

4.5 The Outlook for GPS Technology

Kaula (1986) has summarized the policy of the U.S. National Geodetic Survey (NGS) concerning the development and operation of the GPS. Highlights of this policy, and underlying expectations and assumptions pertinent to airborne gravity are as follows:

- 1) The GPS system of 18 spacecraft probably will not become fully operational before 1990. The existing experimental system of 6 spacecraft will continue to operate but will probably degrade significantly in performance by 1990.
- 2) The accuracy attainable by the system as currently configured is probably around 1 in 10 million. An ultimate accuracy of 1 in 100 million should be achieved (for distances greater than about 1000 km).
- 3) The main limitations on GPS accuracy are radiation pressure and other effects on the orbit; ionospheric refraction; tropospheric refraction; and multipathing (particularly relevant to moving vehicles).
- 4) The main drivers for economy are the locations of moving vehicles. Such dynamic applications of GPS may be its leading contribution to the saving of survey costs. NGS should carry out experiments in airborne and surface dynamic positioning techniques.
- 5) USGS is the prime U.S. agency for study of local and regional crustal motion and any NGS work in this area should

be in collaboration with USGS. USGS also has an interest in dynamic GPS to support it's mapping activities.

5.0 Other Means of Determining Vertical Motion

5.1 Radar Altimetry

The NRL (Brozena, Telecon 870218) currently relies on a powerful radar altimeter supplemented by real-time input from the pressure altimeter for determining vertical motion over water, ice and near-shore wet lands. Although the nominal resolution and accuracy of the system is about 0.3m NRL achieves a resolution of 1-2 cm at a flying height of 500 m through a hardware modification. The radar altimeter, which is described in detail by Brozena et al. (1986), was assembled by the NRL from off-the-shelf hardware and should be readily reproducible (Brozena, Telecon 1987). An extremely narrow pulse width is used (3 nsec) at a pulse repetition rate of up to 10 khz. The most expensive part of the apparatus is the 20-w magnetron amplifier which costs \$20,000. Brozena estimates the entire system could be assembled for less than \$100,000.

Some of the NRL enhancement techniques might be applicable to the NAE radar altimeter, which is described later. The Canada Centre for Remote Sensing (CCRS) in Ottawa also have a radar altimeter installed in their CV-580. (Contacts there for further information on this subject are: Dr. Hawkins (233-4374) and Jack Gibson.)

5.2 Laser Altimetry

A laser altimeter manufactured by Optech Inc., (the representative in Toronto is Joe Liadsky) is typical of current technology. Optech model 501 was recommended as being best suited to measuring altitude precisely. Resolution and accuracy are rated as 10 and 20 cm respectively. A resolution approaching 1 cm is possible through another output from the device and by averaging many pulses since the specifications refer to just one individual pulse. Pulse rate is variable up to 2000 shots per second. The beam width is 2.4 mradian

which produces an image on the surface of diameter 1.2m at 500m. The Model 501 costs \$25K.

A Canadian source of expertise in the use of laser altimeters over water is Bob O'Neil at Canada Centre for Remote Sensing. O'Neil advises that if the surface is very flat specular reflection can occur which would miss the aircraft completely on return. The CCRS has a more powerful version of the basic model 501 altimeter installed in their CV 580, in addition to a radar altimeter, which they use for shallow water depth measurements (to 20-30m).

The NRL was initially enthusiastic about the use of laser altimetry but then lost interest due to orientation problems and frequent data interruptions due to cloud cover. We find the reference to orientation problems puzzling since calculation indicates that at 500 m an incremental aiming error of 5-min of arc superimposed on a steady error in aiming of 1 degree would produce only an incremental range error of 1.2 cm.

5.3 Pressure Altimetry

Other than GPS, pressure altimetry is the only source of precise altitude readily usable over land. The NRL has found pressure altimetry to be very useful and have developed it to a high degree. According to Brozena (Telecon, 1987) results of flight tests comparing pressure, radar and laser altimetry may be available this year.

6.0 Airborne Gravity Measurements Over Land

6.1 Introduction

The primary difference between sea and land airborne gravity surveys lies in the use made of radar or laser altimetry. On a sea survey, the altimetry can be used to derive vertical accelerations because the sea surface is assumed to be flat. On a land survey, the vertical position must be determined from another source and the altimetry gives ground elevations.

There are two advantages in land surveys. First, the topography can be

measured and Bouguer anomalies computed directly on land, as opposed to the case over the sea where water depths cannot be observed from an aircraft. Second, the survey produces a measured topographic profile with an accuracy of better than a meter. This is a valuable product by itself, not only for terrain corrections but for other surveying needs such as control for mapping.

6.2 Inertial Gravimetry: The A.P.T.S System

The Airborne Profiling of Terrain Systems (APTS) is described by Stoltz (p 137, 1985) and by Donna (p341, 1985). It consists of an aircraft-mounted inertial platform and a laser tracker which ranges to fixed ground retroreflectors. In a survey with multiple passes over a small (30 km by 30 km) area and extensive postprocessing, they are able to estimate the gravity field to 2 mGal.

The system measures ranges at 1600 samples/sec with an accuracy of typically 3 cm. Accelerometer values are sampled at 200 samples/sec. These are both averaged and recorded at 12.5 samples/sec. Other parameters, such as pressure, temperature, gyro torques and estimated positions are recorded at a slower rate. All these data produce a considerable storage requirement of about 10 Megabytes for a 3-hour flight.

The data are processed by a Kalman filter which simultaneously adjusts the instrumental errors, retroreflector positions and gravity field parameters. A typical flight with 3 surveyed and 12 unsurveyed retroreflectors may have 36 position unknowns, 28 instrument errors and 18 gravity field unknowns, for a total of 82 unknowns.

The system does not estimate gravity points as such, but estimates parameters of a collocation model describing the gravity field. This means that the gravity can be described at any point in the survey area by this model. To construct the model, various assumptions must be made about the variability of the gravity field.

The gravity model approach is more useful than estimating a number of discrete gravity stations for a number of reasons. The instrument measures a continuous line of gravity, and because of the upward

continuation of gravity, each measurement has a contribution not only from the point underneath but from surrounding points. Also, the gravity model allows a simultaneous adjustment of gravity measurements, crossovers and instrument biases, of which there may be many. Finally, the model allows the direct production of a contour or applicon map, bypassing the usual gridding process which introduces additional errors in the data.

The gravity model can be evaluated for each survey area in one adjustment. An area of, say, 200x200 km with a 5-km grid will have 1600 grid point unknowns as well as instrument biases. Solving a set of equations this large is not trivial, but it has been done in gravity network adjustment, and with large mainframe computers the job is not difficult.

The APTS system has been used for its primary purpose, topographic profiling, for a number of years, and it has been used experimentally for gravity work. Its disadvantage is the requirement to lay out ground retroreflectors which are almost as closely spaced as gravity stations. The only way around this seems to be the use of some satellite ranging system to provide positioning accurate to at least 1 cm. This is presently beyond the capability of GPS. However the APTS system lends itself to adding extra flight lines to improve accuracy. In the simplest case, repeating a flight line will double the amount of information and should reduce the error estimate by 2. When using a gravity model the same effect occurs when the line separation is decreased. For a 5-km grid, lines should normally be flown 5 km apart. However, if lines were to be flown 2.5 km apart, the information would be doubled and the error reduced by 2. If the 5-km lines were augmented by another set flown at right angles at the same 5-km spacing the amount of information should be doubled and also systematic errors would be estimated much more accurately. The actual design of a survey will depend on the instrument errors and the desired accuracy, but in cases where the accuracy is just below the permissible tolerance, it can be improved by increasing the flight time.

7.0 Current Status of the Geophysics Division's Air/Sea Gravimeter

7.1 Administration

The Geophysics Division's dynamic gravimeter is administered by the Gravity Section. The chief user of the gravimeter is R. V. Cooper, who directs its maintenance and repair by the Instrumentation Group (mainly by C. Gagnon under the general supervision of N. Courtier). Cooper's activities in this respect have been exclusively related to gravity surveys at sea. Currently there is no individual or group responsible for the overall preparation of this instrumentation for airborne use.

7.2 Technical Description

The dynamic gravimeter is a LaCoste and Romberg straight-line gravimeter (SL-1) supported on a 3-axis platform. The meter and platform are controlled by an LSI-11/23 computer with software in EPROM and a 9-track tape drive for data logging.

The meter is highly damped and can operate even with large short-term accelerations superimposed on gravity. To drive the beam from the centre to one stop requires an impulse of 450,000 mGal sec (ie. a 1-g acceleration for 1/2 sec or a 4500-mGal change for 100 sec). This corresponds to a velocity change of 4.5 m/sec, such as when an aircraft starts a climb of 800 feet/min. To track changes in gravity, the spring tension can be slewed at up to 20 mGal/sec. The actual slew rate depends on the response time set in the computer, and if this is not as fast as the gravity change, the difference between spring tension and gravity, or total correction, can become large. With the old analog controller, this could not exceed 50 mGal without loss of data, and Brozena et al. (1986) report that with the IBM PC controller this limit is about 100 mGal. With the LSI-11 controller, we have operated at up to 1000 mGal total correction with an error of about 1/2 percent.

The accuracy of the gravity reading depends on the accuracy of measurement of beam position. Since 1 mv/min is 1 mGal, a reading with our 12 bit A/D at full range (10 volt scale) has a 5 mv error. Taking many samples and averaging reduces the error, as does changing the gain where possible, but it still takes over a minute to get gravity to 1

mGal. With a 16 bit A/D we could reduce this time to 15 sec.

The 3-axis platform is held in position by 3 servo loops, each with a gyro to sense platform rotation and a torque motor to correct for it. The platform is thus held fixed with respect to the gyros. To level the platform, each horizontal axis gyro is precessed with a signal from it's corresponding accelerometer. The amount of accelerometer signal used determines the period of the platform. At short periods, such as 4 minutes, the platform levels quickly but goes off level equally quickly with horizontal accelerations. Longer periods make it less sensitive but mean that the platform takes longer to return to level after severe maneuver such as turns. Level errors result in gravity errors, but these can be corrected by post-processing if the amount of off-level and the horizontal accelerations during the off-level period are known.

Data are logged on a 9-track tape drive, normally every 10 sec. One 600 foot tape holds 2 days of data in standard LaCoste format. This system is relatively heavy and bulky, and data once written are awkward to retrieve. A further problem with this system is that the tape position is lost whenever there is a power failure, and the only way to ensure no data are overwritten is to mount a new tape after each power interruption.

7.3 Valliant's Development Plan

H. D. Valliant in early 1986 initiated a plan to upgrade the hardware and software for airborne measurements. He also wanted to make the software understandable and easier to modify in the field. A summary of his plans follows:

- 1) To implement changes required for air operations: Set up a team of three people, review controller hardware and software, off-line processing techniques, GPS, etc. Develop complete understanding of software, hardware, theory and operating procedures.
- 2) To increase processing speed: Replace the existing LSI-11/23 with a 11/73 and floating point unit for a threefold (+)

increase in processing speed.

- 3) To allow field changes to software for flexibility, fine tuning, link to autopilot, etc.: Run the controller program in RAM under the operating system RSX.
- 4) To support an operating system: Replace the 9-track tape drive with a 40 Mbyte hard disk and floppy drive. Store airborne data on disk.
- 5) To facilitate operator interaction: Replace the old hard copy terminal and multi-channel strip chart with a graphics terminal and printer combination.
- 6) To support RAM-based operation: Install an uninterruptible power supply to cover up to 60 sec power outages.
- 7) For clarity and to facilitate program development in field: Rewrite and document the control program.
- 8) To reduce weight: Remove the 60-hz circuits in the controller originally used to drive the conventional chopper motor.
- 9) To reduce noise: Replace the current A/D card in the CPU chassis with a converter outside the computer chassis on the mother board next to the cable socket.
- 10) To improve reliability and to support 9) above: Make a printed circuit mother board for controller cards.

7.4 Implementation of Valliant's Plans

The team Valliant formed consisted of himself, R. Beach and N. Courtier. All existing software was collected on the instrumentation group's minicomputer but before any familiarization started Valliant resigned. A decision was made to purchase some or all of the required

hardware as it was felt that the upgrades required for airborne work would also provide a significant improvement for sea surveys. Installation of acquired hardware is scheduled for July 1987.

Hardware acquired:

- a) Ruggedized LSI card cage, 40-Mbyte Winchester disk and floppy. A 19-inch wide by 20-inch deep chassis was built by our machine shop and components have been partially installed.
- b) Ruggedized VT220 terminal with graphics.
- c) LSI 11/73 with license to operate RSX.
- d) Transport case for electronics.
- e) 128 Kbyte memory board.

Hardware outstanding:

- a) Uninterruptible power supply
- b) Printer plotter for controller.
- c) Hard disk/floppy for off-line computer.
- d) Video terminal and printer for off-line computer.

7.5 Recent Work

SL-1 was set up in the 3-axis configuration in room 10 to observe the response of the platform at a period of 84 minutes. Oscillations in the platform were observed up to 60 arc-sec amplitude. Changing gyros made no improvement. No problems could be detected in the electronics, amplifiers or in the A/D and D/A converters. Possible causes could

include bad bearings or compensation maladjustment. It was generally felt that the problem was of recent origin although there is a possibility that it is due to lack of resolution in the A/D conversion process and has always existed. Noise from the heater circuits was significantly reduced by the introduction of zero-crossing relays.

7.6 Recommended Work and Hardware Improvements.

L&R strongly suggest that the meter seals be replaced, with the current seals the meter could be almost irreparably damaged if inverted during transit by air. A new CRO card and improved associated wiring are available. As we need to change the bearings we should consider replacing the seals (\$16,000 [US]) and possibly the CRO card (\$400 [US]) at the same time. If the bearings are changed the meter should be run through L&R's test machines to set up the platform constants.

A short list of possible work elements arranged according to increasing effort with the general objective of achieving an operational airborne gravimetry system is presented below.

	Est. Time
LAB TESTS	(Person-Months)
1. Familiarization with system. (Will require some documentation of existing software)	3
2. Evaluate platform stability.	1
3. Purchase outstanding hardware.	0
DYNAMIC (ROAD) TESTS	
1. Package new computer for controller.	2
2. Reconfigure controller software.	3
3. Replace gravimeter seals and carry out other L&R recommendations. (Requires return to L&R)	2
4. Carry out tests and report.	3
DYNAMIC (AIR) TESTS	
1. Rewrite controller software.	6
2. Integrate navigation and environmental data into	

on-line data file.	3
3. Write and evaluate prototype off-line data processing software (contract?).	6
4. Carry out tests and report.	3

Installation of an RSX operating system will make it much easier to install modifications for airborne operation, such as increasing the data logging rate and interfacing to a GPS receiver. Further desirable upgrades, not included above, are to acquire a 16-bit A/D converter and to install rotation sensors on the gimbal axes. The converter will increase the accuracy of gravity and accelerometer readings and preferably should be installed inside the gravimeter to minimize noise. The rotation sensors will be needed if we are to use full accuracy of GPS. A synchro mounted on each axis would do the job, as would an accurate potentiometer. Synchros may be more accurate, but pots would be less bulky and easier to interface.

8.0 NAE Flight Test Facilities Pertinent to Airborne Gravity

Discussions were held with Dr Barry Leach regarding the technical feasibility of airborne gravity measurements using NAE flight facilities. The NAE's Convair 580 is an extensively equipped flight research aircraft which is probably as suitable as NRL's P3-A Orion aircraft for this work. The following notes regarding the autopilot and the radar altimeter used in this aircraft are based on comments and a personal note from Dr. Leach.

8.1 Sperry SP-20 Autopilot

This system represents very old technology. It includes it's own built-in aneroid barometer and expects a static pressure source to use for height hold mode. No information is contained in the SP-20 manuals concerning frequency response or time response of the system. The system is known to be highly nonlinear so that, for example, the behaviour found for one set of flight conditions cannot be used to predict the

behaviour under other conditions. There is no provision for direct inputting of a signal from a radar or other altimeter source. The autopilot does not work very well in the heading-hold mode according to pilots' reports. NAE is seriously looking at a more modern system to replace the SP-20 and this presumably would have provision for external input, making it more suitable for airborne gravimetry work.

8.2 Honeywell AN/APN-194(V) Radar Altimeter.

This is a specially modified unit being used for the Spotlight Search and Rescue (SAR) Project, but the system should be available for airborne gravimetry if required. Both the serial digital output (with 1-foot resolution) and the standard DC voltage signal output (for modern autopilots) could be useful. Dr. Leach believes that this radar altimeter could provide a very adequate height hold reference for the autopilot.

8.3 Pressure Altimeter.

Digital static pressure and outside air temperature are presently provided in the CV-580 from which barometric height could be computed. Although nothing comparable to the sophisticated boom/pitot system used on NRL's Orion is available, NAE does have extensive experience in the lab with swivelling statics and air data booms and it is likely that something could be devised in the way of a pressure port which would avoid the bow shock and turbulence produced by the aircraft.

8.4 General Comments

Although certain areas of the aircraft are committed to existing equipment (the approximate center of motion is occupied) tracks are provided in the floor of the fuselage, on each side of the longitudinal center line, for securing new test apparatus. It is believed that the Gravity Division's Air/Sea Gravimeter would fit nicely on one side, about ten feet from the center of motion.

The current outlook suggests that flying time would not be available

on this aircraft until December 1987. Quite apart from actual gravity measurements it is clear that NAE have a long standing interest in many of the basic flight problems which are important to successful airborne gravity surveys. For example: optimum auto pilot performance, precise altitude determination using GPS and/or some form of altimetry, and flight motion data for the lower frequencies. These are all areas where it would be beneficial to gather data before launching an actual gravimeter test.

9.0 Conclusions

The principal problem in the measurement of gravity from an aircraft is in correcting the gravimeter response for the vertical motion of the aircraft. Figures 20 and 21 show the maximum permissible error in this correction under given conditions of motion spectra, grid width, aircraft speed and allowable error in gravity determination. The permissible rms error in height refers to the difference between true height and measured height and the assumption is made that this difference has the same power spectral density (PSD) function shape as the true height variations. The three curves represent three height-variation PSD models: (1) a pure sinusoid with a period equal to twice the grid width (the worst case); (2) a 'white' spectrum shape (ie. PSD is a constant) and (3) the same spectrum shape as that shown in Figure 11 which is based on actual NRL flight data observed under 'turbulent' conditions.

The results are presented in Figure 20 as a function of grid width, assuming an aircraft speed of 300 km/hr, and in Figure 21 as a function of aircraft speed assuming a grid width of 5 km. The figures illustrate that the performance of a particular airborne gravity system can be estimated if the rms height error within the gravity pass band can be estimated, without too much regard for the spectral properties of the height variations. In particular, for aircraft speeds less than about 350 km/hr and a grid width of 5 km the permissible rms error in height determination is approximately 1 cm. This is less than the contribution of instrumental errors alone to vertical positioning by GPS (1.5 cm) and

much less than the 5 cm considered feasible overall by Mader et al. (1986) on the basis of flight tests. On the other hand it is just within the 1 cm height resolution claimed by Brozena et al. (1986) using a radar altimeter over water.

On the basis of the above it is reasonable to conclude that fixed-wing airborne gravity to 5-km resolution and 2-mGal accuracy, independent of ground support such as a dense network of laser targets, is probably just feasible over the sea and probably not generally feasible over land using current and anticipated technology, unless it is economic to add extra flight time. If gravity measurements were made during the course of a magnetic survey, for example, with 1-km line spacing to estimate a 5-km gravity grid, errors would be reduced by more than half. Measurements over the sea on the other hand would benefit from a mix of GPS, inertial navigation and radar and pressure altimetry, using strategies of the type currently being developed in the U.S. by the NRL, and might not require significant extra flight time.

Given that airborne gravity is feasible (at least over water) it is likely that sooner or later this Division will be called upon to play an active or advisory role in such measurements, either with respect to operations in Canada or in third world countries. Some of our immediate technical options in this regard are discussed below.

The Geophysics Division, in collaboration with NAE, has the potential for pursuing successfully a development program along the lines of that of NRL and the basis for such an aggressive program could be determined from details given in this report and from file data (see for example Valliant's recommendations w.r.t an airborne metrology group). At the other extreme we believe that our minimum-effort option is to: (1) maintain the Air/Sea Gravimeter System at state-of-the-art performance capability; (2) maintain staff awareness w.r.t current technology. Some objectives consistent with this latter option have already been proposed for the current fiscal year: (Milestones 6.4.1.06: 13-17). Other desirable steps would be to invite Brozena to address a Division group (perhaps including representatives from the aeromagnetic section and from NAE) and to consider some form of collaboration with NRL and/or NAE to advance our knowledge with respect to dynamic GPS and other airborne position and motion measurements. Further in this connection we would

like to make the point that given an adequate model of the airborne-gravity problem (incorporating: expected motion statistics, GPS performance statistics including multipath effects, gravimeter system response, and surface gravity statistics) much could be done by way of computer simulation before fully instrumented flight trials would be needed. Surprisingly this aspect seems to have been overlooked by proponents of airborne gravity in Canada. Experience with such a model would serve to reveal deficiencies in instrumentation, data or techniques much more economically than calibration flights of the entire air-gravity system. Where model data are found lacking, for example in the areas of dynamic positioning by GPS or the behaviour of radar or pressure altimeters, we could benefit from sharing flight costs with other groups interested in these particular problems.

Finally it should be mentioned that an important omission from this study has been the subject of airborne gravity gradiometry. Like inertial gravimetry, gradiometry employs optimal estimation techniques to determine gravity anomalies at the surface, in this case from gravity gradient measurements. Gradiometry may offer important advantages in regard to tolerable vehicle motion and should be considered in future planning.

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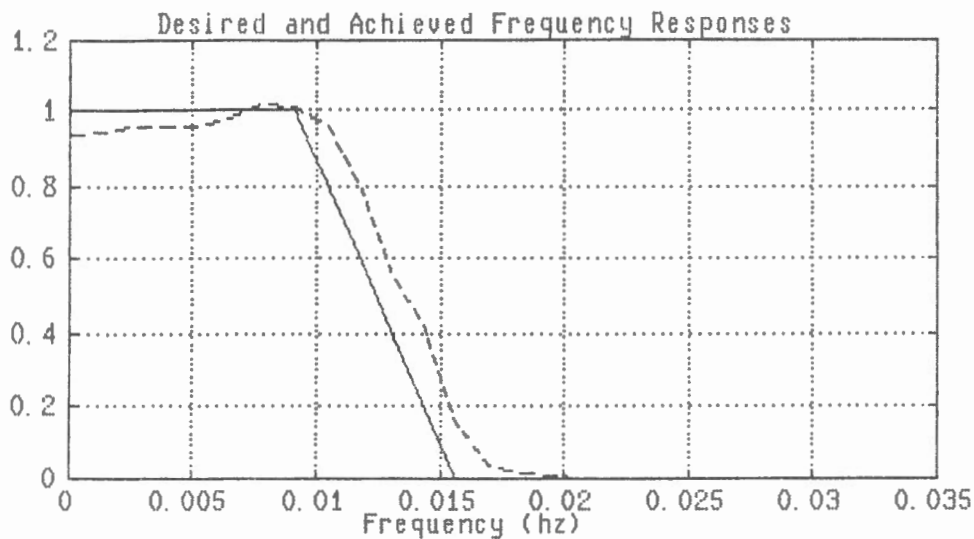


Figure 1.
Desired (solid trace) and Achieved Gravity Filter Responses.

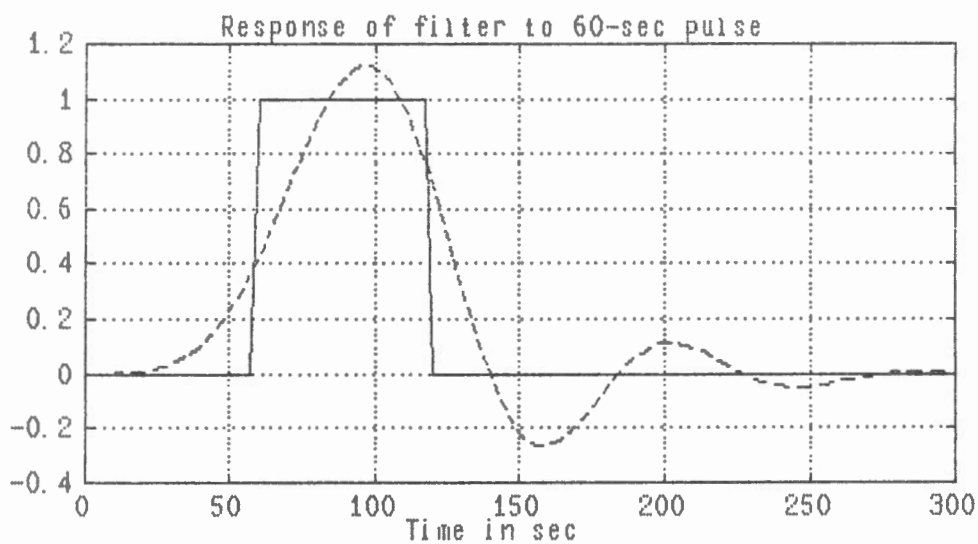


Figure 2
The Response (dashed trace) of the Gravity Filter to a Unit-Pulse Gravity Change Representing one grid width. (The filter response has been corrected for a delay of 66 seconds)

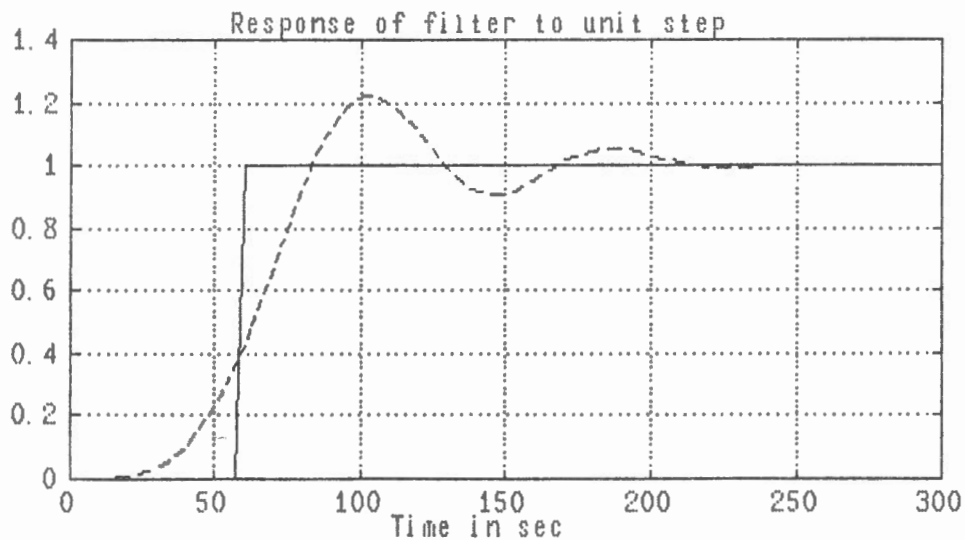


Figure 3
The Response (dashed trace) of the Gravity Filter to a Unit-Step Gravity Change. (Corrected for a delay of 66 seconds)

MODEL GRAVITY FIELD



Figure 4 Contoured Gravity Over a Region Located in Northern Ontario (Nagy, 1986). Profiles A and B were selected from this region and used as input for the gravity filter described in the text. The margins of this figure define an area of 125 x 125 km.

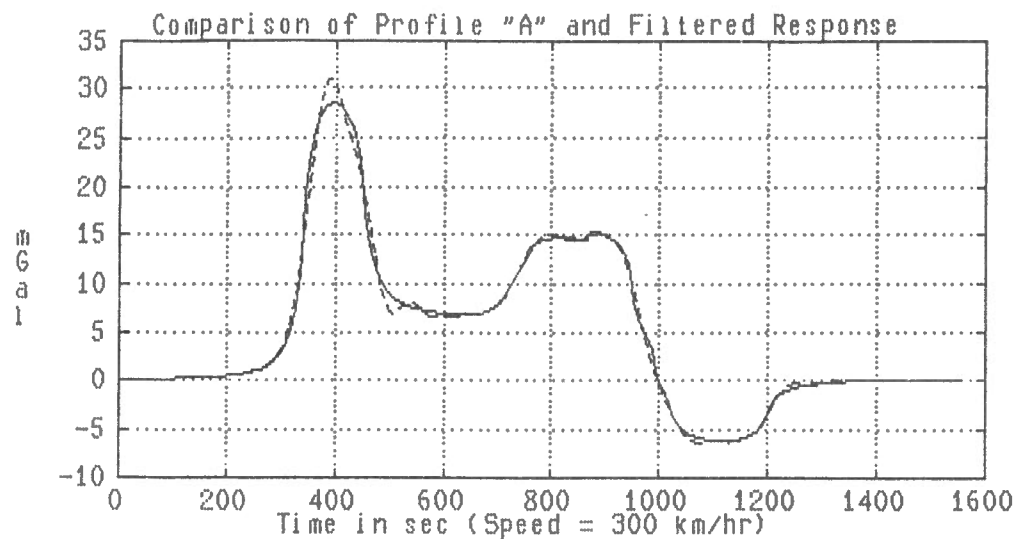


Figure 5
 Comparison of Profile "A"
 (solid trace) and Filtered
 Response. (Corrected for
 delay)

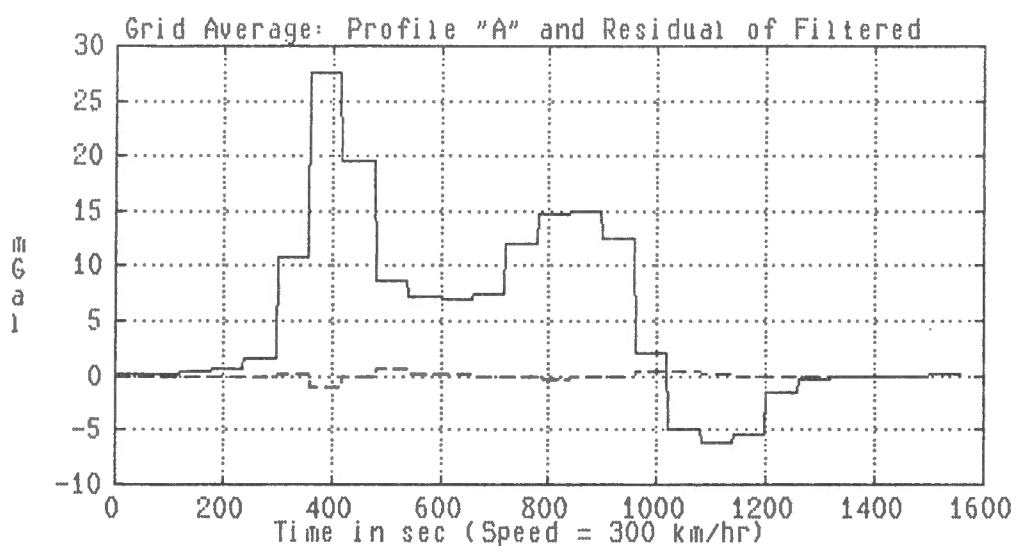


Figure 6
 Gravity on Profile "A" as
 a 5-km-Grid Average (solid)
 and Residual After Subtracting
 Filtered Response - (dashed).

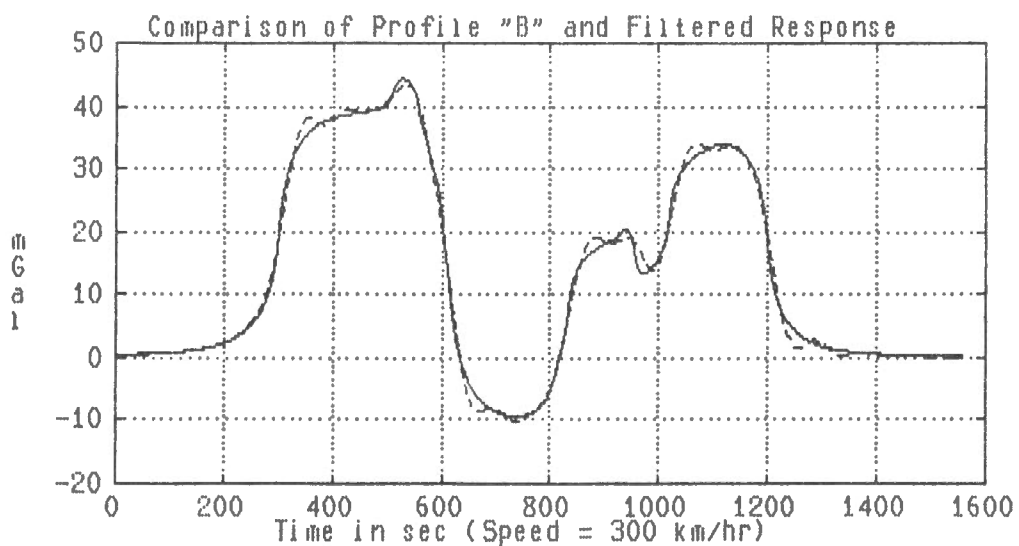


Figure 7
 Comparison of Profile "B"
 (solid trace) and Filtered
 Response. (Corrected for
 delay)

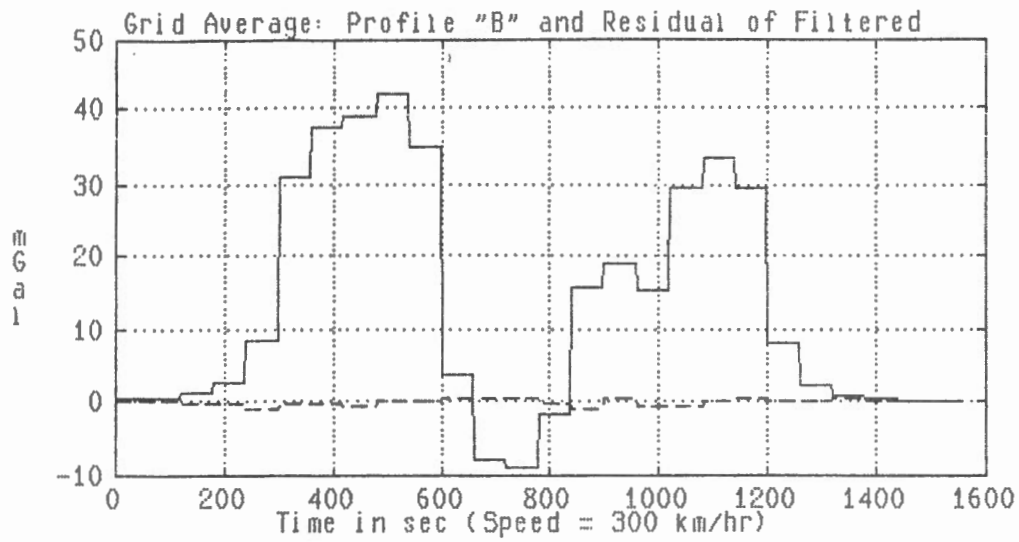


Figure 8

Gravity on Profile "B" as a 5-km-Grid Average (solid) and Residual After Subtracting Filtered Response (dashed).

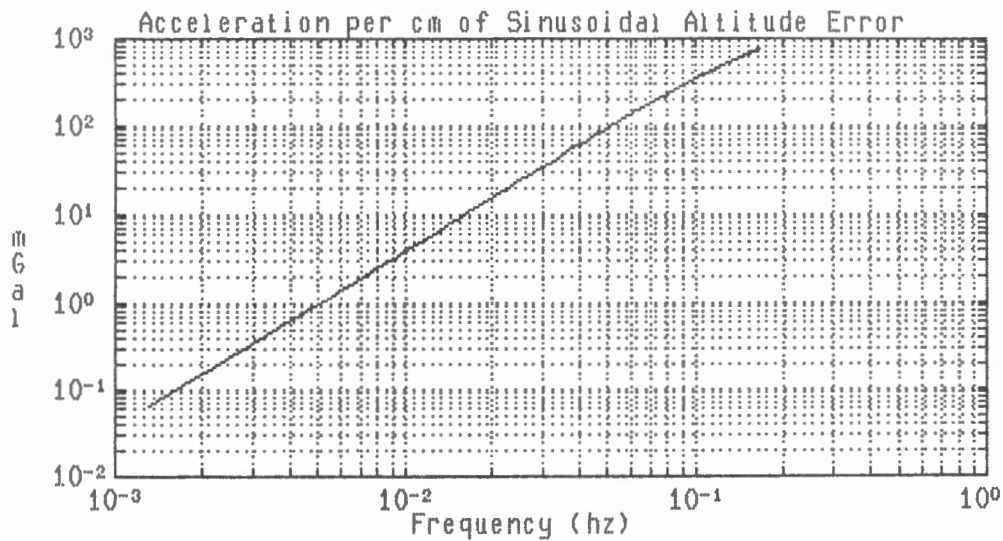


Figure 9

Acceleration per Cm of Sinusoidal Vertical Motion

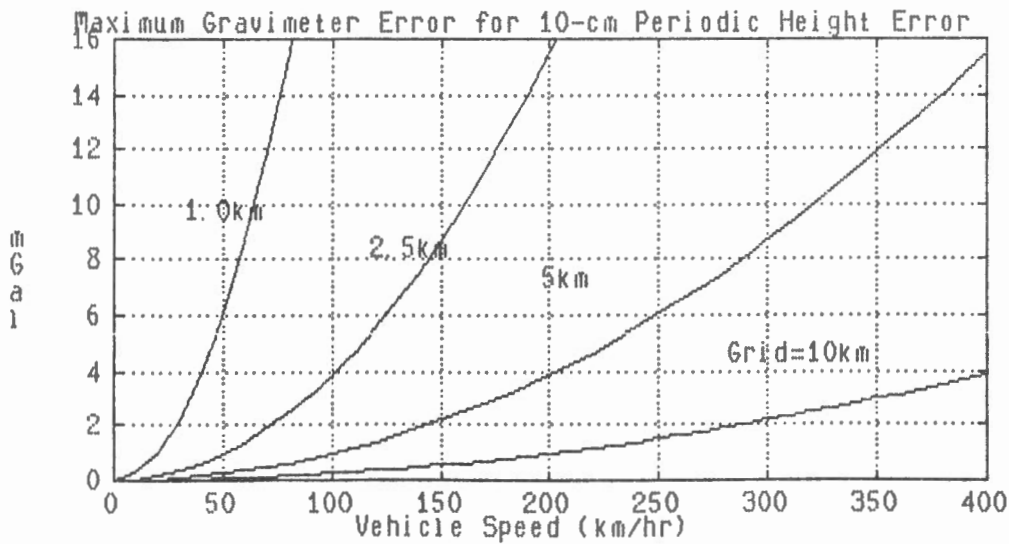


Figure 10

Maximum Gravimeter Error for 10-cm Periodic Height Error.

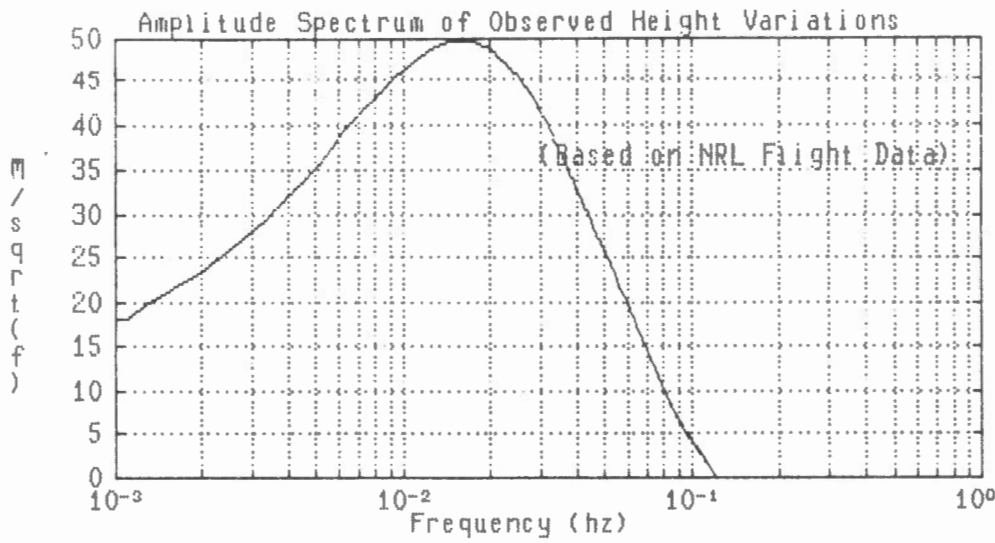


Figure 11

Amplitude Spectra of Height Variations Observed During a P3-A Turbulent Flight After the Passage of a Cold Front (Brozena, 1987). (According to Brozena the motion of the aircraft felt extreme)

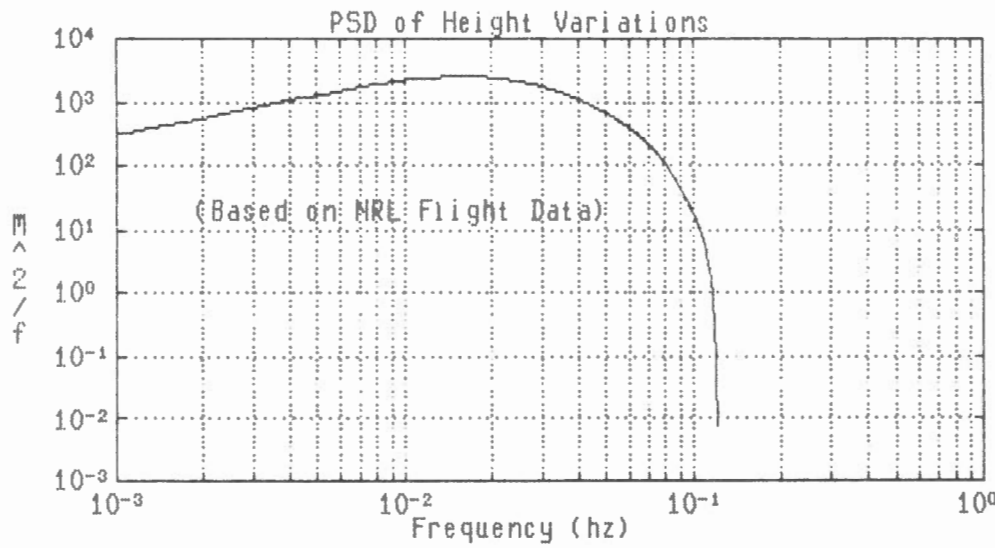


Figure 12

Equivalent Power Spectrum of the Data Represented by Figure 11.

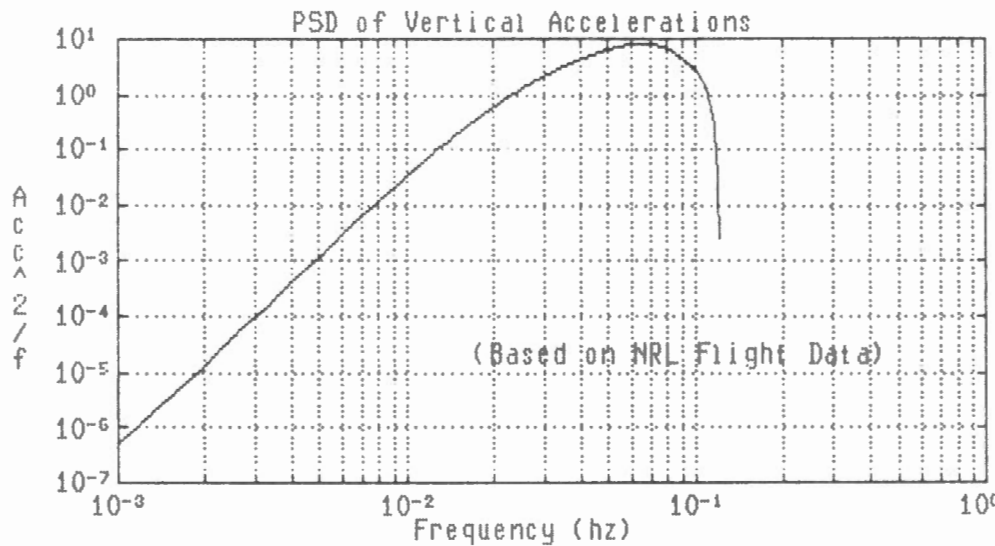


Figure 13

Acceleration Power Spectrum of the Data Represented by Figure 11.

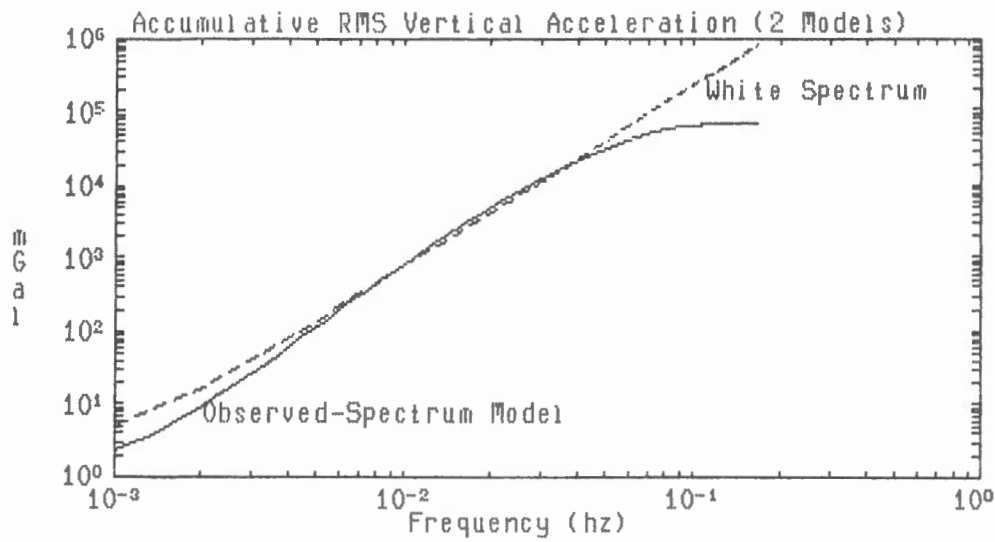


Figure 14

Accumulative RMS Vertical Acceleration (solid trace) For the Data Represented by Figure 11. Also Shown is the Accumulative Acceleration for a White PSD with Approximately the Same Power in the Gravity Passband (dashed trace).

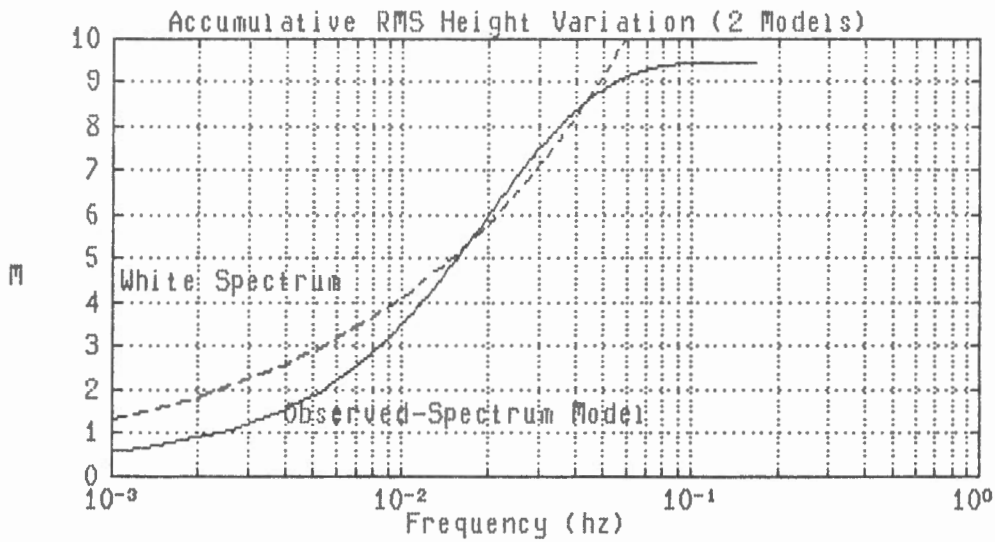


Figure 15

The Data of Figure 14 Expressed as Equivalent RMS Height Variation.

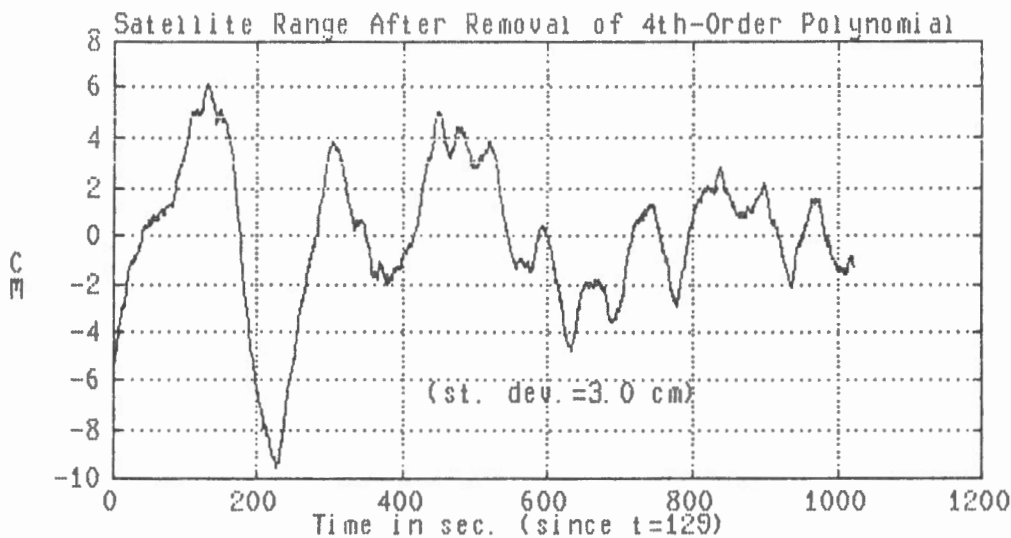


Figure 16

Satellite Range Measurement After Removal of a 4th-Order Polynomial.

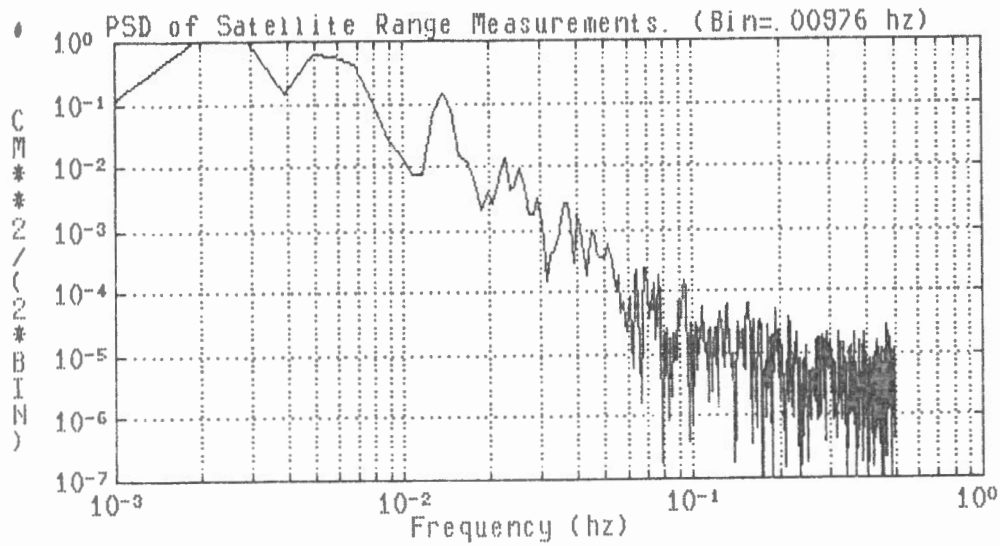


Figure 17

PSD of Satellite Range Measurement Shown in Figure 16.

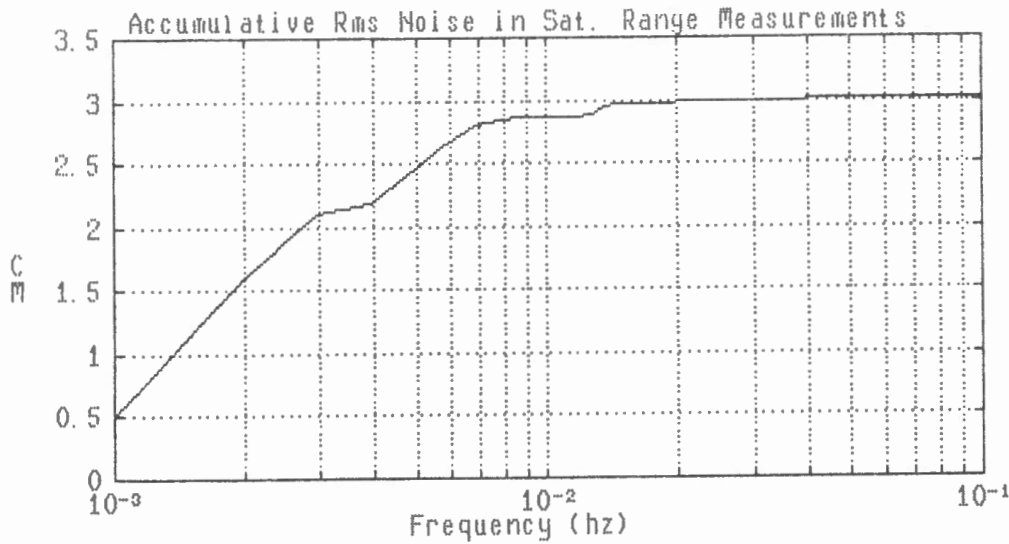


Figure 18

Accumulative RMS Noise in Satellite Range Measurement Shown in Figure 16.

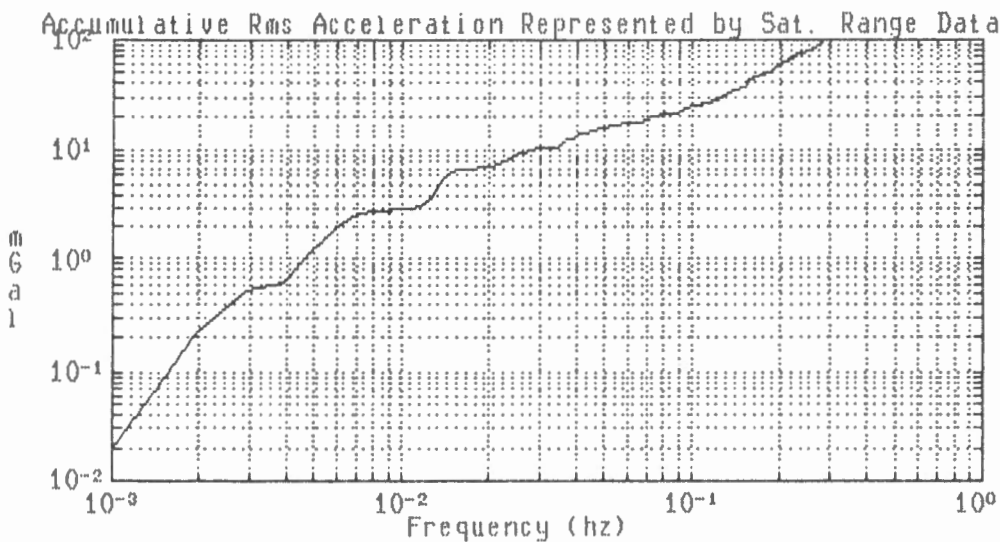


Figure 19

Accumulative RMS Acceleration Represented by Ranging Data Shown in Figure 17.

PERMISSIBLE RMS ERROR IN HEIGHT FOR 2-MGAL GRAVITY ERROR

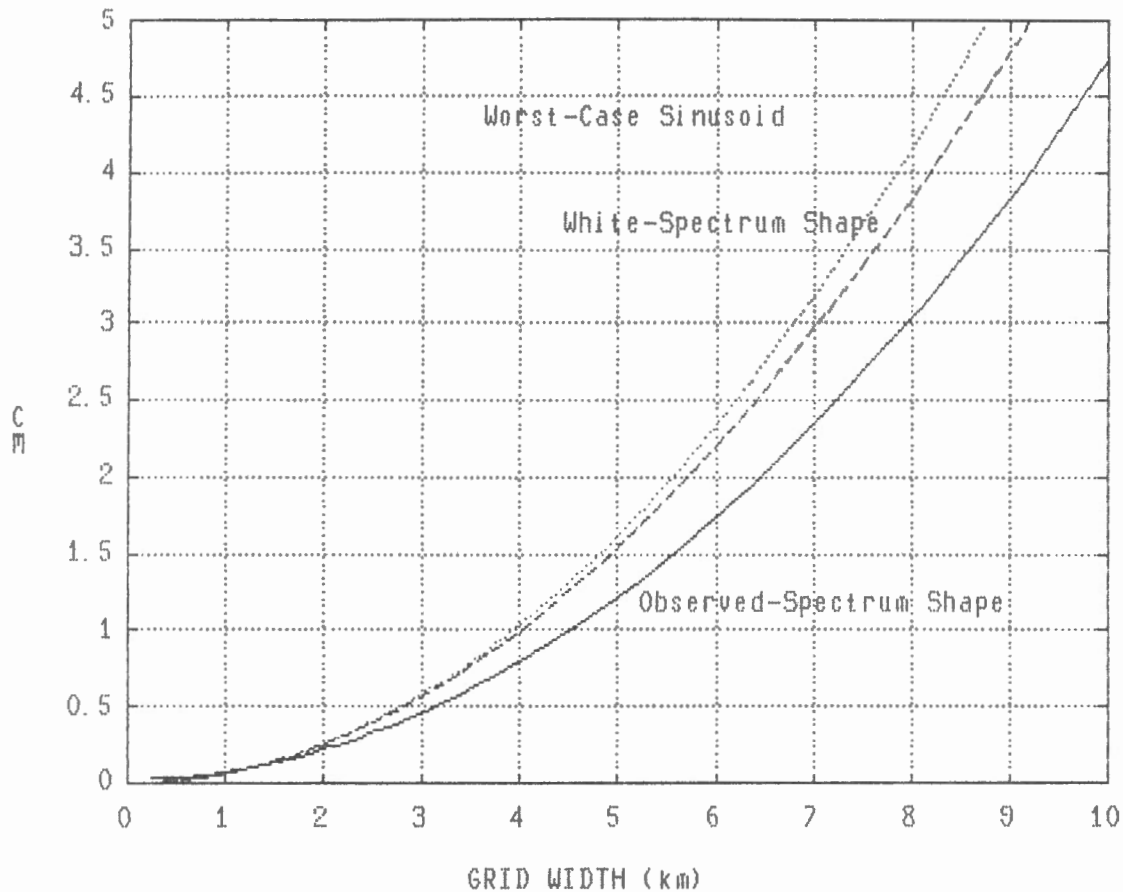


Figure 20

Maximum Permissible RMS Error in Height Determination for an Aircraft Speed of 300 km/hr.

PERMISSIBLE RMS ERROR IN HEIGHT FOR 2-MGAL GRAVITY ERROR

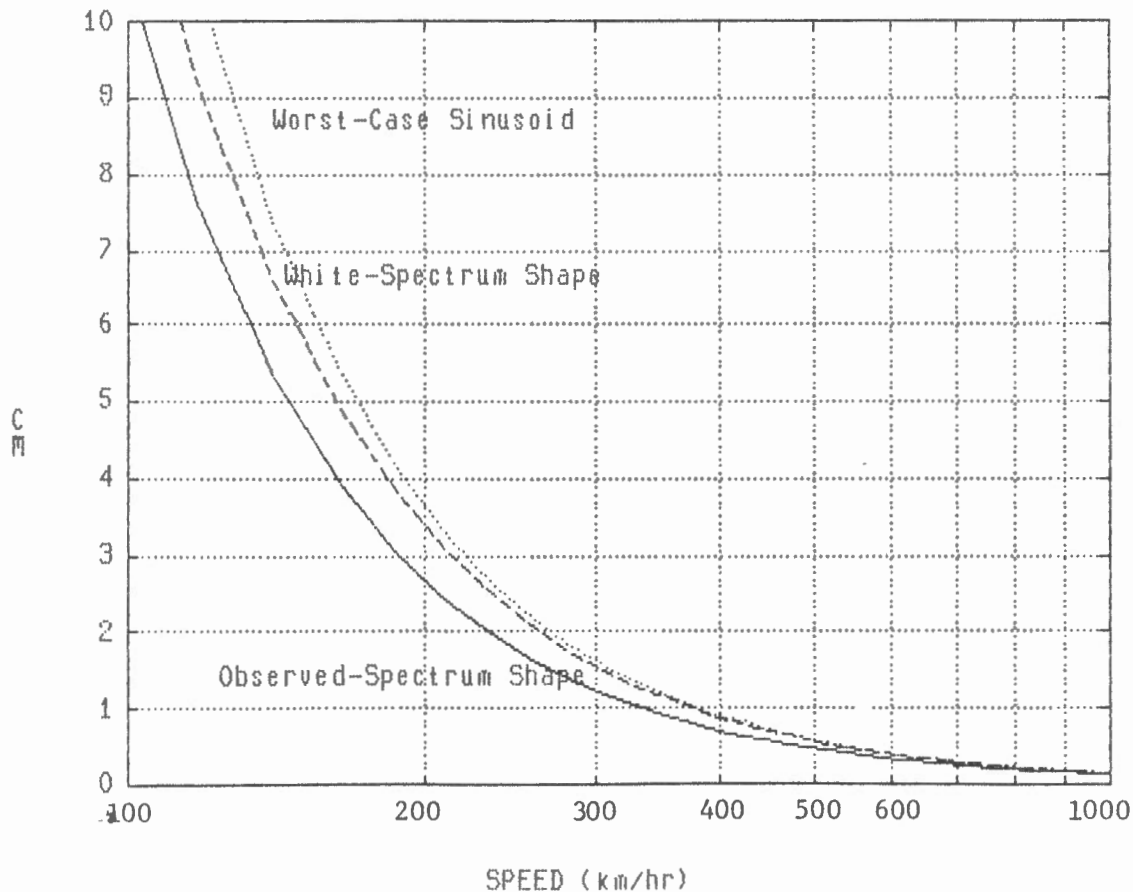


Figure 21

Maximum Permissible RMS Error in Height Determination for a Grid Width of 5 km.

