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Changes in Absolute Gravity of Environmental Origin

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Mike
1. I think you should speak to Luc about the style of abstracts.
2. A detailed look at a problem with inconclusive results.

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Changes in Absolute Gravity

of Environmental Origin

ABSTRACT

Changes in the absolute determination of gravity at the Bureau Internationale des Poids et Mesures (BIPM) in Sèvres (near Paris), France over a period of four years (August 1967 - July 1971) are analyzed in terms of correlation with changes of other environmental variables such as local atmospheric pressure, water table level, variation of latitude and Earth rotation rate. The present study proves a significant correlation with atmospheric pressure (-0.7 ± 0.5 μGal/mb) and with UTO length of day variation (-10. ± 5. μGal/ms ,where the error bounds are 2σ units). In both cases the correlation significance is estimated by a variance-ratio F test where the actually published standard errors and deduced covariances have been taken into account. The same test is used to prove the non-correlation with latitude variation (due to polar motion) and water table level changes (River Seine water level). The results are discussed and elastic homogeneous incompressible earth hypotheses are envisaged.

This is not a good abstract.

Introduction

A total of seventeen absolute determinations of gravity made by A. Sakuma (1971a,b) in the years 1967-1971 with his permanent instrument at BIPM are used here in conjunction with the corresponding values of four environmental variables i.e. the variation of latitude, the River Seine water level, the local atmospheric pressure and the UTO length of day. The environmental variables are average values over the time taken for each gravity determination which often amounts to several weeks (cf table 1 first column). These variables have been chosen because of their experimentally known or theoretically suspected influence on gravity. It was not possible to include all the variables one could think of, first because the data are difficult to obtain and secondly because our sample of gravity values is small, thus limiting the number of possible factors one can analyze at the same time.

The principle of the method consists of working with the differences of the measurements and of the corresponding variables, fitting them by least squares in a linear model. The advantage of such a procedure is to minimize the effect of a sudden permanent change in gravity (for example due to ground subsidence). The gravity dependence on a variable can be tested by comparing the variances obtained from two least squares fits; one including the variable, the other one excluding it. The actual comparison is made via a variance-ratio F test, as explained later.

Before entering into the mathematical details of the method, I first would like to explain how the various data are prepared.

i) An Averaged instantaneous latitude $\bar{\phi}$ is computed using the pole coordinates published by the Bureau Internationale de l'Heure (BIH) according to the following formula (Mueller, 1969; p. 87):

$$\overline{\Phi} = \phi_{CIO} + \bar{x}_p \cos \Lambda - \bar{y}_p \sin \Lambda , \qquad (1)$$

where ϕ_{CIO} is a fixed latitude for the station, as referred to the Conventional International Origin (CIO), (Mueller, 1969; p. 351). Λ is the longitude of the station, positive Eastwards ($\Lambda = 2°13'$). $\bar{x}_p$ and $\bar{y}_p$ are the averaged raw values of the pole coordinates over each period of gravity measurements (BIH, 1967-1971; table 6).

ii) An averaged River Seine water level $\bar{S}$ is computed for each gravity measurement period from daily measurements in two locations, one upstream (Pont de Garigliano) and one downstream (Pont de Suresnes). The averaged water level at Pont de Sèvres, the closest to BIPM, is linearly interpolated from these two gauge stations.

iii) An averaged local atmospheric pressure $\bar{P}$ is computed from three hour interval pressure measurements made at two meteorological stations, Villacoublay 7 km South of BIPM and Montsouris 9 km East of BIPM. The pressure at these two stations, approximately 11 km apart, are well correlated and show an average systematic difference of 12.7 mb (1 mb = 100 Pa) mainly due to the height difference (approximately 100 m) between the two stations. I also computed a set of daily averages using only the 6, 9, 12, 15 and 18 hours measurements every day. They prove to be systematically smaller by 0.1 mb on the average and introduce no significant changes in my results.

iv) Finally, the averaged UT0 length of day $\bar{l}$ for each gravity measurement period is computed by the following formula:

$$\bar{l} = 86400.002592 + \left[(UT0\text{-}UTC)_i - (UT0\text{-}UTC)_f\right]/n, \qquad (2)$$

where 86400.002592 is the measure in atomic time seconds of 24 hours UTC (Universal Time Coordinated), n the number of days between the initial value (i) and the final value (f) of (UT0-UTC), corresponding to a gravity measurement period. (UT0-UTC) at the initial and final date is evaluated by (Mueller, 1969; p. 164):

$$(UT0\text{-}UTC)_i = (x_p^i \sin\Lambda + y_p^i \cos\Lambda)\ \mathrm{tg}\phi + (UT1\text{-}UTC)_i, \qquad (3)$$

where i has to be replaced by f for the final date. x_p, y_p, (UT1-UTC) are the raw values in the annual reports of the BIH (1967 1971), table 6 and also table 4 for 1967. ϕ is the BIPM latitude (ϕ = 48° 50' North) and Λ as in (1) above.

The data utilized in the present work are summarized in Table 1.

Basic Theory and Results

i) Method 1:

Before applying the least squares fit procedure, the data in Table 1 are modified so that only the measurement differences are fitted to a linear mathematical model of the following type:

$$\Delta\hat{g}_i = \hat{g}_{i+1} - \hat{g}_i = \hat{a}(\bar{\Phi}_{i+1} - \bar{\Phi}_i) + \hat{b}(\bar{S}_{i+1} - \bar{S}_i) + \hat{c}(\bar{P}_{i+1} - \bar{P}_i) + \hat{d}(\bar{l}_{i+1} - \bar{l}_i), \qquad (4)$$

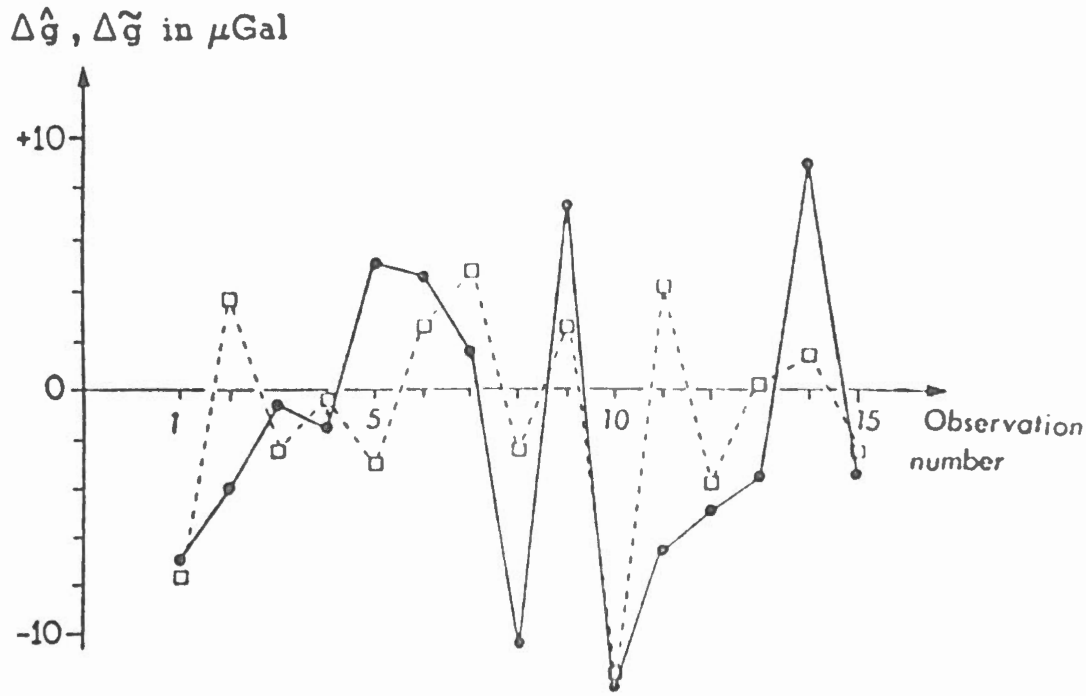
with index i (i=1, ..., 16) referencing values in the i^{th} line of table 1; symbols ϕ, S, P and l as previously defined in the Introduction, and $\hat{a}$, $\hat{b}$, $\hat{c}$,

Table 1: Absolute gravity value for the years 1967-1971 at BIPM and the corresponding averaged latitude, River Seine water level, atmospheric pressure and UTO length of day (l.o.d).

Date	Gravity ($\pm 2\sigma$) (1)	Latitude ($\bar{\phi}-\phi_{CIO}$) (2)	Water level (3)	Pressure (4)	l.o.d. (5)
Aug. Sept. 1967	62 ± 13	- 3	.351	1000.5	1.97
Apr. 1968	55 ± 25	- 9	.272	999.7	2.79
Aug. Sept. 1968	51 ± 19	+ 53	.362	999.1	2.45
June July 1969	50.5 ± 8.2	+ 112	.290	1003.4	2.40
Aug. Sept. 1969	49.0 ± 5.4	+ 109	.367	1001.8	2.54
Oct. Nov. 1969	54.1 ± 10.6	+ 8	.379	1001.0	2.91
Dec. 1969	58.7 ± 5.8	117	.434	999.5	2.75
Jan. 1970	60.3 ± 9.0	- 165	.444	993.0	2.72
Feb. 1970	50.0 ± 9.1	- 172	1.626	996.1	2.77
Aug. 1970	57.4 ± 2.0	+ 194	.374	1000.4	2.22
Oct. 1970	45.3 ± 2.2	+ 154	.384	1005.6	3.06
Dec. (end) 1970	86.3 ± 9.4	- 46	.369	1001.8	2.57
Jan. 1971	79.6 ± 6.0	- 111	.395	994.8	2.64
Feb. 1971	74.7 ± 9.7	- 195	.366	1005.6	2.31
May 1971	71.0 ± 8.7	- 187	.405	996.1	2.92
June 1971	80.1 ± 6.2	- 79	.431	998.7	2.59
July 1971	76.7 ± 5.5	+ 57	.402	1003.8	2.54

Legend:
(1) 980 925 600 + tabulated value - gravity in μGal at Sèvres A2.
(2) in 0".001.
(3) 26 + tabulated value = River Seine water level in m (above mean sea level).
(4) in millibars.
(5) 86 400 000 + tabulated value - UTO l.o.d. in ms.

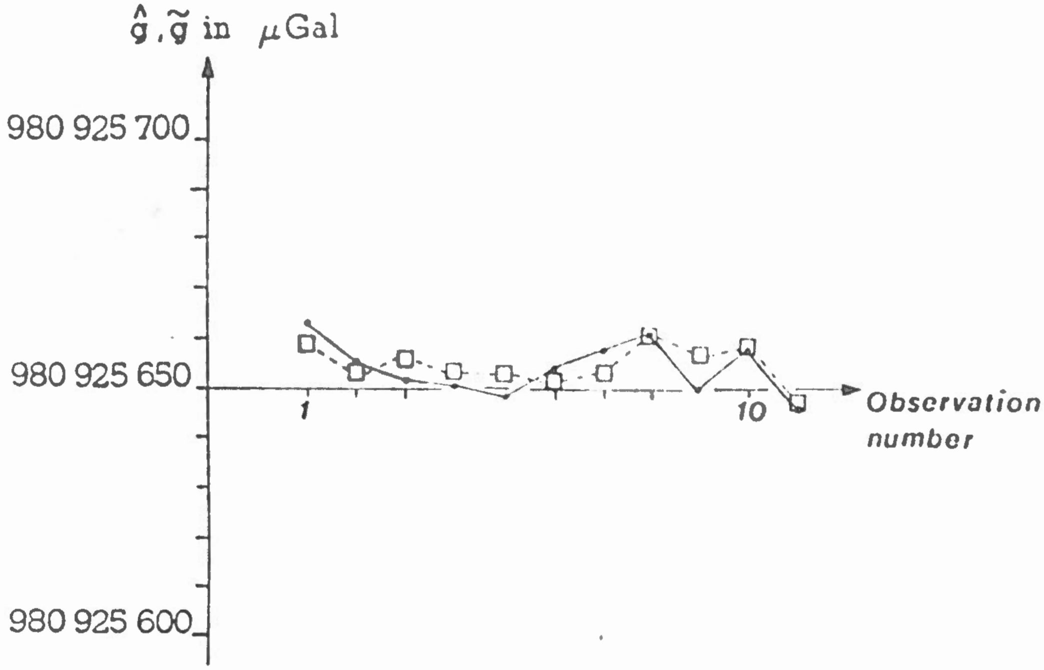
Figure 1: Observed (•) vs least squares compensated (□) absolute gravity differences.



Legend: • represents $\Delta\tilde{g}_i$, i = 1, ..., 15

□ represents $\Delta\hat{g}_i$ as computed by formula (4).

Figure 2: Eleven first observed (•) vs least squares compensated (□) absolute gravity values.



Legend: Plot of $\hat{g}_i$ as computed from formula (11) vs $\tilde{g}_i$ from Table 1 above.

$\hat{d}$ four parameters to be determined by the least squares condition $\hat{V}^T W \hat{V}$ minimum. W is a (16,16) weight matrix and $\hat{V}$ the (16,1) column vector of the residuals $\hat{V}_i = \Delta\hat{g}_i - \Delta\tilde{g}_i$, with $\Delta\tilde{g}_i = \tilde{g}_{i+1}\ \tilde{g}_i$ where $\tilde{g}_i$ is taken from Table 1. The weight matrix W is computed as the inverse of Σ where Σ is the a priori variance-covariance matrix of the observables $\Delta\tilde{g}_i$ (for the geodesy minded reader, the a priori variance factor is chosen equal to unity). The diagonal elements of Σ are the variances:

$$\sigma^2_{\Delta\tilde{g}_i} = \sigma^2_{\tilde{g}_{i+1}} + \sigma^2_{\tilde{g}_i}, \tag{5}$$

where $\sigma_{\tilde{g}_i}$ is the published error (Sakuma, 1971a,b) divided by two; i.e. I consider the overall published accuracy to be of order 2σ which corresponds to a confidence interval of 95%. Furthermore the Δg_i are not independent since $\Delta\tilde{g}_i$ and $\Delta\tilde{g}_{i+1}$ both contain $\tilde{g}_{i+1}$. Assuming the $\tilde{g}_i$ independent, one finds after an easy derivation the following covariances:

$$\sigma_{i\,i+1} = -\sigma^2_{\tilde{g}_{i+1}}, \tag{6}$$

all the other elements of the variance covariance matrix of the observables being zero.

Denoting by A the (16,4) design matrix containing the four environmental variable values as in (4), one obtains the (4,1) parameter vector $\hat{X}$, where $\hat{X}^T = (\hat{a}, \hat{b}, \hat{c}, \hat{d})$, by the following formula (Mikhail, 1976; p. 114):

$$\hat{X} = (A^T W A)^{-1} A^T W \Delta\tilde{g}. \tag{7}$$

The validity of mathematical model (4) can be tested by a confidence interval on the expression $\hat{V}^T \Sigma^{-1} \hat{V} = r$ (sum of squares of the weighted residuals when Σ is diagonal), using the fact that it has a $\chi^2(\nu)$ distribution, thus yielding the following probability statement:

$$P\left[\chi^2_{\alpha/2} \leq r \leq \chi^2_{1-\alpha/2}\right] = 1-\alpha, \tag{8}$$

where χ^2_p is defined by: $p=\int_0^{\chi^2_p} f(x)dx$, $f(x)$ being the probability density function of the χ^2 distribution, and α being a significance threshold (say, $\alpha= 5\%$); $\nu=n-u$ is the number of degrees of freedom, n the number of observations and u the number of parameters.

<u>The first conclusion</u> in the case where all the data are used (n=16) is that mathematical model (4) is inadequate. The confidence interval is : $4.40 \leq r \leq 23.34$ whereas $r = 117.9$, thus outside the allowed interval. The least squares fit can be improved by removing data number 11 which corresponds to a difference of 41 µGal between determination 11 and 12 in Table 1. The interpretation is that such a high difference cannot be explained only in terms of the four environmental variables I consider here.

Now computing the adjustment after rejection of this observation mentioned above (n=15, u=4) shows that <u>mathematical model (4) is adequate</u>; the confidence interval is: $3.8 \leq r \leq 21.9$ whereas $r = 14.57$, thus inside the allowed interval.

The next step is to test the significance of each of the parameters $\hat{a}$, $\hat{b}$, $\hat{c}$ and $\hat{d}$ of mathematical model (4) as summarized in Table 2. First testing the significance of $\hat{b}$, using the variance ratio F test described below, I remove the water level variable $\bar{S}_{i+1}-\bar{S}_i$ in (4) and compute the least squares fit for n=15 and u= 3 yielding r = 16.189 and compare it with the fit for n = 15, u = 4. The variance-ratio F test is based on the assumption that:

$$\rho = (r_1-r_2)(n-u_2) / (r_2 (u_2-u_1)) \quad , \qquad (9)$$

is a particular value of a ρ' random variable having an $F_{u_2-u_1,\ n-u_2}$ Fisher distribution (Hamilton, 1964; p. 139) in the case where mathematical model one (u_1 parameters, where $u_2 > u_1$) is true. This means that the following probability statement holds:

$$P\left\{ \rho \geqq F_{1-\alpha;\ u_2-u_1,\ n-u_2} \right\} = \alpha \qquad (9)\ \text{bis}$$

where α is a significance level, (in practice, say α = 5%). When the two least squares fit have been computed, ρ is calculated by formula (9). If $\rho \geqq F_{1-\alpha;\ u_2-u_1,\ n-u_2}$, the ρ' random variable of which ρ is a particular value cannot be considered to have a Fisher distribution and the hypothesis that mathematical model one is true, is rejected. The u_2-u_1 additional parameters are then considered to be significant.

Intuitively the test is based on the idea that if an additional parameter produces a sufficiently large relative decrease in the r values (i.e. the sum of squares of the weighted residuals when Σ is diagonal), it is significant. Formula (9) bis accounts for both: the number n of observations (as it enters into ρ and into $F_{1-\alpha;\ u_2-u_1,\ n-u_2}$) and the gaussian fluctuations of the random errors. ρ is not affected by the overall scale of random errors, it is a relative value. At a fixed variance-ratio (i.e. $(r_1-r_2)/r_2$) and a fixed number (i.e. u_2-u_1) of additional parameters, the significance test is easier to pass with a large number of observations than with a small one, since ρ is proportional to $(n-u_2)$ and $F_{1-\alpha;\, u_2-u_1,\, n-u_2}$ decreases and tends to a finite positive limit as n tends to infinity. In other words, for small sample of data, the relative decrease in variance (r values) has to be greater than for large sample in order to make a parameter significant.

At last comment about the F test used in the present study has to do with the basic assumption that all observations are gaussian random variables. It is normal practice to make this assumption. When the observation sample is large ($n \geq 100$), it can be checked by a χ^2 test for example. In the present case, the sample is small ($n=17$) and it is not possible to check the gaussian distribution with a reasonable degree of confidence. It is simply assumed to be so and the possibility remains open (although with a small probability) of a failure of the F test due to a non-gaussian distribution of the observations.

Returning to the numerical calculation in the present case ($u_1=3, u_2=4$, $n=15$) $\rho = 1.22 \leq F_{0.95;\, 1,11} = 4.84$, thus parameter $\hat{b}$ in (4) corresponding to the River Seine water level dependence is not significant.

The latitude dependence can be tested after removing variables S and ϕ from (4) and computing a new least squares fit for $n=15$ and $u=2$; I find $r = 16.789$. With $n=15$, $u_1=2$, $u_2=3$, $r_1 = 16.769$ and $r_2 = 16.189$: $\rho=0.43 \leq F_{0.95;\, 1,\, 12}=4.75$ showing that parameter $\hat{a}$ corresponding to the latitude variation due to polar motion in (4) is not significant.

Testing the pressure dependence needs one more calculation with n=15 and u=1, where the only variable is the length of day $\bar{1}$ in (4). I find r = 25.828. Thus with n=15, $u_2=2$, $u_1=1$; $\rho=7.02 \geq F_{0.95;\,1,\,13}=4.67$ and the parameter $\hat{c}$ corresponding to the local atmospheric pressure variation, in (4), is significant.

Length of day dependence is tested using an u=1 least squares fit with atmospheric pressure only. I obtain r = 39.974. For n=15, $u_1=1$, $u_2=2$, $r_1 = 39.974$ and $r_2 = 16.769$: $\rho=18.0 \geq F_{0.95;\,1,\,13}=4.67$ and the parameter $\hat{d}$ corresponding to the length of day variations in (4) is significant. Since the corresponding ρ ($\rho=18.0$) is much greater than for the atmospheric pressure ($\rho=7.02$), one can say that a greater part of absolute gravity variations Δg is accounted for by length of day variation than by pressure variation.

The present study leads to the following formula where absolute gravity variations Δg (in μGal) are expressed as a linear combination of local atmospheric pressure variations ΔP (in millibars) and UTO length of day variations $\Delta 1$ (in milliseconds) (error bounds are $\pm$ 2σ values):

$$\boxed{\Delta g = (-0.7 \pm 0.5)\ \Delta P + (-10. \pm 5.)\Delta\, 1.} \qquad (10)$$

Figure 1 shows the least squares compensated $\Delta\hat{g}_i$ vs the observed $\Delta\tilde{g}_i$.

Table 2: Summary of significance tests

ν	φ	S	P	ℓ	r	ρ	Comment
11	+	+	+	+	14.573		
12	+	–	+	+	16.189	1.22	S not significant
13	–	–	+	+	16.769	0.43	φ not significant
14	–	–	–	+	25.828	7.02	P significant
14	–	–	+	–	39.974		
13	–	–	+	+	16.769	18.0	ℓ significant

Legend : r and ρ as defined in (8) and (9) above

+: parameter included in the least squares fit

–: parameter not included in the least squares fit.

ii) Method 2:

One more step in the present analysis can be done by using two new mathematical models acting on the measurements themselves (cf Table 1) instead of only their differences as in (4). Since a gravity difference had to be eliminated, it creates two independent subsets of observations. Subset 1 comprises the 11 first observations and subset 2 includes the 6 remaining ones. For each of the subsets, the mathematical models are respectively:

$$\hat{g}_i = \hat{m}_1 + \hat{e}_1 \, (\bar{P}_i - P_o) + \hat{f}_1 \, (\bar{l}_i - l_o) \quad (i=1, \ldots, 11), \qquad (11)$$

$$\hat{g}_i = \hat{m}_2 + \hat{e}_2 \, (\bar{P}_i - P_o) + \hat{f}_2 \, (\bar{l}_i - l_o) \quad (i=12, \ldots, 17), \qquad (12)$$

where P_o = 1000 mb and l_o =86 400 000 ms, $\bar{l}_i$, $\bar{P}_i$ and $\hat{g}_i$ being expressed in ms, mb and μGal respectively. $\hat{m}_j$, $\hat{e}_j$, $\hat{f}_j$ (j=1,2) are the parameters to be determined by the least squares conditions $\hat{V}_j^T W_j \hat{V}_j$ minimum (j=1,2) where $\hat{V}_j$ (j=1,2) are the (11,1) and (6,1) residuals vector for mathematical models (11) and (12) respectively. In these two cases, the weight matrices W_j(j=1,2) are diagonal (uncorrelated observations in contradistinction to the differences which are correlated). The results are in agreement with formula (10) for the first subset. The least squares compensated gravity values $\hat{g}_i$ (i=1, ..., 11) are plotted in comparison to the observed values $\hat{g}_i$ (i=1, .., 11) in Figure 2. Subset 2, on the contrary, shows no significant pressure and length of day dependence. This is due to the small number of observations (only 6) and various causes which would be negligible in a larger sample.

Discussion:

The value (-0.7 ± 0.5) μGal/mb is in good agreement with previously estimated factors of -0.35 μGal/mb by Warburton and Goodkind (1977) and -0.45 μGal/mb by Sakuma (1971c). Regarding the other factor (-10.± 0.5)μGal/ms, various Earth deformation models were investigated (Pariisky 1978, Molodenskiy et al. 1975) to account for both, the length of day and the absolute gravity changes. Since there exists a very strong correlation between length of day variations and angular momentum transfer between the atmosphere and the solid Earth (Barnes et al., 1982), there is probably some interference between Earth deformation effects and gravitational attraction changes of the atmosphere on a world-wide scale which could account for the unexpected strong correlation that I find.

Another point is that according to the formulae given below for an elastic incompressible Earth, there should be a distinguishable latitude variation dependence of about 19 μGal/arcsec at ϕ=45°. The present study, as explained above, does not show any significant dependence on latitude variations, although the total latitude range is 0.″39 corresponding to 7 μGal. For an elastic incompressible earth, the following formula was derived by Lambeck (1973) using Love's theory:

$$\Delta g = \Omega^2 r(1+h_2-3/2k_2)\ (\sin 2\phi\ d\phi - 2(d\Omega/\Omega)\ (\cos^2\phi - 2/3)), \qquad (13)$$

where h_2 and k_2 are Love's numbers, Ω the Earth rotation rate, r its radius, ϕ the latitude of the station where gravity change Δg takes place. For most practical purposes $1 + h_2 - 3/2k_2 = 1.16$.

The term in $d\Omega/\Omega$ corresponding to length of day variations has a maximum contribution of $1.55\Omega r d\Omega$ and is therefore negligible (0.06 μGal for 1 ms change in length of day). Formula (13) cannot account at all for the factor -10. ± 5. μGal/ms found in the present study (cf formula (10)).

Conclusion

A sample of seventeen absolute determinations of gravity at Sèvres A2 (BIPM) has been analyzed in comparison to four environmental factors viz. the River Seine water level changes, the latitude variation due to polar motion, the local atmospheric pressure variation and the UTO length of day changes. Only two of these factors prove to be significant i.e. the atmospheric pressure variation with -0.7 ± 0.5 μGal/mb and the UTO length of day variations with -10. ± 5. μGal/ms. The gravity values are corrected for

Earth tides using 378 waves and their local phase shifts. Theoretical results for elastic homogeneous incompressible earth are not in good agreement with the experimental results. Both latitude variation dependence (which should be of order 1 μGal/arcsec, thus significant), and length of day dependence (0.06μGal/ms) which should be negligible are contrary to the experimental evidence presented here. Although a variety of earth deformation models can be compared to the data, recent meteorological studies done elsewhere (cf dicussion above) show a strong correlation between length of day and atmospheric angular momentum variations. Computation of the gravitational attraction of the atmospheric masses on a world-wide scale as a function in time should throw more light on the actual dependence of gravity with length of day changes.

Acknowledgements

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