Report on the Absolute Determination of Gravity from Free Fall Measurements

November 1982

Luc M.A. Jeudy

Internal Report 83-4

Earth Physics Branch

Introduction

First I present a brief account of the results of the first International Calibration of Absolute Gravimeters held in Sèvres (France) at BIPM (October, November 1981). In the second part of the present report I give a short explanation of my data analysis method. The mathematical details can be found in a paper by myself and co-workers (in preparation).

I. Results of the first International Calibration of Absolute Gravimeters, Sèvres (BIPM), October/November 1981.

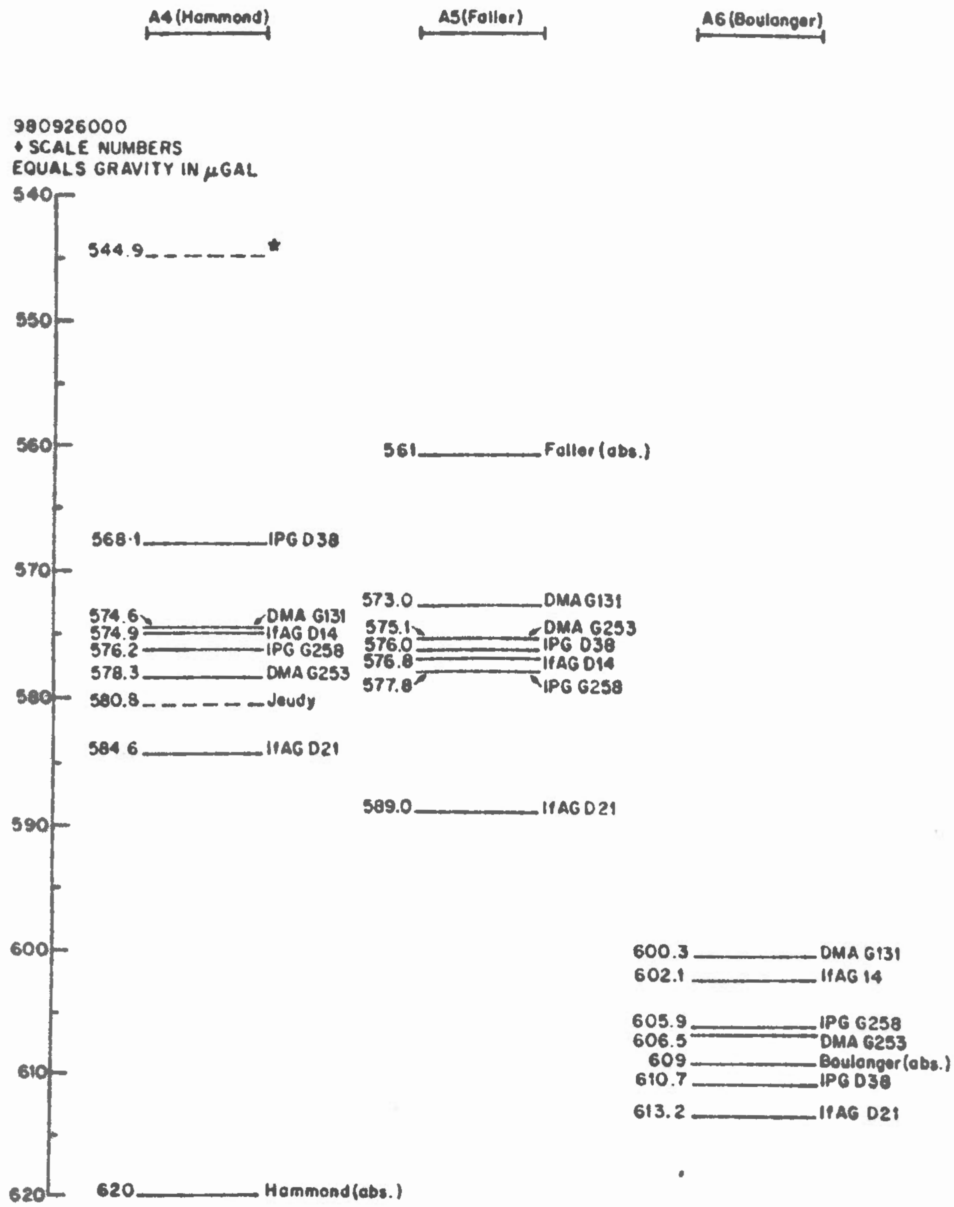
Organized by the Special Study Group 3-40, the first International Calibration of Absolute Gravimeters took place in October, November 1981 at the Bureau International des Poids et Mesures (BIPM), Sèvres, France (Boulanger et al., 1982). Four absolute gravimeters based on the principle of free fall were present: J.A. Hammond's, J.E. Faller's, Yu.D. Boulanger's and A. Sakuma's gravimeters occupying respectively stations A4, A5, A6 and the fundamental station A. In addition Yu.D. Boulanger's gravimeter also occupied station A3 some time later. Stations A4, A5 and A6 are located in a corridor of the basement of the laboratory. Station A3 and the fundamental station A are located approximately at ground level. The Institut für Physikalische Geodäsie (IPG), Darmstadt, West Germany, the Institut für Angewandte Geodäsie (IfAG), Frankfurt, West Germany and the Defence Mapping Agency (DMA), Cheyenne, U.S.A. provided each two Lacoste-Romberg gravimeters (model D or G) for the measurements of the gravity differences between the fundamental station A and every other station and also between stations A4-A5, A5-A6 and A3-A6 (Becker and Groten, 1982). These instruments were also used to determine at stations A3, A4, A5 and A6 the vertical gravity gradients (necessary for the absolute determination of gravity from free fall measurements). On Fig. 1 are represented the values of gravity in stations A4, A5 and A6 deduced from the least squares adjusted relative gravimetric ties A-A4, A-A5 and A-A6 (each relative gravimeter taken separately) and from Sakuma's absolute value of gravity at the fundamental station A. The adjustment takes into account all the measured relative gravity differences for each instrument in turn, including the gravity differences between A4-A5, A5-A6 and A3-A6. Summarizing, the gravity value $g^r_{A_k}$ at station A_k (k=3,4,5,6) for the relative gravimeter r is given by:

$$g^r_{A_k} = g_A + \Delta g^r_{A-A_k}, \qquad (1)$$

where g_A is Sakuma's determination at the fundamental station A and $\Delta g^r_{A-A_k}$ the relative gravity difference measured by instrument r between station A and A_k. Figure 1 also shows the absolute value (denoted by abs.) obtained by each of the participants. As an indication, I also show the absolute value of gravity (station A4) determined from one set of free fall measurements (one "drop") using spectral analysis by least squares and least squares fit (dashed line denoted by "Jeudy"). The solution for that data set by ordinary least squares is at the dashed line "*". Within $\pm$ 3 μGal, it is the value which Hammond would have obtained for that particular data set.

It is also interesting to have a diagram showing the results in terms of averaged gravity values and standard errors for each station as shown on Figure 2. Figure 3 represents Boulanger's result for the station A3 occupied a few days after station A6. That last diagram calls for some comments. Since the gravity difference between the fundamental station A and station A3 has changed by 10 μGal between 1977/78 and 1981 (cf Poitevin and Marson measurements on Fig. 3), it was suggested by Marson that it may be caused by the change in position of a suspended table near station A3. A special study of that problem, proposed at the IAG assembly in Tokyo (May 1982), is going on (Groten 1982).

Figure 1: Absolute Determination of Gravity at Stations A4, A5 and A6, Sèvres (BIPM), October/November 1981.



Legend: Each relative gravimetric tie to the fundamental station A computed by formula (1) is represented by a continuous horizontal line. At one end a number indicates the measured value in μGal minus 980 926 000. μGal. At the other end, the organization initials and instrument type and number are indicated. In the case of absolute measurements (denoted by abs.), these indications are replaced by the name of the scientist. The dashed lines indicate at station 4 the results of absolute determination computed from only one set of data (one drop). Indicated by (*) is the value obtained by least squares fitting of a polynomial of degree two; it is the usual method for the case of quasi-continuous recording of free fall measurements. Indicated by "Jeudy" is the value obtained by my method using least squares fit and spectral analysis.

(end of legend of figure 1)

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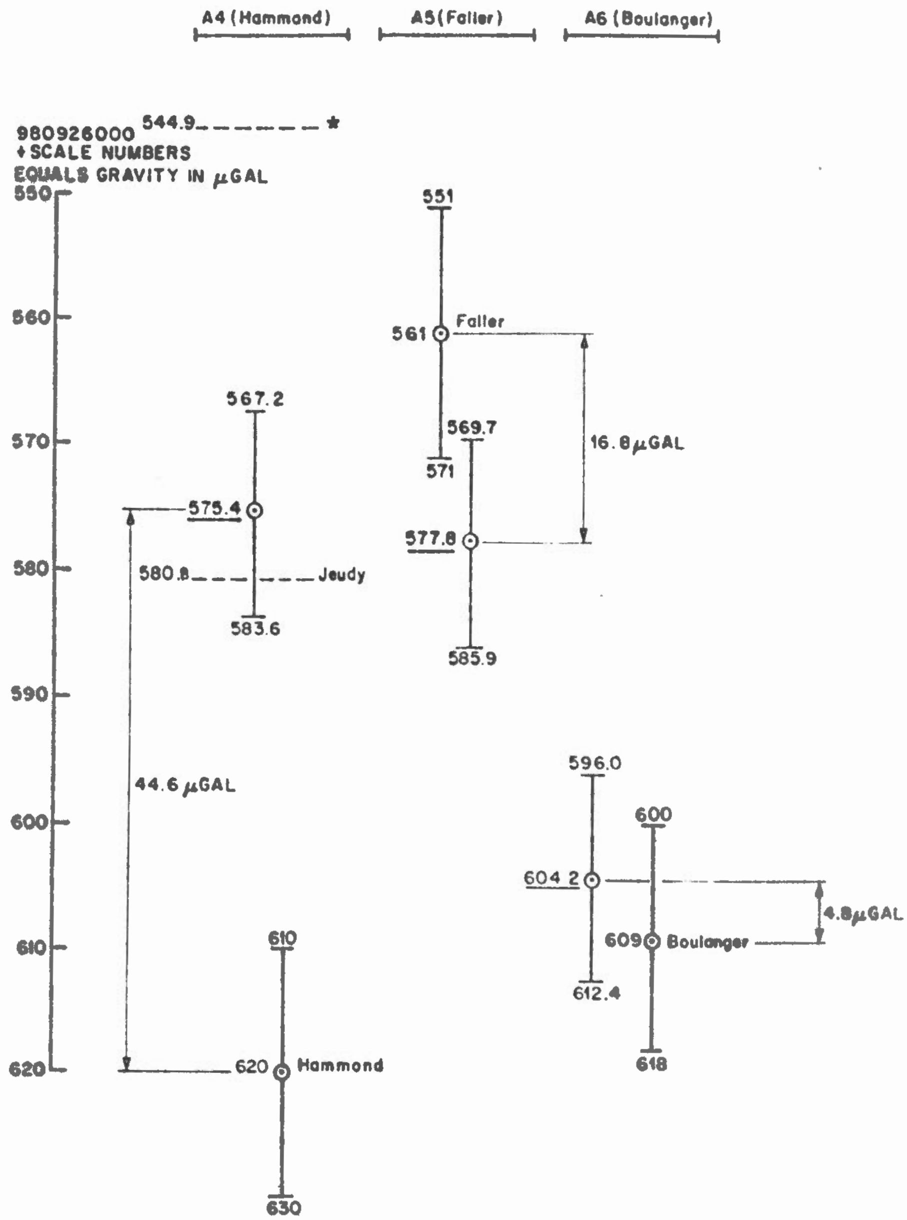
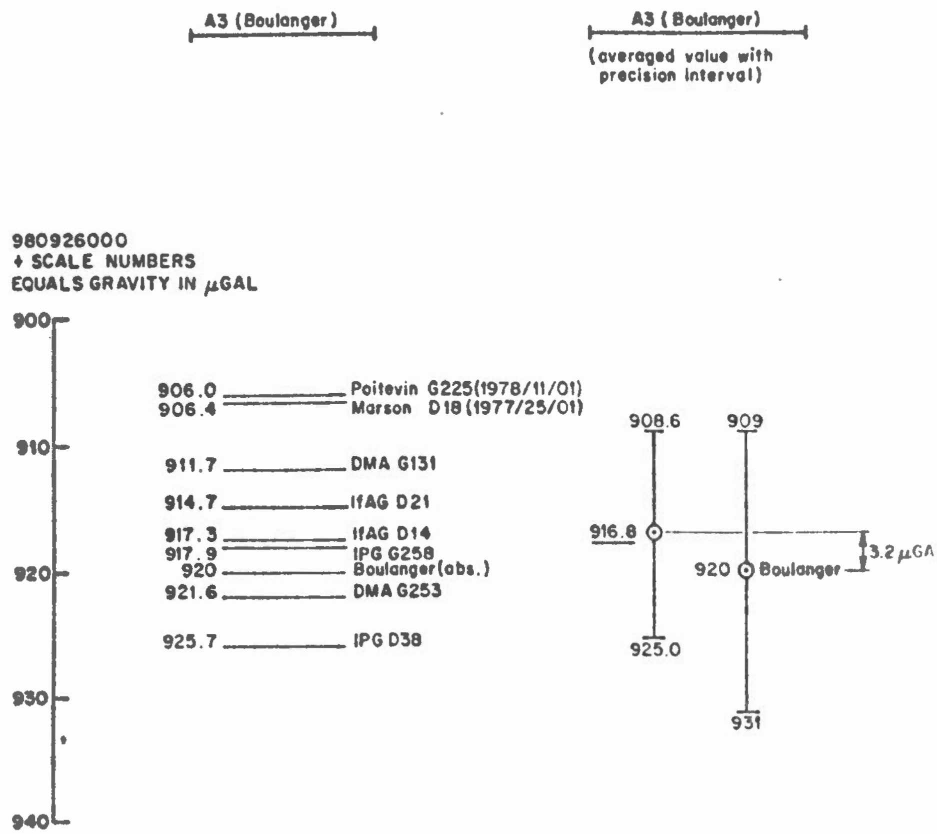
Figure 2: Compatibility of the Absolute Determinations, the Determination at the Fundamental Station being the Reference.



Legend: The underlined values are the averages of the relative gravimetric ties plus the absolute value of gravity at the fundamental station. The vertical lines indicate the precision of each result. Also, as a reference, the dashed lines of Figure 1 are reproduced here (cf explanation in legend of Fig. 1).

Figure 3: Results at Station A3.



Legend: The same explanations as in Figure 1 are valid for the left part of the present diagram. For the right part, the explanations are the same as in Figure 2.

Table 1: Example of absolute gravity computation from real free fall data*

*(kindly provided by Dr. J.A. Hammond, October 28, 1981 at BIPM)

Second degree coefficient multiplied by two, as obtained after removal of four perturbing frequencies (6.2, 29.3, 6114.4 and 68.4 Hz) and correction for the light velocity effect and the gravity gradient effect:		
2 x 490 463 187.6	=	980 926 375.2 μGal
Laser wavelength correction:		
-(1-632 991.435 5/632 991.470 0) x g	=	-53.5 μGal
Reduction to the floor using the measured gravity gradient (2.541μGal/cm):		
111.4 x 2.541	=	+ 283.1 μGal
Earth tide correction:		- 24.0 μGal
Absolute gravity final value:	=	980 926 580.8 μGal

II. Computation of absolute gravity from free fall measurements using spectral analysis:

My method is not basically different from a regular least squares fit without spectral analysis. First of all, it is a least squares method which by definition means that the effect of random errors is systematically minimized. Nevertheless such an assumption holds only if the mathematical model used to represent the data is adequate, that is only if the departures of real data from the ones predicted by the mathematical model are random. Unfortunately, in practice, this is almost never the case and the original mathematical model must be modified.

The modifications must be relatistic and at the same time include the widest possible class of perturbing effects. Since we know that the most serious perturbations are coming from microseisms and since their effects on the measurements can be represented by sine and cosine functions in first approximation, I decided to modify the original mathematical model (i.e. a polynomial of degree two) by adding to it a number of these functions with amplitudes and frequencies to be fitted by least squares to the data. Since an initial value has to be specified for each frequency before the least squares fit can be computed, the question was: "How to find these initial values?". The answer is: "Determine these initial values by spectral analysis if no other information is available".

Among the various existing techniques, I had to choose the most general one which is the method of spectral analysis by least squares. That choice is motivated by the fact that the data is unequally spaced with respect to the independent variable. We have two variables, one is the length variable (z) the other the time variable (t). It is necessary to choose the time variable as the independent variable, because the sine and cosine functions included in the modified mathematical model are functions in time. The practical fact that in a few existing free fall gravimeters the length variable (z) is digitalized in equal intervals, produces unequal intervals in the digitalization of the time variable.

I now would like to explain how the method actually works and how, through an iterative process, the original mathematical model is improved step by step until no more significant improvement can be made. I consider the general case where no information about possible existing perturbing waves in the data is available. The first step (1) consists of fitting the original data to a polynomial of degree two (at that stage there is no difference with the usual method). In the second step (2) the residuals are analyzed by spectral analysis in a given frequency band (it can be a low frequency band i.e. 0 to 500 Hz and/or a set of non-overlapping bands around suspected perturbing frequencies). The third step (3) consists of picking up the probably most significant frequency in the spectrum computed at step two (in practice we may adopt the rule of taking the frequency having the highest peak, the significance is actually tested at a later step). The fourth step (4) consists of modifying the mathematical model by adding to it a sine and a cosine function having as approximate frequency the frequency found at step three. The fifth step (5) consists of fitting the original data to the modified mathematical model. In that least squares fit three additional parameters are adjusted i.e. the two amplitudes for the sine and cosine functions and the frequency. The sixth step (6) consists of testing by an F-test on the variance ratio the significance of the frequency found at step three. If the test is positive, the frequency is significant and the whole process can be run again to search for a further significant frequency (in the same bands but possibly also in other bands); starting at step two the residuals obtained at step five are utilized. The data analysis ends when no additional significant frequency can be found. We see that each time a new frequency is being looked for, a new set of residuals is analyzed i.e. a set of residuals where the disturbing frequencies already found have been removed. In this way significant frequencies which would not appear in the old spectrum may show up in the next or some subsequent one.

Before explaining how gravity is actually computed, let us consider various situations where the procedure can be speeded up. First of all, more than one frequency can be picked up at step three above, provided they are clearly distinct from the noise level. In this case the statistical test at step six will test the significance of the entire set of new parameters, not taking each frequency separately. Another case is the situation where the disturbing frequencies are already known for a given instrument (it may be the current situation in production work where each instrument would have its optimum mathematical model; it could be part of the calibration information). In this case the analysis starts immediately at step four without spectral analysis. The original mathematical model is modified by adding as many sine and cosine functions as there are known perturbing frequencies. In only one run the entire set of parameters is determined.

For the actual gravity value computation, I now want to explain the final stage of the method. Applying the procedure described above, we determine precisely the parameters associated with the perturbing frequencies and the polynomial of degree two. For gravity determination, the most useful parameter is the second degree coefficient of the polynomial. Its value is not correct at that stage because it may still be affected by very low frequency perturbations (i.e. those which do not show up in the spectral analysis because their periods are long compared to the sample length). To overcome this difficulty a method which proves adequate for the moment consists of subtracting from the original data the disturbing waves precisely estimated by the procedure described so far, the result being called corrected data. By fitting this set of corrected data to a polynomial of degree four a good value is provided for the second degree coefficient. This coefficient multiplied by two equals the gravity value provided the appropriate corrections are added. Part of these corrections can be applied right at the beginning to the original data as is done for the light velocity correction and gravity gradient correction. The rest of the corrections, such as the laser wavelength correction, the reduction to the floor and the earthtide correction, can be added easily afterwards. An example of the computations is shown in Table 1 p. 8.

III Conclusion

So far my method was applied to the only set of real data available to me i.e. a set of measurements from one drop made by J.A. Hammond in Paris (October, 1981) at the First International Calibration of Absolute Gravimeters

(Boulanger et al. 1982). Figure 1 clearly shows the improvement brought by my method to the gravity value computed from that set. Using least squares fit of a polynomial of degree two (the usual method) yields the value indicated by the dashed line (*), and using my method yields the value indicated by "Jeudy". I have to process many other sets of data before any reliable conclusion can be drawn. What is clear from the analysis of synthetized data is that a perturbing frequency can induce a systematic error in the result and that my method removes that error. One may argue that over many drops these systematic effects will cancel out, but it can also happen that the average of many drops is still significantly affected as for example, if the perturbing frequency has some systematic behaviour (like having approximately the same phase for each drop).

Finally I intend to develop improved software by introducing damping coefficients for each frequency. This improvement is necessary since the published data clearly shows a very significant damping effect (Hammond & Iliff, 1978). It would then be possible to account not only for steady amplitude perturbing waves but also for transients.

References

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