The Need for an Absolute Gravity Apparatus in the Gravity Division

by

A. Lambert & R.K. McConnell

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1. INTRODUCTION

Recent reports on progress in measuring absolute gravity by the free motion method claim that the precision achieved is now two orders of magnitude better than the precision of absolute determinations prior to 1966. Sakuma (1971) at the Bureau International des Poids et Mesures, Sevres, France, using a symmetrical free motion method claims a precision of a few parts in 10^9 ($\sim$1 microgal). Transportable absolute gravity meters such as the free fall apparatus of Hammond & Faller (1971) have been developed which presently yields absolute determinations with a precision of a few parts in 10^8 ($\sim$10 microgals). The transportable version of the Sakuma apparatus now being tested by the Italian Geodetic Commission has a similar measurement precision. These recent advances in the technology of absolute gravity measurements open up new and interesting possibilities for measurements of changes in gravity with time both through direct measurements or indirectly by providing a calibration standard for relative gravimeters. Such investigations include:

1) Studies of long term (greater than 10 year) changes in gravity due to post-glacial rebound, deep earth processes in the mantle and core and changes in the dynamic constants of the solar system, polar motion or the rotation rate of the earth.

2) Studies of medium term (1 to 10 year) changes in gravity due to water reservoir loading, variations in ground water and filling of underground waste disposal reservoirs.

3) Studies of short term (less than 1 year) changes in gravity due to local seismic displacements, seasonal groundwater variations and the addition or removal of surface material or man-made structures.

2. Measurement of Short and Medium Term Gravity Changes

Medium and short term gravity changes could be measured directly with relatively little development work on one of the existing transportable apparatuses (Faller's or the Italian Geodetic Commission's). Due to the low information rate of this device (in effect, one observation every few days) it might take several years to acquire sufficient data to draw any sound conclusions. Since the recently introduced microgravimeter has a higher internal accuracy than the transportable apparatus as well as a much higher information rate (ten to fifty observations per day) it is proposed to use this instrument as a primary tool in measuring short and medium term gravity changes and to use the absolute apparatus as a calibration standard for the microgravimeter. Thus, although the microgravimeter cannot measure gravity directly it could be used to measure absolute gravity changes. It would permit, for example, the acquisition of data in sufficient quantities for the preparation of a series of anomaly maps charting the variation in gravity with time over a given area.

3. Measurement of Long Term Gravity Changes

The studies of long-term gravity variations at a single station would likely require the development of our own apparatus or the modification of a fixed station apparatus such as that of Sakuma. This would

be a major undertaking, requiring additional manpower, improved laboratory facilities and several years of development time. Important phenomena such as polar wobble, changes in the length of day and long period tides could be monitored by a permanent absolute gravity apparatus having a 1 microgal precision. Such measurements would have implications in studies of Love numbers, core mantle coupling, changes in the moments of inertia of the earth, interplanetary torque and tidal energy dissipation. Theoretical estimates of the magnitude of the gravitational effects of some of these phenomena are given in Appendix III.

4. Calibration of Geodetic, Dynamic and Earth Tide Gravimeters

A transportable absolute apparatus is also required to improve the calibration of several types of gravimeters presently used for land, sea and earth tide measurements.

The geodetic meter has been used extensively in the establishment of the National Gravity Net and the International Gravity Standardization Net (IGSN71) both of which provide standard reference gravity values for the resource exploration industry and geodetic work. Although these systems now have acceptable internal consistency the existence of systematic errors at high and low latitudes is suspected due to non-linearity of the geodetic meter. This problem can only be resolved by the establishment of a long range absolute calibration line having a sufficient number of control points to detect and correct gravimeter non-linearities.

A similar problem exists for the dynamic gravimeter used for sea surface gravity measurements. Due to a lack of seaports at high latitudes it is almost impossible in some cases to obtain local base control for sea gravity surveys in arctic waters. At present, the datum for such surveys is based on instrument readings taken at southern ports like

Halifax or Vancouver, thus even a small error in a dynamic gravimeter calibration at southerly latitudes can produce a considerable survey datum error when extrapolated to high latitudes. It is proposed to use the same absolute calibration line for this type of instrument as for the geodetic gravimeter.

A further application of the transportable apparatus lies in the calibration of earth tide recording gravimeters. At present, quantitative analysis of earth tide data is rendered somewhat uncertain by the absence of an accurate absolute calibration of the recording device.

Details of the calibration problem for several types of relative gravimeters is contained in Appendix II.

5. Economic Considerations

The economics of operation of an absolute apparatus are not too formidable. With an investment of ten to twenty thousand dollars, we may have the opportunity of participating in the final stages of development of one of the existing absolute instruments. This would permit us to evaluate the device before any decision to purchase (at a cost of $50,000 - $100,000) was made and might also offer the opportunity of establishing one absolute calibration line for the relative gravimeters. Subsequent yearly operating costs mostly for field travel would be in the $20,000 to $30,000 range.

Appendix I

Brief Survey of Existing Techniques and Sources of Error in Absolute g Measurements

The best determinations of absolute gravity reported to date, Sakuma (1971) and Hammond and Faller (1971), have been made using the free motion method. The following discussion will be restricted to the two types of apparatus used by these workers. According to Sakuma (1971) the three experimental factors which are important in achieving a measurement of gravity with a precision better than one part in 10^8 are:

a) Use of symmetrical free motion where an object is projected upwards and measurements are made on both the upward and downward motions.

b) Use of interferometric devices for length standards.

c) Use of some sort of vibration-acceleration control of the apparatus.

Sakuma takes advantage of all three of these experimental factors (Figure 1). Faller and Hammond incorporate factors (b) and (c) but use the free fall method for greater mechanical simplicity and ease of transport (Figure 2).

Of the two methods being considered here, the symmetrical free motion method ultimately offers the best possibility to measure with a 1 μgal precision. Assuming that one has a length standard accurate to 5 parts in 10^9 and a time scale accurate to 5 parts in 10^{10}, and given a certain fringe discriminating capability, the actual uncertainties in time measurement depend inversely on the velocity of the dropped object at the beginning and the end of the time interval. Sakuma (1971) shows that accuracies of time and distance measurement being equal the symmetric

experiment makes the best use of the available accuracy by a factor of ten. This advantage is due to the generally higher velocities and longer time intervals of the symmetric experiment. Apart from these metrological advantages the actual uncertainties in length and time measurements are generally smaller for the symmetric free motion method than for the free fall method. The reference length standard used by Sakuma in the symmetric method at present gives better length stability ($\pm$one part in 10^9 per year) than the He-Ne laser ($\pm$ one part in 10^8 per day) used by Hammond and Faller. Also, there is an advantage in timing in the symmetric method due to the fact that the time intervals are defined by two signals of equal form and sharpness (white light interference pattern from a xenon flash lamp) due to the equal velocity of the object passing the beginning and end stations.

There are a number of systematic errors in the free fall method which tend to cancel in the symmetric method. In order to compare the two methods consider the list of systematic effects given in Table 1 of Hammond and Faller (1971).

Laser Wavelength

The measurement of lengths H_1 and H_2 in the free fall method (Figure 2) are critically dependent on the laser wavelength which is presently stated as being uncertain to ± 2 parts in 10^8 (an improvement can be anticipated here). In the symmetric method a reference end standard is used to give H with a precision limited only by the precision of the standard meter definition - a few parts in 10^9.

Direction and Collimation

We can assume that there are similar uncertainties in both methods.

Time Intervals

There is the possibility of a systematic error in the measurement of the time at which the dropped object passes the reference points due to a time constant in the timing system. This error cancels out in the symmetric method. Hammond and Faller, in their free fall method, have the ability to start measurements at arbitrary positions and to vary the length of the time intervals, so that theoretically they could eliminate systematic timing errors.

Gravitational Gradient

It is important to correct the results of both methods for the gravitational gradient γ between the reference points and to transfer the value of g to a bench mark. Herb Valliant's recording gravimeter calibrator may be useful for defining the gradient with a precision of better than 5% which is necessary to achieve a precision in g smaller than 10 μgals.

Velocity of Light

Both methods must be corrected for a phase shift of the interference patterns due to Doppler shifts associated with the moving corner cube reflectors (the dropped object). The uncertainty is small ($\sim$1 μgal).

Air Resistance

Although the free fall apparatus is operated at a high vacuum (5×10^{-7} torr). the uncertainty due to air resistance is quite significant. The first order resistance of the residual air (proportional to the velocity of the moving body) is cancelled out in the symmetric free motion method.

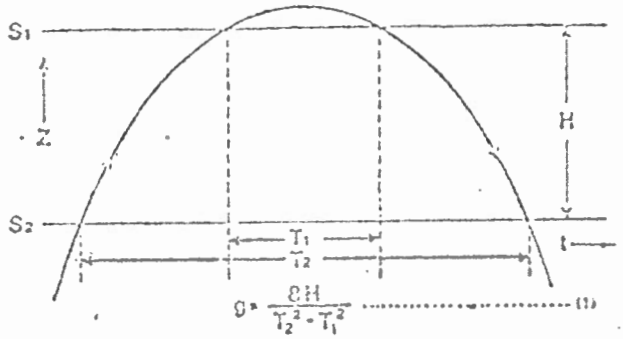
Electrostatic and Magnetic

Resistance caused by eddy currents due to inhomogeneity of the magnetic field along the trajectory may have an effect on the free fall results but these cancel out in the symmetrical experiment.

Seismometer Magnetic

The solenoid which is used to release the dropped object in Hammond and Faller apparatus produces a magnetic field which interacts with the large metal spring of the inertial suspension system (see Figure 2) to produce spurious accelerations of the reference corner cube - in spite of attempts to shield the spring. There is also the possibility of accelerations being induced in the suspension system by thermal effects. The symmetric free motion method of Sakuma (1971) utilizes a servo control system to stabilize the interferometer and a long period seismometer to monitor the residual accelerations. It is possible that the seismometer in this system, although it too is magnetically shielded, could also be perturbed by magnetic effects associated with the launching mechanism. There are few published details on Sakuma's launching system.

It is difficult to evaluate the transportability of Sakuma's apparatus from the literature. No mention is made of the weight and size of the launching system. Presumably, it is quite combersome compared to the dropping mechanism of the free fall apparatus. Also, the mechanical stabilization system and the optical system are more complex. Additional care must be taken with Sakuma's method to isolate the interferometer system from the mechanical shocks caused by the launching of the object. Operation of the Hammond-Faller apparatus merely requires a site consisting of a stable floor in a room with a nine foot high ceiling.



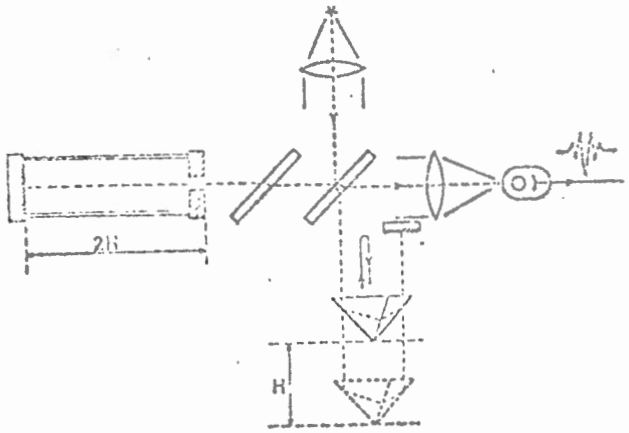
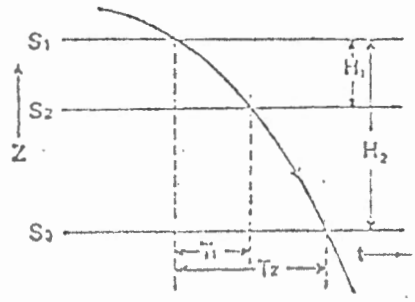


Figure 1



$$G = \frac{2}{T_2 - T_1}\left[\frac{H_2}{T_2} - \frac{H_1}{T_1}\right] \quad \text{.....(2)}$$

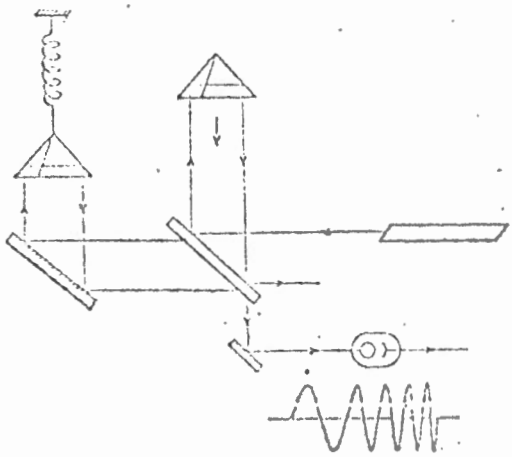


Figure 2

Table 1. Typical Corrections and Systematic Effects and Their Estimated Errors

Source	Correction mgal	Estimated Uncertainty mgal
Laser wavelength	-0.094	±0.020
Direction and collimation	-0.005	±0.005
Time intervals	0.00	±0.010
Gravitational Gradient	+0.432	±0.010
Velocity of light	-0.028	±0.001
Air resistance	+0.010	±0.005
Electrostatic and magnetic	0.00	±0.010
Seismometer magnetic	0.00	±0.030
Net Correction	+0.315	±0.041

Data for BIPM site, Sevres A.

Appendix II

Application of Absolute g to Gravimeter Calibration

The types of gravimeters which require absolute calibration are shown below:

Type	Manufacturer	Typical Operating Range
Earth Tide	LaCoste & Romberg	±0.2 mgals
Earth Tide	North American (modified)	±0.2 mgals
Micro	LaCoste & Romberg	200 mgals
Dynamic	LaCoste & Romberg	7000 mgals
Geodetic	LaCoste & Romberg	7000 mgals

With the exception of the North American earth tide meter all have basically the same type of sensor unit. The response curve of a typical LaCoste & Romberg sensor is shown in Figure 1. The calibration problem consists of determining the value of the scale factor at any point on the dial response curve. Any pair of readings R_1 and R_2 may then be converted to a gravity difference according to $\Delta g = \int_{R_1}^{R_2} K dR$

For the geodetic, and dynamic gravimeters the form of the sensor curve is determined in the manufacturer's laboratory by adding weights to the gravimeter beam. Recent field studies have shown that for some LaCoste & Romberg meters at least, the response curve changes form slightly with time. There is also some evidence to suggest that the weight calibration procedure does not simulate exactly the response of the system to actual gravity changes. In either case, use of the manufacturer's response curve (dial factor table) may not linearize the response of the meter to the required accuracy. Another problem with the weight calibration procedure is that it is not sufficiently sensitive to determine the slope of the response curve to better than 1% over intervals of 0.2 mgals or less. Thus,

TYPICAL LACOSTE-ROMBERG SENSOR RESPONSE

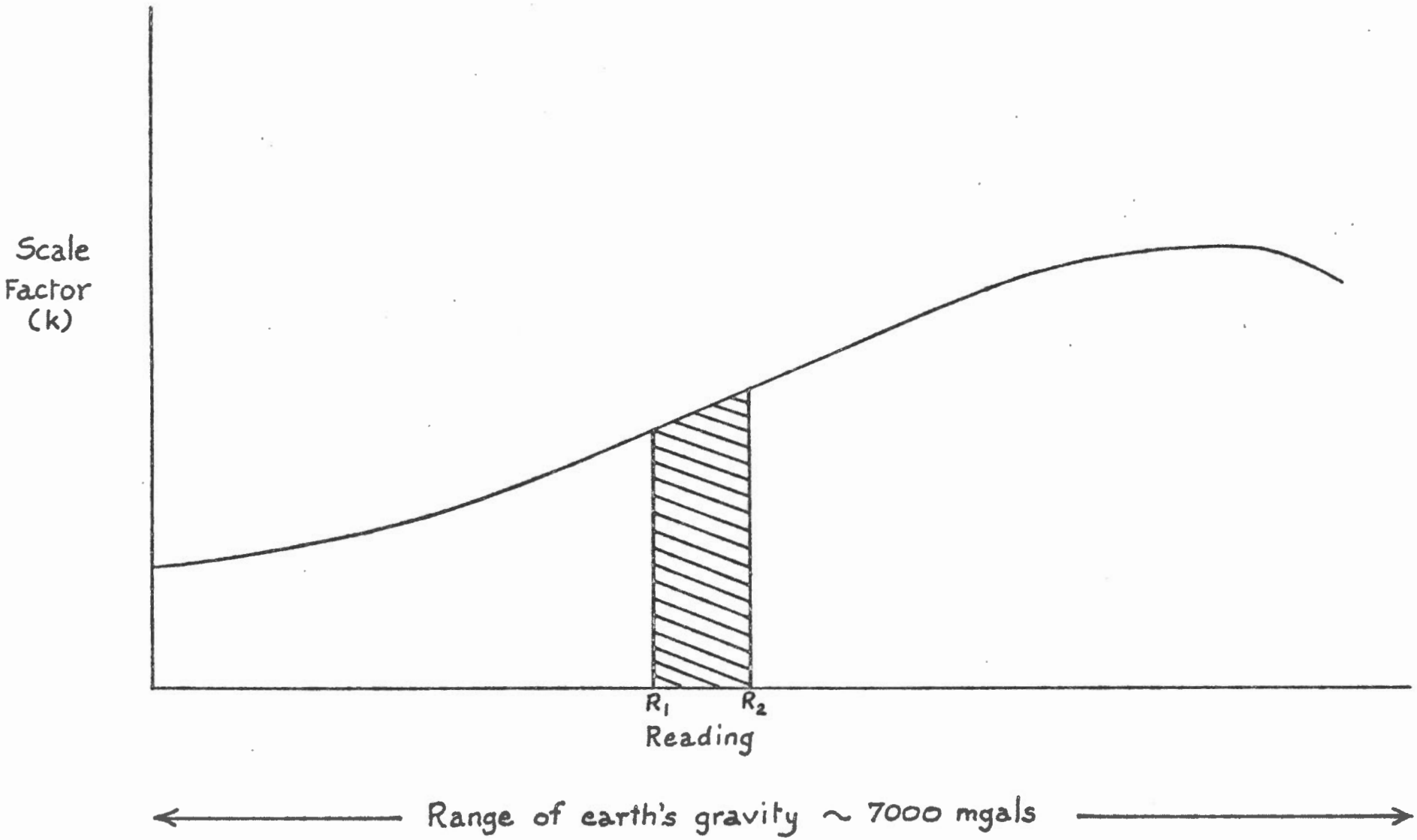


Fig. 1

at present, the calibration of an earth tide meter involves unchecked assumptions about the fine structure of the dial response curve.

The microgravimeter has an essentially flat (within a few parts in 10^4) response curve through its 200 mgal range although it may be necessary on some meters to correct for a small periodic screw error to achieve this accuracy. Thus, with repeated measurements it is possible to achieve standard errors for gravity differences of the order of 3 microgals. This figure represents only internal consistency at present, since the accuracy of the determination of the slope of the response curve and the rate at which it changes as the meter is reset is unknown.

A transportable absolute apparatus could be expected to provide an accuracy of 0.02 mgals. For the geodetic and dynamic gravimeter an absolute calibration line having points spaced at 300 mgal intervals would permit the determination of the dial response curve to about 1 part in 10^4. This would be sufficient to resolve the linearity problem in large scale reference networks used for geodetic and exploration purposes. A similar calibration line with more closely spaced points covering the range of each proposed microgravimeter survey would provide the slope of the microgravimeter dial response curve to about the same accuracy. Thus, in order to reduce calibration uncertainty in microgravimeter work to about 1 microgal the survey would have to be restricted to gravity variations of 10 mgals or less.

For earth tide gravimeters a Hammond-Faller absolute apparatus could be used to give a value for the calibration factor with a precision of $\sim \pm 1\%$ The standard deviation from the mean in a typical 50-drop set of data obtained over a time interval of 30 minutes is ± 0.1 milligals or smaller (Hammond and Faller, 1970). We are free to omit determinations of g where

a large drop to drop scatter is indicated and to utilize only those values taken at low ambient noise levels. An estimate of the calibration factor for a recording gravimeter $\hat{k}$ (complex number) can be obtained by a least squares regression of selected observed values of absolute g (from 50 drops) on corresponding hourly values from the recording gravimeter. The variance of the uncorrelated noise which we can expect in the regression scheme can be estimated from published results of Hammond and Faller (1971). The standard deviation of the mean of a series of 50-drop gravity values corrected (to first order) for the tidal variation is given as 0.01 to 0.02 milligals for a typical station. Using the theory of multivariate normal distributions, we can compute a confidence interval for the calibration factor estimate $\hat{k}$. Assuming that we are dealing with stationary random processes, the confidence intervals for the complex calibration factor with confidence coefficient $1-\alpha$ is determined by a quantity $\hat{r}$ such that

$$|\hat{k} - k|^2 \leq \hat{r}^2$$

$$\hat{r}^2 = \frac{2}{n-2} \quad F_{2,\, n-2;\, \alpha} \frac{\sigma_n^2}{\sigma_T^2}$$

where n = number of degrees of freedom of the estimate (number of data points)

$F_{2,\, n-2;\, \alpha}$ = 100α percentage point of an F distribution with 2 and n - 2 degrees of freedom

σ_n^2 = the mean variance of the uncorrelated noise

σ_T^2 = the mean variance of the observed gravity tide.

Bendat and Piersol (1971) p. 202.

We can estimate the confidence limits on k at the 95% level from one month of data by substituting appropriate values in the above expression.

$\sigma_N^2 = 100\ \mu gal^2$ Hammond and Faller give $\sigma_N \simeq 0.01$ mgal.

$\sigma_T^2 = 4000\ \mu gal^2$ typical data at mid latitudes

$\alpha = 0.05$ 95% confidence limits

$n = 720$ one month of hourly values

$F_{2,\ 720-2;0.05} = 3.0$ from tables

We get $\hat{r}^2 = 2.09 \times 10^{-4}$

which gives $\hat{r} = \pm 1.44 \times 10^{-2}$

$$\hat{\phi} = \sin^{-1} \frac{1.44 \times 10^{-2}}{\sim 1.0} = \pm 0.82^{\circ}$$

therefore, the amplitude and phase limits on the calibration are given by

$$\hat{k} - 0.014 \leq |k| \leq \hat{k} + 0.014 \quad \text{where } |k| \simeq 1.0$$

$$\hat{\phi} - 0.8^{\circ} \leq |\phi| \leq \hat{\phi} + 0.8^{\circ}$$

When the Hammond-Faller apparatus is operated fora long period of time at a single site most of the corrections and systematic effects listed in Table 1 should remain relatively constant over periods of days and weeks. These effects will therefore have negligable influence on the determination of the short period gravity tide (periods $\sim$ 1 day). Most of the uncertainty in the observed tide will be due to the random errors caused by seismic accelerations ($\pm$0.01 mgal to $\pm$0.5 mgal), laser wavelength instability ($\pm$2 parts in 10^8 per day$\equiv\pm$0.02 mgals), long period accelerations of the inertial suspension system due to thermal drifts ($\sim$0.01 mgals) and uncertainty in timing ($\pm$2 nano seconds = $\pm$0.01 mgals). It is reasonable to expect that the precision of earth tide calibration with the Hammond-Faller apparatus could be improved from 1% to 0.1% without substantially increasing the numberof observations but by reducing the above random errors. Lasers

with better stability are now available. Seismically quiet sites could be found or alternatively the accelerations could be monitored by observing the unfiltered output from a recording gravimeter in order to determine suitably quiet times. The accelerations monitored by a recording gravimeter (or long period seismometer) could also be used to supply numerical corrections for each drop. Further refinement could be achieved by operating the apparatus on a table stabilized by piezo electric crapaudines which would cut down the accelerations by a further order of magnitude (Sakuma's technique).

Appendix III

Estimated Magnitude of the Gravitational Effects of some Geophysical Phenomena

1) Polar Wobble (movement of the instantaneous pole of rotation around the mean pole).

The potential due to rotation of the earth about the instantaneous pole is given to first order by

$$U = \tfrac{1}{2} a^2 w^2 \cos^2 \theta$$

where a = mean radius of earth

W = angular velocity of rotation

θ = latitude of place

The rotation component of gravity is given by

$$g_R = aw^2 \cos^2 \theta$$

The change in gravity due to a small change in the position of the instantaneous pole of rotation (change of latitude $d\theta$) is given to first order by $dg_R = 2\, aw^2 \cos\theta \sin\theta \, d\theta$

For a change in pole position of 1 arc second the following changes (30 m ~ 1 arc-sec) in gravity would be expected:

a) At the latitude of Ottawa (45.5° N Lat)

$$dg_R = (2)\ (6.4 \times 10^8 \text{ cm})(7.2 \times 10^{-5} \text{ rad. sec}^{-1}) \times (0.5)\ (0.463 \times 10^{-5} \text{ rad.})$$

$$= 15.4\ \mu\text{gals}$$

b) At the latitude of Alert (82.5°N Lat)

$$dg_R = 4.0\ \mu\text{gals}$$

Data collected by the International Latitude Service (P.Z.T.'s) shows fluctuations of the position of the instantaneous pole of rotation (period $\sim$ 1 year) of from 0.1 arc seconds to as large as 0.8 arc seconds during the last 50 years.

2) Changes in length of day (l.o.d.)

Again, rotation component of gravity is given by

$$g_R = aw^2 \cos^2\theta$$

The change in the rotational component due to a certain fractional change in angular velocity $\frac{dw}{w}$ is given by

$$dg_R = 2aw^2 \cos^2\theta \frac{dw}{w}$$

For an increase in the l.o.d. of 1 millisecond $\frac{dw}{w}$ 1.156×10^{-8} the expected changes in gravity would be:

a) Ottawa $dg_R = (2)\ (6.4 \times 10^8 \text{ cm})\ (7.2 \times 10^{-5} \text{ rad. sec}^{-1})^2 \times (0.5)\ (1.156 \times 10^{-8})$

3.84 μgals

b) Alert $dg_R = 0.1$ μgals

Changes in l.o.d. of the order of milliseconds have been observed by astronomical methods to occur over an interval of just a few years. There are also superimposed periodic variations in l.o.d. of the order of tenths of milliseconds of arc due to the long period earth tides.

3) Long Period Tides

The tidal variations in gravity having periods longer than months have so far eluded observation because of the long term instability (drift) of spring gravimeters. Reliable values for the amplitudes and phases of long period earth tides would yield information on the frequency dependence

of the tidal Love numbers. The following is a table giving the theoretical amplitudes of the major long period tidal constituents as they would be measured on a rigid earth.

Constituent	Period	Amplitude at Alert, N.W.T.
Lunar Nodal Term	18.6 yrs.	5.4 μgals
Sa	1 yr.	0.97 μgals
Ssa	1/2 yr.	6.0 μgals
Mm	27.55 days	6.8 μgals
Msf	14.55 days	0.7 μgals
Mf	13.66 days	13.0 μgals

The above amplitudes are multiplied by the latitude factor $(1/2 - 3/2 \sin^2 \phi)$ so that at 45° N they are reduced by a factor of four.

REFERENCES

Sakuma, A. (1971). Observations Experimentales de la Constance de la Pesanteur au Bureau International des Poids et Mesure, Bulletin Geodesique, No. 100, 159-163.

Sakuma, A. (1971). Une Tendance de la Variation de la Pesanteur Observee au Beureau International des Poids et Mesures, Sevres, France, Bulletin d'Information, Bureau Gravimetrique International, No. 27, 33-35.

Okuda, T. (19). Local Non-Polar Variation of Latitude Deduced from the ILS Data for the Period 1933-1965.

Sakuma, A. (1971). Recent Developments in the Absolute Measurement of Gravitational Acceleration, in Proceedings of the Int. Conference on Precision Measurement and Fundamental Constants, Nat. Bur. Stand. Spec. Publ. 343, 447-456.

Sakuma, A. (1963). Etat Actuel de la Nouvelle Determination Absolue de la Pesanteur au Brueau International des Poids et Mesure. Bulletin Geodesique No. 69, 249-260.

Faller, J.E. and J.A. Hammond (1970). A Laser-Interferometer System for the Absolute Determination of the Acceleration of Gravity, Ph.D. thesis submitted by J.A. Hammond to University of Colorado, Boulder, Col.

Hammond, J.A. and J.E. Faller (1971). A Laser-Interferometer System for the Absolute Determination of the Acceleration due to Gravity, in Proc. Int. Conference on Precision Measurement and Fundamental Constants, Nat. Bur. Stand. Spec. Publ. 343, 457-463.

Hammond, J.A. and J.E. Faller (1971). Results of Absolute Gravity Determinations at a Number of Different Sites, J. Geophys. Res. 76 (32), 7850-7854.

Valliant, H.D.(1971). The Canadian Pendulum Apparatus, Design and Operation, Publ. Earth Physics Branch, 41, No. 4

Valliant, H.D. (1971). A Canadian Network of Gravity Measurements with Pendulums, Geophys. J.R. Astr. Soc. 23, 543-555.

LaCoste, L.J.B. and A. Romberg (1945). Patent Application for gravimeter.

Report on French LaCoste & Romber D-meter trial, La Gravimetrie de Haute Precision - Progress Recents.

Bursa, M. (1972). Variations of the Earth's Gravity Field due to the Free Notation, Studia Geophysica et Geodaetica 1b (2), 122-125.

Bendat, J.S. and Piersol, A.G. (1971). Measurement and Analysis of Random Data, John Wiley and Sons.