GENERATION OF DAMPED RESPONSE SPECTRA FROM AN UNDAMPED RESPONSE SPECTRUM BY CONSTRAINED INTERPOLATION

by

George A. McMechan

SEISMOLOGICAL SERVICE OF CANADA

INTERNAL REPORT 77-8

Division of Seismology and Geothermal Studies

Earth Physics Branch

Department of Energy, Mines and Resources

OTTAWA

May 1977

ABSTRACT

The calculation of damped response spectra can be greatly simplified if only approximate spectra are required. The procedure proposed here is based on the observations that the shape of damped response spectra are similar to that of the undamped spectrum of the same record, and that a highly damped spectrum is smoother, and of smaller average value than a spectrum corresponding to a lower percentage damping. Guided by these general properties of damped spectra, a fast, approximate algorithm has been empirically devised for the production of damped spectra from an undamped spectrum. The technique is basically one of constrained interpolation. It is general in that it is independent of the physical significance of the data to which it is applied. As the number of constraint points increases, the approximations converge to a desired solution, so that any required degree of precision can be obtained. Computational costs, which are typically of the order of 10% of the cost of direct integration, are low enough that the approach described here should be competative with all other techniques for most applications.

Introduction

A commonly calculated function in earthquake engineering is the damped pseudo-relative velocity response spectrum (PSRV). Standard presentations usually include PSRVs corresponding to 0, 2, 5, 10, and 20 percent of critical damping. Typical examples and listings of computer programs for evaluating the PSRV integral are presented by Trifunac and Lee (1973). The calculations are long and the physical significance of the precise shape and position of the individual PSRV peaks are uncertain, and not generally used in subsequent calculations. The question then presents itself as to whether one cannot find some method of adequately approximating PSRV values in a less expensive manner.

A similar problem has been presented by Johnson (1973). He found an empirical algorithm that produces approximations to 5 percent critical PSRVs as a function of magnitude and epicentral distance. Wiggins (1964) averaged the observed response spectra of a number of records to construct an empirical description of the dependence of response spectra on magnitude and epicentral distance. The production of PSRV estimates by digital filtering has also been shown to be a viable alternative to direct integration of accelerograms (Beaudet and Wolfson (1970), Udwadia and Trifunac (1973)).

The general properties of PSRVs are illustrated in Figure 1. All the PSRV curves for a single record are similar in shape. The main differences between the PSRV curves for various damping values are a) the average absolute values decrease as damping increases, and b) the PSRVs become smoother as damping increases. The purpose of this paper is to show that these trends can be simply and explicitly defined and the resulting algorithm used in the production of PSRVs.

While a number of other empirical relations have been used by others for the production of approximate PSRVs, the present approach has a number of distinct advantages:

1) it is independent of event magnitude,

2) it is independent of the source spectrum,

3) it is independent of the structure between the source and receiver,

4) it is independent of epicentral distance,

5) it is valid for any frequency range required,

6) it incorporates real observations of spectral character,

7) unusual events require no special treatment,

8) the approximations can be made as close as desired to the exact solution, and,

9) the accuracy / cost ratio is higher than that of other approximate methods and the exact computations.

Precision of PSRVs

$PSRV_n$ curves can be calculated for any desired periods (T) for any desired percentage (n) of critical damping. There are 91 points on each spectrum in Figure 1. The reliability of these points is uncertain. If the number of points on any of the curves in Figure 1 was to be increased, one would find that the spectra contain a significant amount of fine structure, and some of the points that appear to be at spectral peaks are actually on the side of a larger peak (W.G. Milne, personal communication, 1974). Hudson (1972) has shown that the ground motions generated by an earthquake can vary significantly in a small area and that it is difficult to attribute the observed variations to any single site parameter. Hence, the value of a single, precisely calculated set of PSRV curves is, as yet, undefined in terms of the evaluation of seismic risk.

Figure 2 contains another example. The two solid curves were determined by joining every second point in the undamped spectrum ($PSRV_o$) in Figure 1, first for one set of points, then for the intermediate set. These two curves, therefore, should be equally valid spectral estimates and give a qualitative indication of the overall spectral reliability. In the limit, as the number of calculated PSRV values becomes very large, only a broad envelope, similar to the two dotted curves in Figure 2, is left. This concept is not new. Small, sharp, local peaks in any spectrum are of questionable validity (Blackman and Tukey (1958), Fryer (1973), Currie (1974)) so that the calculation of reliable spectra commonly includes some form of prewhitening or smoothing. The results of Lynch (1969), Blume (1969), Newmark, Blume and Kapur (1973), and Udwadia and Trifunac (1974) suggest that an envelope drawn roughly as in Figure 2 corresponds approximately to $\pm\ \sigma$.

Since it is apparently unrealistic to attempt to determine PSRVs to a precision of greater than ≈ 0.2 on a log scale (as a rough observation from the typical example in Figure 2), one is left with a relatively simple problem. A fast, simple empirical algorithm for constructing PSRV values that lie within that, or any other desired, uncertainty is described in the following section.

Constrained Interpolation

The task of finding a simple algorithm which describes a non-linear phenomenon is generally difficult. However, if the desired solution is known or closely approximated at some control points, a linearized interpolation between these points may be adequate to describe the entire solution provided that the control points are sufficiently close together. Since each point in a PSRV can be calculated explicitly and separately, control values can be obtained for any number of points.

The production of an approximate $PSRV_n$ begins with the calculation of the $PSRV_o$. For the purpose of the present paper, it is not necessary to calculate values for the large number of periods (91 in Trifunac and Lee (1973)) usually used since the $PSRV_n$s are smoother than the $PSRV_o$. Every second or third point is often sufficient, and there is, of course, a corresponding saving in computation costs.

The next step is to choose a number of control points such that the average spectral behavior is approximately linear between the points. Obvious choices are the two end points and the spectral maximum. For some well behaved spectra, 2 or 3 control points are sufficient, and the use of 5 points ensures satisfactory results for almost all cases. $PSRV_n$ values are calculated for the control points by the usual integration method. These integrations are expensive so it is desireable to keep the number of control points as small as is consistent with the precision desired. From the arguments in the previous section, it is expected that the use of more than 7 points is rarely justified. The saving in computational costs in this step is one of the main advantages to the use of an approximate method. If only 5 rather than 91 integrals are required, the computational costs are reduced by approximately 94%. The lost precision which accompanies this is, as previously described, of doubtful significance.

Once control values have been determined for a particular damped spectrum, the major part of the required computational effort is complete as all intermediate values are obtained by interpolation. This interpolation is constrained so that the resulting curve passes through the control points and its shape is similar to that of the $PSRV_o$ over the same period range, but smoother.

Figure 3a illustrates the interpolation algorithm used. Points 3 and 4 are two adjacent control points for a $PSRV_n$ and the curve 1-2 is the $PSRV_o$. In order to interpolate between points 3 and 4, the curve 1-2 is weighted by the dimensionless linear weighting functions w_1 and w_2 as follows:

$$PSRV_n(T_i) = PSRV_o(T_i) - (A_o - C_n)w_1(T_i) - (B_o - D_n)w_2(T_i)$$

where:

- $PSRV_n(T_i)$ = the estimate of an n% critical damped PSRV value at period T_i
- $PSRV_o(T_i)$ = the calculated value of $PSRV_o$ at period T_i
- A_o, B_o = the calculated $PSRV_o$ values at the control points
- C_n, D_n = the calculated $PSRV_n$ values at the control points
- T_i = any period between those of the control points
- w_1, w_2 = the linear weighting functions, which have values between 0 and 1
- w_1 = $(L - l(T_i)) / L$
- w_2 = $l(T_i) / L$
- L = distance between the two control points
- $l(T_i)$ = distance from the first control point to the point T_i.

L and $l(T_i)$ are in units of $\log_{10}T$, w_1 and w_2 are dimensionless, and all other quantities are in units of $\log_{10}$ PSRV.

The final step is to smooth the $PSRV_n$ curve. The amount of smoothing increases as the damping increases. The required filter operators were constructed from a simple triangular filter (0.23, 0.54, 0.23). The amounts of smoothing for 2, 5, 10, and 20 percent critical damping were empirically found to be approximately equivalent to applying this filter 3, 7, 11, and 15 times respectively. A filter approximating the smoothing for any percentage critical damping between 0 and 20 can be determined by interpolation in Figure 3b.

This approach is general in that it can be applied to any set of curves which comprise a group with similar characteristics. As any number of control points can be used, the approximate solution can be made to approach a desired solution with any accuracy required.

Examples

In order to test the applicability of constrained interpolation, seven data sets were taken from Volume III of the California Institute of Technology Earthquake Engineering Research Laboratory Report EERL 72-80. These data, which are described in Table 1, were chosen because they represent a rather wide variety of PSRV shapes. Both vertical and horizontal components are included, although a consistent effect of orientation was neither expected nor observed.

Figures 4 to 10 contain results for seven examples. In each figure there are five solid curves, which, from top to bottom, are for 0, 2, 5, 10, and 20 percent critical damping. These curves were obtained from the EERL report described above and were determined by evaluating the PSRV integral for each of 91 period values on each curve. Superimposed on the bottom four curves are the results of spectral estimation by constrained interpolation between 5 equally spaced control points. Any other reasonable criterion could be used to define control points without significantly affecting the results. The use of equally spaced points simply ensures that all parts of the PSRV curves are similarily constrained, and allows identical treatment of all cases. In these examples, no problem was experienced in obtaining estimates that lie within the physically reasonable limits described earlier. The approximate estimates are closest to the precise estimates for those PSRV curves which appear typical, such as those in Figures 4, 6, 7 and 10. The results in Figures 5, 9, and to a lesser degree, in Figure 8, are not as close to the precise values, but the general behaviors of the PSRV.s in these figures are peculiar, and rare. This problem can be simply overcome by using more control points for any case in which a high degree of non-linearity is suspected.

Conclusions

A fast method for obtaining approximate damped PSRV curves from an undamped PSRV curve is described. The method involves the calculation of precise PSRV values at a few control points with constrained interpolation to estimate intermediate values. Computational savings, which are typically of the order of 90% of the cost of direct integration, are sufficient that an empirical approach such as that described here should be competetive with more precise methods for many applications.

REFERENCES

Beaudet, P.R. and S.J. Wolfson (1970). Digital filters for response spectra, Bull. Seism. Soc. Am., 60, 1001-1013.

Blackman, R.B. and J.W. Tukey (1958). The measurement of power spectra, Dover Publications Inc., New York, 190 pp.

Blume, J.A. (1969). Response of highrise buildings to ground motion from underground nuclear explosions, Bull. Seism. Soc. Am., 59, 2343-2370.

Currie, R.G. (1974). Period and Q_w of the Chandler Wobble, Geophys. J. 38, 179-185.

Fryer, G.J. (1973). Detection of weak seismic arrivals in the presence of microseismic noise, M.S. Thesis, University of Hawaii, 32 pp.

Hudson, D.E. (1972). Local distribution of strong earthquake ground motions, Bull. Seism. Soc. Am., 62, 1765-1786.

Johnson, R.A. (1973). An earthquake spectrum prediction technique, Bull. Seism. Soc. Am., 63, 1255-1274.

Lynch, R.D. (1969). Response spectra for Pahute Mesa nuclear events, Bull. Seism. Soc. Am., 59, 2295-2309.

Newmark, N.M., J.A. Blume and K.K. Kapur (1973). Seismic design spectra for nuclear power plants. Journal of the Power Division, ASCE, 99, No. PO2.

Trifunac, M.D. and V. Lee (1973). Routine computer processing of strong-motion accelerograms, EERL 73-03, California Institute of Technology, Pasedena, 360 pp.

Udwadia, F.E. and M.D. Trifunac (1973). Damped fourier spectrum and response spectra, Bull. Seism. Soc. Am., 63, 1775-1783.

Udwadia, F.E. and M.D. Trifunac (1974). Characterization of response spectra through the statistics of oscillator response, Bull. Seism. Soc. Am., 64, 205-219.

Wiggins, J.H. (1964). Construction of strong motion spectra from magnitude and distance data, Bull. Seism. Soc. Am., 54, 1257-1269.

Table Caption

Table 1 : List of the accelerograms used in the examples in Figures 4 to 10.

TABLE I

Figure	Date	Time	CIT Identification	Component
4	May 18, 1940	2037 PST	IIIA001	S00E
5	May 18, 1940	2037 PST	IIIA001	vert
6	Oct 07, 1951	2011 PST	IIIA002	vert
7	Jul 21, 1952	0453 PDT	IIIA005	S48E
8	Dec 21, 1954	1156 PST	IIIA008	N79E
9	Feb 09, 1956	0725 PST	IIIA012	vert
10	Mar 22, 1957	1144 PST	IIIA017	N26E

Figure Captions

Figure 1 : A typical set of PSRV curves.

Figure 2 : Uncertainty in the undamped PSRV curve in Figure 1.

Figure 3a: Constrained interpolation. This figure illustrates the geometry of the quantities in the interpolation formula.

Figure 3b: The apparent smoothness of a damped response spectrum increases as the % critical damping increases. The smoothing factor (S) of a particular filter operator is defined as the smoothing produced by repeated application (S times) of the 3-point operator described in the text.

Figure 4 : Figures 4 to 10 are similar. The solid curves are the results of precise calculations and the symbols are the results of constrained interpolation. For clarity, only the undamped PSRV curve is plotted at its actual position. The 2% damped curve is displaced downward by 0.1 $\log_{10}$ PSRV units, the 5% curve by 0.2, the 10% curve by 0.3, and the 20% curve by 0.4. The control points are indicated by the arrows.

Figure 5 : Precise and approximate PSRV estimates. See Table 1 and the caption beneath Figure 4.

Figure 6 : Precise and approximate PSRV estimates. See Table 1 and the caption beneath Figure 4.

Figure 7 : Precise and approximate PSRV estimates. See Table 1 and the caption beneath Figure 4.

Figure 8 : Precise and approximate PSRV estimates. See Table 1 and the caption beneath Figure 4.

Figure 9 : Precise and approximate PSRV estimates. See Table 1 and the caption beneath Figure 4.

Figure 10: Precise and approximate PSRV estimates. See Table 1 and the caption beneath Figure 4.

RESPONSE SPECTRUM

IMPERIAL VALLEY EARTHQUAKE MAY 18, 1940 - 2037 PST .

IIIA001 40.001.0 EL CENTRO SITE IMPERIAL VALLEY IRRIGATION DISTRICT COMP SOOE

DAMPING VALUES ARE 0, 2, 5, 10 AND 20 PERCENT OF CRITICAL

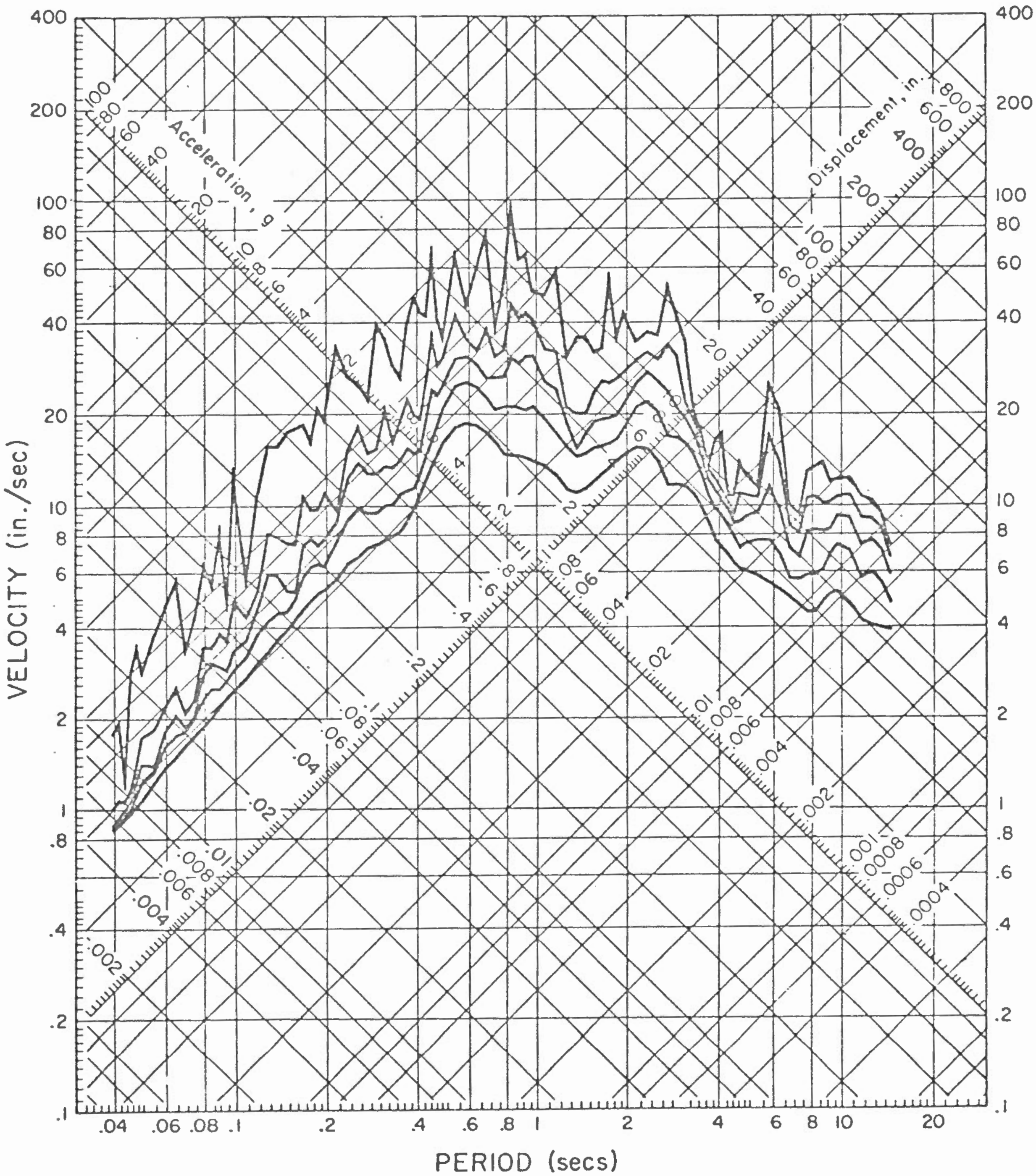


Fig 1

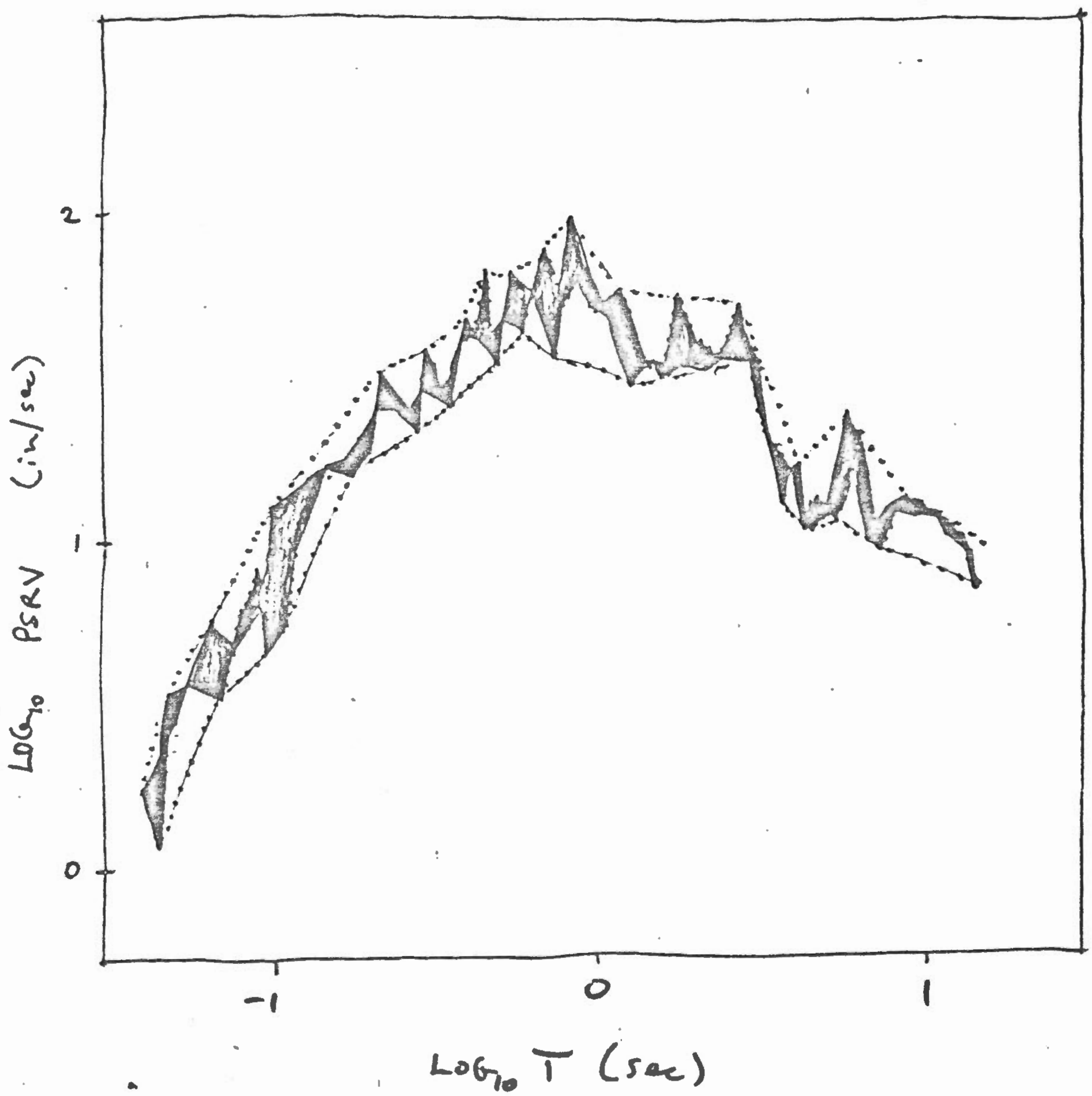
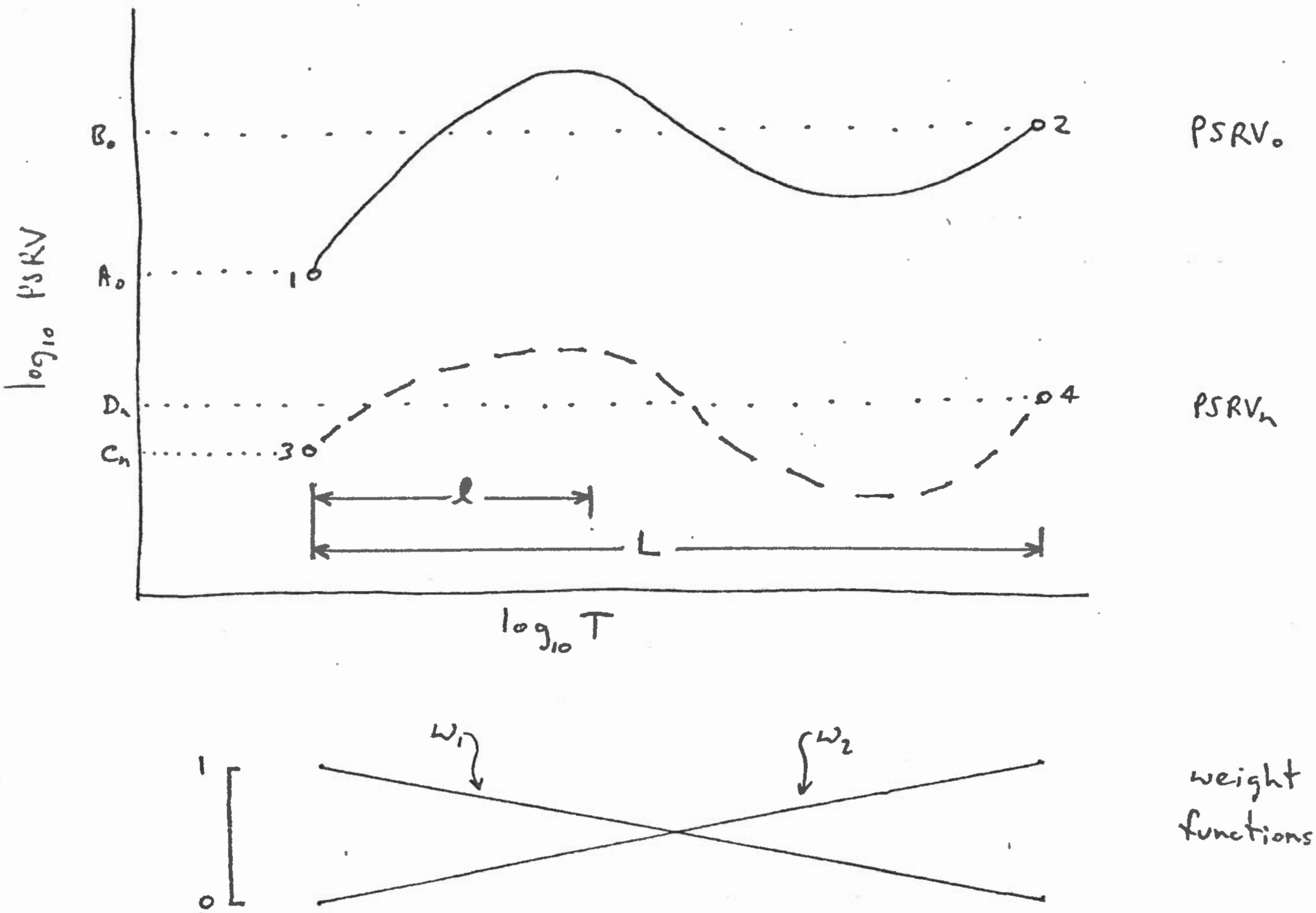


Figure 7



B₀
A₀
D
C
log₁₀ PSRV
1
2
3
4
PSRV₀
PSRVₙ
ℓ
L
log₁₀ T
w₁
w₂
1
0
weight functions

Figure 3a

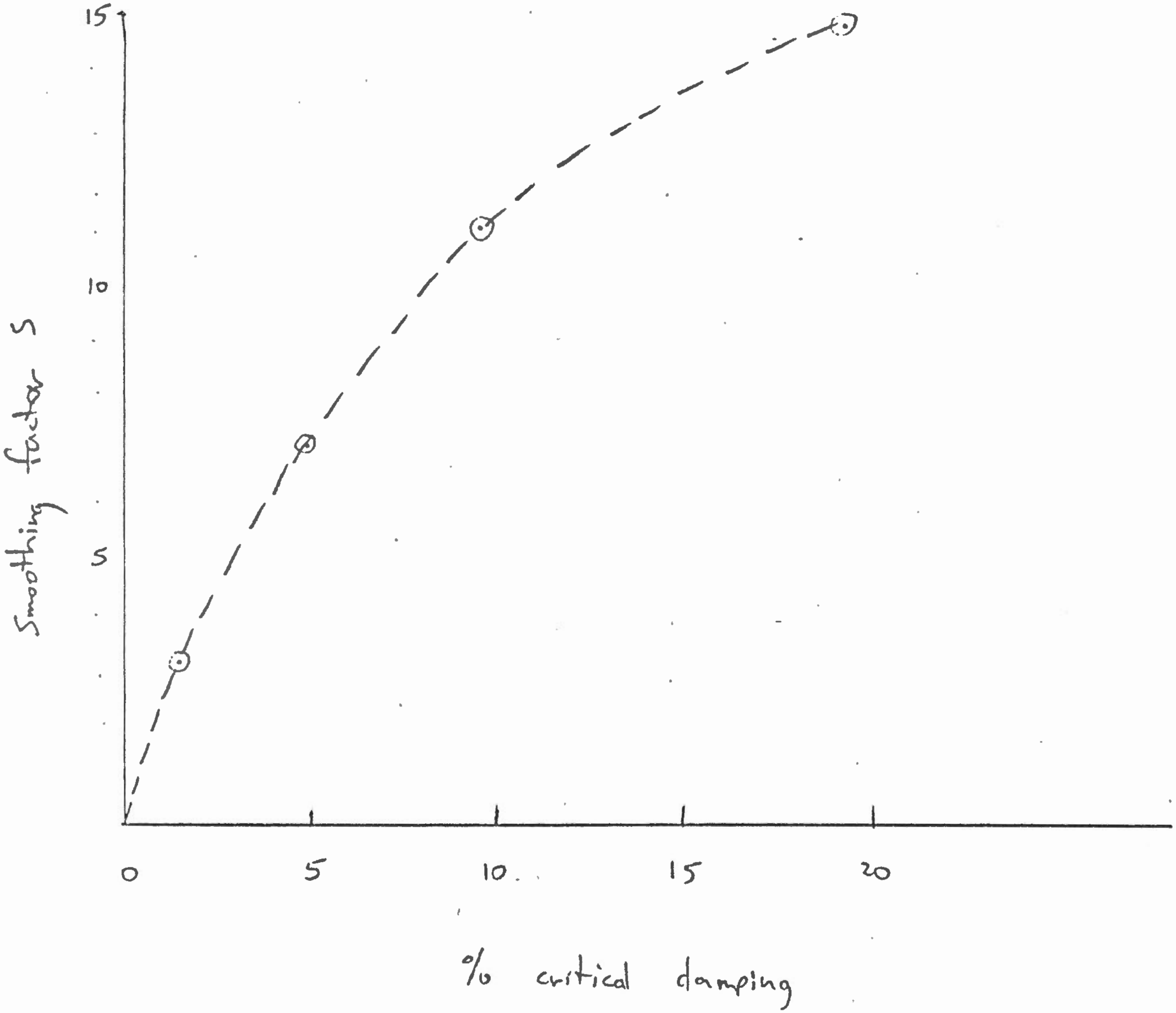
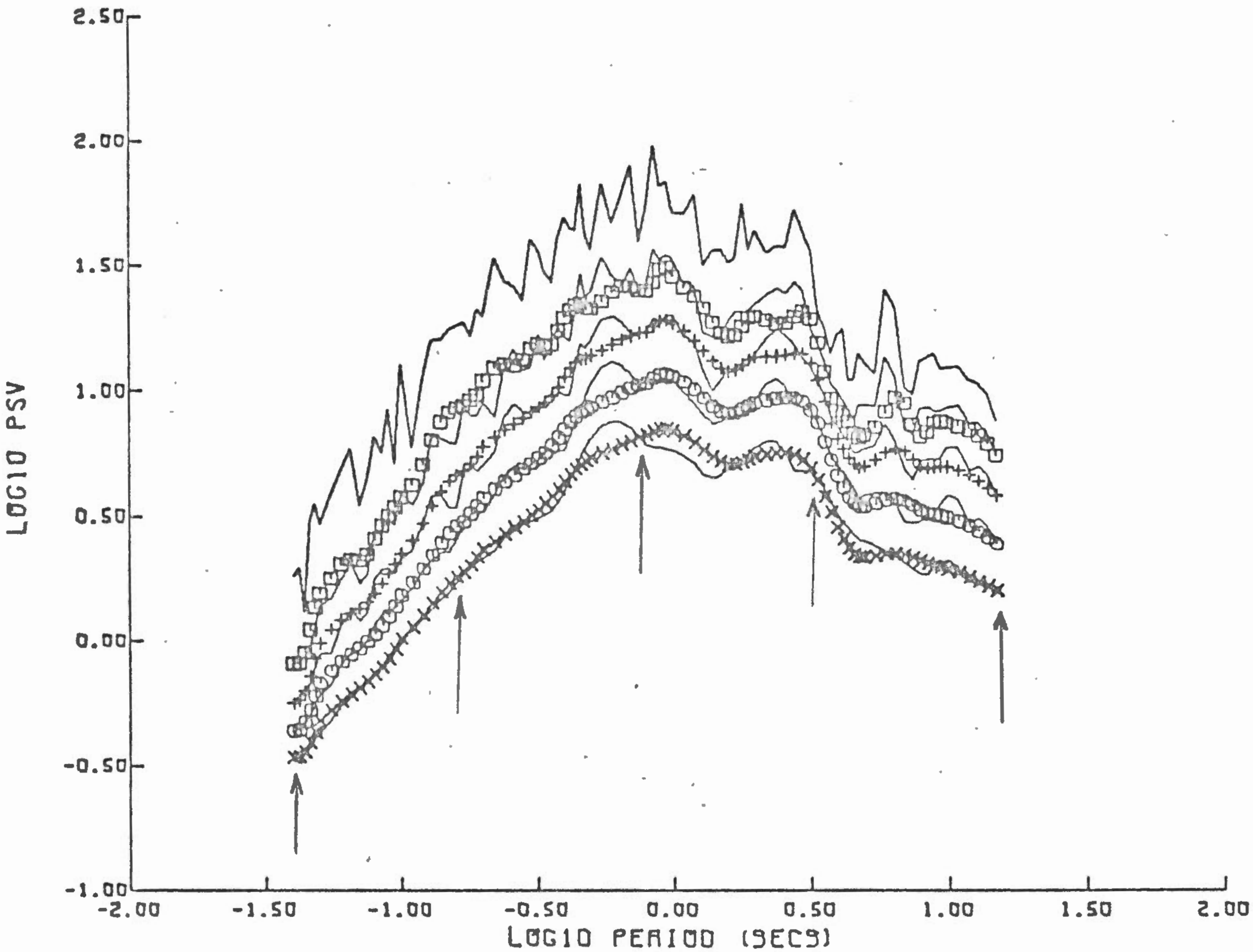
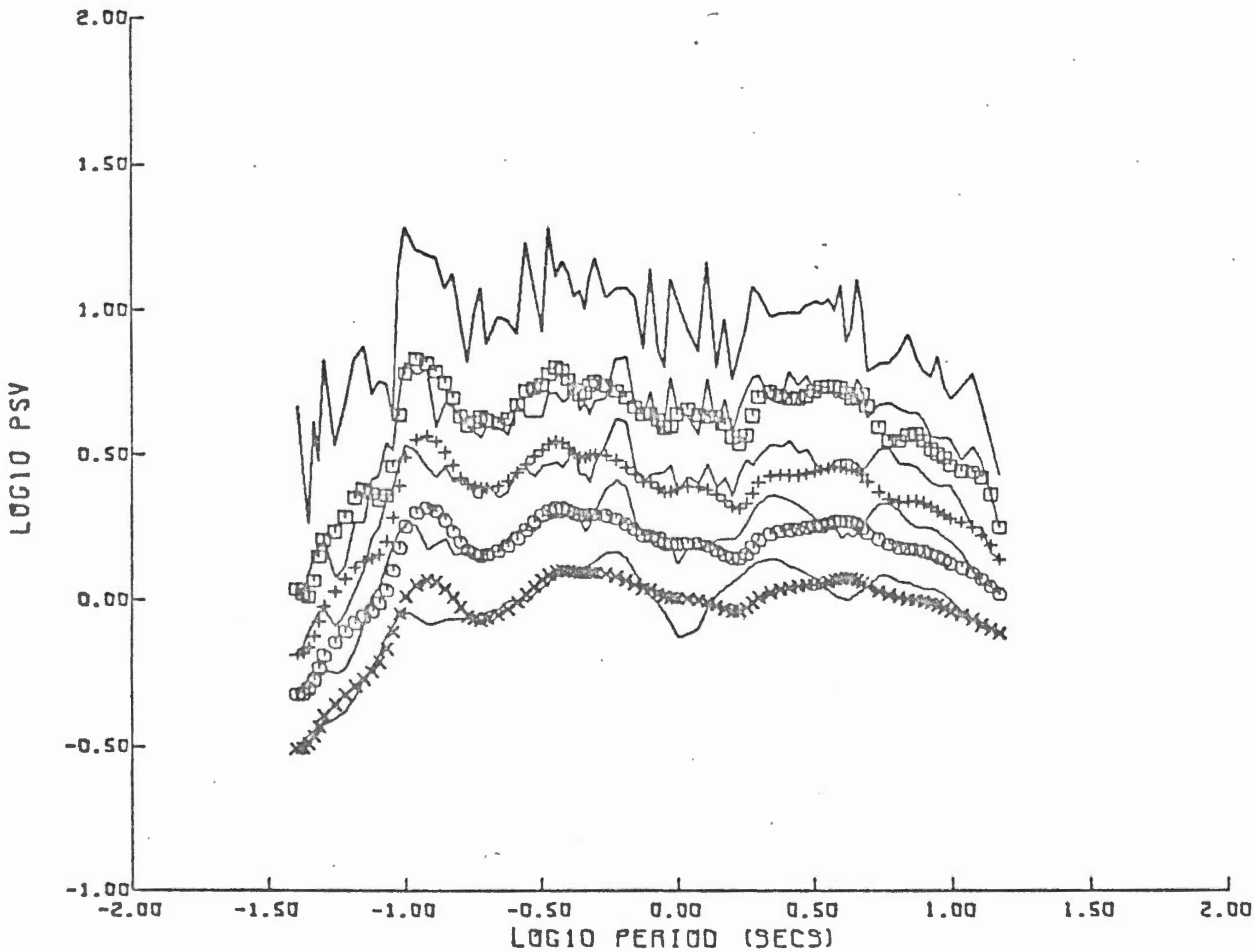


Fig 3b



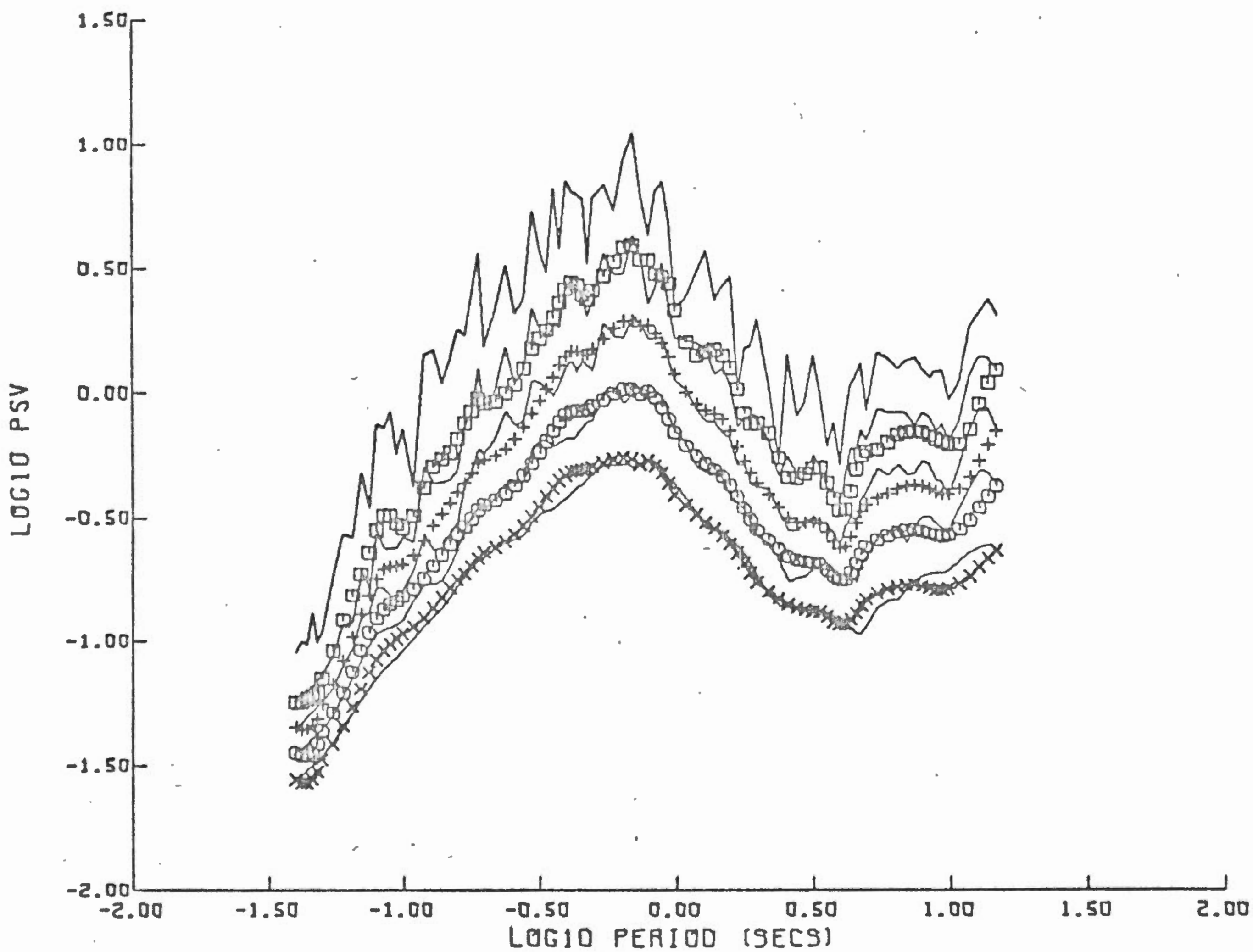
PSEUDO-VELOCITY SPECTRA PREDICTIONS

Fig



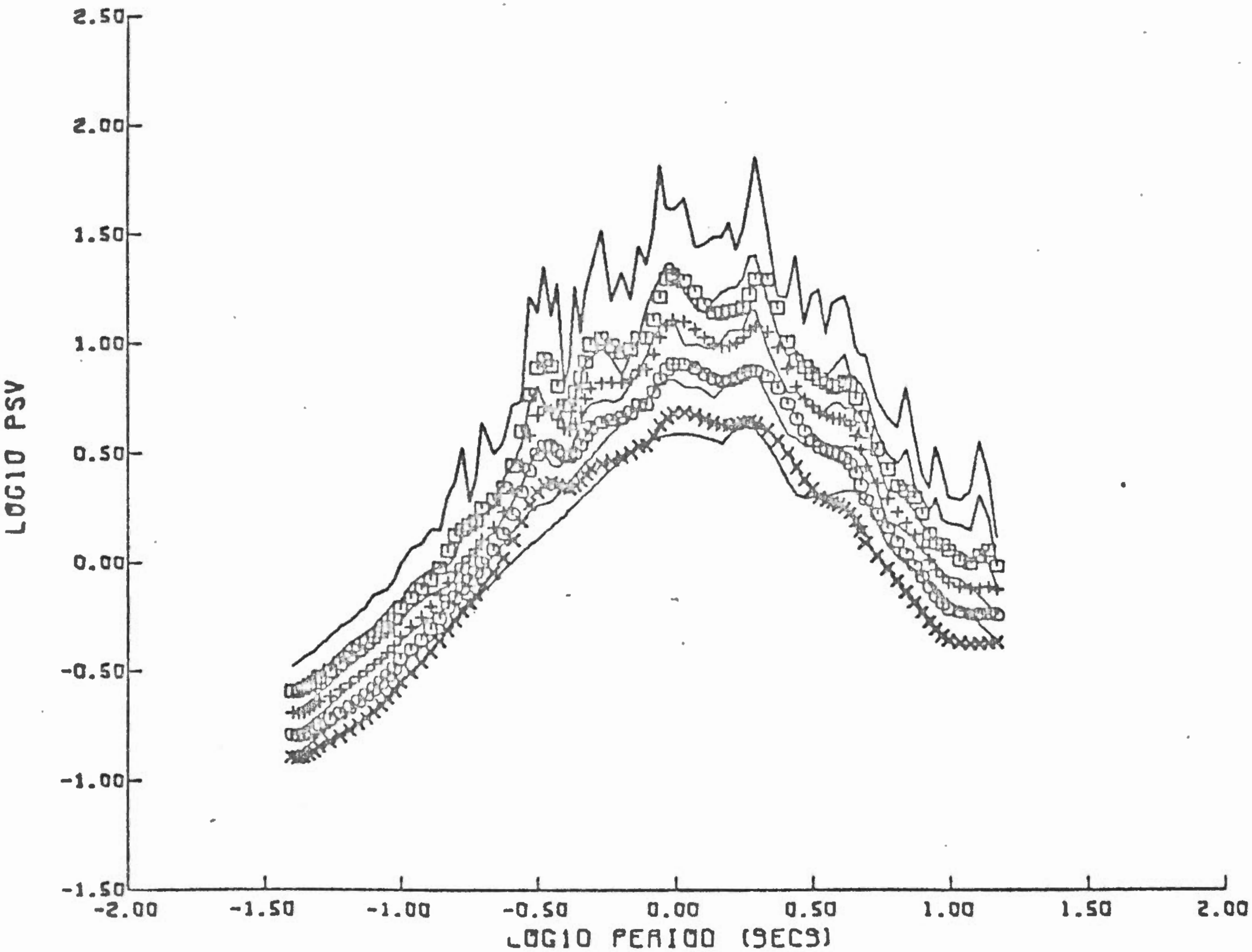
PSEUDO-VELOCITY SPECTRA PREDICTIONS

Fig 5



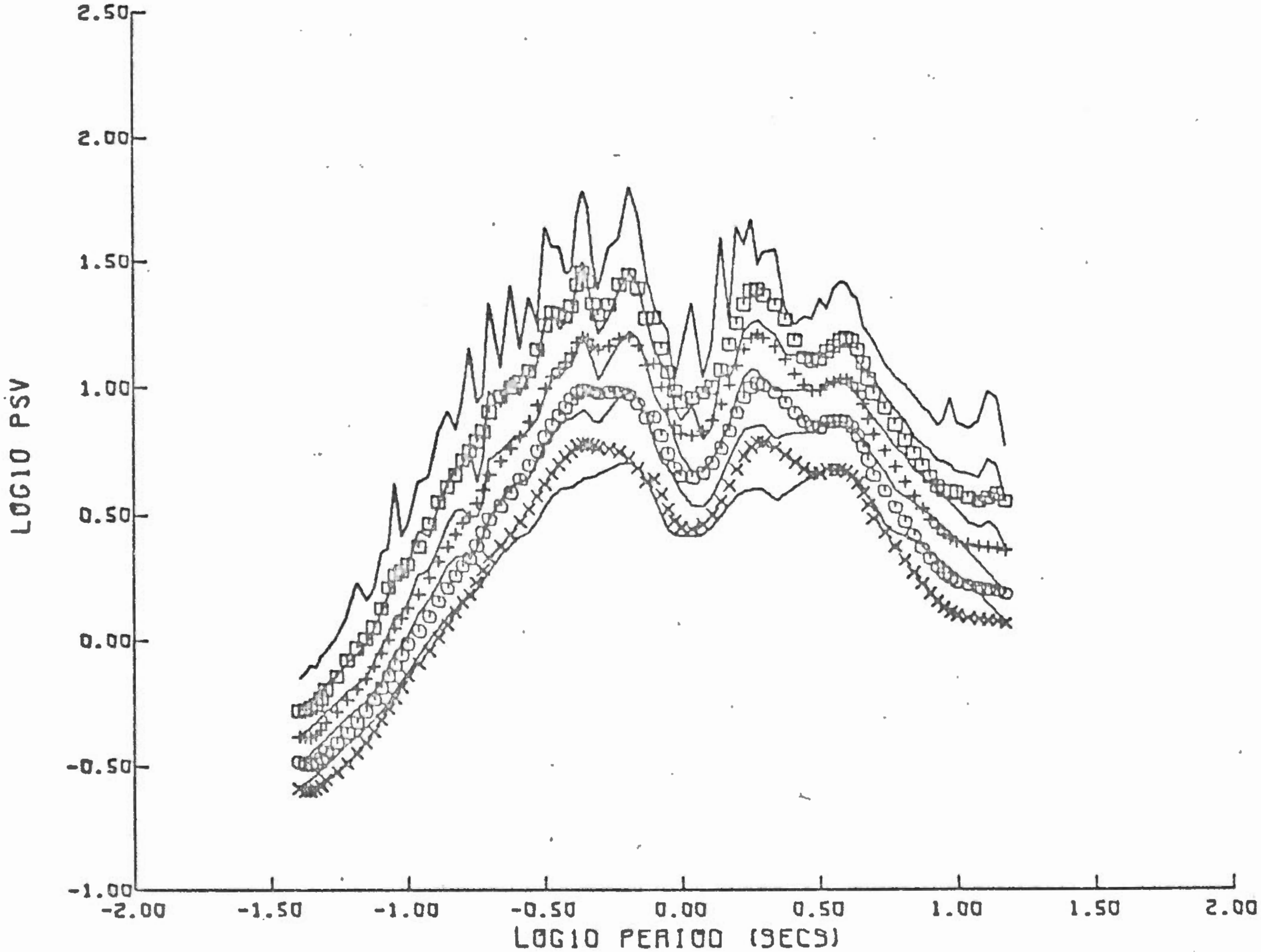
PSEUDO-VELOCITY SPECTRA PREDICTIONS

Fig 6



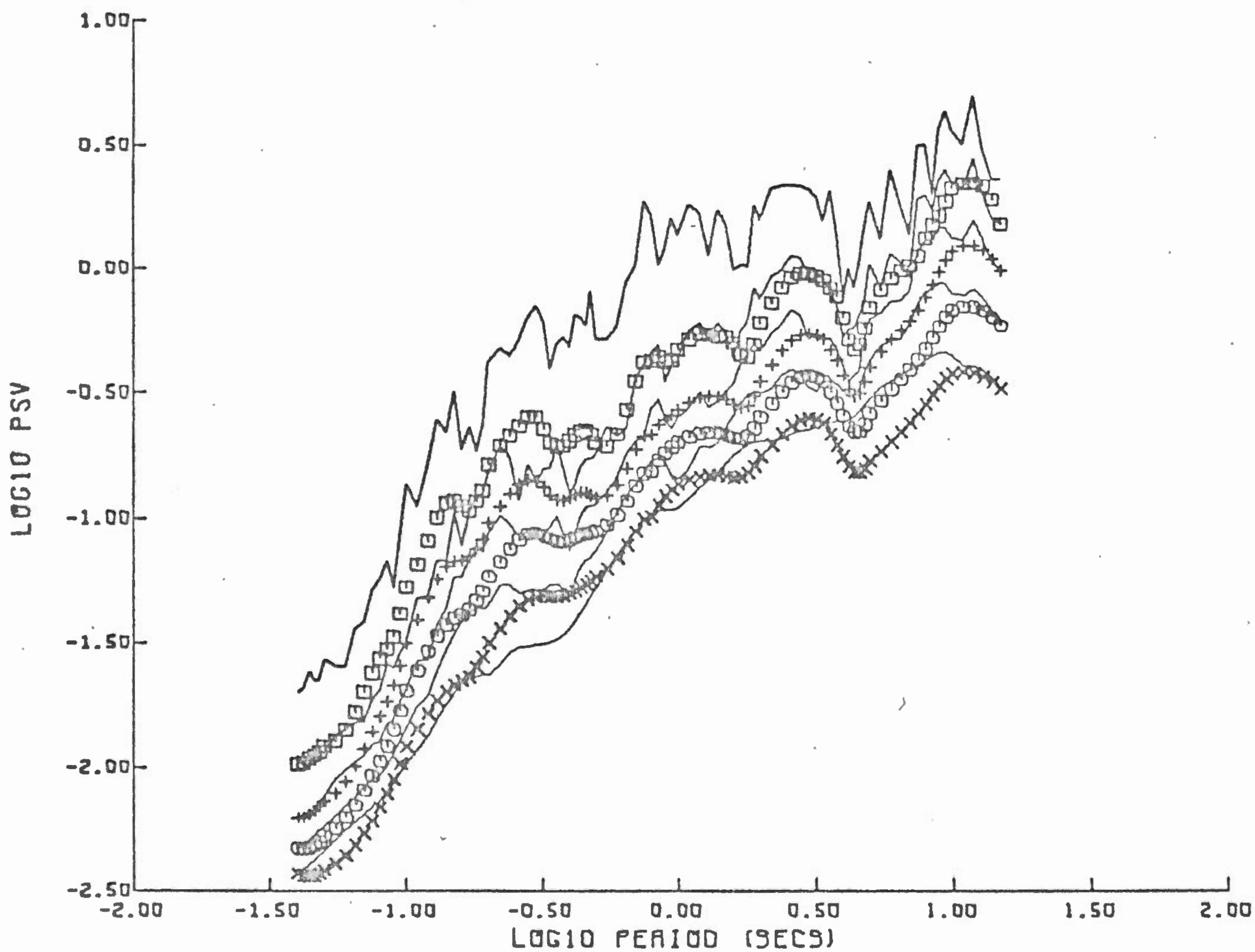
PSEUDO-VELOCITY SPECTRA PREDICTIONS

Fig 7



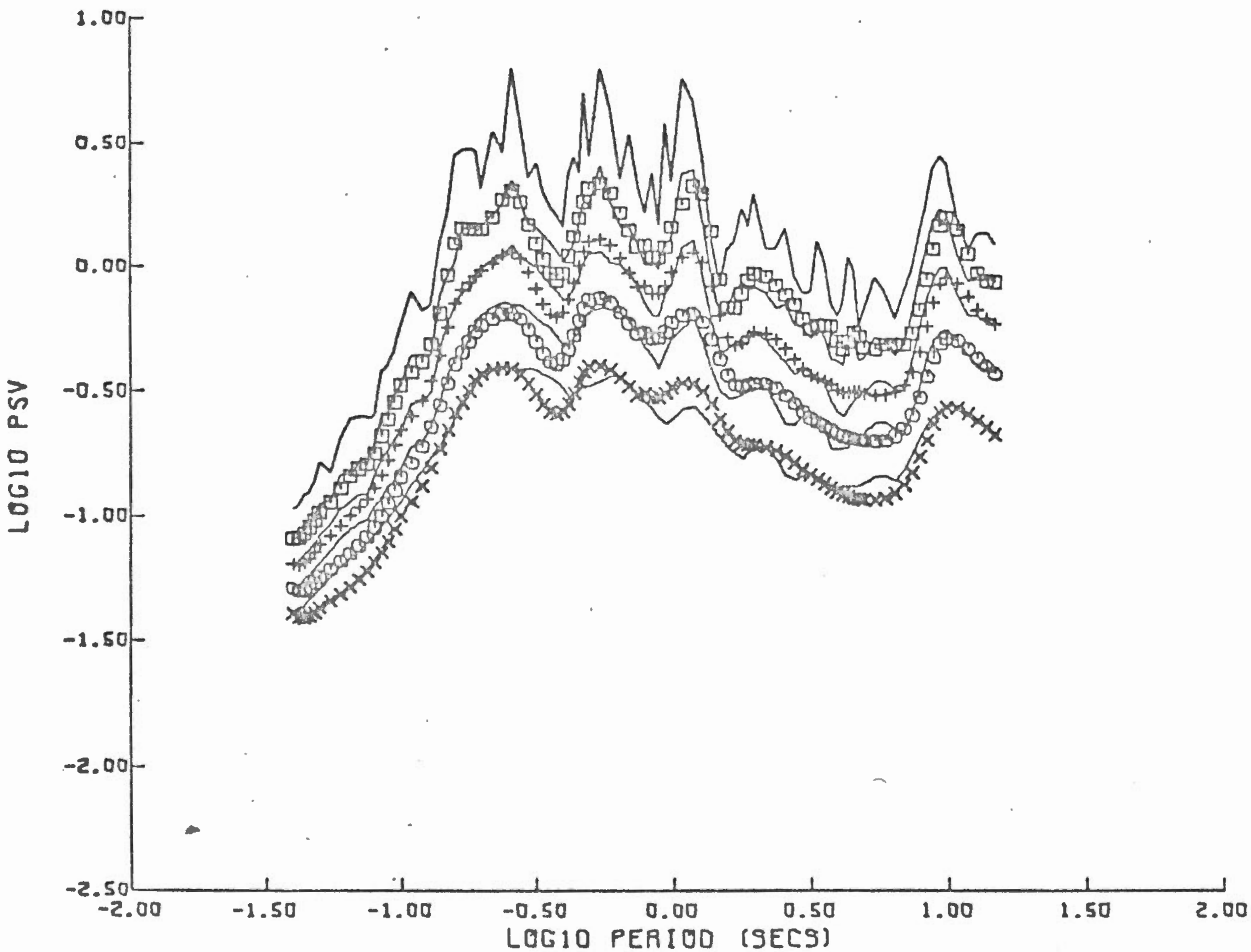
PSEUDO-VELOCITY SPECTRA PREDICTIONS

Fig 8



PSEUDO-VELOCITY SPECTRA PREDICTIONS

Fig 9



PSEUDO-VELOCITY SPECTRA PREDICTIONS

Fig 10