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R. MELDRUM STEWART, M.A., *Director*

Vol. IX

Astrophysics

No. 1

THE CEPHEID PROBLEM

BY

F. HENROTEAU, D. Sc.

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THE CEPHEID PROBLEM

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Chapter 1

THE GENERAL CHARACTERISTICS OF CEPHEID VARIATION

The nature of Cepheid variation is a very well known problem, whose study has received a great impetus from the work of such astronomers as Curtiss, Duncan, Guthnick, Hagen, Hertzsprung, Ludendorff, Luizet, Nijland, Perrine, Shapley and others. In the present state of our knowledge it appears that the variations of stars of the δ Cephei type, and their underlying causes, have a great many characteristics in common with those of the β Canis Majoris type, as has already been pointed out by the writer. Not only these two types, but also others, ought perhaps to be included in the general term Cepheids.

The present article is based on observations covering several years. The work of observing, as well as of measurement and reduction, has been shared equally between Mr. J. F. Frédette and the writer, while during the past year Mr. R. Callander also took a large share of the work. Thanks are also due to Mr. Frédette for many valuable practical suggestions. It is proposed first to review briefly the characteristics of Cepheids and all the allied types, emphasizing especially the analogies existing between them; fresh studies of stars of the β Canis Majoris type which have a bearing on the problem, as well as results of the photographic study of some Cepheids, will also be included; at the close will be found some suggestions concerning the causes of Cepheid variation, as well as some remarks on what Shapley indicated as the period-luminosity relation in this type of variable. What it is proposed to include under the general term Cepheids would then be represented by the following types:—

1. δ Cephei type.
2. ζ Geminorum type.
3. Cumulids.
    A. Antalgol type, such as RR Lyrae.
    B. Cluster variables.
4. β Canis Majoris type.
5. α Orionis type.

Below is given a summary of the characteristics of these different subdivisions grouping together, however, the stars of the δ Cephei, ζ Geminorum and Antalgol types, the others being treated separately.

The δ Cephei, ζ Geminorum and Antalgol types

Although the characteristics of these types are fairly familiar it will be of interest to present them here succinctly; such a presentation will be of service in a comparison with the characteristics of the other types, especially those of the β Canis Majoris type. The number of stars included in these classes is fairly large, as will be seen from the accompanying table, which includes more particularly those situated in the northern hemisphere, with their respective elements.

TABLE OF CEPHEIDS AND THEIR ELEMENTS

No.	Name	α 1855·0	δ 1855·0	Spectral Class	Julian date of maximum or minimum	Period days	Visual Magnitudes	M—m days
		h m s	° '					
1	SY Cassiop........	0 7 26	+57 37	G5	M2417911·615	4·07098	9·3—10·2	1·25
2	SW Androm.......	0 16 8	+28 36	A	8132·805	0·44185	9·2—10·0	0·052
3	TU Cassiop........	0 18 31	+50 29	Fo—F6	9302·12	2·139	7·3— 8·4	0·54
4*	α Urs. Min......	1 6 31	+88 32	F8	8985·936	3·9681	2·3— 2·4	1·984
5	RR Ceti...........	1 24 41	+ 0 36	F	7501·455	0·553022	8·4— 9·0	0·080
6	RW Cassiop........	1 27 49	+57 1	F	7062·5	14·80	8·8—10·2	5·8
7	V Arietis.........	2 7 12	+11 34	N	8267·121	0·99248	8·3— 9·0	0·368
8	SU Cassiop........	2 39 7	+68 17	A8—F5	7287·30	1·9498	5·9— 6·3	0·90
9	RW Camelop.......	3 42 32	+58 13	K5	7857·4	16·402	8·5— 9·2	6·8
10*	SZ Tauri..........	4 28 49	+18 15	F4—G2	8724·12	3·1484	7·2— 7·7	—
11	SV Persei.........	4 39 36	+42 2	F8	7830·4	11·13	8·6— 9·2	—
12	RX Eridani........	4 43 11	−16 0	F5	9853·2826	0·4593	8·8— 9·6	—
13	SU Aurigae........	4 46 45	+30 20	F8	7973·734	0·470143	8·4— 9·0	0·225
14	U Leporis........	4 50 4	−21 30	A	5020·3	{ 0·58144 Variable	9·1—10·0	—
15	RX Aurigae........	4 51 22	+39 44	G5	5083·43	11·6263	7·2— 8·1	—
16	SX Aurigae........	5 1 27	+41 59	A	m2420446·654	{ 1·53234 Variable	8·4— 9·3	—
17	SY Aurigae........	5 2 19	+42 39	G5	2417833·4	10·137	8·8— 9·5	—
18	Y Aurigae........	5 18 19	+42 19	M?	5420·64	3·8590	8·6— 9·6	0·73
19	RZ Geminorum....	5 53 52	+22 14	F2?	8313·313	5·52943	9·0— 9·8	1·728
20	SS Geminorum....	5 59 49	+22 38	K	8288	44·6	8·2— 9·3	20
21	RS Orionis........	6 13 57	+14 44	F5?	8274·65	7·5665	8·2— 8·9	2·89
22	T Monocerotis...	6 17 24	+ 7 10	F4—F8	M2410011·200	27·0122	6·0— 6·8	5·1
23	RT Aurigae........	6 19 15	+30 35	A8—Go	7173·3	3·7282	5·0— 5·9	1·22
24	W Geminorum....	6 26 39	+15 26	F3—Go	3266·34	7·91603	6·4— 7·7	2·91
25*	ζ Geminorum....	6 55 30	+20 47	Go	0640·60	10·15382	3·7— 4·1	5·23
26*	RU Camelop.......	7 5 59	+69 56	R	7610·96	22·172	7·9— 9·0	9·4
27	RR Geminorum....	7 12 18	+31 9	F5?	6223·286	0·3972927	9·7—10·6	0·050
28	X Puppis........	7 26 30	−20 36	G5	5021	25·953	8·0— 9·0	5·0
29	Z Cancri........	8 14 6	+15 27	Mc	8006	74	8·5— 9·2	37
30*?	W Urs. Maj......	9 33 32	+56 37	F8p	m 6129·19264	0·333640	7·9— 8·5	—
31	Z Leonis........	9 43 48	+27 35	Mb	M 8060·0	56·36	7·9— 9·6	—
32	ST Urs. Maj......	11 19 54	+45 59	Mb	m 8229·0	8·8?	6·7— 7·8	—
33	SU Draconis.......	11 29 38	+68 8	A2	M 9708·276	0·6604347	8·9— 9·6	0·15
34	SW Draconis.......	12 10 38	+70 19	F4	8086·2962	0·56965	8·3—10·4	0·125
35	W Virginis........	13 18 33	− 2 37	Pec	2402708·2666	17·2711	8·7—11·0	8·20
36	RV Urs. Maj......	13 27 34	+54 44	F5?	2417861·434	0·468058	9·4—10·3	0·16
37	V Urs. Min.......	13 36 7	+75 3	Mb	9216	71	7·5— 8·7	32
38	RS Boötis.........	14 27 21	+32 23	B8—Fo	8115·626	0·377333	9·2—10·2	0·056
39	RY Boötis.........	14 43 12	+23 38	F5	9229·1	9·0 ±	7·1— 7·4	3
40	RW Draconis.......	16 32 54	+58 8	A?	7407·27917	0·442938	9·9—11·0	0·05
41	Y Ophiuchi.......	17 44 52	− 6 6	F5—Go	2408694·25	17·1207	6·2— 7·0	6·22
42*	W Serpentis......	18 1 31	−15 34	Go	2419223·0	14·13	8·5— 9·6	—
43	WZ Sagittarii......	18 8 28	−19 7	Kp	9229·4	21·7	7·7— 9·2	7
44	Y Sagittarii......	18 12 51	−18 55	F8	0175·10	5·7734	5·8— 6·6	2·10
45	XX Sagittarii......	18 16 20	−16 52	G?	9241·45	6·43	8·3— 9·6	2
46	U Sagittarii......	18 23 21	−19 13	F7	M2414935·3	6·74467	7·0— 8·0	3·3
47	Y Scuti...........	18 30 9	− 8 29	G5	m 7734·8	10·347	8·7— 9·2	3·95
48	RU Scuti..........	18 34 17	− 4 15	K	M 9217·5	20·3	7·9— 9·9	9
49	RZ Lyrae..........	18 38 14	+32 39	A?	8450·235	0·5112750	9·9—10·3	0·05
50*	YZ Sagittarii......	18 41 6	−16 53	F9	9645·9	9·553	7·2— 7·7	—

TABLE OF CEPHEIDS AND THEIR ELEMENTS—*Concluded*

No.	Name	α 1855·0	δ 1855·0	Spectral Class	Julian date of maximum or minimum	Period days	Visual Magnitudes	M—m days
		h m s	° ′					
51	S Scuti..........	18 42 28	− 8 4	N	5979	23	6·4— 7·3?	—
52	SZ Aquilae........	18 57 18	+ 1 6	Gp	7740·811	17·1362	8·2— 9·2	6·12
53	TT Aquilae........	19 0 52	+ 1 4	F9p	1873·865	13·753	7·3— 7·9	5·30
54	RR Lyrae.........	19 20 51	+42 30	B9—F2	9697·764	0·566826	6·8— 7·7	0·12
55	U Aquilae........	19 21 33	− 7 20	F7	0170·325	7·02387	6·2— 6·9	2·3
56	XZ Cygni.........	19 29 29	+56 5	A	7201·25417	0·46659	8·7— 9·3	0·104
57	U Vulpeculae.....	19 30 17	+20 1	FGKK5	4200·253	7·98950	6·9— 7·6	3·464
58	SU Cygni........	19 39 0	+28 55	F5	4202·820	3·845612.	6·7— 7·3	1·29
59	TW Aquilae.......	19 44 18	+13 37	K?	9288	96	10·6—12·7	—
60	η Aquilae........	19 45 5	+ 0 38	A8—G5	2396168·732	7·176382	3·7— 4·3	—
61	S Sagittae.......	19 49 26	+16 15	F4—G3	2409863·324	8·381613	5·4— 6·1	2·60
62	X Vulpeculae.....	19 51 27	+26 10	K?	2417040·732	6·31896	8·5— 9·1	2·05
63	TX Aquilae.......	19 59 23	+ 3 27	F?	9294	32·7	9·3—10·5	18
64	XX Cygni.........	20 0 25	+58 32	A ·	6563·41065 { Variable	0·13486522	11·4—12·1	0·042
65	RW Aquilae........	20 5 12	+15 38	F	5587·60	7·87	8·3— 9·3	—
66	R Sagittae.......	20 7 27	+16 17	G	2400358·5	70·56	8·5—10·3	—
67	SZ Cygni.........	20 28 10	+46 6	Ko	2415097·08	15·1126	8·6— 9·9	5·4
68	V Vulpeculae.....	20 30 32	+26 6	G9	m 6411·4	37·79	8·3— 9·0	—
69	X Cygni.........	20 37 44	+35 4	Go	M2410190·678	16·38543	6·2— 7·4	6·1
70	T Vulpeculae.....	20 45 19	+27 42	F7	2409849·079	4·435521	5·5— 6·4	1·361
71	UY Cygni.........	20 50 23	+29 53	K?	2415346·3933	0·5607103	9·7—10·5	0·08
72	VX Cygni.........	20 51 52	+39 37	K	4935·0	20·1306	9·1—10·3	6·2
73	RV Capricorni.....	20 53 25	−15 48	A	7436·87	0·4676	9·2—10·7	0·075
74*	VY Cygni.........	20 58 43	+39 24	K	6370·9507	7·85926	8·6— 9·4	—
75	SW Aquarii........	21 7 50	− 0 32	F?	9686·3396	0·45932	9·9—10·8	0·12
76	VZ Cygni.........	21 45 53	+42 27	F5	7061·980	4·86384	8·4— 9·2	1·06
77	RY Pegasi.........	21 59 28	+32 48	Md	7801·00	25	10·0—10·6	—
78	Y Lacertae.......	22 3 30	+50 20	F	7615·76	4·3254	9·1— 9·6	1·06
79	δ Cephei........	22 23 48	+57 40	F2—G3	2393659·856	5·366386	3·6— 4·3	1·619
80	W Cephei........	22 30 56	+57 41	K	2412778·1	6·44	Pec.	—
81	RZ Cephei........	22 34 10	+64 6	A	9742·273	0·30864	8·6— 9·3	0·08
82	Z Lacertae.......	22 35 9	+56 4	F5?	7844·4	10·89	8·5— 9·3	4·5
83	RR Lacertae.......	22 35 41	+55 41	F	7882·6	6·412	8·7— 9·4	1·1
84	V Lacertae.......	22 42 44	+55 33	G2	6666·76	4·98269	8·5— 9·4	1·65
85	X Lacertae.......	22 43 9	+55 40	G2	m 6672·45	5·44269	8·2— 8·6	—
86	SW Cassiop........	23 0 59	+57 46	G2	M 7809·2	5·44	9·0— 9·9	—
87	RU Aquarii.......	23 16 48	−18 7	Pec.	7845	64·6	8·7— 9·7	—
88	RS Cassiop........	23 30 32	+61 38	G5	7414·36	6·295	9·1—10·7	1·8
89*	RY Cassiop........	23 45 0	+57 56	G5	7354·44	12·328	9·3—10·2	4·7
90	U Pegasi.........	23 50 34	+15 8	F?	m2415021·2469	0·1873835	9·3— 9·9	—
91	X Sagittarii......	17 39 42	−27 47	F2—G	M2402854·389	7·01188	4·4— 5·0	2·896
92	W Sagittarii......	17 57 2	−29 35	F5	2849·45	7·5946	4·3— 5·1	3·00

The stars marked with an asterisk are of the ζ Geminorum type, while those with periods smaller than a day may be considered as Cumulids or Antalgol stars. Peculiarities in individual stars are indicated in the following remarks:—

10. Shapley thought the variability of this star could be explained by the rotation of a Jacobian ellipsoid (A.N. 4653).

14. The period of U Leporis is about $0^d.58144$ but according to Innes is variable (A. J. 468 and 486).

16. The period of SX Aurigae was given as $1^d.53234$ but is suspected to be variable.

17. The descending branch of the light curve is shorter than the ascending, which is contrary to the known behaviour of stars of the δ Cephei type.

26. Shapley thinks that RU Camelop. can be explained as an ellipsoidal variable (Laws Obs. Bul., No. 21).

29. Z Cancri is inclined rather to the type of long-period variable.

30. W Ursae Majoris, for a long time considered a Cepheid, has been found to be a star of the β Lyrae type with the two minima equal (period 8 hours). Both spectral components show very wide lines due to rapid rotation (Contributions from the Mt. Wilson Observatory, Vol. VIII).

35. There are unexpected irregularities in the light curve of W Virginis. Bright lines are present in the spectrum, and the variations of these bright lines probably alter the simple Cepheid character of the star, as would probably be shown if the variation of the continuous spectrum alone were considered.

42. The maximum light of W Serpentis seems to vary, while the minimum is very sharply defined.

51. S Scuti is possibly an irregular variable. Merrill has shown that when a variable star is of class N it is usually irregular.

61. The light curve of S Sagittae was found to vary greatly in appearance. Similar variations in radial velocity are suspected by Curtiss. Its variation will be referred to in the course of this article.

81. Irregularities in the light curve. According to Shapley the mean velocity of the star is in excess of 1000 km. per sec.

The principal characteristics of the stars of the δ Cephei and Antalgol types are the following:—

1. They show a regular variation having usually a constant or perhaps (as has been found in a few cases) a very slightly variable period.

2. Their periods range from a few hours to several days, as may be seen from the above table. It is probable that they gradually merge into the long-period type.

3. The visual range of variation is rarely over 1·5 magnitudes, often in the neighbourhood of 0·8 magnitudes. Photometry of precision is revealing some very small ranges as for example in the case of Polaris.

4. The variation of light is continuous; the ascending branch of the curve is almost always shorter than the descending one, as may be deduced from the column $M—m$ of the above table. (In very rare cases such as that of SY Aurigae the reverse occurs, see remark 17 above). When the two branches of the curve are of the same length the star is classified as a Geminid (ζ Geminorum), in which case it might be considered as a star of the β Lyrae type with two equal minima, or as an ellipsoidal variable. W Ursae Majoris, for example, is certainly a star of the β Lyrae type (Algol type) with its two minima equal, as has already been pointed out in remark 30. Unlike the Cepheids it is a dwarf star of absolute magnitude 3·5.

According to Dr. Shapley the variables RU Camelopardalis, SZ Tauri and the southern variable S Antliae may be explained by the rotation of a Jacobian ellipsoid having a strong darkening at the limb.[1] Such a supposition has also been made by Stebbins in the case of π^5 Orionis.[2] On the other hand SZ Tauri possesses most of the characteristics generally attributed to Cepheid variables, some of which would disagree with the theory of a rotating Jacobian ellipsoid.

5. The light curve is usually smooth and flowing but in many cases there are evidences of secondary maxima and minima. A good deal of discussion has occurred as to whether these secondary oscillations are real or due to errors of observation. The photographic light curves published by Plummer and Martin in Monthly Notices show a great many oscillations, while similar light curves determined by F. C. Jordan with the large telescope of the Allegheny Observatory do not show them at all. Accurate light curves determined with the photo-electric cell show that secondary humps on the descending branch really exist; on the hypothesis of a binary system these humps might coincide with the peri-astron and apastron passages. The photo-electric light curve of η Aquilae as determined by C. C. Wylie is a good illustration of these secondary humps (see figure, Ap. J., Vol. 56, p. 229).

6. The photographic range of variation as far as determined is usually greater than the visual, frequently one and a half to two or even in some cases three times as large. In other words there is a continuous variation of the colour-index synchronous with the variation of light. The stars are therefore much redder at minimum than at maximum.

Schwarzschild was one of the first to discover this difference of range between the photographic and visual curves[3]; he found the photographic range of η Aquilae to be double the visual. Wirtz found similar properties for δ Cephei and ζ Geminorum,[4] and Wilkens for X Cygni and other stars.[5] The photographic work of Martin and Plummer[6] suggests similar results for the Antalgol variables. Contrary to all other experience, however, Parkhurst and Jordan find that in the case of XX Cygni[7] the photographic range is less than the visual.

There has appeared recently a series of observations made by Galissot at the Lyons Observatory[8] with a Nordmann's heterochrome photometer (using red, green and blue light), showing that the colour index of ζ Geminorum may be considered as constant throughout the variation. The curves which he obtained, published by Prof. Mascart, director of the observatory, show very small variations between the three colours (smaller than the possible errors of observation). Prof. Mascart concludes therefore that the variation of ζ Geminorum must be attributed to a cause similar to that which produces the Algol variables, or some other cause distinct from that giving rise to the Cepheids. On the other hand the determinations of the light curve and colour index curve of the same star were also made recently by Prof. Guthnick,[9] who finds a decided variation in the colour index.

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[1] Laws Obs. Bul. No. 21, p. 73.
[2] Ap. J., Vol. 51, p. 218 (1920).
[3] Pub. der Kuffnerschen Sternwarte, Vol. 5C, p. 100 (1900).
[4] A. N., Vol. 154, p. 327, 1901.
[5] A. N., Vol. 172, p. 316, 1906.
[6] M. N., Vol. 73, p. 166, p. 440, 74, p. 225.
[7] Ap. J., Vol. 23, p. 84, 1906.
[8] Bul. de l'observatoire de Lyon, Vol. 4, p. 99, 1922.
[9] A. N. Jubilaumsnummer Tafel 2.

7. The variation in colour index is accompanied (as might have been expected) by a variation in spectral class. The class is usually earlier (indicating of course higher temperature) at maximum, and later at minimum brightness; a shift of the maximum intensity in the spectra of Cepheids was discovered by Albrecht[1] and confirmed by Kiess[2] and other Lick observers, while a change of spectrum was first discovered by Albrecht and Duncan.[3] Shapley however determined the variation in spectral type definitely for several Cepheids; according to Albrecht the range of variation in spectral class is about one class interval and is independent of the period.[4] Adams and Joy have noted a marked distinction in spectral types of Cepheids, depending on whether obtained from hydrogen lines alone or from the general spectrum.[5] The range of variation in class is much more considerable as obtained from the hydrogen lines. The variation in spectral class is also accompanied by a variation in the sharpness and width of the lines. Prof. Adams, for example, measuring the widths of the lines in δ Cephei finds the following results[6]:—

Average width at maximum 0·214 A

Average width at minimum 0·403

These results recall very much the changes of line widths in the spectra of the stars of the β Canis Majoris type.[7]

8. All the Cepheids show variation of radial velocity with the period of the light c anges. The epoch of minimum radial velocity (most rapid approach) coincides closely with that of maximum light, and that of maximum radial velocity with minimum light. These changes of radial velocity were at first attributed to the fact that the Cepheids might be spectroscopic binaries; on this assumption it is found that the orbits possess very peculiar properties; their eccentricities are usually very large, which is contrary to what usually occurs for ordinary spectroscopic binaries of similar periods; the values of the angular distance of periastron from the receding node present a maximum frequency in the first quadrant and practically never occur in the third quadrant; in ordinary spectroscopic binaries these values are distributed at random; usually the value

of the mass function $\dfrac{m_1^3 \sin^3 i}{(m+m_1)^2}$

(where m is the mass of the primary, m_1 that of the secondary and i the inclination between the plane of the orbit and the plane perpendicular to the line of sight) is extraordinarily small. Perrine points out that the larger the value of the mass function the larger the secondary humps in the light curve. The value of $a \sin i$ is also small, the greatest value found being approximately 2,000,000 km. and the least 45,000; this led Shapley to say that, interpreted as spectroscopic binaries, the Cepheids move in orbits whose apparent radii average less than one-tenth the radii of the stars themselves.[8] All these characteristics and others led first Shapley, then Ludendorff, Eddington and others to reject the binary theory of Cepheids and adopt pulsation or other factors as the cause of Cepheid variation.

[1] L. O. B., Vol. 4, p. 131.
[2] L. O. B., Vol. 7, p. 140.
[3] L. O. B., Vol. 5, p. 93.
[4] Ap. J., Vol. 54, pp. 161-190.
[5] Proc. Nat. Ac. Sc. Washington, 4 p. 129-132 (1918).
[6] The Observatory, Vol. 42, p. 167 (1919).
[7] L. O. B., Vol. 9, p. 158 (1918).
   Ap. J., Vol. 40, p. 459.

For many Cepheids the total range of velocity variation is so small that secondary oscillations and other irregularities of considerable importance may easily be lost in the accidental errors. In the cases of ζ Geminorum[1] and W Sagittarii[2] the velocity curves show marked humps, indicating, if orbital motion is assumed, considerable departure from simple elliptic motion. In the cases of Y Ophiuchi and T Vulpeculae Albrecht suspects that some spectral lines give abnormal velocities[3] while in the case of W Virginis, which exhibits emission lines, the Mt. Wilson observers find that marked differences are shown in the radial velocities given by the dark and bright lines.[4] The apparent brightness of Cepheids being usually small, very few complete investigations of the radial velocities have been made; usually only sufficient observations have been secured during a short interval of time to determine one velocity curve (of the same period as that of light variation). In some cases however, such as α Ursae Minoris and Y Sagittarii.[5] there are direct evidences of a variation of the centre of mass of the short-period system; Ludendorff has also pointed out from the observations of Albrecht that a variation of amplitude and perhaps of center-of-mass velocity probably exists in the case of Y Ophiuchi[6]; the system of δ Cephei is thought by Bélopolsky to be triple[7], while there are indications from Belopolsky's and Wright's observations that η Aquilae is also a triple system. Analogy with the fact that all stars of the β Canis Majoris type which have been investigated, and which are most probably a special kind of Cepheids, behave like triple systems, suggests that Cepheids when more thoroughly investigated will also prove to be such, or rather suggests the superposition of a physical phenomenon (pulsation or any other action accounting for the variation of brightness) on a purely mechanical phenomenon, the revolution of the star around a companion in an orbit.

Prof. P. Guthnick from his photo-electric cell investigations finds that the light variations of many stars seem to be due to a combination of an Algol type variation, or variation due to eclipse (orbital motion) with a plain Cepheid variation (most likely not orbital)[8]; these two types of variation have different periods. The idea is hence suggested that in the case of an ordinary Cepheid no eclipse is produced, (unless a very small partial eclipse), but that the Cepheid variation is bound up with the presence of a second body which produces some kind of tidal effect upon the primary,

In a great many instances where the star is too faint or has not been investigated with a spectrograph, variations in the shape and the period of the light curve are strong indications of variations in the centre-of-mass velocity of the short-period oscillation. A remarkable case is that of S Sagittae, whose many light curves determined by several observers are so different from one another; the curves generally show a secondary minimum, perhaps indicative of partial eclipse; this secondary minimum in different curves is more or less pronounced, while the values of the two maxima are not always equal

[1] Ap. J., Vol. 13, p. 94.
[2] L. O. B., Vol. 3, p. 36.
[3] L. O. B., No. 118.
[4] Mt. Wilson report, 1921, p. 269.
[5] Ap. J., Vol. 56, p. 373, 1922.
[6] A. N., Vol. 203, p. 368.
[7] Mitt. Pulk, Vol. 3, p. 70, 1909.
[8] Veroff der K. Sternw. zu Berlin Babelsberg Band II, Heft 3, p. 129.

and are separated by different intervals of time. A very good representation of light curves of S Sagittae obtained by different observers has been given by Luizet,[1] while a comparison of the visual and photo-visual curves has been given by F. C. Jordan:[2]

Assuming the radial velocity curves of Cepheids to be due to orbital motion (which is probably a wrong assumption), the orbits given in the accompanying table have been determined.

[1] Sur l'Etoile variable S Fleche Bul. Soc. Astr. de France, 1907, p. 277.
[2] Ap. J., Vol. 50, p. 195.

Star	α 1900·0	δ 1900·0	P	T Julian date	ω	e	K	$\dfrac{m_1{}^2 \sin^2 i}{(m+m_1)^2}$	a. sin i	γ	Remarks
	h m	° ′			°		km.		km.	km.	
α Urs. Min.....	1 22·6	+88 46	3ᵈ·9683	2415398·50	113·6	·20	3·00	·00001	160,400	Var.	Hartmann.
			3·9683	4890·04	80·0	·13	3·04	·00001	164,500	Var.	Hobe.
			11·9 years?	....	293	·35	2·98	·0098	166,800,000	−14·8	Centre of mass of short period.
SU Cass.........	2 43·0	+68 28	1ᵈ·9495	....			11·0	·0003	295,000		
SZ Tauri.......	4 31·4	+18 20	3·1481				10·9	·00039	460,000		
RT Aurigae......	6 22·1	+30 34	3·7282	M+3ᵈ·423	95·0	·368	17·96	·0018	856,000	+21·43	Duncan (1908).
			3·72806	M+3ᵈ·686	115·5	·428	11·97	....	554,700	+19·96	Kiess (1917).
ʒ Gemin.......	6 58·2	+20 43	10·154	m+1·313	333	·22	13·2	·0023	1,797,800	+ 6·8	Campbell, unexplained oscilla-
S Muscae......	12 7·4	−69 26	....				16 ?			+ 8?	tion.
R Triang A....	15 10·8	−66 8					17 ?			−18 ?	
S Triang A....	15 52·2	−63 30					14 ?			+ 7 ?	
S Normae.....	16 10·6	−57 39					12 ?			0 ?	
RV Scorpii......	16 51·8	−33 27					16 ?			−25 ?	
X Sagittarii...	17 41·3	−27 48	7·01185	2416723·05	93·6	·40	15·2	·0011	1,334,000	−13·50	Moore.
Y Ophiuchi....	17 47·3	− 6 7	17·1207	M+2ᵈ·6	209·2	·10	8·5	·0011	1,999,000	− 5·0	Albrecht. According to Luden-dorff vel. curve of Y Ophiuchi varies in amplitude. A. N. 203 p. 368.
W Sagittarii....	17 58·6	−29 35	7·59460	M+6·20	70·0	·32	19·5	·00499	1,930,000	−28·6	Curtiss Secondary oscillations.
Y Sagittarii....	18 15·5	−18 54	5·773268	M+ 4·51	43·0	·21	19·3	·004	1,500,000	+ 3·6	Duncan (1908).
				M+ 5·05	74·5	·42	20·6	·003	1,354,000	− 5·9	Duncan (1921).
RR Lyrae.......	19 22·3	+42 36	0·566826	M+ 0·508	96·85	·271	22·2	·00057	166,500	−68·7	Kiess.
U Aquilae......	19 24·0	− 7 15	7·02387	....						......	Albrecht.
SU Cygni.......	19 40·8	+29 1	3·844	M+ 2·5	345·8	·21	25	·0058	1,350,000	−33·4	Madrill.
η Aquilae......	19 47·4	+ 0 45	7·176	m+ 1·92	90	·163	16·3	·0031	....	−13·7	Bélopolsky.
				M+ 6·210	68·91	·489	20·59	·0043	1,545,000	−14·16	Wright.
S Sagittae.....	19 51·5	+16 22	8·3832	....	69·9	·35	19	·0049	2,000,000	−12·5	Madrill.
X Cygni.......	20 39·5	+35 14	16·38543	M+14·685	101·1	·246	28·03	·034	6,121,000	+ 9·32	Duncan.
T Vulpeculae ..	20 47·2	+27 52	4·43578	M+ 3·76	111	·43	17·6	·0018	969,180	− 1·3	Albrecht.
δ Cephei......	22 25·4	+57 54	5·366386	m+ 1·07	88	·46	20·5	·0034	1,300,000	+ 0·5	Bélopolsky.
				m+ 1·002	82·8	·355	19·81	·0037	1,370,000	Var.	Bélopolsky.
				2417888·428	85·385	·484	19·675	·0028	1,270,600	−16·83	Moore.

The spectroscopic investigations of many of these Cepheids are incomplete, while only five orbits have been determined a second time, those of α Ursae Minoris, RT Aurigae, Y Sagittarii, η Aquilae and δ Cephei. Of these at least the first three, and perhaps all, have a strong resemblance to triple systems. It can be seen that considerable work has yet to be done on the orbits of Cepheids before it will be possible to present a satisfactory theory of these variables.

One of the most important characteristics of the Cepheid variables is that they all appear to be giant stars or even super-giants, stars of exceedingly small density and of enormous volume, though not necessarily massive. From a consideration of their proper motions Prof. Hertzsprung was the first to point out that they were giants.[1] Direct determinations of parallaxes of Cepheids show that they must be at a considerable distance and hence very luminous.

TRIGONOMETRIC PARALLAXES OF CEPHEID VARIABLES

Star	Parallax	Observer
	"	
α Ursae Min	+0·041	Flint et al.
SU Cassiop	+0·010	Mt. Wilson.
RX Aurigae	+0·001	Mt. Wilson.
T Monocerotis	−0·009	McCormick.
RT Aurigae	+0·031	Mt. Wilson—Sproul.
	+0·003	McCormick.
ς Gemin	+0·015	Allegheny-Sproul.
	+0·0011	McCormick.
RR Lyrae	+0·008	Mt. Wilson.
U Vulpeculae	+0·009	McCormick.
η Aquilae	+0·004	McCormick.
S Sagittae	+0·005	McCormick.
X Cygni	+0·006	Sproul.
T Vulpeculae	+0·018	McCormick.
δ Cephei	+0·011	Allegheny.

The probable errors of these parallaxes vary from ±0″·003 to ±0″·011, that of α Ursae Minoris being perhaps somewhat larger. Quantitatively, they do not mean much, but qualitatively, they show that the Cepheids must be very distant. The investigations of the spectra of these stars show them to be in a very highly ionized state. The lines are usually narrow, well-defined, sharp—they are all of what is called the c characteristic, a characteristic which usually corresponds, as in α Cygni, to exceedingly small density. The proper motions of 74 Cepheid variables have been studied by R. E. Wilson[2]; they have been found exceedingly small, only two exceeding 0″·2 per annum, and the mean of the remaining 72 being 0″·025 per annum. Taking account of the radial velocities of 25 of these stars, the mean parallax of the group is found to be 0″·0030, in good agreement with the parallaxes determined directly.

[1] A. N., Vol. 196, p. 203.
[2] Pop. Ast., Vol. 31, p. 258, 1923. See also A. J., Vol. 35, p. 35.

From spectral characteristics there are very few stars that seem to be super-giants and they have been classified into *Cepheids* and *Pseudo-Cepheids*. The spectra of these two classes are very much the same, with very sharp and narrow lines, very strong enhanced lines corresponding to ionized elements. In particular, attention may be called to the following enhanced lines which are also prominent in the spectrum of the solar chromosphere and in stars of the α Cygni type.

Sr 4077...	?	4375
Sr 4215...	*Fe*	4385
Fe 4233...	*Ti*	4534
Y 4246...	*Fe*	4584
Ti 4290...		

The stars of the first class are known to vary considerably in light, while those of the second have certainly no large variation, if any. Many of the latter, however, if investigated with a photo-electric cell might prove to be variable also, but with a very small amplitude; they might be similar to α Ursae Minoris, whose radial velocity curve and magnitude variation are not very marked. Following is a list of the principal Pseudo-Cepheids as discovered by Prof. Adams:—

TABLE OF PSEUDO-CEPHEIDS

No.	Star	α 1900	δ 1900·0	Vis. Mag.	Spectral Class	Annual Proper Motion	Adams Spectrosc. parallax	Trig. parallax	Radial Vel.
		h m	° ′			″	″	″	km.
1	14 Persei...............	2 37·6	+43 52	5·6	Gop	0·004	0·003	....	− 3·8
2	α Persei...............	3 17·2	+49 30	1·9	F5	0·039	0·023	0·017	Var.
3	58 Persei...............	4 29·8	+41 4	4·5	G4p	0·025	0·002	0·028	Var.
4	β Camelop..............	4 54·5	+60 18	4·2	F7p	0·013	0·004	....	− 1·4
5	ε Aurigae...............	4 54·8	+43 41	3·4	F5p	0·014	0·008	0·060	Var.
6	α Leporis...............	5 28·3	−17 54	2·7	F4p	0·003	0·021	0·014	+24·3
7	46 Aurigae...............	6 17·2	+49 20	5·1	K2p	0·014	0·002	....	Var.
8	δ Can. Maj.............	7 4·3	−26 14	2·0	G2p	0·005	0·010	....	Var. ?
9	ξ Puppis...............	7 45·1	−24 37	3·5	G6p	0·007	0·003	....	Var.
10	ρ Puppis...............	8 3·3	−24 1	2·9	F7p	0·100	0·010	0·031	+46·0
11	29 Monocer.............	8 3·6	− 2 42	4·4	G3p	0·025	0·002	....	+29·0
12	298 G. Puppis............	8 18·6	−26 2	5·9	F5p	0·017	0·003	....	
13	β Draconis..............	17 28·2	+52 23	3·0	Gop	0·016	0·005	0·005	−20·5
14	*V* Herculis..............	17 54·7	+30 12	4·5	F2p	0·004	0·012	....	−21·6
15	π Sagittarii.............	19 3·8	−21 11	3·0	F4p	0·040	0·017	0·017	−10·1
16	α¹ Capricorni...........	20 12·1	−12 49	4·6	Gop	0·015	0·002	−0·012	−25·5
17	γ Cygni................	20 18·6	+39 56	2·3	Gop	0·003	0·009	0·014	
18	ρ Capricorni A..........	20 23·2	−18 9	5·1	A9	0·026	0·010	0·024	+21·0
19	41 Cygni...............	20 25·3	+30 2	4·1	F6	0·010	0·009	−0·018	−18·4
20	ξ Cygni...............	21 1·3	+43 32	3·9	K2p	0·007	0·004	−0·003	Var.
21	ζ Capricorni...........	21 21·0	−22 51	3·9	G1p	0·023	0·003	....	Var.
22	β Aquarii...............	21 26·3	− 6 1	3·1	F9	0·017	0·009	−0·016	+ 5·6
23	γ Capricorni...........	21 34·6	−17 7	3·8	F4	0·188	0·013	0·018	Var.
24	α Aquarii...............	22 0·6	− 0 48	3·2	Go	0·015	0·006	0·010	+ 6·8
25	5 Lacertae...............	22 25·4	+47 12	4·6	K2p	0·021	0·003	0·005	−11·5
26	B. D. + 56° 2923......	22 55·9	+56 25	5·5	G2p	0·009	0·002	....	
27	89 Aquarii..............	23 4·6	−23 0	4·9	F8	0·013	0·008	....	− 4·4
28	ρ Cassiopeiae..........	23 49·4	+56 57	4·8	G5p	0·007	0·002	0·032	−42·6

The variations of radial velocity of these stars are in most cases fairly small. A period of $4^d \cdot 0938$ has been found for α Persei by Hnatek in Vienna[1]; the semi-amplitude of velocity variation is $0 \cdot 83$ km. The stars 2, 5 and 28 are suspected by Guthnick to have a small variation of light; an elaborate study of these three is greatly to be desired, especially on account of their fairly large trigonometric parallaxes. It seems quite possible that an exhaustive investigation of the Pseudo-Cepheids would show them to be Cepheid variables with small variations of light; α Cygni although of class A2 and, on account of this, not classified by Adams among the Pseudo-Cepheids, is nevertheless a typical star of this kind; it has been found by Guthnick to vary in light[2]; its radial velocity is also variable. α Cygni is thus probably a Cepheid variable. The consideration of the Cepheids and the Pseudo-Cepheids leads us then to this: the Cepheids are all stars of very low density; but are all stars of low-density Cepheids, or are they merely Cepheids when some kind of action, for example that of a satellite, plays its part?

10. There are a few more possible characteristics of the stars of the δ Cephei type which might be mentioned here, for example Ludendorff's relation[3]

$$2\,K = 47 \cdot 3\,A,$$

where K is the semi-amplitude of velocity variation and A the range of visual magnitude. This relation however is only approximate and seems to be true only for the brightest Cepheids; if the stars of the β Canis Majoris type are considered to be Cepheids the relation is far from true.

Dr. R. H. Curtiss[4] has discussed several other possible characteristics of the Cepheid variables; most of these are of very considerable interest, but the data that he had at hand were in most cases not sufficient to establish them with certainty.

Another remarkable fact is that the stars of this type exhibit a progressive tendency towards more advanced spectral class as the length of the period of light variation increases (while the range of variation of spectral class of any one Cepheid is approximately one class interval and is independent of the period).

11. RR Lyrae is the brightest, and may be considered as the typical, Antalgol star; it is the only one whose radial velocity curve has been determined[5]; a redetermination of this velocity curve is however needed. Hertzsprung has shown definitely[6] that the amplitude of its light variation, as well as the period of this variation, is not constant, the oscillation, as has already been pointed out by Shapley,[7] having most likely an approximate period of $40^d \cdot 6$. This recalls the results that have been found for the stars of the β Canis Majoris type; 12 Lacertae, which shows considerable variations in the amplitude of its short-period velocity curve, also shows similar variations in its short-period light curve, as determined so accurately by Guthnick[8]. W Baade[9] found similar results for the Antalgol star SS Cancri, and according to Hertzsprung it seems likely that all Antalgol stars show similar characteristics, *i.e.* a variation of amplitude in the short-period curve,

[1] A. N., Vol. 192, p. 245.
[2] Veroff der K. Sternwarte zu Berlin-Babelsberg, Band 1, p. 61.
[3] A. N., Vol. 193, p. 301.
[4] Pub. Detroit Obs., Vol. 1, p. 104.
[5] C. C. Kiess, L. O. B., Vol. 7, p. 140, 1913.
[6] Bul. of the Astr. Inst. of the Netherl., No. 24, 1922.
[7] Contributions from the Mt. Wilson Observatory, No. 112, 1916.
[8] A. N. Jubilaumsnummer Tafel 3.
[9] Mitt. d Hamburger Sternw. in Bergedorf 5, No. 14, 23, 1922.

this variation having most likely a period of several days or months, or, as has been remarked above, a short-period physical phenomenon and an orbit of considerably longer period. Such a phenomenon as this double variation is no doubt most easily discovered in very short-period Cepheids; in the case of longer periods most of the light curves are, so to speak, mean curves, or the combination of observations which have been taken during a long interval of time and whose phases have been computed by assuming a constant period for the variation. It would be of great advantage to have an international agreement between observatories of different longitudes, if possible encircling the earth, to follow the same Cepheids during the same lapse of time so that a set of continuous curves could be obtained; similar instruments and methods ought however to be adopted by the observers; it is practically certain that results of great value would be obtained.

The annual proper motion of RR Lyrae is $0''\cdot25$; this is considerable when compared with the proper motions of most of the stars of the δ Cephei type, and led Hertzsprung to suspect that the star is not distant. The parallax as directly measured by Van Maanen at the Mt. Wilson Observatory is $+0''\cdot006\pm0''\cdot006$, which indicates that Hertzsprung's suggestion is probably not correct, but that the star has a very large motion in space. Its mean radial velocity, $-68\cdot7$ km. as determined by Kiess, certainly indicates a large motion. Most of the Antalgol stars so far measured have very high velocities, the highest, according to Shapley, being that of RZ Cephei which is of the order of 1,100 km. per second. Probably these high velocities are due to the small masses of the stars investigated, as was pointed out in a previous article[1].

12. A very remarkable fact is that most of the stars of the δ Cephei type have a low galactic latitude, while those of the Antalgol type have galactic latitudes distributed at random.

13. It will be unnecessary to give here the different hypotheses that have been brought forward to explain Cepheid variation, a very good account of most of these theories having been given by D. Brunt in his article "The Problem of the Cepheid Variables[2]." Duncan's hypothesis, for instance[3], is rather remarkable.

THE CLUSTER VARIABLES

The cluster variables are exceedingly faint stars which are found in the globular clusters and in the Small Magellanic Cloud. They have properties very similar to those of the Antalgol type and constitute, no doubt, a particular variety of Cepheids. There is not a very large number of globular clusters in the heavens. On the Franklin Adams charts Melotte[4] counted 83 of these objects in the whole heavens, while Bailey in his catalogue of brighter clusters and nebulae counted 54[5], some having a diameter of less than 5 minutes of arc, while the largest like ω Centauri cover a larger area than that of the full moon.

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[1] Pub. Dom. Obs., Vol. VIII, p. 80.

[2] The Observatory, Vol. 36, p. 59, 1913.

[3] L. O. B., Vol. 5, p. 82.

[4] Journal of the British Astr. Ass., Vol. 25, p. 341, 1915.

[5] H. A., Vol. 60, p. 199, 1908.

In the large globular cluster Messier 3 ($13^h 37^m \cdot 6 + 28°53'$) about 132 stars of the 900 brightest ones have been found to be variables of the Cepheid type. The periods are in all cases very short, most of them in the neighbourhood of half a day[1].

The following table gives the approximate number of variables of the Cepheid type found in six of the principal globular clusters:—

Cluster	Number of Variables
ω Centauri	128
Messier 3	132
Messier 5	85
Messier 15	51
47 Tucanae	6
Messier 13	7
The latter two have probably only periods of several days.	

There is apparently a remarkable difference between the first four and the last two. Most of these variables are very faint, being of about the 14th magnitude. They show a range of approximately one magnitude or less; their periods range from about $0^d \cdot 3$ to $0^d \cdot 7$, although periods of several days have been discovered, for instance in Messier 13; long periods constitute, however, marked exceptions. The mean colour-index for those of short period is approximately $+0^m \cdot 4$, which would indicate that the mean spectral class of the short-period cluster variable is about F3 or F4; the colour-index is larger for those with periods of several days, placing the stars between spectral classes G and K. As in the stars of the δ Cephei type the amplitude of the photographic light curve is usually larger than that of the photo-visual. The most important contributors to the study of globular clusters and their variables have been Prof. S. I. Bailey and Dr. Shapley. Their extensive work is to be found in the Harvard Annals and the Contributions from the Mt. Wilson Observatory.

Fainter globular clusters than those mentioned above have been investigated at Mt. Wilson; in the faint globular cluster N. G. C. 7006, for example, Cepheids of the 19th apparent magnitude have been recorded. [Note added (January, 1925) while going through press.][2]

A very important discovery was made recently by Baade and Larink at the Bergedorf Observatory in Hamburg.[3] They investigated a large number of cluster variables, determined their periods and compared them with the periods obtained formerly by Prof. Bailey. In some cases the periods were the same; in other cases they had increased or decreased by sometimes large amounts in comparison with the length of the period. Variations as large as $0^d \cdot 01$ or $0^d \cdot 02$ were found, while smaller changes of about $0^d \cdot 001$ were also present. The question arises whether these variations of period are continuous or sudden. From mathematical considerations and careful comparisons with the work of Bailey, Baade and Larink concluded that the variations must be sudden. This phenomenon of variation of period is apparently very important, and seems to be present in a great many Cepheids and variable stars. The variations may be real or apparent; if for instance the star has a constant period but is describing a certain orbit the observed times of maximum light would necessarily shift slightly according to the position in the orbit.

[1] H. A., Vol. 78-1.
[2] Very faint Cepheids have just been discovered in two spiral nebulae. (Mt. Wilson).
[3] Astronomische Abhandlungen der Hamburger Sternwarte in Bergedorf II, 6 Hamburg, 1922.

In short-period variables such a change would no doubt be very marked and in a globular cluster we would expect to find, as Baade and Larink have found, periods that have increased, decreased or remained constant. Changes of period, as we have seen, are certain in the case of RR Lyrae; according to Kron the mean period of XX Cygni is decreasing by about a tenth of a second a year, while Roberts finds that the mean period of S Arae is decreasing by four one-hundredths of a second a year.[1]

δ Cephei for a long time was thought to have a slightly variable period; this variation was first pointed out by Chandler[2] and later maintained by Nijland,[3] but has been completely rejected by Luizet[4]; Bélopolsky, however, finds an oscillation in the spectroscopic period.[5] W. J. S. Lockyer found[6] that, while the mean period of η Aquilae is constant, there is an oscillation in the epoch of maximum through an amplitude of ten hours. The question of the variation of period in η Aquilae has been recently treated by C. C. Wylie.[7] Shapley has also found oscillations in the periods of several cluster variables,[8] while the fact of a variation of period in the short-period radial velocity curve of σ Scorpii (decrease and increase), has now been established without doubt by the writer.

While we might ascribe these changes of period to only an apparent cause (the equation of light), this could perhaps scarcely be the case with long-period variables. Luyten in an extensive memoir[9] has shown that the long-period variables R Aquilae, R Ursae Majoris and T Geminorum decrease in period. Contrary to the opinion of Prof. Turner of Oxford, who thinks that such decreases of period (which occur also in R Hydrae) are sudden, Luyten thinks them progressive and continuous.

It has already been remarked that Antalgol stars have a large mean radial velocity. Individual radial velocities of variables in clusters have not been obtained, but it is interesting to note that the mean radial velocities of clusters are also large. Following are some of these radial velocities, as determined mainly by Dr. Slipher.[10]

n. g. c.	Messier	Radial Velocity
		km.
5024	53	−170
5272	3	−125
5904	5	+ 10
6205	13	−300
6333	9	+225
6341	92	−160
6626	28	0
6934		−350
7078	15	− 95
7089	2	− 10
Large Magellanic Cloud		+260
Small " "		+150

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[1] Ap. J., Vol. 33, p. 200.
[2] A. J., Vol. 13, p. 101, 1893.
[3] A. N., Vol. 161, p. 229.
[4] Annales de l'Universite de Lyon, Nouvelle serie fascicule 33.
[5] Mitt. Pulk., Vol. 3, p. 63, 1909. See also The Observatory, Vol. 42, p .33 ℰ 1919.
[6] Dissertation Gottingen, 1896.
[7] Ap. J., Vol. 56, p. 229.
[8] Pop. Ast., Vol. 22, p. 144, 1914.
[9] Annales de l'Observatoire de Leyde.
[10] Jour. R. A. S. C., Vol. II.

The variables of the Small Magellanic Cloud

A very remarkable discovery, which has already had far-reaching consequences, is that made by Miss Leavitt for the variables of the Small Megallanic Cloud.

In 1904 and 1905 Miss Leavitt discovered nearly a thousand variable stars in the Small Magellanic Cloud from photographs made at Harvard's observing station at Arequipa, Peru; she also found more than eight hundred in the Large Magellanic Cloud. Fifty - nine of those in the Small Magellanic Cloud were measured in 1904, using a provisional scale of magnitudes,[1] and the periods of seventeen were published. They resemble the variables found in globular clusters, diminishing slowly in brightness, remaining near minimum for the greater part of the time, and increasing very rapidly to a brief maximum. The following table is a reproduction of Miss Leavitt's table giving the periods of 25 of the stars arranged in increasing order. The different columns contain the Harvard number, the brightness at maximum and at minimum as read from the light curve, the epoch expressed in days following J. D. 2,410,000 and the length of the period expressed in days. A remarkable relation between the brightness of these variables and the length of their periods will be noticed; it is found by plotting the results[2] that there is a linear relation between the magnitudes of the variables and the logarithms of their periods; since all these stars may be considered at very nearly the same distance an equivalent relation holds for the absolute magnitudes.

PERIODS OF VARIABLE STARS IN THE SMALL MAGELLANIC CLOUD

(Miss Leavitt's Table)

H	Max.	Min.	Epoch	Period
			d	d
1505	14·8	16·1	0·02	1·25336
1436	14·8	16·4	0·02	1·6637
1446	14·8	16·4	1·38	1·7620
1506	15·1	16·3	1·08	1·87502
1413	14·7	15·6	0·35	2·17352
1460	14·4	15·7	0·00	2·913
1422	14·7	15·9	0·6	3·501
842	14·6	16·1	2·61	4·2897
1425	14·3	15·3	2·8	4·547
1742	14·3	15·5	0·95	4·9866
1646	14·4	15·4	4·30	5·311
1649	14·3	15·2	5·05	5·323
1492	13·8	14·8	0·6	6·2926
1400	14·1	14·8	4·0	6·650
1355	14·0	14·8	4·8	7·483
1374	13·9	15·2	6·0	8·397
818	13·6	14·7	4·0	10·336
1610	13·4	14·6	11·0	11·645
1365	13·8	14·8	9·6	12·417
1351	13·4	14·4	4·0	13·08
827	13·4	14·3	11·6	13·47
822	13·0	14·6	13·0	16·75
823	12·2	14·1	2·9	31·94
824	11·4	12·8	4·0	65·8
821	11·2	12·1	97·0	127·0

[1] H. A., Vol. 60, No. 4, Table VI.
[2] H. C., No. 173.

It is to be noted that the shortest period given in the above table is longer than one day. From a recent investigation, however, Shapley[1] finds that the faintest variables in the Small Magellanic Cloud are Cepheids of the Antalgol type having a mean period of $0^d \cdot 64$; they apparently verify the period-luminosity law given by Miss Leavitt.

According to Prof. Bailey[2] Miss Leavitt's law does not apply to clusters as rigorously as to the Small Magellanic Cloud; there is however in each cluster a certain relation between the magnitudes and the periods. Shapley also found that in the case of cluster variables, the linear formula given by Miss Leavitt did not hold.

In a bold attempt to connect the galactic Cepheids with the Cepheids found in clusters and in the Small Magellanic Cloud, Shapley derived an empirical curve expressing a relation between period and luminosity for all Cepheids. Choosing eleven Cepheids whose radial velocity curves had been determined[3] and whose proper motions were given in the Preliminary General Catalogue of Boss, and assuming that the data were complete for each star and in most respects homogeneous, that there was no evidence of preferential motion and that the average peculiar motions of such stars are small compared with their parallactic drifts, Shapley computed a mean parallax for the group; and by using it he was able to determine the absolute values of the coördinates of his empirical period-luminosity curve, which curve is given on page 96 of Vol. VIII of the Contributions from the Mt. Wilson Observatory. It should be stated, however, that Curtis [4] and some other astronomers have failed to agree with these conclusions.

Shapley also found an interesting relation between the periods and the colour-indices for the variables of the Small Magellanic Cloud[5]; there is, no doubt, a close relation between this law and Miss Leavitt's period-luminosity law, and this would naturally be taken as an indication that a similar period-colour-index relation should be found for the stars of the δ Cephei type. A look at our table of the principal stars of this type shows, however, that although there is a tendency for these stars to have a larger colour-index (or to be of a redder spectral class) according as the period is longer, there is no relation *with one degree of freedom* between the two; Antalgol stars may be usually of class A but some are of class F and even K; to quote another example at random, V Lacertae (84) is of class G_2 with a period of $4^d \cdot 98269$ while RR Lacertae (83) is of class F with the longer period of $6^d \cdot 412$. A period-luminosity relation exists for the stars of the Small Magellanic Cloud, as does also probably a period-colour-index relation; since, however, such a period-colour-index relation does not exist for the stars of the δ Cephei type, it is logical to infer also that a period-luminosity relation does not apply rigorously to them. It is not, however, the intention to suggest that Shapley's theory should be discarded completely, as will be seen in the conclusion of this article.

Shapley made an extensive use of his empirical period-luminosity relation to determine the parallaxes of all the known galactic Cepheids.[6] He extended it, then, to the deter-

[1] Proc. Nat. Acad. Sc., Vol. 8, p. 69. See also Harv. Bul., No. 765.
[2] H. A., Vol. 78, p. 249.
[3] Contributions from the Mt. Wilson Observatory, Vol. VII₁, p. 85.
[4] The Scale of the Universe Bul. of the Nat. Research Council, No. 11, p. 205.
[5] Contributions from the Mt. Wilson Observatory, Vol. VIII, p. 99.
[6] Contributions from the Mt. Wilson Observatory, Vol. VIII, p. 145.

mination of the parallaxes of Cepheids in clusters or the consequent determination of the parallaxes of these clusters, concluding by a utilization of these results to form an estimate of the Scale of the Universe.[1]

The β Canis Majoris Type

There is growing evidence leading towards the consideration of stars of the β Canis Majoris type as Cepheids. Like the stars of the δ Cephei type they vary in light continuously, though the amplitude of variation is small. The curve of variation of magnitude is parallel to the curve of radial velocity variation; in β Cephei for example the maximum velocity of approach coïncides very nearly with the maximum luminous intensity. The widths of the spectral lines and the character of the spectrum also change during the period of variation.

We have already shown[2] how the stars of the β Canis Majoris type form a sequel to the stars of the δ Cephei type—the greater the density of such stars the shorter the period, following more or less (taking account of the masses) the sequence giant K, giant G, giant F, giant A, B, dwarf A, dwarf F. In the last classes the period is exceedingly short, but the Cepheid type of variation is still found in stars like τ Cygni, δ Aquilae and γ Ursae Minoris (period $2^h 36^m 10^s$). Several years ago β Cephei had already been considered by Guthnick, Crump and Shapley as a Cepheid; its parallax as deduced from the period luminosity curve was given by the latter as $0''\cdot018$. The absolute magnitude given by this curve for τ Cygni is very nearly $-0\cdot3$; the trigonometric parallax obtained by several observers, which may be considered as very well determined, is $0''\cdot042$; the spectroscopic parallax as determined by Dr. W. S. Adams is also $0''\cdot042$; this gives for τ Cygni an absolute magnitude of $+1\cdot9$; it is in total disagreement with the result obtained from the curve.

The principal characteristics of the stars of the β Canis Majoris type are the following:—

1. Very short-period variation of radial velocity, accompanied by a parallel variation of magnitude.

2. In a large number of cases there is a variation of amplitude of the short-period velocity curve, a variation which is also found in the light-curve; this is for example very marked in the case of 12 Lacertae[3] and is no doubt present in the other stars. The variations of δ Ceti, which at first were thought to be erratic, have now been found to have the same characteristics, as will be seen in the second chapter of the present article.

3. There is in the majority of cases a long period of variation of velocity which indicates that the star is moving in an orbit of that period. The variation of amplitude of the short-period curve is no doubt a function of the position of the star in this orbit. In some cases the short-period variation is apparently not constant—which may well be a consequence, real or apparent (due perhaps in some cases to the equation of light), of the motion in the orbit; the short-period variation itself cannot, however, be ascribed to orbital motion.

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[1] Contributions from the Mt. Wilson Observatory, Vol. VIII, p. 191, and many other publications.
[2] Pub. Dom. Obs., Vol. VIII, pp. 78 and 79.
[3] A. N. Jubilaumsnummer Tafel 3.

4. There is a variation of spectral characteristics (width and intensity of lines) in a *constant* period which is very nearly equal to the short-period radial velocity oscillation.

All these variations are similar to the variations found by Shapley and Hertzsprung in RR Lyrae, of which mention has already been made.

The importance of the study of stars of the β Canis Majoris type as a particular case of Cepheid variation is considerable. Most of the stars discovered are comparatively bright and consequently can be investigated with instruments of precision such as spectrographs and photo-electric cells; a certain number have evidently fairly large parallaxes, which has not been found to be the case among the stars of the δ Cephei type. The shortness of their periods allows variations to be disclosed which it would be difficult to find otherwise. A great many of their spectra are apparently very poor, especially toward the end of the dwarf classes; this makes it difficult to ascertain positively the variation when only spectrographic investigations are made (and so far this is precisely the mode of research we have used); photo-electric investigations would be likely to help to a large extent. Following is a list of the stars which are known or suspected to be of the β Canis Majoris type:—

STARS OF THE β CANIS MAJORIS TYPE (KNOWN OR SUSPECTED)

H. R.	Star	α 1900	δ 1900	Visual mag.	Spect.	Discoverer
		h m	° ′			
779	δ Ceti	2 34·4	− 0 6	4·04	B 2	
1149	20 Tauri	3 39·9	+24 4	4·02	B 5	
1320	μ Tauri	4 10·1	+ 8 39	4·32	B 5	
1463	ν Eridani	4 31·3	− 3 33	4·12	B 2	
1641	η Aurigae	4 59·5	+41 6	3·28	B 3	
1810	114 Tauri	5 21·6	+21 51	4·83	B 3	
1931	σ Orionis	5 33·7	− 2 39	3·78	B	
2294	β Canis Majoris	6 18·3	−17 54	1·99	B 1	Albrecht.
2344	10 Monocerotis	6 23·0	− 4 42	4·98	B 3	
2387	4 Canis Majoris	6 27·6	−23 21	4·35	B 1	
2490	42 Camelopardalis	6 40·5	+67 41	5·04	B 3	
2571	15 Canis Majoris	6 49·2	−20 6	4·66	B 1	
4295	β Ursae Majoris	10 55·8	+56 55	2·44	A	Guthnick.
4422	57 Ursae Majoris	11 23·7	+39 54	5·26	A 2	Otto Struve.
5435	γ Boötis	14 28·1	+38 45	3·00	F	Guthnick.
5735	γ Ursae Minoris	15 20·9	+72 11	3·14	A 2	Otto Struve.
6084	σ Scorpii	16 15·1	−25 21	3·08	B 1	Selga.
6453	θ Ophiuchi	17 15·9	−24 54	3·37	B 3	
7178	γ Lyrae	18 55·2	+32 33	3·30	A	Otto Struve.
7298	η Lyrae	19 10·4	+38 58	4·46	B 3	
7372	2 Cygni	19 20·2	+29 26	4·86	B 2	
7377	δ Aquilae	19 20·5	+ 2 55	3·44	F	
7426	8 Cygni	19 28·1	+34 14	4·85	B 3	
7447	ι Aquilae	19 31·6	− 1 31	4·28	B 5	
7977	55 Cygni	20 45·5	+45 35	4·89	B 2	
8130	τ Cygni	21 10·8	+37 37	3·82	F	Paraskévopoulos.
8238	β Cephei	21 27·4	+70 7	3·32	B 1	Frost.
8279	9 Cephei	21 35·2	+61 38	4·87	B 2	
8640	12 Lacertae	22 37·0	+39 43	5·18	B 2	Young.

Detailed studies of several of these stars have already been made, notably by R. K. Young in the Dominion Astrophysical Observatory Publications, C. C. Crump in the Detroit Observatory Publications, P. Guthnick in the Berlin-Babelsberg Observatory Publications and the Astronomische Nachrichten, Father Selga in the Spanish Review of the Astronomical Society of Spain and America, Paraskévopoulos in the Astrophysical Journal, Otto Struve in the Publications of the Astronomical Society of America, and the writer in the Lick Observatory Bulletins and the Publications of the Dominion Observatory. Further investigations of several of these systems are also to be found in the present article.

The α Orionis Type

This title (α Orionis type) may seem at first sight rather inappropriate, considering that α Orionis is classified as an irregular variable. Its range of variation is, however, quite small (in the neighbourhood of half a magnitude); the variation is very slow and the star is very red and very bright, three features, as any variable star observer knows, which make for great uncertainty in the estimate of the magnitude. Even with good instruments, but with variable seeing and transparency, errors of half a magnitude and perhaps more could sometimes occur in the estimation of the magnitude of a bright red star.

The radial velocity of α Orionis is also known to be variable, having an amplitude or variation of about five kilometers. It was shown by Bottlinger[1], and later by Lunt,[2] that the period of radial velocity variation is in the neighbourhood of six years. Assuming the star to be a spectroscopic binary, the following elements have been given by Bottlinger:

$$P = 6 \cdot 0 \text{ years}$$
$$e = 0 \cdot 24$$
$$\omega = 255°$$
$$K = 2 \cdot 45 \text{ km.}$$
$$\gamma = + 21 \cdot 3 \text{ km.}$$
$$T = 1904 \text{ Aug.}$$
$$a \sin i = 70{,}000{,}000 \text{ km.}$$

$$\frac{m_1^3 \sin^3 i}{(m + m_1)^2} = 0 \cdot 0029$$

These elements show all the characteristics of Cepheid variables, a large eccentricity, a very small value of the mass function, and a value of $a \sin i$ which is much smaller than the real diameter of the star as measured with the Mt. Wilson interferometer, (approximately 240,000,000 km.) Several measures of the angular diameter of α Orionis have now been made by Pease with the 20-foot interferometer attached to the 100-inch reflector and variations have been found. Pease has prepared a diagram published on page 346 of Vol. 34 of the Publications of the Astronomical Society of the Pacific showing a possible correlation between the variation of diameter, the variation of radial velocity, and the variations of light as obtained by Barnard and by Osthoff.[3]

[1] A. N., Vol. 187, p. 33.
[2] Ap. J., Vol. 44, p. 250, 1916.
[3] A. N., Vol. 216, p. 187.

The large uncertainties in the determinations of radial velocities and of magnitudes (especially the latter) introduce considerable difficulties, but it seems quite likely that α Orionis should be considered as an extreme case of Cepheid variation, just as γ Ursae Minoris is another extreme. Irregularities in the light-curve would in that case be due to errors of observation, or perhaps to convection currents or formation of spots of cooler material such as occur on our own Sun. There may be a main light curve due to true Cepheid variation on which are superposed accidental variations which might easily occur in an enormous volume of material at a fairly low temperature, such as found in stars of class M (α Orionis is of class M_a).

According to Pickering's classification all variable stars can be subdivided into the following classes:

I.—Novae.

II.—Variable stars with periods of several months or longer, having usually a considerable amplitude of variation—such stars as o Ceti and χ Cygni.

III.—Irregular variables, such as α Orionis and R Coronae Borealis. These may, however, be subdivided into two classes:—1—The α Orionis class, comprising stars of small amplitude of variation, which perhaps on closer examination might prove to be periodic and not irregular; 2—The R Coronae Borealis class, in which the brightness seems to be constant for a long time (several months), then suddenly increases by several magnitudes, to fall again after many oscillations to its previous level, where it remains again for a considerable period. SS Cygni is another star of this type.

IV.—Stars of the δ Cephei type.

V.—Those of the Algol and β Lyrae types.

All the stars of class II show remarkable variations in their spectra, especially the appearance and disappearance of bright lines. Some Cepheid variables such as W Virginis and RU Camelopardalis have now been found to have variable bright lines in their spectra; they would constitute perhaps transition stars between the true δ Cephei type and the long-period variables of class II. It is however possible that these stars of class II owe their variation to an entirely different cause, perhaps similar to what produces the undecennial variation in our own Sun. On the other hand some stars of fairly long period have already been classified as being of the δ Cephei type. Among these might be mentioned the following:—

Star	Period	Spectral Class
	d	
RX Cephei	130·0	Go
SS Gemin	44·87	G5
U Monocerotis	56·0	G5
RS Puppis	41·31	Ko
l Carinae	35·523	Go
U Carinae	38·740	Go
R Sagittae	70·56	Cont

And it is quite logical to suppose that the periods go on increasing with decrease of density, accompanied probably by a decrease of temperature (that is, extending to class M) *without however showing bright lines*.   Variations in the brightness and radial velocity of such stars might perhaps be difficult to detect, but there are strong arguments to support the belief that stars like α Orionis are extreme cases of Cepheid variation.

Chapter II

THE SPECTROSCOPIC SYSTEM DELTA CETI

The star δ Ceti ($\alpha = 2^h34^m \cdot 4$; $\delta = -0°6'$, class B2) has already been investigated at some length.[1]   It appeared then that this system was rather complicated, as evidenced by the different radial velocity curves published.   From further observations taken in 1922, however, it appears that the system is not as complicated as thought at first, and that it is merely a star of the β Canis Majoris type with a short-period variation of 3^h 52^m (the shortest known of class B), with a widely variable amplitude of short-period radial velocity oscillation, and a longer period of oscillation of mean velocity (as deduced from individual short-period curves).   The curves given previously were rather arbitrarily drawn through the different points given by the observations; some of these points being much better determined than others it is more than probable that the curves are not altogether true representations of the radial velocity oscillations (also the early spectrograms were of too long exposure).   The results are, however, of considerable value, since they can now be used to advantage in combination with the later observations.

It is to be remarked that the spectral lines of δ Ceti are more diffuse than those of β Canis Majoris;   this appears to agree with the theories and the graph given in the article, "A Spectrographic Study of Stars of Classes A and F."[2]   Its spectral lines being more diffuse it is likely that δ Ceti has a greater density and hence its period of pulsation, if pulsation it is, would be shorter.

On account of the shortness of the period and the rather small amplitude, the study of this star with our equipment was somewhat difficult.   It was placed on our programme to be observed every clear night from the middle of September to the end of December, 1922.   In the following table are found the radial velocities obtained during that period; all the spectrograms were measured as usual on a direct measuring engine in both the direct and reverse positions; a large number of them were also remeasured on the spectro-comparator, giving results in fair agreement with those obtained from the direct measures.

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[1] Pub. Dom. Obs., Vol. V, p. 413.
[2] Pub. Dom. Obs., Vol. VIII, No. 5.

RADIAL VELOCITIES OF δ CETI OBTAINED IN 1922

Date, 1922	Julian Day	Velocity	Probable error	Number of lines	Length of exposure
		km.	±	.	minutes
Sept. 21..................................	2423319·736	+ 8·7	2·8	9	36
	·763	− 3·0	5·9	5	36
	·789	− 3·2	0·5	2	35
	·814	− 1·9	4·3	8	35
Sept. 24..................................	322·726	+11·2	2·6	9	40
Sept. 25..................................	323·719	+13·0	1·3	8	29
	·740	+10·3	1·1	8	26
	·759	+ 2·9	2·3	8	26
	·778	− 3·6	2·0	7	26
	·798	− 3·3	1·3	8	26
	·817	− 5·8	1·3	9	26
	·837	+ 0·8	2·1	7	26
	·856	−12·7	1·8	5	26
Sept. 29..................................	327·697	+ 3·6	2·8	6	30
	·717	+ 2·0	3·2	8	26
	·736	+11·5	1·0	8	24
	·754	+10·0	3·4	9	24
	·772	+ 6·8	1·7	7	24
	·790	+ 1·2	4·0	9	24
	·808	+ 3·4	2·6	7	24
Oct. 3..................................	331·721	−16·3	3·6	6	34
	·748	−23·7	5·8	9	30
Oct. 11..................................	339·716	− 6·7	2·9	9	36
	·741	− 9·8	3·3	8	30
	·769	− 6·8	2·4	7	32
Oct. 12..................................	340·649	+ 3·8	1·8	8	30
	·694	−19·5	3·0	6	20
	·720	−13·5	2·5	8	20
Oct. 13..................................	341·590	+ 1·3	3·4	9	38
	·616	+ 9·4	2·9	10	28
	·635	−11·2	2·8	6	24
	·653	−20·8	1·6	7	24
	·671	−14·6	2·4	8	22
	·694	−19·3	1·6	8	22
	·712	−23·2	2·3	9	22
	·728	−10·5	4·5	9	22
	·747	− 9·6	6·7	6	26
	·767	+ 3·4	2·8	10	30
	·790	− 0·2	2·6	9	30
	·811	− 1·0	4·0	8	28
	·833	+ 1·0	3·9	9	30
Oct. 17..................................	345·596	− 3·3	2·4	8	28
	·616	+ 1·5	2·0	10	24
	·631	+ 5·2	4·2	3	15
Oct. 18..................................	346·576	+ 4·5	2·4	7	32
	·598	+ 4·7	1·8	8	26
	·617	+10·4	2·6	11	26
	·636	+ 7·2	2·8	10	24
	·653	− 4·5	3·2	10	22
	·669	− 4·0	4·0	7	18
	·692	− 4·7	2·9	9	20
	·706	− 6·0	3·0	9	18
	·719	+ 1·0	2·2	10	18

RADIAL VELOCITIES OF δ CETI OBTAINED IN 1922—*Continued*

Date, 1922	Julian Day	Velocity	Probable error	Number of lines	Length of exposure
		km.	±		minutes
Oct. 18....................................	2423346·733	+ 5·0	3·0	9	18
	·747	+ 7·8	2·4	7	18
	·760	+ 5·9	3·6	9	18
	·774	+ 3·6	2·0	9	18
	·788	+ 9·0	1·6	9	18
	·802	−18·2	7·4	2	18
Oct. 19....................................	347·584	+13·3	3·2	5	20
	·626	− 1·6	2·1	7	20
	·641	− 8·9	2·6	8	18
	·654	− 9·9	2·1	10	18
	·668	− 4·9	2·4	9	20
	·691	0·0	3·0	10	18
	·704	− 3·5	2·6	7	18
	·717	+ 1·9	2·4	11	18
	·744	+ 6·0	2·1	8	18
	·758	+10·9	2·1	12	18
	·772	+ 2·4	1·3	11	18
	·786	+ 1·2	3·3	9	18
	·800	− 1·7	2·8	10	18
	·839	− 2·9	2·7	11	26
	·858	+ 2·0	2·3	9	22
	·873	− 2·7	2·6	6	20
	·890	+ 0·4	1·8	7	24
Oct. 20....................................	348·560	− 1·6	1·6	8	26
	·578	− 2·2	1·2	6	20
	·603	− 6·7	2·0	8	16
	·615	−10·7	3·6	8	16
	·626	− 0·6	2·4	8	16
	·638	− 5·4	1·3	5	16
	·650	− 4·6	2·6	7	16
	·662	+ 2·9	2·5	6	16
	·691	+ 1·7	2·4	9	16
	·702	+ 8·6	1·3	8	14
	·712	− 2·2	2·4	8	14
	·723	− 0·6	1·6	7	14
	·733	+ 3·8	1·8	8	14
	·744	− 8·1	1·8	8	14
	·760	−12·5	2·9	8	14
	·772	− 8·5	2·0	10	16
	·785	− 6·3	1·4	8	16
	·797	+ 1·1	3·8	5	16
	·810	− 8·6	4·4	3	18
	·825	+ 6·0	3·8	3	20
	·840	− 2·6	3·3	5	18
	·856	− 2·7	2·7	6	24
	·875	+10·6	3·4	5	20
Nov. 9....................................	368·633	− 2·5	2·1	9	30
	·655	− 1·8	2·0	7	26
	·707	+ 5·7	2·4	9	20
	·726	−17·6	2·6	5	20
	·743	+ 1·9	1·9	9	22
	·759	− 3·4	1·7	4	20
	·776	− 3·2	2·9	5	26

RADIAL VELOCITIES OF δ CETI OBTAINED IN 1922—*Continued*

Date, 1922	Julian Day	Velocity	Probable error	Number of lines	Length of exposure
		km.	±		minutes
Nov. 9.............................	2423368·796	− 2·6	2·4	2	26
	·817	− 6·5	2·7	5	30
Nov. 12.............................	371·587	+15·0	4·1	3	40
	·649	+ 6·9	2·5	4	40
Nov. 13.............................	372·660	+ 2·9	2·7	9	40
	·717	+ 5·2	4·2	8	40
	·748	+27·0	3·4	2	40
	·777	+ 1·5	2·4	10	40
Nov. 16.............................	375·551	+ 6·7	3·2	6	36
	·574	+10·6	2·0	6	26
	·592	+14·9	1·6	9	20
	·607	+14·7	2·4	6	20
	·622	+13·1	1·9	7	20
	·638	+ 8·3	1·8	7	23
	·656	− 0·5	1·6	5	25
	·699	+ 2·9	2·4	5	27
	·719	+ 0·1	2·2	5	28
Dec. 3.............................	392·515	+ 2·1	3·6	5	30
	·537	+ 9·6	2·1	9	30
	·560	+ 9·5	2·4	6	30
	·581	+ 1·3	2·4	6	28
	·600	+ 2·5	3·9	3	26
	·619	− 4·0	2·5	2	26
Dec. 10.............................	399·488	+ 2·5	2·0	8	30
	·508	+ 2·3	1·6	10	24
	·527	− 5·0	1·9	9	30
Dec. 12.............................	401·490	+ 2·4	1·6	7	28
	·509	+11·1	2·0	7	24
	·526	+12·0	1·6	7	22
	·542	+13·9	1·3	6	22
	·560	+13·4	2·6	8	28
Dec. 15.............................	404·423	− 0·2	1·8	9	30
	·439	+ 6·9	2·2	4	14
	·469	+11·5	2·4	9	22
	·486	+11·0	2·1	9	22
	·503	+11·8	0·9	7	22
	·539	+ 3·6	4·0	3	10
Dec. 18.............................	407·441	+ 1·8	1·5	6	20
	·456	+ 3·3	2·2	6	18
	·471	+ 4·2	2·1	7	20
	·486	+16·1	2·2	7	20
	·501	− 1·2	8·0	3	20
	·517	+ 9·5	0·6	2	20
	·535	+12·2	4·7	3	32
	·556	+ 1·6	4·0	7	24
	·574	− 1·8	1·8	7	24
	·591	− 0·5	2·6	7	20
	·607	+ 7·7	2·0	8	22
	·624	+ 0·5	1·9	8	22
	·640	+ 9·5	1·7	8	22
	·658	+ 9·0	2·7	10	24
Dec. 19.............................	408·420	+ 0·3	2·5	6	30
	·440	+ 3·2	1·9	7	24

RADIAL VELOCITIES OF δ CETI OBTAINED IN 1922—*Concluded*

Date, 1922	Julian Day	Velocity	Probable error	Number of lines	Length of exposure
		km.	±		minutes
Dec. 19................................	2423408·457	+20·6	2·0	6	22
	·475	+ 5·1	2·4	5	26
	·494	+20·7	2·0	2	24
	·515	+10·5	2·8	3	30
	·536	+ 4·6	3·0	6	28
	·553	+ 0·9	0·9	10	20
	·567	+ 8·7	3·5	5	18
	·582	+ 4·4	3·2	6	20
	·597	+ 0·7	2·0	6	20
	·612	+ 5·7	2·8	7	16
	·639	+ 2·1	3·3	4	20
Dec. 27................................	416·424	+ 1·1	1·4	9	26
	·444	+ 3·4	1·7	11	28
	·464	+ 3·2	0·7	10	24
	·481	+ 4·2	1·1	8	22
	·497	+ 3·0	2·0	6	20
	·512	+ 5·5	2·4	6	20
	·529	+ 6·2	3·2	3	28
	·551	+ 9·6	1·1	7	34
	·573	+ 5·0	2·3	6	24
	·590	+ 2·0	1·6	8	22
	·607	+ 4·5	1·7	7	22
	·625	+ 0·7	1·5	5	22
	·642	+ 9·0	4·1	2	22
Dec. 29................................	418·418	+ 2·1	4·1	4	22
	·438	+ 7·8	2·4	8	30
	·461	+12·2	1·4	8	30
	·483	+ 7·7	2·1	7	32
	·508	+14·0	1·8	8	32
	·531	+ 2·0	1·0	12	32
	·555	+ 0·9	1·3	12	32
	·578	+ 4·1	1·8	9	30
	·601	+10·1	1·1	9	32
	·625	+ 6·4	1·0	7	34
	·649	+12·3	1·6	8	30

Combining these observations with those obtained in 1921 (see Pub. Dom. Obs., Vol. V, No. 11) it is found that the period $0^d \cdot 16122$ fits all the observations. If we take the observed maximum of radial velocity J. D. $2423346 \cdot 610$ as origin and add or subtract multiples of this period we obtain the following Julian dates of maxima for days when observations were secured in 1921 and 1922:—

COMPUTED EPOCHS OF MAXIMA OF δ CETI

A.—1921	B.—1922
J. D. 2422989·669	J. D. 2423319·686
990·475	·847
993·699	322·749
996·763	323·717
2423001·599	327·747
·760	339·839
002·567	340·645
·728	341·612
028·523	·773
030·458	345·643
·619	346·610
031·586	·771
032·553	347·577
039·485	·739
·647	·900
043·516	348·545
045·451	·706
·612	·867
049·482	368·697
·643	371·599
053·512	372·728
	375·630
	392·558
	399·490
	401·586
	404·488
	407·551
	·712
	408·518
	·680
	416·418
	·578
	418·514
	·675

On plotting the different velocity curves for the above dates (all the curves for 1921 were given in the previous article) it is found that these maxima are verified within the limits of error. Attention should be called to the curve of December 14, 1921; the observations giving this curve have been found to be very poor, and certainly one of them upon which the maximum depends, is not at all reliable. Four of the curves obtained in 1922 are shown in Fig. 1. There is no doubt a good deal of uncertainty in these curves; the following conclusions, however, appear to be justified:—

1. The period $0^d \cdot 16122$, approximately $3^h \, 52^m$, may be considered as correct.

2. The amplitude probably varies considerably.

3. There is a variation of the mean velocity as given by the individual curves; without more powerful equipment, however, it is practically hopeless to determine the period of this variation.
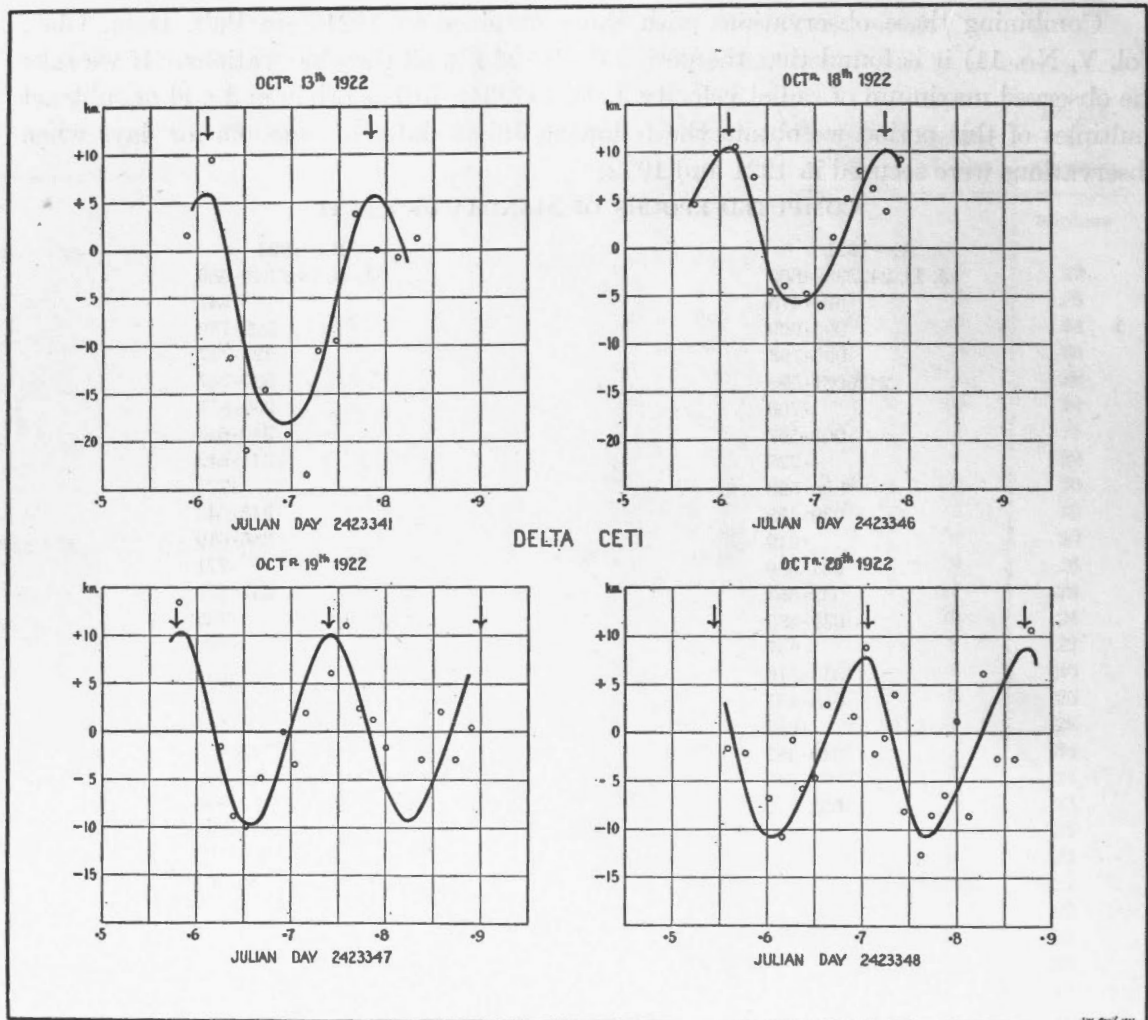
FIGURE 1.   RADIAL VELOCITY CURVES OF DELTA CETI

4. There is, apparently, nothing abnormal or erratic about the radial velocity variations of δ Ceti as at first supposed. It is a typical star of the β Canis Majoris type with the characteristic double variation. It has the shortest period at present known of any stars of this type belonging to class B.

Chapter III

THE SPECTROSCOPIC SYSTEM TAU CYGNI

The star τ Cygni was classified among the very short-period binaries, or what we have called the β Canis Majoris type, by J. S. Paraskévopoulos; he gave for its period $0^d.1425$ or $3^h 25^m.2$. The importance of studying this star further appeared immediately to the writer, and many spectrograms of it were secured in 1921. Unfortunately the spectrograms are very poor and difficult to measure, the lines being wide and hazy and sometimes hard to identify with known spectral lines. On account of the extreme shortness of the period it was necessary to make the exposures as short as possible, and hence to use a wide slit; the measures are, consequently, of rather poor quality.

The results, however, are of great interest because they seem to indicate that the mean velocity is also variable, with a period of twenty to twenty-five days, showing that τ Cygni, like other stars of the β Canis Majoris type already investigated, acts as if it were a spectroscopic triple system. The radial velocities obtained here for τ Cygni in 1921 are as follows:—

RADIAL VELOCITIES OF τ CYGNI

Date	Julian Day	Velocity	Date	Julian Day	Velocity
1921		km.	1921		km.
Aug. 2............	2422904·566	−36·2	Aug. 16............	2422918·567	−32·0
	·589	−26·9		·588	−28·4
	·611	−23·8		·610	−43·5
Aug. 4............	906·566	−31·8		·651	−30·9
	·588	−14·5		·673	−27·9
	·647	−15·0	Aug. 18............	920·538	−24·5
	·670	−16·5		·559	−35·7
	·692	−36·9		·601	−51·6
Aug. 5............	907·558	−24·6		·622	−33·6
	·581	−18·4		·663	−27·9
	·624	−20·3	Aug. 21............	923·535	−33·0
	·682	−33·9		·556	−26·9
Aug. 8............	910·621	−24·2	Aug. 22............	924·573	−23·2
Aug. 9............	911·567	−16·9		·594	−31·0
Aug. 12............	914·560	−32·3		·615	−24·8
	·582	−26·3	Aug. 24............	926·528	−31·9
	·603	−26·0		·549	−22·8
	·625	−16·7		·570	−29·1
	·662	−14·2		·591	−28·0
	·683	−37·4		·658	−24·6
	·708	−29·3	Aug. 26............	928·558	−25·4
Aug. 14............	916·589	−29·4		·599	−46·7
	·611	−28·8		·622	−42·1
	·663	−18·6			
	·685	−31·2			

On plotting the above observations it appears that a period of $0^d \cdot 1432$ would connect the different maxima; this period gives maxima on the following Julian dates:—

MAXIMA OF τ CYGNI

Date	Julian date of maximum	Estimated mean velocity for each day	Date	Julian date of maximum	Estimated mean velocity for each day
1921		km.	1921		km.
Aug. 2............	2422904·626	−37	Aug. 16............	2422918·660	−36
4............	906·631	−26	18............	920·664	−38
5............	907·633	−30	21............	923·528	−40 ?
8............	910·640		22............	924·674	—
9............	911·500	−20 ?	24............	926·536	−34 ?
12............	914·650	−25	26............	928·540	−32 ?
14............	916·655	−28			

[1] Ap. J., Vol. 53, p. 145, 1921.
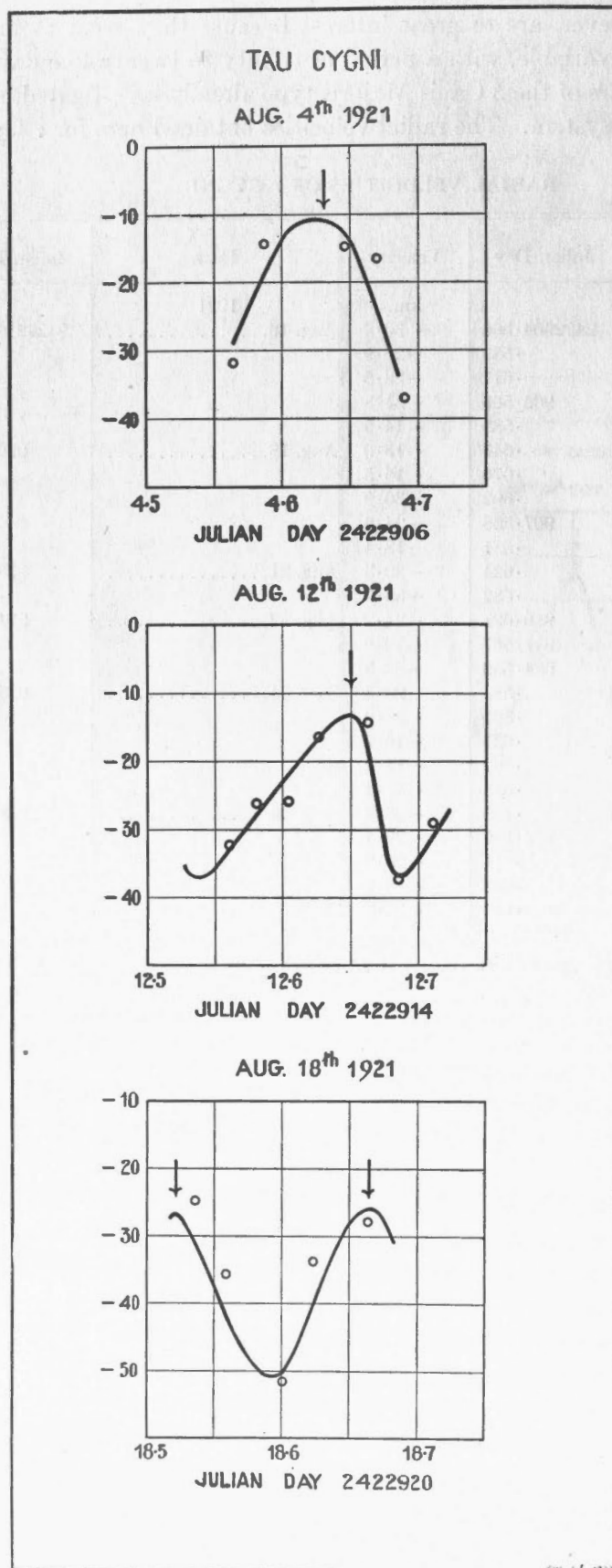
FIGURE 2. RADIAL VELOCITY CURVES OF TAU CYGNI
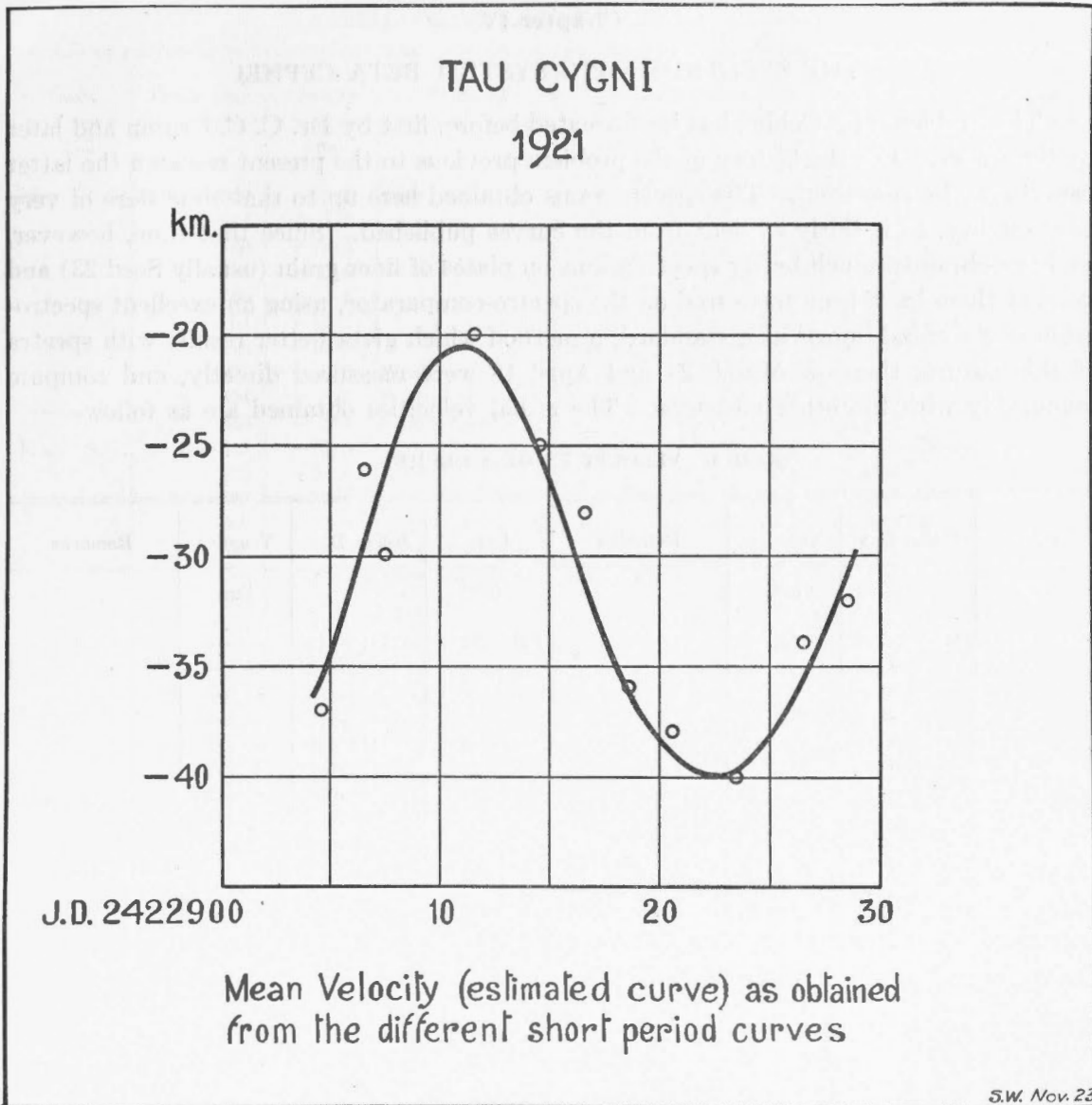
FIGURE 3. MEAN VELOCITY CURVE OF TAU CYGNI

Three of the curves given by the observations are shown in Fig. 2; the arrows indicate the maxima as computed with the period $0^d \cdot 1432$. From all the curves rough estimates of the mean velocity for each day have been made, as given in the third column of the preceding table; these results give the curve shown in Fig. 3. There is no doubt a great uncertainty in the determination of this curve, but it shows that the radial velocity of τ Cygni has most probably a double periodicity, as is the case with σ Scorpii, β Canis Majoris and other similar stars.

Chapter IV

THE SPECTROSCOPIC SYSTEM BETA CEPHEI

The problem of β Cephei has been treated before, first by Dr. C. C. Crump and later by the writer. For the history of the problem previous to the present research the latter paper[1] may be consulted. The spectrograms obtained here up to that time were of very poor quality, as is fairly evident from the curves published. Since that time, however, we have obtained much better spectrograms on plates of finer grain (usually Seed 23) and most of them have been measured on the spectro-comparator, using an excellent spectrogram of β Canis Majoris as a standard, a method which gives better results with spectra of this nature; those of March 24 and April 18 were measured directly, and compare favourably with the other measures. The radial velocities obtained are as follows:—

RADIAL VELOCITIES OF β CEPHEI

Date	Julian Day	Velocity	Remarks	Date	Julian Day	Velocity	Remarks
1922		km.		1922		km.	
Feb. 16...	2423102·774	−33·6		Feb .24...	2423110·813	−19·4	
	·798	−25·4			·833	− 8·4	
	·820	−22·8			·853	− 0·2	
	·842	−24·9			·876	− 1·6	
	·865	− 8·4		Mar. 2...	116·569	− 3·8	
	·887	− 2·0			·585	+ 7·7	
	·909	− 9·6			·601	+ 2·5	
	·931	−18·9			·616	+13·1	
Feb. 20...	106·678	− 7·3			·633	− 8·0	
	·701	+ 2·7			·653	−10·0	Weak plate.
	·717	− 0·9			·705	−23·8	
	·731	− 5·8			·722	−22·7	
	·747	− 0·4			·741	−20·5	
	·762	−10·0			·763	− 0·8	
	·776	−17·1			·785	− 5·2	
	·790	−13·8			·855	−25·6	Poor plate.
	·804	−17·2			·872	−36·2	
	·818	−11·3		Mar. 3...	117·568	+ 0·3	
	·832	− 9·1			·586	+ 4·0	
	·867	+ 1·8			·603	− 6·7	
	·881	+ 1·6			·621	−31·7	Poor plate.
	·895	− 8·1			·640	−37·2	
	·909	−10·8			·657	−21·1	
Feb. 23...	109·758	+ 1·3			·678	−19·3	
	·782	−12·0			·699	−15·0	Poor plate.
	·803	−20·7			·716	− 4·0	
	·822	−14·6			·733	− 4·3	
	·840	−15·1			·752	−11·3	
	·888	− 1·4			·774	−11·6	
	·902	−10·6			·796	−32·6	Weak plate.
	·917	− 0·8			·821	−23·8	
Feb. 24...	110·730	−14·0			·850	−26·2	
	·746	−28·6	Poor plate.		·868	−25·7	Very coarse grain.
	·761	−28·2		Mar. 5...	119·589	−13·6	
	·778	−29·0			·604	−12·6	
	·796	−23·6			·619	− 8·4	

[1] Pub. Dom. Obs., Vol. V, p. 77.

RADIAL VELOCITIES OF β CEPHEI—*Continued*

Date	Julian Day	Velocity	Remarks	Date	Julian Day	Velocity	Remarks
1922		km.		1922		km.	
Mar. 5...	2423119·633	−16·3	Poor plate.	Mar. 22...	2423136·764	−14·7	
	·647	+ 4·7			·785	− 6·4	
	·705	−19·4			·806	− 7·1	
	·722	−33·4			·826	−13·3	
	·740	−22·5			·847	−23·9	
	·761	−17·7			·868	−25·2	
	·782	−16·3		Mar. 24...	138·524	− 4·2	
	·803	− 3·6	Weak.		·545	− 5·6	
	·824	− 3·5	Poor plate.		·564	−17·9	
	·868	− 5·1	Peculiar plate.		·580	−26·3	
Mar. 8...	122·558	−20·5			·597	−38·0	Poor plate.
	·573	−23·8			·615	−34·4	
	·587	−32·4			·658	−25·0	
	·600	−34·9			·710	− 6·4	
	·646	−12·0	Very poor plate.		·728	−13·5	
	·675	−10·2	Very poor plate.	Mar. 29...	143·547	−20·0	
	·704	+ 5·4			·573	−33·2	
	·729	−14·0			·585	−25·8	
	·751	−30·7			·604	−17·0	
	·812	−27·3	Very poor plate.	April 18...	163·540	−30·8	
	·831	−19·2	Poor plate.		·574	−27·3	
	·849	− 4·9			·592	−13·5	
	·867	−15·0	Poor plate.		·607	+ 1·1	
	·885	+ 4·1			·622	+ 0·8	
Mar. 10...	124·566	− 6·2			·638	−14·8	
	·583	−11·7	Very poor plate.	July 5...	241·755	− 3·4	
	·599	− 2·7					
	·643	− 9·5		1923			
	·753	− 1·5	Poor plate.	Mar. 5...	484·783	+ 2·1	Poor plate.
	·785	− 1·1			·812	+ 4·1	
	·807	+ 6·7	Very poor plate.		·837	− 5·2	
	·828	−30·3	Very poor plate.		·860	−13·8	
	·856	−18·8			·881	−14·0	
	·876	−41·7			·902	−17·4	
Mar. 15...	129·550	− 5·4			·923	−11·2	
	·568	−21·2	Weak plate.	Mar. 7...	486·748	− 1·8	
Mar. 17...	131·600	−13·3		Mar. 8...	487·493	+ 3·9	
	·618	− 7·5			·515	− 7·5	
	·639	−10·3			·536	−13·6	
	·658	−13·1			·556	−17·0	
	·703	−39·9	Poor plate.		·575	−15·8	
	·726	−28·8	Poor plate.		·593	−21·2	
Mar. 18...	132·719	−17·9	Poor plate.		·611	−14·4	
	·741	+ 0·8			·630	+ 2·8	
	·759	− 5·2			·650	+ 4·2	
	·776	− 7·7			·697	− 0·4	
	·796	− 7·1			·719	− 4·8	
	·817	− 7·1			·743	−19·7	
	·837	−21·8			·812	−16·9	
	·860	−27·0			·831	− 9·7	
Mar. 22...	136·551	− 4·1			·850	− 1·1	
	·567	− 2·7			·867	− 2·4	
	·613	− 1·1	Poor plate.		·891	− 2·8	
	·725	−25·2			·900	− 3·0	
	·744	−20·7			·917	−13·8	

RADIAL VELOCITIES OF β CEPHEI *Concluded*

Date	Julian Day	Velocity	Remarks	Date	Julian Day	Velocity	Remarks
1923		km.		1923		km.	
Mar. 14...	2423493·738	− 2·5		Mar. 26...	505·564	+ 7·1	
	·758	+ 3·7			·606	− 4·0	
	·778	+ 1·7			·630	−20·3	
	·803	− 5·8			·653	−28·8	
	·824	−15·1			·701	−11·5	
	·866	−20·4			·776	+ 2·1	Poor plat
	·888	− 5·6			·798	− 0·3	
	·909	− 7·9			·881	−17·5	
Mar. 16...	495·722	−13·5			·905	− 3·7	
	·746	− 8·1	Poor plate.	Mar. 28...	507·769	− 5·2	
	·772	−11·7	Poor plate.		·794	0·0	Poor plate.
	·793	− 7·6	Very poor plate.		·816	+ 2·8	Poor plate.
	·815	0·0			·838	+ 1·4	
	·837	+14·8			·859	+ 5·3	
	·860	+ 1·7			·880	+ 0·9	
	·905	− 3·9			·903	−14·8	
Mar. 19...	498·783	−11·5			·924	−29·5	
	·808	− 7·8	Poor plate.	April 6...	516·563	− 2·2	
	·830	−12·1		April 26...	536·548	−14·1	
	·854	− 5·2					
	·876	+ 3·4					
	·920	+ 2·0					

A number of curves representing the above velocities are given in Figs. 4-8; the large differences of amplitude, as previously found by Dr. Crump are very evident. Of particular interest are five curves, where spectrograms were obtained through almost the whole night; they are those of 1922 March 2, 3, 5 and 22, and 1923 March 8, where the second maximum is apparently lower than the first; this may, however, be due to accidental error.
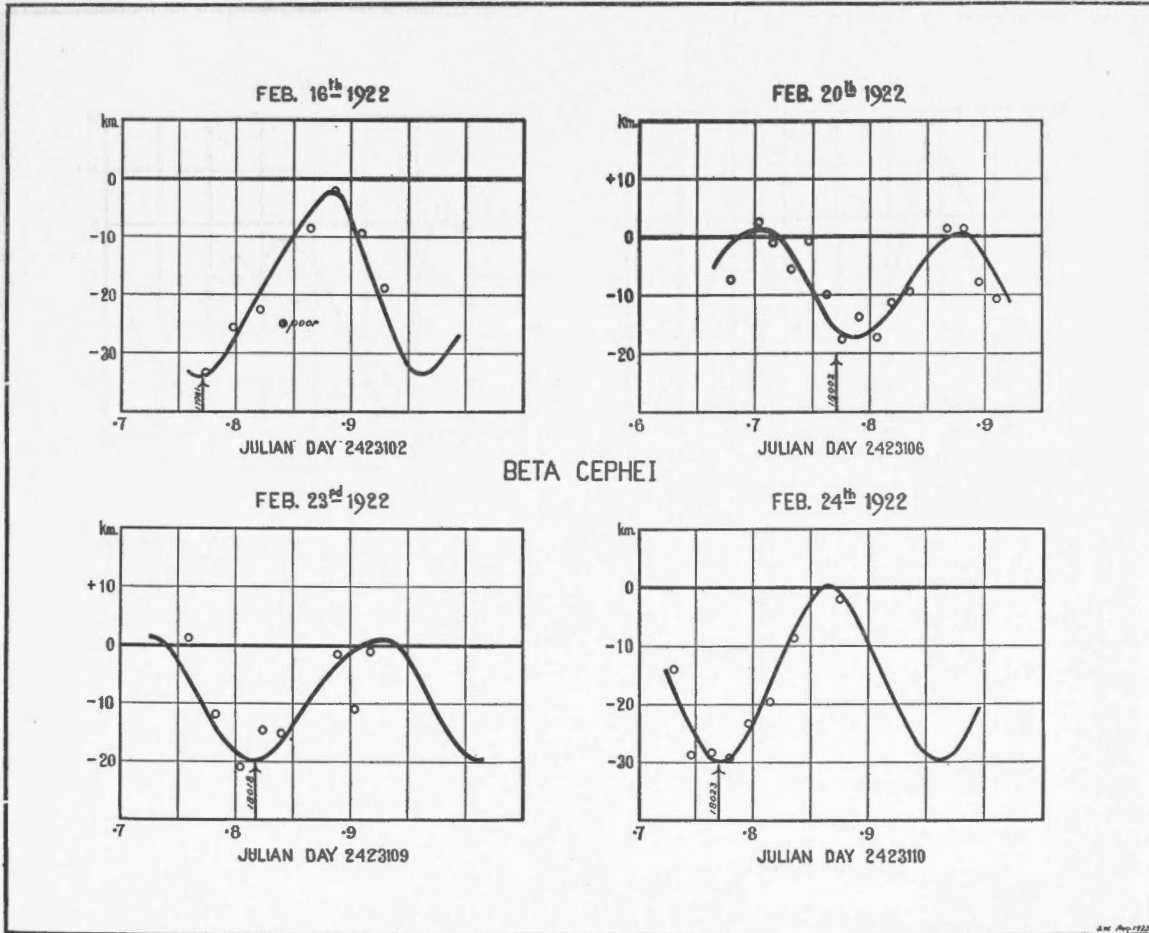
FIGURE 4. RADIAL VELOCITY CURVES OF BETA CEPHEI
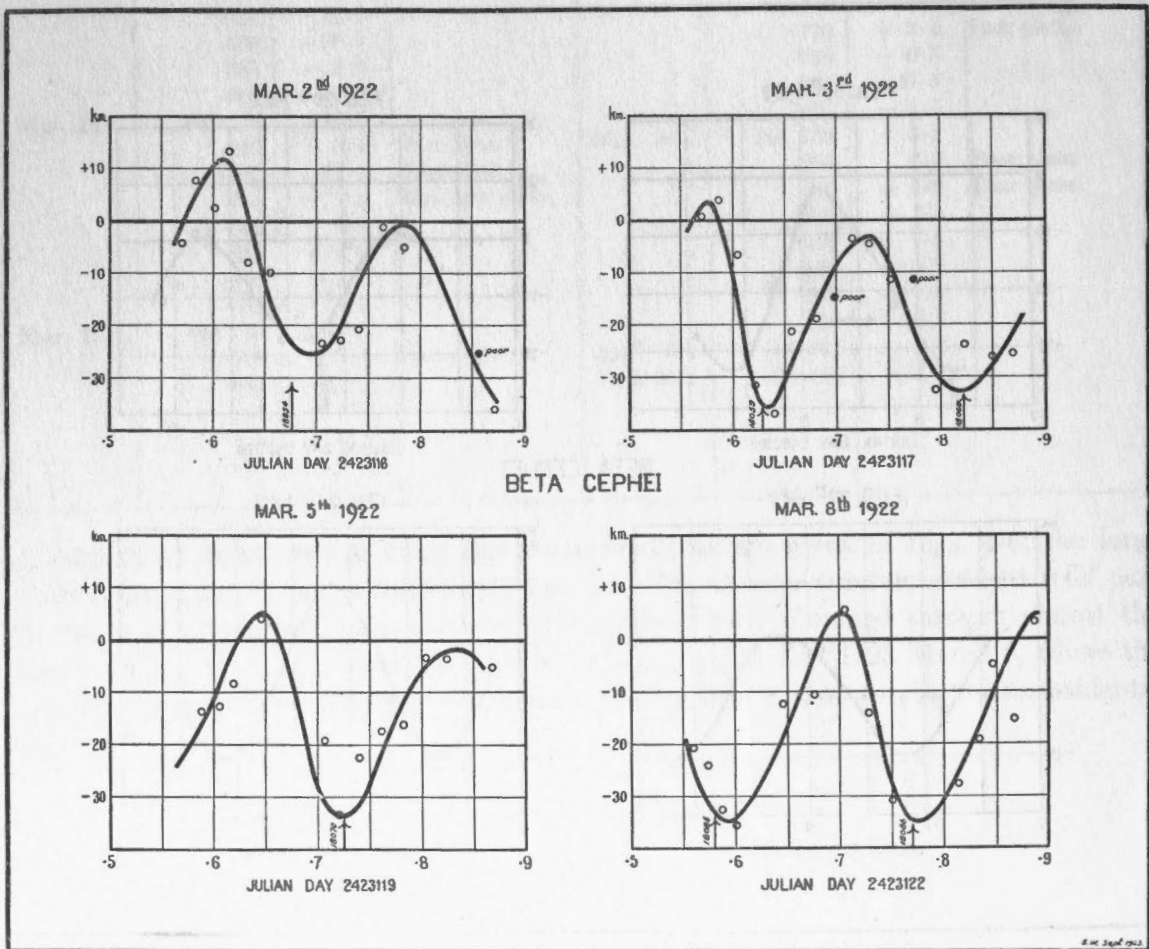
FIGURE 5. RADIAL VELOCITY CURVES OF BETA CEPHEI
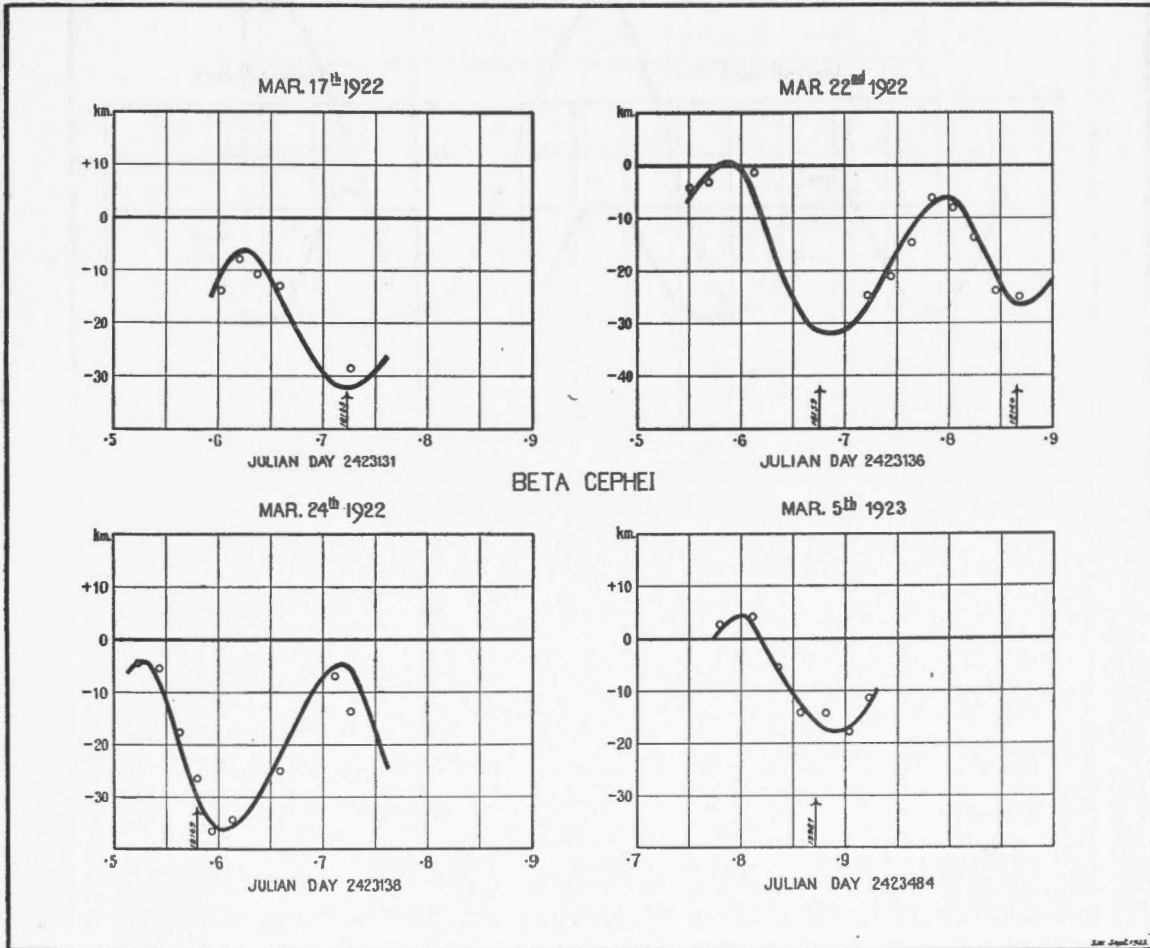
FIGURE 6. RADIAL VELOCITY CURVES OF BETA CEPHEI
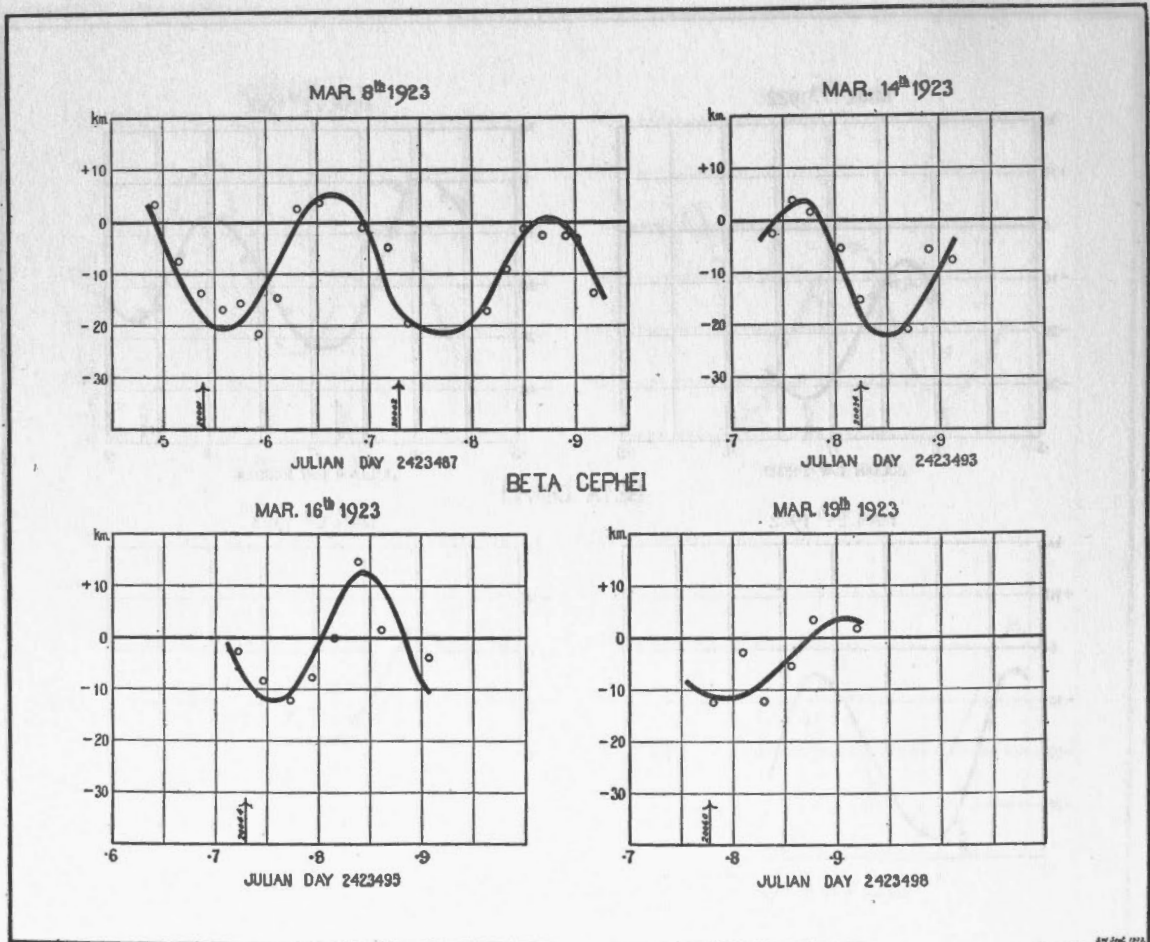
FIGURE 7.   RADIAL VELOCITY CURVES OF BETA CEPHEI
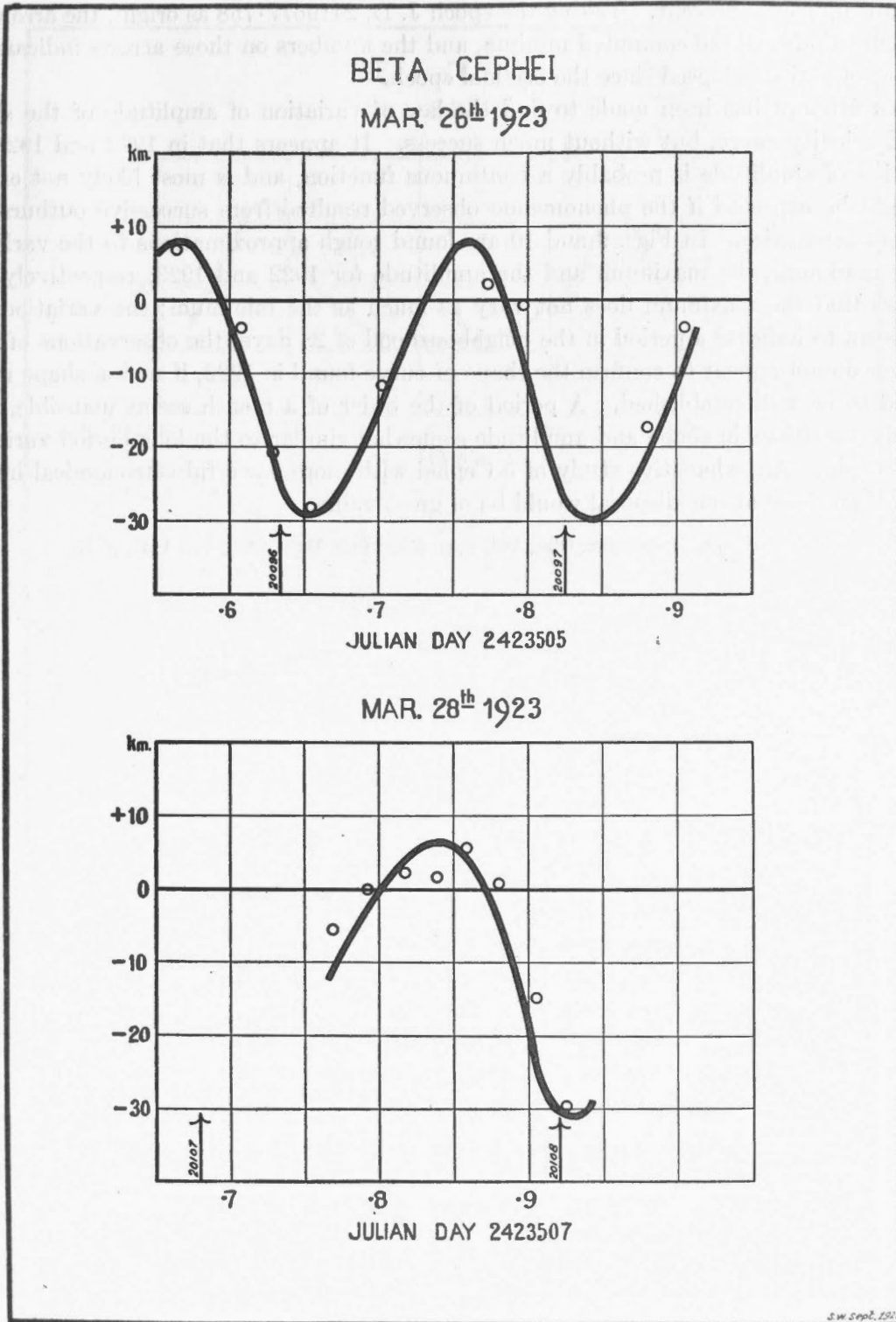
FIGURE 8. RADIAL VELOCITY CURVES OF BETA CEPHEI

An outstanding fact is that the period given by Dr. Crump ($0^d \cdot 1904795$) has remained the same up to the present. Taking the epoch J. D. $2419677 \cdot 758$ as origin, the arrows on the figures indicate the computed minima, and the numbers on those arrows indicate the number of periods elapsed since the original epoch.

An attempt has been made to find the law of variation of amplitude of the short-period velocity curve, but without much success. It appears that in 1922 and 1923 the variation of amplitude is probably a continuous function, and is most likely not erratic as might be expected if the phenomenon observed resulted from successive outbursts of different intensities. In Figs. 9 and 10 are found rough approximations to the variation of the minimum, the maximum and the amplitude for 1922 and 1923, respectively. It appears that the maximum does not vary as much as the minimum; the variations for 1922 seem to indicate a period in the neighbourhood of 28 days; the observations of 1923 however do not appear to confirm the shape of curve found in 1922, if such a shape might be said to be well established. A period of the order of a month seems plausible, with possibly variations in shape and amplitude somewhat similar to the long-period variation of σ Scorpii[2]. An exhaustive study of β Cephei with more powerful astronomical instruments than those at our disposal would be of great value.

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[2] L. O. B., Vol. 9, p. 173.   Pub. Dom. Obs., Vol. V, p. 301.   Pub. Dom. Obs., Vol. VIII, p. 45.
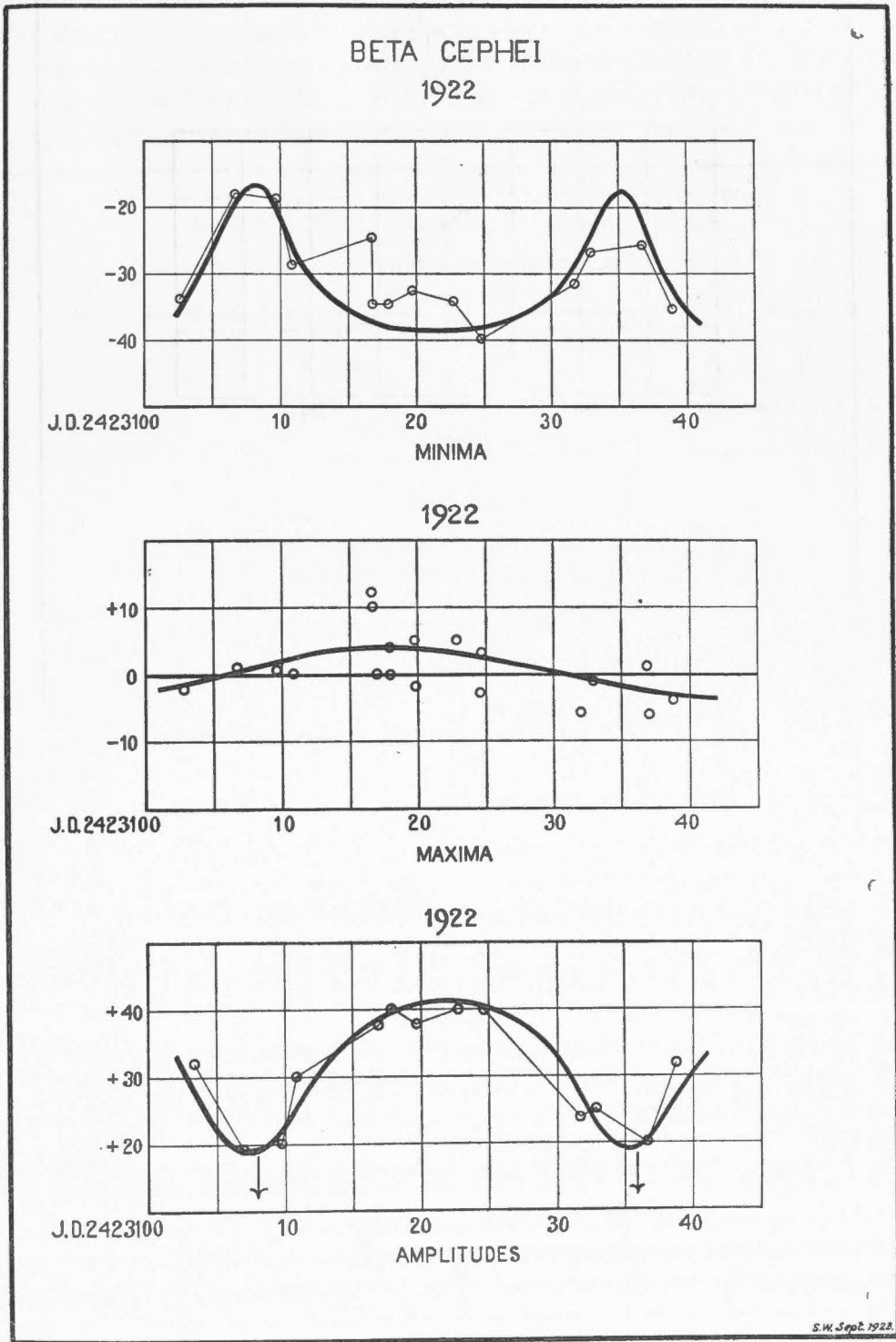
FIGURE 9. VARIATION OF AMPLITUDE OF BETA CEPHEI 1922
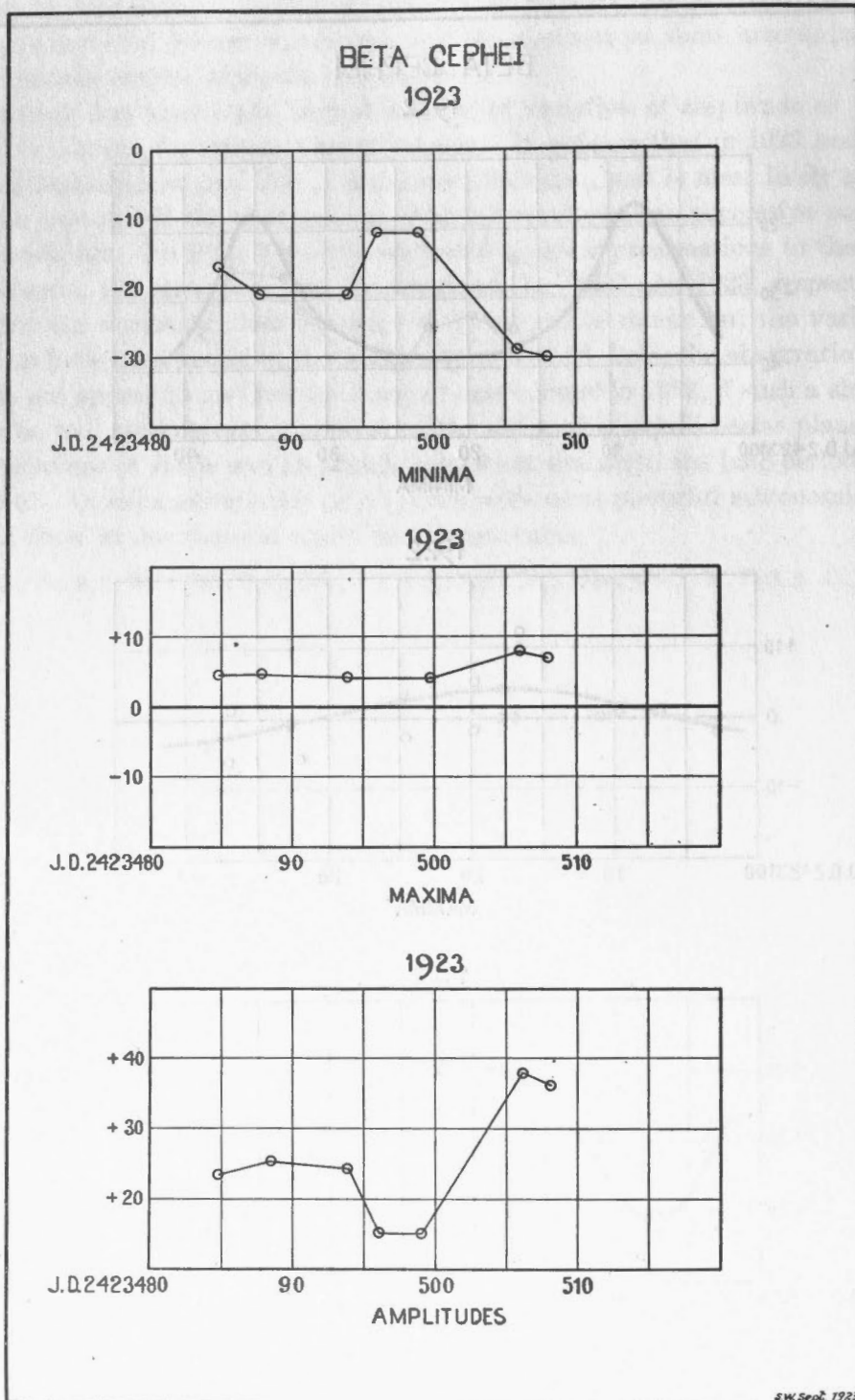
FIGURE 10. VARIATION OF AMPLITUDE OF BETA CEPHEI 1923

On April 18th, 1922, a series of direct extrafocal photographs of β Cephei was taken with the 6-inch Brashear doublet and the photographic light-curve determined. A comparison of the two curves (radial velocity and magnitude) determined simultaneously is shown in Fig. 11. The maximum luminous intensity of the star coincides, at least approximately, with the maximum radial velocity of approach, which is a typical characteristic of the Cepheid type of variation.
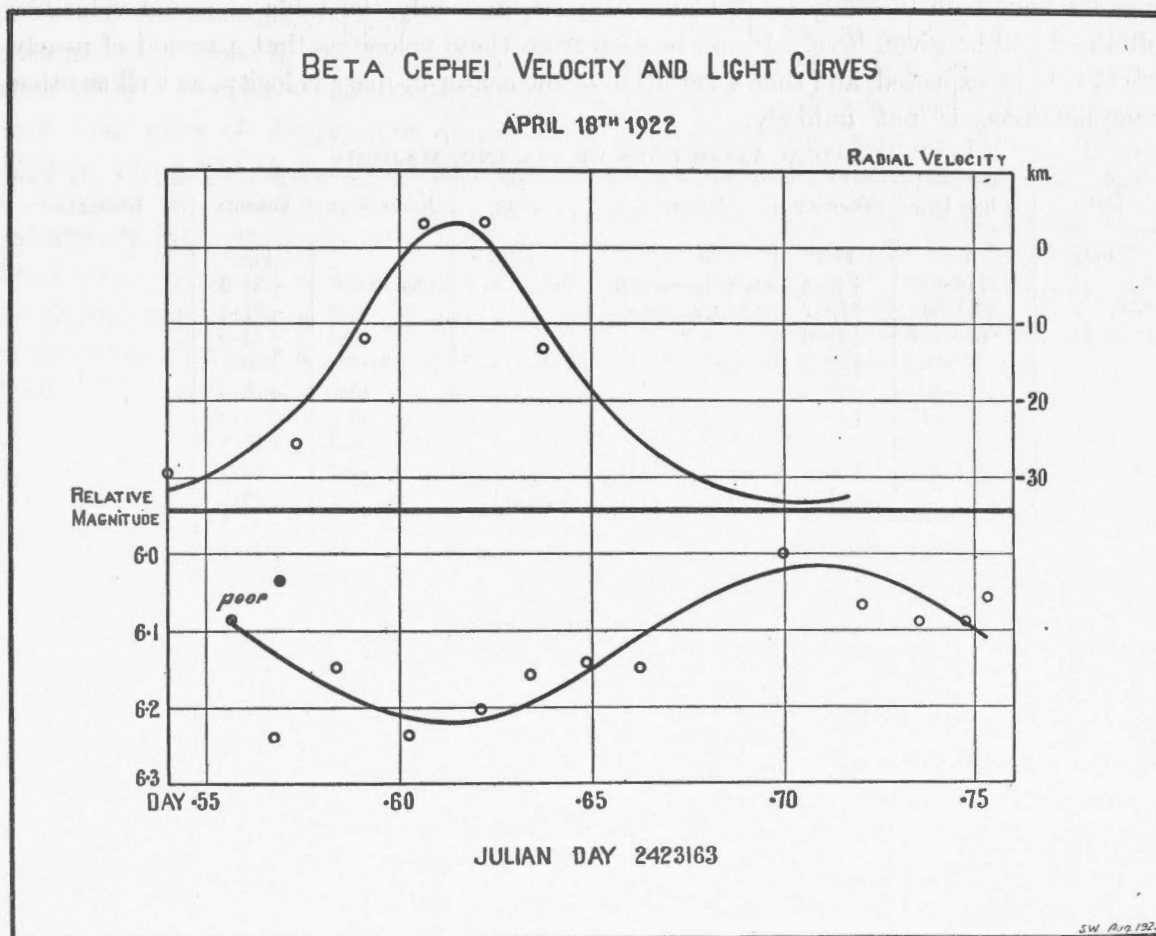
FIGURE 11. RADIAL VELOCITY AND LIGHT CURVES OF BETA CEPHEI

Chapter V

THE SPECTROSCOPIC SYSTEM 15 CANIS MAJORIS

The star 15 Canis Majoris ($\alpha = 6^{\text{h}}\ 49^{\text{m}} \cdot 2$, $\delta = +20°6'$, class B 1), visual magnitude 4·66, announced by Dr. Campbell to be a spectroscopic binary, had already been suspected to be of the β Canis Majoris type.[1] A fairly large number of spectrograms of it were secured at the beginning of 1923, and they indicate without doubt that the radial velocity variation is exceedingly rapid. A period of perhaps less than three hours is expected; the star is too faint for adequate investigation with our instrumental equipment;

[1] Pub. Dom. Obs., Vol. V, p. 362.

being usually at a low altitude fairly long exposures were necessary (about 40 minutes or a little more), which might amount to a large fraction of the period of variation. Most of the spectrograms were secured on rapid but rather coarse-grained plates (Seed Graflex) and were measured on the Hartmann Spectrocomparator, using the same standard spectrogram of β Canis Majoris which was used in measuring the plates of β Cephei.

It seems likely that observatories with a more powerful equipment than ours could make a very fruitful study of 15 Canis Majoris, and only the table of radial velocities obtained will be given here. It can be seen from these velocities that a period of nearly $0^d \cdot 15$ is to be expected, and that a variation of the center-of-mass velocity, as well as other complications, is not unlikely.

RADIAL VELOCITIES OF 15 CANIS MAJORIS

Date	Julian Day	Velocity	Remarks	Date	Julian Day	Velocity	Remarks
1921		km.		1923		km.	
Jan. 24...	2422714·654	+46·4	Direct measure.	Feb. 4...	2423455·597	+31·2	
Dec. 7...	3031·841	+38·7	Direct measure.		·635	+31·4	
Dec. 14...	038·724	+18·6			·689	+12·9	
	·762	+34·2		Feb. 5...	456·553	+30·7	
	·802	−13·2			·585	+10·9	
	·847	+20·0			·615	+22·0	
1922				Feb. 5...	456·644	+19·7	
Dec. 5...	394·741	+25·8			·697	+19·7	
	·785	+21·8		Feb. 11...	462·557	+22·1	
	·828	+17·3			·586	+30·2	
Dec. 12...	401·764	+10·1			·615	+37·1	
	·811	+22·0			·645	+21·7	
1923					·701	+17·9	
Jan. 22...	442·583	+ 5·3		Feb. 15...	466·502	+43·4	
	·617	+21·0			·565	+ 6·9	
	·649	+36·7	Direct measure +30·6.		·594	+16·6	
					·624	+24·8	
	·736	+16·8	Direct measure +18·5.		·655	+26·4	
				Feb. 16...	467·510	+ 7·6	
Jan. 25...	445·626	+20·0		Feb. 18...	469·512	+28·4	
	·658	+14·1			·537	+25·0	
	·696	+21·3			·560	+20·9	
	·736	+26·7			·580	+27·4	
Jan. 29...	449·550	+10·9		Feb. 23...	474·540	+31·5	
	·618	+26·4			·567	+38·3	
	·651	+ 0·1			·592	+20·2	
	·715	+23·5			·616	+21·7	
Jan. 30...	450·548	+26·3			·644	+30·7	
	·581	0·0					

To these velocities may be added those obtained and published by the Lick observers[1] which are as follows:—

RADIAL VELOCITIES OF 15 CANIS MAJORIS OBTAINED AT LICK OBSERVATORY

Date	Julian Day	Velocity	Remarks	Date	Julian Day	Velocity	Remarks
1910		km.		1910		km.	
Jan. 24...	2418696·599	+57·0		Dec. 27...	2419033·753	+53·8	
Nov. 12...	988·979	+31·2		1911			
Dec. 1...	2419007·756	+29·5		Feb. 16...	084·715	+36·5	
Dec. 15...	021·890	+27·6					

[1] L. O. B., Vol. 6, p. 145.

Chapter VI

ALPHA URSAE MINORIS

The star α Ursae Minoris (Polaris) has already been studied spectroscopically by Campbell,[1] Frost,[2] Hartmann,[3] Bélopolsky[4] and Küstner.[5] It was found to act as a short-period binary with a period of $3^d \cdot 9681$, and, moreover, was discovered by Campbell to have a variable centre-of-mass velocity, so that at first it was considered to be a triple system. Miss Hobe at the Lick Observatory gave a period of $11 \cdot 9$ years for the variation of this centre-of-mass velocity,[6] but our observations do not verify this period. Very long series of observations during short intervals of time had not hitherto been obtained and it was thought that although the amplitude is small, a considerable number of our one-prism spectrograms would give valuable information. The variation of light had already been suspected by several observers, e.g. Seidel[7], Schmidt[8] and Pannekoek.[9] Hertzsprung, using his method of side images produced by placing a grating in front of the objective of a camera and using the period deduced from radial velocity observations, succeeded in obtaining a photographic light curve.[10] The amplitude of this light curve is only $0^m \cdot 171$ and the elements of variation are:—

$$\text{Max.} = \text{J. D. } 2418985 \cdot 856 + 3^d \cdot 9681 \text{ E.}$$

Hertzsprung, not only from this light curve but also from the character of the spectrum, established without any doubt that Polaris is a Cepheid variable. This was verified again from the Harvard photographic observations by King,[11] and from the Harvard visual observations by E. C. Pickering.[12]

A very good light-curve of Polaris was determined by Prof. Stebbins[13] with a selenium photometer. This curve has an amplitude of $0^m \cdot 078$, considerably smaller than that obtained by Hertzsprung, but the light affecting the selenium cell has a longer wave-length than that affecting the ordinary photographic plate (much nearer the mean wave-length of visual light), and it is a well-known property of Cepheids that their photographic amplitude is usually larger than the visual. The elements given by Stebbins, which were adopted by Hartwig as final elements, are as follows:—

$$\text{Max.} = \text{J. D. } 2418985 \cdot 936 + 3^d \cdot 9681 \text{ E.}$$

It is these elements that have been used in the present investigation.

Prof. Gramatzki in Germany,[14] using a new kind of visual photometer, has redetermined the visual light-curve; he finds elements slightly different, a difference which however would not affect the results to any appreciable extent. His elements are:—

$$\text{Max.} = \text{J. D. } 2422954 \cdot 2147 \text{ helioc. G. M. T.} + 3^d \cdot 96835 \text{ E.}$$

[1] Ap. J., Vol. 10, p. 180; Vo. 21, p. 191; and Vol. 25, pp. 59; and L. O. B., Vol. 1, pp. 23; and Vol. 4, p. 98.
[2] Ap. J., Vol. 10, p. 184.
[3] Ap. J., Vol. 14, p. 52.
[4] A. N., Vol. 152, p. 201.
[5] Ap. J., Vol. 27, p. 304.
[6] L. O. B., Vol. 6, p. 18.
[7] Abh. Akad, Wiss. Munchen, Vol. 6, p. p. 568 and 603; Vol. 9, p. pp. 117 and 160.
[8] A. N., Vol. 46, p. 293.
[9] A. N., Vol. 194, p. 359.
[10] A. N., Vol. 189, p. 89.
[11] H. A., Vol. 59, p. 249.
[12] H. C., No. 174.
[13] A. N., Vol. 192, p. 189.
[14] A. N., Vol. 217, p. 454.

RADIAL VELOCITIES OF POLARIS

The following radial velocities for Polaris have been obtained here:—

Date	Julian Day	Phase	Velocity	Remarks
1923		d	km.	
April 29....	2423539·627	2·280	−15·6	Gramatzki's elements give phase 2ᵈ·065.
	·649	2·298	−13·3	
May 1....	541·585	0·280	−19·8	
	·603	0·298	−19·5	Mean of three measures.
	·620	0·305	−16·5	
	·637	0·322	−18·0	
May 2....	542·580	1·265	−15·1	
	·597	1·282	−13·8	
	·615	1·300	−15·4	
	·632	1·317	−15·2	
May 3....	543·777	2·462	−14·0	
May 4....	544·594	3·279	−16·5	
	·611	3·296	−14·8	
May 6....	546·561	1·278	−14·1	
	·578	1·295	−15·7	
	·596	1·313	−14·5	
	·613	1·330	−14·7	
May 7....	547·612	2·329	−12·5	
May 10....	550·610	1·359	−15·9	
May 13....	553·585	0·365	−18·8	
	·603	0·384	−16·1	Mean of two measures.
May 18....	558·560	1·373	−14·6	Mean of two measures.
	·619	1·432	−14·9	
	·641	1·454	−16·2	
May 21....	561·598	0·442	−19·7	
May 23....	563·604	2·448	−14·0	Mean of two measures.
	·620	2·465	−14·3	Mean of two measures.
	·666	2·510	−15·8	
	·683	2·527	−13·9	Mean of two measures.
June 15....	586·679	1·715	−14·2	
June 18....	589·599	0·667	−19·3	
	·628	0·696	−16·4	
	·677	0·745	−19·8	
June 24....	595·597	2·697	−14·1	Mean of two measures.
	·662	2·762	−17·8	
	·679	2·779	−15·3	
June 25....	596·793	3·893	−18·6	
June 27....	598·573	1·705	−16·7	
	·590	1·722	−17·4	
	·703	1·835	−15·2	
	·750	1·882	−16·0	Mean of two measures.
	·837	1·969	−15·6	Mean of two measures.
June 29....	600·606	3·738	−20·5	Mean of four measures −20·0; −19·6; −21·4 and −20·9.
	·640	3·772	−18·9	
	·660	3·792	−18·0	
	·674	3·806	−18·8	
	·687	3·819	−18·3	
	·701	3·833	−18·2	
	·781	3·913	−18·1	
	·840	0·004	−18·0	
Aug. 23....	655·849	3·427	−17·2	

RADIAL VELOCITIES OF POLARIS—*Concluded*

Date	Julian Day	Phase	Velocity	Remarks
1923			km.	
		d		
Sept. 10....	2423673·606	1·344	−17·5	
	·623	1·361	−19·4	
	·685	1·423	−17·4	
	·699	1·437	−18·5	
	·769	1·507	−16·8	
Sept. 13....	676·669	0·439	−20·3	Gramatzki's elements give phase 0^d·215.
	·681	0·451	−24·5	

Only the spectrograms considered good have been measured, all on the spectro-comparator, using a good spectrogram of daylight as standard. All the observations between April 29 and June 29 are plotted on the graph shown in Fig. 12, the open circles indicating the observations from April 29 to May 13 inclusive, the squares those from May 14 to June 18, and the darkened circles those from June 24 to June 29. From the graph there seems to be an indication that the open circles give a mean curve of larger amplitude than the squares, while that from the darkened circles is perhaps smaller. The amplitude of the general mean curve is about 4 kilometres, while the mean amplitude given by Dr. Campbell in his Second Catalogue of Spectroscopic Binaries is 6·08 kilometres. There is thus a possibility of a variation of amplitude similar to what has been found for stars of the β Canis Majoris type, but a large number of very good high-dispersion spectrograms would be needed to confirm this.
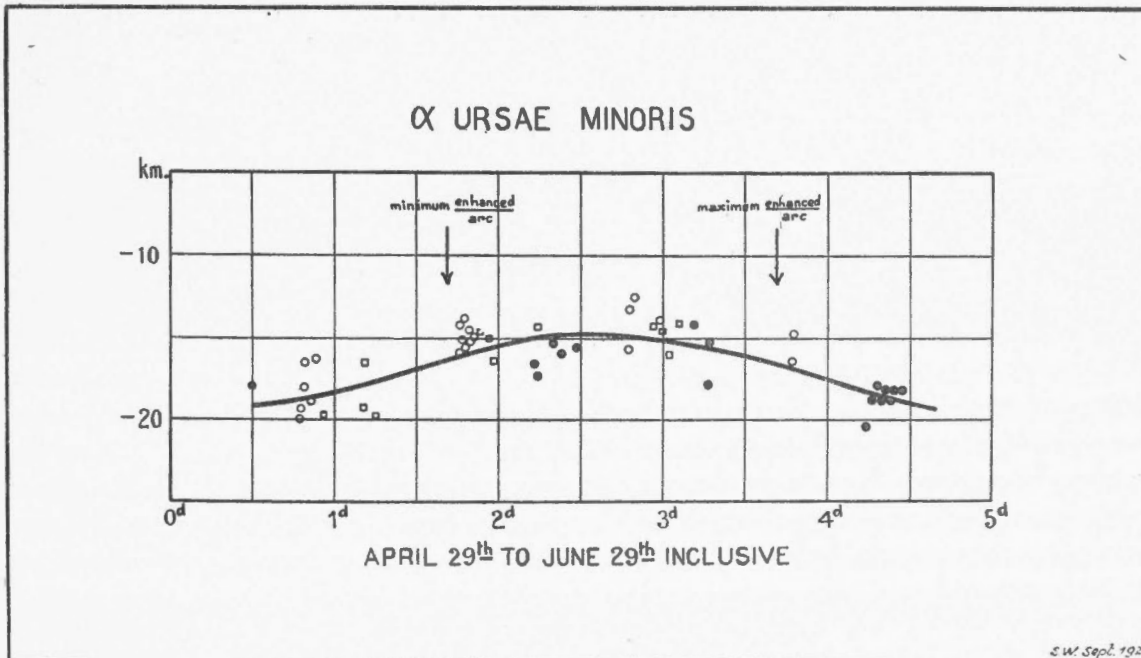
FIGURE 12. RADIAL VELOCITY CURVE OF ALPHA URSAE MINORIS

The radial velocity curve is in good agreement with the theory of Cepheid variation, the maximum velocity of approach coinciding with the maximum of light. The few observations obtained in September give on the whole lower velocities than those obtained before, indicating that the centre of mass of the system has changed by an appreciable amount between May and September. It may be, indeed, that what has been interpreted above as a possible change of amplitude is merely an indication of a decreasing value of the centre-of-mass velocity between April and June.

The centre-of-mass velocity given by these observations disagrees entirely with the provisional elements of the orbit of the centre of mass given by Miss Hobe in Dr. Campbell's catalogue referred to above. This might of course be due to a systematic difference between our radial velocities and those of the Lick Observatory, but this is scarcely likely, since standard velocity stars measured here, (with the same standard daylight spectrogram), have given velocities agreeing very well with those obtained at Mt. Hamilton. The variation found between the observations of June and those of September being fairly large (as compared with the total range of variation of the centre-of-mass velocity 5·96 km. given by Miss Hobe) it seems that a shorter period than 11·9 years has to be looked for.

An examination of all our plates of α Ursae Minoris shows definitely the existence of a variation in spectral type. Three separate investigations, by J. F. Frédette, R. Callander, and the writer, seem to indicate that the minimum value of the ratio between the intensities of enhanced and arc lines occurs at about phase $1^d \cdot 2$, and the maximum at about phase $3^d \cdot 2$; the maximum is more vaguely determined, however, than the minimum and might without difficulty be interpreted to occur at phase $0^d \cdot 0$, coinciding with the maximum of light. It may be remarked that in the Cepheids in which changes of spectral type have been detected (by Shapley) the maximum and minimum values of this ratio appear usually to coincide very nearly with the maximum and minimum of light. The possible divergence from this rule in the case of Polaris would therefore be very interesting if verified.

Chapter VII

DELTA CEPHEI

Delta Cephei, the most important Cepheid variable, is too well known to require an extended bibliography here; many such can be found in the ordinary text-books on astronomy. It was discovered to be variable in 1784 by Goodricke,[1] and was first studied by him and Pigott; they gave as the period of variation $5^d 8^h 37^m$. About thirty years later it was observed by Westphal (1817-1818)[2] and from 1840 to 1856 by Argelander.[3] After this it was observed by a fairly large number of astronomers; among the visual light curves that have been published may be mentioned those of Schur,[4] Beljawsky,[5] Markwick,[6]

[1] Phil. Trans., Vol. 76, p. 50.
[2] Naturf. Ges. Neue Schriften, Heft 2.
[3] A. N., Vol. 18, p. 133, and Vol. 44, p. 195.
[4] A. N., Vol. 137, p. 297.
[5] A. N., Vol. 165, p. 225.
[6] Mem. British Astr. Ass., Vol. 11, Table IV, and Jour. Br. Astr. Ass., Vol. 17, p. 211.

Felix de Roy,[1] Luizet,[2] Miss Clerke,[3] Bemporad,[4] Padova,[5] Lau,[6] Stratonow[7] and Stebbins.[8] Only three photographic light-curves have been published; those by Wirtz[9] and by Meyermann[10] were determined by the extrafocal method; Jordan determined both the photographic and the photovisual light curves.[11]

The best light-curves are perhaps those determined by Guthnick, first with a rubidium[12] and later with a potassium[13] photo-electric cell. He finds no secondary oscillations in either the ascending or the descending branch.

R. H. Curtiss supports the suggestion that since 1850 the light range of δ Cephei has been measurably variable.[14] The question of a slight variation of period has been widely discussed; an interesting paper on this subject is that published recently by Danjon.[15]

Considering the importance of the star it may be said that its spectrographic study has been more or less neglected. Bélopolsky obtained a few series of spectrograms of it during the course of several years[16] and Dr. Moore determined its spectroscopic orbit at the end of 1907.[17] Since 1908 apparently no long series of spectrograms have been obtained. For the purpose of comparison there is given below a collected table of radial velocities previously published.

[1] Bul. Soc. Astr. de France, 1905, p. 414.
[2] Les Céphéides considérées comme étoiles doubles (Ann. Univers. Lyon Nouvelle Série, Vol. 1, 1912, fascicule 33).
[3] The Observatory, Vol. 19, p. 114.
[4] Mem. Spettr Italiani, Vol. 39, p. 74.
[5] Item, Vol. 40, p. 102, and (2) Vol. 1, p. 141.
[6] Bul. Astr., Vol. 23, p. 20.
[7] Taschkent, Pub., Vol. 5, p. 32.
[8] A. N., Vol. 154, p. 334.
[9] Ap. J., Vol. 27, p. 192.
[10] A. N., Vol. 175, p. 1.
[11] Ap. J., Vol. 50, p. 201.
[12] A. N., Vol. 208, p. 172.
[13] A. N. Jubilaumsnummer Zum Hundert. Best tafel 2.
[14] Pub. Amer. Astr. Soc., 26th and 28th meetings.
[15] L'Astronomie, Vol. 37 (1923), p. 346.
[16] Mitteilungen der Nikolai-Hauptsternwarte zu Pulkowa, Band III, p. 69.
[17] L. O. B., Vol. 7, p. 153.

RADIAL VELOCITIES OF δ CEPHEI OBTAINED AT OTHER OBSERVATORIES

Author	Date	Julian Day	Phase	Velocity	Remarks
				km.	
Bélopolsky....	1894	Not given	0·13	+3·7	The spectrograms of 1894, '95, '97 and '98 were obtained with a one-prism spectrograph and measured on a direct measuring engine. Phases counted from epochs of light-minimum as given in the Annuaire du Bureau des Longitudes.
			0·67	+ 8·2	
			1·04	− 8·5	
			1·50	−27·1	
			2·29	−29·8	
			2·42	−23·2	
			2·79	−26·3	
			3·00	−17·3	
			3·29	−16·9	
			3·38	−15·5	
			3·50	−13·4	
			3·79	−12·3	
	1894	Not given	4·13	− 4·5	
			4·50	+ 0·2	
			4·79	+ 2·0	
			4·92	+ 4·0	
			5·00	+ 6·2	
	1895	Not given	0·50	− 6·9	
			0·71	− 7·6	
			1·00	−17·4	
			1·63	−38·5	
			1·92	−32·1	
			2·33	−28·2	
			2·71	−35·1	
			3·67	−24·6	
			4·04	−16·9	
			4·92	− 1·9	
			5·04	− 6·7	
			5·33	− 1·6	
	1897	Not given	1·15	−21·5	
			1·25	−39·3	
			1·47	−39·3	
			1·64	−31·5	
			3·07	−24·3	
			3·93	−19·6	
			5·12	−23·7	
	1898	Not given	0·63	− 8·3	
			0·75	− 4·2	
			1·00	−21·2	
			1·17	−15·4	
			1·83	−40·6	
			2·13	−35·5	
			2·17	−41·8	
			2·25	−29·1	
			2·63	−29·2	
			2·87	−29·6	
			3·13	−27·8	
			3·25	−24·7	
			3·79	−13·4	
			3·87	−19·2	
			4·75	− 7·5	

RADIAL VELOCITIES OF δ CEPHEI OBTAINED AT OTHER OBSERVATORIES—*Continued*

Author	Date	Julian Day	Phase	Velocity	Remarks
				km.	
Bélopolsky....	1902	Not given	1·61	−42·8	The spectrograms of 1902, '03, '04, '05 and '08 were
			1·65	−38·9	obtained with a three-prism spectrograph and
			1·75	−41·7	measured on a spectro-comparator.
			2·10	−33·6	
			3·10	−28·2	
			4·49	−12·8	
	1903	Not given	0·22	− 4·6	
			0·46	(− 1·9)	
			0·85	− 8·2	
				(5·7)	
			1·18	−28·6	
			1·22	−32·0	
			2·23	−39·7	
			2·83	−30·3	
			3·22	−28·7	
			4·22	−15·4	
			4·56	−14·7	
			·5·16	− 6·8	
			5·22	− 9·9	
	1904	Not given	0·63	+ 3·5	
			1·15	−21·9	
			2·14	−33·6	
			3·14	−25·2	
			3·80	−16·6	
			4·80	− 8·4	
	1905	Not given	0·30	+ 2·2	
			3·91	−18·1	
	1908	Not given	1·12	−17·6	
			1·23	−30·5	
			2·11	−32·0	
			3·12	−30·7	
			3·49	−20·7	
			3·85	(−16·2)	
			4·11	−15·7	
			4·91	− 6·7	
Lick Observers.....	1896				
	Nov. 12	2413876·647	1·615	−22·36	Beginning at this point the phases have been com-
	1897				puted from the epochs of light-maximum derived
	Nov. 11	2414240·695	0·749	−29·10	from the formula Max=J. D. 2393659·856 +
	1898				5^d·366386 (Luizet's elements).
	Oct. 25	2414588·774	0·013	−34·18	
J. H. Moore...	1907				
	Sept. 18	2417837·698	2·273	−14·20	
		·761	2·336	−15·44	
		·836	2·411	−12·25	
	Sept. 19	838·692	3·267	− 6·78	
		·850	3·425	− 4·97	
	Sept. 23	842·743	1·952	−18·51	
		·826	2·035	−17·09	
	Sept. 25	844·735	3·944	+ 2·22	
		·827	4·036	+ 3·49	

RADIAL VELOCITIES OF δ CEPHEI OBTAINED AT OTHER OBSERVATORIES—*Concluded*

Author	Date	Julian Day	Phase	Velocity	Remarks
				km.	
	1907				
J. H. Moore...	Sept. 26	2417845·724	4·933	−29·01	
		·809	5·018	−30·50	
	Sept. 29	848·716	2·558	−11·97	
		·811	2·653	−10·25	
	Oct. 6	855·755	4·231	+ 3·24	
		·796	4·272	+ 2·65	
	Oct. 7	856·716	5·192	−36·00	
		·778	5·254	−35·90	
		·836	5·312	−35·59	
	Oct. 9	858·797	1·906	−19·55	
	Oct. 10	859·670	2·779	− 9·84	
	Oct. 12	861·605	4·715	−16·32	
	Oct. 13	862·727	0·470	−32·30	
		·769	0·512	−31·64	
	Oct. 17	866·667	4·410	+ 2·78	
		·760	4·503	− 0·73	
	Oct. 18	867·736	0·113	−35·29	
		·782	0·159	−34·55	
	Oct. 19	868·615	0·992	−26·76	
	Oct. 30	879·721	1·365	−24·30	
	Nov. 10	890·677	1·588	−21·99	
	Nov. 13	893·645	4·556	− 3·86	
		·687	4·598	− 6·40	
		·737	4·648	−10·08	
	Nov. 14	894·711	0·256	−34·69	
	Nov. 24	904·659	4·837	−24·69	
	Nov. 29	909·663	4·475	− 1·98	
	Dec. 12	922·623	1·336	−25·21	
	1908				
	Jan. 5	946·611	3·858	− 0·31	
Küstner.......	1911				
	Aug. 24	2419273·43	5·18	−36·1	A. N., Vol. 198, p. 441.
	1912				
	Sept. 24	670·36	5·00	−30·4	
	Sept. 26	672·32	1·49	−30·9	

On plotting radial velocity curves from the above observations it becomes evident that Bélopolsky's observations have much larger errors than those of Moore; the latter may be said to be excellent.

The radial velocities obtained here are given below; the phases are computed as above from the epochs of light maximum as obtained from Luizet's formula, Max. = J. D. 2393659·856 + 5^d·366386 E.

RADIAL VELOCITIES OF δ CEPHEI OBSERVED AT OTTAWA

Date		Julian Day	Phase	Velocity	Remarks
	1923		d	km.	
July	4...............	2423605·697	1·407	−22·6	Remeasure −21·4
	8...............	609·710	0·054	−33·9	−35·9
	10...............	611·615	1·959	−19·2	−17·5
	12...............	613·783	4·127	+ 2·9	+ 1·7
	19...............	620·751	0·362	−27·0	−28·4
	22...............	623·621	3·232	+ 5·3	Fair plate + 5·4
	23...............	624·692	4·303	+ 5·8	+ 9·0
	29...............	630·733	4·978	−27·7	
	31...............	632·666	1·544	−24·3	
Aug.	9...............	641·604	5·116	−35·9	
	15...............	647·556	0·335	−35·7	
		·602	0·381	−31·1	
	20...............	652·570	5·349	−34·1	
	22...............	654·611	2·024	−13·8	
	23...............	655·583	2·996	− 7·4	
		·693	3·106	− 0·6	
	26...............	658·636	0·682	−26·4	
	30...............	662·576	4·622	−11·4	
		·649	4·695	−16·2	
		·728	4·774	−23·4	
		·806	4·852	−20·6	
Aug.	31...............	663·657	0·337	−35·2	
Sept.	5...............	668·535	5·215	−37·7	
		·585	5·265	−34·8	
	6...............	669·553	0·866	−24·2	
	10...............	673·822	5·135	−34·1	
		·864	5·177	−35·0	
	12...............	675·554	1·501	−19·1	
	13...............	676·542	2·489	−10·1	
		·612	2·559	− 5·7	
	14...............	677·560	3·507	− 1·6	

DELTA CEPHEI

km
+10

o mean of two
measures

0

−10

−20

−30

0^d 1^d 2^d 3^d 4^d 5^d

RADIAL VELOCITY CURVES

The broken line is the radial velocity-curve determined by Dr Moore in 1907.

Mag.

4·5

5·0

0^d 1^d 2^d 3^d 4^d 5^d

PHOTOGRAPHIC LIGHT CURVE

+20

+10

0^d 1^d 2^d 3^d 4^d 5^d

RATIO OF INTENSITY $\frac{enhanced\ line}{arc\ line}$ OR VARIATION OF SPECTRAL CLASS

S.W. Oct. 1923.
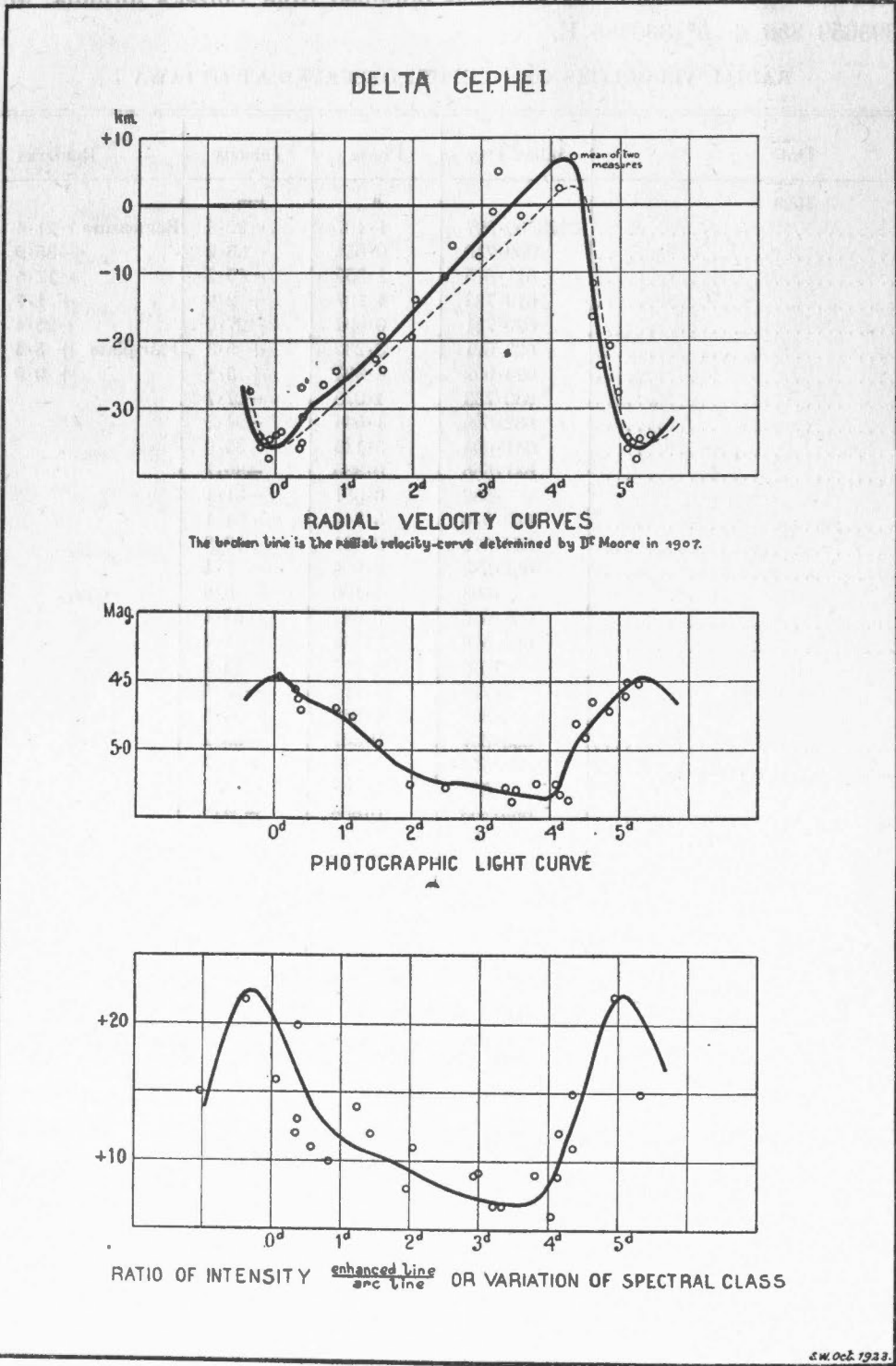
FIGURE 13. RADIAL VELOCITY, MAGNITUDE AND SPECTRAL CLASS OF DELTA CEPHEI

It may be remarked that these spectrograms, taken as they were with a single prism spectrograph on fairly coarse-grained plates (Seed 30), and with comparatively long exposures, will of course have larger probable errors than the Lick spectrograms. They were measured on the spectro-comparator, using the same standard spectrogram of daylight as that used for measuring the plates of Polaris. The radial velocity-curve obtained from the above velocities (*see* Fig. 13) appears slightly different from that of Moore (shown as a broken line in the figure). A change of amplitude and perhaps of center-of-mass velocity has apparently occurred. If the variation of light is a direct function of the variation of radial velocity (as for instance in the case of 12 Lacertae, where the photo-electric observations of Guthnick[1] show the amplitudes of light and velocity curves to vary simultaneously), the suggestion of R. H. Curtiss that the light range of δ Cephei has been measurably variable would receive additional weight.

It is also worth noting that the minimum of our velocity curve, which is well determined, and corresponds to the maximum light, falls a trifle earlier than the predicted one, and that by a quantity of the same order of magnitude as predicted by Danjon (equation of light); this fact lends further colour to the supposition that the star moves in a long-period orbit. Bélopolsky, from his observations, obtained for this orbit a period of 6·36 years; Moore, on the contrary, from a comparison of his own observations with the three older Lick observations given in the above table, concluded that the general velocity curve did not show any variation, and that no long-period orbit existed. A point apparently overlooked by Bélopolsky is the possibility of a variation of amplitude, he having considered only a variation of center-of-mass velocity; his observations, although having perhaps the largest probable errors of any of the observations above, suggest, when replotted, both slight variations of amplitude and of center-of-mass velocity. Moore's conclusions (the value of which, as he himself points out, is based entirely on three older observations, which may have fairly large probable errors) may perhaps require revision. The definite determination of the possible long-period orbit with the observations so far available, especially in view of the possibility of a variation of amplitude of the short-period curve, seems at present impossible.

During the period covered by the Ottawa spectrograms there were also made, with a short-focus camera, plates of the surrounding star-field. These have been used to determine the photographic magnitude of δ Cephei, taking as standards the photographic magnitudes of the three following stars as given in the Revised Harvard Photometry[2]:—

Star	Photo. Mag.
ε Cephei	4·65
λ Cephei	5·29
B. D.+56° 2765	5·86

---

[1] A. N. Jubilaumsnummer zum Hundert, Best. Tafel 3.
[2] H. A., Vol. 50.

The method of estimation of the magnitudes is described in Chapter IX of the present paper; the results are given in the following table:—

ESTIMATED PHOTOGRAPHIC MAGNITUDES OF δ CEPHEI

Date	Julian Day	Phase	Photographic magnitude	Date	Julian Day	Phase	Photographic magnitude
1923		d		1923		d	
July 6.......	2423607·76	3·47	5·35	Aug. 30.......	2423662·56	4·61	4·65
8.......	609·70	0·04	4·45	31.......	663·62	0·30	4·55
10.......	611·64	1·98	5·25	Sept. 3.......	666·69	3·37	5·27
12.......	613·77	4·11	5·30	4.......	667·56	4·24	5·35
19.......	620·75	0·36	4·60	5.......	668·56	5·24	4·50
Aug. 3.......	635·60	4·48	4·90	6.......	669·56	0·87	4·70
5.......	637·60	1·11	4·75	10.......	673·54	4·85	4·70
8.......	640·59	4·10	5·25	....	·82	5·13	4·50
9.......	641·59	5·10	4·60	12.......	675·55	1·50	4·95
13.......	645·66	3·81	5·25	13.......	676·55	2·50	5·25
15.......	647·60	0·38	4·70	14.......	677·55	3·50	5·29
19.......	651·60	4·38	4·80				

These magnitudes (which in this case are *merely estimates*, and cannot compare in accuracy with the magnitudes obtained for fainter variables in Chapter IX, where large numbers of comparison stars were available) give the light curve shown in Fig. 13. It is seen that the maximum brightness coincides almost exactly with the minimum radial velocity, while the minimum brightness slightly precedes the maximum radial velocity. This had already been found by Dr. Moore on comparison of his velocity curve with the photometric light-curve of Prof. Stebbins. In the present case the comparison of the two curves presents the advantage that they were obtained during the same period of time.

Although very nearly of the same spectral class as the Sun, the spectrum of δ Cephei presents marked differences; and these differences appear most marked when the star is near its maximum brightness. δ Cephei is a giant G star, while the Sun is a dwarf of the same class; since the density of the former is much the smaller, although the masses and temperatures may not be very different, a greater degree of dissociation of the elements is to be expected. Hence ionized elements will be preponderant and the spectral lines of ionized atoms will be strong (enhanced lines), while the spectral lines of neutral atoms will be weak (arc lines).

It was found in 1913 by Inna Lehmann[1] at Poulkowa that the appearance of the spectrum of δ Cephei varied in the course of the period. All Bélopolsky's three-prism spectrograms were compared by her with a chosen standard spectrogram of the same star; the intensities of eight spectral lines, probably all enhanced lines, were estimated on each spectrogram, taking as the unit of intensity for each line its intensity on the standard plate; the comparisons were made on a spectro-comparator. In this way a curve of variation of intensity was deduced, and was found to show perfect parallelism with the curve of variation of light as obtained by Stebbins.

[1] Pulkowo Mitteilungen, Band V, p. 177.

In 1916 Adams and Shapley[1] pointed out again the marked changes in the spectrum between maximum and minimum luminosity; the most marked of these are as follows:—

Spectral line	Maximum	Minimum
Hydrogen line $H\gamma$	Strong	Much weakened.
Enhanced lines of *Fe, Ti, Sr.* and *Cr*	Strong	Much weakened.
λ 4481, enhanced *Mg*	Very strong	Much weakened.
λ 4227, calcium	Strong	Strengthened.
Low temperature lines of *Ca, Fe, Ti* and *Cr*	Weak	Strengthened.
Continuous spectrum	Strong in violet	Weakened in violet.

This is equivalent to a variation of spectral class, similar to that shown in many Cehpeids and probably present in all. On examination of our spectrograms it was found that the titanium arc line, λ 4534·953, and the titanium enhanced line, λ 4534·139, show a very marked variation of their relative intensities. The comparison of this close pair of lines on different spectrograms offers perhaps the best criterion for determination of the variation of spectral class. On a large number of our spectrograms this pair of lines was examined; in each case the intensity of the arc line was assumed as 10 and the intensity of the enhanced line estimated with respect to it. The estimated intensities of the enhanced line are given in the following table:—

RELATIVE INTENSITIES OF ENHANCED AND ARC LINES

Date	Phase	Enhanced *Ti* λ 4534·139 Intensity	Arc *Ti* λ 4534·953 Intensity	Date	Phase	Enhanced *Ti* λ 4534·139 Intensity	Arc *Ti* λ 4534·953 Intensity
1923	d			1923	d		
July 4	1·407	12	10	July 30	0·593	11	10
6	3·354	7	10	31	1·544	7	10
8	0·054	16	10	Aug. 5	1·236	14	10
10	1·959	8	10	8	4·144	9	10
11	2·964	9	10	13	3·801	9	10
12	4·022	6	10	15	0·335	12	10
12	4·127	12	10	15	0·381	20	10
19	0·362	13	10	19	4·353	11	10
22	3·232	7	10	20	5·349	15	10
23	4·303	15	10	22	2·024	11	10
25	0·828	10	10	23	2·996	9	10
29	4·978	24	10				

These are of course mere estimates, but when the relative intensities are plotted according to phase they give the curve in Fig. 13, showing very definitely the variation of spectral class.

[1] Proc. Nat. Acad. of Sc. of the U.S.A., Vol. 2, 1916, p. 136.

Chapter VIII

THE SPECTROSCOPIC SYSTEM GAMMA URSAE MINORIS

γ Ursae Minoris ($\alpha = 15^h\ 20^m \cdot 9\ \delta = +72°11'$, class A2) was discovered by Otto Struve to be a star of the β Canis Majoris type;[1] its period is the shortest of this type at present known. From 200 spectrograms secured at the Yerkes Observatory between February and August, 1922, Struve finds a period of $2^h\ 36^m\ 10^s$ or $0^d \cdot 108449$; he also finds the shape and range of the velocity curve to vary considerably, not only from one night to another, but even during the same night. Its brightness has also been found by Guthnick to be variable, the variation having an amplitude of approximately $0 \cdot 04$ magnitude; in May, 1922, Dr. Bottlinger, at Babelsberg, found the variations to occur in very short intervals of time ($2 \cdot 3$ hours, approximately), but no definite period could be established.

Though the star was still under investigation by Struve, it was, with his consent, placed also on our observing programme. The spectrum is exceedingly poor, the lines few, wide and diffuse, and it proved extremely difficult to measure the spectrograms, many of which, indeed, had to be rejected. On account of the extreme shortness of the period it was necessary to make short exposures, and hence to use a wide slit, a circumstance which quite possibly introduced considerable accidental errors due to poor guiding. Too much importance should therefore not be attached to the results. The exposures averaged twenty minutes, Seed 23 plates being generally used; the spectrograms were measured on a Toepfer engine. The radial velocities obtained are given in the following table:—

RADIAL VELOCITIES OF γ URSAE MINORIS

Date	Julian Day	Velocity	Remarks	Date	Julian Day	Velocity	Remarks
1922		km.		1922		km.	
Sept. 17...	2423315·531	−27·1	Poor.	Feb. 15...	2423466·784	−13·2	
	·554	−47·0			·798	−26·9	
	·603	−19·8			·810	−36·3	
	·681	−26·6			·824	−28·3	
	·704	−20·1			·837	−15·7	
	·728	−21·3			·850	−22·2	
	·742	−31·5			·863	−13·4	
	·756	−43·9			·892	− 8·2	
	·784	−36·5			·904	− 8·5	
	·798	−36·9		Feb. 22...	473·625	−29·6	
Sept. 21...	319·544	−39·7			·640	−30·0	
	·569	−43·7			·655	−26·7	
	·585	−50·7			·701	−25·2	
	·601	−43·5			·717	−18·6	
	·616	−16·6			·733	− 7·5	Poor.
	·674	−37·1			·768	−34·8	
	·689	−45·1			·810	− 0·7	
1923	·704	−37·9			·827	− 9·0	
Jan. 26...	446·719	− 2·4	Poor.		·849	−14·4	
	·744	−14·5	Poor.		·867	−28·2	
	·792	− 7·6			·883	−26·2	
	·814	− 5·5			·897	−26·6	
	·837	−31·0			·913	−29·7	
	·859	−29·2		Feb. 25...	476·543	−12·1	
	·881	−25·9	Poor.		·570	−43·1	
	·903	−27·9	Poor.		·584	−37·4	

[1] Pub. Amer. Astr. Soc., 28th meeting, p. 391.

To these may be added the following, which have been kindly communicated by Dr. Struve.

Date	Julian Day	Velocity	Remarks	Date	Julian Day	Velocity	Remarks
1922		km.		1922		km.	
May 29...	2423204·699	−15·7		May 29...	2423204·775	+21·6	
	·712	−13·6			·783	+ 5·8	
	·718	− 0·3					
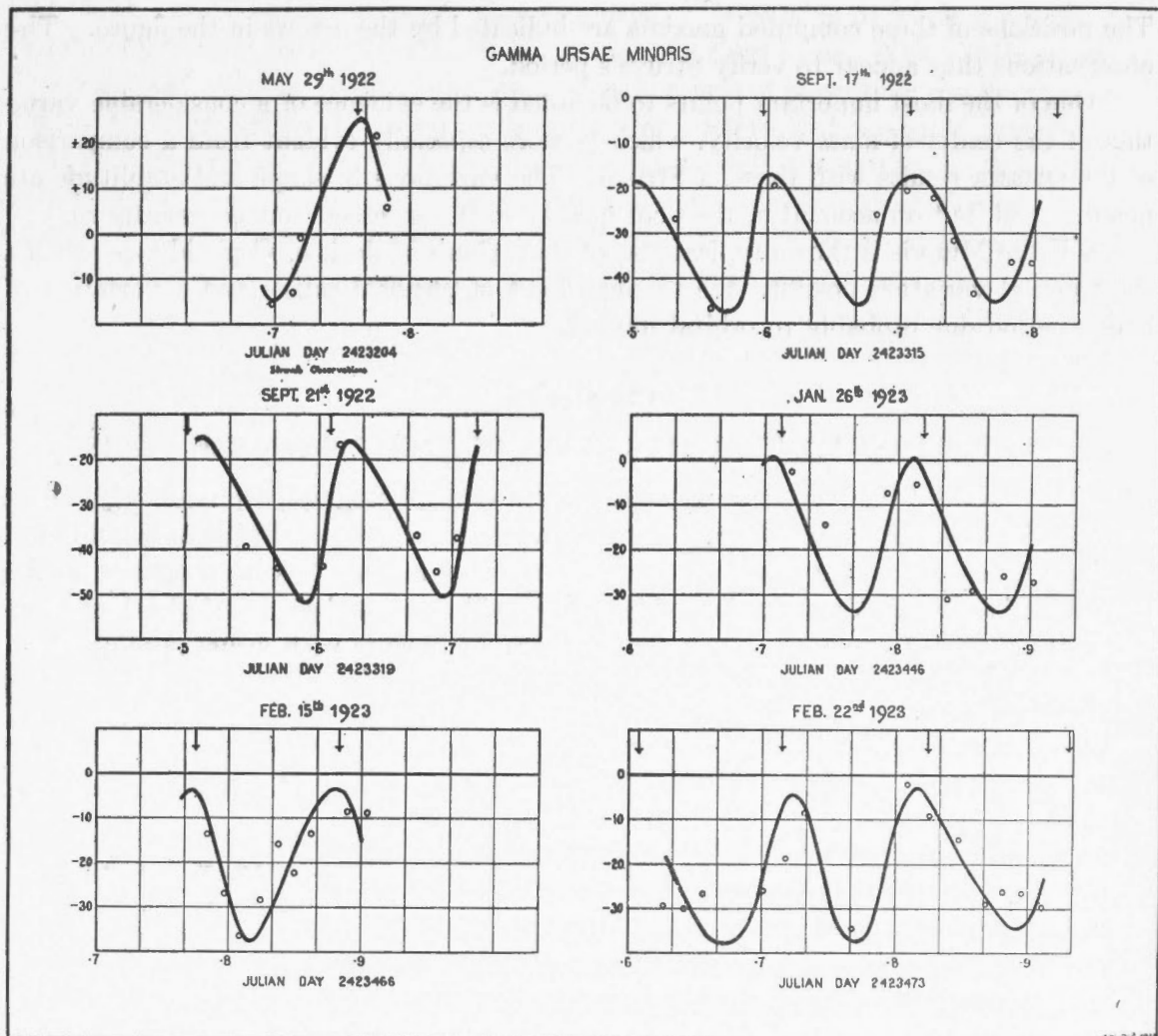
FIGURE 14. RADIAL VELOCITY CURVES OF GAMMA URSAE MINORIS

The above observations are shown in graphic form in Fig. 14. Struve's observations give a maximum at J.D. 2423204·762, and his formula, Max. = J.D. 2423204·762 + 0^d·108499 E, gives the following maxima on the dates of the Ottawa observations:—

J.D.	2423315·488	J.D.	2423446·712
	·597		·820
	·705		466·774
	·813		·883
	319·501		473·607
	·609		·716
	·718		·824

The positions of these computed maxima are indicated by the arrows in the figure. The observations thus appear to verify Struve's period.

One of the most important points to be noted is the evidence of a considerable variation of the center-of-mass velocity, which is more especially evident from a comparison of the Ottawa results with those of Struve. The variations in shape and amplitude are possibly real, but on account of the poor quality of the spectrum not necessarily so.

γ Ursae Minoris is thus a typical star of the β Canis Majoris or Cepheid type, with a short-period variation possibly due to some kind of physical cause, and a variation of longer period due probably to orbital motion.

Chapter IX

THE PHOTOGRAPHIC LIGHT-CURVES OF EIGHT VARIABLE STARS

In the present chapter is introduced a method of photographic photometry which has the advantage of being rapid and at the same time, for a certain range of magnitudes, as accurate as any of the best methods in use. This method has been applied to the variable stars given in the following table. They will be treated separately in the following paragraphs, with comparisons of the photographic results with visual ones obtained elsewhere.

VARIABLE STARS OBSERVED

Star	α 1855·0		δ 1855·0	
	h	m	°	′
X Cygni	20	37·7	+35	4·0
SZ Cygni	20	28·2	+46	6·5
TX Cygni	20	54·8	+42	2·0
UY Cygni	20	50·4	+29	52·6
VX Cygni	20	51·9	+39	37·2
VY Cygni	20	58·7	+39	23·7
WZ Cygni	20	47·6	+38	16·9
XZ Cygni	19	29·5	+56	4·6

A large number of plates of each variable has been secured with a short-focus camera, (lens two inches aperture and thirteen inches focal length). The plates, which are cabinet size (6½ by 4¾ inches), cover a wide area of the sky, and have the advantage both of having a large number of star images per unit area and of showing a large range in the diameters of these star images.

The image of the variable star is compared with the images of a number of stars whose photographic magnitudes have been accurately determined; often some twenty to forty stars are used, which on account of the small scale of the photograph can all be found in the field of view of the microscope under which the plate is examined. One or two star images on each plate are chosen having as nearly as possible the same intensity as the variable, that is, covering the same area, and with the same shape and density; often two stars are used, one a trifle brighter and the other a trifle fainter than the variable, so that a careful interpolation gives the true magnitude of the latter. In short, this is nothing else but Argelander's method, applied by him directly to the determination of photometric magnitudes, but here applied to the photographic plate. The latter, offering a field which can be examined much more comfortably, is capable of yielding results of considerable accuracy, especially with the advantage of a short focus. With a little experience the method is rapid; eighty to one hundred estimations of magnitude can be made in a day; if the magnitudes of the comparison stars are determined correctly, and if the differences of magnitude between them are sufficiently small, the method is practically free from systematic errors. For the brighter stars, down to about the fifth magnitude, the method is less accurate, because few comparison stars are to be found within the field of the microscope, but for stars between the eighth and twelfth magnitudes it attains a very high degree of accuracy, especially for variables which like the Cepheids are located in the Milky Way, where surrounding stars of the same order of magnitude are abundant. For stars fainter than these a camera of greater aperture and longer focal length would be of advantage.

The greatest difficulty in using this method is in determining accurately the photographic magnitudes of the comparison stars, since there is no reliable durchmusterung of the northern sky giving the photographic magnitudes of the stars. Kapteyn has given the photographic magnitudes of a considerable number of southern stars, down to about the tenth magnitude, in the Cape Photographic Durchmusterung; he also gives the photographic magnitudes (on the international scale) of stars to a considerable degree of faintness in his 108 selected areas of the northern hemisphere.[1] A certain number of astrographic catalogues (of the Carte Photographique du Ciel) furnish photographic magnitudes, or the diameters of star images with the necessary constants to compute these magnitudes; these are, however, far from covering the whole sky, and are also, in some cases, scarcely accurate enough for use in the investigation of variable stars. The new Henry Draper Catalogue of Harvard gives photographic magnitudes for all the stars it contains; for the stars south of $-19°$ declination the magnitudes given are derived (applying certain corrections to reduce to the international scale) from the Cape Photographic Durchmusterung, while for stars north of that declination they are in most cases derived by adding to the photometric magnitude the colour-index corresponding to the spectral class. The magnitudes so obtained are, however, far from reliable, being often in error by several tenths of a magnitude and sometimes by even a whole magnitude. The most reliable sources of photographic magnitudes on the international scale are the North Polar Sequence[2] and its associated Harvard Standard Regions, as well as Miss Leavitt's Standards of Magnitude for the Astrographic Catalogue.[3]

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[1] H. A., Vol. 101.
[2] H. A., Vol. 71.
[3] H. A., Vol. 85, No. 1.

To determine the photographic magnitudes, on the international scale, of stars in the field of a variable, the practice was followed of photographing successively on the same plate, on a clear moonless night, both the field of the variable and one of Miss Leavitt's fields, taking care that the two fields were approximately at the same altitude. The photographs were made with the six-inch Brashear doublet camera, which gives excellent definition; with careful guiding it was found possible to obtain sharply defined and perfectly round images. The diameters of the star images of the two fields were then measured accurately with a micrometer microscope, and, using the diameters of the star-images of Miss Leavitt's field as abscissae and her magnitudes as ordinates, a curve connecting the magnitudes with the diameters of the star-images was determined. This curve was used to deduce the photographic magnitudes of the stars in the field of the variable from the measured diameters of their images. The number of stars in each field being quite large the use of such a curve was found simpler and to some extent more accurate than the use of any of the standard formulae connecting the magnitudes and diameters of the star images.

The plates taken with the short-focus camera mentioned above required of course somewhat prolonged exposures, the aperture-ratio being slightly less than $f/6$; they were obtained, however, in the course of the regular programme of spectrographic observations; the small camera was mounted on the tube of the equatorial, near the objective, and plates exposed as opportunity offered during the progress of a series of spectrograms of the star in question.

The several variables mentioned above are discussed individually in the following pages.

X Cygni

The variability of X Cygni was discovered by Chandler in 1886.[1] It is a typical Cepheid of class Go whose photometric elements are given by the formula:—

$$\text{Max.} = \text{J. D. } 2410190 \cdot 678 + 16^d \cdot 38543 \text{ E.}$$

Visual light-curves have been given by Pickering[2] and by Luizet[3]; Wilkens obtained the photographic light-curve by the extra-focal method,[4] while a comparison of the photographic and photo-visual light-curves is afforded by the results of Jordan.[5] According to Luizet the luminosities at maximum and at minimum show a certain amount of variation, small but real; an investigation of such variations with a view to determining their laws would be of interest. It may be noted that variations of this nature have been suspected in the case of δ Cephei by R. H. Curtiss, and have been recognized in other Cepheids, viz.—RR Lyrae and stars of the β Canis Majoris type. The investigation of such variations, which are indicative of an effect due to a relatively long-period orbital motion, is possibly important in relation to the formation of an adequate theory of Cepheid variation.

[1] A. J., Vol. 7, p. 32.
[2] H. A., Vol. 46, p. 156.
[3] A. N., Vol. 193, p. 85.
[4] A. N., Vol. 172, p. 325.
[5] A. J., Vol. 50, p. 191.

Luizet secured 795 observations of X Cygni between 1898 and 1912 by the use of field glasses and Argelander's method. His observations, combined into normal places and transformed into visual magnitudes (they were given originally in a system of arbitrary grades, making comparisons with other observations difficult) are given in the following table:—

LUIZET'S VISUAL MAGNITUDES OF X CYGNI

Phase	Mag.	Phase	Mag.	Phase	Mag.
d		d		d	
0·327	6·40	6·182	7·00	11·430	7·15
0·763	6·40	6·515	6·97	11·880	7·11
1·274	6·54	6·934	7·04	12·319	7·03
1·762	6·60	7·330	7·04	12·689	6·96
2·182	6·59	7·636	7·10	13·013	7·01
2·635	6·64	8·032	7·13	13·333	6·99
2·962	6·64	8·454	7·16	13·732	6·89
3·373	6·70	8·808	7·15	14·144	6·78
3·857	6·74	9·156	7·21	14·553	6·56
4·174	6·80	9·528	7·23	14·942	6·47
4·722	6·86	9·913	7·23	15·468	6·39
5·055	6·83	10·507	7·23	15·888	6·36
5·470	6·83	11·038	7·21	16·248	6·43
5·847	6·88				

The following table gives the photographic magnitudes of the comparison stars in the field of X Cygni, as measured from our plates by the method outlined above:—

FIELD OF X CYGNI

B. D.	α 1855·0	δ 1855·0	.Phtg. Mag.	Spect. Class	Phtg. Mag. in Draper Catal.
	h m	° '			
4109..............	20 34·2	+34 56	9·31		
4111..............	20 34·3	+34 52	8·74	Ko	7·86
4114..............	20 35·2	+34 31	8·40	F2	7·51
4217..............	20 35·9	+35 13	8·23	B9	8·2
4218..............	20 35·9	+35 24	8·90	K2	8·59
4219..............	20 36·0	+35 2	8·69	Go	7·98
4220..............	20 36·1	+35 23	9·80		
4221..............	20 36·2	+35 14	9·39	F5	8·8
4224..............	20 36·6	+35 37	9·39	F8	9·3
4127..............	20 36·7	+34 56	6·50	B3	6·33
4229..............	20 37·1	+35 52	8·48		
4231..............	20 37·4	+35 14	8·96	Ao	8·8
4232..............	20 37·6	+35 13	8·06	Ao	8·6
4004..............	20 37·6	+33 21	7·10	Ao	7·8
4234..............	20 37·7	+35 4	Var.	Gop	
4237..............	20 37·9	+35 4	9·63		
4009..............	20 38·1	+33 33	7·95	Ao	8·6
4240..............	20 38·5	+35 1	9·39		
4022..............	20 41·0	+33 51	8·82	A5	9·3
4268..............	20 41·8	+35 45	8·31	A3	8·3

In the last column have been added the photographic magnitudes of many of the stars as given in the Draper Catalogue, where they have been obtained by adding the colour-index corresponding to the given spectral class to the visual magnitude. It is to be noted that the discrepancies are in many cases large; it would appear, as has already been pointed out by many astronomers, among them Prof. H. N. Russell, that the law connecting visual magnitude, spectral class and photographic magnitude is only approximate, and in some cases far from true. In particular cases a good deal no doubt depends on whether the star is a giant or a dwarf, and it might be well, for the same spectral class, to look for a relation between visual magnitude, photographic magnitude and density.

The photographic magnitudes obtained here for X Cygni are given in the following table:—

PHOTOGRAPHIC MAGNITUDES OF X CYGNI

Date	Julian Day	Phase	Phtg. Mag.	Wt.
1920		d		
Aug. 19..	2422556·7	11·4	8·75	2
20..	557·6	12·3	8·23	2
1922				
July 13..	2423249·7	16·2	6·75	1
19..	255·7	5·8	8·60	2
20..	256·7	6·8	8·69	2
20..	256·8	6·9	8·69	2
24..	260·7	10·8	8·75	2
24..	260·8	10·9	8·80	2
25..	261·7	11·8	8·48	2
25..	261·8	11·9	8·69	2
26..	262·7	12·8	8·23	2
28..	264·7	14·8	7·10	2
28..	264·7	14·8	6·80	1
31..	267·8	1·5	7·05	1
Aug. 18..	285·7	3·1	7·55	1
18..	285·7	3·1	7·60	1
23..	290·7	8·1	8·75	2
25..	292·6	10·0	8·85	2
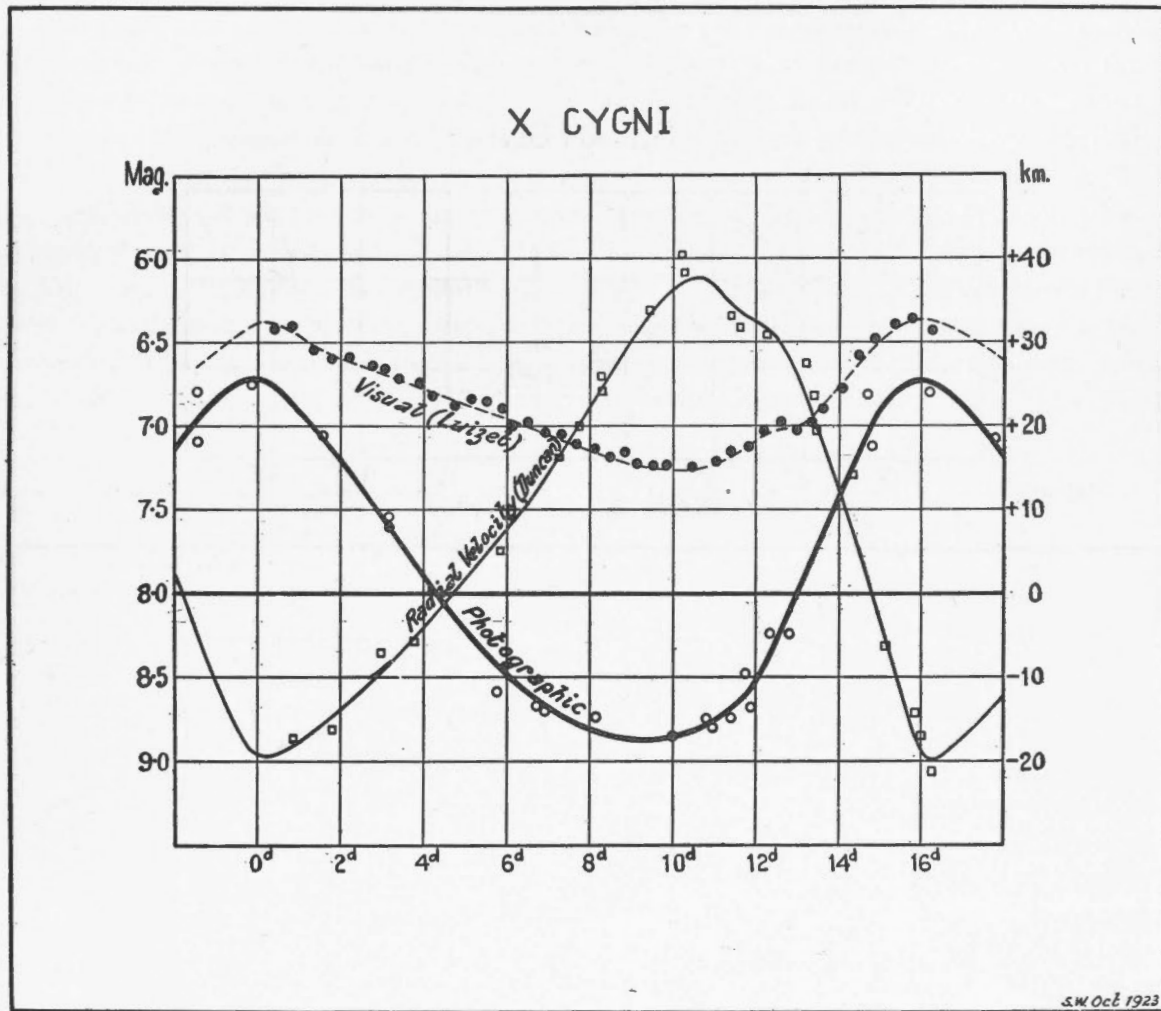
FIGURE 15. VARIATIONS OF X CYGNI

In Fig. 15 the broken line shows the visual observations of Luizet, the heavy continuous line the Ottawa photographic curve. For the sake of comparison there is shown on the third curve a plot of the radial velocities as obtained by Duncan[1] from plates taken with the 60-inch Mt. Wilson reflector. A small hump just succeeding the 12^d epoch is suggested by the curves. The large difference in range between the visual and photographic curves is perhaps the outstanding effect.

SZ Cygni

SZ Cygni was found to be variable by Williams[2] in 1900. It has been followed visually by many observers and two visual light-curves have been published, one by Lau,[4] the other by Luizet.[3] Luizet's results, transformed from his arbitrary grades into magnitudes, are shown in Fig. 16, together with the photographic light-curve as determined here.

[1] Contributions from the Mt. Wilson Observatory, Vol. 9, p. 396.
[2] A. N., Vol. 152, p. 77.
[3] Bul. Soc. Astr. France, 1907, p. 95.
[4] Bul. Astr., Vol. 25, p. 212.
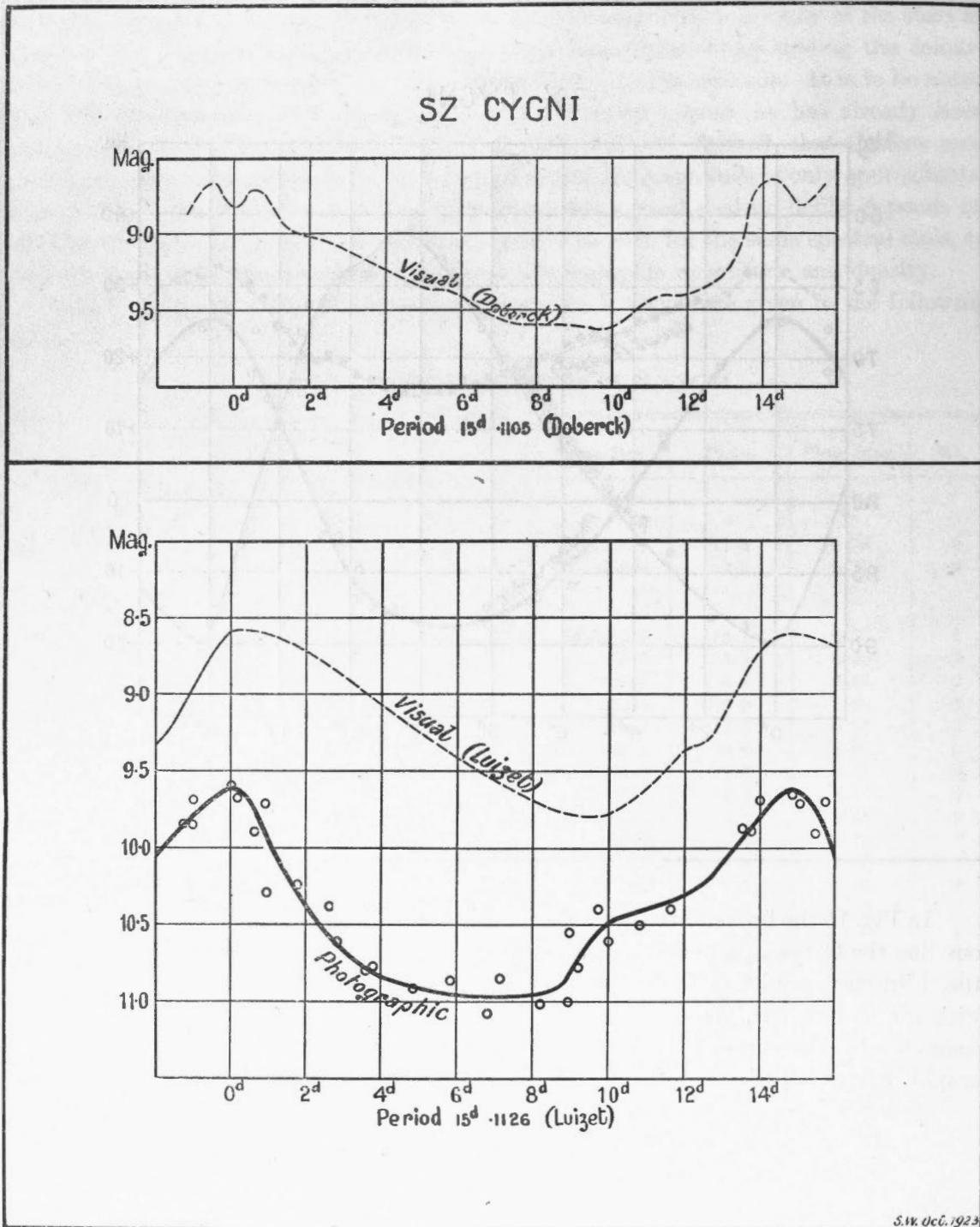
FIGURE 16. LIGHT CURVES OF SZ CYGNI

Luizet draws attention to the probability of the reality of the marked hump shown on the ascending branch. Comparison of the photographic light-curve with the visual one of Luizet shows that the average colour-index corresponds very well to the given spectral class, which is given as Ko. In other respects, however, there are marked differences;

the photographic curve shows a more decided hump on the ascending branch and a much more rapid descent immediately after maximum than does Luizet's curve. Another remarkable feature is that the photographic range of magnitude is hardly larger than the visual. It becomes an interesting question, which may repay further study, whether this is by virtue of the possible fact that the ratio of these ranges is a function of the spectral class, or whether it is an individual peculiarity of this particular star. The images, it may be noted, were in most cases located not far from the edge of the plate, but it seems unlikely that this would have had a very marked effect on the determination of the photographic magnitudes, especially since the comparison stars were close to the variable. The photographic magnitudes of the comparison stars as obtained here, and the resulting magnitudes of SZ Cygni, are given in the following tables:—

FIELD OF SZ CYGNI

B. D.	α 1855·0		δ 1855·0		Phtg. Mag.	Remarks
	h	m	°	′		
2956.........	20	26·1	+46	18	9·55	
2958.........	20	26·3	+46	17	8·10	
2960.........	20	26·5	+46	14	8·90	
2961.........	20	26·6	+46	24	9·42	
2965.........	20	27·9	+46	5	10·77	Var. suspected.
	20	28·1	+46	21	10·88	
2966.........	20	28·2	+46	6	Var.	
2967.........	20	28·4	+46	22	9·87	
2968.........	20	28·5	+46	23	9·35	
	20	28·5	+46	37	10·34	
2969.........	20	28·6	+46	40	8·80	
2971.........	20	28·8	+46	14	10·60	
	20	28·8	+46	26	10·76	
2975.........	20	29·0	+46	30	9·14	
2976.........	20	29·1	+46	26	10·31	
	20	29·1	+45	57	10·85	
	20	29·2	+45	56	11·12	
	20	29·2	+46	2	10·53	
2978.........	20	29·2	+46	14	9·49	
	20	29·3	+46	20	11·73	
	20	29·3	+46	26	12·06	
	20	29·4	+46	4	12·00	
2982.........	20	30·0	+46	20	7·90	
3220.........	20	30·0	+45	54	10·22	
	20	30·2	+46	0	11·48	
2984.........	20	30·3	+46	13	10·40	
	20	30·3	+46	2	11·01	
	20	30·4	+46	16	10·53	
	20	30·5	+46	20	10·57	
	20	30·7	+45	53	10·75	Var. suspected.
	20	30·8	+46	32	9·75	
3226.........	20	31·2	+45	50	9·66	
3227.........	20	31·2	+45	59	9·87	
	20	31·4	+46	9	10·60	
2988.........	20	31·4	+46	15	9·60	
3228.........	20	31·5	+45	47	9·79	
	20	31·6	+46	0	10·88	

PHOTOGRAPHIC MAGNITUDES OF SZ CYGNI

Date	Julian Day	Phase	Phtg. Mag.	Wt.	Date	Julian Day	Phase	Phtg. Mag.	Wt.
1920		d			1920		d		
July 20...	2422526·6	9·2	10·76	2	Sept. 20...	2422588·6	10·8	10·51	1
Aug. 20...	557·6	10·0	10·60	1	23...	591·5	13·7	9·87	1
24...	561·6	14·0	9·70	1	Oct. 6...	604·6	11·7	10·40	2
25...	562·6	15·0	9·60	2	10...	608·6	0·6	9·90	2
25...	562·8	0·1	9·66	2	11...	609·7	1·7	10·26	1
26...	563·6	0·9	10·30	1	12...	610·5	2·5	10·40	3
Sept. 1...	569 8	7·1	10·88	1	Dec. 2...	661·6	8·2	11·01	1
3...	571·6	8·9	11·00	1	1922				
10...	578·6	0·8	9·70	1	July 20...	2423256·7	13·9	9·87	2
13...	581·5	3·7	10·80	1	24...	260·7	2·8	10·60	1
15...	583·6	5·8	10·88	1	25...	261·7	3·8	10·80	3
18...	586·8	9·0	10·55	2	26...	262·7	4·8	10·94	1
19...	587·6	9·8	10·40	3	28...	264·7	6·8	11·10	1

The above magnitudes were those used in plotting the photographic curve. The phases were computed from the formula:—

$$\text{Max.} = \text{J. D. } 2415097 \cdot 08 + 15^d \cdot 1126 \, E.$$

Doberck[1] in 1920 gave the following slightly different elements based on his visual observations:—

$$\text{Max.} = \text{J. D. } 2421836 \cdot 37 + 15^d \cdot 1105 \, E.$$

As may be seen from Fig. 16, he obtains a close double maximum and a hump at phase about 11^d very similar to the one indicated by the photographic observations. The use of his formula would increase the phases of the Ottawa observations by about a day; the latter therefore agree better with the period $15^d \cdot 1126$. The existence of a double maximum is doubtful, while the existence of a hump on the ascending branch appears probable. If we reduce Doberck's observations with the period $15^d \cdot 1126$ there appear to be indications of a slow displacement of the hump. In 1917 Luizet places it at about phase $12^d \cdot 2$; several years later Doberck's observations would place it at about $10^d \cdot 2$, while our observations place it at about $9^d \cdot 6$. If this displacement should be confirmed it would constitute a new and highly interesting problem to elucidate, for which a long series of observations would be required.

TX Cygni

TX Cygni was discovered to be variable by Williams in 1900.[2] It was followed visually by several observers, among whom were Hartwig,[3] Yendell,[4] van der Bilt and v. Zeipel.[5] No light curve, however, has been published; the table given below of v. Zeipel's observations, made with a Zöllner photometer at Upsala Observatory, has been formed by computing the phases from the formula:—

$$\text{Max.} = \text{J. D. } 2417010 \cdot 5 + 14^d \cdot 71 \, E.$$

which is found in Hartwig's ephemerides.

[1] A. J., Vol. 32, p. 164.
[2] A. N., Vol. 154, p. 147.
[3] Viertaljahrschrift, Vol. 36, p. 269.
[4] A. J., No. 563.
[5] A. N., Vol. 177, p. 376.

PHOTOMETRIC OBSERVATIONS OF TX CYGNI BY v. ZEIPEL

Date	Julian Day	Phase	Magnitude	Remarks
1907		d		
Jan. 16..	2417592·245	8·055	10·09	
17..	593·243	9·053	9·98	
Sept. 13..	832·374	12·824	9·80	
15..	834·383	0·123	8·83	
17..	836·373	2·113	9·18	
21..	840·337	6·077	9·71	
Oct. 16..	865·335	1·655	9·33	
Nov. 29..	909·367	1·557	9·27	Poor.
30..	910·301	2·491	9·25	
Dec. 29..	939·285	2·055	9·33	
30..	940·279	3·049	9·19	
31..	941·294	4·064	9·42	
1908				
Jan. 1..	942·319	5·089	9·67	
7..	948·297	11·067	9·95	
13..	954·266	2·326	9·25	
19..	960·242	8·302	9·95	
24..	965·255	13·315	9·67	
27..	968·266	1·616	9·29	
30..	971·273	4·623	9·42	
Feb. 4..	976·194	9·544	9·97	Poor.
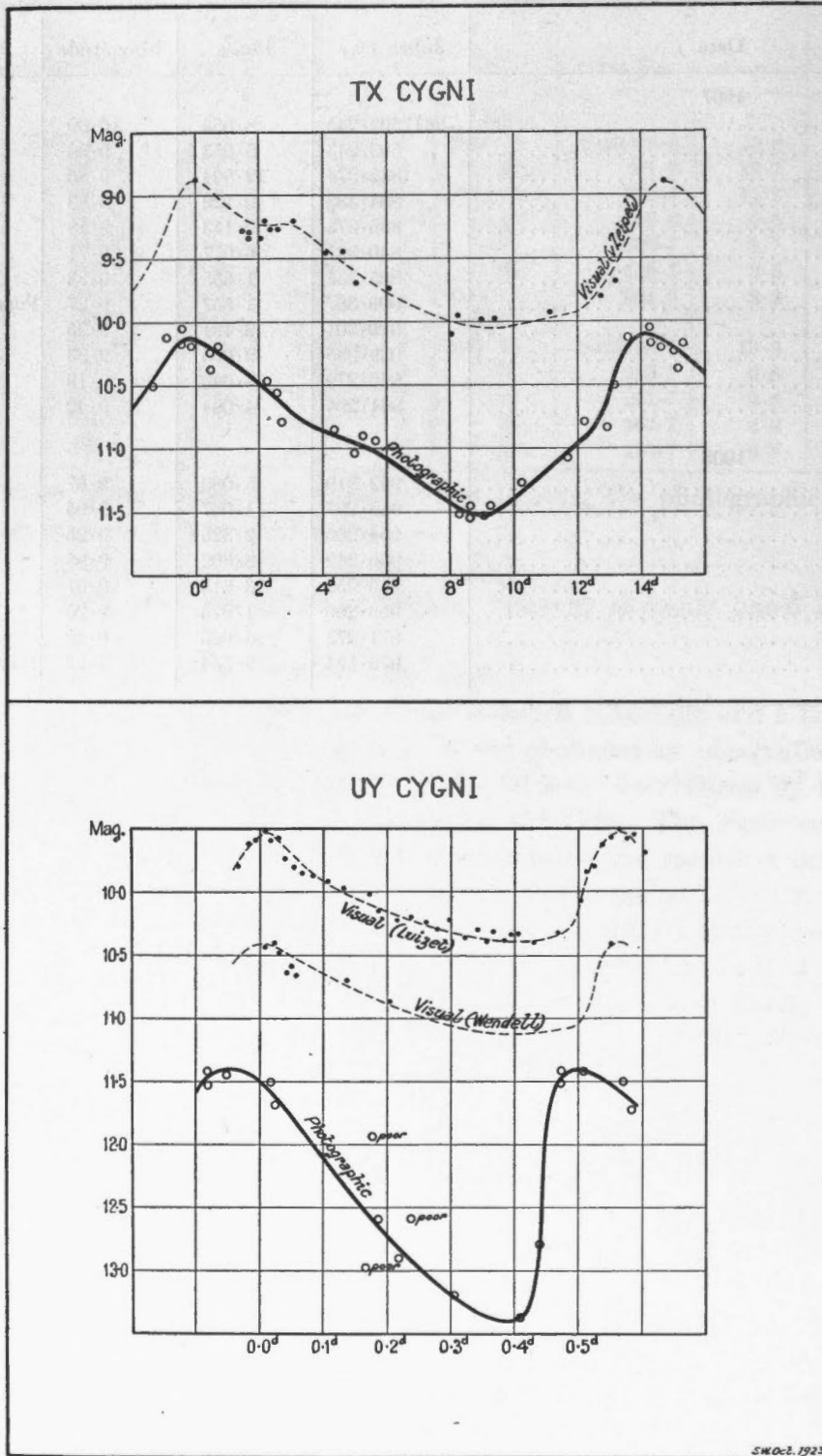
FIGURE 17. LIGHT CURVES OF TX CYGNI AND UY CYGNI

These observations give the visual light-curve shown in Fig. 17. The photographic magnitudes of the comparison stars as determined here with the resulting photographic magnitudes of TX Cygni are given in the two following tables:—

FIELD OF TX CYGNI

B. D.	α 1855·0	δ 1855·0	Phtg. Mag	B. D.	α 1855·0	δ 1855·0	Phtg. Mag.
	h m	° ′			h m	° ′	
3918.........	20 51·9	+42 1	10·02	3964........	20 55·4	+41 59	10·59
	52·0	+42 4	10·78		55·6	+41 32	11·14
3944........	52·2	+41 53	9·01		55·6	+42 0	12·02
3921........	52·6	+42 4	10·15		55·7	+42 35	10·93
3950........	53·2	+41 54	9·43		55·9	+42 6	11·00
3926........	53·4	+42 9	10·23	3941........	56·1	+42 11	10·93
	53·5	+42 15	11·22	3940........	56·1	+42 27	10·52
3954........	53·6	+41 49	10·38		56·1	+41 58	11·54
3955........	53·9	+41 30	9·84		56·1	+41 29	11·49
3929........	53·9	+42 22	10·48		56·3	+41 46	10·68
3930........	53·9	+42 23	10·63		56·3	+42 18	11·13
3959........	54·2	+41 28	10·49		56·4	+42 14	10·80
	54·2	+41 37	11·08	3942........	56·6	+42 29	10·01
	54·3	+41 40	10·97	3944........	56·7	+42 17	9·61
	54·6	+42 25	11·38	3943........	56·7	+42 7	10·10
3933..........	54·6	+42 33	10·82		56·8	+42 15	11·00
	54·7	+42 33	11·58	3967........	56·9	+42 0	10·20
	54·8	+42 2	Var.	3968........	57·0	+41 56	11·05
3963........	55·1	+41 37	9·09	3948........	57·0	+42 2	10·08

PHOTOGRAPHIC MAGNITUDES OF TX CYGNI

Date	Julian Day	Phase	Phtg. Mag.	Wt.	Date	Julian Day	Phase	Phtg. Mag.	Wt.
1920		d			1922		d		
July 20...	2422526·6	14·6	10·18	2	July 20...	2423256·8	9·3	11·47	1
Aug. 19...	556·7	0·5	10·22	3	24...	260·7	13·2	10·50	1
Oct. 6...	604·6	4·3	10·87	1	25...	261·7	14·2	10·05	3
7...	605·6	5·3	10·91	1	25...	261·8	14·3	10·15	3
10...	608·6	8·3	11·50	1	26...	262·7	0·5	10·38	2
12...	610·5	10·2	11·30	1	28...	264·7	2·5	10·57	2
1921					31...	267·8	5·6	11·43	2
Aug. 4...	906·7	12·2	10·80	3	Aug. 18...	285·7	8·7	11·46	1
5...	907·6	13·1	10·84	2	18...	285·7	8·7	11·51	1
22...	924·6	0·7	10·19	1	23...	290·7	13·7	10·11	3
24...	926·6	2·7	10·80	1	29...	296·7	5·0	11·02	2
1922					1923				
July 13...	2423249·7	2·2	10·48	1	Aug. 9...	641·7	11·7	11·05	5
20...	256·7	9·2	11·51	1					

These magnitudes give the photographic curve in Fig. 17. It will be seen that the maximum is slightly displaced, indicating probably a slight correction ($-0^d \cdot 0014$) to the assumed period $14^d \cdot 71$; the more exact period would therefore be $14^d \cdot 7086$. The mean difference between the visual and photographic magnitudes indicates that the star belongs probably to spectral class K. Here again, as in the case of SZ Cygni, the difference

between the amplitudes of the visual and photographic light-curves is not large, the latter being a trifle greater. The contrast between these two and X Cygni is marked; the three stars have periods of the same order of magnitude; it seems difficult to believe however, that X and TX can have about the same absolute magnitude. Another remarkable feature is the difference in shape of the ascending branches of the visual and photographic curves, a difference very similar to that exhibited between the descending branches of SZ Cygni.

UY Cygni

UY Cygni was found to be variable by Williams in 1902,[1] and recognized by him to be of very short period. Visual observations have been published by Williams,[2] Hartwig,[3] Graff,[4] Luizet[5] and Wendell.[6] The normal places corresponding to the observations of Luizet are given in the following table:—

LUIZET'S VISUAL MAGNITUDES OF UY CYGNI

Phase	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase	Magnitude
d		d		d		d	
0·0070	9·50	0·1354	9·96	0·2978	10·20	0·4280	10·39
0·0156	9·59	0·1706	10·00	0·3216	10·35	0·4686	10·33
0·0276	9·57	0·1922	10·11	0·3406	10·28	0·5004	10·08
0·0396	9·74	0·2142	10·20	0·3548	10·36	0·5148	9·83
0·0512	9·78	0·2364	10·17	0·3680	10·28	0·5296	9·77
0·0666	9·83	0·2600	10·21	0·3820	10·35	0·5414	9·60
0·0854	9·84	0·2774	10·28	0·3950	10·31	0·5522	9·56
0·1094	9·90			0·4028	10·30		

The phases in heliocentric time have been computed from the formula of Williams:—

$$\text{Max.} = \text{J. D. } 2415346 \cdot 3933 + 0^d \cdot 5607103 \text{ E.}$$

[1] A. N., Vol. 158, p. 45.
[2] Monthly Notices, Vol. 63, p. 304, and Vol. 65, p. 586.
[3] Vierteljahrschrift, Vol. 37, p. 284; Vol. 38, p. 240.
[4] A. N., Vol. 197, p. 253.
[5] Bul. Astr., Vol. 24, p. 342.
[6] H. A., Vol. 69, p. 126.

The photographic magnitudes of the comparison stars, as obtained here, are given in the following table:—

FIELD OF UY CYGNI

B. D.	α 1855·0	δ 1855·0	Phtg. Mag.	B. D.	α 1855·0	δ 1855·0	Phtg. Mag.
	h　m	°　′			h　m	°　′	
4213........	20　45·9	+29　31	9·40	4231........	20　48·5	+30　13	10·70
	20　46·2	+29　29	12·20	4232........	20　48·5	+30　19	10·84
	20　46·3	+30　5	10·10	4230........	20　48·6	+29　16	10·45
4216........	20　46·4	+29　39	11·00		20　48·8	+29　24	13·0
4217........	20　46·4	+29　49	10·10		20　49·0	+29　33	13·4
4218........	20　46·4	+29　42	9·65		20　49·1	+29　33	11·12
4219........	20　46·6	+29　34	10·50		20　49·1	+29　47	11·43
4221........	20　47·0	+29　7	7·79	4236........	20　49·3	+30　11	11·40
	20　47·0	+29　57	12·7	4234........	20　49·3	+29　13	8·70
	20　47·2	+30　16	11·40		20　49·5	+29　34	12·3
	20　47·3	+30　7	11·77	4238........	20　49·6	+29　45	10·79
4218........	20　47·5	+30　24	7·87		20　49·6	+29　40	13·3
	20　47·5	+30　19	11·30	4240........	20　50·3	+29　44	9·18
	20　47·6	+29　12	11·40		20　50·4	+29　53	Var.
	20　47·6	+30　23	10·30		20　50·6	+29　44	12·6
	20　47·7	+29　59	13·3	4244........	20　50·7	+29　54	11·94
	20　47·8	+30　1	11·77	4244........	20　50·7	+30　8	9·55
4223........	20　48·0	+30　13	10·40	4247........	20　50·9	+29　59	11·20
4226........	20　48·1	+29　9	10·69	4248........	20　50·9	+29　53	10·75
4227........	20　48·1	+29　10	9·64	4251........	20　51·5	+29　58	10·55
4228........	20　48·3	+29　7	10·31	4255........	20　52·4	+29　37	9·91
	20　48·4	+29　39	13·4	4256........	20　52·4	+29　50	9·99
	20　48·4	+29　36	13·1				

The next table gives the resulting photographic magnitudes of UY Cygni. The dates given are heliocentric, and, as there were a few fairly long exposures, have been corrected also for the error introduced by the length of exposure. To determine these corrections a preliminary curve was plotted with the mean epochs of the exposures as abscissae. On this curve the area comprised between the axis of X, the vertical lines passing through the abscissae corresponding to beginning and end of exposure, and the curve was divided into equal areas by a vertical line, and the time indicated on the axis of X by this vertical line was taken as the true time when the star had the measured photographic magnitude.

PHOTOGRAPHIC MAGNITUDES OF UY CYGNI

Date	Corrected Helio-centric Julian Day	Phase	Phtg. Mag.	Date	Corrected Helio-centric Julian Day	Phase	Phtg. Mag.
1922		d		1922		d	
July 20.....	2423256·708	0·374	13·40	July 28.....	2423264·796	0 052	11·67
20.....	256·801	0·468	11·50	31.....	267·781	0·233	12·90
24.....	260·729	0·471	11·43	Aug. 18.....	285·671	0·181	11·94
25.....	261·698	0·318	13·20	18.....	285·741	0·251	12·60
25.....	261·796	0·416	12·80	23.....	290·751	0·214	12·60
26.....	262·682	0·181	13·00	29.....	296·741	0·036	11·50
28.....	264·698	0·515	11·40				

The phases have been computed from the formula of Williams given above. These results, as well as those of Luizet, are exhibited in graphical form in Fig. 17. It will be seen that according to the formula of Williams, which satisfied Luizet's observations exactly, the maximum is slightly displaced. If the photographic and visual maxima coincide it appears likely that the period has decreased a little. A shorter period, $0^d \cdot 5607065$, would perhaps serve as a compromise, but it would not satisfy the early observations, covering the years 1902 to 1906, quite as well as $0^d \cdot 5607103$, as shown in the following table (heliocentric time in Julian days).

<div align="center">

PERIOD OF UY CYGNI

</div>

Author	Observed Maxima	Computed with $0^d \cdot 5607103$	Computed with $0^d \cdot 5607065$
Williams...	2415984·483	·482	·477
Luizet...	2417383·457	·454	·440
"	457·466	·468	·453
"	466·441	·439	·425
Ottawa...	2423260·204	·258	·205

There is apparently a systematic error in the magnitudes of the comparison stars used by Luizet for his curve. The observations of Wendell, which are also shown in Fig. 17, although few, are probably on a better absolute scale of visual magnitude; they indicate a curve of shape similar to that obtained by Luizet. The comparison of Wendell's visual curve with our photographic one indicates a large colour-index, which would normally indicate that the star is of rather advanced spectral class K to M, a classification which would be rather remarkable in view of the short period. The difference in range between visual and photographic curves is also remarkable.

VX Cygni

VX Cygni was found to be variable by Williams in 1903.[2] It was observed visually by Hartwig, Seares,[3] van der Bilt, Beljawsky, Doberck[4] and Williams.[5] It is from the observations of the latter that the visual light curve shown in Fig. 18 is derived. The best elements, as given in Hartwig's ephemerides for 1922, are:—

$$\text{Max.} = \text{J. D. } 2414935 \cdot 0 + 20^d \cdot 1306 \text{ E.}$$

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[1] Note added in January, 1925, while going through press). VY Cygni has been recently found, at Mt. Wilson, to be of the spectral class Fo near maximum brightness; hence it is probably more advanced than Fo for any other brightness. Pub. Ast. Soc. Pac., Vol. 36, 1924, p. 139.

[2] A. N., Vol. 163, p. 301.

[3] Laws Bul., No. 10.

[4] Journal des observateurs, Vol. 3, pp. 105-108.

[5] A. N., Vol. 168, p. 25.
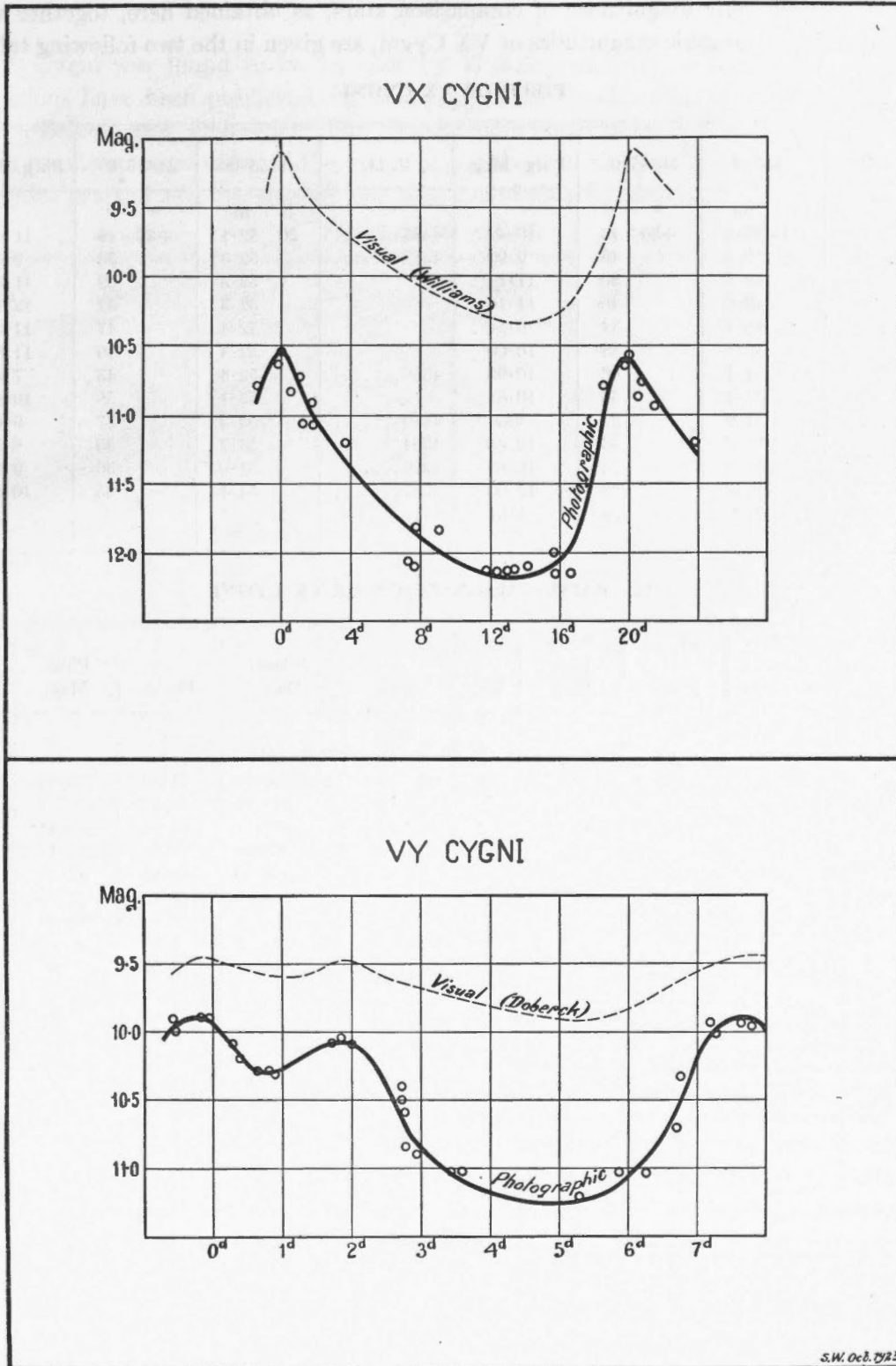
FIGURE 18. LIGHT CURVES OF VX CYGNI AND VY CYGNI

The photographic magnitudes of comparison stars, as obtained here, together with the resulting photographic magnitudes of VX Cygni, are given in the two following tables:

FIELD OF VX CYGNI

B. D.	α1855·0	δ1855·0	Phtg. Mag.	B. D.	α1855·0	δ1855·0	Phtg. Mag.
	h m	° ′			h m	° ′	
	20 49·4	+39 46	10·42	4381...	20 52·1	+39 49	11·06
4365........	49·5	48	9·90	4385........	52·3	33	9·47
	49·8	50	11·12		52·3	52	11·36
	49·8	48	11·16		52·3	52	12·15
4370........	50·1	34	9·64		52·4	47	11·31
4372........	50·1	48	10·00		52·4	49	11·80
4374........	51·1	46	10·92	4386........	52·4	43	7·37
4376........	51·4	56	10·61		53·1	58	10·83
4379........	51·9	37	Var.	4389........	53·3	57	9·67
4380........	51·9	53	10·48	4394........	53·7	30	8·30
	51·9	25	10·80	4396........	53·9	36	9·98
	52·0	39	12·00	4397........	54·1	55	10·50
4381........	52·1	29	9·61				

PHOTOGRAPHIC MAGNITUDES OF VX CYGNI

Date	Julian Day	Phase	Phtg. Mag.	Wt.	Date	Julian Day	Phase	Phtg. Mag.	Wt.
1920					1922				
		d					d		
Aug. 19...	2422556·65	12·28	12·15	1	July 24...	2423260·72	11·78	12·15	1
Oct. 6...	604·57	19·94	10·63	2	25...	261·69	12·75	12·15	2
10...	608·61	3·85	11·21	2	25...	261·79	12·85	12·15	1
Dec. 31...	611·68	6·92	10·84	3	26...	262·68	13·74	12·15	1
1921					28...	264·69	15·75	12·00	1
Aug. 4...	906·66	20·07	10·55	1	28...	264·79	15·85	12·15	1
5...	907·65	0·93	11·06	1	31...	267·78	18·84	10·80	2
26...	928·59	1·74	11·06	1	Aug. 23...	290·75	1·55	10·92	1
1922					29...	296·74	7·54	12·08	2
July 13...	2423249·66	0·72	10·73	2	1923				
20...	256·70	7·76	12·08	2	Sept. 16...	679·62	7·94	12·03	3
20...	256·80	7·86	11·85	2	17...	680·69	9·01	11·85	2

These observations give the photographic light-curve, shown in Fig. 18, which appears to be perfectly regular. The average colour-index corresponds to a fairly advanced spectral class, K, which is the class to which the star belongs. The photographic range is $1^m·75$, while the visual is $1^m·40$; here again the ratio of the two ranges is not far from unity.

VY Cyqni

VY Cygni was found to be variable by Williams in 1903. Several long series of observations have been published by Williams,[1] Luizet[2] who has given a visual light-curve, v. Zeipel[3] and Doberck.[4] The photographic magnitudes of the comparison stars and the resulting magnitudes of the variable are given in the two following tables. The magnitudes marked with an asterisk are Miss Leavitt's standards.

FIELD OF VY CYGNI

B. D.	1855·0		1855·0		Phtg. Mag.	B. D.	α1855·0		δ1855·0		Phtg. Mag.
	h	m	°	′			h	m	°	′	
4400........	20	54·4	+39	41	7·10*	4424........	20	59·0	+39	12	11·00
4318........		56·3	+38	41	7·74*	4425........		59·2	+39	25	11·39*
4418........		56·9	+39	34	10·50	4426........		59·4	+39	34	10·88
4417........		57·4	+39	00	10·84	4428........		59·6	+39	51	9·29*
4418........		57·6	+39	25	9·23*			59·6	+39	31	11·06*
4419........		57·6	+39	29	10·58			59·6	+39	0	11·00
4420........		57·6	+39	49	9·34			59·9	+39	3	11·51
4421........		57·7	+39	40	8·16*		21	0·0	+39	23	10·32
		58·0	+39	46	10·90			0·3	+39	15	11·88
		58·1	+38	59	11·37	4341........		0·3	+38	45	8·19*
		58·3	+39	15	11·57	4433........		0·3	+39	6	11·02
		58·5	+39	24	11·99*	4434........		0·4	+39	42	9·81*
		58·5	+39	14	11·86	4435........		0·5	+39	26	10·11*
		58·6	+39	20	11·37	4439........		1·3	+39	46	10·30
		58·6	+39	51	11·00	4440........		1·3	+39	44	9·93
4423........		58·7	+39	24	Var.	4441........		1·6	+39	52	9·80
		58·8	+39	20	11·81*			1·6	+39	35	11·21
		58·8	+39	27	12·60*	4442........		1·7	+39	23	10·05

PHOTOGRAPHIC MAGNITUDES OF VY CYGNI

Date	Julian Day	Phase	Phtg. Mag.	Remarks	Date	Julian Day	Phase	Phtg. Mag.	Remarks
1920		d			1922		d		
Aug. 19...	2422556·651	1·990	10·11		July 20...	2423256·797	2·834	10·40	
Oct. 6...	604·573	2·768	10·58		24...	260·725	6·762	10·70	
10...	608·613	6·808	10·32		25...	261·694	7·731	9·90	
12...	610·549	0·886	10·32		25...	261·792	7·829	9·93	
1921					26...	262·686	0·866	10·31	
Aug. 4...	906·655	6·271	11·03		28...	264·695	2·875	10·85	
5...	907·649	7·265	10·00		28...	264·782	2·962	10·90	
22...	924·574	0·618	10·30	Poor.	31...	267·777	5·957	11·03	
1922					Aug. 18...	285·667	0·275	10·08	
July 13...	2423249·657	3·551	11·01		18...	285·737	0·345	10·20	
19...	255·697	1·734	10·08		23...	290·747	5·355	11·21	
19...	255·781	1·818	10·05		25...	292·596	7·204	9·93	
20...	256·704	2·741	10·50		29...	296·737	3·487	11·01	

[1] A. N., Vol. 164, p. 43.
[2] A. N., Vol. 176, p. 37.
[3] A. N., Vol. 177, p. 377.
[4] A. J., Vol. 32, p. 188.

These observations give the photographic light-curve shown in Fig. 18. In the same figure is given the visual light-curve as deduced from the observations of Doberck.

The double maximum is a remarkable feature of this light-curve. It is well marked in the curves deduced from the visual observations of Williams, v. Zeipel and Doberck. Luizet considers the curve flat at maximum; that is, the secondary minimum is not shown. Visually this secondary minimum is difficult to detect, but the photographic observations accentuate it considerably. The second maximum is merely a large hump on the descending branch, such as is found in many Cepheids. The star could not be of the β Lyrae type, since the maxima are too near one another, and could not be explained by the revolution around each other of two very close bodies.

VY Cygni is quite possibly a star far advanced in the process of separation into two bodies. It appears to form a connecting link between stars with a smooth light-curve, like VX Cygni, and those of the β Lyrae type, like WZ Cygni, consisting of two bodies.

The difference of amplitude between visual and photographic curves is also quite remarkable.

A plot has been made of the results of the observations of each of the different observers (except those of Luizet which were plotted by himself) and the dates of maximum found for each. These maxima do not verify Luizet's formula,

$$\text{Max.} = \text{J. D. } 2416370 \cdot 9507 + 7^{d} \cdot 85926 \text{ E.}$$

it being found that the elements,

$$\text{Max.} = \text{J. D. } 2416370 \cdot 9507 + 7^{d} \cdot 85732 \text{ E.}$$

give much better agreement. The latter period was therefore used in computing the phases above. The observed and computed dates of maximum corresponding to each group of observations are given in the following table:—

JULIAN DATES OF FIRST MAXIMUM OF VY CYGNI

Observer	Observed J. D.	Computed J. D.	O–C
			d
Williams	2416402·47	2416402·379	+0·09
Luizet	2417408·9	2417408·117	+0·78
Luizet	2417479·2	2417478·832	+0·37
v. Zeipel	2417549·33	2417549·548	−0·22
Doberck	2421745·36	2421745·358	0·00
Ottawa	2423261·53	2423261·820	−0·29

The residuals given in the last column, though possibly due to errors of observation, may on the other hand be real. If this is the case they point to a fluctuation of the period, which it would require further observations to bring out clearly.

WZ Cygni

The variability of WZ Cygni was discovered by Williams in 1905.[1] It was observed by Williams and Shapley.[2]

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[1] A. N., Vol. 169, p. 365.
[2] Contributions from the Princeton Univ. Obs., No. 3, p. 54.

In the two following tables are given the photographic magnitudes of our comparison stars and the resulting magnitudes of WZ Cygni.

FIELD OF WZ CYGNI

B. D.	α 1855·0	δ 1855·0	Phtg. Mag.	B. D.	α 1855·0	δ 1855·0	Phtg. Mag.
	h m	° ′			h m	° ′	
	20 46·6	+39 12	10·98	4262........	20 47·6	+38 17	Var.
4257........	46·6	+39 22	9·13		47·6	+38 21	11·80
	46·6	+38 18	11·23		47·6	+38 11	11·07
	46·7	+38 16	11·00		47·7	+38 8	10·80
	46·7	+38 32	11·84		47·8	+38 19	12·13
	46·8	+38 33	11·57		47·8	+38 7	11·33
	46·8	+38 21	11·20	4265........	48·1	+38 6	10·73
	46·9	+38 29	10·24	4266........	48·5	+38 30	9·69
	46·9	+38 25	10·05		48·6	+38 13	10·80
4259........	47·1	+38 34	10·05	4267........	48·6	+38 29	10·20
4260........	47·2	+38 16	9·35	4269........	48·8	+38 30	10·24
4261........	47·2	+38 22	10·63		48·9	+38 29	11·88
	47·4	+38 20	12·40		49·0	+38 17	10·52
	47·6	+38 10	11·10				

PHOTOGRAPHIC MAGNITUDES OF WZ CYGNI

Date	Geocentric Julian Day	Heliocentric Phase	Phtg. Mag.	Date	Geocentric Julian Day	Heliocentric Phase	Phtg. Mag.
1920		d		1922		d	
Aug. 19.....	2422556·651	0·449	10·52	Oct. 1.....	2423694·590	0·416	10·63
20.....	557·647	0·276	10·73	2.....	695·585	0·262	10·73
Oct. 6.....	604·573	0·444	10·39	5.....	698·569	0·324	10·76
10.....	608·613	0·393	10·62		·601	0·356	10·73
12.....	610·549	0·575	11·37		·635	0·390	10·73
1922					·669	0·424	10·63
July 20.....	2423256·704	0·314	10·73		·702	0·457	10·57
20.....	256·797	0·407	10·70		·739	0·494	10·45
24.....	260·725	0·244	10·62		·774	0·529	10·63
25.....	261·694	0·044	10·98		·814	0·569	11·07
25.....	261·792	0·141	10·52	Oct. 7.....	700·519	0·520	10·70
26.....	262·686	0·451	10·70		·551	0·552	10·90
28.....	264·695	0·123	10·81		·583	0·584	11·32
28.....	264·782	0·220	10·38		·614	0·031	11·14
31.....	267·777	0·282	10·80		·645	0·062	10·95
Aug. 18.....	285·667	0·053	11·03		·678	0·095	10·73
18.....	285·737	0·124	10·52		·711	0·128	10·65
1923					·745	0·162	10·50
Sept. 16.....	679·624	0·082	10·73		·780	0·197	10·52
30.....	693·597	0·027	11·23		·818	0·235	10·60

The formula Max.=J. D. 2414936·5487+0^d·584464 E was used in computing the phases.
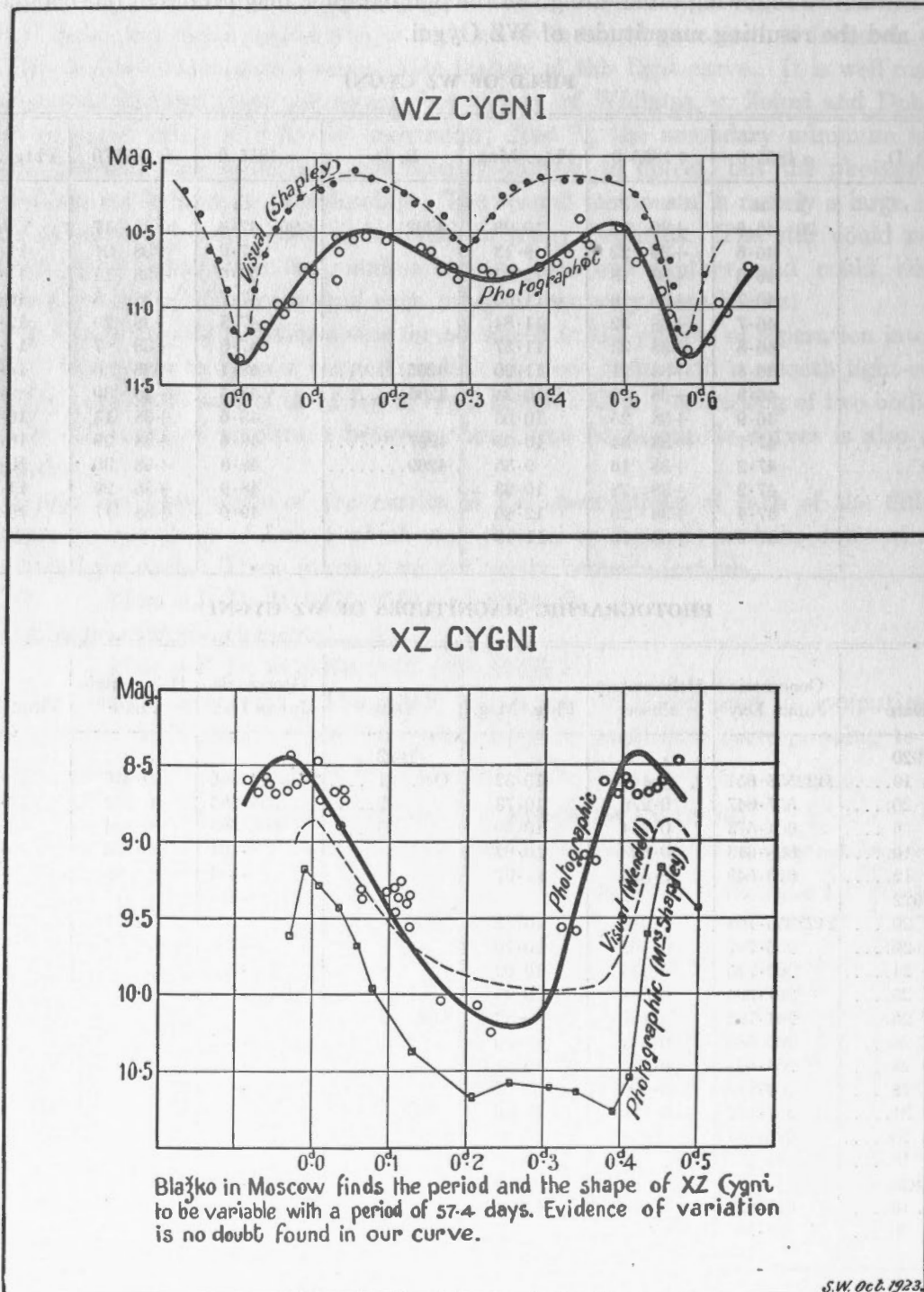
FIGURE 19. LIGHT CURVES OF WZ CYGNI AND XZ CYGNI

These magnitudes give the photographic light curve shown in Fig. 19. In the same figure is found the visual light-curve as deduced from the observations of Shapley. From comparison of the curves it is seen that the average colour-index of the star is small;

this is in agreement with the fact that it is of spectral class A. The shapes of the curves however are different, and there is apparently a general slanting of the photographic curve with respect to the visual; the amplitudes are also slightly different. There are very few variable stars of the β Lyrae type whose photographic light curves have been determined, and for only two of them, apparently, has the photographic curve been compared to the visual. In the case of β Lyrae itself Swarzschild finds only very small differences between the shapes of the curves,[1] and Martin and Plummer in the case of TT Aurigae[2] find that they practically coincide.

The above differences in amplitude and shape, if definitely verified, would somewhat complicate the physical explanation of variations of the β Lyrae type. They might be partially explained by differences in spectral type of the components or by tidal effects. If the suggestions brought forward in the discussion of VY Cygni above should prove to have any value we might expect intermediate stages between quasi-Cepheid variation and that of the true β Lyrae type, of which the present case might be an example. It may be remarked that most stars of the β Lyrae type are of early spectral class, with periods of the same order as might be expected, with not unreasonable suppositions as to density, to produce Jacobian ellipsoids or closely allied forms involving ultimate separation by fission.

XZ Cygni

The variation of XZ Cygni was discovered by Madame Ceraski at the Observatory of Moscow[3] in 1905, and independently by Miss Leavitt, at Harvard in 1907. It was observed visually by Enebo,[4] Wendell,[5] v. Zeipel,[6] Yendell[7] and Blaẑko,[8] while photographic light-curves have been determined by Martin and Plummer,[9] and by Mrs. Shapley;[10] the latter determined also a photo-visual light-curve. The light-curve of Martin and Plummer shows a large number of minor secondary oscillations, as most of their light-curves do. According to Jordan these oscillations are likely due either to errors of observation or more probably to the method of reduction. Mrs. Shapley's magnitudes were determined by the methods of measurement and reduction of Prof. Seares, regularly employed in photometry with the 60-inch reflector;[11] five of her plates were also exposed on the North Polar Standards to determine the magnitudes of the comparison stars. Her curve, as well as our own photographic curve and the visual one of Wendell, is reproduced in Fig. 19. It will be seen that there is a systematic difference between her curve and the Ottawa one, though the amplitude and shape are nearly the same. The scale of relative magnitudes is apparently the same for the two curves, but there is a systematic difference between their absolute values. The Ottawa curve agrees more nearly, as to the scale of absolute magnitude, with that of Martin and Plummer.

[1] Kuffner Pub., Vol. 5, C., p. 123.
[2] Monthly Notices, Vol. 76, p. 395.
[3] A. N., Vol. 168, p. 324.
[4] A. N., Vol. 171, p. 219, and Enebo's Special Publications, Vol. 1, p. 37, and Vol. 11, p. 40.
[5] H. A., Vol. 69, pp. 125 and 165.
[6] A. N., Vol. 4, p. 293.
[7] A. J., Vol. 28, p. 126.
[8] A. N., Vol. 216, p. 112.
[9] Monthly Notices, Vol. 74, pp. 225-233.
[10] Contributions from the Mt. Wilson Observatory, Vol. 7, p. 52.
[11] Contributions from the Mt. Wilson Observatory, Vol. 4, p. 293.

The phases were computed from the formula:—

$$\text{Max.} = \text{J. D. } 2417201 \cdot 25417 + 0^d \cdot 46659 \text{ E};$$

the displacement of the maximum indicates a slight correction to this period. The plot of the individual observations also indicates departures from the mean curve rather larger than the usual accidental errors; there is nothing surprising in this, as variations in the shape and period have apparently been detected by Blažko in Moscow. Blažko observed the star on 98 nights and obtained 87 well determined maxima; he gives for the elements the formula:—

$$\text{Max.} = \text{J. D. } 2417201 \cdot 2350 + 0^d \cdot 4665892 \text{ E}$$
$$+ 0 \cdot 002 \ (E/100)^2 + 0 \cdot 0079 \sin 2^\circ \cdot 927 \ (E + 41 \cdot 5)$$
$$+ 0 \cdot 0024 \sin 5^\circ \cdot 854 \ (E + 1)$$

His period varies between $11^h 11^m 18^s \cdot 39$ and $11^h 12^m 28^s \cdot 23$, and the principal inequality has a period of $57 \cdot 4$ days, covering 123 of the short periods. He finds the shape of the curve also to vary with a period of $57 \cdot 4$ days, the extreme shapes showing the same character as those of RW Draconis.

In the two following tables are given the photographic magnitudes of the comparison stars as obtained here, together with the resulting magnitudes of the variable. The plates of 1923 were made with the 6-inch Brashear doublet camera; the derived magnitudes were obtained by measuring the diameters of the stellar images and deriving from them curves expressing the relation between these diameters and the corresponding magnitudes.

FIELD OF XZ CYGNI

B. D.	α 1855·0	δ 1855·0	Phtg. Mag.	B. D.	α 1855·0	δ 1855·0	Phtg. Mag.
	h m	° ′			h m	° ′	
	19 23·3	+55 36	9·22	2218........	19 28·8	+55 19	9·36
2201........	24·3	+55 40	8·60		29·5	+56 4	Var.
2203........	24·9	+55 47	9·99	2220........	30·1	+55 21	9·53
	25·1	+56 45	10·11	2259........	30·2	+56 7	10·65
	25·3	+55 55	9·84	2260........	30·8	+56 20	10·27
	25·6	+55 59	10·24	2261........	30·9	+56 9	7·13
	26·1	+55 58	10·11	2263........	31·4	+56 18	10·08
2247........	26·6	+56 50	9·00	2266........	31·8	+56 14	10·03
	26·9	+56 3	11·08		31·9	+55 38	9·81
	27·2	+56 54	9·82	2267........	32·0	+56 6	10·54
	27·3	+56 6	10·55	2268........	32·0	+56 41	10·08
2251........	27·5	+56 20	9·51	2223........	32·1	+55 59	10·66
2252........	27·5	+56 53	8·12		32·4	+55 33	8·80
2254........	27·8	+56 48	8·66	2269........	32·5	+56 4	9·59
	28·0	+56 42	10·60	2273........	33·4	+56 26	8·55
2216........	28·3	+55 56	7·95				

PHOTOGRAPHIC MAGNITUDES OF XZ CYGNI

Date	Heliocentric Julian Day	Phase	Phtg. Mag.	Date	Heliocentric Julian Day	Phase	Phtg. Mag.
1920		d		1920		d	
Aug. 24.....	2422561·561	0·121	9·36	Nov. 28.....	2422657·593	0·035	8·66
26.....	563·835	0·062	9·31	1923			
30.....	567·568	0·062	9·36	July 6.....	2423612·770	0·103	9·45
·628		0·122	9·55	Sept. 4.....	667·612	0·354	9·08
Sept. 1.....	569·585	0·213	10·08		·665	0·407	8·62
·803		0·431	8·66		·750	0·025	8·68
3.....	571·561	0·323	9·56	5.....	668·556	0·364	9·12
10.....	578·631	0·394	8·66		·571	0·379	8·60
13.....	581·542	0·039	8·73		·593	0·401	8·58
14.....	582·541	0·104	9·32		·608	0·416	8·67
18.....	586·747	0·111	9·27		·630	0·438	8·43
19.....	587·601	0·032	8·90		·659	0·467	8·48
20.....	588·611	0·109	9·36		·676	0·018	8·80
21.....	589·530	0·095	9·34	6.....	669·566	0·441	8·64
22.....	590·531	0·162	10·03		·582	0·457	8·60
23.....	591·533	0·231	10·27		·604	0·013	8·72
Oct. 1.....	599·571	0·337	9·56		·619	0·028	8·69
5.....	603·553	0·120	9·38				

CONCLUSION

The present state of our knowledge of Cepheid variation is scarcely adequate to explain all the phenomena involved. The ordinary binary theory may almost certainly be definitely ruled out of court, while on the pulsation theory there are certain points not accounted for.

As has been mentioned in this article and elsewhere, it is at least plausible to suppose that short-period variables of the β Canis Majoris and allied types should be considered as forming a part of the same sequence as the true Cepheids, and their behaviour cannot be overlooked in any complete theory of Cepheid variation. We are then confronted with the fact that these short-period variables exhibit definite peculiarities (such as variation of amplitude and of centre-of-mass velocity) which appear to definitely indicate the existence of a satellite, and which cannot be satisfactorily explained on the assumption of simple pulsation alone; there is also to be considered the further fact that in some of the true Cepheids, such as δ Cephei itself, the presence of similar fluctuations is suspected, though perhaps not definitely proved.

Are we then to assume that in the short-period variables, and in them alone, there is present a satellite whose tidal action is superimposed upon the true Cepheid variation of the primary? Or is it more logical to suppose that the satellite is present in all cases, and forms one of the necessary conditions and causes of Cepheid variation, the secondary perturbations in the case of the longer-period stars being so small as in most cases to have escaped notice or be practically evanescent?

It is usually assumed that the mean density of a Cepheid is a function of its period of light variation. Assuming also that in general the effective temperature of a star is a function of its mean density and mass (volume or luminosity might replace one of these variables), and that the color-index is a function of the temperature alone, it follows that in Cepheids the mean color-index is a function of the period and of the mean absolute magnitude.

If for certain groups of Cepheids, such as those of the Small Magellanic Cloud, the period is a function of the absolute magnitude alone, the color-index must be a function of the period alone and vice-versa.

Shapley has pointed out for the galactic Cepheids[1] that the length of the period has a tendency to increase with the spectral class, although W. W. Campbell[2] had already noticed that it was not strictly so.

Our table of Cepheids shows that some short-periods may be of fairly advanced spectral class, and the case of UY Cygni of very short-period and very large color-index indicates that the latter is not a function of the period alone.

The above and similar facts point to the conclusion that Shapley's period-luminosity relation should perhaps be regarded more or less as a curve of statistical averages, and that the true relation, applicable to particular cases, should involve colour-index as well. Whether, and by how much, this would affect conclusions as to the scale of the universe can scarcely be determined until it is known in what way color-index enters into the relation, if at all.

[1] Ap. J., Vol. 40, p. 463 (1914).
[2] L. O. B., Vol. 6, p. 51 (1910).

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THE SPECTROSCOPIC SYSTEM SIGMA SCORPII

Third Paper

BY F. HENROTEAU, D.SC.

The second paper on σ Scorpii concerned the observations of 1922. A number of additional observations were obtained in 1923 and 1924. These indicate that σ Scorpii is almost without doubt a real triple system, revolving in a fairly short-period orbit of $33^d.0$, but the centre of mass of this orbit most probably revolves around a third body in a period of several years. If we except cases like α Ursae Minoris[1], where the shorter period of radial velocity oscillation may be ascribed to some kind of pulsation (Cepheid), and like κ Pegasi[2], where the longer period of revolution was discovered as a visual binary, σ Scorpii would be the rare case of a triple system discovered entirely from measures of spectrograms. To this are added the interesting facts that it may be considered as a Cepheid, that it has stationary H and K lines of calcium, and that elements of its longer period orbit can be obtained from the equation of light indicated by advance or retardation of the observed maxima of the six-hour period of Cepheid variation: σ Scorpii is thus a system at present unique of its kind.

All the spectrograms of 1923 and 1924 have been measured on the spectro-comparator. This method of measuring them was found more reliable, quicker and giving results of greater homogeneity. An excellent spectrogram of β Canis Majoris was used as a standard, the same that was used for all the measures of β Cephei, δ Ceti and other stars of early class B.

A complete re-examination of former measures was made. The conclusion was arrived at that the 33-day period curves of 1920 and 1921 were not based on enough observations to be determined accurately; the measures of 1921 are good although perhaps not as good as those of 1922-23-24; the plates of 1920 were all remeasured on the spectro-comparator; the results obtained are probably more reliable and indicate a curve of larger amplitude than that found previously. Although there is not such a large variation of amplitude of the long-period curve as apparently obtained in the first paper, a variation of shape and amplitude certainly exists, which could be ascribed to the perturbing effect of a third body. When observations will have been secured for a few more years, they will offer a most interesting problem of sidereal mechanics.

Equal shares in the observations, measures and reductions of 1923-24 have been taken by Messrs. J. F. Frédette, R. Callander and the writer.

A list of the radial velocities of 1923-24 is given here, followed by a new list of the velocities of 1920 as obtained by remeasuring the spectrograms on the spectro-comparator.

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[1] Pub. Dom. Obs. Vol. 9, No. 1.
[2] L.O.B. Vol. 9, p. 120.

RADIAL VELOCITIES OF σ SCORPII

Date	Julian Day	Velocity km.	Remarks
1923 April 9.................................	2423519·752	−48·1	
	·775	−44·5	
	·798	−78·4	Poor plate.
	·819	−47·0	
	·836	−34·8	Mean of 2 measures.
	·874	− 2·5	
	·900	+ 9·1	
April 11.................................	521·733	−39·0	
	·762	−55·1	
	·789	−25·6	
	·812	−31·4	
	·833	−14·8	
	·851	−13·4	
	·872	+25·6	Poor plate.
	·898	+15·8	
April 13.................................	523·736	−54·3	
	·764	−52·2	
	·789	−29·7	
	·812	−20·1	
	·837	− 2·8	
April 14.................................	524·758	−59·0	Poor plate.
	·783	−30·7	
	·808	− 6·3	
April 17.................................	527·710	−42·0	
	·735	−31·2	
	·756	−15·1	
	·776	− 4·4	Poor plate.
	·796	+20·8	
	·816	+16·0	
April 18.................................	528·733	−24·0	
	·757	− 2·9	
	·779	+ 4·4	
	·806	+22·0	
April 23.................................	533·710	+35·6	Poor plate.
	·737	+79·4	Good plate.
	·769	+25·5	
	·799	+15·1	
April 24.................................	534·710	+63·3	Poor plate.
	·759	+16·8	
	·780	+23·5	
	·799	+14·9	
	·817	+ 1·0	
	·837	− 3·9	Good plate.
	·858	− 8·4	
	·883	+25·7	
April 25.................................	535·702	+72·5	Poor plate.
	·745	+36·1	
	·765	+12·9	
	·784	−10·1	
	·833	−18·9	Poor plate.
	·858	− 6·9	
	·883	+47·1	Thin plate.
April 26.................................	536·739	+24·5	
	·767	+33·5	
	·797	+ 7·0	

RADIAL VELOCITIES OF σ SCORPII—*Continued*

Date	Julian Day	Velocity km.	Remarks
1923 April 29	2423539·704	− 1·5	
	·730	+ 4·0	
	·753	−13·9	
May 1	541·703	− 8·0	
	·726	−16·7	
	·749	−18·9	
	·770	−21·7	
	·790	−11·9	
	·811	+ 5·4	
	·833	+19·2	
	·858	+38·0	
May 2	542·660	− 0·9	Poor plate.
	·715	−25·4	
	·733	−24·6	
	·751	−15·3	
	·769	−10·5	
	·787	− 7·5	Dense plate.
	·806	+ 4·1	
	·826	+ 1·3	Poor plate.
May 4	544·706	−28·7	Poor plate.
	·728	−26·8	Poor plate.
	·753	−13·9	
	·778	− 6·3	
	·802	+11·5	Thin plate.
	·828	+24·9	
May 5	545·708	−36·5	
	·727	−18·4	
	·745	−14·5	
	·763	+17·4	Very poor plate.
	·781	+22·6	
	·800	+44·3	
	·821	+25·0	Thin plate.
May 6	546·699	−29·3	
	·719	−17·9	
	·740	− 7·8	
	·760	+11·5	
	·781	+21·8	
	·803	+38·5	Thin plate.
May 9	549·700	− 9·9	Thin plate.
	·722	− 8·9	
	·744	+14·9	Poor plate.
	·767	+31·1	
May 10	550·656	−38·9	
	·697	− 9·3	
	·715	− 0·3	
	·769	+17·4	
May 13	553·655	−22·5	
	·677	− 3·1	
	·698	+ 2·9	
	·719	+26·6	
	·740	+29·3	Poor plate.
	·760	− 3·9	Thin plate.
	·782	−27·0	
	·806	−25·2	Thin plate.

RADIAL VELOCITIES OF σ SCORPII—*Continued*

Date	Julian Day	Velocity km.	Remarks
1923 May 14	2423554·680	+ 3·7	Thin plate.
	·704	+16·6	
May 17	557·791	−67·4	Very poor plate.
May 18	558·684	+17·2	
	·710	−18·1	
May 21	561·636	+36·9	Thin plate
	·668	−31·3	
	·694	−35·0	
	·716	−43·0	
	·736	−44·4	Thin plate.
	·756	−54·5	
	·778	−26·7	
May ..22	562·609	+38·1	
	·664	−13·2	
	·684	−29·6	
	·703	−35·3	
	·724	−52·2	
	·745	−48·4	
	·767	−27·6	
	·791	−22·8	
1924 Mar. 31	876·771	− 8·4	
	·794	+21·4	
	·819	+46·3	
	·844	+37·0	
	·867	− 6·8	
April 4	880·759	+20·6	
	·788	+41·6	
April 9	885·801	−58·4	
	·819	−61·0	
	·836	−57·8	
	·853	−56·6	
	·869	−40·8	
April 10	886·751	−27·9	
	·777	−45·2	
	·802	−57·8	
	·826	−53·8	
	·849	−61·8	
	·872	−34·4	
	·894	−24·7	
April 11	887·730	−36·5	
April 14	890·756	−53·3	
	·780	−54·5	
	·810	−26·9	
	·837	− 4·1	
	·866	− 6·3	
April 15	891·733	−67·2	
	·759	−66·1	
	·783	−27·0	
	·807	−14·1	
	·831	− 3·1	
	·858	+ 4·7	
	·885	+29·4	
April 16	892·729	−47·8	
	·759	−30·3	
	·786	−17·4	

RADIAL VELOCITIES OF σ SCORPII—*Continued*

Date	Julian Day	Velocity km.	Remarks
1924 April 16.............................	2423892·813	+20·9	
	·840	+19·0	
	·870	+15·1	
April 19.............................	895·728	− 9·6	
	·753	− 8·5	
	·776	+14·4	
	·797	+41·8	
	·817	+36·1	
	·837	+24·6	
	·859	+23·0	
	·881	+ 4·7	
April 20.............................	896·854	+37·0	
	·879	+10·4	
April 23.............................	899·720	+27·7	
	·747	+54·7	
	·773	+87·9	
	·797	+55·2	
	·820	+23·5	
	·844	+29·5	
	·867	+ 5·6	
April 24.............................	900·794	+25·9	
	·814	+17·4	
	·834	0·0	
	·854	− 0·9	
April 25.............................	901·841	− 1·6	
	·862	− 7·2	
April 26.............................	902·709	+37·3	
	·728	+52·2	
	·747	+67·8	
	·765	+49·1	
	·785	+19·0	
	·806	+ 1·8	
	·827	− 7·6	
	·849	− 7·7	
	·874	−17·0	
April 28.............................	904·727	+60·5	
	·822	−12·9	
May 2.............................	908·701	− 4·6	
	·726	− 9·5	
	·748	−12·6	
	·768	−32·0	
	·789	−28·2	
	·810	−19·7	
	·833	−12·4	
	·858	+12·4	
May 4.............................	910·716	−15·0	
	·738	−42·7	
	·759	−39·7	
	·780	−24·9	
	·801	−27·4	
May 5.............................	911·699	−13·8	
	·719	−26·5	
	·743	−42·0	
May 6.............................	912·700	−31·7	
	·721	−43·0	

RADIAL VELOCITIES OF σ SCORPII—*Continued*

Date	Julian Day	Velocity km.	Remarks
1924 May 6...........................	2423912·741	−43·6	
	·760	−49·7	
	·778	−23·3	
	·800	− 4·3	
	·823	+12·9	
May 8.............................	914·751	−36·1	
	·775	−14·2	
	·802	+ 0·4	
May 11.............................	917·717	−25·8	
	·774	− 3·6	
May 16.............................	922·785	−17·3	
	·806	−22·7	
May 18.............................	924·671	+ 3·1	
	·690	+14·6	
	·704	+23·0	
	·718	+13·6	
	·732	+ 8·4	
	·746	− 3·6	
	·760	−16·5	
	·775	−26·3	
May 19.............................	925·619	−16·5	
	·659	+ 8·7	
	·677	+14·4	
	·688	+24·9	
	·700	+23·4	
	·710	+23·8	
	·731	−11·9	
	·744	−19·1	
May 20.............................	926·667	+22·1	
	·685	+28·8	
	·696	+30·1	
	·707	+ 8·9	
	·719	− 1·4	
	·730	+ 6·3	
	·742	−12·7	
	·756	− 9·5	
May 22.............................	928·623	+46·6	
	·663	+72·6	
	·706	+ 2·5	
	·726	−18·6	
May 25.............................	931·716	− 9·0	
	·733	− 9·2	
	·751	+ 0·8	
	·769	+ 0·8	
May 29.............................	935·598	+14·2	
	·622	+ 4·7	
	·660	+ 2·3	
	·677	+ 2·6	
	·742	+17·2	
	·767	+33·6	
	·794	+29·2	
May 30.............................	936·633	− 6·2	
	·657	− 8·7	
	·678	−20·7	
	·699	− 0·4	

RADIAL VELOCITIES OF σ SCORPII—*Continued*

Date	Julian Day	Velocity km.	Remarks
1924 May 30....................................	2423936·718	+ 1·3	
	·738	+13·3	
	·760	+25·5	
June 3....................................	940·585	−14·9	
	·613	−34·6	
	·660	−19·3	
	·685	+ 5·3	
June 5....................................	942·583	−25·8	
	·612	−21·3	
	·665	+12·7	
	·690	+12·4	
	·716	+12·1	
June 8....................................	945·614	−27·4	
	·658	+10·3	
	·676	+27·9	
	·697	+29·9	
	·721	+13·2	
	·747	−12·7	
June 9....................................	946·630	+ 3·6	
	·651	+20·8	
	·674	+37·8	
	·690	+29·3	
	·706	+ 4·4	
	·723	+ 2·7	
	·744	−15·2	
	·771	−48·8	
June 10....................................	947·604	− 1·4	
June 13....................................	950·601	+13·8	
	·617	+15·3	
	·677	−23·6	
	·703	−42·5	
June 16....................................	953·569	− 0·4	
	·592	+25·3	
	·612	+17·8	
	·650	−27·4	
	·669	−35·6	
	·689	−52·5	
	·709	−40·6	
June 18....................................	955·564	+19·0	
	·584	+26·0	
	·599	− 8·8	
1920 April 13....................................	2422428·772	−49·8	
	·789	−53·8	
	·804	−39·6	
	·820	−34·9	
	·834	−35·2	
	·846	−22·8	
April 16....................................	431·727	−41·7	
	·769	−62·8	
	·785	−51·6	
	·804	−58·3	
	·826	−55·0	
April 18....................................	433·712	−51·7	
	·773	−47·2	
	·799	−43·4	

RADIAL VELOCITIES OF σ SCORPII–*Concluded*

Date	Julian Day	Velocity km.	Remarks
1920 April 18................................	2422433·822	−22·7	
April 19................................	434·724	−47·2	Poor plate
	·777	−37·0	
	·800	−20·1	Poor plate
	·833	+ 0·6	Very poor
April 25................................	440·713	−19·2	
	·768	+17·9	
	·788	+25·2	
	·813	− 7·4	
	·838	−25·7	Poor
May 4................................	449·684	+29·6	Poor
	·707	+ 8·8	
	·729	− 4·3	Poor
	·760	−23·8	
	·783	−22·7	
	·811	− 9·9	Very poor
May 5................................	450·683	+ 6·1	
	·707	+ 6·1	
	·729	+ 4·5	
	·768	−12·0	Poor
	·792	−10·8	
	·817	+ 5·0	Very weak
May 10................................	455·676	−32·8	
	·699	−34·0	
	·722	−29·1	Weak
	·767	−16·4	
May 11................................	456·673	−31·1	
	·696	−37·7	
	·718	−38·5	
	·764	− 7·4	
	·786	+25·9	Very poor
	·811	+10·7	Very poor

The above velocities furnish a fairly large number of nearly complete velocity curves which may be considered, within the limits of error, of constant amplitude.   Their center-of -mass lines, if they might be called so, give the following velocities for the nights indicated.

VALUES OF CENTER-OF-MASS VELOCITY DEDUCED FROM SHORT-PERIOD CURVES

Date	Julian Day	Velocity km.	Date	Julian Day	Velocity km.
1920 April 13....	2422428·8	−12	1924 Mar. 31....	2423876·8	+ 5
" 16....	431·8	−23	April 9....	885·7	−22
" 18....	433·8	−20	" 10....	886·7	−22
" 19....	434·8	−20	" 14....	890·9	−15
" 25....	440·8	−18	" 15....	891·9	−15
May 4....	449·7	+18	" 16....	892·9	− 6
" 5....	450·7	+23	" 19....	895·8	+ 8
" 10....	455·7	+ 3	" 20....	896·8	+22
" 11....	456·8	+ 2	" 23....	899·8	+35
1923 April 9....	2423519·9	−18	" 24....	900·8	+30
" 11....	521·9	−18	" 25....	901·8	+32
" 13....	523·8	−26	" 26....	902·7	+24
" 14....	542·8	−28	" 28....	904·7	+22
" 17....	527·8	−22	May 2....	908·9	+ 8
" 18....	528·8	−18	" 4....	910·6	+ 2
" 23....	533·8	+28	" 5....	911·6	− 2
" 24....	534·8	+32	" 6....	912·9	− 7
" 25....	535·8	+24	" 8....	914·8	−12
" 26....	536·8	+32	" 18....	924·7	−15
" 29....	539·7	+16	" 19....	925·7	−15
May 1....	541·8	+11	" 20....	926·7	− 7
" 2....	542·8	+ 8	" 22....	928·7	+13
" 4....	544·8	0	" 25....	931·6	+32
" 5....	545·8	+ 2	" 29....	935·6	+31
" 6....	546·8	+ 1	" 30....	936·8	+22
" 9....	549·7	−11	June 3....	940·8	+10
" 10....	550·7	−12	" 5....	942·7	+10
" 13....	553·8	−15	" 8....	945·7	− 8
" 14....	554·7	−22	" 9....	946·7	−10
" 18....:	558·7	−24	" 13....	950·6	−20
" 21....	561·7	−16	" 16....	953·6	−20
" 22....	562·7	−13	" 18....	955·6	−19

The velocities of 1923-24 furnish very good curves of radial velocity variation of the centre of mass. It is found, however, that the new velocities of 1920 indicate a curve of larger amplitude than that previously found; also it is considered that the velocities of 1920 and 1921 are too few to determine curves with certainty. The different curves of 1918-20-21-22-23-24 are given in Fig. 1. The curve of 1918 has been redetermined from new values of centre-of-mass velocities derived from the Lick original measures; these new values were obtained in exactly the same way as the more recent Ottawa values.

A considerable variation of the amplitude of these curves is doubtful, but the fact remains that there is some variation of shape and amplitude which is apparently due to the perturbations of a third body. Indeed it is cleary shown that the centre-of-mass velocity of these different curves varies, a strong indication of a revolution in an orbit of a period of several years.
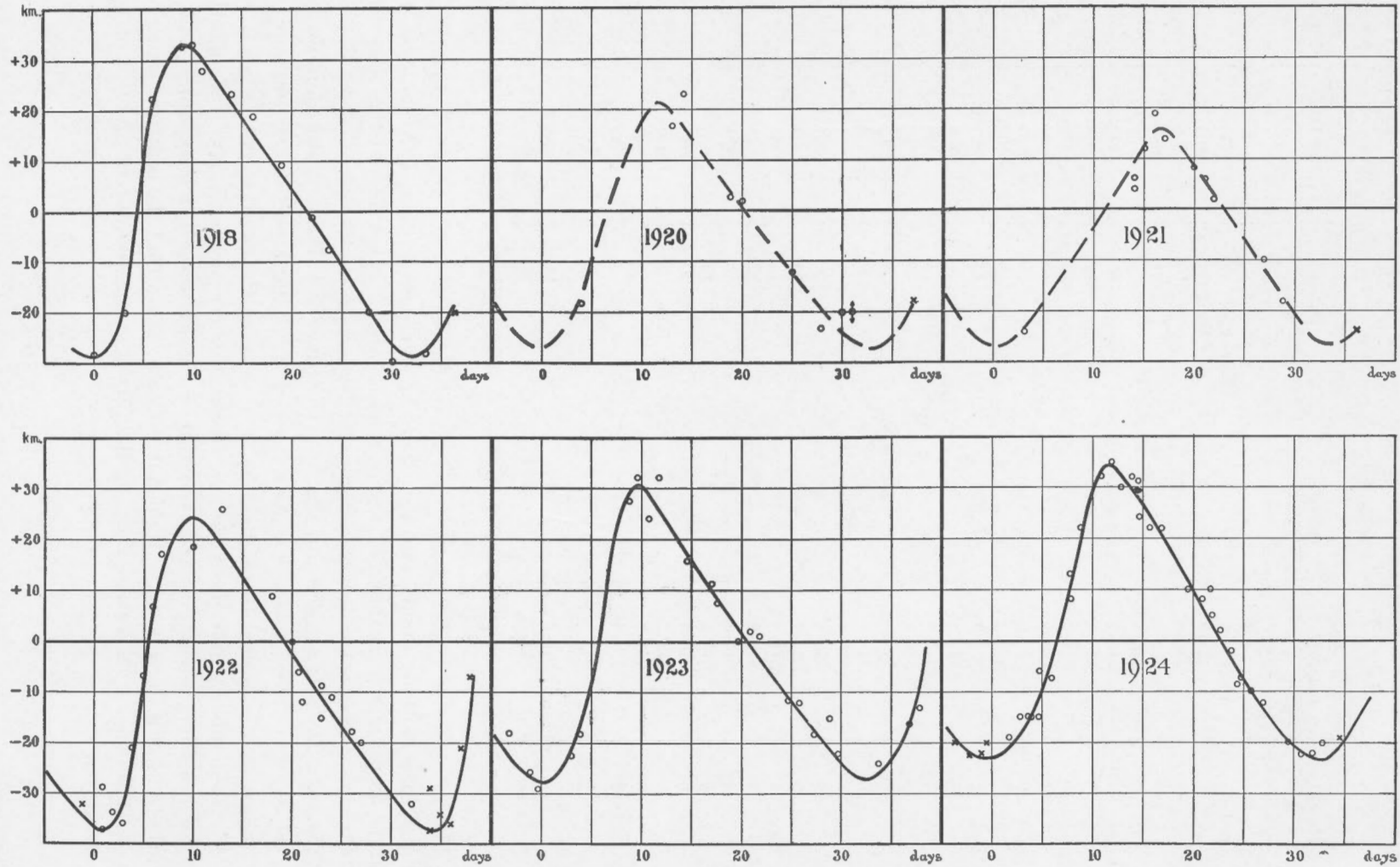
SIGMA SCORPII.

Fig. 1—Radial Velocity Variation of Center of Mass

The period connecting the different minima of the long-period curves from 1915 to 1924 is now found to be $33^d.0$. The observed value of the very short period of velocity variation does not appear to be constant; this may, however, be explained on the assumption that the true period is constant but appears variable by reason of the displacement of the body in an orbit of several years' period, so that the observed times of maximum and minimum require correction for equation of light. The true very short period cannot be computed exactly until we know more about the very long-period orbit, but if we adopt as origin the date of maximum J. D. 2420644.830 and the period $0^d.246834$ the following tables give the observed and computed times of maxima and the residuals obtained. The mean of the residuals of each year is given at the bottom of each table.

MAXIMA OF SHORT-PERIOD VELOCITY CURVES

Observed	Computed	O–C	Observed	Computed	O–C
	1915			1921	
2420644·829	·830	− ·001	2422813·742	·760	− ·018
671·728	·734	− 6	815·694	·735	− 41
674·688	·696	− 8	819·630	·684	− 54
701·600	·602	− 2	820·636	·672	− 36
			821·622	·659	− 37
		− ·004	826·819	·843	− 24 (?)
			828·841	·817	+ 24
	1918		835·692	·729	− 37
			846·578	·589	− 11
2421687·950	·950	·000	847·552	·577	− 25
688·933	·937	− 4	849·536	·551	− 15
695·968	·968	0			
699·918	·917	+ 1			− ·025
701·900	·892	+ 8			
705·840	·841	− 1		1922	
707·950	·944	+ 6			
713·860	·867	− 7	2423147·930	·974	− ·044
716·955	·948	+ 7	149·930	·948	− 18
719·916	·911	+ 5	150·897	·936	− 39
724·850	·848	+ 2	152·875	·910	− 35
726·821	·822	− 1	161·773	·796	− 23
			163·741	·771	− 30
		+ ·001	166·940	·980	− 40
			168·915	·954	− 39
	1920		169·885	·942	− 57
			172·882	·904	− 22
			175·855	·866	− 11
2422428·672	·699	− ·027	183·728	·765	− 37
431·655	·661	− ·005	185·716	·739	− 23
433·626	·636	− ·010	186·690	·727	− 37
434·864	·870	− ·006	188·665	·701	− 36
440·776	·794	− ·018	189·650	·689	− 39
449·653	·680	− ·027	196·798	·847	− 49
450·650	·668	− ·018	197·798	·834	− 36
455·844	·851	− ·007	198·781	·821	− 40
456·830	·838	− ·008	200·770	·796	− 26
		− ·014			− ·034

MAXIMA OF SHORT-PERIOD VELOCITY CURVES

Observed	Computed	O–C.	Observed	Computed	O–C
	1923			1924	
2423519·910	·952	− ·042	2423876·831	·874	− ·043
521·885	·927	− 42	885·955	·007	− 52
523·860	·902	− 42	886·703	·748	− 45
524·846	·889	− 43	890·900	·944	− 44
527·809	·851	− 42	891·875	·931	− 56
528·805	·838	− 33	892·854	·919	− 65
534·718	·762	− 44	895·830	·881	− 51
535·705	·750	− 45	899·780	·830	− 50
541·876	·921	− 45	902·746	·792	− 46
542·865	·908	− 43	908·665	·716	− 51
544·837	·883	− 46	910·639	·691	− 52
545·820	·870	− 50	912·863	·912	− 49
546·813	·857	− 44	924·713	·760	− 47
549·775	·819	− 44	925·695	·748	− 53
550·764	·807	− 43	926·691	·735	− 44
553·727	·769	− 42	928·649	·709	− 60
558·662	·705	− 43	935·567	·621	− 54
561·620	·667	− 47	936·808	·855	− 47
562·615	·655	− 40	940·754	·817	− 63
			942·748	·792	− 44
		− ·043	945·696	·741	− 45
			946·680	·728	− 48
			950·625	·678	− 53
			953·600	·640	− 40
					− ·050

These residuals show a variation of the equation of light from year to year. If, however, the maxima of radial velocity of 1915 had been shifted by about one hour through some errors of observation, one might find an approximately constant period which would give no such variation. Such a shift, however, (about one-sixth of the period) does not seem probable, and since the series of 33-day period curves indicate the existence of an orbit of several years' period, an effect depending on the equation of light is to be expected in the six-hour period.

It is to be remarked that a circular orbit of twelve years' period with a semi-amplitude of radial velocity variation of about 10 km. would give a value of $a \sin i$ of the same order of magnitude as that of an equation of light of $0^d.05$. We have, approximately, from the spectroscopic orbit

$$a \sin i = 602{,}293{,}800 \text{ km.}$$

and from the equation of light

$$a \sin i = 648{,}000{,}000 \text{ km.}$$

The parallax of σ Scorpii is no doubt very small; according to Professor Kapteyn, as obtained from star-streaming, it is $0''.0086$ (Boss 4158)[1]. The apparent radius of a visual twelve-year period orbit would be approximately $0''.04$, a quantity impossible to detect by direct measurement, unless perhaps by interferential methods.

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[1] Contributions from the Mt. Wilson Observatory, Vol. 4, p. 417.
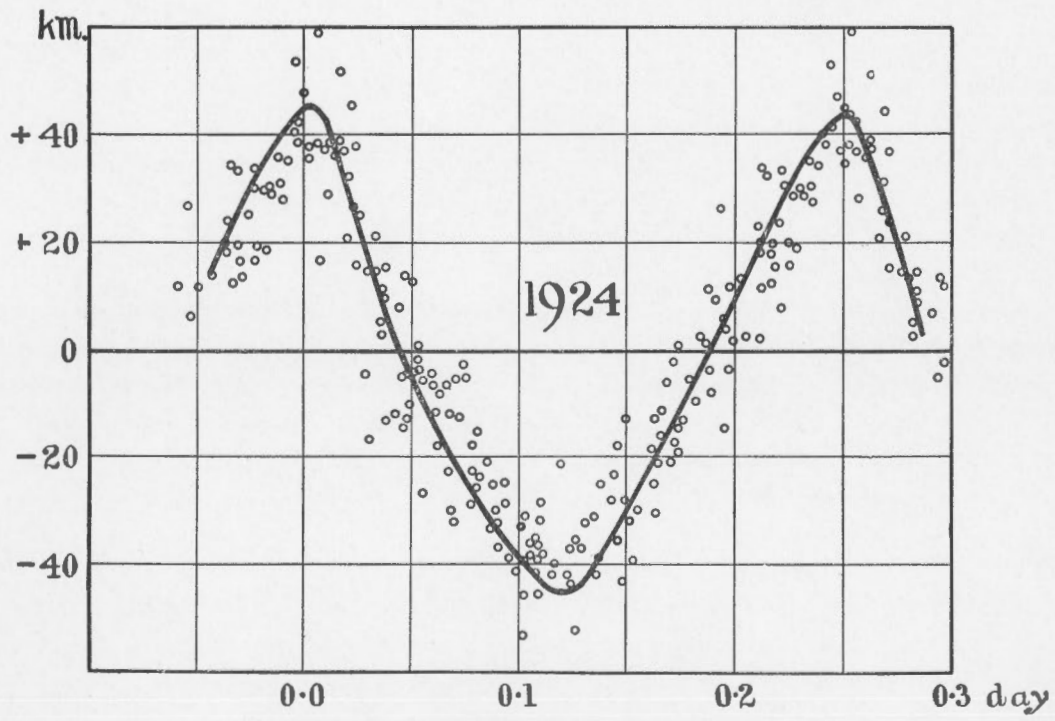
SIGMA SCORPII
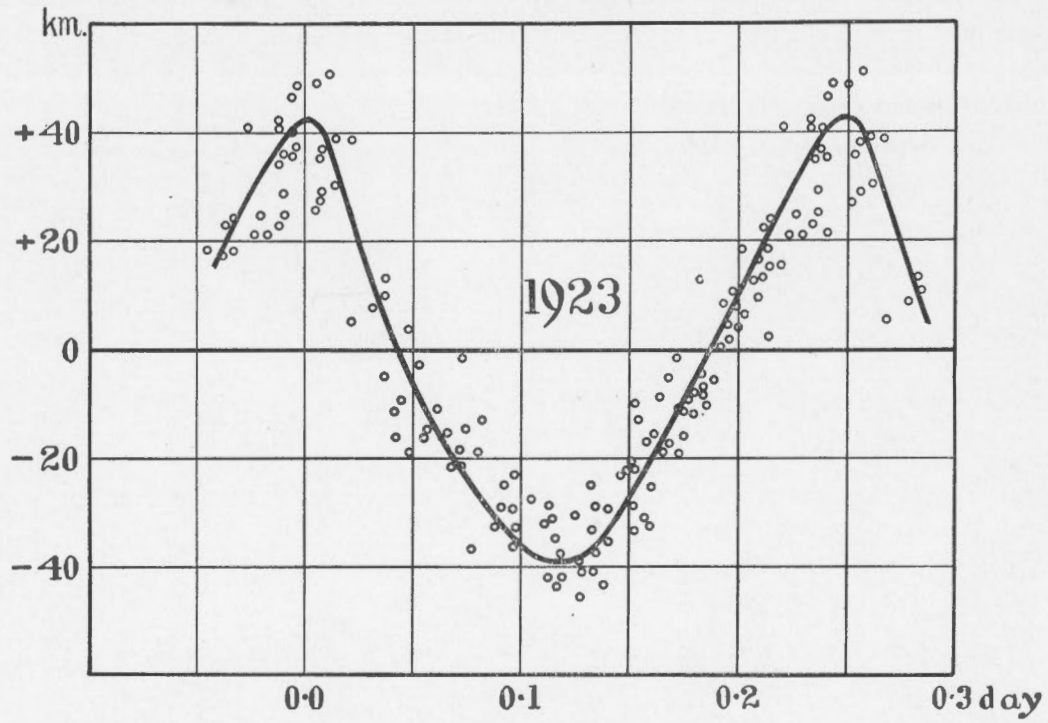
Fig. 2—Short Period Velocity Curve

Taking the two groups of observations of 1923 and 1924 one may superpose for each group the different short-period velocity curves so that all of them have the same but undetermined mean velocity.   This furnishes the curves shown in Fig. 2.

Dominion Observatory, Ottawa
          September 10, 1924.

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The discovery of the variation of the light of ζ Geminorum is usually attributed to Schmidt in 1844. Before this, however, the star had been suspected of variability as mentioned by Wurm in the Berliner Jahrbuch for 1790 (p. 166). Several light curves of it have since been published, among which are those of E. C. Pickering,[1] Markwick,[2] Luizet,[3] Hornig,[4] and Guthnick.[5] The amplitude of the photographic light curve as found by Wirtz[6] is much larger than the visual, and Professor Guthnick also, using his photo-electric photometer and colour-screen, found a decided variation in its colour index.

The variability of the radial velocity of ζ Geminorum was discovered independently by Bélopolsky[7] and by Campbell.[8] The latter derived a mean radial velocity curve from a series of 44 spectrograms taken in 1898, 1899 and 1900. This curve indicates considerable departure from Keplerian orbital motion.

The radial velocities of this star which have been published up to the present are given in the following tables:—

RADIAL VELOCITIES AT OTHER OBSERVATORIES

(Bélopolsky's velocities)

Date		Julian Day	Velocity km.	Remarks
1897 Dec.	31	2414290·415	+32·7	German miles reduced
1898 Jan.	2	292·436	+21·2	to kilometers.
	20	310·353	+37·5	See A.N. Vol. 149, p.
	21	311·394	+35·3	239.
	29	319·336	+22·3	
	31	321·353	+20·5	
Feb.	17	338·353	+14·5	
Mar.	21	370·252	+22·4	
	25	374·244	+ 5·0	
	26	375·244	+ 0·3	
	27	376·244	− 3·0	
	28	377·240	−20·0	
1899 Mar.	25	739·240	+ 5·2	
	27	741·248	+ 3·7	
	28	742·277	+ 2·5	

[1] H. A., Vol. 46, 1903, p. 155.
[2] Jour. B. A. A., Vol. 17, 1907, p. 208.
[3] A. N., Vol. 160, p. 364.
[4] A. N., Vol. 201, p. 155.
[5] A. N., Jubilatimsnummer, 1921.
[6] A. N., Vol. 154, p. 336.
[7] A. N., Vol. 149, p. 239.
[8] Ap. J., Vol. 13, 1901, p. 90.

RADIAL VELOCITIES AT OTHER OBSERVATORIES—*Concluded*

(Campbell's velocities)

Date	Julian Day	Velocity km.	Remarks
1898 Nov. 11	2414605·975	+19·9	
1899 Jan. 24	679·846	+ 0·7	
25	680·875	− 1·8	
27	682·841	− 2·4	
28	683·841	+ 5·4	
29	684·767	+13·0	
30	685·826	+20·5	
	·879	+20·1	
Feb. 6	692·692	− 2·9	
	·737	− 2·6	
7	693·675	+ 3·6	
13	699·667	+ 1·7	
15	701·763	− 4·3	
	·817	− 4·7	
21	707·680	+17·9	
	·758	+16·3	
22	708·688	+ 7·0	
	·763	+ 4·7	
April 5	750·650	+ 2·5	
10	755·684	+12·6	
12	757·701	+23·5	
13	758·701	+13·3	
Sept. 12	910·021	+23·5	
13	911·029	+14·6	
22	920·000	+24·2	
27	925·000	− 3·8	
Oct. 4	932·038	+ 5·2	
23	951·996	+10·2	
25	953·029	+ 2·9	
Nov. 30	989·017	+11·2	
Dec. 25	2415014·858	+ 1·3	
26	015·004	+ 0·7	
	·768	0·0	
27	016·962	− 6·2	
28	017·008	− 5·9	
1900 Jan. 10	030·809	+18·9	
15	035·680	+ 0·5	
21	041·688	+21·2	
29	049·767	+ 8·8	
30	050·754	+16·2	
Feb. 6	057·861	− 6·7	
7	058·716	− 0·4	
11	062·667	+21·0	
	·779	+19·4	

(Küstner's velocities)

Date	Julian Day	Velocity km.	Remarks
1912 Jan. 19	2419421·43	+ 5·6	
Feb. 28	461·34	− 1·7	

(Lunt's velocities)

Date	Julian Day	Velocity km.	Remarks
1909 Feb. 12	2418350	+13·7	See Ap. J., Vol. 50, p.
1914 Jan. 23	2420156	+ 7·9	169.
1916 Feb. 1	2420895	− 5·9	

Following are the radial velocities of ζ Geminorum obtained at Ottawa. The plates were measured against standard spectrograms of daylight and of Polaris; of these the latter is the more similar to that of the star, and consequently gives more reliable measures.

OTTAWA RADIAL VELOCITIES OF ζ GEMINORUM OBTAINED IN 1924

Date	Julian Day	Velocity km. Daylight	Velocity km. Polaris	Remarks
1924 Jan. 14......................................	2423799·717	− 3·1	+ 0·2	
	·752	+ 0·2	− 0·4	
15......................................	800·587	− 1·6	+ 0·3	
20......................................	805·653	+ 3·5	+ 8·2	
23......................................	808·746	− 3·0	− 1·0	
	·796	− 6·9	− 8·5	
27......................................	812·580	+10·4		
31......................................	816·571	+ 1·6	+ 2·7	
	·656	− 0·9	+ 2·3	
Feb. 3......................................	819·630	− 7·6	− 6·8	
	·715	− 4·5	− 6·8	
8......................................	824·549	+18·0	+21·9	
	·647	+17·7	+21·2	
11......................................	827·569	− 2·8	− 0·2	
12......................................	828·660	− 3·4	− 1·6	
13......................................	829·566	− 7·7	− 5·0	
	·622	− 6·7	− 4·1	
15......................................	831·533	+ 5·7	+ 5·1	
	·591	+ 4·4	+10·3	
	·654	+ 4·2	+ 9·3	
18......................................	834·560	+18·4	+20·2	
	·658	+17·3	+20·2	
21......................................	837·598	+ 3·4	+ 4·6	
22......................................	838·517	− 2·5	+ 2·0	
	·578	− 2·2	+ 0·6	
	·653	− 0·5	+ 2·1	
24......................................	840·533	− 5·5	+ 0·1	
	·583	− 2·3	− 1·8	
26......................................	842·558	+13·8	+10·0	
	·632	+15·2	+18·2	Poor
28......................................	844·602	....	+20·3	
29......................................	845·554	+ 7·5	+ 8·4	
	·661	+ 6·3	+11·4	
Mar. 12......................................	857·558	+ 0·7	+ 2·4	
	·629	+ 1·7	+ 1·0	
13......................................	858·541	− 1·6	+ 0·5	
	·606	− 4·5	+ 1·1	
17......................................	862·551	+10·7	+10·7	
19......................................	864·637	+25·2	+23·6	
20......................................	865·581	+21·6	+21·2	
23......................................	868·618	− 3·4	+ 0·5	
24......................................	869·542	+ 0·4	− 5·2	
31......................................	876·563	+ 6·7	+10·7	
April 2......................................	878·558	+ 0·7	+ 4·7	

The velocities in the last column, together with the elements J.D. 2410639·339 + 10^d·15375 E, have been utilized for the radial velocity curve shown in Fig. 1.
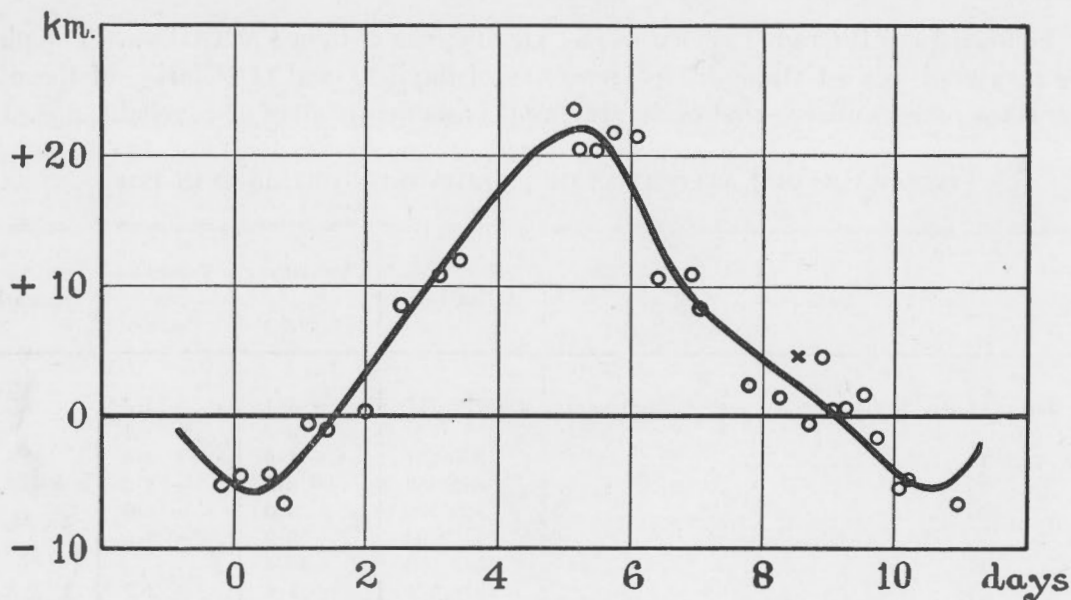
FIG. 1. Radial Velocity curve of Zeta Geminorum, 1924, Spectro-comparator measures.
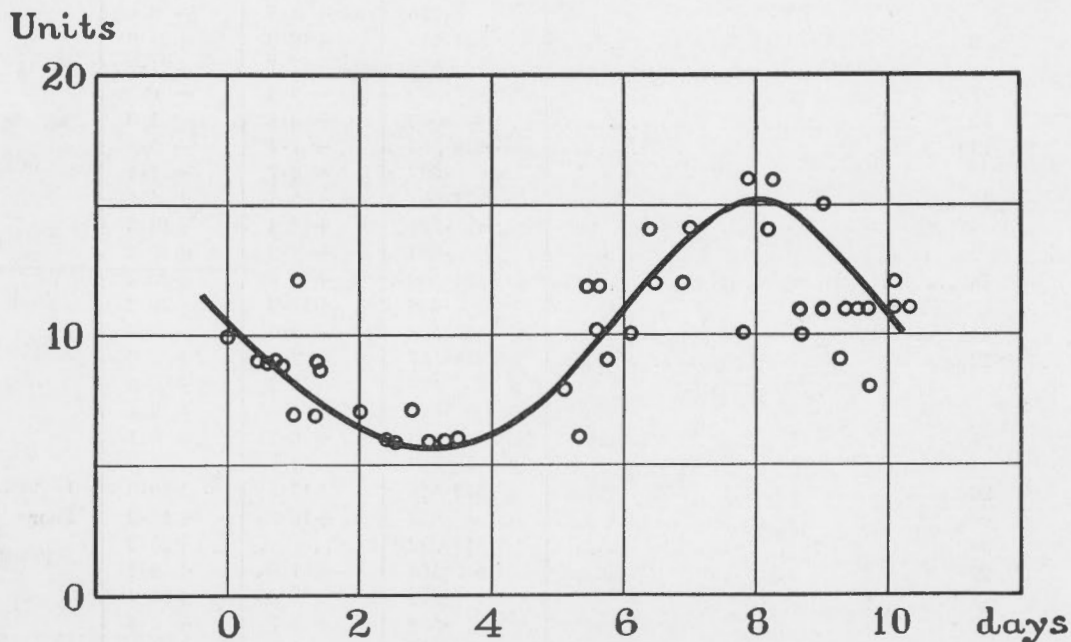

FIG. 2. Zeta Geminorum. Variation of ratio of intensities of two close enhanced and arc lines.

VARIATION OF PERIOD

Several observers have pointed out the probability of a variation in the period of ζ Geminorum. Speaking of photometric observations Professor Mascart[1] says: "En particulier pour ζ Gémeaux la longueur de la période est depuis longtemps soupçonnée d'irrégularités assez brusques et sans allure systématique reconnue; la longue serie d'observations de Luizet révèle des variations de phases considérables; les observations de Moye en 1919-1920 sont concordantes tandis que en 1921, la période paraît brusquement

[1] Bul. de l'Obs. de Lyon, Vol. 4, 1922, p. 101.

PUBLICATIONS

OF THE

𝔇𝔬𝔪𝔦𝔫𝔦𝔬𝔫 𝔄𝔰𝔱𝔯𝔬𝔭𝔥𝔶𝔰𝔦𝔠𝔞𝔩 𝔒𝔟𝔰𝔢𝔯𝔳𝔞𝔱𝔬𝔯𝔶

VICTORIA

Vol. III, No. 8

THE ORBITS OF FIVE SPECTROSCOPIC BINARIES

BY W. E. HARPER

The stars whose orbits are determined are given in the following table.

Star	α 1900		δ 1900		Vis. Mag.	Type
	h	m	°	′		
45 Aurigae...............................	6	13·7	+53	30	5·41	F5
19 Leo Minoris...........................	9	51·6	41	32	5·19	F4
105 Herculis.............................	18	15·1	24	24	5·49	K5
22 Vulpeculae............................	20	11·2	23	12	5·38	G7
π Cephei..............................	23	04·7	+74	51	4·56	G5

45 AURIGAE

Abstract.—Twenty-eight spectrograms of single-prism dispersion and measured on the spectro-comparator against Procyon as a standard serve to determine the orbital elements. Early Lick observations assist in fixing the period at 6·5013 days. The eccentricity is ·019.

This star (1900 α = 6^h 13^m.7, δ = +53° 30′) whose visual magnitude is 5.41 and type F5, was announced as a spectroscopic binary by Campbell in the *Publications of the Astronomical Society of the Pacific* for June, 1922.

Twenty-eight spectrograms obtained with the single-prism spectrograph whose dispersion at Hγ is 29 angstroms per millimetre, were made during 1922, 1923, 1924 and 1925, and these form the basis of the determination of the orbit. The plates of 1923,

April 2, and 1924, April 7, were given half weight, as in the former a very narrow spectrum was secured and in the latter the focus was not of the best. Further, the plate of 1924, November 22, has not been used at all in the determination, owing to the fact that a few minutes' exposure of an A-type star was accidentally made on the same plate.

The spectrum contains numerous well-defined lines due principally to iron, titanium, manganese, chromium and calcium. On one plate, number 8541, there were measured 153 lines between λ3941 and λ4600. By selecting a plate of different exposure other lines could be measured and the total increased. In the list given in *Harvard Annals*, Volume 28, page 79, which lists the lines in stars of division c, there are 183 recorded between the same limits. The great majority, though not all, of the 153 measured in this star are found in that particular list.

Through the kindness of the Lick observers the data of their seven observations were furnished, and for convenience of reference these are tabulated here. All their plates were made with the three-prism Mills spectrograph and should be very reliable. The plates have assisted materially in fixing the period accurately, but otherwise have not been made use of in the determination. A systematic difference of −3.67 km. has been added to each velocity before the residuals as shown were obtained. Our own plates were measured on the spectrocomparator against the standard plate of Procyon, number 3375, and the velocities deduced are given in the table following. The tables list the phases of the observations as based upon the final elements, and the residuals from the final curve are given in the last column. The determination does not appear wholly satisfactory, as the probable error of a plate is ±1.89 km. per sec., larger than one might expect from single-prism plates when the character of the spectrum is considered. In the final curve the way the observations fall above and below the curve around the γ-point is suggestive of the presence of the spectrum of the second component. The last observation was made on a fine-grained plate to see if the fainter spectrum could be recorded, but no certain evidence was obtained.

LICK OBSERVATIONS OF 45 AURIGAE

Plate Number	Date	Julian Date	Phase	Velocity	Lines	O–C
9503	1917 Nov. 1	2,421,534·036	6 291	+24·35	30	−2·14
10507	1919 Jan. 25	1,984·866	2·030	+ 3·60	32	−1·85
10901	Sept. 25	2,227·948	4·564	−19·43	31	+1·75
10934	Oct. 11	2,243·942	1·054	+31·86	30	+1·94
10992	Nov. 10	2,273·047	4·154	−27·64	31	−0·20
11030	Nov. 24	2,287·052	5·156	− 6·65	30	−0·58
11082	Dec. 29	2,422,322·935	2·032	+ 6·58	27	+1·19

VICTORIA OBSERVATIONS OF 45 AURIGAE

Plate Number	Date	Julian Date	Phase	Velocity	Wt.	O–C
8348......................	1922 Nov. 11	2,423,370·963	3·350	−31·2	1	−1·5
8495......................	1923 Jan. 10	3,430·865	4·741	−19·0	1	+2·0
8536......................	" 28	3,448·782	3·154	−29·6	1	−2·6
8541......................,	" 31	3,451·754	6·126	+19·9	1	+0·6
8578......................	Feb. 24	3,475·780	4·147	−27·3	1	+3·7
8603......................	" 26	3,477·764	6·131	+23·0	1	+3·6
8637......................	Mar. 2	3,481·770	3·635	−32·4	1	0·0
8677......................	" 19	3,498·695	1·056	+24·4	1	−1·8
8722......................	" 26	3,505·728	1·588	+12·8	1	−1·4
8723......................	" 26	3,505·745	1·605	+11·9	1	−1·9
8762......................	" 30	3,509·771	5·631	+ 5·3	1	+0·3
8775......................	April 2	3,512·653	2·012	+ 6·6	½	+4·4
8776......................	" 2	3,512·666	2·025	+ 6·2	½	+4·2
8808......................	" 9	3,519·719	2·576	−18·7	1	−5·0
8809......................	" 9	3,519·732	2·589	−16·6	1	−2·3
9706......................	1924 Mar. 3	3,848·702	6·494	+28·8	1	+2·2
9721......................	" 10	3,855·721	·511	+32·9	1	+2·1
9742......................	" 14	3,859·712	4·502	−26·3	1	−0·3
9828......................	April 7	3,883·681	2·466	− 5·5	½	+5·2
9829......................	" 7	3,883·708	2·493	− 3·4	½	+8·1
10859.....................	Oct. 10	4,069·020	5·768	+ 6·4	1	−2·6
10942.....................	Nov. 22	4,112·908	4·147	−24·5		
11101.....................	Dec. 23	4,143·918	2·651	−14·2	1	+1·5
11175.....................	1925 Jan. 23	4,174·794	1·020	+25·3	1	−1·4
11214.....................	Feb. 14	4,196·730	3·452	−33·5	1	−2·3
11226.....................	" 24	4,205·734	5·955	+10·0	1	−4·4
11260.....................	Mar. 6	4,216·730	3·948	−33·0	1	−0·5
11320.....................	" 16	2,424,226·702	·918	+28·0	1	−0·2

From the first few plates it was seen that the period was in the neighbourhood of 6·5 days. Our later observations defined this more closely and with the aid of the Lick results a definite value of 6·5013 days was established. Preliminary elements were then adopted as follows.

PRELIMINARY ELEMENTS

$$P = 6·5013 \text{ days}$$
$$e = ·04$$
$$\omega = 354°$$
$$K = 32·3 \text{ km.}$$
$$\gamma = -0·7 \text{ km.}$$
$$T = \text{J.D. } 2,423,634·597$$

The observations were grouped into 16 normal places and observation equations were built up according to the usual Lehman-Filhés form, connecting the residuals with the elements K, e, ω and T. In these equations

$$x = \delta \gamma$$
$$y = \delta K$$
$$z = K \cdot \delta e$$
$$u = K \cdot \delta \omega$$
$$v = 31·292 \, \delta T$$

NORMAL PLACES

	Mean Phase	Mean Velocity	Wt.	O–C Prel.	O–C Final		Mean Phase	Mean Velocity	Wt.	O–C Prel.	O–C Final
1	1·038	+24·85	2	−3·7	−1·66	9	4·502	−26·30	1	−0·8	−0·28
2	1·597	+12·40	2	−3·0	−1·92	10	4·741	−19·00	1	+1·6	+2·01
3	2·019	+ 6·40	1	+3·8	+4·47	11	5·631	+ 5·30	1	+0·5	+0·34
4	2·479	− 4·20	1	+7·0	+6·97	12	5·861	+ 8·20	2	−3·8	−3·64
5	2·605	−16·50	3	−1·8	−1·81	13	6·129	+21·40	2	+1·7	+2·29
6	3·252	−30·40	2	−2·5	−1·74	14	6·494	+28·80	1	+0·9	+2·23
7	3·543	−32·95	2	−2·0	−1·09	15	·511	+32·90	1	0·0	+2·14
8	4·047	−30·15	2	+0·8	+1·74	16	·918	+28·00	1	−2·3	−0·19

OBSERVATION EQUATIONS

1............................	1·000x	+ ·904y	+ ·389z	+ ·499u	+ ·537v	+3·7 = 0
2............................	1·000	+ ·497	− ·679	+ ·885	+ ·914	+3·0
3............................	1·000	+ ·101	−1·002	+ ·994	+ ·994	−3·8
4............................	1·000	− ·326	− ·644	+ ·926	+ ·897	−7·0
5............................	1·000	− ·433	− ·450	+ ·877	+ ·843	+1·8
6............................	1·000	− ·843	+ ·648	+ ·466	+ ·435	+2·5
7............................	1·000	− ·934	+ ·939	+ ·223	+ ·209	+2·0
8............................	1·000	− ·936	+ ·856	− ·224	− ·203	−0·8
9............................	1·000	− ·767	+ ·211	− ·595	− ·556	+0·8
10...........................	1·000	− ·617	− ·227	− ·758	− ·719	−1·6
11...........................	1·000	+ ·170	− ·946	− ·995	−1·010	−0·5
12...........................	1·000	+ ·394	− ·695	− ·939	− ·970	+3·8
13...........................	1·000	+ ·631	− ·219	− ·811	− ·852	−1·7
14...........................	1·000	+ ·885	+ ·511	− ·538	− ·574	−0·9
15...........................	1·000	+1·040	+ ·998	− ·025	− ·023	0·0
16...........................	1·000	+ ·959	+ ·602	+ ·390	+ ·422	+2·3

From these were formed the normal equations

$$25·000x - ·428y + ·631z + 2·228u + 2·100v + 19·700 = 0$$
$$12·613 - 2·668 - 1·479 - 1·419 + 5·328$$
$$10·991 - ·367 - ·256 + 7·712$$
$$12·287 + 12·338 + 5·291$$
$$12·405 + 5·513$$

from which there resulted the corrections as follows:

$$\delta\gamma = - 0·82 \text{ km.}$$
$$\delta K = - 0·56 \text{ km.}$$
$$\delta e = - 0·021$$
$$\delta\omega = -23°·40$$
$$\delta T = - 0·431 \text{ days}$$

The corrections for ω and T are large, but uncertainty in their determination is always present when the eccentricity is as low as in this case, resulting as it does in almost identical coefficients in the observation and normal equations for these two elements. Had either element been considered fixed, as is often done, the correction to the other would have been vanishingly small.

The sum of the squares of the residuals for the normal places was reduced from 184·5 to 156·6, or about 15 per cent, and satisfactory agreement existed between equation and ephemeris residuals.

FINAL ELEMENTS

$$P = 6·5013 \text{ days}$$
$$e = ·019 \pm ·026$$
$$\omega = 330°·60 \pm 40°·85$$
$$\gamma = -1·52 \text{ km.} \pm 0·57 \text{ km.}$$
$$K = 31·74 \text{ km.} \pm 1·08 \text{ km.}$$
$$T = \text{J.D. } 2,423,634·166 \pm 0·741 \text{ day}$$
$$a \sin i = 2,837,000 \text{ km.}$$

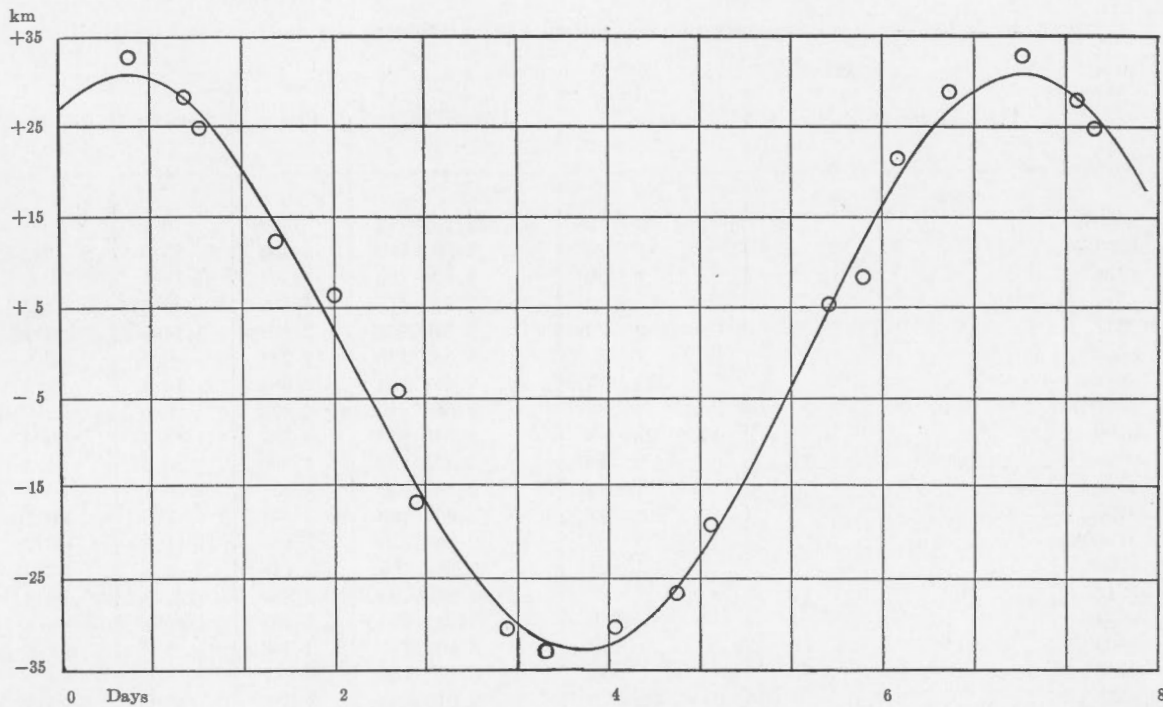The graph shown (Fig. 1) represents the final elements with the observations as grouped.

Fig. 1—Radial Velocity Curve of 45 Aurigae

In our absolute magnitude work we found from the ratios of six pairs of lines an absolute magnitude of +1·5 for this star corresponding to a prallax of 0″·016.

In the foregoing the dates are given in Greenwich Mean Time as used prior to 1925.

Dominion Astrophysical Observatory,
        Victoria, B.C.

                March, 1925.

19 LEO MINORIS

Abstract.—Thirty-three spectrograms of this star obtained in 1923, 1924 and 1925 with the single-prism spectrograph serve to determine the orbital elements. Its spectrum is F4 and the plates were measured on the spectrocomparator against Procyon as a standard. The probable error of a plate is ±1·2 km./sec. The period is 9·283 days and the eccentricity small.

The star 19 Leo Minoris, Boss 2665 (1900 $\omega = 9^h 51^m \cdot 6$, $\delta = +41° 32'$), visual magnitude 5·19 and type F4, was noted to be a spectroscopic binary from the first two plates made in 1923. A total of 33 plates have been obtained with the single-prism spectrograph, and these serve as the basis for the determination of the orbital elements. The lines of the F4 spectrum are well defined and the plates were measured against Procyon on the spectrocomparator. As the star is bright the plates are all well exposed and, while two are slightly inferior to the rest, they have all been weighted equally in the solution. The probable error of a plate on the basis of the final elements is ± 1·2 km./sec. and is quite satisfactory.

OBSERVATIONS OF 19 LEO MINORIS

Plate Number	Date	Julian Date	Phase	Velocity	Residual O–C
8680	1923 Mar. 19	2,423,498·773	·178	+ 6·2	+0·4
8702	" 24	3,503·779	5·184	−25·3	−0·2
8726	" 26	3,505·796	7·201	−11·7	−0·2
8779	April 2	3,512·717	4·839	−26·5	−1·3
8811	" 9	3,519·763	2·602	−11·7	+0·1
8866	" 23	3,533·725	7·281	− 9·9	+0·6
8943	May 7	3,547·694	2·684	−14·1	−1·4
8977	" 22	3,562·716	8·423	− 0·9	−1·3
9709	1924 Mar. 3	3,848·843	6·777	−15·3	+0·4
9725	" 10	3,855·839	4·490	−26·6	−1·8
9744	" 14	3,859·780	8·431	+ 0·1	−0·4
9767	" 17	3,862·793	2·161	− 7·6	−0·1
9785	" 22	3,867·740	7·108	−11·7	+0·7
9799	" 24	3,869·780	9·148	+ 8·0	+3·3
9813	" 28	3,873·795	3·880	−22·8	−0·4
9831	April 8	3,884·765	5·567	−24·4	−0·4
9838	" 14	3,890·737	2·256	− 6·6	+1·8
9850	" 21	3,897·758	9·277	+ 5·6	+0·5
9935	May 9	3,915·693	8·646	− 1·2	−3·5
9978	" 19	3,925·705	·092	+ 3·4	−2·1
10008	" 23	3,929·700	4·087	−26·0	−2·5
10055	June 2	3,939·705	4·809	−22·9	+2·4
10108	" 16	3,953·724	·262	+ 8·0	+2·5
10164	" 26	3,963·722	·977	− 0·8	−3·6
11105	Dec. 24	4,144·008	4·886	−27·3	−2·0
11141	1925 Jan. 9	4,160·955	3·267	−14·7	+3·5
11185	" 24	4,174·964	7·993	− 2·6	+0·6
11263	Mar. 6	4,216·775	3·389	−17·7	+1·4
11310	" 14	4,224·840	2·171	− 8·6	−1·0
11323	" 16	4,226·799	4·130	−23·1	+0·6
11346	" 22	4,232·912	·960	+ 1·7	−1·3
11388	" 30	4,240·851	8·899	+ 4·6	+0·9
11415	April 6	2,424,247·661	6·426	−16·7	+2·0

The period was quite early seen to be about 9·3 days, and a value of 9·283 was established from the first two years' observations. It has not been necessary to change this for the 1925 series. The preliminary elements adopted from graphical methods follow.

PRELIMINARY ELEMENTS

P = 9·283 days
e = ·10
ω = 345°
γ = −10·75 km.
K = 16·0 km.
T = J.D. 2,423,498·500

The observations were grouped into 17 normal places and observation equations built up in the usual way connecting the elements with the residuals for the normal places.

NORMAL PLACES

	Mean Phase		Mean Velocity	Wt.	Residual O–C	
	Prel.	Final			Prel.	Final
1.	8·994	8·899	+ 4·6	1	−0·2	+0·9
2.	·079	9·267	+ 5·7	3	−0·8	+0·7
3.	·315	·220	+ 7·1	2	+0·3	+1·9
4.	1·064	·969	+ 0·4	2	−3·6	−2·5
5.	2·291	2·196	− 7·6	3	+0·3	+0·3
6.	2·738	2·643	−12·9	2	−0·4	−0·4
7.	3·423	3·328	−16·2	2	+2·3	+2·5
8.	4·127	4·032	−24·0	3	−1·2	−0·8
9.	4·585	4·490	−26·6	1	−2·1	−1·8
10.	4·940	4·845	−25·6	3	−0·5	−0·3
11.	5·279	5·184	−25·3	1	−0·2	−0·2
12.	5·662	5·567	−24·4	1	0·0	−0·4
13.	6·521	6·426	−16·7	1	+3·3	+2·0
14.	7·038	6·943	−13·5	2	+2·1	+0·5
15.	7·336	7·241	−10·8	2	+1·7	+0·2
16.	8·088	7·993	− 2·6	1	+1·2	+0·6
17.	8·595	8·500	− 0·7	3	−2·3	−1·9

In the observation equations the following substitutions have been made in the Lehmann-Filhés coefficients.

x = δγ
y = δ K
z = K·δe
u = K·δω
v = 10·994.δ T

OBSERVATION EQUATIONS OF 19 LEO MINORIS

							Wt.
1...................	$1 \cdot 000x$	$+ \cdot 974y$	$+ \cdot 725z$	$- \cdot 506u$	$- \cdot 578v$	$+ 0 \cdot 2 = 0$	1
2...................	$1 \cdot 000$	$+1 \cdot 078$	$+1 \cdot 239$	$- \cdot 220$	$- \cdot 235$	$+ 0 \cdot 8$	3
3...................	$1 \cdot 000$	$+1 \cdot 097$	$+ \cdot 966$	$- \cdot 026$	000	$- 0 \cdot 3$	2
4...................	$1 \cdot 000$	$+ \cdot 920$	$+ \cdot 064$	$+ \cdot 542$	$+ \cdot 644$	$+ 3 \cdot 6$	2
5...................	$1 \cdot 000$	$+ \cdot 181$	$- \cdot 998$	$+ \cdot 970$	$+ \cdot 961$	$- 0 \cdot 3$	3
6...................	$1 \cdot 000$	$- \cdot 107$	$- \cdot 759$	$+ \cdot 953$	$+ \cdot 893$	$+ 0 \cdot 4$	2
7...................	$1 \cdot 000$	$- \cdot 482$	$- \cdot 043$	$+ \cdot 789$	$+ \cdot 694$	$- 2 \cdot 3$	2
8...................	$1 \cdot 000$	$- \cdot 756$	$+ \cdot 681$	$+ \cdot 496$	$+ \cdot 427$	$+ 1 \cdot 2$	3
9...................	$1 \cdot 000$	$- \cdot 860$	$+ \cdot 949$	$+ \cdot 263$	$+ \cdot 234$	$+ 2 \cdot 1$	1
10..................	$1 \cdot 000$	$- \cdot 899$	$+ \cdot 996$	$+ \cdot 068$	$+ \cdot 076$	$+ 0 \cdot 5$	3
11..................	$1 \cdot 000$	$- \cdot 899$	$+ \cdot 903$	$- \cdot 120$	$- \cdot 077$	$+ 0 \cdot 2$	1
12..................	$1 \cdot 000$	$- \cdot 854$	$+ \cdot 640$	$- \cdot 335$	$- \cdot 259$	$0 \cdot 0$	1
13..................	$1 \cdot 000$	$- \cdot 302$	$- \cdot 853$	$- \cdot 943$	$- \cdot 890$	$- 2 \cdot 1$	2
14..................	$1 \cdot 000$	$- \cdot 109$	$-1 \cdot 013$	$-1 \cdot 004$	$- \cdot 989$	$- 1 \cdot 7$	2
15..................	$1 \cdot 000$	$+ \cdot 434$	$- \cdot 642$	$- \cdot 968$	$-1 \cdot 052$	$- 1 \cdot 2$	1
16..................	$1 \cdot 000$	$+ \cdot 773$	$+ \cdot 132$	$- \cdot 763$	$- \cdot 866$	$+ 2 \cdot 3$	3
17..................	$1 \cdot 000$	$- \cdot 580$	$- \cdot 321$	$- \cdot 762$	$- \cdot 670$	$- 3 \cdot 3$	1

From these there resulted the following normal equations:—

$$
\begin{aligned}
33 \cdot 000x + \quad 1 \cdot 380y + \quad 5 \cdot 128z - \quad \cdot 153u - \quad \cdot 609v + \quad 6 \cdot 700 &= 0 \\
18 \cdot 061 + \quad 1 \cdot 154 - \quad 2 \cdot 799 - \quad 2 \cdot 882 + \quad 13 \cdot 026 & \\
21 \cdot 716 - \quad \cdot 234 - \quad \cdot 349 + \quad 19 \cdot 376 & \\
14 \cdot 878 + \quad 14 \cdot 723 + \quad 7 \cdot 752 & \\
14 \cdot 696 + \quad 7 \cdot 340 &
\end{aligned}
$$

which gave corrections as follows:—

$$
\begin{aligned}
\delta \gamma &= - \cdot 03 \text{ km.} \\
\delta K &= - \cdot 76 \text{ km.} \\
\delta e &= - \cdot 052 \\
\delta \omega \cdot &= + 6° \cdot 09 \\
\delta T &= + \cdot 095 \text{ days}
\end{aligned}
$$

The sum of the squares of the residuals for the normal places was reduced from $91 \cdot 2$ to $56 \cdot 6$ and satisfactory agreement existed between equation and ephemeris residuals, the numerical average being $0 \cdot 04$ km.

The final elements, then, with their probable errors are the following:

FINAL ELEMENTS

$$
\begin{aligned}
P &= \quad 9 \cdot 283 \text{ days} \\
e &= \quad \quad \cdot 048 \pm \cdot 020 \\
\omega &= 351° \cdot 09 \pm 14° \cdot 95 \\
\gamma &= -10 \cdot 78 \pm 0 \cdot 26 \text{ km.} \\
K &= 15 \cdot 24 \pm 0 \cdot 35 \text{ km.} \\
T &= \text{J.D. } 2,423,498 \cdot 595 \pm 0 \cdot 383 \text{ day} \\
a \sin i &= 1,943,130 \text{ km.}
\end{aligned}
$$

The graph (Fig. 2) represents the final elements and the grouped observations are shown.
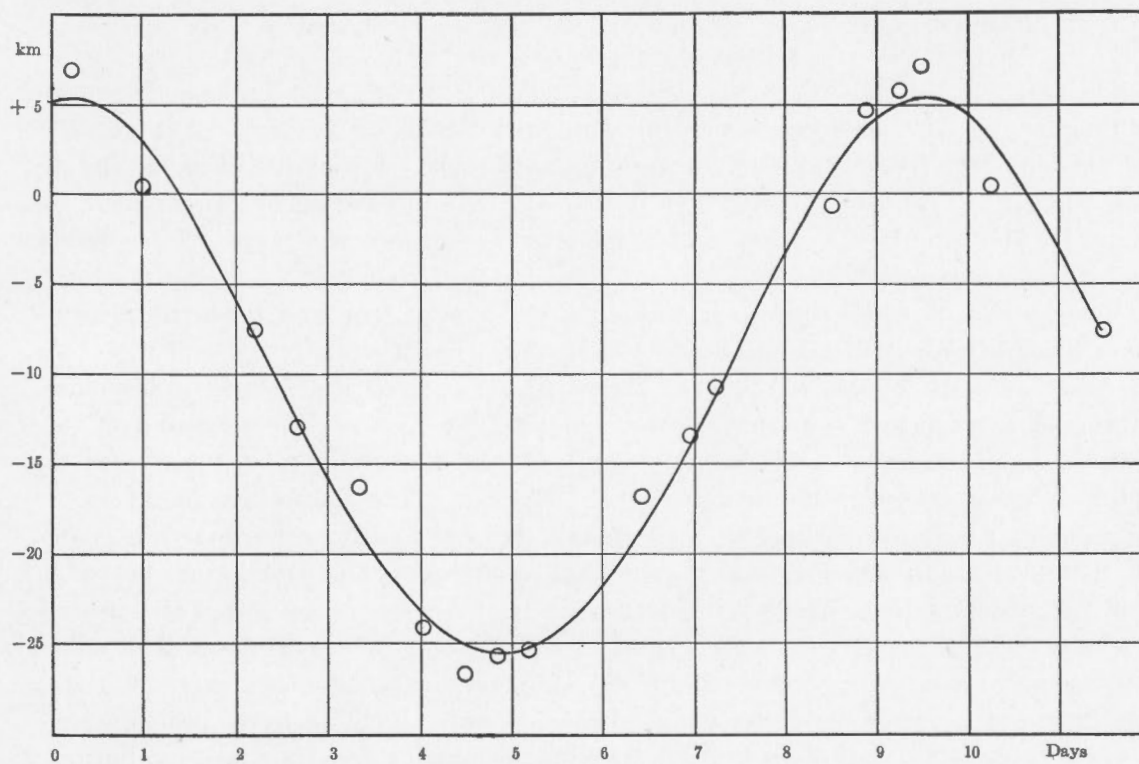

FIG. 2.—Radial Velocity Curve of 19 Leo Minoris

All times are expressed in Greenwich Mean Time as used prior to 1925.

Dominion Astrophysical Observatory,
      Victoria, B.C.
           April 10, 1925.

105 HERCULIS

Abstract.—Twenty-five spectrograms of this K-type star were obtained with the single-prism spectrograph since April, 1922, and upon these a solution of the orbit is based.   The period adopted is 478 days, but this is uncertain to the extent of a few days, as the observations cover only two cycles.   The eccentricity is ·398 with semi-amplitude 16·07 km./sec.

The star 105 Herculis (1900 $\alpha = 18^h \ 15^m \cdot 1, \ \delta = +24° \ 24'$) whose visual magnitude is 5·49 and type K5, was first observed here on April 17, 1922.   A total of 25 plates has been secured with the single-prism spectrograph and these serve as the basis for the determination of the orbit of this spectroscopic binary.   In our parallax work we gave the type as K2, but the best exposed plate, made probably since that type was assigned, shows it to be closer to K5 which Harvard gives for the star.

The plates as a whole are much inferior to our average run and the writer accepts the responsibility for such being the case.   The star seems unusually red and it so happened that on many of the nights when the star was observed it was more or less hazy, resulting in a weakened spectrum in the violet.   The plates were measured on the spectrocomparator against Arcturus as a standard, and the velocities deduced with the weights assigned are given in the table of observations.   The phases are based on the final value of periastron passage with the period 478 days and the residuals are scaled from the curve shown which represents the final elements.   The elements are not to be considered as definite for, owing to the fact that only two cycles of the period are covered, the period must be uncertain to the extent of a few days.   Observations will be taken here in future years to improve the value of the period and by means of which it is hoped to lessen the probable errors of the elements.   These are rather high due to the probable error of a plate being $\pm 2 \cdot 0$ km. per second and also to the small range in velocity variation.

OBSERVATIONS OF 105 HERCULIS

Plate Number	Date			Julian Date	Phase	Velocity	Wt.	O–C
7470	1922	April	17	2,423,162·001	99·35	− 2·7	3	−0·7
7622		May	28	3,203·867	141·22	− 7·8	3	−5·0
8254		Oct.	27	3,355·650	293·00	−21·2	2	−7·0
8284		Nov.	2	3,361·625	298·98	−11·4	2	+2·7
8308		"	8	3,367·572	304·92	−11·4	3	+3·4
8709	1923	Mar.	25	3,504·007	441·36	−34·3	3	−0·3
8875		April	23	3,533·971	471·32	−26·6	3	+3·5
8950		May	7	3,547·906	7·26	−21·8	2	+2·3
8971		"	19	3,559·942	19·29	−19·5	3	−0·6
9026		June	6	3,577·891	37·24	−11·6	2	−1·0
9084		"	25	3,596·846	56·20	− 7·3	3	−1·6
9190		July	20	3,621·827	81·18	− 4·3	3	−1·6
9304		Aug.	17	3,649·742	109·09	− 0·7	3	+1·3
9493		Oct.	8	3,701·613	160·96	− 3·4	3	+0·2
9941	1924	May	9	3,915·937	375·29	−22·0	3	+2·1
10065		June	2	3,939·950	399·30	−28·3	3	−0·2
10198		"	30	3,967·895	427·24	−38·8	1	−6·0
10242		July	7	3,974·780	434·13	−34·1	3	−0·5
10534		Aug.	25	4,023·699	5·05	−21·6	3	+3·6
10634		Sept.	8	4,037·729	19·08	−23·7	3	−5·2
10906		Oct.	27	4,086·592	67·94	+ 0·1	3	+4·0
10908		"	31	4,090·587	71·94	− 2·8	3	+0·6
10944		Nov.	24	4,114·558	95·91	+ 3·4	2	+5·4
11430	1925	April	6	4,247·978	229·33	− 8·2	3	−0·2
11503		"	13	2,424,254·981	236·33	− 8·6	2	−0·3

NORMAL PLACES

—	Mean Phase Final	Mean Velocity	Wt.	O–C	
				Prel.	Final
1.	102·14	−0·42	·8	+2·40	+1·62
2.	151·09	5·60	·6	−2·18	−2·50
3.	232·13	8·36	·5	− ·30	− ·10
4.	299·82	14·20	·7	− ·17	+ ·15
5.	375·29	22·00	·3	+1·94	+2·12
6.	399·30	28·30	·3	− ·35	− ·22
7.	436·24	34·86	·7	−1·72	−1·18
8.	471·32	26·60	·3	+2·09	+3·47
9.	13·16	21·64	1·1	−1·26	− ·57
10.	37·24	11·60	·2	− ·43	−1·01
11.	56·20	7·30	·3	− ·61	−1·59
12.	73·69	−2·67	·9	+1·69	+ ·67

PRELIMINARY ELEMENTS

$$P = 478 \text{ days} \qquad \gamma = -14\cdot30 \text{ km.}$$
$$e = \cdot38 \qquad K = 15\cdot35 \text{ km.}$$
$$\omega = 230° \qquad T = \text{J.D. } 2,423,536\cdot09$$

From graphical methods the foregoing preliminary elements were adopted and then a set of observation equations according to the form of Lehmann-Filhés was built up connecting the five elements γ, K, e, ω and T with the residuals for the grouped observations. In these observation equations, weighted as above, the following substitutions were made for the sake of homogeneity.

$$x = \delta\gamma \qquad u = -K.\delta\omega$$
$$y = \delta K \qquad v = \cdot25495 \, \delta T$$
$$z = K.\delta e$$

OBSERVATION EQUATIONS FOR 105 HERCULIS

1.	1·000x	+ ·748y	− ·420z	− ·415w	− ·080v	−2·40 = 0
2.	1·000	+ ·709	− ·961	+ ·013	+ ·139	+2·18
3.	1·000	+ ·407	− ·659	+ ·470	+ ·292	+ ·30
4.	1·000	+ ·018	+ ·141	+ ·676	+ ·414	+ ·17
5.	1·000	− ·628	+1·106	+ ·634	+ ·628	−1·94
6.	1·000	− ·889	+1·060	+ ·475	+ ·668	+ ·35
7.	1·000	−1·227	− ·232	− ·105	+ ·260	+1·72
8.	1·000	− ·937	− ·778	−1·010	−1·372	−2·09
9.	1·000	− ·396	+ ·750	−1·277	−1·735	+1·26
10.	1·000	+ ·204	+1·420	−1·183	−1·185	+ ·43
11.	1·000	+ ·496	+ ·940	− ·962	− ·693	+ ·61
12.	1·000	+ ·648	+ ·340	− ·740	− ·376	−1·69

These gave the following normal equations:—

$$6\cdot700x \quad - \quad \cdot018y \quad + \quad \cdot806z \quad - \quad 2\cdot258u \quad - \quad 2\cdot078v \quad - \quad \cdot109 = 0$$
$$3\cdot136 \quad - \quad \cdot795 \quad - \quad \cdot074 \quad + \quad \cdot326 \quad - \quad 2\cdot488$$
$$3\cdot237 \quad - \quad 1\cdot229 \quad - \quad 1\cdot485 \quad - \quad \cdot043$$
$$3\cdot914 \quad + \quad 4\cdot071 \quad + \quad \cdot229$$
$$4\cdot906 \quad - \quad \cdot755$$

which resulted in the following corrections to the elements:—

$$\delta\gamma = - \cdot 06 \text{ km.}$$
$$\delta K = + \cdot 72 \text{ km.}$$
$$\delta e = + \cdot 018$$
$$\delta \omega = + 4°\cdot 47$$
$$\delta T = + 4 \cdot 56 \text{ days}$$

The sum of the squares of the residuals for the normal places was reduced a small amount from 16·54 to 13·53 and good agreement existed between equation and ephemeris residuals, none being greater than ·06.

The following, then, are the elements as determined for the present.

FINAL ELEMENTS

$$P = 478 \text{ days}$$
$$e = \cdot 398 \pm \cdot 038$$
$$\omega = 234°\cdot 47 \pm 5°\cdot 11$$
$$\gamma = -14\cdot 36 \text{ km.} \pm 0\cdot 41 \text{ km.}$$
$$K = 16\cdot 07 \text{ km.} \pm 0\cdot 57 \text{ km.}$$
$$T = \text{J.D. } 2,423,540\cdot 65 \pm 4\cdot 75$$
$$a \sin i = 96,904,000 \text{ km.}$$
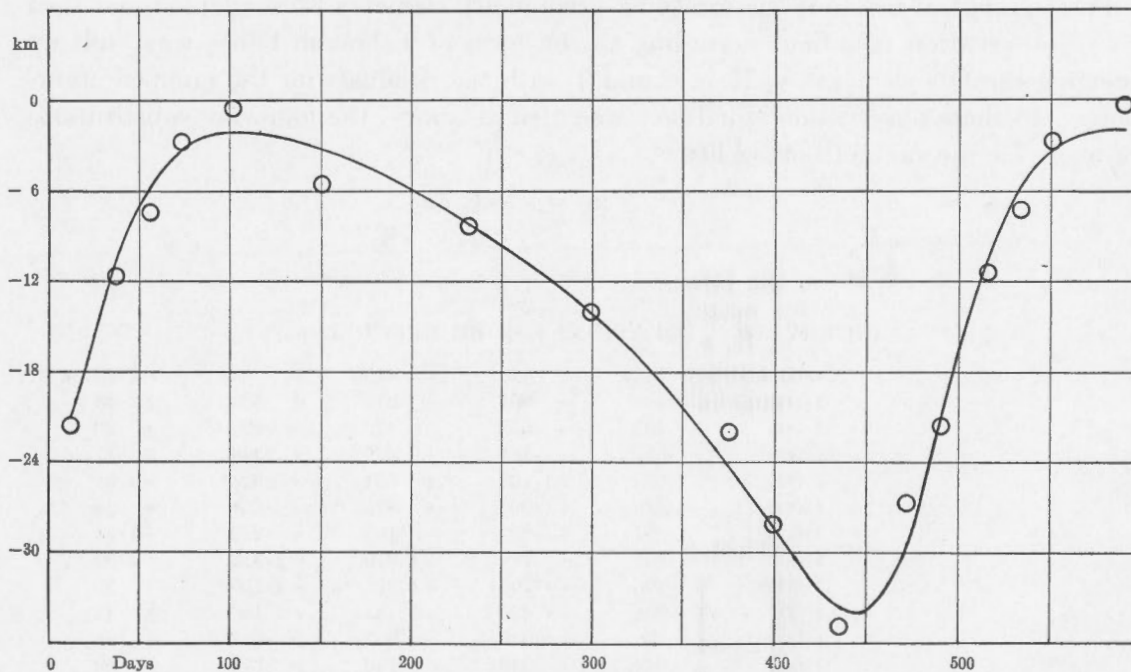
FIG. 3.—Radial Velocity Curve of 105 Herculis

In the foregoing the times are given in Greenwich Mean Time in the sense used prior to 1925.

Dominion Astrophysical Observatory,
        Victoria, B.C.
                April 24, 1925.

22 VULPECULAE

Abstract.—Twenty-seven spectrograms of this star whose visual magnitude is 5·4 and type G7 are made the basis of a determination of its spectroscopic orbit. The period is 251 days, the eccentricity ·042 and the probable error of a plate ±1·0 km. per sec.

Observations were begun on this star, whose right ascension is $20^h 11^m \cdot 2$ and declination $+23° 12'$, in the year 1922 and twenty-seven spectrograms have been secured on which to base a determination of the orbital elements. The value obtained for the period was 251·0 days, and the observations, while not uniformly distributed over the period, are nevertheless sufficiently so to ensure a fair determination of the elements.

The spectrum is of type G7 and the plates have been measured on the spectro-comparator using α Boötis, No. 2702, as a standard plate. The table following contains the phases reckoned from the time of periastron, the velocities, the weights assigned the plates and the residual for each from the curve representing the final elements. Through a slip the weight 1 was used for the first plate, whereas it should have been 2, but any correction would be negligible.

OBSERVATIONS OF 22 VULPECULAE

Plate	Date			Julian Date	Phase	Velocity	Wt.	O–C
7852...............	1922	July	28	2,423,264·866	100·016	−24·1	1	+1·5
8009...............		Aug.	28	3,295·739	130·889	− 8·0	3	−0·4
8027...............		Sept.	1	3,299·802	134·952	− 2·9	3	+2·8
8052...............		"	7	3,305·815	140·965	− 5·8	3	−2·4
8078...............		"	10	3,308·766	143·916	− 2·4	2	−0·1
8136...............		"	16	3,314·798	149·948	− 3·6	2	−3·1
8213...............		Oct.	9	3,337·699	170·849	+ 4·3	3	+1·8
8255...............		"	27	3,355·697	190·847	+ 0·6	3	+1·8
8337...............		Nov.	11	3,370·571	205·721	− 6·8	3	+0·8
8379...............		"	14	3,373·650	208·800	−12·1	2	−2·5
8954...............	1923	May	7	3,547·989	132·139	− 6·7	2	+0·5
9086...............		June	25	3,596·908	181·058	+ 1·8	3	+0·3
9192...............		July	20	3,621·871	206·021	− 7·6	3	+0·3
9306...............		Aug.	17	3,649·798	233·948	−27·1	3	−0·7
9495...............		Oct.	8	3,701·667	34·817	−50·6	3	+0·4
9526...............		"	12	3,705·671	38·821	−49·6	2	+1·7
9539...............		"	22	3,715·594	48·744	−51·8	3	−1·1
10066...............	1924	June	2	3,939·964	22·114	−48·4	2	+0·4
10247...............		July	7	3,974·884	57·034	−50·7	3	−2·5
10331...............		"	21	3,988·844	70·994	−41·5	3	+1·0
10536...............		Aug.	25	4,023·758	105·908	−19·8	3	+2·1
10623...............		Sept.	6	4,035·753	117·903	−15·4	3	−0·8
10725...............		"	15	4,044·718	126·868	− 9·3	3	+0·7
10778...............		"	26	4,055·694	137·844	− 4·4	3	+0·1
10850...............		Oct.	9	4,068·723	150·873	− 1·6	3	−1·2
10910...............		"	31	4,090·670	172·820	+ 2·3	2	−0·2
10945...............		Nov.	24	2,424,114·600	196·750	− 2·1	2	+1·3

NORMAL PLACES OF 22 VULPECULAE

—	Mean Phase	Mean Velocity	Wt.	Residual O–C	
				Prel.	Final
1.	100·016	−24·10	1	+1·67	+1·52
2.	105·908	−19·80	3	+2·30	+2·10
3.	117·903	−15·40	3	−0·48	−0·76
4.	129·694	− 8·16	8	+0·38	+0·08
5.	137·920	− 4·37	9	+0·37	+0·07
6.	148·621	− 2·40	7	−1·63	−1·63
7.	172·837	+ 3·50	5	+0·73	+0·89
8.	185·952	+ 1·20	6	+0·19	+0·64
9.	202·133	− 4·92	5	0·00	+0·73
10.	207·135	− 9·40	5	−1·89	−1·07
11.	233·948	−27·10	3	−1·47	−0·70
12.	22·114	−48·40	2	+0·26	+0·43
13.	36·419	−50·20	6	+0·97	+1·06
14.	52·889	−51·25	5	−1·94	−1·85
15.	70·994	−41·50	3	+0·98	+1·04

The observations were grouped into 15 normal places on the basis of phase. A value of the period of 251·0 days seemed most satisfactory and while there are only four cycles covered by the observations and this value may be in error by as much as a day, it was not considered that any great improvement would result by including the period in the least-squares solution. Moreover, as the eccentricity was small it was necessary to consider either T or ω fixed, and the former was the element taken as fixed.

PRELIMINARY ELEMENTS

P = 251·0 days γ = −23·52 km.
e = ·05   K = 27·0 km.
ω = 120°   T = J.D. 2,423,415·850

With these, observation equations were built up connecting the residuals with the four elements, e, ω, γ and K and for homogeneity the following substitutions were made:

x = $\delta\gamma$ z = K δe
y = δ K   u = −K.$\delta\omega$

OBSERVATION EQUATIONS OF 22 VULPECULAE

1.	1·000x	− ·083y	+ ·140z	− ·955u	−1·67 = 0
2.	1·000	+ ·053	− ·027	− ·954	−2·30
3.	1·000	+ ·319	− ·336	− ·896	+0·48
4.	1·000	+ ·555	− ·578	− ·771	−0·38
5.	1·000	+ ·696	− ·701	− ·650	−0·37
6.	1·000	+ ·843	− ·785	− ·454	+1·63
7.	1·000	+ ·974	− ·435	+ ·090	−0·73
8.	1·000	+ ·909	+ ·155	+ ·402	−0·19
9.	1·000	+ ·689	+1·029	+ ·743	·00
10.	1·000	+ ·593	+1·235	+ ·829	+1·89
11.	1·000	− ·078	+ ·811	+1·042	+1·47
12.	1·000	+ ·931	−1·186	+ ·467	−0·26
13.	1·000	−1·024	− ·386	+ ·097	−0·97
14.	1·000	− ·955	+ ·280	− ·323	+1·94
15.	1·000	− ·702	+ ·698	− ·693	−0·98

These resulted in the following normal equations:—

$$7 \cdot 100x \; + \; 1 \cdot 932y \; - \; \cdot 540z \; - \; 1 \cdot 047u \; + \; 1 \cdot 031 \; = \; 0$$
$$3 \cdot 995 \; - \; \cdot 508 \; - \; \cdot 178 \; + \; \cdot 286$$
$$3 \cdot 322 \; + \; 1 \cdot 935 \; + \; 1 \cdot 498$$
$$2 \cdot 907 \; + \; 1 \cdot 541$$

from which the following corrections were obtained:—

$$\delta\gamma \;\; = \;\; - \cdot 23 \text{ km.}$$
$$\delta K \;\; = \;\; - \cdot 01 \text{ km.}$$
$$\delta e \;\; = \;\; - \cdot 008$$
$$\delta \omega \;\; = \;\; + 1° \cdot 0$$

The following, then, are the final elements obtained, with their probable errors attached.

FINAL ELEMENTS

$$P \; = \;\; 251 \cdot 0 \text{ days}$$
$$e \; = \;\; \cdot 042 \; \pm \; \cdot 015$$
$$\omega \; = \;\; 121° \cdot 0 \; \pm \; 0° \cdot 93$$
$$\gamma \; = \;\; -23 \cdot 75 \text{ km.} \; \pm \; 0 \cdot 24 \text{ km.}$$
$$K \; = \;\; 26 \cdot 99 \text{ km.} \; \pm \; 0 \cdot 31 \text{ km.}$$
$$T \; = \;\; \text{J.D. } 2,423,415 \cdot 850$$
$$a \sin i \; = \;\; 93,072,000 \text{ km.}$$
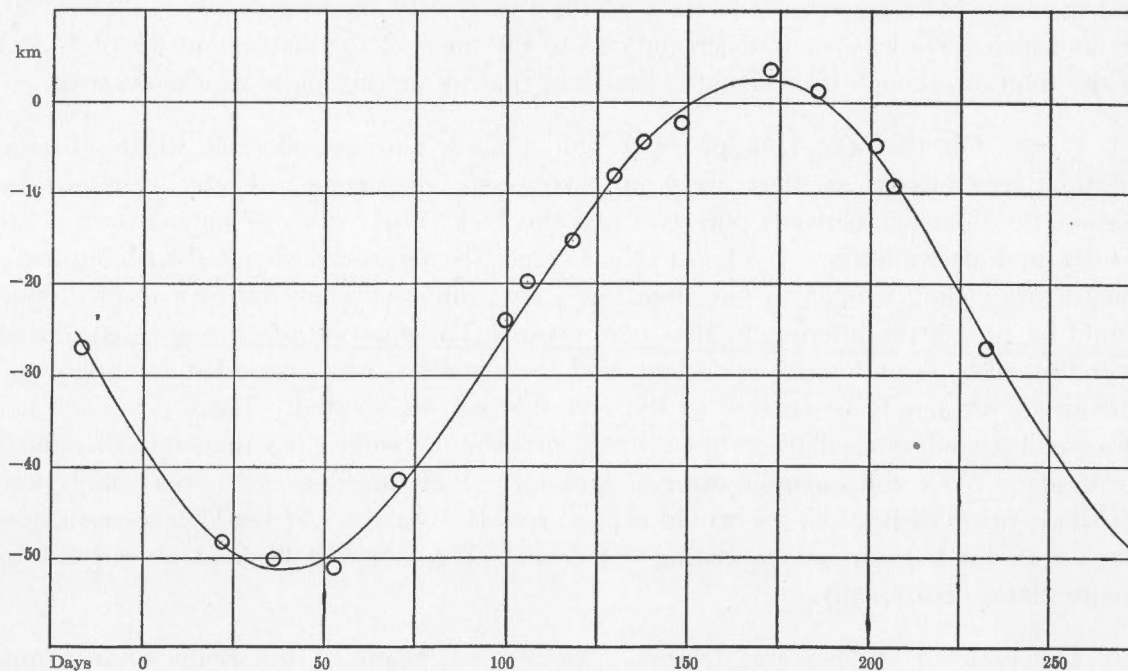
FIG. 4.—Radial Velocity Curve of 22 Vulpeculae

The curve (Fig. 4) accompanying represents the final elements, and on it are shown the observations as grouped.

Dominion Astrophysical Observatory,
    Victoria, B.C.
        Dec. 8, 1924.

π CEPHEI

Abstract.—Twenty-three spectrograms taken with the single-prism spectrograph and measured on the spectrocomparator against the sky as a standard, were made the basis of the determination of the elements. Five observations taken 25 years ago at the Lick Observatory were made use of also and fixed the period as 556·2 days. The semi-amplitude is 23·02 km. and the eccentricity 0·281.

The star π Cephei (1900 $\alpha = 23^h\ 04^m \cdot 7, \delta = +74°\ 51'$, visual magnitude 4·56 and type G5) was announced[1] as a spectroscopic binary from the Lick Observatory from five plates taken in the years 1899 and 1900. The results of four plates made at the Bonn Observatory in the years 1909 and 1912 have since been published[2] as well as two of Hnatek[3] at Vienna in 1913. For convenience of reference all are given in the table of observations.

The star is a long period visual binary listed as number 12196 in Burnham's catalogue. While the separation at present is less than a second of arc, yet the yellow component to which the present discussion relates is over two magnitudes brighter than the fainter and hence the measures should not be vitiated to any appreciable degree by the light from the fainter star.

The star has been observed here with the single-prism instrument since the beginning of 1922. The measures have been made on the comparator against the sky as a standard and in many cases against Arcturus also, the two results agreeing closely. Plate 8030, about which there was some uncertainty as to the focus of the instrument was not used in the solution, though its residual is less than that of another plate which was used.

It was felt that the Lick plates would, if used, add considerable weight to the solution, particularly as they were of three-prism dispersion. There seems to be systematic difference between ourselves and the Lick Observatory as judged from other results, and after adding −1·5 km. to their values they were incorporated with our own. Such a proceeding is open to question, but a determination from our own results alone would be very little different to that here given. The observations are not distributed over the curve as uniformly as desired, and the elements, while regarded as closely approximate, are not to be treated as the best that can be secured. There is a suspicion of a small secondary oscillation, but the measures are not sufficiently numerous or refined to decide. While the probable error of a plate, ±1·58 km. per sec., is reasonably low for single-prism dispersion, one would expect from the character of the lines for measurement a probable error not exceeding ±1·0 km. The star will be kept on our list to secure plates occasionally.

The table of observations follows. No use was made of the Bonn and Vienna single-prism results, though the residuals from our curve are tabulated and are exceedingly small for the Bonn Observatory.

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[1] Ap. J. Vol. 14 p. 138
[2] A.N. 4750.
   A.N. 197, p. 187

OBSERVATIONS OF π CEPHEI

Observatory Plate No.	Date			Julian Date	Phase	Velocity	O–C
Lick III.....................	1899	Aug.	8	2,414,875·9	193·37	−33·0	+0·6
"		"	23	4,890·9	208·37	−36·0	−1·4
"		"	29*	4,896·9	214·37	−37·0	−2·0
"	1900	Oct.	7	5,300·9	62·17	− 5·0	+1·0
"		Dec.	24	5,378·9	140·17	−23·0	+4·0
Bonn I.....................	1909	Oct.	24	8,604·32	28·39	+ 3·1	−1·6
"	1912	Sept.	26	9,672·39	540·26	+ 8·9	−0·4
"		Oct.	9	9,685·34	553·21	+11·0	+1·2
"		Oct.	11	2,419,687·36	555·23	+ 8·8	−1·0
Vienna I.....................	1913	Oct.	31	2,420,072·332	384·00	−20·3	+7·9
"		Nov.	21	0,093·308	404·98	−28·5	−3·5
Victoria—							
6975......................	1922	Jan.	6	2,423,061·594	36·07	+ 3·8	+1·4
8030......................		Sept.	1	3,299·869	274·34	−30·8	+4·9
8258......................		Oct.	27	3,355·772	330·24	−37·2	−3·3
8286......................		Nov.	2	3,361·672	336·14	−34·3	−0·9
8307......................		"	5	3,364·772	339·24	−32·5	+0·7
8311......................		"	8	3,367·664	342·14	−27·6	+5·4
8433......................		"	27	3,386·687	361·16	−28·4	+2·4
8497......................	1923	Jan.	13	3,433·594	408·07	−24·8	−0·1
9312......................		Aug.	17	3,649·969	68·24	−10·6	−2·0
9501......................		Oct.	8	3,701·848	120·12	−18·8	+4·2
9566......................		"	26	3,719·772	138·04	−25·6	+0·9
10250.....................	1924	July	7	3,974·959	393·23	−27·7	−1·0
10334.....................		"	21	3,988·961	407·23	−26·0	−1·3
10571.....................		Aug.	29	4,027·895	446·17	−20·0	−3·3
10610.....................		Sept.	3	4,032·925	451·20	−13·6	+1·9
10627.....................		"	6	4,035·909	454·18	−12·3	+1·9
10636.....................		"	8	4,037·876	456·15	−15·4	−1·4
10727.....................		"	15	4,044·821	463·09	−10·8	+1·0
10853.....................		Oct.	9	4,068·832	487·10	− 5·8	−1·3
10989.....................		Nov.	28	4,118·723	537·00	+ 7·5	−1·3
11030.....................		Dec.	8	4,128·716	546·99	+11·0	+1·3
11092.....................		"	23	4,143·638	5·71	+10·7	+1·4
11197.....................	1925	Feb.	9	2,424,191·590	53·66	− 7·3	−3·6

All the plates were considered of equal weight and some occurring at or about the same phase were grouped as shown in the table of normal places. The mean phases there listed are based on the final elements.

NORMAL PLACES FOR π CEPHEI

	Phase	Velocity	Wt.	O–C			Phase	Velocity	Wt.	O–C	
				Prel.	Final					Prel.	Final
1	36·07	+ 3·8	1	−0·3	+1·4	10	377·20	−28·0	2	+0·7	+1·1
2	57·92	− 6·9	2	−2·4	−1·9	11	407·65	−25·4	2	−1·3	−0·7
3	68·24	−10·6	1	−2·2	−2·0	12	448·67	−16·8	2	−1·2	−1·0
4	129·08	−22·2	2	+4·5	+2·8	13	455·17	−13·8	2	+0·1	+0·2
5	140·17	−24·5	1	+4·3	+2·5	14	463·09	−10·8	1	+1·0	+1·0
6	193·37	−34·5	1	+0·5	−0·9	15	487·10	− 5·8	1	−1·0	−1·3
7	211·37	−38·0	2	−2·0	−3·2	16	537·00	+ 7·5	1	−1·8	−1·3
8	333·19	−35·8	2	−2·5	−2·1	17	546·99	+11·0	1	+0·2	+1·3
9	340·69	−30·0	2	+2·7	+3·1	18	5·71	+10·7	1	−0·3	+1·4

Graphical methods in use here gave the following preliminary values of the elements.

$$P = 556\cdot2 \text{ days}$$
$$e = \cdot30$$
$$\omega = 15°$$
$$K = 24\cdot0 \text{ km.}$$
$$\gamma = -19\cdot75 \text{ km.}$$
$$T = \text{J.D. } 2,414,138\cdot0$$

In the observation equations which were built up the following substitutions were made.

$$x = \delta\gamma$$
$$y = \delta K$$
$$z = K.\delta e$$
$$u = -K.\delta\omega$$
$$v = [9\cdot49460]\,\delta T$$

OBSERVATION EQUATIONS FOR π CEPHEI

							Wt.
1............	1·000x	+ ·996y	+ ·086z	+ ·786u	+1·124v	+0·3 = 0	1
2............	1·000	+ ·634	− ·867	+1·017	+1·290	+2·4	2
3............	1·000	+ ·472	−1·111	+1·061	+1·253	+2·2	1
4............	1·000	− ·289	− ·624	+ ·893	+ ·653	+4·5	2
5............	1·000	− ·378	− ·393	+ ·822	+ ·550	−4·3	1
6............	1·000	− ·635	+ ·520	+ ·458	+ ·220	−0·5	1
7............	1·000	− ·677	+ ·725	+ ·332	+ ·139	−2·0	2
8............	1·000	− ·566	+ ·697	− ·438	− ·262	+2·5	2
9............	1·000	− ·540	+ ·625	− ·480	− ·286	+2·7	2
10............	1·000	− ·374	+ ·175	− ·670	− ·420	−0·7	2
11............	1·000	− ·180	− ·307	− ·805	− ·558	+1·3	2
12............	1·000	+ ·175	− ·952	− ·915	− ·786	+1·2	2
13............	1·000	+ ·245	−1·031	− ·921	− ·825	−0·1	2
14............	1·000	+ ·332	−1·107	− ·921	− ·873	−1·0	1
15............	1·000	+ ·624	−1·125	− ·865	− ·988	+1·0	1
16............	1·000	+1·212	+ ·385	− ·308	− ·591	+1·8	1
17............	1·000	+1·272	+ ·760	− ·111	− ·305	−0·2	1
18............	1·000	+1·282	+1·010	− ·207	+ ·218	+0·3	1

There resulted the following normal equations:—

$$27\cdot000x + 2\cdot033y - 4\cdot093z - 2\cdot845u - 1\cdot502v + 2\cdot400 = 0$$
$$10\cdot629 - 3\cdot002 + \cdot945 + 1\cdot145 + 9\cdot334$$
$$15\cdot176 + 1\cdot097 + \cdot262 + 1\cdot180$$
$$14\cdot669 + 13\cdot491 - 6\cdot200$$
$$13\cdot505 - 1\cdot913$$

from which the following corrections resulted:

$$\delta\gamma = + \cdot12 \text{ km.}$$
$$\delta K = - \cdot98 \text{ km.}$$
$$\delta e = - \cdot019$$
$$\delta\omega = -9°\cdot30$$
$$\delta T = -11\cdot67 \text{ days}$$

so that the final elements, with their probable errors attached are as follows:—

FINAL ELEMENTS

$$P = 556 \cdot 2 \text{ days}$$
$$e = \cdot 281 \pm \cdot 020$$
$$\omega = 5° \cdot 70 \pm 4° \cdot 21$$
$$K = 23 \cdot 02 \text{ km.} \pm 0 \cdot 59 \text{ km.}$$
$$\gamma = -19 \cdot 63 \text{ km.} \pm 0 \cdot 37 \text{ km.}$$
$$T = \text{J.D. } 2,414,126 \cdot 33 \pm 5 \cdot 83 \text{ days}$$
$$a \sin i = 168,970,000 \text{ km.}$$
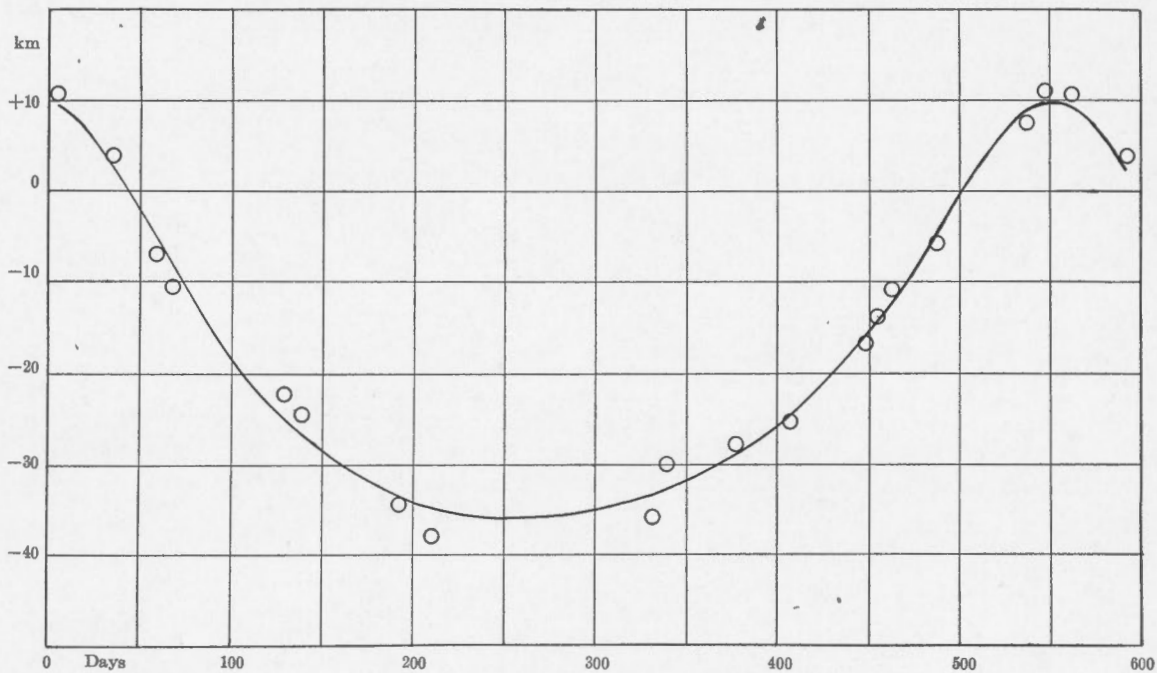
FIG. 5.—Radial Velocity Curve of π Cephei

The curve shown (Fig. 5) represents the final elements.

Dominion Astrophysical Observatory,
        Victoria. B.C.

                Feb. 27, 1925.
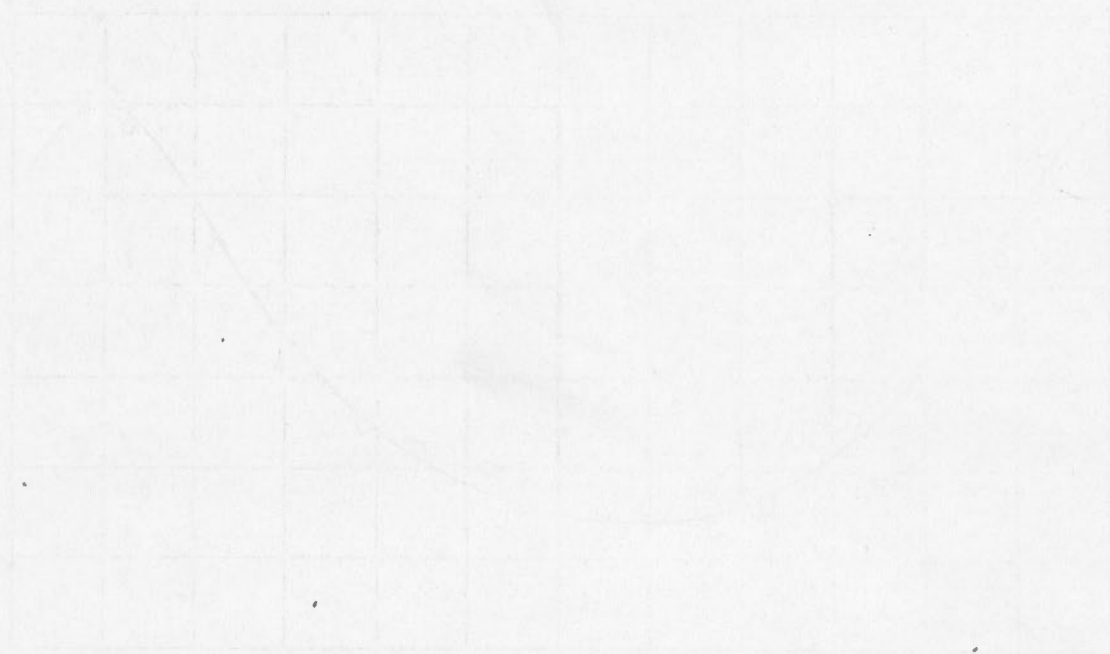
en avance." Chandler has given a period of $10^d \cdot 15382$, Guthnick a period of $10^d \cdot 15457$, while in the Vierteljahrsschrift catalogue the elements of variation are given as:—

$$\text{J.D. } 2410639 \cdot 339 + 10^d \cdot 15375 \text{ E} + 0^d \cdot 62 \sin (0° \cdot 112 \text{ E} + 116° \cdot 5).$$

On account of the uncertainty of ordinary visual and photographic observations and the difficulty of determining accurate maxima and minima, it does not seem advisable to place much confidence in periods determined from photometric observations unless these cover long intervals of time and are of precision such as obtained with the photo-electric photometer.

Radial velocities are more reliable. Jacobsen, who obtained twenty-one of these (as yet unpublished) between March 5 and May 15, 1923, found that a period of $10^d \cdot 15258$ connected Campbell's observations with his. The same period connects Campbell's observations with ours, but it does not appear to fit Küstner's observations and certainly not those of Lunt. There is thus found a variation of period within a few years.

VELOCITIES FROM INDIVIDUAL LINES

A new method of analyzing variable radial velocity data by a systematic study of line displacements from different elements and at different levels of a star's atmosphere has been developed recently by Dr. W. Carl Rufus of the Detroit Observatory. This method has already given important results in the case of some of the brighter Cepheid variables, in which a motion of pulsation of the star appears to be indicated. Among the papers treating of this may be mentioned:—"Atmospheric Pulsation of Cepheids, A Method of Attack," by Carl Rufus;[1] "Atmospheric Pulsation of η Aquilae," by W. Carl Rufus;[2] "Atmospheric Pulsation of the Cepheid Variable η Aquilae," by W. Carl Rufus;[3] "Résumé of remarks concerning recent studies of Cepheid variables at the Detroit Observatory," by R. H. Curtiss;[4] "Velocity-curves from groups of lines of different chromospheric heights in the atmosphere of W. Sagittarii," by R. H. Curtiss;[5] "Radial Velocities of S. Sagittae," abstract by J. A. Aldrich.[6]

Considering the important influence that these results might have on the theory of Cepheids it was thought that an investigation of ζ Geminorum would be of value, especially for lines originating in different levels of its atmosphere.

All the plates of ζ Geminorum were, therefore, measured again directly on a Toepfer measuring engine, in both direct and reversed positions as usual. The velocities obtained for individual lines, after correction for the earth's motion, are given in the following tables. In these tables each plate is indicated by its Julian date, under which is given the phase computed from the formula

$$\text{J.D. } 2410639 \cdot 339 + 10^d \cdot 15375 \text{ E}$$

In the first column the approximate wave-lengths of the spectral lines are given, preceded by the symbols of the elements which are thought to produce them; in the second column are given the heights of these elements in the chromosphere of the Sun as determined by Professor S. A. Mitchell[7]. Resulting curves obtained for lines of high, medium and low level, as well as for the mean of all the lines, are given in figures 3, 4, 5 and 6. There is apparently no marked variation of amplitude from one curve to another.

[1] Pop. Ast., Vol. 32, 1924, p. 22.
[2] Pop. Ast., Vol. 32, 1924, p. 228.
[3] Proc. Nat. Acad. Sc., Vol. 10, 1924, p. 264.
[4] Pop. Ast., Vol. 32, 1924, p. 471.
[5] Pop. Ast., Vol. 32, 1924, p. 547.
[6] Pop. Ast., Vol. 32, 1924, p. 218.
[7] Ap. J., Vol. 38, 1913, p. 407.

Julian Day		799·7		805·7		808·7		808·8		819·7		824·5		824·6	
Phase		1·12		7·05		10·15		0·05		0·80		5·64		5·74	
Wave lengths	Chromospheric height km.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.
Fe. 4191·7	550	+6·8	1	......	...							......		+17·3	2
Zr. 4209·1	550	−1·5	2	+20·1	2					− 7·3	1	+18·4	1	+21·3	2
Sr. 4215 7	6000	+0·2	2	......				− 4·6	1	− 2·5	1	+17·3	2	+21·3	2
Ca. 4226·9	5000	0·0	2	+ 9·4	1							......		+18·0	2
Fe. 4233·8	400	+8·5	1	......				− 4·6	1			......		+25·7	3
Fe. 4260·6	600	+5·2	5	+13·9	3			−11·2	1	− 9·0	2	+14·6	3	+19·8	7
Fe. 4308·0	750	+0·3	2	+ 4·0	2					+ 0·3	2	+25·9	2	+25·8	2
Ti. 4313·0	600	+1·0	1	+ 1·4	3	− 5·8	1	− 8·2	3	− 8·9	1	+22·6	2	+23·6	2
Sc. 4321·0	800	−3·2	6	+ 7·1	5	− 1·2	1	−10·2	2	− 0·9	1	+18·3	3	+20·4	3
Fe. 4325·9	900	......		− 2·1	2					− 2·4	1	+19·2	2	+24·7	2
Y. 4375·1	550	−0·3	3	+10·7	3					− 4·4	3	+27·4	3	+27·3	3
Fe. 4383·7	1600	−2·8	5	+ 8·3	2			− 7·6	2			+22·8	2	......	...
Fe. 4404·9	800	−5·2	3	+12·1	4	+ 3·2	3	......	4	− 2·0	2	+23·5	4	+22·2	4
Fe. 4415·3	500	−1·9	3	+ 9·5	2	+ 3·0	1	− 5·4	2	− 5·8	4	+20·9	3	+23·2	2
Ca. 4455·0	500	−7·6	2	+ 9·1	2			− 9·0	4	......				......	...
? 4464·6	500	+1·7	1	+ 1·3	6	− 0·5	2	− 6·8	3	+ 0·5	2	+14·7	3	+18·3	2
Ti. 4468·7	1500	+2·3	3	+ 8·1	3			+ 7·5	3	+ 1·1	2	+25·2	3	+25·1	4
Ti. 4501·4	1600	−8·6	1	+12·6	1	+ 9·6	4	+ 5·6	2	−17·3	3	+24·0	2	+22·6	2
Fe. 4508·5	600	−1·3	1	+13·7	1	− 6·0	2	− 1·0	4	− 1·0	1	+17·6	4	+21·3	2
Ti. 4549·7	1300	+3·9	4	+ 6·2	5	+ 3·2	4	− 9·8	3	+ 6·0	5	+26·2	4	+19·5	4
Ti. 4563·9	1200	−6·5	1	+ 9·1	2	+ 0·7	1	− 0·7	3	− 0·6	1	+20·0	2	+13·3	3
Ti. 4572·2	1200	−1·2	3	+ 9·2	2	− 7·2	2	− 8·6	3	− 8·4	2	+ 9·6	4	+25·5	2

Julian Day		828·7		831·6		831·7		834·6		834·7		838·6	
Phase		9·75		2·53		2·59		5·50		5·60		9·52	
Wave lengths	Chromospheric height km.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.
Fe. 4191·7	550	....		....				....				− 0·5	1
Zr. 4209·1	550	....		− 6·0	2	+ 4·0	3	+21·4	3	+20·4	2	+ 5·7	1
Sr. 4215·7	6000	....		....		+ 9·0	1	+20·4	1	+25·6	1	+ 2·4	2
Ca. 4226·9	5000	....		....				....		+22·2	1	− 5·0	3
Fe. 4233·8	400	....		....		− 0·2	1	....				+ 4·7	1
Fe. 4260·6	600	....		+10·6	2	+ 5·1	1	+29·3	3	+20·1	3	+ 5·9	4
Fe. 4308·0	750	....		+13·0	1	+ 7·4	3	+17·5	3	+23·0	1	− 0·5	5
Ti. 4313·0	600	....		+15·1	1	+ 8·3	4	+24·0	3	+25·2	1	− 0·8	4
Sc 4321·0	800	+ 9·6	3	+12·0	2	+ 8·5	4	+18·7	2	+18·7	2	+ 3·9	6
Fe. 4325·9	900	+ 0·5	1	+ 5·0	1	+ 3·8	2	+19·6	4	+21·9	4	+ 4·7	3
Y. 4375·1	550	....		+12·8	1	−11·5	3	+21·0	2	+25·7	2	+ 5·6	6
Fe. 4383·7	1600	....		....		− 0·2	4	+26·9	1	+25·8	3	+ 6·7	1
Fe. 4404·9	800	− 8·2	4	+ 9·8	2	+ 1·3	5	+25·4	4	+19·4	4	+ 0·1	6
Fe. 4415·3	500	− 4·9	1	+ 7·2	1	− 0·1	5	+13·2	2	+20·4	3	+ 2·2	2
Ca. 4455·0	500	− 7·8	1	+ 1·0	4	+ 8·2	3	+ 8·5	2	+20·0	4	− 0·4	2
? 4464·6	500	− 0·9	2	....		+ 2·9	4	+13·1	1	+11·9	2	+ 3·0	1
Ti. 4468 7	1500	....		+12·3	4	+ 9·7	4	+29·8	4	+20·0	4	+ 3·6	2
Ti. 4501·4	1600	− 3·3	...	....		− 3·2	2	+22·5	1	....		+ 2·0	4
Fe. 4508·5	600	+ 5·4	3	+14·6	1	+ 5·5	4	....		+16·5	1	+ 3·1	4
Ti. 4549·7	1300	− 7·2	2	+ 3·6	4	− 0·5	4	+24·7	2	+24·8	6	+ 3·6	3
Ti. 4563·9	1200	....		+ 6·4	1	+ 2·4	3	+22·5	1	+16·0	1	+10·5	2
Ti. 4572·2	1200	+ 1·0	1	....		+ 1·2	4	+22·8	2	+20·2	2	+ 6·8	1

Julian Day		840·5		840·6		842·6		845·6		845·7		857·6	
Phase		1·32		1·37		3·35		6·34		6·45		8·19	
Wave lengths	Chromospheric height km.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.
Fe. 4191·7	550	....	....	....	....	....	....	+ 3·3	1	....	....	....	....
Zr. 4209·1	550	....	....	....	....	....	....	+ 7·6	3	+ 8·4	2	....	....
Sr. 4215·7	6000	− 2·2	1	+ 3·5	1	....	....	+16·7	2	+12·4	3	....	....
Ca. 4226·9	5000	....	....	....	....	....	....	+12·4	1	+14·2	1	....	....
Fe. 4233·8	400	....	....	....	....	....	....	+16·0	3	+12·7	3	....	....
Fe. 4260·6	600	− 4·2	3	+ 0·9	2	....	....	+14·3	3	+14·0	4	....	....
Fe. 4308·0	750	− 2·2	3	− 8·9	3	....	....	+ 9·4	3	+10·3	2	+ 8·6	1
Ti. 4313·0	600	+ 4·2	6	+ 5·2	2	....	....	+ 8·2	3	+ 3·5	5	+ 0·8	2
Sc. 4321·0	800	0·0	3	− 0·1	1	+14·8	1	+11·7	3	+ 9·2	4	− 2·4	3
Fe. 4325·9	900	....	....	− 7·2	2	....	....	+12·6	3	+10·1	4	....	....
Y. 4375·1	550	− 0·8	2	− 2·0	1	....	....	+ 9·2	3	+12·5	3	+ 1·6	1
Fe. 4383·7	1600	....	....	....	....	....	....	+15·1	4	+10·2	1	....	....
Fe. 4404·9	800	+ 9·0	4	− 5·4	6	+22·6	2	+ 9·8	2	+10·8	2	+ 2·1	1
Fe. 4415·3	500	+ 5·5	3	− 2·0	3	+35·5	2	+10·8	2	+12·9	4	− 4·1	2
Fe. 4455·0	500	+ 4·0	2	+ 0·2	2	+ 9·4	2	− 0·1	1	+ 0·9	2	− 9·2	1
? 4464·6	500	+18·5	4	+14·7	2	+17·8	2	....	....	+11·7	2	− 1·0	1
Ti. 4468·7	1500	− 4·4	3	+ 5·4	3	....	....	+ 7·6	4	+ 3·6	4	−17·7	1
Ti. 4501·4	1600	+ 6·5	1	− 7·6	1	+15·9	1	+15·0	1	+ 7·2	4	−13·6	1
Fe. 4508·5	600	+ 6·4	1	+ 2·4	1	....	....	+ 7·3	2	+ 9·6	2	....	....
Ti. 4549·7	1300	+ 8·3	4	− 2·3	4	+18·1	1	+10·6	2	− 0·1	5	− 2·8	1
Ti. 4563·9	1200	− 4·6	2	− 2·1	1	....	....	....	....	+16·0	1	+ 1·4	1
Ti. 4572·2	1200	− 5·8	1	+ 2·1	3	+ 1·5	2	....	....	− 3·6	2	− 7·7	1

| Julian Day		857·6		858·6		868·6		869·5		876·6		878·6	
Phase		8·26		9·17		9·10		10·02		6·88		8·88	
Wave lengths	Chromospheric height km.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.	Vel. km.	Wt.
Fe. 4191·7	550	....	....	....	....	....	....	− 5·4	3	+ 9·2	1	− 0·9	2
Zr. 4209·1	550	....	....	+ 1·4	3	....	....	− 2·2	2	+14·7	4	+13·6	1
Sr. 4215·7	6000	....	....	+ 9·5	1	....	....	− 4·4	3	+ 7·5	2	+ 2·3	4
Ca. 4226·9	5000	....	....	....	....	....	....	− 8·8	2	+10·3	1	+ 5·1	3
Fe. 4233·8	400	....	....	....	....	....	....	− 6·3	2	+ 7·8	3	+ 0·4	3
Fe. 4260·6	600	....	....	+ 0·6	1	− 9·5	3	− 6·3	3	+14·4	2	+ 3·8	3
Fe. 4308·0	750	....	....	+ 8·5	1	− 3·0	1	− 4·1	4	+12·0	3	+ 7·5	2
Ti. 4313·0	600	....	....	− 2·7	1	+ 4·6	1	− 5·4	6	+ 6·3	4	+10·8	2
Sc. 4321·0	800	+ 1·9	2	− 3·6	1	+ 1·4	1	− 5·2	7	+ 5·5	6	+ 2·1	6
Fe. 4325·9	900	....	....	− 1·7	2	− 3·3	2	− 5·5	4	+10·8	4	+ 1·2	4
Y. 4375·1	550	....	....	+ 6·1	2	+11·5	1	− 2·4	4	....	....	+ 7·5	6
Fe. 4383·7	1600	....	....	....	....	....	....	−11·8	2	+14·6	2	+ 4·0	6
Fe. 4404·9	800	+ 7·9	2	+ 1·9	3	+ 3·9	2	− 7·9	4	+13·0	3	+ 3·2	3
Fe. 4415·3	500	+ 7·6	1	+ 4·1	1	....	....	− 7·0	4	+ 6·8	3	+15·1	2
Ca. 4455·0	500	+12·8	1	+ 0·5	2	....	....	− 9·7	1	+10·7	4	+ 0·9	1
? 4464·6	500	− 3·6	1	+ 2·6	4	− 5·2	2	−16·3	2	+ 0·6	6	+13·0	1
Ti. 4468·7	1500	....	....	....	....	....	....	−10·7	3	+ 7·4	3	− 5·0	2
Ti. 4501·4	1600	+ 5·6	1	− 3·4	2	+ 1·4	1	− 4·9	3	+12·4	1	+ 7·3	1
Fe. 4508·5	600	+ 2·9	1	+ 0·3	2	− 8·9	2	− 8·9	2	....	....	− 1·7	1
Ti. 4549·7	1300	− 4·3	2	+ 4·8	4	− 7·2	3	− 7·2	3	+ 5·4	3	+ 5·4	5
Ti. 4563·9	1200	....	....	+ 1·2	3	− 3·0	3	− 3·0	3	+ 8·4	1	+ 8·4	2
Ti. 4572·2	1200	− 1·2	3	− 6·6	1	− 5·5	3	− 5·5	3	+12·7	5	+12·7	2
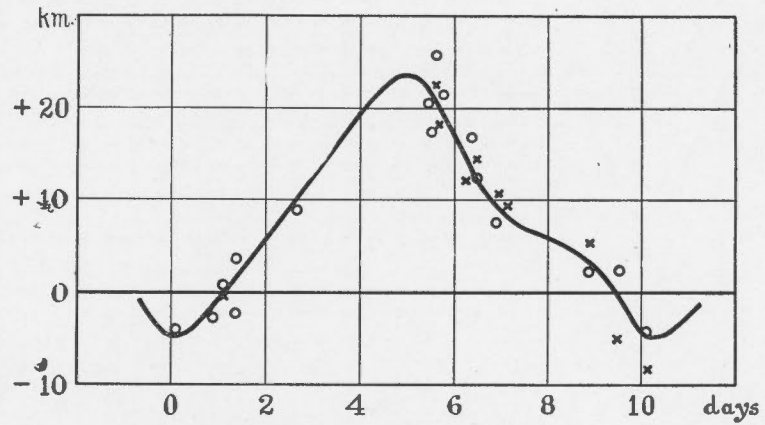
Fig. 3. Zeta Geminorum.   Direct measures of two lines
O = 4215.73   6000 km. level in Sun's chromosphere.
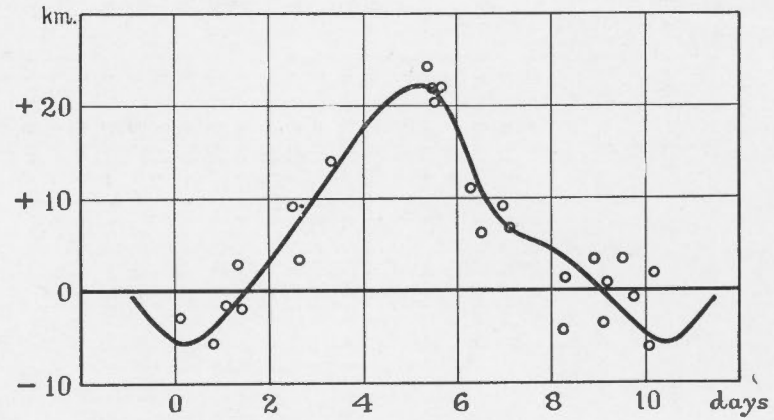X = 4226.90   5000 km. level in Sun's chromosphere.

Fig. 4. Zeta Geminorum.   Direct measures of ten lines
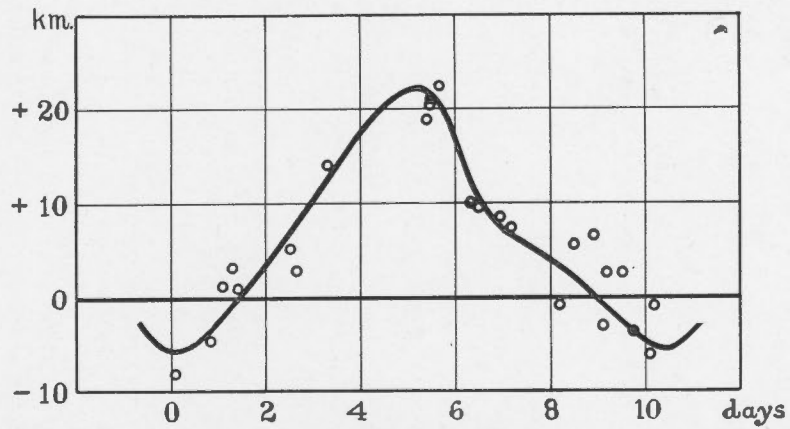800—1600 km. level in Sun's chromosphere.

Fig. 5. Zeta Geminorum.   Direct measures of ten lines all below 800 km. level in
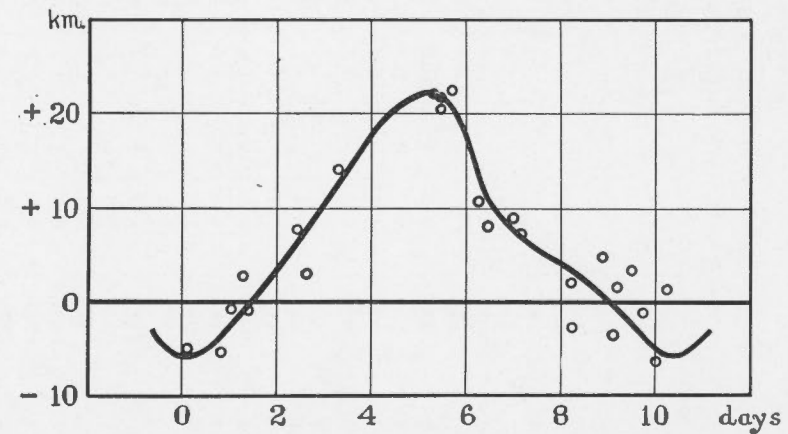Sun's chromosphere.

Fig. 6. Zeta Geminorum.   Direct measures of twenty lines up to 1600 km. level in
Sun's chromosphere.

VARIATION OF SPECTRAL CLASS

The intensity of the titanium enhanced line 4534·139 relative to that of the titanium arc line 4534·953 has been estimated in the spectrograms of ζ Geminorum. These two lines, from the fact that they are so close together and so sensitive to variation of spectral class, afford perhaps the best criterion for the determination of this variation. They have already given satisfactory results in the cases of δ Cephei and α Ursae Minoris.[1] While both lines actually vary, the intensity of the arc line was throughout assumed as 10, and that of the enhanced line estimated in terms of it. The intensities given in the following table are therefore essentially ratios.

Date, 1924	Phase	Enhanced Ti 4534·139 Intensity	Date, 1924	Phase	Enhanced Ti 4534·139 Intensity
Jan. 14.............	1·12	12	Feb. 22.............	9·48	11
14.............	1·13	7	22.............	9·52	11
15.............	2·00	7	22.............	9·60	11
20.............	7·05	14	24.............	1·32	9
23.............	10·15	11	24.............	1·37	7
23.............	0·05	10	26.............	3·35	6
31.............	7·83	10	26.............	3·42	6
31.............	7·91	16	28.............	5·39	6
Feb. 3.............	0·64	9	29.............	6·34	14
3.............	0·80	9	29.............	6·45	12
8.............	5·64	10	Mar. 12.............	8·19	14
8.............	5·74	9	12.............	8·26	16
11.............	8·65	11	13.............	9·17	11
12.............	9·75	8	13.............	9·23	9
13.............	0·50	9	17.............	3·03	6
13.............	0·60	9	19.............	5·12	8
15.............	2·51	6	20.............	6·07	10
15.............	2·53	6	23.............	9·10	15
15.............	2·59	7	24.............	10·02	12
18.............	5·50	12	24.............	10·08	11
18.............	5·60	12	31.............	6·88	12
21.............	8·62	10			

All these estimates were made at one sitting, and with no knowledge of the phases corresponding to individual spectrograms. The results are plotted on the accompanying curve (Fig. 2). This curve shows that for ζ Geminorum the maximum and minimum of the enhanced lines occur, respectively, about one-quarter of the period before and after maximum light. This result is in agreement with that found by the writer for α Ursae Minoris, which is also a Geminid, but in direct contrast with what was obtained for δ Cephei, and for many other variables of the δ Cephei type.[2]

[1] Pub. Dom. Obs., Vol. IX, No. 1, pp.. 52 and 61.
[2] Mt. Wilson Contrib., Vol. 7, p. 1.

CONCLUSION

To sum up it may be said that in ζ Geminorum all lines apparently give the same radial velocities, while for η Aquilae, which was also investigated by us as well as by Professor Rufus, this is not the case; that in ζ Geminorum and α Ursae Minoris, which are the two brightest Geminids so far investigated, the maximum ionization seems to occur about one-quarter of the period before maximum light and the minimum ionization one-quarter of the period after. Although both the Geminids and stars strictly of the δ Cephei type are giant stars, or stars of very low density, the phenomena occurring in the first are somewhat different from those in the second.

The co-operation of Messrs. J. F. Frédette and R. Callander in the work of observing and measuring is here acknowledged.

DOMINION OBSERVATORY, OTTAWA,
            March, 1925.

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The star ν Eridani ($\alpha = 4^h\ 31^m \cdot 3$, $\delta = -3°\ 33'$) was discovered by the writer to be of the β Canis Majoris type.[1] As early as 1920 the short-period oscillation of radial velocity was recognized, as well as the large variation of amplitude of the oscillation, this being from 26 to 70 kilometres. The necessity of studying this star more thoroughly was recognized, and when the study of δ Ceti was completed as far as possible, ν Eridani was put on the observing list, and spectrograms were made every clear night between October 15, 1923, and January 23, 1924, as well as between October 1, 1924, and January 14, 1925.

The radial velocity determinations as obtained between these dates are given in Table I. The spectrograms were measured on the spectro-comparator, using the same standard spectrogram of β Canis Majóris as was used previously for other early class B stars.

It should be noted that the dates throughout have been entered on the basis of the day beginning at noon; this is to conform with the resolution adopted at the Cambridge meeting of the International Astronomical Union (1925) that the Julian day would still be counted as formerly.

The velocity curves plotted from these measures show not only a considerable variation in amplitude, but also marked differences in the inclinations of their different branches. Sometimes a very rapid rise is found, at other times a more gradual one.

If any variation of centre-of-mass velocity from curve to curve is present it is small and difficult to detect, being masked by the great variation in amplitude.

In spite of a long and systematic search, no period could be found to connect the different maxima or minima of the individual velocity curves.

Figures 1 and 2 exhibit graphically the distribution of radial velocities with respect to time on twelve different dates. The successive points have been connected by straight lines.

For the curves which are well determined amplitudes have been estimated, and a period of $7^d \cdot 9444$ was found which may possibly be that of variation of amplitude. The data used for this purpose are exhibited in Table II, where the first column gives the approximate Julian date of the observed amplitude, the second column its phase computed from the formula, Maximum = J.D. $2424066 \cdot 7 + 7 \cdot 9444E$, and the third the estimated amplitude. These have been plotted according to phase in figure 3, the curve having been drawn more or less arbitrarily to satisfy the observations.

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[1] Pub. Dom. Obs., Vol. V, p. 56.

TABLE I—RADIAL VELOCITIES OF ν ERIDANI

Date	Julian Day	Velocity km.	Date	Julian Day	Velocity km.
1923 Oct. 15....	2423708·774	− 6·6	1923 Nov. 1....	2423725·824	+ 3·8
	·807	+12·5	Nov. 2....	2423726·642	− 2·1
	·837	+26·2		·676	− 3·3
	·871	+27·8		·712	+ 6·4
	·903	+23·2		·735	+26·0
Oct. 16....	2423709·732	+46·5		·758	+33·3
	·762	+27·2		·783	+40·1
	·791	+ 3·3		·803	+ 8·6
	·820	−12·1	Nov. 8....	2423732·649	+59·7
	·849	+ 6·4		·669	+44·9
Oct. 17....	2423710·712	− 5·3		·701	−14·0
	·746	+26·0		·718	− 9·9
	·776	+32·3		·740	−14·4
	·803	+37·4		·769	+18·8
	·831	+21·5		·797	+39·6
	·858	− 1·4		·822	+58·0
	·890	+ 6·9	Nov. 13....	2423737·642	+31·2
	·924	+17·7		·664	+25·6
Oct. 21....	2423714·785	+23·5		·717	+ 3·8
	·837	− 7·2		·746	− 1·8
	·860	− 1·4		·799	+22·1
	·881	+ 5·4	Nov. 18....	2423742·659	+10·4
	·904	+19·4		·714	+40·4
Oct. 22....	2423715·697	−11·9		·740	+26·6
	·727	−10·1	Nov. 22....	2423746·604	+ 7·1
	·752	+10·4		·633	+14·3
	·774	+29·9		·662	+26·3
	·794	+34·2		·712	+19·4
	·812	+48·0		·741	+10·8
	·831	+38·1		·769	+10·4
	·851	+ 5·4		·818	+ 9·9
	·877	−12·3	Nov. 25....	2423749·590	+22·0
Oct. 26....	2423719·671	+25·2		·624	+50·2
	·697	+16·3		·654	+40·7
	·723	+ 0·9		·712	−11·6
	·746	+ 4·1		·740	+12·3
	·767	+19·6	Dec. 3....	2423757·671	−12·6
	·790	+19·8	Dec. 9....	2423763·557	+24·9
	·815	+20·2		·581	+ 0·2
	·841	+27·1		·610	+ 6·1
	·865	+20·8		·636	+13·0
	·892	+ 8·6		·664	+21·9
Oct. 28....	2423721·703	+30·2		·736	+ 8·5
	·724	+12·4		·767	+ 0·4
Oct. 29....	2423722·772	+14·4		·799	+ 4·2
	·797	− 8·6	Dec. 11....	2423765·543	+ 1·7
	·820	− 3·0		·569	+22·6
	·847	+16·2		·599	+40·8
	·876	+21·8		·661	−12·9
Nov. 1....	2423725·648	− 0·4		·715	− 5·2
	·667	+17·2	Dec. 14....	2423768·548	+16·4
	·716	+55·6		·585	+17·8
	·739	+51·0		·612	+13·1
	·760	+13·1		·638	− 2·2
	·783	− 9·2		·664	+ 0·7
	·803	− 8·6		·705	+24·7

TABLE I—RADIAL VELOCITIES OF ν ERIDANI—*Continued*

Date	Julian Day	Velocity km.	Date	Julian Day	Velocity km.
1923 Dec. 14....	2423768·734	+25·6	1924 Jan. 21....	2423806·571	+49·6
	·763	+24·5	Jan. 23....	2423808·450	+27·1
	·791	+ 7·7		·472	+33·1
Dec. 17....	2423771·541	+10·3		·497	+32·2
	·567	− 5·0		·528	+14·8
	·596	−10·1		·556	− 0·1
	·623	+ 5·5		·585	+ 7·8
	·649	+33·8		·614	+15·1
	·678	+42·1		·643	+30·0
	·707	+29·8		·675	+37·4
	·733	− 9·2		·703	+10·2
	·761	−13·0	Oct. 1....	2424060·752	+37·9
	·790	+ 0·8		·776	+22·9
Dec. 18....	2423772·632	−12·7		·797	+14·2
Dec. 19....	2423773·548	+ 7·1		·817	− 3·6
	·578	+19·6		·835	−15·3
	·609	+28·4		·855	− 4·4
	·640	+ 2·5		·876	+ 8·2
	·672	− 4·7		·896	+24·6
	·705	− 4·3		·914	+27·2
Dec. 23....	2423777·594	+15·1	Oct. 2....	2424061·760	+ 9·4
	·654	− 7·2		·807	+29·1
	·704	+22·6		·831	+25·6
	·731	+41·2		·858	+ 6·7
	·754	+38·8		·888	−10·4
	·778	+ 0·9	Oct. 3....	2424062·728	+15·5
Dec. 29....	2423783·616	+18·2		·753	+ 8·2
	·641	+19·4		·776	+15·3
	·668	+ 5·2		·799	+15·8
	·725	+ 1·1		·822	+21·4
1924 Jan. 1....	2423786·497	−21·0		·848	+26·0
	·524	−12·2		·876	+14·9
	·552	+22·0		·903	+ 4·6
	·581	+41·9	Oct. 5....	2424064·767	+11·5
	·610	+45·7		·792	− 9·5
Jan. 3....	2423788·519	+44·8		·820	−14·9
	·549	+33·7		·920	+29·9
	·597	−18·7	Oct. 7....	2424066·756	− 1·2
	·618	− 3·6		·797	+23·5
	·639	+ 4·2		·819	+48·8
	·697	+49·7		·842	+33·0
	·751	− 1·4		·865	+ 8·6
Jan. 7....	2423792·475	+36·8		·888	− 9·1
	·500	+23·2	Oct. 8....	2424067·724	+18·0
	·528	+ 7·9		·746	−10·7
	·548	+ 5·0		·767	−11·2
	·625	+16·5		·790	− 7·9
	·651	+25·5		·813	+11·2*
Jan. 14....	2423799·491	+16·9		·839	+29·8
	·518	+ 8·6		·887	+37·0
	·544	+ 4·0		·907	+12·8
Jan. 21....	2423806·476	−10·9	Oct. 9....	2424068·717	+31·1
	·501	+ 9·2		·743	+32·7
	·526	+26·9		·765	+37·1
	·547	+41·5		·785	+ 6·2

*Remeasured +16·5.

TABLE I—RADIAL VELOCITIES OF ν ERIDANI—*Continued*

Date	Julian Day	Velocity km.	Date	Julian Day	Velocity km.
1924 Oct. 9....	2424068·810	− 1·0	Oct. 23....	2424082·719	+ 3·7
	·830	+ 4·1		·744	+11·1
	·854	+ 7·1		·766	+16·1
	·879	+17·2		·816	+35·0
Oct. 10....	2424069·710	+ 8·6		·847	+ 8·8
	·736	+28·9	Oct. 24....	2424083·703	+18·0
	·760	+27·9		·731	+10·3
	·782	+15·8		·760	− 3·2
	·801	+10·7		·792	+22·4
	·826	+ 7·0		·824	+11·9
	·851	− 2·4		·880	+11·3
	·876	+ 5·3	Oct. 26....	2424085·699	+16·0
	·903	+17·4		·726	+20·6
Oct. 12....	2424071·712	+10·0		·751	+ 1·4
	·733	− 7·7		·773	− 8·4
	·753	−16·4		·794	+ 0·1
	·772	− 1·7		·815	− 4·3
	·793	+17·2		·835	+12·6
	·814	+28·0		·857	+29·2
	·835	+22·4		·906	+18·3
	·856	+34·3	Oct. 27....	2424086·710	+10·9
	·876	+15·1		·734	+33·1
	·897	− 6·5		·756	+42·1
	·919	− 8·4		·779	+40·9
Oct. 13....	2424072·715	+33·6		·806	+ 5·8*
	·741	+34·7		·834	− 8·4
	·760	+ 9·1		·862	−11·0
	·780	−11·5	Oct. 28....	2424087·769	+22·0
	·800	−10·0		·796	+48·5
	·822	+ 1·9		·821	+52·2
	·843	+17·9		·844	+14·6
	·868	+29·0		·867	−11·1
Oct. 16....	2424075·787	+ 5·1		·890	−18·8
	·808	+23·2		·915	− 4·6
	·827	+29·1	Oct. 29....	2424088·721	+ 0·2
	·846	+23·5		·738	−16·1
	·863	+24·9		·759	− 8·7
	·881	+14·4		·776	−14·9
	·900	+11·4		·793	− 2·1
	·919	+ 5·1		·810	+17·7
	·937	+ 0·7		·829	+34·4
Oct. 17....	2424076·715	+16·5		·849	+47·4
	·740	+11·9		·869	+44·9
	·762	+12·9		·890	+13·0
	·781	+ 9·0		·915	− 9·5
	·798	+ 5·2	Nov. 7....	2424097·697	+48·2
	·817	+17·3	Nov. 9....	2424099·703	−20·0
Oct. 19....	2424078·749	+27·4		·726	− 2·4
	·771	+42·7		·748	+20·0
	·792	+43·3		·770	+29·4
	·810	+25·2		·792	+37·7
	·849	− 6·0		·815	+28·6
	·872	− 6·9		·837	+ 4·8
	·894	+ 9·9		·859	−11·3
	·917	+31·4		·884	− 4·2

*Remeasured +1·9.

TABLE I—RADIAL VELOCITIES OF ν ERIDANI—*Continued*

Date	Julian Day	Velocity km.	Date	Julian Day	Velocity km.
1924 Nov. 10....	2424100·739	− 8·1	1924 Nov. 30....	2424120·641	− 3·7
	·763	+ 1·8		·659	+ 2·3
	·787	+10·6		·705	+ 0·6
	·810	+23·2		·725	+14·6
	·835	+35·6		·803	+ 8·8
	·862	+33·3	Dec. 4....	2424124·581	+18·8
Nov. 12....	2424102·597	+15·1		·601	+34·6
	·618	+18·0		·620	+29·6
	·637	+ 2·2		·657	+ 5·1
	·655	+ 9·9		·709	+ 6·7
	·697	+12·7		·729	+ 5·0
	·714	+26·1		·752	+22·9
	·733	+15·6		·772	+22·9
	·752	+19·6		·794	+26·0
	·772	+21·0	Dec. 11....	2424131·555	+16·7
	·791	+12·3		·574	+20·9
	·811	+ 0·4		·593	+10·5
	·833	+ 0·4		·612	+ 1·9
	·855	+ 9·4		·630	− 2·5
	·878	+23·4		·645	+ 5·3
Nov. 16....	2424106·660	+ 5·4		·660	+ 9·3
	·700	+37·2	Dec. 14....	2424134·622	+23·6
	·715	+44·0		·644	+47·6
	·731	+56·4		·676	+29·5
	·752	+30·4		·709	− 4·2
	·776	−15·7		·748	− 1·4
	·803	−19·5		·770	+ 3·7
	·828	+ 2·1	Dec. 15....	2424135·713	+47·7
	·851	+15·1	Dec. 19....	2424139·569	+ 6·1
	·874	+39·1		·591	+11·2
Nov. 17....	2424107·597	+55·6		·614	+ 2·1
	·615	+46·2		·635	+ 7·1
	·632	+24·3		·658	+14·1
	·645	−13·2	Dec. 21....	2424141·533	+16·4
	·660	−21·3	Dec. 28....	2424148·612	+ 1·1
	·699	−14·2		·633	+ 6·6
	·714	+ 1·5		·653	+15·0
	·731	+16·4		·701	+29·5
	·748	+29·1		·721	+22·8
	·767	+48·6		·745	+ 1·6
	·791	+40·5	Dec. 31....	2424151·524	+17·7
	·812	− 7·9		·549	−12·0
	·831	+ 0·9		·567	− 5·4
	·849	−17·4		·585	− 8·8
Nov. 18....	2424108·629	+29·5		·603	+ 9·1
Nov. 27....	2424117·699	+36·8		·619	+29·7
	·715	+19·4		·635	+34·2
	·731	− 4·2		·649	+39·2
	·748	− 4·8		·703	+ 6·7
	·766	− 0·1		·731	−16·5
	·784	+ 9·5	1925 Jan. 6....	2424157·509	+ 0·5
	·805	+17·8	Jan. 11....	2424162·501	−17·7
Nov. 28....	2424118·705	+15·2		·520	− 9·7
	·720	+13·0		·540	− 0·6
Nov. 30....	2424120·601	+30·8		·559	+28·3
	·622	+13·2		·578	+44·6

TABLE I—RADIAL VELOCITIES OF ν ERIDANI—*Concluded*

Date	Julian Day	Velocity km.	Date	Julian Day	Velocity km.
1925 Jan. 11....	2424162·598	+50·6	Jan. 14....	2424165·520	+ 8·8
	·619	+40·0		·541	+17·9
	·639	−12·1		·562	+18·2
	·658	−13·7		·582	+28·6
	·707	− 4·7		·624	+20·4
				·646	− 5·7

TABLE II—AMPLITUDES OF INDIVIDUAL CURVES

Julian Day	Phase	Observed Amplitude	Julian Day	Phase	Observed Amplitude
	d.	km.		d.	km.
2423709·7................	0·5	55	2424067·9................	1·2	56
710·7................	1·5	45	068·7................	2·0	40
714·7................	5·5	43	069·8................	3·1	28
715·8................	6·6	60	071·9................	5·2	54
719·8................	2·7	30	072·7................	6·0	50
725·7................	0·6	63	078·8................	4·2	58
726·7................	1·6	43	087·9................	5·3	72
732·8................	7·7	78	088·9................	6·3	68
768·8................	4·0	30	106·8................	0·4	77
771·1................	6·9	54	107·8................	1·4	74
785·6................	4·9	66	124·6................	2·3	34
788·7................	0·1	70	151·7................	5·6	54
808·5................	4·0	40	162·7................	0·7	56

Of all stars of the β Canis Majoris type studied up to the present, ν Eridani is the one that shows the greatest amplitude variation of the radial velocity curve. It contrasts sharply with σ Scorpii, where there is apparently no amplitude variation, but a considerable variation of centre-of-mass velocity.

If we assume the short-period effect to be a physical one, and that in addition there is a secondary revolving about the primary in a period of $7^{d}\cdot9444$, it follows that what has been called above the variation in centre-of-mass velocity represents simply the velocity of the primary in its orbit about the centre of mass of the system. Since this variation is too small to be easily detected, it is evident (on the assumption that the short-period effect is physical) that if a secondary exists it must be either very small or, more probably, very close to the primary; on the latter supposition it would cause strong tidal action on the primary, which might go far towards explaining the irregularities both in amplitude and in the shape of the curve.

The tentative explanation along these lines, which may be taken for what it is worth, would then be that the primary has an unsymmetrical figure of equilibrium, which may perhaps be most easily visualized as a Jacobian ellipsoid. The rotation of this unsymmetrical figure would give a short-period radial velocity curve, which would be
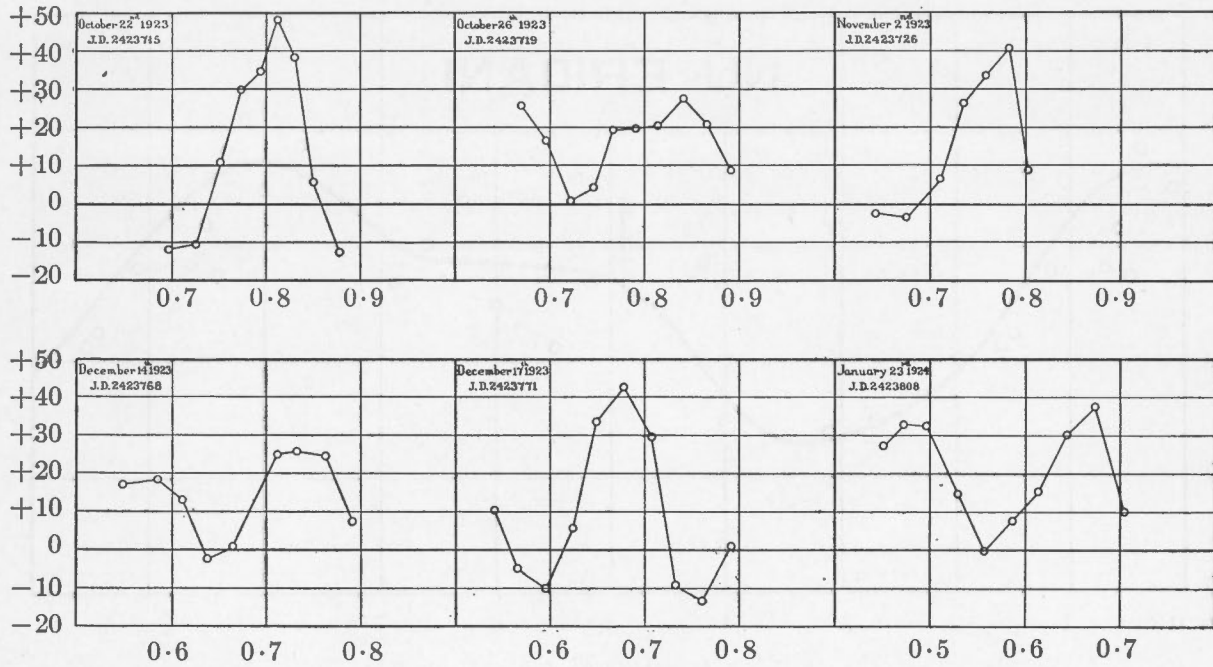
FIG. 1   Radial Velocities of Nu Eridani, 1923
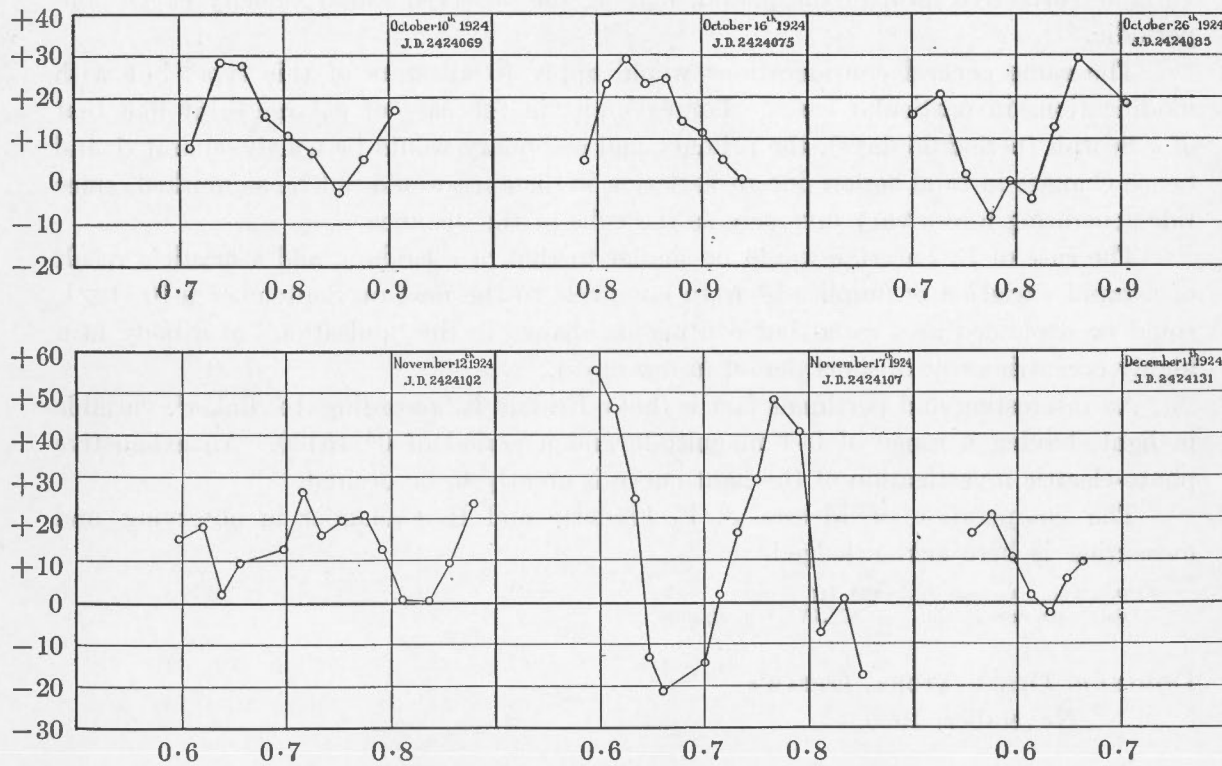
FIG. 2   Radial Velocities of Nu Eridani, 1924
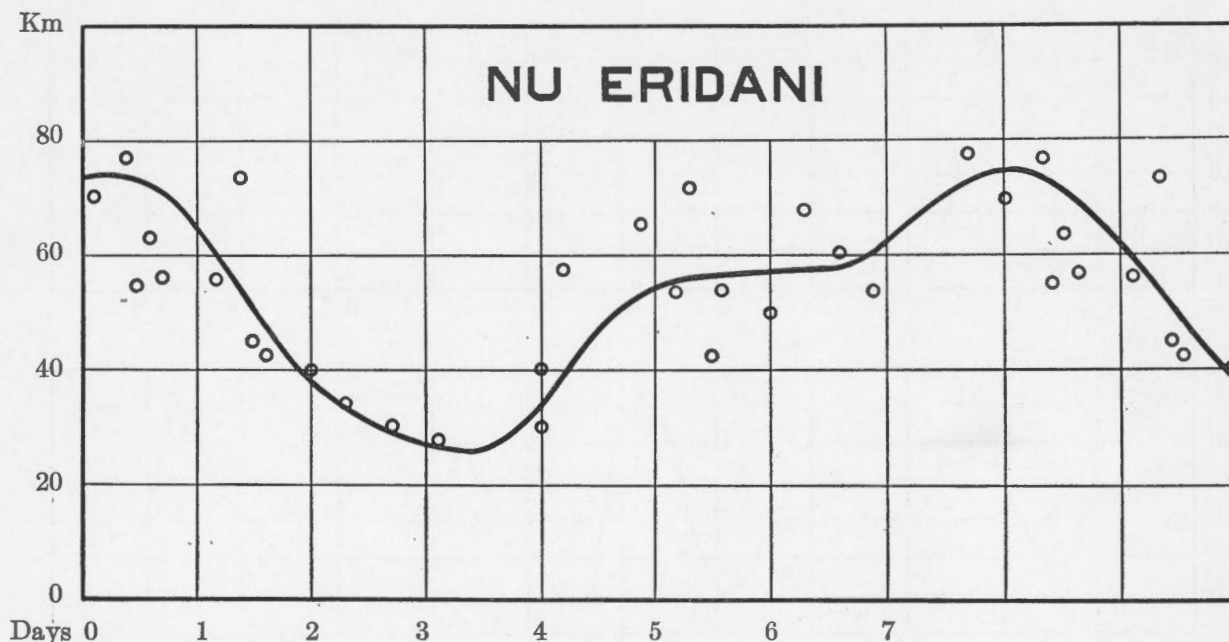
Fig. 3 Amplitudes of Nu Eridani

regular if uncomplicated by tidal action; the effect of the latter, however, caused by a near companion in a highly elliptical orbit, would give rise to a variable amplitude, whose maximum would correspond to periastron; the same causes would give rise to a variable corrective term in the period, making the observed radial velocity curve non-periodic.

The same general considerations would apply to all stars of this type, but with modifications in particular cases. For example, in the case of a large orbit like that of σ Scorpii (period 33 days), the primary and secondary would be widely separated, and hence changes in tidal action due to variation in distance would not be so marked, since tide-generating forces vary inversely as the cube of the distance.

The case of 12 Lacertae would be similar to that of ν Eridani, and Christie's result of a rapid variation of amplitude[2] from one cycle to the next on September 9-10, 1924, could be explained as a rapid but continuous change in the "pulsation" of a body in a highly eccentric orbit of short period (a few days).

An interesting and pertinent fact is that ν Eridani is, according to Baker[3], variable in light, having a range of 0·1 magnitude and a period of $0^d \cdot 15430$. An exhaustive photo-electric investigation of the light curve is greatly to be desired.

The co-operation of Messrs. J. F. Frédette and R. Callander in observing and measuring is here acknowledged.

[2] Pub. Dom. Astroph. Obs., Vol. III, p. 223.
[3] Pub. Astr. Soc. Pacific, Vol. XXXVIII, p. 93, 1926.

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FORMER RESEARCH ON THE LIGHT-CURVE

In 1874 Pigott[1] found a light variation of this well-known Cepheid; it is mentioned, however, that Justus Byrgius[2] had noticed some changes in brightness in 1612.

The light-curve has been studied by many observers for more than a century, and a complete bibliography of the observations may be found in "Geschichte und Literatur des Lichtwechsels" by Müller and Hartwig[3]. Since the publication of this valuable compendium many important studies have been made. The following are given particular mention as representative and expert studies;—

(1) Grouiller[4] in his "Monographie de l'étoile variable η Aigle" arrives at the important conclusion that the variation of η Aquilæ cannot be regarded as perfectly regular. Fluctuations occur in the successive maxima and minima of brightness and there is also a slight oscillation of their epochs on each side of a mean value. It would, however, be very difficult to study these variations by means of visual observations.

(2) Wylie[5] in 1920 determined a light-curve with a photo-electric photometer, introducing a new era in the investigation of Cepheids; the accuracy of the curve obtained will make it possible after further investigation to find out definitely whether there are alterations in the shape of the curve and if necessary the nature of these alterations. Wylie's light-curve, a marvel of modern scientific method, may be found on page 225 of the Astrophysical Journal mentioned. The "secondary maximum" is confirmed as a pronounced halt in the decrease of light, from phase $1^d.8$ to $2^d.4$. Another fluctuation is indicated with maximum about phase $4^d.3$. Comparison with previous observations suggests a shortening of the interval from minimum to maximum since 1900 and a gradually increasing period.

(3) Nÿland[6] from a large number of observations does not think that the aspect of the light-curve has changed appreciably in an interval of twenty years.

(4) Very different are the results of Hopmann[7], who from his colorimetric observations considers the secondary hump to be of variable amplitude. He estimates that this amplitude has approximately an 11-year period, being a maximum in the years 1900, 1910, 1920 and a minimum in 1896, 1908, 1918, 1928.

(5) The conclusions of Friedrich Becker[8], based on a large number of observations by Plassmann, confirm Hopmann's deductions. Seven mean curves were determined, each from fourteen normal places and for different epochs as follows:—

[1] Phil. Trans. Vol. 75, 1785, p. 129.
[2] Berl. Jahrbuch 1817, p. 117.
[3] Vol. 2, p. 229.
[4] B.A., 2e série Tome 1, p. 331.
[5] Ap. J, Vol. 56, p. 217.
[6] Recherches de l'observatoire d'Utrecht Vol. 8 lère partie, p. 40.
[7] A.N., Vol 222, p. 1.
[8] A.N., Vol. 225, p. 1. *See also* Die Himmelswelt, Vol. 35, 1925, p. 160.

Curve	Year	Number of observations	Probable error of one observation
			m
I..	1895–1899	196	±0·085
II...	1899–1903	194	0·094
III.....•...	1903–1907	200	0·084
IV...	1908–1912	233	0·092
V..	1913–1917	237	0·092
VI...	1918–1921	250	0·109
VII..	1921–1924	224	0·094

The fourteen normal places for each curve are as follows (phase and magnitude). The results are exhibited graphically in fig. 1[1].

I		II		III		IV		V		VI		VII	
d	m	d	m	d	m	d	m	d	m	d	m	d	m
0·318	3·68	0·164	3·65	0·314	3·74	0·305	3·78	0·303	3·80	0·263	3·78	0·317	3·71
0·734	3·73	0·663	3·81	0·792	3·83	0·876	3·93	0·707	3·84	0·722	3·84	0·751	3·91
1·223	3·85	1·311	3·87	1·444	3·90	1·296	4·05	1·362	4·01	1·244	3·94	1·160	3·99
1·747	3·97	2·007	3·97	1·841	4·03	1·839	4·05	1·829	4·08	1·771	4·01	1·727	4·02
2·250	3·95	2·486	3·95	2·325	4·03	2·329	4·06	2·347	4·06	2·180	3·96	2·201	4·07
2·918	4·17	2·904	4·12	2·753	4·08	2·895	4·12	2·830	4·13	2·576	4·08	2·823	4·12
3·353	4·24	3·365	4·29	3·206	4·17	3·404	4·22	3·453	4·19	3·047	4·25	3·202	4·25
3·757	4·38	3·866	4·37	3·703	4·23	3·954	4·29	3·853	4·33	3·537	4·24	3·641	4·29
4·282	4·49	4·205	4·45	4·304	4·30	4·356	4·44	4·442	4·43	3·982	4·34	4·228	4·43
4·722	4·51	4·638	4·45	4·918	4·43	4·822	4·47	5·045	4·46	4·439	4·33	4·713	4·44
5·172	4·59	5·038	4·53	5·501	4·40	5·391	4·48	5·566	4·44	5·025	4·50	5·247	4·50
5·694	4·48	5·553	4·54	5·905	4·30	5·826	4·29	6·193	4·23	5·679	4·43	5·838	4·36
6·146	4·21	6·158	4·25	6·390	4·10	6·321	4·12	6·617	3·94	6·330	4·10	6·403	4·09
6·875	3·80	6·846	3·74	6·840	3·79	6·831	3·91	7·018	3·76	6·887	3·84	6·965	3·84

[1]See also A.N. Vol. 225, p. 1.
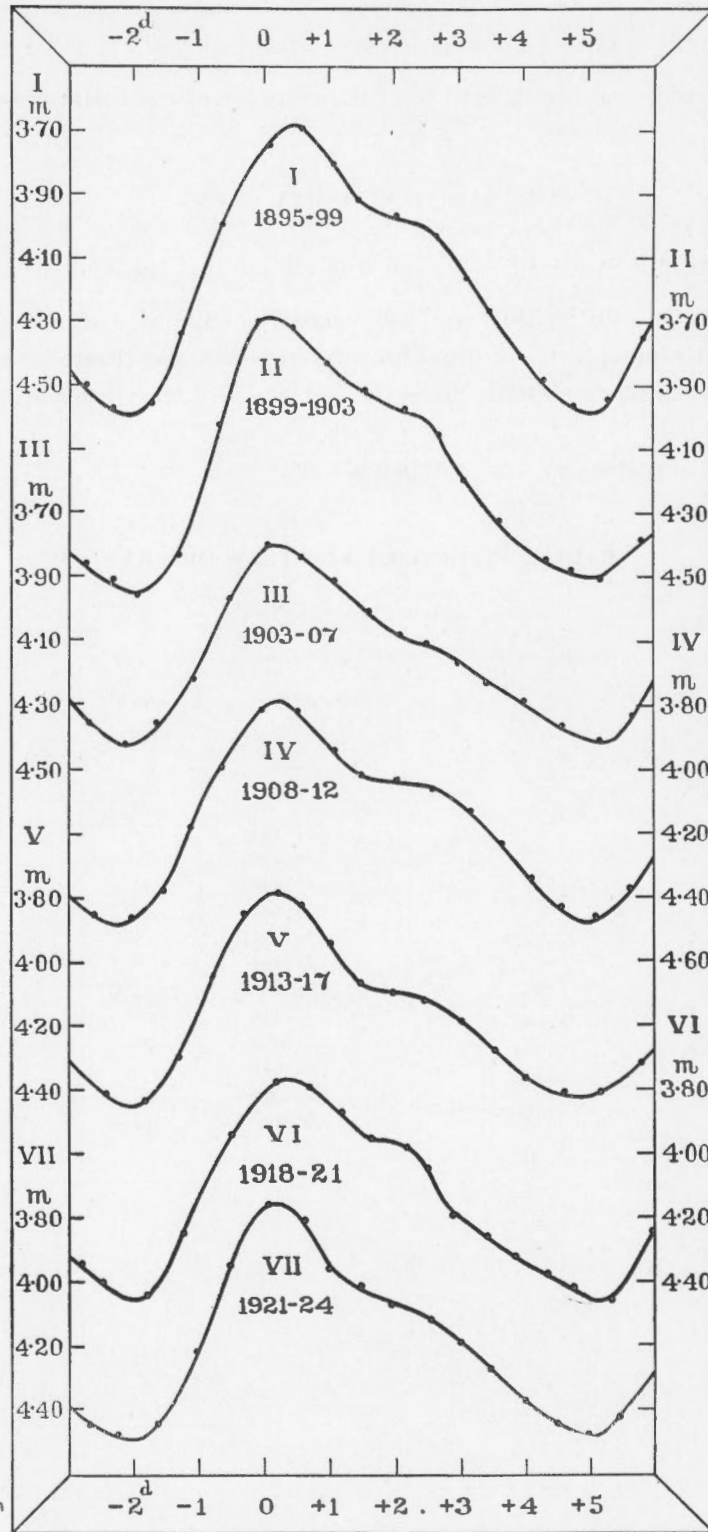
Fig. 1—Light curves of η Aquilæ obtained by Becker from
Plassmann's observations

The changes in shape are decided; several of the curves show large humps, while in other cases the hump is not so marked or even almost absent.

These results indicate a rich field for further research with the photo-electric photometer.

THE RADIAL VELOCITY CURVE

The spectrographic study of η Aquilæ was abandoned for some years.

Studied by Bélopolsky[1] in 1897 and by Wright[2] in 1898, it was not until twenty years later that it was resumed with the making of forty-two spectrograms at the University of Michigan. It was more recently investigated at the Lick Observatory by Jacobsen[3].

The radial velocities so far published are:—

RADIAL VELOCITIES AT OTHER OBSERVATORIES

(Bélopolsky's velocities)

Date	Julian Day G.M.T.	Velocity km.	Remarks
1897 July 10	2414116·344	−28·7	
11	117·344	−28·9	
12	118·385	−20·0	
13	119·344	−11·7	
17	123·344	−25·5	
21	127·344	− 9·6	
22	128·344	+ 0·2	
25	131·302	−32·0	
25	·344	−29·0	
26	132·302	−24·4	
30	136·344	+ 4·4	
Aug. 2	139·302	−27·1	
13	150·302	+ 1·0	

[1] Ap. J, Vol. 6, 1897, p. 393.
[2] Ap. J., Vol. 9, 1899, p. 59.
[3] L.O.B., Vol. 12, 1926, p. 147.

RADIAL VELOCITIES AT OTHER OBSERVATORIES—Continued

(Wright's Velocities)

	Date	Julian Day G.M.T.	Velocity km.	Remarks
1898	June 21	2414462·882	−25·3	
	27	468·901	−30·6	
	28	469·887	−25·2	
July	4	475·930	−30·4	
	5	476·858	−24·5	
	12	483·903	−27·7	
	19	490·867	−28·6	
	26	497·821	−28·8	
Aug.	2	504·729	−29·8	
	6	508·798	− 0·3	
	7	509·727	+ 9·0	
	7	·791	+11·4	Campbell +11·4
	8	510·724	−14·1	
	15	517·728	− 9·2	
	19	521·753	−14·6	
	21	523·758	+ 2·7	
	25	527·685	−23·6	
	25	·742	−20·6	
	26	529·687	−16·6	
	28	530·697	+ 1·7	Campbell +1·5
	29	531·672	+ 8·0	
	30	532·676	−28·8	
Sept.	9	542·726	−19·5	
	17	550·643	−12·4	
	18	551·671	− 6·7	
Oct.	9	572·640	−14·3	Campbell −13·5
	10	573·637	+ 0·8	
	17	580·733	+ 0·1	

(Küstner's velocities)

	Date	Julian Day G.M.T.	Velocity km.	Remarks
1909	Aug. 12	2418531·44	−26·8	A. N. n° 4750
1912	Oct. 5	2419651·29	−18·0	

(Lunt's velocities)

	Date	Julian Day G.M.T.	Velocity km.	Remarks
1910	Aug. 22	2418906	−13·9	Ap. J. Vol. 50 p. 171
1911	Sept. 6	2419286	−20·7	
1912	Aug. 23	2419638	−17·5	

RADIAL VELOCITIES AT OTHER OBSERVATORIES—Concluded

(Jacobsen's velocities)

	Date	Julian Day G.M.T.	Velocity km.	Remarks
1923	June 30	2423601·889	− 0·17	
	July 1	602·975	+ 5·63	
	2	603·963	−29·57	
	4	605·943	−22·83	
	6	607·948	−14·25	
	9	610·962	−24·85	
	10	611·937	−30·82	
	11	612·933	−24·19	
	12	613·947	−18·01	
	14	615·958	− 3·67	
	21	622·925	− 6·51	
	24	625·913	−31·47	
	Aug. 2	634·917	−23·39	
	3	635·897	−15·87	
	19	651·918	− 2·59	
	20	652·824	+ 9·10	
	30	662·853	−26·32	
	Sept. 6	669·810	−28·80	
	16	679·747	−13·90	
	23	686·770	−13·60	
	28	691·748	−25·10	
	30	693·743	−14·66	
	Oct. 10	703·706	− 1·86	
	15	708·641	−12·94	
	16	709·628	+ 1·64	
	18	711·626	−29·96	
	28	721·609	−16·58	
	Nov. 2	726·661	−30·61	
1925	Nov. 27	2424481·629	−29·85	Astronomical date changed
	Dec. 6	490·616	−18·33	by 12 hours, but Julian
	9	493·621	+ 8·76	Day unchanged.
	16	500·618	−15·80	

Following are the radial velocities of η Aquilæ obtained at Ottawa. The plates were measured against a standard spectrogram of Polaris, the same that was used for other Cepheids.

OTTAWA RADIAL VELOCITIES OF η AQUILÆ

	Date	Julian Day G.M.T.	Velocity km.	Remarks
1924	Aug. 7	2424005·645	−25·2	
	8	006·601	−29·4	
	10	008·700	−13·0	
	11	009·585	−15·2	
	12	010·699	− 5·1	
	14	012·581	−16·5	

OTTAWA RADIAL VELOCITIES OF η AQUILÆ—Concluded

Date		Julian Day G.M.T.	Velocity km.	Remarks
1924	Aug. 14	2424012·668	−19·6	
	15	013·692	−32·0	
	18	016·569	−13·1	
	24	022·677	−20·4	
	25	023·645	− 9·9	
	26	024·585	− 8·2	
	27	025·593	+ 0·2	
	29	027·586	−31·5	
	Sept. 2	031·601	− 8·9	
	3	032·588	+ 4·3	
	4	033·569	+14·3	
	5	034·581	−32·3	
	11	040·565	+ 6·2	
	14	043·551	−26·8	
	17	046·552	− 2·5	
	18	047·566	+ 8·2	
	19	048·551	−20·5	
	26	055·550	−12·9	
1925	June 24	325·717	− 8·7	Astronomical date changed
	27	328·707	−23·3	by 12 hours, but Julian
	29	330·742	−14·8	Day unchanged.
	July 1	332·697	−14·7	
	3	334·718	+ 6·5	
	6	337·702	−23·0	
	8	339·701	−11·8	
	9	340·707	− 1·4	
	11	342·694	−15·7	
	13	344·743	−23·0	
	14	345·726	−18·9	
	20	351·693	−25·8	
	27	358·708	−28·4	
	Aug. 4	366·678	−18·2	
	5	367·616	−16·7	
	8	370·641	+13·2	
	12	374·640	−12·5	
	17	379·628	−30·2	
	21	383·605	− 4·6	
	22	384·610	+ 9·3	
	25	387·591	−25·8	
	26	388·647	−18·9	
	27	389·558	−14·2	
	27	·635	−15·4	
	28	390·589	− 6·0	
	31	393·637	−30·7	
	Sept. 1	394·616	−22·6	
	2	395·572	−21·8	
	9	402·551	−20·0	
	15	408·542	−32·3	
	15	·613	−29·0	
	22	415·545	−35·1	
	Oct. 5	428·581	+ 0·3	

The **Ottawa** velocities, together with the elements J.D. 2414827·150 ± 7^d.176678 E, have been utilized for the curves shown in figs. 2 and 3.
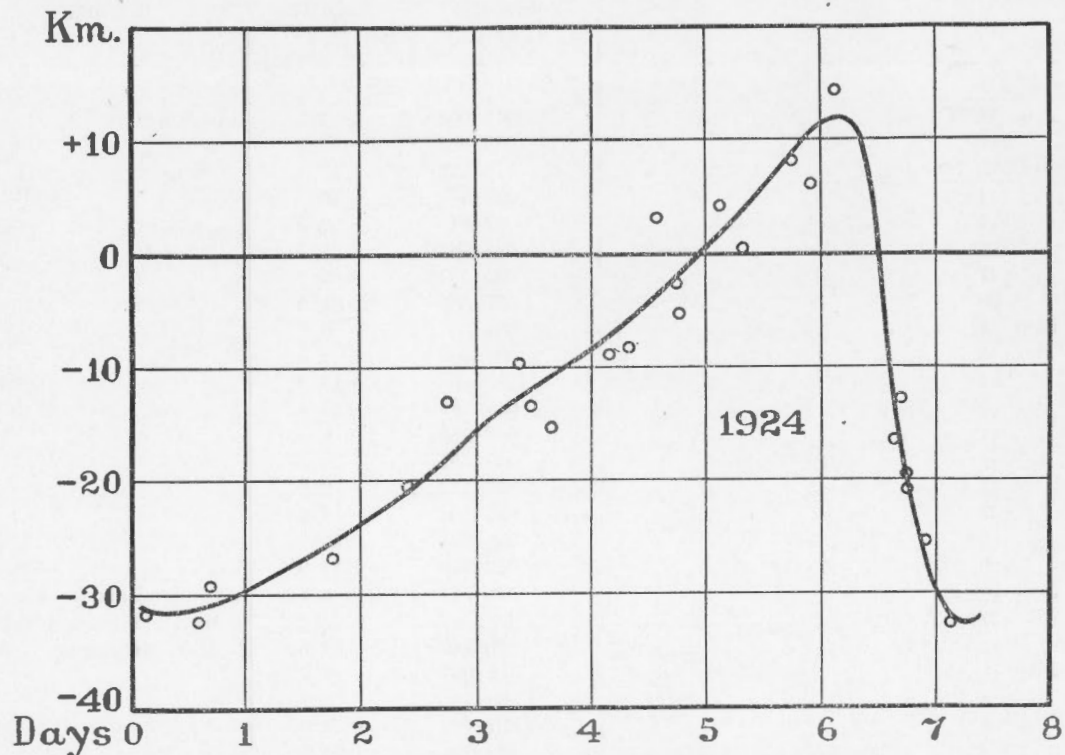
Fig. 2—Radial Velocity curve of η Aquilæ in 1924
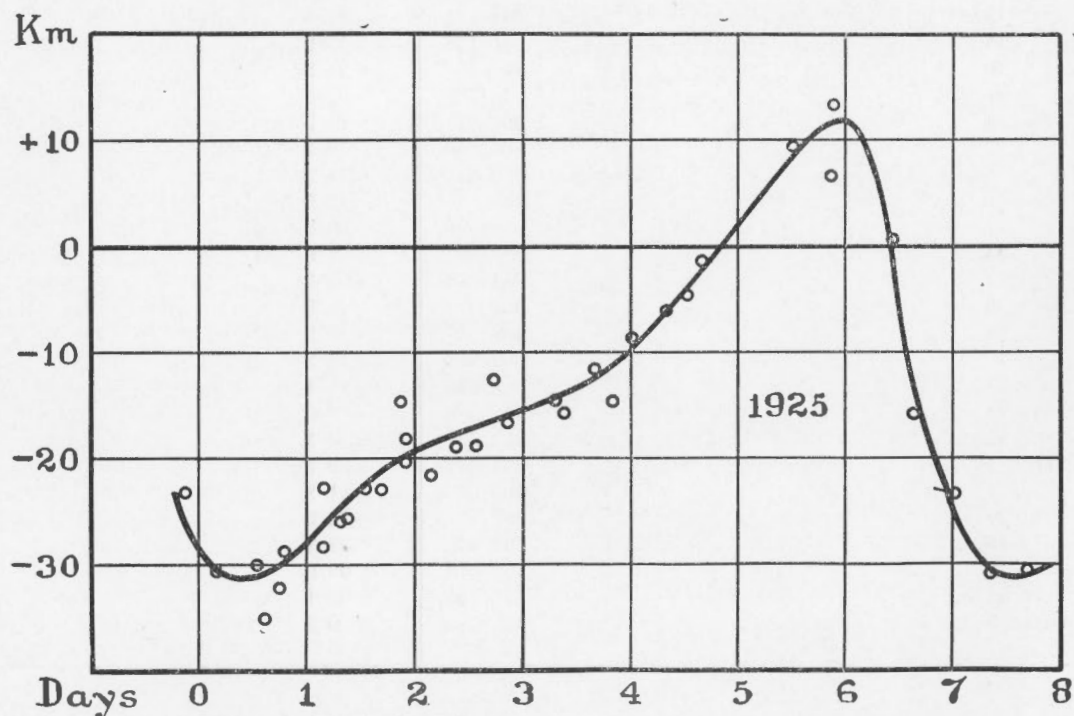
Fig. 3—Radial Velocity curve of η Aquilæ in 1925

The curve in fig. 3 is very similar to that found by Jacobsen in 1923 and shows a marked hump on the ascending branch probably corresponding to the hump on the descending branch of the light-curve. There are not, however, enough observations to determine this hump with any great degree of certainty on the curve of 1924.

VELOCITIES FROM INDIVIDUAL LINES

Research on the radial velocities given by individual lines in the spectra of Cepheids was mentioned in the writer's paper on ζ Geminorum[1]; this work was initiated by R. H. Curtiss, W. Carl Rufus and J. A. Aldrich. In the particular study of η Aquilæ by Dr. W. Carl Rufus[2] it was found that lines of different levels did not give the same velocity curve. Dr. Rufus obtains what he calls velocity-difference curves, such as velocity of high level minus velocity of intermediate level, or high-minus-low curve, or hydrogen-minus-intermediate curve. The following extracts from Rufus' article will give a clear idea of his results:—

"Velocity-difference curves correlated with the light variation and other periodic changes characterize the method here applied. The radial velocities of concentric layers of the star's atmosphere are determined separately by isolating certain elements, and by grouping lines of the spectrum originating at assumed levels based upon the determination of their heights in the sun by St. John[3] and by Mitchell[4]. The approach of two layers indicates compression of the intervening gases and recession indicates expansion. Resulting changes in temperature, density, radiation, and absorption, may be correlated with the light variation and changes of spectrum to explain many of the anomalous characteristics of Cepheid variation. Inasmuch as spectroscopic work is limited to surface radiations characterized by atmospheric effects, a new method of analysis to determine changes of atmospheric conditions should yield results of fundamental significance."

In η Aquilae "the hydrogen-minus-intermediate curve has an amplitude greater than 20 km., which exceeds one-half the amplitude of the velocity curve itself. Special attention is called to the similarity of this velocity-difference curve to Wylie's photometric light-curve, and to a spectral variation curve based upon Shapley's data[5]. The intermediate-minus-low curve resembles the reverse of the light-curve with synchronous secondary features. The 'humps' of the velocity-difference curves are synchronous with the 'stillstand' of the light-curve, which is not true in the case of the velocity curves. The 'stillstand' of the light variation seems to be due to a stage of comparative rest in the atmosphere of the star."

In a further study of ζ Geminorum Rufus[6] finds phenomena analogous to what he found in η Aquilæ. Twenty-six selected lines were grouped at three levels, six high, six intermediate and fourteen low. High-minus-low and high-minus-intermediate curves were formed by plotting the velocity differences with respect to phases. The amplitude of these curves is greater than one-third that of the velocity curve itself.

[1] Pub. Dom. Obs., Vol. 9, p. 111.
[2] Proc. Nat. Acad. Sc., Vol. 10, 1924, p. 264.
[3] Ap. J., Vol. 37, 1913, p. 322 and Vol. 38, 1913, p. 341.
[4] Ap. J., Vol. 38, 1913, p. 407.
[5] Ap. J. Vol. 44, 1916, p. 287.
[6] Pop. Ast., Vol. 34, 1926, p. 242.

These results do not agree with those obtained at Ottawa, as can be gathered from the curves given on page 114 of the present volume; all these curves are practically identical and do not suggest the existence of velocity-difference curves.

δ Cephei and η Aquilæ were investigated by Jacobsen[1] at the Lick Observatory by the use of a specially built 3-prism spectrograph giving the violet region. The velocity differences for δ Cephei show no relation to phase. In η Aquilæ there seems to be a high-minus-low effect of the nature found by Rufus but of considerably smaller amplitude. The following table gives Jacobsen's results:—

| | δCephei | | | ηAquilæ | |
Phase	High minus low	Intermediate minus low	Phase	High minus low	Intermediate minus low
	km.	km.		km.	km.
0ᵈ·21	+3·0	+6·1	0ᵈ·24	−2·0	+5·9
0·68	+0·4	+5·5	2·09	−0·7	+4·6
2·53	+5·5	+8·8	2·35	−1·6	+3·6
3·52	+1·1	+1·8	6·09	+3·6	+5·4
3·56	+3·6	+8·2	6·23	+6·2	+5·1
4·78	+0·8	+5·1	7·15	−4·0	+2·7

Jacobsen considers his results, however, as only preliminary.

The results obtained here for individual lines of η Aquilæ agree with those of Jacobsen but not so well with those of Rufus. It appears that the dispersion of the Detroit spectrograms is not so great as those of Lick and Ottawa, and that the slit-width used was somewhat greater. The possibility is suggested that the results of Rufus may be due to blends of ionized and neutral lines; it is known that the relative intensities and widths of many such pairs vary considerably during the variation-cycle of the star; such variations in the case of a close unresolved pair would produce just such apparent changes in radial velocity as observed by Rufus.

Measures of individual lines were made on a direct measuring microscope by Mr. J. F. Frédette of this Observatory and on the spectro-comparator by the writer. The direct measures have been combined into normal places which are in the following tables:—

---

[1] L.O.B., Vol. 12, p. 151.

Sr⁺4215·7 (1924 and 1925) 6000 km. level in Sun			Fe 4260·6 (1925) 600 km. level in Sun		
Phase	Velocity	Weight	Phase	Velocity	Weight
	km.			km.	
7ᵈ·15	−28·8	11	7ᵈ·14	−22·2	3
0·68	−21·5	10	0·67	−29·5	7
1·21	−13·9	6	1·25	−33·1	7
1·75	−11·0	8	1·75	−24·9	14
2·29	−15·2	5	2·23	−16·0	5
2·69	− 5·0	8	2·71	−17·8	10
3·51	− 5·3	3	3·66	−14·7	6
4·24	− 2·7	4	4·11	−10·4	4
4·80	+ 8·7	7	4·60	− 8·9	6
5·93	+17·2	7	5·82	+ 7·6	7
6·60	+ 1·6	8	6·59	− 8·5	6
6·77	−13·7	7			

Fe 4325·9 (1925) 900 km. level in Sun			Hγ 4340·6 (1925)		
Phase	Velocity	Weight	Phase	Velocity	Weight
	km.			km.	
7ᵈ·11	−23·3	1	7ᵈ·20	−28·4	2
0·72	−25·3	8	0·70	−33·0	3
1·25	−17·8	5	1·19	−28·4	6
1·78	−21·1	11	1·79	−19·9	8
2·21	−19·6	3	2·19	−24·8	4
2·71	− 9·9	9	2·65	−13·7	7
3·56	−12·0	4	3·63	− 6·4	3
4·11	− 2·0	4	4·11	− 4·8	4
4·68	− 0·7	1	4·60	− 6·0	
5·81	+10·3	6	5·73	− 1·8	5
6·55	− 6·3	4	6·55	+ 1·4	2

Mg⁺4481·4 (1925)			Ti 4549·7 (1924 and 1925) 1300 km. level in Sun		
Phase	Velocity	Weight	Phase	Velocity	Weight
	km.			km.	
7ᵈ·20	−39·9	2	7ᵈ·14	−23·9	18
0·67	−21·6	6	0·65	−27·2	16
1·25	−16·2	6	1·27	−17·8	6
1·76	−20·0	8	1·78	−19·6	17
2·26	−18·2	2	2·31	−17·2	6
2·70	−18·6	4	2·76	−13·4	14
3·62	−20·5	5	3·52	− 9·9	14
4·04	−18·1	2	4·23	− 6·6	9
4·52	− 2·8	1	4·78	− 0·8	12
5·75	+ 9·0	3	5·80	+11·1	13
6·49	− 6·8	4	6·60	− 4·5	12

In addition, for the mean of the three lines Ti 4395·2, Fe 4415·3 and Ti 4549·7 whose respective levels in the sun are 2500 km., 500 km., and 1300 km., we have the following values:—

Phase	Velocity	Weight
	km.	
7ᵈ·02	−21·4	35
0·67	−26·2	26
1·23	−18·2	14
1·77	−17·4	40
2·25	−17·0	13
2·73	−10·7	34
3·54	− 8·2	24
4·16	− 7·3	23
4·70	− 1·5	22
5·77	+13·4	26
6·56	− 1·0	20

The first six series of measures give the velocity curves in fig. 4 and the last series the curve in fig. 5.

The ionized lines Sr⁺4215·7 and Mg⁺4481·4 give velocity curves having an amplitude of nearly 50 km. with pronounced secondary humps. The low-level lines also give a pronounced hump, while this hump is not so marked for intermediate levels agreeing with Rufus' results.

The neutral lines give amplitudes of nearly 40 km.

From comparison with spectro-comparator measures we place little confidence in the real existence of a flattened maximum in the Hγ curve.

The five separate lines Sr⁺4215·7; Hγ; Fe 4415·3; Ti⁺4534·1 and Ti 4534·9 measured on the spectro-comparator give the following velocities corresponding to phases.
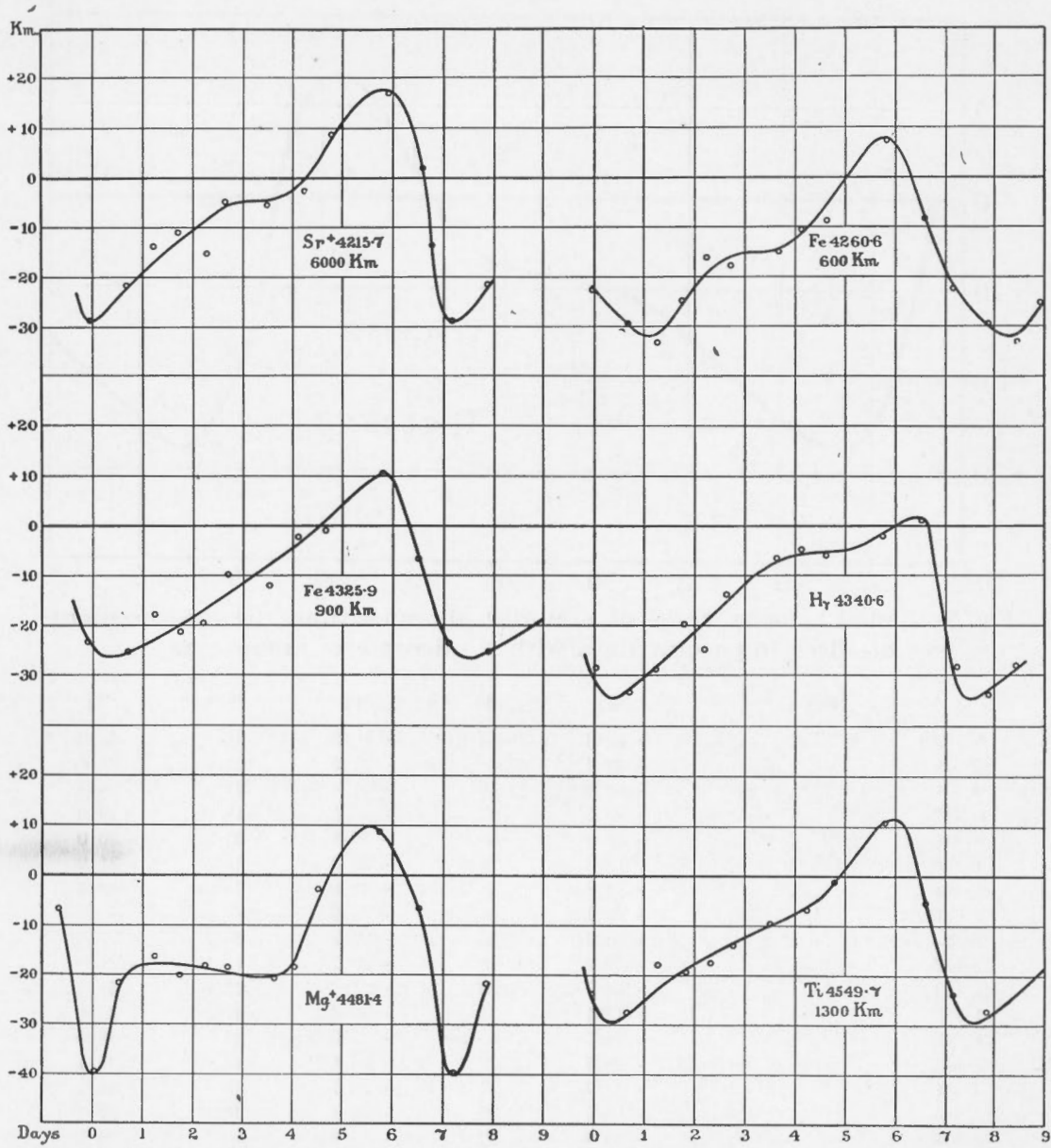
Fig. 4—Radial Velocity curves of η Aquilæ as obtained from individual lines measured with a micrometer microscope
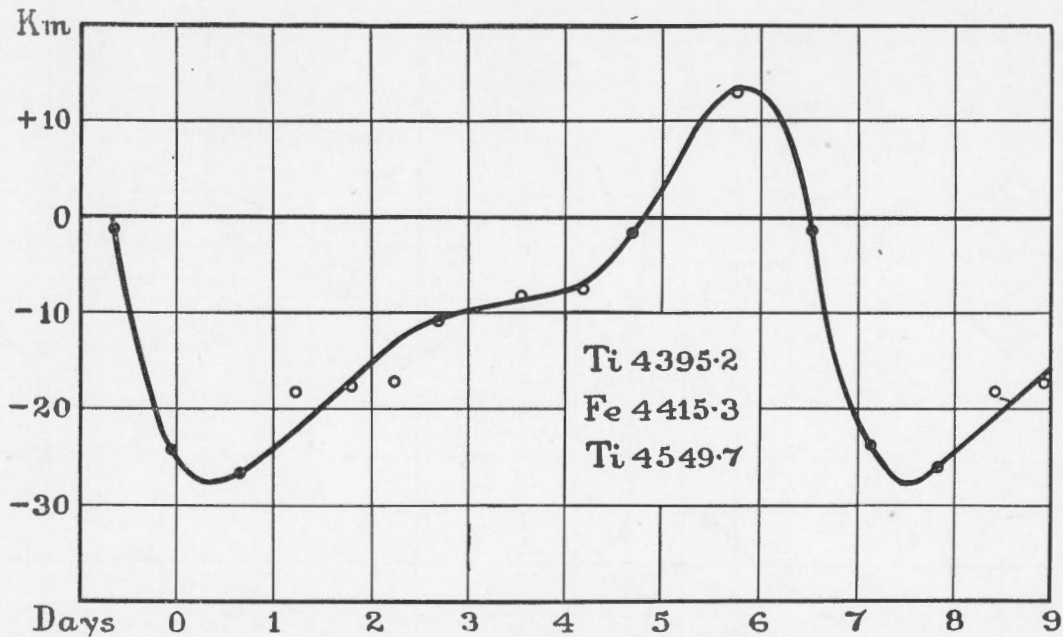
Fig. 5—Radial Velocity curve of η Aquilæ obtained from the measurement of three important lines with a micrometer microscope

Phase	Sr⁺	Hγ	Fe	Ti⁺	Ti
0ᵈ·195...................	−40·3	−45·8	−40·3	−33·9	−42·9
0·538...................	−24·4	−35·7	−27·5	−23·0	−34·6
0·570...................	−42·8	−39·1	−31·4	−31·4	−26·3
0·747...................	−36·2	−37·6	−33·5	−48·0	−49·3
0·818...................	−32·2	−43·3	−28·8	−33·9	−30·0
1·148...................	−12·3	−28·0	−28·3	−32·5	−31·2
1·174...................	−33·8	−55·5	−29·0	−38·3	−46·1
1·310...................	−22·1	−36·7	−26·9	−	−
1·325...................	−36·2	−25·9	−26·3	−20·1	−27·9
1·535...................	−20·9	−43·5	−27·2	−16·3	−27·9
1·671...................	−22·7	−26·8	−21·6	−22·0	−31·1
1·887...................	−32·1	−31·7	−29·4	−37·2	−39·8
1·932...................	−28·0	−45·5	−37·3	−29·1	−33·0
1·941...................	−28·7	−23·1	−23·9	−37·7	−35·1
2·130...................	−32·7	−27·5	−20·7	−16·4	−31·9
2·381...................	−18·0	−32·8	−19·4	−29·4	−37·1
2·518...................	−26·4	−22·7	−19·3	−10·3	−25·7
2·727...................	−24·6	−22·7	−17·3	−14·7	−18·6
2·879...................	−23·1	−24·7	−14·9	−12·4	−22·8
3·290...................	−18·2	−37·5	−14·8	−16·7	−14·1
3·369...................	−17·4	−38·8	−19·8	−24·6	−31·1
3·670...................	−26·7	−19·9	−22·5	−16·5	−19·1
3·842...................	−19·5	−21·2	−18·2	−27·6	−27·6
4·038...................	− 9·6	−23·3	− 7·7	+ 1·2	−11·7
4·323...................	−18·7	−40·2	−11·7	−19·7	−26·2
4·515...................	−16·2	−16·4	−16·4	− 5·6	−18·5
4·676...................	− 7·1	−11·7	− 7·8	− 5·7	−12·1
5·520...................	+ 8·0	−11·1	+ 7·1	+ 1·8	+ 1·8
5·863...................	+ 5·1	+ 1·4	+ 3·5	+ 2·5	+ 5·1
5·904...................	+ 4·0	+10·7	+ 7·4	+11·9	− 1·0
6·434...................	− 7·1	−11·8	− 4·7	− 2·2	−13·8
6·683...................	−10·7	− 8·8	−14·2	−16·5	−21·7
7·028...................	−37·2	−44·2	−26·0	−18·1	−32·3

The radial velocity curves obtained from these values for Sr⁺, Hγ and Fe are given in fig. 6.
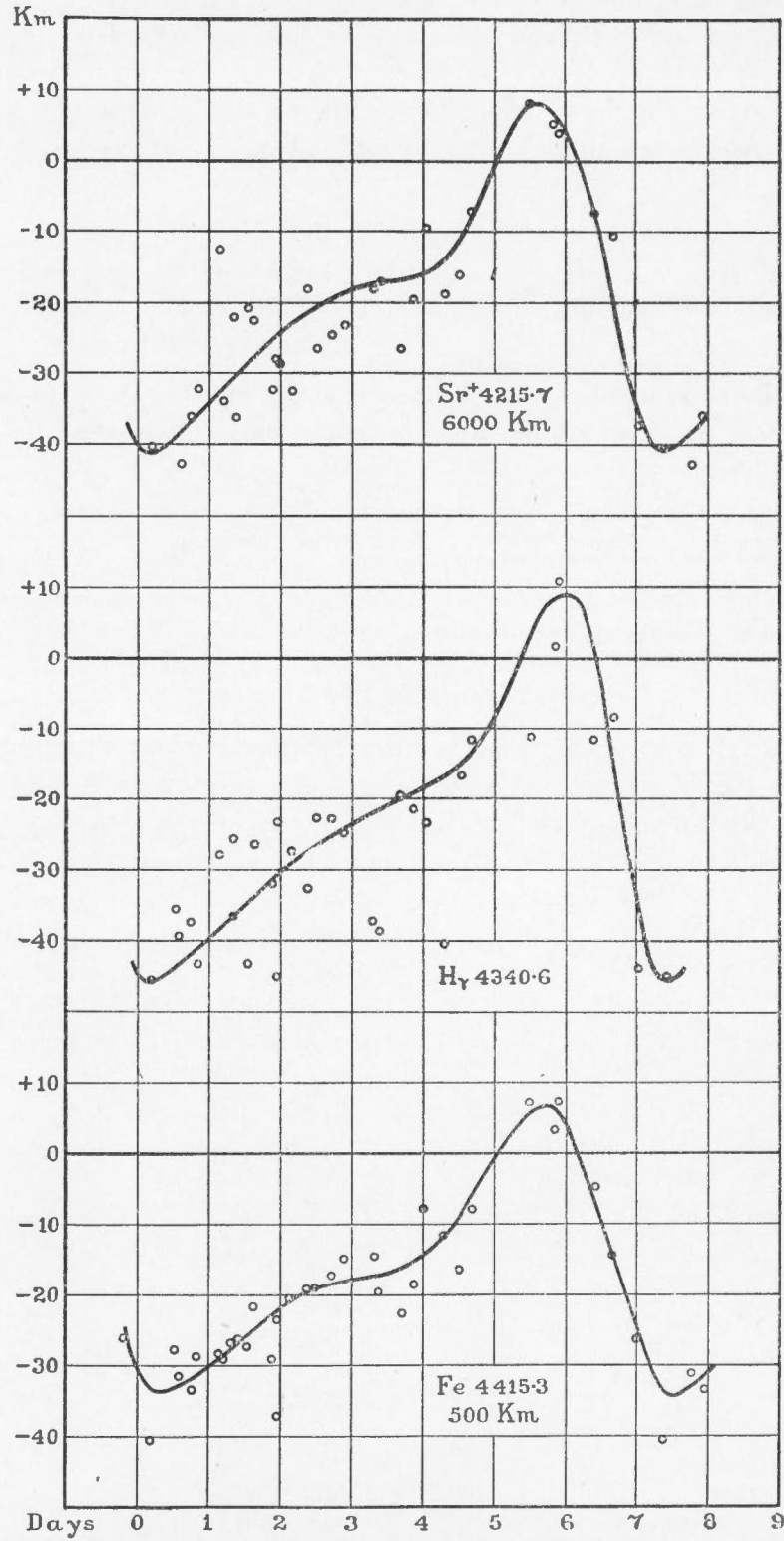
Fig. 6—Radial Velocity curves of η Aquilæ as obtained from individual lines measured on the spectro-comparator

The Fe line is excellent, Sr$^+$ is broader but good, while Hγ is difficult to measure. The lines Ti$^+$ and Ti are poor and close together, a circumstance which might influence the accuracy of their measurement; the curves which they furnish indicate amplitudes of nearly 50 km or more for Ti$^+$ and of nearly 40 km. for Ti. These two titanium lines are used by the writer for the determination of the amount of ionization in Cepheids.

The Sr$^+$ curve has an amplitude of 50 km., as has the Hγ curve. The Fe curve has an amplitude of 40 km.

If it is remarked that all so called high-level lines are likely to be ionized lines, and low-level neutral lines, it is seen that our results agree with those of Jacobsen who also finds that the high-level lines give an amplitude about ten kilometers larger than the low-level lines.

It must be borne in mind that the ionized lines in Cepheids are prominent but much less numerous than the neutral lines, which explains that the mean radial-velocity curve has the same amplitude as the neutral-line curves.

The net result of the present radial velocity investigation is as follows:—

1. *The neutral lines give a radial-velocity curve having an average amplitude of 40 km.*
2. *The ionized lines give a radial-velocity curve having an average amplitude of 50 km.*
3. *The secondary hump is present in all.*

VARIATION OF SPECTRAL CLASS

The relative intensity of the titanium lines Ti$^+$4534·139 and Ti 4534·953 has been estimated on the spectrograms of η Aquilæ. While both lines actually vary, the intensity of the arc line was throughout assumed as 10, and that of the enhanced line estimated in terms of it. The following table gives these relative intensities.

Date	Phase	Intensity	Weight
1924 Aug. 7	6^d·90	12	3
10	2·78	9	2
11	3·66	12	3
12	4·78	8	3
14	6·66	13	5
14	6·75	15	5
15	0·60	12	3
18	3·48	9	2
24	2·41	14	1
25	3·37	10	3
26	4·31	7	3
27	5·32	10	3
Sept. 2	4·15	8	3
3	5·14	7	4
4	6·12	10	3
5	7·13	17	3
11	5·94	15	4
17	4·75	8	3
18	5·77	11	1
19	6·75	14	4
26	6·70	18	3

	Date	Phase	Intensity	Weight
1925	June 24	4·04	10	4
	27	7·03	18	4
	29	1·89	10	5
July	1	3·84	7	3
	3	5·86	11	5
	6	1·67	14	4
	8	3·67	9	1
	9	4·68	7	4
	11	6·68	18	5
	13	1·54	10	5
	14	2·52	11	5
	20	1·31	11	2
	27	1·15	16	4
Aug.	4	1·94	9	3
	5	2·88	13	3
	12	2·73	8	4
	17	0·54	13	3
	21	4·52	8	3
	22	5·52	8	4
	25	1·33	12	4
	27	3·29	10	3
	27	3·37	7	4
	28	4·32	10	4
	31	0·20	15	1
Sept.	1	1·17	13	1
	2	2·13	6	3
	9	1·93	10	5
	15	0·75	16	5
	15	0·82	15	4
	22	0·57	16	4
Oct.	5	6·43	14	4

If the values of weight 1 are omitted these measures give the curve of ionization change in fig. 7 where the crosses indicate the 1924 observations and the circles those of 1925. The weights of the observations are entered in the figure.
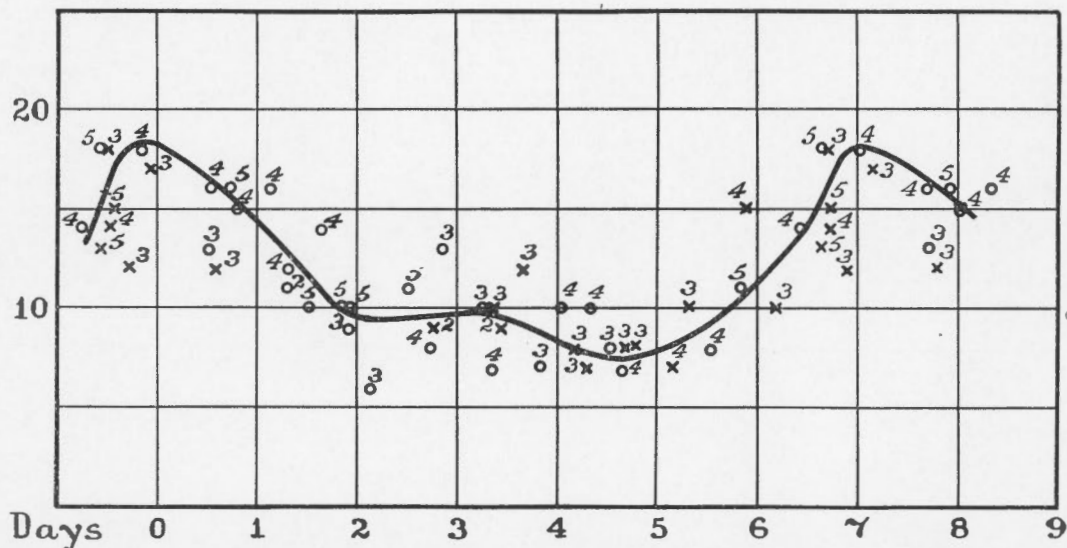
Fig. 7—Ionization curve for η Aquilæ or relative intensity of the two lines
Ti⁺ 4534·139 and Ti 4534·953

The ionization curve resembles the light-curve more than the radial-velocity curve. From phase zero on the ionization drops quickly and is followed by a "stillstand" corresponding to the hump of the light-curve; it reaches a minimum at about phase five days just as the light does, and this about one day before the maximum radial velocity.

The co-operation of Messrs. J. F. Frédette and R. Callander in the work of observing and measuring is here acknowledged.

DOMINION OBSERVATORY, OTTAWA.

November 1926.

DEPARTMENT OF THE INTERIOR
CANADA

Hon. Charles Stewart, *Minister*   W. W. Cory, C.M.G., *Deputy Minister*

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THE CASTOR SYSTEM

BY DOUGLAS A. BARLOW

The Castor System[1] consists of a visual double star, of which the components α_1 and α_2 Geminorum are both spectroscopic binaries, and a distant companion C. The position is, R.A. $7^h 28^m \cdot 2$; decl. $32° 6'$; magnitudes α_1 $2 \cdot 85$, α_2 $1 \cdot 99$, and C $9 \cdot 0$. Both α_1 and α_2 are of type A; however, the absorption in the case of the fainter is somewhat more complete, and consequently the number of measurable lines in its spectrum is greater than in that of the brighter. The companion is separated by $73''$ from the primary system; it is of type $dMIe$.

On October 12, 1926, the first of a series of spectrograms was made with the one-prism spectrograph of the 15-inch equatorial of the Dominion Observatory. All plates used were Eastman 40, with the exception of the two standards, which were Eastman 33, the latter being a slower plate of finer grain. One hundred and three spectrograms were made between the above date and May 9, 1927. The plates were measured with a spectrocomparator, using standards of the respective stars.

Dr. Heber D. Curtis in his discussion[2] makes the statement that, "Although the lines in α_2 are not so distinctly defined, yet, when the proper exposure is made, the measurements seem to be somewhat more accurate on this star than on α_1". The writer finds that when measuring on the comparator α_1 is to be preferred to α_2 for accuracy. Exposures of from sixteen to twenty-four minutes and from eight to twelve minutes were required on α_1 and α_2, respectively.

α_1 GEMINORUM

The binary character of this component was discovered by Professor Belopolsky at Pulkowa in 1896[3]; 118 radial velocity determinations were made during the years 1894 to 1898, and an orbit determined. In the years 1904 and 1905, Heber D. Curtis investigated the star very thoroughly with the Mills spectrograph and obtained an orbit. At intervals between 1903 and 1917 J. Lehmann-Balanowskaja also made an exhaustive study of the star.[4]

The present discussion is based on the measurement of forty-two measurable spectrograms out of the forty-six obtained, four being discarded on account of weakness of stellar spectrum.

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[1] Handbuch der Astrophysik, Band VI, p. 460.
[2] L.O.B., Vol. 4, p. 55.
[3] Bulletin, Academy of Sciences of St. Petersburg, Dec., 1896.
    Memoirs, Academy of Sciences of St. Petersburg, 11, No. 4, 1900.
    Ap. J., Vol. 5, p. 1, 1897.
[4] Bulletin of the Central Observatory of Russia at Pulkowa, Vol. 9, No. 93, p. 365.

RADIAL VELOCITIES OF α_1 GEMINORUM

No.	Date		Julian Date	Velocity
	1926			km.
14717	Oct.	18	2424806·727	+30·96
22		19	807·851	−14·81
29		22	810·765	−16·59
30		22	·782	−15·18
40		28	816·724	−15·51
56	Nov.	6	825·663	−21·98
62		8	827·712	+20·80
92	Dec.	3	852·712	−15·71
14802		7	856·709	+30·07
22		23	872·665	−27·14
30		30	879·647	+18·73
	1927			
40	Jan.	8	888·699	+28·13
42		8	·715	+ 2·37
49		10	890·657	−21·93
63		21	891·654	−16·59
75		25	905·656	− 6·83
83		27	907·705	−21·01
94	Feb.	3	914·717	+14·61
14923	Mar.	1	940·711	−14·07
34		4	943·658	−10·11
42		10	949·553	− 2·85
47		11	950·654	+22·94
56		16	955·676	+ 8·50
60		17	956·664	+16·64
62		18	957·743	−28·02
65		20	959·715	+15·59
66		20	·729	+13·97
67		20	·758	+ 8·54
68		23	962·504	+11·95
70		23	·556	+17·07
71		23	·576	+ 9·94
72		23	·617	+ 5·35
74		23	·702	+ 5·50
75		23	·728	+ 6·60
76		23	·765	+ 7·23
84		25	964·686	+21·21
87		26	965·528	+11·76
90		26	·724	+ 0·16
91		27	966·653	−28·80
92		28	967·508	+13·43
96		29	968·712	− 6·90
97		30	969·518	−27·04

These were combined in normal places as follows, the phases listed being determined according to the adopted periastron and period.

No.	Phase	Velocity	Weight
1	0·023	8·50	0·4
2	0·102	14·02	0·7
3	0·182	19·47	0·5
4	0·407	29·55	0·6
5	0·618	30·07	0·3
6	0·858	22·94	0·4
7	0·928	20·80	0·3
8	1·002	14·30	0·7
9	1·057	13·51	0·6
10	1·115	11·67	0·9
11	1·194	6·88	0·8
12	1·273	− 3·54	0·5
13	1·346	− 6·90	0·4
14	1·537	−15·53	0·9
15	1·633	−16·05	0·6
16	1·806	−21·50	0·7
17	1·932	−27·14	0·2
18	2·090	−28·02	0·3
19	2·181	−27·92	0·7
20	2·355	−21·93	0·3
21	2·537	−14·89	0·5
22	2·680	− 4·16	1·0

From these normal places (omitting numbers 6 to 11, for reasons which follow), repeated orbits were obtained by experiment, using the third Henroteau method,[1] and testing each by summing the squares of the residuals until the elements given below were obtained. The period given was deduced from the observations of Curtis and the writer.

$$P = 2^{d}\cdot9283$$
$$e = 0\cdot06$$
$$\omega = 290°\cdot0$$
$$K = 28\cdot2 \text{ km.}$$
$$\gamma = 0\cdot22 \text{ km.}$$
$$T = \text{J.D. } 2424888\cdot302$$
$$a \sin i = 1,129,400 \text{ km.}$$

Figure 1 shows this orbit with the normal places.

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[1] Journal R.A.S.C., Vol. 21, p. 265 .
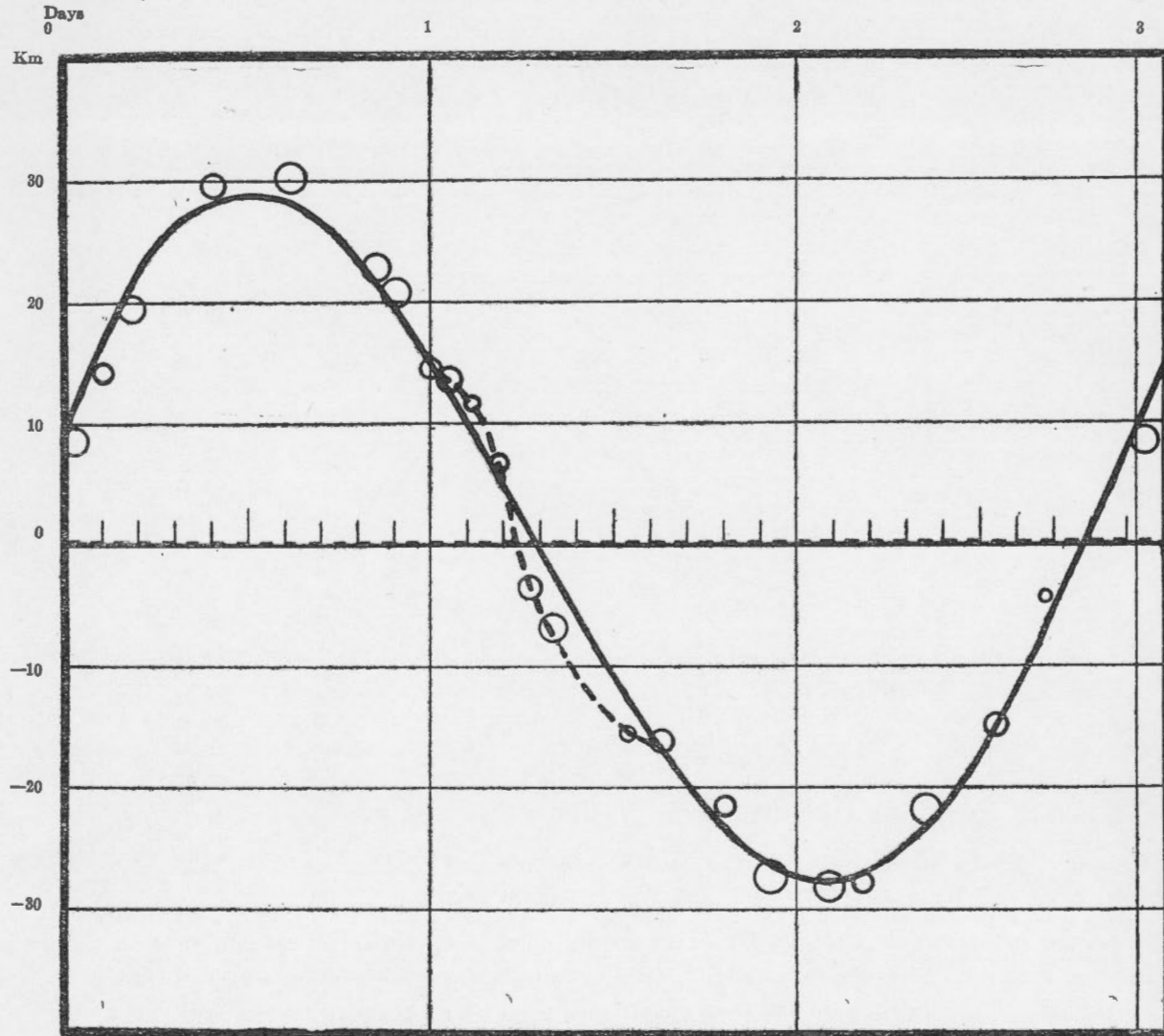
Fig. 1—Orbit of α_1 Geminorum

Following is a list of orbits which have been determined,—complete in so far as the writer is aware.

Author	Epoch	K	γ	ω	e	$a \sin i$	T J.D.
		km.	km.				
Belopolsky	1896	33·43	−4·1	82·0	0·12	1,336,400	2,413,559·00
L.–B.	1903·2, I	31·99	−1·2	14·4	0·02	1,287,500	6,136·24
Curtis	1904·7	31·76	−1·0	102·5	0·01	1,279,900	6,828·057
L.–B.	1905·7, II	30·13	−1·0	250·2	0·09	1,208,500	6,870·29
L.–B.	1910·0, III	29·19	−1·8	351·3	0·03	1,174,800	8,364·58
L.–B.	1912·9, IV	26·84	−2·6	86·5	0·05	1,080,200	9,454·72
L.–B.	1914·8, V	28·67	0·8	283·3	0·05	1,153,400	2,420,176·70
L.–B.	1916·8 VI	26·47	−9·3	35·7	0·04	1,065,100	0,927·24
Barlow	1926·9	28·2	0·2	290·0	0·06	1,129,400	4,888·302

Dr. Belopolsky considered a rotation in the line of apsides probable, having arrived at this idea by a classification of his observations into three groups which gave different elements. He assigned a period of 1,502 days, but he did not consider the observational data sufficient for a definitive decision as to the reality of this change. The writer agrees with Mrs. Lehmann-Balanowskaja in her conclusion that, considering the clearly small eccentricity of the orbit and the consequent opportunity for error, the attempt to define the variation may be wisely abandoned for the present. Dr. Belopolsky gives as period 2·934050 days.

Dr. Curtis' orbit was obtained from a series of 32 plates; his period 2·928285 days, was determined by using the maxima and minima given by Belopolsky in 1896-9 in connection with his own of 1904-5.

Between 1904 and 1917, 147 plates were secured at Pulkowa. These were divided into six sets and six orbits were determined by Mrs. Lehmann-Balanowskaja. By using the ascending and descending nodes she computed a mean period of 2·928308 days. She suggests that there may be a continuous variation in the amplitude, and offers the following relation:

$$K = 29 \cdot 13 - 0 \cdot 34 \ (t - 1910 \cdot 0)$$

Referring to fig. 1, particular attention is called to the peculiarity in the descending branch of the curve. This made its appearance early in the investigation, and care was taken to accumulate a greater number of observations at this phase, with the result shown, the points in that vicinity being each the weighted mean of a number of observations. On account of this peculiarity, normal places 6 to 11 were omitted from the process of determination of elements, as being under the possible influence of a slight eclipse.

Evidence in favour of the eclipse is found in a publication by P. Guthnick and R. Prager,[6] "Photoelektrische Untersuchungen an Spektroskopischen Doppelsternen und an Planeten". An account is given of the installation of a photo-electric photometer and investigations therewith, including that of Castor. The measurements of magnitude were of the entire system, with results as follows:

M (Maximum), 1914 April 18·78 G.M.T. 2·92825 E

M − m (Minimum), 1^{d}·5

M 0^{m}·35; m 0^{m}·44

This change in combined magnitude corresponds to a change of 0^{m}·28 in the fainter component, whose magnitude is 2^{m}·85

The Guthnick light curve tends to raise some doubts as to the apparent simplicity of the system. The occurrence of an eclipse is neither definitely indicated, nor definitely excluded. Furthermore, it is clear that if this epoch of minimum be taken to be that of eclipse, we have a useful check upon the number of periods elapsed between the epochs of Curtis and the writer, that is a check upon the period given.

Taking the natural phase of eclipse, that is, when $v + \omega = 90°$, the following epochs are obtained:

Curtis, J. D. 2,416,830·895

Barlow,   2,424,889·583.
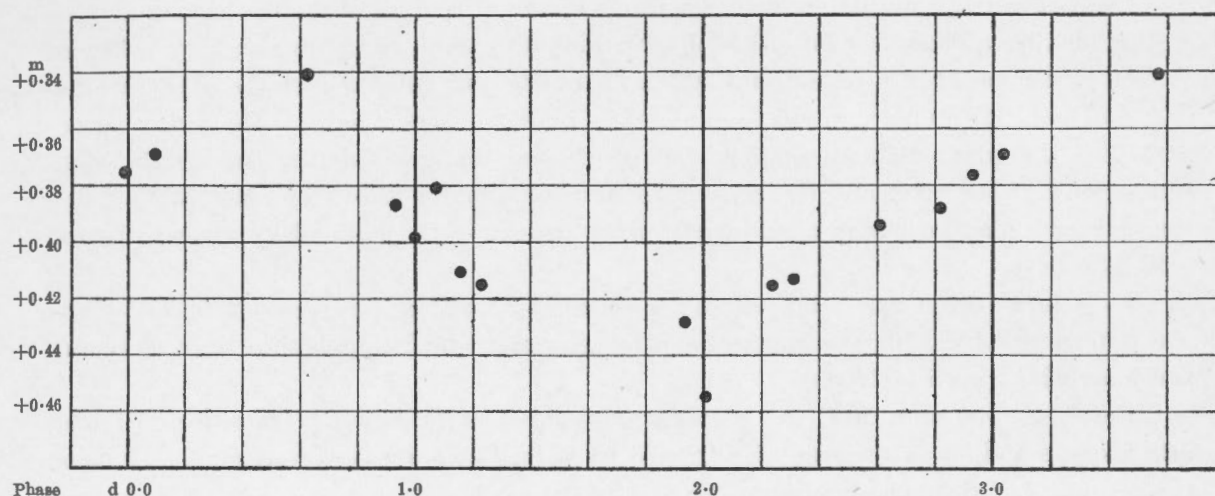
[6] Berlin-Babelsburg Publications, Band 1, Heft 1.

Fig. 2—Guthnick Light—Curve of α_1 Geminorum

($m = \alpha_1$ Geminorum—α Leonis)

The epoch of light minimum of the Guthnick curve is J.D. 2,420,245·280.

A period of $2^d \cdot 928302$ is indicated. The improvement on the period $2^d \cdot 928308$ being thus practically negligible. The difference between the Guthnick light-minimum and the date of assumed eclipse ($v + \omega = 90°$) for that particular revolution is $0^d \cdot 015$, which, in view of the curve, and the lack of accuracy with which the time of minimum is determinable, is remarkably small. The writer feels safe in adopting a period of $2 \cdot 9283$ days.

α_2 GEMINORUM

The variation in the radial velocity of this brighter component of Castor was discovered in October, 1905, by Dr. Curtis, and from a series of 40 observations an orbit was computed.[7]

Between the years 1909 and 1916 thirty-eight plates were made by A. A. Belopolsky at Pulkowa, and the investigation was pursued by V. Rossowskaja.[8]

The writer knows of no subsequent investigations other than the present.

Advantage was taken of all clear weather between October 12, 1926, and May 9, 1927, to make the 48 observations listed below. All measurements were made on the comparator, except that of 15064. The measurable lines in this spectrum, are as a rule, decidedly faint. There are obvious discrepancies from the smooth curve, but great care was taken in measurement, and many plates were re-measured with no appreciable change. The fact that the range is less than half that of α_1 renders these discrepancies more obvious.

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[7] L. O. B., Vol. IV, p. 55, 1897.
[8] Pulkowa Bulletin, Vol. 10, p. 192.

RADIAL VELOCITIES OF α_2 GEMINORUM

No.	Date		Julian Date	Velocity
	1926			
14690	Oct.	12	2424800·728	+17·91
91		12	·736	+ 9·17
92		12	·750	+14·33
14700		14	802·728	+ 0·99
01		14	·740	+ 9·32
02		14	·744	+ 9·32
18		18	806·737	−10·66
21		19	807·835	+ 0·69
23		19	·864	+ 3·85
28		22	810·752	+ 7·21
31		22	·864	+ 8·22
41		28	816·734	− 7·67
47		30	818·728	+13·37
55	Nov.	6	825·655	− 9·99
93	Dec.	3	852·729	−14·97
14803		7	856·728	+ 7·73
04		7	·741	+ 3·61
09		15	864·751	+13·03
21		23	872·654	+ 8·68
41		23	·709	− 6·50
	1927			
53	Jan.	11	891·550	+16·62
64		21	901·698	+15·29
68		23	903·658	+ 2·44
76		25	905·749	+ 5·76
82		27	907·686	− 5·89
93	Feb.	3	914·702	+ 2·04
98		5	916·624	− 8·38
14908		11	922·763	+ 2·72
10		13	924·660	− 3·37
19		28	939·726	+ 7·66
24	Mar.	1	940·722	+ 4·33
35		4	943·706	− 4·74
39		5	944·647	−11·87
48		11	950·665	+ 4·57
50		12	951·578	− 0·19
55		16	955·665	+11·57
64		19	959·494	+ 6·99
79		24	963·586	−13·46
85		25	964·710	+ 6·59
86		26	965·509	+15·56
95		29	968·664	+ 4·56
99		30	969·658	+ 2·90
15064	April	28	998·536	− 3·38
72	May	3	2425003·557	+16·07
73		3	·615	+14·90
86		9	009·574	− 9·39
87		9	·588	− 9·36
88		9	·598	− 8·60

These were combined in normal places as follows:

No.	Phase	Velocity	Weight
1	0·027	+ 3·85	0·2
2	0·072	+ 6·69	0·2
3	0·252	+ 9·96	0·3
4	0·795	+16·03	0·3
5	1·625	+13·86	0·6
6	2·096	+13·85	0·7
7	2·860	+ 6·89	0·9
8	3·572	+ 3·23	0·5
9	4·122	+ 5·21	1·0
10	4·474	+ 4·57	0·2
11	5·020	+ 2·90	0·2
12	5·398	+ 1·05	0·3
13	6·198	− 3·38	0·4
14	6·739	− 4·74	0·2
15	7·084	− 6·50	0·2
16	7·328	− 8·38	0·1
17	7·643	− 8·55	0·3
18	8·075	−11·09	1·0
19	8·595	− 9·99	0·1
20	8·897	− 7·67	0·1
21	9·222	+ 0·69	0·1

From these normal places a curve was drawn and a first orbit obtained by the third Henroteau method. With the sum of the squares as criterion, this was adjusted by experiment until the following orbit was obtained.

$$P = 9^d \cdot 2236$$
$$e = 0 \cdot 50$$
$$K = 13 \cdot 4 \text{ km.}$$
$$\gamma = 2 \cdot 66 \text{ km.}$$
$$\omega = 267° \cdot 8$$
$$T = \text{J.D. } 2,424,890 \cdot 849$$
$$a \sin i = 1,103,900 \text{ km.}$$

Dr. Curtis gives 9·218826 days for the period and Rossowskaja 9·2189. The period given in the list of elements was obtained from the epochs of Curtis and the writer.

Following are the three orbits for comparison purposes:

Author	Epoch	K	γ	ω	e	$a \sin i$	T
Curtis	1904·7	13·557	6·20	265·353	0·5033	1,485,000	2,416,746·385
Rossowskaja	1910·3	13·92	6·18	266·98	0·5	1,523,000	2,418,786·307
Barlow	1926·9	13·4	2·66	267·8	0·50	1,103,900	2,424,890·849
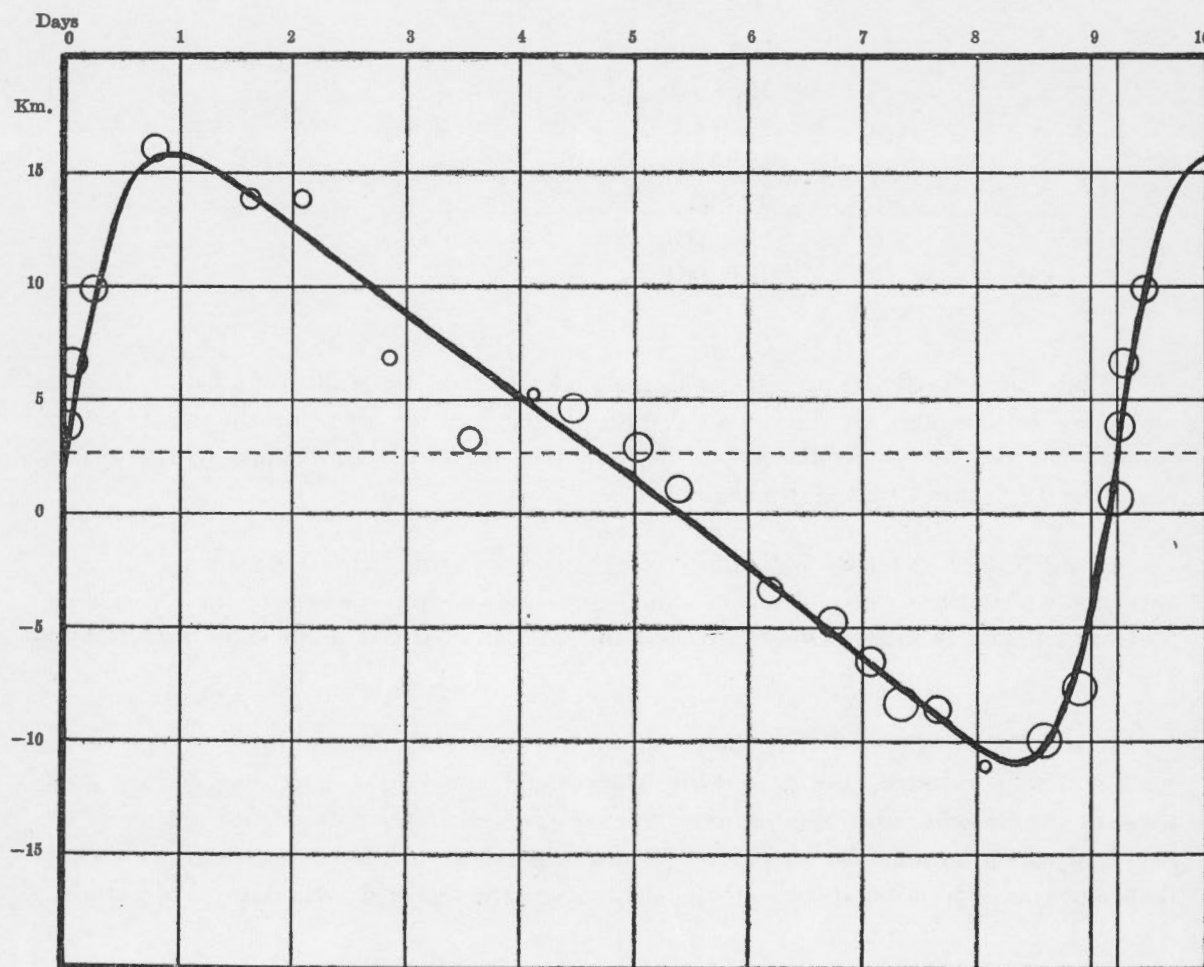
Fig. 3—Orbit of α_2 Geminorum

Dr. Curtis obtained a curve of great smoothness, with no large residuals. Commenting upon the extremes in eccentricity of the two components, he says: "This extraordinary difference seems, by the accepted theories of stellar evolution, to indicate that the brighter component is the older, and that the fainter is, spectroscopically speaking, a binary of relatively recent origin". One would expect the effect of tides to be noticeable in a system of such high eccentricity; but without data of greater accuracy than at present available nothing definite can be determined.

THE VISUAL ORBIT

Castor was discovered to be a double star by Bradley and Pound in 1719. Since then, although it has been frequently observed, its orbit has not been definitely determined. Many orbits have been published, all different; for the present discussion, the elements given by Rabe[9] are adopted:

[9] Astr. Nach., Vol. 216, p. 49.

$$
\begin{aligned}
P &= 306 \cdot 28 \text{ years} \\
T &= 1954 \cdot 728 \text{ A.D.} \\
a &= 6'' \cdot 060 \\
n &= 1° \cdot 1754 \\
e &= 0 \cdot 5593 \\
i &= 113° \cdot 207 \\
\omega &= 278° \cdot 031 \\
\Omega &= 32° \cdot 546
\end{aligned}
$$

When Curtis made his investigation, Doberck had recently published a list of elements and these he adopted for discussion. With these, and his own for the spectroscopic systems, he derived a parallax of $0'' \cdot 05$. On this basis, the total mass of the system would be $12 \cdot 7$ times that of the Sun.

Using Rabe's elements with those of Curtis, the writer finds a parallax of $0'' \cdot 075$, agreeing well with a later determination from photographs made at the Allegheny, McCormick, and Dearborn Observatories, namely $0'' \cdot 077 \pm 0 \cdot 004$, and a value of 12,189,000 km. for $a_1 \, a_2$.

It will be remembered that these figures are functions, among other things, of the centre-of-mass velocity, and that while those determined by Curtis may be accepted, those of the present determination rest, in each case, entirely on the measurement of the standard spectrogram. The figures which might result from the use of these would not, then, serve as criteria for Rabe's orbital elements or the recent determinations of parallax.

Castor C

This companion is distant $73''$ from Castor, in position angle $165°$, and it has the same parallax and proper motion. The apparent magnitude is $9 \cdot 0$. From a spectrogram made on March 15, 1916, at Mount Wilson, it was discovered to be a binary star, having two spectra of about equal brightness and closely similar, with both absorption and emission lines. Thirty-five spectrograms were made by A. H. Joy and R. F. Sanford[10] with results as follows:

$$
\begin{aligned}
P &= 0^d \cdot 814266 \\
K_2 &= 114 \cdot 0 \text{ km. sec.} \\
K_1 &= 126 \cdot 7 \text{ km. sec.} \\
T &= \text{J.D. } 2,423,746 \cdot 524 \\
\gamma &= 4 \cdot 3 \text{ km. sec.} \\
(a_1 + a_2) \sin i &= 2,695,000 \text{ km.} \\
m \sin^3 i &= 0 \cdot 63 \; \odot \\
m_2 \sin^3 i &= 0 \cdot 57 \; \odot
\end{aligned}
$$

---

[10] Mount Wilson Obs. Contributions, Ap. J., Vol. 64, 1926.

H. Van Gent published a paper giving an account of a photometric investigation of Castor C[11] showing it to be an eclipsing system; the results of his computation are as follows, assuming the orbit to be circular:

$$\text{Min. J.D. } 2,424,595 \cdot 4105 + 0 \cdot 81430 \ E$$
$$\pm \ 0 \cdot 0015 \ \pm \ 0 \cdot 00024$$

radius of each component, $r = 406,000$ km.

distance between centres, $a = 2,581,000$ km.

average mass, $\dfrac{M_1 + M_2}{2} = 0 \cdot 518 \ \odot$

density $d = 2 \cdot 596 \ \odot$

This density is much above the average for spectroscopic binaries. A surface temperature of 3,500° has been computed, distinguishing the components as dwarf stars.[12]

In concluding, the writer wishes to express his acknowledgment of the co-operation of Dr. Henroteau and Mr. J. F. Frédette in taking some of the spectra, and of the advice and assistance of the former throughout.

DOMINION OBSERVATORY, OTTAWA.
March, 1928.

[11] Bulletin Astr. Inst. Netherlands, Vol. 3, No. 97, 1926
[12] L'Astronomie, October 1927, p. 482.

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A STUDY OF ETA AQUILÆ

SECOND PAPER

BY

F. HENROTEAU, D.SC., AND A. VIBERT DOUGLAS, PH.D.

In a first paper on η Aquilae,[1] a succinct history of this important variable has been given, as well as all the radial velocities so far published, and the results obtained at the Dominion Observatory on velocities obtained from individual lines and on the variation of spectral class.

The present paper is a continuation of the research; it is a part of an extensive plan of investigation of all the brighter Cepheids with all the modern means we have at our disposal. Seventeen additional one-prism spectrograms were secured, which have been measured on a Hartmann spectrocomparator. These and previous ones have been analyzed with a Moll thermopile microphotometer to investigate the structure and intensity of individual spectral lines. A photo-electric photometer attached to the 15-inch equatorial was also used to determine a light-curve. As, however, this was the first work done with this instrument, and was largely experimental, the accuracy is not on a par with that of later measures; its discussion is not included in the present paper.

The observing was done at the Dominion Observatory by F. Henroteau with the assistance of Douglas Barlow, while the Moll microphotograms were made at McGill University in Montreal, by A. Vibert Douglas. The reducing work was divided nearly equally between the two, with the assistance of Douglas Barlow and of Miriam S. Burland, both of the Dominion Observatory. Some of the spectrograms were also taken by J. F. Frédette.

The present paper comprises: (1) The radial velocities, (2) The variation of ionization, band intensity and other characteristics of the spectrum.

[1] Dom. Obs. Pub. Vol. IX, No. 5, p. 229.

THE RADIAL VELOCITIES

Table I gives the Ottawa velocities for 1927. They have been measured on the spectrocomparator, using the same standard of Polaris as was used to measure the plates of 1924 and 1925.

TABLE I

Date	Julian Day	Phase	Velocity
		d	km.
1927 June 16..	2425047·752	1·013	−32·2
21..	052·719	5·980	− 0·6
24..	055·719	1·803	−30·6
July 2..	063·734	2·641	−22·8
9..	070·680	2·410	−18·1
11..	072·701	4·431	− 8·4
13..	074·624	6·354	−17·7
15..	076·620	1·174	−29·4
19..	080·625	5·179	− 4·2
21..	082·622	0·000	−37·4
28..	089·687	7·064	−34·0
30..	091·654	1·854	−30·5
Aug. 4..	096·654	6·854	−29·6
18..	110·667	6·514	−21·1
25..	117·627	6·297	−10·7
26..	118·667	0·161	−31·8
27..	119·655	1·149	−28·2

The phases have been computed from the formula Max. =J. D. 2414827·150 ± 7^d·176678E.
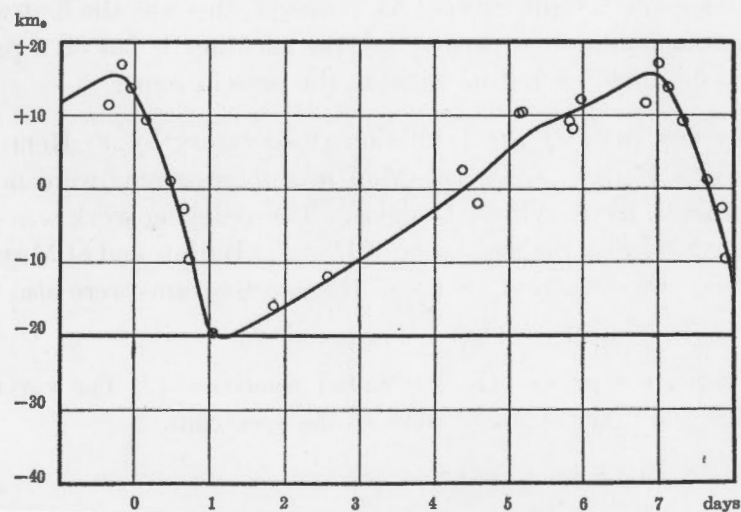
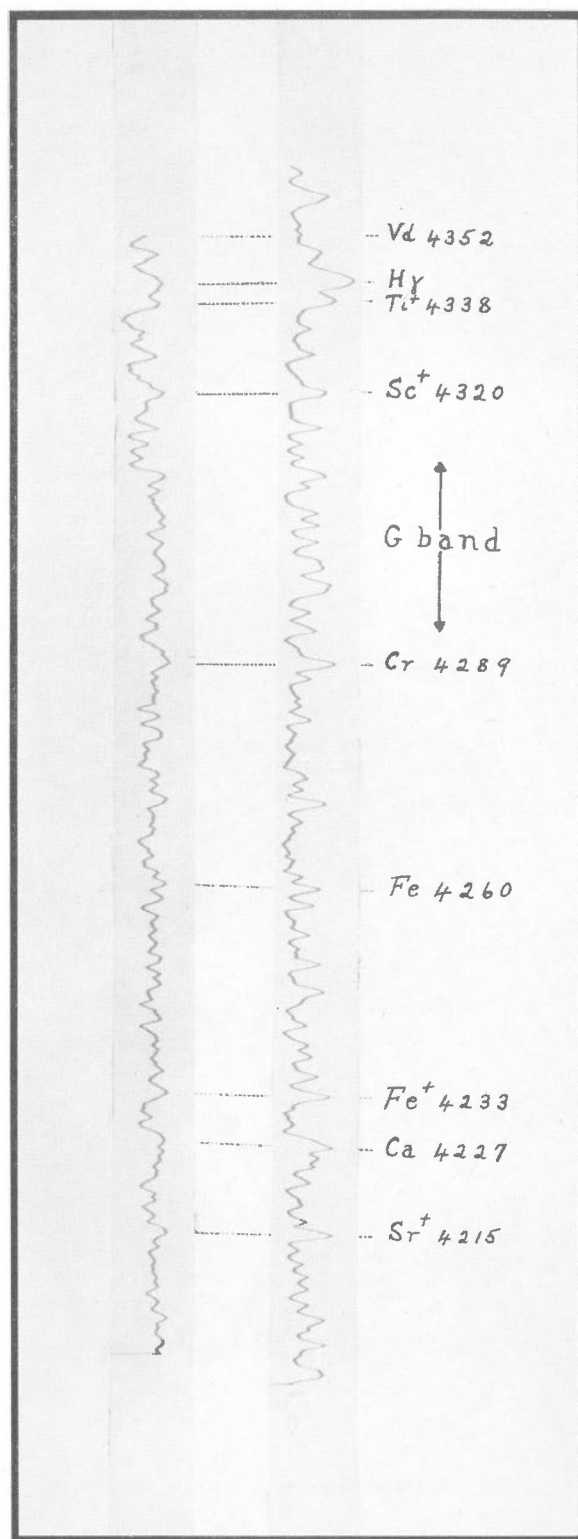Fig. 1.—Radial Velocity Curve of η Aquilae for 1927

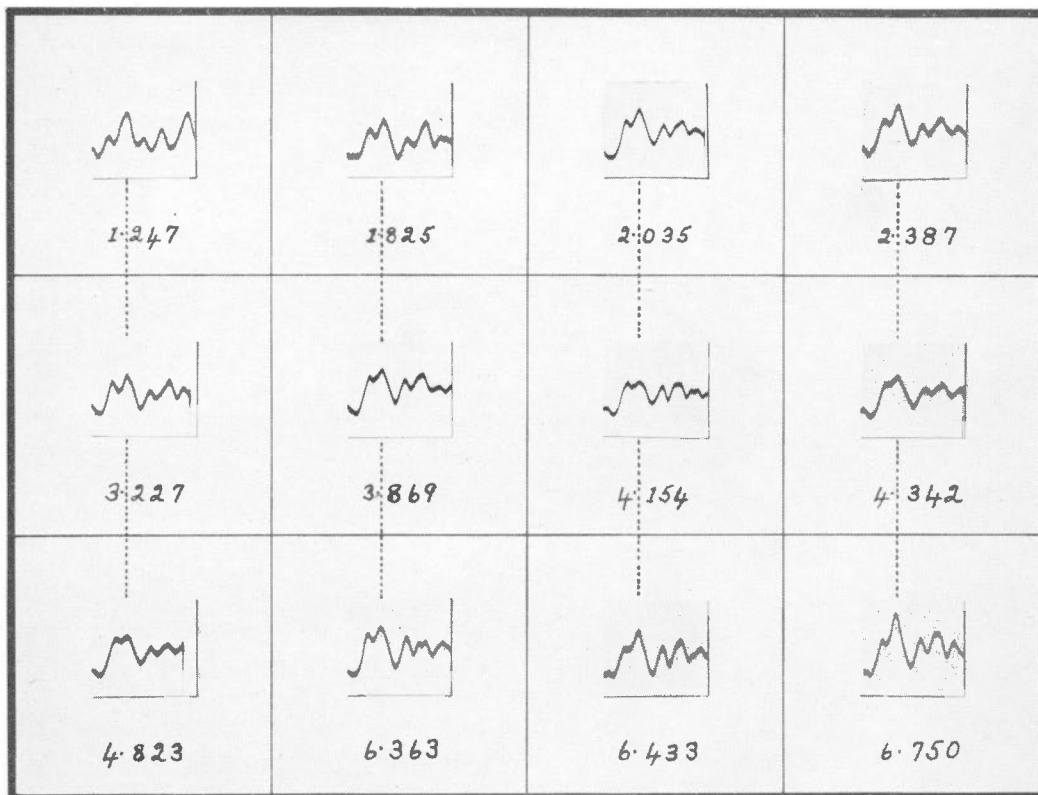Fig. 2.—Microphotograms of η Aquilae for Maximum and Minimum Light

Fig. 3.—Phase Variation of Pair of Titanium Lines.

The velocity curve for 1927 is given in fig. 1. It is interesting to compare this curve with those of 1924 and 1925 which are on page 136 of the present volume. Its amplitude is now 36 km., while in the other years it was about 44 km., and, in particular, the maximum radial velocity comes now about 12 km. below the maxima of 1924 and 1925.

No doubt there is a change in the amplitude and the centre-of-mass velocity of the radial velocity curve. This is in agreement with the results found by F. Henroteau for other Cepheids and for stars of the β Canis Majoris type.[1] It is also paralleled by the results obtained by F. Becker[2] based on Plassmann's observations, showing that the light-curve of η Aquilae had a variable amplitude (see fig. 1, p. 131 of the present volume).

THE VARIATION OF IONIZATION, BAND INTENSITY AND OTHER CHARACTERISTICS OF THE SPECTRUM

Observations[3] by Shapley have shown that the spectral classification of η Aquilae changes from F2 at Phase $6^d \cdot 8$ to G9 at Phase $4^d \cdot 9$. The outstanding changes in the spectral characteristics to be looked for are therefore a gradual decrease in the intensity of lines due to ionized atoms followed by a rapid increase, a similar variation in the Balmer lines of hydrogen, a reverse variation in the intensity of lines due to neutral atoms, and still greater variation in the intensity of spectral bands. Obviously variation of temperature and variation of pressure will affect both lines and bands, but very likely in different proportions. This may be extremely important, as by a combination of the line intensity curve and the band intensity curve it will be possible with the aid of a suitable theory to determine both temperature and pressure.

Microphotograms were obtained with a Moll instrument of most of the spectrograms of η Aquilae secured at the Dominion Observatory. Fig. 2 is a reproduction of a portion of two of these, the right showing the range $H\gamma$ to $\lambda\, 4210$ at maximum phase, the left showing the same portion of the spectrum near minimum phase. The changes in the intensities of $H\gamma$, Ti^+ 4338, Sc^+ 4320, Fe^+ 4233, Sr^+ 4215 and of the G band are very marked. Fig. 3 shows the pair of titanium lines Ti^+ $4533 \cdot 97$ and Ti $4534 \cdot 78$ as obtained from several spectrograms of different phases. The changes in relative intensity are at once apparent.

A large number of lines were studied in the following way: a smooth curve representing the continuous background spectrum was drawn in and the distances from this line to the various maxima of the microphotogram curve were considered to be a measure of the intensities of the lines corresponding to these maxima. Some neutral line was selected closely adjacent to each of the ionized lines under consideration and the ratio of the neutral line to the ionized line determined in each case. In the case of $H\gamma$ several neutral lines were tried out as comparisons, and in investigating the G band its maximum height above the smooth curve was compared with the height of the chromium line, $\lambda\, 4289$, just beyond the band.

[1] Jour. R. A. S. C. Vol. 21, 1927, p. 268.
[2] A.N. Vol. 225, p. 1. See also Die Himmelswelt, Vol. 35, 1925, p. 160.
[3] H. C. 313, 1927.

Table II gives the ionized lines which have been found to vary in intensity as compared with the respective comparison neutral lines also given. The scale of the ratio of intensities is assumed to be comparable from spectrogram to spectrogram throughout the entire cycle of variation.

TABLE II

Atomic number of ionized element	Lines of ionized element		Lines of neutral element	
1...	$H\gamma$	4340·4	Fe	4325·7
			Vd	4330·0
			Vd	4352·8
21..	Sc^+	4246·8	Ba	4242·6
		4314·1	Ca	4318·6
		4320·7	Ca	4318·6
		4415·5	Ca	4425·4
22..	Ti^+	4307·8	Ca	4318·6
		4337·9	Vd	4330·0
		4533·9	Ti	4534·7
		4563·7	Cr	4565·5
		4571·9	Cr	4565·5
26..	Fe^+	4233·1	Fe	4229·7
		4555·9	Vd	4560·7
38..	Sr^+	4215·5	Tb	4213·5
39..	Y^+	4309·6	Ca	4318·6
		4374·9	Vd	4379·2
		4398·0	Fe	4404·7
		4422·6	Ca	4425·4
56..	Ba^+	4554·0	Vd	4560·7
57..	La^+	4558·4	Vd	4560·7
		4580·0	Cr	4565·5

In Table III are given the measurements for nine of the above lines and the resulting ratios.

TABLE III

Julian Date	Phase	Ti+ 4534	Ti+ 4338	Hγ	G band 4308	Ti 4534·78	Vd 4330	Vd 4352	Fe 4325	Cr 4289	Ti+ 4534 / Ti 4534·7	Ti+ 4338 / Vd 4330	Hγ / Vd 4330	Hγ / Vd 4352	Hγ / Fe 4325	G band / Cr 4289
2424005·645	6·904	0.84	0.46	0.48	0.14	0.58	0.30	0.43	0.50	0.45	1.45	1.53	1.60	1.12	0.96	0.31
008·700	2·782	0.50	.....	.....	.....	0.38	.....	.....	.....	.....	1.32					
009·585	3·667	{0.50 / 0.55}	.....	.....	.....	{0.43 / 0.50}	.....	.....	.....	.....	{1.16 / 1.10}					
010·699	4·781	0.39	.....	.....	.....	0.37	.....	.....	.....	.....	1.05					
012·581	6·663	0.95	0.55	0.65	0.10	0.70	0.42	0.55	0.54	0.50	1.36	1.31	1.55	1.18	1.20	0.20
012·668	6·750	{0.98 / 1.03}	0.85	0.95	0.28	{0.55 / 0.55}	0.54	0.63	0.75	0.90	{1.78 / 1.87}	1.57	1.76	1.51	1.26	0.31
013·692	0·598	{0.73 / 0.73}	0.40	0.51	0.09	{0.42 / 0.45}	0.33	0.35	0.38	0.40	{1.74 / 1.62}	1.21	1.54	1.45	1.34	0.22
016·569	3·475	{0.35 / 0.53}	.....	.....	.....	{0.30 / 0.40}	.....	.....	.....	.....	{1.16 / 1.32}					
022·677	2·406	0.25	.....	.....	.....	0.25	.....	.....	.....	.....	1.00					
023·645	3·374	{0.40 / 0.40}	.....	.....	.....	{0.32 / 0.35}	.....	.....	.....	.....	{1.25 / 1.14}					
024·585	4·314	{0.35 / 0.35}	.....	.....	.....	{0.40 / 0.39}	.....	.....	.....	.....	{0.87 / 0.90}					
025·593	5·322	0.50	.....	.....	.....	0.42	.....	.....	.....	.....	1.19					
027·586	0·139	{0.65 / 0.68}	0.38	0.44	0.08	{0.35 / 0.38}	0.20	0.33	0.30	0.45	{1.86 / 1.79}	1.90	2.20	1.33	1.47	0.18
031·601	4·154	0.55	.....	.....	.....	0.55	.....	.....	.....	.....	1.00					
032·588	5·141	{0.87 / 0.83}	0.75	0.85	0.35	{0.89 / 0.78}	0.65	.....	0.80	0.77	1.00	1.15	1.31	.....	1.06	0.45
033·569	6·122	0.65	.....	.....	.....	0.56	.....	.....	.....	.....	1.16					
034·581	7·134	{0.70 / 0.60}	1.09	1.66	0.15	{0.36 / 0.28}	0.65	0.84	0.84	1.08	{1.94 / 2.14}	1.68	2.55	1.98	1.97	0.14
040·565	5·942	{0.55 / 0.60}	0.23	0.33	.....	{0.45 / 0.52}	0.15	0.25	0.18	.....	{1.22 / 1.15}	1.53	2.20	1.32	1.83	
043·551	1·751	{0.44 / 0.35}	.....	.....	.....	{0.34 / 0.28}	.....	.....	.....	.....	{1.29 / 1.25}					
046·552	4·752	{0.55 / 0.56}	0.38	0.40	0.15	{0.55 / 0.53}	0.35	0.48	0.37	0.25	{1.00 / 1.05}	1.08	1.14	0.83	1.08	0.60
047·566	5·766	{0.41 / 0.43}	.....	.....	.....	{0.38 / 0.41}	.....	.....	.....	.....	{1.08 / 1.05}					
048·551	6·751	{1.10 / 1.05}	0.45	0.45	0.10	{0.63 / 0.68}	0.31	0.38	0.38	0.43	{1.75 / 1.54}	1.45	1.45	1.18	1.18	0.23

TABLE III—Concluded

Julian Date	Phase	Ti+ 4534	Ti+ 4338	Hγ	G band 4308	Ti 4534·78	Vd 4330	Vd 4352	Fe 4325	Cr 4289	Ti+ 4534 / Ti 4534·7	Ti+ 4338 / Vd 4330	Hγ / Vd 4330	Hγ / Vd 4352	Hγ / Fe 4325	G band / Cr 4289
2424053·555	4·579	0·40 / 0·50	.....	.....	.....	0·33 / 0·42	.....	.....	.....	.....	1·21 / 1·29					
055·550	6·574	0·90 / 0·87	0·30	0·38	0·10	0·58 / 0·55	0·29	0·34	0·39	0·39	1·55 / 1·58	1·03	1·31	1·12	0·97	0·26
326·217	4·538	0·75 / 0·70	0·75	0·85	0·40	0·63 / 0·58	0·45	0·78	0·80	0·70	1·19 / 1·20	1·66	1·88	1·09	1·06	0·57
329·207	0·352	0·82 / 0·70	1·02	1·40	0·20	0·43 / 0·35	0·55	0·84	0·82	1·10	1·91 / 2·00	1·85	2·54	1·67	1·71	0·18
331·242	2·387	0·76 / 0·69	0·87	1·10	0·25	0·55 / 0·53	0·58	0·85	0·83	0·90	1·38 / 1·30	1·50	1·90	1·29	1·32	0·28
333·197	4·342	0·65 / 0·68	.....	.....	.....	0·57 / 0·65	.....	.....	.....	.....	1·14 / 1·04					
335·218	6·363	0·78 / 0·73	1·03	1·10	0·32	0·60 / 0·63	0·59	1·02	0·90	0·85	1·30 / 1·16	1·75	1·86	1·08	1·22	0·38
338·202	2·171	0·70 / 0·65	0·48	0·45	0·10	0·42 / 0·43	0·33	0·48	0·42	0·40	1·66 / 1·51	1·45	1·36	0·94	1·07	0·25
340·201	4·170	0·50	.....	.....	.....	0·45	.....	.....	.....	.....	1·11					
341·207	5·176	0·33 / 0·80	0·53	0·59	0·15	0·29 / 0·77	0·23	0·45	0·45	0·38	1·14 / 1·04	2·30	2·56	1·31	1·31	0·39
343·194	7·163	0·70 / 0·73	0·88	0·95	0·18	0·25 / 0·25	0·53	0·75	0·75	0·80	3·00 / 2·92	1·66	1·79	1·26	1·26	0·22
345·243	2·035	0·73 / 0·76	0·80	1·00	0·25	0·58 / 0·58	0·48	0·73	0·67	0·80	1·26 / 1·29	1·66	2·08	1·36	1·49	0·31
346·226	3·018	0·60	1·35	1·68	0·30	0·40	0·75	1·04	1·08	1·25	1·50	1·80	2·24	1·62	1·56	0·13
352·193	1·809	0·65	0·65	0·83	0·20	0·50	0·43	0·58	0·53	0·58	1·30	1·51	1·93	1·43	1·56	0·34
359·208	1·648	0·65	0·77	1·00	0·20	0·52	0·37	0·65	0·63	0·70	1·25	2·08	2·70	1·54	1·58	0·28
367·178	2·441	0·52 / 0·45	0·34	0·52	0·06	0·42 / 0·38	0·35	0·38	0·30	0·31	1·24 / 1·18	0·97	1·48	1·37	1·73	0·19
368·116	3·379	0·60	0·30	0·40	0·15	0·45	0·16	0·30	0·25	0·30	1·33	1·87	2·50	1·25	1·60	0·50
371·141	6·404	0·52 / 0·50	0·29	0·38	0·12	0·48 / 0·40	0·25	0·33	0·35	0·35	1·08 / 1·25	1·16	1·52	1·15	1·08	0·34
375·140	3·227	0·55 / 0·63	0·60	0·70	0·15	0·45 / 0·53	0·42	0·50	0·58	0·50	1·22 / 1·19	1·43	1·67	1·40	1·21	0·30
380·128	1·038	0·65 / 0·70	0·45	0·50	.....	0·40 / 0·42	.....	.....	.....	.....	1·62 / 1·66					
384·105	5·015	0·48 / 0·75	0·45	0·45	0·25	0·45 / 0·70	.....	.....	.....	0·45	1·06 / 1·07	.....	.....	.....	.....	0·56
385·110	6·020	0·70	0·70	0·79	.....	0·68	0·57	0·72	0·60	.....	1·03	1·23	1·38	1·09	1·32	

2424388·091	1·825	0·57 / 0·57 / 0·55				0·43 / 0·35 / 0·36					1·32 / 1·63 / 1·53					
389·147	2·881	0·40 / 0·48	0·35	0·40		0·35 / 0·40		0·36			1·14 / 1·20			1·11		
390·058	3·792	0·35				0·33					1·06					
390·135	3·869	0·70 / 0·75	0·48	0·55	0·20	0·60 / 0·60				0·45	1·16 / 1·25					0·44
391·089	4·823	0·75 / 0·75	0·79	0·85	0·37	0·69 / 0·71	0·51	0·78	0·78·	0·78	1·08 / 1·06	1·55	1·67	1·09	1·09	0·47
394·137	0·695	0·25				0·20					1·25					
395·116	1·674	0·55 / 0·50	0·32	0·40		0·40 / 0·38					1·37 / 1·32					
396·072	2·630	0·50 / 0·55	0·40	0·59	0·08	0·36 / 0·40	0·37	0·44	0·44	0·40	1·39 / 1·37	1·08	1·59	1·34	1·34	0·20
403·051	2·432	0·77 / 0·80	0·85	1·00	0·30	0·55 / 0·55	0·53	0·90	0·75	0·95	1·40 / 1·45	1·60	1·88	1·11	1·33	0·32
409·042	1·247	0·75 / 0·70	0·88	1·08	0·20	0·38 / 0·34	0·38	0·68	0·58	0·65	1·97 / 2·06	2·32	2·84	1·59	1·86	0·31
409·113	1·318	0·70	0·85	1·23		0·40	0·52	0·80	0·75		1·75	1·63	2·36	1·54	1·64	
415·545	0·573	0·68 / 0·70	0·75	1·00	0·20	0·42 / 0·42				0·75	1·62 / 1·66					0·26
428·581	6·433	0·75 / 0·70	0·68	0·81	0·20	0·48 / 0·45	0·58	0·67	0·60	0·67	1·56 / 1·55	1·17	1·40	1·21	1·35	0·30
2425047·752	1·259	1·10	1·70	2·50	0·65	0·90	0·53	1·35	1·20	1·60	1·22	3·21	4·72	1·85	2·08	0·40
052·719	6·226	1·15	1·00	1·35	0·50	1·10	0·58	1·05	0·95	1·00	1·04	1·72	2·33	1·28	1·42	0·50
055·719	2·049	0·95	1·15	1·40	0·50	0·95	0·60	1·15	0·80	1·05	1·00	1·92	2·33	1·22	1·75	0·48
063·734	2·888	1·15	1·40	1·80	0·60	1·00	0·88	1·26	1·05	1·15	1·15	1·59	2·04	1·43	1·71	0·52
070·680	2·658	1·05 / 0·90	1·25	1·55	0·50	1·10 / 0·95		1·07		1·05	0·95 / 0·95			1·68		0·48
072·701	4·679	0·75	0·55	0·60		0·80	0·25	0·51	0·45		0·94	2·20	2·40	1·18	1·33	
074·624	6·602	1·13	0·85	1·00	0·20	1·13		0·72		0·55	1·00			1·39		0·36
076·620	1·421	1·05	1·22	1·60	0·50	1·00	0·60	0·94	1·00	1·05	1·05	2·03	2·66	1·70	1·60	0·48
080·625	5·426	0·57	0·57	0·62	0·25	0·60	0·28	0·46	0·42	0·45	0·95	2·04	2·21	1·34	1·47	0·55
082·622	0·247	0·70	1·20	1·65	0·15	0·40	0·62	0·85	0·80	1·05	1·75	1·94	2·66	1·94	2·06	0·14
089·687	0·136	0·85	1·52	2·00	0·20	0·58	0·83	1·10	1·08	1·50	1·46	1·84	2·41	1·82	1·85	0·13
091·654	2·102	0·44	1·05	1·30	0·40	0·44	0·52	0·84	0·80	1·10	1·00	2·02	2·50	1·55	1·62	0·36
096·654	7·102	1·05 / 0·80	1·60	2·35	0·15	0·58 / 0·50		1·00		1·90	1·81 / 1·60			2·35		0·79
110·667	6·763	0·85	1·15	1·50	0·35	0·60	0·62	1·00	1·00	1·10	1·42	1·86	2·42	1·50	1·50	0·32
117·627	6·546	0·65	0·50	0·60	0·10	0·45	0·42	0·51	0·51	0·45	1·44	1·19	1·43	1·17	1·17	0·22
118·667	0·410	0·65	1·30	1·60	0·30	0·60	0·40	0·86	0·65	0·85	1·08	3·25	4·00	1·86	2·46	0·35
119·655	1·398	0·35	0·36	0·38	0·10	0·35	0·21	0·35	0·30	0·25	1·00	1·71	1·81	1·08	1·26	0·40

TABLE IV

Phase	Ti+ 4534 Ti 4534·7			Ti+ 4338 Vd 4330			Hγ Vd 4352			Hγ Vd 4330			Hγ Fe 4325			G band Cr 4289		
	x	y	Σp	x	y	Σp	x	y	Σp	x	y	Σp	x	y	Σp	x	y	Σp
0—0·512	0·252	1·616	362	0·267	2·206	542	0·368	1·782	398	0·344 0·325	2·839 2·50	709 549	0·266	1·752	709	0·337	0·186	495
0·512—1·025	0·602	1·599	167	0·598	1·210	40	0·598	1·450	35	0·598	1·540	51	0·598	1·340	51	0·582	0·241	115
1·025—1·538	1·279	1·413	455	1·316	2·390	501	1·317	1·647	412	1·314 1·346	3·345 2·544	679 429	1·314	1·806	679	1·314	0·407	355
1·538—2·050	1·854	1·250	448	1·914	1·816	337	1·917	1·358	311	1·904	2·006	423	1·904	1·611	423	1·911	0·366	313
2·050—2·563	2·359	1·390	291	2·290	1·618	359	2·305	1·266	345	2·296	1·968	437	2·296	1·434	437	2·294	0·304	366
2·563—3·075	2·777	1·182	427	2·911	1·615	315	2·834	1·504	417	2·904	2·057	407	2·904	1·594	407	2·840	0·349	385
3·075—3·588	3·354	1·247	203	3·278	1·576	90	3·284	1·343	80	3·282	1·972	110	3·282	1·352	110	3·284	0·375	80
3·588—4·101	3·786	1·205	169	No	values		No	values		No	values		No	values		3·869	0·440	45
4·101—4·613	4·357	1·102	323	4·538	1·660	75	4·538	1·090	78	4·538	1·880	85	4·538	1·060	85	4·538	0·570	70
4·613—5·126	4·808	1·024	307	4·761	1·654	172	4·762	1·045	177	4·761	1·792	185	4·761	1·166	185	4·869	0·519	148
5·126—5·638	5·251	1·047	248	5·239	1·754	185	5·302	1·325	91	5·236	1·939	206	5·236	1·255	206	5·229	0·464	160
5·638—6·151	5·984	1·108	235	6·000	1·304	93	5·999	1·149	97	5·997	1·622	112	5·997	1·470	112	No	values	
6·151—6·664	6·452	1·318	560	6·418	1·437	445	6·444	1·206	519	6·414	1·758	527	6·414	1·255	527	6·441	0·345	476
6·664—7·176	6·917	1·821	606	6·927	1·664	488	6·962	1·658	503	6·938	2·113	599	6·938	1·487	599	6·924	0·251	476

These ratios have been used to obtain their curves of variation with respect to phase. Normal points, however, have been derived by dividing the period into fourteen equal sections, assuming for the weight of each ratio the measured intensity of the first line of the pair and taking for all ratios comprised in one section the respective values of the abscissa and of the ordinate

$$x = \frac{p_1 x_1 + p_2 x_2 + \ldots \ldots p_n x_n}{p_1 + p_2 + \ldots \ldots p_n} \qquad\qquad y = \frac{p_1 y_1 + p_2 y_2 + \ldots \ldots p_n y_n}{p_1 + p_2 + \ldots \ldots p_n}$$

where $x_1, x_2, \ldots \ldots, x_n$ are the phases corresponding to the different ratios, $x_1, x_2, \ldots \ldots,$ y_n the values of these ratios and $p_1, p_2, \ldots \ldots p_n$ their assumed weights.

The normal points for the ratios in Table III are given with their respective weights in Table IV and from these the curves given in figs. 4, 5, 6, 7, 8 and 9 have been plotted.
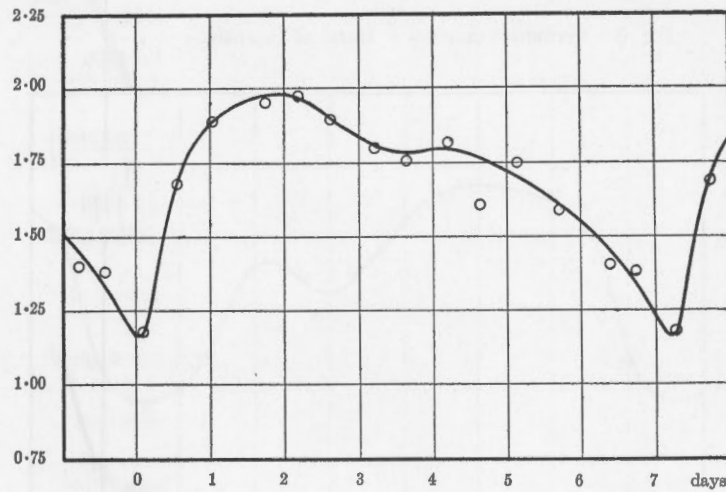
Fig. 4.—Periodic Variation of Ratio of Intensities $\dfrac{\text{Ti}^+\ 4535}{\text{Ti}\ 4534\cdot7}$
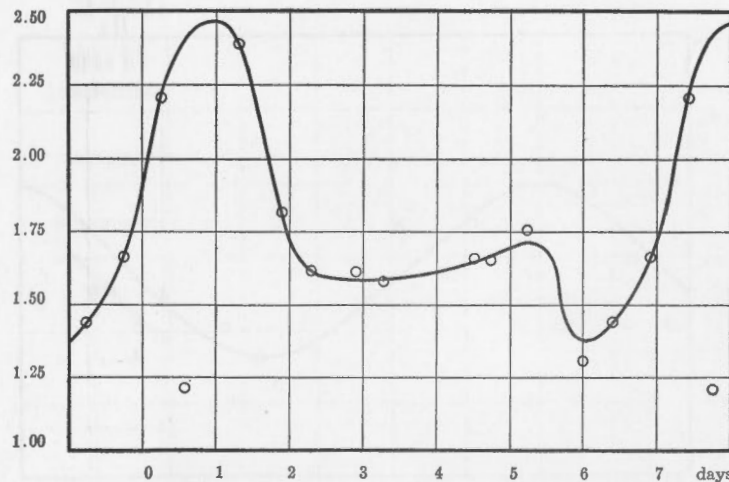
Fig. 5.—Periodic Variation of Ratio of Intensities $\dfrac{\text{Ti}^+\ 4338}{\text{Vd}\ 4330}$
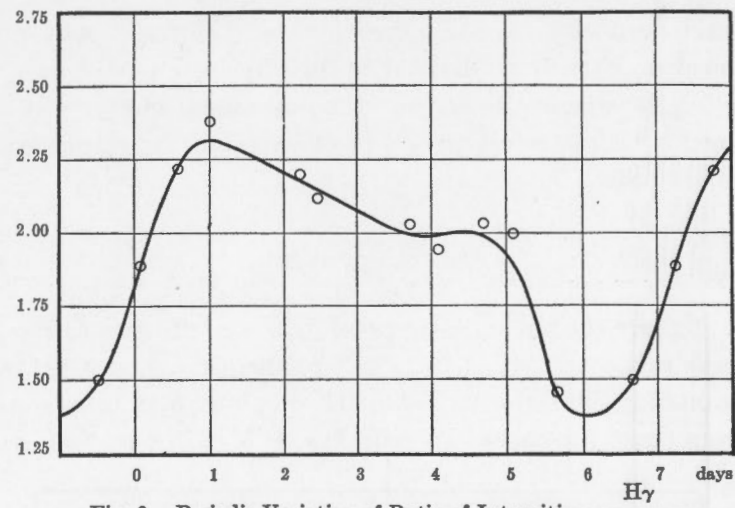
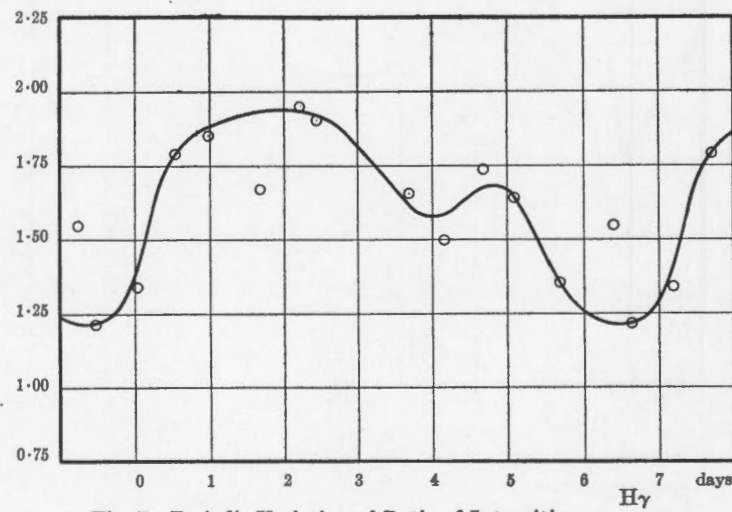Fig. 6.—Periodic Variation of Ratio of Intensities $\dfrac{H\gamma}{Vd\ 4330}$

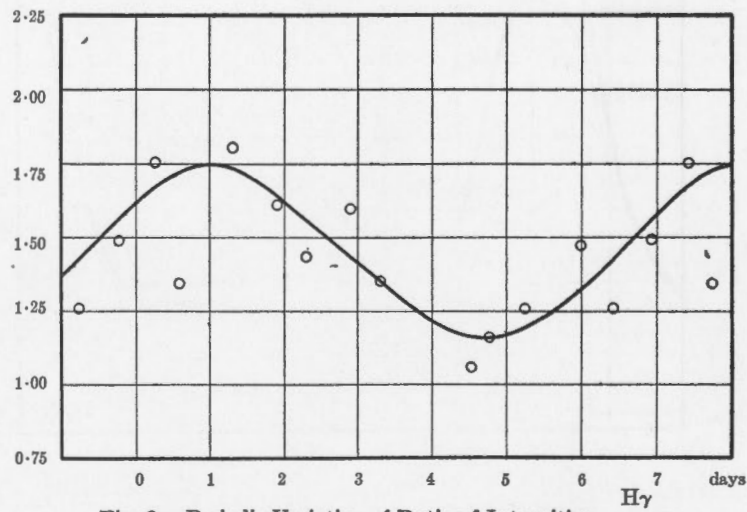Fig. 7—Periodic Variation of Ratio of Intensities $\dfrac{H\gamma}{Vd\ 4352}$

Fig. 8.—Periodic Variation of Ratio of Intensities $\dfrac{H\gamma}{Fe\ 4325}$
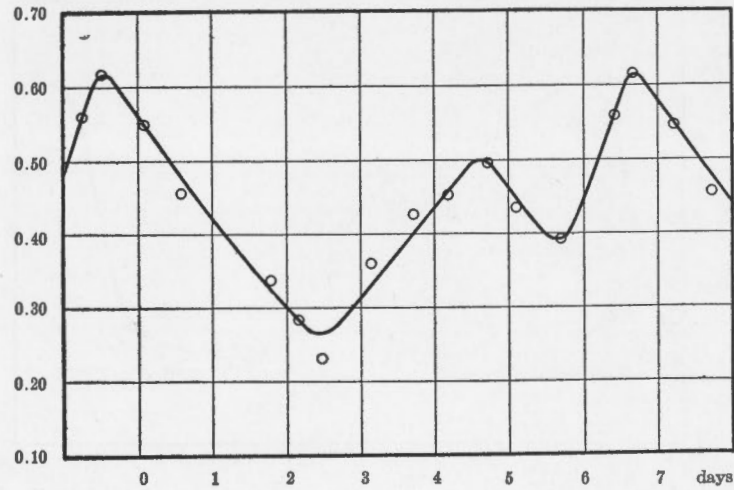
Fig. 9.—Periodic Variation of Ratio of Intensities $\dfrac{\text{G band}}{\text{Cr 4289}}$

Fig. 4 can be compared with much advantage with the corresponding curve of ratios of titanium lines published on p. 145 of this volume. That curve, however, was obtained from eye estimates.

Fig. 9, the curve for the G band, shows exactly the reverse tendency to the curve of ionization. The G band is strongest at minimum light, as was to be expected.

Secondary oscillations are also found in these various curves, just as they were established in the light-curve and in some of the radial velocity curves.
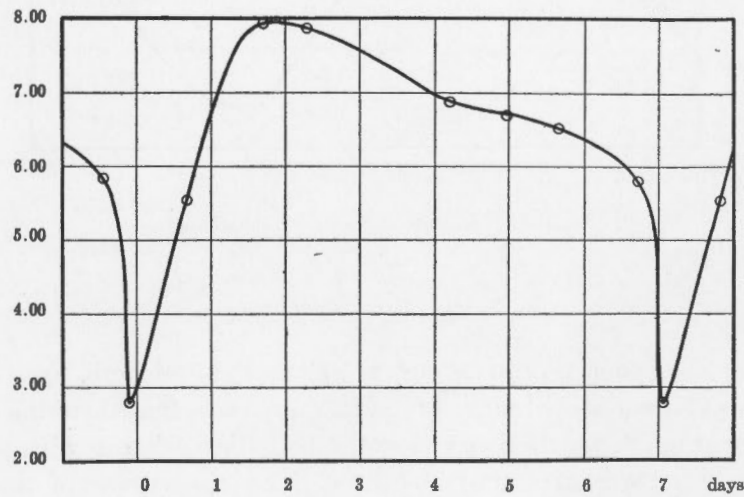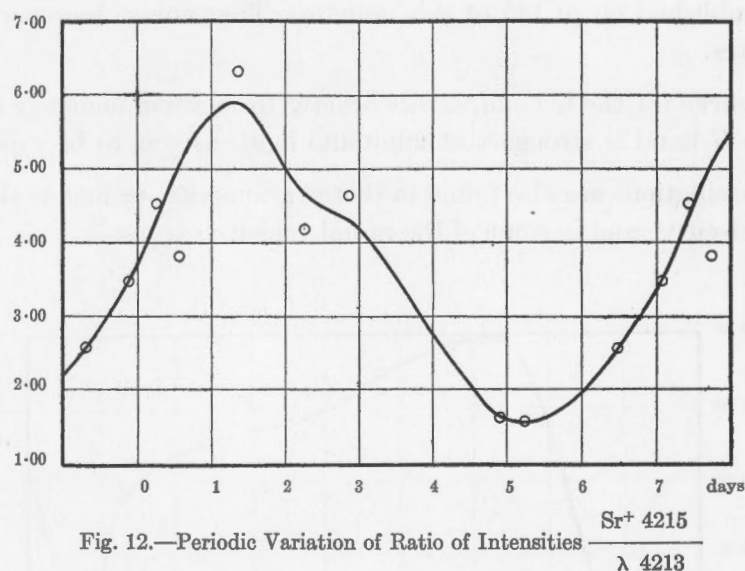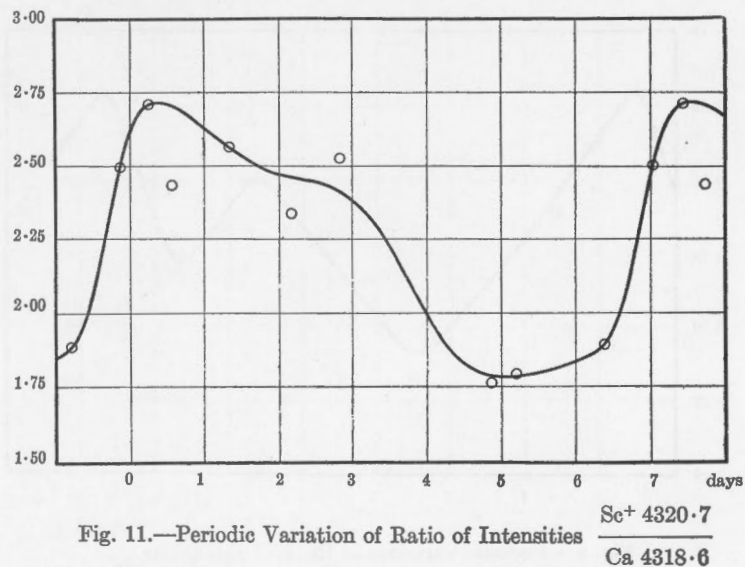

Fig. 10.—Periodic Variation of Ratio of Intensities $\dfrac{\text{Fe}^+ \ 4555 \cdot 9}{\text{Vd 4561}}$

Fig. 11.—Periodic Variation of Ratio of Intensities $\dfrac{\text{Sc}^+\ 4320\cdot7}{\text{Ca}\ 4318\cdot6}$

Fig. 12.—Periodic Variation of Ratio of Intensities $\dfrac{\text{Sr}^+\ 4215}{\lambda\ 4213}$

Only three of the other seventeen curves are reproduced here, figs. 10, 11 and 12. They are based upon fewer measurements and in most cases the scattering is considerably greater than that of fig. 4, but in every case the periodic variation with phase is clearly evident. The distinctive characteristics exhibited by the curves of lines of different elements have been used in another investigation [1] in which astrophysical estimates of the *ionization potentials* of *iron, yttrium* and *lanthanum* have been obtained as follows:—

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[1] Nature, June 9, 1928, p. 906.

Element	Astrophysical estimate of I.P.	Previous determinations of I.P.
Fe..	6·6	5·9, 8·15 (Sommerfeld, Gieseler, Grotrian). 7·5 (Menzel.)
Y...	6·6	6·6 (Meggers, Russell)
La..	4·9	– –

A glance at the curves in fig. 2 will show at once that very many lines change in intensity with phase. Some of these are so heavily blended that there seems no advantage to be gained by plotting the variations. It is not exclusively the ionized and hydrogen lines which undergo these changes. Certain neutral lines, both ultimate and penultimate, are similarly, but to a less degree (see fig. 7), strengthened at maximum light. Examples of this are Ca 4227 and Fe 4260 and the ultimate vanadium line λ 4352, whose periodicity in another cepheid variable led one investigator [1] to regard it as an ionized line, attributing it to Ti^+. Other neutral lines fall in intensity at maximum light and are strengthened noticeably at minimum. This is the case with Ti 4534·78 (see fig. 3), and in fact the periodicity shown in fig. 4 is due in larger measure to the rising and falling of the arc line than to the waning and rising of the ionized line. This is not the case, however, with figs. 5, 10, 11 and 12, which are curves of ionization intensity in the strictest sense.

DOMINION OBSERVATORY,

OTTAWA, September, 1928.

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[1] Ap. J. October, 1927, p. 180.