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THE OTTAWA PHOTOGRAPHIC ZENITH TELESCOPE
BY
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METHOD AND FORMULAE USED IN P.Z.T. PLATE MEASUREMENT
BY
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BY
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## Contents

Page
The Ottawa Photographic Zenith Telescope ..... 319
Method and Formulae Used in P.Z.T. Plate Measurement ..... 345
A Program Machine for Automatic Operation of the Ottawa P.Z.T. ..... 351

# The Ottawa Photographic Zenith Telescope 

## BY

M. M. Thomson


#### Abstract

The Ottawa photographic zenith telescope, utilizing a 10 -inch objective of 14 feet focal length, was first tested in June 1951, and has been operated continuously since October 1951. More than 500 plates have been secured for the purpose of determining fundamental time and variation of latitude. Mean error of a clock correction from a single star is $\pm{ }^{8} 027$. Mean error of a latitude determination from a single star is $\pm 0^{\prime \prime} 29$. A description of the Ottawa instrument, methods used for reducing observations, and a discussion of results is presented.


## HISTORICAL

The origin of the modern P.Z.T. dates back to the days of the Astronomer Royal, Airy, who designed a reflex zenith telescope about the year 1850. Its purpose was to continue observations of the zenith star $\gamma$ Draconis which had been studied for parallax and aberration, and for the determination of the latitude of London. In 1888 Küstner announced the variation of latitude, resulting in increased attention to precise latitude observing. Twenty years later F. E. Ross (1) of the United States Coast and Geodetic Survey used the Airy principle in the development of a photographic zenith telescope (P.Z.T.) for the measurement of latitude variation. In the latter half of the 1920's F. B. Littell and J. E. Willis (2) of the United States Naval Observatory adapted the P.Z.T. to time determination and soon established its superiority over visual observations via the conventional transit telescope.

Preliminary investigations into the use of a P.Z.T. at Ottawa were commenced prior to World War 2, but the project did not become active until 1946 when a recommendation from the executive committee of the International Astronomical Union was received by R. Meldrum Stewart, Dominion Astronomer, advocating the location of such an instrument in Ottawa. Following Mr. Stewart's retirement in the same year, his successor, C. S. Beals, asked the Positional Astronomy Division to prepare concrete plans for the instrument, and public funds were appropriated for the project in 1949-50.

## GENERAL DESCRIPTION

A photographic zenith telescope is mounted rigidly in a vertical position and its range of operation is limited to those stars which culminate close to the zenith in their apparent daily motion from east to west. It is a reflex instrument. Light enters through an objective at the top to a mercury basin at the bottom, then is reflected back on itself and comes to a focus on a small photographic plate located close to and just below the objective. As a consequence of the reflex characteristic, a P.Z.T. with a focal length of, say, 14 feet requires little more than 8 feet of head-room.

The components of the Canadian P.Z.T. are contained in three sections, the rotary, the main tube and the mercury basin. The rotatable head, or rotary, is so called because it is turned through half a circle between each exposure. Contained within it are the
objective lens and the photographic plate. In order to secure point images, the plate is moved at stellar speed by a precise lead screw. Control for the lead screw is a 1000 -cycle motor attached to the side of the rotary which, with its associated gearing, permits the screw to be turned in either direction. No matter which way the rotary is oriented, head east or head west, the plate may always be driven from west to east. The 1000 cycle frequency is derived from a quartz clock. The rotary is of aluminum, for lightness, except for a steel ring at its base. Gear teeth on the side of the ring provide a means of rotation by the reversing motor, while the smooth base of the ring rests on and is


Fig. 1. Ottawa Photographic Zenith Telescope (P.Z.T.) designed with 10 -inch objective lens with a focal length of 168 inches ( 14 feet). The light from a star passes through the lens and is reflected by a mercury mirror at the base to focus on a $2 \times 2$ inch photographic plate located about $\frac{1}{2}$ inch below the lens.

Field of view is 30 minutes of arc.
The telescope is supported by two 8 -inch "I" beams and mounted on two cement piers isolated from the building to eliminate vibration.


Frg. 2. Ottawa P.Z.T. head in the "motor west" position. A vertical shaft from the gear box engages the gear which reverses the head to "motor east" position and then back again between successive exposures. Access doors permit loading of the photographic plate and access to wires. The lens cell is held to the rotary head in strict adjustment by three push pull screws. Two microscopes on the rotary in conjunction with finely etched stationary marks provide accurate adjustment of the 180 -degree reversal.
constrained by roller bearings in the top of the main tube. The main tube is securely fastened to two heavy I beams which are supported on two cement piers. Also mounted on the I beams is the $\frac{1}{8}$ th h.p. reversible a.c. motor and associated gear box which is connected by a vertical transmission rod to the gear ring of the rotary to provide the $180^{\circ}$ reversal. The lower portion of the main tube may be raised to gain access to the mercury basin which is independently attached to the same cement piers by two lengths of angle iron. The basin containing the mercury reflector is itself floating in mercury to cushion it from vibrations, and bence defines the zenith accurately. An exhaust fan connected by a rubber duct to the vent at the base of the mercury basin keeps the air in motion down through the telescope at all times except during an exposure.

## DESIGN OF CANADIAN P.Z.T.

The design of the Canadian P.Z.T. was based on drawings prepared by the United States Naval Observatory for their new instrument now at Richmond, Florida. Modifications in the plans were necessitated due to differences in the focal length, the objective size, and the location in latitude.

It had been remarked by the advisory group at Washington that the minimum focal length of a P.Z.T. should be about 12 feet. Scintillation and refraction irregularities would place an upper limit of about 20 feet. The original Washington instrument has a focal length of 17 feet and an 8 -inch objective. The two new P.Z.T.'s then in prospect were for Richmond, Florida, focal length 12 feet 6 inches with an 8 -inch objective, and for Greenwich, England, 11 feet 3 inches with a 10 -inch objective. In order to provide a plate scale a little greater than one minute of are per millimeter, a focal length of 14 feet was settled upon for Ottawa. The plate scale is therefore close to 48 seconds of are per millimeter.


Fig. 3. Ottawa P.Z.T. with head partly turned to show the 1000 -cycle motor which is the nerve center of the telescope. With a 1000 -cycle control from a quartz crystal source, the motor serves to drive the plate carriage back and forth, and to operate all the switches which control the opening and closing of the exposure shutter, the direction of plate carriage drive, and the reversal of the head between exposures.


Fig. 4. Ottawa P.Z.T. gear box, reversing motor, and switching mechanism for the automatic reversal of the rotor. An 18 -inch boom, driven at its center pin by an arm within the gear box, serves to (a) throw a motor limiting toggle switch with its spring loaded jaws, (b) throw one of two safety switches in case of over run. Spring loaded plungers return the boom to neutral as soon as reversal commences. Also shown is the chassis containing relays, rectifiers and cable terminal strips.

A survey of zenith stars for the Ottawa instrument indicated that it would be necessary to observe stars as faint as the ninth and tenth magnitudes. For this reason an objective of 10 inches in diameter was chosen. The spectral region between 4000 and $5000 \AA$ is utilized with plates sensitive to the blue.

The casting and figuring of the objective were assigned to Perkin-Elmer Company of Glenbrook, Connecticut, with the following specifications:

1. Back focal length 14 feet ( 168 inches).
2. To be used with plates sensitive only to wave lengths shorter than $5000 \AA$
3. Back focal length to be approximately 0.5 inches greater than the equivalent focal length. (The rear nodal point to be behind the final glass surface at least one centimeter). Equivalent focal length to be 167 inches.
4. Diameter $10.4 \pm 0.04$ inches ( $264 \pm 1 \mathrm{~mm}$ ), clear aperture 10 inches ( 254 mm ).
5. Chromatic aberration. The foci for wave lengths of $4860 \AA$ ( $F$, hydrogen) and $4050 \AA$ (h, mercury) on the axis, as determined photographically by autocollimation to be within 0.04 inches ( 1 mm ) of each other. (The value secured by auto-collimation is double the actual permissible chromatic difference).
6. Spherical aberration. Under test by means of the Foucault knife-edge test by auto-collimation as near the axis as is practicable, with light of $4360 \AA$ wave length, the foci of all zones to be within a range of 0.16 inch ( 4 mm ). (This again is double the extreme permissible longitudinal spherical aberration).
7. Coma. To be no noticeable comatic flare of star image by auto-collimation with the object and image each 1.6 inches from the axis ( 38 mm ).
8. Glass. The glass to be of such quality that no striae are visible when the knife edge test by auto-collimation is used. No bubbles or other light absorbing or scattering particles larger than 0.02 inch $(0.5 \mathrm{~mm})$ to be tolerated. Freedom from all such particles is desirable, but as many as 12 below the size specified will be tolerated (total in all components).
9. Light Transmission. The transmission of the assembled lens measured at the focus for wave lengths of $4860 \AA$ and $4050 \AA$ to be at least 70 per cent.
10. The objective to be furnished mounted in a suitable metal cell of stainless steel, having a co-efficient of expansion closely approximating that of glass; handles to be fitted to facilitate removal. Springs to be incorporated into the cell which will press against the sides of the objective elements in such a manner as to positively prevent any motion of the elements within the cell.


Fig. 5. Ottawa P.Z.T. floating mercury reflector and focussing rod. Mounted on the floor near the pier is a fan which draws a current of air through the telescope between star observations to prevent air striations. The hood enclosing the mirror is seen in the "up" position, but is normally down during observations.


Fig. 6. Ottawa P.Z.T. showing "I" beams, gear box with switching arm, reversing motor and chassis containing relays, rectifiers, condensers and cable terminal strips.
11. A flat photographic field $\frac{1}{2}^{\circ} \times \frac{1}{2}^{\circ}$ required.
12. All glass surfaces to be coated to reduce reflecting losses.

The completed lens was tested in all respects in the optical section of the National Research Council, and was found to be well within required tolerances. The nodal point is 18.29 mm behind the vertex of the rear lens surface. Coating of all surfaces was inadvertently omitted at the time of construction and the performance on faint stars has been so satisfactory that it has not been added since.

The other precision components associated with the objective and rotary include the photographic plate carriage, the 1000 -cycle motor, and the lead screw. They were made in the Observatory workshop using, with minor modifications, the drawings for the Richmond, Florida, P.Z.T.

According to the theory of the instrument the plane of the photographic plate must contain the Gaussian or second nodal point of the lens. Then any small tilt of the whole rotary about a horizontal axis will cause no displacement of the image on the plate. In the case of the Ottawa P.Z.T. the nodal point is 18.29 mm below the lower surface, so that sufficient space is available for easy loading of the plate holder. The plate is constrained in the holder by spring-loaded plungers against two edges and by a flat spring in the back cover. The holder slips under spring clips and then is bolted securely to the carriage.


Fig. 7
OTTAWA P•Z•T.

Smooth motion for the plate carriage is provided by tripod mounting with a ball bearing at each foot riding on a hardened steel flat embedded in the aluminum of the rotary. The carriage must move freely in the east-west direction only. Constraint in north-south direction is provided at each end of the carriage by spring tension against two hardened flats which are mounted vertically. Fastened to one end of the carriage is the solenoid which operates the shutter through an arc of 70 degrees. In the closed position the shutter rests at an angle of about $20^{\circ}$ to the horizontal, and when open is vertically down. At the opposite end to the solenoid, the carriage is held in contact with the lead screw by spring tension.

The lead screw is made of steel, and turns in a brass nut. Consideration was given at the time of construction to making the screw of hardened steel, but no facilities for hardening were available at the Observatory. So far there has been no evidence of uneven wear in the screw but a future modification may well include a hardened screw for the plate drive.

The pitch of the screw for the Ottawa P.Z.T. was determined from the following considerations:-

Latitude of Ottawa $=45^{\circ} 23^{\prime} 38^{\prime \prime} .7$
Focal length $\quad=167.47$ inches
Period of revolution $=2$ seconds mean solar time.

$$
\begin{aligned}
\text { Pitch } & =167.47 \times \tan \frac{1^{\prime}}{} \times \cos 45^{\circ} 23^{\prime} 38^{\prime \prime} .7 \times 1.0027379 \\
& =0.017151 \text { inch. }
\end{aligned}
$$

When refraction is included, the required pitch of the screw is 0.0171466 .

$$
\left(1 \cdot 0027379=\frac{366 \cdot 2422}{365 \cdot 2422}=\text { factor for conversion of mean solar to sidereal interval }\right)
$$

By manipulating the gears in the lathe of the Observatory machine shop a ratio was secured of:-

$$
\frac{25}{243} \times \frac{1}{6}=0.017147 \text { inch. }
$$

The expected resolving power of the Toepfer plate measuring machine was of the order of 0.001 mm , which is equal to $0^{5} 004$. In actual practice, bisection of a good stellar image is made with an uncertainty of about $\pm 0.002 \mathrm{~mm}$. Bench tests for periodic and progressive errors in the lead screw made before the P.Z.T. was assembled, have indicated that such errors are too small to be detected.

Vickers Limited of Montreal formed all the castings, after first redrawing the Richmond P.Z.T. plans to allow for the 10 -inch objective and the 14 -foot focal length. The rotary is of aluminum except for the gear ring, while the main tube and mercury basin components are of cast iron.

## LOCATION

The location of the new P.Z.T. was dictated by the desire to keep it near the established meridian ( $5^{\mathrm{h}} 02^{\mathrm{m}} 51^{\mathrm{s}} 940$ ) for initial testing. Some thought was given to a properly designed, well ventilated, building close by on the Observatory grounds, but finally the observing pier and south collimator pier of the Cooke transit were levelled, and the
supporting I beams of the P.Z.T. placed between them. This location in our present transit house is about 10 feet east of the meridian circle, or equivalent to a difference of $0^{\mathrm{s}} 014$ in R.A.

TEST OF P.Z.T.
Initial testing of the new instrument involved levelling, focusing, the adjustment in azimuth, and adjustment of the 180-degree reversal of the rotary. As a preliminary to levelling the instrument, it was first ascertained by bench tests that the steel ring at the base of the rotary and the plane of the plate carriage were parallel. Two spirit levels were then mounted on the side of the rotary which permitted the whole telescope to be levelled on its three-point base. An additional level adapted to test the objective relative to the rotary was used in conjunction with the three push pull mounting screws of the cell to bring it level with the plane of the plate carriage.

Focusing is accomplished by raising or lowering the mercury reflecting surface. A preliminary adjustment was secured by measuring with a steel tape 83.7 inches from the emulsion surface of the photographic plate to the mercury surface. Subsequent adjustments were made using stellar images. It is not necessary to have a numerical measure of best focus if some means is available for preserving it from night to night. An invar rod, pointed at the bottom, and with the bottom section bent and in a swivel so that it may be turned away from the mercury surface, preserves the exact distance from the mercury reflector to the photographic plate.

Azimuth orientation was provided initially by simply aligning the mounting beams parallel to the transit room wall and the slit opening by visual estimation. Successive adjustments were provided by star trailing. The shutter is held open permitting a star to trail in to the center of the plate. The plate is then driven for the maximum interval available, (about 40 seconds), after which the star trails off to the far edge. Any error in azimuth causes a north-south component in the motion of the plate carriage. The trail in to and out from the stellar image will form two parallel lines instead of two sections of the same straight line, and the stellar image will be elongated in the north-south direction.

The rotary is turned through exactly 180 degrees between each exposure. A precise indication of the amount of rotation is provided by two microscopes fixed on opposite sides of the rotary and focused on fiducial marks attached to the main tube. The micrometer eyepieces measure 4.5 seconds of arc per division, which is the tolerance within which the stops are adjusted.

## CONTROL

Control of P.Z.T. exposures at first was effected by a small console located in the Time Room. One toggle switch started and stopped the exposure sequence, while a second switch turned the telescope exhaust fan off and on. The procedure was to turn the fan off one minute before each exposure, and turn it on again immediately afterwards. Failure to turn it off caused a disturbance to the mercury surface resulting in blurred and distorted images. Three colored pilot lights indicated when the rotary was located with head west or head east, or was in transition from one position to the other. It was desirable to commence each exposure on the exact second, because in some cases a
deliberate advance or delay had been arranged to avoid stars of the same zenith distance being superimposed. In December 1952 a P.Z.T. Program Machine, using synchronously driven punched tape, was placed in service and now provides automatic control of each night's observing. Only occasional attention needs to be given, and time may be spent at other duties. (see page 351).


Fig. 8. Controls for the Ottawa P.Z.T. are located in the Time Room. The decade counter measures the fraction of a second between P.Z.T. impulses and second beats of the primary quartz clock. The two switches control the photographic sequence and the fan at the base of the telescope.

## TIMING

Timing of the P.Z.T. exposures is effected by a contact (Figure 9) operated by a cam on the carriage drive screw. The raised portion of the carn causes the circuit to be alternately closed and opened each half revolution. One half revolution of the screw is accomplished in one second of mean solar time, being driven by the 1000 -cycle motor with frequency derived from a quartz clock.

The makes and breaks are fed into a thyratron discharge tube which operates on both impulses to give the starting impulse for a decade counter. Alternate P.Z.T. impulses can be adjusted to within one or two milliseconds of uniformity. They will then bear the same phase relationship to the seconds beats of the control clock as long as the P.Z.T. 1000 -cycle motor continues to run.

The Ottawa P.Z.T. 1000 -cycle motor has been allowed to rotate continuously with no detectable wear. However the practice now is to start the motor afresh each night it is used. The motor, with its associated gear box, is serviced in both spring and autumn as a matter of routine practice. During the winter months, especially when temperatures at Ottawa are around $-18^{\circ} \mathrm{C}$ or colder, a few watts of heat have been fed into the motor box and the whole box insulated.


FIG. 9
The decade counter, started by the P.Z.T. impulse, is stopped by the seconds impulse from the control crystal clock, thus providing an accurate time interval in the sense from P.Z.T. to clock. While the decade counter provides time intervals accurate to one part in a million, it is read only to one part in a thousand, i.e., to the millisecond. Complete timing of a star transit is secured by recording the P.Z.T. impulses on a drum chronograph together with seconds beats from the control clock. Thus the hour, minute and second as well as the part of a second of each transit are obtained. However when looking for differences of a few milliseconds, the hour and minute become unnecessary. Indeed the whole seconds are ignored in the final stages of reduction.

## THE PHOTOGRAPH

Four exposures, at exactly 30 seconds interval (mean time), are made of each star. Approximately 20 seconds are required for the actual exposure time, the remaining 10 seconds being used for the rotation of the rotary. As nearly as possible the four exposures are symmetrically disposed with respect to the meridian, (Figure 11a), except when a deliberate off-set is necessary to avoid superimposing two stars of the same zenith distance. Also an occasional pair of stars has to be staggered, one early and the other late by a few seconds, because they come too close together in transit time. Between each exposure the rotary head is turned through 180 degrees. Ideally, then, images 1 and 4 will be close to a line parallel to the meridian and symmetrical with respect to the prime vertical (Figure 11b). The same is true of images 3 and 2. The average distance normal to the prime vertical between 1 and 4, and also between 3 and 2 , is a measure of double the zenith distance. Furthermore, since the exposures are made with a moving plate so as to give point images, they may be considered as instantaneous exposures at half-minute intervals. The distance on the plate in right ascension from image 1 to 3 , or from 4 to 2 , represents exactly one minute of mean solar time.


Fig. 10. Plate 398A exposed on September 1, 1953. Groups XI and XII, comprising 24 program stars together with a considerable number of volunteers, are here recorded. Magnitudes range from 5 to 9 (photographic), and the groups were taken "head east" and "head west" respectively. The four bright images forming the pattern near the centre of the plate are from the star $\rho$ Cygni, magnitude 5.22 (photographic). BD $44^{\circ} 4263$, magnitude 9.1 (photographic) is indicated by the arrows.

For the purpose of deriving the mathematical relationship between time of transit and the positions of the star images on the plate, it is geometrically equivalent to suppose that the star moves at a uniform rate up to the meridian and then instantly reverses and moves back in the opposite direction at the same rate.


FIG. 11 a


FIG. 11 b


FIG. 12
Let us restrict our attention to Figure 12. The arrows indicate the direction of the stellar motion at the times that the two disc-like images are impressed on the plate. The dotted line represents the meridian. Let v (in $\mathrm{mm} / \mathrm{sec}$ ) represent the velocity of the star (and plate); $t_{1}$ be the contact time of image 1 and $t_{4}$ the corresponding contact time (when the carriage is in the same position relative to the meridian) for image 4. Also let $x_{1}$ be the distance of image 1 from the meridian and $x_{4}$ the distance of image 4 from the meridian. Then the time of meridian passage of the star may be expressed as:-

$$
\begin{aligned}
t_{0} & =t_{1}+x_{1} / v=t_{4}-x_{4} / v \\
\text { or } t_{0} & =\left(t_{1}+t_{4}\right) / 2+\left(x_{1}-x_{4}\right) / 2 v
\end{aligned}
$$

The important point about this equation is that the value of $t_{0}$ depends on $t_{1}$ and $t_{4}$ which are determined by a chronograph, and $x_{1}-x_{4}$ which is measured directly on the plate.

In order to use the information on the plate, accurate measurements must be made by some arbitrary scale in the two coordinates which are parallel to the meridian and to the prime vertical. From the diagrams, it may be seen that the prime vertical is very nearly parallel to the directions from 1 to 3 and from 4 to 2 .

## MEASURING MACHINE

Pending completion of a new projection type instrument, a Toepfer measuring engine has been used at Ottawa. The Toepfer has two slides at right angles to each other. The lower slide, to which the plate is clamped, is actuated by a screw with a half mm pitch which has a range of 50 mm . The other slide which contains the microscope with its fiducial lines, has a pitch of 1 mm and a range of 50 mm . The ratio in pitch between the two screws has been measured as 1.9990 . Also the directions of the two slides differ from 90 degrees by $10^{\prime} 15^{\prime \prime}$, the tangent of which is 0.00298 . Using the fact that the distance from image 1 to 3 and from 4 to 2 in each case represents one minute of mean solar time, the scale (in arc at the zenith) of the horizontal screw, denominated $\mathrm{S}_{\alpha}^{\prime \prime}$, is $24^{\prime \prime} .22994$ per revolution. Using the ratio 1.9990 , the scale value for the other slide, which is used to measure zenith distance, is $\mathrm{S}_{\delta}^{\prime \prime}=48^{\prime \prime} 43565$. Alternatively one may say that the measure of 1 minute mean solar time in right ascension at the zenith is equal to $26 \cdot 155$ revolutions of the half mm screw; $2 \times 26 \cdot 155$ revolutions is designated 2 D for a zenith star. For a star not in the zenith, the scale value varies by the ratio of the cosine of the star's declination to the cosine of the latitude ( $45^{\circ} 23^{\prime} 38^{\prime \prime} .7$ ). The change in value is small but it distinguishes north stars from south stars. Also it is used in determining the scale value of a particular plate. Accordingly Table 1 has been constructed with the measured value of $4 \times$ Z.D. as argument, against which are tabulated $\mathrm{S}_{\alpha}^{\prime \prime} \times$ sec $\delta / 60 \mathrm{r}$ and 2D. " r " is the conversion factor 1.0027379 mean solar to sidereal.

## GEOMETRY OF PLATE REDUCTION

In Figure 13 consideration is centered on a south star which has been observed early. First exposure was made with head west. In the measuring machine it is placed emulsion up, i.e., toward the microscope, with the first image at the lower left and at the high reading in both $\alpha$ and $\delta$ scales. After the plate has been carefully aligned, there remains the small angle B between the axis of the horizontal screw and the prime vertical. OA is the direction of the prime vertical.

First it is required to find B.

$$
\begin{aligned}
\tan \mathrm{B} & =\left\{\frac{(1+4)^{\delta}}{2}-\frac{(3+2)^{\delta}}{2}+\frac{\zeta}{2}\right\} \mathrm{S}_{\delta}^{\prime \prime} \div 26 \cdot 155 \mathrm{~S}_{\alpha}^{\prime \prime} \\
& =\frac{1}{2}\left\{(1+4)^{\delta}-(3+2)^{\delta}+\zeta\right\} \div 13 \cdot 078 \\
& =(-\kappa) \div 13 \cdot 078
\end{aligned}
$$

$\kappa$ takes on the opposite sign to $\frac{1}{2}\left\{(1+4)^{8}-(3+2)^{\delta}+\zeta\right\}$
In practice $\kappa$ is kept small, less than 0.010 . But even at the maximum of the table where $\kappa=0.020, \tan B$ becomes 0.00152 , and may be replaced by $B$ with no sensible error.
$\zeta$ is a small correction applied to each star depending on the skew, i.e., the value $\left\{(1-4)^{\alpha}+(3-2)^{\alpha}\right\}$. It arises from the fact that the star path is curved, and not a straight line, as assumed in this simplified picture. It appears in the more rigorous discussion which follows (see page 349). For the latitude of Ottawa,

$$
\zeta=0.00078\left\{(1-4)^{\alpha}+(3-2)^{\alpha}\right\}
$$

The value $\kappa$, the orientation correction factor, is determined by taking the average value of $\frac{1}{2}\left\{(1+4)^{\delta}-(3+2)^{\delta}+\zeta\right\}$ for all the stars. Some judgment is exercised in eliminating a star by reason of a faulty image, or generally poor seeing.

An orientation factor $\kappa$ means that the measured values of right ascension displacement, $(1-4)^{\alpha}$ and $(3-2)^{\alpha}$ are not correct. In Figure $13,(1-4)^{\alpha}$ is EM, while the required value is DM .
$\mathrm{DM}=(1-4)^{\alpha}+2(1-4)^{\delta} \mathrm{B}$ (where $\alpha$ and $\delta$ indicate direct scale readings). ( B is small enough that $\cos \mathrm{B}=1$ and $\sin \mathrm{B}=\mathrm{B}$ with no error). $2(1-4)^{8}$ may be taken as $\left\{(1-4)^{\delta}+(3-2)^{\delta}\right\}$ which is a tabulated value. Also tabulated is the value $\left\{(1-4)^{\alpha}+\right.$ $\left.(3-2)^{\alpha}\right\}$ which is essentially double the quantity $(1-4)^{\alpha}$. The total plate correction to be applied to $\left\{(1-4)^{\alpha}+(3-2)^{\alpha}\right\}$ is hence $2 \times\left\{(1-4)^{\delta}+(3-2)^{\delta}\right\} B=4 \mathrm{ZD}(-\kappa) / 13 \cdot 078$.


FIG. 13
From auxiliary measurements, using the same reticle face up and face down, the angle between the Toepfer slides was found to be less than $90^{\circ}$ by $10^{\prime} 15^{\prime \prime}$, an angle whose tangent is $0 \cdot 00298$. To the plate correction factor of $(-\kappa) / 13.078$ must be added $0 \cdot 00596$, the measuring machine constant. The total plate correction is then $4 \mathrm{Z}(0.00596-\kappa / 13 \cdot 084)$.

Table III provides the correction to the $\delta$ screw readings $\left\{(1-4)^{\delta}+(3-2)^{\delta}\right\}$ due to the combined effect of the $\kappa$ factor and the angle of deviation between Toepfer slides. In Fig. 13, the required value is:

$$
\mathrm{OD}=(1-4)^{\delta}-\frac{1}{2}(1-4)^{\alpha} \mathrm{B}
$$

Both $(1-4)^{\alpha}$ and B have polarity. In the above example (Fig. 13), $(1-4)^{\alpha}$ is positive and B is positive. So the measured value of OE must be reduced by a small quantity. Table III is entered with arguments $\kappa$ and $\left\{(1-4)^{\alpha}+(3-2)^{\alpha}\right\}$. The correction to $\left\{(1-4)^{\delta}+(3-2)^{\delta}\right\}$ is $\frac{1}{2}\left\{(1-4)^{\alpha}+(3-2)^{\alpha}\right\}(-\kappa) / 13 \cdot 078$.

Table IV provides the correction to the zenith distance measures which are required due to a change in plate scale. On any plate the measured values of 2 D for all the stars are compared with the tabulated values for the same zenith distances. The mean of the residuals of each star value, in the sense observed minus tabulated, is an indication of the amount that the overall scale of the plate differs from the adopted value. To recapitulate, the adopted value of the $\alpha$ screw:-
$\mathrm{D}=60$ seconds mean solar time $=26 \cdot 155$ revolutions.
Ratio of the two screws $1: 1.9990$
1 rev. of dec. screw $=48$ " 4356 (as given above).
The tabulated value $\left\{(1-4)^{8}+(3-2)^{\delta}\right\}$ is 4 times the actual zenith distance; hence the quarter scale value of $S_{\delta}^{\prime \prime}$ is required, namely $12 \cdot 108928$.

Change in plate scale is the quantity:-

$$
\begin{aligned}
& \Sigma\left\{\frac{(1-3)^{\alpha}+(4-2)^{\alpha}-2 \mathrm{D}}{\mathrm{n} \times 2 \mathrm{D}}\right\} \times 12 \cdot 1089 \text { (per rev. change in scale) } \\
= & \Sigma\left\{\frac{(1-3)^{\alpha}+(4-2)^{\alpha}-2 \mathrm{D}}{\mathrm{n}}\right\} \times 0.00023146 \text { per } 0.001 \text { rev. change in scale. }
\end{aligned}
$$

If the observed value of 2 D exceeds the tabulated, it means that there are fewer seconds of are per revolution of the dec. screw. Hence the legend that the correction in scale is applied to $12 \cdot 1089$ with the opposite sign to $\Sigma\left\{\frac{(1-3)^{\alpha}+(4-2)^{\alpha}-2 \mathrm{D}}{\mathrm{n}}\right\}$.

The scale of the plate is initially dependent for its determination on the one known value, namely the 60 second time interval between images 1 and 3 , and images 2 and 4 (Figure 11b). And as indicated above, variations in the plate scale will be reflected in the average value of 2D. However, once the catalogue of PZT stars has been completely observed and revised, small variations in plate scale are preferably referred to stellar declinations themselves. While the Ottawa 2D measure is equivalent to a zenith distance of about 5.5 minutes of arc, the root mean square zenith distance of the stars measured on a plate is more nearly 9 minutes of arc. A least square solution for scale correction of a plate of $n$ stars may be obtained from a set of conditional equations of the form
where

$$
\varphi_{0}+\mathrm{S} r_{\mathrm{i}}=\varphi_{\mathrm{i}} \quad \mathrm{i}=1,2, \ldots \mathrm{n}
$$

$S=$ scale correction
$r=$ revolutions of declination micrometer (north stars + ve; south -ve .)
$\varphi_{0}=$ corrected average latitude of the night
$\varphi_{i}=\varphi_{\text {observed }}$ for each star
Thus

$$
\mathrm{S}=\frac{\mathrm{n} \Sigma r_{\mathrm{i}} \varphi_{\mathrm{i}}-\Sigma r_{\mathrm{i}} \Sigma \varphi_{\mathrm{i}}}{\mathrm{n} \Sigma r_{\mathrm{i}}^{2}-\Sigma r_{\mathrm{i}} \Sigma r_{\mathrm{i}}}
$$

also

$$
\begin{aligned}
\varphi_{0} & =\frac{\Sigma \varphi_{\mathrm{i}} \Sigma r_{\mathrm{i}}^{2}-\Sigma r_{\mathrm{i}} \Sigma r_{i} \varphi_{\mathrm{i}}}{\mathrm{n} \mathrm{\Sigma r}_{\mathrm{i}}^{2}-\Sigma r_{\mathrm{i}} \Sigma r_{\mathrm{i}}} \\
& ={ }_{\mathrm{n}}^{1} \Sigma\left(\varphi_{\mathrm{i}}-\mathrm{S} r_{\mathrm{i}}\right)
\end{aligned}
$$

where $\left(\varphi_{\mathrm{i}}-\mathrm{S} r_{\mathrm{i}}\right)$ is the corrected latitude derived from star i .


FIG. 14. $X_{3}$ Quartz Clock Behaviour from PZT Observations

The P.Z.T. daily aberration correction, $-0^{8} 021$, when combined with the longitude difference $+0^{\text {s }} 014$ provides a constant correction $-0^{s} 007$ to the observed time.

## PERFORMANCE

The performance of the Ottawa P.Z.T. is illustrated in Figure 14 which indicates the behaviour of a Western Electric quartz oscillator type D175730. The smooth curve was derived from a grouping of star results into periods of a month or more, and solving by least squares. Individual circles represent the clock correction for the date in question derived from stars observed over a 4-hour interval. Normally 24 transits are included. Cloudy weather reduced the number on a few nights, there being only 4 on August 1, but on July 17, August 6, 20, 22 and 27, observing was continued with an additional plate, increasing the number of transits to between 30 and 40. The data obtained on any one night is thus considerable. Yet the deviation from the adopted curve on two consecutive nights can be as great as $0^{8} 052$, as indicated on August 2 and 3.

The errors attendant on P.Z.T. performance are tabulated as mean errors, computed from Ottawa results for 1953.

|  | R.A. | Dec. |
| :---: | :---: | :---: |
| Uncertainty of catalogue position of star | $\pm{ }^{8} 013$ | $\pm$ "14 |
| Plate error from criterion $\mathrm{x}_{1}+\mathrm{x}_{2}-\mathrm{x}_{3}-\mathrm{x}_{4}=0$ | $\pm \cdot 014$ | $\pm \cdot 15$ |
| Toepfer measuring engine. | $\pm \cdot 005$ | $\pm \cdot 05$ |
| Single observation compared to mean of night | $\pm .027$ | $\pm \cdot 29$ |
| Mean of the night from scatter of individual stars | $\pm .006$ | $\pm .06$ |
| Mean of the night from $\Delta T$ scatter on successive nights | $\pm \cdot 017$ | $\pm \cdot 18$ |

The uncertainty of catalogue position is derived from the residuals to the star positions in 1953 after they have been corrected by the P.Z.T. observations in 1952. Plate error includes all the errors attendant on image displacement due to poor seeing, emulsion shift, scaling uncertainty, and instrumental errors. Ideally the distance from image 1 to image 3 is the same as the distance from image 4 to image 2 in right ascension, being a measure of 60 seconds of mean time. The error in declination in this case is simply a value in are corresponding to the value in time. The contribution of the Toepfer measuring engine to the errors was derived from resetting on stellar images. The mean error of a single setting with the engine is $\pm 2.2$ microns, corresponding to an uncertainty of $\pm 0^{s} .010$ or $\pm 0^{\prime \prime} 11$. When distributed over the 4 images measured for one star position, the measuring error reduces to $\pm 0 \% 00$ or $\pm 0.05$. It would be expected from this that the value of the night to night scatter should also be of the order of $\pm 0^{s} 006= \pm 0!06$. Instead it is three times as great. Independent comparisons among crystal oscillators relieve the clock of any responsibility for such a night to night scatter. It is suggested by Tanner and Miller (3) that such discrepancies may be due to varying inclinations of layers of equal density of air above the P.Z.T. or to small temperature gradients in the air inside the instrument. The error attendant on the timing of each exposure has not been assessed, but it is felt that its contribution to the total error is small.

The present location of the P.Z.T. must be considered as temporary. New meridian instruments are now usually mounted in buildings of small heat capacity, and plans are being formed to provide such a building for the Ottawa instrument.

TABLE 1
SOUTH STARS


TABLE 11

CORTECTIONS TO a SCREW READINGS
$(1-4)^{a}+(3-2)^{a}$
$K$ (in units of .001 )
If $K$ is Negative - South Stars Positive
North Stars Negative
If $K$ is Positive - South Stars Negative
North Stars Positive

| $\begin{aligned} & (1-4)^{8} \\ & (3-2)^{8} \end{aligned}$ | ${ }^{\delta}$ Screw Orient. ${ }^{8}$ Scuth + North - | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 210 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 6 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 12 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 |
| 2 | 12 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 | 6 |
| 3 | 18 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 45 | 5 | 6 | 6 | 6 | 7 | 7 | 8 | 8 | 9 | 9 |
| 4 | 24 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 | 6 | 7 | 7 | 8 | 9 | 9 | 10 | 10 | 11 | 12 | 12 |
| 5 | 30 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 13 | 14 | 15 | 15 |
| 6 | 36 | 1 | 2 | 3 | 4 | 5 | 6 | 6 | 7 | 8 | 89 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 17 | 18 |
| 7 | 42 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 8 | 48 | 1 | 2 | 4 | 5 | 6 | 7 | 9 | 10 | 11 | 12 | 13 | 15 | 16 | 17 | 18 | 20 | 21 | 22 | 23 | 24 |
| 9 | 54 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 12 | 14 | 15 | 17 | 18 | 19 | 21 | 22 | 23 | 25 | 26 | 28 |
| 10 | 60 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 | 15 | 17 | 18 | 20 | 21. | 23 | 24 | 26 | 28 | 29 | 31 |
| 11 | 66 | 2 | 3 | 5 | 7 | 8 | 10 | 12 | 13 | 15 | 17 | 19 | 20 | 22 | 24 | 25 | 27 | 29 | 30 | 32 | 34 |
| 12 | 72 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 | 18 | 20 | 22 | 24 | 26 | 28 | 29 | 31 | 33 | 35 | 37 |
| 13 | 77 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 |
| 14 | 83 | 2 | 4 | 6 | 9 | 11 | 13 | 15 | 17 | 19 | 21 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 39 | 41 | 43 |
| 15 | 89 | 2 | 5 | 7 | 9 | 11 | 14 | 16 | 18 | 21 | 23 | 25 | 28 | 30 | 32 | 34 | 37 | 39 | 41 | 44 | 46 |
| 16 | 95 | 2 | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 22 | 24 | 27 | 29 | 32 | 34 | 37 | 39. | 42 | 4 | 47 | 49 |
| 17 | 101 | 3 | 5 | 8 | 10 | 13 | 16 | 18 | 21 | 23 | 26 | 29 | 31 | 34 | 36 | 39 | 42 | 44 | 47 | 49 | 52 |
| 18 | 107 | 3 | 6 | 8 | 11 | 14 | 17 | 19 | 22 | 25 | 28 | 30 | 33 | 36 | 39 | 42 | 44 | 47 | 50 | 52 | 55 |
| 19 | 113 | 3 | 6 | 9 | 12 | 15 | 17 | 20 | 23 | 26 | 29 | 32 | 35 | 38 | 41 | 44 | 47 | 49 | 52 | 55 | 58 |
| 20 | 119 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 28 | 31 | 34 | 37 | 40 | 43 | 46 | 49 | 52 | 55 | 58 | 61 |
| 21 | 125 | 3 | 6 | 10 | 13 | 15 | 19 | 22 | 26 | 29 | 32 | 35 | 39 | 42 | 45 | 48 | 51 | 55 | 58 | 61 | 64 |
| 22 | 131 | 3 | 7 | 10 | 13 | 17 | 20 | 24 | 27 | 30 | 34 | 37 | 40 | 44 | 47 | 50 | 54 | 57 | 61 | 64 | 67 |
| 23 | 137 | 4 | 7 | 11 | 14 | 18 | 21 | 25 | 28 | 32 | 35 | 39 | 42 | 46 | 49 | 53 | 56 | 60 | 63 | 67 | 70 |
| 24 | 143 | 4 | 7 | 11 | 15 | 18 | 22 | 26 | 29 | 33 | 37 | 40 | 44 | 48 | 51 | 55 | 59 | 62 | 66 | 70 | 73 |
| 25 | 149 | 4 | 8 | 11 | 15 | 19 | 23 | 27 | 31 | 34 | 38 | 42 | 46 | 50 | 54 | 57 | 61 | 65 | 69 | 73 | 77 |
| 26 | 155 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 | 52 | 56 | 60 | 64 | 68 | 72 | 76 | 80 |
| 27 | 161 | 4 | 8 | 12 | 17 | 21 | 25 | 29 | 33 | 37 | 41 | 45 | 50 | 54 | 58 | 62 | 66 | 70 | 74 | 78 | 83 |
| 28 | 167 | 4 | 9 | 13 | 17 | 21 | 26 | 30 | 34 | 39 | 43 | 47 | 51 | 56 | 60 | 64 | 69 | 73 | 77 | 81 | 86 |
| 29 | 173 | 4 | 9 | 13 | 18 | 22 | 27 | 31 | 36 | 40 | 44 | 49 | 53 | 58 | 62 | 67 | 71 | 75 | 80 | 84 | 89 |
| 30 | 179 | 5 | 9 | 14 | 18 | 23 | 28 | 32 | 37 | 41 | 46 | 50 | 55 | 60 | 64 | 69 | 73 | 78 | 83 | 87 | 92 |
| 31 | 185 | 5 | 9 | 14 | 19 | 24 | 28 | 33 | 38 | 43 | 47 | 52 | 57 | 62 | 66 | 71 | 76 | 81 | 85 | 90 | 95 |
| 32 | 191 | 5 | 10 | 15 | 20 | 24 | 29 | 34 | 39 | 44 | 49 | 54 | 59 | 64 | 69 | 73 | 78 | 83 | 88 | 93 | 98 |
| 33 | 197 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 56 | 61 | 66 | 71 | 76 | 81 | 86 | 91 | 96 | 01 |
| 34 | 203 | 5 | 10 | 16 | 21 | 26 | 31 | 36 | 42 | 47 | 52 | 57 | 62 | 68 | 73 | 78 | 83 | 88 | 94 | 99 | $\mathrm{O}_{4}$ |
| 35 | 209 | 5 | 11 | 16 | 21 | 27 | 32 | 37 | 43 | 48 | 54 | 59 | 64 | 70 | 75 | 80 | 86 | 91 | 96 | 02 | 07 |
| 36 | 225 | 6 | 11 | 17 | 22 | 28 | 33 | 39 | 44 | 50 | 55 | 61 | 66 | 72 | 77 | 83 | 88 | 94 | 99 | 05 | 10 |
| 37 | 221 | 6 | 11 | 17 | 23 | 28 | 34 | 40 | 45 | 51 | 57 | 62 | 68 | 74 | 79 | 85 | 91 | 02 | 02 | 08 | 13 |
| 38 | 226 | 6 | 12 | 17 | 23 | 29 | 35 | 41 | 47 | 52 | 56 | 64 | 70 | 76 | 81 | 87 | 93 | 99 | 05 | 10 | 16 |
| 39 | 232 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 | 78 | 84 | 90 | 95 | 01 | O7 | 13 | 19 |
| 40 | 238 | 6 | 12 | 18 | 24 | 31 | 37 | 43 | 49 | 55 | 61 | 67 | 73 | 80 | 86 | 92 | 98 | $\mathrm{O}_{4}$ | 10 | 16 | 22 |

table 11 B

CORRECTIONS TO a SCREW READINGS
$(1-4)^{\alpha}+(3-2)^{\alpha}$

| K ( | n units | Of | . 001 |  |  |  |  |  |  |  | $\begin{array}{r} \text { If } K \text { is Negative }- \text { South Stars Positive } \\ \text { North Stars Negative } \\ \text { If } K \text { is Positive - South Stars Negative } \\ \\ \hline \end{array}$ |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} (1-4) \\ +4 \\ (3-2)^{8} \end{gathered}$ | Screw Orient. South + North - | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| , 41 | 244 | 6 | 13 | 19 | 25 | 31 | 38 | 44 | 50 | 56 | 63 | 69 | 75 | 82 | 88 | 94 | 00 | 07 | 13 | 19 | 25 |
| 42 | 250 | 6 | 13 | 19 | 26 | 32 | 39 | 45 | 51 | 58 | 64 | 71 | 77 | 84 | 90 | 96 | 03 | 09 | 16 | 22 | 29 |
| 43 | 256 | 7 | 13 | 20 | 26 | 33 | 39 | 46 | 53 | 59 | 66 | 72 | 79 | 86 | 92 | 99 | 05 | 12 | 18 | 25 | 32 |
| 4 | 262 | 7 | 13 | 20 | 27 | 34 | 40 | 47 | 54 | 61 | 67 | 74 | 81 | 88 | 94 | 01 | 08 | 14 | 21 | 28 | 35 |
| 45 | 268 | 7 | 14 | 21 | 28 | 34 | 41 | 48 | 55 | 62 | 69 | 76 | 83 | 90 | 96 | 03 | 10 | 17 | 24 | 31 | 38 |
| 46 | 274 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 | 92 | 99 | 06 | 13 | 20 | 27 | 34 | 41 |
| 47 | 280 | 7 | 14 | 22 | 29 | 36 | 43 | 50 | 58 | 65 | 72 | 79 | 86 | 93 | 01 | 08 | 15 | 22 | 29 | 37 | 44 |
| 48 | 286 | 7 | 15 | 22 | 29 | 37 | 44 | 51 | 59 | 66 | 73 | 81 | 88 | 95 | 03 | 10 | 18 | 25 | 32 | 40 | 47 |
| 49 | 292 | 7 | 15 | 22 | 30 | 37 | 45 | 52 | 60 | 67 | 75 | 82 | 90 | 97 | 05 | 12 | 20 | 27 | 35 | 42 | 50 |
| 50 | 298 | 8 | 25 | 23 | 31 | 38 | 46 | 54 | 61 | 69. | 77 | 84 | 92 | 99 | 07 | 15 | 22 | 30 | 38 | 45 | 53 |
| 51 | 304 | 8 | 16 | 23 | 31 | 39 | 47 | 55 | 62 | 70 | 78 | 86 | 94 | 01 | 09 | 17 | 25 | 33 | 40 | 48 | 56 |
| 52 | 310 | 8 | 17 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 95 | 03 | 11 | 19 | 27 | 35 | 43 | 51 | 59 |
| 53 | 316 | 8 | 17 | 24 | 32 | 41 | 49 | 57 | 65 | 73 | 81 | 89 | 97 | 05 | 14 | 22 | 30 | 38 | 46 | 54 | 62 |
| 54 | 322 | 8 | 17 | 25 | 33 | 41 | 50 | 58 | 66 | 74 | 83 | 91 | 99 | 07 | 16 | 24 | 32 | 40 | 49 | 57 | 65 |
| 55 | 328 | 8 | 17 | 25 | 34 | 42 | 50 | 59 | 67 | 76 | 84 | 93 | 01 | 09 | 18 | 26 | 35 | 43 | 51 | 60 | 68 |
| 56 | 334 | 9 | 17 | 26 | 34 | 43 | 51 | 60 | 69 | 77 | 86 | 94 | 03 | 11 | 20 | 29 | 37 | 46 | 54 | 63 | 71 |
| 57 | 340 | 9 | 17 | 26 | 35 | 44 | 52 | 61 | 70 | 79 | 87 | 96 | 05 | 13 | 22 | 31 | 40 | 48 | 57 | 66 | 74 |
| 58 | 346 | 9 | 18 | 27 | 36 | 44 | 53 | 62 | 71 | 80 | 89 | 98 | 06 | 15 | 24 | 33 | 42 | 51 | 60 | 69 | 78 |
| 59 | 352 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 08 | 17 | 26 | 35 | 44 | 53 | 63 | 72 | 81 |
| 60 | 358 | 9 | 18 | 28 | 37 | 46 | 55 | 64 | 73 | 83 | 92 | 01 | 10 | 19 | 29 | 38 | 47 | 56 | 65 | 74 | 84 |
| 61 | 364 | 9 | 19 | 28 | 37 | 47 | 56 | 65 | 75 | 84 | 93 | 03 | 12 | 21 | 31 | 40 | 49 | 59 | 68 | 77 | 87 |
| 62 | 370 | 9 | 19 | 28 | 38 | 47 | 57 | 66 | 76 | 85 | 95 | 04 | 14 | 23 | 33 | 42 | 52 | 61 | 71 | 80 | 90 |
| 63 | 376 | 10 | 19 | 29 | 39 | 48 | 58 | 67 | 77 | 87 | 96 | 06 | 16 | 25 | 35 | 45 | 54 | 64 | 74 | 83 | 93 |
| 64 | 381 | 10 | 20 | 29 | 39 | 49 | 59 | 69 | 78 | 88 | 98 | 08 | 18 | 27 | 37 | 47 | 57 | 66 | 76 | 86 | 96 |
| 65 | 387 | 10 | 20 | 30 | 40 | 50. | 60 | 70 | 80 | 90 | 99 | 09 | 19 | 29 | 39 | 49 | 59 | 69 | 79 | 89 | 99 |
|  | 393 | 10 | 20 | 30 | 40 | 51 | 61 | 71 | 81 | 91 | 01 | 11 | 21 | 31 | 4 | 51 | 62 | 72 | 82 | 92 | 02 |
| 67 | 399 | 10 | 21 | 31 | 41 | 51 | 62 | 72 | 82 | 92 | 03 | 13 | 23 | 33 | 44 | 54 | 64 | 74 | 85 | 95 | 05 |
| 68 | 405 | 10 | 21 | 31 | 42 | 52 | 62 | 73 | 83 | 94 | $\mathrm{O}_{4}$ | 14 | 25 | 35 | 46 | 56 | 66 | 77 | 87 | O8 | 08 |
| 69 | 411 | 11 | 21 | 32 | 42 | 53 | 63 | 74 | 84 | 95 | 06 | 16 | 27 | 37 | 48 | 58 | 69 | 79 | 90 | 01 | 11 |
| 70 | 417 | 11 | 21 | 32 | 43 | 54 | 64 | 75 | 86 | 96 | 07 | 18 | 29 | 39 | 50 | 61 | 71 | 82 | 93 | $\mathrm{O}_{4}$ | 14 |
| 71 | 423 | 11 | 22 | 33 | 43 | 54 | 65 | 76 | 87 | 98 | 09 | 20 | 30 | 41 | 52 | 63 | 74 | 85 | 96 | 06 | 17 |
| 72 | 429 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 10 | 21 | 32 | 43 | 54 | 65 | 76 | 87 | 98 | 09 | 20 |
| 73 | 435 | 11 | 22 | 34 | 45 | 56 | 67 | 78 | 89 | 01 | 12 | 23 | 34 | 45 | 56 | 68 | 79 | 90 | 01 | 12 | 23 |
| $74$ | 441 | 11 | 23 | 34 | 45 | 57 | 68 | 79 | 91 | 02 | 13 | 25 | 36 | 47 | 59 | 70 | 81 | 93 | $\mathrm{O}_{4}$ | 15 | 26 |
| 75 | 447 | 11 | 23 | 34 | 46 | 57 | 69 | 80 | 92 | 03 | 15 | 26 | 38 | 49 | 61 | 72 | 84 | 95 | 07 | 18 | 30 |
| 76 | 453 | 12 | 23 | 35 | 47 | 58 | 70 | 81 | 93 | 05 | 16 | 28 | 40 | 51 | 63 | 74 | 86 | 98 | 09 | 21 | 33 |
| 77 | 459 | 12 | 24. | 35 | 47 | 59 | 71 | 82 | 94 | 06 | 18 | 30 | 41 | 53 | 65 | 7 | 89 | 00 | 12 | 24 | 36 |
| 78 | 465 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 95 | 07 | 19 | 31 | 43 | 55 | 67 | 79 | 91 | 03 | 15 | 27 | 39 |
| 79 | 471 | 12 | 24 | 36 | 48 | 60 | 73 | 85 | 97 | 09 | 21 | 33 | 45 | 57 | 69 | 81 | 93 | 06 | 18 | 30 | 42 |
| 80 | 477 | 12 | 24 | 37 | 49 | 61 | 73 | 86 | 98 | 10 | 22 | 35 | 47 | 59 | 72 | 84 | 96 | 08 | 20 | 33 | 45 |

TABLE 111

CORRECTIONS TO $\delta$ SCREW READINGS
$K$ POS.
SOUTH STARS SAME SIGN AS $(1-4)^{\alpha}$ NORTH STARS OPPOSITE SIGN AS $(1-4)^{\alpha}$

K NDG.
SOUTH STAR OPPOSITE SIGN TO $(1-4)^{\alpha}$ NORTH STAR SAME SIGN AS $(1-4)^{a}$

$$
\text { Unit }=0 R_{001}
$$



TABLE IV
CORRECTION TO $\delta$ SCREW FACTOR Unit = 0.001

| 21 | .000 | .001 | .002 | .003 | .004 | .005 | .006 | .007 | .008 | .009 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| .000 | .0002 | .0004 | .0007 | .0009 | .0011 | .0014 | .0016 | .0018 | .0021 |  |
| .010 | .0023 | .0025 | .0028 | .0030 | .0032 | .0035 | .0037 | .0039 | .0041 | .0044 |
| .020 | .0046 | .0048 | .0051 | .0053 | .0055 | .0058 | .0060 | .0062 | .0065 | .0067 |
| .030 | .0069 | .0072 | .0074 | .0076 | .0079 | .0081 | .0083 | .0085 | .0088 | .0090 |
| .040 | .0092 | .0085 | .0097 | .0099 | .0102 | .0104 | .0106 | .0109 | .0111 | .0113 |

Screw factor is 12.1089
Correction is opposite sign to 2D correction.

## GENERAL

Acknowledgement is here made to Dr. H. R. Morgan of the United States Naval Observatory for his courtesy in supplying the best available positions for our list of stars, based on the FK3 system. Meridian circle observations of them were started in Ottawa in 1951 and continued through 1953. P.Z.T. results for 1952 were thus required to smooth the star list internally both in R.A. and in Declination. In the former the greatest divergence in position was of the order of 0.2 second of time, while in declination it was 1.0 second of arc. Stars which exhibited a large deviation were not used for time or latitude determinations during 1952, but their residuals were applied to the catalogue positions to make them useful for 1953. In fact the residuals for each star were considered as corrections to catalogue, and applied before the catalogue was used the second year.

In July 1952 the star list was divided into 12 two-hour groups with approximately 12 stars per group and an even weighting of north and south stars. A few changes were made in the list to provide a fairly uniform distribution and to replace the few that proved to be too faint or too bright. Useful magnitudes lie between 5 th and 9 th.

Two groups normally constitute a night's work, yielding about 24 stars if complete. At the summer solstice an interval of 7 hours is the maximum available from civil twilight to civil twilight. At winter solstice the interval is about 14 hours. Star groups 10 to 3, observed between July and December, thus remain in observation longer than groups 4 to 9 which are observed between January and June.

The first observations were taken in September 1951, and the next few months were considered as an experimental period, in order to become familiar with an entirely new procedure of observing and computing. Mr. Paul Sollenberger of the United States Naval Observatory very kindly placed his experience at our disposal in isolating the various sources of difficulty. Observations with the broken Cooke Transit for time were carried on to the beginning of 1952 when it became clear that the observations with the P.Z.T. were giving a better determination of time than the previous method of observing. All the early errors in method and computing have been corrected and in some cases the plates were remeasured.

The photographic plate used in the Ottawa P.Z.T. program is blue sensitive Kodak $103-0$ and may be handled conveniently in the light of a red filter Wratten Series 2. It is ordered to size $22^{\frac{3}{2}}$ inches square. D-19 developer at $68^{\circ} \mathrm{F}$ with 4 minutes of development time provides adequate images. A small tank designed for the development of 35 mm film is used, together with a U-shaped stainless steel clip which holds the plate in a horizontal position about a quarter of an inch from the base of the tank.

No fogging of the plate due to city lights is noticeable during the half hour or more of total exposure time. Direct moonlight is screened from the lens by a moon shade which is mounted horizontally about 3 feet above the P.Z.T. On moonlit nights some fogging does occur due to haze and also due to any clouds which might drift over during an exposure.

Normal vibrations, due to traffic on nearby Carling Avenue, and around the Observatory building, seem to have little effect on the steadiness of the star image. Observing carried on with the 6 -inch meridian instrument in the adjacent room, and operation of
the fan in the P.Z.T. room to reduce the temperature differential between inside and outside the building, have no detrimental effect on stellar images. The west end of the Observatory, which houses the transit instruments is made of the same solid stone and masonry as the remainder of the building. Care is taken to have the observing room opened as much as possible to the outside air but a temperature difference of less than two degrees centigrade is seldom attained.

## REFERENCES

1. Special Publication No. 7, United States Coast and Geodetic Survey, 1915.
2. A. J. $40,7,1929 ;$ A. J. $43,9,1933$.
3. Contributions Dom. Obs. Vol. 2, No. 18, 1954.

# Method and Formulae Used in P.Z.T. Plate Measurement <br> BY <br> R. W. Tanner 

## SKY COORDINATES

In the tangent plane to the celestial sphere at the zenith, take the zenith as origin with the positive x direction to the east and the positive y direction to the south (Figure 15). A star at $\alpha, \delta$, has rectangular coordinates in seconds of are

$$
\begin{aligned}
& \mathrm{x}=-15 \mathrm{hr} \cos \delta \\
& \mathrm{y}=-\left(\mathrm{Z}+\mathrm{k} \mathrm{r}^{2} \mathrm{~h}^{2}\right) \\
& \text { Zenith }
\end{aligned}
$$

FIG. 15
where $h$ is the star's west hour angle in seconds of mean solar time; $Z \mathrm{Z}$ is the north zenith distance in seconds of arc $; r=1.0027379$ is the conversion to give the number of sidereal seconds in a mean solar interval; $k$, the curvature factor, has the value $2.727 \times 10^{-4}$ for the latitude of Ottawa ( $\varphi$ ). It follows that $\mathrm{Z}=\delta-\varphi$. These expressions for x and y are the true coordinates. Measured distances are affected by refraction and must be increased by 1.000286 in the photographic range to obtain true distances. However, the scale factor for conversion from millimeters to seconds is determined from the plate itself and includes this factor.

## PLATE COORDINATES

The telescope projects the image of this coordinate system into its focal plane. Figure 16 indicates the view as seen from below. x z y represent the coordinate plane while SENW are the directions with respect to the ground.


FIG. 16

The photographic plate presents itself to this image in two aspects, $\mathbf{A}$ and $\mathbf{B}$. The plate holder supplies an identifying " $v$ " on the emulsion on the (ground) north edge with the head west, which is the direction of the rotary when loading. The two aspects are shown in Figure 17. In rotating, the head goes through north to reach east, and returns through north.


FIG. 17
In aspect A, head west, the plate travels away from the head, i.e., from west to east; in B, head east, it returns to the head over the same path. At fixed points in the path signals are given, and the mechanism and optics of the telescope are such that at corresponding signal instants on either path, the plate is in one of two positions differing by $180^{\circ}$ due to rotation about the image of the zenith. A set of exposures $A_{1} B_{2} A_{3} B_{4}$ constitutes a head west cycle; $\mathrm{B}_{1} \mathrm{~A}_{2} \mathrm{~B}_{3} \mathrm{~A}_{4}$ a head east cycle.

Assuming at first that the direction of motion of the plate is truly east and west in both aspects (no azimuth or rotation error), choose a coordinate system X O Y fixed to the plate, coinciding with the xz y system at one of the A signals. At the corresponding B signal $X$ and $x$ will have opposite signs, as also will $Y$ and $y$. Hence for head west initial exposure:-

$$
\begin{array}{llll}
\mathbf{X}_{1}=\mathrm{x}_{1} & \mathrm{Y}_{1}=\mathrm{y}_{1} & \mathbf{X}_{2}=-\mathrm{x}_{2} & \mathbf{Y}_{2}=-\mathrm{y}_{2} \\
\mathrm{X}_{3}=\mathrm{x}_{3} & \mathbf{Y}_{3}=\mathrm{y}_{3} & \mathbf{X}_{4}=-\mathrm{x}_{4} & \mathbf{Y}_{4}=-\mathbf{y}_{4}
\end{array}
$$

The star image moves east or west with respect to the plate at the rate $15 \mathrm{r}\left(\cos \delta_{0}-\cos \delta\right)$ per second of time where $\delta_{0}$ is the declination for which the plate speed is correct. For a 20 -second exposure, and a $15^{\prime}$ difference between $\delta_{0}$ and $\delta$ this motion is less than one second of are (approximately $0!9$ ). The star image also moves north and south with respect to the plate at the rate of $2 \mathrm{kr}^{2} \mathrm{~h}^{\prime \prime}$ per second of time because of its curved path. During a 20 -second exposure at an extreme hour angle of 60 seconds, this motion is also less than a second of arc (approximately 0.7 ). These motions are smaller than seeing displacements and the images may be treated as instantaneous exposures at the signal instant.

## TIMING

An interval timer measures the fraction of a second from the telescope signal to the next clock second. If these intervals are $p$ and $q$ for the aspects $A$ and $B$, and $T_{1}$ is the whole second immediately following the mean of the first exposure, the instantaneous exposures occur at:-
head west

$$
\begin{array}{r}
\mathrm{T}_{1}-\mathrm{p} \\
\mathrm{~T}_{1}+30-\mathrm{q} \\
\mathrm{~T}_{1}+60-\mathrm{p} \\
\mathrm{~T}_{1}+90-\mathrm{q}
\end{array}
$$

head east

$$
\begin{array}{r}
\mathrm{T}_{1}-\mathrm{q} \\
\mathrm{~T}_{1}+30-\mathrm{p} \\
\mathrm{~T}_{1}+60-\mathrm{q} \\
\mathrm{~T}_{1}+90-\mathrm{p}
\end{array}
$$

The central time of a transit is hence $\mathrm{T}_{0}=\mathrm{T}_{1}+45-(\mathrm{p}+\mathrm{q}) / 2$.

If $s$ be defined as $s=(p-q) / 2$, then the four exposure times for head west are:-

$$
\begin{array}{ll}
\mathrm{T}_{0}-45-\mathrm{s} & \mathrm{~T}_{0}+15-\mathrm{s}  \tag{1}\\
\mathrm{~T}_{0}-15+\mathrm{s} & \mathrm{~T}_{0}+45+\mathrm{s}
\end{array}
$$

For head east, $s$ takes the opposite sign.
Since $\mathrm{h}=\mathrm{T}+\Delta \mathrm{T}-\alpha$, where T is time by the observatory clock, and if by definition $\mathrm{h}_{0}=\mathrm{T}_{0}+\Delta \mathrm{T}-\alpha$ then the four hour angles may be expressed for head west as:-

$$
\begin{array}{ll}
\mathrm{h}_{1}=\mathrm{h}_{0}-45-\mathrm{s} & \mathrm{~h}_{3}=\mathrm{h}_{0}+15-\mathrm{s} \\
\mathrm{~h}_{2}=\mathrm{h}_{0}-15+\mathrm{s} & \mathrm{~h}_{4}=\mathrm{h}_{0}+45+\mathrm{s} \tag{2}
\end{array}
$$

with opposite sign of s for head east.

## IDEAL REDUCTION

(a) Scale

The sum ( $\mathrm{X}_{1}-\mathrm{X}_{2}-\mathrm{X}_{3}+\mathrm{X}_{4}$ ) (head west), is equal to ( $\mathrm{x}_{1}+\mathrm{x}_{2}-\mathrm{x}_{3}-\mathrm{x}_{4}$ ). With values of h from (1) and (2) the result is:-

$$
\begin{equation*}
\mathbf{X}_{1}-\mathbf{X}_{2}-\mathbf{X}_{3}+\mathbf{X}_{4}=1800 \mathrm{r} \cos \delta \tag{3}
\end{equation*}
$$

with opposite sign for head east. When $\delta$ is known, this gives the scale of the plate in seconds of arc per revolution of the measuring screw.
(b) Clock Correction.

The sum $\left(-X_{1}+X_{2}-X_{3}+X_{4}\right)$ for head west is equal to
or

$$
\begin{align*}
\left(-\mathrm{x}_{1}-\mathrm{x}_{2}-\mathrm{x}_{3}-\mathrm{x}_{4}\right) & =60 \mathrm{r} \mathrm{~h}_{0} \cos \delta \\
\mathrm{~h}_{0} & =\left(-\mathrm{X}_{1}+\mathrm{X}_{2}-\mathrm{X}_{3}+\mathrm{X}_{4}\right) /(60 \mathrm{r} \cos \delta) . \tag{4}
\end{align*}
$$

The scale being known from (3), this gives $h_{0}$ in seconds of time.
Then $\Delta T$ the clock correction is:-

$$
\begin{equation*}
\Delta \mathrm{T}=\mathrm{h}_{0}+\alpha-\mathrm{T}_{0} \tag{5}
\end{equation*}
$$

$\mathrm{T}_{0}$ being defined as above.

## (c) Zenith Distance

The sum

$$
\begin{aligned}
\left(-Y_{1}+Y_{2}-Y_{3}+Y_{4}\right) & =\left(-y_{1}-y_{2}-y_{3}-y_{4}\right) \text { (for head west) } \\
& =4 Z+\mathrm{kr}^{2}\left(h_{1}^{2}+h_{2}^{2}+h_{3}^{2}+h_{4}^{2}\right) .
\end{aligned}
$$

The second term of the last member is the reduction to the meridian.
For $\mathrm{h}_{0}=0$ it amounts to $1!23$. Hence

$$
\begin{equation*}
\mathrm{Z}+\because 31=\left(-\mathrm{Y}_{1}+\mathrm{Y}_{2}-\mathrm{Y}_{3}+\mathrm{Y}_{4}\right) / 4 \tag{6}
\end{equation*}
$$

whence $\varphi=\delta-\mathrm{Z}$ or $\delta=\varphi+\mathrm{Z}$.

## AZIMUTH AND ROTATION ERRORS

Suppose in aspect A the actual direction of motion of the plate makes a small angle $\epsilon$ with the image of the prime vertical. If the exposures were truly instantaneous this would be of no consequence. For $\epsilon$ small enough to replace the cosine by unity and the tangent by the angle, there is evidently no effect on the X coordinate. The effect on the Y coordinate for a 20 -second exposure is $(300 \mathrm{r} \cos \varphi) \epsilon$, about $\left(200^{\prime \prime}\right) \epsilon$. Hence even for $\epsilon=10^{\prime},\left(3 \times 10^{-3}\right.$ radians $),\left(200^{\prime \prime}\right) \epsilon=0^{\prime \prime} 6$, which is the same order as the curvature
mentioned in section 2 above. In practice, $\epsilon$ is readily reduced to a value less than $10^{\prime}$. Further it is seen that if all four exposures are uniform during the 20 seconds, the mean positions of the images will be independent of $\epsilon$.

If, in rotation to the $B$ aspect, the angle turned differs from $180^{\circ}$ by $\eta$, errors in $X$ and Y occur of amounts $\eta \mathrm{Y}$ and $\eta \mathrm{X}$ respectively. Since $\eta$ is held to $5^{\prime \prime}$ or less, $\left(2.4 \times 10^{-5}\right.$ radians), the effect reaches $0 \% 02$ only at the edge of the plate where Y becomes $900^{\prime \prime}$, and so is neglected.

## ACTUAL MEASURES

The actual measures are made with the two screws of a measuring machine which together constitute another coordinate system $\mathrm{X}^{\prime} \mathbf{O}^{\prime} \mathrm{Y}^{\prime}$ which closely approximates the ideal one. $\mathrm{O}^{\prime}$ represents the origin of each screw. Let $\mathrm{i}_{\mathbf{x}}$ be the angle between the positive directions of $O X$ and $O^{\prime} X^{\prime}$ and $i_{y}$ be the angle between $O Y$ and $O^{\prime} Y^{\prime}$, in the positive sense


FIG. 18
as shown in Figure 18. Further, let the scale of the $\mathrm{X}^{\prime}$ screw in seconds of arc be $\mathrm{S}_{\alpha}^{\prime \prime}$ per revolution, including refraction, and the scale of the $\mathrm{Y}^{\prime}$ screw be $\mathrm{S}_{\delta}^{\prime \prime}$ per revolution. It is assumed that the plate is always adjusted in the measuring machine with the prime vertical as nearly as possible coincident with the $O^{\prime} X^{\prime}$ axis. The value ( $i_{x}+i_{y}$ ) is known from auxiliary measurements. $i_{x}$ is determined by the measures themselves. Both are sufficiently small to replace the sine by the angle and the cosine by unity. The relation between the two coordinate systems is:-

$$
\begin{align*}
& \left(\mathrm{X}^{\prime}-\mathrm{X}_{0}^{\prime}\right) \mathrm{S}_{\alpha}^{\prime \prime}=+\mathrm{X}-\mathrm{Yi}_{\mathrm{Y}}  \tag{7}\\
& \left(\mathrm{Y}^{\prime}-\mathrm{Y}_{0}^{\prime}\right) \mathrm{S}_{\delta}^{\prime \prime}=+\mathrm{Y}-\mathrm{Xi}_{\mathrm{i}_{\mathrm{x}}}
\end{align*}
$$

where $\mathrm{X}_{0}^{\prime} \mathrm{Y}_{0}^{\prime}$ are the coordinates of O with respect to $\mathrm{O}^{\prime}$. The measured coordinates are in revolutions; the ideal ones are in seconds of are; $i_{x}$ and $i_{y}$ are in radians. The subsequent discussion will assume head west only, head east involving a change in sign of the measures.

## Scale

The measured sum

$$
\left(X_{1}^{\prime}-X_{2}^{\prime}-X_{3}^{\prime}+X_{4}^{\prime}\right)=\left\{\left(\mathbf{X}_{1}-X_{2}-X_{3}+X_{4}\right)-i_{y}\left(Y_{1}-Y_{2}-Y_{3}+Y_{4}\right)\right\} / \mathbb{S}_{\alpha}^{\prime \prime}
$$

The coefficient of $-i_{y}$ is

$$
\begin{align*}
\mathrm{y}_{1}+\mathrm{y}_{2}-\mathrm{y}_{3}-\mathrm{y}_{4} & =-\mathrm{kr}^{2}\left(\mathrm{~h}_{1}^{2}+\mathrm{h}_{2}^{2}-\mathrm{h}_{3}^{2}-\mathrm{h}_{4}^{2}\right) \\
& =240 \mathrm{~h}_{0} \mathrm{kr}^{2} \tag{8}
\end{align*}
$$

which vanishes for a symmetrical set of exposures, $\left(h_{0}=0\right)$, and is small in any case. Its product with $i_{y}$ is therefore negligible. Hence, from (3) it follows:-

$$
\begin{equation*}
\mathrm{S}_{\alpha}^{\prime \prime}=1800 \mathrm{r} \cos \delta /\left(\mathrm{X}_{1}^{\prime}-\mathrm{X}_{2}^{\prime}-\mathrm{X}_{3}^{\prime}+\mathrm{X}_{4}^{\prime}\right) \tag{9}
\end{equation*}
$$

gives the scale factor to reduce the measured $X^{\prime}$ quantities to seconds of arc. The ratio $S_{\alpha}^{\prime \prime} / \mathbb{S}_{\delta}^{\prime \prime}$ is independently determined, so that $\mathbb{S}_{8}^{\prime \prime}$ may also be regarded as known.

## Inclination

The angle $i_{x}$ between the $\alpha$ screw and the prime vertical is found as follows: The measured sum

$$
\left(\mathrm{Y}_{1}^{\prime}-\mathrm{Y}_{2}^{\prime}-\mathrm{Y}_{3}^{\prime}+\mathrm{Y}_{4}^{\prime}\right)=\left\{\left(\mathrm{Y}_{1}-\mathrm{Y}_{2}-\mathrm{Y}_{3}+\mathrm{Y}_{4}\right)-\mathrm{i}_{\mathrm{x}}\left(\mathrm{X}_{1}-\mathrm{X}_{2}-\mathrm{X}_{3}+\mathrm{X}_{4}\right)\right\} / \mathrm{S}_{8}^{\prime \prime}
$$

The right hand member with the aid of (3) and (8),:-

$$
=\left(240 \mathrm{~h}_{0} \mathrm{kr}^{2}-\mathrm{i}_{\mathrm{z}} 1800 \mathrm{r} \cos \delta\right) / \mathrm{S}_{\delta}^{\prime \prime}
$$

With negligible error $i_{x}$ may conveniently be expressed in terms of measured quantities as:-

$$
\mathrm{i}_{\mathrm{x}}=\frac{4 \mathrm{krsec} \delta\left(-\mathrm{X}_{1}^{\prime}+\mathrm{X}_{2}^{\prime}-\mathrm{X}_{3}^{\prime}+\mathrm{X}_{4}^{\prime}\right)-\left(\mathrm{S}_{8}^{\prime \prime} / \mathrm{S}_{\alpha}^{\prime \prime}\right)\left(\mathrm{Y}_{1}^{\prime}-\mathrm{Y}_{2}^{\prime}-\mathrm{Y}_{3}^{\prime}+\mathrm{Y}_{4}^{\prime}\right)}{\left(\mathrm{X}_{1}^{\prime}-\mathrm{X}_{2}^{\prime}-\mathrm{X}_{3}^{\prime}+\mathrm{X}_{4}^{\prime}\right)}
$$

The first term of the numerator in the right hand member, involving the skew and curvature, is a small corrective term. $4 \mathrm{kr} \sec \delta$ is taken as 4 kr sec $\varphi=1.56 \times 10^{-3}$. Also $1800 \mathrm{r} \cos \varphi$ in revolutions is used for the denominator, since all the stars in the plate are used to obtain an average value of $i_{\mathbf{x}}$. $i_{y}$ follows since ( $i_{x}+i_{y}$ ) is known from auxiliary measures. In practice $i_{x}$ is determined with the following expression:-

$$
\mathrm{i}_{\mathrm{x}}=\frac{\mathrm{S}_{\delta}^{\prime \prime}}{1800 \mathrm{r} \cos \varphi^{\prime \prime}} \frac{1}{\mathrm{n}} \sum^{\mathrm{n}}\left\{\left(\mathrm{~S}_{\alpha}^{\prime \prime} / \mathrm{S}_{\delta}^{\prime \prime}\right) 4 \mathrm{kr} \sec \varphi\left(-\mathrm{X}_{1}^{\prime}+\mathrm{X}_{2}^{\prime}-\mathrm{X}_{3}^{\prime}+\mathrm{X}_{4}^{\prime}\right)-\left(\mathrm{Y}_{1}^{\prime}-\mathrm{Y}_{2}^{\prime}-\mathrm{Y}_{3}^{\prime}+\mathrm{Y}_{4}^{\prime}\right)\right\}
$$

where $n$ is the number of stars.
It will be noted that

$$
\begin{aligned}
\left(\mathrm{S}_{\alpha}^{\prime \prime} / \mathrm{S}_{8}^{\prime \prime}\right) 4 \mathrm{kr} \sec \varphi\left(-\mathrm{X}_{1}^{\prime}+\mathrm{X}_{2}^{\prime}-\mathrm{X}_{3}^{\prime}+\mathrm{X}_{4}^{\prime}\right) & =0.00078\left(-\mathrm{X}_{1}^{\prime}+\mathrm{X}_{2}^{\prime}-\mathbf{X}_{3}^{\prime}+\mathrm{X}_{4}^{\prime}\right) \\
& =\zeta,
\end{aligned}
$$

which is the quantity referred to on page 333.

## Zenith Distance

The measured sum

$$
\begin{aligned}
\left(-\mathrm{Y}_{1}^{\prime}+\mathrm{Y}_{2}^{\prime}-\mathrm{Y}_{3}^{\prime}+\mathrm{Y}_{4}^{\prime}\right) & =\left(1 / \mathrm{S}_{8}^{\prime \prime}\right)\left\{\left(-\mathrm{Y}_{1}+\mathrm{Y}_{2}-\mathrm{Y}_{3}+\mathrm{Y}_{4}\right)-\mathrm{i}_{\mathbf{x}}\left(-\mathrm{X}_{1}+\mathrm{X}_{2}-\mathrm{X}_{3}+\mathrm{X}_{4}\right)\right\} \\
& =\left(1 / \mathrm{S}_{\delta}^{\prime \prime}\right)\left\{\left(4 \mathrm{Z}+1^{\prime \prime} 23\right)-\mathrm{i}_{\mathrm{x}}(60 \mathrm{r} \cos \delta) \mathrm{h}_{0}\right\}
\end{aligned}
$$

whence

$$
\left(\mathrm{Z}+0^{\prime \prime} 31\right)=\left(\mathrm{S}_{\mathrm{d}}^{\prime \prime} / 4\right)\left\{\left(-\mathrm{Y}_{1}^{\prime}+\mathrm{Y}_{2}^{\prime}-\mathrm{Y}_{3}^{\prime}+\mathrm{Y}_{\mathrm{s}}^{\prime}\right)+\left(\mathrm{S}_{\alpha}^{\prime \prime} / \mathrm{S}_{\mathrm{s}}^{\prime \prime} \mathrm{i}_{\mathrm{x}}\left(-\mathrm{X}_{1}^{\prime}+\mathrm{X}_{2}^{\prime}-\mathrm{X}_{\mathrm{s}}^{\prime}+\mathrm{X}_{\mathrm{a}}^{\prime}\right)\right\}\right.
$$

## Clock Correction.

The measured sum (the skew)

$$
\left(-\mathbf{X}_{1}^{\prime}+\mathbf{X}_{2}^{\prime}-\mathbf{X}_{3}^{\prime}+\mathbf{X}_{4}^{\prime}\right)=\left(\mathbf{1} / \mathrm{S}_{\alpha}^{\prime \prime}\right)\left\{\left(-\mathbf{X}_{1}+\mathbf{X}_{2}-\mathbf{X}_{3}+\mathbf{X}_{4}\right)-\mathrm{i}_{y}\left(-\mathbf{Y}_{1}+\mathbf{Y}_{2}-\mathbf{Y}_{3}+\mathrm{Y}_{4}\right)\right\}
$$

The first half of the right hand member is $(60 \mathrm{r} \cos \delta) \mathrm{h}_{0}$, while the coefficient of $-\mathrm{i}_{\mathrm{y}}$ is 4 Z . Hence:-

$$
\begin{equation*}
\mathbf{h}_{0}=\left(\mathrm{S}_{\alpha}^{\prime \prime} / 60 \mathrm{r} \cos \delta\right)\left\{\left(-\mathrm{X}_{1}^{\prime}+\mathbf{X}_{2}^{\prime}-\dot{X}_{3}^{\prime}+\mathrm{X}_{4}^{\prime}\right)+\left(\mathrm{S}_{\delta}^{\prime \prime} / \mathrm{S}_{\alpha}^{\prime \prime}\right) \mathrm{i}_{\mathrm{y}}\left(-\mathrm{Y}_{1}^{\prime}+\mathrm{Y}_{2}^{\prime}-\mathrm{Y}_{3}^{\prime}+\mathrm{Y}_{4}^{\prime}\right)\right\} \tag{10}
\end{equation*}
$$

## Check Sum

It may be noted that the measured sum ( $\mathbf{X}_{1}^{\prime}+\mathbf{X}_{2}^{\prime}-\mathbf{X}_{3}^{\prime}-\mathbf{X}_{4}^{\prime}$ ) should vanish, as it invoives only products of small terms. The residual gives an indication of the dispersion due to "seeing" and errors of measurement.

# A Program Machine for Automatic Operation of the Ottawa P.Z.T. 

BY

V. E. Hollinsworth


#### Abstract

A program tape mechanism is described which uses exposed 35 mm film perforated to operate a number of contacts at definite time periods when driven by a synchronous motor. Contact operating times are observable on a clock which is geared to the tape drive. An operating period of 17 hours is obtainable with a 1000 -foot reel of film and the accuracy of contacts at this rate is better than $\pm 0.5$ seconds.


## INTRODUCTION

When the Ottawa P.Z.T. was first placed in service its operation required the close attention of an astronomer to start and stop the photographic mechanism at the correct instant in accordance with the program of stars to be photographed. This was tedious work over a period of many hours and was subject to errors and omissions as a consequence. Therefore a device which would perform these duties automatically was considered desirable and as a result the Program Machine described below was designed and constructed at the Dominion Observatory and placed in operation in December 1952.

Figure 19 gives a general view of the equipment. It will be seen that a frame or chassis supports two reels which contain the perforated film. A standard 16-tooth sprocket, driven through two gears by a Bodine synchronous motor, is used to pull the film under the contacts. Take up of film is provided by rotation of the left hand reel in an anti-clockwise direction by means of a steel spring belt. A similar belt is used on the right hand wheel to act as a brake. The film is rewound by turning the crank on the shaft of another 16 -tooth sprocket, shown at the right, after interchanging the two belts.

The contact assembly and the punch for perforating the film may be seen on the top of the chassis. The die into which the punch is pressed is a permanent part of the bed plate. When it is not used for perforating the fim it forms recesses into which the 4 spring contact fingers may drop after passing through the holes in the film, thereby permitting greater separation of the contacts than would be the case if the film thickness alone were used for this purpose. The contact points are pure silver and are readily adjusted by turning the horizontal screws in or out as required. The tubular caps which cover the screw heads serve as lock nuts.

An essential part of the machine is the clock. Its sweep second shaft is coupled to the 1 r.p.m. output shaft of the synchronous motor through a clutch. The film drive sprocket is also connected to this same motor shaft through two 48 -pitch gears having 180 and 96 teeth. It will therefore be seen that the film drive and the clock are tied together so that any point on the film is represented by a time indication on the clock. With the motor de-clutched the film may be cranked backward or forward and the clock hands will follow exactly.


Frg. 19. Front view of P.Z.T. Program Machine.
The punched tape is made of movie film. Sprocket and gear connection to the clock provide positive timing. Pilot lights indicate the various operations of the P.Z.T.


Fig. 20. Top view of P.Z.T. Program Machine.
The synchronous motor is connected to the clock by means of a clutch. To the right of the motor rests the punch. When the contact arms are lifted clear, the punch fits into the dowelled location visible at the end of the arms, and is used to punch properly spaced sequence holes in the film.

The gear ratio chosen advances the film one sprocket hole every two seconds or at the rate of $56 \frac{1}{4}$ feet per hour. The clutch was installed so that the motor could be uncoupled to permit rewinding of the film and to facilitate the initial setting of the clock to correct time each time the machine is used. By applying a reasonable finger pressure to the gears, and thereby slipping the clutch, the clock, and film, may be advanced or retarded for exact synchronization with true time without uncoupling the motor.

For the automatic operation of the P.Z.T. four contacts are required. Two are used to start and stop the sequence of events necessary to take four exposures of a star and two operate the fan which ventilates the telescope tube.

Since the contacts close only momentarily, while the holes in the tape pass under them, relays which will prolong the operation are required. Two polar (magnetized) relays each having two coils were found ideal for this purpose. The contacts on these remain set in the position to which they were moved, by current in one coil, until moved in the opposite direction by current in the other coil. Thus the sequence "start" contact activates the relay momentarily but this remains in the operating position until reversed by the "stop" contact. The fan is operated in a similar manner.

The relays are mounted inside the chassis and power for operating them is obtained by rectifying the output of a 12 -volt transformer. Pilot lights are installed on the front panel to indicate that the remotely operated equipment is functioning. Four toggle switches allow manual operation if desired as well as control of power to relays and motor.

Since the P.Z.T. is operated on sidereal or star time rather than on mean time the motor has to be supplied with a frequency of 60 cycles per sidereal second. This is available from amplifiers at the Dominion Observatory which are operated through mean time to sidereal time converters. The mean time is obtained from the crystal controlled master clocks. Hence the derived sidereal time has a very high order of accuracy. Therefore once the tape and clock are properly set the equipment controlled by it will operate on the exact second for long periods of time.

Two reels of film are all that are required for a whole year's star program of 24 hours since one alone will run for more than 12 hours.

Actual operation of the P.Z.T. by this equipment has proved that it is a reliable and useful device in this application and the astronomers have been relieved of an onerous task.

