DEPARTMENT OF THE INTERIOR
CANADA

HON. CHARLES STEWART, *Minister* W. W. CORY, C.M.G., *Deputy Minister*

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Second Paper

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In a first paper on η Aquilae,[1] a succinct history of this important variable has been given, as well as all the radial velocities so far published, and the results obtained at the Dominion Observatory on velocities obtained from individual lines and on the variation of spectral class.

The present paper is a continuation of the research; it is a part of an extensive plan of investigation of all the brighter Cepheids with all the modern means we have at our disposal. Seventeen additional one-prism spectrograms were secured, which have been measured on a Hartmann spectrocomparator. These and previous ones have been analyzed with a Moll thermopile microphotometer to investigate the structure and intensity of individual spectral lines. A photo-electric photometer attached to the 15-inch equatorial was also used to determine a light-curve. As, however, this was the first work done with this instrument, and was largely experimental, the accuracy is not on a par with that of later measures; its discussion is not included in the present paper.

The observing was done at the Dominion Observatory by F. Henroteau with the assistance of Douglas Barlow, while the Moll microphotograms were made at McGill University in Montreal, by A. Vibert Douglas. The reducing work was divided nearly equally between the two, with the assistance of Douglas Barlow and of Miriam S. Burland, both of the Dominion Observatory. Some of the spectrograms were also taken by J. F. Frédette.

The present paper comprises: (1) The radial velocities, (2) The variation of ionization, band intensity and other characteristics of the spectrum.

[1] Dom. Obs. Pub. Vol. IX, No. 5, p. 229.

75272—1½

THE RADIAL VELOCITIES

Table I gives the Ottawa velocities for 1927. They have been measured on the spectrocomparator, using the same standard of Polaris as was used to measure the plates of 1924 and 1925.

TABLE I

Date	Julian Day	Phase	Velocity
		d	km.
1927 June 16	2425047·752	1·013	−32·2
21	052·719	5·980	− 0·6
24	055·719	1·803	−30·6
July 2	063·734	2·641	−22·8
9	070·680	2·410	−18·1
11	072·701	4·431	− 8·4
13	074·624	6·354	−17·7
15	076·620	1·174	−29·4
19	080·625	5·179	− 4·2
21	082·622	0·000	−37·4
28	089·687	7·064	−34·0
30	091·654	1·854	−30·5
Aug. 4	096·654	6·854	−29·6
18	110·667	6·514	−21·1
25	117·627	6·297	−10·7
26	118·667	0·161	−31·8
27	119·655	1·149	−28·2

The phases have been computed from the formula Max. =J. D. 2414827·150 ± 7^d·176678E.

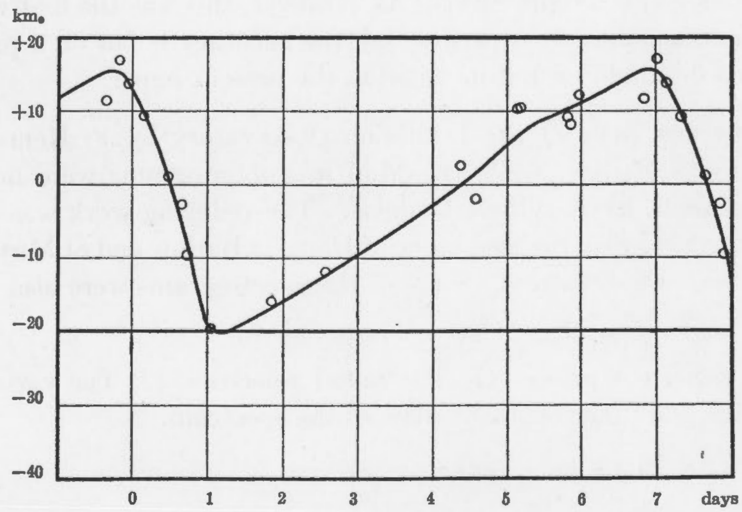

Fig. 1.—Radial Velocity Curve of η Aquilae for 1927

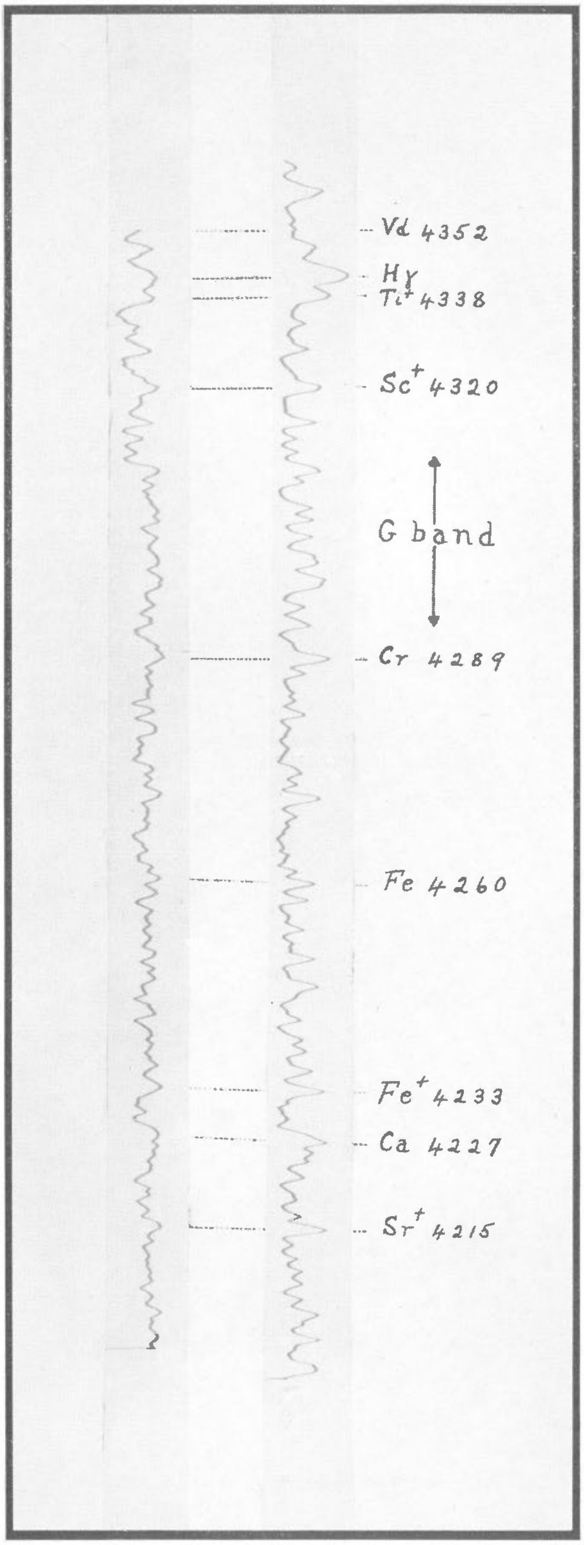

Fig. 2.—Microphotograms of η Aquilae for Maximum and Minimum Light

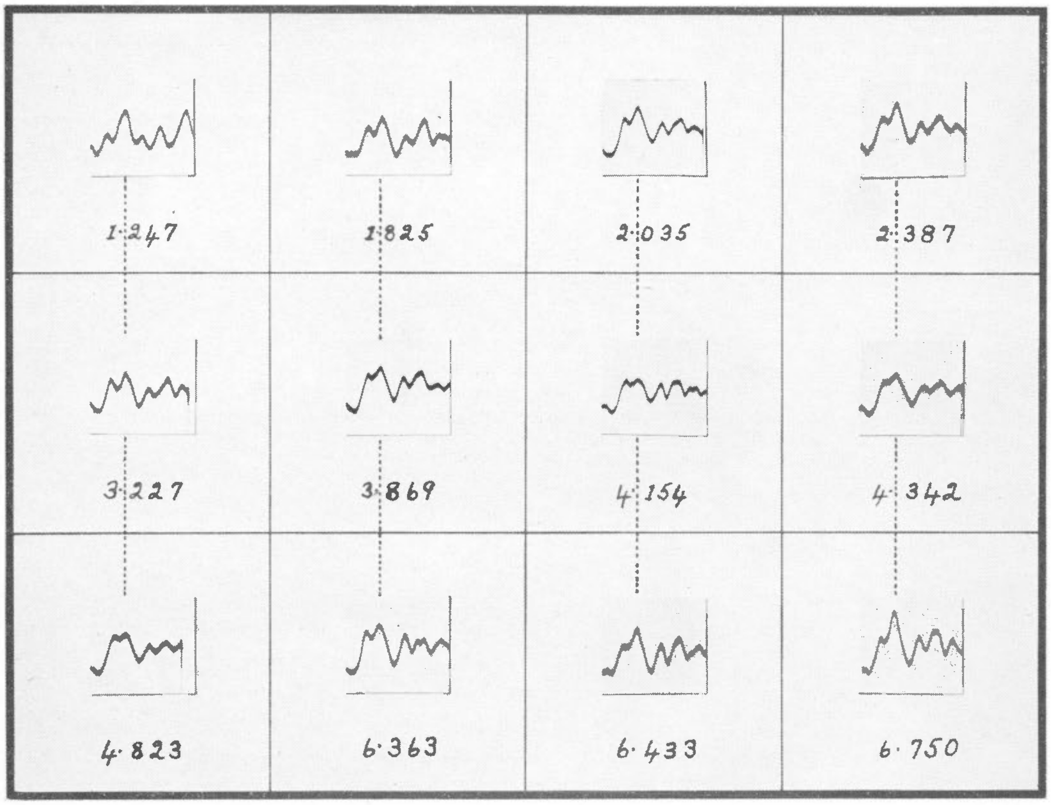

Fig. 3.—Phase Variation of Pair of Titanium Lines.

The velocity curve for 1927 is given in fig. 1. It is interesting to compare this curve with those of 1924 and 1925 which are on page 136 of the present volume. Its amplitude is now 36 km., while in the other years it was about 44 km., and, in particular, the maximum radial velocity comes now about 12 km. below the maxima of 1924 and 1925.

No doubt there is a change in the amplitude and the centre-of-mass velocity of the radial velocity curve. This is in agreement with the results found by F. Henroteau for other Cepheids and for stars of the β Canis Majoris type.[1] It is also paralleled by the results obtained by F. Becker[2] based on Plassmann's observations, showing that the light-curve of η Aquilae had a variable amplitude (see fig. 1, p. 131 of the present volume).

THE VARIATION OF IONIZATION, BAND INTENSITY AND OTHER CHARACTERISTICS OF THE SPECTRUM

Observations[3] by Shapley have shown that the spectral classification of η Aquilae changes from F2 at Phase $6^d \cdot 8$ to G9 at Phase $4^d \cdot 9$. The outstanding changes in the spectral characteristics to be looked for are therefore a gradual decrease in the intensity of lines due to ionized atoms followed by a rapid increase, a similar variation in the Balmer lines of hydrogen, a reverse variation in the intensity of lines due to neutral atoms, and still greater variation in the intensity of spectral bands. Obviously variation of temperature and variation of pressure will affect both lines and bands, but very likely in different proportions. This may be extremely important, as by a combination of the line intensity curve and the band intensity curve it will be possible with the aid of a suitable theory to determine both temperature and pressure.

Microphotograms were obtained with a Moll instrument of most of the spectrograms of η Aquilae secured at the Dominion Observatory. Fig. 2 is a reproduction of a portion of two of these, the right showing the range $H\gamma$ to λ 4210 at maximum phase, the left showing the same portion of the spectrum near minimum phase. The changes in the intensities of $H\gamma$, Ti^+ 4338, Sc^+ 4320, Fe^+ 4233, Sr^+ 4215 and of the G band are very marked. Fig. 3 shows the pair of titanium lines Ti^+ 4533·97 and Ti 4534·78 as obtained from several spectrograms of different phases. The changes in relative intensity are at once apparent.

A large number of lines were studied in the following way: a smooth curve representing the continuous background spectrum was drawn in and the distances from this line to the various maxima of the microphotogram curve were considered to be a measure of the intensities of the lines corresponding to these maxima. Some neutral line was selected closely adjacent to each of the ionized lines under consideration and the ratio of the neutral line to the ionized line determined in each case. In the case of $H\gamma$ several neutral lines were tried out as comparisons, and in investigating the G band its maximum height above the smooth curve was compared with the height of the chromium line, λ 4289, just beyond the band.

[1] Jour. R. A. S. C. Vol. 21, 1927, p. 268.
[2] A.N. Vol. 225, p. 1. See also Die Himmelswelt, Vol. 35, 1925, p. 160.
[3] H. C. 313, 1927.

Table II gives the ionized lines which have been found to vary in intensity as compared with the respective comparison neutral lines also given. The scale of the ratio of intensities is assumed to be comparable from spectrogram to spectrogram throughout the entire cycle of variation.

TABLE II

Atomic number of ionized element	Lines of ionized element		Lines of neutral element	
1	$H\gamma$	4340·4	*Fe*	4325·7
			Vd	4330·0
			Vd	4352·8
21	Sc^{+}	4246·8	*Ba*	4242·6
		4314·1	*Ca*	4318·6
		4320·7	*Ca*	4318·6
		4415·5	*Ca*	4425·4
22	Ti^{+}	4307·8	*Ca*	4318·6
		4337·9	*Vd*	4330·0
		4533·9	*Ti*	4534·7
		4563·7	*Cr*	4565·5
		4571·9	*Cr*	4565·5
26	Fe^{+}	4233·1	*Fe*	4229·7
		4555·9	*Vd*	4560·7
38	Sr^{+}	4215·5	*Tb*	4213·5
39	Y^{+}	4309·6	*Ca*	4318·6
		4374·9	*Vd*	4379·2
		4398·0	*Fe*	4404·7
		4422·6	*Ca*	4425·4
56	Ba^{+}	4554·0	*Vd*	4560·7
57	La^{+}	4558·4	*Vd*	4560·7
		4580·0	*Cr*	4565·5

In Table III are given the measurements for nine of the above lines and the resulting ratios.

TABLE III

Julian Date	Phase	*Ti*+ 4534	*Ti*+ 4338	*H*γ	*G* band 4308	*Ti* 4534·78	*Vd* 4330	*Vd* 4352	*Fe* 4325	*Cr* 4289	*Ti*+ 4534 / *Ti* 4534·7	*Ti*+ 4338 / *Vd* 4330	*H*γ / *Vd* 4330	*H*γ / *Vd* 4352	*H*γ / *Fe* 4325	*G* band / *Cr* 4289
2424005·645	6·904	0·84	0·46	0·48	0·14	0·58	0·30	0·43	0·50	0·45	1·45	1·53	1·60	1·12	0·96	0·31
008·700	2·782	0·50	.....	.....	.....	0·38	.....	.....	.....	.....	1·32					
009·585	3·667	0·50 0·55	.....	.....	.....	0·43 0·50	.....	.....	.....	.....	1·16 1·10					
010·699	4·781	0·39	.....	.....	.....	0·37	.....	.....	.....	.....	1·05					
012·581	6·663	0·95	0·55	0·65	0·10	0·70	0·42	0·55	0·54	0·50	1·36	1·31	1·55	1·18	1·20	0·20
012·668	6·750	0·98 1·03	0·85	0·95	0·28	0·55 0·55	0·54	0·63	0·75	0·90	1·78 1·87	1·57	1·76	1·51	1·26	0·31
013·692	0·598	0·73 0·73	0·40	0·51	0·09	0·42 0·45	0·33	0·35	0·38	0·40	1·74 1·62	1·21·	1·54	1·45	1·34	0·22
016·569	3·475	0·35 0·53	.....	.....	.....	0·30 0·40	.....	.....	.....	.....	1·16 1·32					
022·677	2·406	0·25	.....	.....	.....	0·25	.....	.....	.....	.....	1·00					
023·645	3·374	0·40 0·40	.....	.....	.....	0·32 0·35	.....	.....	.....	.....	1·25 1·14					
024·585	4·314	0·35 0·35	.....	.....	.....	0·40 0·39	.....	.....	.....	.....	0·87 0·90					
025·593	5·322	0·50	.....	.....	.....	0·42	.....	.....	.....	.....	1·19					
027·586	0·139	0·65 0·68	0·38	0·44	0·08	0·35 0·38	0·20	0·33	0·30	0·45	1·86 1·79	1·90	2·20	1·33	1·47	0·18
031·601	4·154	0·55	.....	.....	.....	0·55	.....	.....	.....	.....	1·00					
032·588	5·141	0·87 0·83	0·75	0·85	0·35	0·89 0·78	0·65	.....	0·80	0·77	1·00	1·15	1·31	.....	1·06	0·45
033·569	6·122	0·65	.....	.....	.....	0·56	.....	.....	.....	.....	1·16					
034·581	7·134	0·70 0·60	1·09	1·66	0·15	0·36 0·28	0·65	0·84	0·84	1·08	1·94 2·14	1·68	2·55	1·98	1·97	0·14
040·565	5·942	0·55 0·60	0·23	0·33	.....	0·45 0·52	0·15	0·25	0·18	.....	1·22 1·15	1·53	2·20	1·32	1·83	
043·551	1·751	0·44 0·35	.....	.....	.....	0·34 0·28	.....	.....	.....	.....	1·29 1·25					
046·552	4·752	0·55 0·56	0·38	0·40	0·15	0·55 0·53	0·35	0·48	0·37	0·25	1·00 1·05	1·08	1·14	0·83	1·08	0·60
047·566	5·766	0·41 0·43	.....	.....	.....	0·38 0·41	.....	.....	.....	.....	1·08 1·05					
048·551	6·751	1·10 1·05	0·45	0·45	0·10	0·63 0·68	0·31	0·38	0·38	0·43	1·75 1·54	1·45	1·45	1·18	1·18	0·23

TABLE III—Concluded

Julian Date	Phase	*Ti*+ 4534	*Ti*+ 4338	*H*γ	*G* band 4308	*Ti* 4534·78	*Vd* 4330	*Vd* 4352	*Fe* 4325	*Cr* 4289	*Ti*+ 4534 / *Ti* 4534·7	*Ti*+ 4338 / *Vd* 4330	*H*γ / *Vd* 4330	*H*γ / *Vd* 4352	*H*γ / *Fe* 4325	*G* band / *Cr* 4289
2424053·555	4·579	0·40 0·50	.....	.....	.....	0·33 0·42	.....	.....	.....	.....	1·21 1·29					
055·550	6·574	0·90 0·87	0·30	0·38	0·10	0·58 0·55	0·29	0·34	0·39	0·39	1·55 1·58	1·03	1·31	1·12	0·97	0·26
326·217	4·538	0·75 0·70	0·75	0·85	0·40	0·63 0·58	0·45	0·78	0·80	0·70	1·19 1·20	1·66	1·88	1·09	1·06	0·57
329·207	0·352	0·82 0·70	1·02	1·40	0·20	0·43 0·35	0·55	0·84	0·82	1·10	1·91 2·00	1·85	2·54	1·67	1·71	0·18
331·242	2·387	0·76 0·69	0·87	1·10	0·25	0·55 0·53	0·58	0·85	0·83	0·90	1·38 1·30	1·50	1·90	1·29	1·32	0·28
333·197	4·342	0·65 0·68	.....	.....	.....	0·57 0·65	.....	.....	.....	.....	1·14 1·04					
335·218	6·363	0·78 0·73	1·03	1·10	0·32	0·60 0·63	0·59	1·02	0·90	0·85	1·30 1·16	1·75	1·86	1·08	1·22	0·38
338·202	2·171	0·70 0·65	0·48	0·45	0·10	0·42 0·43	0·33	0·48	0·42	0·40	1·66 1·51	1·45	1·36	0·94	1·07	0·25
340·201	4·170	0·50	.....	.....	.....	0·45	.....	.....	.....	.....	1·11					
341·207	5·176	0·33 0·80	0·53	0·59	0·15	0·29 0·77	0·23	0·45	0·45	0·38	1·14 1·04	2·30	2·56	1·31	1·31	0·39
343·194	7·163	0·70 0·73	0·88	0·95	0·18	0·25 0·25	0·53	0·75	0·75	0·80	3·00 2·92	1·66	1·79	1·26	1·26	0·22
345·243	2·035	0·73 0·76	0·80	1·00	0·25	0·58 0·58	0·48	0·73	0·67	0·80	1·26 1·29	1·66	2·08	1·36	1·49	0·31
346·226	3·018	0·60	1·35	1·68	0·30	0·40	0·75	1·04	1·08	1·25	1·50	1·80	2·24	1·62	1·56	0·13
352·193	1·809	0·65	0·65	0·83	0·20	0·50	0·43	0·58	0·53	0·58	1·30	1·51	1·93	1·43	1·56	0·34
359·208	1·648	0·65	0·77	1·00	0·20	0·52	0·37	0·65	0·63	0·70	1·25	2·08	2·70	1·54	1·58	0·28
367·178	2·441	0·52 0·45	0·34	0·52	0·06	0·42 0·38	0·35	0·38	0·30	0·31	1·24 1·18	0·97	1·48	1·37	1·73	0·19
368·116	3·379	0·60	0·30	0·40	0·15	0·45	0·16	0·30	0·25	0·30	1·33	1·87	2·50	1·25	1·60	0·50
371·141	6·404	0·52 0·50	0·29	0·38	0·12	0·48 0·40	0·25	0·33	0·35	0·35	1·08 1·25	1·16	1·52	1·15	1·08	0·34
375·140	3·227	0·55 0·63	0·60	0·70	0·15	0·45 0·53	0·42	0·50	0·58	0·50	1·22 1·19	1·43	1·67	1·40	1·21	0·30
380·128	1·038	0·65 0·70	0·45	0·50	.....	0·40 0·42	.....	.....	.....	.....	1·62 1·66					
384·105	5·015	0·48 0·75	0·45	0·45	0·25	0·45 0·70	.....	.....	.....	0·45	1·06 1·07	.....	.....	.....	.....	0·56
385·110	6·020	0·70	0·70	0·79	.....	0·68	0·57	0·72	0·60	.....	1·03	1·23	1·38	1·09	1·32	

2424388·091	1·825	0·57, 0·57, 0·55	......	......	......	0·43, 0·35, 0·36	......	......	......	......	1·32, 1·63, 1·53					
389·147	2·881	0·40, 0·48	0·35	0·40	......	0·35, 0·40	......	0·36	......	......	1·14, 1·20	......	......	1·11		
390·058	3·792	0·35	......	......	......	0·33	......	......	......	......	1·06					
390·135	3·869	0·70, 0·75	0·48	0·55	0·20	0·60, 0·60	......	......	......	0·45	1·16, 1·25	......	......	......	......	0·44
391·089	4·823	0·75, 0·75	0·79	0·85	0·37	0·69, 0·71	0·51	0·78	0·78	0·78	1·08, 1·06	1·55	1·67	1·09	1·09	0·47
394·137	0·695	0·25	......	......	......	0·20	......	......	......	......	1·25					
395·116	1·674	0·55, 0·50	0·32	0·40	......	0·40, 0·38	......	......	......	......	1·37, 1·32					
396·072	2·630	0·50, 0·55	0·40	0·59	0·08	0·36, 0·40	0·37	0·44	0·44	0·40	1·39, 1·37	1·08	1·59	1·34	1·34	0·20
403·051	2·432	0·77, 0·80	0·85	1·00	0·30	0·55, 0·55	0·53	0·90	0·75	0·95	1·40, 1·45	1·60	1·88	1·11	1·33	0·32
409·042	1·247	0·75, 0·70	0·88	1·08	0·20	0·38, 0·34	0·38	0·68	0·58	0·65	1·97, 2·06	2·32	2·84	1·59	1·86	0·31
409·113	1·318	0·70	0·85	1·23	......	0·40	0·52	0·80	0·75	......	1·75	1·63	2·36	1·54	1·64	
415·545	0·573	0·68, 0·70	0·75	1·00	0·20	0·42, 0·42	......	......	......	0·75	1·62, 1·66	......	......	......	......	0·26
428·581	6·433	0·75, 0·70	0·68	0·81	0·20	0·48, 0·45	0·58	0·67	0·60	0·67	1·56, 1·55	1·17	1·40	1·21	1·35	0·30
2425047·752	1·259	1·10	1·70	2·50	0·65	0·90	0·53	1·35	1·20	1·60	1·22	3·21	4·72	1·85	2·08	0·40
052·719	6·226	1·15	1·00	1·35	0·50	1·10	0·58	1·05	0·95	1·00	1·04	1·72	2·33	1·28	1·42	0·50
055·719	2·049	0·95	1·15	1·40	0·50	0·95	0·60	1·15	0·80	1·05	1·00	1·92	2·33	1·22	1·75	0·48
063·734	2·888	1·15	1·40	1·80	0·60	1·00	0·88	1·26	1·05	1·15	1·15	1·59	2·04	1·43	1·71	0·52
070·680	2·658	1·05, 0·90	1·25	1·55	0·50	1·10, 0·95	......	1·07	......	1·05	0·95, 0·95	......	......	1·68	......	0·48
072·701	4·679	0·75	0·55	0·60	......	0·80	0·25	0·51	0·45	......	0·94	2·20	2·40	1·18	1·33	......
074·624	6·602	1·13	0·85	1·00	0·20	1·13	......	0·72	......	0·55	1·00	......	......	1·39	......	0·36
076·620	1·421	1·05	1·22	1·60	0·50	1·00	0·60	0·94	1·00	1·05	1·05	2·03	2·66	1·70	1·60	0·48
080·625	5·426	0·57	0·57	0·62	0·25	0·60	0·28	0·46	0·42	0·45	0·95	2·04	2·21	1·34	1·47	0·55
082·622	0·247	0·70	1·20	1·65	0·15	0·40	0·62	0·85	0·80	1·05	1·75	1·94	2·66	1·94	2·06	0·14
089·687	0·136	0·85	1·52	2·00	0·20	0·58	0·83	1·10	1·08	1·50	1·46	1·84	2·41	1·82	1·85	0·13
091·654	2·102	0·44	1·05	1·30	0·40	0·44	0·52	0·84	0·80	1·10	1·00	2·02	2·50	1·55	1·62	0·36
096·654	7·102	1·05, 0·80	1·60	2·35	0·15	0·58, 0·50	......	1·00	......	1·90	1·81, 1·60	......	......	2·35	......	0·79
110·667	6·763	0·85	1·15	1·50	0·35	0·60	0·62	1·00	1·00	1·10	1·42	1·86	2·42	1·50	1·50	0·32
117·627	6·546	0·65	0·50	0·60	0·10	0·45	0·42	0·51	0·51	0·45	1·44	1·19	1·43	1·17	1·17	0·22
118·667	0·410	0·65	1·30	1·60	0·30	0·60	0·40	0·86	0·65	0·85	1·08	3·25	4·00	1·86	2·46	0·35
119·655	1·398	0·35	0·36	0·38	0·10	0·35	0·21	0·35	0·30	0·25	1·00	1·71	1·81	1·08	1·26	0·40

TABLE IV

Phase	*Ti*$^+$ 4534 / *Ti* 4534·7			*Ti*$^+$ 4338 / *Vd* 4330			*H*γ / *Vd* 4352			*H*γ / *Vd* 4330			*H*γ / *Fe* 4325			*G* band / *Cr* 4289		
	x	*y*	Σ *p*	*x*	*y*	Σ*p*	*x*	*y*	Σ *p*	*x*	*y*	Σ *p*	*x*	*y*	Σ *p*	*x*	*y*	Σ *p*
0—0·512..........	0·252	1·616	362	0·267	2·206	542	0·368	1·782	398	{0·344 0·325	{2·839 2·50	{709 549}	0·266	1·752	709	0·337	0·186	495
0·512—1·025..........	0·602	1·599	167	0·598	1·210	40	0·598	1·450	35	0·598	1·540	51	0·598	1·340	51	0·582	0·241	115
1·025—1·538..........	1·279	1·413	455	1·316	2·390	501	1·317	1·647	412	{1·314 1·346	{3·345 2·544	{679 429}	1·314	1·806	679	1·314	0·407	355
1·538—2·050..........	1·854	1·250	448	1·914	1·816	337	1·917	1·358	311	1·904	2·006	423	1·904	1·611	423	1·911	0·366	313
2·050—2·563..........	2·359	1·390	291	2·290	1·618	359	2·305	1·266	345	2·296	1·968	437	2·296	1·434	437	2·294	0·304	366
2·563—3·075..........	2·777	1·182	427	2·911	1·615	315	2·834	1·504	417	2·904	2·057	407	2·904	1·594	407	2·840	0·349	385
3·075—3·588..........	3·354	1·247	203	3·278	1·576	90	3·284	1·343	80	3·282	1·972	110	3·282	1·352	110	3·284	0·375	80
3·588—4·101..........	3·786	1·205	169	No	values		No	values		No	values		No	values		3·869	0·440	45
4·101—4·613..........	4·357	1·102	323	4·538	1·660	75	4·538	1·090	78	4·538	1·880	85	4·538	1·060	85	4·538	0·570	70
4·613—5·126..........	4·808	1·024	307	4·761	1·654	172	4·762	1·045	177	4·761	1·792	185	4·761	1·166	185	4·869	0·519	148
5·126—5·638..........	5·251	1·047	248	5·239	1·754	185	5·302	1·325	91	5·236	1·939	206	5·236	1·255	206	5·229	0·464	160
5·638—6·151..........	5·984	1·108	235	6·000	1·304	93	5·999	1·149	97	5·997	1·622	112	5·997	1·470	112	No	values	
6·151—6·664..........	6·452	1·318	560	6·418	1·437	445	6·444	1·206	519	6·414	1·758	527	6·414	1·255	527	6·441	0·345	476
6·664—7·176..........	6·917	1·821	606	6·927	1·664	488	6·962	1·658	503	6·938	2·113	599	6·938	1·487	599	6·924	0·251	476

These ratios have been used to obtain their curves of variation with respect to phase. Normal points, however, have been derived by dividing the period into fourteen equal sections, assuming for the weight of each ratio the measured intensity of the first line of the pair and taking for all ratios comprised in one section the respective values of the abscissa and of the ordinate

$$x = \frac{p_1x_1 + p_2x_2 + \ldots\ldots p_nx_n}{p_1 + p_2 + \ldots\ldots p_n} \qquad y = \frac{p_1y_1 + p_2y_2 + \ldots\ldots p_ny_n}{p_1 + p_2 + \ldots\ldots p_n}$$

where $x_1, x_2, \ldots\ldots, x_n$ are the phases corresponding to the different ratios, $x_1, x_2, \ldots\ldots$, y_n the values of these ratios and $p_1, p_2, \ldots\ldots p_n$ their assumed weights.

The normal points for the ratios in Table III are given with their respective weights in Table IV and from these the curves given in figs. 4, 5, 6, 7, 8 and 9 have been plotted.

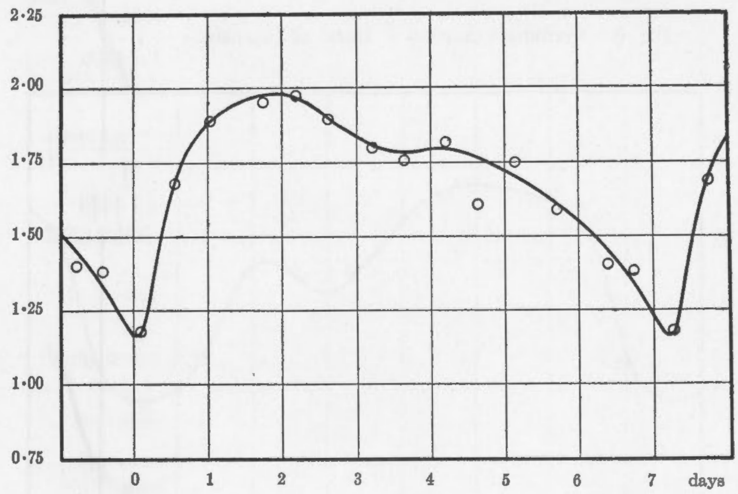

Fig. 4.—Periodic Variation of Ratio of Intensities $\frac{Ti^+ 4535}{Ti\ 4534 \cdot 7}$

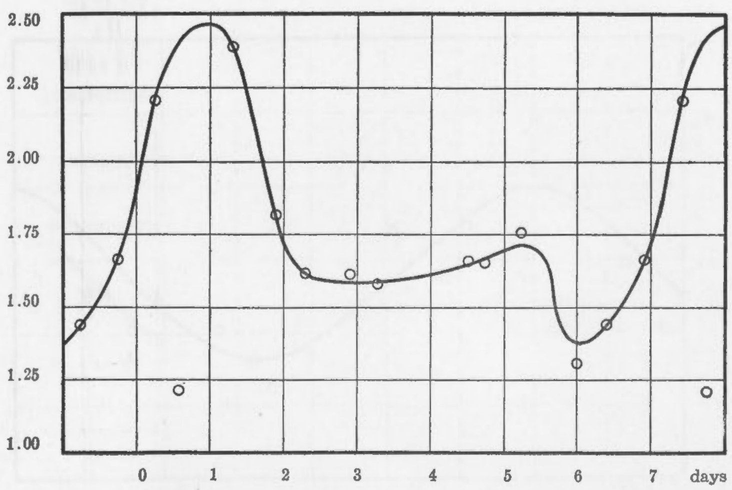

Fig. 5.—Periodic Variation of Ratio of Intensities $\frac{Ti^+ 4338}{Vd\ 4330}$

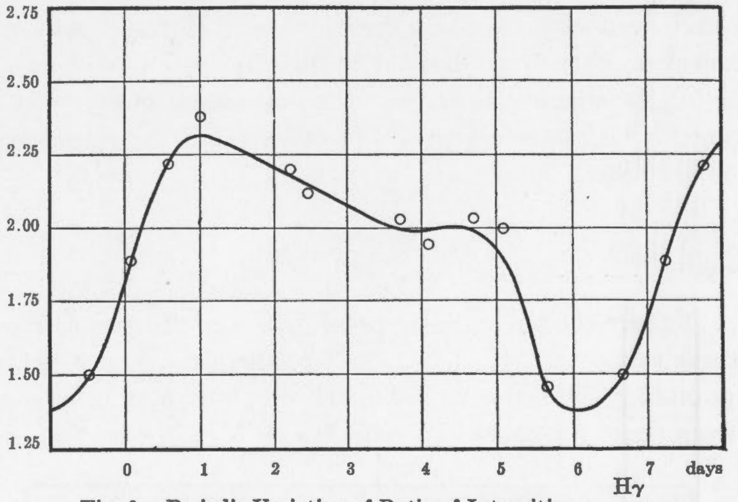

Fig. 6.—Periodic Variation of Ratio of Intensities $\frac{H\gamma}{\text{Vd } 4330}$

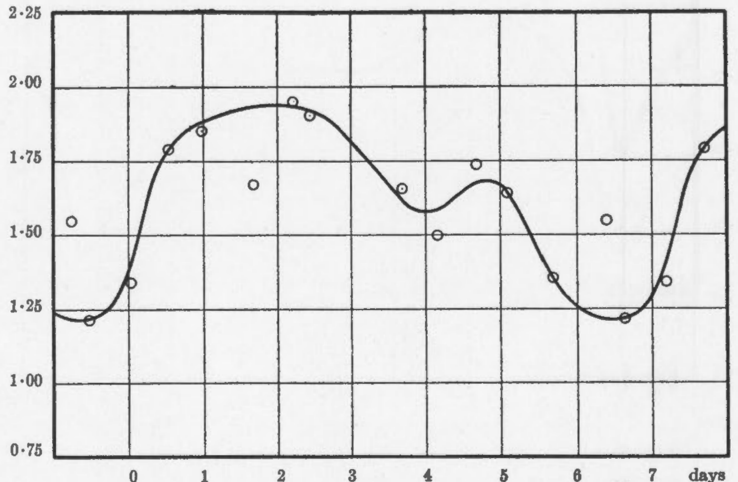

Fig. 7—Periodic Variation of Ratio of Intensities $\frac{H\gamma}{\text{Vd } 4352}$

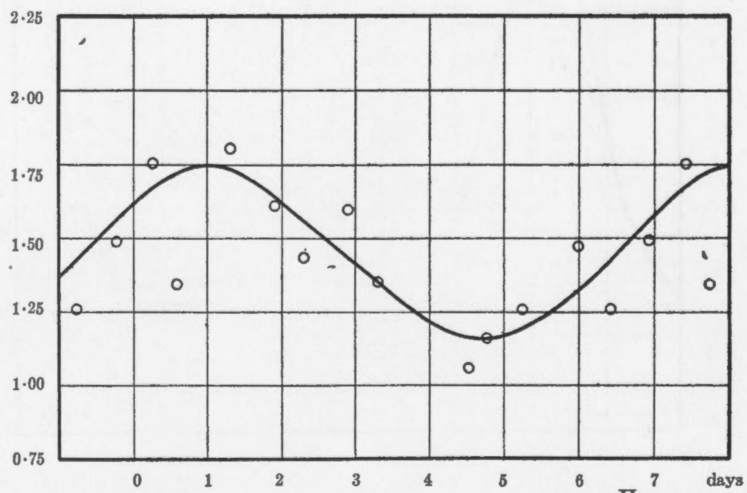

Fig. 8.—Periodic Variation of Ratio of Intensities $\frac{H\gamma}{\text{Fe } 4325}$

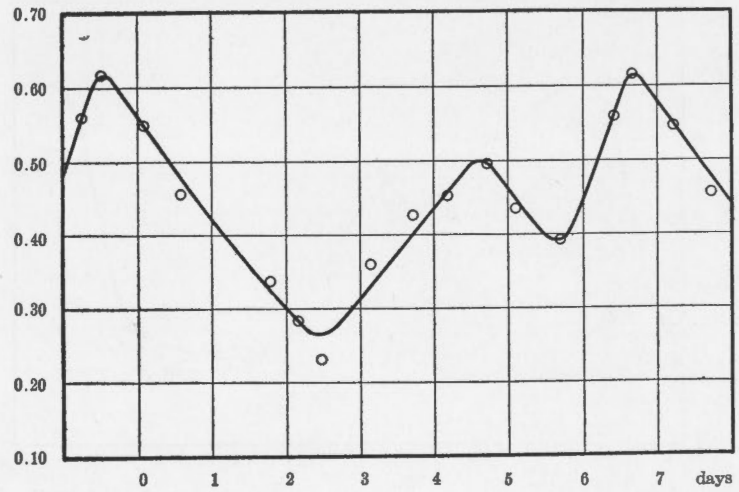

Fig. 9.—Periodic Variation of Ratio of Intensities $\frac{\text{G band}}{\text{Cr 4289}}$

Fig. 4 can be compared with much advantage with the corresponding curve of ratios of titanium lines published on p. 145 of this volume. That curve, however, was obtained from eye estimates.

Fig. 9, the curve for the *G* band, shows exactly the reverse tendency to the curve of ionization. The *G* band is strongest at minimum light, as was to be expected.

Secondary oscillations are also found in these various curves, just as they were established in the light-curve and in some of the radial velocity curves.

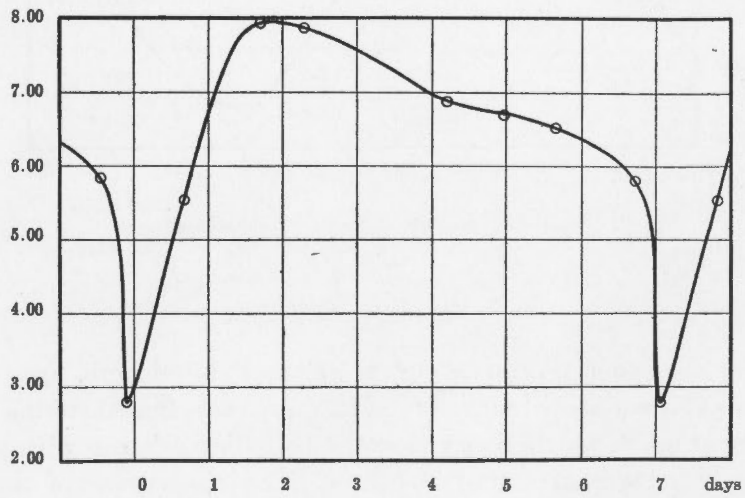

Fig. 10.—Periodic Variation of Ratio of Intensities $\frac{\text{Fe}^{+}\ 4555 \cdot 9}{\text{Vd 4561}}$

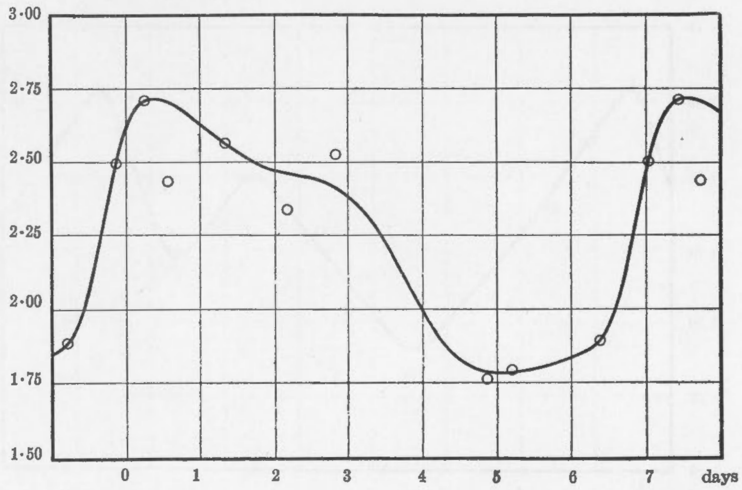

Fig. 11.—Periodic Variation of Ratio of Intensities $\frac{Sc^{+}\ 4320\cdot 7}{Ca\ 4318\cdot 6}$

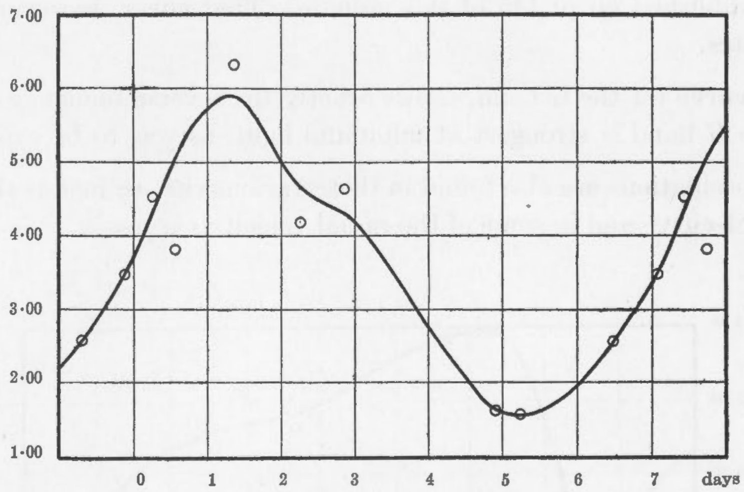

Fig. 12.—Periodic Variation of Ratio of Intensities $\frac{Sr^{+}\ 4215}{\lambda\ 4213}$

Only three of the other seventeen curves are reproduced here, figs. 10, 11 and 12. They are based upon fewer measurements and in most cases the scattering is considerably greater than that of fig. 4, but in every case the periodic variation with phase is clearly evident. The distinctive characteristics exhibited by the curves of lines of different elements have been used in another investigation [1] in which astrophysical estimates of the *ionization potentials* of *iron*, *yttrium* and *lanthanum* have been obtained as follows:—

[1] Nature, June 9, 1928, p. 906.

Element	Astrophysical estimate of I.P.	Previous determinations of I.P.
Fe	6·6	5·9, 8·15 (Sommerfeld, Gieseler, Grotrian). 7·5 (Menzel.)
Y	6·6	6·6 (Meggers, Russell)
La	4·9	- -

A glance at the curves in fig. 2 will show at once that very many lines change in intensity with phase. Some of these are so heavily blended that there seems no advantage to be gained by plotting the variations. It is not exclusively the ionized and hydrogen lines which undergo these changes. Certain neutral lines, both ultimate and penultimate, are similarly, but to a less degree (see fig. 7), strengthened at maximum light. Examples of this are *Ca* 4227 and *Fe* 4260 and the ultimate vanadium line λ 4352, whose periodicity in another cepheid variable led one investigator [1] to regard it as an ionized line, attributing it to Ti^{+}. Other neutral lines fall in intensity at maximum light and are strengthened noticeably at minimum. This is the case with *Ti* 4534·78 (see fig. 3), and in fact the periodicity shown in fig. 4 is due in larger measure to the rising and falling of the arc line than to the waning and rising of the ionized line. This is not the case, however, with figs. 5, 10, 11 and 12, which are curves of ionization intensity in the strictest sense.

DOMINION OBSERVATORY,

OTTAWA, September, 1928.

[1] Ap. J. October, 1927, p. 180.