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DEPARTMENT OF THE INTERIOR
CANADA
HON. CHARLES STEWART, *Minister* W. W. CORY, C.M.G., *Deputy Minister*

PUBLICATIONS

OF THE

Dominion Observatory

OTTAWA

R. MELDRUM STEWART, M.A., *Director*

Vol. IX

Astrophysics

No. 1

THE CEPHEID PROBLEM

BY

F. HENROTEAU, D. Sc.

OTTAWA
F. A. ACLAND
IMPRIMEUR DE SA TRÈS EXCELLENTE MAJESTÉ LE ROI
1925

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Chapter 1

THE GENERAL CHARACTERISTICS OF CEPHEID VARIATION

The nature of Cepheid variation is a very well known problem, whose study has received a great impetus from the work of such astronomers as Curtiss, Duncan, Guthnick, Hagen, Hertzsprung, Ludendorff, Luizet, Nijland, Perrine, Shapley and others. In the present state of our knowledge it appears that the variations of stars of the δ Cephei type, and their underlying causes, have a great many characteristics in common with those of the β Canis Majoris type, as has already been pointed out by the writer. Not only these two types, but also others, ought perhaps to be included in the general term Cepheids.

The present article is based on observations covering several years. The work of observing, as well as of measurement and reduction, has been shared equally between Mr. J. F. Frédette and the writer, while during the past year Mr. R. Callander also took a large share of the work. Thanks are also due to Mr. Frédette for many valuable practical suggestions. It is proposed first to review briefly the characteristics of Cepheids and all the allied types, emphasizing especially the analogies existing between them; fresh studies of stars of the β Canis Majoris type which have a bearing on the problem, as well as results of the photographic study of some Cepheids, will also be included; at the close will be found some suggestions concerning the causes of Cepheid variation, as well as some remarks on what Shapley indicated as the period-luminosity relation in this type of variable. What it is proposed to include under the general term Cepheids would then be represented by the following types:—

1. δ Cephei type.
2. ζ Geminorum type.
3. Cumulids.
    - A. Antalgol type, such as RR Lyrae.
    - B. Cluster variables.
4. β Canis Majoris type.
5. α Orionis type.

Below is given a summary of the characteristics of these different subdivisions grouping together, however, the stars of the δ Cephei, ζ Geminorum and Antalgol types, the others being treated separately.

The δ Cephei, ζ Geminorum and Antalgol types

Although the characteristics of these types are fairly familiar it will be of interest to present them here succinctly; such a presentation will be of service in a comparison with the characteristics of the other types, especially those of the β Canis Majoris type. The number of stars included in these classes is fairly large, as will be seen from the accompanying table, which includes more particularly those situated in the northern hemisphere, with their respective elements.

TABLE OF CEPHEIDS AND THEIR ELEMENTS

No.	Name	α 1855·0	δ 1855·0	Spectral Class	Julian date of maximum or minimum	Period days	Visual Magnitudes	M—m days
		h m s	° ′					
1	SY Cassiop........	0 7 26	+57 37	G5	M2417911·615	4·07098	9·3—10·2	1·25
2	SW Androm.......	0 16 8	+28 36	A	8132·805	0·44185	9·2—10·0	0·052
3	TU Cassiop........	0 18 31	+50 29	Fo—F6	9302·12	2·139	7·3— 8·4	0·54
4*	α Urs. Min.......	1 6 31	+88 32	F8	8985·936	3·9681	2·3— 2·4	1·984
5	RR Ceti...........	1 24 41	+ 0 36	F	7501·455	0·553022	8·4— 9·0	0·080
6	RW Cassiop........	1 27 49	+57 1	F	7062·5	14·80	8·8—10·2	5·8
7	V Arietis.........	2 7 12	+11 34	N	8267·121	0·99248	8·3— 9·0	0·368
8	SU Cassiop........	2 39 7	+68 17	A8—F5	7287·30	1·9498	5·9— 6·3	0·90
9	RW Camelop.......	3 42 32	+58 13	K5	7857·4	16·402	8·5— 9·2	6·8
10*	SZ Tauri..........	4 28 49	+18 15	F4—G2	8724·12	3·1484	7·2— 7·7	—
11	SV Persei..........	4 39 36	+42 2	F8	7830·4	11·13	8·6— 9·2	—
12	RX Eridani.........	4 43 11	−16 0	F5	9853·2826	0·4593	8·8— 9·6	—
13	SU Aurigae........	4 46 45	+30 20	F8	7973·734	0·470143	8·4— 9·0	0·225
14	U Leporis........	4 50 4	−21 30	A	5020·3	0·58144 Variable	9·1—10·0	—
15	RX Aurigae........	4 51 22	+39 44	G5	5083·43	11·6263	7·2— 8·1	—
16	SX Aurigae........	5 1 27	+41 59	A	m2420446·654	1·53234 Variable	8·4— 9·3	—
17	SY Aurigae........	5 2 19	+42 39	G5	2417833·4	10·137	8·8— 9·5	—
18	Y Aurigae........	5 18 19	+42 19	M?	5420·64	3·8590	8·6— 9·6	0·73
19	RZ Geminorum....	5 53 52	+22 14	F2?	8313·313	5·52943	9·0— 9·8	1·728
20	SS Geminorum....	5 59 49	+22 38	K	8288	44·6	8·2— 9·3	20
21	RS Orionis........	6 13 57	+14 44	F5?	8274·65	7·5665	8·2— 8·9	2·89
22	T Monocerotis...	6 17 24	+ 7 10	F4—F8	M2410011·200	27·0122	6·0— 6·8	5·1
23	RT Aurigae........	6 19 15	+30 35	A8—Go	7173·3	3·7282	5·0— 5·9	1·22
24	W Geminorum....	6 26 39	+15 26	F3—Go	3266·34	7·91603	6·4— 7·7	2·91
25*	ζ Geminorum....	6 55 30	+20 47	Go	0640·60	10·15382	3·7— 4·1	5·23
26*	RU Camelop.......	7 5 59	+69 56	R	7610·96	22·172	7·9— 9·0	9·4
27	RR Geminorum....	7 12 18	+31 9	F5?	6223·286	0·3972927	9·7—10·6	0·050
28	X Puppis........	7 26 30	−20 36	G5	5021	25·953	8·0— 9·0	5·0
29	Z Cancri........	8 14 6	+15 27	Mc	8006	74	8·5— 9·2	37
30*?	W Urs. Maj......	9 33 32	+56 37	F8p	m 6129·19264	0·333640	7·9— 8·5	—
31	Z Leonis.........	9 43 48	+27 35	Mb	M 8060·0	56·36	7·9— 9·6	—
32	ST Urs. Maj......	11 19 54	+45 59	Mb	m 8229·0	8·8?	6·7— 7·8	—
33	SU Draconis.......	11 29 38	+68 8	A2	M 9708·276	0·6604347	8·9— 9·6	0·15
34	SW Draconis.......	12 10 38	+70 19	F4	8086·2962	0·56965	8·3—10·4	0·125
35	W Virginis........	13 18 33	− 2 37	Pec	2402708·2666	17·2711	8·7—11·0	8·20
36	RV Urs. Maj......	13 27 34	+54 44	F5?	2417861·434	0·468058	9·4—10·3	0·16
37	V Urs. Min.......	13 36 7	+75 3	Mb	9216	71	7·5— 8·7	32
38	RS Boötis.........	14 27 21	+32 23	B8—Fo	8115·626	0·377333	9·2—10·2	0·056
39	RY Boötis.........	14 43 12	+23 38	F5	9229·1	9·0 ±	7·1— 7·4	3
40	RW Draconis.......	16 32 54	+58 8	A?	7407·27917	0·442938	9·9—11·0	0·05
41	Y Ophiuchi.......	17 44 52	− 6 6	F5—Go	2408694·25	17·1207	6·2— 7·0	6·22
42*	W Serpentis......	18 1 31	−15 34	Go	2419223·0	14·13	8·5— 9·6	—
43	WZ Sagittarii......	18 8 28	−19 7	Kp	9229·4	21·7	7·7— 9·2	7
44	Y Sagittarii......	18 12 51	−18 55	F8	0175·10	5·7734	5·8— 6·6	2·10
45	XX Sagittarii......	18 16 20	−16 52	G?	9241·45	6·43	8·3— 9·6	2
46	U Sagittarii......	18 23 21	−19 13	F7	M2414935·3	6·74467	7·0— 8·0	3·3
47	Y Scuti..........	18 30 9	− 8 29	G5	m 7734·8	10·347	8·7— 9·2	3·95
48	RU Scuti..........	18 34 17	− 4 15	K	M 9217·5	20·3	7·9— 9·9	9
49	RZ Lyrae.........	18 38 14	+32 39	A?	8450·235	0·5112750	9·9—10·3	0·05
50*	YZ Sagittarii......	18 41 6	−16 53	F9	9645·9	9·553	7·2— 7·7	—

TABLE OF CEPHEIDS AND THEIR ELEMENTS—*Concluded*

No.	Name	α 1855·0	δ 1855·0	Spectral Class	Julian date of maximum or minimum	Period days	Visual Magnitudes	M—m days
		h m s	° ′					
51	S Scuti..........	18 42 28	− 8 4	N	5979	23	6·4— 7·3?	—
52	SZ Aquilae..........	18 57 18	+ 1 6	Gp	7740·811	17·1362	8·2— 9·2	6·12
53	TT Aquilae..........	19 0 52	+ 1 4	F9p	1873·865	13·753	7·3— 7·9	5·30
54	RR Lyrae..........	19 20 51	+42 30	B9—F2	9697·764	0·566826	6·8— 7·7	0·12
55	U Aquilae..........	19 21 33	− 7 20	F7	0170·325	7·02387	6·2— 6·9	2·3
56	XZ Cygni..........	19 29 29	+56 5	A	7201·25417	0·46659	8·7— 9·3	0·104
57	U Vulpeculae.....	19 30 17	+20 1	FGKK5	4200·253	7·98950	6·9— 7·6	3·464
58	SU Cygni..........	19 39 0	+28 55	F5	4202·820	3·845612.	6·7— 7·3	1·29
59	TW Aquilae..........	19 44 18	+13 37	K?	9288	96	10·6—12·7	—
60	η Aquilae..........	19 45 5	+ 0 38	A8—G5	2396168·732	7·176382	3·7— 4·3	—
61	S Sagittae..........	19 49 26	+16 15	F4—G3	2409863·324	8·381613	5·4— 6·1	2·60
62	X Vulpeculae.....	19 51 27	+26 10	K?	2417040·732	6·31896	8·5— 9·1	2·05
63	TX Aquilae..........	19 59 23	+ 3 27	F?	9294	32·7	9·3—10·5	18
64	XX Cygni..........	20 0 25	+58 32	A ·	6563·41065	0·13486522 Variable	11·4—12·1	0·042
65	RW Aquilae..........	20 5 12	+15 38	F	5587·60	7·87	8·3— 9·3	—
66	R Sagittae..........	20 7 27	+16 17	G	2400358·5	70·56	8·5—10·3	—
67	SZ Cygni..........	20 28 10	+46 6	Ko	2415097·08	15·1126	8·6— 9·9	5·4
68	V Vulpeculae.....	20 30 32	+26 6	G9	m 6411·4	37·79	8·3— 9·0	—
69	X Cygni..........	20 37 44	+35 4	Go	M2410190·678	16·38543	6·2— 7·4	6·1
70	T Vulpeculae.....	20 45 19	+27 42	F7	2409849·079	4·435521	5·5— 6·4	1·361
71	UY Cygni..........	20 50 23	+29 53	K?	2415346·3933	0·5607103	9·7—10·5	0·08
72	VX Cygni..........	20 51 52	+39 37	K	4935·0	20·1306	9·1—10·3	6·2
73	RV Capricorni.....	20 53 25	−15 48	A	7436·87	0·4676	9·2—10·7	0·075
74*	VY Cygni..........	20 58 43	+39 24	K	6370·9507	7·85926	8·6— 9·4	—
75	SW Aquarii..........	21 7 50	− 0 32	F?	9686·3396	0·45932	9·9—10·8	0·12
76	VZ Cygni..........	21 45 53	+42 27	F5	7061·980	4·86384	8·4— 9·2	1·06
77	RY Pegasi..........	21 59 28	+32 48	Md	7801·00	25	10·0—10·6	—
78	Y Lacertae..........	22 3 30	+50 20	F	7615·76	4·3254	9·1— 9·6	1·06
79	δ Cephei..........	22 23 48	+57 40	F2—G3	2393659·856	5·366386	3·6— 4·3	1·619
80	W Cephei..........	22 30 56	+57 41	K	2412778·1	6·44	Pec.	—
81	RZ Cephei..........	22 34 10	+64 6	A	9742·273	0·30864	8·6— 9·3	0·08
82	Z Lacertae..........	22 35 9	+56 4	F5?	7844·4	10·89	8·5— 9·3	4·5
83	RR Lacertae..........	22 35 41	+55 41	F	7882·6	6·412	8·7— 9·4	1·1
84	V Lacertae..........	22 42 44	+55 33	G2	6666·76	4·98269	8·5— 9·4	1·65
85	X Lacertae..........	22 43 9	+55 40	G2	m 6672·45	5·44269	8·2— 8·6	—
86	SW Cassiop..........	23 0 59	+57 46	G2	M 7809·2	5·44	9·0— 9·9	—
87	RU Aquarii..........	23 16 48	−18 7	Pec.	7845	64·6	8·7— 9·7	—
88	RS Cassiop..........	23 30 32	+61 38	G5	7414·36	6·295	9·1—10·7	1·8
89*	RY Cassiop..........	23 45 0	+57 56	G5	7354·44	12·328	9·3—10·2	4·7
90	U Pegasi..........	23 50 34	+15 8	F?	m2415021·2469	0·1873835	9·3— 9·9	—
91	X Sagittarii......	17 39 42	−27 47	F2—G	M2402854·389	7·01188	4·4— 5·0	2·896
92	W Sagittarii......	17 57 2	−29 35	F5	2849·45	7·5946	4·3— 5·1	3·00

The stars marked with an asterisk are of the ζ Geminorum type, while those with periods smaller than a day may be considered as Cumulids or Antalgol stars. Peculiarities in individual stars are indicated in the following remarks:—

10. Shapley thought the variability of this star could be explained by the rotation of a Jacobian ellipsoid (A.N. 4653).

14. The period of U Leporis is about $0^{d}58144$ but according to Innes is variable (A. J. 468 and 486).

16. The period of SX Aurigae was given as $1^{d}53234$ but is suspected to be variable.

17. The descending branch of the light curve is shorter than the ascending, which is contrary to the known behaviour of stars of the δ Cephei type.

26. Shapley thinks that RU Camelop. can be explained as an ellipsoidal variable (Laws Obs. Bul., No. 21).

29. Z Cancri is inclined rather to the type of long-period variable.

30. W Ursae Majoris, for a long time considered a Cepheid, has been found to be a star of the β Lyrae type with the two minima equal (period 8 hours). Both spectral components show very wide lines due to rapid rotation (Contributions from the Mt. Wilson Observatory, Vol. VIII).

35. There are unexpected irregularities in the light curve of W Virginis. Bright lines are present in the spectrum, and the variations of these bright lines probably alter the simple Cepheid character of the star, as would probably be shown if the variation of the continuous spectrum alone were considered.

42. The maximum light of W Serpentis seems to vary, while the minimum is very sharply defined.

51. S Scuti is possibly an irregular variable. Merrill has shown that when a variable star is of class N it is usually irregular.

61. The light curve of S Sagittae was found to vary greatly in appearance. Similar variations in radial velocity are suspected by Curtiss. Its variation will be referred to in the course of this article.

81. Irregularities in the light curve. According to Shapley the mean velocity of the star is in excess of 1000 km. per sec.

The principal characteristics of the stars of the δ Cephei and Antalgol types are the following:—

1. They show a regular variation having usually a constant or perhaps (as has been found in a few cases) a very slightly variable period.

2. Their periods range from a few hours to several days, as may be seen from the above table. It is probable that they gradually merge into the long-period type.

3. The visual range of variation is rarely over 1·5 magnitudes, often in the neighbourhood of 0·8 magnitudes. Photometry of precision is revealing some very small ranges as for example in the case of Polaris.

4. The variation of light is continuous; the ascending branch of the curve is almost always shorter than the descending one, as may be deduced from the column *M—m* of the above table. (In very rare cases such as that of SY Aurigae the reverse occurs, see remark 17 above). When the two branches of the curve are of the same length the star is classified as a Geminid (ζ Geminorum), in which case it might be considered as a star of the β Lyrae type with two equal minima, or as an ellipsoidal variable. W Ursae Majoris, for example, is certainly a star of the β Lyrae type (Algol type) with its two minima equal, as has already been pointed out in remark 30. Unlike the Cepheids it is a dwarf star of absolute magnitude 3·5.

According to Dr. Shapley the variables RU Camelopardalis, SZ Tauri and the southern variable S Antliae may be explained by the rotation of a Jacobian ellipsoid having a strong darkening at the limb.[1] Such a supposition has also been made by Stebbins in the case of π^5 Orionis.[2] On the other hand SZ Tauri possesses most of the characteristics generally attributed to Cepheid variables, some of which would disagree with the theory of a rotating Jacobian ellipsoid.

5. The light curve is usually smooth and flowing but in many cases there are evidences of secondary maxima and minima. A good deal of discussion has occurred as to whether these secondary oscillations are real or due to errors of observation. The photographic light curves published by Plummer and Martin in Monthly Notices show a great many oscillations, while similar light curves determined by F. C. Jordan with the large telescope of the Allegheny Observatory do not show them at all. Accurate light curves determined with the photo-electric cell show that secondary humps on the descending branch really exist; on the hypothesis of a binary system these humps might coincide with the periastron and apastron passages. The photo-electric light curve of η Aquilae as determined by C. C. Wylie is a good illustration of these secondary humps (see figure, Ap. J., Vol. 56, p. 229).

6. The photographic range of variation as far as determined is usually greater than the visual, frequently one and a half to two or even in some cases three times as large. In other words there is a continuous variation of the colour-index synchronous with the variation of light. The stars are therefore much redder at minimum than at maximum.

Schwarzschild was one of the first to discover this difference of range between the photographic and visual curves[3]; he found the photographic range of η Aquilae to be double the visual. Wirtz found similar properties for δ Cephei and ζ Geminorum,[4] and Wilkens for X Cygni and other stars.[5] The photographic work of Martin and Plummer[6] suggests similar results for the Antalgol variables. Contrary to all other experience, however, Parkhurst and Jordan find that in the case of XX Cygni[7] the photographic range is less than the visual.

There has appeared recently a series of observations made by Galissot at the Lyons Observatory[8] with a Nordmann's heterochrome photometer (using red, green and blue light), showing that the colour index of ζ Geminorum may be considered as constant throughout the variation. The curves which he obtained, published by Prof. Mascart, director of the observatory, show very small variations between the three colours (smaller than the possible errors of observation). Prof. Mascart concludes therefore that the variation of ζ Geminorum must be attributed to a cause similar to that which produces the Algol variables, or some other cause distinct from that giving rise to the Cepheids. On the other hand the determinations of the light curve and colour index curve of the same star were also made recently by Prof. Guthnick,[9] who finds a decided variation in the colour index.

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[1] Laws Obs. Bul. No. 21, p. 73.

[2] Ap. J., Vol. 51, p. 218 (1920).

[3] Pub. der Kuffnerschen Sternwarte, Vol. 5C, p. 100 (1900).

[4] A. N., Vol. 154, p. 327, 1901.

[5] A. N., Vol. 172, p. 316, 1906.

[6] M. N., Vol. 73, p. 166, p. 440, 74, p. 225.

[7] Ap. J., Vol. 23, p. 84, 1906.

[8] Bul. de l'observatoire de Lyon, Vol. 4, p. 99, 1922.

[9] A. N. Jubilaumsnummer Tafel 2.

7. The variation in colour index is accompanied (as might have been expected) by a variation in spectral class. The class is usually earlier (indicating of course higher temperature) at maximum, and later at minimum brightness; a shift of the maximum intensity in the spectra of Cepheids was discovered by Albrecht[1] and confirmed by Kiess[2] and other Lick observers, while a change of spectrum was first discovered by Albrecht and Duncan.[3] Shapley however determined the variation in spectral type definitely for several Cepheids; according to Albrecht the range of variation in spectral class is about one class interval and is independent of the period.[4] Adams and Joy have noted a marked distinction in spectral types of Cepheids, depending on whether obtained from hydrogen lines alone or from the general spectrum.[5] The range of variation in class is much more considerable as obtained from the hydrogen lines. The variation in spectral class is also accompanied by a variation in the sharpness and width of the lines. Prof. Adams, for example, measuring the widths of the lines in δ Cephei finds the following results[6]:—

Average width at maximum 0·214 A
Average width at minimum 0·403

These results recall very much the changes of line widths in the spectra of the stars of the β Canis Majoris type.[7]

8. All the Cepheids show variation of radial velocity with the period of the light changes. The epoch of minimum radial velocity (most rapid approach) coincides closely with that of maximum light, and that of maximum radial velocity with minimum light. These changes of radial velocity were at first attributed to the fact that the Cepheids might be spectroscopic binaries; on this assumption it is found that the orbits possess very peculiar properties; their eccentricities are usually very large, which is contrary to what usually occurs for ordinary spectroscopic binaries of similar periods; the values of the angular distance of periastron from the receding node present a maximum frequency in the first quadrant and practically never occur in the third quadrant; in ordinary spectroscopic binaries these values are distributed at random; usually the value of the mass function $\dfrac{m_1^3 \sin^3 i}{(m+m_1)^2}$

(where m is the mass of the primary, m_1 that of the secondary and i the inclination between the plane of the orbit and the plane perpendicular to the line of sight) is extraordinarily small. Perrine points out that the larger the value of the mass function the larger the secondary humps in the light curve. The value of $a \sin i$ is also small, the greatest value found being approximately 2,000,000 km. and the least 45,000; this led Shapley to say that, interpreted as spectroscopic binaries, the Cepheids move in orbits whose apparent radii average less than one-tenth the radii of the stars themselves.[8] All these characteristics and others led first Shapley, then Ludendorff, Eddington and others to reject the binary theory of Cepheids and adopt pulsation or other factors as the cause of Cepheid variation.

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[1] L. O. B., Vol. 4, p. 131.
[2] L. O. B., Vol. 7, p. 140.
[3] L. O. B., Vol. 5, p. 93.
[4] Ap. J., Vol. 54, pp. 161-190.
[5] Proc. Nat. Ac. Sc. Washington, 4 p. 129-132 (1918).
[6] The Observatory, Vol. 42, p. 167 (1919).
[7] L. O. B., Vol. 9, p. 158 (1918).
Ap. J., Vol. 40, p. 459.

For many Cepheids the total range of velocity variation is so small that secondary oscillations and other irregularities of considerable importance may easily be lost in the accidental errors. In the cases of ζ Geminorum[1] and W Sagittarii[2] the velocity curves show marked humps, indicating, if orbital motion is assumed, considerable departure from simple elliptic motion. In the cases of Y Ophiuchi and T Vulpeculae Albrecht suspects that some spectral lines give abnormal velocities[3] while in the case of W Virginis, which exhibits emission lines, the Mt. Wilson observers find that marked differences are shown in the radial velocities given by the dark and bright lines.[4] The apparent brightness of Cepheids being usually small, very few complete investigations of the radial velocities have been made; usually only sufficient observations have been secured during a short interval of time to determine one velocity curve (of the same period as that of light variation). In some cases however, such as α Ursae Minoris and Y Sagittarii.[5] there are direct evidences of a variation of the centre of mass of the short-period system; Ludendorff has also pointed out from the observations of Albrecht that a variation of amplitude and perhaps of center-of-mass velocity probably exists in the case of Y Ophiuchi[6]; the system of δ Cephei is thought by Bélopolsky to be triple[7], while there are indications from Belopolsky's and Wright's observations that η Aquilae is also a triple system. Analogy with the fact that all stars of the β Canis Majoris type which have been investigated, and which are most probably a special kind of Cepheids, behave like triple systems, suggests that Cepheids when more thoroughly investigated will also prove to be such, or rather suggests the superposition of a physical phenomenon (pulsation or any other action accounting for the variation of brightness) on a purely mechanical phenomenon, the revolution of the star around a companion in an orbit.

Prof. P. Guthnick from his photo-electric cell investigations finds that the light variations of many stars seem to be due to a combination of an Algol type variation, or variation due to eclipse (orbital motion) with a plain Cepheid variation (most likely not orbital)[8]; these two types of variation have different periods. The idea is hence suggested that in the case of an ordinary Cepheid no eclipse is produced, (unless a very small partial eclipse), but that the Cepheid variation is bound up with the presence of a second body which produces some kind of tidal effect upon the primary,

In a great many instances where the star is too faint or has not been investigated with a spectrograph, variations in the shape and the period of the light curve are strong indications of variations in the centre-of-mass velocity of the short-period oscillation. A remarkable case is that of S Sagittae, whose many light curves determined by several observers are so different from one another; the curves generally show a secondary minimum, perhaps indicative of partial eclipse; this secondary minimum in different curves is more or less pronounced, while the values of the two maxima are not always equal

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[1] Ap. J., Vol. 13, p. 94.
[2] L. O. B., Vol. 3, p. 36.
[3] L. O. B., No. 118.
[4] Mt. Wilson report, 1921, p. 269.
[5] Ap. J., Vol. 56, p. 373, 1922.
[6] A. N., Vol. 203, p. 368.
[7] Mitt. Pulk, Vol. 3, p. 70, 1909.
[8] Veroff der K. Sternw. zu Berlin Babelsberg Band II, Heft 3, p. 129.

and are separated by different intervals of time. A very good representation of light curves of S Sagittae obtained by different observers has been given by Luizet,[1] while a comparison of the visual and photo-visual curves has been given by F. C. Jordan.[2]

Assuming the radial velocity curves of Cepheids to be due to orbital motion (which is probably a wrong assumption), the orbits given in the accompanying table have been determined.

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[1] Sur l'Etoile variable S Fleche Bul. Soc. Astr. de France, 1907, p. 277.

[2] Ap. J., Vol. 50, p. 195.

ORBIT ELEMENTS OF CEPHEID VARIABLES (ASSUMING BINARY THEORY)

Star	α 1900·0	δ 1900·0	P	T Julian date	ω	e	K	$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$	a. sin i	γ	Remarks
	h m	° ′			°		km.		km.	km.	
α Urs. Min.	1 22·6	+88 46	3^{d}·9683	2415398·50	113·6	·20	3·00	·00001	160,400	Var.	Hartmann.
			3·9683	4890·04	80·0	·13	3·04	·00001	164,500	Var.	Hobe.
			11·9 years?		293	·35	2·98	·0098	166,800,000	−14·8	Centre of mass of short period.
SU Cass.	2 43·0	+68 28	1^{d}·9495				11·0	·0003	295,000		
SZ Tauri	4 31·4	+18 20	3·1481				10·9	·00039	460,000		
RT Aurigae	6 22·1	+30 34	3·7282	M+3^{d}·423	95·0	·368	17·96	·0018	856,000	+21·43	Duncan (1908).
			3·72806	M+3^{d}·686	115·5	·428	11·97		554,700	+19·96	Kiess (1917).
ζ Gemin.	6 58·2	+20 43	10·154	m+1·313	333	·22	13·2	·0023	1,797,800	+ 6·8	Campbell, unexplained oscillation.
S Muscae	12 7·4	−69 26					16 ?			+ 8 ?	
R Triang A.	15 10·8	−66 8					17 ?			−18 ?	
S Triang A.	15 52·2	−63 30					14 ?			+ 7 ?	
S Normae	16 10·6	−57 39					12 ?			0 ?	
RV Scorpii	16 51·8	−33 27					16 ?			−25 ?	
X Sagittarii	17 41·3	−27 48	7·01185	2416723·05	93·6	·40	15·2	·0011	1,334,000	−13·50	Moore.
Y Ophiuchi	17 47·3	− 6 7	17·1207	M+2^{d}·6	209·2	·10	8·5	·0011	1,999,000	− 5·0	Albrecht. According to Ludendorff vel. curve of Y Ophiuchi varies in amplitude. A. N. 203 p. 368.
W Sagittarii	17 58·6	−29 35	7·59460	M+6·20	70·0	·32	19·5	·00499	1,930,000	−28·6	Curtiss Secondary oscillations.
Y Sagittarii	18 15·5	−18 54	5·773268	M+ 4·51	43·0	·21	19·3	·004	1,500,000	+ 3·6	Duncan (1908).
				M+ 5·05	74·5	·42	20·6	·003	1,354,000	− 5·9	Duncan (1921).
RR Lyrae	19 22·3	+42 36	0·566826	M+ 0·508	96·85	·271	22·2	·00057	166,500	−68·7	Kiess.
U Aquilae	19 24·0	− 7 15	7·02387								Albrecht.
SU Cygni	19 40·8	+29 1	3·844	M+ 2·5	345·8	·21	25	·0058	1,350,000	−33·4	Madrill.
η Aquilae	19 47·4	+ 0 45	7·176	m+ 1·92	90	·163	16·3	·0031		−13·7	Bélopolsky.
				M+ 6·210	68·91	·489	20·59	·0043	1,545,000	−14·16	Wright.
S Sagittae	19 51·5	+16 22	8·3832		69·9	·35	19	·0049	2,000,000	−12·5	Madrill.
X Cygni	20 39·5	+35 14	16·38543	M+14·685	101·1	·246	28·03	·034	6,121,000	+ 9·32	Duncan.
T Vulpeculae	20 47·2	+27 52	4·43578	M+ 3·76	111	·43	17·6	·0018	969,180	− 1·3	Albrecht.
δ Cephei	22 25·4	+57 54	5·366386	m+ 1·07	88	·46	20·5	·0034	1,300,000	+ 0·5	Bélopolsky.
				m+ 1·002	82·8	·355	19·81	·0037	1,370,000	Var.	Bélopolsky.
				2417888·428	85·385	·484	19·675	·0028	1,270,600	−16·83	Moore.

The spectroscopic investigations of many of these Cepheids are incomplete, while only five orbits have been determined a second time, those of α Ursae Minoris, RT Aurigae, Y Sagittarii, η Aquilae and δ Cephei. Of these at least the first three, and perhaps all, have a strong resemblance to triple systems. It can be seen that considerable work has yet to be done on the orbits of Cepheids before it will be possible to present a satisfactory theory of these variables.

One of the most important characteristics of the Cepheid variables is that they all appear to be giant stars or even super-giants, stars of exceedingly small density and of enormous volume, though not necessarily massive. From a consideration of their proper motions Prof. Hertzsprung was the first to point out that they were giants.[1] Direct determinations of parallaxes of Cepheids show that they must be at a considerable distance and hence very luminous.

TRIGONOMETRIC PARALLAXES OF CEPHEID VARIABLES

Star	Parallax	Observer
	″	
α Ursae Min.	+0·041	Flint et al.
SU Cassiop.	+0·010	Mt. Wilson.
RX Aurigae	+0·001	Mt. Wilson.
T Monocerotis	−0·009	McCormick.
RT Aurigae	+0·031	Mt. Wilson—Sproul.
	+0·003	McCormick.
ζ Gemin.	+0·015	Allegheny-Sproul.
	+0·0011	McCormick.
RR Lyrae	+0·008	Mt. Wilson.
U Vulpeculae	+0·009	McCormick.
η Aquilae	+0·004	McCormick.
S Sagittae	+0·005	McCormick.
X Cygni	+0·006	Sproul.
T Vulpeculae	+0·018	McCormick.
δ Cephei	+0·011	Allegheny.

The probable errors of these parallaxes vary from ±0″·003 to ±0″·011, that of α Ursae Minoris being perhaps somewhat larger. Quantitatively, they do not mean much, but qualitatively, they show that the Cepheids must be very distant. The investigations of the spectra of these stars show them to be in a very highly ionized state. The lines are usually narrow, well-defined, sharp—they are all of what is called the *c* characteristic, a characteristic which usually corresponds, as in α Cygni, to exceedingly small density. The proper motions of 74 Cepheid variables have been studied by R. E. Wilson[2]; they have been found exceedingly small, only two exceeding 0″·2 per annum, and the mean of the remaining 72 being 0″·025 per annum. Taking account of the radial velocities of 25 of these stars, the mean parallax of the group is found to be 0″·0030, in good agreement with the parallaxes determined directly.

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[1] A. N., Vol. 196, p. 203.

[2] Pop. Ast., Vol. 31, p. 258, 1923. See also A. J., Vol. 35, p. 35.

From spectral characteristics there are very few stars that seem to be super-giants and they have been classified into *Cepheids* and *Pseudo-Cepheids*. The spectra of these two classes are very much the same, with very sharp and narrow lines, very strong enhanced lines corresponding to ionized elements. In particular, attention may be called to the following enhanced lines which are also prominent in the spectrum of the solar chromosphere and in stars of the α Cygni type.

Sr 4077
Sr 4215
Fe 4233
Y 4246
Ti 4290
? 4375
Fe 4385
Ti 4534
Fe 4584

The stars of the first class are known to vary considerably in light, while those of the second have certainly no large variation, if any. Many of the latter, however, if investigated with a photo-electric cell might prove to be variable also, but with a very small amplitude; they might be similar to α Ursae Minoris, whose radial velocity curve and magnitude variation are not very marked. Following is a list of the principal Pseudo-Cepheids as discovered by Prof. Adams:—

TABLE OF PSEUDO-CEPHEIDS

No.	Star	α 1900	δ 1900·0	Vis. Mag.	Spectral Class	Annual Proper Motion	Adams Spectrosc. parallax	Trig. parallax	Radial Vel.
		h m	° ′			″	″	″	km.
1	14 Persei	2 37·6	+43 52	5·6	Gop	0·004	0·003	....	− 3·8
2	α Persei	3 17·2	+49 30	1·9	F5	0·039	0·023	0·017	Var.
3	58 Persei	4 29·8	+41 4	4·5	G4p	0·025	0·002	0·028	Var.
4	β Camelop.	4 54·5	+60 18	4·2	F7p	0·013	0·004	....	− 1·4
5	ε Aurigae	4 54·8	+43 41	3·4	F5p	0·014	0·008	0·060	Var.
6	α Leporis	5 28·3	−17 54	2·7	F4p	0·003	0·021	0·014	+24·3
7	46 Aurigae	6 17·2	+49 20	5·1	K2p	0·014	0·002	....	Var.
8	δ Can. Maj	7 4·3	−26 14	2·0	G2p	0·005	0·010	....	Var. ?
9	ξ Puppis	7 45·1	−24 37	3·5	G6p	0·007	0·003	....	Var.
10	ρ Puppis	8 3·3	−24 1	2·9	F7p	0·100	0·010	0·031	+46·0
11	29 Monocer.	8 3·6	− 2 42	4·4	G3p	0·025	0·002	....	+29·0
12	298 G. Puppis	8 18·6	−26 2	5·9	F5p	0·017	0·003	....	
13	β Draconis	17 28·2	+52 23	3·0	Gop	0·016	0·005	0·005	−20·5
14	*V* Herculis	17 54·7	+30 12	4·5	F2p	0·004	0·012	....	−21·6
15	π Sagittarii	19 3·8	−21 11	3·0	F4p	0·040	0·017	0·017	−10·1
16	α^1 Capricorni	20 12·1	−12 49	4·6	Gop	0·015	0·002	−0·012	−25·5
17	γ Cygni	20 18·6	+39 56	2·3	Gop	0·003	0·009	0·014	
18	ρ Capricorni A	20 23·2	−18 9	5·1	A9	0·026	0·010	0·024	+21·0
19	41 Cygni	20 25·3	+30 2	4·1	F6	0·010	0·009	−0·018	−18·4
20	ξ Cygni	21 1·3	+43 32	3·9	K2p	0·007	0·004	−0·003	Var.
21	ζ Capricorni	21 21·0	−22 51	3·9	G1p	0·023	0·003	....	Var.
22	β Aquarii	21 26·3	− 6 1	3·1	F9	0·017	0·009	−0·016	+ 5·6
23	γ Capricorni	21 34·6	−17 7	3·8	F4	0·188	0·013	0·018	Var.
24	α Aquarii	22 0·6	− 0 48	3·2	Go	0·015	0·006	0·010	+ 6·8
25	5 Lacertae	22 25·4	+47 12	4·6	K2p	0·021	0·003	0·005	−11·5
26	B. D. + 56° 2923	22 55·9	+56 25	5·5	G2p	0·009	0·002	....	
27	89 Aquarii	23 4·6	−23 0	4·9	F8	0·013	0·008	....	− 4·4
28	ρ Cassiopeiae	23 49·4	+56 57	4·8	G5p	0·007	0·002	0·032	−42·6

The variations of radial velocity of these stars are in most cases fairly small. A period of $4^d \cdot 0938$ has been found for α Persei by Hnatek in Vienna[1]; the semi-amplitude of velocity variation is 0·83 km. The stars 2, 5 and 28 are suspected by Guthnick to have a small variation of light; an elaborate study of these three is greatly to be desired, especially on account of their fairly large trigonometric parallaxes. It seems quite possible that an exhaustive investigation of the Pseudo-Cepheids would show them to be Cepheid variables with small variations of light; α Cygni although of class A2 and, on account of this, not classified by Adams among the Pseudo-Cepheids, is nevertheless a typical star of this kind; it has been found by Guthnick to vary in light[2]; its radial velocity is also variable. α Cygni is thus probably a Cepheid variable. The consideration of the Cepheids and the Pseudo-Cepheids leads us then to this: the Cepheids are all stars of very low density; but are all stars of low-density Cepheids, or are they merely Cepheids when some kind of action, for example that of a satellite, plays its part?

10. There are a few more possible characteristics of the stars of the δ Cephei type which might be mentioned here, for example Ludendorff's relation[3]

$$2\,K = 47 \cdot 3\,A,$$

where K is the semi-amplitude of velocity variation and A the range of visual magnitude. This relation however is only approximate and seems to be true only for the brightest Cepheids; if the stars of the β Canis Majoris type are considered to be Cepheids the relation is far from true.

Dr. R. H. Curtiss[4] has discussed several other possible characteristics of the Cepheid variables; most of these are of very considerable interest, but the data that he had at hand were in most cases not sufficient to establish them with certainty.

Another remarkable fact is that the stars of this type exhibit a progressive tendency towards more advanced spectral class as the length of the period of light variation increases (while the range of variation of spectral class of any one Cepheid is approximately one class interval and is independent of the period).

11. RR Lyrae is the brightest, and may be considered as the typical, Antalgol star; it is the only one whose radial velocity curve has been determined[5]; a redetermination of this velocity curve is however needed. Hertzsprung has shown definitely[6] that the amplitude of its light variation, as well as the period of this variation, is not constant, the oscillation, as has already been pointed out by Shapley,[7] having most likely an approximate period of $40^d \cdot 6$. This recalls the results that have been found for the stars of the β Canis Majoris type; 12 Lacertae, which shows considerable variations in the amplitude of its short-period velocity curve, also shows similar variations in its short-period light curve, as determined so accurately by Guthnick[8]. W Baade[9] found similar results for the Antalgol star SS Cancri, and according to Hertzsprung it seems likely that all Antalgol stars show similar characteristics, *i.e.* a variation of amplitude in the short-period curve,

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[1] A. N., Vol. 192, p. 245.
[2] Veroff der K. Sternwarte zu Berlin-Babelsberg, Band 1, p. 61.
[3] A. N., Vol. 193, p. 301.
[4] Pub. Detroit Obs., Vol. 1, p. 104.
[5] C. C. Kiess, L. O. B., Vol. 7, p. 140, 1913.
[6] Bul. of the Astr. Inst. of the Netherl., No. 24, 1922.
[7] Contributions from the Mt. Wilson Observatory, No. 112, 1916.
[8] A. N. Jubilaumsnummer Tafel 3.
[9] Mitt. d Hamburger Sternw. in Bergedorf 5, No. 14, 23, 1922.

this variation having most likely a period of several days or months, or, as has been remarked above, a short-period physical phenomenon and an orbit of considerably longer period. Such a phenomenon as this double variation is no doubt most easily discovered in very short-period Cepheids; in the case of longer periods most of the light curves are, so to speak, mean curves, or the combination of observations which have been taken during a long interval of time and whose phases have been computed by assuming a constant period for the variation. It would be of great advantage to have an international agreement between observatories of different longitudes, if possible encircling the earth, to follow the same Cepheids during the same lapse of time so that a set of continuous curves could be obtained; similar instruments and methods ought however to be adopted by the observers; it is practically certain that results of great value would be obtained.

The annual proper motion of RR Lyrae is $0''\cdot25$; this is considerable when compared with the proper motions of most of the stars of the δ Cephei type, and led Hertzsprung to suspect that the star is not distant. The parallax as directly measured by Van Maanen at the Mt. Wilson Observatory is $+0''\cdot006 \pm 0''\cdot006$, which indicates that Hertzsprung's suggestion is probably not correct, but that the star has a very large motion in space. Its mean radial velocity, $-68\cdot7$ km. as determined by Kiess, certainly indicates a large motion. Most of the Antalgol stars so far measured have very high velocities, the highest, according to Shapley, being that of RZ Cephei which is of the order of 1,100 km. per second. Probably these high velocities are due to the small masses of the stars investigated, as was pointed out in a previous article[1].

12. A very remarkable fact is that most of the stars of the δ Cephei type have a low galactic latitude, while those of the Antalgol type have galactic latitudes distributed at random.

13. It will be unnecessary to give here the different hypotheses that have been brought forward to explain Cepheid variation, a very good account of most of these theories having been given by D. Brunt in his article "The Problem of the Cepheid Variables[2]." Duncan's hypothesis, for instance[3], is rather remarkable.

THE CLUSTER VARIABLES

The cluster variables are exceedingly faint stars which are found in the globular clusters and in the Small Magellanic Cloud. They have properties very similar to those of the Antalgol type and constitute, no doubt, a particular variety of Cepheids. There is not a very large number of globular clusters in the heavens. On the Franklin Adams charts Melotte[4] counted 83 of these objects in the whole heavens, while Bailey in his catalogue of brighter clusters and nebulae counted 54[5], some having a diameter of less than 5 minutes of arc, while the largest like ω Centauri cover a larger area than that of the full moon.

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[1] Pub. Dom. Obs., Vol. VIII, p. 80.

[2] The Observatory, Vol. 36, p. 59, 1913.

[3] L. O. B., Vol. 5, p. 82.

[4] Journal of the British Astr. Ass., Vol. 25, p. 341, 1915.

[5] H. A., Vol. 60, p. 199, 1908.

In the large globular cluster Messier 3 ($13^{h}\,37^{m}\cdot 6 + 28^{\circ}53'$) about 132 stars of the 900 brightest ones have been found to be variables of the Cepheid type. The periods are in all cases very short, most of them in the neighbourhood of half a day[1].

The following table gives the approximate number of variables of the Cepheid type found in six of the principal globular clusters:—

Cluster	Number of Variables
ω Centauri	128
Messier 3	132
Messier 5	85
Messier 15	51
47 Tucanae	6
Messier 13	7

The latter two have probably only periods of several days.

There is apparently a remarkable difference between the first four and the last two. Most of these variables are very faint, being of about the 14th magnitude. They show a range of approximately one magnitude or less; their periods range from about $0^{d}\cdot 3$ to $0^{d}\cdot 7$, although periods of several days have been discovered, for instance in Messier 13; long periods constitute, however, marked exceptions. The mean colour-index for those of short period is approximately $+0^{m}\cdot 4$, which would indicate that the mean spectral class of the short-period cluster variable is about F3 or F4; the colour-index is larger for those with periods of several days, placing the stars between spectral classes G and K. As in the stars of the δ Cephei type the amplitude of the photographic light curve is usually larger than that of the photo-visual. The most important contributors to the study of globular clusters and their variables have been Prof. S. I. Bailey and Dr. Shapley. Their extensive work is to be found in the Harvard Annals and the Contributions from the Mt. Wilson Observatory.

Fainter globular clusters than those mentioned above have been investigated at Mt. Wilson; in the faint globular cluster N. G. C. 7006, for example, Cepheids of the 19th apparent magnitude have been recorded. [Note added (January, 1925) while going through press.][2]

A very important discovery was made recently by Baade and Larink at the Bergedorf Observatory in Hamburg.[3] They investigated a large number of cluster variables, determined their periods and compared them with the periods obtained formerly by Prof. Bailey. In some cases the periods were the same; in other cases they had increased or decreased by sometimes large amounts in comparison with the length of the period. Variations as large as $0^{d}\cdot 01$ or $0^{d}\cdot 02$ were found, while smaller changes of about $0^{d}\cdot 001$ were also present. The question arises whether these variations of period are continuous or sudden. From mathematical considerations and careful comparisons with the work of Bailey, Baade and Larink concluded that the variations must be sudden. This phenomenon of variation of period is apparently very important, and seems to be present in a great many Cepheids and variable stars. The variations may be real or apparent; if for instance the star has a constant period but is describing a certain orbit the observed times of maximum light would necessarily shift slightly according to the position in the orbit.

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[1] H. A., Vol. 78-1.

[2] Very faint Cepheids have just been discovered in two spiral nebulae. (Mt. Wilson).

[3] Astronomische Abhandlungen der Hamburger Sternwarte in Bergedorf II, 6 Hamburg, 1922.

In short-period variables such a change would no doubt be very marked and in a globular cluster we would expect to find, as Baade and Larink have found, periods that have increased, decreased or remained constant. Changes of period, as we have seen, are certain in the case of RR Lyrae; according to Kron the mean period of XX Cygni is decreasing by about a tenth of a second a year, while Roberts finds that the mean period of S Arae is decreasing by four one-hundredths of a second a year.[1]

δ Cephei for a long time was thought to have a slightly variable period; this variation was first pointed out by Chandler[2] and later maintained by Nijland,[3] but has been completely rejected by Luizet[4]; Bélopolsky, however, finds an oscillation in the spectroscopic period.[5] W. J. S. Lockyer found[6] that, while the mean period of η Aquilae is constant, there is an oscillation in the epoch of maximum through an amplitude of ten hours. The question of the variation of period in η Aquilae has been recently treated by C. C. Wylie.[7] Shapley has also found oscillations in the periods of several cluster variables,[8] while the fact of a variation of period in the short-period radial velocity curve of σ Scorpii (decrease and increase), has now been established without doubt by the writer.

While we might ascribe these changes of period to only an apparent cause (the equation of light), this could perhaps scarcely be the case with long-period variables. Luyten in an extensive memoir[9] has shown that the long-period variables R Aquilae, R Ursae Majoris and T Geminorum decrease in period. Contrary to the opinion of Prof. Turner of Oxford, who thinks that such decreases of period (which occur also in R Hydrae) are sudden, Luyten thinks them progressive and continuous.

It has already been remarked that Antalgol stars have a large mean radial velocity. Individual radial velocities of variables in clusters have not been obtained, but it is interesting to note that the mean radial velocities of clusters are also large. Following are some of these radial velocities, as determined mainly by Dr. Slipher.[10]

n. g. c.	Messier	Radial Velocity
		km.
5024	53	−170
5272	3	−125
5904	5	+ 10
6205	13	−300
6333	9	+225
6341	92	−160
6626	28	0
6934		−350
7078	15	− 95
7089	2	− 10
Large Magellanic Cloud		+260
Small " "		+150

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[1] Ap. J., Vol. 33, p. 200.

[2] A. J., Vol. 13, p. 101, 1893.

[3] A. N., Vol. 161, p. 229.

[4] Annales de l'Universite de Lyon, Nouvelle serie fascicule 33.

[5] Mitt. Pulk., Vol. 3, p. 63, 1909. See also The Observatory, Vol. 42, p. 339, 1919.

[6] Dissertation Gottingen, 1896.

[7] Ap. J., Vol. 56, p. 229.

[8] Pop. Ast., Vol. 22, p. 144, 1914.

[9] Annales de l'Observatoire de Leyde.

[10] Jour. R. A. S. C., Vol. II.

The variables of the Small Magellanic Cloud

A very remarkable discovery, which has already had far-reaching consequences, is that made by Miss Leavitt for the variables of the Small Megallanic Cloud.

In 1904 and 1905 Miss Leavitt discovered nearly a thousand variable stars in the Small Magellanic Cloud from photographs made at Harvard's observing station at Arequipa, Peru; she also found more than eight hundred in the Large Magellanic Cloud. Fifty - nine of those in the Small Magellanic Cloud were measured in 1904, using a provisional scale of magnitudes,[1] and the periods of seventeen were published. They resemble the variables found in globular clusters, diminishing slowly in brightness, remaining near minimum for the greater part of the time, and increasing very rapidly to a brief maximum. The following table is a reproduction of Miss Leavitt's table giving the periods of 25 of the stars arranged in increasing order. The different columns contain the Harvard number, the brightness at maximum and at minimum as read from the light curve, the epoch expressed in days following J. D. 2,410,000 and the length of the period expressed in days. A remarkable relation between the brightness of these variables and the length of their periods will be noticed; it is found by plotting the results[2] that there is a linear relation between the magnitudes of the variables and the logarithms of their periods; since all these stars may be considered at very nearly the same distance an equivalent relation holds for the absolute magnitudes.

PERIODS OF VARIABLE STARS IN THE SMALL MAGELLANIC CLOUD

(Miss Leavitt's Table)

H	Max.	Min.	Epoch	Period
			d	d
1505	14·8	16·1	0·02	1·25336
1436	14·8	16·4	0·02	1·6637
1446	14·8	16·4	1·38	1·7620
1506	15·1	16·3	1·08	1·87502
1413	14·7	15·6	0·35	2·17352
1460	14·4	15·7	0·00	2·913
1422	14·7	15·9	0·6	3·501
842	14·6	16·1	2·61	4·2897
1425	14·3	15·3	2·8	4·547
1742	14·3	15·5	0·95	4·9866
1646	14·4	15·4	4·30	5·311
1649	14·3	15·2	5·05	5·323
1492	13·8	14·8	0·6	6·2926
1400	14·1	14·8	4·0	6·650
1355	14·0	14·8	4·8	7·483
1374	13·9	15·2	6·0	8·397
818	13·6	14·7	4·0	10·336
1610	13·4	14·6	11·0	11·645
1365	13·8	14·8	9·6	12·417
1351	13·4	14·4	4·0	13·08
827	13·4	14·3	11·6	13·47
822	13·0	14·6	13·0	16·75
823	12·2	14·1	2·9	31·94
824	11·4	12·8	4·0	65·8
821	11·2	12·1	97·0	127·0

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[1] H. A., Vol. 60, No. 4, Table VI.

[2] H. C., No. 173.

It is to be noted that the shortest period given in the above table is longer than one day. From a recent investigation, however, Shapley[1] finds that the faintest variables in the Small Magellanic Cloud are Cepheids of the Antalgol type having a mean period of $0^d\cdot64$; they apparently verify the period-luminosity law given by Miss Leavitt.

According to Prof. Bailey[2] Miss Leavitt's law does not apply to clusters as rigorously as to the Small Magellanic Cloud; there is however in each cluster a certain relation between the magnitudes and the periods. Shapley also found that in the case of cluster variables, the linear formula given by Miss Leavitt did not hold.

In a bold attempt to connect the galactic Cepheids with the Cepheids found in clusters and in the Small Magellanic Cloud, Shapley derived an empirical curve expressing a relation between period and luminosity for all Cepheids. Choosing eleven Cepheids whose radial velocity curves had been determined[3] and whose proper motions were given in the Preliminary General Catalogue of Boss, and assuming that the data were complete for each star and in most respects homogeneous, that there was no evidence of preferential motion and that the average peculiar motions of such stars are small compared with their parallactic drifts, Shapley computed a mean parallax for the group; and by using it he was able to determine the absolute values of the coördinates of his empirical period-luminosity curve, which curve is given on page 96 of Vol. VIII of the Contributions from the Mt. Wilson Observatory. It should be stated, however, that Curtis [4] and some other astronomers have failed to agree with these conclusions.

Shapley also found an interesting relation between the periods and the colour-indices for the variables of the Small Magellanic Cloud[5]; there is, no doubt, a close relation between this law and Miss Leavitt's period-luminosity law, and this would naturally be taken as an indication that a similar period-colour-index relation should be found for the stars of the δ Cephei type. A look at our table of the principal stars of this type shows, however, that although there is a tendency for these stars to have a larger colour-index (or to be of a redder spectral class) according as the period is longer, there is no relation *with one degree of freedom* between the two; Antalgol stars may be usually of class A but some are of class F and even K; to quote another example at random, V Lacertae (84) is of class G_2 with a period of $4^d\cdot98269$ while RR Lacertae (83) is of class F with the longer period of $6^d\cdot412$. A period-luminosity relation exists for the stars of the Small Magellanic Cloud, as does also probably a period-colour-index relation; since, however, such a period-colour-index relation does not exist for the stars of the δ Cephei type, it is logical to infer also that a period-luminosity relation does not apply rigorously to them. It is not, however, the intention to suggest that Shapley's theory should be discarded completely, as will be seen in the conclusion of this article.

Shapley made an extensive use of his empirical period-luminosity relation to determine the parallaxes of all the known galactic Cepheids.[6] He extended it, then, to the deter-

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[1] Proc. Nat. Acad. Sc., Vol. 8, p. 69. See also Harv. Bul., No. 765.

[2] H. A., Vol. 78, p. 249.

[3] Contributions from the Mt. Wilson Observatory, Vol. VIII, p. 85.

[4] The Scale of the Universe Bul. of the Nat. Research Council, No. 11, p. 205.

[5] Contributions from the Mt. Wilson Observatory, Vol. VIII, p. 99.

[6] Contributions from the Mt. Wilson Observatory, Vol. VIII, p. 145.

mination of the parallaxes of Cepheids in clusters or the consequent determination of the parallaxes of these clusters, concluding by a utilization of these results to form an estimate of the Scale of the Universe.[1]

The β Canis Majoris Type

There is growing evidence leading towards the consideration of stars of the β Canis Majoris type as Cepheids. Like the stars of the δ Cephei type they vary in light continuously, though the amplitude of variation is small. The curve of variation of magnitude is parallel to the curve of radial velocity variation; in β Cephei for example the maximum velocity of approach coincides very nearly with the maximum luminous intensity. The widths of the spectral lines and the character of the spectrum also change during the period of variation.

We have already shown[2] how the stars of the β Canis Majoris type form a sequel to the stars of the δ Cephei type—the greater the density of such stars the shorter the period, following more or less (taking account of the masses) the sequence giant K, giant G, giant F, giant A, B, dwarf A, dwarf F. In the last classes the period is exceedingly short, but the Cepheid type of variation is still found in stars like τ Cygni, δ Aquilae and γ Ursae Minoris (period $2^h\ 36^m\ 10^s$). Several years ago β Cephei had already been considered by Guthnick, Crump and Shapley as a Cepheid; its parallax as deduced from the period luminosity curve was given by the latter as $0''\cdot018$. The absolute magnitude given by this curve for τ Cygni is very nearly $-0\cdot3$; the trigonometric parallax obtained by several observers, which may be considered as very well determined, is $0''\cdot042$; the spectroscopic parallax as determined by Dr. W. S. Adams is also $0''\cdot042$; this gives for τ Cygni an absolute magnitude of $+1\cdot9$; it is in total disagreement with the result obtained from the curve.

The principal characteristics of the stars of the β Canis Majoris type are the following:—

1. Very short-period variation of radial velocity, accompanied by a parallel variation of magnitude.

2. In a large number of cases there is a variation of amplitude of the short-period velocity curve, a variation which is also found in the light-curve; this is for example very marked in the case of 12 Lacertae[3] and is no doubt present in the other stars. The variations of δ Ceti, which at first were thought to be erratic, have now been found to have the same characteristics, as will be seen in the second chapter of the present article.

3. There is in the majority of cases a long period of variation of velocity which indicates that the star is moving in an orbit of that period. The variation of amplitude of the short-period curve is no doubt a function of the position of the star in this orbit. In some cases the short-period variation is apparently not constant—which may well be a consequence, real or apparent (due perhaps in some cases to the equation of light), of the motion in the orbit; the short-period variation itself cannot, however, be ascribed to orbital motion.

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[1] Contributions from the Mt. Wilson Observatory, Vol. VIII, p. 191, and many other publications.
[2] Pub. Dom. Obs., Vol. VIII, pp. 78 and 79.
[3] A. N. Jubilaumsnummer Tafel 3.

4. There is a variation of spectral characteristics (width and intensity of lines) in a *constant* period which is very nearly equal to the short-period radial velocity oscillation.

All these variations are similar to the variations found by Shapley and Hertzsprung in RR Lyrae, of which mention has already been made.

The importance of the study of stars of the β Canis Majoris type as a particular case of Cepheid variation is considerable. Most of the stars discovered are comparatively bright and consequently can be investigated with instruments of precision such as spectrographs and photo-electric cells; a certain number have evidently fairly large parallaxes, which has not been found to be the case among the stars of the δ Cephei type. The shortness of their periods allows variations to be disclosed which it would be difficult to find otherwise. A great many of their spectra are apparently very poor, especially toward the end of the dwarf classes; this makes it difficult to ascertain positively the variation when only spectrographic investigations are made (and so far this is precisely the mode of research we have used); photo-electric investigations would be likely to help to a large extent. Following is a list of the stars which are known or suspected to be of the β Canis Majoris type:—

STARS OF THE β CANIS MAJORIS TYPE (KNOWN OR SUSPECTED)

H. R.	Star	α 1900	δ 1900	Visual mag.	Spect.	Discoverer
		h m	° ′			
779	δ Ceti	2 34·4	− 0 6	4·04	B 2	
1149	20 Tauri	3 39·9	+24 4	4·02	B 5	
1320	μ Tauri	4 10·1	+ 8 39	4·32	B 5	
1463	ν Eridani	4 31·3	− 3 33	4·12	B 2	
1641	η Aurigae	4 59·5	+41 6	3·28	B 3	
1810	114 Tauri	5 21·6	+21 51	4·83	B 3	
1931	σ Orionis	5 33·7	− 2 39	3·78	B	
2294	β Canis Majoris	6 18·3	−17 54	1·99	B 1	Albrecht.
2344	10 Monocerotis	6 23·0	− 4 42	4·98	B 3	
2387	4 Canis Majoris	6 27·6	−23 21	4·35	B 1	
2490	42 Camelopardalis	6 40·5	+67 41	5·04	B 3	
2571	15 Canis Majoris	6 49·2	−20 6	4·66	B 1	
4295	β Ursae Majoris	10 55·8	+56 55	2·44	A	Guthnick.
4422	57 Ursae Majoris	11 23·7	+39 54	5·26	A 2	Otto Struve.
5435	γ Boötis	14 28·1	+38 45	3·00	F	Guthnick.
5735	γ Ursae Minoris	15 20·9	+72 11	3·14	A 2	Otto Struve.
6084	σ Scorpii	16 15·1	−25 21	3·08	B 1	Selga.
6453	θ Ophiuchi	17 15·9	−24 54	3·37	B 3	
7178	γ Lyrae	18 55·2	+32 33	3·30	A	Otto Struve.
7298	η Lyrae	19 10·4	+38 58	4·46	B 3	
7372	2 Cygni	19 20·2	+29 26	4·86	B 2	
7377	δ Aquilae	19 20·5	+ 2 55	3·44	F	
7426	8 Cygni	19 28·1	+34 14	4·85	B 3	
7447	ι Aquilae	19 31·6	− 1 31	4·28	B 5	
7977	55 Cygni	20 45·5	+45 35	4·89	B 2	
8130	τ Cygni	21 10·8	+37 37	3·82	F	Paraskévopoulos.
8238	β Cephei	21 27·4	+70 7	3·32	B 1	Frost.
8279	9 Cephei	21 35·2	+61 38	4·87	B 2	
8640	12 Lacertae	22 37·0	+39 43	5·18	B 2	Young.

Detailed studies of several of these stars have already been made, notably by R. K. Young in the Dominion Astrophysical Observatory Publications, C. C. Crump in the Detroit Observatory Publications, P. Guthnick in the Berlin-Babelsberg Observatory Publications and the Astronomische Nachrichten, Father Selga in the Spanish Review of the Astronomical Society of Spain and America, Paraskévopoulos in the Astrophysical Journal, Otto Struve in the Publications of the Astronomical Society of America, and the writer in the Lick Observatory Bulletins and the Publications of the Dominion Observatory. Further investigations of several of these systems are also to be found in the present article.

The α Orionis Type

This title (α Orionis type) may seem at first sight rather inappropriate, considering that α Orionis is classified as an irregular variable. Its range of variation is, however, quite small (in the neighbourhood of half a magnitude); the variation is very slow and the star is very red and very bright, three features, as any variable star observer knows, which make for great uncertainty in the estimate of the magnitude. Even with good instruments, but with variable seeing and transparency, errors of half a magnitude and perhaps more could sometimes occur in the estimation of the magnitude of a bright red star.

The radial velocity of α Orionis is also known to be variable, having an amplitude of variation of about five kilometers. It was shown by Bottlinger[1], and later by Lunt,[2] that the period of radial velocity variation is in the neighbourhood of six years. Assuming the star to be a spectroscopic binary, the following elements have been given by Bottlinger:

$$P = 6{\cdot}0 \text{ years}$$
$$e = 0{\cdot}24$$
$$\omega = 255°$$
$$K = 2{\cdot}45 \text{ km.}$$
$$\gamma = +21{\cdot}3 \text{ km.}$$
$$T = 1904 \text{ Aug.}$$
$$a \sin i = 70{,}000{,}000 \text{ km.}$$
$$\frac{m_1^3 \sin^3 i}{(m + m_1)^2} = 0{\cdot}0029$$

These elements show all the characteristics of Cepheid variables, a large eccentricity, a very small value of the mass function, and a value of $a \sin i$ which is much smaller than the real diameter of the star as measured with the Mt. Wilson interferometer, (approximately 240,000,000 km.) Several measures of the angular diameter of α Orionis have now been made by Pease with the 20-foot interferometer attached to the 100-inch reflector and variations have been found. Pease has prepared a diagram published on page 346 of Vol. 34 of the Publications of the Astronomical Society of the Pacific showing a possible correlation between the variation of diameter, the variation of radial velocity, and the variations of light as obtained by Barnard and by Osthoff.[3]

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[1] A. N., Vol. 187, p. 33.
[2] Ap. J., Vol. 44, p. 250, 1916.
[3] A. N., Vol. 216, p. 187.

The large uncertainties in the determinations of radial velocities and of magnitudes (especially the latter) introduce considerable difficulties, but it seems quite likely that α Orionis should be considered as an extreme case of Cepheid variation, just as γ Ursae Minoris is another extreme. Irregularities in the light-curve would in that case be due to errors of observation, or perhaps to convection currents or formation of spots of cooler material such as occur on our own Sun. There may be a main light curve due to true Cepheid variation on which are superposed accidental variations which might easily occur in an enormous volume of material at a fairly low temperature, such as found in stars of class M (α Orionis is of class M_a).

According to Pickering's classification all variable stars can be subdivided into the following classes:

I.—Novae.

II.—Variable stars with periods of several months or longer, having usually a considerable amplitude of variation—such stars as o Ceti and χ Cygni.

III.—Irregular variables, such as α Orionis and R Coronae Borealis. These may, however, be subdivided into two classes:—1—The α Orionis class, comprising stars of small amplitude of variation, which perhaps on closer examination might prove to be periodic and not irregular; 2—The R Coronae Borealis class, in which the brightness seems to be constant for a long time (several months), then suddenly increases by several magnitudes, to fall again after many oscillations to its previous level, where it remains again for a considerable period. SS Cygni is another star of this type.

IV.—Stars of the δ Cephei type.

V.—Those of the Algol and β Lyrae types.

All the stars of class II show remarkable variations in their spectra, especially the appearance and disappearance of bright lines. Some Cepheid variables such as W Virginis and RU Camelopardalis have now been found to have variable bright lines in their spectra; they would constitute perhaps transition stars between the true δ Cephei type and the long-period variables of class II. It is however possible that these stars of class II owe their variation to an entirely different cause, perhaps similar to what produces the undecennial variation in our own Sun. On the other hand some stars of fairly long period have already been classified as being of the δ Cephei type. Among these might be mentioned the following:—

Star	Period	Spectral Class
	d	
RX Cephei	130·0	Go
SS Gemin.	44·87	G5
U Monocerotis	56·0	G5
RS Puppis	41·31	Ko
l Carinae	35·523	Go
U Carinae	38·740	Go
R Sagittae	70·56	Cont

And it is quite logical to suppose that the periods go on increasing with decrease of density, accompanied probably by a decrease of temperature (that is, extending to class M) *without however showing bright lines.* Variations in the brightness and radial velocity of such stars might perhaps be difficult to detect, but there are strong arguments to support the belief that stars like α Orionis are extreme cases of Cepheid variation.

Chapter II

THE SPECTROSCOPIC SYSTEM DELTA CETI

The star δ Ceti ($\alpha = 2^h 34^m \cdot 4$; $\delta = -0°6'$, class B2) has already been investigated at some length.[1] It appeared then that this system was rather complicated, as evidenced by the different radial velocity curves published. From further observations taken in 1922, however, it appears that the system is not as complicated as thought at first, and that it is merely a star of the β Canis Majoris type with a short-period variation of 3^h 52^m (the shortest known of class B), with a widely variable amplitude of short-period radial velocity oscillation, and a longer period of oscillation of mean velocity (as deduced from individual short-period curves). The curves given previously were rather arbitrarily drawn through the different points given by the observations; some of these points being much better determined than others it is more than probable that the curves are not altogether true representations of the radial velocity oscillations (also the early spectrograms were of too long exposure). The results are, however, of considerable value, since they can now be used to advantage in combination with the later observations.

It is to be remarked that the spectral lines of δ Ceti are more diffuse than those of β Canis Majoris; this appears to agree with the theories and the graph given in the article, "A Spectrographic Study of Stars of Classes A and F."[2] Its spectral lines being more diffuse it is likely that δ Ceti has a greater density and hence its period of pulsation, if pulsation it is, would be shorter.

On account of the shortness of the period and the rather small amplitude, the study of this star with our equipment was somewhat difficult. It was placed on our programme to be observed every clear night from the middle of September to the end of December, 1922. In the following table are found the radial velocities obtained during that period; all the spectrograms were measured as usual on a direct measuring engine in both the direct and reverse positions; a large number of them were also remeasured on the spectro-comparator, giving results in fair agreement with those obtained from the direct measures.

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[1] Pub. Dom. Obs., Vol. V, p. 413.
[2] Pub. Dom. Obs., Vol. VIII, No. 5.

RADIAL VELOCITIES OF δ CETI OBTAINED IN 1922

Date, 1922	Julian Day	Velocity	Probable error	Number of lines	Length of exposure
		km.	±		minutes
Sept. 21	2423319·736	+ 8·7	2·8	9	36
	·763	− 3·0	5·9	5	36
	·789	− 3·2	0·5	2	35
	·814	− 1·9	4·3	8	35
Sept. 24	322·726	+11·2	2·6	9	40
Sept. 25	323·719	+13·0	1·3	8	29
	·740	+10·3	1·1	8	26
	·759	+ 2·9	2·3	8	26
	·778	− 3·6	2·0	7	26
	·798	− 3·3	1·3	8	26
	·817	− 5·8	1·3	9	26
	·837	+ 0·8	2·1	7	26
	·856	−12·7	1·8	5	26
Sept. 29	327·697	+ 3·6	2·8	6	30
	·717	+ 2·0	3·2	8	26
	·736	+11·5	1·0	8	24
	·754	+10·0	3·4	9	24
	·772	+ 6·8	1·7	7	24
	·790	+ 1·2	4·0	9	24
	·808	+ 3·4	2·6	7	24
Oct. 3	331·721	−16·3	3·6	6	34
	·748	−23·7	5·8	9	30
Oct. 11	339·716	− 6·7	2·9	9	36
	·741	− 9·8	3·3	8	30
	·769	− 6·8	2·4	7	32
Oct. 12	340·649	+ 3·8	1·8	8	30
	·694	−19·5	3·0	6	20
	·720	−13·5	2·5	8	20
Oct. 13	341·590	+ 1·3	3·4	9	38
	·616	+ 9·4	2·9	10	28
	·635	−11·2	2·8	6	24
	·653	−20·8	1·6	7	24
	·671	−14·6	2·4	8	22
	·694	−19·3	1·6	8	22
	·712	−23·2	2·3	9	22
	·728	−10·5	4·5	9	22
	·747	− 9·6	6·7	6	26
	·767	+ 3·4	2·8	10	30
	·790	− 0·2	2·6	9	30
	·811	− 1·0	4·0	8	28
	·833	+ 1·0	3·9	9	30
Oct. 17	345·596	− 3·3	2·4	8	28
	·616	+ 1·5	2·0	10	24
	·631	+ 5·2	4·2	3	15
Oct. 18	346·576	+ 4·5	2·4	7	32
	·598	+ 4·7	1·8	8	26
	·617	+10·4	2·6	11	26
	·636	+ 7·2	2·8	10	24
	·653	− 4·5	3·2	10	22
	·669	− 4·0	4·0	7	18
	·692	− 4·7	2·9	9	20
	·706	− 6·0	3·0	9	18
	·719	+ 1·0	2·2	10	18

RADIAL VELOCITIES OF δ CETI OBTAINED IN 1922—*Continued*

Date, 1922	Julian Day	Velocity	Probable error	Number of lines	Length of exposure
		km.	±		minutes
Oct. 18..	2423346·733	+ 5·0	3·0	9	18
	·747	+ 7·8	2·4	7	18
	·760	+ 5·9	3·6	9	18
	·774	+ 3·6	2·0	9	18
	·788	+ 9·0	1·6	9	18
	·802	−18·2	7·4	2	18
Oct. 19..	347·584	+13·3	3·2	5	20
	·626	− 1·6	2·1	7	20
	·641	− 8·9	2·6	8	18
	·654	− 9·9	2·1	10	18
	·668	− 4·9	2·4	9	20
	·691	0·0	3·0	10	18
	·704	− 3·5	2·6	7	18
	·717	+ 1·9	2·4	11	18
	·744	+ 6·0	2·1	8	18
	·758	+10·9	2·1	12	18
	·772	+ 2·4	1·3	11	18
	·786	+ 1·2	3·3	9	18
	·800	− 1·7	2·8	10	18
	·839	− 2·9	2·7	11	26
	·858	+ 2·0	2·3	9	22
	·873	− 2·7	2·6	6	20
	·890	+ 0·4	1·8	7	24
Oct. 20..	348·560	− 1·6	1·6	8	26
	·578	− 2·2	1·2	6	20
	·603	− 6·7	2·0	8	16
	·615	−10·7	3·6	8	16
	·626	− 0·6	2·4	8	16
	·638	− 5·4	1·3	5	16
	·650	− 4·6	2·6	7	16
	·662	+ 2·9	2·5	6	16
	·691	+ 1·7	2·4	9	16
	·702	+ 8·6	1·3	8	14
	·712	− 2·2	2·4	8	14
	·723	− 0·6	1·6	7	14
	·733	+ 3·8	1·8	8	14
	·744	− 8·1	1·8	8	14
	·760	−12·5	2·9	8	14
	·772	− 8·5	2·0	10	16
	·785	− 6·3	1·4	8	16
	·797	+ 1·1	3·8	5	16
	·810	− 8·6	4·4	3	18
	·825	+ 6·0	3·8	3	20
	·840	− 2·6	3·3	5	18
	·856	− 2·7	2·7	6	24
	·875	+10·6	3·4	5	20
Nov. 9..	368·633	− 2·5	2·1	9	30
	·655	− 1·8	2·0	7	26
	·707	+ 5·7	2·4	9	20
	·726	−17·6	2·6	5	20
	·743	+ 1·9	1·9	9	22
	·759	− 3·4	1·7	4	20
	·776	− 3·2	2·9	5	26

RADIAL VELOCITIES OF δ CETI OBTAINED IN 1922—*Continued*

Date, 1922	Julian Day	Velocity	Probable error	Number of lines	Length of exposure
		km.	±		minutes
Nov. 9	2423368·796	− 2·6	2·4	2	26
	·817	− 6·5	2·7	5	30
Nov. 12	371·587	+15·0	4·1	3	40
	·649	+ 6·9	2·5	4	40
Nov. 13	372·660	+ 2·9	2·7	9	40
	·717	+ 5·2	4·2	8	40
	·748	+27·0	3·4	2	40
	·777	+ 1·5	2·4	10	40
Nov. 16	375·551	+ 6·7	3·2	6	36
	·574	+10·6	2·0	6	26
	·592	+14·9	1·6	9	20
	·607	+14·7	2·4	6	20
	·622	+13·1	1·9	7	20
	·638	+ 8·3	1·8	7	23
	·656	− 0·5	1·6	5	25
	·699	+ 2·9	2·4	5	27
	·719	+ 0·1	2·2	5	28
Dec. 3	392·515	+ 2·1	3·6	5	30
	·537	+ 9·6	2·1	9	30
	·560	+ 9·5	2·4	6	30
	·581	+ 1·3	2·4	6	28
	·600	+ 2·5	3·9	3	26
	·619	− 4·0	2·5	2	26
Dec. 10	399·488	+ 2·5	2·0	8	30
	·508	+ 2·3	1·6	10	24
	·527	− 5·0	1·9	9	30
Dec. 12	401·490	+ 2·4	1·6	7	28
	·509	+11·1	2·0	7	24
	·526	+12·0	1·6	7	22
	·542	+13·9	1·3	6	22
	·560	+13·4	2·6	8	28
Dec. 15	404·423	− 0·2	1·8	9	30
	·439	+ 6·9	2·2	4	14
	·469	+11·5	2·4	9	22
	·486	+11·0	2·1	9	22
	·503	+11·8	0·9	7	22
	·539	+ 3·6	4·0	3	10
Dec. 18	407·441	+ 1·8	1·5	6	20
	·456	+ 3·3	2·2	6	18
	·471	+ 4·2	2·1	7	20
	·486	+16·1	2·2	7	20
	·501	− 1·2	8·0	3	20
	·517	+ 9·5	0·6	2	20
	·535	+12·2	4·7	3	32
	·556	+ 1·6	4·0	7	24
	·574	− 1·8	1·8	7	24
	·591	− 0·5	2·6	7	20
	·607	+ 7·7	2·0	8	22
	·624	+ 0·5	1·9	8	22
	·640	+ 9·5	1·7	8	22
	·658	+ 9·0	2·7	10	24
Dec. 19	408·420	+ 0·3	2·5	6	30
	·440	+ 3·2	1·9	7	24

RADIAL VELOCITIES OF δ CETI OBTAINED IN 1922—*Concluded*

Date, 1922	Julian Day	Velocity	Probable error	Number of lines	Length of exposure
		km.	±		minutes
Dec. 19	2423408·457	+20·6	2·0	6	22
	·475	+ 5·1	2·4	5	26
	·494	+20·7	2·0	2	24
	·515	+10·5	2·8	3	30
	·536	+ 4·6	3·0	6	28
	·553	+ 0·9	0·9	10	20
	·567	+ 8·7	3·5	5	18
	·582	+ 4·4	3·2	6	20
	·597	+ 0·7	2·0	6	20
	·612	+ 5·7	2·8	7	16
	·639	+ 2·1	3·3	4	20
Dec. 27	416·424	+ 1·1	1·4	9	26
	·444	+ 3·4	1·7	11	28
	·464	+ 3·2	0·7	10	24
	·481	+ 4·2	1·1	8	22
	·497	+ 3·0	2·0	6	20
	·512	+ 5·5	2·4	6	20
	·529	+ 6·2	3·2	3	28
	·551	+ 9·6	1·1	7	34
	·573	+ 5·0	2·3	6	24
	·590	+ 2·0	1·6	8	22
	·607	+ 4·5	1·7	7	22
	·625	+ 0·7	1·5	5	22
	·642	+ 9·0	4·1	2	22
Dec. 29	418·418	+ 2·1	4·1	4	22
	·438	+ 7·8	2·4	8	30
	·461	+12·2	1·4	8	30
	·483	+ 7·7	2·1	7	32
	·508	+14·0	1·8	8	32
	·531	+ 2·0	1·0	12	32
	·555	+ 0·9	1·3	12	32
	·578	+ 4·1	1·8	9	30
	·601	+10·1	1·1	9	32
	·625	+ 6·4	1·0	7	34
	·649	+12·3	1·6	8	30

Combining these observations with those obtained in 1921 (see Pub. Dom. Obs., Vol. V, No. 11) it is found that the period $0^d\cdot16122$ fits all the observations. If we take the observed maximum of radial velocity J. D. 2423346·610 as origin and add or subtract multiples of this period we obtain the following Julian dates of maxima for days when observations were secured in 1921 and 1922:—

COMPUTED EPOCHS OF MAXIMA OF δ CETI

A.—1921	B.—1922
J. D. 2422989·669	J. D. 2423319·686
990·475	·847
993·699	322·749
996·763	323·717
2423001·599	327·747
·760	339·839
002·567	340·645
·728	341·612
028·523	·773
030·458	345·643
·619	346·610
031·586	·771
032·553	347·577
039·485	·739
·647	·900
043·516	348·545
045·451	·706
·612	·867
049·482	368·697
·643	371·599
053·512	372·728
	375·630
	392·558
	399·490
	401·586
	404·488
	407·551
	·712
	408·518
	·680
	416·418
	·578
	418·514
	·675

On plotting the different velocity curves for the above dates (all the curves for 1921 were given in the previous article) it is found that these maxima are verified within the limits of error. Attention should be called to the curve of December 14, 1921; the observations giving this curve have been found to be very poor, and certainly one of them upon which the maximum depends, is not at all reliable. Four of the curves obtained in 1922 are shown in Fig. 1. There is no doubt a good deal of uncertainty in these curves; the following conclusions, however, appear to be justified:—

1. The period $0^d\cdot16122$, approximately $3^h\ 52^m$, may be considered as correct.
2. The amplitude probably varies considerably.
3. There is a variation of the mean velocity as given by the individual curves; without more powerful equipment, however, it is practically hopeless to determine the period of this variation.

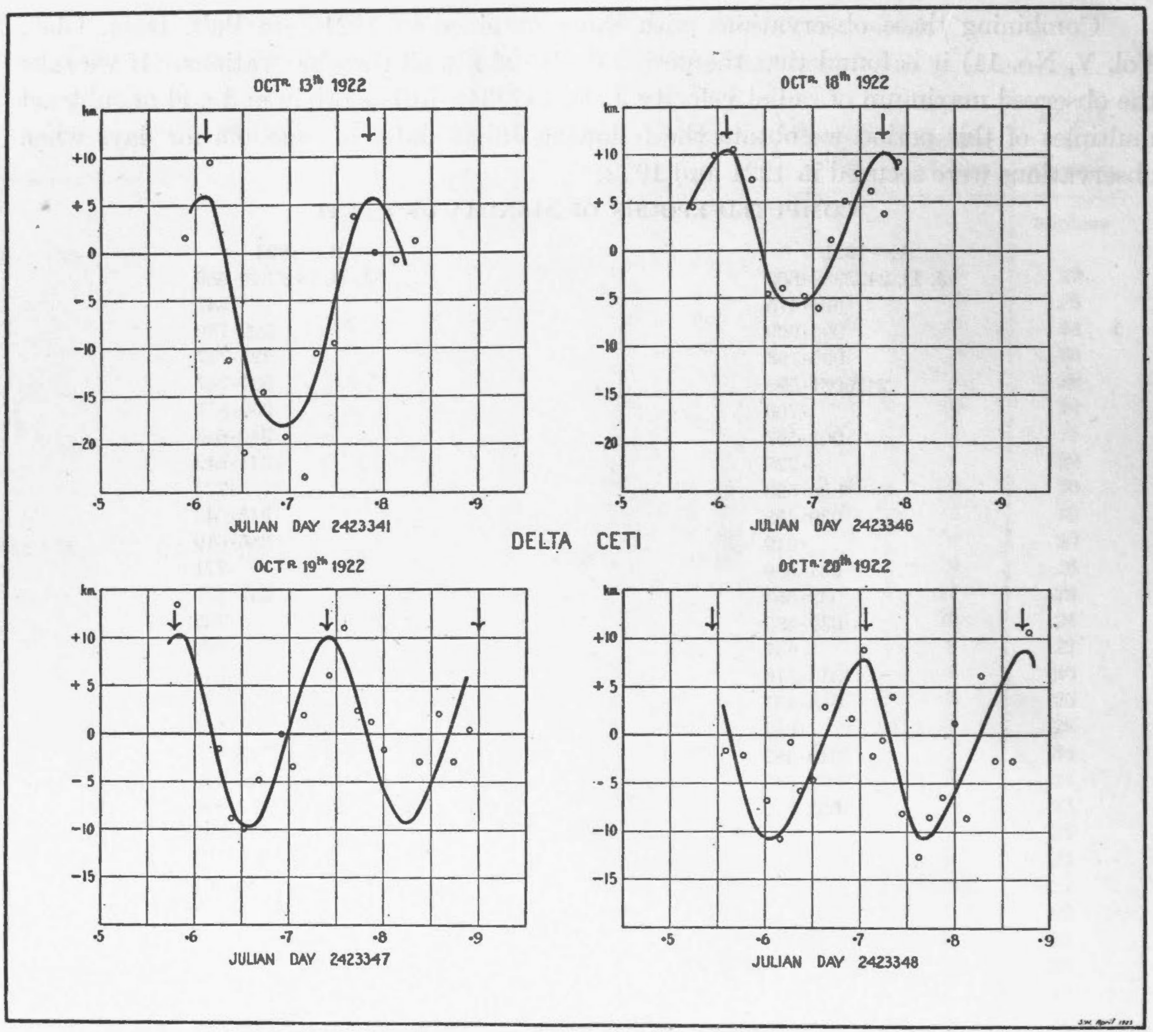

FIGURE 1. RADIAL VELOCITY CURVES OF DELTA CETI

4. There is, apparently, nothing abnormal or erratic about the radial velocity variations of δ Ceti as at first supposed. It is a typical star of the β Canis Majoris type with the characteristic double variation. It has the shortest period at present known of any stars of this type belonging to class B.

Chapter III

THE SPECTROSCOPIC SYSTEM TAU CYGNI

The star τ Cygni was classified among the very short-period binaries, or what we have called the β Canis Majoris type, by J. S. Paraskévopoulos; he gave for its period $0^d{\cdot}1425$ or $3^h\ 25^m{\cdot}2$. The importance of studying this star further appeared immediately to the writer, and many spectrograms of it were secured in 1921. Unfortunately the spectrograms are very poor and difficult to measure, the lines being wide and hazy and sometimes hard to identify with known spectral lines. On account of the extreme shortness of the period it was necessary to make the exposures as short as possible, and hence to use a wide slit; the measures are, consequently, of rather poor quality.

The results, however, are of great interest because they seem to indicate that the mean velocity is also variable, with a period of twenty to twenty-five days, showing that τ Cygni, like other stars of the β Canis Majoris type already investigated, acts as if it were a spectroscopic triple system. The radial velocities obtained here for τ Cygni in 1921 are as follows:—

RADIAL VELOCITIES OF τ CYGNI

Date	Julian Day	Velocity	Date	Julian Day	Velocity
1921		km.	1921		km.
Aug. 2................	2422904·566	−36·2	Aug. 16................	2422918·567	−32·0
	·589	−26·9		·588	−28·4
	·611	−23·8		·610	−43·5
Aug. 4................	906·566	−31·8		·651	−30·9
	·588	−14·5		·673	−27·9
	·647	−15·0	Aug. 18................	920·538	−24·5
	·670	−16·5		·559	−35·7
	·692	−36·9		·601	−51·6
Aug. 5................	907·558	−24·6		·622	−33·6
	·581	−18·4		·663	−27·9
	·624	−20·3	Aug. 21................	923·535	−33·0
	·682	−33·9		·556	−26·9
Aug. 8................	910·621	−24·2	Aug. 22................	924·573	−23·2
Aug. 9................	911·567	−16·9		·594	−31·0
Aug. 12................	914·560	−32·3		·615	−24·8
	·582	−26·3	Aug. 24................	926·528	−31·9
	·603	−26·0		·549	−22·8
	·625	−16·7		·570	−29·1
	·662	−14·2		·591	−28·0
	·683	−37·4		·658	−24·6
	·708	−29·3	Aug. 26................	928·558	−25·4
Aug. 14................	916·589	−29·4		·599	−46·7
	·611	−28·8		·622	−42·1
	·663	−18·6			
	·685	−31·2			

On plotting the above observations it appears that a period of $0^d \cdot 1432$ would connect the different maxima; this period gives maxima on the following Julian dates:—

MAXIMA OF τ CYGNI

Date	Julian date of maximum	Estimated mean velocity for each day	Date	Julian date of maximum	Estimated mean velocity for each day
1921		km.	1921		km.
Aug. 2................	2422904·626	−37	Aug. 16................	2422918·660	−36
4................	906·631	−26	18................	920·664	−38
5................	907·633	−30	21................	923·528	−40 ?
8................	910·640		22................	924·674	—
9................	911·500	−20 ?	24................	926·536	−34 ?
12................	914·650	−25	26................	928·540	−32 ?
14................	916·655	−28			

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[1] Ap. J., Vol. 53, p. 145, 1921.

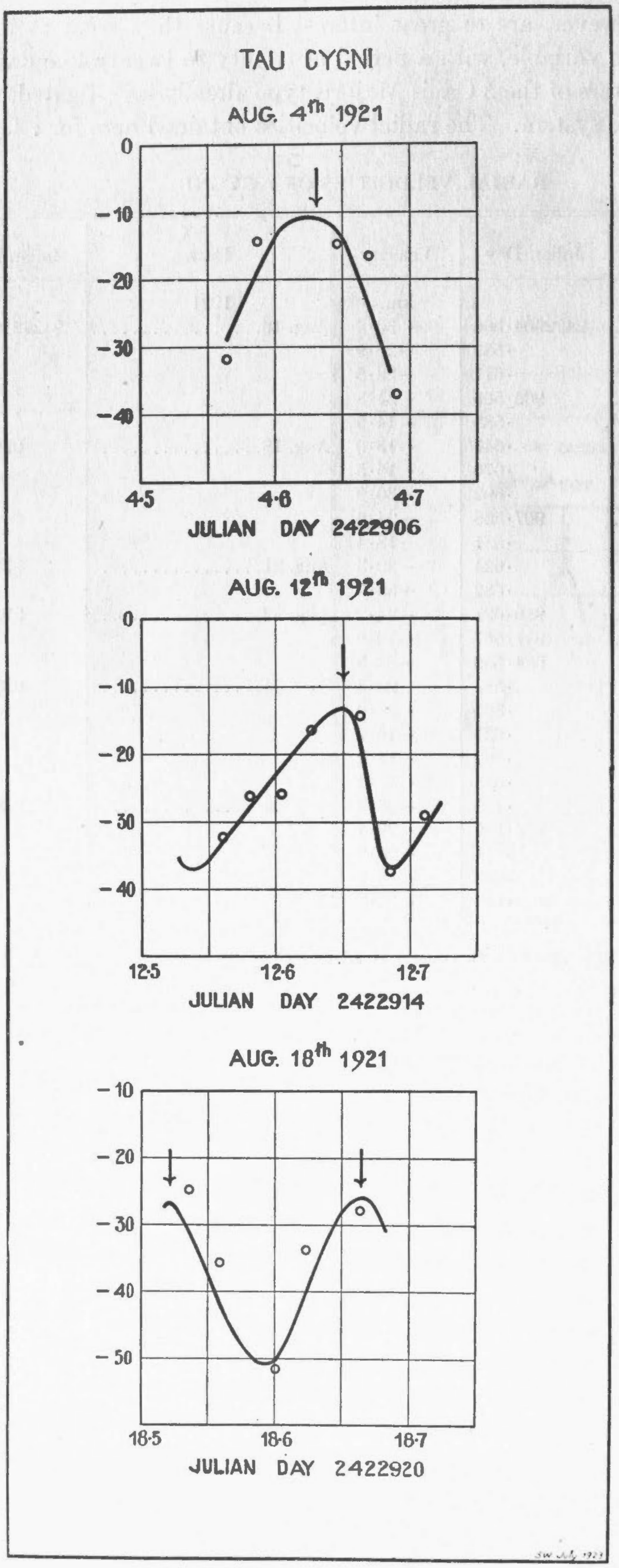

Figure 2. Radial Velocity Curves of Tau Cygni

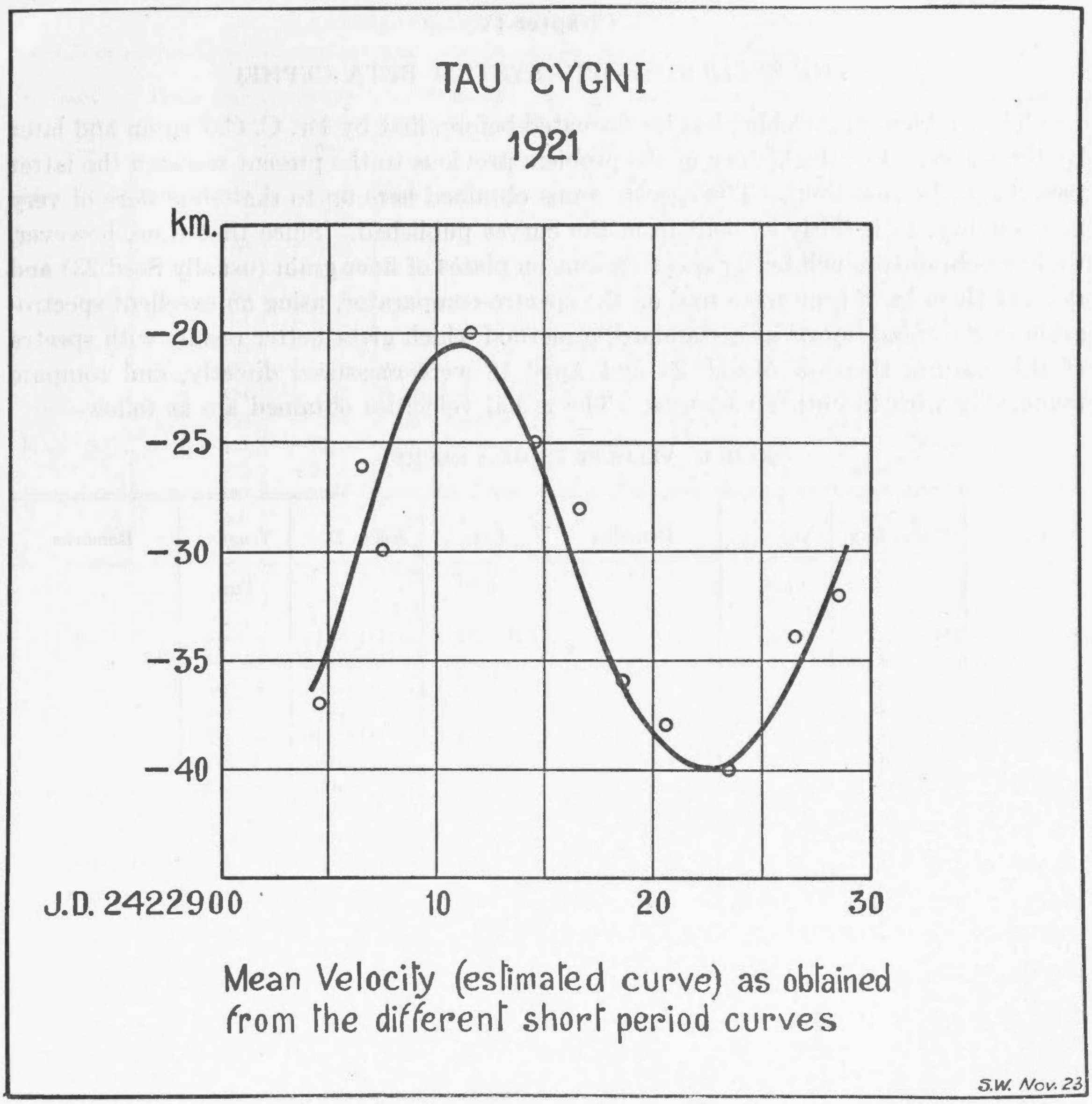

FIGURE 3. MEAN VELOCITY CURVE OF TAU CYGNI

Three of the curves given by the observations are shown in Fig. 2; the arrows indicate the maxima as computed with the period $0^d \cdot 1432$. From all the curves rough estimates of the mean velocity for each day have been made, as given in the third column of the preceding table; these results give the curve shown in Fig. 3. There is no doubt a great uncertainty in the determination of this curve, but it shows that the radial velocity of τ Cygni has most probably a double periodicity, as is the case with σ Scorpii, β Canis Majoris and other similar stars.

Chapter IV

THE SPECTROSCOPIC SYSTEM BETA CEPHEI

The problem of β Cephei has been treated before, first by Dr. C. C. Crump and later by the writer. For the history of the problem previous to the present research the latter paper[1] may be consulted. The spectrograms obtained here up to that time were of very poor quality, as is fairly evident from the curves published. Since that time, however, we have obtained much better spectrograms on plates of finer grain (usually Seed 23) and most of them have been measured on the spectro-comparator, using an excellent spectrogram of β Canis Majoris as a standard, a method which gives better results with spectra of this nature; those of March 24 and April 18 were measured directly, and compare favourably with the other measures. The radial velocities obtained are as follows:—

RADIAL VELOCITIES OF β CEPHEI

Date	Julian Day	Velocity	Remarks	Date	Julian Day	Velocity	Remarks
1922		km.		1922		km.	
Feb. 16...	2423102·774	−33·6		Feb .24...	2423110·813	−19·4	
	·798	−25·4			·833	− 8·4	
	·820	−22·8			·853	− 0·2	
	·842	−24·9			·876	− 1·6	
	·865	− 8·4		Mar. 2...	116·569	− 3·8	
	·887	− 2·0			·585	+ 7·7	
	·909	− 9·6			·601	+ 2·5	
	·931	−18·9			·616	+13·1	
Feb. 20...	106·678	− 7·3			·633	− 8·0	
	·701	+ 2·7			·653	−10·0	Weak plate.
	·717	− 0·9			·705	−23·8	
	·731	− 5·8			·722	−22·7	
	·747	− 0·4			·741	−20·5	
	·762	−10·0			·763	− 0·8	
	·776	−17·1			·785	− 5·2	
	·790	−13·8			·855	−25·6	Poor plate.
	·804	−17·2			·872	−36·2	
	·818	−11·3		Mar. 3...	117·568	+ 0·3	
	·832	− 9·1			·586	+ 4·0	
	·867	+ 1·8			·603	− 6·7	
	·881	+ 1·6			·621	−31·7	Poor plate.
	·895	− 8·1			·640	−37·2	
	·909	−10·8			·657	−21·1	
Feb. 23...	109·758	+ 1·3			·678	−19·3	
	·782	−12·0			·699	−15·0	Poor plate.
	·803	−20·7			·716	− 4·0	
	·822	−14·6			·733	− 4·3	
	·840	−15·1			·752	−11·3	
	·888	− 1·4			·774	−11·6	
	·902	−10·6			·796	−32·6	Weak plate.
	·917	− 0·8			·821	−23·8	
Feb. 24...	110·730	−14·0			·850	−26·2	
	·746	−28·6	Poor plate.		·868	−25·7	Very coarse grain.
	·761	−28·2		Mar. 5...	119·589	−13·6	
	·778	−29·0			·604	−12·6	
	·796	−23·6			·619	− 8·4	

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[1] Pub. Dom. Obs., Vol. V, p. 77.

RADIAL VELOCITIES OF β CEPHEI—*Continued*

Date	Julian Day	Velocity	Remarks	Date	Julian Day	Velocity	Remarks
1922		km.		1922		km.	
Mar. 5...	2423119·633	−16·3	Poor plate.	Mar. 22...	2423136·764	−14·7	
	·647	+4·7			·785	−6·4	
	·705	−19·4			·806	−7·1	
	·722	−33·4			·826	−13·3	
	·740	−22·5			·847	−23·9	
	·761	−17·7			·868	−25·2	
	·782	−16·3		Mar. 24...	138·524	−4·2	
	·803	−3·6	Weak.		·545	−5·6	
	·824	−3·5	Poor plate.		·564	−17·9	
	·868	−5·1	Peculiar plate.		·580	−26·3	
Mar. 8...	122·558	−20·5			·597	−38·0	Poor plate.
	·573	−23·8			·615	−34·4	
	·587	−32·4			·658	−25·0	
	·600	−34·9			·710	−6·4	
	·646	−12·0	Very poor plate.		·728	−13·5	
	·675	−10·2	Very poor plate.	Mar. 29...	143·547	−20·0	
	·704	+5·4			·573	−33·2	
	·729	−14·0			·585	−25·8	
	·751	−30·7			·604	−17·0	
	·812	−27·3	Very poor plate.	April 18...	163·540	−30·8	
	·831	−19·2	Poor plate.		·574	−27·3	
	·849	−4·9			·592	−13·5	
	·867	−15·0	Poor plate.		·607	+1·1	
	·885	+4·1			·622	+0·8	
Mar. 10...	124·566	−6·2			·638	−14·8	
	·583	−11·7	Very poor plate.	July 5...	241·755	−3·4	
	·599	−2·7					
	·643	−9·5		1923			
	·753	−1·5	Poor plate.	Mar. 5...	484·783	+2·1	Poor plate.
	·785	−1·1			·812	+4·1	
	·807	+6·7	Very poor plate.		·837	−5·2	
	·828	−30·3	Very poor plate.		·860	−13·8	
	·856	−18·8			·881	−14·0	
	·876	−41·7			·902	−17·4	
Mar. 15...	129·550	−5·4			·923	−11·2	
	·568	−21·2	Weak plate.	Mar. 7...	486·748	−1·8	
Mar. 17...	131·600	−13·3		Mar. 8...	487·493	+3·9	
	·618	−7·5			·515	−7·5	
	·639	−10·3			·536	−13·6	
	·658	−13·1			·556	−17·0	
	·703	−39·9	Poor plate.		·575	−15·8	
	·726	−28·8	Poor plate.		·593	−21·2	
Mar. 18...	132·719	−17·9	Poor plate.		·611	−14·4	
	·741	+0·8			·630	+2·8	
	·759	−5·2			·650	+4·2	
	·776	−7·7			·697	−0·4	
	·796	−7·1			·719	−4·8	
	·817	−7·1			·743	−19·7	
	·837	−21·8			·812	−16·9	
	·860	−27·0			·831	−9·7	
Mar. 22...	136·551	−4·1			·850	−1·1	
	·567	−2·7			·867	−2·4	
	·613	−1·1	Poor plate.		·891	−2·8	
	·725	−25·2			·900	−3·0	
	·744	−20·7			·917	−13·8	

RADIAL VELOCITIES OF β CEPHEI *Concluded*

Date	Julian Day	Velocity	Remarks	Date	Julian Day	Velocity	Remarks
1923		km.		1923		km.	
Mar. 14...	2423493·738	− 2·5		Mar. 26...	505·564	+ 7·1	
	·758	+ 3·7			·606	− 4·0	
	·778	+ 1·7			·630	−20·3	
	·803	− 5·8			·653	−28·8	
	·824	−15·1			·701	−11·5	
	·866	−20·4			·776	+ 2·1	Poor plat
	·888	− 5·6			·798	− 0·3	
	·909	− 7·9			·881	−17·5	
Mar. 16...	495·722	−13·5			·905	− 3·7	
	·746	− 8·1	Poor plate.	Mar. 28...	507·769	− 5·2	
	·772	−11·7	Poor plate.		·794	0·0	Poor plate.
	·793	− 7·6	Very poor plate.		·816	+ 2·8	Poor plate.
	·815	0·0			·838	+ 1·4	
	·837	+14·8			·859	+ 5·3	
	·860	+ 1·7			·880	+ 0·9	
	·905	− 3·9			·903	−14·8	
Mar. 19...	498·783	−11·5			·924	−29·5	
	·808	− 7·8	Poor plate.	April 6...	516·563	− 2·2	
	·830	−12·1		April 26...	536·548	−14·1	
	·854	− 5·2					
	·876	+ 3·4					
	·920	+ 2·0					

A number of curves representing the above velocities are given in Figs. 4-8; the large differences of amplitude, as previously found by Dr. Crump are very evident. Of particular interest are five curves, where spectrograms were obtained through almost the whole night; they are those of 1922 March 2, 3, 5 and 22, and 1923 March 8, where the second maximum is apparently lower than the first; this may, however, be due to accidental error.

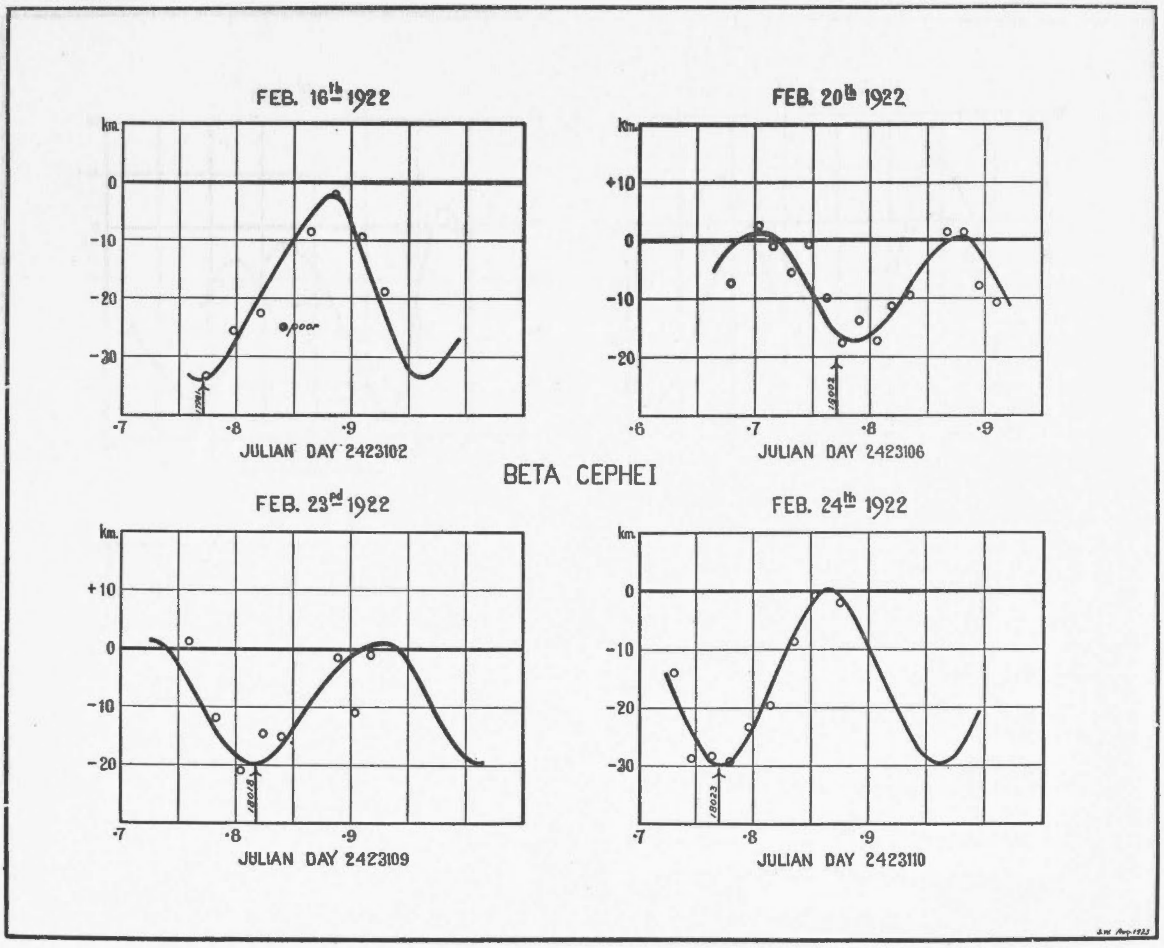

FIGURE 4. RADIAL VELOCITY CURVES OF BETA CEPHEI

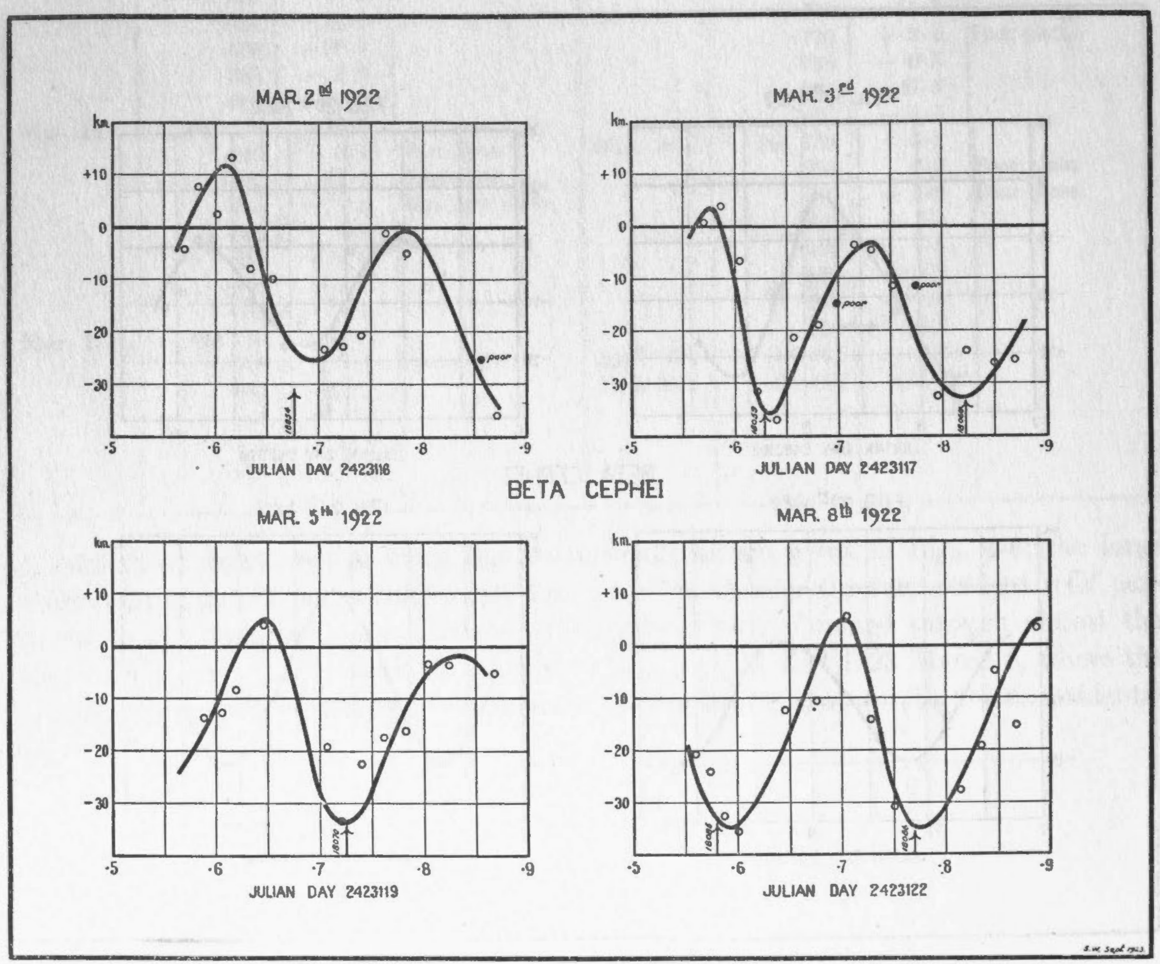

FIGURE 5. RADIAL VELOCITY CURVES OF BETA CEPHEI

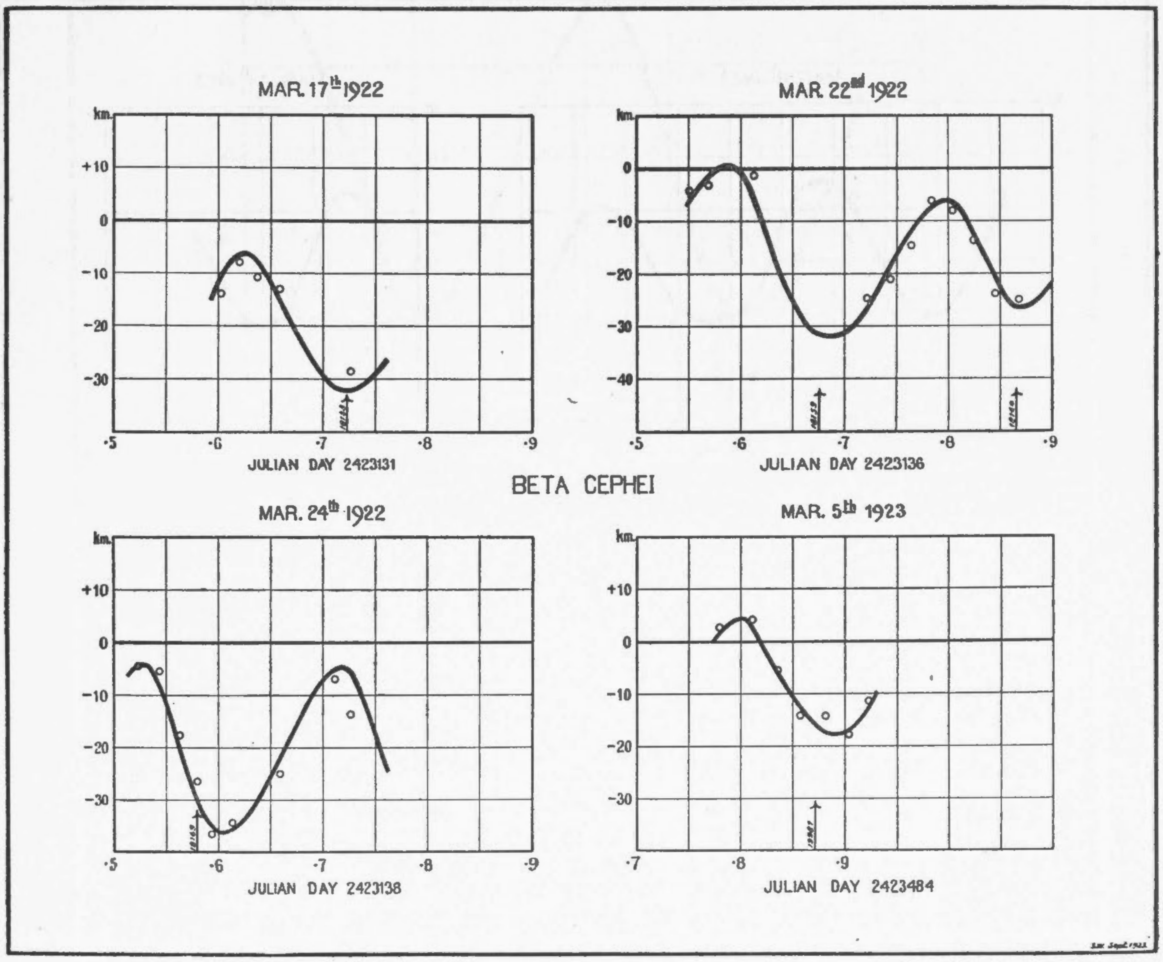

FIGURE 6. RADIAL VELOCITY CURVES OF BETA CEPHEI

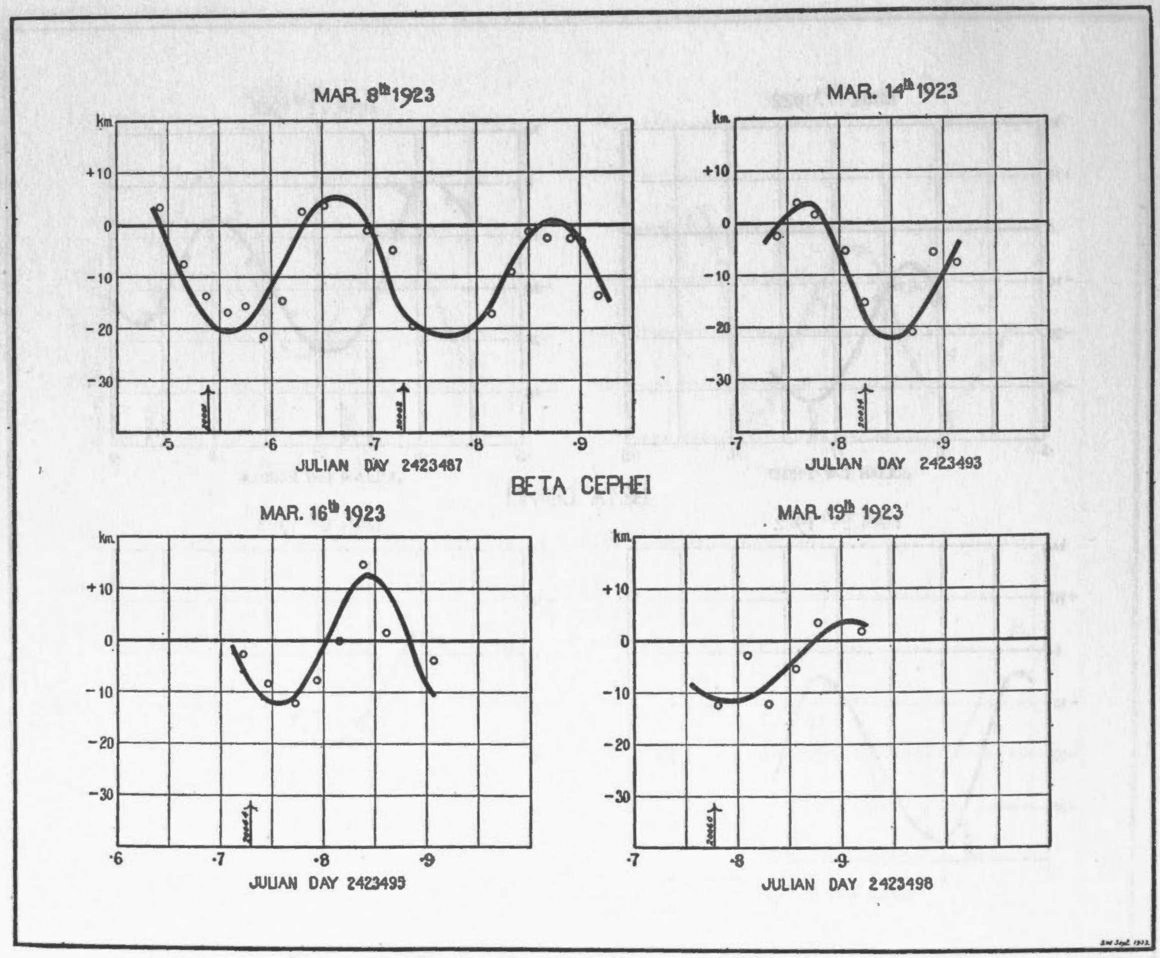

FIGURE 7. RADIAL VELOCITY CURVES OF BETA CEPHEI

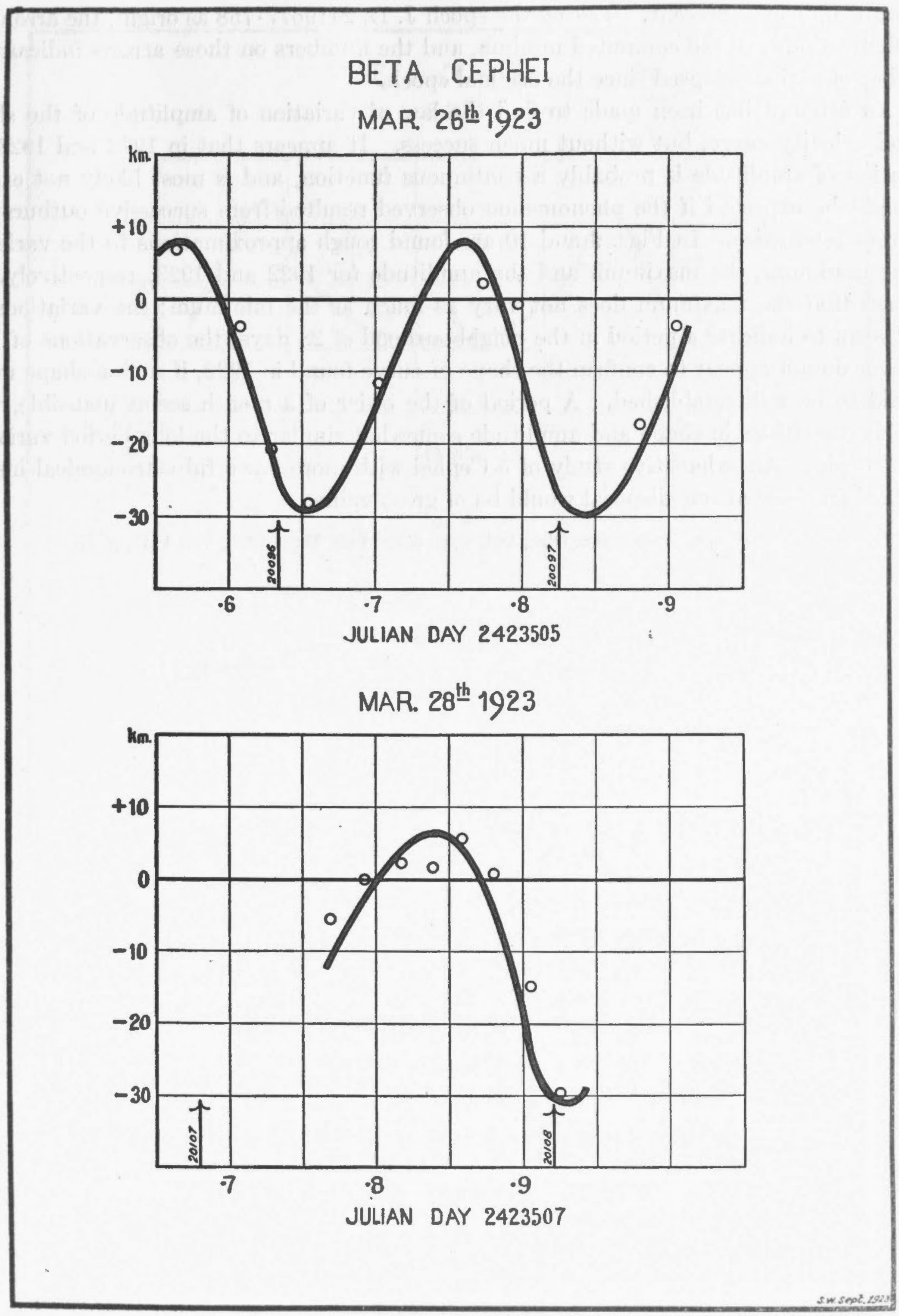

FIGURE 8. RADIAL VELOCITY CURVES OF BETA CEPHEI

An outstanding fact is that the period given by Dr. Crump ($0^d \cdot 1904795$) has remained the same up to the present. Taking the epoch J. D. 2419677·758 as origin, the arrows on the figures indicate the computed minima, and the numbers on those arrows indicate the number of periods elapsed since the original epoch.

An attempt has been made to find the law of variation of amplitude of the short-period velocity curve, but without much success. It appears that in 1922 and 1923 the variation of amplitude is probably a continuous function, and is most likely not erratic as might be expected if the phenomenon observed resulted from successive outbursts of different intensities. In Figs. 9 and 10 are found rough approximations to the variation of the minimum, the maximum and the amplitude for 1922 and 1923, respectively. It appears that the maximum does not vary as much as the minimum; the variations for 1922 seem to indicate a period in the neighbourhood of 28 days; the observations of 1923 however do not appear to confirm the shape of curve found in 1922, if such a shape might be said to be well established. A period of the order of a month seems plausible, with possibly variations in shape and amplitude somewhat similar to the long-period variation of σ Scorpii[2]. An exhaustive study of β Cephei with more powerful astronomical instruments than those at our disposal would be of great value.

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[2] L. O. B., Vol. 9, p. 173. Pub. Dom. Obs., Vol. V, p. 301. Pub. Dom. Obs., Vol. VIII, p. 45.

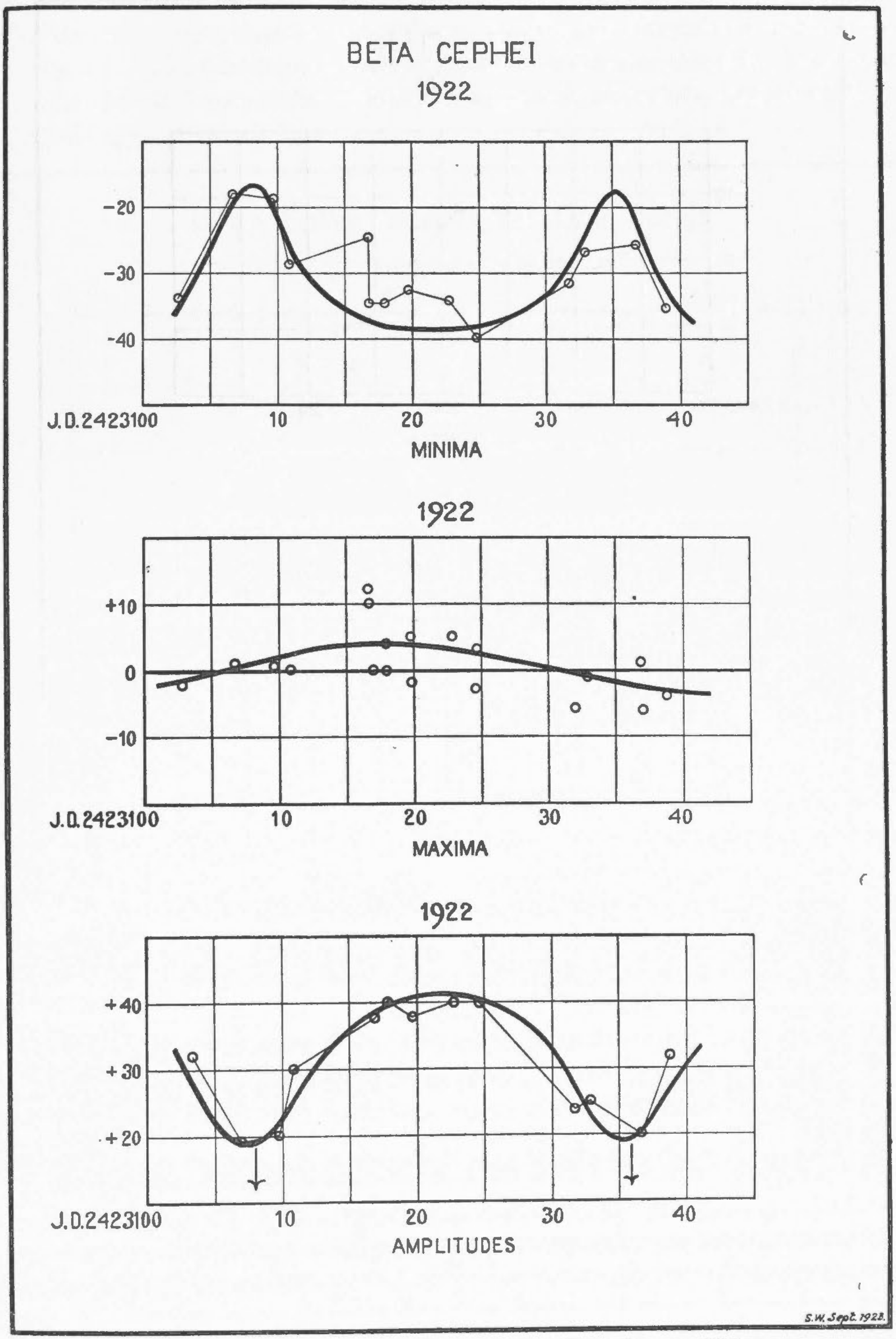

FIGURE 9. VARIATION OF AMPLITUDE OF BETA CEPHEI 1922

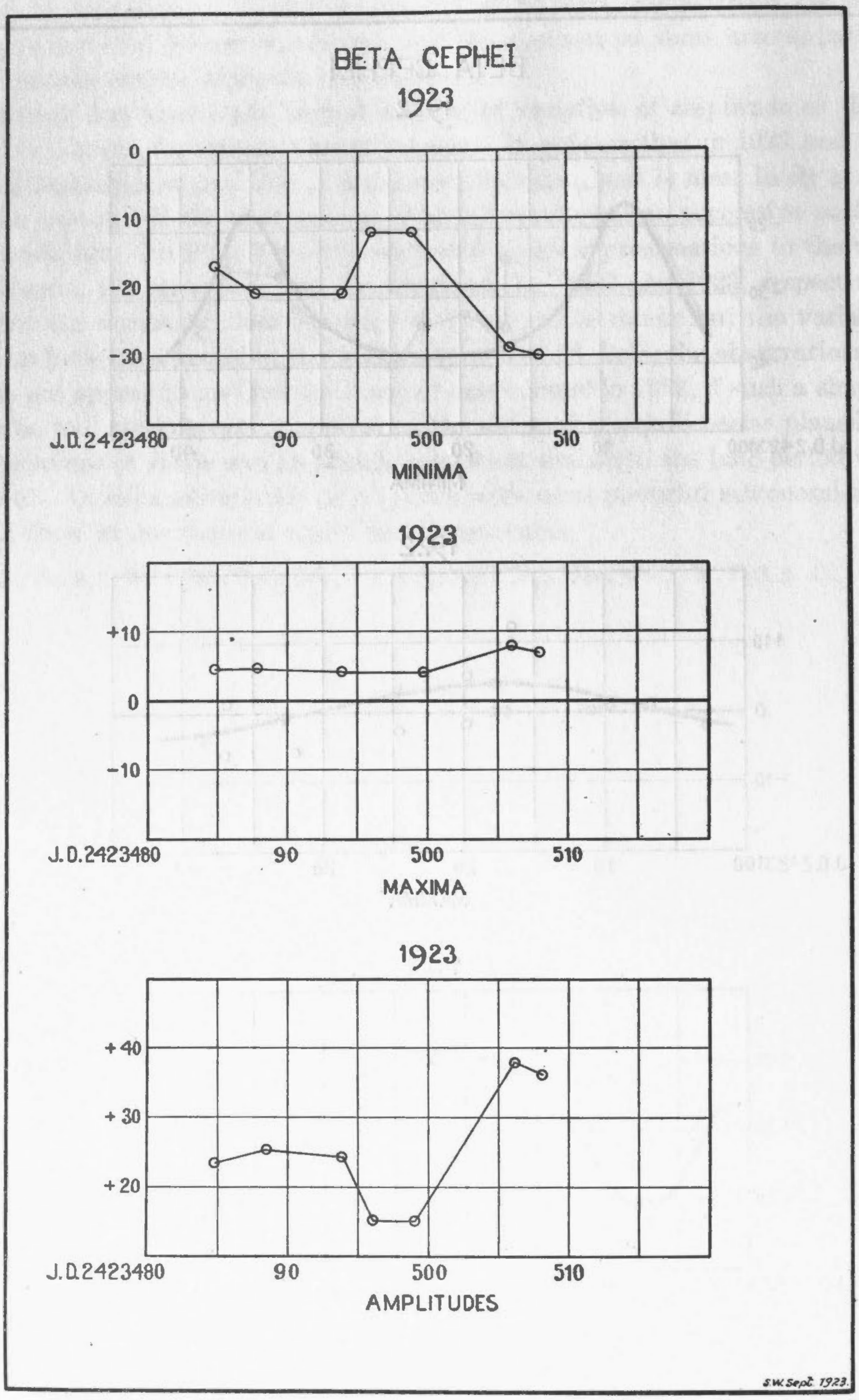

Figure 10. Variation of Amplitude of Beta Cephei 1923

On April 18th, 1922, a series of direct extrafocal photographs of β Cephei was taken with the 6-inch Brashear doublet and the photographic light-curve determined. A comparison of the two curves (radial velocity and magnitude) determined simultaneously is shown in Fig. 11. The maximum luminous intensity of the star coincides, at least approximately, with the maximum radial velocity of approach, which is a typical characteristic of the Cepheid type of variation.

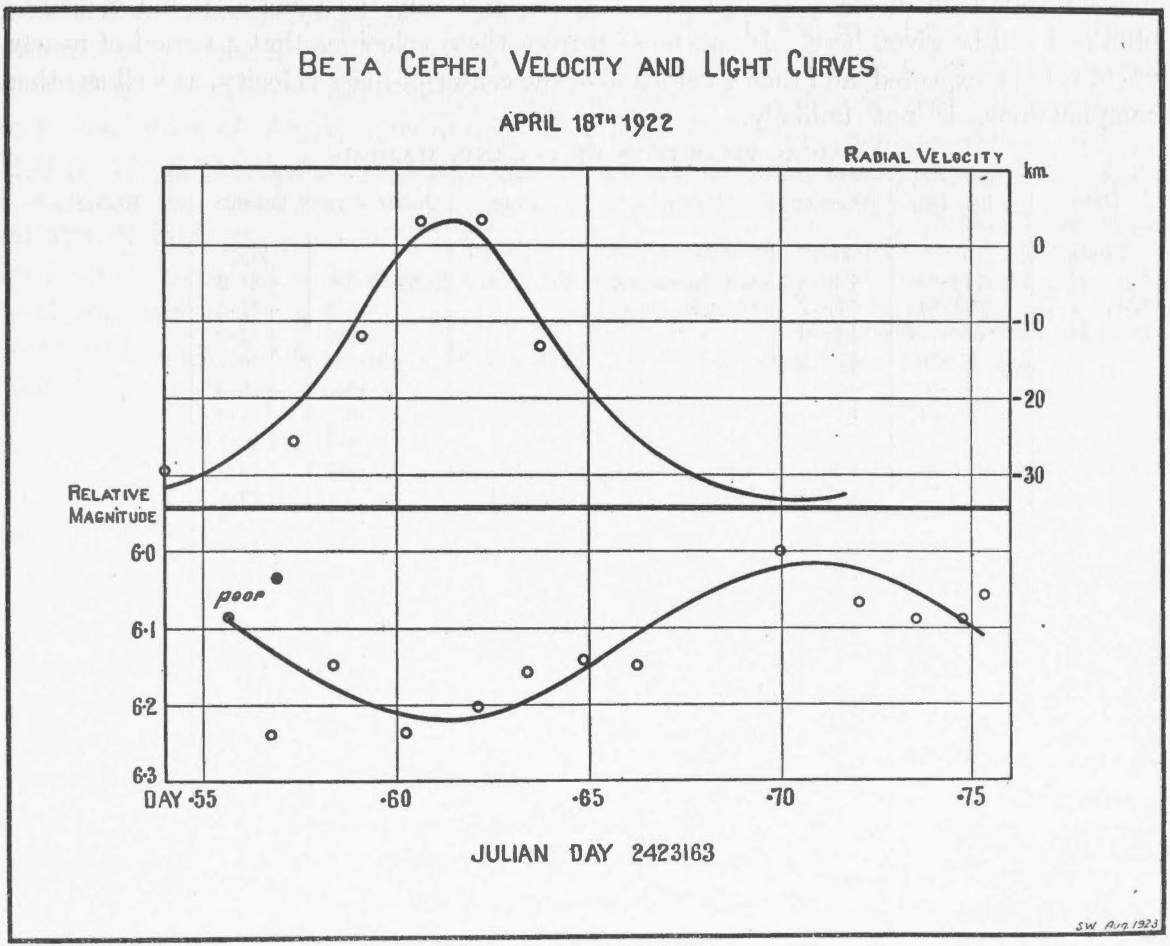

FIGURE 11. RADIAL VELOCITY AND LIGHT CURVES OF BETA CEPHEI

Chapter V

THE SPECTROSCOPIC SYSTEM 15 CANIS MAJORIS

The star 15 Canis Majoris ($\alpha = 6^h\ 49^m{\cdot}2$, $\delta = +20°6'$, class B 1), visual magnitude 4·66, announced by Dr. Campbell to be a spectroscopic binary, had already been suspected to be of the β Canis Majoris type.[1] A fairly large number of spectrograms of it were secured at the beginning of 1923, and they indicate without doubt that the radial velocity variation is exceedingly rapid. A period of perhaps less than three hours is expected; the star is too faint for adequate investigation with our instrumental equipment;

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[1] Pub. Dom. Obs., Vol. V, p. 362.

being usually at a low altitude fairly long exposures were necessary (about 40 minutes or a little more), which might amount to a large fraction of the period of variation. Most of the spectrograms were secured on rapid but rather coarse-grained plates (Seed Graflex) and were measured on the Hartmann Spectrocomparator, using the same standard spectrogram of β Canis Majoris which was used in measuring the plates of β Cephei.

It seems likely that observatories with a more powerful equipment than ours could make a very fruitful study of 15 Canis Majoris, and only the table of radial velocities obtained will be given here. It can be seen from these velocities that a period of nearly $0^{d}\cdot 15$ is to be expected, and that a variation of the center-of-mass velocity, as well as other complications, is not unlikely.

RADIAL VELOCITIES OF 15 CANIS MAJORIS

Date	Julian Day	Velocity	Remarks	Date	Julian Day	Velocity	Remarks
1921		km.		1923		km.	
Jan. 24...	2422714·654	+46·4	Direct measure.	Feb. 4...	2423455·597	+31·2	
Dec. 7...	3031·841	+38·7	Direct measure.		·635	+31·4	
Dec. 14...	038·724	+18·6			·689	+12·9	
	·762	+34·2		Feb. 5...	456·553	+30·7	
	·802	−13·2			·585	+10·9	
	·847	+20·0			·615	+22·0	
1922				Feb. 5...	456·644	+19·7	
Dec. 5...	394·741	+25·8			·697	+19·7	
	·785	+21·8		Feb. 11...	462·557	+22·1	
	·828	+17·3			·586	+30·2	
Dec. 12...	401·764	+10·1			·615	+37·1	
	·811	+22·0			·645	+21·7	
1923					·701	+17·9	
Jan. 22...	442·583	+ 5·3		Feb. 15...	466·502	+43·4	
	·617	+21·0			·565	+ 6·9	
	·649	+36·7	Direct measure +30·6.		·594	+16·6	
					·624	+24·8	
	·736	+16·8	Direct measure +18·5.		·655	+26·4	
				Feb. 16...	467·510	+ 7·6	
Jan. 25...	445·626	+20·0		Feb. 18...	469·512	+28·4	
	·658	+14·1			·537	+25·0	
	·696	+21·3			·560	+20·9	
	·736	+26·7			·580	+27·4	
Jan. 29...	449·550	+10·9		Feb. 23...	474·540	+31·5	
	·618	+26·4			·567	+38·3	
	·651	+ 0·1			·592	+20·2	
	·715	+23·5			·616	+21·7	
Jan. 30...	450·548	+26·3			·644	+30·7	
	·581	0·0					

To these velocities may be added those obtained and published by the Lick observers[1] which are as follows:—

RADIAL VELOCITIES OF 15 CANIS MAJORIS OBTAINED AT LICK OBSERVATORY

Date	Julian Day	Velocity	Remarks	Date	Julian Day	Velocity	Remarks
1910		km.		1910		km.	
Jan. 24...	2418696·599	+57·0		Dec. 27...	2419033·753	+53·8	
Nov. 12...	988·979	+31·2		1911			
Dec. 1...	2419007·756	+29·5		Feb. 16...	084·715	+36·5	
Dec. 15...	021·890	+27·6					

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[1] L. O. B., Vol. 6, p. 145.

Chapter VI

ALPHA URSAE MINORIS

The star α Ursae Minoris (Polaris) has already been studied spectroscopically by Campbell,[1] Frost,[2] Hartmann,[3] Bélopolsky[4] and Kûstner.[5] It was found to act as a short-period binary with a period of $3^{d}\cdot9681$, and, moreover, was discovered by Campbell to have a variable centre-of-mass velocity, so that at first it was considered to be a triple system. Miss Hobe at the Lick Observatory gave a period of 11·9 years for the variation of this centre-of-mass velocity,[6] but our observations do not verify this period. Very long series of observations during short intervals of time had not hitherto been obtained and it was thought that although the amplitude is small, a considerable number of our one-prism spectrograms would give valuable information. The variation of light had already been suspected by several observers, *e.g.* Seidel[7], Schmidt[8] and Pannekoek.[9] Hertzsprung, using his method of side images produced by placing a grating in front of the objective of a camera and using the period deduced from radial velocity observations, succeeded in obtaining a photographic light curve.[10] The amplitude of this light curve is only $0^{m}\cdot171$ and the elements of variation are:—

$$\text{Max.} = \text{J. D. } 2418985\cdot856 + 3^{d}\cdot9681 \text{ E.}$$

Hertzsprung, not only from this light curve but also from the character of the spectrum, established without any doubt that Polaris is a Cepheid variable. This was verified again from the Harvard photographic observations by King,[11] and from the Harvard visual observations by E. C. Pickering.[12]

A very good light-curve of Polaris was determined by Prof. Stebbins[13] with a selenium photometer. This curve has an amplitude of $0^{m}\cdot078$, considerably smaller than that obtained by Hertzsprung, but the light affecting the selenium cell has a longer wave-length than that affecting the ordinary photographic plate (much nearer the mean wave-length of visual light), and it is a well-known property of Cepheids that their photographic amplitude is usually larger than the visual. The elements given by Stebbins, which were adopted by Hartwig as final elements, are as follows:—

$$\text{Max.} = \text{J. D. } 2418985\cdot936 + 3^{d}\cdot9681 \text{ E.}$$

It is these elements that have been used in the present investigation.

Prof. Gramatzki in Germany,[14] using a new kind of visual photometer, has redetermined the visual light-curve; he finds elements slightly different, a difference which however would not affect the results to any appreciable extent. His elements are:—

$$\text{Max.} = \text{J. D. } 2422954\cdot2147 \text{ helioc. G. M. T.} + 3^{d}\cdot96835 \text{ E.}$$

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[1] Ap. J., Vol. 10, p. 180; Vo. 21, p. 191; and Vol. 25, pp. 59; and L. O. B., Vol. 1, pp. 23; and Vol. 4, p. 98.
[2] Ap. J., Vol. 10, p. 184.
[3] Ap. J., Vol. 14, p. 52.
[4] A. N., Vol. 152, p. 201.
[5] Ap. J., Vol. 27, p. 304.
[6] L. O. B., Vol. 6, p. 18.
[7] Abh. Akad, Wiss. Munchen, Vol. 6, p. p. 568 and 603; Vol. 9, p. pp. 117 and 160.
[8] A. N., Vol. 46, p. 293.
[9] A. N., Vol. 194, p. 359.
[10] A. N., Vol. 189, p. 89.
[11] H. A., Vol. 59, p. 249.
[12] H. C., No. 174.
[13] A. N., Vol. 192, p. 189.
[14] A. N., Vol. 217, p. 454.

RADIAL VELOCITIES OF POLARIS

The following radial velocities for Polaris have been obtained here:—

Date	Julian Day	Phase	Velocity	Remarks
1923		d	km.	
April 29....	2423539·627	2·280	−15·6	Gramatzki's elements give phase 2d·065.
	·649	2·298	−13·3	
May 1....	541·585	0·280	−19·8	
	·603	0·298	−19·5	Mean of three measures.
	·620	0·305	−16·5	
	·637	0·322	−18·0	
May 2....	542·580	1·265	−15·1	
	·597	1·282	−13·8	
	·615	1·300	−15·4	
	·632	1·317	−15·2	
May 3....	543·777	2·462	−14·0	
May 4....	544·594	3·279	−16·5	
	·611	3·296	−14·8	
May 6....	546·561	1·278	−14·1	
	·578	1·295	−15·7	
	·596	1·313	−14·5	
	·613	1·330	−14·7	
May 7....	547·612	2·329	−12·5	
May 10....	550·610	1·359	−15·9	
May 13....	553·585	0·365	−18·8	
	·603	0·384	−16·1	Mean of two measures.
May 18....	558·560	1·373	−14·6	Mean of two measures.
	·619	1·432	−14·9	
	·641	1·454	−16·2	
May 21....	561·598	0·442	−19·7	
May 23....	563·604	2·448	−14·0	Mean of two measures.
	·620	2·465	−14·3	Mean of two measures.
	·666	2·510	−15·8	
	·683	2·527	−13·9	Mean of two measures.
June 15....	586·679	1·715	−14·2	
June 18....	589·599	0·667	−19·3	
	·628	0·696	−16·4	
	·677	0·745	−19·8	
June 24....	595·597	2·697	−14·1	Mean of two measures.
	·662	2·762	−17·8	
	·679	2·779	−15·3	
June 25....	596·793	3·893	−18·6	
June 27....	598·573	1·705	−16·7	
	·590	1·722	−17·4	
	·703	1·835	−15·2	
	·750	1·882	−16·0	Mean of two measures.
	·837	1·969	−15·6	Mean of two measures.
June 29....	600·606	3·738	−20·5	Mean of four measures −20·0; −19·6; −21·4 and −20·9.
	·640	3·772	−18·9	
	·660	3·792	−18·0	
	·674	3·806	−18·8	
	·687	3·819	−18·3	
	·701	3·833	−18·2	
	·781	3·913	−18·1	
	·840	0·004	−18·0	
Aug. 23....	655·849	3·427	−17·2	

RADIAL VELOCITIES OF POLARIS—*Concluded*

Date	Julian Day	Phase	Velocity	Remarks
1923		d	km.	
Sept. 10....	2423673·606	1·344	−17·5	
	·623	1·361	−19·4	
	·685	1·423	−17·4	
	·699	1·437	−18·5	
	·769	1·507	−16·8	
Sept. 13....	676·669	0·439	−20·3	Gramatzki's elements give phase 0d·215.
	·681	0·451	−24·5	

Only the spectrograms considered good have been measured, all on the spectrocomparator, using a good spectrogram of daylight as standard. All the observations between April 29 and June 29 are plotted on the graph shown in Fig. 12, the open circles indicating the observations from April 29 to May 13 inclusive, the squares those from May 14 to June 18, and the darkened circles those from June 24 to June 29. From the graph there seems to be an indication that the open circles give a mean curve of larger amplitude than the squares, while that from the darkened circles is perhaps smaller. The amplitude of the general mean curve is about 4 kilometres, while the mean amplitude given by Dr. Campbell in his Second Catalogue of Spectroscopic Binaries is 6·08 kilometres. There is thus a possibility of a variation of amplitude similar to what has been found for stars of the β Canis Majoris type, but a large number of very good high-dispersion spectrograms would be needed to confirm this.

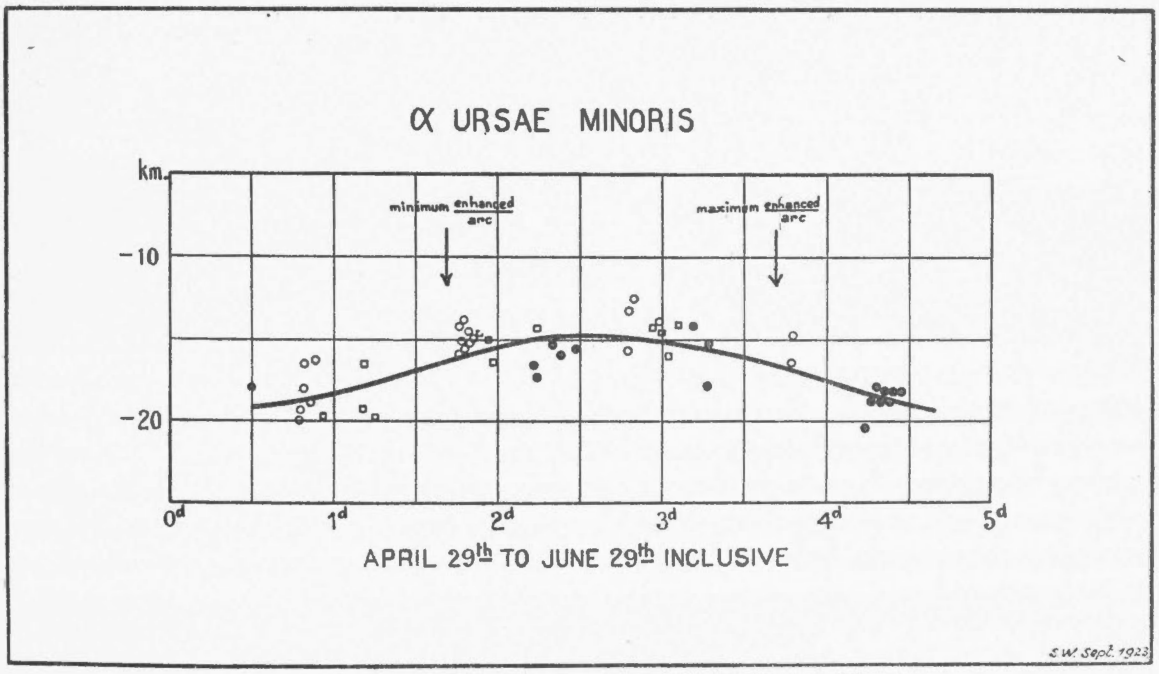

FIGURE 12. RADIAL VELOCITY CURVE OF ALPHA URSAE MINORIS

The radial velocity curve is in good agreement with the theory of Cepheid variation, the maximum velocity of approach coinciding with the maximum of light. The few observations obtained in September give on the whole lower velocities than those obtained before, indicating that the centre of mass of the system has changed by an appreciable amount between May and September. It may be, indeed, that what has been interpreted above as a possible change of amplitude is merely an indication of a decreasing value of the centre-of-mass velocity between April and June.

The centre-of-mass velocity given by these observations disagrees entirely with the provisional elements of the orbit of the centre of mass given by Miss Hobe in Dr. Campbell's catalogue referred to above. This might of course be due to a systematic difference between our radial velocities and those of the Lick Observatory, but this is scarcely likely, since standard velocity stars measured here, (with the same standard daylight spectrogram), have given velocities agreeing very well with those obtained at Mt. Hamilton. The variation found between the observations of June and those of September being fairly large (as compared with the total range of variation of the centre-of-mass velocity 5·96 km. given by Miss Hobe) it seems that a shorter period than 11·9 years has to be looked for.

An examination of all our plates of α Ursae Minoris shows definitely the existence of a variation in spectral type. Three separate investigations, by J. F. Frédette, R. Callander, and the writer, seem to indicate that the minimum value of the ratio between the intensities of enhanced and arc lines occurs at about phase $1^d\cdot2$, and the maximum at about phase $3^d\cdot2$; the maximum is more vaguely determined, however, than the minimum and might without difficulty be interpreted to occur at phase $0^d\cdot0$, coinciding with the maximum of light. It may be remarked that in the Cepheids in which changes of spectral type have been detected (by Shapley) the maximum and minimum values of this ratio appear usually to coincide very nearly with the maximum and minimum of light. The possible divergence from this rule in the case of Polaris would therefore be very interesting if verified.

Chapter VII

DELTA CEPHEI

Delta Cephei, the most important Cepheid variable, is too well known to require an extended bibliography here; many such can be found in the ordinary text-books on astronomy. It was discovered to be variable in 1784 by Goodricke,[1] and was first studied by him and Pigott; they gave as the period of variation $5^d\ 8^h\ 37^m$. About thirty years later it was observed by Westphal (1817-1818)[2] and from 1840 to 1856 by Argelander.[3] After this it was observed by a fairly large number of astronomers; among the visual light curves that have been published may be mentioned those of Schur,[4] Beljawsky,[5] Markwick,[6]

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[1] Phil. Trans., Vol. 76, p. 50.
[2] Naturf. Ges. Neue Schriften, Heft 2.
[3] A. N., Vol. 18, p. 133, and Vol. 44, p. 195.
[4] A. N., Vol. 137, p. 297.
[5] A. N., Vol. 165, p. 225.
[6] Mem. British Astr. Ass., Vol. 11, Table IV, and Jour. Br. Astr. Ass., Vol. 17, p. 211.

Felix de Roy,[1] Luizet,[2] Miss Clerke,[3] Bemporad,[4] Padova,[5] Lau,[6] Stratonow[7] and Stebbins.[8] Only three photographic light-curves have been published; those by Wirtz[9] and by Meyermann[10] were determined by the extrafocal method; Jordan determined both the photographic and the photovisual light curves.[11]

The best light-curves are perhaps those determined by Guthnick, first with a rubidium[12] and later with a potassium[13] photo-electric cell. He finds no secondary oscillations in either the ascending or the descending branch.

R. H. Curtiss supports the suggestion that since 1850 the light range of δ Cephei has been measurably variable.[14] The question of a slight variation of period has been widely discussed; an interesting paper on this subject is that published recently by Danjon.[15]

Considering the importance of the star it may be said that its spectrographic study has been more or less neglected. Bélopolsky obtained a few series of spectrograms of it during the course of several years[16] and Dr. Moore determined its spectroscopic orbit at the end of 1907.[17] Since 1908 apparently no long series of spectrograms have been obtained. For the purpose of comparison there is given below a collected table of radial velocities previously published.

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[1] Bul. Soc. Astr. de France, 1905, p. 414.

[2] Les Céphéides considérées comme étoiles doubles (Ann. Univers. Lyon Nouvelle Série, Vol. 1, 1912, fascicule 33).

[3] The Observatory, Vol. 19, p. 114.

[4] Mem. Spettr Italiani, Vol. 39, p. 74.

[5] Item, Vol. 40, p. 102, and (2) Vol. 1, p. 141.

[6] Bul. Astr., Vol. 23, p. 20.

[7] Taschkent, Pub., Vol. 5, p. 32.

[8] A. N., Vol. 154, p. 334.

[9] Ap. J., Vol. 27, p. 192.

[10] A. N., Vol. 175, p. 1.

[11] Ap. J., Vol. 50, p. 201.

[12] A. N., Vol. 208, p. 172.

[13] A. N. Jubilaumsnummer Zum Hundert. Best tafel 2.

[14] Pub. Amer. Astr. Soc., 26th and 28th meetings.

[15] L'Astronomie, Vol. 37 (1923), p. 346.

[16] Mitteilungen der Nikolai-Hauptsternwarte zu Pulkowa, Band III, p. 69.

[17] L. O. B., Vol. 7, p. 153.

RADIAL VELOCITIES OF δ CEPHEI OBTAINED AT OTHER OBSERVATORIES

Author	Date	Julian Day	Phase	Velocity	Remarks
				km.	
Bélopolsky....	1894	Not given	0·13	+3·7	The spectrograms of 1894, '95, '97 and '98 were obtained with a one-prism spectrograph and measured on a direct measuring engine. Phases counted from epochs of light-minimum as given in the Annuaire du Bureau des Longitudes.
			0·67	+ 8·2	
			1·04	− 8·5	
			1·50	−27·1	
			2·29	−29·8	
			2·42	−23·2	
			2·79	−26·3	
			3·00	−17·3	
			3·29	−16·9	
			3·38	−15·5	
			3·50	−13·4	
			3·79	−12·3	
	1894	Not given	4·13	− 4·5	
			4·50	+ 0·2	
			4·79	+ 2·0	
			4·92	+ 4·0	
			5·00	+ 6·2	
	1895	Not given	0·50	− 6·9	
			0·71	− 7·6	
			1·00	−17·4	
			1·63	−38·5	
			1·92	−32·1	
			2·33	−28·2	
			2·71	−35·1	
			3·67	−24·6	
			4·04	−16·9	
			4·92	− 1·9	
			5·04	− 6·7	
			5·33	− 1·6	
	1897	Not given	1·15	−21·5	
			1·25	−39·3	
			1·47	−39·3	
			1·64	−31·5	
			3·07	−24·3	
			3·93	−19·6	
			5·12	−23·7	
	1898	Not given	0·63	− 8·3	
			0·75	− 4·2	
			1·00	−21·2	
			1·17	−15·4	
			1·83	−40·6	
			2·13	−35·5	
			2·17	−41·8	
			2·25	−29·1	
			2·63	−29·2	
			2·87	−29·6	
			3·13	−27·8	
			3·25	−24·7	
			3·79	−13·4	
			3·87	−19·2	
			4·75	− 7·5	

RADIAL VELOCITIES OF δ CEPHEI OBTAINED AT OTHER OBSERVATORIES—*Continued*

Author	Date	Julian Day	Phase	Velocity	Remarks
				km.	
Bélopolsky....	1902	Not given	1·61	−42·8	The spectrograms of 1902, '03, '04, '05 and '08 were obtained with a three-prism spectrograph and measured on a spectro-comparator.
			1·65	−38·9	
			1·75	−41·7	
			2·10	−33·6	
			3·10	−28·2	
			4·49	−12·8	
	1903	Not given	0·22	− 4·6	
			0·46	(− 1·9)	
			0·85	− 8·2	
				(5·7)	
			1·18	−28·6	
			1·22	−32·0	
			2·23	−39·7	
			2·83	−30·3	
			3·22	−28·7	
			4·22	−15·4	
			4·56	−14·7	
			5·16	− 6·8	
			5·22	− 9·9	
	1904	Not given	0·63	+ 3·5	
			1·15	−21·9	
			2·14	−33·6	
			3·14	−25·2	
			3·80	−16·6	
			4·80	− 8·4	
	1905	Not given	0·30	+ 2·2	
			3·91	−18·1	
	1908	Not given	1·12	−17·6	
			1·23	−30·5	
			2·11	−32·0	
			3·12	−30·7	
			3·49	−20·7	
			3·85	(−16·2)	
			4·11	−15·7	
			4·91	− 6·7	
Lick Observers.....	1896				
	Nov. 12	2413876·647	1·615	−22·36	Beginning at this point the phases have been computed from the epochs of light-maximum derived from the formula Max=J. D. 2393659·856 + 5^d·366386 (Luizet's elements).
	1897				
	Nov. 11	2414240·695	0·749	−29·10	
	1898				
	Oct. 25	2414588·774	0·013	−34·18	
J. H. Moore...	1907				
	Sept. 18	2417837·698	2·273	−14·20	
		·761	2·336	−15·44	
		·836	2·411	−12·25	
	Sept. 19	838·692	3·267	− 6·78	
		·850	3·425	− 4·97	
	Sept. 23	842·743	1·952	−18·51	
		·826	2·035	−17·09	
	Sept. 25	844·735	3·944	+ 2·22	
		·827	4·036	+ 3·49	

RADIAL VELOCITIES OF δ CEPHEI OBTAINED AT OTHER OBSERVATORIES—*Concluded*

Author	Date	Julian Day	Phase	Velocity	Remarks
				km.	
	1907				
J. H. Moore...	Sept. 26	2417845·724	4·933	−29·01	
		·809	5·018	−30·50	
	Sept. 29	848·716	2·558	−11·97	
		·811	2·653	−10·25	
	Oct. 6	855·755	4·231	+ 3·24	
		·796	4·272	+ 2·65	
	Oct. 7	856·716	5·192	−36·00	
		·778	5·254	−35·90	
		·836	5·312	−35·59	
	Oct. 9	858·797	1·906	−19·55	
	Oct. 10	859·670	2·779	− 9·84	
	Oct. 12	861·605	4·715	−16·32	
	Oct. 13	862·727	0·470	−32·30	
		·769	0·512	−31·64	
	Oct. 17	866·667	4·410	+ 2·78	
		·760	4·503	− 0·73	
	Oct. 18	867·736	0·113	−35·29	
		·782	0·159	−34·55	
	Oct. 19	868·615	0·992	−26·76	
	Oct. 30	879·721	1·365	−24·30	
	Nov. 10	890·677	1·588	−21·99	
	Nov. 13	893·645	4·556	− 3·86	
		·687	4·598	− 6·40	
		·737	4·648	−10·08	
	Nov. 14	894·711	0·256	−34·69	
	Nov. 24	904·659	4·837	−24·69	
	Nov. 29	909·663	4·475	− 1·98	
	Dec. 12	922·623	1·336	−25·21	
	1908				
	Jan. 5	946·611	3·858	− 0·31	
Küstner.......	1911				
	Aug. 24	2419273·43	5·18	−36·1	A. N., Vol. 198, p. 441.
	1912				
	Sept. 24	670·36	5·00	−30·4	
	Sept. 26	672·32	1·49	−30·9	

On plotting radial velocity curves from the above observations it becomes evident that Bélopolsky's observations have much larger errors than those of Moore; the latter may be said to be excellent.

The radial velocities obtained here are given below; the phases are computed as above from the epochs of light maximum as obtained from Luizet's formula, Max. = J. D. 2393659·856 + 5^d·366386 E.

RADIAL VELOCITIES OF δ CEPHEI OBSERVED AT OTTAWA

Date	Julian Day	Phase	Velocity	Remarks
1923		d	km.	
July 4	2423605·697	1·407	−22·6	Remeasure −21·4
8	609·710	0·054	−33·9	−35·9
10	611·615	1·959	−19·2	−17·5
12	613·783	4·127	+ 2·9	+ 1·7
19	620·751	0·362	−27·0	−28·4
22	623·621	3·232	+ 5·3	Fair plate + 5·4
23	624·692	4·303	+ 5·8	+ 9·0
29	630·733	4·978	−27·7	
31	632·666	1·544	−24·3	
Aug. 9	641·604	5·116	−35·9	
15	647·556	0·335	−35·7	
	·602	0·381	−31·1	
20	652·570	5·349	−34·1	
22	654·611	2·024	−13·8	
23	655·583	2·996	− 7·4	
	·693	3·106	− 0·6	
26	658·636	0·682	−26·4	
30	662·576	4·622	−11·4	
	·649	4·695	−16·2	
	·728	4·774	−23·4	
	·806	4·852	−20·6	
Aug. 31	663·657	0·337	−35·2	
Sept. 5	668·535	5·215	−37·7	
	·585	5·265	−34·8	
6	669·553	0·866	−24·2	
10	673·822	5·135	−34·1	
	·864	5·177	−35·0	
12	675·554	1·501	−19·1	
13	676·542	2·489	−10·1	
	·612	2·559	− 5·7	
14	677·560	3·507	− 1·6	

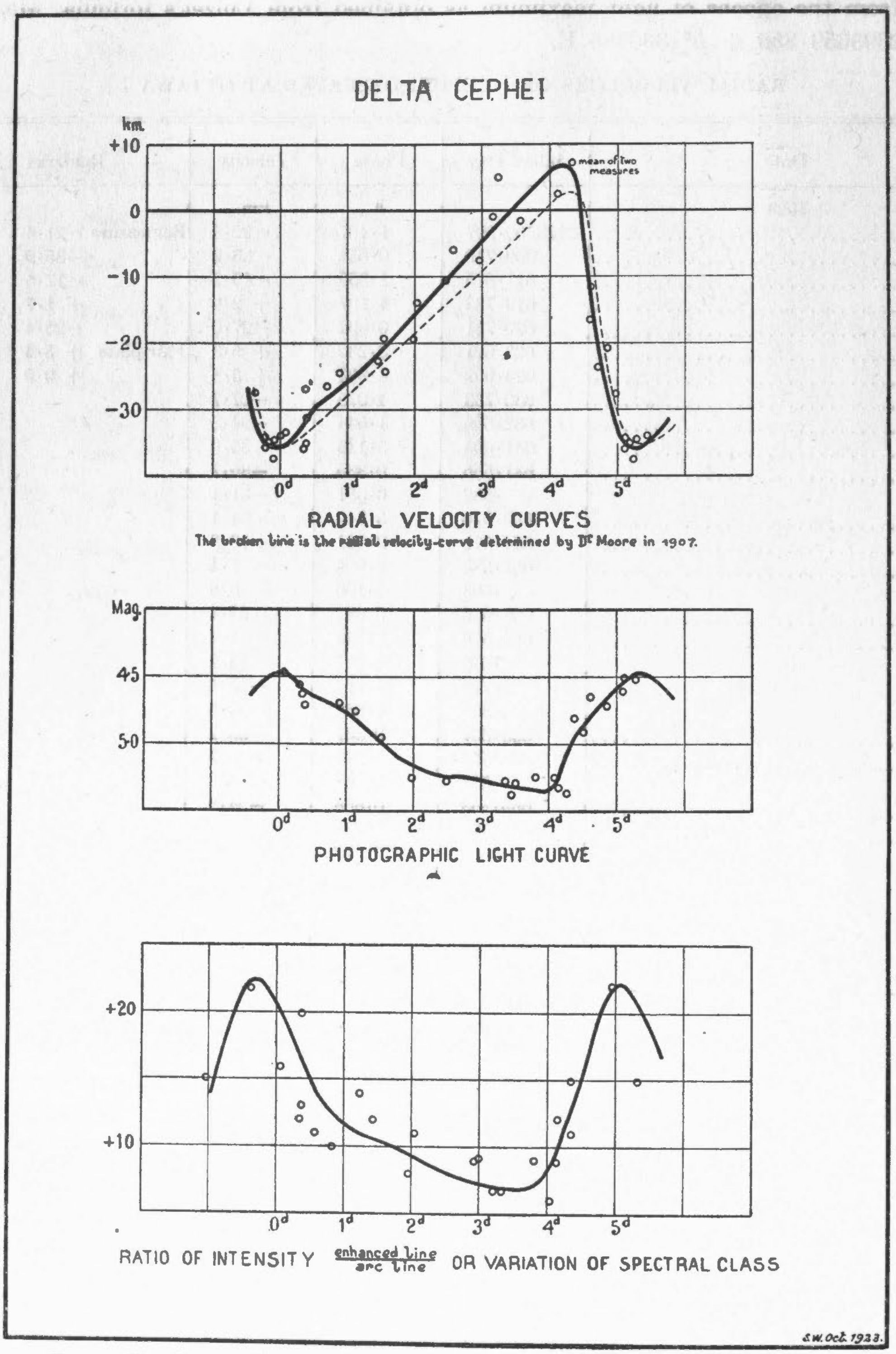

FIGURE 13. RADIAL VELOCITY, MAGNITUDE AND SPECTRAL CLASS OF DELTA CEPHEI

It may be remarked that these spectrograms, taken as they were with a single prism spectrograph on fairly coarse-grained plates (Seed 30), and with comparatively long exposures, will of course have larger probable errors than the Lick spectrograms. They were measured on the spectro-comparator, using the same standard spectrogram of daylight as that used for measuring the plates of Polaris. The radial velocity-curve obtained from the above velocities (*see* Fig. 13) appears slightly different from that of Moore (shown as a broken line in the figure). A change of amplitude and perhaps of center-of-mass velocity has apparently occurred. If the variation of light is a direct function of the variation of radial velocity (as for instance in the case of 12 Lacertae, where the photoelectric observations of Guthnick[1] show the amplitudes of light and velocity curves to vary simultaneously), the suggestion of R. H. Curtiss that the light range of δ Cephei has been measurably variable would receive additional weight.

It is also worth noting that the minimum of our velocity curve, which is well determined, and corresponds to the maximum light, falls a trifle earlier than the predicted one, and that by a quantity of the same order of magnitude as predicted by Danjon (equation of light); this fact lends further colour to the supposition that the star moves in a long-period orbit. Bélopolsky, from his observations, obtained for this orbit a period of 6·36 years; Moore, on the contrary, from a comparison of his own observations with the three older Lick observations given in the above table, concluded that the general velocity curve did not show any variation, and that no long-period orbit existed. A point apparently overlooked by Bélopolsky is the possibility of a variation of amplitude, he having considered only a variation of center-of-mass velocity; his observations, although having perhaps the largest probable errors of any of the observations above, suggest, when replotted, both slight variations of amplitude and of center-of-mass velocity. Moore's conclusions (the value of which, as he himself points out, is based entirely on three older observations, which may have fairly large probable errors) may perhaps require revision. The definite determination of the possible long-period orbit with the observations so far available, especially in view of the possibility of a variation of amplitude of the short-period curve, seems at present impossible.

During the period covered by the Ottawa spectrograms there were also made, with a short-focus camera, plates of the surrounding star-field. These have been used to determine the photographic magnitude of δ Cephei, taking as standards the photographic magnitudes of the three following stars as given in the Revised Harvard Photometry[2]:—

Star	*Photo. Mag.*
ε Cephei	4·65
λ Cephei	5·29
B. D.+56° 2765	5·86

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[1] A. N. Jubilaumsnummer zum Hundert, Best. Tafel 3.
[2] H. A., Vol. 50.

The method of estimation of the magnitudes is described in Chapter IX of the present paper; the results are given in the following table:—

ESTIMATED PHOTOGRAPHIC MAGNITUDES OF δ CEPHEI

Date	Julian Day	Phase	Photographic magnitude	Date	Julian Day	Phase	Photographic magnitude
1923				1923			
		d				d	
July 6.......	2423607·76	3·47	5·35	Aug. 30.......	2423662·56	4·61	4·65
8.......	609·70	0·04	4·45	31.......	663·62	0·30	4·55
10.......	611·64	1·98	5·25	Sept. 3.......	666·69	3·37	5·27
12.......	613·77	4·11	5·30	4.......	667·56	4·24	5·35
19.......	620·75	0·36	4·60	5.......	668·56	5·24	4·50
Aug. 3.......	635·60	4·48	4·90	6.......	669·56	0·87	4·70
5.......	637·60	1·11	4·75	10.......	673·54	4·85	4·70
8.......	640·59	4·10	5·25	....	·82	5·13	4·50
9.......	641·59	5·10	4·60	12.......	675·55	1·50	4·95
13.......	645·66	3·81	5·25	13.......	676·55	2·50	5·25
15.......	647·60	0·38	4·70	14.......	677·55	3·50	5·29
19.......	651·60	4·38	4·80				

These magnitudes (which in this case are *merely estimates*, and cannot compare in accuracy with the magnitudes obtained for fainter variables in Chapter IX, where large numbers of comparison stars were available) give the light curve shown in Fig. 13. It is seen that the maximum brightness coincides almost exactly with the minimum radial velocity, while the minimum brightness slightly precedes the maximum radial velocity. This had already been found by Dr. Moore on comparison of his velocity curve with the photometric light-curve of Prof. Stebbins. In the present case the comparison of the two curves presents the advantage that they were obtained during the same period of time.

Although very nearly of the same spectral class as the Sun, the spectrum of δ Cephei presents marked differences; and these differences appear most marked when the star is near its maximum brightness. δ Cephei is a giant G star, while the Sun is a dwarf of the same class; since the density of the former is much the smaller, although the masses and temperatures may not be very different, a greater degree of dissociation of the elements is to be expected. Hence ionized elements will be preponderant and the spectral lines of ionized atoms will be strong (enhanced lines), while the spectral lines of neutral atoms will be weak (arc lines).

It was found in 1913 by Inna Lehmann[1] at Poulkowa that the appearance of the spectrum of δ Cephei varied in the course of the period. All Bélopolsky's three-prism spectrograms were compared by her with a chosen standard spectrogram of the same star; the intensities of eight spectral lines, probably all enhanced lines, were estimated on each spectrogram, taking as the unit of intensity for each line its intensity on the standard plate; the comparisons were made on a spectro-comparator. In this way a curve of variation of intensity was deduced, and was found to show perfect parallelism with the curve of variation of light as obtained by Stebbins.

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[1] Pulkowo Mitteilungen, Band V, p. 177.

In 1916 Adams and Shapley[1] pointed out again the marked changes in the spectrum between maximum and minimum luminosity; the most marked of these are as follows:—

Spectral line	Maximum	Minimum
Hydrogen line $H\gamma$	Strong	Much weakened.
Enhanced lines of *Fe*, *Ti*, *Sr.* and *Cr*	Strong	Much weakened.
λ 4481, enhanced *Mg*	Very strong	Much weakened.
λ 4227, calcium	Strong	Strengthened.
Low temperature lines of *Ca*, *Fe*, *Ti* and *Cr*	Weak	Strengthened.
Continuous spectrum	Strong in violet	Weakened in violet.

This is equivalent to a variation of spectral class, similar to that shown in many Cehpeids and probably present in all. On examination of our spectrograms it was found that the titanium arc line, λ 4534·953, and the titanium enhanced line, λ 4534·139, show a very marked variation of their relative intensities. The comparison of this close pair of lines on different spectrograms offers perhaps the best criterion for determination of the variation of spectral class. On a large number of our spectrograms this pair of lines was examined; in each case the intensity of the arc line was assumed as 10 and the intensity of the enhanced line estimated with respect to it. The estimated intensities of the enhanced line are given in the following table:—

RELATIVE INTENSITIES OF ENHANCED AND ARC LINES

Date	Phase	Enhanced *Ti* λ 4534·139 Intensity	Arc *Ti* λ 4534·953 Intensity	Date	Phase	Enhanced *Ti* λ 4534·139 Intensity	Arc *Ti* λ 4534·953 Intensity
1923	d			1923	d		
July 4	1·407	12	10	July 30	0·593	11	10
6	3·354	7	10	31	1·544	7	10
8	0·054	16	10	Aug. 5	1·236	14	10
10	1·959	8	10	8	4·144	9	10
11	2·964	9	10	13	3·801	9	10
12	4·022	6	10	15	0·335	12	10
12	4·127	12	10	15	0·381	20	10
19	0·362	13	10	19	4·353	11	10
22	3·232	7	10	20	5·349	15	10
23	4·303	15	10	22	2·024	11	10
25	0·828	10	10	23	2·996	9	10
29	4·978	24	10				

These are of course mere estimates, but when the relative intensities are plotted according to phase they give the curve in Fig. 13, showing very definitely the variation of spectral class.

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[1] Proc. Nat. Acad. of Sc. of the U.S.A., Vol. 2, 1916, p. 136.

Chapter VIII

THE SPECTROSCOPIC SYSTEM GAMMA URSAE MINORIS

γ Ursae Minoris ($\alpha=15^h\ 20^m\cdot 9$ $\delta=+72°11'$, class A2) was discovered by Otto Struve to be a star of the β Canis Majoris type;[1] its period is the shortest of this type at present known. From 200 spectrograms secured at the Yerkes Observatory between February and August, 1922, Struve finds a period of $2^h\ 36^m\ 10^s$ or $0^d\cdot 108449$; he also finds the shape and range of the velocity curve to vary considerably, not only from one night to another, but even during the same night. Its brightness has also been found by Guthnick to be variable, the variation having an amplitude of approximately 0·04 magnitude; in May, 1922, Dr. Bottlinger, at Babelsberg, found the variations to occur in very short intervals of time (2·3 hours, approximately), but no definite period could be established.

Though the star was still under investigation by Struve, it was, with his consent, placed also on our observing programme. The spectrum is exceedingly poor, the lines few, wide and diffuse, and it proved extremely difficult to measure the spectrograms, many of which, indeed, had to be rejected. On account of the extreme shortness of the period it was necessary to make short exposures, and hence to use a wide slit, a circumstance which quite possibly introduced considerable accidental errors due to poor guiding. Too much importance should therefore not be attached to the results. The exposures averaged twenty minutes, Seed 23 plates being generally used; the spectrograms were measured on a Toepfer engine. The radial velocities obtained are given in the following table:—

RADIAL VELOCITIES OF γ URSAE MINORIS

Date	Julian Day	Velocity	Remarks	Date	Julian Day	Velocity	Remarks
1922		km.		1922		km.	
Sept. 17...	2423315·531	−27·1	Poor.	Feb. 15...	2423466·784	−13·2	
	·554	−47·0			·798	−26·9	
	·603	−19·8			·810	−36·3	
	·681	−26·6			·824	−28·3	
	·704	−20·1			·837	−15·7	
	·728	−21·3			·850	−22·2	
	·742	−31·5			·863	−13·4	
	·756	−43·9			·892	− 8·2	
	·784	−36·5			·904	− 8·5	
	·798	−36·9		Feb. 22...	473·625	−29·6	
Sept. 21...	319·544	−39·7			·640	−30·0	
	·569	−43·7			·655	−26·7	
	·585	−50·7			·701	−25·2	
	·601	−43·5			·717	−18·6	
	·616	−16·6			·733	− 7·5	Poor.
	·674	−37·1			·768	−34·8	
	·689	−45·1			·810	− 0·7	
1923	·704	−37·9			·827	− 9·0	
Jan. 26...	446·719	− 2·4	Poor.		·849	−14·4	
	·744	−14·5	Poor.		·867	−28·2	
	·792	− 7·6			·883	−26·2	
	·814	− 5·5			·897	−26·6	
	·837	−31·0			·913	−29·7	
	·859	−29·2		Feb. 25...	476·543	−12·1	
	·881	−25·9	Poor.		·570	−43·1	
	·903	−27·9	Poor.		·584	−37·4	

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[1] Pub. Amer. Astr. Soc., 28th meeting, p. 391.

To these may be added the following, which have been kindly communicated by Dr. Struve.

Date	Julian Day	Velocity	Remarks	Date	Julian Day	Velocity	Remarks
1922		km.		1922		km.	
May 29...	2423204·699	−15·7		May 29...	2423204·775	+21·6	
	·712	−13·6			·783	+ 5·8	
	·718	− 0·3					

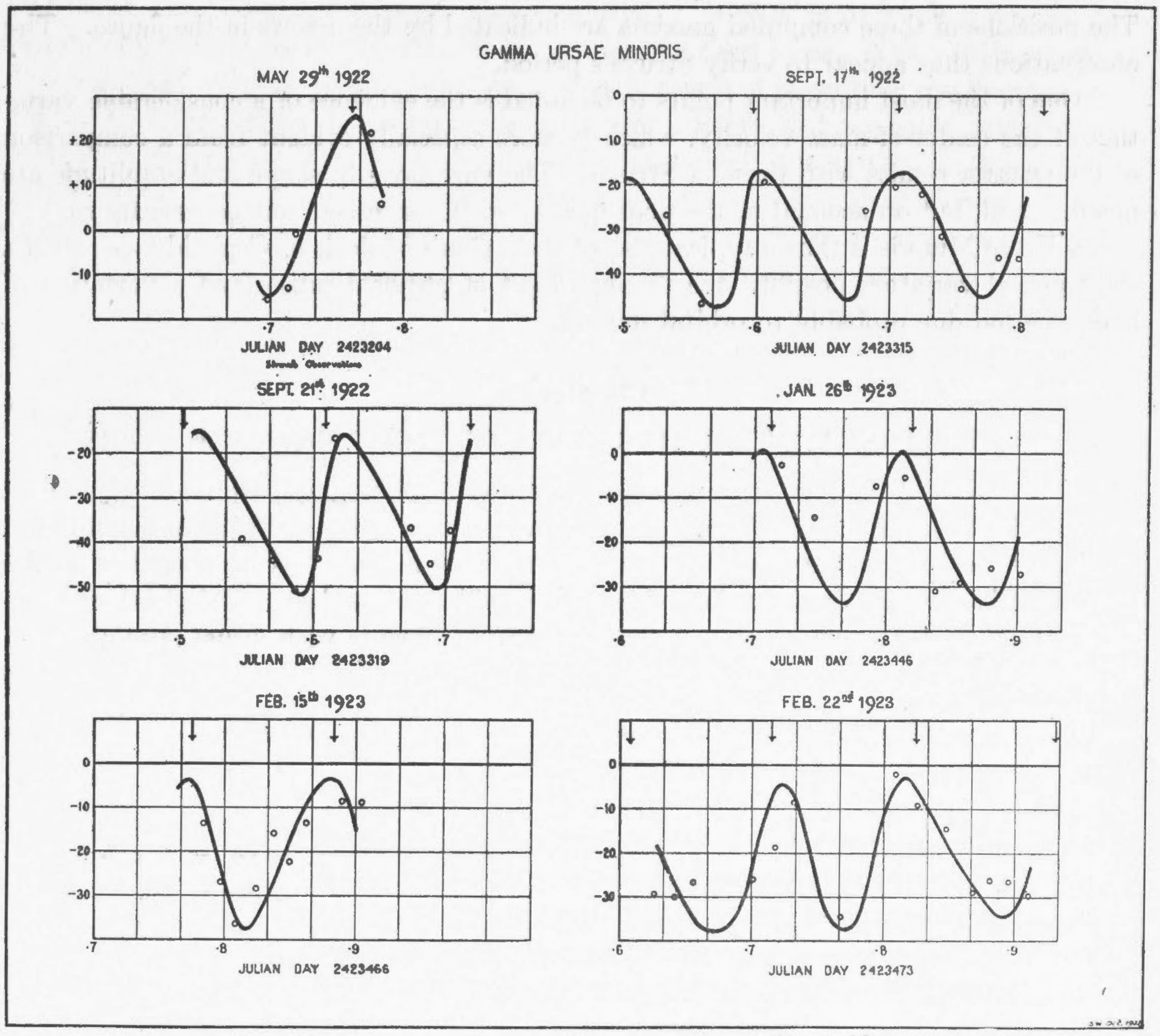

FIGURE 14. RADIAL VELOCITY CURVES OF GAMMA URSAE MINORIS

The above observations are shown in graphic form in Fig. 14. Struve's observations give a maximum at J.D. 2423204·762, and his formula, Max. = J.D. 2423204·762 + 0^d·108499 E, gives the following maxima on the dates of the Ottawa observations:—

J.D. 2423315·488
·597
·705
·813
319·501
·609
·718

J.D. 2423446·712
·820
466·774
·883
473·607
·716
·824

The positions of these computed maxima are indicated by the arrows in the figure. The observations thus appear to verify Struve's period.

One of the most important points to be noted is the evidence of a considerable variation of the center-of-mass velocity, which is more especially evident from a comparison of the Ottawa results with those of Struve. The variations in shape and amplitude are possibly real, but on account of the poor quality of the spectrum not necessarily so.

γ Ursae Minoris is thus a typical star of the β Canis Majoris or Cepheid type, with a short-period variation possibly due to some kind of physical cause, and a variation of longer period due probably to orbital motion.

Chapter IX

THE PHOTOGRAPHIC LIGHT-CURVES OF EIGHT VARIABLE STARS

In the present chapter is introduced a method of photographic photometry which has the advantage of being rapid and at the same time, for a certain range of magnitudes, as accurate as any of the best methods in use. This method has been applied to the variable stars given in the following table. They will be treated separately in the following paragraphs, with comparisons of the photographic results with visual ones obtained elsewhere.

VARIABLE STARS OBSERVED

Star	α 1855·0		δ 1855·0	
	h	m	°	′
X Cygni	20	37·7	+35	4·0
SZ Cygni	20	28·2	+46	6·5
TX Cygni	20	54·8	+42	2·0
UY Cygni	20	50·4	+29	52·6
VX Cygni	20	51·9	+39	37·2
VY Cygni	20	58·7	+39	23·7
WZ Cygni	20	47·6	+38	16·9
XZ Cygni	19	29·5	+56	4·6

A large number of plates of each variable has been secured with a short-focus camera, (lens two inches aperture and thirteen inches focal length). The plates, which are cabinet size (6½ by 4¾ inches), cover a wide area of the sky, and have the advantage both of having a large number of star images per unit area and of showing a large range in the diameters of these star images.

The image of the variable star is compared with the images of a number of stars whose photographic magnitudes have been accurately determined; often some twenty to forty stars are used, which on account of the small scale of the photograph can all be found in the field of view of the microscope under which the plate is examined. One or two star images on each plate are chosen having as nearly as possible the same intensity as the variable, that is, covering the same area, and with the same shape and density; often two stars are used, one a trifle brighter and the other a trifle fainter than the variable, so that a careful interpolation gives the true magnitude of the latter. In short, this is nothing else but Argelander's method, applied by him directly to the determination of photometric magnitudes, but here applied to the photographic plate. The latter, offering a field which can be examined much more comfortably, is capable of yielding results of considerable accuracy, especially with the advantage of a short focus. With a little experience the method is rapid; eighty to one hundred estimations of magnitude can be made in a day; if the magnitudes of the comparison stars are determined correctly, and if the differences of magnitude between them are sufficiently small, the method is practically free from systematic errors. For the brighter stars, down to about the fifth magnitude, the method is less accurate, because few comparison stars are to be found within the field of the microscope, but for stars between the eighth and twelfth magnitudes it attains a very high degree of accuracy, especially for variables which like the Cepheids are located in the Milky Way, where surrounding stars of the same order of magnitude are abundant. For stars fainter than these a camera of greater aperture and longer focal length would be of advantage.

The greatest difficulty in using this method is in determining accurately the photographic magnitudes of the comparison stars, since there is no reliable durchmusterung of the northern sky giving the photographic magnitudes of the stars. Kapteyn has given the photographic magnitudes of a considerable number of southern stars, down to about the tenth magnitude, in the Cape Photographic Durchmusterung; he also gives the photographic magnitudes (on the international scale) of stars to a considerable degree of faintness in his 108 selected areas of the northern hemisphere.[1] A certain number of astrographic catalogues (of the Carte Photographique du Ciel) furnish photographic magnitudes, or the diameters of star images with the necessary constants to compute these magnitudes; these are, however, far from covering the whole sky, and are also, in some cases, scarcely accurate enough for use in the investigation of variable stars. The new Henry Draper Catalogue of Harvard gives photographic magnitudes for all the stars it contains; for the stars south of −19° declination the magnitudes given are derived (applying certain corrections to reduce to the international scale) from the Cape Photographic Durchmusterung, while for stars north of that declination they are in most cases derived by adding to the photometric magnitude the colour-index corresponding to the spectral class. The magnitudes so obtained are, however, far from reliable, being often in error by several tenths of a magnitude and sometimes by even a whole magnitude. The most reliable sources of photographic magnitudes on the international scale are the North Polar Sequence[2] and its associated Harvard Standard Regions, as well as Miss Leavitt's Standards of Magnitude for the Astrographic Catalogue.[3]

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[1] H. A., Vol. 101.

[2] H. A., Vol. 71.

[3] H. A., Vol. 85, No. 1.

To determine the photographic magnitudes, on the international scale, of stars in the field of a variable, the practice was followed of photographing successively on the same plate, on a clear moonless night, both the field of the variable and one of Miss Leavitt's fields, taking care that the two fields were approximately at the same altitude. The photographs were made with the six-inch Brashear doublet camera, which gives excellent definition; with careful guiding it was found possible to obtain sharply defined and perfectly round images. The diameters of the star images of the two fields were then measured accurately with a micrometer microscope, and, using the diameters of the star-images of Miss Leavitt's field as abscissae and her magnitudes as ordinates, a curve connecting the magnitudes with the diameters of the star-images was determined. This curve was used to deduce the photographic magnitudes of the stars in the field of the variable from the measured diameters of their images. The number of stars in each field being quite large the use of such a curve was found simpler and to some extent more accurate than the use of any of the standard formulae connecting the magnitudes and diameters of the star images.

The plates taken with the short-focus camera mentioned above required of course somewhat prolonged exposures, the aperture-ratio being slightly less than *f*/6; they were obtained, however, in the course of the regular programme of spectrographic observations; the small camera was mounted on the tube of the equatorial, near the objective, and plates exposed as opportunity offered during the progress of a series of spectrograms of the star in question.

The several variables mentioned above are discussed individually in the following pages.

X Cygni

The variability of X Cygni was discovered by Chandler in 1886.[1] It is a typical Cepheid of class Go whose photometric elements are given by the formula:—

$$\text{Max.} = \text{J. D. } 2410190\cdot678 + 16^{d}\cdot38543 \text{ E.}$$

Visual light-curves have been given by Pickering[2] and by Luizet[3]; Wilkens obtained the photographic light-curve by the extra-focal method,[4] while a comparison of the photographic and photo-visual light-curves is afforded by the results of Jordan.[5] According to Luizet the luminosities at maximum and at minimum show a certain amount of variation, small but real; an investigation of such variations with a view to determining their laws would be of interest. It may be noted that variations of this nature have been suspected in the case of δ Cephei by R. H. Curtiss, and have been recognized in other Cepheids, viz.—RR Lyrae and stars of the β Canis Majoris type. The investigation of such variations, which are indicative of an effect due to a relatively long-period orbital motion, is possibly important in relation to the formation of an adequate theory of Cepheid variation.

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[1] A. J., Vol. 7, p. 32.

[2] H. A., Vol. 46, p. 156.

[3] A. N., Vol. 193, p. 85.

[4] A. N., Vol. 172, p. 325.

[5] A. J., Vol. 50, p. 191.

Luizet secured 795 observations of X Cygni between 1898 and 1912 by the use of field glasses and Argelander's method. His observations, combined into normal places and transformed into visual magnitudes (they were given originally in a system of arbitrary grades, making comparisons with other observations difficult) are given in the following table:—

LUIZET'S VISUAL MAGNITUDES OF X CYGNI

Phase	Mag.	Phase	Mag.	Phase	Mag.
d		d		d	
0·327	6·40	6·182	7·00	11·430	7·15
0·763	6·40	6·515	6·97	11·880	7·11
1·274	6·54	6·934	7·04	12·319	7·03
1·762	6·60	7·330	7·04	12·689	6·96
2·182	6·59	7·636	7·10	13·013	7·01
2·635	6·64	8·032	7·13	13·333	6·99
2·962	6·64	8·454	7·16	13·732	6·89
3·373	6·70	8·808	7·15	14·144	6·78
3·857	6·74	9·156	7·21	14·553	6·56
4·174	6·80	9·528	7·23	14·942	6·47
4·722	6·86	9·913	7·23	15·468	6·39
5·055	6·83	10·507	7·23	15·888	6·36
5·470	6·83	11·038	7·21	16·248	6·43
5·847	6·88				

The following table gives the photographic magnitudes of the comparison stars in the field of X Cygni, as measured from our plates by the method outlined above:—

FIELD OF X CYGNI

B. D.	α 1855·0	δ 1855·0	Phtg. Mag.	Spect. Class	Phtg. Mag. in Draper Catal.
	h m	° '			
4109.............	20 34·2	+34 56	9·31		
4111.............	20 34·3	+34 52	8·74	Ko	7·86
4114.............	20 35·2	+34 31	8·40	F2	7·51
4217.............	20 35·9	+35 13	8·23	B9	8·2
4218.............	20 35·9	+35 24	8·90	K2	8·59
4219.............	20 36·0	+35 2	8·69	Go	7·98
4220.............	20 36·1	+35 23	9·80		
4221.............	20 36·2	+35 14	9·39	F5	8·8
4224.............	20 36·6	+35 37	9·39	F8	9·3
4127.............	20 36·7	+34 56	6·50	B3	6·33
4229.............	20 37·1	+35 52	8·48		
4231.............	20 37·4	+35 14	8·96	Ao	8·8
4232.............	20 37·6	+35 13	8·06	Ao	8·6
4004.............	20 37·6	+33 21	7·10	Ao	7·8
4234.............	20 37·7	+35 4	Var.	Gop	
4237.............	20 37·9	+35 4	9·63		
4009.............	20 38·1	+33 33	7·95	Ao	8·6
4240.............	20 38·5	+35 1	9·39		
4022.............	20 41·0	+33 51	8·82	A5	9·3
4268.............	20 41·8	+35 45	8·31	A3	8·3

In the last column have been added the photographic magnitudes of many of the stars as given in the Draper Catalogue, where they have been obtained by adding the colour-index corresponding to the given spectral class to the visual magnitude. It is to be noted that the discrepancies are in many cases large; it would appear, as has already been pointed out by many astronomers, among them Prof. H. N. Russell, that the law connecting visual magnitude, spectral class and photographic magnitude is only approximate, and in some cases far from true. In particular cases a good deal no doubt depends on whether the star is a giant or a dwarf, and it might be well, for the same spectral class, to look for a relation between visual magnitude, photographic magnitude and density.

The photographic magnitudes obtained here for X Cygni are given in the following table:—

PHOTOGRAPHIC MAGNITUDES OF X CYGNI

Date	Julian Day	Phase	Phtg. Mag.	Wt.
1920		d		
Aug. 19	2422556·7	11·4	8·75	2
20	557·6	12·3	8·23	2
1922				
July 13	2423249·7	16·2	6·75	1
19	255·7	5·8	8·60	2
20	256·7	6·8	8·69	2
20	256·8	6·9	8·69	2
24	260·7	10·8	8·75	2
24	260·8	10·9	8·80	2
25	261·7	11·8	8·48	2
25	261·8	11·9	8·69	2
26	262·7	12·8	8·23	2
28	264·7	14·8	7·10	2
28	264·7	14·8	6·80	1
31	267·8	1·5	7·05	1
Aug. 18	285·7	3·1	7·55	1
18	285·7	3·1	7·60	1
23	290·7	8·1	8·75	2
25	292·6	10·0	8·85	2

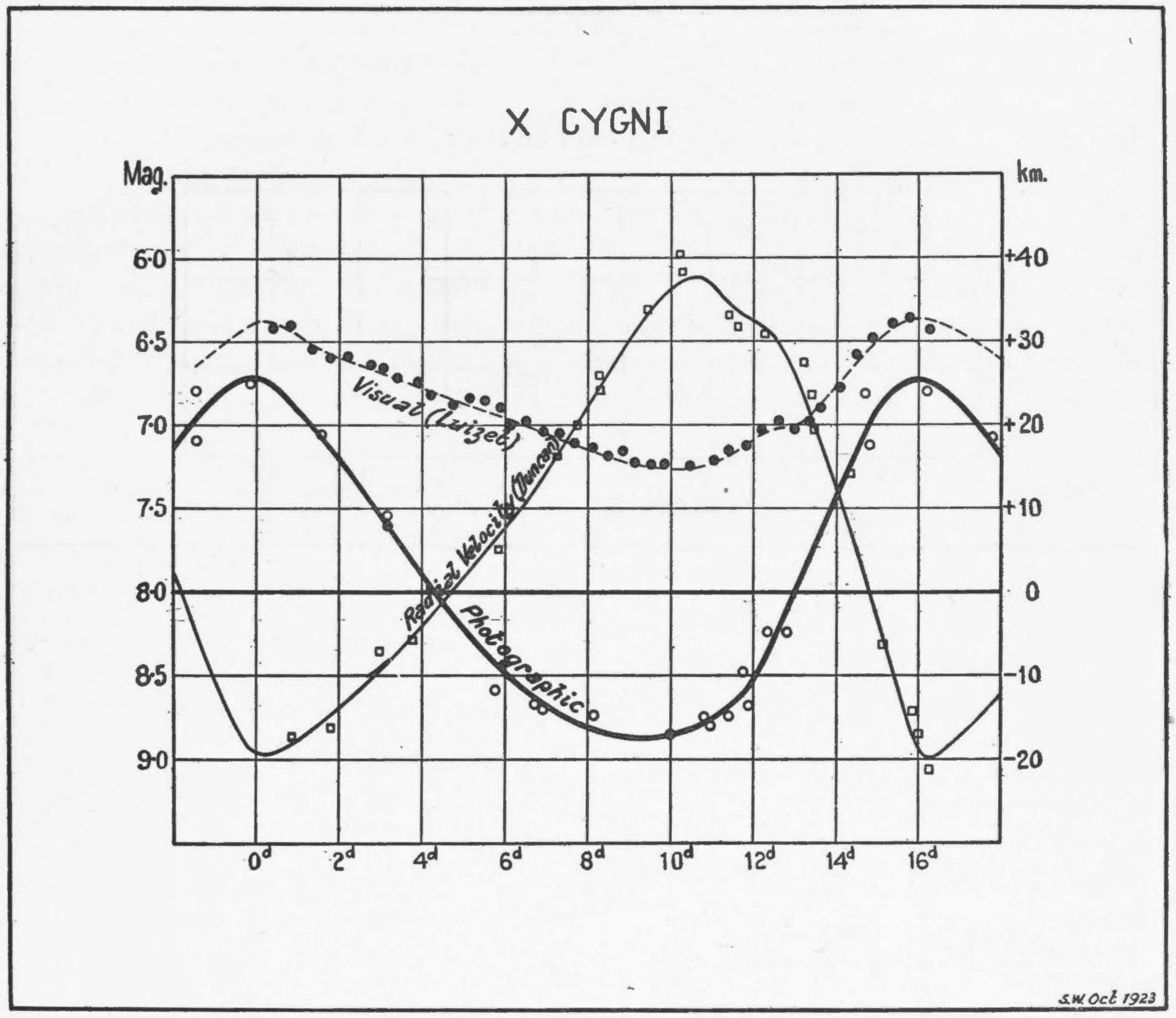

FIGURE 15. VARIATIONS OF X CYGNI

In Fig. 15 the broken line shows the visual observations of Luizet, the heavy continuous line the Ottawa photographic curve. For the sake of comparison there is shown on the third curve a plot of the radial velocities as obtained by Duncan[1] from plates taken with the 60-inch Mt. Wilson reflector. A small hump just succeeding the 12^d epoch is suggested by the curves. The large difference in range between the visual and photographic curves is perhaps the outstanding effect.

SZ Cygni

SZ Cygni was found to be variable by Williams[2] in 1900. It has been followed visually by many observers and two visual light-curves have been published, one by Lau,[4] the other by Luizet.[3] Luizet's results, transformed from his arbitrary grades into magnitudes, are shown in Fig. 16, together with the photographic light-curve as determined here.

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[1] Contributions from the Mt. Wilson Observatory, Vol. 9, p. 396.
[2] A. N., Vol. 152, p. 77.
[3] Bul. Soc. Astr. France, 1907, p. 95.
[4] Bul. Astr., Vol. 25, p. 212.

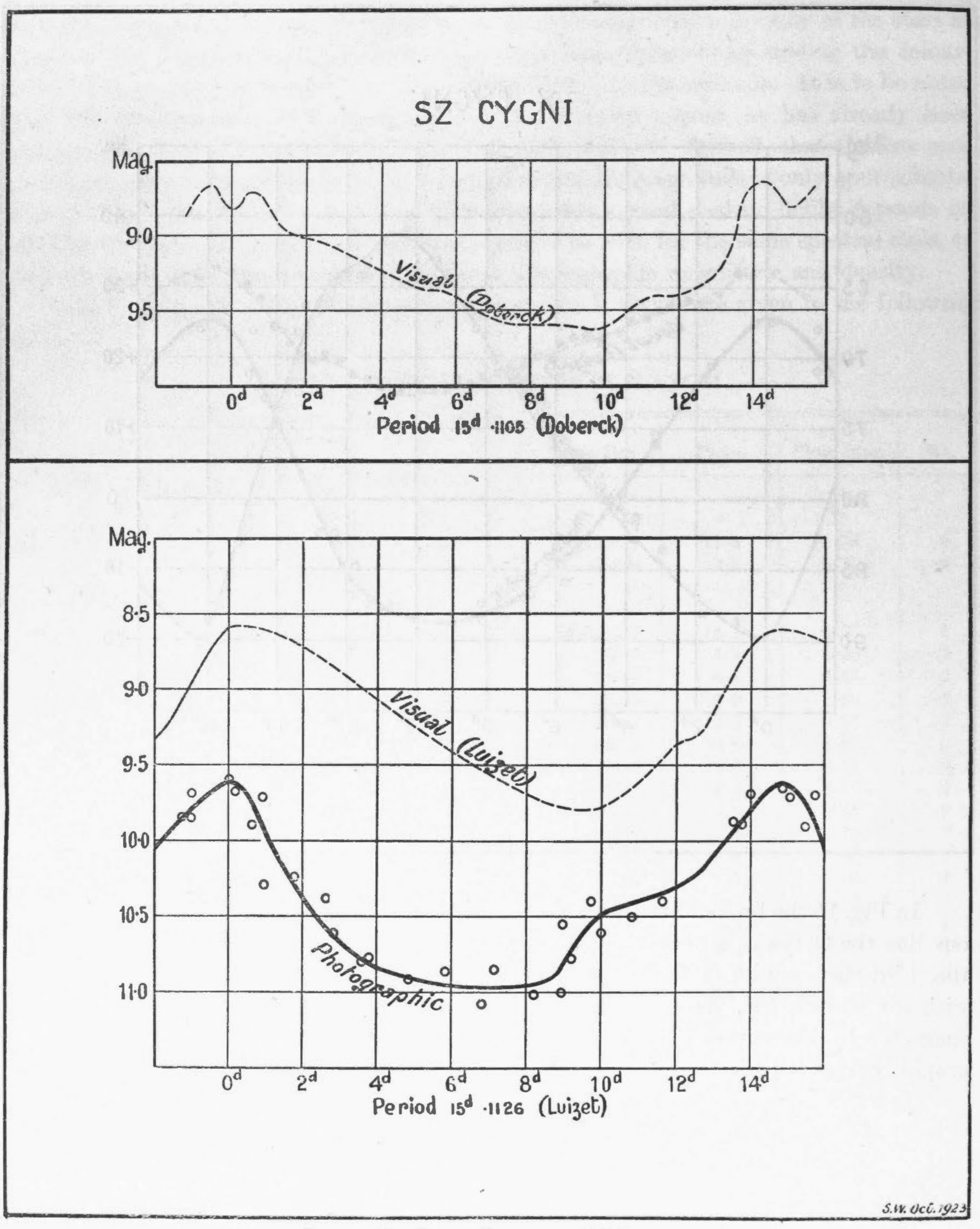

FIGURE 16. LIGHT CURVES OF SZ CYGNI

Luizet draws attention to the probability of the reality of the marked hump shown on the ascending branch. Comparison of the photographic light-curve with the visual one of Luizet shows that the average colour-index corresponds very well to the given spectral class, which is given as Ko. In other respects, however, there are marked differences;

the photographic curve shows a more decided hump on the ascending branch and a much more rapid descent immediately after maximum than does Luizet's curve. Another remarkable feature is that the photographic range of magnitude is hardly larger than the visual. It becomes an interesting question, which may repay further study, whether this is by virtue of the possible fact that the ratio of these ranges is a function of the spectral class, or whether it is an individual peculiarity of this particular star. The images, it may be noted, were in most cases located not far from the edge of the plate, but it seems unlikely that this would have had a very marked effect on the determination of the photographic magnitudes, especially since the comparison stars were close to the variable. The photographic magnitudes of the comparison stars as obtained here, and the resulting magnitudes of SZ Cygni, are given in the following tables:—

FIELD OF SZ CYGNI

B. D.	α 1855·0		δ 1855·0		Phtg. Mag.	Remarks
	h	m	°	′		
2956	20	26·1	+46	18	9·55	
2958	20	26·3	+46	17	8·10	
2960	20	26·5	+46	14	8·90	
2961	20	26·6	+46	24	9·42	
2965	20	27·9	+46	5	10·77	Var. suspected.
	20	28·1	+46	21	10·88	
2966	20	28·2	+46	6	Var.	
2967	20	28·4	+46	22	9·87	
2968	20	28·5	+46	23	9·35	
	20	28·5	+46	37	10·34	
2969	20	28·6	+46	40	8·80	
2971	20	28·8	+46	14	10·60	
	20	28·8	+46	26	10·76	
2975	20	29·0	+46	30	9·14	
2976	20	29·1	+46	26	10·31	
	20	29·1	+45	57	10·85	
	20	29·2	+45	56	11·12	
	20	29·2	+46	2	10·53	
2978	20	29·2	+46	14	9·49	
	20	29·3	+46	20	11·73	
	20	29·3	+46	26	12·06	
	20	29·4	+46	4	12·00	
2982	20	30·0	+46	20	7·90	
3220	20	30·0	+45	54	10·22	
	20	30·2	+46	0	11·48	
2984	20	30·3	+46	13	10·40	
	20	30·3	+46	2	11·01	
	20	30·4	+46	16	10·53	
	20	30·5	+46	20	10·57	
	20	30·7	+45	53	10·75	Var. suspected.
	20	30·8	+46	32	9·75	
3226	20	31·2	+45	50	9·66	
3227	20	31·2	+45	59	9·87	
	20	31·4	+46	9	10·60	
2988	20	31·4	+46	15	9·60	
3228	20	31·5	+45	47	9·79	
	20	31·6	+46	0	10·88	

PHOTOGRAPHIC MAGNITUDES OF SZ CYGNI

Date	Julian Day	Phase	Phtg. Mag.	Wt.	Date	Julian Day	Phase	Phtg. Mag.	Wt.
1920		d			1920		d		
July 20...	2422526·6	9·2	10·76	2	Sept. 20...	2422588·6	10·8	10·51	1
Aug. 20...	557·6	10·0	10·60	1	23...	591·5	13·7	9·87	1
24...	561·6	14·0	9·70	1	Oct. 6...	604·6	11·7	10·40	2
25...	562·6	15·0	9·60	2	10...	608·6	0·6	9·90	2
25...	562·8	0·1	9·66	2	11...	609·7	1·7	10·26	1
26...	563·6	0·9	10·30	1	12...	610·5	2·5	10·40	3
Sept. 1...	569 8	7·1	10·88	1	Dec. 2...	661·6	8·2	11·01	1
3...	571·6	8·9	11·00	1	1922				
10...	578·6	0·8	9·70	1	July 20...	2423256·7	13·9	9·87	2
13...	581·5	3·7	10·80	1	24...	260·7	2·8	10·60	1
15...	583·6	5·8	10·88	1	25...	261·7	3·8	10·80	3
18...	586·8	9·0	10·55	2	26...	262·7	4·8	10·94	1
19...	587·6	9·8	10·40	3	28...	264·7	6·8	11·10	1

The above magnitudes were those used in plotting the photographic curve. The phases were computed from the formula:—

$$\text{Max.} = \text{J. D. } 2415097{\cdot}08 + 15^{d}{\cdot}1126 \text{ E.}$$

Doberck[1] in 1920 gave the following slightly different elements based on his visual observations:—

$$\text{Max.} = \text{J. D. } 2421836{\cdot}37 + 15^{d}{\cdot}1105 \text{ E.}$$

As may be seen from Fig. 16, he obtains a close double maximum and a hump at phase about 11^{d} very similar to the one indicated by the photographic observations. The use of his formula would increase the phases of the Ottawa observations by about a day; the latter therefore agree better with the period $15^{d}{\cdot}1126$. The existence of a double maximum is doubtful, while the existence of a hump on the ascending branch appears probable. If we reduce Doberck's observations with the period $15^{d}{\cdot}1126$ there appear to be indications of a slow displacement of the hump. In 1917 Luizet places it at about phase $12^{d}{\cdot}2$; several years later Doberck's observations would place it at about $10^{d}{\cdot}2$, while our observations place it at about $9^{d}{\cdot}6$. If this displacement should be confirmed it would constitute a new and highly interesting problem to elucidate, for which a long series of observations would be required.

TX Cygni

TX Cygni was discovered to be variable by Williams in 1900.[2] It was followed visually by several observers, among whom were Hartwig,[3], Yendell,[4] van der Bilt and v. Zeipel.[5] No light curve, however, has been published; the table given below of v. Zeipel's observations, made with a Zöllner photometer at Upsala Observatory, has been formed by computing the phases from the formula:—

$$\text{Max.} = \text{J. D. } 2417010{\cdot}5 + 14^{d}{\cdot}71 \text{ E.}$$

which is found in Hartwig's ephemerides.

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[1] A. J., Vol. 32, p. 164.
[2] A. N., Vol. 154, p. 147.
[3] Viertaljahrschrift, Vol. 36, p. 269.
[4] A. J., No. 563.
[5] A. N., Vol. 177, p. 376.

PHOTOMETRIC OBSERVATIONS OF TX CYGNI BY v. ZEIPEL

Date	Julian Day	Phase	Magnitude	Remarks
1907		d		
Jan. 16	2417592·245	8·055	10·09	
17	593·243	9·053	9·98	
Sept. 13	832·374	12·824	9·80	
15	834·383	0·123	8·83	
17	836·373	2·113	9·18	
21	840·337	6·077	9·71	
Oct. 16	865·335	1·655	9·33	
Nov. 29	909·367	1·557	9·27	Poor.
30	910·301	2·491	9·25	
Dec. 29	939·285	2·055	9·33	
30	940·279	3·049	9·19	
31	941·294	4·064	9·42	
1908				
Jan. 1	942·319	5·089	9·67	
7	948·297	11·067	9·95	
13	954·266	2·326	9·25	
19	960·242	8·302	9·95	
24	965·255	13·315	9·67	
27	968·266	1·616	9·29	
30	971·273	4·623	9·42	
Feb. 4	976·194	9·544	9·97	Poor.

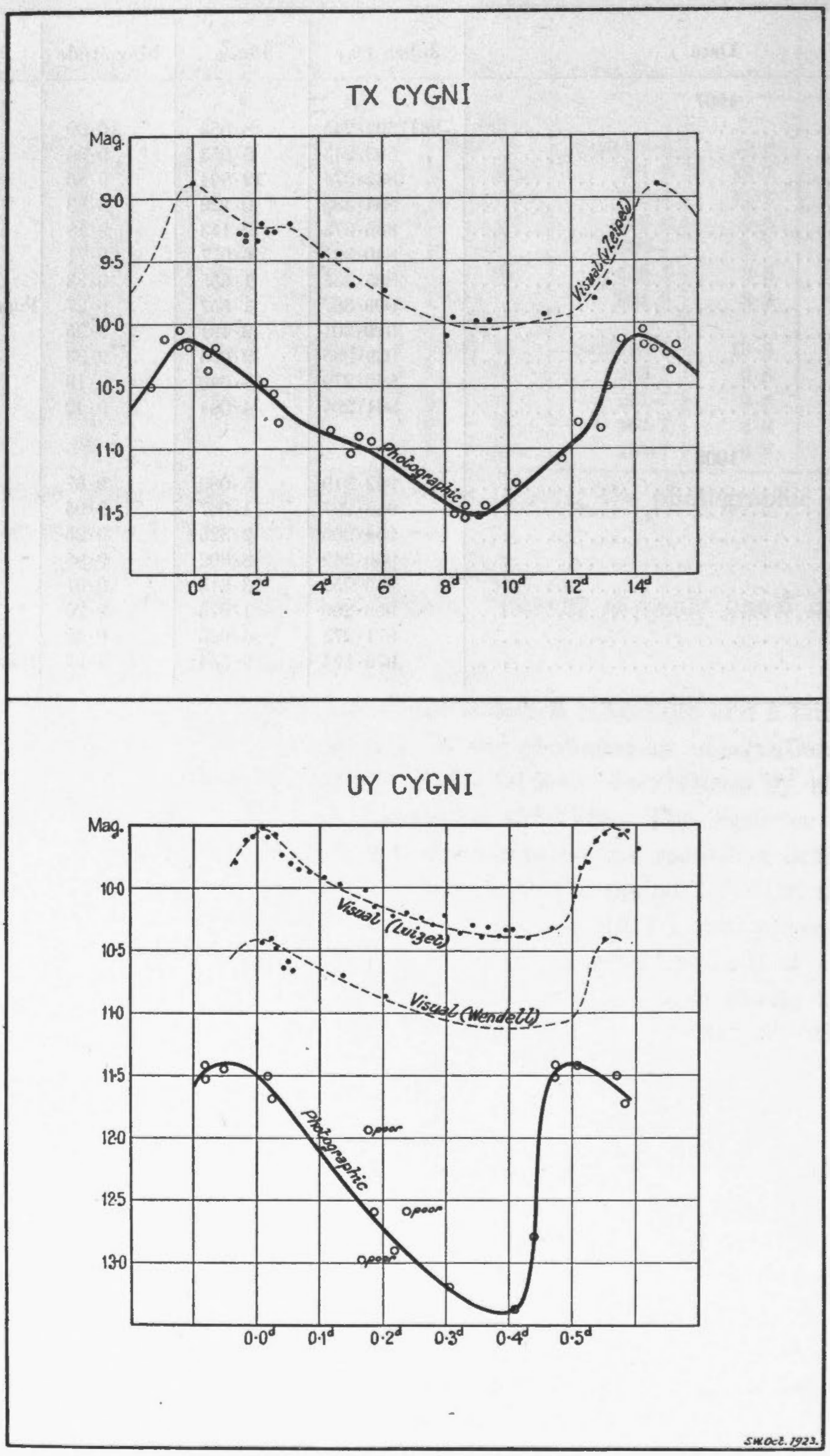

FIGURE 17. LIGHT CURVES OF TX CYGNI AND UY CYGNI

These observations give the visual light-curve shown in Fig. 17. The photographic magnitudes of the comparison stars as determined here with the resulting photographic magnitudes of TX Cygni are given in the two following tables:—

FIELD OF TX CYGNI

B. D.	α 1855·0	δ 1855·0	Phtg. Mag	B. D.	α 1855·0	δ 1855·0	Phtg. Mag.
	h m	° ′			h m	° ′	
3918	20 51·9	+42 1	10·02	3964	20 55·4	+41 59	10·59
	52·0	+42 4	10·78		55·6	+41 32	11·14
3944	52·2	+41 53	9·01		55·6	+42 0	12·02
3921	52·6	+42 4	10·15		55·7	+42 35	10·93
3950	53·2	+41 54	9·43		55·9	+42 6	11·00
3926	53·4	+42 9	10·23	3941	56·1	+42 11	10·93
	53·5	+42 15	11·22	3940	56·1	+42 27	10·52
3954	53·6	+41 49	10·38		56·1	+41 58	11·54
3955	53·9	+41 30	9·84		56·1	+41 29	11·49
3929	53·9	+42 22	10·48		56·3	+41 46	10·68
3930	53·9	+42 23	10·63		56·3	+42 18	11·13
3959	54·2	+41 28	10·49		56·4	+42 14	10·80
	54·2	+41 37	11·08	3942	56·6	+42 29	10·01
	54·3	+41 40	10·97	3944	56·7	+42 17	9·61
	54·6	+42 25	11·38	3943	56·7	+42 7	10·10
3933	54·6	+42 33	10·82		56·8	+42 15	11·00
	54·7	+42 33	11·58	3967	56·9	+42 0	10·20
	54·8	+42 2	Var.	3968	57·0	+41 56	11·05
3963	55·1	+41 37	9·09	3948	57·0	+42 2	10·08

PHOTOGRAPHIC MAGNITUDES OF TX CYGNI

Date	Julian Day	Phase	Phtg. Mag.	Wt.	Date	Julian Day	Phase	Phtg. Mag.	Wt.
1920		d			1922		d		
July 20	2422526·6	14·6	10·18	2	July 20	2423256·8	9·3	11·47	1
Aug. 19	556·7	0·5	10·22	3	24	260·7	13·2	10·50	1
Oct. 6	604·6	4·3	10·87	1	25	261·7	14·2	10·05	3
7	605·6	5·3	10·91	1	25	261·8	14·3	10·15	3
10	608·6	8·3	11·50	1	26	262·7	0·5	10·38	2
12	610·5	10·2	11·30	1	28	264·7	2·5	10·57	2
1921					31	267·8	5·6	11·43	2
Aug. 4	906·7	12·2	10·80	3	Aug. 18	285·7	8·7	11·46	1
5	907·6	13·1	10·84	2	18	285·7	8·7	11·51	1
22	924·6	0·7	10·19	1	23	290·7	13·7	10·11	3
24	926·6	2·7	10·80	1	29	296·7	5·0	11·02	2
1922					1923				
July 13	2423249·7	2·2	10·48	1	Aug. 9	641·7	11·7	11·05	5
20	256·7	9·2	11·51	1					

These magnitudes give the photographic curve in Fig. 17. It will be seen that the maximum is slightly displaced, indicating probably a slight correction ($-0^{d}\cdot 0014$) to the assumed period $14^{d}\cdot 71$; the more exact period would therefore be $14^{d}\cdot 7086$. The mean difference between the visual and photographic magnitudes indicates that the star belongs probably to spectral class K. Here again, as in the case of SZ Cygni, the difference between the amplitudes of the visual and photographic light-curves is not large, the latter being a trifle greater. The contrast between these two and X Cygni is marked; the three stars have periods of the same order of magnitude; it seems difficult to believe however, that X and TX can have about the same absolute magnitude. Another remarkable feature is the difference in shape of the ascending branches of the visual and photographic curves, a difference very similar to that exhibited between the descending branches of SZ Cygni.

UY Cygni

UY Cygni was found to be variable by Williams in 1902,[1] and recognized by him to be of very short period. Visual observations have been published by Williams,[2] Hartwig,[3] Graff,[4] Luizet[5] and Wendell.[6] The normal places corresponding to the observations of Luizet are given in the following table:—

LUIZET'S VISUAL MAGNITUDES OF UY CYGNI

Phase	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase	Magnitude
d		d		d		d	
0·0070	9·50	0·1354	9·96	0·2978	10·20	0·4280	10·39
0·0156	9·59	0·1706	10·00	0·3216	10·35	0·4686	10·33
0·0276	9·57	0·1922	10·11	0·3406	10·28	0·5004	10·08
0·0396	9·74	0·2142	10·20	0·3548	10·36	0·5148	9·83
0·0512	9·78	0·2364	10·17	0·3680	10·28	0·5296	9·77
0·0666	9·83	0·2600	10·21	0·3820	10·35	0·5414	9·60
0·0854	9·84	0·2774	10·28	0·3950	10·31	0·5522	9·56
0·1094	9·90			0·4028	10·30		

The phases in heliocentric time have been computed from the formula of Williams:—

$$\text{Max.} = \text{J. D. } 2415346{\cdot}3933 + 0^{d}{\cdot}5607103\ \text{E.}$$

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[1] A. N., Vol. 158, p. 45.

[2] Monthly Notices, Vol. 63, p. 304, and Vol. 65, p. 586.

[3] Vierteljahrschrift, Vol. 37, p. 284; Vol. 38, p. 240.

[4] A. N., Vol. 197, p. 253.

[5] Bul. Astr., Vol. 24, p. 342.

[6] H. A., Vol. 69, p. 126.

The photographic magnitudes of the comparison stars, as obtained here, are given in the following table:—

FIELD OF UY CYGNI

B. D.	α 1855·0	δ 1855·0	Phtg. Mag.	B. D.	α 1855·0	δ 1855·0	Phtg. Mag.
	h m	° ′			h m	° ′	
4213........	20 45·9	+29 31	9·40	4231........	20 48·5	+30 13	10·70
	20 46·2	+29 29	12·20	4232........	20 48·5	+30 19	10·84
	20 46·3	+30 5	10·10	4230........	20 48·6	+29 16	10·45
4216........	20 46·4	+29 39	11·00		20 48·8	+29 24	13·0
4217........	20 46·4	+29 49	10·10		20 49·0	+29 33	13·4
4218........	20 46·4	+29 42	9·65		20 49·1	+29 33	11·12
4219........	20 46·6	+29 34	10·50		20 49·1	+29 47	11·43
4221........	20 47·0	+29 7	7·79	4236........	20 49·3	+30 11	11·40
	20 47·0	+29 57	12·7	4234........	20 49·3	+29 13	8·70
	20 47·2	+30 16	11·40		20 49·5	+29 34	12·3
	20 47·3	+30 7	11·77	4238........	20 49·6	+29 45	10·79
4218........	20 47·5	+30 24	7·87		20 49·6	+29 40	13·3
	20 47·5	+30 19	11·30	4240........	20 50·3	+29 44	9·18
	20 47·6	+29 12	11·40		20 50·4	+29 53	Var.
	20 47·6	+30 23	10·30		20 50·6	+29 44	12·6
	20 47·7	+29 59	13·3	4244........	20 50·7	+29 54	11·94
	20 47·8	+30 1	11·77	4244........	20 50·7	+30 8	9·55
4223........	20 48·0	+30 13	10·40	4247........	20 50·9	+29 59	11·20
4226........	20 48·1	+29 9	10·69	4248........	20 50·9	+29 53	10·75
4227........	20 48·1	+29 10	9·64	4251........	20 51·5	+29 58	10·55
4228........	20 48·3	+29 7	10·31	4255........	20 52·4	+29 37	9·91
	20 48·4	+29 39	13·4	4256........	20 52·4	+29 50	9·99
	20 48·4	+29 36	13·1				

The next table gives the resulting photographic magnitudes of UY Cygni. The dates given are heliocentric, and, as there were a few fairly long exposures, have been corrected also for the error introduced by the length of exposure. To determine these corrections a preliminary curve was plotted with the mean epochs of the exposures as abscissae. On this curve the area comprised between the axis of X, the vertical lines passing through the abscissae corresponding to beginning and end of exposure, and the curve was divided into equal areas by a vertical line, and the time indicated on the axis of X by this vertical line was taken as the true time when the star had the measured photographic magnitude.

PHOTOGRAPHIC MAGNITUDES OF UY CYGNI

Date	Corrected Helio-centric Julian Day	Phase	Phtg. Mag.	Date	Corrected Helio-centric Julian Day	Phase	Phtg. Mag.
1922		d		1922		d	
July 20.....	2423256·708	0·374	13·40	July 28.....	2423264·796	0 052	11·67
20.....	256·801	0·468	11·50	31.....	267·781	0·233	12·90
24.....	260·729	0·471	11·43	Aug. 18.....	285·671	0·181	11·94
25.....	261·698	0·318	13·20	18.....	285·741	0·251	12·60
25.....	261·796	0·416	12·80	23.....	290·751	0·214	12·60
26.....	262·682	0·181	13·00	29.....	296·741	0·036	11·50
28.....	264·698	0·515	11·40				

The phases have been computed from the formula of Williams given above. These results, as well as those of Luizet, are exhibited in graphical form in Fig. 17. It will be seen that according to the formula of Williams, which satisfied Luizet's observations exactly, the maximum is slightly displaced. If the photographic and visual maxima coincide it appears likely that the period has decreased a little. A shorter period, $0^d\cdot5607065$, would perhaps serve as a compromise, but it would not satisfy the early observations, covering the years 1902 to 1906, quite as well as $0^d\cdot5607103$, as shown in the following table (heliocentric time in Julian days).

PERIOD OF UY CYGNI

Author	Observed Maxima	Computed with $0^d\cdot5607103$	Computed with $0^d\cdot5607065$
Williams	2415984·483	·482	·477
Luizet	2417383·457	·454	·440
"	457·466	·468	·453
"	466·441	·439	·425
Ottawa	2423260·204	·258	·205

There is apparently a systematic error in the magnitudes of the comparison stars used by Luizet for his curve. The observations of Wendell, which are also shown in Fig. 17, although few, are probably on a better absolute scale of visual magnitude; they indicate a curve of shape similar to that obtained by Luizet. The comparison of Wendell's visual curve with our photographic one indicates a large colour-index, which would normally indicate that the star is of rather advanced spectral class K to M, a classification which would be rather remarkable in view of the short period. The difference in range between visual and photographic curves is also remarkable.

VX Cygni

VX Cygni was found to be variable by Williams in 1903.[2] It was observed visually by Hartwig, Seares,[3] van der Bilt, Beljawsky, Doberck[4] and Williams.[5] It is from the observations of the latter that the visual light curve shown in Fig. 18 is derived. The best elements, as given in Hartwig's ephemerides for 1922, are:—

$$\text{Max.} = \text{J. D. } 2414935\cdot0 + 20^d\cdot1306 \text{ E.}$$

([1] Note added in January, 1925, while going through press). VY Cygni has been recently found, at Mt. Wilson, to be of the spectral class Fo near maximum brightness; hence it is probably more advanced than Fo for any other brightness. Pub. Ast. Soc. Pac., Vol. 36, 1924, p. 139.

[2] A. N., Vol. 163, p. 301.

[3] Laws Bul., No. 10.

[4] Journal des observateurs, Vol. 3, pp. 105-108.

[5] A. N., Vol. 168, p. 25.

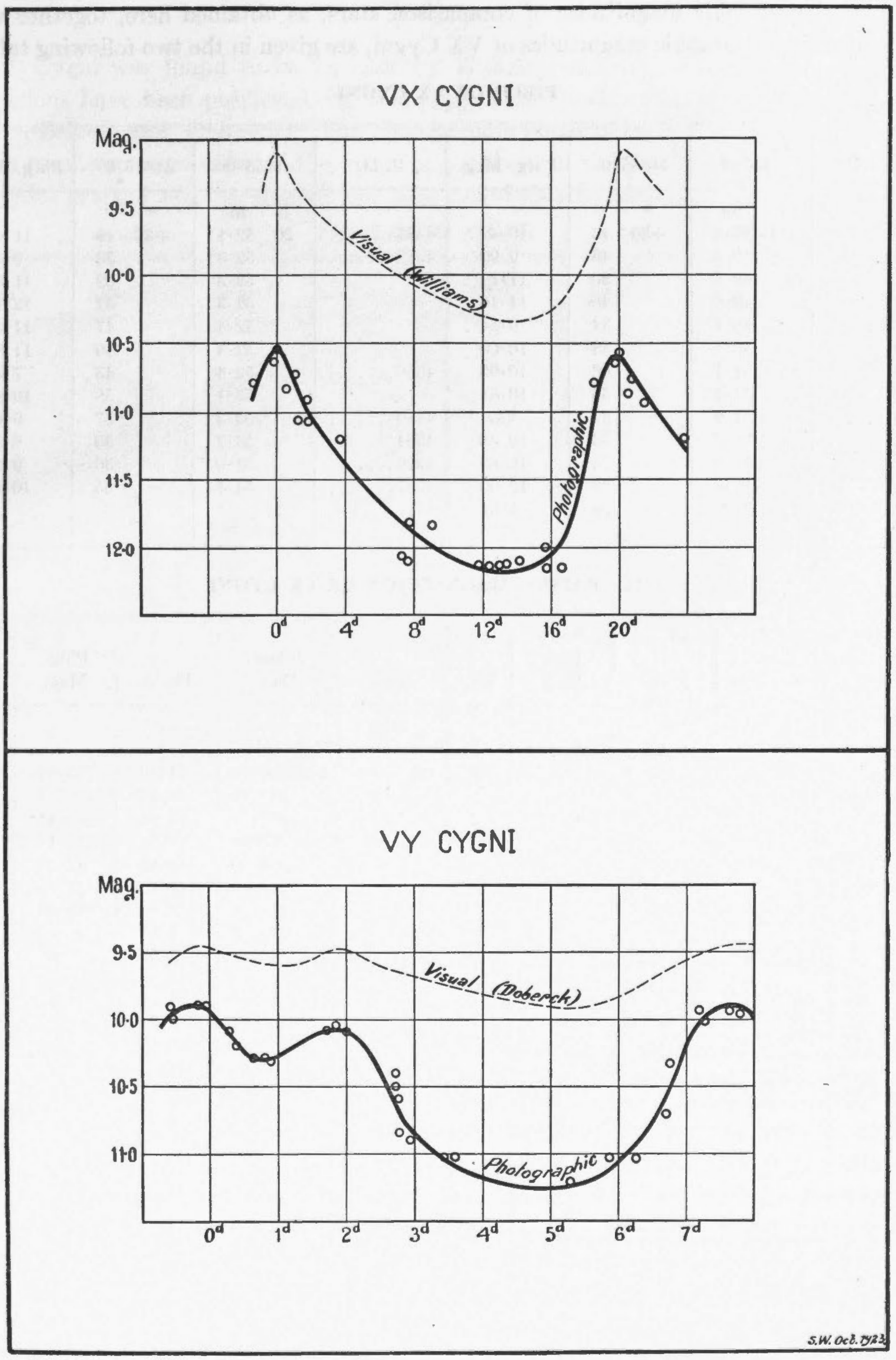

FIGURE 18. LIGHT CURVES OF VX CYGNI AND VY CYGNI

The photographic magnitudes of comparison stars, as obtained here, together with the resulting photographic magnitudes of VX Cygni, are given in the two following tables:

FIELD OF VX CYGNI

B. D.	α1855·0	δ1855·0	Phtg. Mag.	B. D.	α1855·0	δ1855·0	Phtg. Mag.
	h m	° ′			h m	° ′	
	20 49·4	+39 46	10·42	4381........	20 52·1	+39 49	11·06
4365........	49·5	48	9·90	4385........	52·3	33	9·47
	49·8	50	11·12		52·3	52	11·36
	49·8	48	11·16		52·3	52	12·15
4370........	50·1	34	9·64		52·4	47	11·31
4372........	50·1	48	10·00		52·4	49	11·80
4374........	51·1	46	10·92	4386........	52·4	43	7·37
4376........	51·4	56	10·61		53·1	58	10·83
4379........	51·9	37	Var.	4389........	53·3	57	9·67
4380........	51·9	53	10·48	4394........	53·7	30	8·30
	51·9	25	10·80	4396........	53·9	36	9·98
	52·0	39	12·00	4397........	54·1	55	10·50
4381........	52·1	29	9·61				

PHOTOGRAPHIC MAGNITUDES OF VX CYGNI

Date	Julian Day	Phase	Phtg. Mag.	Wt.	Date	Julian Day	Phase	Phtg. Mag.	Wt.
1920					1922				
		d					d		
Aug. 19...	2422556·65	12·28	12·15	1	July 24...	2423260·72	11·78	12·15	1
Oct. 6...	604·57	19·94	10·63	2	25...	261·69	12·75	12·15	2
10...	608·61	3·85	11·21	2	25...	261·79	12·85	12·15	1
Dec. 31...	611·68	6·92	10·84	3	26...	262·68	13·74	12·15	1
1921					28...	264·69	15·75	12·00	1
Aug. 4...	906·66	20·07	10·55	1	28...	264·79	15·85	12·15	1
5...	907·65	0·93	11·06	1	31...	267·78	18·84	10·80	2
26...	928·59	1·74	11·06	1	Aug. 23...	290·75	1·55	10·92	1
1922					29...	296·74	7·54	12·08	2
July 13...	2423249·66	0·72	10·73	2	1923				
20...	256·70	7·76	12·08	2	Sept. 16...	679·62	7·94	12·03	3
20...	256·80	7·86	11·85	2	17...	680·69	9·01	11·85	2

These observations give the photographic light-curve, shown in Fig. 18, which appears to be perfectly regular. The average colour-index corresponds to a fairly advanced spectral class, K, which is the class to which the star belongs. The photographic range is $1^m{\cdot}75$, while the visual is $1^m{\cdot}40$; here again the ratio of the two ranges is not far from unity.

VY Cygni

VY Cygni was found to be variable by Williams in 1903. Several long series of observations have been published by Williams,[1] Luizet[2] who has given a visual light-curve, v. Zeipel[3] and Doberck.[4] The photographic magnitudes of the comparison stars and the resulting magnitudes of the variable are given in the two following tables. The magnitudes marked with an asterisk are Miss Leavitt's standards.

FIELD OF VY CYGNI

B. D.	1855·0	1855·0	Phtg. Mag.	B. D.	α1855·0	δ1855·0	Phtg. Mag.
	h m	° ′			h m	° ′	
4400........	20 54·4	+39 41	7·10*	4424........	20 59·0	+39 12	11·00
4318........	56·3	+38 41	7·74*	4425........	59·2	+39 25	11·39*
4418........	56·9	+39 34	10·50	4426........	59·4	+39 34	10·88
4417........	57·4	+39 00	10·84	4428........	59·6	+39 51	9·29*
4418........	57·6	+39 25	9·23*		59·6	+39 31	11·06*
4419........	57·6	+39 29	10·58		59·6	+39 0	11·00
4420........	57·6	+39 49	9·34		59·9	+39 3	11·51
4421........	57·7	+39 40	8·16*		21 0·0	+39 23	10·32
	58·0	+39 46	10·90		0·3	+39 15	11·88
	58·1	+38 59	11·37	4341........	0·3	+38 45	8·19*
	58·3	+39 15	11·57	4433........	0·3	+39 6	11·02
	58·5	+39 24	11·99*	4434........	0·4	+39 42	9·81*
	58·5	+39 14	11·86	4435........	0·5	+39 26	10·11*
	58·6	+39 20	11·37	4439........	1·3	+39 46	10·30
	58·6	+39 51	11·00	4440........	1·3	+39 44	9·93
4423........	58·7	+39 24	Var.	4441........	1·6	+39 52	9·80
	58·8	+39 20	11·81*		1·6	+39 35	11·21
	58·8	+39 27	12·60*	4442........	1·7	+39 23	10·05

PHOTOGRAPHIC MAGNITUDES OF VY CYGNI

Date	Julian Day	Phase	Phtg. Mag.	Remarks	Date	Julian Day	Phase	Phtg. Mag.	Remarks
1920		d			1922		d		
Aug. 19...	2422556·651	1·990	10·11		July 20...	2423256·797	2·834	10·40	
Oct. 6...	604·573	2·768	10·58		24...	260·725	6·762	10·70	
10...	608·613	6·808	10·32		25...	261·694	7·731	9·90	
12...	610·549	0·886	10·32		25...	261·792	7·829	9·93	
1921					26...	262·686	0·866	10·31	
Aug. 4...	906·655	6·271	11·03		28...	264·695	2·875	10·85	
5...	907·649	7·265	10·00		28...	264·782	2·962	10·90	
22...	924·574	0·618	10·30	Poor.	31...	267·777	5·957	11·03	
1922					Aug. 18...	285·667	0·275	10·08	
July 13...	2423249·657	3·551	11·01		18...	285·737	0·345	10·20	
19...	255·697	1·734	10·08		23...	290·747	5·355	11·21	
19...	255·781	1·818	10·05		25...	292·596	7·204	9·93	
20...	256·704	2·741	10·50		29...	296·737	3·487	11·01	

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[1] A. N., Vol. 164, p. 43.
[2] A. N., Vol. 176, p. 37.
[3] A. N., Vol. 177, p. 377.
[4] A. J., Vol. 32, p. 188.

These observations give the photographic light-curve shown in Fig. 18. In the same figure is given the visual light-curve as deduced from the observations of Doberck.

The double maximum is a remarkable feature of this light-curve. It is well marked in the curves deduced from the visual observations of Williams, v. Zeipel and Doberck. Luizet considers the curve flat at maximum; that is, the secondary minimum is not shown. Visually this secondary minimum is difficult to detect, but the photographic observations accentuate it considerably. The second maximum is merely a large hump on the descending branch, such as is found in many Cepheids. The star could not be of the β Lyrae type, since the maxima are too near one another, and could not be explained by the revolution around each other of two very close bodies.

VY Cygni is quite possibly a star far advanced in the process of separation into two bodies. It appears to form a connecting link between stars with a smooth light-curve, like VX Cygni, and those of the β Lyrae type, like WZ Cygni, consisting of two bodies.

The difference of amplitude between visual and photographic curves is also quite remarkable.

A plot has been made of the results of the observations of each of the different observers (except those of Luizet which were plotted by himself) and the dates of maximum found for each. These maxima do not verify Luizet's formula,

$$\text{Max.} = \text{J. D. } 2416370{\cdot}9507 + 7^{d}{\cdot}85926 \text{ E.}$$

it being found that the elements,

$$\text{Max.} = \text{J. D. } 2416370{\cdot}9507 + 7^{d}{\cdot}85732 \text{ E.}$$

give much better agreement. The latter period was therefore used in computing the phases above. The observed and computed dates of maximum corresponding to each group of observations are given in the following table:—

JULIAN DATES OF FIRST MAXIMUM OF VY CYGNI

Observer	Observed J. D.	Computed J. D.	O–C
			d
Williams	2416402·47	2416402·379	+0·09
Luizet	2417408·9	2417408·117	+0·78
Luizet	2417479·2	2417478·832	+0·37
v. Zeipel	2417549·33	2417549·548	−0·22
Doberck	2421745·36	2421745·358	0·00
Ottawa	2423261·53	2423261·820	−0·29

The residuals given in the last column, though possibly due to errors of observation, may on the other hand be real. If this is the case they point to a fluctuation of the period, which it would require further observations to bring out clearly.

WZ Cygni

The variability of WZ Cygni was discovered by Williams in 1905.[1] It was observed by Williams and Shapley.[2]

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[1] A. N., Vol. 169, p. 365.

[2] Contributions from the Princeton Univ. Obs., No. 3, p. 54.

In the two following tables are given the photographic magnitudes of our comparison stars and the resulting magnitudes of WZ Cygni.

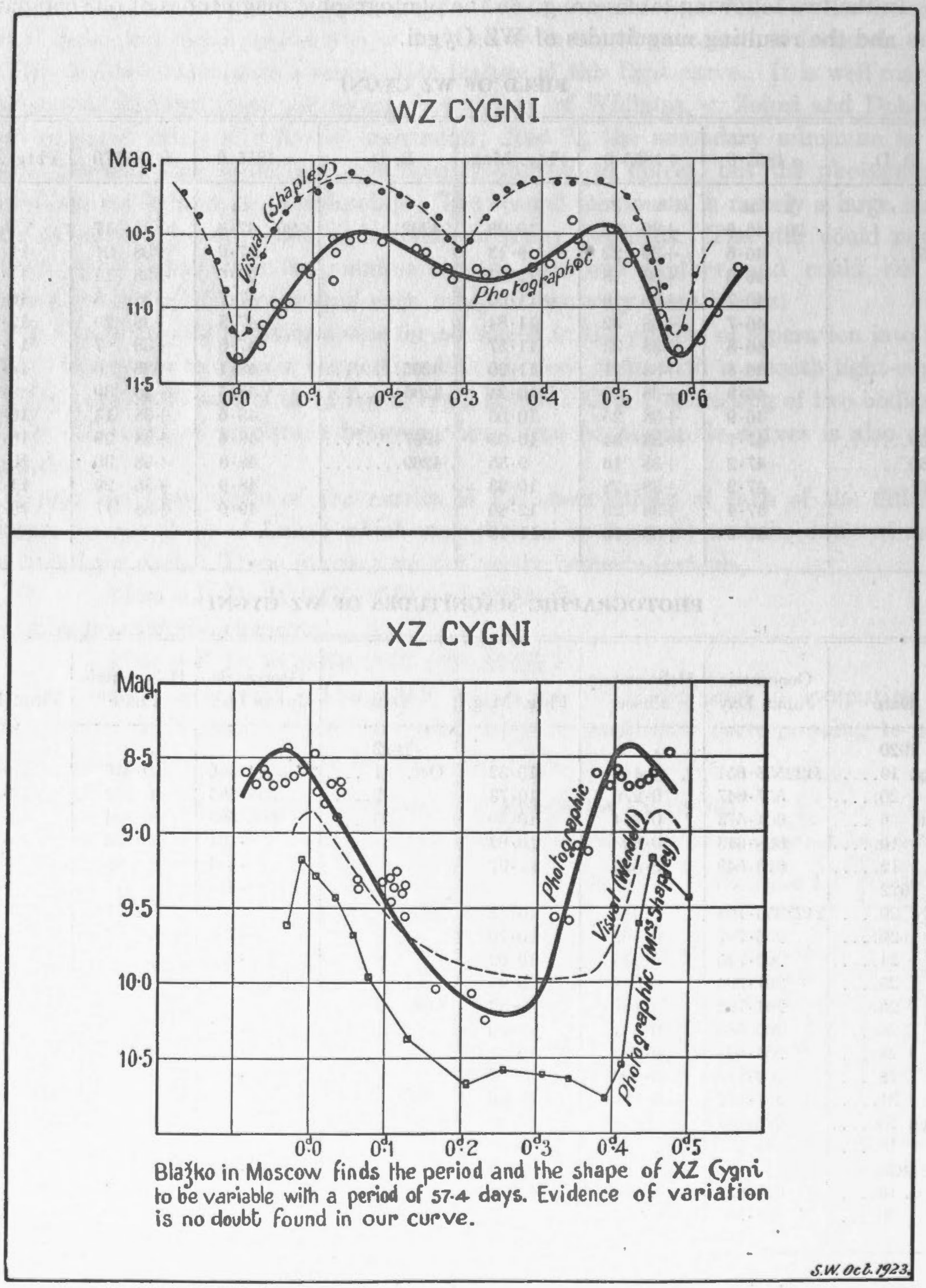
FIELD OF WZ CYGNI

B. D.	α 1855·0	δ 1855·0	Phtg. Mag.	B. D.	α 1855·0	δ 1855·0	Phtg. Mag.
	h m	° ′			h m	° ′	
	20 46·6	+39 12	10·98	4262........	20 47·6	+38 17	Var.
4257........	46·6	+39 22	9·13		47·6	+38 21	11·80
	46·6	+38 18	11·23		47·6	+38 11	11·07
	46·7	+38 16	11·00		47·7	+38 8	10·80
	46·7	+38 32	11·84		47·8	+38 19	12·13
	46·8	+38 33	11·57		47·8	+38 7	11·33
	46·8	+38 21	11·20	4265........	48·1	+38 6	10·73
	46·9	+38 29	10·24	4266........	48·5	+38 30	9·69
	46·9	+38 25	10·05		48·6	+38 13	10·80
4259........	47·1	+38 34	10·05	4267........	48·6	+38 29	10·20
4260........	47·2	+38 16	9·35	4269........	48·8	+38 30	10·24
4261........	47·2	+38 22	10·63		48·9	+38 29	11·88
	47·4	+38 20	12·40		49·0	+38 17	10·52
	47·6	+38 10	11·10				

PHOTOGRAPHIC MAGNITUDES OF WZ CYGNI

Date	Geocentric Julian Day	Heliocentric Phase	Phtg. Mag.	Date	Geocentric Julian Day	Heliocentric Phase	Phtg. Mag.
1920		d		1922		d	
Aug. 19.....	2422556·651	0·449	10·52	Oct. 1.....	2423694·590	0·416	10·63
20.....	557·647	0·276	10·73	2.....	695·585	0·262	10·73
Oct. 6.....	604·573	0·444	10·39	5.....	698·569	0·324	10·76
10.....	608·613	0·393	10·62		·601	0·356	10·73
12.....	610·549	0·575	11·37		·635	0·390	10·73
1922					·669	0·424	10·63
July 20.....	2423256·704	0·314	10·73		·702	0·457	10·57
20.....	256·797	0·407	10·70		·739	0·494	10·45
24.....	260·725	0·244	10·62		·774	0·529	10·63
25.....	261·694	0·044	10·98		·814	0·569	11·07
25.....	261·792	0·141	10·52	Oct. 7.....	700·519	0·520	10·70
26.....	262·686	0·451	10·70		·551	0·552	10·90
28.....	264·695	0·123	10·81		·583	0·584	11·32
28.....	264·782	0·220	10·38		·614	0·031	11·14
31.....	267·777	0·282	10·80		·645	0·062	10·95
Aug. 18.....	285·667	0·053	11·03		·678	0·095	10·73
18.....	285·737	0·124	10·52		·711	0·128	10·65
1923					·745	0·162	10·50
Sept. 16.....	679·624	0·082	10·73		·780	0·197	10·52
30.....	693·597	0·027	11·23		·818	0·235	10·60

The formula Max. = J. D. 2414936·5487 + 0^{d}·584464 E was used in computing the phases.



Blaƶko in Moscow finds the period and the shape of XZ Cygni to be variable with a period of 57·4 days. Evidence of variation is no doubt found in our curve.

S.W. Oct. 1923.

FIGURE 19. LIGHT CURVES OF WZ CYGNI AND XZ CYGNI

These magnitudes give the photographic light curve shown in Fig. 19. In the same figure is found the visual light-curve as deduced from the observations of Shapley. From comparison of the curves it is seen that the average colour-index of the star is small;

this is in agreement with the fact that it is of spectral class A. The shapes of the curves however are different, and there is apparently a general slanting of the photographic curve with respect to the visual; the amplitudes are also slightly different. There are very few variable stars of the β Lyrae type whose photographic light curves have been determined, and for only two of them, apparently, has the photographic curve been compared to the visual. In the case of β Lyrae itself Swarzschild finds only very small differences between the shapes of the curves,[1] and Martin and Plummer in the case of TT Aurigae[2] find that they practically coincide.

The above differences in amplitude and shape, if definitely verified, would somewhat complicate the physical explanation of variations of the β Lyrae type. They might be partially explained by differences in spectral type of the components or by tidal effects. If the suggestions brought forward in the discussion of VY Cygni above should prove to have any value we might expect intermediate stages between quasi-Cepheid variation and that of the true β Lyrae type, of which the present case might be an example. It may be remarked that most stars of the β Lyrae type are of early spectral class, with periods of the same order as might be expected, with not unreasonable suppositions as to density, to produce Jacobian ellipsoids or closely allied forms involving ultimate separation by fission.

XZ Cygni

The variation of XZ Cygni was discovered by Madame Ceraski at the Observatory of Moscow[3] in 1905, and independently by Miss Leavitt, at Harvard in 1907. It was observed visually by Enebo,[4] Wendell,[5] v. Zeipel,[6] Yendell[7] and Blažko,[8] while photographic light-curves have been determined by Martin and Plummer,[9] and by Mrs. Shapley;[10] the latter determined also a photo-visual light-curve. The light-curve of Martin and Plummer shows a large number of minor secondary oscillations, as most of their light-curves do. According to Jordan these oscillations are likely due either to errors of observation or more probably to the method of reduction. Mrs. Shapley's magnitudes were determined by the methods of measurement and reduction of Prof. Seares, regularly employed in photometry with the 60-inch reflector;[11] five of her plates were also exposed on the North Polar Standards to determine the magnitudes of the comparison stars. Her curve, as well as our own photographic curve and the visual one of Wendell, is reproduced in Fig. 19. It will be seen that there is a systematic difference between her curve and the Ottawa one, though the amplitude and shape are nearly the same. The scale of relative magnitudes is apparently the same for the two curves, but there is a systematic difference between their absolute values. The Ottawa curve agrees more nearly, as to the scale of absolute magnitude, with that of Martin and Plummer.

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[1] Kuffner Pub., Vol. 5, C., p. 123.
[2] Monthly Notices, Vol. 76, p. 395.
[3] A. N., Vol. 168, p. 324.
[4] A. N., Vol. 171, p. 219, and Enebo's Special Publications, Vol. 1, p. 37, and Vol. 11, p. 40.
[5] H. A., Vol. 69, pp. 125 and 165.
[6] A. N., Vol. 4, p. 293.
[7] A. J., Vol. 28, p. 126.
[8] A. N., Vol. 216, p. 112.
[9] Monthly Notices, Vol. 74, pp. 225-233.
[10] Contributions from the Mt. Wilson Observatory, Vol. 7, p. 52.
[11] Contributions from the Mt. Wilson Observatory, Vol. 4, p. 293.

The phases were computed from the formula:—

$$\text{Max.} = \text{J. D. } 2417201{\cdot}25417 + 0^{d}{\cdot}46659\ E;$$

the displacement of the maximum indicates a slight correction to this period. The plot of the individual observations also indicates departures from the mean curve rather larger than the usual accidental errors; there is nothing surprising in this, as variations in the shape and period have apparently been detected by Blažko in Moscow. Blažko observed the star on 98 nights and obtained 87 well determined maxima; he gives for the elements the formula:—

$$\begin{aligned}\text{Max.} = \text{J. D. } & 2417201{\cdot}2350 + 0^{d}{\cdot}4665892\ E\\ & +0{\cdot}002\ (E/100)^{2} + 0{\cdot}0079 \sin 2^{\circ}{\cdot}927\ (E+41{\cdot}5)\\ & +0{\cdot}0024 \sin 5^{\circ}{\cdot}854\ (E+1)\end{aligned}$$

His period varies between $11^{h}\ 11^{m}\ 18^{s}{\cdot}39$ and $11^{h}\ 12^{m}\ 28^{s}{\cdot}23$, and the principal inequality has a period of 57·4 days, covering 123 of the short periods. He finds the shape of the curve also to vary with a period of 57·4 days, the extreme shapes showing the same character as those of RW Draconis.

In the two following tables are given the photographic magnitudes of the comparison stars as obtained here, together with the resulting magnitudes of the variable. The plates of 1923 were made with the 6-inch Brashear doublet camera; the derived magnitudes were obtained by measuring the diameters of the stellar images and deriving from them curves expressing the relation between these diameters and the corresponding magnitudes.

FIELD OF XZ CYGNI

B. D.	α 1855·0	δ 1855·0	Phtg. Mag.	B. D.	α 1855·0	δ 1855·0	Phtg. Mag.
	h m	° ′			h m	° ′	
	19 23·3	+55 36	9·22	2218........	19 28·8	+55 19	9·36
2201........	24·3	+55 40	8·60		29·5	+56 4	Var.
2203........	24·9	+55 47	9·99	2220........	30·1	+55 21	9·53
	25·1	+56 45	10·11	2259........	30·2	+56 7	10·65
	25·3	+55 55	9·84	2260........	30·8	+56 20	10·27
	25·6	+55 59	10·24	2261........	30·9	+56 9	7·13
	26·1	+55 58	10·11	2263........	31·4	+56 18	10·08
2247........	26·6	+56 50	9·00	2266........	31·8	+56 14	10·03
	26·9	+56 3	11·08		31·9	+55 38	9·81
	27·2	+56 54	9·82	2267........	32·0	+56 6	10·54
	27·3	+56 6	10·55	2268........	32·0	+56 41	10·08
2251........	27·5	+56 20	9·51	2223........	32·1	+55 59	10·66
2252........	27·5	+56 53	8·12		32·4	+55 33	8·80
2254........	27·8	+56 48	8·66	2269........	32·5	+56 4	9·59
	28·0	+56 42	10·60	2273........	33·4	+56 26	8·55
2216........	28·3	+55 56	7·95				

PHOTOGRAPHIC MAGNITUDES OF XZ CYGNI

Date	Heliocentric Julian Day	Phase	Phtg. Mag.	Date	Heliocentric Julian Day	Phase	Phtg. Mag.
1920		d		1920		d	
Aug. 24.....	2422561·561	0·121	9·36	Nov. 28.....	2422657·593	0·035	8·66
26.....	563·835	0·062	9·31	1923			
30.....	567·568	0·062	9·36	July 6.....	2423612·770	0·103	9·45
	·628	0·122	9·55	Sept. 4.....	667·612	0·354	9·08
Sept. 1.....	569·585	0·213	10·08		·665	0·407	8·62
	·803	0·431	8·66		·750	0·025	8·68
3.....	571·561	0·323	9·56	5.....	668·556	0·364	9·12
10.....	578·631	0·394	8·66		·571	0·379	8·60
13.....	581·542	0·039	8·73		·593	0·401	8·58
14.....	582·541	0·104	9·32		·608	0·416	8·67
18.....	586·747	0·111	9·27		·630	0·438	8·43
19.....	587·601	0·032	8·90		·659	0·467	8·48
20.....	588·611	0·109	9·36		·676	0·018	8·80
21.....	589·530	0·095	9·34	6.....	669·566	0·441	8·64
22.....	590·531	0·162	10·03		·582	0·457	8·60
23.....	591·533	0·231	10·27		·604	0·013	8·72
Oct. 1.....	599·571	0·337	9·56		·619	0·028	8·69
5.....	603·553	0·120	9·38				

CONCLUSION

The present state of our knowledge of Cepheid variation is scarcely adequate to explain all the phenomena involved. The ordinary binary theory may almost certainly be definitely ruled out of court, while on the pulsation theory there are certain points not accounted for.

As has been mentioned in this article and elsewhere, it is at least plausible to suppose that short-period variables of the β Canis Majoris and allied types should be considered as forming a part of the same sequence as the true Cepheids, and their behaviour cannot be overlooked in any complete theory of Cepheid variation. We are then confronted with the fact that these short-period variables exhibit definite peculiarities (such as variation of amplitude and of centre-of-mass velocity) which appear to definitely indicate the existence of a satellite, and which cannot be satisfactorily explained on the assumption of simple pulsation alone; there is also to be considered the further fact that in some of the true Cepheids, such as δ Cephei itself, the presence of similar fluctuations is suspected, though perhaps not definitely proved.

Are we then to assume that in the short-period variables, and in them alone, there is present a satellite whose tidal action is superimposed upon the true Cepheid variation of the primary? Or is it more logical to suppose that the satellite is present in all cases, and forms one of the necessary conditions and causes of Cepheid variation, the secondary perturbations in the case of the longer-period stars being so small as in most cases to have escaped notice or be practically evanescent?

It is usually assumed that the mean density of a Cepheid is a function of its period of light variation. Assuming also that in general the effective temperature of a star is a function of its mean density and mass (volume or luminosity might replace one of these variables), and that the color-index is a function of the temperature alone, it follows that in Cepheids the mean color-index is a function of the period and of the mean absolute magnitude.

If for certain groups of Cepheids, such as those of the Small Magellanic Cloud, the period is a function of the absolute magnitude alone, the color-index must be a function of the period alone and vice-versa.

Shapley has pointed out for the galactic Cepheids[1] that the length of the period has a tendency to increase with the spectral class, although W. W. Campbell[2] had already noticed that it was not strictly so.

Our table of Cepheids shows that some short-periods may be of fairly advanced spectral class, and the case of UY Cygni of very short-period and very large color-index indicates that the latter is not a function of the period alone.

The above and similar facts point to the conclusion that Shapley's period-luminosity relation should perhaps be regarded more or less as a curve of statistical averages, and that the true relation, applicable to particular cases, should involve colour-index as well. Whether, and by how much, this would affect conclusions as to the scale of the universe can scarcely be determined until it is known in what way color-index enters into the relation, if at all.

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[1] Ap. J., Vol. 40, p. 463 (1914).
[2] L. O. B., Vol. 6, p. 51 (1910).

DOMINION OBSERVATORY

OTTAWA, CANADA

March, 1924.