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Spectroscopic Investigations of the Sun

PART I

GENERAL OUTLINE OF OBSERVATIONS, INSTRUMENTS, AND METHODS—SECTIONS 8 and 9

BY

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GENERAL OUTLINE OF OBSERVATIONS, INSTRUMENTS AND METHODS

Section 8—Tables for Computing the Heliographic Co-ordinates, and the Component to Earth of the Rotational Velocity, of an Observed Point on the Sun

BY

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The following tables facilitate the computation of heliographic co-ordinates and components of the velocity of the sun's rotation directed to the observer. The Ottawa series of spectroscopic observations of the solar rotation is so extensive that it is imperative to employ some such method of lessening considerably the time involved in the computations. A graduated sphere and co-ordinate frame, described in the next Section, was devised for similar purposes to those for which the tables are applicable.

The computation of heliographic positions of observed points on the solar disc is based on the adopted determinations[1] due to Carrington,

"$I = 7° 15'$, and $N = 73° 40'$ for 1850.0", where I is the inclination of the solar equator to the ecliptic, and N is the longitude of the ascending node on the solar equator. Subsequent determinations do not differ seriously from these values.

From I, N and $\odot$ (longitude of the sun), B_o, the heliographic latitude of the centre of the solar disc, is determined from the following equation:

$$\sin B_o = \sin I \sin (\odot - N) \tag{1}$$

However, daily values of B_o are now published in the ephemerides[2].

In observing it is necessary to know P, the angle between the solar and terrestrial axes projected on the solar disc. This is determined by computing the following two angles (though now as in the case of B_o it may be read from an ephemeris):

p, the angle on the solar disc between the projected solar axis and the normal to the ecliptic, and

p', the angle on the solar disc between the projected terrestrial axis and the normal to the ecliptic.

Then
$$\tan p = \tan I \cos (\odot - N) \tag{2}$$
$$\tan p' = \tan \epsilon \cos \odot \tag{3}$$

or

$$\cos p = \cos I \sec B_o \tag{2a}$$
$$\cos p' = \cos \epsilon \sec \delta \tag{3a}$$

where ϵ is the obliquity of the ecliptic, and δ is the declination of the sun.

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[1] R. C. Carrington, "Observations of Solar Spots", 1854–1861.

[2] "Ephemeris for Physical Observations of the Sun" in *The Nautical Almanac*, and in *The American Ephemeris and Nautical Almanac*.

(The angle p is also required in the computation of the components of the earth's orbital velocity to observed points on the sun, for which purpose it is convenient to tabulate it with reference to B_o, as done by means of equation (2a) in the preceding Section 7.)

In practice, the direction of the "East-West line" is determined from the drifting of the solar image due to the earth's rotation or the rotating on its polar axis of the coelostat or telescope. The direction of the "equatorial diameter" of the solar disc is measured $P°$ from the East-West line.

The position on the solar disc of an observed point is recorded by measuring:

a, the position-angle between the equatorial diameter and the radius of the disc on which the observed point lies, and

r, the distance from the centre of the disc along that radius, expressed as a decimal of the length of the radius.

The two measurements, a and r, together with B_o for the recorded time of the observation, and s, the angular semi-diameter of the sun, serve as follows in determining the heliographic co-ordinates and the angle between the line to the observer and the direction of the solar rotation of the observed point:

ρ, the heliocentric angle between the observer and the observed point on the solar disc, may be computed from the equation

$$\rho = \sin^{-1} r - rs \qquad (4)$$

However, this computation is rendered unnecessary by the useful table, issued in 1878 by Warren de la Rue, of values of Log cos ρ and Log sin ρ for values of r progressing by thousandths of the radius from centre to limb of the disc, using the average value of s (which suffices for our purposes). The values of sin ρ and cos ρ, used in computing the tables below, were derived from this table.

ϕ, the heliographic latitude, is computed by the equation

$$\sin \phi = \sin B_o \cos \rho + \cos B_o \sin \rho \sin a \qquad (5)$$

This computation is greatly expedited by the use of Table XVII of sin B_o cos ρ, and Table XVIII of cos B_o sin ρ, arranged with arguments of B_o and r.

In most series of observations of the solar rotation, the values of the position-angle a are selected from multiples of 5°, so that a table of the selected values of sin a should be convenient for performing (by machine) the multiplication in the last term. Since B_o is at most 7°·25, the integral values of B_o used in computing the tables suffice for interpolation; and likewise the values of r, in hundredths of the radius, permit of easy and accurate interpolation, particularly in Table XVIII, where 0·001 in the values of r corresponds to 0·0010 in the product throughout the table.

In rotation observations two points equidistant from the centre are usually observed on a diameter of the solar disc, so that it is convenient in the use of equation (5) to regard sin ϕ as positive, the quadrant being indicated, and the two values of sin ϕ apply as follows:

$$-\sin B_o \cos \rho + \cos B_o \sin \rho \sin a$$

for points in the half of the disc bounded by the equatorial diameter and containing the equator, and

$$+ \sin B_o \cos \rho + \cos B_o \sin \rho \sin a$$

for points in the other half of the disc.

λ, the heliographic longitude, measured from the plane containing the axis of the sun and the radius vector, is computed from the equation

$$\sin \lambda = \sin \rho \cos a \sec \phi \tag{6}$$

For this computation, $\sin \rho$ is taken from Table XVIII and multiplied by the product $\cos a \sec \phi$ from Table XIX.

In Table XIX the values of ϕ progress by increments varying from 1° to 0°·1 to insure sufficiently accurate interpolation. The values of a progress by increments of 5°. When other than these standard values of a occur, the values of $\sec \phi$ included in Table XIX will be convenient for the multiplication. The values of ϕ are sufficiently extended to cover the interpolation for the greatest values of ϕ possible for each value of a. These maximum values occur at the values of ρ in the following equation:

$$\tan \rho = \cot 7°{\cdot}25 \sin a \tag{7}$$

γ_o, the angle between the radius vector and the pole of the plane containing the solar axis and the observed point[3], may be determined from the equation

$$\cos \gamma_o = \cos B_o \sin \lambda \tag{8}$$

$$= \cos B_o \sin \rho \cos a \sec \phi \tag{9}$$

It is readily computed by multiplying the selected value of $\cos B_o \sin \rho$ from Table XVIII by $\cos a \sec \phi$ from Table XIX.

The angle γ_o is approximately the inclination between the direction of the solar rotation at the observed point and the line from the point to the observer, since s,the angular semi-diameter of the sun, is very small.

γ, the angle between the pole of the plane normal to the line from the observer to the observed point, and the pole of the plane containing the solar axis and the observed point, may be computed from the equation

$$\cos \gamma = \cos (B_o \pm rs \sin a) \sin (\lambda + rs \cos a) \tag{10}$$

where $B_o + rs \sin a$ applies to points in the half of the solar disc bounded by the equatorial diameter which contains the equator, and $B_o - rs \sin a$, to points in the other half.

The component toward the observer of the velocity of the sun's rotation, V, at the observed point, is

$$V \cos \gamma \tag{11}$$

[3] N. C. Dunér, in his pioneer spectroscopic investigations of the solar rotation, computed a table of values of this angle for the special case of observed points on the solar limb.

The difference between $\cos\gamma$ and $\cos\gamma_o$ is small and for points very near the limb or the axis it may be neglected; but for points on other parts of the disc the difference is appreciable. Cos γ may be derived rapidly from $\cos\gamma_o$ by the addition of a small number tabulated in Table XX. This addendum is derived as follows:

Since B_o is at most $7°\cdot25$ and $rs \sin a$ has a maximum value of about 16′, the error in using $\cos B_o$ for $\cos(B_o \pm rs \sin a)$ cannot exceed 1 part in about 1650, and this largest error occurs only for points near the poles of the sun. (If desired, however, the modified B_o may be used instead of B_o in selecting the product from Table XVIII.) Hence, approximately,

$$\begin{aligned}\cos\gamma &= \cos B_o \sin(\lambda + rs\cos a)\\ &= \cos B_o(\sin\lambda + \cos\lambda\, r\cos a\sin s)\\ &= \cos B_o(\sin\lambda + \cos\lambda\sin\rho\cos a\sin s)\\ &= \cos B_o(\sin\lambda + \sin\lambda\cos\lambda\cos\phi\sin s), \text{ from equation (6).}\end{aligned}$$

Hence, since $\cos B_o$ is never less than $0\cdot9920$, it follows to a sufficiently close approximation that

$$\cos\gamma = \cos B_o \sin\lambda + \sin\lambda\cos\lambda\cos\phi\sin s, \text{ or}$$
$$\cos\gamma = \cos\gamma_o + \sin\lambda\cos\lambda\cos\phi\sin s, \qquad (12)$$

in which the (average) value of $\sin s$ is $0\cdot0047$. The addendum never exceeds $0\cdot0023$, having its maximum values when $\lambda = 45°$.

The tables were computed by machine multiplication of the natural trigonometric functions. The values of the factors used and the computations were checked by Mr. John L. O'Connor. It is believed that the use of the tables will yield the various desired angles to the nearest tenth of a degree, which is as accurate as the usual observations justify.

Dominion Observatory,
Ottawa, Canada,
December, 1933.

TABLE XVII—VALUES OF $\sin B_0 \cos \rho$, FOR VALUES OF B_0 AND r

r	cos ρ	B_0 1°	2°	3°	4°	5°	6°	7°	r	cos ρ	B_0 1°	2°	3°	4°	5°	6°	7°
0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·
010	99995	0174	0349	0523	0698	0872	1045	1219	510	86139	0150	0301	0451	0601	0751	0900	1050
20	99979	0174	0349	0523	0697	0871	1045	1218	20	85542	0149	0299	0448	0597	0746	0894	1043
30	99956	0174	0349	0523	0697	0871	1045	1218	30	84930	0148	0296	0445	0592	0740	0888	1035
40	99922	0174	0349	0523	0697	0871	1044	1218	40	84302	0147	0294	0441	0588	0735	0881	1027
50	99876	0174	0349	0523	0697	0871	1044	1217	50	83657	0146	0292	0438	0584	0729	0874	1020
60	99821	0174	0348	0522	0696	0870	1043	1217	60	82995	0145	0290	0434	0579	0723	0868	1011
70	99756	0174	0348	0522	0696	0869	1043	1216	70	82315	0144	0287	0431	0574	0717	0860	1003
80	99683	0174	0348	0522	0695	0869	1042	1215	80	81619	0142	0285	0427	0569	0711	0853	0995
90	99598	0174	0348	0521	0695	0868	1041	1214	90	80902	0141	0282	0423	0564	0705	0846	0986
100	99504	0174	0347	0521	0694	0867	1040	1213	600	80168	0140	0280	0420	0559	0699	0838	0977
10	99399	0173	0347	0520	0693	0866	1039	1211	10	79413	0139	0277	0416	0554	0692	0830	0968
20	99284	0173	0347	0520	0693	0865	1038	1210	20	78639	0137	0274	0412	0549	0685	0822	0958
30	99159	0173	0346	0519	0692	0864	1037	1208	30	77845	0136	0272	0407	0543	0678	0814	0949
40	99024	0173	0346	0518	0691	0863	1035	1207	40	77028	0134	0269	0403	0537	0671	0805	0939
50	98878	0173	0345	0518	0690	0862	1034	1205	50	76190	0133	0266	0399	0532	0664	0796	0929
60	98723	0172	0345	0517	0689	0860	1032	1203	60	75329	0131	0263	0394	0525	0657	0787	0918
70	98558	0172	0344	0516	0688	0859	1030	1201	70	74444	0130	0260	0390	0519	0649	0778	0907
80	98381	0172	0343	0515	0686	0857	1028	1199	80	73536	0128	0257	0385	0513	0641	0769	0896
90	98195	0171	0343	0514	0685	0856	1026	1197	90	72602	0127	0253	0380	0506	0633	0759	0885
200	97999	0171	0342	0513	0684	0854	1024	1194	700	71642	0125	0250	0375	0500	0624	0749	0873
10	97791	0171	0341	0512	0682	0852	1022	1192	10	70655	0123	0247	0370	0493	0616	0739	0861
20	97573	0170	0341	0511	0681	0850	1020	1189	20	69639	0122	0243	0364	0486	0607	0728	0849
30	97344	0170	0340	0509	0679	0848	1018	1186	30	68593	0120	0239	0359	0479	0598	0717	0836
40	97105	0169	0339	0508	0677	0846	1015	1183	40	67515	0118	0236	0353	0471	0588	0706	0823
50	96855	0169	0338	0507	0676	0844	1012	1180	50	66406	0116	0232	0348	0463	0579	0694	0809
60	96592	0169	0337	0506	0674	0842	1010	1177	60	65262	0114	0228	0342	0455	0569	0682	0795
70	96321	0168	0336	0504	0672	0840	1007	1174	70	64081	0112	0224	0335	0447	0559	0670	0781
80	96037	0168	0335	0503	0670	0837	1004	1170	80	62861	0110	0219	0329	0439	0548	0657	0766
90	95741	0167	0334	0501	0668	0834	1001	1167	90	61601	0107	0215	0322	0430	0537	0644	0751
300	95436	0167	0333	0500	0666	0832	0998	1163	800	60298	0105	0210	0316	0421	0526	0630	0735
10	95117	0166	0332	0498	0664	0829	0994	1159	10	58948	0103	0206	0309	0411	0514	0616	0718
20	94789	0165	0331	0496	0661	0826	0991	1155	20	57549	0100	0201	0301	0401	0502	0602	0701
30	94450	0165	0330	0494	0659	0823	0987	1151	30	56097	0098	0196	0294	0391	0489	0586	0684
40	94096	0164	0328	0492	0656	0820	0984	1147	40	54587	0095	0191	0286	0381	0476	0571	0665
50	93732	0164	0327	0491	0654	0817	0980	1142	50	53015	0093	0185	0277	0370	0462	0554	0646
60	93356	0163	0326	0489	0651	0814	0976	1138	60	51374	0090	0179	0269	0358	0448	0537	0626
70	92967	0162	0224	0487	0649	0810	0972	1133	70	49658	0087	0173	0260	0346	0433	0519	0605
80	92566	0162	0323	0484	0646	0807	0968	1128	80	47858	0084	0167	0250	0334	0417	0500	0583
90	92151	0161	0322	0482	0643	0803	0963	1123	90	45965	0080	0160	0241	0321	0401	0480	0560
400	91725	0160	0320	0480	0640	0799	0959	1118	900	43966	0077	0153	0230	0307	0383	0460	0536
10	91287	0159	0319	0478	0637	0796	0954	1113	10	41850	0073	0146	0219	0292	0365	0437	0510
20	90834	0159	0317	0475	0634	0792	0949	1107	20	39582	0069	0138	0207	0276	0345	0414	0482
30	90369	0158	0315	0473	0630	0788	0945	1101	30	37162	0065	0130	0195	0259	0324	0388	0453
40	89890	0157	0314	0470	0627	0783	0940	1095	40	34530	0060	0121	0181	0241	0301	0361	0421
50	89396	0156	0312	0468	0624	0779	0934	1089	50	31645	0055	0110	0166	0221	0276	0331	0386
60	88891	0155	0310	0465	0620	0775	0929	1083	60	28432	0050	0099	0149	0198	0248	0297	0347
70	88369	0154	0308	0463	0616	0770	0924	1077	70	24751	0043	0087	0130	0173	0216	0259	0302
80	87833	0153	0307	0460	0613	0766	0918	1070	80	20347	0036	0071	0106	0142	0177	0213	0248
90	87283	0152	0305	0457	0609	0761	0912	1064	90	14565	0025	0051	0076	0102	0127	0152	0178
500	86718	0151	0303	0454	0605	0756	0906	1057	000	00467	0001	0002	0002	0003	0004	0005	0006

TABLE XVIII—VALUES OF $\cos B_o \sin \rho$, FOR VALUES OF B_o AND r

r	sin ρ	B_o 1°	2°	3°	4°	5°	6°	7°	r	sin ρ	B_o 1°	2°	3°	4°	5°	6°	7°
0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·
010	00995	0099	0099	0099	0099	0099	0099	0099	510	50795	5079	5076	5073	5067	5060	5052	5042
20	01991	0199	0199	0199	0199	0198	0198	0198	20	51793	5179	5176	5172	5167	5160	5151	5141
30	02986	0299	0298	0298	0298	0297	0297	0296	30	52790	5278	5276	5272	5266	5259	5250	5240
40	03981	0398	0398	0398	0397	0397	0396	0395	40	53787	5378	5375	5371	5366	5358	5349	5339
50	04977	0498	0497	0497	0496	0496	0495	0494	50	54786	5478	5475	5471	5465	5458	5449	5438
60	05972	0597	0597	0596	0596	0595	0594	0593	60	55784	5578	5575	5571	5565	5557	5548	5537
70	06966	0696	0696	0696	0695	0694	0693	0691	70	56782	5677	5675	5670	5664	5657	5647	5636
80	07963	0796	0796	0795	0794	0793	0792	0790	80	57779	5777	5774	5770	5764	5756	5746	5735
90	08958	0896	0895	0895	0894	0892	0891	0889	90	58777	5877	5874	5870	5863	5855	5845	5834
100	09954	0995	0995	0994	0993	0992	0990	0988	600	59776	5977	5974	5969	5963	5955	5945	5933
10	10950	1095	1094	1093	1092	1091	1089	1087	10	60774	6076	6074	6069	6063	6054	6044	6032
20	11945	1194	1194	1193	1192	1190	1188	1186	20	61773	6176	6174	6169	6162	6154	6143	6131
30	12939	1294	1293	1292	1291	1289	1287	1284	30	62771	6276	6273	6269	6262	6253	6243	6230
40	13935	1393	1393	1392	1390	1388	1386	1383	40	63771	6376	6373	6368	6362	6353	6342	6330
50	14931	1493	1492	1491	1489	1487	1485	1482	50	64769	6476	6473	6468	6461	6452	6441	6429
60	15926	1592	1592	1590	1589	1587	1584	1581	60	65769	6576	6573	6568	6561	6552	6541	6528
70	16920	1692	1691	1690	1688	1686	1683	1679	70	66768	6676	6673	6668	6661	6651	6640	6627
80	17917	1791	1791	1789	1787	1785	1782	1778	80	67767	6776	6773	6767	6760	6751	6740	6726
90	18913	1891	1890	1889	1887	1884	1881	1877	90	68767	6876	6873	6867	6860	6850	6839	6825
200	19909	1991	1990	1988	1986	1983	1980	1976	700	69767	6976	6972	6967	6960	6950	6938	6925
10	20904	2090	2089	2088	2085	2082	2079	2075	10	70767	7076	7072	7067	7059	7050	7038	7024
20	21900	2190	2189	2187	2185	2182	2178	2174	20	71766	7176	7172	7167	7159	7149	7137	7123
30	22895	2289	2288	2286	2284	2281	2277	2272	30	72766	7276	7272	7267	7259	7249	7237	7222
40	23891	2389	2388	2386	2383	2380	2376	2371	40	73768	7376	7372	7367	7359	7349	7336	7322
50	24887	2488	2487	2485	2483	2479	2475	2470	50	74769	7476	7472	7467	7459	7448	7436	7421
60	25883	2588	2587	2585	2582	2578	2574	2569	60	75769	7576	7572	7567	7558	7548	7535	7520
70	26879	2687	2686	2684	2681	2678	2673	2668	70	76770	7676	7672	7666	7658	7648	7635	7620
80	27875	2787	2786	2784	2781	2777	2772	2767	80	77771	7776	7772	7766	7758	7747	7734	7719
90	28871	2887	2885	2883	2880	2876	2871	2866	90	78773	7876	7872	7867	7858	7847	7834	7819
300	29866	2986	2985	2983	2979	2975	2970	2964	800	79776	7976	7973	7967	7958	7947	7934	7918
10	30862	3086	3084	3082	3079	3074	3069	3063	10	80777	8076	8073	8067	8058	8047	8033	8018
20	31859	3185	3184	3182	3178	3174	3168	3162	20	81781	8177	8173	8167	8158	8147	8133	8117
30	32855	3285	3283	3281	3277	3273	3267	3261	30	82783	8277	8273	8267	8258	8247	8233	8217
40	33851	3385	3383	3380	3377	3372	3367	3360	40	83788	8378	8374	8367	8358	8347	8333	8316
50	34847	3484	3483	3480	3476	3471	3466	3459	50	84791	8478	8474	8467	8458	8447	8433	8416
60	35843	3584	3582	3579	3576	3571	3565	3558	60	85795	8578	8574	8568	8559	8547	8532	8516
70	36840	3683	3682	3679	3675	3670	3664	3657	70	86800	8679	8675	8668	8659	8647	8632	8615
80	37836	3783	3781	3778	3774	3769	3763	3755	80	87805	8779	8775	8768	8759	8747	8732	8715
90	38833	3883	3881	3878	3874	3869	3862	3854	90	88810	8880	8876	8869	8859	8847	8832	8815
400	39829	3982	3980	3977	3973	3968	3861	3953	900	89817	8980	8976	8969	8960	8947	8932	8915
10	40825	4082	4080	4077	4073	4067	4060	4052	10	90824	9081	9077	9070	9060	9048	9033	9015
20	41823	4182	4180	4177	4172	4166	4159	4151	20	91831	9182	9177	9171	9161	9148	9133	9115
30	42818	4281	4279	4276	4271	4265	4258	4250	30	92839	9283	9278	9271	9261	9249	9233	9215
40	43816	4381	4379	4376	4371	4365	4358	4349	40	93849	9383	9379	9372	9362	9349	9333	9315
50	44813	4481	4479	4475	4470	4464	4457	4448	50	94862	9485	9480	9473	9463	9450	9434	9416
60	45809	4580	4578	4575	4570	4563	4556	4547	60	95874	9586	9582	9574	9564	9551	9535	9516
70	46807	4680	4678	4674	4669	4663	4655	4646	70	96888	9687	9683	9676	9665	9652	9636	9617
80	47804	4780	4777	4774	4769	4762	4754	4745	80	97908	9789	9785	9777	9767	9753	9737	9718
90	48801	4879	4877	4873	4868	4862	4853	4844	90	98933	9892	9887	9880	9869	9856	9839	9820
500	49798	4979	4977	4973	4968	4961	4953	4943	000	99999	9998	9994	9986	9976	9962	9945	9925

a	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85
sin a	0872	1736	2588	3420	4226	5000	5736	6428	7071	7660	8192	8660	9063	9397	9659	9848	9962

TABLE XIX—VALUES OF $\cos \alpha \sec \phi$ FOR STANDARD VALUES OF α, AND THE POSSIBLE VALUES OF ϕ

ϕ	$\sec \phi$	α																
		5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°
°	1·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·
0	0000	9962	9848	9659	9397	9063	8660	8192	7660	7071	6428	5736	5000	4226	3420	2588	1736	0872
1	0002	9963	9850	9661	9398	9064	8662	8193	7662	7072	6429	5737	5001	4227	3421	2589	1737	0872
2	0006	9968	9854	9665	9403	9069	8666	8196	7665	7075	6432	5739	5003	4229	3422	2590	1738	0872
3	0014	9976	9862	9673	9410	9076	8672	8203	7671	7081	6437	5744	5007	4232	3425	2592	1739	0873
4	0024	9986	9872	9683	9420	9085	8681	8211	7679	7088	6444	5750	5012	4237	3429	2595	1741	0874
5	0038	1·	9886	9696	9433	9098	8693	8223	7690	7098	6452	5758	5019	4242	3433	2598	1743	0875
6	0055	0017	9902	9713	9449	9113	8708	8237	7703	7110	6463	5767	5028	4249	3439	2602	1746	0876
7	0075	0037	9922	9732	9467	9131	8725	8253	7718	7124	6476	5779	5038	4258	3446	2608	1750	0878
8	0098	0060	9945	9754	9489	9152	8745	8272	7736	7141	6491	5792	5049	4268	3454	2614	1754	0880
9	0125	0086	9971	9780	9514	9176	8768	8294	7756	7159	6508	5807	5062	4279	3463	2620	1758	0882
10	0154	.....	1·	9808	9542	9203	8794	8318	7779	7180	6527	5824	5077	4291	3473	2628	1763	0885
11	0187	.....	0032	9840	9573	9233	8822	8345	7804	7203	6548	5843	5094	4305	3484	2637	1769	0888
12	0223	.....	0068	9875	9607	9266	8854	8374	7832	7229	6571	5864	5112	4321	3497	2646	1775	0891
13	0263	.....	0107	9913	9644	9301	8888	8407	7862	7257	6597	5887	5132	4337	3510	2656	1782	0895
14	0306	.....	.....	9955	9685	9341	8925	8442	7895	7288	6625	5911	5153	4356	3525	2667	1790	0898
15	0353	.....	.....	1·	9728	9383	8966	8480	7931	7321	6655	5938	5176	4375	3541	2680	1798	0902
16	0403	.....	.....	0049	9776	9428	9009	8522	7969	7356	6687	5967	5202	4397	3558	2693	1806	0907
17	0457	.....	.....	0101	9826	9477	9056	8566	8010	7394	6722	5998	5228	4419	3576	2706	1816	0911
18	0515	.....	.....	.....	9880	9529	9106	8613	8055	7435	6759	6031	5257	4444	3596	2721	1826	0916
19	0576	.....	.....	.....	9938	9585	9159	8663	8102	7479	6798	6066	5288	4470	3617	2737	1837	0922
20	0642	.....	.....	.....	1·	9645	9216	8717	8152	7525	6840	6104	5321	4497	3640	2754	1848	0928
21	0712	.....	.....	.....	0065	9708	9276	8774	8205	7574	6885	6144	5356	4527	3664	2772	1860	0934
22	0785	.....	.....	.....	0135	9775	9340	8835	8262	7626	6933	6186	5393	4558	3689	2891	1873	0940
23	0864	.....	.....	.....	.....	9846	9408	8899	8322	7682	6983	6231	5432	4591	3716	2812	1886	0947
24	0946	.....	.....	.....	.....	9921	9480	8967	8385	7740	7036	6279	5473	4526	3744	2833	1901	0954
25	1034	.....	.....	.....	.....	1·	9556	9038	8452	7802	7092	6329	5517	4663	3774	2856	1916	0962
26	1126	.....	.....	.....	.....	0084	9635	9114	8523	7867	7152	6382	5563	4702	3805	2880	1932	0970
27	1223	.....	.....	.....	.....	.....	9720	9194	8597	7936	7214	6437	5612	4743	3839	2905	1949	0978
28	1326	.....	.....	.....	.....	.....	9808	9277	8676	8009	7280	6496	5663	4786	3874	2931	1967	0987
29	1434	.....	.....	.....	.....	.....	9902	9366	8759	8085	7349	6558	5717	4832	3910	2959	1985	0997
30	1547	.....	.....	.....	.....	.....	1.	9459	8845	8165	7422	6623	5774	4880	3949	2989	2005	1006
31	1666	.....	.....	.....	.....	.....	0103	9556	8937	8249	7499	6692	5833	4930	3990	3019	2026	1017
32	1792	.....	.....	.....	.....	.....	.....	9659	9033	8338	7580	6764	5896	4983	4033	3052	2048	1028
33	1924	.....	.....	.....	.....	.....	.....	9767	9134	8431	7664	6839	5962	5039	4078	3086	2071	1039
34	2062	.....	.....	.....	.....	.....	.....	9881	9240	8529	7753	6919	6031	5098	4126	3122	2095	1051
35	2208	.....	.....	.....	.....	.....	.....	1·	9352	8632	7847	7002	6104	5159	4175	3160	2120	1064
36	2361	.....	.....	.....	.....	.....	.....	0125	9469	8740	7945	7090	6180	5224	4228	3199	2146	1077
37	2521	.....	.....	.....	.....	.....	.....	.....	9592	8854	8049	7182	6261	5292	4283	3241	2174	1091
38	2690	.....	.....	.....	.....	.....	.....	.....	9721	8973	8157	7279	6345	5363	4340	3284	2204	1106
39	2868	.....	.....	.....	.....	.....	.....	.....	9857	9099	8271	7381	6434	5438	4401	3330	2234	1122
40	3054	.....	.....	.....	.....	.....	.....	.....	1·	9231	8391	7488	6527	5517	4465	3379	2267	1138
41	3250	.....	.....	.....	.....	.....	.....	.....	0150	9369	8517	7600	6625	5600	4532	3429	2301	1155
42	3456	.....	.....	.....	.....	.....	.....	.....	.....	9515	8650	7718	6728	5687	4602	3483	2337	1173
43	3673	.....	.....	.....	.....	.....	.....	.....	.....	9669	8789	7843	6837	5779	4677	3539	2374	1192
44	3902	.....	.....	.....	.....	.....	.....	.....	.....	9830	8936	7974	6951	5875	4755	3598	2414	1212
45	4142	.....	.....	.....	.....	.....	.....	.....	.....	1·	9090	8112	7071	5977	4837	3660	2456	1233

TABLE XIX—*Continued*

ϕ	sec ϕc	α 50°	55°	60°	65°	70°	75°	80°	85°	ϕ	sec ϕ	α 55°	60°	65°	70°	75°	80°	85°
°	1·	0·	0·	0·	0·	0·	0·	0·	·0	°	1·	0·	0·	0·	0·	0·	0·	0·
45·0	4142	9090	8112	7071	5977	4837	3660	2456	1233	53·0	6616	9531	8308	7022	5683	4301	2885	1448
·5	4267	9171	8183	7134	6030	4880	3693	2477	1244	·5	6812	9643	8406	7105	5750	4351	2919	1465
46·0	4396	9253	8257	7198	6084	4924	3726	2500	1255	54·0	7013	9758	8507	7190	5819	4403	2954	1483
·5	4527	9338	8334	7264	6140	4969	3760	2523	1266	·5	7221	9877	8610	7278	5890	4457	2990	1501
47·0	4663	9425	8410	7331	6197	5015	3795	2546	1278	55·0	7435	1·	8717	7368	5963	4512	3028	1520
·5	4802	9515	8490	7401	6256	5063	3831	2570	1290	·5	7655	0127	8828	7461	6038	4570	3066	1539
48·0	4945	9606	8572	7472	6316	5111	3868	2595	1303	56·0	7883	.....	8941	7558	6116	4628	3105	1559
·5	5092	9701	8656	7546	6378	5162	3906	2621	1315	·5	8118	.....	9059	7657	6197	4689	3146	1579
49·0	5243	9798	8743	7621	6442	5213	3945	2647	1329	57·0	8361	.....	9180	7760	6280	4752	3188	1600
·5	5398	9897	8832	7699	6507	5266	3985	2674	1342	·5	8612	.....	9306	7866	6366	4817	3232	1622
50·0	5557	1·	8923	7779	6575	5321	4027	2702	1356	58·0	8871	.....	9435	7975	6454	4884	3277	1645
·5	5721	0105	9017	7861	6644	5377	4069	2730	1370	7 ·5	9139	.....	9569	8088	6546	4954	3323	1668
51·0	5890	.....	9114	7945	6716	5435	4113	2759	1385	59·0	9416	.....	9708	8206	6641	5025	3372	1692
·5	6064	.....	9214	8032	6789	5494	4158	2789	1400	·5	9703	.....	9851	8327	6739	5100	3421	1717
52·0	6243	.....	9316	8121	6864	5555	4204	2821	1416	60·0	2· 0000	.....	1·	8452	6840	5176	3473	1743
·5	6427	.....	9422	8213	6942	5618	4252	2853	1432	·5	0308	.....	0154	8582	6946	5256	3526	1770

ϕ	sec ϕ	α 60°	65°	70°	75°	80°	85°	ϕ	sec ϕ	α 70°	75°	80°	85°	ϕ	sec ϕ	α 75°	80°	85°
°	2·	1·	0·	0·	0·	0·	0·	°	2·	0·	0·	0·	0·	°	2·	0·	0·	0·
60·2	0122	0061	8504	6882	5208	3494	1754	65·2	3841	8154	6170	4140	2078	70·2	9521	7641	5126	2573
·4	0245	0123	8556	6924	5240	3516	1765	·4	4022	8216	6217	4171	2094	·4	9811 3·	7716	5177	2598
·6	0371	.....	8609	6967	5272	3537	1776	·6	4207	8279	6265	4204	2110	·6	0106	7792	5228	2624
·8	0498	.....	8663	7011	5305	3559	1787	·8	4395	8344	6314	4236	2126	·8	0408	7870	5280	2650
61·0	0627	.....	8717	7055	5339	3582	1798	66·0	4586	8409	6363	4269	2143	71·0	0716	7950	5334	2677
·2	0758	.....	8773	7099	5372	3605	1809	·2	4780	8475	6414	4303	2160	·2	1030	8031	5388	2705
·4	0890	.....	8829	7145	5407	3628	1821	·4	4978	8543	6465	4337	2177	·4	1352	8115	5444	2733
·6	1025	.....	8886	7191	5442	3651	1833	·6	5180	8612	6517	4372	2195	·6	1681	8200	5501	2761
·8	1162	.....	8943	7238	5477	3675	1844	·8	5385	8682	6570	4408	2213	·8	2017	8287	5560	2791
62·0	1301	.....	9002	7285	5513	3599	1857	67·0	5593	8753	6624	4444	2231	72·0	2361	8376	5619	2821
·2	1441	.....	9062	7333	5549	3723	1869	·2	5805	8826	6679	4481	2249	·2	2712	8467	5680	2851
·4	1585	.....	9122	7382	5587	3748	1881	·4	6022	8900	6735	4519	2268	·4	3072	8560	5743	2883
·6	1730	.....	9183	7432	5624	3773	1894	·6	6242	8975	6792	4557	2287	·6	3440	8655	5807	2915
·8	1877	.....	9246	7482	5662	3799	1907	·8	6466	9052	6850	4596	2307	·8	3817	8753	5872	2947
63·0	2027	.....	9309	7534	5701	3825	1920	68·0	6695	9130	6909	4636	2327	73·0	4203	8852	5939	2981
·2	2179	.....	9373	7586	5740	3851	1933	·2	6928	9210	6969	4676	2347	·2	4598	8955	6008	3016
·4	2333	.....	9439	7638	5780	3878	1947	·4	7165	9291	7031	4717	2368	·4	5003	9060	6078	3051
·6	2490	.....	9505	7692	5821	3905	1960	·6	7407	9374	7093	4759	2389	·6	5418	9167	6150	3087
·8	2650	.....	9572	7747	5862	3933	1974	·8	7653	9458	7157	4802	2410	·8	5843	9277	6224	3124
64·0	2812	.....	9641	7802	5904	3961	1988	69·0	7904	9544	7222	4846	2432	74·0	6280	9390	6300	3162
·2	2976	.....	9710	7858	5947	3990	2003	·2	8161	9631	7289	4890	2454	·2	6727	9506	6378	3201
·4	3144	.....	9781	7916	5990	4019	2017	·4	9422	9721	7356	4935	2477	·4	7186	9624	6457	3241
·6	3314	.....	9853	7974	6034	4048	2032	·6	8689	9812	7425	4982	2500	·6	7657	9746	6539	3282
·8	3486	.....	9926	8033	6079	4078	2047	·8	8961	9905	7496	5029	2524	·8	8140	9871	6623	3324
65·0	3662	.....	1·	8093	6124	4109	2062	70·0	9238	1·	7567	5077	2548	75·0	8637	1·	6709	3368

TABLE XIX—*Concluded*

ϕ	sec ϕ	a		ϕ	sec ϕ	a		ϕ	sec ϕ	a		ϕ	sec ϕ	a	ϕ	sec ϕ	a
		80°	85°			80°	85°			80°	85°			85°			85°
°	3·	0·	0·	°	4·	0·	0·	°	5·	0·	0·	°	6·	0·	°	8·	0·
75·1	8890	6753	3390	77·1	4793	7778	3904	79·1	2883	9183	4609	81·1	4637	5634	83·1	3238	7255
·2	9147	6798	3412	·2	5137	7838	3934	·2	3367	9267	4651	·2	5366	5697	·2	4457	7361
·3	9408	6843	3435	·3	5486	7899	3965	·3	3860	9353	4694	·3	6111	5762	·3	5711	7471
·4	9672	6889	3478	·4	5841	7960	3996	·4	4362	9440	4738	·4	6874	5829	·4	7004	7583
·5	9939	6935	3481	·5	6202	8023	4027	·5	4874	9529	4783	·5	7655	5897	·5	8337	7699
·6	4·0211	6983	3505	·6	6569	8087	4059	·6	5396	9619	4828	·6	8454	5966	·6	9711	7819
·7	0486	7030	3529	·7	6942	8151	4091	·7	5928	9712	4875	·7	9273	6038	·7	9·1129	7943
·8	0765	7079	3553	·8	7321	8217	4124	·8	6470	9806	4922	·8	7·0112	6111	·8	2593	8070
·9	1048	7128	3578	·9	7706	8284	4158	·9	7023	9902	4970	·9	0972	6186	·9	4105	8202
76·0	1336	7178	3603	78·0	8097	8352	4192	80·0	7588	1· 5019		82·0	1853	6263	84·0	5668	8338
·1	1627	7229	3628	·1	8496	8421	4227	·1	8164	0100	5070	·1	2757	6341	·1	7283	8479
·2	1923	7280	3654	·2	8901	8492	4262	·2	8751	.....	5121	·2	3684	6422	·2	8955	8625
·3	2223	7332	3680	·3	9313	8563	4298	·3	9351	.....	5173	·3	4635	6505	·3	10·0685	8776
·4	2528	7385	3707	·4	9732	8636	4335	·4	9963	.....	5226	·4	5611	6590	·4	2477	8932
·5	2837	7439	3734	·5	5·0159	8710	4372	·5	6·0589	.....	5281	·5	6613	6678	·5	4334	9094
·6	3150	7493	3761	·6	0593	8785	4410	·6	1227	.....	5337	·6	7642	6767	·6	6261	9262
·7	3469	7548	3789	·7	1034	8862	4448	·7	1880	.....	5393	·7	8700	6860	·7	8260	9436
·8	3792	7605	3817	·8	1484	8940	4487	·8	2546	.....	5452	·8	9787	6954	·8	11·0336	9617
·9	4121	7662	3846	·9	1942	9020	4527	·9	3228	.....	5511	·9	8·0905	7052	·9	2493	9805
77·0	4454	7719	3875	79·0	2408	9101	4568	81·0	3925	.....	5572	83·0	2055	7152	85·0	4737	1·

Summary of the symbols used and the method of employing the Tables:

a, is the position-angle of observed point measured from equatorial diameter of solar disc.

r, is the distance of observed point from centre of solar disc, the radius being unity.

ρ, is the heliocentric angle between observer and observed point. s, is the angular semi-diameter of sun.

ϕ, is the heliographic latitude, and λ is the heliographic longitude of observed point.

γ_o, is the angle between the plane of the solar limb and the plane containing observed point and solar axis.

γ, is the angle between plane normal to line-of-sight and plane containing observed point and solar axis.

$$\sin\phi = (\sin B_o \cos\rho) + (\cos B_o \sin\rho)\sin a$$
Table XVII Table XVIII

$$\sin\lambda = \sin\rho\,(\cos a \sec\phi)$$
Table XVIII Table XIX

$$\cos\gamma_o = (\cos B_o \sin\rho)(\cos a \sec\phi)$$

$$\cos\gamma = \cos\gamma_o + \text{Addendum of Table XX}$$

$V\cos\gamma$ = Component to observer of the solar rotational velocity, V, at the observed point.

TABLE XX—VALUES OF THE ADDENDUM, $\sin\lambda\cos\lambda\cos\phi\sin s$, FOR VALUES OF λ AND ϕ.

λ	ϕ																
° °	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°
5, 85	0004	0004	0004	0004	0004	0004	0003	0003	0003	0003	0002	0002	0002	0001	0001	0001	0000
10, 80	8	8	8	7	7	7	7	6	6	5	5	4	3	3	2	1	1
15, 75	12	11	11	11	11	10	10	9	8	8	7	6	5	4	3	2	1
20, 70	15	15	14	14	14	13	12	11	11	10	9	7	6	5	4	3	1
25, 65	18	18	17	17	16	15	15	14	13	11	10	9	8	6	5	3	2
30, 60	20	20	19	19	18	17	17	15	14	13	12	10	9	7	5	4	2
35, 55	22	22	21	21	20	19	18	17	16	14	13	11	9	7	6	4	2
40, 50	23	23	22	22	21	20	19	18	16	15	13	11	10	8	6	4	2
45, 45	0023	0023	0023	0022	0021	0020	0019	0018	0017	0015	0013	0012	0010	0008	0006	0004	0002

GENERAL OUTLINE OF OBSERVATIONS, INSTRUMENTS AND METHODS

SECTION 9.—A GRADUATED SPHERE AND CO-ORDINATE FRAME FOR READING THE HELIOGRAPHIC LATITUDE AND LONGITUDE, AND THE INCLINATION OF THE DIRECTION OF THE SOLAR ROTATION CORRESPONDING TO AN OBSERVED POINT ON THE SOLAR DISC

BY

RALPH E. DE LURY

A graduated sphere which may be rotated on an axis through its equator, together with a co-ordinate frame capable of rotation in a plane through the centre of the sphere corresponding to the plane of which the radius vector of the earth's orbit is the pole, has been constructed in the machine-shop of the observatory, primarily for the purpose of reading directly, for an observed point on the solar disc, the heliographic latitude and longitude and the inclination of the direction of the sun's rotation to the line of observation. It was designed to eliminate much of the tedious computation involved in the large Ottawa series of observations of the solar rotation, many of which are perhaps unique in including the simultaneous observation of seven points on the solar disc together with comparison spectra[1].

The instrument is illustrated in the accompanying fig. 24. The sphere is a brass shell turned to an outside diameter of 228 mm., about the average diameter of the observed solar disc. The latitude lines are 1 degree apart, and the longitude lines 2 degrees apart. The ends of the axis through the equator of the sphere rest in a sturdy ring supported by four legs on a base plate. Inset in the ring in another ring graduated in degrees, which may be read to tenths with the aid of a vernier engraved on the former ring. The graduated position-angle ring has four blocks mounted at 0°, 90°, 180°, and 270°. On these blocks are mounted two co-ordinate plates intersecting above the centre of the sphere. The inner diameter of these plates is 228·6 mm. (9 inches), thus providing a clearance of 0·3 mm. between the sphere and the plates. The co-ordinate plates are graduated in degrees around the inner edge, numbered from the centre outward. The plates are 2 mm. thick, one of them, the "equatorial plate", with its middle vertically above the centre of the sphere, and the other, the "polar plate", with an edge vertically above the sphere's centre. The edges of the plates thus correspond to the positions of the four limb observations, two of which are on a diameter of the solar disc, while the other two are offset 1 mm. The intermediate points observed are offset again by 1 mm. from the equatorial plate, and two movable pointers, 1 mm. in thickness, correspond to the positions of these points. The finish is of dull nickel.

In operation, the sphere is turned (by means of handles on a shaft having a friction disc which bears against the bottom of the sphere) so that its polar axis is inclined at an angle B_o, the heliographic latitude of the centre of the solar disc at the time of observa-

[1] See fig. 9, preceding Section 3, p. 15.

tion, to the plane of the position-angle ring, which plane corresponds to the plane through the centre of the sun having as its pole the radius vector. The position-angle ring with its pair of co-ordinate plates is rotated to the position-angle α of the observation. The frame is thus set for reading the heliographic co-ordinates.

The distance r from the centre of the solar disc, expressed as a decimal of the radius of the disc, being known for each point of an observation, the heliocentric angle ρ, between the observer and the observed point, may be read for each point from the accompanying Table XXI. The value of ρ in each case is used as a "pointer" on the degree scale on the inner edge of the proper co-ordinate plate, for reading the latitude ϕ, and the longitude λ on the sphere, corresponding to each observed point. A duplicate set of readings may be made at position-angle α in the other quadrant, and a similar pair of readings by tilting the opposite pole of the solar axis to the angle B_o.

TABLE XXI—HELIOCENTRIC ANGLE, ρ, CORRESPONDING TO DISTANCE, r, FROM THE CENTER OF THE SOLAR DISC

r	ρ	r	ρ	r	ρ	r	ρ	r	ρ	r	ρ	r	ρ	r	ρ	r	ρ
0·	°	0·	°	0·	°	0·	°	0·	°	0·	°	0·	°	0·	°	0·	°
·020	1·14	310	17·98	510	30·53	710	45·06	901	64 05	921	66·83	941	69·97	961	73·69	981	78·55
40	2·28	20	18·58	20	31·19	20	45·86	2	·18	22	·97	42	70·14	62	·90	82	·85
60	3·42	30	19·18	30	31·86	30	46·69	3	·31	23	67·12	43	·31	63	74·11	83	79·16
80	4·57	40	19·78	40	32·54	40	47·53	4	·45	24	·27	44	·48	64	·32	84	·47
100	5·71	50	20·39	50	33·22	50	48·39	5	·58	25	·42	45	·65	65	·54	85	·80
20	6·86	60	21·00	60	33·91	60	49·26	6	·72	26	·57	46	·83	66	·76	86	80·14
40	8·01	70	21·62	70	34·60	70	50·15	7	·85	27	·73	47	71·01	67	·98	87	·49
60	9 17	80	22·23	80	35·30	80	51·05	8	·99	28	·88	48	·19	68	75·21	88	·85
80	10·32	90	22·85	90	36·00	90	51·97	9	65·13	29	68·03	49	·37	69	·44	89	81·23
200	11·48	400	23·67	600	36·71	800	52·92	910	·26	930	68·19	950	71·55	970	75·67	990	·62
10	12·07	10	24·09	10	37·43	10	53·88	11	·40	31	·34	51	·74	71	·91	91	82·04
20	12·65	20	24·72	20	38·15	20	54·87	12	·54	32	·50	52	·92	72	76·15	92	·48
30	13·24	30	25·35	30	38·88	30	55·88	13	·68	33	·66	53	72·11	73	·40	93	·96
40	13·82	40	25·99	40	39·62	40	56·92	14	·82	34	·82	54	·30	74	·65	94	83·45
50	14·41	50	26·62	50	40·37	50	57·98	15	·96	35	·98	55	·49	75	·90	95	84·01
60	15·01	60	27·26	60	41·12	60	59·09	16	66·10	36	69·14	56	·69	76	77·16	96	·60
70	15·59	70	27·91	70	41·89	70	60·23	17	·25	37	·30	57	·88	77	·43	97	85·29
80	16·18	80	28·56	80	42·66	80	61·41	18	·39	38	·47	58	73·08	78	·70	98	86·11
90	16·77	90	29·21	90	43·45	90	62·64	19	·53	39	·63	59	·28	79	·98	99	87·17
300	17·38	500	29·87	700	44·26	900	63·92	920	66·68	940	69·80	960	73·48	980	78·26	1·000	89·73

Careful reading with the aid of a hand magnifying lens gives sufficiently accurate latitude values, but the longitude readings for points at the higher latitudes need to be supplemented by computation, using the equation

$$\sin \lambda = \sin \rho \cos \alpha \sec \phi,$$

employing the values of ϕ read from the sphere, and multiplying the values of $\sin \rho$, corresponding to the values of r, from Table XVIII, by the values of $\cos \alpha \sec \phi$ from Table XIX of the preceding Section 8.

Having obtained the longitude, λ, of an observed point, the angle, γ_o, between the radius vector and the pole of the plane containing the solar axis and the observed point, may be read directly by two methods:

(1) from the degree scale on the edge of the polar plate which has been rotated to cut the equator at longitude $90° - \lambda$, the inclination of the axis of the sphere being left undisturbed at angle B_o, as is evident from equation (8) of the preceding Section, namely

$$\cos \gamma_o = \cos B_o \sin \lambda ;$$

(2) since γ_o is also the angle between the plane of the position-angle ring and the longitude plane containing the observed point, it may be read from the degree scale of the polar plate at the point where the longitude line, λ, is at the maximum angle on the plate from the plane of the position-angle ring, easily determined by rotating the co-ordinate frame back and forth about the maximum point. The value of $\cos \gamma_o$ is now increased by the selected value of the addendum of Table XX, preceding Section, to yield the required $\cos \gamma$, that is, the cosine of the angle between the direction of the solar rotation at the observed point and the line from the point to the observer, the required component of the rotational velocity, V, being $V \cos \gamma$.

Less simply, the angle γ may also be read directly by the two methods of reading γ_o:

(1) with settings $B_o \pm rs \sin a$ and $90° - \lambda - r s \cos a$ (s being the angular semi-diameter of the sun, B_o, which never exceeds $7°{\cdot}25$, may be used instead of $B_o \pm r s \sin a$ without appreciable error, as explained in the preceding Section);

(2) by measuring on the polar plate the maximum angle between the longitude line $\lambda + r s \cos a$, and the plane of the position-angle ring, the sphere being set with its axis tilted $B_o \pm r s \sin a$, or with sufficient accuracy it may be left at the setting B_o.

The following example illustrates the application of the sphere to the determinations of one of the simpler earlier observations, in comparison with the computations by logarithms and by the use of the tables of the preceding Section:

Plate L 833 (1)–(7), August 10, 1911, $11^h\ 20^m\ 35^s$, E.S.T.

$$r = 0{\cdot}930, \text{ and } B_o = 6°{\cdot}4.$$

(Two observed points within the solar limb, at equal distances r from the centre of the solar disc, on diameters of the disc at position-angle a, at intervals of 15° from 0° to 90°.)

I. BY LOGARITHMS:

α	(1) 0°	(2) 15°		(3) 30°		(4) 45°		(5) 60°		(6) 75°	
Log sin α.....	.....	9·41300	.....	9·69897	.....	9·84949	.....	9·93753	.....	9·98494	
Log cos B_0.....	9·99729										
Log sin ρ.....	9·96773										
	9·96502	9·37802	.....	9·66399	.....	9·81451	.....	9·90255	.....	9·94996	
	·9226	·2388	.....	·4613	.....	·6524	.....	·7990	.....	·8912	
Log sin B_0.....	9·04715										
Log cos ρ.....	9·5701										
	8·61725										
sin ϕ.....	·0414	·2802	·1974	·5027	·4199	·6938	·6110	·8404	·7576	·9326	·8498
ϕ.....	2° 22′	16° 16′	11° 23′	30° 11,	24° 50′	43° 56′	37° 40′	57° 11′	49° 15′	68° 51′	58° 11′
Log sec ϕ.....	10·00037	10·01774	·00862	10·06327	·04214	10·14258	·10151	10·26604	·18525	10·44272	·27802
Log cos α.....	10·00000	9·98494	·98494	9·93753	·93753	9·84949	·84949	9·69897	·69897	9·41300	·41300
Log sin ρ.....	9·96773										
Log sin λ.....	9·96810	9·97041	·96129	9·96853	·94740	9·95980	·91873	9·93274	·85195	9·82345	·65875
λ.....	68° 18′	69° 7′	66° 10′	68° 27′	62° 22	65° 44′	56° 2′	58° 56′	45° 20′	41° 45′	27° 7′
Log cos γ_0.....	9·96539	9·96770	·95858	9·96582	·94469	9·95709	·91602	9·93003	·84924	9·82074	·6560
cos γ_0.....	·9234	·9283	·9090	·9243	·8804	·9059	·8242	·8512	·7067	·6618	·4529
Addendum.....	·0016	·0016	·0016	·0016	·0017	·0013	·0017	·0012	·0015	·0008	·0010
cos γ.....	·9250	·9299	·9106	·9259	·8821	·9072	·8259	·8524	·7082	·6626	·4539

II. USING TABLES:

sin α.....	0·0000	0·2588	.....	·5000	.....	·7071	.....	·8660	.....	·9659	
cos B_0 sin ρ sin α	0·0000	·2388	.....	·4613	.....	·6524	.....	·7990	.....	·8911	
sin B_0 cos ρ.....	0·0414										
sin ϕ	0·0414	·2802	·1974	·5027	·4199	·6938	·6110	·8404	·7576	·9325	·8497
ϕ.....	2° 22′	16° 16′	11° 23′	30° 11′	24° 50′	43° 56′	37° 40′	57° 11—	49° 15′	68° 50′	58° 11′
cos α sec ϕ.....	1·0009	1·0063	·9854	1·0019	·9541	·9818	·8934	·9225	·7660	·7167	·4909
sin λ	·9292	·9352	·9148	·9302	·8858	·9115	·8294	·8564	·7111	·6654	·4557
λ.....	68° 19′	69° 6′	66° 11′	68° 28′	62° 21′	65° 43′	56° 2′	58° 55′	45° 19′	41° 43′	27° 7′
cos γ_0.....	·9234	·9284	·9091	·9244	·8804	·9058	·8243	18511	·7067	·6612	·4529
Addendum	·0016	·0016	·0016	·0016	·0017	·0013	·0017	·0012	·0015	·0008	·0010
cos γ	·9250	·9300	·9107	·9260	·8821	·9071	·8260	·8523	·7082	6620	·4539

III. USING SPHERE:

ϕ.....	2·4	16·3	11·4	30·2	24·8	44·0	37·6	57·3	49·3	68·9	58·2
	2·35	16·3	11·4	30·2	24·85	43·9	37·7	57·2	49·3	68·85	58·2
λ.....	68·2	69·1	66·3	68·4	62·3	65·8	56·0	58·9	45·3	41·7	27·3
	68·3	69·1	66·2	68·5	62·4	65·7	56·0	58·9	45·3	41·7	27·1
$r\ s$ cos α.....	15′	14′	14′	13′	13′	12′	12′	7′	7′	4′	4′
$\lambda + r\ s \cos \alpha$.....	68° 36′	69° 20′	66° 29′	68° 40′	62° 34′	65° 57′	56° 12′	59° 1′	45° 25′	41° 46′	27° 16′
γ.....	22° 15′	21° 40′	24° 16′	22° 11′	28° 8′	24° 48′	34° 21′	31° 35′	44° 56′	48° 32′	62° 56′
cos γ.....	·9255	·9293	·9116	·9260	·8819	·9078	·8256	·8519	·7079	·6622	·4550

The means of duplicate readings on the sphere are in close agreement with the computed values of ϕ, λ, and γ, as indicated by the above comparative results. In the solar rotation work it is not necessary to determine the longitudes, except for one or two plates in a series, so that the values of the addenda may be read from Table XX. A satisfactory and speedy method in this work is to read the values of ϕ from the sphere, and from these values, selecting the products $\cos \alpha \sec \phi$ and multiplying by $\cos B_0 \sin \rho$ from Tables XIX and XVIII of the preceding Section, to determine $\cos \gamma_0$, which with the addendum of Table XX yields $\cos \gamma$. The use of either the tables or the sphere effects a great saving of time in comparison with the use of logarithms.

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FIG. 24.—Graduated Sphere and Co-ordinate Frame for Heliographic Readings.