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A Spectrographic Study of Stars of Classes A and F

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A SPECTROGRAPHIC STUDY OF STARS OF CLASSES A AND F

BY F. HENROTEAU, PH.D.

The purpose of the present study was to see whether any stars of the β Canis Majoris type could be found among the later spectral classes. The discovery at the Yerkes Observatory that τ Cygni (class F) is of this type shows that very short-period radial velocity oscillations are not limited to stars of class B. Since stars of class B, however, are to be classified among the giants or semi-giants, and also since the absolute magnitude of τ Cygni is 1·9 (semi-giant), it seemed advisable to search among the stars of neighbouring absolute magnitudes (stars probably of low density and large volume where pulsations or tidal effects would happen on a large scale) to find whether among them there might be others of the β Canis Majoris type.

The stars given in the following table were observed: the absolute magnitudes given are those determined by Dr. W. S. Adams at the Mount Wilson Observatory.[1]

It must be mentioned that the spectrograms of these stars secured here were made with a wide slit. With such a slit a great gain was obtained in the time of exposure, though perhaps to some extent at the expense of accuracy.

Most of the spectrograms were measured on a Hartmann spectro-comparator, using a spectrogram of α Cygni as standard. This standard spectrogram was divided into thirteen sections numbered from 1 to 13, corresponding to micrometer readings (half millimeter unit scale) $35+(n-1)\times 2{\cdot}5$, where n is the number of the section. In the detailed measures of the plates f is the constant by which to multiply the sum of all the differences, V the velocity indicated by the standard spectrogram, Vs the velocity obtained, not corrected for the motions of the earth, while Va and Vd are as usual the corrections due, respectively, to the earth's revolution around the Sun and the earth's rotation.

The assistance of Mr. J. F. Frédette and of Mr. W. A. Thorn is acknowledged.

TABLE OF STARS INVESTIGATED

H.R.	Star	R.A. (1900)	Decl. (1900)	Visual Magnitude	Absolute Magnitude	Spectrum
		h m	° ′			
2484	ξ Geminorum	6 39·7	+13 0	3·40	2·0	F5
3569	ι Ursae Majoris	8 52·4	+48 26	3·12	1·9	A5
3888	υ Ursae Majoris	9 43·9	+59 31	3·89	1·5	F
4031	ζ Leonis	10 11·1	+23 55	3·65	−0·1	F
4662	γ Corvi	12 10·7	−16 59	2·78	—	B8
5055	ζ_2 Ursae Majoris	13 19·9	+55 26	3·96	1·9	A2
5435	γ Bootis	14 28·1	+38 45	3·00	1·8	F
5487	μ Virginis	14 37·8	− 5 13	3·95	2·4	F5
5733	μ Bootis	15 20·7	+37 44	4·47	2·2	F
6095	γ Herculis	16 17·5	+19 23	3·79	1·4	F
7377	δ Aquilae	19 20·5	+ 2 55	3·44	1·8	F

The individual studies of these stars follow.

[1]Contributions from the Mount Wilson Obs., No. 199.

ξ GEMINORUM

The spectrograms of ξ Geminorum are none too good for measurement. They give the following results:

OTTAWA RADIAL VELOCITIES OF ξ GEMINORUM

Plate	Date	Velocity	Remarks
		km.	
9499	1921 Feb. 18·548	+14·5	
9500	·570	+14·4	
9504	Feb. 20·540	+17·0	poor.
9506	·584	+ 9·6	weak.
9509	·654	+12·4	

The spectrum of ξ Geminorum is rather complex. Professor Küstner [2] in Bonn suspected that two spectra are superposed; he considered the star as being possibly a close visual double. Our velocities are too poor to indicate a short-period radial velocity variation; the velocity in February, 1921, is, however, very different from velocities obtained formerly, which are given in the following table. This gives weight to the supposition that the star is a close double, which would probably be an interesting object for measuring by the interferometer method.

RADIAL VELOCITIES OF ξ GEMINORUM

(Other Observatories)

Place	Author	Date	Velocity	Remarks
			km.	
Bonn	Küstner	1910 Mar. 6·33	+29·7	Küstner considers the velocity as constant.
		1911 Jan. 15·43	+31·6	
		April 4·32	+24·6	
		7·31	+30·2	
		8·32	+29·0	
		13·30	+20·5	
		14·30	+23·6	
		15·30	+25·0	
Vienna	Hnatek	1913 Mar. 7·300	+29·5	
		11·392	+20·4	
		12·287	+25·8	

[2] A.N. No. 4750.

ι URSAE MAJORIS

The spectrum of ι Ursae Majoris is very poor. The lines are very wide and diffuse and the radial velocities are far from reliable.

The velocities obtained here are:

RADIAL VELOCITIES OF ι URSAE MAJORIS

Plate	Date	Velocity
		km.
10260	1922 Feb. 7 ·739	+14·4
10261	·765	− 9·0
10262	·787	− 5·5
10263	·810	+11·6
10265	·853	+ 4·5
10266	·876	+14·3
10268	·919	+ 1·2
10269	·942	+ 4·0

This gives a mean velocity of + 4·4 km.

The velocities obtained formerly by Hnatek are:—

Date	Velocity
	km.
1913 Mar. 7 ·447	+10·2
12 ·449	+11·9
14 ·422	+12·8

υ URSAE MAJORIS

The spectrum of υ Ursae Majoris is among the poorest encountered. The velocities obtained here for this star have therefore to be given a very small weight. They are:—

RADIAL VELOCITIES OF υ URSAE MAJORIS

Plate	Date	Velocity
		km.
10278	1922 Feb. 13 ·722	−11·1
10279	·751	−23·5
10280	·781	+ 7·4
10281	·810	− 0·7

This gives a mean velocity of −7·0 km.

The following velocities have been given by Hnatek:[3]

Date	Velocity
	km.
1913 May 25 ·402	+26·5
26 ·382	+27·4

υ Ursae Majoris is a visual double star suspected to be a physical system.

ζ LEONIS

ζ Leonis was discovered to be a spectroscopic binary by Mr. Wilson at the Lick Observatory.[4] The following velocities were obtained there.

LICK OBSERVATORY VELOCITIES OF ζ LEONIS

Date	Velocity
	km.
1899 Feb. 21 ·933	− 8·0
1905 Feb. 28 ·738	−26·7
1906 Jan. 26 ·965	−20·2
1910 Nov. 30 ·083	− 5·5
Dec. 5 ·105	− 7·6
1911 Jan. 3 ·019	−10·7

To this has to be added a velocity of −17·3 km. obtained by J. B. Cannon[5] on 1917, March 26·731.

[3] A. N. Vol. 196, p. 384.
[4] L. O. B., Vol. VI, p. 145.
[5] Pub. Dom. Obs., Vol. IV, p. 255.

The velocities obtained here in 1922 are the following:—

OTTAWA VELOCITIES OF ζ LEONIS

Plate	Date	Velocity
		km.
10542	1922 April 21 ·551	−54·2
10543	·574	−57·5
10546	·637	−50·5
10547	·656	−55·5
10553	23 ·529	−48·8
10554	·549	−55·2
10555	·569	−49·4
10556	·590	−59·0
10557	·610	−55·3
10558	·630	−55·0
10559	·650	−53·1

ζ Leonis is evidently not a short-period binary. The large negative velocity obtained here is remarkable and indicates that the amplitude of the velocity variation is at least 50 kilometers.

Following are the detailed measures of two of the plates:—

Plate 10553
1922 April 23·529
f = 0·05206
V = −31·64

Section	Difference Star-Comparison	
	Direct	Reversed
3	0	+ 3
4	− 2	+ 8
5	0	0
6	+17	+ 8
7	+17	0
8	0	+10
9	+12	+27
10	+ 5	+19
11	+ 8	+14
12	+14	+22
13	− 5	0
Sum	+66	+111

Vs	−22·43
Va	−26·45
Vd	+ 0·04
Radial Velocity	−48·84

Plate 10556
1922 April 23·590
f = 0·05206
V = −31·64

Section	Difference Star-Comparison	
	Direct	Reversed
3	+ 1	−13
4	0	+ 7
5	−12	−14
6	− 6	+12
7	− 3	+ 3
8	−14	+ 1
9	− 5	− 3
10	+ 6	+ 3
11	0	− 7
12	+ 3	+18
13	+ 1	+ 7
Sum	−29	+14

Vs	−32·42
Va	−26·46
Vd	− 0·07
Radial velocity	−58·95

γ CORVI

γ Corvi was found to be a spectroscopic binary by Dr. W. W. Campbell and Dr. H. D. Curtis.

The radial velocities obtained here for this star are:—

RADIAL VELOCITIES OF γ CORVI

Plate	Date	Velocity
		km.
10509	1922 April 10·581	0·0
10510	·603	− 7·4
10511	16·535	−16·4
10512	·550	−10·2
10513	·565	−21·6
10514	·581	−15·1
10515	·594	−18·2
10516	·605	−12·3
10517	·618	−15·6
10518	·628	−13·6
10519	·638	−20·4
10520	·649	−20·1

The star is probably not of period shorter than one day. The period must, however, be fairly short. The radial velocities published formerly are:—

Place	Authors	Date	Velocity
			km.
Yerkes	Frost and Adams	1902 April 2·688	− 7·4
		3·688	− 7·0
		19·710	− 6·1
Lick	Campbell and Curtis	1902 Dec. 30	+ 2
		1903 Feb. 23	0
		May 6	−20
		May 11	− 6
		May 17	+ 4

The detailed velocities of two of our plates are given here:—

Plate 10510 1922 April 10·603 f=0·06172 V=−31·64			Plate 10519 1922 April 16·638 f=0·06172 V=−31·64		
Section	Difference Star-Comparison		Section	Difference Star-Comparison	
	Direct	Reversed		Direct	Reversed
3	+42	+37	3	+ 5	+16
4	+23	+28	4	+ 5	+ 9
5	+30	+26	5	+16	+15
6	+18	+20	6	+20	+18
7	+17	+ 9	7	+27	+20
8	+18	+28	8	+30	+21
9	+35	+32	9	+26	+13
10	+23	+34	10	+14	+11
11	+28	+25	11	+19	+25
Sum.............	+234	+239	Sum.............	+162	+148
Vs	− 2·44		Vs	−12·51	
Va	− 5·06		Va	− 7·95	
Vd	+ 0·12		Vd	+ 0·02	
Radial velocity	− 7·38		Radial velocity	−20·44	

ζ_2 URSAE MAJORIS

The lines in our spectrograms of ζ_2 Ursae Majoris are rather diffuse, although fairly good for spectra of class A. The velocities obtained are:—

RADIAL VELOCITIES OF ζ_2 URSAE MAJORIS

Plate	Date	Velocity
		km.
10477	1922 April 2·561..	−21·2
10478	·590..	−21·8
10479	·618..	−17·7
10480	·647..	−23·5
10481	·675..	−21·0
10482	·703..	−31·0
10489	April 3·540..	−27·4
10490	·569..	−25·6

Considering the large velocity of plate 10482 as accidental it seems likely that the star is a spectroscopic binary of fairly short period, but not shorter than one day.

The velocities obtained formerly by Ludendorff at Potsdam[6] are:—

Date	Velocity
	km.
1905 April 20·396	−13·0
May 16·375	−12·0
1906 April 10·400	−14·0
May 4·379	− 9·6
5·375	− 7·3
6·366	−13·0
7·377	−14·0
14·362	− 7·9
1907 April 20·341	−10·0
25·370	−16·4
26·341	−11·7
28·341	−15·1
May 12·362	−17·7
14·345	−13·4

Frost and Lee also give a range of velocity of −17 to +10 km.[7]

The detailed measures of two of our plates are:—

Plate 10477
1922 April 2·561
f=0·05206
V=−31·64

Section	Difference Star-Comparison	
	Direct	Reversed
3	+19	+20
4	+ 8	+12
5	+ 7	+10
6	− 1	+11
7	+ 7	+ 7
8	+ 7	+12
9	+20	+28
10	+15	+26
11	+18	+27
12	+15	+26
13	+15	+34
Sum	+130	+213

Vs	−13·78
Va	− 7·55
Vd	+ 0·17
Radial velocity	−21·16

Plate 10478
1922 April 2·590
f=0·05206
V=−31·64

Section	Difference Star-Comparison	
	Direct	Reversed
3	+19	+23
4	+14	+ 9
5	+ 3	+23
6	+17	+18
7	+ 2	+ 9
8	+ 9	+ 7
9	0	+28
10	+20	+35
11	+20	+27
12	+13	+18
13	+ 1	+16
Sum	+118	+213

Vs	−14·41
Va	− 7·56
Vd	+ 0·15
Radial velocity	−21·82

[6] A.N. Vol. 177, p. 9.
[7] A. N. Vol. 177, p. 171.

γ BOOTIS

Professor P. Guthnick,[8] Director of the Babelsberg Observatory, has investigated γ Bootis with the photo-electric cell. He found it to be a variable of a very small amplitude, $0^{m}\cdot 05$, and very short period, $0^{d}\cdot 290313$. The probabilities are then that γ Bootis is of the β Canis Majoris type. Unfortunately its spectrum is very poor, the lines being wide and diffuse. Only one radial velocity, -35 km., was given before for this star by Dr. Campbell.[9]

The velocities we have obtained are the following:—

RADIAL VELOCITIES OF γ BOOTIS

Plate	Date	Velocity	Phase, taking 1922 Mar. 25·762 as origin
		km.	
10472	1922 Mar. 25·809	−48·8	$0^{d}\cdot 047$
10648	May 14·590	−48·3	0·184
10658	·833	−46·5	0·137
10659	May 15·573	−63·6	0·006
10660	·581	−60·8	0·014
10662	·610	−49·1	0·043

A variation of radial velocity is possible. The phases are given for the period $0^{d}\cdot 290313$; a curve could pass through the plotted observations; there are, however, too few observations to say with certainty whether the radial velocity variation admits of the above period; the spectrum is also too poor.

Following are the detailed measures of two of the plates:—

Plate 10472
1922 Mar. 25·809
f = 0·11457
V = −31·64

Section	Difference Star-Comparison	
	Direct	Reversed
6	−40	−38
7	−44	−16
8	−42	− 9
9	−38	+17
10	+ 3	+20
Sum	−161	−26
Vs	−53·07	
Va	+ 4·25	
Vd	0·00	
Radial velocity	−48·82	

Plate 10660
1922 May 15·581
f = 0·05818
V = −31·64

Section	Difference Star-Comparison	
	Direct	Reversed
4	−20	+22
5	−26	+40
6	−14	− 2
7	−34	−11
8	−32	− 4
9	−30	−10
10	−59	−24
11	−23	− 9
12	−47	− 4
13	−15	− 1
Sum	−300	− 3
Vs	−49·27	
Va	−11·61	
Vd	+ 0·13	
Radial velocity	−60·75	

These measures are average ones; they show that the spectrograms are in the main very poor.

[8] Veröff. der U. Sternwarte zu Berlin-Babelsberg Bd. II Heft 3/p. 128.

[9] L. O. B. Vol. 7, p. 116.

μ VIRGINIS

Only three spectrograms have been secured of this star; they give the following velocities:—

RADIAL VELOCITIES OF μ VIRGINIS

Plate	Date	Velocity
		km.
10595	1922 April 30·549	−21·6
10596	·577	− 5·5
10597	·603	−11·1

A short-period radial velocity variation might be suspected but more plates will be needed to ascertain this, especially since the lines are wide and diffuse.

The star was announced to be a spectroscopic binary by Dr. Campbell[10] (discovered by Mr. Burns). The Lick Observatory velocities are the following:—

Date	Velocity
	km.
1898 April 15·861	+ 6
1899 Feb. 15·013	0
May 2·802	+ 1
1904 May 23·786	− 2
1905 April 10·840	+17
1906 April 2·974	+ 5
1907 April 8·921	+11
1910 Feb. 11·097	+ 2
1911 April 20·815	+ 1
30·852	+ 6

μ BOOTIS

The spectrum of μ Bootis is very poor and difficult to measure. The lines are wide and diffuse.

Five spectrograms were secured on one night, but only three were measurable. They give the following velocities:—

[10] L. O. B. Vol. VI, p. 146

RADIAL VELOCITIES OF μ BOOTIS

Plate	Date	Velocity
		km.
10289	1922 Feb. 15·812	−29·2
10290	·853	−17·2
10291	·895	−18·8

Velocities obtained formerly are:—

Place	Author	Date	Velocity
			km.
Allegheny	F. C. Jordan[11]	1909 Mar. 26·790	−13·3
		April 15·771	−19·4
		May 11·751	−10·6
		June 6·693	−15·1

A real change in radial velocity is doubtful.

γ HERCULIS

Dr. W. W. Campbell gave one radial velocity, −39 km., for this star.

The velocities obtained here are:—

RADIAL VELOCITIES OF γ HERCULIS

Plate	Date	Velocity	Remarks
		km.	
10590	1922 April 28·549	−40·2	weak
10591	·578	−49·3	poor
10592	·604	−62·0	very poor
10593	·627	−46·2	

The spectrum of γ Herculis having very wide and diffuse lines a variation of radial velocity is doubtful.

[11] Pub. Allegheny Obs. Vol. II, p. 124.

The detailed measures are:—

Plate 10590
1922 April 28·549
f=0·05206
V=−31·64

Section	Difference Star-Comparison Direct	Difference Star-Comparison Reversed
3	−12	+ 2
4	−36	−19
5	−11	−50
6	−38	−31
7	−15	−15
8	−20	− 5
9	− 6	+ 3
10	−10	−13
11	−22	−19
12	+21	−19
13	− 7	− 2
Sum	−156	−168

Vs	−48·5
Va	+ 8·0
Vd	+ 0·3
Radial velocity	−40·2

Plate 10591
1922 April 28·578
f=0·05206
V=−31·64

Section	Difference Star-Comparison Direct	Difference Star-Comparison Reversed
3	−50	−39
4	−28	−23
5	−33	−47
6	− 3	−44
7	−24	−17
8	−47	−27
9	−12	−34
10	−14	−31
11	+ 8	−12
12	+13	− 7
13	−15	−13
Sum	−205	−294

Vs	−57·6
Va	+ 8·0
Vd	+ 0·3
Radial velocity	−49·3

Plate 10592
1922 April 28·604
f=0·05206
V=−31·64

Section	Difference Star-Comparison Direct	Difference Star-Comparison Reversed
3	−66	−36
4	−41	−42
5	−33	−32
6	−13	−38
7	−22	−42
8	−24	−52
9	−29	−20
10	−27	−35
11	−24	−56
12	−33	−33
13	−41	− 5
Sum	−353	−391

Vs	−70·3
Va	+ 8·0
Vd	+ 0·3
Radial velocity	−62·0

Plate 10593
1922 April 28·627
f=0·05206
V=−31·64

Section	Difference Star-Comparison Direct	Difference Star-Comparison Reversed
3	−68	−26
4	−49	−28
5	−58	−17
6	−30	−18
7	−19	−14
8	−26	− 9
9	− 8	− 6
10	+ 4	+ 2
11	0	−24
12	+16	− 9
13	−30	−23
Sum	−268	−172

Vs	−54·5
Va	+ 8·0
Vd	+ 0·3
Radial velocity	−46·2

δ AQUILAE

The lines in the spectrum of δ Aquilae are rather diffuse and difficult to measure. However, it shows a decided range of velocity variation. The star was discovered to be of variable velocity by Dr. Campbell and Dr. Curtis at the Lick Observatory.[12] It was afterwards investigated by Mr. Parker, who secured and measured thirty-four spectrograms, but failed to find a period for the radial velocity variation. Spectrograms have been secured here on five different nights and their measures indicate a very short period of about $0^h \cdot 1571$; it can thus be classified among the stars of the β Canis Majoris type. At the request of the writer, Mr. Otto Struve at the Yerkes Observatory, after securing and measuring a series of spectrograms, confirmed the fact that its radial velocity variation is of very short period.

The case of δ Aquilae is thus similar to that of τ Cygni. The spectra are almost identical; their absolute magnitudes are 1·8 and 1·9, respectively, and they are both situated in dense regions of the Milky Way.

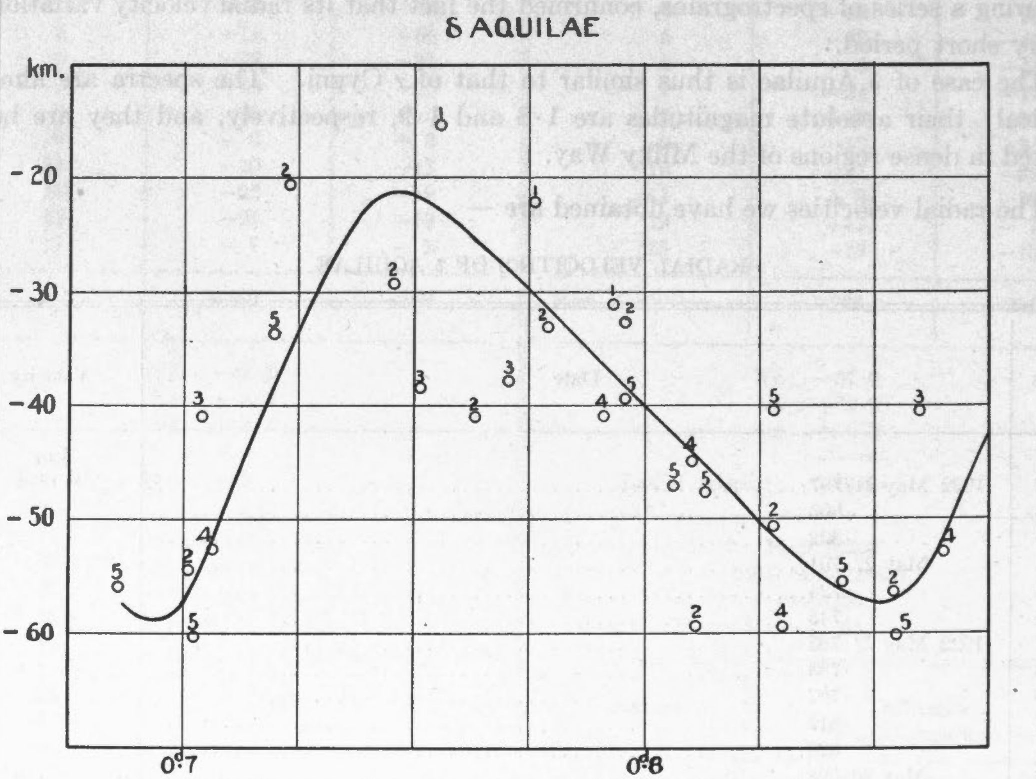
The radial velocities we have obtained are:—

RADIAL VELOCITIES OF δ AQUILAE

Plate	Date	Velocity
		km.
10672	1922 May 21·787	−15·4
10673	·806	−22·7
10674	·822	−31·2
10704	May 27·701	−55·8
10705	·724	−20·8
10706	·745	−29·6
10707	1922 May 27·763	−41·0
10708	·783	−34·0
10709	·797	−32·9
10710	·812	−59·5
10711	·828	−50·5
10724	May 30·688	−41·0
10725	·710	−29·8
10726	·733	−38·8
10727	·754	−37·5
10739	June 14·699	−41·1
10740	·720	−45·4
10741	·741	−59·4
10742	·773	−53·3
10743	·794	−53·3
10744	June 16·661	−47·4
10745	·681	−40·5
10746	·697	−55·5
10747	·712	−60·1
10748	·729	−33·9
10752	·806	−39·5
10753	·824	−48·0

[12] Ap. J. Vol. XVIII, p. 308.

If the above velocities are plotted, taking the time as abscissa and using the period 0d·1571, one can see, account being taken of the errors of observation, that they have a tendency to follow a curve of rather steep ascending branch (see figure). In the figure the five different nights are represented by the numbers 1 to 5.



Complete Observations of 1922 plotted on those of May 27th.

The detailed measures of the plates of May 27th are given in the following tables:—

Plate 10704
1922 May 27·701
f=0·05206
V=−31·64

Section	Difference Star-Comparison Direct	Difference Star-Comparison Reversed
3	−61	−57
4	−54	−60
5	−65	−27
6	−52	−31
7	−67	−17
8	−61	−41
9	−29	−21
10	−39	−20
11	−35	−12
12	−15	− 7
13	−43	−27
Sum.............	−521	−320

Vs	−75·42
Va	+19·39
Vd	+ 0·25
Radial velocity	−55·78

Plate 10705
1922 May 27·724
f=0·05206
V=−31·64

Section	Difference Star-Comparison Direct	Difference Star-Comparison Reversed
3	−26	+ 4
4	−26	− 8
5	−21	− 4
6	−22	−11
7	−17	− 3
8	−19	−15
9	− 9	− 1
10	− 8	+ 1
11	−10	+14
12	0	+16
13	0	− 4
Sum.............	−158	−11

Vs	−40·44
Va	+19·39
Vd	+ 0·21
Radial velocity	−20·84

Plate 10706
1922 May 27·745
f=0·05206
V=−31·64

Section	Difference Star-Comparison Direct	Difference Star-Comparison Reversed
3	−19	−14
4	−28	−25
5	−30	−18
6	−41	0
7	−31	− 8
8	−39	− 1
9	− 8	− 1
10	−17	−12
11	−16	− 4
12	−10	− 1
13	−14	− 1
Sum.............	−253	−85

Vs	−49·24
Va	+19·38
Vd	+ 0·18
Radial velocity	−29·58

Plate 10707
1922 May 27·763
f=0·05206
V=−31·64

Section	Difference Star-Comparison Direct	Difference Star-Comparison Reversed
3	−30	−26
4	−57	−21
5	−53	− 4
6	−20	− 5
7	−33	−10
8	−27	− 6
9	−43	−13
10	−40	−26
11	−33	−22
12	−34	−16
13	−36	− 1
Sum.............	−406	−150

Vs	−60·58
Va	+19·38
Vd	+ 0·15
Radial velocity	−41·05

Plate 10708
1922 May 27·781
f = 0·05206
V = −31·64

Section	Difference Star-Comparison Direct	Reversed
3	−36	−21
4	−44	−24
5	−47	−14
6	−41	−14
7	−28	−19
8	−26	−18
9	−28	− 6
10	−19	− 7
11	+ 1	+ 2
12	− 8	− 1
13	− 9	−12
Sum.............	−285	−134

Vs −53·45
Va +19·37
Vd + 0·11

Radial velocity −33·97

Plate 10709
1922 May 27·797
f = 0·05206
V = −31·64

Section	Difference Star-Comparison Direct	Reversed
3	−37	−14
4	−30	− 2
5	−49	− 9
6	−36	− 2
7	−33	− 9
8	−15	−12
9	−13	−24
10	−30	−13
11	−17	− 2
12	−18	−10
13	− 8	−15
Sum.............	−286	−112

Vs −52·36
Va +19·37
Vd + 0·08

Radial velocity −32·91

Plate 10710
1922 May 27·812
f = 0·05206
V = −31·64

Section	Difference Star-Comparison Direct	Reversed
3	−41	−37
4	−57	−43
5	−70	−53
6	−56	−49
7	−39	−27
8	−70	−22
9	−33	−54
10	−38	−14
11	−39	−42
12	−47	−34
13	−35	− 8
Sum.............	−525	−383

Vs −78·91
Va +19·36
Vd + 0·05

Radial velocity −59·50

Plate 10711
1922 May 27·828
f = 0·05206
V = −31·64

Section	Difference Star-Comparison Direct	Reversed
3	−39	−33
4	−39	−32
5	−44	−33
6	−29	−46
7	−38	−41
8	−52	−44
9	−30	−29
10	−41	−20
11	−30	−22
12	−31	−25
13	−28	− 8
Sum.............	−401	−333

Vs −69·85
Va +19·36
Vd + 0·02

Radial velocity −50·47

Following are given the velocities obtained previously both here and elsewhere

RADIAL VELOCITIES OF δ AQUILAE (Former Values)

Place	Author	Date	Velocity	Remarks
			km.	
Lick		1900 May 22	−25	
		1902 July 31	−35	
		1903 May 12	2	
		May 27	−32	
Potsdam	Ludendorff	1904 Sept. 5·317	−31	
		" 6·321	−60	
		" 16·279	−47	
		" 18·271	−33	
Ottawa	Parker	1906 Aug. 6·73	−41·8	
		" 15·65	−45·2	
		" 24·65	−45·1	
		Sept 10·64	−25·0	
		" 27·61	−29·0	
		Oct. 23·57	−39·7	
		1907 May 31·79	−29·9	
		June 10·80	−42·8	
		July 2·76	−19·5	
		" 8·75	−28·0	
		" 9·68	−15·6	
		" 10·68	−18·5	
		" 25·68	−16·5	
		Aug. 3·61	−25·7	
		" 5·68	−29·4	
		Sept. 6·65	−25·4	
		" 18·58	−27·0	Mean of two measures.
		1908 May 18·83	−21·9	
		" 22·85	−40·1	
		June 3·83	−29·9	Mean of two measures.
		" 5·95	−35·5	
		" 24·77	−29·5	
		" 26·78	−39·8	
		" 27·75	−40·9	
		July 3·77	−21·9	
		" 8·75	−36·4	
		" 10·77	−26·1	
		" 11·77	−34·6	
		" 13·78	−47·0	
		" 31·69	−30·7	
		" 31·72	−39·1	
		Aug. 5·75	−29·7	
		" 15·73	−31·9	
		" 27·62	−39·5	

The present investigation was not very fruitful in detecting new stars of the β Canis Majoris type that are not of class B. At present only three such stars are known; τ Cygni discovered by Paraskévopoulos, γ Ursae Minoris, discovered by Otto Struve, and δ Aquilae. (Possible additions might be γ Bootis and β Ursae Majoris).[13]

[13] L. O. B. Vol. IX, p. 22.

The remarkable fact about the above three stars is the extreme shortness of their periods, three hours more or less. For γ Ursae Minoris, Struve gives the period as $2^h\ 36^m\ 10^s$; compared with similar stars of class B, where known periods range from about $4^h\ 30^m$ to beyond 6^h, it is much shorter. Also the lines in these three stars are rather wide and diffuse while in early class B stars they attain a higher degree of sharpness, the spectrum of β Canis Majoris, for instance, being among the best.

In a recent investigation on a method of deriving the distances of the A-type stars[14] by W. S. Adams and A. H. Joy, it is said:—

"There seems to be little doubt that great intrinsic luminosity is associated with sharp and narrow lines in all spectral types and that this is due to the *low density* in the atmosphere of such stars. In stars of various spectral types, such as β Orionis, α Cygni, the Cepheid variables and α Orionis, this effect is very marked, and the change in the character of the lines of the variable star *o* Ceti from maximum when they are well defined, to minimum when they are diffuse and vague, is an excellent illustration of the same phenomenon."

Taking account of this, if we consider the variations in Cepheids as being due to the same cause as the variations in stars of the β Canis Majoris type we must come to the conclusion that the longer the period of radial velocity oscillation, the smaller the density. Along these lines the following table seems to point naturally in that direction.

Type of Variable	Spectral Class	Length of Period (approximate)	Character of Lines
Cepheid	Giant G	35 to 5 days	Sharp.
"	Giant F	16 to 3 days or less	Sharp, perhaps a little more diffuse for some short periods.
β Canis Majoris	B	8^h to 4^h30^m	Fairly sharp, sometimes a little diffuse.
"	A and F	4^h to 2^h30^m	Wide and diffuse.

The period decreases with the advance in the age of the star, (adopting Russell's theory of evolution) while for real spectroscopic binaries it is usually the reverse that occurs.[15] The above table suggests, then, the probability that:—

The period is a function of one variable, the mean density of the star alone; or perhaps a function of two variables, mean density and mass of the star, the variation of the mean density being by far the most important factor in making the period vary.

To explain this theoretically one might consider for instance that a rotating star assumes the form of a Jacobian ellipsoid not very far from the point of bifurcation between McLaurin and Jacobian ellipsoids, such as was assumed in a theoretical study of σ Scorpii.[16] For such a Jacobian ellipsoid we would have, if we call ω the angular velocity of rotation, ρ the density of the ellipsoid and f the universal constant of gravitation.[17]

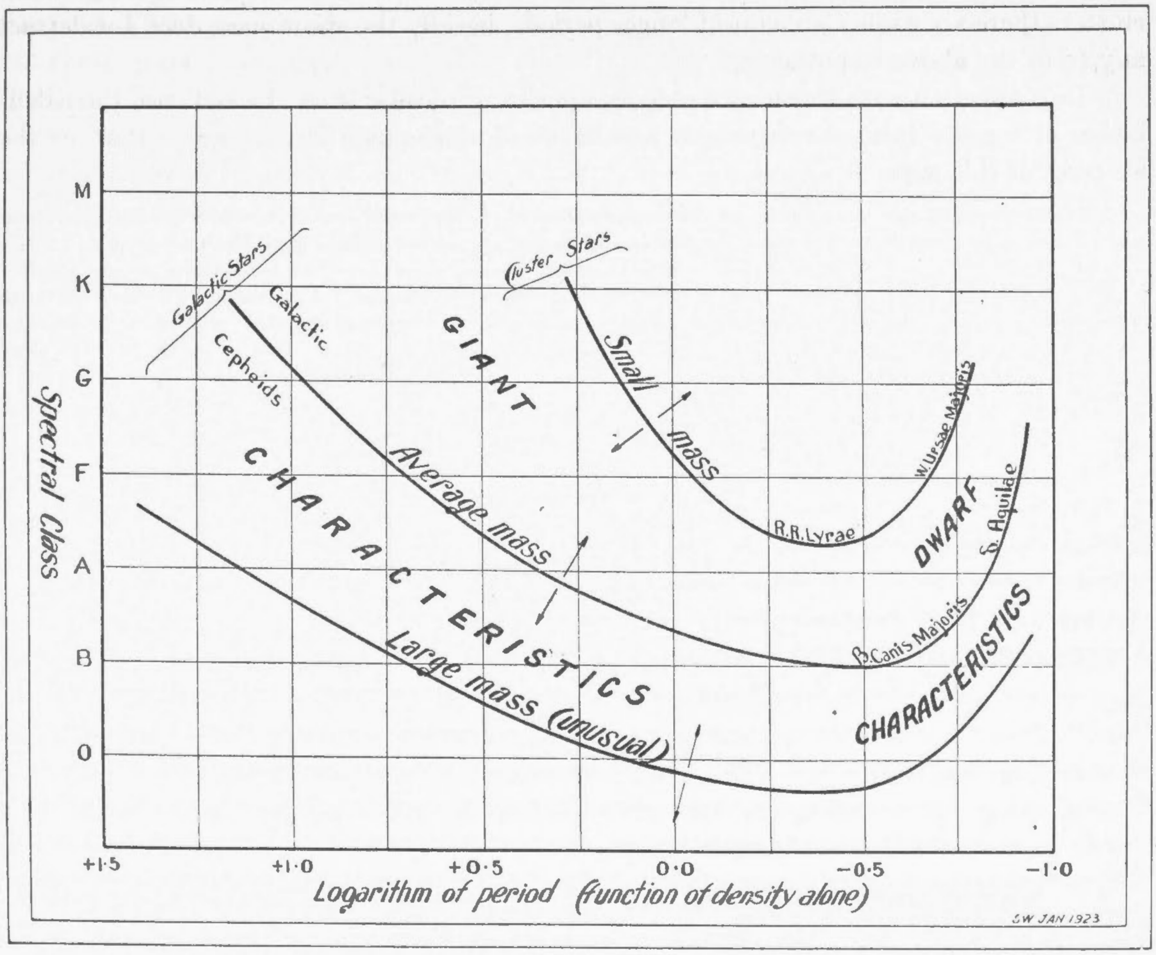
$$\frac{\omega^2}{2\pi f \rho} = 0\cdot18709, \text{ very nearly} \qquad (1)$$

If we assume the radial velocity oscillations as being due to some kind of pulsation it is very likely that the period is a function of the density alone. It was shown by Eddington that if we have two similar globes of fluid[18] executing oscillations under their own gravitation, their periods will be inversely proportional to the square roots of their densities, so that a formula similar to formula (1) can be adopted, or:—

[14] Proceedings of the Nat. Ac. of Sc. Vol. 8, p. 176.
[15] L. O. B. Vol. VI, p. 38.
[16] Pub. Dom. Obs. Vol. V, p. 314.
[17] Tisserand Mécanique Céleste, Vol. II, p. 104.
[18] The Observatory, Vol. 41, 1918, p. 379.

$$\frac{1}{P^2} = C\rho \qquad (2)$$

where P is the period of oscillation, C a constant and ρ the density of the body. The period-luminosity law as found by Miss Leavitt for the Cepheids of the smaller magellanic cloud and extended by Shapley to the case of Cepheids in star clusters, adds a large observational weight to the above theories. If we consider the spectral class of a star as a definite measure on a scale, we have to suppose that this spectral class is a function both of the density and of the mass of the star. We know already from Eddington's work on the interior of the stars that only stars of larger mass can attain the B type in their evolution and stars of still larger mass the O type, (according to J. S. Plaskett's discovery,[19] an O star has been found the most massive). The following hypothesis as illustrated by the accompanying graph is then very logical. This graph is self explanatory; the curve representing the life-history of a variable star (as represented in the figure) would have an equation of the form $f(x, y, \alpha)$, where x is the period of variation, y is the spectral class and α a variable parameter or the mass of the star. This leads immediately to the following conclusion:—



[19] M. N. Vol. 82, 1922, p. 447.

Let us assume for the sake of argument that the variability in these stars is due to pulsation. Any other theory where the period is very nearly a function of the density alone would do. A pulsating star of small mass which would have an evolution, let us say M K G F G K M, without ever attaining the stage O or B, or even A, would, however, decrease in period as any other Cepheid would. When it had attained a rather considerable density, for which the formula (2) would give a period of a fraction of a day, it might still exhibit the spectral characteristics of a giant F because near the top of the giant stage of its evolution, but *it would be a star of small mass and most likely of low total brightness.* Indeed, if it had a larger mass, the graph indicates that it would be near class B.

Cepheids in clusters as well as a very few Cepheids of our galactic system (those of the cluster type) have very short periods, periods such that the graph for galactic stars of average mass would place them near early class B. But these Cepheids are nearly class F; hence, according to our hypothesis, they would be of small mass, perhaps much smaller than the average mass of a galactic star. Accordingly, stars in globular clusters would be very small stars; their atmospheres would be highly ionized and their spectra would exhibit strong enhanced lines and weak arc lines, but they would be of low total brightness, because their masses and their volumes would be very small. The fact that in the globular clusters there are some Cepheids of longer periods, besides the short ones, does not detract any from the above hypothesis.

In connection with this idea a recent paper by Professor H. N. Russell[20] on the calculation of masses from spectroscopic parallaxes is of singular importance. Part of the abstract of this paper is as follows:—

"*Calculation of masses of stars from spectroscopic parallaxes.* It appears probable that the line intensities upon which spectroscopic parallaxes are based are *functions of the temperature and density of the star's atmosphere.* If this were exactly true, all stars of the same surface brightness and density would have the same spectroscopic absolute magnitude, and *the masses, computed from the spectroscopic parallaxes would come out the same for all the stars* of such a group (whatever the dispersion among their actual masses) and equal to the geometrical mean of the latter. To obtain a reliable measure of the dispersion in mass among binary stars, parallaxes must be determined in some other way.

Spectroscopic and dynamical parallaxes.—It follows that the spectroscopic parallaxes and the dynamical parallaxes (derived on the assumption that the mass of a binary system is equal to the mean value for stars of its spectral type and absolute magnitude) are *systematically equivalent to one another* and really rest on the same physical relationship and assumptions."

In other words, although it has been shown that the dispersion in mass for the majority of the stars is probably not large, there might be exceptions, such as stars of very small masses in clusters, which by Adams' method or by Shapley's assumption of period-luminosity curve would be classified as giant stars and supposed to be much larger than they really are.

Schouten[21] finds the distance of the globular clusters about seven times smaller than does Shapley.[22] The former bases his results on the assumption that the luminosity curve for stars in the clusters is identical with that found for the stars of the whole sky. The latter bases his distances on his estimations of the absolute luminosities of the Cepheid variables found in the clusters; the values he gives for these luminosities may be entirely too high.

[20] Ap. J. Vol. 55, 1922, p. 238.

[21] Proceedings of the Ac. of Sc. of Amsterdam, Vol. 20, p. 1108 and Vol. 21, p. 36, 1918.

[22] Contributions from the Mt. Wilson Obs., Nos. 151 and 152.

Kapteyn and Van Rhyn[23] from the observed proper motions of the galactic Cepheids of so-called cluster type compute large parallaxes and relatively low luminosities for these stars. They conclude, therefore, that the analogous short-period Cepheids in clusters may also be dwarfs, and consequently that the distances assigned by Shapley to globular clusters may be about eight times too great.

The following mean radial velocities of short-period Cepheids have also to be noted:—

Star	α 1855·0	Period	Radial Velocity	Author
	h m s	d	km.	
SU Draconis	11 29 38	0·6604347	−193	Adams.
SW Draconis	12 10 38	0·56965	− 74	"
RS Bootis	14 27 21	0·377333	− 51	"
RR Lyrae	19 20 17	0·566826	− 69	Kiess[24]

These high radial velocities are more suggestive of stars of small mass and hence of rather small intrinsic luminosity. It may be supposed, however, as was done by Shapley,[25] that these stars have small parallaxes, large intrinsic luminosities and great peculiar velocities; then the fact that they are so widely distributed over the sky, combined with their small parallaxes, would cast some doubt on the possibility of their forming a cluster similar to the Taurus or the Ursae Majoris clusters. An investigation of the space motion of 1,646 stars by Adams and Strömgren has shown that the average velocities of stars of types F, G, K and M vary with absolute magnitude to a marked degree, the fainter stars moving more rapidly than the brighter ones. The giant stars show an especially regular increase of velocity with decreasing brightness. If the kinetic theory of gases be applied to the sidereal universe, assuming an approximate equidistribution of energy, the most massive stars should have the lowest velocities.

It is quite likely that for the average galactic star of mean mass on the descending branch of evolution, the value of the density would soon be too large and hence the period of variation would soon be too small (if pulsation, would soon be stopped); so that in classes G and K of the galactic stars we cannot expect to find stars of the β Canis Majoris type. A certain number of stars of classes G and K have been investigated here in order to find whether any of them were of the β Canis Majoris type, but none have been found.

To sum up, in the light of the comparison between the periods of the Cepheids and those of the stars of the β Canis Majoris type, a relation such as that indicated by the accompanying diagram seems to be logical. A whole family of such curves would of course be required for a complete representation, each one being lower than the preceding one with increase of mass. For the stars of our galactic system these curves would usually be comprised between narrow limits, but for stars of globular clusters, it might not be so.

[23] Bul. of the Ast. Inst. of the Netherlands No. 8, 1922, Mar. 13.

[24] L. O. B. Vol. VII, p. 146.

[25] Contributions from the Mount Wilson Obs., No. 153, p. 10.

It is of great interest to consider here the following extract from an article published recently by Dr. Paul W. Merrill [26] of the Mt. Wilson Observatory.

"Measurements of the velocities of the Md stars by spectroscopic methods have shown them to have the highest speeds of any class of stars. The average residual radial velocity (random motion of an individual star) is 31 km. per second, and the average space motion is twice this. Some of the highest velocities measured are given in the following table:—

	Velocity km.	Period days	Range magnitude
R Arietis	+ 102	186	5·9
X Monocerotis	+ 146	Irreg.	2·0
T Ursae Maj	− 102	257	6·7
S Librae	+ 285	192	5·4
R Draconis	− 144	246	5·7
T Herculis	− 130	165	6·4
W Lyrae	− 182	197	4·9
RT Cygni	− 127	190	5·3
Z Cygni	− 173	263	6·0

The periods of these stars are less than the average for all long-period variables. Why the shorter period stars should have higher velocities than the others is a question we cannot answer."

An extension of our diagram as given above, considering the long-period variables as Cepheids, as many astronomers do, would answer this last question. Indeed if we consider the average period of variation for stars of class Md of average galactic mass it would be in the neighbourhood of 400 days or more, so that if on the diagram we take as abscissa the logarithm of a period in the neighbourhood of 100 days and as ordinate the one that puts the star in class Md, the corresponding point would be on a curve of small mass—the star would then probably have a larger space motion.

My appreciation is due to Mr. R. Meldrum Stewart for his kindness in reading my articles and discussing with me important points.

[26] Some Results of Spectroscopic Observations of Long Period Variable Stars. Published by the Amer. Ass. of Var. Star Observers, May 25, 1922.

DOMINION OBSERVATORY,
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