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The Spectroscopic System Sigma Scorpii

By

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THE SPECTROSCOPIC SYSTEM SIGMA SCORPII

BY F. HENROTEAU, PH.D.

The present study in which Mr. J. F. Frédette greatly assisted generalizes the researches already made by myself at the Lick Observatory in 1918. Although a much greater amount of data will be necessary to investigate this system thoroughly the phenomena studied seem to be new and altogether unlike what have been found in other systems of stars investigated spectroscopically.

σ Scorpii was discovered by Dr. V. M. Slipher* to be a spectroscopic binary and shown by Father Selga† to be of very short period, so that it can be classified among the stars of the β Canis Majoris type. Father Selga also suspected a variation of the center-of-mass velocity of the very short-period orbit. A more complete study of the system was made by the writer ‡ at the Lick Observatory in 1918, the centre-of-mass velocity curve being found to be periodical and indicating that this centre of mass most likely moves in an elliptical orbit of rather high eccentricity (0.33) whose period is $34^d.08$. The system thus seems to be a spectroscopic triple, if we assume the short-period velocity curve as also indicating orbital motion.

σ Scorpii thus needed to be further investigated and in spite of its very large southern declination which rendered its study rather difficult here at Ottawa two new centre-of-mass velocity curves have been determined, one in 1920 and the other in 1921 showing, as seen from the figures, most characteristic variations.

All the radial velocities which have to our knowledge been obtained up to the present in different observatories, as well as our own velocities, are given in the following table:

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* Lowell Obs. Bul. No. 1 p. 57, 1904 and No. 2, p. 1, 1909.
† Revista de la Soc. Astr. de España y America, Vol. 6, p. 41, 1916.
‡ Lick Obs. Bul. Vol. 9, p. 173.

RADIAL VELOCITIES OF σ SCORPII

Place	Author	Date	Julian Day	Velocity	Remarks
				km.	
Lick....	....	1902 July 2....	2415933·753	−28	Three prisms
Lowell....	....	1904 June 18....	6650·750	−25	One prism
		June 25....	657·760	+25	"
		June 26....	658·748	+17	"
		July 7....	669·710	− 5	"
Lick....	....	1905 Mar. 31....	936·047	− 7·6	Three prisms
Santiago....	....	1907 May 11....	7707·660	−47	"
		1909 June 22....	8480·675	−17·8	"
Lowell....	V. M. Slipher....		·748	−31·5	One prism
		June 23....	481·776	+ 5·2	"
		June 25....	483·785	+60·6	"
		June 28....	486·778	+83·8	"
		July 8....	496·748	−33·6	"
		July 10....	498·748	−51·3	"
		July 12....	500·748	−43·2	"
		July 14....	502·763	−12·3	"
		July 15....	503·738	−16·9	"
		July 22....	510·697	−10·1	"
		July 27....	515·678	+66·9	"
		July 28....	516·659	+ 3·2	"
		July 29....	517·702	+12·8	"
		July 30....	518·683	+19·9	"
		1911 May 31....	9188·823	+14·4	"
Santiago....	....	1912 Mar. 16....	478·788	+76·4	Three prisms
		....	·788	+81·0	"
Lowell....	....	Aug. 2....	617·696	− 2·9	One prism
Lick....	Father Selga....	1914 Sept. 11....	20387·645	+13·8	"
		Sept. 23....	399·623	−25·6	"
		Sept. 25....	401·615	−21·3	"
		Sept. 28....	404·620	+13·6	"
		Sept. 29....	405·611	+14·0	"
		Oct. 5....	411·607	+16·8	"
		Oct. 10....	416·599	−30·0	"
		Oct. 11....	417·602	−45·2	"
		Oct. 12....	418·599	−60·8	"
		Oct. 14....	420·592	−56·7	"
		1915 Jan. 23....	521·083	−49·3	"
		Mar. 2....	559·047	−46·5	"
		....	·089	−28·0	"
		Mar. 3....	560·039	−36·9	"
		....	·066	−31·4	"
		Mar. 7....	564·079	+45·1	"
		Mar. 8....	565·067	+24·0	"
			·084	+53·1	"
		Mar. 11....	568·028	+31·7	"
			·047	+35·7	"
		Mar. 16....	573·066	− 3·6	"
		Mar. 18....	575·969	+26·5	"
		Mar. 19....	576·070	−24·3	"
		Mar. 21....	578·055	−39·2	"
		Mar. 22....	579·073	−27·9	"
		Mar. 24....	581·058	−40·8	"
		April 10....	598·867	+32·6	"
		April 11....	599·042	−22·0	"
			·904	+31·7	"

RADIAL VELOCITIES OF σ SCORPII—*Continued*

Place	Author	Date	Julian Day	Velocity	Remarks
				km.	
Lick............	Father Selga.......	1915 April 13....	2420601·874	+30·9	One prism
			·914	− 2·9	"
		April 14....	602·003	−25·0	"
			·869	+39·2	"
			·928	− 5·0	"
		April 15....	603·014	−15·0	"
			·882	+52·1	"
			·927	−14·6	"
			·963	−30·9	"
		April 16....	604·002	−18·1	"
			·038	− 3·6	"
			·981	−23·0	"
		April 17....	605·012	− 7·7	"
			·037	+ 1·3	"
		April 18....	606·941	−27·7	"
			·986	+ 4·0	"
		April 19....	607·024	+21·1	"
			·945	−20·5	"
			·993	− 0·8	"
		May 25....	643·763	−21·5	"
			·784	−11·6	"
			·802	+ 9·8	"
			·823	+25·2	"
		May 26....	644·707	−42·8	"
			·730	−34·0	"
			·749	−16·4	"
			·764	−22·6	"
			·782	+ 5·2	"
			·806	+22·9	"
			·836	+35·3	"
			·856	+ 8·1	"
			·893	−11·2	"
			·930	−32·1	"
			·963	−41·7	"
		June 7....	656·867	−14·3	"
			·881	− 0·7	"
			·893	+ 4·3	"
			·907	+ 9·1	"
			·923	+12·3	"
			·940	+18·2	"
		June 22....	671·684	+20·1	"
			·699	+30·3	"
			·713	+34·1	"
			·727	+41·6	"
			·742	+41·5	"
		....	·756	+27·4	"
			·768	+11·9	"
			·780	+ 6·1	"
			·794	+ 4·5	"
			·807	− 7·8	"
			·823	−19·2	"
			·836	−26·3	"
			·851	−39·1	"
			·866	−24·3	"
			·880	−24·9	"

RADIAL VELOCITIES OF σ SCORPII—*Continued*

Place	Author	Date	Julian Day	Velocity	Remarks
				km.	
Lick............	Father Selga.......	1915 June 22....	2420671·895	− 7·7	One prism
			·911	− 0·1	"
		June 25....	674·664	+30·0	"
			·677	+38·2	"
			·691	+41·3	"
			·703	+36·8	"
			·718	+24·5	"
			·754	− 2·4	"
			·766	−14·8	"
			·785	−32·4	"
			·797	−45·3	"
			·810	−41·8	"
			·824	−32·4	"
			·837	−28·9	"
		July 7....	686·783	+ 6·9	"
			·792	+10·9	"
			·801	+13·9	"
			·810	+ 3·0	"
		July 22....	701·668	+ 6·6	"
			·678	+ 5·8	"
			·688	+ 0·4	"
			·699	− 6·2	"
			·711	−21·6	"
			·722	−31·3	"
			·730	−19·3	"
			·740	− 9·2	"
			·750	−10·0	"
			·764	− 3·7	"
			·774	+ 2·4	"
			·784	+10·9	"
			·795	+ 9·5	"
			·805	+16·2	"
			·814	+27·4	"
			·825	+27·3	"
Lick............	F. Henroteau.......	1918 April 3....	2421687·869	+20·5	Three prisms
			·909	+47·4	"
			·943	+76·3	"
			·977	+57·7	"
		April 4....	688·010	+22·4	"
			·041	+ 3·6	"
			·868	+31·0	"
			·902	+46·7	"
			·935	+69·7	"
			·970	+44·6	"
		April 5....	689·004	+16·7	"
			·043	− 8·3	"
		April 11....	695·854	+44·1	"
			·892	+ 6·9	"
			·934	−12·7	"
			·970	−31·5	"
		April 12....	696·002	−14·1	"
			·032	+ 1·9	"
		April 15....	699·836	+ 7·3	"
			·871	−21·0	"
			·903	−33·0	"

RADIAL VELOCITIES OF σ SCORPII—*Continued*

Place	Author	Date	Julian Day	Velocity	Remarks
				km.	
Lick............	F. Henroteau.......	1918 April 15....	2421699·935	−39·3	Three prisms
			·969	−20·8	"
		April 16....	700·003	+ 0·6	"
		April 17....	701·844	−24·6	"
			·876	−38·0	"
			·907	−48·2	"
			·939	−27·8	"
			·969	− 8·4	"
		April 18....	702·004	+25·1	"
		April 21....	705·810	−51·7	"
			·866	−50·3	"
			·897	−32·5	"
			·929	−10·3	"
		April 23....	707·840	−57·5	"
			·876	−38·1	"
			·906	−16·0	"
			·935	+ 4·0	"
			·967	+ 4·9	"
			·998	−29·1	"
		April 26....	710·845	−32·5	"
		April 29....	713·823	− 0·2	"
			·856	+19·3	"
			·886	+ 4·1	"
			·917	−17·4	"
			·951	−36·1	"
			·987	−51·0	"
		May 2....	716·826	+59·1	"
			·857	+55·8	"
			·895	+ 8·3	"
			·925	− 8·3	"
			·957	−16·6	"
			·990	− 4·8	"
		May 5....	719·850	+22·5	"
			·875	+11·9	"
			·904	− 7·2	"
			·934	− 1·8	"
			·970	+23·8	"
		May 10....	724·816	− 6·0	"
			·845	−17·5	"
			·875	− 9·1	"
			·906	+ 8·5	"
			·938	+37·0	"
			·970	+60·6	"
		May 12....	726·790	− 9·3	"
			·820	−23·4	"
			·851	−11·6	"
			·881	+ 9·0	"
			·922	+39·2	"
			·953	+58·6	"
		May 29....	743·765	−19·9	"
			·797	−46·7	"
			·834	−67·8	"
			·880	−57·9	"
			·916	−32·7	"

RADIAL VELOCITIES OF σ SCORPII—*Continued*

Place	Author	Date	Julian Day	Velocity	Remarks
				km.	
Ottawa.........	F. Henroteau.......	1920 April 13....	2422428·789	−49·7	One prism
			·804	−45·2	"
			·820	−45·7	"
			·834	−30·5	"
			·846	−28·3	"
		April 16....	431·727	−41·4	"
			·764	−53·4	"
			·785	−38·3	"
			·804	−53·1	"
			·826	−34·4	"
		April 18....	433·712	−38·0	"
			·773	−59·6	"
			·799	−38·8	"
			·822	−13·1	"
		April 19....	434·724	−49·5	"
			·775	−30·5	"
			·800	−10·4	"
		April 25....	440·712	−15·9	"
			·768	+33·2	"
			·788	+31·7	"
			·813	+17·7	"
			·838	− 9·3	"
		May 4....	449·707	+20·6	"
			·729	+10·3	"
			·760	−21·5	"
			·783	−29·8	"
		May 5....	450·683	+29·7	"
			·707	+ 7·0	"
			·729	− 5·5	"
			·768	−25·8	"
			·792	−30·3	"
			·817	−39·1	"
		May 10....	455·676	−22·4	"
			·700	−23·5	"
			·722	−33·6	"
			·767	− 7·1	"
		May 11....	456·673	−29·4	"
			·696	−40·9	"
			·718	−41·7	"
			·764	−17·7	"
		1921 May 3....	813·711	+36·9	"
			·740	+47·7	"
			·767	+31·7	"
			·798	+ 6·8	"
			·825	−22·3	"
		May 4....	814·700	+27·4	"
			·729	+35·2	"
		May 5....	815·676	+53·0	"
			·707	+61·4	"
			·737	+18·6	"
			·769	− 4·8	"
			·798	−20·9	"
		May 6....	816·771	− 3·9	"
		May 9....	819·654	+29·8	"
			·686	+ 2·0	"

RADIAL VELOCITIES OF σ SCORPII—*Concluded*

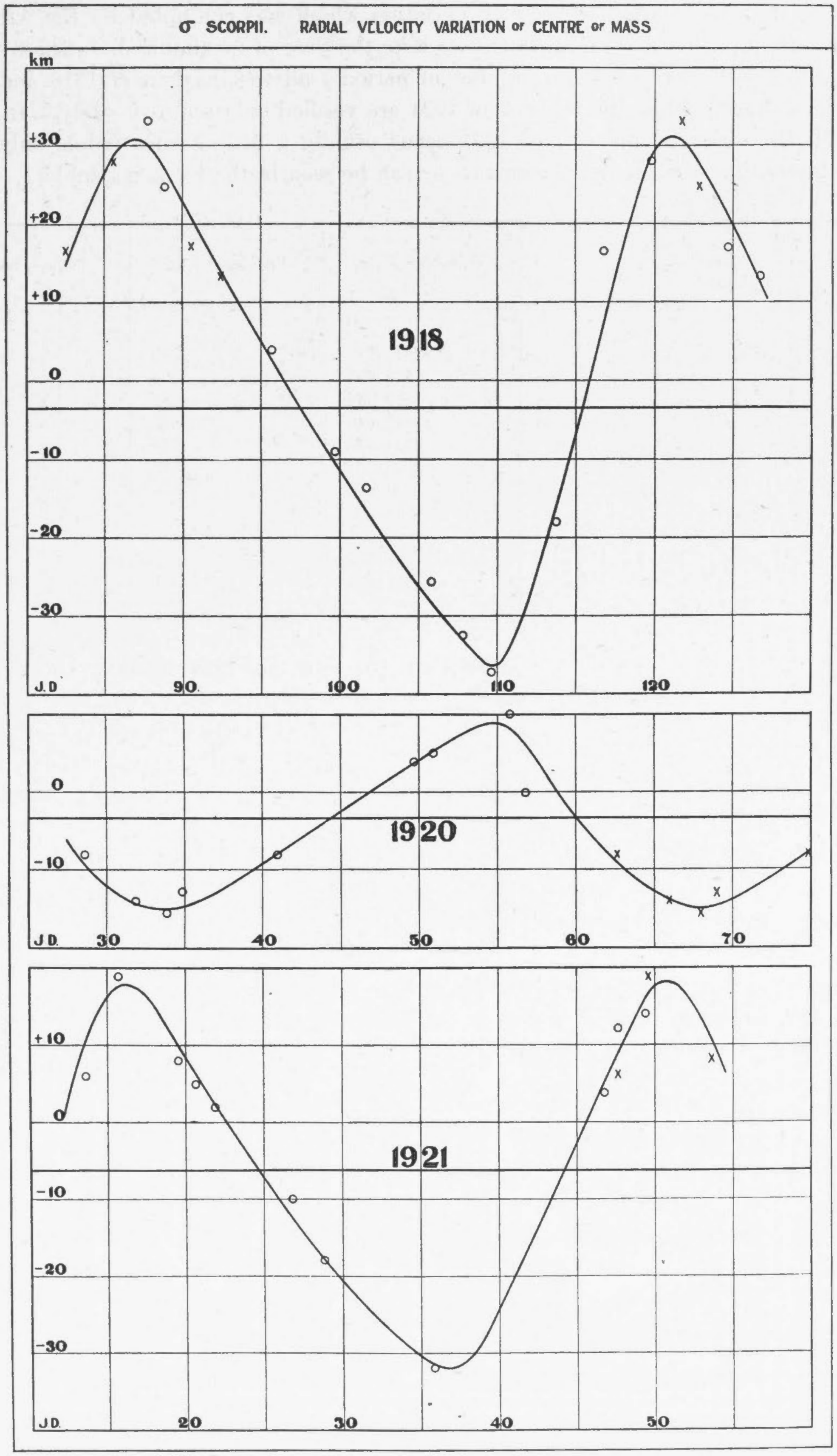
Place	Author	Date	Julian Day	Velocity	Remarks
				km.	
Ottawa.........	F. Henroteau.......	1921 May 9....	2422819·713	−19·4	One prism
			·742	−33·7	"
			·770	−34·3	"
		May 10....	820·659	+39·7	"
			·689	− 4·4	"
			·716	−18·6	"
			·746	−37·6	"
			·774	−34·2	"
		May 11....	821·651	+23·3	"
			·681	− 5·9	"
			·709	−20·3	"
			·740	−39·9	"
			·769	−33·2	"
		May 16....	826·669	−40·0	"
			·700	−51·9	"
			·727	−31·3	"
			·757	−10·2	"
			·785	+22·6	"
		May 18....	828·654	−34·3	"
			·688	−54·3	"
			·720	−60·0	"
		May 25....	835·612	−27·3	"
			·658	− 0·8	"
			·687	+ 8·0	"
			·715	+ 3·1	"
			·745	−18·2	"
		May 26....	836·608	−32·6	"
			·629	−26·6	"
		June 1....	842·682	+13·1	"
			·705	−24·0	"
			·729	−30·7	"
		June 3....	844·646	+25·0	"
			·670	−15·6	"
			·693	−26·0	"
		June 5....	846·618	− 6·3	"
			·642	+16·9	"
			·665	−29·9	"
			·688	−35·4	"
			·711	−36·1	"
		June 6....	847·615	+ 0·8	"
			·637	−12·5	"
			·660	−25·6	"
			·684	−29·2	"
			·708	−12·7	"
		June 8....	849·610	−10·6	"
			·658	−31·2	"
			·682	−21·4	"

The above velocities furnish us a rather large number of nearly complete velocity curves which may be considered, within the limits of error, of constant amplitude. Their center-of-mass lines give us the following radial velocities for the nights indicated.

VALUES OF CENTER-OF-MASS VELOCITY OBTAINED FROM SHORT-PERIOD CURVES.

Date	Julian Day	Velocity
		km.
1915 April 14	2420602	+ 2·0
April 15	603	+ 5·0
May 26	644	− 6·0
June 7	656	−26·0
June 22	671	+ 1·6
June 25	674	− 2·2
July 7	686	−32·0
July 22	701	+14·0
1918 April 3	1687	+33·0
April 4	688	+24·8
April 11	695	+ 4·0
April 15	699	− 8·0
April 17	701	−13·5
April 21	705	−25·6
April 23	707	−32·2
April 29	713	−17·0
May 2	716	+16·4
May 5	719	+28·0
May 10	724	+17·1
May 12	726	+13·5
May 29	743	−37·5
1920 April 13	2428	− 8·0
April 16	431	−14·0
April 18	433	−15·5
April 19	434	−13·0
April 25	440	− 8·0
May 4	449	+ 4·0
May 5	450	+ 5·0
May 10	455	+10·0
May 11	456	0·0
1921 May 3	813	+ 6·0
May 5	815	+19·0
May 9	819	+ 8·0
May 10	820	+ 6·0
May 11	821	+ 2·0
May 16	826	−10·0
May 18	828	−18·0
May 25	835	−32·0
June 5	846	+ 4·0
June 6	847	+12·0
June 8	849	+14·0

The above velocities furnish us the three accompanying curves of radial velocity variation of the center of mass, assuming that the period of variation is in the neighbourhood of 34 days. The variation of amplitude of these velocity curves from 76 km. in 1918 to 24 km. in 1920 and then 50 km. in 1921 is of the greatest interest, as well as the apparent reversal from a minimum at the end of the long branch of the curve in 1918 to a maximum at the end of the long branch of the curve in 1920.

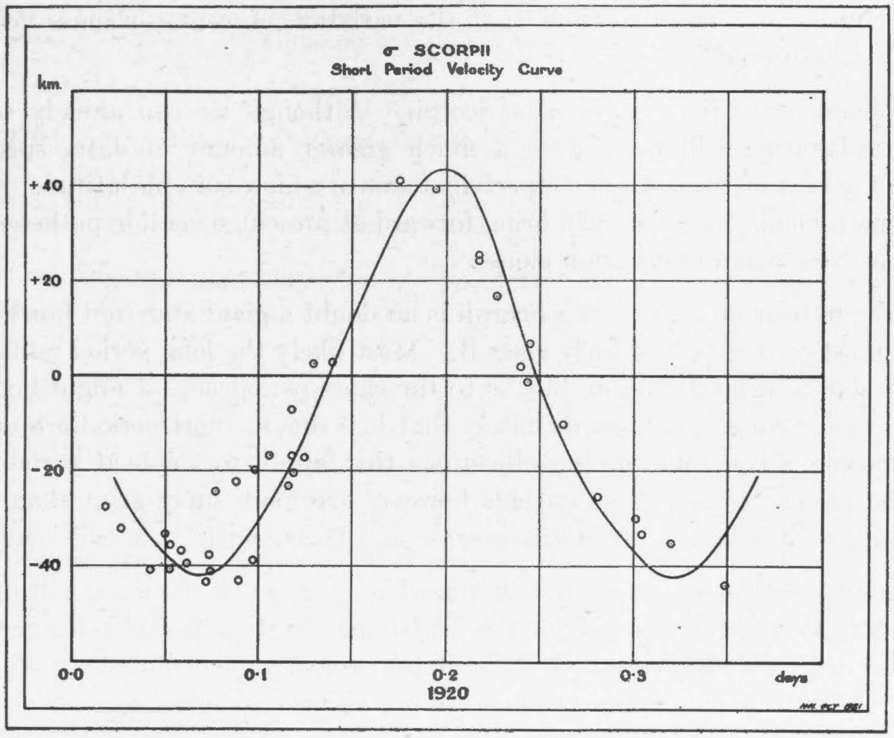

σ SCORPII. RADIAL VELOCITY VARIATION OF CENTRE OF MASS
km
+30
+20
+10
0
-10
-20
-30
1918
J.D. 90. 100 110 120
0
-10
1920
J.D. 30 40 50 60 70
+10
0
-10
-20
-30
1921
J.D. 20 30 40 50

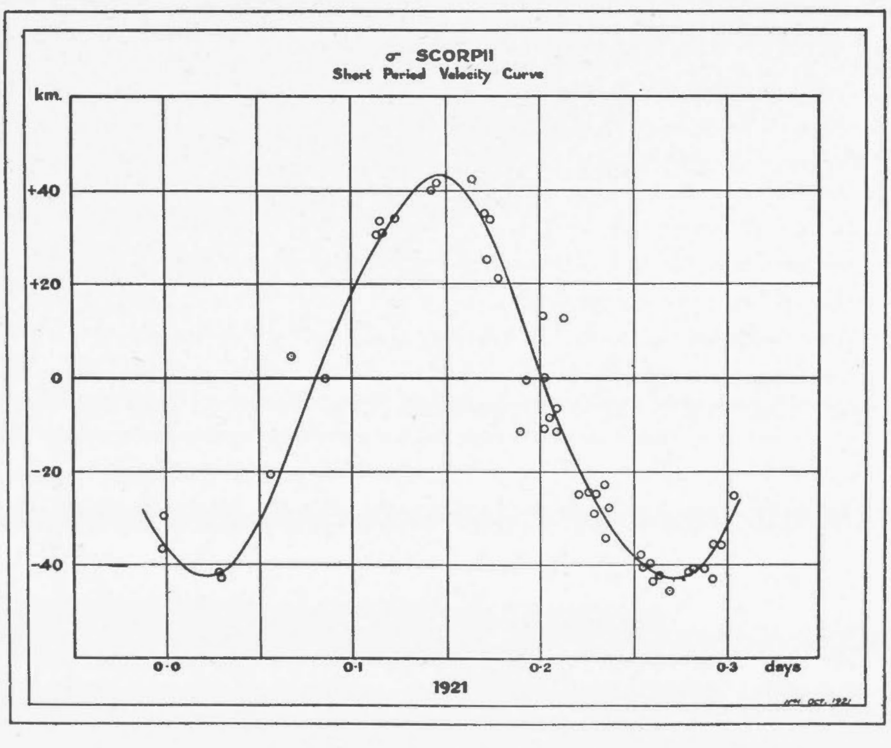
As to the short period of velocity variation which was computed by the writer in 1918 as being $0^d \cdot 246834$ we find that if we take the date of maximum J.D. 2420644·830 as origin (adding to it a certain number of periods) all the maxima of 1918 and 1920 are verified exactly while the maxima of 1921 are verified only approximately. In other words all the observed maxima of 1921 occur usually a little earlier (while only once a little later) than the predicted maxima, as can be seen in the following table.

Date 1921	Observed J.D.	Predicted J.D.	Obs.-Pred.
May 3	2422813·745	·760	−·015
May 5	815·694	·735	−·041
May 9	819·627	·684	−·057
May 10	820·646	·672	−·026
May 11	821·625	·659	−·034
May 16	826·821	·843	−·022
May 18	828·840	·817	+·023
May 25	835·693	·729	−·036
June 5	846·578	·589	−·011
June 6	847·555	·577	−·022
June 8	849·538	·551	−·013

The differences observed are larger than the errors of observation. However, as the velocity curve of May 18 is based on only three observations it may be that the difference for that date could be eliminated; if we then take the residuals of the differences from the mean of these differences the residuals would not be larger than the errors of observation. This would then indicate *that the short period variation has decreased a little* from 1920 to 1921, this decrease being of the order of $0^d \cdot 00002$.

Taking now separately the observations of 1920 and of 1921, we may superpose the different velocity curves so that all of them have the same but undetermined mean velocity. We then obtain the accompanying curves, respectively, for each year showing that the short-period velocity curve has a constant amplitude.


σ SCORPII
Short Period Velocity Curve
km.
+40
+20
0
-20
-40
0·0
0·1
0·2
0·3
days
1920


σ SCORPII
Short Period Velocity Curve
km.
+40
+20
0
-20
-40
0·0
0·1
0·2
0·3
days
1921

These curves also serve to show that the variation of center-of-mass velocity has been rightly interpreted.

What happens in the system of σ Scorpii? Although we can already see that a thorough explanation will necessitate a much greater amount of data, spectrograms taken with the best instruments and especially taken at a more suitable latitude (preferably in the southern hemisphere), we will bring forward at present several hypotheses based on the action of Newtonian gravitation alone.

We have to bear in mind that σ Scorpii is no doubt a giant star (not however super-giant) like most of the stars of early class B. Most likely the long period radial velocity variation is due to orbital motion, but as to the short period one, it might be explained in several ways. We can not say definitely that it is due to short period orbital motion, because in some ways, some of the objections that apply to Cepheid variables would hold for the present case. The Cepheids however are more super-giant than the early class B stars, their volumes are much greater and their densities much smaller.

As the question of pulsation in such stars has been much discussed during the last few years, pulsation might be considered as explaining the short period velocity variation; but as theoretically, a binary system or the hypothesis of a Jacobian ellipsoid as brought forward in the next pages, would produce similar observed effects, we will not consider any pulsation theory in the present paper.

A plausible way to explain the variation of range of the long period velocity curve is to suppose that the plane of the orbit is seen under different angles, sometimes nearly edgewise, the variations of the positions of the plane (variation of the inclination or revolution of the line of nodes) being due to strong perturbations. For the three years, 1918, 1920 and 1921 indeed the elements of the orbit such as period, eccentricity . . . do not seem to change much.

We could now suppose that the short period radial velocity variation is due to the rotation of an unsymmetrical body such as a Jacobian ellipsoid. If we have a gaseous Jacobian ellipsoid with a very small nucleus, or with a rather intense darkening at the limb, it is evident then that the absorption lines in our spectra will be produced by only a very small portion of the gaseous surface, that portion which is the nearest to us, and the rotation of the ellipsoid instead of being indicated by a wide line, will be indicated by a much narrower line having a periodic oscillation, such as the oscillations due to orbital motions. The period of oscillation in such a case would be one-half the period of rotation.

We shall first have to investigate from the knowledge we have of early class B stars, whether in the case of σ Scorpii such a Jacobian ellipsoid could be a stable figure or not.

It has been proved in celestial mechanics* that if we call ω the angular velocity of a rotating ellipsoid, ρ its density (the mass being supposed homogeneous) f the universal constant of attraction, then:

* Tisserand, Mécanique Céleste, Vol. 2, p. 104.

If $\dfrac{\omega^2}{2\pi f \rho} > 0 \cdot 22467$, no ellipsoid can be a figure of equilibrium.

If $0 \cdot 18709 < \dfrac{\omega^2}{2\pi f \rho} < 0 \cdot 22467$ two ellipsoids of revolution could be figures of equilibrium for each set of values of ω and ρ.

If $\dfrac{\omega^2}{2\pi f \rho} < 0 \cdot 18709$ two ellipsoids of revolution and a Jacobian ellipsoid could be figures of equilibrium.

If we call $T_\circ$ the time of revolution of the Earth $\rho_\circ$ its density, we have $\dfrac{\omega^2}{2\pi f \rho} = 0 \cdot 001150 \left(\dfrac{T_\circ}{T}\right)^2 \dfrac{\rho_\circ}{\rho}$, T being the time of revolution of the body. The density of the Earth is four times that of the Sun, $T_\circ = 86164$ seconds or $0^d \cdot 997269$, $\rho_\circ = 4$. In the case of σ Scorpii we would evidently have the time of rotation double the period of the velocity curve or $0^d \cdot 493668$; hence $\dfrac{T_\circ}{T} = 2 \cdot 02$

$$\frac{\omega^2}{2\pi f \rho} = \frac{0 \cdot 00115 \times 16 \cdot 32}{\rho} = \frac{0 \cdot 018768}{\rho}$$

In order to have a Jacobian ellipsoid we must have

$$\frac{0 \cdot 018768}{\rho} < 0 \cdot 18709$$

$$\text{or } 0 \cdot 018768 < 0 \cdot 18709 \rho$$

$$\text{or } \rho > 0 \cdot 1 \text{ (approximately)}$$

the density of the Sun being unity. From the study of Algol variables of early class B* it is estimated that their densities are generally a little superior to $0 \cdot 1$; it would not be illogical to suppose then that such would be the case for σ Scorpii, hence the principal star of σ Scorpii could be a Jacobian ellipsoid.

We shall at present, for obvious reasons, consider σ Scorpii not a triple system but *a double system in which the principal star is a Jacobian ellipsoid.* The fact that this principal star is not spherical will introduce rather large perturbations in the orbital motion of the satellite, which will mostly be shown in a revolution of the line of nodes (or intersection of the plane of the orbit of the satellite with the equatorial plane of the principal body) and a revolution of the line of apsides of the orbit of the satellite. The revolution of the line of nodes means simply that the inclination of the plane of the orbit to the plane perpendicular to the line of sight varies, that is, the amplitude of the long-period velocity curve varies (which has been found to be the case).

Let us now consider the possible dimensions of the principal body of σ Scorpii by using the data we have obtained, as well as by making certain logical assumptions.

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* See Plaskett's determinations, Jour. R.A.S.C. 1920, p. 423.

It is known that if

$$\frac{\omega^2}{2\,\pi f \rho} < 0\cdot 14200$$

the figure of equilibrium might be pear shaped, that is if $0\cdot 018768 < 0\cdot 14200\,\rho$ or $\rho > 0\cdot 132$, adopting the above rotation period for σ Scorpii.

We shall adopt for σ Scorpii

$$0\cdot 1 < \rho < 0\cdot 132$$

We might quote here some densities of early class B stars which are Algol variables.

Star	Spectrum	ρ
u Herculis	B 3	0·095
V Puppis	B 1	0·042
U Coronae	B 3	0·175
Y Cygni	B 2	0·170

or if we consult the table of densities according to spectra given by H. Shapley* in his article "The Orbits of Eighty-seven Eclipsing Binaries" we see that the assumed value of ρ for σ Scorpii is certainly very near its real value.

Following Darwin's theories and computations† we can now find what would be the ratios of the three semi-axes, a, b, c of the Jacobian ellipsoid for values of ρ comprised between $0\cdot 1$ and $0\cdot 132$ and for the period of rotation that we have found (c coinciding with the axis of rotation).

Let us take first the two cases where $\rho = 0\cdot 100$ and $\rho = 0\cdot 132$ and the angular velocity found for σ Scorpii which gave respectively:

$$\frac{\omega^2}{2\,\pi f \rho} = 0\cdot 187 \text{ and } \frac{\omega^2}{2\,\pi f \rho} = 0\cdot 142$$

If we call $r_o = \sqrt[3]{a\,b\,c}$ it is found that in the first case

$$\frac{a}{r_o} = 1\cdot 1972,\ \frac{b}{r_o} = 1\cdot 1972,\ \frac{c}{r_o} = 0\cdot 6977$$

and in the second case

$$\frac{a}{r_o} = 1\cdot 8858,\ \frac{b}{r_o} = 0\cdot 8150,\ \frac{c}{r_o} = 0\cdot 6507$$

while for intermediate values we shall have the following table according to Darwin's computations‡

* Ap. J. Vol. 38, 1913, p. 173.
† Collected Works, Vol. 3, p. 119.
‡ Collected Works, Vol. 3, p. 130.

$\frac{\omega^2}{2\pi f\rho}$	$\frac{a}{r_o}$	$\frac{b}{r_o}$	$\frac{c}{r_o}$	Moment of momentum
0·18712	1·1972	1·1972	0·6977	0·30375
0·1870	1·216	1·179	0·697	0·304
0·186	1·279	1·123	0·696	0·306
0·1812	1·3831	1·0454	0·6916	0·3134
0·1659	1·6007	0·9235	0·6765	0·3407
0·14200	1·88583	0·81498	0·65066	0·3898

We shall now consider a theoretical system of two bodies of which the principal one is a Jacobian ellipsoid; it will not be necessarily identical with σ Scorpii since a certain number of elements in that system are unknown to us but it will be in all respects very similar to it, satisfying our data and all the knowledge we have of σ Scorpii and of early class B stars. The Jacobian ellipsoid considered will come immediately after the point of bifurcation between Maclaurin's and Jacobi's ellipsoids, or rather for theoretical purpose, will be the ellipsoid at the point of bifurcation. Such a hypothesis is necessary in order to conform with the rather small amplitude of the short-period radial velocity curve compared to the rather large size of the body. The density of the body will be taken as $\rho=0\cdot1$ that of the Sun being unity; by comparison with Algol variables of early class B we will suppose its mean radius $r_o = \sqrt[3]{a\,b\,c} = 3{,}000{,}000$ km. or as in the computations we will take the mean distance between the Sun and the Earth as unit of length, or 150,000,000 km., we shall have $r_o=0\cdot02$ and to agree with Darwin's values the dimensions of the three semi-axes of the ellipsoid will be

$$a=0\cdot023944$$
$$b=0\cdot023944$$
$$c=0\cdot013954$$

The mass of the principal body will then be 12·5000, that of the Sun being unity.

Let us now refer the whole system to rectangular coördinates whose origin is at the center of the ellipsoid and whose z axis is given by the direction of the rotation axis of the ellipsoid. We found before that the elements of the long-period orbit of σ Scorpii in 1918 were*:

$$\left.\begin{aligned} k &= 33 \text{ km.} \\ e &= 0\cdot33 \\ P &= 34^{d}\cdot08 \\ \omega &= 270^\circ \\ T &= 2421715\cdot35 \text{ J.D.} \\ \gamma &= -3\cdot2 \text{ km.} \\ a \sin i &= 14{,}600{,}000 \text{ km.} \\ \frac{m_1^3 \sin^3 i}{(m+m_1)^2} &= 0\cdot107 \end{aligned}\right\}$$

* Lick Obs. Bul. Vol. 9, p. 177.

If we assume in the last elements i to be 30° we find

$$a = 29{,}200{,}000 \text{ km.}$$

and having assumed already that the mass of the primary is 12·5 we find for the mass of the secondary 6·8415.

If we suppose the primary at the origin we find now that the secondary revolves around it in an ellipse whose semi-major axis is 82,550,873 km. or expressed in astronomical units 0·550339.

We shall now from the above considerations and from a few more assumptions have the following elements of our theoretical system as referred to the center of the ellipsoid and our rectangular system of coördinates:

Period of revolution $P = 34^d \cdot 08$

Angle of line of nodes with Ox (assumed)

$$\Omega = 90°$$

Inclination of orbit's plane with respect to equator of ellipsoid (assumed)

$$i = 40°$$

Angle of line of apsides with line of nodes (assumed)

$$\omega = 90°$$

Semi-major axis of orbit of secondary around primary expressed in astronomical units and deduced from considerations above,

$$a = 0 \cdot 550339$$

Eccentricity (derived from observations)

$$e = 0 \cdot 33$$

Time of periastron passage (chosen as origin of time)

$$T = 0^d \cdot 0$$

It is to be remarked that, the eccentricity being given, the semi-minor axis of the ellipse will be:

$$b = 0 \cdot 519509$$

$$\text{and } c = ae = 0 \cdot 181612$$

The last notations a, b, c must not be confounded with those giving the dimensions of the ellipsoid.

Our theoretical problem is then the following:

Being given the above binary system in which the central body is ellipsoidal, find what will be the perturbations of the elements of the orbit in the course of one revolution. By extending the results to several revolutions, see if some cases are possible which would account for the variations observed in the long-period radial velocity curve of σ Scorpii.

If we assume the central body spherical there will be no disturbing force. For each point in the orbit the disturbing force will then be the difference between the gravitational attraction of the ellipsoid and the gravitational attraction of the sphere.

To compute the perturbations of the elements we shall use Lagrange's perturbation formulae giving the derivatives of these elements with respect to the time, which are:

$$\frac{d\,\Omega}{d\,t} = \frac{r \sin u}{n\,a^2\sqrt{1-e^2}\sin i}\,W$$

$$\frac{d\,i}{d\,t} = \frac{r \cos u}{n\;a^2\sqrt{1-e^2}}\,W$$

$$\frac{d\,\omega}{d\,t} = \frac{-\sqrt{1-e^2}\,\cos\,v}{n\,a\,e}\,R + \frac{\sqrt{1-e^2}}{n\,a\,e}\left[1+\frac{r}{p}\right]\sin v.\,S$$

$$-\frac{r \sin u\,\cot i}{n\;a^2\sqrt{1-e^2}}\,W$$

$$\frac{d\,a}{d\,t} = \frac{2\,e \sin v}{n\sqrt{1-e^2}}\,\mathrm{R} + \frac{2\,a\sqrt{1-e^2}}{n\,r}\,S$$

$$\frac{de}{d\,t} = \frac{\sqrt{1-e^2}\sin v}{n\,a} + \frac{\sqrt{1-e^2}}{n\,a^2\,e}\left[\frac{a^2\,(1-e^2)}{r} - r\right]S$$

$$\frac{d\,\sigma}{d\,t} = -\frac{1}{n\,a}\left[\frac{2\,r}{a} - \frac{1-e^2}{e}\cos v\right]R - \frac{(1-e^2)}{n\,a\,e}\left[1+\frac{\mathrm{r}}{p}\right]\sin\,v.\,S$$

where we have:

W, the component of the disturbing acceleration perpendicular to the plane of the orbit, positive toward positive r.

S, the component of the disturbing acceleration perpendicular to the radius vector in the plane of the orbit, its positive direction making an angle less than 90° with the direction of motion.

R, the component of the disturbing acceleration along the radius vector, positive away from central body.

Ω, angle between positive x-axis and line of nodes.

ω, angle between node and periastron.

a, semi-major axis.

e, eccentricity.

$\sigma = -n\,T$.

T, time of periastron passage.

n, mean motion of the body, or $n = \frac{2\,\pi}{P}$, where P is the period, or $n = \frac{2\,\pi}{34^{d}\cdot 08}$

u, angle between direction of node and radius vector.

v, angle between direction of periastron and radius vector, $u=v+\omega$.

r, radius vector.

p, semi-parameter of the ellipse described by the satellite, or value of r when $v=90°$.

It is to be remembered that in the following computations the unit of time will be the mean solar day, the unit of mass the mass of the Sun, and the unit of distance the semi-major axis of the Earth's orbit or 150,000,000 km. In such a case the Gaussian Constant k has been adopted to be

$$k = 0 \cdot 01720209895.$$

We shall now compute the accelerations due to the attractions of the ellipsoidal body for eighteen positions in the orbit; they will be obtained by the three components parallel to the axes of the ellipsoid.

For each of these positions, by subtracting from these accelerations those due to the same central mass supposed to be spherical, we will obtain W, S, and R (after two transformations of coördinates).

Using Lagrange's perturbation formulae we obtain

$$\frac{d\Omega}{dt},\quad \frac{di}{dt},\quad \frac{d\omega}{dt},\quad \frac{da}{dt},\quad \frac{de}{dt},\ \text{and}\ \frac{d\sigma}{dt}$$

for the eighteen positions considered. Taking then for each of these a rectangular system of plane coördinates, the times being taken as abscissae and the values of

$$\frac{d\Omega}{dt}\ \left(\text{or}\ \frac{di}{dt}\ldots\ldots\right)$$

as ordinates, we will obtain eighteen points that will be joined by a curve. If then we measure, for instance, (with a planimeter) the area comprised between the curve, the two ordinates for $t=0$ and $t=t_1$ and the axis of abscissae, this area will represent the perturbations respectively of Ω, i, ω.. after the time $t=t_1$. We shall take $t_1 = P = 34^d \cdot 08$ to obtain the total perturbations after one revolution of the satellite.

If we call m_1 the mass of the satellite and x', y', z', its coördinates in the adopted system of rectangular coördinates, remembering that a, b, c designate the semi-axes of the ellipsoid and ρ its density, we will adopt for the expressions of the components of the disturbing acceleration of m_1 with respect to the centre of the ellipsoid:

$$X' = \left(\frac{1}{m} + \frac{1}{m_1}\right)\left[-4\,\pi\,\rho\,m_1\,b\,c\,k^2\,x' \int_0^{\frac{a}{\sqrt{a^2+\kappa}}} \frac{u^2\,du}{\sqrt{[a^2-(a^2-b^2)\,u^2]\,[a^2-(a^2-c^2)\,u^2]}}\right]$$

$$Y' = \left(\frac{1}{m} + \frac{1}{m_1}\right)\left[-4\,\pi\,\rho\,m_1\,c\,a\,k^2\,y' \int_0^{\frac{b}{\sqrt{b^2+\kappa}}} \frac{u^2\,du}{\sqrt{[b^2-(b^2-c^2)u^2]\,[b^2-(b^2-a^2)\,u^2]}}\right]$$

$$Z' = \left(\frac{1}{m} + \frac{1}{m_1}\right)\left[-4\,\pi\,\rho\,m_1\,a\,b\,k^2\,z' \int_0^{\frac{c}{\sqrt{c^2+\kappa}}} \frac{u^2\,du}{\sqrt{[c^2-(c^2-a^2)\,u^2]\,[c^2-(c^2-b^2)\,u^2]}}\right]$$

where m is the mass of the ellipsoid and where κ is defined by the equation

$$\frac{x'^2}{a^2+\kappa} + \frac{y'^2}{b^2+\kappa} + \frac{z'^2}{c^2+\kappa} - 1 = 0$$

These equations are a direct consequence of Ivory's theorem in celestial mechanics, which is: Two confocal ellipsoids attract particles which are exterior to both of them in the same direction and with forces which are proportional to their masses.

In the present case, the ellipsoid is an oblate ellipsoid of revolution. Hence $a=b$ and the above equations can be integrated and become

$$\frac{X'}{x'} = \frac{Y'}{y'} = \left(\frac{1}{m} + \frac{1}{m_1}\right)\left\{-2\pi\rho m_1 k^2 \frac{\sqrt{1-e^2}}{e^3}\left[\frac{-ae}{\sqrt{a^2+\kappa}}\sqrt{1-\frac{a^2 e^2}{a^2+\kappa}} + \sin^{-1}\left(\frac{ae}{\sqrt{a^2+\kappa}}\right)\right]\right\}$$

$$\frac{Z'}{z'} = \left(\frac{1}{m} + \frac{1}{m_1}\right)\left\{-4\pi\rho m_1 \frac{k^2}{e^3}\left[\frac{ce}{\sqrt{c^2+\kappa}} - \sqrt{1-e^2} \times \tan^{-1}\left(\frac{ce}{\sqrt{(1-e^2)(c^2+\kappa)}}\right)\right]\right\}$$

e being the eccentricity of the principal section of the ellipsoid.

These formulae can be greatly simplified, however, because the value of the upper limit of integration is always very small. The expressions $\sin^{-1}$ and $\tan^{-1}$ can be developed into series which are exceedingly convergent and by making simplifications we obtain the following formulae which give the values of the attractions to eight significant figures:

$$\frac{X'}{x'} = \frac{Y'}{y'} = -2\pi\rho k^2 \frac{\sqrt{1-e^2}}{e^3}\left[\frac{2\alpha^3}{3} + \frac{\alpha^5}{5} + \frac{23\alpha^7}{280}\right] m_1 \left(\frac{1}{m} + \frac{1}{m_1}\right)$$

$$\text{where } \alpha = \sqrt{\frac{a^2-c^2}{a^2+\kappa}} \text{ and } e = \sqrt{\frac{a^2-c^2}{a^2}}$$

$$\text{or in the present case } \alpha = \frac{0\cdot019457673}{\sqrt{a^2+\kappa}}, \quad \rho = \frac{12\cdot5}{(0\cdot02)^3}$$

$$\frac{Z'}{z'} = -4\pi\rho k^2 \frac{\sqrt{1-e^2}}{e^3}\left[\frac{\beta^3}{3} - \frac{\beta^5}{5} + \frac{\beta^7}{7}\right] m_1 \left(\frac{1}{m} + \frac{1}{m_1}\right)$$

$$\text{where } \beta = \sqrt{\frac{c^2 e^2}{(1-e^2)(c^2+\kappa)}} = \sqrt{\frac{a^2-c^2}{c^2+\kappa}} = \frac{0\cdot019457673}{\sqrt{c^2+\kappa}}$$

Applying all the above formulae and noting that the computations when made for one-half of the ellipse can easily furnish the values for the other half, the following results for the chosen set of values of v have been obtained.

True Anomaly v	$\frac{d\Omega}{dt}$	$\frac{di}{dt}$	$\frac{d\omega}{dt}$	$\frac{da}{dt}$	$\frac{de}{dt}$	$\frac{d\sigma}{dt}$	Mean Anomaly	Time
° ′							°	d
0 00·0	−0·0003771	0·0000000	+0·00005528	0·00000000	0·00000000	+0·00011107	0·000	0·0000
10 43·0	−0·0003593	+0·0000438	+0·00003914	−0·00000319	+0·00005146	+0·00004180	5·120	0·4847
21 51·0	−0·0003081	+0·0000796	+0·00000710	−0·00001117	+0·00010862	−0·00014095	10·517	0·9956
33 46·8	−0·0002293	+0·0000986	−0·00003920	−0·00002591	+0·00012273	−0·00033677	16·536	1·5654
47 07·2	−0·0001364	+0·0000944	−0·00008752	−0·00004540	+0·00007107	−0·00033785	23·715	2·2450
62 43·8	−0·0000513	+0·0000640	−0·00008087	−0·00005879	−0·00002337	−0·00024884	33·064	3·1301
81 55·8	−0·0000036	+0·0000164	−0·00002892	−0·00005188	−0·00009626	+0·00009416	46·400	4·3925
106 47·4	−0·0000100	−0·0000213	+0·00004953	−0·00001645	−0·00009653	+0·00027685	68·505	6·4851
139 39·0	−0·0000391	−0·0000214	+0·00006890	−0·00000134	−0·00004478	+0·00010409	109·821	10·3964
180 00·0	−0·0000481	0·0000000	+0·00005029	0·00000000	0·00000000	−0·00002521	180·000	17·0400
220 21·0	−0·0000391	+0·0000214	+0·00006890	+0·00000134	+0·00004478	+0·00010409	250·179	23·6836
253 12·6	−0·0000100	+0·0000213	+0·00004953	+0·00001645	+0·00009653	+0·00027685	291·495	27·5949
278 04·2	−0·0000036	−0·0000164	−0·00002892	+0·00005188	+0·00009626	+0·00009416	313·600	29·6875
297 16·2	−0·0000513	−0·0000640	−0·00008087	+0·00005879	+0·00002337	−0·00024884	326·936	30·9499
312 52·8	−0·0001364	−0·0000944	−0·00008752	+0·00004540	−0·00007107	−0·00033785	336·285	31·8350
326 13·2	−0·0002293	−0·0000986	−0·00003920	+0·00002591	−0·00012273	−0·00033677	343·464	32·5146
338 09·0	−0·0003081	−0·0000796	+0·00000710	+0·00001117	−0·00010862	−0·00014095	349·483	33·0844
349 16·2	−0·0003593	−0·0000438	+0·00003914	+0·00000319	−0·00005146	+0·00004180	354·880	33·5953

We can now obtain the curves of variation of $\frac{d\Omega}{dt}$, $\frac{di}{dt}$, ... according to time. The period of our theoretical binary system being $34^d{\cdot}08$, and since the values of $\frac{di}{dt}$, $\frac{da}{dt}$ and $\frac{de}{dt}$ between 180° and 360° are the same but of opposite sign to those between 180° and 0°, we shall evidently have:

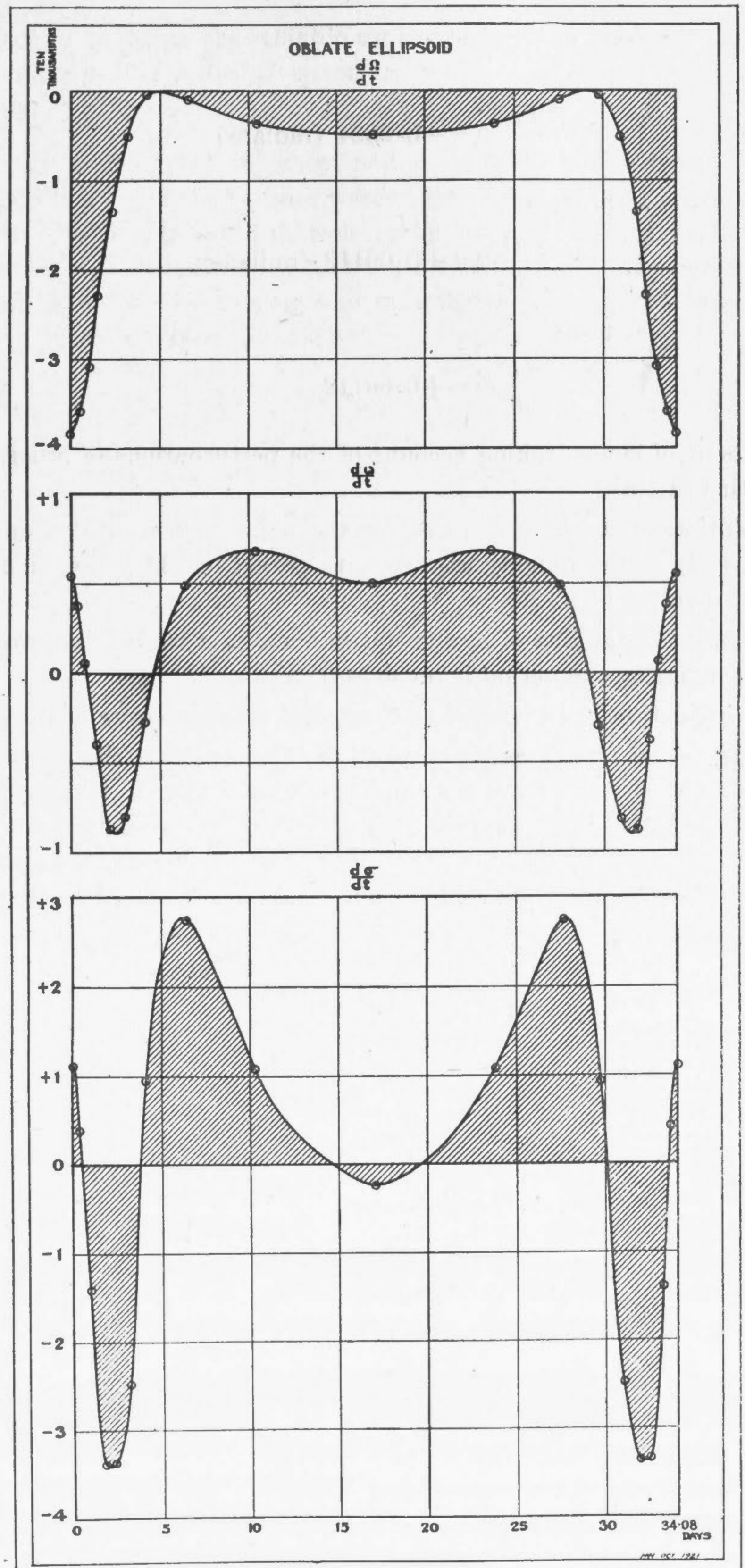
$$\int_0^{34^d\cdot 08} \frac{di}{dt}\, dt = 0$$

$$\int_0^{34^d\cdot 08} \frac{da}{dt}\, dt = 0$$

$$\int_0^{34^d\cdot 08} \frac{de}{dt}\, dt = 0$$

so that considering the perturbations of the first order only, after a certain number of complete revolutions of the satellite there will be no perturbation of the inclination, of the semi-major axis or of the eccentricity.

Considering now $\frac{d\Omega}{dt}$, $\frac{d\omega}{dt}$ and $\frac{d\sigma}{dt}$ they will give us the accompanying curves of first order perturbations.


OBLATE ELLIPSOID
TEN THOUSANDTHS
$\frac{d\Omega}{dt}$
0
-1
-2
-3
-4
$\frac{d\omega}{dt}$
+1
0
-1
$\frac{d\sigma}{dt}$
+3
+2
+1
0
-1
-2
-3
-4
0
5
10
15
20
25
30
34·08
DAYS

By measuring the areas of these curves we obtain:

$$\int_0^{34^d \cdot 08} \frac{d\Omega}{dt} dt = -0 \cdot 0024 \text{ (radians)}$$

$$\int_0^{34^d \cdot 08} \frac{d\omega}{dt} dt = +0 \cdot 00112 \text{ (radians)}$$

$$\int_0^{34^d \cdot 08} \frac{d\sigma}{dt} dt = +0 \cdot 00112$$

this means, without of course taking account of the perturbations of orders higher than the first, and other factors,

(1) A revolution of the line of nodes (or the plane of the orbit around the small axis of the ellipsoid) in the roughly approximate period of 244 years, since during the period $34^d \cdot 08$ it revolves $0° \ 8' \cdot 25$.

(2) A revolution of the line of apsides in its plane in a roughly approximate period of 524 years, since during the period it revolves $0° \ 3' \cdot 85$.

(3) A slow oscillation in the period of the radial velocity curve of the satellite.

Qualitatively, these results seem to explain the observations; the theoretical system that we have considered is no doubt not far from being similar to what σ Scorpii would be under similar hypotheses. Quantitatively, the theoretical results do not agree, for it seems that the revolution of the line of nodes in the case of σ Scorpii is much more rapid. We have tried to modify the conditions in our theoretical system, so that the periods of variations would agree better with the observations, but so far in vain.

If for instance we take the inclination of the orbit to be 80° instead of 40° and compute the values of W, S and R for $v = 180°$ and $v = 0°$ we have:

v	$\frac{d\Omega}{dt}$	
180°	$-0 \cdot 00095763$	
0°	$-0 \cdot 00016994$	

Comparing these results with the same results plotted on our former curve of $\frac{d\Omega}{dt}$, and knowing that the ordinates are zero for the same abscissae it is not difficult to see that the area of the present curve or $\int_0^{34^d \cdot 08} \frac{d\Omega}{dt} dt$ will be approximately eight times the area of the former curve—or in other words the revolution of the line of nodes will be about eight times as fast. However, this does not constitute a great improvement.

The effect of changing the relative masses of the two bodies and the consideration as such of different theoretical systems have not done much to improve our results quantitatively.

Let us now assume that the short period velocity curve is due to the existence of a short period binary system; in other words let us assume that σ Scorpii is really a triple system. The following theoretical triple system would not be very far from satisfying the observations. We shall assume as before that the total mass $m_1 + m_2$ of the close binary system is $12 \cdot 5$, remembering also that the elements of the short period spectroscopic orbit in 1918 have been given as*

$$\left.\begin{aligned}
k &= 41 \cdot 2 \text{ km.} \\
e &= 0 \cdot 11 \\
P &= 0^{d} \cdot 246834 \\
\omega &= 15^\circ \\
T &= 2421687 \cdot 972 \text{ J.D.} \\
\gamma &\quad \text{variable} \\
a \sin i &= 138500 \text{ km.} \\
\frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} &= 0 \cdot 00176
\end{aligned}\right\}$$

We shall however assume that this short period orbit is circular, $e = 0 \cdot 0$, which will simplify the computations but not alter the order and magnitude of the perturbations obtained. Let us refer the whole system to the center of mass of the close binary system; then if we call a_1, a_2 and a_3, respectively, the semi-major axes of m_1, m_2 and m_3 around that center we will adopt the following values, partly deduced from the observations:

$$\begin{aligned}
m_1 &= 7 \cdot 1639906 \\
m_2 &= 5 \cdot 3360094 \\
m_3 &= 6 \cdot 8415 \\
m_1 + m_2 &= 12 \cdot 5 \\
a_1 &= 0 \cdot 007576423 \\
a_2 &= 0 \cdot 010171912 \\
a_3 &= 0 \cdot 550339
\end{aligned}$$

and we will assume the angle between the two orbit planes to be

$$i = 37^\circ$$

The other elements of the large orbit have been assumed to be the same as in the case of the ellipsoid.

We shall take the plane of the large orbit as plane of reference or xy plane, the center of mass of the close binary system being the origin, the major axis of the large orbit will be found along the Ox axis, defining this way our system of rectangular coördinates

* Lick Obs. Bul. Vol 9, p. 176.

The perturbations of the large orbit will be computed, for one revolution of the third body, by applying again Lagrange's equations given above. The components of the disturbing acceleration W, R and S will be computed for a certain number of consecutive positions of the three bodies—the disturbing force is equal to the attraction of the two bodies m_1 and m_2 on m_3 minus the attraction of $m_1 + m_2$ supposed to be condensed at the center of mass of the close binary system.

In order, however, to simplify the computations which are very long, a semi-graphical method of obtaining the perturbations will be used as follows. Eighteen points, as in the case of the oblate ellipsoid, will be taken on the large ellipse, with the same mean and true anomalies that have already been computed. We will then take on the small circles on which the masses m_1 and m_2 move, a rigid determined position of the two bodies and for this position and the eighteen points determine:

$$\frac{d\Omega}{dt}, \quad \frac{da}{dt}, \quad \frac{d\omega}{dt}, \quad \frac{de}{dt}, \quad \frac{di}{dt} \quad \text{and} \quad \frac{d\sigma}{dt}$$

Let us now suppose that for this rigid position the curve of variation of $\frac{d\Omega}{dt}$ has been determined. If we assume now that m_1 and m_2 move, they will occupy this determined position at times t_1, t_2, t_3 . . . . whose intervals are equal to the period $0^d{\cdot}246834$ and the points A, B, C, D . . . . that correspond to these times are points of the true curve of variation of $\frac{d\Omega}{dt}$ in our theoretical triple system. If now in the same manner and for another rigid position of m_1 and m_2 another curve of variation of $\frac{d\Omega}{dt}$ is determined, they will give for the times t'_1, t'_2 . . . . . at which the bodies are in this position, the points A', B', C', D' . . . . which are other points of the curve of variation of $\frac{d\Omega}{dt}$. If we proceed in a similar manner for other rigid positions we will see that the curve $A\ A'\ A''$ . . . $B\ B'\ B''$ . . . $C\ C'\ C''$ . . is the real curve of variation of $\frac{d\Omega}{dt}$

Let us now consider the rectangular system of coördinates $Oxyz$ as defined above that is, the center of mass of m_1 and m_2 or focus of the large ellipse will be taken as origin, Ox will coincide with the major axis of this ellipse and Oy will be in its plane (in the present case will coincide with the line of nodes, or intersection of the planes of the two orbits). Let zOx rotate 37° around Oy, to obtain a new system of coördinates $Ox'yz'$, yOx' being the plane of the small orbit.

In the plane yOx' let us consider the equations of the two circles which are:

$$x'^2 + y^2 = a_1^2$$
$$x'^2 + y^2 = a_2^2$$

Let us now consider six rigid positions of m_1 on the first circle and the six corresponding positions of m_2 on the second circle which are:

First Circle			Second Circle		
x'	y	z'	x'	y	z'
$+a_1$	0	0	$-a_2$	0	0
$a_1 \cos 60°$	$a_1 \sin 60°$	0	$-a_2 \cos 60°$	$-a_2 \sin 60°$	0
$a_1 \cos 120°$	$a_1 \sin 120°$	0	$-a_2 \cos 120°$	$-a_2 \sin 120°$	0
$-a_1$	0	0	$+a_2$	0	0
$a_1 \cos 240°$	$a_1 \sin 240°$	0	$-a_2 \cos 240°$	$-a_2 \sin 240°$	0
$a_1 \cos 300°$	$a_1 \sin 300°$	0	$-a_2 \cos 300°$	$-a_2 \sin 300°$	0

We will obtain their coördinates in the system $Oxyz$ by using the formulae of transformation:

$$x = x' \cos 37°$$
$$z = x' \sin 37°$$

For each of these six pairs of points combined with each of the eighteen points of the ellipse, the disturbing forces will be computed. If we call x_1, y_1, z_1; x_2, y_2, z_2; x_3, y_3, z_3 the coördinates, respectively, of m_1, m_2, m_3 we have:

Attraction parallel to Ox of m_1 on m_3:

$$-k^2 \frac{m_1 m_3 (x_3 - x_1)}{\rho_2^3} \qquad \text{(A)}$$

Attraction parallel to Ox of m_2 on m_3:

$$-k^2 \frac{m_2 m_3 (x_3 - x_2)}{\rho_1^3} \qquad \text{(B)}$$

with

$$\rho_2 = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$$

$$\rho_1 = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$$

Attraction parallel to Ox of $m_1 + m_2$ (supposed to be at the center of mass of m_1 and m_2):

$$-k^2 \frac{(m_1 + m_2) m_3 x_3}{r^3} \qquad \text{(C)}$$

with

$$r = \sqrt{x_3^2 + y_3^2 + z_3^2}$$

The component of the disturbing acceleration parallel to Ox will then be:

$$\left\{ \text{(A)} + \text{(B)} - \text{(C)} \right\} \frac{m_1 + m_2 + m_3}{(m_1 + m_2)\, m_3}$$

Similar formulæ will be used for the components parallel to Oy and Oz. The computations have been made for the six positions of m_1 and m_2 given above, and the following results have been obtained:

FIRST POSITION

v	$\frac{d\Omega}{dt}$	$\frac{d\omega}{dt}$	$\frac{d\sigma}{dt}$	$\frac{di}{dt}$	$\frac{da}{dt}$	$\frac{de}{dt}$
° ′						
0 00·0	0	−0·001002	+0·000038	0	−0·000043	+0·000034
10 43·8	0	−0·000803	+0·000043	0	+0·000180	−0·000068
21 51·0	0	−0·000180	+0·000600	0	+0·000338	−0·000179
33 46·8	0	+0·000725	+0·000684	0	+0·000380	−0·000332
47 07·2	0	+0·001133	+0·000818	0	+0·000310	−0·000409
62 43·8	0	+0·001051	+0·000768	0	+0·000125	−0·000437
81 55·8	0	+0·000295	+0·000504	0	−0·000047	−0·000285
106 47·4	0	−0·000325	+0·000106	0	−0·000085	−0·000022
139 39·0	0	−0·000142	−0·000184	0	−0·000028	+0·000083
180 00·0	0	+0·000065	−0·000121	0	−0·000174	+0·000010

SECOND POSITION

v	$\frac{d\Omega}{dt}$	$\frac{d\omega}{dt}$	$\frac{d\sigma}{dt}$	$\frac{di}{dt}$	$\frac{da}{dt}$	$\frac{de}{dt}$
° ′						
0 00·0	+0·000302	+0·000193	−0·000206	0·000000	+0·000381	−0·000299
10 43·8	+0·000328	+0·002888	−0·001023	−0·000037	+0·000446	−0·000588
21 51·0	+0·000318	+0·003413	−0·000555	−0·000077	+0·000445	−0·000882
33 46·8	+0·000273	+0·003187	+0·000302	−0·000110	+0·000408	−0·001028
47 07·2	+0·000195	+0·002559	+0·001044	−0·000126	+0·000326	−0·000974
62 43·8	+0·000099	+0·001583	+0·001256	−0·000115	+0·000228	−0·000672
81 55·8	+0·000018	+0·000672	+0·000683	−0·000075	+0·000134	−0·000245
106 47·4	−0·000011	+0·000196	−0·000088	−0·000022	+0·000050	+0·000022
139 39·0	+0·000012	+0·000020	−0·000075	+0·000006	+0·000003	+0·000012
180 00·0	+0·000038	−0·000059	+0·000053	0·000000	−0·000012	+0·000049

THIRD POSITION

v	$\frac{d\Omega}{dt}$	$\frac{d\omega}{dt}$	$\frac{d\sigma}{dt}$	$\frac{di}{dt}$	$\frac{da}{dt}$	$\frac{de}{dt}$
° ′						
0 00·0	+0·000302	+0·000193	−0·000206	0·000000	−0·000381	+0·000299
10 43·8	+0·000248	+0·001247	−0·000867	−0·000028	−0·000269	+0·000082
21 51·0	+0·000173	+0·000562	−0·000459	−0·000042	−0·000138	−0·000031
33 46·8	+0·000093	−0·000136	+0·000002	−0·000038	−0·000004	+0·000017
47 07·2	+0·000023	−0·000371	+0·000106	−0·000015	+0·000068	+0·000125
62 43·8	−0·000017	−0·000370	−0·000158	+0·000020	+0·000049	+0·000215
81 55·8	−0·000012	−0·000394	−0·000467	+0·000024	−0·000046	+0·000200
106 47·4	+0·000027	−0·000323	−0·000038	+0·000053	−0·000070	+0·000026
139 39·0	+0·000050	−0·000416	+0·000248	+0·000026	−0·000056	+0·000029
180 00·0	+0·000038	−0·000061	+0·000053	0·000000	−0·000012	+0·000049

FOURTH POSITION

v	$\frac{d\Omega}{dt}$	$\frac{d\omega}{dt}$	$\frac{d\sigma}{dt}$	$\frac{di}{dt}$	$\frac{da}{dt}$	$\frac{de}{dt}$
° ′						
0 00·0	0	−0·001002	+0·000477	0	+0·000043	−0·000034
10 43·8	0	−0·000694	+0·000524	0	+0·000256	−0·000134
21 51·0	0	+0·000014	+0·000635	0	+0·000402	−0·000153
33 46·8	0	+0·000841	+0·000777	0	+0·000432	−0·000382
47 07·2	0	+0·001358	+0·000876	0	+0·000324	−0·000490
62 43·8	0	+0·001181	+0·000839	0	+0·000122	−0·000501
81 55·8	0	+0·000328	+0·000578	0	−0·000061	−0·000332
106 47·4	0	+0·000130	+0·000638	0	−0·000008	−0·000224
139 39·0	0	+0·000132	+0·000307	0	+0·000029	−0·000112
180 00·0	0	+0·000065	−0·000121	0	+0·000002	−0·000010

FIFTH POSITION

v	$\frac{d\Omega}{dt}$	$\frac{d\omega}{dt}$	$\frac{d\sigma}{dt}$	$\frac{di}{dt}$	$\frac{da}{dt}$	$\frac{de}{dt}$
° ′						
0 00·0	+0·000298	+0·000204	−0·000210	0·000000	+0·000376	−0·000294
10 43·8	+0·000322	+0·002850	−0·001011	−0·000037	+0·000437	−0·000578
21 51·0	+0·000312	+0·003235	−0·000495	−0·000076	+0·000443	−0·000850
33 46·8	+0·000268	+0·003132	+0·000293	−0·000108	+0·000399	−0·001009
47 07·2	+0·000191	+0·002516	+0·001023	−0·000124	+0·000318	−0·000958
62 43·8	+0·000097	+0·001563	+0·001237	−0·000114	+0·000224	−0·000619
81 55·8	+0·000017	+0·000668	+0·000681	−0·000074	+0·000177	−0·000245
106 47·4	−0·000011	+0·000197	−0·000085	−0·000022	+0·000050	+0·000021
139 39·0	+0·000012	+0·000021	−0·000077	+0·000006	+0·000003	−0·000007
180 00·0	+0·000038	−0·000059	+0·000053	0·000000	−0·000012	+0·000049

SIXTH POSITION

v	$\frac{d\Omega}{dt}$	$\frac{d\omega}{dt}$	$\frac{d\sigma}{dt}$	$\frac{di}{dt}$	$\frac{da}{dt}$	$\frac{de}{dt}$
° ′						
0 00·0	+0·000298	+0·000204	−0·000210	0·000000	−0·000376	+0·000294
10 43·8	+0·000245	+0·001250	−0·000866	−0·000028	−0·000267	+0·000081
21 51·0	+0·000173	+0·000443	−0·000410	−0·000042	−0·000132	−0·000015
33 46·8	+0·000095	−0·000121	−0·000010	−0·000038	−0·000005	+0·000014
47 07·2	+0·000025	−0·000362	+0·000106	−0·000016	+0·000067	+0·000121
62 43·8	−0·000016	−0·000366	−0·000154	+0·000019	+0·000050	+0·000211
81 55·8	−0·000012	−0·000405	−0·000479	+0·000050	−0·000048	+0·000205
106 47·4	+0·000027	−0·000776	−0·000466	+0·000054	−0·000153	+0·000146
139 39·0	+0·000050	−0·000550	+0·000060	+0·000026	−0·000083	+0·000135
180 00·0	+0·000038	−0·000059	+0·000053	0·000000	−0·000012	+0·000049

For the positions between 180° and 360° we obtain the same values as between 180° and 0°, with the same signs for $\frac{d\Omega}{dt}$, $\frac{d\omega}{dt}$ and $\frac{d\sigma}{dt}$ and with opposite signs for $\frac{di}{dt}$, $\frac{da}{dt}$ and $\frac{de}{dt}$. The perturbations of the first order after one revolution of the third body are thus negligible for i, a and e. Obtaining the curves of variation of $\frac{d\Omega}{dt}$, $\frac{d\omega}{dt}$ and $\frac{d\sigma}{dt}$, which present a considerable number of oscillations, it is found that in the main the perturbations are of the same magnitude as in the case where the short period radial velocity curve is explained by a Jacobian ellipsoid.

The present method of computing the perturbations in the case of a three-body system is not however very satisfactory; it would be better and perhaps simpler to try to solve the problem directly, following already developed theoretical methods when the third body is at a great distance from the two others (the last ones being rather near each other) but since some doubt is cast on the possibility of σ Scorpii being really a triple system, and also a considerable number of further observations are desirable, it seems preferable at present not to undertake to solve that question.

A factor of importance which we have not yet mentioned in this paper is the rather remarkable change in the widths of the spectral lines, a change which has the same period as the short period radial velocity variation*. This change, perhaps, could best be explained with the Jacobian ellipsoid hypothesis. We have also to remember that σ Scorpii on direct photographs of the sky seems to be in the midst of extensive nebular clouds.

It is possible, considering the high temperature of the star and the great ionization of the gases that compose it, that strong electric or magnetic action would have to be considered (producing perhaps by a Zeeman or a Stark effect, the variation of width in the spectral lines). It may be, also, considering that σ Scorpii is perhaps plunged in an enormous nebula, that this nebula would play an important part in producing the observed variations. It is however better for the present to suspend any further speculation and wait until more data concerning σ Scorpii and other stars of the β Canis Majoris type have been obtained.

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* Lick Obs. Bul., Vol. 9, p. 177.

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