

PUBLICATIONS
OF THE
DOMINION OBSERVATORY
OTTAWA, CANADA

Vol. IV, No. 16

ORBIT OF THE SPECTROSCOPIC BINARY *h* DRACONIS

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This star ($\alpha=16^h 56^m$, $\delta=+65^\circ 16'$, photographic magnitude 5.3, type F5) was announced as a spectroscopic binary by Campbell in *Astrophysical Journal*, X, page 178. His four observations, given in the table of measures, show a range of 20 km. The star was placed on our observing programme eleven years ago, but it was not followed up until this year when 25 spectrograms were obtained with the single-prism spectrograph. This is somewhat fewer than the number of observations generally used in determining an orbit, but the lines are very good for measurement and none of the plates show large residuals from the final curve, the probable error of a plate being ± 1.3 km. per second.

About 15 or 20 lines were measured on the plates. Their wave-lengths are given in the following table, which shows also how many times they were measured and their residuals, taken in the sense, mean velocity of plate minus line-velocity.

LINES IN *h* DRACONIS

λ	Element	Times Measured	Residual		λ	Element	Times Measured	Residual	
			Numerical	Algebraic				Numerical	Algebraic
4572.106	Ti	6	7.6	-1.8	4271.755	Fe	24	5.0	+0.4
4549.809	Fe-Ti	22	7.3	+0.5	4260.604	Fe	16	4.9	+0.4
4531.419	Cr-Fe	3	0.6	-0.4	4250.593	Fe	9	5.0	-0.4
4501.517	Ti	3	3.3	-0.2	4236.121	Fe	12	6.6	+4.1
4481.848	Mg	4	11.6	-2.3	4233.475	Mn-Fe	15	6.8	0.0
4455.072	Ca-Zr-Mn	9	5.0	-1.0	4227.055	Ca	14	4.5	-1.3
4415.389	Fe	21	5.1	-0.2	4215.727	Fe-Sr	6	3.8	-0.6
4404.897	Fe	24	6.4	+0.2	4202.308	Fe	7	3.7	+0.3
4395.446	Ti-V	6	3.5	-0.9	4198.702	Fe	10	6.2	-2.8
4383.690	Fe	5	3.6	+1.2	4143.897	Fe	19	6.3	-0.8
4352.048	Cr-Mg	12	8.9	+0.2	4101.890	H	5	13.0	-3.6
4340.634	H	19	7.2	+0.5	4071.929	Fe	15	2.7	-0.5
4325.799	Fe	20	6.2	-0.5	4063.770	Fe	17	5.6	+0.9
4308.044	Fe	21	6.7	-1.2	4045.966	Fe	17	7.1	+0.3
4289.904	Cr-Ti	3	1.8	+1.8	4005.356	Fe	3	3.1	+0.8
4282.793	Fe	2	3.6	+0.8					

MEASURES OF δ DRACONIS

Plate	Date	Julian Date	Phase	Velocity	Weight	O-C
	1899					
Lick.....	June 26.....	2,414,832.8*	19.05	-26.	+1.1
Lick.....	July 11.....	847.8	34.05	36.	+2.0
Lick.....	" 16.....	852.8	39.05	32.	+0.7
Lick.....	" 24.....	860.8	47.05	16.	-0.9
	1918					
8451.....	Feb. 17.....	2,421,642.907	3.44	1.5	19	+1.1
8471.....	Mar. 10.....	663.775	24.31	34.6	11	0.0
8482.....	" 27.....	680.710	41.24	33.2	11	-4.2
8498.....	April 9.....	693.704	2.52	2.0	17	+0.8
8501.....	" 14.....	698.720	7.54	4.2	11	+1.6
8506.....	" 19.....	703.651	12.47	13.9	16	+0.5
8510.....	" 22.....	706.647	15.47	20.7	10	-0.1
8512.....	" 24.....	708.688	17.51	25.4	15	-0.9
8517.....	" 26.....	710.667	19.49	26.4	13	+1.5
8520.....	May 8.....	722.602	31.42	40.1	14	-1.5
8521.....	" 8.....	722.656	31.48	40.3	11	-1.7
8522.....	" 14.....	728.690	37.51	35.8	12	-0.8
8525.....	" 20.....	734.657	43.48	20.4	12	+3.7
8537.....	June 5.....	750.647	7.76	6.6	2	-0.6
8539.....	" 7.....	752.693	9.80	10.3	12	-0.8
8580.....	" 18.....	763.656	20.77	28.6	7	+1.3
8593.....	" 25.....	770.610	27.72	35.0	3	+2.5
8611.....	July 9.....	784.604	41.71	27.6	12	+0.3
8617.....	" 12.....	787.684	44.79	22.8	22	-2.0
8622.....	" 17.....	792.699	49.81	9.7	17	-1.1
8639.....	Aug. 25.....	831.685	37.08	32.9	5	+2.5
8640.....	" 26.....	832.573	37.97	29.8	15	+4.5
8645.....	" 29.....	835.651	41.05	30.5	22	-1.1
8649.....	Sept. 13.....	850.550	4.24	5.3	10	-2.6
8650.....	" 17.....	2,421,854.664	8.35	-7.1	11	-0.1

* The decimal of a day is assumed

MEASURES OF κ DRACONIS

λ	8451		8471		8482		8498		8501		8506		8510	
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.
4572	+ 6.3	$\frac{1}{2}$	-21.4	1										
4549	- 6.4	$\frac{1}{2}$			-24.5	$\frac{1}{2}$	-10.0	1	-17.5	1	- 5.7	1	- 8.9	1
4531	- 3.8	$\frac{1}{2}$											17.9	1
4481					18.8	1			+11.8	1				
4455	-13.4	$\frac{3}{4}$			15.9	$\frac{1}{2}$			- 1.7	1	- 9.1	1		
4415	-11.2	1	24.1	1			- 3.0	1	- 7.1	1	-16.4	$1\frac{1}{2}$	21.8	1
4404	-17.8	$\frac{1}{2}$	17.8	1	39.7	1	- 6.9	1	+ 2.2	1	-11.8	$\frac{1}{2}$	22.6	$\frac{1}{2}$
4395	- 4.6	$\frac{1}{2}$	38.7	1										
4383							+ 5.8	1			-16.0	1		
4352											+ 4.1	$\frac{1}{2}$	36.3	1
4340	+ 2.5	1	51.2	1	38.0	$\frac{1}{2}$	- 3.3	1	-22.5	1	-12.3	1		
4325	- 3.6	1	44.0	1			+ 4.3	1	+ 9.8	1	-16.5	1		
4308	- 7.2	1	38.8	1	38.3	1	+ 1.9	1	+11.4	1	-16.1	1		
4289	- 4.7	1					- 4.2	1						
4282	- 0.8	1												
4271	- 8.4	$1\frac{1}{2}$	44.8	1	32.0	1	+ 3.9	1	- 7.5	1	- 2.2	1	25.3	1
4260	- 6.5	1			31.8	1	+ 2.3	1			-14.3	1	23.5	1
4250											-17.2	$\frac{1}{2}$		
4236	+ 1.1	1	41.5	1	34.5	$\frac{1}{2}$								
4233	+ 1.7	1			35.8	1	+ 3.8	1						
4227	+ 8.2	1					- 0.5	1			- 8.9	1	11.1	$\frac{1}{2}$
4215											- 5.6	1		
4198							+ 4.4	1	+ 3.5	1	-10.9	$\frac{1}{2}$	5.4	1
4143	- 7.2	$\frac{1}{2}$	25.1	$\frac{1}{2}$	26.9	1	- 1.7	1	- 7.3	1			15.1	1
4071	- 7.7	1	34.9	$\frac{1}{2}$			- 4.1	1			-11.7	1	-10.7	1
4063	- 3.8	$\frac{1}{2}$	31.0	$\frac{1}{2}$	36.4	1	+ 2.4	1			-21.9	1		
4045	+ 2.1	1	-38.3	$\frac{1}{2}$	-41.8	1	- 1.0	1			- 9.3	1		
4005	- 0.1	1												
Weighted mean	- 3.61		-35.16		-32.54		- 0.35		- 2.26		-11.64		-18.17	
V_0	+ 2.28		+ 0.77		- 0.52		- 1.48		- 1.82		- 2.15		- 2.34	
V_d	+ 0.08		+ 0.12		+ 0.13		+ 0.12		+ 0.11		+ 0.13		+ 0.13	
Curv.	- 0.28		- 0.28		- 0.28		- 0.28		- 0.28		- 0.28		- 0.28	
Radial Velocity	- 1.5		-34.6		-33.2		- 2.0		- 4.2		-13.9		-20.7	

MEASURES OF δ DRACONIS—Continued

λ	8512		8517		8520		8521		8522		8525		8537	
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.
4572									-30.2	1	-18.7	1		
4549	-14.9	$\frac{1}{2}$	-10.2	1	-30.6	1	-31.9	1					-1.4	$\frac{1}{2}$
4531							35.7	1						
4481	29.9	$\frac{1}{2}$	35.0	$\frac{1}{2}$										
4455					38.7	1	40.4	1	31.3	1	9.3	$\frac{1}{2}$		
4415	16.1	1	27.5	1	38.8	1			26.4	1	19.3	1	+5.0	$\frac{1}{2}$
4404	36.0	1	26.7	1	36.1	1	39.7	1	37.1	1			-7.4	1
4395	21.9	1			39.9	1								
4383			26.8	1					36.2	1				
4352											0.0	1		
4340			37.6	$\frac{1}{2}$	46.8	1	36.6	1						
4325	26.2	$\frac{1}{2}$	17.6	$\frac{1}{2}$	28.0	1	42.2	1	29.4	1				
4308	24.9	1	46.2	1	28.1	1	25.5	1	30.3	1	13.3	1	+9.9	$\frac{1}{2}$
4271	23.6	1	18.8	1	33.6	1	40.5	1	27.8	1	9.5	1		
4260	19.0	1	24.8	1							25.5	$\frac{1}{2}$		
4250	14.7	1	14.6	1							19.6	$\frac{1}{2}$		
4236	25.9	$\frac{1}{2}$	13.4	1							20.8	$\frac{1}{2}$		
4233	42.0	$\frac{1}{2}$	37.8	$\frac{1}{2}$			34.7	1	37.6	1	13.2	1		
4227	22.8	1	27.4	$\frac{1}{2}$	35.1	1	47.1	1						
4215	26.1	1												
4202					31.3	1								
4198							-31.6	1	47.0	1				
4143	7.7	1	12.2	1					25.8	1	27.7	1		
4071	20.5	$\frac{1}{2}$			33.3	1					18.3	1		
4063	9.3	$\frac{1}{2}$			46.9	1					19.2	1		
4045	-32.9	1	-16.1	$\frac{1}{2}$	-46.7	1			-25.1	1	-24.4	$\frac{1}{2}$		
Weighted mean	-22.77		-23.60		-36.71		-36.90		-32.02		-16.43		-2.20	
V_a	-2.47		-2.65		-3.24		-3.24		-3.52		-3.76		-4.20	
V_d	+0.12		+0.12		+0.12		+0.09		+0.07		+0.07		+0.09	
Curv.	-0.28		-0.28		-0.28		-0.28		-0.28		-0.28		-0.28	
Radial Velocity	-25.4		-26.4		-40.1		-40.3		-35.8		-20.4		-6.6	

MEASURES OF *h* DRACONIS—Continued

λ	8539		8580		8593		8611		8617		8622		8639	
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.
4572											- 2.7	$\frac{1}{2}$		
4549	-21.4	1	-28.2	1	-35.1	$\frac{1}{2}$	-21.4	1	-12.5	1	- 5.3	$\frac{1}{2}$	-49.1	$\frac{1}{2}$
4501	- 8.1	$\frac{1}{2}$											25.3	$\frac{1}{2}$
4415	- 8.8	1	29.4	$\frac{1}{2}$			20.1	1	24.6	1	- 0.7	1	15.1	$\frac{1}{2}$
4404	- 3.1	1	8.5	$\frac{1}{2}$	13.7	$\frac{1}{2}$	31.5	1	26.6	1	- 5.1	1	29.1	$\frac{1}{2}$
4395													24.2	$\frac{1}{2}$
4352			18.7	$\frac{1}{2}$	54.7	$\frac{1}{4}$	23.5	$\frac{1}{2}$	30.4	1	- 3.1	1		
4340	- 0.2	$\frac{1}{2}$	10.6	$\frac{1}{2}$	24.0	$\frac{1}{2}$	27.7	$\frac{1}{2}$	8.6	1	+ 2.8	$\frac{1}{2}$		
4325	+14.3	$\frac{1}{2}$	37.0	$\frac{1}{2}$	32.6	$\frac{1}{4}$			9.2	1	- 3.7	1	33.0	$\frac{1}{2}$
4308	+ 3.9	$\frac{1}{2}$	24.9	$\frac{1}{2}$					11.9	1	+ 2.4	1		
4289	- 6.4	1												
4271	- 9.9	1	14.4	1	35.8	$\frac{1}{2}$	9.6	1	23.5	$1\frac{1}{2}$	- 8.5	1	36.3	$\frac{1}{2}$
4260	- 8.8	1					8.3	$\frac{1}{2}$	17.7	1	+ 1.3	1	25.8	$\frac{1}{2}$
4250	+ 1.6	1			-31.1	$\frac{1}{4}$			20.1	$\frac{1}{2}$				
4236			37.7	1			29.2	1	23.9	1	-14.6	1		
4233			-22.0	$\frac{1}{2}$					8.6	1	- 4.5	1		
4227							27.1	1	15.7	1			33.8	$\frac{1}{2}$
4215									20.0	1	-10.1	1		
4202	- 1.5	1					29.3	$\frac{1}{2}$	16.2	1				
4198							17.3	$\frac{1}{2}$	20.3	1				
4143	0.0	$\frac{1}{2}$					27.8	$\frac{1}{2}$	16.7	1	-16.0	1	22.6	$\frac{1}{2}$
4101									13.1	1	+ 3.2	1	54.0	$\frac{1}{2}$
4071							25.3	1	20.1	1	- 4.5	1	38.1	$\frac{1}{2}$
4063	- 0.3	1					15.0	1	21.2	1	- 6.2	1	38.5	$\frac{1}{2}$
4045							-31.9	1	20.1	1	-14.2	1	- 3.2	$\frac{1}{2}$
4005									-19.9	1	- 9.7	$\frac{1}{2}$		
Weighted mean	- 5.82		- 23.97		- 30.51		- 23.17		- 18.30		- 5.40		- 30.51	
<i>V</i> _o	- 4.24		- 4.34		- 4.34		- 4.13		- 3.99		- 3.90		- 1.92	
<i>V</i> _r	+ 0.02		+ 0.03		+ 0.10		+ 0.02		- 0.15		- 0.10		- 0.15	
Curv.	- 0.28		- 0.28		- 0.28		- 0.28		- 0.28		- 0.28		- 0.28	
Radial Velocity	- 10.3		- 28.6		- 35.0		- 27.6		- 22.8		- 9.7		- 32.9	

MEASURES OF λ DRACONIS—*Concluded*

λ	8640		8645		8649		8650							
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.
4572					-19.3	$\frac{1}{2}$								
4549	-38.3	$\frac{1}{2}$	-32.7	1	+0.5	$\frac{1}{2}$	-0.1	$\frac{1}{2}$						
4501					-6.7	$\frac{1}{2}$								
4455							-9.1	$\frac{1}{2}$						
4415	27.0	1	34.1	1	-3.1	$\frac{1}{2}$								
4404	18.6	1	28.0	1	-6.3	$\frac{1}{2}$	-0.1	$\frac{3}{4}$						
4395					+1.5	$\frac{1}{2}$								
4383			29.4	1										
4352	25.5	$\frac{1}{2}$	22.4	$\frac{1}{2}$	-1.4	$\frac{1}{2}$	-4.9	$\frac{1}{2}$						
4340	22.1	$\frac{1}{2}$	28.3	1	+2.0	$\frac{1}{2}$	-4.6	$\frac{3}{4}$						
4325	33.1	1	26.5	1	-4.8	$\frac{3}{4}$	-17.0	$\frac{3}{4}$						
4308	22.4	1	29.6	1	-7.4	$\frac{3}{4}$	-16.3	$\frac{3}{4}$						
4282			33.0	1										
4271	29.4	1	25.4	1	-10.1	$\frac{3}{4}$	-9.1	$\frac{1}{2}$						
4260	34.6	1	39.3	1			-4.7	$\frac{1}{2}$						
4250			28.5	$\frac{1}{2}$			-15.7	$\frac{1}{2}$						
4236			29.0	1										
4233	35.9	1	26.7	1	+12.5	$\frac{1}{2}$	-1.9	$\frac{3}{4}$						
4227			28.7	1	-2.4	$\frac{1}{2}$	+5.0	$\frac{3}{4}$						
4215	25.7	1	23.2	1										
4202	30.3	$\frac{1}{2}$	32.3	1			-7.6	$\frac{3}{4}$						
4198			24.3	1			-0.7	$\frac{1}{2}$						
4143	32.3	1	33.4	1	-10.2	$\frac{1}{2}$	-6.1	$\frac{1}{2}$						
4101	8.2	$\frac{1}{2}$	20.2	1										
4071	22.1	1	22.7	1	-0.9	1								
4063	36.8	1	23.1	1	-9.4	$\frac{1}{2}$	-7.8	1						
4045	-19.0	1	-31.3	1			-6.9	1						
Weighted mean	-27.52		-28.48		-4.32		-6.42							
V_s	-1.87		-1.66		-0.59		-0.28							
V_z	-0.09		-0.12		-0.09		-0.13							
Curv.	-0.28		-0.28		-0.28		-0.28							
Radial Velocity	-29.8		-30.5		-5.3		-7.1							

Our own observations when plotted gave a period around 50 days. If the correct number of cycles has been taken to connect up the Lick observations with our own, the period determined, 51.710 days, should be correct at least to the second decimal place. With this period, the Ottawa observations were grouped according to phase into 11 normal places and graphical elements determined as follows:

$$\begin{aligned}
 P &= 51.710 \text{ days} \\
 e &= .15 \\
 \omega &= 330^\circ \\
 K &= 18.25 \text{ km.} \\
 \gamma &= -22.62 \text{ km.} \\
 T &= \text{J. D. } 2,414,813.93
 \end{aligned}$$

NORMAL PLACES

	Final Mean Phase	Mean Velocity	Weight	Final O-C		Final Mean Phase	Mean Velocity	Weight	Final O-C
1	19.94	-27.17	2.0	+1.32	7	49.81	-9.70	1.5	-1.13
2	25.04	34.70	1.5	+0.59	8	3.01	1.74	3.5	+0.98
3	31.45	40.19	2.5	-1.62	9	6.84	5.60	3.5	-0.66
4	38.57	32.68	4.0	+0.98	10	11.33	12.36	2.5	+0.05
5	41.29	29.48	3.5	-0.50	11	16.69	-23.52	2.5	-0.51
6	44.32	-21.95	3.5	+0.12					

Using the differential formula of Lehmann-Filhés, observation equations were built up connecting the residuals with the elements whose values were to be improved upon, namely, γ , K , e , ω and T . In the equations, weighted as above, substitutions were made as follows:

$$\begin{aligned}
 x &= \delta\gamma \\
 y &= \delta K \\
 z &= K \cdot \delta e \\
 u &= K \cdot \delta\omega \\
 v &= [0.36069] \delta T
 \end{aligned}$$

OBSERVATION EQUATIONS FOR δ DRACONIS

	x	y	z	u	v	
1.....	1.000	-0.321	-0.071	-0.818	+0.683	-1.57=0
2.....	1.000	-0.687	+0.767	-0.501	+0.417	-0.46
3.....	1.000	-0.870	+0.890	+0.049	+0.020	+1.69
4.....	1.000	-0.630	-0.355	+0.724	-0.586	-1.44
5.....	1.000	-0.385	-0.885	+0.932	-0.852	-0.17
6.....	1.000	-0.006	-1.064	+1.066	-1.107	-0.78
7.....	1.000	+0.777	+0.303	+0.838	-0.993	+1.26
8.....	1.000	+1.028	+0.924	+0.134	-0.076	-0.30
9.....	1.000	+0.990	-0.078	-0.435	+0.588	+1.05
10.....	1.000	+0.556	-0.967	-0.830	+0.882	-0.12
11.....	1.000	-0.030	-0.625	-0.912	+0.808	+0.35

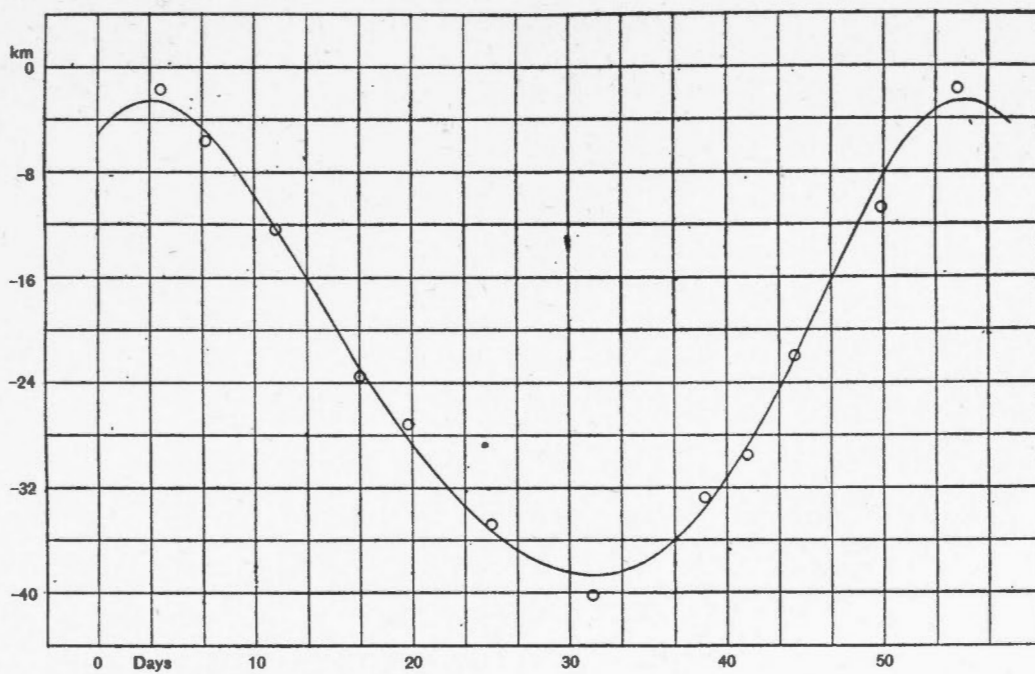
NORMAL EQUATIONS

$$\begin{array}{rclclclcl}
 3.050x & + & .182y & - & .557z & + & .349u & - & .266v & - & .304 & = & 0 \\
 & & 1.368y & + & .155z & - & .332u & + & .352v & + & .531 & = & 0 \\
 & & & & 1.658z & - & .386u & + & .371v & + & .795 & = & 0 \\
 & & & & & & 1.639u & - & 1.587v & - & .568 & = & 0 \\
 & & & & & & & & 1.569v & + & .569 & = & 0
 \end{array}$$

From these there resulted small corrections to the elements with a corresponding reduction of Σpv for the normal places from 29.6 to 22.2. The final elements are:

$$\begin{aligned}
 P &= 51.710 \text{ days} \\
 e &= .128 \pm .035 \\
 \omega &= 329^{\circ}.32 \pm 14^{\circ}.09 \\
 K &= 17.96 \text{ km.} \pm .70 \text{ km.} \\
 \gamma &= -22.59 \text{ km.} \pm .47 \text{ km.} \\
 T &= \text{J. D. } 2,414,813.75 \pm 1.996 \text{ days} \\
 a \sin i &= 12,625,000 \text{ km.} \\
 \frac{m_1^3 \sin^3 i}{(m + m_1)^2} &= .03 \odot
 \end{aligned}$$

Dominion Observatory
Ottawa
October, 1918.

Radial Velocity Curve of λ Draconis

