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### ORBIT OF THE SPECTROSCOPIC BINARY X AURIGÆ

AND A NOTE ON 12 LACERTÆ

BY REYNOLD K. YOUNG, Ph. D.

The binary character of  $\chi$  Aurigæ ( $\alpha=5^h$   $26^m$ ,  $\delta=+32^\circ$  7', mag. 4.88, type B1) was announced by Frost and Adams in the Astrophysical Journal in 1903. An orbit has been determined from eighty-eight single-prism spectrograms secured at this observatory during the years 1913, 1914, 1915, and 1916. An idea of the spectrum may be obtained from Table I, which gives the wave-lengths of the lines used in making the reductions, the number of times each line was measured and other data necessary for correcting the wave-lengths by the usual method of making the weighted mean of the residuals equal to zero. The residuals quoted are taken in the sense, observed minus the mean of the plate. In addition to the lines given in this table, the H and K lines of calcium are present and generally were measured, but they were not included with the rest of the lines for reasons which will appear later. On well-exposed plates all of the lines given are fairly sharp. Those due to silicon and some of those due to helium are faint, which accounts for the small number of times they were measured.

The velocities obtained at the Yerkes and Ottawa observatories follow in Tables II and III. The headings of the columns in these tables are self-explanatory. The residuals, as given under O-C, were obtained graphically from the final curve, and so may occasionally be a tenth of a kilometre or so in error.

TABLE I

Element	Wave-Length	Arithmetic Residual	Algebraic Residual	Total Weight	Number of Times Measured
Helium	3964 · 875	7.5	+2.1	101	32
Helium	4009 - 417	6.8	+2.1	8	23
Helium	4026 - 352	5.7	+2.9	391	79
Hydrogen	4101 · 890	5.8	-0.6	301	64
Helium	4121.016	8.3	-0.9	$5\frac{1}{2}$	19
Silicon	4128-211	10.0	-3.2	9	29
Silicon		6.2	-2.7	71	24
Helium	4143 - 928	7.7	-3.1	221/2	61
Carbon	4267 - 301	7.1	+2.8	131	40
Hydrogen	4340 - 634	6.7	-3.0	481	83
Helium	4388 • 100	7.2	-1.3	37	. 76
Helium	4471 - 676	6.1	+3.2	63	86
Magnesium		5.7	-1.2	34	72

Y SI OTTANIA TABLE II

#### YERKES OBSERVATIONS OF X AURIGÆ

Date -	Julian Day	Velocity	0-C
1903 AND STATE OF STA		THO THE	911
ept. 5	2,416,363 · 825	+24.2	+12.8
et. 10	398 - 782	+12.2	- 2.4
Oct. 16	404 · 844	+13.8	- 1.2

felt sque 86. A gata 77 Navy est end a se manufer de restainte caust est fille de des parts de la felt de la f

# OTTAWA OBSERVATIONS OF X AURIGÆ

Plate	Observer*	lin d	Date	Julian Day	Velocity	Weight	0-C	Velocity H and K Lines	Weight
			1010	i i i i i i i i i i i i i i i i i i i				7 10 10 10 10	align 1
PROT	T. '	3.7	1913	0.100.000.010					1.00
5791	P1	Nov.	4	2,420,076 · 915	-19.8	1	+0.5	-10.6	1
5798	Y	Nov.	5	077 - 872	-17.5	1	+ 2.7	-11.8	1
5804	Y	Nov.	6	078 · 719	-20.6	1	-0.5		
5818	P1	Dec.	8	110.774	-19.2	1 2	- 3.4	-12.7	1
5834	Y	Dec.	18	120.628	- 9.8	1/2	+4.5	- 9.5	1
5849	C	Dec.	1914	124.780	-22.3	1/2	- 8.7	- 1.9	1/2
5873	Y	Jan.	1	134.729	-14.6	1 2	2.7		
5894	P	Jan.	21	154.705	-14.5	1	- 5.9	+ 2.7	1
5922	Pı	Feb.	9	173 - 689	-15.2	1 1	- 9.7	1	3
5925	Y	Feb.	11	175 - 663	+ 1.5	1	+ 6.7		
5939	Y	Feb.	15	179.589	- 2.5	1	+ 2.0		
5957	C-P1	Feb.	23	187 - 665	+ 6.7	1	+ 9.9	+ 1.8	1
5973	Y	Mar.	11	203 · 576	- 8.4	1	- 7.7	+ 1.3	1 1
6397	Y	Sept.	17	393.889	4 7.6	1	-10.2	- 6.0	1
6460	Y	Oct.	1	407 - 839	+17.6	1	- 0.3	+ 8.9	1
6470	C	Oct.	2	408 - 833	+29.1	1	+11.3	+15.2	-
6483	H	Oct.	4	410.822	+18.0	1	+ 0.1	+12.5	1
6486	Y	Oct.	6	412.914	+22.2	1	+ 4.3	+16.9	1
6489	Y	Oct.	11	417.887	+21.1	î	+ 3.3	+11.0	1
6508	H	Oct.	20	426.791	+19.0	1	+ 1.3	411.0	1
6526	Y	Oct.	22	428-911	+24.7	î	+ 7.1	+13.6	
6562	H	Nov.	17	454 · 824	+17.8	î	+1.2	+ 9.9	1
6587	P-Y	Nov.	28	465.748	+ 5.2	1	-12.5	+ 3.2	-
6593	C-P1	Dec.	4	471.757	+20.8	1	+5.8	+12.3	1
6599	Y	Dec.	5	472.759	+16.4	1	+ 1.4	+12.3	1
6604	Y-H	Dec.	6	473 - 627	+13.3	1	- 1.4	+7.1	1
6617	C-P1	Dec.	11	478.733	+10.7	1	-3.7	+10.5	1 2
6627	Y	Dec.	15	482 · 637	+14.6	1	+ 0.6	+10·5 - 3·7	1
6639	Pı	Dec.	16	483 - 757	+15.8	1	+ 2.0	- 3·7 - 0·1	1

TABLE III
OTTAWA OBSERVATIONS OF X AURIGÆ—Continued

Plate	Observer*	0-4	Date	Julian Day	Velocity	Weight	0-C	Velocity H and K Lines	Weight
						-			
		3 130	1914			18.5			
6643	H	Dec.		2,420,484 · 654	+14.5	1	+ 0.8	+11.2	1
6646	Y	Dec.	20	487 · 496	+15.3	1	+ 1.9	+ 3.4	1/2
6652	Y	Dec.	22	489 · 592	+10.4	1	- 2.8	+ 6.2	1
6667	Y	Dec.	30: 1915	497 · 571	+16.2	1	+ 3.9	+ 8.9	1
6680	C	Jan.	4	502.744	+11.7	1	+ 0.2	+ 6.9	1
6702	Y	Jan.	10	508 · 592	+ 1.4	1	- 9.1	+ 7.2	1
6731	Y	Jan.	24	522 · 609	+ 7.5	1	-0.5	+ 3.6	1 2
6750	H-Y	Jan.	28	. 526 - 721	+ 8.1	1	+ 1.0	+ 6.7	1
6783	C	Feb.	12	541 · 640	+6.2	1	+ 2.1	+ 9.3	1
6788	H	Feb.	17	546 · 599	+11.2	1	+ 8.3	+ 8.8	1
6819	Y	Feb.	28	557 · 604	- 2.8	1	- 3.2	- 1.1	1
6844	Y	Mar.	7	564 · 573	+ 1.7	1	+ 3.2	- 2.6	1
6879	Y		19	576 - 555	-11.3	1 .	- 6.8	+ 4.8	1
6896	Y		30	587 · 528	- 4.8	1	+ 2.3	0.0	1
6919	Y	Apr.	13	601 · 536	- 6.8	1	+ 4.1	- 5.6	1
6956	Y	Apr.	28	616 · 556	-17.3	1	- 2.7	-18.0	1/2
7135	Y	July	29	707 · 858	-21.8	1/2	+ 0.7		1
7152	H	Aug.	10	720 · 851	-24.5	1	- 3.2		1
7175	Y	Aug.	26	736.826	-18.7	1	+ 0.9		1
7199	Y	Sept.	2	743 · 883	-21.6	1 1	-2.8 + 1.1		
7227	Y P11	Sept.	9	750 · 906 758 · 875	-16.8 $-11.8$	1	+5.0		
7264	H		17	762.900	-13.9	1	+ 2.2	- 3.7	1/2
7280 7316	Y	-	21	771.881	-13.9 $-22.2$	1	- 7.4	-11.7	1
7323	Y	Oct.	9	780 · 816	-7.1	1	+ 6.3	-12.9	1
7345	C	Oct.	15	786 · 816	- 2.2	î	+10.3	120	
7364	Y	Oct.	24	795 · 892	$-12 \cdot 2$	1	-1.2	- 0.6	1/2
7373	Y	Nov.	3	805 · 622	- 9.8	1	- 0.3	- 6.6	1
7384	Y	Nov.	6	808 · 862	-14.6	1	- 5.7		
7401	Н	Nov.		814 · 740	-5.3	1	+2.6	+10.1	1
7415	C	Nov.	17	819 - 625	$-16 \cdot 6$	1/2	-9.5		
7429	Y-C	Nov.		826 - 708	- 1.4	1	+ 4.4	- 2.4	$\frac{1}{2}$
7440	Y	Dec.	3	835 · 570	+6.4	1/2	+10.7		
7447	C	Dec.	10	842 · 622	- 4.9	1	-1.7	+ 1.5	1
7454	P	Dec.	20	852 - 649	- 9.2	1	-7.7	- 3.9	1/2
7458	Y	Dec.	28 1916	860 · 651	+ 3.0	1	+ 3.2	- 8.0	1/2
7475	C	Jan.	7	870.514	-5.8	1	-7.0	+ 4.5	1
7481	H	Jan.	13	$876 \cdot 569$	-2.0	1	-4.0	+ 1.8	1/2
7494	Y	Jan.	28	891.574	+ 2.6	1	-1.6	+ 0.3	1
7502	H	Feb.	3	897 · 728	+12.6	1	+7.6	+12.6	1
7504	Y		10	904 · 584	+6.2	1	+ 0.2	+11.6	1
7510	P	Feb.	19	913 · 669	+13.5	1	+ 6-3		
7512	Y	Feb.	20	914 · 501	+ 9.4	1	+ 3.1	+ 5.3	1
7517	C	Heh	23	917.629	+ 6.5	1	-1.1	+ 8.5	1

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TABLE III
OTTAWA OBSERVATIONS OF X AURIGÆ—Concluded

Plate	Observer*	25 (3	Date	Julian Day	Velocity	Weight	0-C	Velocity H and K Lines	Weight
			1916						
7532	C	Mar.	1	2,420,924 · 501	+ 4.8	1	- 3.7	- 2.0	The 1
7536	н	Mar.	2	925 · 624	+ 8.5	1	- 0.1	+ 3.9	1
7539	Y	Mar.	5	928 · 534	+ 0.9	1	- 8.1	+ 6.1	1
7542	C	Mar.	10	933 · 535	+10.8	1	- 1.3	+10.1	1
7549	Y	Mar.	17	940 - 597	+11.5	1	+ 1.2	+ 4.2	1
7550	Y	Mar.	17	940 · 642	+ 8.8	1	- 1.5	+ 5.5	1
7558	Y-H	Mar.	19	942 - 637	+ 8.5	1	- 2.1	+ 9.2	1
7562	Y	Mar.	21	944.510	+14.0	1	+ 3.2	+11.7	1
7571	H	Mar.	23	946 · 618	+14.8	1	+ 3.8	+ 6.5	1
7585	Y	Mar.	30	953 · 575	+ 7.8	1	- 4.0	+ 7.8	1
7594	Y	Apr.	2	956 · 572	+ 9.0	1	- 3.0	+ 3.7	1
7601	H	Apr.	5	959 - 525	+16.5	1	+ 4.2	+13.0	1
7624	Y	May	2	$986 \cdot 548$	+ 9.3	1	- 5.5	+ 0.3	1
7632	H	May	4	$988 \cdot 555$	+26.1	1 2	+11.2	+17.4	1

<sup>\*</sup>C=Cannon; H=Harper; P1=Parker; P=Plaskett, J. S.; P11=Plaskett, H. H.; Y=Young

#### MEASURES OF X AURIGÆ

9,63	5791	5798	5804	5818	5834	5849	5873
1W 1 1 107	Vel. W	. Vel. Wt	Vel. Wt.	Vel Wt.	Vel. Wt.	Vel. Wt.	Vel. Wt
3933 · 825 3964 · 875	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	-29·9 ½		-15.4 1	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	+ 2.8 1/2	
4009 · 417 4026 · 352 4101 · 890	$ \begin{array}{c cccc} -49.0 & \frac{1}{2} \\ -33.4 & 1 \\ -35.6 & \frac{1}{2} \end{array} $	-18·0 ½ -40·4 ½	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	- 3.1	-15.3 1	+ 3.8 ½
4121·016 4128·211 4131·047	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	-45.0 1	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$				
4143 · 928 4267 · 301 4340 · 634	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	-48.2 1	-10·8 ½ -18·8 ½	- 9.4 1	-15·0 ½	- 2.3 1
4388·100 4471·676 4481·400	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c cccc} -40 \cdot 6 & 1 \\ -31 \cdot 6 & \frac{1}{2} \\ -42 \cdot 3 & \frac{1}{2} \end{array} $	$ \begin{array}{c cccc} -47.8 & \frac{1}{2} \\ -44.3 & \frac{1}{2} \\ -44.6 & \frac{1}{2} \end{array} $	-34.5		$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
Weighted			1				
mean Va Vd	$ \begin{array}{r} -38.40 \\ +18.94 \\ -0.11 \end{array} $	$ \begin{array}{r} - 35.60 \\ + 18.57 \\ - 0.15 \end{array} $	$ \begin{array}{r} -38.48 \\ +18.21 \\ -0.07 \end{array} $	$\begin{array}{r} -21.88 \\ +3.07 \\ -0.07 \end{array}$	- 7·47 - 2·06 0·00	$ \begin{array}{rrr}  & - & 17.68 \\  & - & 4.22 \\  & - & 0.16 \end{array} $	$ \begin{array}{rrrr}  & - & 2 \cdot 42 \\  & - & 9 \cdot 30 \\  & - & 0 \cdot 12 \end{array} $
Curv.	- 0.28	- 0.28	- 0.28	- 0.28	- 0.28	- 0.28	- 0.28
Radial			20.4	10.0		00.0	10.1
Velocity H and K	- 19·8 - 10·6	-17.5 $-11.8$	- 20.6	$-19.2 \\ -12.7$	- 9·8 - 9·5	- 22·3 - 1·9	- 12.1

	5894	592	2	5925		5939		5957		5973		6397	
λ 11 281	Vel.	Vt. Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt
3933 · 825 3964 · 875 3968 · 625 4026 · 352 4101 · 890 4121 · 016 4128 · 211 4131 · 047 4143 · 928 4267 · 301 4340 · 634 4388 · 100 4471 · 676 4481 · 400	$+21 \cdot 4$ $+11 \cdot 4$ $+ 4 \cdot 7$ $-7 \cdot 9$ $+22 \cdot 4$ $+12 \cdot 5$ $-5 \cdot 0$	\$ \$ +11 +24 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2	7 1	+27·2 +37·6 +12·4 +30·1 +43·4 +17·7 +21·2 +17·6	में के निर्व के निर्व निर्व निर्व निर्व निर्व निर्व	+24·2 +32·5 +16·0 + 5·7 + 4·8 +22·1 +23·4 +28·9 +37·6 +27·1 +43·7 +10·1	1 2	+30·4 +28·1 +28·1 +36·4 +27·1 +44·5 +44·1	nder oder oder oder	+31·3  +22·8  +22·6  +13·6  +21·5  +19·3  +16·5  +33·7  +15·2		$-35 \cdot 0$ $-25 \cdot 5$ $+ 6 \cdot 9$ $- 5 \cdot 6$ $-27 \cdot 0$ $-20 \cdot 1$ $-22 \cdot 3$ $-24 \cdot 9$	
Weighted mean Va Vd Curv.	+ 4·11 - 18·4 0·00 - 0·23	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	0·26 0·00 0·20 0·28	_		+ 24· - 26· - 0· - 0·	60 10	+ 35· - 28· - 0· - 0·	08	+ 21· - 29· - 0· - 0·	54 15	- 21· + 29· + 0· - 0·	22 11
Radial Velocity H and K	- 14·5 + 2·7	- 15	.2	+ 1.	5	- 2.	5	+ 6· + 1·		- 8· + 1·	_	+ 7.	

A TOLO	6460		6470		6483		6486		6489	,	6508	3	6520	6
λ	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt
3933 · 825 3964 · 875 3968 · 625 4009 · 417 4026 · 352 4101 · 890 4121 · 016 4128 · 211 4131 · 047 4143 · 928 4267 · 301 4340 · 634 4388 · 100 4471 · 676 4481 · 400	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	1	-12·7 -10·2 + 0·9 - 6·5 - 4·7	100 100 100 100 100 100 100 100 100 100	-17·5 -14·0 -22·7 -16·6 -10·5 -19·1 -3·7 -5·0	1 1 1 2 1 2 1 2 2 1 2 2 2 2 2 2 2 2 2 2	$ \begin{array}{r} -8.7 \\ -9.8 \\ +3.4 \\ -22.3 \\ \\ -1.9 \\ +4.6 \\ +6.2 \\ -9.9 \end{array} $		-15·1		-22·9 + 3·4 -16·5 -17·6 + 4·7	abbe toles to the sales to the	- 9·5 -20·5 -1·7 + 3·8 - 9·4 +12·3 -1·7 + 7·4 - 9·0 - 7·0 + 7·4	
Weighted mean Va Vd Curv.	- 10· + 28· + 0· - 0·	18 12	+ 1· + 28· + 0· - 0·	05 14	- 9· + 27· + 0· - 0·	75	- 3· + 26· - 0· - 0·	39 07	- 5· + 26· 0· - 0·	43 00	- 5· + 24· + 0· - 0·	19 14	+ 1· + 23· - 0· - 0·	· 50 · 14
Radial Velocity H and K	+ 17· + 8·	-	+ 29· + 15·		+ 18· + 12·		+ 22· + 16·		+ 21 · + 11 ·		+ 19	0	+ 24 + 13	

9555 \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	6562	2	6587	1031	6593		6599		6604		6617	7	6627	
171 101	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt
3933 · 825 3964 · 875 3968 · 625 4009 · 417 4026 · 352 4101 · 890 4121 · 016 4128 · 211 4131 · 047 4143 · 928 4267 · 301 4340 · 634 4388 · 100 4471 · 676 4481 · 400	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		$ \begin{array}{r} -4.8 \\ +14.0 \\ \\ +18.9 \\ +0.8 \\ +6.5 \\ \\ -4.8 \\ +1.9 \\ -16.4 \\ \\ -13.6 \\ \\ -24.9 \\ -11.3 \end{array} $	र न्योर	+11·2 +3·5 +12·9 +11·3 +21·4 +6·6 +23·8  +28·0 +22·4 +18·1 +4·7 +9·9 +21·9		+ 8·0 +18·2 0·0 +16·2 0·0 +15·3 - 1·9 - 6·4 - 4·5 +35·1 +33·6 +22·5		+ 3·2 +23·9 +18·9 +18·3 - 1·9 - 2·9 - + 4·5 +11·7 + 6·8 + 8·7	- refer - refer refer refer - refer refer -	+ 8.8 + 8.2 + 9.9 + 22.6 + 6.5 + 1.0 + 10.2 + 19.9 + 8.7 + 6.3		- 7·2 + 5·8 + 5·8 - 13·1 +31·7  + 1·0 +31·0 + 6·8 +10·5 +11·2 +14·0	
Weighted mean Va Va Curv.	+ 8 - 0	·39 ·83 ·07 ·28	+ 8-	00	+ 15· + 5· - 0· - 0·	28 04	+ 11· + 4· - 0· - 0·	77 06	+ 4.	41 32 16 28	+ 1	· 27 · 78 · 04 · 28	+ 15.	42 12
Radial Velocity H and K	+ 17 + 9		+ 5· + 3·		+ 20· + 12·	377	+ 16· + 17·		+ 13· + 7·		+ 10 + 10		+ 14· - 3·	

15.00	6639		6643		6646		6652		6667		6680		6702	2
λ 	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt
3933 · 825 3964 · 875 3968 · 625 4009 · 417 4026 · 352 4101 · 890 4121 · 016 4128 · 211 4131 · 047 4143 · 928 4267 · 301 4340 · 634 4388 · 100 4471 · 676 4481 · 400	$\begin{array}{c} -0.8 \\ +17.3 \\ +3.3 \\ \end{array}$ $\begin{array}{c} +17.4 \\ +23.4 \\ \end{array}$ $\begin{array}{c} +13.5 \\ +17.1 \\ +26.0 \\ +9.4 \\ +18.7 \\ +2.6 \end{array}$		+15·2 +20·6 +10·7 + 9·4 +25·3 +14·9 		+ 6·4	्रान्तिः । ज्योत्वा ज्येत्वः । ज्योत्वः	+10·4 +15·6 +10·1 +17·2 +16·6  + 3·8 + 7·6  +20·3 + 1·2 +18·0 +22·5	1-103 1-104	+16·8 +17·4 +28·3 +35·7 +28·0 +18·9 +15·3 +21·3 +16·0 +17·0 +26·3 +31·7 +23·8	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	+ 9·6	्राची की	+23·2 +15·6 +16·5 ······ +25·3 + 2·8 ····· +22·9 +26·8 + 5·8 +13·9 + 5·7 + 9·4 +18·7 +17·5	1 14 14 14 15 1
Weighted mean Va Vd Curv.	+ 17 · · · · · · · · · · · · · · · · · ·	96 15	+ 16 - 1 0	42	+ 0.	91	+ 14· - 3· + 0· - 0·	99	+ 24· - 8· + 0· - 0·	09 15	+ 23. - 10. - 0. - 0.	62 15	+ 15 · - 13 · + 0 · - 0 ·	56
Radial Velocity H and K	+ 15 - 0		+ 14· + 11·		+ 15· + 3·		+ 10· + 6·		+ 16· + 8·		+ 11· + 6·		+ 1· + 7·	

Lone	6731		6750		6783		6788		6819		6844		6879	
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt
			1 5											
3933 - 825	+22.4	3	+27.2	1	+32.0	1	+36.0	1	+30.4	1	+27.2	1	+30.4	1
3964 · 875			+21.4	1.			+33.7	1						
3968 - 625			+29.7	1	+42-1	1	+36.3	1	+19.0	1			+38.8	1
4009 - 417	+24.9	1/2	+37.8	1							+41.2			
$4026 \cdot 352$	+44.4	1	+42.7	1	+27.0	1	+41.9	1	$+26 \cdot 2$	1	+34.0	1 2	+24.4	1
4101 - 890					+ 9.3	1 4	+41.0	1/2	+29.9	1	+21.4	1 2	+ 6.5	1
4121 · 016	+37.0	1	$+35 \cdot 1$	1										
4128 - 211											+43.9	1		
4131-047			+25.8	1										
4143 · 928	+19.3	1/2	+21.3	1	+48.3	1			+13.5	1/2				
4267 · 301			+24.6	1					+26.7	_	+43.8		+21.4	
4340 - 634	+24.9	1/2	$+23 \cdot 8$	1	+30.5		+30.5	1/4	+21.5		$+22 \cdot 6$	1	+20.4	
4388 - 100	+21.1	1/2	$+35 \cdot 1$	1/2	+31.7		$+35 \cdot 1$	1	+42.7	_	+28.1	1	+ 7.0	-
4471 - 676	+37.3	1/2	+36.1	1 2	+31.1		+42.3	1/2	$+31 \cdot 1$	_	+34.8		+18.0	_
4481 · 400	+23.2	1/2	+27.5	1/2	+44.4	1 2	+36.9	2	+13.1	14	+29.4	1 2	+27.5	1/2
Weighted					1									
mean	+ 27	1.00	+ 29.		+ 32	2/20	+ 38		+ 26		+ 31.	17.4	+ 18-	
$V_a$	- 19		- 20		- 25		- 26		- 28		- 29	77.7	- 29	
Vd	- 0.		- 0.		- 0.		- 0.		- 0.		- 0.		- 0-	_
Curv.	- 0.	28	- 0.	28	- 0.	28	- 0.	28	- 0.	28	- 0.	28	- 0.	28
							•							
Radial														
Velocity	+ 7.		+ 8.		+ 6.		+ 11.	2	- 2.	8	+ 1.	7	- 11.	3
H and $K$	+ 3.	6	+ 6.	7	+ 9.	3	+ 8.	8	- · 1·	1	- 2.	6	+ 4.	8

	6896		6919		6956		7135		7152		7178	5	7199	)
<b>λ</b>	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	. Vel.	Wt
3933 · 825 3964 · 875 3968 · 625 4009 · 417 4026 · 352 4101 · 890 4131 · 047 4143 · 928 4267 · 301 4340 · 634 4388 · 100 4471 · 676 4481 · 400	+28·0 +13·2 +30·5 +18·9 +34·0 +21·5 +21·0 +27·0  +19·0 +22·2 +33·6 +17·5	1	+32·4	14 14 15 15 15 15 15 15 15 15 15 15 15 15 15	+3.6 $+19.9$ $+6.5$ $-6.4$ $+11.3$ $-2.3$ $0.0$ $+5.0$	्रानुस्य वर्गस्य । वर्गस्य वर्गस्य वर्गस्य वर्गस्य वर्गस्य वर्गस्य	-41·6 -45·8 -33·7	 1/2	-44·3 -47·0 -54·8	1	$-38 \cdot 1$ $-66 \cdot 4$ $-40 \cdot 4$ $-63 \cdot 0$ $-45 \cdot 5$ $-27 \cdot 2$ $-45 \cdot 5$	ार्च न्द्रंच न्द्रंश न्द्रंश न्द्रंश	-52·0 -45·7 -50·0 -41·5 -58·6	1 1 1 2
Weighted mean Va Vd Curv.	+ 24· - 28· - 0· - 0·	37 16	+ 18·3 - 25·3 - 0·3	59 22	+ 4· - 21· - 0· - 0·	04 28	- 41· + 19· + 0· - 0·	95 27	- 48. + 23. + 0. - 0.	82	- 46 + 27 + 0 - 0	48 23	- 50· + 28· + 0· - 0·	47 16
Radial Velocity H and K	- 4· 0·	_	- 6·3 - 5·	-	- 17· - 18·	-	- 21	8	- 24	5	- 18	7	_ 21.	6

Table 1	7227		7264		7280	7	7316		7323		7345		7364	
<b>λ</b>	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt
3933 · 825 3964 · 875 4009 · 417 4026 · 352 4101 · 890 4121 · 016 4128 · 211 4131 · 047 4143 · 928 4267 · 301 4340 · 634 4388 · 100 4471 · 676 4481 · 400	-35·5 -53·9 -50·0 -45·0 -44·3 -46·4		$-32 \cdot 9$ $-27 \cdot 9$ $-47 \cdot 8$ $-37 \cdot 7$ $-61 \cdot 1$	1 2 1	-32·6 -40·7 -48·1 -25·9 -56·0 -35·2 -61·1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	-52·9 -55·7 -52·9 -50·7 -42·0 -48·9 -61·1		-28·0 -48·0 -30·1 -37·7 -33·9 -35·1 -30·5	्र वर्षक व्यक्ति । । । । । । । । । । । । । । । । । । ।	-28·6	্ কার্কির পার্কির   । পার্কির পার্কির পার্কির পার্কির পার্কির পার্কির পার্কির পার্কির	-23·1 -36·7 -37·3 -38·0 -42·4 -21·3 -36·1 -38·5 -33·9 -30·4 -40·8 -31·5	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
Weighted mean Va Vd Curv.	- 45 + 29 + 0 - 0	·06 ·10	- 40 + 29 + 0 - 0	·23 ·13	- 42· + 29· + 0· - 0·	11 04	- 50 + 28 + 0 - 0	34	- 33 + 26 + 0 - 0 ·	90	- 27 + 25 + 0 - 0	57	- 34 + 23 - 0 - 0	·00 ·07
Radial Velocity H and K	- 16	.8	- 11	.8	- 13 · · · 3 ·		- 22 - 11		- 7· - 12·		- 2	2	- 12 - 0	

λ	7373		7384		740	1	7418	5	742	9	7440	0	74	47
ANT THE	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel	Wt.	Vel.	Wt
3933 · 825 3964 · 875 3968 · 625 4026 · 352 4101 · 890 4143 · 928 4267 · 301 4340 · 634 4388 · 100 4471 · 676 4481 · 400	-25·2 - 8·232·442·9 -37·0 -19·9 -31·3	1 1 1 2	-36·6 -29·4 -45·0 -53·0 -25·8 -29·2 -35·0	1 1 1	- 0·6 -10·7 -19·1 -12·1 -13·9 -30·0 -31·5 -24·8 - 3·7	1 12	-31·6 -31·2 -32·3 -28·2	122	-12·8 -17·8 -17·8 -5·1 -26·1 -3·4 -28·1 -6·8	122 123 14	+11·3 - 7·4 -12·5	1 2	- 8·	3 ½ 7 ½ 7 ½
Weighted mean Ve Vd Curv.					- 21 + 15 + 0 - 0	-92	- 30 + 13 + 0 - 0	76	- 11 + 10 + 0 - 0	.42	+ 7 + 0	· 94 · 41 · 23 · 28	+ :	7·11 2·28 0·04 0·28
Radial Velocity H and K	- 9 - 6	100	- 14	6	- 5 + 10		- 16	.6		·4 ·4	+ 6	•4		4:9 1:5

λ	7454	7458	7475	7481	7494	7502	7504
<b>^</b>	Vel. W	t. Vel. Wt	Vel. Wt.	Vel. Wt.	Vel. Wt.	Vel. Wt.	Vel. W
3933·825 3964·875 3968·625 4009·417 4009·417 4010·890 4121·016 4128·211 4131·047 4143·928 4267·301 4340·634 4388·100 4471·676 4481·400	- 0·8 - 7·4 - 3·5 -15·8 + 5·3 -16·4 - 3·5 - 4·3 1	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	+29·6
Weighted mean Va Vd	- 6·15 - 2·85 + 0·07 - 0·28	+ 10·15 - 6·97 + 0·04 - 0·28	+ 6·01 - 11·76 + 0·19 - 0·28	+ 12.98 - 14.71 0:00 - 0.28	+ 22·62 - 20·81 + 0·02 - 0·28	+ 35·87 - 22·72 - 0·23 - 0·28	+ 31·52 - 24·94 - 0·06 - 0·28
Radial Velocity	- 9·2 - 3·9	+ 3·0 - 8·0	- 5·8 + 4·5	- 2·0 + 1·8	+ 2·6 + 0·3	+ 12·6 + 12·6	+ 6·2 + 11·6

λ	7510		7512		7517		7527	Ť	7532		7536		7539	
ENT Chris	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt
3933 · 825 3964 · 875 3968 · 625 4009 · 417 4026 · 352 4101 · 890 4128 · 211 4131 · 047 4143 · 928 4267 · 301 4340 · 634 4388 · 100 4471 · 676 4481 · 400	+30·4 +31·3 +43·0  +42·3 +45·7  +29·5  +41·3 +32·8 +47·9 +47·6	1 1 12	+34·4 +30·0 +42·9 +31·9 +25·4 +42·9	1	+32·0 +41·6 +41·2 +35·3 +25·8 +19·1 +30·9 +37·4 +31·6 +44·8 +30·7	्राप्ति न्त्रीय न्त्रीय । न्त्रीय न्त्रीय न्त्रीय न्त्रीय । न्त्रीय न्त्रीय	+31·2 +29·6 +41·3 +38·2 +30·1 	न्हरंग न्हेंच	+27·2  +45·3  +33·6  +47·0  +22·6  +22·2  +37·3  +31·3	milita relata relata relata relata relata relata relata	+28·8 +38·0 +42·7 +37·3 	migra	+32·8 +38·8 +31·4 +33·4 +24·9 +26·1 +24·6 +46·6 +32·6	
Weighted mean Va Va Curv.							+ 31· - 28· - 0· - 0·	82 06	+ 34· - 28· 0· - 0·	94 00	+ 38· - 29· - 0· - 0·	04 19	+ 30 - 29 - 0 - 0	29
Radial Velocity H and K	+ 13	-5	+ 9· + 5·		+ 6· + 8·		+ 1· + 5·	-	+ 4· - 2·	-	+ 8.	-	+ 0.+ 6.	-

MEASURES OF X AURIGÆ—Continued

λ	7542		7549		7550		7558		7562	*	7571		7585	
# • 6.7	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt
3933 · 825 3964 · 875	+40.0	1	+34·4 +42·8	1 1	+36·0 +42·8	12 14	+39.2	1	+40·0 +49·3	THE REAL PROPERTY.	+32·0 +49·3		+37·6 +39·5	
3968 · 625 4009 · 417			$+33.9 \\ +28.3$	1 1 1	+35.1	1			$+42 \cdot 1 \\ +49 \cdot 7$	1 1	+40.2	1	+35.6	2
4026 · 352 4101 · 890 4121 · 016	+54·9 +36·4	- 1	+34·0 +43·9	100 100	+44·1 +48·0	100	+34·4 +35·5	-	+38·8 +39·2	12 12	+51·5 +44·3	1 2 1 2	+39.6 +37.3 +38.4	1 3
4128·211 4131·047			+42·0 +47·8	14 12	+44.8				+31.5	-			+52·5 +38·7	1
4143 · 928 4267 · 301 4340 · 634	+44.1		$+39 \cdot 2  +51 \cdot 3  +34 \cdot 5$	10 14 1	$+32 \cdot 9$ $+35 \cdot 1$	1 1 2	+48·8 +30·0		$+48.8 \\ +55.6 \\ +36.4$	1	$+35.3 \\ +52.4 \\ +40.7$	-	$+30.4 \\ +37.4 \\ +28.3$	1 2
4388 · 100 4471 · 676 4481 · 400	+25·7 +47·3	1/2	+42·7 +48·5 +36·3	1/2	$+23 \cdot 4$ $+37 \cdot 4$ $+35 \cdot 7$	N -14 -17 -17	$+25.8 \\ +52.2 \\ +29.4$	1	+21·8 +58·5 +42·6	1	+39·2 +45·4 +43·2	19 14	+17.·6 +47·3 +42·6	19 19
			.,										l ielu	
Veighted mean Va Vd	+ 40· - 29· - 0·		+ 41·· - 29· - 0·	47	+ 38· - 29· - 0·	47	+ 38· - 29· - 0·	46	+ 43· - 29· - 0·	28	+ 44· - 29· - 0·	11	+ 36· - 28· - 0·	26
Curv.	- 0.	28	- 0.	28	- 0.	28	- 0.	28	- 0.	28	- 0.	28	- 0.	28
Radial Velocity	+ 10.	Q	+ 11.	5	+ 8-		+ 8.		1 14	0	+ 14.		+ 7.	0
H and K	+ 10.		+ 4.	- 1	+ 5.	_	+ 8.	_	+ 14.		+ 6.		+ 7.	_

MEASURES OF X AURIGÆ-Concluded

one he lo	7594	910	7601	al la	7624	ingus Le w Incla	7632	rios derio	lo sil l a svita do stas	daker uali:	is tole nik smil o vazni		rately. On swit The m	Thou state
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt
3933 · 825 3964 · 875 3968 · 625 4009 · 417 4026 · 352 4101 · 890 4121 · 016 4128 · 211 4131 · 047 4143 · 928 4267 · 301 4340 · 634 4388 · 100 4471 · 676 4481 · 400	+40·0 +46·9 +24·0 +33·5 +31·0 +28·0 +36·5 +24·8 +35·4 +34·8 +56·7 +29·4 +51·5 +38·6 +42·6	1	+38·4 +48·6 +45·4  +55·8 +55·0  +31·5 +36·3 +15·0  +55·4 +45·6	1 1 1 1 1	+20·0	1 1	+32·8 	14d . 14d 140 14d 14d 14d 14d . 162 162 162 162						
Weighted mean Vs Vd Curv.	+ 37· - 27· - 0· - 0·	80	+ 44· - 27· - 0· - 0·	22 19										
Radial Velocity H and K	+ 9· + 3·		+ 16· + 13·		+ 9· + 0·		+ 26· + 17·		4 * * * * * * * * *		• • • • • • • •			

The period is remarkably long for an early type star, and it so happens that the observations taken at the Yerkes Observatory fall almost at the top of the curve, so near this point, in fact, that in the preliminary elements it was impossible to decide on which branch of the curve they should lie. For this reason, and also because there is generally a systematic difference between radial velocities obtained at two observatories, the period was determined from the Ottawa observations alone. As it turned out, the inclusion of the Yerkes Observatory plates would have yielded practically the same result, for the final orbit gives a mean residual for their plates of +3.1 kilometres, which is just about the difference usually found between measures made by the two observatories. Seventeen normal places were formed as given. It is unfortunate that the negative minimum is not more strongly determined, but the lack of observations is due

to the star and sun being in conjunction at that time. If the star were followed for another revolution this defect could be remedied. In order to determine the period accurately, the star should be observed at some epoch in the future which can be so chosen as to define the negative minimum as well.

The preliminary elements chosen follow the normal places and the successive steps in the least-square solution are recorded.

#### NORMAL PLACES

No.	Julian Day	Phase from Preliminary	Velocity	Weight	O-C Preliminary	O-C Final
		- 1 1 1- 1- 1- 1- 1- 1- 1- 1- 1- 1			8 2 - 10 200 - 1 2 - 100 - 1	ALTE TEL
1	2,420,077.8	108.8	-19.3	• •3	+1.2	+1.1
2	129 - 1	160 · 1	$-16 \cdot 1$	-3	-3.8	-3-1
3	185.2	216-2	- 2.3	-4	+0.6	+1.4
4	413.8	444.8	+19.2	-8	+0.9	+1.8
5	467.9	498.9	+14.0	-6	-2.3	-1:0
3	492.1	523 · 1	+12.5	-8	-1.4	-0.1
7	534.4	565 · 4	+ 8.3	-4	+1.6	+2.4
3	571.6	602.6	- 4.3	.4	-1.6	-1.3
)	609.0	640.0	$-12 \cdot 1$	·2	+1.3	-0.6
)	730.3	101.3	-21.6	•4-	-0.1	-0.9
L	761 - 1	132 - 1	$-16 \cdot 2$	-4	+0.8	+0.4
2	800.4	171-4	- 9.2	-6	+1.2	+1.0
3	839.9	210.9	- 3.5	-4	+0.3	+0.2
1	879 - 4	250 - 4	+ 2.1	.5	-0.2	-0.2
5	920.7	291.7	+ 6.9	.9	-0.9	-1.0
3	948 • 1	319.1	+11.4	-8	+0.5	+0.4
7	987.2	358.2	+14.9	.2	+0.2	+0.3

#### PRELIMINARY ELEMENTS

P = 660 days T = J. D. 2,420,629  $\omega = 135^{\circ}$  e = 0.20 K = 21.5  $\gamma = -0.16$  $\mu = 0^{\circ}.545455$ 

JATIONS	ONS	COUATI	TION	OBSERV.
JATIO	U.	QUATI	TIUN	OBSERV.

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	x	y	Z	p	q	r	-n	Weight
test kills	medies to	Walmyv.	Holland		AS LET THE		TECHTANEO	1901 \$ \$160 000 000 000
1	1.000	946	+ .531	+ .452	629	347	-1.21	0.3
2	1.000	565	+1.007	+ .765	787	394	+3.78	0.3
3	1.000	128	+ .639	+ .859	734	326	-0.62	0.4
4	1.000	+.859	714	135	004	001	-0.90	0.8
5	1.000	+.769	+ .077	557	+ .360	+.058	+2.34	0.6
6	1.000	+ .654	+ .528	747	+ .574	+.079	+1.39	0.8
7	1.000	+.319	+1.105	-1.029	+1.000	+.095	-1.61	0.4
8	1.000	120	+ .901	-1.141	+1.295	+.074	+1.55	0.4
9	1.000	<b>-</b> ⋅616	<ul><li>− ·133</li></ul>	-1.021	+1.250	+.025	-1:31	0.2
10	1.000	992	+ .381	+ .384	574	+.058	+0.10	0.4
11	1.000	<b>783</b>	+ .869	+ .625	738	+.097	-0.80	0.4
12	1.000	475	+ .989	+ .801	− ·787	+.135	-1.18	0.6
13	1.000	<b>-</b> ⋅167	+ .701	+ .859	743	+ · 157	-0.26	0.4
14	1.000	+.114	+ .217	+ .826	661	+.165	+0.18	0.5
15	1:000	+.370	308	+ .719	559	+.163	+0.89	0.9
16	1.000	+.514	606	+ .614	485	+.155	-0.52	0.8
17	1.000	+.693	896	+ .410	355	+.127	-0.17	0.2

Where 
$$x = d\gamma$$
  
 $y = dK$   
 $z = Kde$   
 $p = Kd\omega$   
 $q = \frac{K\mu}{(1 - e^2)^{\frac{3}{2}}} dT$   
 $r = \frac{1000 K}{(1 - e^2)^{\frac{3}{2}}} d\mu$ 

#### NORMAL EQUATIONS

Whence 
$$x = +0.013$$
 or  $d\gamma = +0.01$  km.  
 $y = -0.969$  or  $dK = -0.97$  km.  
 $z = -0.625$  or  $dK = -0.97$  km.  
 $dK = -0.97$  km.

#### FINAL ELEMENTS

 $P = 655 \cdot 16 \pm 5 \cdot 26 \text{ days}$   $T = J. D. 2,420,629 \cdot 78 \pm 9 \cdot 56 \text{ days}$   $\omega = 135^{\circ} \cdot 52 \pm 5^{\circ} \cdot 2$   $e = 0 \cdot 171 \pm 0 \cdot 026$   $K = 20 \cdot 53 \text{ km.} \pm 0 \cdot 57 \text{ km.}$   $\gamma = -0 \cdot 15 \text{ km.} \pm 0 \cdot 35 \text{ km.}$   $a \sin i = 182,300,000 \text{ km.}$   $\frac{m_1^3 \sin^3 i}{(m + m_1)^2} = 0 \cdot 56 \odot$ 

#### THIRD BODY

It will be noticed that the binary character was announced from three plates, which the orbit shows to fall near the epoch of positive maximum (cf. radial velocity curve)\*. In the interval covered by these plates the variation should be small and the velocities increasing. One is forced to conclude that there is another period superimposed upon the one found, or else that the announced variation means nothing. The spectrograms have been remeasured at the Yerkes Observatory, and Professor Frost finds no reason to doubt that the range indicated by the velocities is real.

We have been unable to find a period for this variation. Several periods would harmonize some of the larger residuals and so reduce the sum  $\Sigma pv^2$  a little, but the fact that there are several which look equally good (or bad) is enough to render one suspicious of the reality of any. No period found would satisfy the residuals from the main orbit well enough to warrant its adoption.

Considering the elements of the system as published, the probable error of a single observation is 3.5 kilometres. This is larger than we usually find for good line stars by about 20 per cent.

In the Astrophysical Journal, March, 1915, Dr. Schlesinger published a method for determining the periodicity in a series of observations from a consideration of the distribution of the observed velocities. Thus, for example, if we are dealing with a circular orbit, more velocities should be found near positive maximum and negative minimum than near the  $\gamma$  line. If the residuals are due entirely to error of observation, the velocities around the  $\gamma$  line will be most numerous. Curves are published giving the expected distribution of the velocities for orbits of various forms. In these curves some allowance has been made for the effect of probable error of measurement upon the results.

Some time previous to the publication of this article, the writer tried to make use of the same method in getting a period for the binary 12 Lacertæ. In this case the velocities were distributed very much as one would expect if the observed range were due to error of measurement alone and yet, since the total observed range was over sixty kilometres, there could be no doubt of the binary nature of the star. The conclusion

<sup>\*</sup>The normal place from the observations taken at the Yerkes Observatory is indicated by a circle and cross.

was that the expected distribution was affected very much by errors of measurement. In applying the method to the present star I have taken some care to find out just what distribution of velocities we should expect when both factors are taken into account.

Let us suppose that we are dealing with a circular orbit whose range is from plus ten kilometres to minus ten kilometres, and that the probable error of measurement is r. Let the velocities, if there were zero error of measurement, which would lie between the limits plus ten to plus nine be denoted by a, plus nine to plus eight be denoted by b, plus eight to plus six by c, ..., plus two to zero by f. Let those from zero to minus two be denoted by f', and so on till those between minus nine and minus ten are denoted by a'. Secondly, let us suppose the range in the observations is due to error of measurement alone, and let the velocities which would lie between the limits zero to plus two be denoted by f, between the limits plus two to plus four by f, etc. to infinity. To distinguish the negative from the positive add the subscript minus. Thus f indicates the velocities lying between minus two and zero.

The numbers a, b, c, are easily computed analytically or graphically. The graphical method has the advantage that it can be applied to any radial velocity curve, while the analytic expression for a, b, c, for other than circular orbits is very cumbersome. The values of 1, 2, 3, . . . . . . . can be readily computed for any assumed probable error.

Now let all the velocities a, b, c, etc., be operated on by errors of measurement, which errors are distributed as indicated by 1, 2, 3, etc. The product of a and 1 (which may be written  $a_1$ ) will yield velocities lying between +9 and +12, and if we have chosen our interval small enough we may regard the velocities as distributed uniformly within this interval, so that  $\frac{2}{3}$   $a_1$  will be the number lying between +12 and +10.  $\frac{1}{3}$   $a_2$  will also lie between the same limits, and if we select all these combinations we will obtain the following series as giving the number of velocities between +12 and +10.

To obtain the velocities lying between +10 to +8 all that it is necessary to do is to depress the subscripts by one,  $\frac{2}{3}$   $a_1$  becomes  $\frac{2}{3}$   $a_1$ .,  $\frac{1}{3}$   $a_2$  becomes  $\frac{1}{3}$   $a_1$ , etc. To obtain the velocities lying between +14 and +12 increase the subscripts by one. The number of velocities lying between limits x and x+2 are obtained by increasing the subscripts x-10.

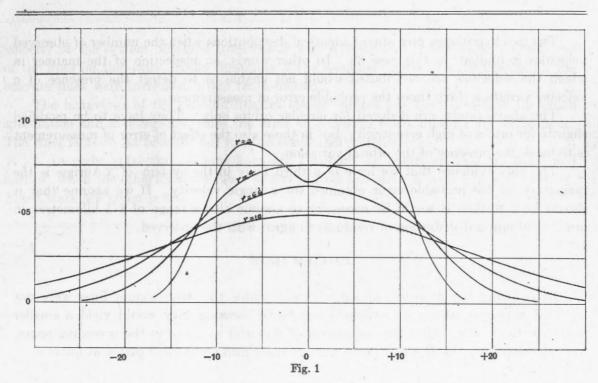
2

These series have been evaluated for  $r=2,\,4,\,6\cdot66,\,10$  kilometres, *i.e.*, for the ratios of probable error of measurement to total range equal  $\frac{1}{10},\,\frac{1}{5},\,\frac{1}{3},\,\frac{1}{2}$ . The results are tabulated below and represented graphically in fig. 1.

	the broader of the state of the state of the state of	die stille and	a Chadwide of	a tengen set	Transfer est
	Velocities in km.	r=2 km.	r=4 km.	r=6.6 km.	τ=10 km
17450	-alon at manage and a first time only a m	or country to			d 190
From	+ 0 to + 2	-0680	-0753	-0637	-0479
46	+ 2 to + 4	.0795	.0748	-0623	-0474
66	+ 4 to + 6	.0833	.0723	.0591	-0461
66	+ 6 to + 8	-0878	.0674	.0554	-0441
66	+ 8 to +10	.0793	.0604	-0503	-0415
66	+10 to +12	.0571	.0495	.0442	-0387
66	+12 to +14	-0295	.0380	-0384	-0356
66	+14 to +16	.0104	-0266	-0318	.0321
66	+16 to +18	.0025	.0173	-0258	-0284
66	+18 to +20	-0004	.0099	-0201	-0249
44	+20 to +22		•0053	-0150	-0215
46	+22 to +24		.0024	-0111	-0185
66	+24 to +26		.0009	-0079	.0153
66	+26 to +28		-0002	-0052	-0124
66	+28 to +30		-0001	-0031	-0100
66	+30 to +32			-0019	-0079
66	+32 to +34			∙0010	-0061
	Sum	. 4978	·5004	·4963	-4784

In the tabulated results the numbers are given to four decimal places, and as each was obtained by the summation of several factors, each of which was only computed to four places, the last figure is not in general correct. The sum of the columns given at the bottom if extended to infinity should total one-half, so that the addition of the columns serves as a partial check on the work.

In the curves the abscissae are kilometres, any point plus x indicates velocities lying between plus x-1 and plus x+1 and the corresponding ordinate shows what fraction of all the velocities will lie between these limits. It will be noticed that curves for the ratios  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{5}$  are very similar to error curves.



Suppose now we are working with such a binary as the one indicated, namely, a range plus ten to minus ten, and error of measurement r kilometres. Let us suppose, further, that we are unable to get a period, and treat the residuals as errors, how much in excess of the true value of r would the value we obtain be? The results we have tabulated for the distribution of the velocities enable us to compute this with a fair degree of accuracy, and we obtain that, if r equal 2 kilometres the computed r will be  $5\cdot 1$ , if r equal 4 kilometres the computed r will be  $6\cdot 1$ , and if r equal  $6\cdot 6$  kilometres the computed r will be  $8\cdot 2$ . The computation becomes a little uncertain for r equal 10 on account of some velocities lying beyond the computed values.

To apply the results to  $\chi$  Aurigæ, we have computed the distribution of the velocities on two bases. In the first the residuals are supposed to be due to error of measurement alone, in the second they are supposed to be due to a probable error of measurement 2.84 kilometres, combined with a binary whose eccentricity is zero and range 8.52 kilometres. In the table below, the first line gives the distribution of the observed residuals, the second, the computed distribution on hypothesis one, and the third the computed distribution on hypothesis two.

km. 0-1	km. 1–2	km. 2–3	km. 3–4	km. 4–5	km. 5-6	km. 6-8	km. 8–10	km. 10-∞
14	15	11	14	6	4	11	7	6
 13	13	12	11	9	8	11	5	6
 13	13	12	11	9	8	11	6	5

The two hypotheses give almost identical distributions when the number of observed velocities is limited, in this case 88. In other words, an inspection of the manner in which the velocities are distributed would not enable us to detect the presence of a velocity variation three times the probable error of measurement.

The above results are deduced for circular orbits only. They have to be modified slightly for orbits of high eccentricity, but in these also the effect of error of measurement is to mask the presence of the orbital variation.

The only evidence that we have of a third body in the system of  $\chi$  Aurigæ is the magnitude of the probable error obtained for a single velocity. If we assume that it should be 2.8, then it would be necessary to assume also a range of 8.5 kilometres in order to obtain a distribution of residuals to agree with the observed.

#### CALCIUM LINES

In reducing the measures of the different plates the two calcium lines were not included with the others, for evidence was found showing they would yield a smaller range for the curve. After the completion of the orbit as given in the preceding pages, the velocities from the H and K lines were grouped into ten normal places as follows.

NORMAL PLACES FOR H AND K LINES

	Julian Day	Phase from Final T	Velocity	Weight	0-C
1	2,420,104 · 4	129.8	- 9.5	1.0	-2.3
2	151.3	176.7	- 0.8	0.8	+2.5
3	202.4	227.8	+ 0.5	0.9	-0.4
4	249.9	275.3	+ 7.6	1.0	+3.2
5	277.0	302.5	+ 5.2	1.3	-0.9
6	302.9	328.3	+10.3	1.2	+2.8
7	411.8	437.2	+ 9.9	1.3	-0.6
8	473.4	498.8	+ 7.2	1.4	-1.9
9	$502 \cdot 1$	527.5	+ 7.2	1.4	-0.1
.0	571.2	596 · 6	+ 0.6	1.5	+0.7

Preliminary elements to satisfy these places were selected, the period, eccentricity and  $\omega$  were taken as previously found for the other lines. A solution was carried through for  $\gamma$  and K with the result,

$$\gamma = + 1.5$$
 km.  $\pm 0.46$   
 $K = 10.47$  km.  $\pm 0.85$ 

The radial velocity curve for the H and K lines is given on the same diagram as that for the other lines and is drawn with the finer line. The difference between the

velocity of the system obtained from the H and K lines on the one hand, and the rest of the lines on the other, would disappear if the assumed values of wave-lengths for the calcium lines were increased 0.022 tenth-metres.

The behaviour of the calcium lines in stellar spectra has been a subject for much speculation and investigation since the orbit of δ Orionis was published by Hartmann.<sup>1</sup> The chief facts so far brought out may be condensed as follows:-

1. In eight stars the calcium lines are constant, or nearly so, and are sharp and narrow, while the other lines are more or less diffuse and show a large range in velocity. These stars are given below.

one and which is name in the later than the same and the same in t		. 1900	Туре	Range of Broad Lines	Remarks	
aveil sulq motions ages all the retain		er Pro-		11 4 65		
femonth v districts , claim at mall.	h	m		km.		
o Persei	3	38.0	B1	224	Lick O	bs. Bull. 181
η Orionis.	5	19.4	B1	290	66	66
V Orionis	5	21.6	B2	288	66	66
δ Orionis	5	26.9	В	216	66	66
VV Orionis	5	28.5	B2	264	P.A.O.	, Vol. III, No. 21
θ <sub>2</sub> Orionis	5	30.5	B1	· 220	Lick O	bs. Bull. 181
β Scorpii	15	59.6	B1	251	P.A.O.	, Vol. II, No. 14
Boss 6142	23	51.0	Oep	230	D.O. (1	unpublished)

All the stars in this list are of type B2 or earlier, and, moreover, if we form a list of all such stars whose orbits have been determined and the K line measured, we find only one, v Orionis, showing a large range and not included above. Future orbits may add other exceptions.

- 2. In several stars, the calcium lines are known to vary differently from the other lines and in four cases, orbits have been determined from the H and K lines, i.e., for ξ Persei<sup>2</sup>, type Oe5; 9 Camelopardalis<sup>3</sup>, type B; χ Aurigæ, type B1; 12 Lacertæ<sup>4</sup>, type Oe5. In the last two the amplitude is probably about one-half as large for the calcium lines as for the others. There can be little doubt but that future investigation will reveal all gradations from those in which the H and K lines are constant to those where they have the same oscillation as the other lines.
- 3. In three stars the calcium lines are sharp and narrow and are shifted about onehalf an Angström unit to the violet. These stars are,  $-\xi$  Persei, type Oe5;  $\phi_1$  Orionis<sup>5</sup>, type B; ρ Leonis<sup>6</sup>, type B. There is evidence in these cases that it is the hydrogen

Ap. J., Vol. XIX, p. 268, 1904.

<sup>&</sup>lt;sup>2</sup>Dom. Obs. Pub., Vol. I, p. 355.

<sup>&</sup>lt;sup>3</sup>Ap. J., Vol. XXXVII, p. 1, 1913.

<sup>4</sup>R.A.S.C.Jr., November, 1915. <sup>6</sup>Ap. J., Vol. XXX, p. 63, 1909.

Dom. Obs. Pub., Vol. 1, p. 337.

and helium lines that are shifted to the red and that the calcium lines may be in their normal positions.

In  $\xi$  Persei, the hydrogen and helium lines are broad and no orbit has been determined from them, although they seem to show a large range. The mean velocity is about plus sixty-five kilometres. An orbit has been determined from the H and K lines, and the velocity of the system is plus fifteen kilometres. The component of the solar motion away from this star is about plus six kilometres, so that unless the star has an abnormally high velocity for an early B-type star we must regard the K lines as giving the true velocity of the star.

In the case of  $\phi_1$  Orionis, no orbit has been published, but if we take the mean of the velocities as representing final values, the calcium lines give plus nineteen, the hydrogen and helium lines plus forty, as compared with a computed velocity from the solar motion plus fifteen. Here again it is the velocity from the H and K lines that seems the most probable.

No orbit has been published for  $\rho$  Leonis, but the mean velocities are, for the calcium lines plus ten, for the other lines plus forty-three, for the solar motion plus five.

- 4. In several stars ( $\kappa$  Cassiopeiæ<sup>7</sup>, type B; 9 Camelopardalis, type B;  $\epsilon$  Orionis<sup>8</sup>, type B), the calcium lines are sharp and the other lines diffuse, but there does not seem to be much difference between the mean velocities as derived from the two sets of lines. The difference between these stars and those in 2 is only one of degree.
- 5. In many stars of early B-type, all the lines are very diffuse and in some cases the spectrum is almost continuous, so that little can be said concerning the behaviour of the calcium.  $\alpha$  Virginis and  $\delta$  Scorpii are representative.
- 6. There seems to be no exception to the rule, that when the calcium lines are sharp and narrow and the other lines broad, the star exhibits a variable radial velocity.

Three theories have been advanced to explain the observations. In the first, the calcium lines are supposed to be due to masses of cooler gas between the observer and the binary. In the second, the gas is supposed to surround the star or stars, and in the third, the phenomena are ascribed to anomalous dispersion.

The first hypothesis accounts for the sharp and narrow character of the H and K lines in class 1 and the fact that they yield constant velocity. Why should the calcium clouds always lie in front of a star of type B2 or earlier? We have examined several later B-type stars without finding anywhere H and K are stationary. There may be some stars later than B2 discovered which will show the effect, but the phenomenon seems to be one connected with early spectral type. In all cases in class 1 the velocities yielded by the calcium lines are very near the velocity of the system. This fact, however, does not argue strongly either against or for the first hypothesis on account of the very low velocities of the early B-type stars. The stars given under 3 require an explanation for the hydrogen and helium lines rather than for the calcium. The phenomenon is too complex to be accounted for by an hypothesis which allows so little latitude for adjustment.

<sup>&</sup>lt;sup>7</sup>Unpublished results (Harper).

<sup>&</sup>lt;sup>8</sup>Ap. J., Vol. XXIX, p. 235, 1908.

The second hypothesis may be presented as follows:-

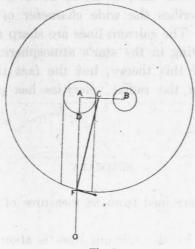


Fig. 2

Let A and B (fig. 2) be two stars or the two nuclei of a binary, and let the gases be condensed about these. It is known that calcium occurs at very high levels in the sun, and let us suppose that the equidensity layers in the calcium vapour about the two stars are nearly spheres. The calcium cloud will rotate about the common centre of gravity of the two stars in the same period as they revolve about each other. If A is the primary star and only one set of lines is observed on the plate, we need concern ourselves only with the light from A. The light from the photosphere gives the continuous spectrum, and as it passes through the cooler gases, hydrogen, helium and calcium, has lines from these elements absorbed. The absorption of the calcium takes place where the element is very rare, which accounts for the narrow character exhibited by these lines. The absorption of helium and hydrogen must be supposed to take place at a much lower level. Assume that it is permissible to speak of effective levels at which this absorption occurs, at a height D for the hydrogen and helium and at a height F for the calcium. At the time when the primary is moving toward us with the greatest velocity, the points at which the calcium absorption is taking place are rotating then as always about the point C and so are moving nearly across the line of sight. Moreover, the different points are all moving at about the same rate toward the observer, thus maintaining the narrow character of the lines. The points at which the hydrogen and helium absorption occurs, exhibit a wide range in velocity and give the wide character of the lines and the high range if the inclination is nearly zero. If the inclination is nearer to ninety degrees, the hydrogen and helium lines would be sharper and give a smaller range. This fits the case of 12 Lacertæ and X Aurigæ. There are doubtless other factors not understood which may contribute to the width of the lines observed in other cases. If the calcium cloud were extensive enough the calcium lines would remain nearly stationary, at least the amplitude would be small enough to be masked by errors of measurement. As the cloud condensed, the amplitude observed in the H and K lines would increase, the exact range depending on the degree of condensation of the calcium.

This theory is capable of considerable adjustment<sup>9</sup> to meet varying demands, and suggests future types of variations that should be discovered.

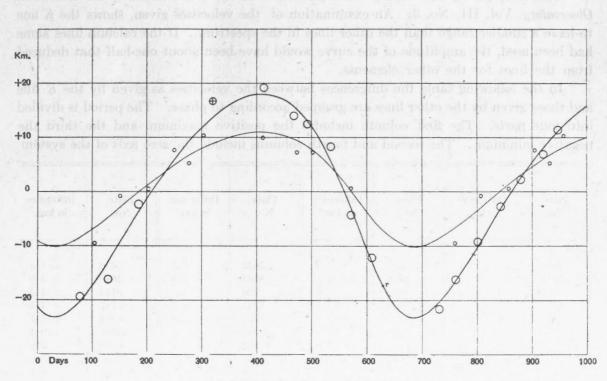
The third hypothesis ascribes the wide character of the lines in these stars to anomalous dispersion effects. The calcium lines are sharp and narrow, due to the small traces of these elements existing in the star's atmosphere. It is not intended to pass on the merits or demerits of this theory, but the fact that it does not satisfactorily account for the periodicity in the radial velocities has prevented its receiving wide acceptance.

Ap. J. Vol. XXXVII, p. 1, 1913.

#### SUMMARY

- 1. An orbit has been determined from 88 measures of radial velocity of x Aurigæ.
- 2. The amplitude shown by the calcium lines is about one-half that for the other lines of the spectrum.
- 3. An analysis of the different cases so far known where the calcium lines behave differently from the other lines shows that the phenomenon is one of type, and is best explained by the presence of calcium in a cloud surrounding the binary. The absorption of calcium takes place at a much higher level than that of the other elements.
- 4. The residuals given from the simple elliptic orbit were examined to ascertain if there might be a third body present. No secondary period was found, but the star should be observed with higher dispersion to test this point further.
- 5. The method of detecting periodicity in a set of observations from a consideration of the distribution of the velocities was tested. The value of this method is greatly interfered with by the effects of errors of measurement. If the ratio of error of measurement to range in the binary is as great as one-fifth, the resulting distribution of velocities is very much the same as would result from errors alone. If the ratio is as large as one-third, the distribution is almost identical. It is in these very cases that the method is most likely to be needed.

Dominion Observatory Ottawa June, 1916.



Radial Velocity Curves of x Aurigae

### NOTE ON THE SPECTROSCOPIC BINARY 12 LACERTÆ BY REYNOLD K. YOUNG, Ph.D.

The elements of this binary were published in the *Publications of the Dominion Observatory*, Vol. III, No. 3. An examination of the velocities given, shows the *K* line to have a smaller range than the other lines in the spectrum. If the calcium lines alone had been used, the amplitude of the curve would have been about one-half that deduced from the lines for the other elements.

In the following table the differences between the velocities as given by the K line and those given by the other lines are grouped according to phase. The period is divided into four parts. The first column includes the positive maximum and the third the negative minimum. The second and fourth columns include the zero axis of the system

Plate No.	Difference in km.	Plate No.	Difference in km.	Plate No.	Difference in km.	Plate No.	Difference in km.
	10.0	5040		<b>7000</b>		2000	
5670	-12.0	5642	- 4.4	5623	+ 7.3	5630	+ 1.4
5735	-22.5	5720	+ 5.5	5692	+ 7.5	5680	+ 3.0
6244	- 3.0	5800	+ 6.9	5728	+ 5.7	5711	+ 1.3
6276 6328	- 0·9 - 3·9	6198 6335	+12.4	5792	+ 8.9	5787	- 4.1
6369		6349	+13.7	6270	+ 7.9	6262	+17.1
	+5.3 $-9.9$	1	$ \begin{array}{c c} -16 \cdot 2 \\ -2 \cdot 9 \end{array} $	6281	+ 1.8	6291	+19.4
6409 6426	- 9·9 - 4·2	6504 6992	- 2.9	6342	+ 8.6	6308	- 2.6
6497	- 4.2	6998	+ 3.8	6531 6542	+ 6.1	6356	+11.0
6521	+12.6	7033	- 9.4	7001	- 4.1	6454 7023	- 5·4 - 8·1
6557	+ 0.4	7043	+11.9	7020	+14·8 +14·2	7178	+ 0.3
7026	- 1.9	7183	+ 9.9	7177	+14.2	7210	+16.0
7176	-13.3	7229	- 4.2	7214	+16.9	7244	- 4.4
7182	+ 1.2	1225	- 4.2	7214	+ 5.2	1244	-
7201	-12.2			7251	+ 9.9		
7211	- 9.8			7266	+ 6.7		
7249	- 2.3			7270	+15.0		* * * * * * * * * * * * * * * * * * * *
7268	+ 4.1			7293	- 2.2		
7302	- 2.5			7300	+10.0		
7310	-16.8			7313	+28.4		
7314	-11.0						
an	- 5.2	Mean	+ 1.9	Mean	+ 9.7	Mean	+ 3.5

The weighted mean of all the residuals is positive, due to the value of the wavelength of K used. If we remove this effect, the resulting means for columns 1, 2, 3, 4 would read -7.3 km., -0.2 km., +7.6 km., +1.4 km. The semi-amplitude of the velocity curve as found for all the lines was 16.92 km. The amplitude for the K line would be about 7.5 km. less.

Ottawa
May, 1916.

MUDLICATION.

# DOMINION OBSERVACIORS

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