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Mean distance of stars whose radial velocities, proper motions and parallaxes have been determined

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# MEAN DISTANCE OF STARS WHOSE RADIAL VELOCITIES, PROPER MOTIONS AND PARALLAXES HAVE BEEN <br> DETERMINED. 

BY REYNOLD K. YOUNG, Ph.D.

The following investigation was undertaken to answer the question,Will the mean distance of the stars as determined from their proper motions and radial velocities agree with the mean distance as formed from the directly measured parallaxes?

The data for the parallaxes were selected from the list published by Kapteyn and Weersma in 1910*. For one hundred and ninety-five stars of this list, radial velocities were obtained from the Mount Wilson $\dagger$ and Lick Observatory $\ddagger$ results. The proper motions of all these stars are very large, and indeed for the most part the stars were selected for parallax measurement on this account. The excessive magnitude of the cross motion may be due

1. to the proximity of the stars,
2. to the excessive speed,
3. to exceptionally large values of the inclination of the motion to the line of sight.

An examination of the data seemed to show that the first factor was by far the most potent. If we reject twenty-eight stars with exceptional velocities, over fifty kilometres per second, the remaining ones have a mean radial velocity very little above the average of the stars in general.

[^0]From one hundred and sixty-seven stars whose radial velocities were less than fifty kilometres per second, a solution for the sun's motion was made with the result:-

$$
\begin{array}{ll}
\begin{array}{l}
\text { Right Ascension of Apex........ }
\end{array} & 270^{\circ} \cdot 0 \\
\text { Declination of Apex........... } & +14^{\circ} \cdot 7 \\
\text { Velocity of Sun.................... }
\end{array}
$$

If the motion of each star is corrected with these values, we find that the average radial velocity for the one hundred and sixty-seven stars is $\mathbf{1 5 . 2}$ kilometres. The list of stars includes nine B-type stars, seventeen A, forty-eight $F$, forty-eight G, thirty-four $K$, seven $M$, and four of unknown type. Campbell's average radial velocities for this distribution of stellar types would be $13 \cdot 8$. The agreement of the two values supports the conclusion that the mean velocity of the stars employed is not greatly in excess of the average.

The proper motions at right angles to the solar motion ( $r$ components) and along the direction of solar motion ( $v$ components) were also computed with the above values for the solar motion. The position of the apex as given by this special group of stars is preferred, because we then demand only that the motions shall be at random among themselves which is a less radical assumption than that they be at random with respect to a given direction, an assumption we would have to make if we had adopted any other position of the apex.

The direction of the solar motion determined from this solution has very little weight when applied to the universe in general, but it is interesting to note in passing that it is the declination which shows the variation from the usually accepted values. Nearly all the stars used are in the northern hemisphere. This was true also of the stars employed by Campbell* in his first solution for the sun's way from two hundred and eighty velocities. Both solutions give low values for the declination of the apex. Whether the small declinations are due to the northern positions of the stars or are to be ascribed to the chance operation of unsymmetrical data is hard to say. The agreement would tend to show that the northern hemisphere gives low values for the declination of the apex.

[^1]The mean distance of the group of stars was evaluated from the formula

$$
\pi_{m}=4.738 \frac{\tau_{m}}{V_{m}}
$$

Where $\pi_{m}$ is the mean'pparallax.
$\tau_{m}$ is the mean component of proper motion at right angles to the solar motion.
$V_{m}$ is the mean radial velocity.
Eighty-three $\tau$ 's are positive and eighty-four negative, and their average value is $0^{\prime \prime} \cdot 339$, which makes $\pi_{m}$ computed $0^{\prime \prime} \cdot 106$. The mean parallax as observed for the same stars is $0^{\prime \prime} .072$. If we reject a dozen stars whose proper motions are large and which were not rejected by the condition that the radial velocity be under fifty kilometres, we obtain a computed mean parallax $0^{\prime \prime} .085$ as opposed to $0^{\prime \prime} \cdot 060$ observed. The rejection of these stars does not alter the ratio of the computed and observed parallaxes materially.

Methods of determining the mean distance of the stars based upon the data from the $v$ components are still available, and while these can not be considered as having as much weight as those based on the $\tau$ components, it is of interest to see how they agree with the results above.

The first method is based upon the magnitude of the parallactic motion. We have determined by a system of trials, that mean distance, such that when each star is corrected for the solar motion, the total sum of the positive $v$ components is equal to the sum of the negative. The mean parallax that will accomplish the result is $0^{\prime \prime} \cdot 105$, a value almost in exact agreement with the value found from the $\tau$ components. Rejecting the same dozen stars as before would yield the value $0^{\prime \prime} \cdot 092$.

When the $v$ components have been corrected on the basis of this mean distance, the corrected values may be treated as the $\tau$ components to yield a third value for the mean distance of the stars.

$$
\pi_{m}=4.738 \times \frac{\mathrm{v}^{1} m}{V_{m}}
$$

where $v^{1}{ }_{m}$ is now the mean value of the corrected $v$ components. From this data the value $0^{\prime \prime} \cdot 133$ was obtained or $0^{\prime \prime} \cdot 098$ if twelve stars were omitted. The collected results are shown in the table below:-

|  | Observed. | Comptere. |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\tau$ Components. | Parallactic motion. | v Components. |
| From 167 stars. | $0^{\prime \prime} .072$ | $0^{\prime \prime} \cdot 106$ | $0^{\prime \prime} \cdot 105$ | $0^{\prime \prime} \cdot 133$ |
| From 155 stars. | 0" ${ }^{\prime \prime} 060$ | $0^{\prime \prime} \cdot 085$ | $0^{\prime \prime} \cdot 092$ | $0^{\prime \prime} \cdot 098$ |

What effect should the phenomenon of star streaming have on the result? In order to ascertain if this were present, the stars were tested both from their radial velocitiés and from their proper motions. For the radial velocities, the mean value for stars within sixty degrees of the vertices of preferential motion was $17 \cdot 6$ kilometres as opposed to $13 \cdot 6$ in the remaining part of the sky. To test the proper motions, each star was corrected for the solar component on the basis of the observed parallax. The stars were then divided into two groups, those within sixty degrees of the vertex and those outside this region. If the stars are moving more rapidly along the line joining the vertices, the proper motions should be greatest at right angles to this line. The average values in the two regions were $0^{\prime \prime} \cdot 584$ and $0^{\prime \prime} \cdot 577$. The method is doubtless crude and the regions rather large but we would have expected the star streaming to have shown more than these numbers indicate.

Let us consider however what the effect of such a preferential motion would be on the results. The directions of this motion as given by Kapteyn are

Right ascension. $.91^{\circ}$ and $271^{\circ}$
Declination.... $+13^{\circ}$ and $-13^{\circ}$
a line which is not very far from the apex of the sun's way used in this solution. The $\tau$ components of proper motion should not be much affected, while the radial velocities should be increased, and we would expect that the application of the formula

$$
\pi_{m}=4.738 \times \frac{\tau_{m}}{V_{m}}
$$

should give too small rather than too large a value for the mean parallax.
In treating the corrected $v$ components by the same formula

$$
\pi_{m}=4 \cdot 738 \frac{v_{m}^{1}}{V_{m}}
$$

the effect will depend on the distribution of the stars. If nearly all the stars were found near the vertices, $V_{m}^{1}$ would be increased while $v_{m}^{1}$ would remain about the same. If on the other hand most of the stars were midway between the vertices, then $v_{m}^{1}$ would be increased and $V_{m}$ remain constant. A symmetrical distribution should leave the results of the application of this formula entitled to as much weight as the value from the $\tau$ components.

As regards the adjusted value which removes the parallactic effect, it depends directly upon the velocity of the sun adopted. The higher the velocity of the sun chosen, the smaller will be the value of the parallax. If we had used a velocity of the sun about twenty-five kilometres per second, the observed and computed values of the mean parallax from this source would have been in agreement.

The parallactic method of determining the mean distance of a group of stars is applicable without a knowledge of the radial velocities and so could have been applied to a much larger number of stars. This solution has been practically carried out by Lewis Boss.* From 559 stars with mean proper motion over $31^{\prime \prime} .9$ per century he determined the parallactic motion and assuming the parallaxes to be correct, reversed the problem to determine the speed of the sun. It is very interesting that the velocity obtained was 24.5 kilometres; a value which is undoubtedly too high, but which is almost in exact agreement with the speed that would have to be assumed for the present list of stars to make the observed and computed mean parallaxes agree.

[^2]Other explanations of the difference between the observed and computed parallaxes suggest themselves. The first is that in choosing stars with large proper motions we have selected stars whose mean angle of inclination to the line of sight is larger than the law of random motion would make it. If this is true the present results would be explained. The operation of this factor would affect relations connecting parallax and proper motions in general, for such relations are derived from stars with large proper motions. Another explanation might be that the stellar system as a whole is rotating, a phenomenon which would affect the proper motions and not the radial velocities. However this latter explanation seems very improbable, as the major part of any rotational effect in proper motions must be eliminated by solutions for the position of the vernal equinox. There remains the possibility that the stars employed contain an unusual number of negative parallaxes.

Dominion Observatory, Ottawa, July, 1915.


[^0]:    *Grönigen Publications, No. 24.
    $\dagger$ Ap. J. Vol. 39.
    $\ddagger$ Lick Observatory Bulletin 214.

[^1]:    *Ap. J. vol. 13, 80.

[^2]:    *Astronomical \#Journal, vol. 26, 118.

