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VELOCITIES OF LONGITUDINAL WAVES IN THE UPPER PART OF THE EARTH'S MANTLE

BY

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RÉSUMÉ. — *L'article concerne seulement l'Europe, et rappelle la complexité de sa structure. Puisqu'on admet maintenant que la courbe de propagation des P est une ligne droite jusqu'à 15° environ de distance épicentrale, la vitesse en profondeur ne peut être tirée par une méthode directe ; mais une solution possible peut être obtenue par tâtonnements. Le gradient de vitesse doit être supposé très petit ou nul dans une couche superficielle. On admet que celle-ci atteint 220 km de profondeur et qu'on trouve là un accroissement brusque de la vitesse et du gradient de vitesse. A partir de 15° la courbe P correspond à des ondes réfractées dans la couche inférieure.*

On adopte les vitesses qui figurent dans la Table 2. Jusqu'à 22° la courbe P correspondante est en bon accord avec la courbe de JEFFREYS *révisée en 1954.*

On examine les propriétés des courbes P et pP pour les séismes ayant leurs foyers à une certaine profondeur dans la couche supérieure. On étudie quelques séismes profonds de Roumanie ayant tous le même foyer, et l'on compare les durées de propagation des P avec les durées calculées. Quelques écarts semblent ne pouvoir s'interpréter autrement que par des différences locales de structure. On étudie également un séisme profond de la mer Tyrrhénienne.

La solution adoptée correspond à une possibilité, mais elle n'est pas unique et de nouvelles observations sont indispensables. Des déterminations plus précises de la variation d'amplitude seraient particulièrement utiles.

SUMMARY. — *The investigation deals with Europe only and recalls the complexity of its structure. Because the P time-distance curve is now taken to be nearly a straight line up to about 15° epicentral distance, the velocity at depth cannot be derived from it by the direct method, but by trial and error a possible solution is obtainable. The velocity gradient has to be taken quite small or zero in an upper layer. This was taken to extent to 220 km depth and an abrupt increase of velocity and velocity gradient to set in at this depth. From about 15° onwards the P curve becomes associated with waves refracted in the deeper layer.*

The velocities given in table 2 were adopted. Up to 22° the corresponding P curve is in good agreement with JEFFREYS' *revised 1954 curve.*

The properties of P and pP curves of shocks having their foci at some depth in the upper layer were considered. Some deep Rumanian earthquakes all from the same focus were examined and their P times compared with those calculated. Some deviations seemed explainable only as due to local differences of structure. A deep earthquake in the Tyrrhenian Sea was also examined.

The adopted solution seems a possible one, but it is not unique and more observations are required. More precise determinations of amplitude variation would be particularly useful.

The constitution of the upper part of the Earth's mantle is a matter of great interest to geophysicists of various fields, and they are hoping for seismology to supply relevant information of a more precise and detailed nature than otherwise obtainable. We observe the transmission times of seismic waves, and are supposed to be able to derive from them the variation of the velocity with depth, which again is a clue to the variation of physical properties.

Time-distance tables were constructed, in particular by GUTENBERG and RICHTER (1934) and by JEFFREYS and BULLEN (1940), and wave velocities were derived from them. It was found, however, that the tables were in error at small distances, and when the necessary corrections were applied wave velocities for the upper mantle could no longer be derived from them. GUTENBERG (1948, 1955) made tentative solutions, but a unique solution is not obtainable. This is due to the fact that the velocity just below the MOHOROVIČIĆ discontinuity is greater than at first assumed, and the curvature of the time-curve up to about 15° so small that the direct method for derivation of the velocity function cannot be applied.

In the course of further studies it was found necessary to distinguish between regions. I shall here consider the European region only.

It was from large explosions that the velocity just below the MOHOROVIČIĆ discontinuity was at first found to be greater than the velocity derived from the JEFFREYS-BULLEN (J. B.) tables. The largest of these, the Heligoland explosion (WILLMORE, 1948), gave us the travel times of P waves out to a distance of 9°; the time-curve was indistinguishable from a straight line of slope 13.6 sec./degree. JEFFREYS, combining this result with some earthquake observations, obtained a corrected P time-curve for Europe (JEFFREYS, 1954).

In recent years numerous large earthquakes were well recorded in Europe at the distances with which I am here concerned. There were a great number of Greek earthquakes and there were also large earthquakes in Algeria. Swiss and other Central European earthquakes were well recorded at the smaller distances. It may seem well worth while to make a study of these earthquakes that are well recorded by a far greater number of stations and with much greater precision than those used in earlier work, and it may seem futile to attempt velocity determination before this has been done.

It is undoubtedly desirable that a comprehensive study of recent European earthquakes should be made. It is not likely, however, to prove so very straightforward. Most of the earthquakes occur in outlying regions, where epicentre determinations are uncertain. On the other hand a great number of stations are in small azimuthal sectors and, therefore, should yield reliable slopes of the time-curves. This is on the supposition that the first wave observed is the same everywhere. It may seem as if we could rely on the first P wave being recorded at the now numerous stations equipped with sensitive short-period seismographs. However, in his study of intermediate earthquakes GALANOPOULOS (1953) mentions that shallow Greek earthquakes, even very large ones, in contrast to the intermediate earthquakes have very small first P waves, and that the subsequent movement increases gradually. It may, therefore, seem uncertain that it is possible to pick up the onset of the same wave everywhere, and if we cannot rely upon this the observations are not very useful for the construction of time-curves. Also, European structure may not be so homogeneous as we would like and Eurasiatic structure less so. We know the composition of the crust to vary a great deal, and the depth of the MOHOROVIČIĆ discontinuity is believed to be at somewhat varying depth. Also, we have in Rumania earthquakes at a depth of about 150 km, showing that the mantle is not in a stable state there. PETERSCHMITT (1956) has found that the Calabrian arc has properties similar to those of the Pacific arcs. A deep earthquake has occurred in the region and several intermediate ones. We have had a very deep earthquake with its epicentre in Spain. All of this points to considerable structural differences in the mantle underlying Europe. It does not reduce the interest attached to a comprehensive study of European earthquakes, on the contrary, but it makes it somewhat doubtful that precise results applying to the whole of Europe are obtainable.

The P time-curve as we have it now (JEFFREYS, 1954) may not be the best approach to a mean time-curve for Europe, but certain of its features are not likely to be greatly modified in future studies. The small or negligible curvature up to about 15° epicentral distance and an appreciable curvature from there onward sare likely to be maintained. The corresponding velocity function necessarily differs considerably from that derivable from the J. B. time-curve, and it is of some interest to see in what respects it differs from it. We cannot actually determine the velocity function, but we can arrive at some of its characteristic features. We can assume a velocity function having these features, derive the time-curve from it and alter the assumption until a good fit to observations is obtained. The velocity function arrived at in this way is one of the many possible solutions, and it may help us to see what kind of solution can be considered.

On a previous occasion I worked out a tentative solution (LEHMANN, 1956) but I have now worked out results more precisely. The time-curve I attempted to approximate was taken to be a straight line of slope 13.6 sec./degree up to 15° epicentral distance and to start to bend there. The difference of the heights at 15° and 22° was taken to be the same as that of the J. B. curve for a surface focus.

The velocity function taken was the one first used by A. MOHOROVIČIĆ (1910) when he attempted to determine the depth of his discontinuity. In later years BULLEN (1945) has drawn attention to the formula that may be written

(1) $$v = ar^{-k} = v_0 \left(\frac{r}{r_0}\right)^{-k},$$

where v is the velocity at distance r from the centre of the sphere to which the formula is applied, the subscript $_0$ indicates surface values, and a and k are constants.

When computations are involved the formula is easier to work with than the WIECHERT formula $v = a - br^2$, while this latter formula lends itself more easily to construction since the rays are circular arcs.

Since it may come useful to others intending to do similar work, I shall go into some detail about how the formula is applied.

The epicentral distance at which a ray of constant $\alpha = \frac{r}{v} \sin i$ emerges is

(2) $$\Delta = 2\alpha \int_{r_u} \frac{1}{r} \frac{1}{\sqrt{u^2 - \alpha^2}} dr$$

where $u = \frac{r}{v}$ and r_u is the distance from the centre to the deepest point of the ray. We have :

(3) $$\alpha = \frac{r_u}{v_u} = \frac{r_0}{v_0} \sin i_0 = \frac{dt}{d\Delta},$$

where t is travel time to distance Δ when Δ is measured in radians, or $\alpha = \frac{dt}{d\Delta} \cdot \frac{180}{\pi}$ when Δ is measured in degrees. From (1) we find

$$u = \frac{1}{a} r^{k+1},$$

and substituting this in (2) we find :

$$\Delta = 2 \int_{r_u}^{r_0} \frac{1}{r} \frac{1}{\sqrt{\frac{1}{a^2 \alpha^2} r^{2(k+1)} - 1}} dr.$$

Putting

$$x = \frac{1}{a\alpha} r^{k+1}, \qquad \frac{dx}{x} = \frac{k+1}{r} dr,$$

we find

$$\Delta = \frac{2}{k+1} \int_{x_u}^{x_0} \frac{1}{\sqrt{x^2 - 1}} \frac{dx}{x}$$

and obtain :

$$\Delta = \frac{2}{k+1} [\sec^{-1} x]_{x_u}^{x_0} = \frac{2}{k+1} \left[\sec^{-1} \frac{1}{a\alpha} r^{k+1} \right]_{r_u}^{r_0};$$

(1) and (3) give us :

$$\frac{r_0}{v_0} = \frac{r_0^{k+1}}{a} \text{ and } \frac{r_u}{v_u} = \alpha = \frac{r_u^{k+1}}{a},$$

and therefore

(4) $$\cos \frac{k+1}{2} \Delta = \frac{\alpha}{u_0}.$$

From $\alpha = \frac{dt}{d\Delta} = u_0 \cos \frac{k+1}{2} \Delta$ we find :

(5) $$t = u_0 \frac{2}{k+1} \sin \frac{k+1}{2} \Delta.$$

From (4) and (5) we obtain Δ and t for any $\alpha \leqslant u_0$ when the surface velocity v_0 and the constant k are known. We proceed as follows. $\frac{dt}{d\Delta}$ (Δ measured in degrees) is taken as independent variable ; it is multiplied by $\frac{180}{\pi}\frac{1}{u_0}$ to give $\cos\frac{k+1}{2}\Delta$. From a trigonometrical table $\frac{k+1}{2}\Delta$ and $\sin\frac{k+1}{2}\Delta$ are obtained and from these Δ and t are found. The calculations are conveniently carried out in a table under the following headings :

$$\frac{dt}{d\Delta} \qquad \cos\frac{k+1}{2}\Delta \qquad \sin\frac{k+1}{2}\Delta \qquad \frac{k+1}{2}\Delta \qquad \Delta \qquad t.$$

We may wish to find the r_u and v_u corresponding to a given value of $\frac{dt}{d\Delta}$.

We have $\frac{r_u}{v_u} = \alpha$ and $v_u = ar_u^{-k}$

hence

$$r_u^{k+1} = a\alpha, \tag{6}$$

and

$$\log r_u = \frac{1}{k+1}(\log a + \log \alpha), \tag{7}$$

$$\log v_u = \frac{1}{k+1}(\log a - \log \alpha). \tag{8}$$

From (6) we can also find the α and $\frac{dt}{d\Delta}$ of the ray having a given r_u.

As a rule the velocity formula is taken to be valid only down to a certain depth r_1 and at this depth the k and possibly also the v changes. The problem is then to find the $[\Delta, t]_{0,1}$ for transmission through the layer $[r_0, r_1]$. Take (Δ_0, t_0) to be the (Δ, t) as found for the sphere of radius r_0 with surface velocity v_0 for $k = k_0$; $(\Delta_{0,1}, t_{0,1})$ the $(\Delta\, t,)$ of a sphere with radius r_1, surface velocity $v_1 = a_0 r_1^{-k_0}$ for $k = k_0$. Then $[\Delta, t]_{0,1} = (\Delta_0 - \Delta_{0,1},\ t_0 - t_{0,1})$. It is to be noted that k has to be taken $= k_0$ also in the sphere of radius r_1 since $v = v_0\left(\frac{r}{r_0}\right)^{-k} = v_1\left(\frac{r}{r_1}\right)^{-k}$.

We have (1)

$$v = ar^{-k},$$

$$\log v = \log a - k \log r,$$

$$\frac{dv}{dr} = -k\frac{v}{r} = -akr^{-k-1}. \tag{9}$$

Hence v increases with decreasing r when k is positive and is constant for $k = 0$.

**

When $k < 0$ v decreases with decreasing r and for $k = -1$ we have $\frac{dv}{dr} = \frac{v}{r}$ which marks critical decrease of velocity. Thus for $k \leqslant -1$ the rays do not emerge. This is easily seen in another way for we obtain from (1)

$$\frac{r}{v} = \frac{r^{k+1}}{a},$$

or for $k = -1$

$$\frac{r}{v} = \frac{1}{a},$$

and since $\alpha = \frac{r}{v} \sin i$, $\sin i$ and i remain constant for a given α. The ray therefore is a logarithmic spiral. Since

$$\alpha = \frac{r^{k+1}}{a} \sin i,$$

we see that for $k < -1$ $\sin i$ and i decrease with decreasing r; the ray therefore goes down into the earth more and more steeply and it never emerges.

When $k \leqslant -1$, Δ and t can still be obtained from our formulae (4) and (5), but they become negative and are therefore without physical meaning. However, u being greater than u_0, $\Delta_{0,1}$ and $t_{0,1}$ are numerically greater than Δ_0 and t_0 and the $\Delta_0 - \Delta_{0,1}$ and $t_0 - t_{0,1}$ are positive. The $[\Delta, t]_{0,1}$ for transmission through the layer r_0, r_1 can therefore be obtained in the usual way. Since layers in which the rate of decrease of velocity is greater than critical are supposed to exist, it is of some importance to be able to calculate times of transmission through them on simple velocity assumptions.

As said already, calculations become more involved when the WIECHERT formula $v = a - br^2 = v_0 - b\,(r^2 - r_0^2)$ is taken than when the formula $v = ar^{-k}$ is used. We derive from it :

$$\cot i \cot \frac{\Delta}{2} = \lambda, \tag{10}$$

where λ is a constant $= \frac{2r_0^2 b}{v_0} + 1 \geqslant 1$ provided $b \geqslant 0$ and

$$t = 2\alpha \frac{1}{\sqrt{\lambda^2 - 1}} \text{arc} \sin \left(\sqrt{\lambda^2 - 1} \sin \frac{\Delta}{2}\right), \tag{11}$$

Taking again $\frac{dt}{d\Delta}$ as independant variable we find

$\sin i = \frac{dt}{d\Delta} \frac{180}{\pi} \cdot \frac{1}{u_0}$ and thereafter $\cot i$ and from (10) $\cot \frac{\Delta}{2}$ and Δ. We then successively find $\sin \frac{\Delta}{2}$, $\sqrt{\lambda^2 - 1} \sin \frac{\Delta}{2}$, $\text{arc} \sin \left(\sqrt{\lambda^2 - 1} \sin \frac{\Delta}{2}\right)$ and t. If the calculations are carried out in a table similar to the one on p. 385, this table has to have 9 columns. When the $(\Delta, t)_{0,1}$ for transmission through the layer (r_0, r_1) are wanted we have to use different values of λ in the two spheres while k remained unaltered, for we have

$$\lambda_0 = \frac{2r_0^2 b}{v_0} + 1 \qquad \lambda_1 = \frac{2r_1^2 b}{v_1} + 1.$$

We shall now attempt to approximate our P curve, calculating it on certain velocity assumptions. The time distance curve we wish to approximate is much more straight than the J. B. curve up to an epicentral distance of about 15°. Our velocity increase in the corresponding layer is therefore necessarily smaller. Corresponding to the J. B. curve the velocity increases from 7.75 km/sec just below the MOHOROVIČIĆ discontinuity to 8.32 km/sec at 220 km depth where the ray emerging at 15° epicentral distance has its deepest point. When there is a smaller velocity increase the rays are more shallow, and when the velocity increase is small enough to make the curve up to 15° seem nearly straight, the ray emerging at 15° cannot come down to a depth much greater than 120 km. This is to say that we have to assume either that the velocity increase responsible for the bend that begins at 15° epicentral distance sets in at a depth not much greater than 120 km or else that the time-curve from 15° onwards is not the continuation of the curve at smaller distances but is a different branch due to a wave refracted in a deeper layer. This latter possibility was considered in my earlier study, but I regard it as a certainty now, for a strong velocity increase cannot be taken to set in at a depth of about 120 km. The chief evidence comes from GUTENBERG'S determination of the velocity at the focal depth of large earthquakes (GUTENBERG, 1953). The velocities found for depths smaller than 200 km vary a great deal, but a marked increase of velocity with depth does not occur until at depths exceeding 200 kms. I have tentatively taken the boundary to be at 220 kms depth.

This leaves us with a layer about 100 km deep from which no rays are observed to emerge in shallow shocks. If they were observable they would be associated with a time-curve of slight curvature forming the continuation of the straight line up to 15°. This is to say that no direct information about the velocity variation in the layer between, say, 120 and 220 kms is obtainable from observations of shallow shocks. Some further information should be obtainable from deep shocks having their foci in the layer, but as we shall see later, even deep shocks will not supply information for depths exceeding about 160 kms.

I calculated the P curve taking the velocity in the crust to be constant = 6.3 km/sec. This is not quite correct, but it makes very little difference to the time-curve for greater distances. Below the MOHOROVIČIĆ discontinuity the velocity was taken to increase rather strongly from 8.0 to 8.12 km/sec. in a layer 20 km deep, and to remain constant from there down to the 220 km level. The deepest ray in the uppermost layer emerges at a distance of 4°6. The strong velocity increase was assumed because amplitudes are found to be relatively large at small distances and to decrease rapidly later on. Actually it may not be necessary to assume a strong velocity increase at small depth in order to account for the large amplitudes since the laws of ray optics do not hold close to a boundary. However, supposing these laws to hold deeper down, at the depths in question, the strong and sometimes rather sudden decrease of amplitudes beyond about 5° is easily accounted for, since the rays are widely spread when they enter a layer of constant or nearly constant velocity from a layer in which there is a much stronger velocity increase. There will be a decrease of energy that may well come near to producing a shadow zone. In my example I had $\frac{dt}{d\Delta} = 13.57$ sec/degree at 5° epicentral distance and 13.50 sec/degree at 15°. Thus the bundle of rays responsible for that part of the time-curve is exceedingly small, so small indeed that there may not be any observations at all except, perhaps, in very large earthquakes. It may be necessary to assume some increase of velocity in the lower layer to account for the observations obtained. However, only a small increase of velocity is possible if the time-curve is to be nearly straight and when transmission times only, not amplitudes, are considered it makes very little difference to the results whether the velocity is taken to increase slightly, to be constant or to decrease slightly. The simplest assumption, that of constant velocity, was therefore maintained.

The ray having its deepest point at the 220 km level then emerges at an epicentral distance of 28°3. The time-curve associated with the rays transmitted in the layer has a slight curvature, the slope at the end-point being 13.22 sec/degree.

We now had to find a velocity function in the lower mantle, below the 220 km level, that would produce a branch intersecting the first branch at about 15° epicentral distance and bending so that the difference of height at 15° and 22° would be the same as for the J. B. curve.

Various attempts were made. At first the velocity at 220 km depth was retained and an abrupt increase of velocity gradient assumed. It resulted in a time-curve having a loop with its lower end at a distance not much smaller than 15°. There was a concentration of energy at the turning point that would necessarily give rise to large amplitudes and since exceptionally large amplitudes are not observed at this distance the assumption was abandoned. An abrupt increase in the velocity itself as well as in velocity gradient was then assumed. Again various attempts were made. The final assumption adopted was that of an increase of velocity from 8.12 km/sec to 8.40 km/sec. at the 220 km boundary and a strong velocity gradient below. In the formula $v = v_0 \left(\frac{r}{r_0}\right)^{-k}$ k was taken $= 3$. It resulted in a time-curve that up to 22° deviated only very slightly from JEFFREYS' final time-curve of 1954 (see table 1). It has a slightly smaller slope below 15° since JEFFREYS (rather arbitrarily as he remarks) adopted the slope 13.66 sec/degree while my average slope is 13.54 sec/degree. JEFFREYS' time exceeds mine by $0^s.4$ at 2° and the deviation increases to $1^s.7$ at 15°. From 17° to 22° the difference does not exceed a few tenths of a second.

TABLE 1

TRAVEL TIMES OF P WAVES

Δ	*t*. H. J.		*t*. I. L.	
0	m	s	m	s
2		35.0		34.6
4	1	2.4	1	2.0
6		29.8		29.1
8		57.1		56.3
10	2	24.4	2	23.4
11		38.1		36.9
12		51.7		50.5
13	3	5.4	3	4.0
14		19.0		17.5
15		32.7		31.0
16		45.5		44.5
17		57.9		57.8
18	4	10.0	4	9.7
19		21.8		21.5
20		33.2		32.8
21		44.1		43.9
22	4	54.7	4	54.6

The intersection of my two branches occurs at 17° instead of at 15° as was intended, but the two branches are very close to one another at 15°, only slightly more than 1^s apart, and the increase of amplitude that seems to take place at about that distance could be due to the wave associated with the second branch.

The velocities derived from the J. B. time-curve (JEFFREYS 1939, p. 511) and my velocities are compared in table 2. JEFFREYS' velocity in the upper part of the mantle is at first smaller than the one here assumed, but it increases so as to reach the same value at 159 km. depth and thereafter becomes greater. At 220 km depth the increase of my velocity makes it become greater than that of JEFFREYS, and it remains greater down to somewhere between 412 and 476 kms

TABLE 2

VELOCITIES OF P WAVES

DEPTH R	VEL. H. J. km/sec	DEPTH km	VEL. I. L. km/sec
0.00	7.75	35	8.00
		55	8.12
.01	7.94	95	8.12
.02	8.12	159	8.12
.03	8.32	220	8.12
		220	8.40
.04	8.56	286	8.68
.05	8.76	349	8.95
.06	8.96	412	9.24
.07	9.52	476	9.54
.08	9.88	539	9.86
.09	10.28	602	10.18
.10	10.53	666	10.53
.11	10.77	729	10.89
.12	10.99	793	11.26

depth, where JEFFREYS velocity increase sets in. From there down to the 666 km depth the velocities are equal. Below that depth my velocity increases more strongly than that of JEFFREYS', but there the velocity formula is no longer applicable since the rate of increase of the actual velocity decreases as is indicated by a straightening of the time-curve and diminishing amplitudes from about 22° epicentral distance.

Up to that distance our solution seems quite satisfactory and the corrected J. B. curve can be joined on to it there.

The question is now whether or not other results derivable on our assumptions are in agreement with observations.

We can calculate time-distance curves for foci at varying depth, but in Europe there are not many shocks deeper than normal with which we can compare. There are intermediate shocks in the Aegean Sea and near Crete, but as a rule the epicentres cannot be well determined and the depths found for them have great uncertainty. I have tried to use the observations of some of them, but they scattered too widely. However, in Rumania several large earthquakes occurred at a depth supposed to be about 150 km. Their epicentres should be determinable with a fair degree of accuracy. They have been very well observed by a considerable number of stations in the range of distance in which we are interested.

I shall compare the observations of the Rumanian shocks with time-curves calculated on our velocity assumptions, but before doing so we may consider in a general way some of the implications of these assumptions.

The P time-distance curves calculated for foci in the upper layer of the mantle, i.e. above the 220 km level, all have two branches as has the P curve for a surface focus, and the points of intersection of the branches will be at a smaller epicentral distance the greater the depth of focus.

The branches associated with the direct waves, for which the rays are entirely above the 220 km level, have inflexion points at the distances where the rays starting horizontally at the focus meet the surface of the earth ; they are at greater epicentral distances the greater the focal depth. They are at greater distances than the corresponding J. B. inflexion points since the rays bend less. The branches are very nearly straight lines from the inflexion points onwards and also for some distance below. The slope of the line depends on the velocity at the depth of focus. The line is intersected by the curved branch due to the refracted wave and for distances greater than that of the point of intersection the direct wave is not likely to be observable, so the line would be cut off, so to speak, and this would happen at a smaller epicentral distance the greater the focal depth. For focal depth 160 km the point of intersection is very close to the inflexion point, at 11°5. The time-curve, however, has still an almost straight section extending from about 5° to 11°5 and the slope of this section will be close to that at the inflexion point. For greater depth the straight section will become smaller and its slope will deviate slightly from that at the point of inflexion. It would become increasingly difficult to determine the slope from observations.

We remarked on the fact that when the velocity was very nearly constant in the upper mantle, no ray from a surface focus penetrating deeper than to about 120 km would be seen to emerge at the surface. We now find that in deep focus earthquakes we shall be able to observe the emergence of rays having their deepest points down to about 160 km, but not below that depth. Thus no direct information about the variation of the velocity between 160 km and 220 km depth is obtainable from observations.

We have taken the velocity to be constant in the upper mantle. It is not unlikely that instead it increases slightly. If so, the inflexion points will be at somewhat smaller distances. Taking the velocity to increase from 8.12 km/sec to 8.2 km/sec. at 220 kms depth the inflexion point for a focus at 100 kms depth will be at 7°2 epicentral distance instead of for constant velocity at 8°2 and the inflexion point for 220 km depth will be at 12°1 instead of at 14°1.

Since our velocity in the upper mantle is at first greater than the J. B. velocity and it increases less with depth, the straight part of the time-curves have smaller slopes than the J. B. curves for small depth of focus, but the slopes decrease less with depth and for 160 km focal depth the slopes are equal.

The pP curves calculated on our velocity assumptions also have two branches. The pP and PP curves have their common starting point at the distance reached by the ray leaving the focus horizontally and reflected at the surface of the earth. This point will be at a considerably greater epicentral distance than that of the J. B. tables because the rays are more straight. The pP curve at first goes backwards a little way, stays at a focal point and then goes forward (see Bullen, 1955 and note at end). When the P ray forming part of pP meets the 220 km boundary and is refracted the pP emerges at a much shorter epicentral distance than the « first » pP ray.

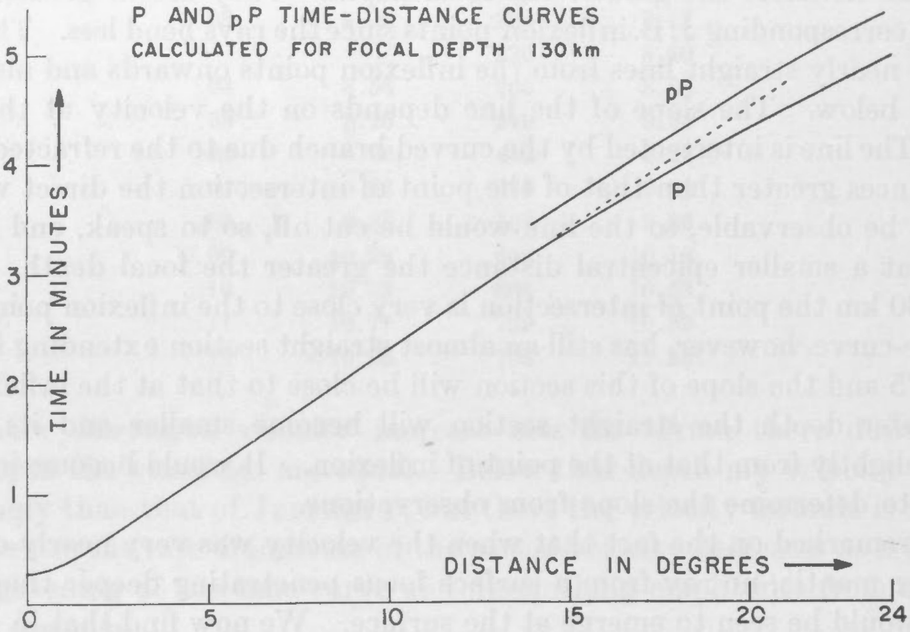

FIG. 1.

In figure 1 are seen the two branches of the P curve calculated for focal depth 130 km and the branch of the refracted pP. The two P branches intersect close to 13° epicentral distance so that is where the curve begins to bend. The inflexion point is at distance $10^{\circ}1$. The first point of the pP curve is at $30^{\circ}3$ epicentral distance, 63^{s} above P, and the focal point is close to this point at a slightly smaller distance. The second branch, the one plotted in the figure, has its lowest point at $14^{\circ}1$, 16^{s} above P. pP—P, however, increases rapidly with distance and at 24° is 25^{s} or 2^{s} smaller than the J. B. pP—P. The smallest distance at which PP appears is about 20° and there it is 10^{s} later than pP.

The J. B. pP and PP would have their common starting point at a distance of approximately 17° and about 16^{s} after P. There would be a focal point with large amplitudes close to this point. This does not seem to have been observed and, indeed, cannot be present at so small an epicentral distance when the velocity gradient in the upper mantle is small.

We shall now consider the Rumanian earthquakes.

The earthquake of 1929, Nov. 1, was used by H. JEFFREYS (1935) when he determined corrections to the original J. B. tables by means of observations of deep focus earthquakes. He corrected the I. S. S. epicentre and found the point 45°.88 N 26°.48 E. The depth he fixed at 142 km ± 8 km.

In the Seismicity of the Earth (GUTENBERG and RICHTER, 1954) 14 Rumanian earthquakes all from approximately the same focus are listed. They are listed in Table 3. The epicentres and depths are those given in the Seismicity of the Earth and in the International Seismological Summary. M is magnitude according to GUTENBERG and RICHTER. The numbers of P (and P′) and the greatest distances at which they were recorded are taken from the I. S. S. For the earthquakes N^os 6, 9 and 13 two distances are given, there being many observations out to the smaller distances and just a few at much greater distances.

P was well recorded by many stations in 8 of these earthquakes viz. in N^os 1, 3, 4, 7, 8, 10, 13, 14. When the transmission times as given in the I. S. S. were taken and corrected for differences in origin time, the times of individual stations of N^os 1,3, 7, 8, 10 and 14 were found to vary very little. Those of N° 4 differed systematically from the others and random errors were rather large in N° 13.

The stations selected were those recording the earthquake of 1948, May 29. Most of these stations also recorded the two large earthquakes of 1940, Oct. 22 and Nov. 10, and there was excellent agreement between the corrected transmission times, also at the greatest distances where the transmission times of Tinemaha at distance 91°.8 for the three shocks were practically the same and those of Mount Wilson (94°.1) also. There can be no doubt about these three shocks having the same focus. Comparison with the other shocks was not quite so effective because they were not recorded at all the same stations. Thus the most distant stations recording the 1934 shock were not in operation in 1940 and 1948. The 1929 shock was recorded by 19 stations at distances between 6° and 11° but only 5 of these operated in 1940, and in 1929 there were 21 observations between 11° and 15° while in 1940 there were 18 and about half of them were not the same. However, where comparison was possible agreement was so close that there can be no doubt about the 6 shocks mentioned and marked by a cross in the table all having the same focus.

Mean values of the observations of individual stations were formed for distances up to 25°. Mean deviations m_2 of the means were also determined and when these were smaller than or equal to 1^s the station and its mean value was retained for further work and entered in table 4. 7 of the stations had mean deviations $0^s.2$ or smaller ; that of Basel is 0^s, but it has been put in parenthesis because

TABLE 3

RUMANIAN EARTHQUAKES

No	Date	Hour	Epicentre G and R		I. S. S.		Depth G and R	I. S. S.	M	Nos. of P Reported	Greatest Distance °
1×	1929 Nov. 1	6	45.9 N	26.5 E	46.0 N	26.1 E	150 km	n	5 3/4	68	46.9
2	1934 Feb. 2	19	45 N	26 E	45.7 N	26.1 E	»	n	5 1/4	18	18.2
3×	» Mar. 29	20	45 3/4 N	26 1/2 E	45.8 N	26.5 N	»	n	6 1/4	75	69.3
4	1935 July 13	0	46 N	26 1/4 E	46.2 N	26.5 E	»	n	5 1/4	40	35.3
5	1938 July 13	20	45 3/4 N	26 3/4 E	45.7 N	26.8 E	»	.025 R	5 1/4	32	23.7
6	1939 Sept. 5	6	45 3/4 N	26 1/2 E	»	»	»	.010 R	5 1/4	25	23.7/94.3
7×	1940 June 24	9	45 3/4 N	26 3/4 E	»	»	»	n	5 1/2	39	29.9
8×	» Oct. 22	6	45 3/4 N	26 1/2 E	»	»	»	.010 R	6 1/2	79	94.7
9	» Nov. 8	12	45 1/2 N	26 E	»	»	»	»	5 1/2	15	16.2/93.
10×	» Nov. 10	1	45 3/4 N	26 1/2 E	»	»	»	»	7.4	139	156.4
11	» Nov. 11	6	46 N	26 3/4 E	»	»	»	»	5 1/2	25	33.9
12	» Nov. 19	20	46 N	26 1/2 E	»	»	»	»	5 1/4	13	16.6
13	1946 Nov. 3	18	45 3/4 N	26 1/2 E	»	»	»	»	5 1/2	41	32.2/91.2
14×	1948 May 29	4	46 N	26 3/4 E	»	»	»	.015 R	5 3/4	60	94.3

TABLE 4

6 RUMANIAN EARTHQUAKES

MEAN VALUES OF THE TRANSMISSION TIMES OF P
COMPARED WITH CALCULATED TIMES, DEPTH 130 KM.

STATION	DISTANCE	AZIMUTH	P OBS.		m_2	O-C
	°	°	*m*	*s*	*s*	*s*
	—	—	—	—	—	—
Bucharest	1.45	198	0	25.5	0.6	—1
Sofia	3.9	221	0	56.0	1.0	—1
Istanbul	5.1	160	1	11.5	1.0	—2
Budapest	5.5	291	1	17.4	.9	—1
Warsaw	7.4	332	1	45.3	.2	2
Trieste	9.0	275	2	5.6	.7	0
Prague	9.2	303	2	10.2	1.0	2
Rome	10.9	255	2	28.3	.2	—2
Potsdam	11.0	312	2	31.8	.2	0
Jena	11.2	303	2	35.2	.5	1
Chur	11.85	282	2	42.5	.4	1
Moscow	12.1	30	2	47.0	.9	0
Stuttgart	12.2	291	2	48.0	.4	0
Zürich	12.5	284	2	51.3	.2	—1
Basel	13.2	285	2	59.0	(.0)	—2
Hamburg	13.25	312	2	59.4	.5	2
Copenhagen..	13.3	323	3	0.5	.2	—2
Neuchatel	13.6	283	3	5.0	.6	—2
Ksara	13.9	146	3	11.0	.6	1
Pulkovo	14.2	7	3	11.0	.7	—3
Uppsala	15.05	342	3	22.5	.6	—2
De Bilt	15.4	304	3	30.0	.5	1
Uccle	15.6	297	3	33.0	.0	2
Helwan	16.35	166	3	39.8	.7	0
Paris.........	16.6	289	3	43.5	.5	1
Baku	17.8	100	4	1.3	.5	5
Kew	18.6	298	4	8.7	.8	3
Bergen	19.3	329	4	16.3	.5	3
Toledo	23.15	267	4	52.4	.4	—1
Sverdlovsk ...	23.7	50	5	0.5	1.0	
Granada......	24.1	261	5	2.8	.7	

there are only 3 observations. For 16 of the 29 stations the mean deviation did not exceed 0.5. One decimal has been retained in the means although it has no great certainty; it is in order not to introduce greater errors than necessary when differences are formed.

In the Seismicity of the Earth and in the I. S. S. the foci of our 6 earthquakes were not all the same. For a comparison of the travel times with those I calculated a small error in the epicentre would not be serious since most of the stations are in one azimuthal quadrant, the NW quadrant. Nevertheless I tried to adjust the epicentre and since I did not wish to do this by means of a travel time table, I had to use pairs of stations having approximately the same travel times. This restricts us to use only a small number of all the observations available and, for reasons appearing in the course of the work, other objections may be raised against applying the method. However, the result obtained seemed to be an improvement on earlier solutions.

The trial epicentre was that of the I. S. S. for the 1940 earthquakes. When the residuals of the largest shock, that of Nov. 10, were inspected it appeared that the epicentre had been taken too far east. When distances from JEFFREYS' epicentre (see p. 393) were calculated this was found to be too far west.

The pairs of stations chosen are shown in Table 5. az_1 and az_2 are the azimuths of the stations, Δ_1' the I. S. S. epicentral distances of the nearer stations of the pair. δt is the transmission time of the first station minus that of the second station as tabulated in table 4. However, the stations of the last two pairs of stations have been taken as representatives of westerly or northeasterly groups of stations and the transmission times have been so determined as to have the residual that is the mean residual of the group. The $\delta\Delta$ are the differences of distance that according to the travel times I calculated for depth 130 km correspond to the differences of transmission times δt, and the δ are found from :

$$\delta = \cos \Delta_1' - \cos (\Delta_1' + \delta\Delta).$$

Since for small $\delta\Delta$ this difference varies very little with small variations of Δ_1', we may put :

$$\cos \Delta_i - \cos \Delta_k = \delta_i,$$

where Δ_i and Δ_k are the distances from the final epicentre. We have :

$$\cos \Delta_i = \sum a_{i,j}\, a_{0,j}, \quad j = 1, 2, 3,$$

where the a_j stand for the co-ordinates usually called a, b, c and subscript i denotes a station, o the epicentre. We have therefore :

$$\sum a_{0,j}\, (a_{i,j} - a_{k,j}) = \delta_i.$$

Taking all the pairs of stations available we obtain a set of equations from which the co-ordinates $a_{0,j}$ of the epicentre can be obtained. The method of least squares, however, cannot be applied to the equations as they stand for we have $\Sigma a_{0,j}^2 = 1$. We therefore divide by $a_{0,3}$ and obtain :

$$\frac{a_{0.1}}{a_{0.3}}(a_{i.1} - a_{k.1}) + \frac{a_{0.2}}{a_{0.3}}(a_{i.2} - a_{k.2}) = \frac{\delta_i}{a_{0.3}} - (a_{i.3} - a_{k.3}).$$

TABLE 5

RUMANIAN EARTHQUAKES

PAIRS OF STATIONS

N°	STATIONS	az_1	az_2	Δ'_1	δt	$\delta\Delta$	δ	Δ_1		Δ_2		$d\Delta$	$d\Delta'$
		°	°	°	sec	min		°	m	°	m	m	°
1	Istanbul-Budapest......	160	291	4.9	—5.9	—27	0.00071	5	11	5	31	—20	—0.3
2	Rome-Potsdam	255	312	11.0	—3.5	—15	84	10	59	11	3	— 4	—0.2
3	Chur-Moscow	282	30	12.0	—4.5	—20	123	11	52	12	4	—12	—0.1
4	Ksara-Pulkovo.........	146	7	13.8	0.0	0	0	13	59	14	9	—10	—0.5
5	Helwan-Paris	289	166	16.2	—3.7	—20	164	16	25	16	38	—13	—0.6
6	Baku-Kew	100	298	17.7	—7.4	—39	351	17	51	18	37	—46	—1.1
7	Sverdlovsk-Granada	50	261	23.7	—2.3	—15	176	23	41	24	7	—26	—0.5
8	Georgetown-Zi-ka-wei ..	307	65	71.6	0.0	0	0	71	27	71	26	1	0.2
9	Tokyo-San Juan	50	285	78.9	0.0	0	0	78	57	78	54	3	—0.2

There we have the unknown $a_{0,3}$ on the right hand side of the equation, but since δ_i is small we may substitute $a_{,,3}$ of the trial epicentre for it. Then the equations can be solved for $\frac{a_{0,1}}{a_{0,3}}$ and $\frac{a_{0,2}}{a_{0,3}}$ by the method of least squares and from these quantities the $a_{0,j}$ and λ and φ of the epicentre can be obtained.

The epicentre found from our pairs of stations is 45°50′ N 26°37′ E. It is a little to the northwest of the I. S. S. epicentre taken for most of the shocks. Distances have been calculated from it to all the stations of the pairs ; they are the

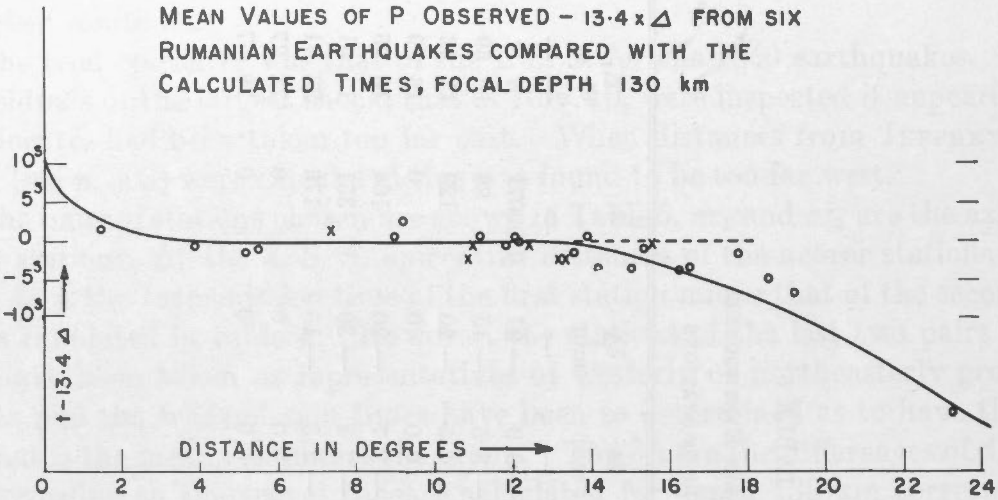

FIG. 2.

Δ_1 and Δ_2 of Table 5. The difference $d\Delta = \Delta_1 - \Delta_2$ should be close to $\delta\Delta$ if we had succeeded in adapting the epicentre to all our pairs of stations. We see, however, that there are considerable differences but that, on the other hand, there is a distinct improvement on the differences $d\Delta'$ of the distances from the trial epicentre.

The epicentre actually taken in the following has latitude 45°49′ N instead of 45°50′. An earlier determination that included an additional pair of stations gave this result, and since the difference is slight, it did not seem necessary to correct the findings based on this value.

The depth was taken to be 130 km. A few trials were made before this depth was fixed on as the one giving the best fit on the velocity assumptions adopted. Obviously it has no great certainty.

It was mentioned in the Rumanian National Report to the I. U. G. G. at Toronto 1957, that the epicentres as determined for the deep Rumanian earthquakes were found to centre on the point 45°.8 N 26°.6 E in good agreement with our present result. A publication by P. IONESCU (1956) was referred to.

The distances of Table 4 are from the epicentre 45°49′ N 26°37′ E, and the residuals are from my trial tables. The inflexion point of the calculated curve is at 10°.1, and the corresponding slope is 13.4 sec/degree. The curve with $(13.4\,\Delta + 5)$ sec subtracted from its ordinates is plotted in figure 2. It is very nearly a straight line from 6° to 13°. The points mark the observed transmission times with $(13.4\,\Delta + 4\,1/2)$ sec subtracted from them. On the whole the fit is not bad, but many of the deviations are larger than would be expected when the accuracy of the observations is considered. The crosses indicate observations, the mean deviations of which do not exceed 0^s.2. Five of them have residuals of $+2^s$ or -2^s and it is seen immediately from the figure that there is no way of fitting a time-curve closely to all of them.

Taking first the straight part of our time-curve we see that there is no marked systematic deviation from it. Yet when a straight line is fitted to the points at distances from 7°.4 to 12°.5 by the method of least squares, we find its slope to be (13.0 ± 0.2) sec/degree, so the slope is not very well determined. The line of slope 13.0 sec/degree passes through the Warsaw point at 7°.4, but it leaves the points at smaller distances so far below as not to be acceptable. The straight line of figure 2 actually seems to give about as good a fit as is obtainable, and we have to conclude that Warsaw and Rome (10°.9) have systematic errors, Warsaw being about 2^s late and Rome about 2^s early. The slope of our line corresponds to the velocity 8.12 km/sec at the depth of focus.

B. Gutenberg (1953), when determining the velocity at the depth of focus of three of the shocks here considered, viz. those of 1934, Mar. 29, 1940, Oct. 22 and Nov. 10, found the values 7.8, 8.0, 8.2 km/sec respectively. It is not surprising that the velocities found for individual earthquakes differ so much when no very accurate determination is obtainable from the mean values of the observations of our 6 shocks.

Beyond 13° we have at first a number of negative residuals from well determined travel times. This could be taken to indicate that the actual travel-time curve bends at a smaller epicentral distance and more strongly than our calculated curve, but at a slightly greater distance we have well determined positive residuals indicating a smaller bend, one of them being the Uccle residual, $+2^s$, at 15°.6. The travel-times of Uccle being exactly the same in all 6 shocks its residual is particularly well determined. Thus no travel-time curve can be fitted closely to the points beyond 13°, so here again we have systematic errors. The Swiss stations, Hamburg, Copenhagen, Pulkovo and Uppsala are early while Uccle and De Bilt are late. Baku is very late and probably has a systematic error. Kew and Bergen are also late, but this may partly be due to the time-curve needing a correction.

We had hoped to be able to draw conclusions as to the validity of our velocity assumption by comparing the well determined means of the Rumanian travel times with our calculated curve ; we do not, however, obtain the precise information we were looking for, but instead the somewhat distracting information that the travel-times do not always depend solely on the distance travelled ; they may differ significantly on different paths.

We adjusted the epicentre of the Rumanian shocks using pairs of stations the travel times of which were approximately equal. It is evident that errors are introduced in the epicentre determination when some of the stations have systematic errors and that such errors may affect the result rather seriously when the number of pairs of stations is small.

I shall not venture a guess as to where or at what depth the structures responsible for the differences of travel time are to be found. I shall mention, however, that in an earlier work (LEHMANN, 1949) I tried to determine possible systematic deviations in the travel times of a number of European stations. For this purpose I made use of some Japanese earthquakes very well observed at epicentral distances from about 70° to 80°. No systematic deviations were found for Zurich, Hamburg and Copenhagen (Basel had not been recording). De Bilt was found to be about 1/2 sec late. Uccle and Paris were very nearly normal with a small tendency for Uccle to be early and for Paris to be late. Since in distant earthquakes the rays pass steeply through the upper mantle and the crust, possible differences of structure in them could not make themselves strongly felt. They would be much more effective at distances small enough for the rays to be rather shallow, and have long paths in the upper mantle. In shallow European earthquakes we always come up against various sources of error as already explained, but in the study referred to a few European earthquakes were considered and it was attempted to eliminate the errors. The earthquakes were the two Yugoslavian earthquakes of March 7 and 8, 1931, and the Greek earthquake of 1932, Sept. 26. For these Uccle, De Bilt and Kew were left with positive residuals that no readjustment of the elements of the earthquakes could remove. Their azimuths were 314° and 319° in the Yugoslavian earthquakes, 315° and 320° in the Greek earthquake, but in the Rumanian earthquakes the azimuths were 297° and 307°.

Returning to the Rumanian earthquakes we find that the I. S. S. under the heading « Supp. » has several readings at short intervals after P, and these have been interpreted as either pP or PP. In the large 1940 shocks we have some "pP" readings 6^s to 30^s after P at quite short epicentral distances where the phase could not exist. From 16° onwards the readings become more frequent ; for distances up to 24° most of them have been interpreted as PP. They are from 5^s to 28^s after P. Obviously the readings give us but little information about the behaviour of pP and PP at the distances concerned. It might be possible to trace the two phases if a collective study of the records were made.

We have, as already mentioned, also deep earthquakes in the Tyrrhenian Sea.

The largest occurred on April 13, 1938. CALOI and GIORGI (1951) determined the epicentre 39°.3 N 15°.2 E and found the depth to be 285 km. The travel-time curve for this depth was calculated on my velocity assumptions and compared with observations of reliable stations in the north-westerly quadrant. On the whole the fit is very good, but there are negative residuals of -3^s at Neuchatel and Basel at about 10° epicentral distance, and the residual $+3^s$ at De Bilt at 14°.6 ; the Uccle residual is 0^s. A small swing precedes the large P onset in most records, and this may give rise to uncertainties of $1^s - 2^s$ in the readings, so it cannot be said to what extent the deviations noted are due to differences of structure on the paths.

The inflection point of the calculated curve is at 7°.6 and the slope at this point is 12.25 sec/degree. From 6° to 11° the curve is very nearly straight and has the mean slope 12.2 sec/degree. There are 12 good stations in the northwesterly quadrant in this range of distance ; their residuals are small and have no apparent systematic trend. However, when a straight line is fitted to the travel-time points by the method of least squares, its slope is found to be 12.1 ± 0.4 sec/degree. Thus the uncertainty is considerable and there may be a significant departure from our calculated slope and from the velocity 8.68 km/sec assumed at the depth of focus. The velocity determined by GUTENBERG (1953) is 8.2 km/sec corresponding to slope 12.95 sec/degree.

In conclusion it can be said that our solution for a velocity function is a possible one on the evidence in hand. The travel time curve for a surface focus is in good agreement with JEFFREYS' revised curve for Europe, and the travel times for deep shocks are not in obvious disagreement with the earthquake observations with which we have compared. The solution, however, is not unique. It is possible to find velocity functions differing in various ways from the one here taken, and yet giving travel times that are in good agreement with our data. If, e.g., we alter somewhat the depth of the discontinuity now taken to be at 220 km and at the same time the velocity increase at this discontinuity we may still obtain good agreement with the data. Also, the abrupt velocity increase may be replaced by a strong velocity increase in a thin layer, and we may have a low velocity layer. The calculation of travel-time curves from a given velocity function is a laborious process when an ordinary calculating machine is used. A modern automatic calculator, however, reduces the time required from many hours to a few minutes. It should, therefore, be possible to have travel-time curves calculated on a variety of velocity assumptions and to come to see more clearly what are the limitations placed on them by the data.

It is obvious from the start that the limitations are not so narrow as we could wish them to be. Much more precise data are required for solutions of any accuracy. A great many good observations actually are at hand that have never been reduced with the object of improving the time-distance curve. It is to be hoped that this will be done, but the precision required for a satisfactory determination of the velocity function is so great that it is not very likely to be obtainable in this way. Explosion work may help to improve the results, but it seems possible that increase of accuracy of observation will partly go to reveal variations of travel times on different paths and that mean travel times of very high precision are not obtainable. This is more likely to apply to a continent of so varied a structure as the Eurasiatic continent than e.g. to the Canadian Shield and the Eastern United States.

Our deductions as to the nature of the velocity function are based largely on amplitude observations, but the information obtained from these is rather vague. It would be extremely valuable to have careful studies made with a view to obtaining a clearer picture of the variation of amplitude with distance.

The intense study of surface waves carried out in later years also provides us with means of investigating the structure of the upper mantle. Dispersion curves have been constructed from observations on modern seismographs tuned to respond to very long waves, and the calculation of dispersion curves on given velocity assumptions, formerly a matter of months, can now be done in some hours. This new approach may prove to be of great value.

Note on the focal point of pP

On p. 392 it was mentioned that the time-distance curve of pP is at first retro-

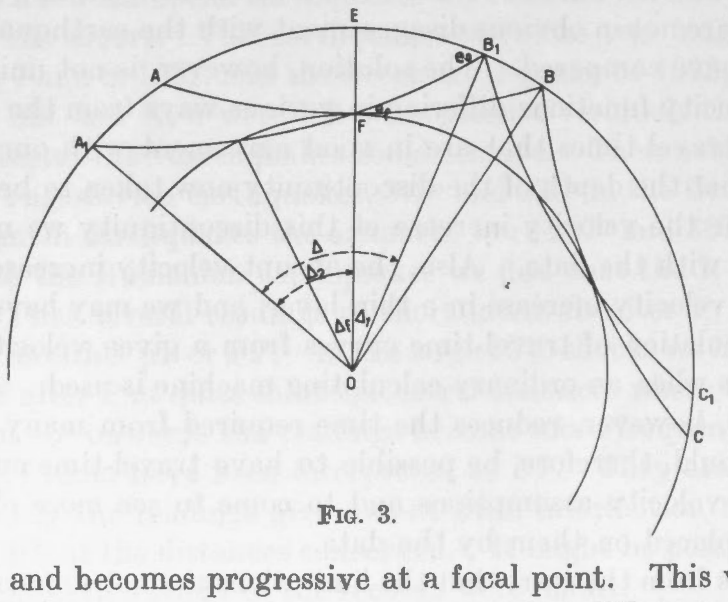

Fig. 3.

grade but turns and becomes progressive at a focal point. This was pointed out by Bullen (1955), but it may be shown in a somewhat different way.

In the figure the rays leaving the focus F horizontally emerge at A and B. The ray reflected at B emerges at C, FBC being the "first" pP and PP ray. All rays leaving the focus upwards are reflected as pP, whereas those leaving it downwards are reflected as PP. Taking the velocity variation to be « ordinary », with continuous variation of velocity and velocity gradient at the depths concerned, all PP rays will emerge outside C, but pP rays having small angles of emergence at the focus will emerge inside C. Let FB_1 have angle of emergence e_f at the focus and let the complementary ray emerge at A_1. Let the angular distances Δ be as indicated in the figure. B_1C_1 being the ray reflected at B_1, the epicentral distance of C_1 is :

$$EC_1 = 2\Delta_1 + \Delta_2. \tag{1}$$

Under the assumptions made Δ_1 and Δ_2 will vary continuously with e_f and so will their first derivatives. We may write :

$$\frac{dEC_1}{de_f} = 2\frac{d\Delta_1}{de_f} + \frac{d\Delta_2}{de_f}.$$

$\frac{d\Delta_1}{de_f}$ and $\frac{d\Delta_2}{de_f}$ have opposite signs but numerically converge towards the same value when $e_f \to 0$, and therefore $\frac{dEC_1}{de_f}$ has at first the same sign as $\frac{d\Delta_1}{de_f}$. Thus, when B_1 moves from B towards the epicentre, C_1 moves in the same direction, but it does not continue in this direction, for EC_1 is known to approach π when e_f approaches $\frac{\pi}{2}$. There is a minimum distance at which it stops and begins to move the other way. At the turning point we have :

$$\frac{dEC_1}{de_f} = 0.$$

and this is a focal point.

The time-distance curves of pP and PP have their common starting point at epicentral distance EC. The PP curve is progressive, but the first branch of pP is retrograde ; it stops at a focal point where the curve becomes progressive. The common point of the pP and PP curves is a point of inflection ; the constant of the corresponding ray is $\alpha = \frac{r_f}{v_f}$ which is a maximum value.

Taking the velocity function to be given by :

$$v = v_0\left(\frac{r}{r_0}\right)^{-k}, \tag{2}$$

we can find the minimum epicentral distance of pP.

Instead of (1) we can write :

(3) $$EC_1 = \frac{1}{2}(3\Delta - \Delta_f).$$

We have : [(4) p. 384] :

$$\cos\frac{k+1}{2}\Delta = \frac{\alpha}{u_0},$$

and therefore :

(4) $$\Delta = \frac{2}{k+1}e,$$

where e is the angle of emergence at the surface of a ray reaching distance Δ. By (3) and (4) we obtain :

(5) $$EC_1 = \frac{1}{k+1}(3e_0 - e_f),$$

where e_0 is the angle of emergence at B_1. For the pP ray emerging at minimum distance we have :

$$\frac{dEC_1}{de_f} = 0,$$

or :

$$\frac{de_0}{de_f} = \frac{1}{3}.$$

Taking α to be the constant of a ray and $u = \frac{r}{v}$ we have :

(6) $$\alpha = u_0 \cos e_0 = u_f \cos e_f,$$

and therefore

$$\frac{de_0}{de_f} = \frac{u_f \sin e_f}{u_0 \sin u_0} = \frac{\sqrt{u_f^2 - \alpha^2}}{\sqrt{u_0^2 - \alpha^2}} = \frac{1}{3}$$

from which we find :

(7) $$8\alpha^2 = 9u_f^2 - u_0^2,$$

determining the constant α of the pP ray emerging at minimum distance Δ_m. This is the result arrived at by BULLEN written in a different notation.

Using (6) and (7) and introducing into (5) we obtain :

$$(k+1)\,\Delta_m = 3\arccos\sqrt{\frac{9u_f^2 - u_0^2}{8u_0^2}} - \arccos\sqrt{\frac{9u_f^2 - u_0^2}{8u^2}}.$$

For the time of travel to distance Δ we have [(5) p. 384] :

$$t = u_0\frac{2}{k+1}\sin\frac{k+1}{2}\Delta,$$

and therefore :

$$(k+1)\,t_m = \sqrt{8(u_0^2 - u_f^2)}.$$

The constant α_f of the ray starting horizontally at F is u_f and we find :

$$8(\alpha_f^2 - \alpha^2) = u_0^2 - u_f^2,$$

u_f differs more from u_0, and therefore α differs more from α_f, the stronger the velocity increase with depth and the deeper the focus. For constant velocity and depth 130 km as taken in the preceding, u_f does not differ much from u_0 ; consequently the first branch of the pP curve is short. We found the "first" point of the pP curve to be at epicentral distance 30°.3 (p. 392) and the focal point is at distance 28°.8.

We have here spoken of pP as propagated in a uniform layer. If there is a discontinuity such as assumed in the preceding, the P part of pP will be refracted when it meets this discontinuity and pP will emerge at a smaller epicentral distance.

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