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TABLES OF EXTENDED DISTANCES FOR PP AND pP

BY

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ABSTRACT

This paper is the third in a series extending Byerly's method of determining the direction of faulting in an earthquake to deep focus earthquakes and permitting the use of secondary P phases. Here tables of extended distances are presented for the reflected rays of PP and pP, for earthquakes of all focal depths down to 0·12 R. The tables are consistent with earlier ones for P, PKP, and PcP, so that the several phases can be used in a single solution.

INTRODUCTION

In carrying out solutions for the direction of faulting in earthquakes, one is frequently handicapped by the scarcity of stations, or by the poor distribution of stations which are available. For this reason it becomes desirable to make as much use as possible of secondary arrivals because these phases, having left the focus at different angles from the first arriving phases, provide points on the projection at the same azimuth but usually at quite different extended distances. Tables for the secondary phases PcP and PKP_2 have already been published[1] and it is the purpose of this paper to present tables of extended distances for the phases PP and pP which involve reflections from the earth's surface in the manner indicated in Figure 2.

DERIVATION OF THE TABLES

These tables of extended distances for PP and pP have been obtained from the tables of extended distances already published[2] for P.

To understand the method of derivation refer to Figure 1, which is reproduced from Figure 1 of the earlier paper[2]. In the figure the ray AFF'B is supposed to be a continuous one. It is the path that would be followed by a P wave travelling between the points A and B. Under these circumstances it was shown that A and B would receive initial P impulses of the same sign, that is either both compressional or both dilatational, from an earthquake occuring at F. The extended distances of points such as A were thus shown in the tables as equal to the extended distances of the related points, such as B, with a negative sign indicating an opposite azimuth.

Pairs of related points, such as A and B, may be selected very simply from the tables by finding pairs of values of extended distances which differ only in sign. For example,

[1] J. H. Hodgson and J. F. J. Allen, "Tables of Extended Distances for PKP and PcP", *Publications of the Dominion Observatory, Vol. XVI, No. 10, 1954.*

[2] J. H. Hodgson and R. S. Storey, "Tables Extending Byerly's Fault Plane Techniques to Earthquakes of any Focal Depth", *Bull. Seism. Soc. Am.*, Vol. 43, 49–61, 1953.

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assuming a focal depth of 0·12 R, we note that the extended distance for 4°0 is −1·341. By interpolation, the extended distance for 76°1 is +1·341. Then epicentral distances of 4°0 and 76°1 define a pair of related points.

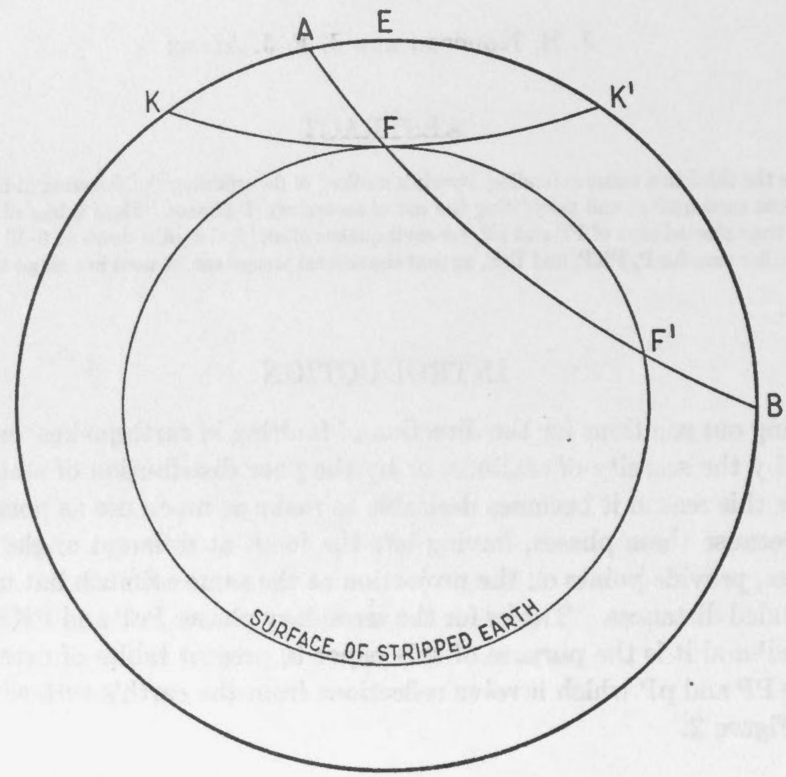

FIGURE 1.

Before leaving Figure 1 some mention should be made of the ray KFK′ drawn tangential to the stripped earth at the focus F. This ray is important in defining the phases pP and PP. By definition rays, such as FA, rising above it, give rise by reflection at the earth's surface to pP phases; rays, such as FB, falling below it, give rise by reflection to PP phases. The reflected phases generated by the ray KFK′ represent the limiting case where pP and PP are the same.

Now consider Figure 2. The ray AFB of this figure is analagous to the ray AFB of Figure 1, A and B being related points. Then the extended distances of A and B have the same absolute value but opposite signs. The ray FB not only gives rise to a P phase at B but also, by reflection, to a PP phase at C. Since extended distance is a function only of the angle at which the ray leaves the focus, these two phases must have the same extended distance. Similarly a P phase recorded at A and a pP phase recorded at D have the same extended distance; as we have seen it is the negative of the extended distance of a P phase at B. To determine one of these extended distances is to determine them all and it only remains to determine the relative epicentral distances of the points A, B, C and D.

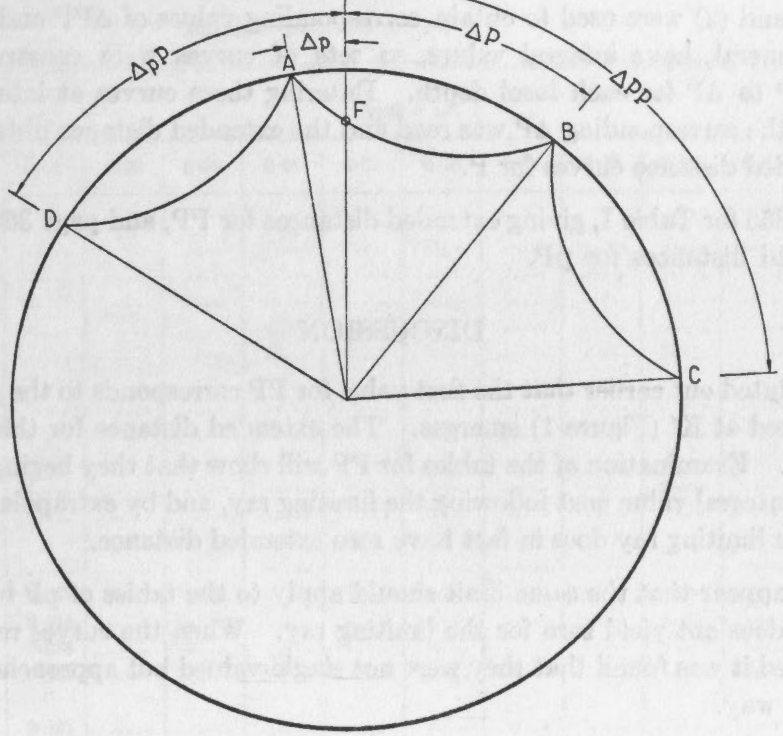

FIGURE 2.

Let these points have the epicentral distances shown in the figure. Then, by inspection, we may write immediately:

$$\Delta PP = 2\Delta P + \Delta p \ldots\ldots\ldots\ldots\ldots (1)$$
$$\Delta pP = \Delta P + 2\Delta p \ldots\ldots\ldots\ldots\ldots (2)$$

The application of these equations to the pair of related points already determined, will illustrate their use. For focal depth 0·12 R we had $\Delta P = 76^\circ\!.1$, $\Delta p = 4^\circ\!.0$, with a common extended distance of 1·341

$$\text{Then } \Delta PP = 152^\circ\!.2 + 4^\circ\!.0 = 156^\circ\!.2$$
$$\Delta pP = 76\cdot 1 + 8^\circ\!.0 = 84^\circ\!.1$$

Then, the extended distance for P at 76°.1 = + 1·341,
the extended distance for PP at 156°.2 = + 1·341,
the extended distance for P at 4°.0 = − 1·341,
and the extended distance for pP at 84°.1 = − 1·341.

The first step in derivation of the tables was to determine sets of related epicentral distances ΔP and Δp. This was accomplished in the manner outlined above except that the curves from which the tables[2] were derived were used instead of the tables themselves.

This reduced the work of interpolation. With the sets of related points determined, equations (1) and (2) were used to obtain corresponding values of ΔPP and ΔpP. These did not, in general, have integral values, so sets of curves were constructed relating ΔPP and ΔpP to ΔP for each focal depth. Entering these curves at integral values of ΔPP or ΔpP, the corresponding ΔP was read and the extended distance obtained from the original extended distance curves for P.

See page 355 for Table I, giving extended distances for PP, and page 360 for Table II, giving extended distances for pP.

DISCUSSION

It was pointed out earlier that the first value for PP corresponds to the point at which a phase reflected at K′ (Figure 1) emerges. The extended distance for this phase will of course be zero. Examination of the tables for PP will show that they begin, for each focal depth, at the integral value next following the limiting ray, and by extrapolation it may be shown that the limiting ray does in fact have zero extended distance.

It would appear that the same limit should apply to the tables of pP but in this case extrapolation does not yield zero for the limiting ray. When the curves relating ΔpP to ΔP were plotted it was found that they were not single-valued but approached the limit in a most erratic way.

This was at first ascribed to errors in the original tables[2] of extended distance for P. It was thought that the method used in derivation of those tables was less accurate than that developed later for use with PKP. To test this, the latter method was applied to the P data over the complete range of focal depth and over a considerable range of epicentral distance. It was found that the two methods gave results which never differed by more than 1 per cent. Apparently the multiple values in the ΔpP $-$ ΔP curve were real. It was concluded that they must indicate a complicated cusp on the pP curve.

We have investigated this matter in some detail and find that there are actually two cusps present in the pP curve. One of them is analagous to the 20° cusp on the P curve. The other occurs at the very beginning of the pP curve. It is due to the fact that the point of emergence of pP, for rays rising above the tangent ray but close to it, is at less epicentral distance than the point of emergence of the pP due to the tangent ray. The extent of the cusp, and the range of angle of the generating ray contributing to it, varies with focal depth but as the depth of focus increases the two cusps come together and interact in a most complicated fashion.

A full description of the phenomenon is reserved for a separate paper. It is sufficient for the present investigation to note that the failure of the pP extended distances to extrapolate smoothly to zero at the point of emergence of the tangent ray is reasonable.

The tables presented herewith are to be used in conjunction with the earlier ones[1, 2], the projections being compatible.

TABLE I

Extended Distances for PP

Δ°	Depth h =													
	Surface	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·11	0·12
0	0·000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
1	0·021	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
2	0·042	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
3	0·061	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4	0·077	0·004	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
5	0·093	0·031	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
6	0·108	0·052	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
7	0·122	0·077	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
8	0·135	0·096	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
9	0·148	0·113	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
10	0·160	0·130	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
11	0·173	0·148	0·007	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
12	0·186	0·165	0·044	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
13	0·201	0·181	0·076	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
14	0·215	0·197	0·100	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
15	0·231	0·213	0·123	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
16	0·246	0·230	0·145	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
17	0·262	0·245	0·162	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
18	0·278	0·261	0·181	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
19	0·294	0·278	0·198	0·034	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
20	0·309	0·294	0·215	0·081	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
21	0·325	0·310	0·234	0·122	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
22	0·341	0·327	0·250	0·149	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
23	0·357	0·343	0·267	0·178	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
24	0·374	0·360	0·287	0·201	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
25	0·389	0·375	0·303	0·223	0·091	.....	.....	.....	.....	.....	.....	.....	.....	.....
26	0·406	0·391	0·320	0·244	0·133	.....	.....	.....	0·031	.....	.....	.....	.....	.....
27	0·422	0·407	0·336	0·264	0·166	.....	.....	.....	0·083	.....	.....	.....	.....	.....
28	0·437	0·422	0·353	0·281	0·195	0·056	.....	.....	0·117	.....	.....	.....	.....	.....
29	0·453	0·438	0·373	0·300	0·222	0·108	.....	0·010	0·159	.....	.....	.....	.....	.....
30	0·470	0·453	0·390	0·320	0·244	0·144	.....	0·045	0·190	.....	.....	.....	.....	.....
31	0·487	0·470	0·409	0·336	0·264	0·180	0·019	0·328	0·220	.....	.....	.....	.....	.....
32	0·504	0·487	0·428	0·357	0·284	0·206	0·064	0·409	0·248	.....	.....	.....	.....	.....
33	0·524	0·506	0·449	0·379	0·311	0·236	0·115	0·453	0·277	0·060	.....	.....	.....	.....
34	0·544	0·526	0·476	0·401	0·341	0·290	0·502	0·485	0·305	0·115	.....	.....	.....	.....
35	0·567	0·548	0·503	0·434	0·385	0·503	0·545	0·511	0·332	0·163	.....	.....	.....	.....
36	0·593	0·575	0·539	0·481	0·515	0·613	0·579	0·537	0·359	0·208	.....	.....	.....	.....
37	0·626	0·610	0·585	0·558	0·679	0·649	0·602	0·560	0·385	0·240	0·090	.....	.....	.....
38	0·672	0·659	0·660	0·733	0·718	0·676	0·626	0·578	0·411	0·270	0·140	.....	.....	.....
39	0·742	0·756	0·754	0·789	0·747	0·700	0·645	0·598	0·435	0·303	0·183	.....	.....	.....

TABLE I *(Continued)*

Extended Distances for PP

Δ°	Depth h =													
	Surface	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·11	0·12
40	0·840	0·850	0·846	0·821	0·776	0·724	0·666	0·615	0·459	0·334	0·217	0·044	.....	.....
41	0·920	0·910	0·887	0·850	0·797	0·742	0·683	0·634	0·478	0·359	0·249	0·122	.....	.....
42	0·966	0·952	0·916	0·871	0·815	0·758	0·699	0·649	0·500	0·382	0·280	0·166	.....	.....
43	1·001	0·982	0·942	0·890	0·832	0·777	0·719	0·665	0·517	0·405	0·304	0·205	.....	.....
44	1·028	1·007	0·963	0·908	0·852	0·792	0·733	0·679	0·538	0·425	0·329	0·233	0·064	.....
45	1·050	1·030	0·981	0·925	0·868	0·805	0·707	0·696	0·554	0·445	0·349	0·255	0·121	.....
46	1·070	1·050	0·999	0·940	0·883	0·819	0·760	0·710	0·572	0·462	0·365	0·276	0·158	0·015
47	1·088	1·067	1·015	0·955	0·898	0·832	0·774	0·724	0·586	0·480	0·382	0·294	0·185	0·070
48	1·104	1·084	1·030	0·969	0·912	0·845	0·786	0·739	0·600	0·497	0·399	0·310	0·211	0·105
49	1·120	1·100	1·045	0·982	0·925	0·862	0·800	0·750	0·616	0·510	0·411	0·326	0·231	0·135
50	1·135	1·115	1·059	0·996	0·939	0·875	0·812	0·763	0·629	0·526	0·424	0·339	0·253	0·166
51	1·150	1·129	1·072	1·010	0·951	0·887	0·824	0·775	0·641	0·539	0·438	0·353	0·270	0·187
52	1·163	1·144	1·086	1·024	0·964	0·900	0·837	0·787	0·652	0·550	0·451	0·364	0·285	0·208
53	1·177	1·156	1·099	1·035	0·975	0·913	0·850	0·798	0·665	0·564	0·462	0·376	0·301	0·227
54	1·190	1·171	1·111	1·048	0·987	0·925	0·860	0·808	0·675	0·574	0·475	0·386	0·315	0·243
55	1·203	1·185	1·123	1·060	0·999	0·939	0·872	0·818	0·685	0·584	0·484	0·399	0·326	0·260
56	1·216	1·197	1·135	1·071	1·010	0·950	0·883	0·827	0·697	0·594	0·493	0·409	0·336	0·271
57	1·227	1·210	1·146	1·082	1·020	0·960	0·893	0·838	0·707	0·605	0·504	0·420	0·347	0·284
58	1·238	1·221	1·157	1·094	1·031	0·970	0·902	0·847	0·716	0·614	0·511	0·429	0·355	0·295
59	1·250	1·233	1·169	1·104	1·041	0·979	0·913	0·855	0·725	0·623	0·520	0·437	0·366	0·307
60	1·259	1·245	1·179	1·115	1·053	0·988	0·922	0·864	0·733	0·631	0·530	0·446	0·375	0·319
61	1·269	1·255	1·189	1·125	1·062	0·997	0·931	0·871	0·743	0·641	0·537	0·456	0·384	0·328
62	1·279	1·265	1·200	1·134	1·072	1·005	0·939	0·879	0·750	0·647	0·545	0·465	0·394	0·340
63	1·290	1·276	1·210	1·144	1·081	1·014	0·946	0·885	0·759	0·655	0·552	0·472	0·400	0·350
64	1·299	1·285	1·219	1·153	1·090	1·022	0·955	0·892	0·766	0·663	0·560	0·481	0·410	0·360
65	1·308	1·295	1·228	1·162	1·099	1·030	0·965	0·899	0·774	0·670	0·568	0·488	0·417	0·370
66	1·317	1·304	1·237	1·170	1·107	1·039	0·972	0·905	0·780	0·676	0·575	0·495	0·425	0·378
67	1·326	1·314	1·246	1·179	1·115	1·047	0·980	0·912	0·787	0·681	0·582	0·504	0·433	0·387
68	1·335	1·325	1·255	1·187	1·123	1·055	0·986	0·920	0·794	0·687	0·589	0·510	0·440	0·395
69	1·344	1·333	1·263	1·195	1·130	1·066	0·993	0·925	0·800	0·695	0·597	0·517	0·446	0·404
70	1·353	1·341	1·271	1·202	1·138	1·070	1·000	0·931	0·808	0·700	0·604	0·525	0·454	0·411
71	1·360	1·349	1·279	1·210	1·145	1·077	1·006	0·938	0·814	0·706	0·610	0·531	0·460	0·420
72	1·368	1·356	1·287	1·217	1·153	1·084	1·012	0·944	0·820	0·712	0·616	0·538	0·467	0·427
73	1·375	1·364	1·295	1·225	1·160	1·091	1·020	0·950	0·826	0·718	0·623	0·545	0·475	0·436
74	1·382	1·371	1·302	1·231	1·167	1·098	1·026	0·956	0·832	0·725	0·630	0·552	0·481	0·444
75	1·390	1·378	1·309	1·238	1·174	1·105	1·034	0·962	0·838	0·730	0·638	0·559	0·489	0·451
76	1·396	1·385	1·316	1·245	1·181	1·112	1·040	0·969	0·843	0·736	0·645	0·566	0·495	0·459
77	1·403	1·391	1·323	1·251	1·187	1·119	1·047	0·975	0·849	0·743	0·650	0·574	0·503	0·466
78	1·410	1·398	1·330	1·259	1·194	1·125	1·054	0·982	0·855	0·750	0·659	0·580	0·510	0·475
79	1·416	1·405	1·337	1·265	1·200	1·132	1·060	0·990	0·860	0·757	0·666	0·588	0·519	0·482

TABLE I *(Continued)*

Extended Distances for PP

Δ°	Depth *h* =													
	Surface	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·11	0·12
80	1·422	1·411	1·344	1·273	1·207	1·139	1·067	0·996	0·866	0·764	0·672	0·595	0·526	0·490
81	1·429	1·418	1·350	1·280	1·214	1·145	1·075	1·003	0·874	0·770	0·680	0·605	0·534	0·499
82	1·435	1·425	1·359	1·287	1·220	1·152	1·082	1·010	0·880	0·777	0·688	0·611	0·543	0·508
83	1·441	1·431	1·365	1·294	1·227	1·160	1·090	1·017	0·887	0·784	0·696	0·620	0·550	0·515
84	1·448	1·437	1·371	1·300	1·234	1·166	1·098	1·024	0·894	0·791	0·704	0·628	0·560	0·525
85	1·455	1·444	1·377	1·308	1·241	1·174	1·105	1·031	0·900	0·800	0·711	0·635	0·567	0·534
86	1·461	1·451	1·385	1·315	1·248	1·181	1·113	1·038	0·907	0·807	0·719	0·644	0·575	0·543
87	1·469	1·459	1·391	1·322	1·255	1·190	1·120	1·046	0·914	0·815	0·728	0·654	0·586	0·551
88	1·476	1·466	1·398	1·330	1·263	1·198	1·126	1·053	0·923	0·822	0·735	0·662	0·595	0·561
89	1·484	1·474	1·405	1·338	1·271	1·206	1·136	1·061	0·930	0·830	0·744	0·670	0·604	0·570
90	1·492	1·481	1·412	1·345	1·280	1·214	1·145	1·070	0·938	0·838	0·751	0·679	0·615	0·580
91	1·500	1·490	1·420	1·354	1·287	1·221	1·152	1·078	0·946	0·846	0·760	0·689	0·623	0·589
92	1·509	1·498	1·428	1·362	1·295	1·230	1·160	1·088	0·955	0·855	0·770	0·698	0·633	0·600
93	1·519	1·506	1·435	1·371	1·304	1·239	1·169	1·096	0·963	0·864	0·779	0·707	0·645	0·609
94	1·527	1·515	1·444	1·380	1·313	1·247	1·178	1·105	0·972	0·872	0·788	0·716	0·654	0·620
95	1·536	1·525	1·452	1·389	1·322	1·255	1·186	1·115	0·980	0·883	0·797	0·725	0·664	0·629
96	1·546	1·535	1·461	1·398	1·330	1·265	1·195	1·123	0·990	0·893	0·806	0·736	0·675	0·640
97	1·556	1·544	1·471	1·408	1·340	1·274	1·205	1·132	1·001	0·902	0·817	0·746	0·685	0·650
98	1·566	1·554	1·480	1·417	1·349	1·284	1·215	1·144	1·011	0·911	0·827	0·756	0·695	0·660
99	1·576	1·564	1·490	1·427	1·359	1·293	1·224	1·154	1·020	0·920	0·836	0·765	0·707	0·671
100	1·587	1·574	1·500	1·437	1·369	1·303	1·234	1·164	1·030	0·931	0·846	0·775	0·717	0·681
101	1·598	1·585	1·511	1·447	1·380	1·314	1·245	1·174	1·041	0·943	0·856	0·787	0·728	0·691
102	1·609	1·595	1·521	1·458	1·390	1·324	1·255	1·185	1·051	0·953	0·866	0·798	0·740	0·704
103	1·620	1·606	1·533	1·471	1·401	1·334	1·265	1·195	1·062	0·964	0·877	0·808	0·752	0·715
104	1·631	1·618	1·544	1·481	1·411	1·344	1·276	1·205	1·075	0·974	0·890	0·820	0·763	0·725
105	1·643	1·629	1·555	1·493	1·422	1·355	1·286	1·215	1·085	0·985	0·900	0·830	0·774	0·735
106	1·654	1·640	1·567	1·504	1·434	1·367	1·297	1·228	1·095	0·995	0·910	0·841	0·784	0·748
107	1·666	1·653	1·579	1·515	1·445	1·379	1·308	1·239	1·106	1·005	0·920	0·854	0·795	0·758
108	1·679	1·665	1·590	1·526	1·455	1·390	1·321	1·250	1·118	1·019	0·931	0·864	0·806	0·768
109	1·690	1·677	1·602	1·539	1·467	1·401	1·332	1·260	1·129	1·030	0·941	0·875	0·816	0·780
110	1·704	1·689	1·615	1·550	1·479	1·413	1·343	1·272	1·140	1·040	0·952	0·885	0·828	0·790
111	1·715	1·702	1·627	1·562	1·490	1·425	1·355	1·284	1·150	1·051	0·965	0·898	0·840	0·800
112	1·727	1·714	1·640	1·574	1·502	1·436	1·366	1·295	1·161	1·063	0·976	0·909	0·850	0·814
113	1·740	1·727	1·652	1·585	1·514	1·448	1·377	1·305	1·175	1·074	0·986	0·919	0·862	0·824
114	1·754	1·740	1·665	1·597	1·526	1·460	1·389	1·317	1·185	1·085	0·996	0·930	0·872	0·834
115	1·767	1·752	1·678	1·610	1·538	1·471	1·400	1·328	1·197	1·098	1·007	0·942	0·885	0·846
116	1·780	1·766	1·690	1·622	1·550	1·483	1·411	1·340	1·208	1·110	1·020	0·953	0·895	0·856
117	1·794	1·780	1·703	1·634	1·562	1·495	1·423	1·350	1·219	1·120	1·030	0·964	0·905	0·867
118	1·808	1·793	1·716	1·646	1·575	1·507	1·435	1·362	1·230	1·131	1·041	0·974	0·917	0·880
119	1·823	1·807	1·730	1·659	1·586	1·519	1·446	1·375	1·241	1·143	1·051	0·985	0·930	0·890

TABLE I *(Continued)*

Extended Distances for PP

Δ°	Depth *h* =													
	Surface	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·11	0·12
120	1·837	1·821	1·743	1·671	1·599	1·530	1·459	1·386	1·255	1·155	1·063	0·997	0·940	0·900
121	1·850	1·835	1·756	1·685	1·611	1·542	1·470	1·400	1·266	1·165	1·075	1·008	0·951	0·911
122	1·864	1·849	1·770	1·698	1·625	1·555	1·482	1·410	1·278	1·176	1·085	1·018	0·963	0·922
123	1·879	1·863	1·786	1·711	1·639	1·566	1·494	1·424	1·290	1·187	1·097	1·029	0·974	0·934
124	1·892	1·877	1·800	1·725	1·650	1·579	1·506	1·435	1·300	1·200	1·108	1·040	0·984	0·945
125	1·906	1·892	1·814	1·738	1·663	1·593	1·517	1·447	1·312	1·211	1·119	1·052	0·994	0·956
126	1·922	1·907	1·828	1·750	1·675	1·605	1·530	1·459	1·325	1·222	1·130	1·063	1·004	0·968
127	1·936	1·921	1·841	1·764	1·689	1·617	1·540	1·471	1·335	1·234	1·140	1·073	1·015	0·980
128	1·951	1·936	1·856	1·778	1·701	1·630	1·553	1·484	1·348	1·245	1·155	1·084	1·025	0·990
129	1·965	1·951	1·870	1·792	1·714	1·643	1·567	1·495	1·361	1·255	1·166	1·095	1·036	1·001
130	1·980	1·965	1·885	1·806	1·727	1·655	1·580	1·510	1·373	1·267	1·177	1·105	1·046	1·012
131	1·995	1·981	1·900	1·819	1·740	1·668	1·592	1·523	1·385	1·279	1·189	1·116	1·057	1·023
132	2·010	1·996	1·914	1·834	1·754	1·680	1·605	1·535	1·396	1·290	1·200	1·130	1·069	1·034
133	2·025	2·010	1·929	1·848	1·767	1·694	1·617	1·547	1·408	1·303	1·214	1·140	1·080	1·047
134	2·039	2·025	1·943	1·862	1·780	1·707	1·630	1·560	1·420	1·315	1·226	1·152	1·090	1·058
135	2·055	2·040	1·958	1·876	1·794	1·720	1·644	1·573	1·432	1·326	1·238	1·164	1·101	1·069
136	2·069	2·055	1·974	1·890	1·808	1·734	1·656	1·586	1·445	1·337	1·250	1·175	1·112	1·080
137	2·085	2·070	1·987	1·905	1·821	1·746	1·670	1·600	1·457	1·349	1·261	1·187	1·124	1·094
138	2·098	2·084	2·002	1·919	1·835	1·760	1·682	1·613	1·471	1·360	1·275	1·199	1·135	1·105
139	2·113	2·099	2·018	1·933	1·850	1·774	1·695	1·625	1·485	1·372	1·288	1·213	1·148	1·116
140	2·128	2·114	2·032	1·948	1·864	1·787	1·712	1·639	1·497	1·386	1·300	1·225	1·159	1·127
141	2·143	2·129	2·047	1·962	1·879	1·800	1·725	1·651	1·510	1·399	1·312	1·237	1·170	1·140
142	2·159	2·144	2·062	1·977	1·894	1·815	1·740	1·665	1·523	1·411	1·325	1·249	1·182	1·153
143	2·174	2·162	2·078	1·992	1·912	1·829	1·754	1·679	1·539	1·424	1·337	1·261	1·194	1·165
144	2·189	2·178	2·094	2·010	1·927	1·844	1·769	1·693	1·551	1·436	1·350	1·276	1·208	1·177
145	2·205	2·194	2·110	2·025	1·943	1·859	1·783	1·706	1·565	1·450	1·362	1·290	1·220	1·190
146	2·221	2·210	2·125	2·041	1·959	1·876	1·798	1·720	1·579	1·465	1·375	1·302	1·233	1·204
147	2·239	2·226	2·142	2·056	1·975	1·891	1·812	1·738	1·592	1·480	1·388	1·315	1·245	1·215
148	2·255	2·243	2·159	2·073	1·991	1·907	1·828	1·752	1·606	1·493	1·401	1·327	1·259	1·229
149	2·272	2·260	2·175	2·089	2·007	1·924	1·846	1·768	1·620	1·508	1·414	1·343	1·271	1·243
150	2·290	2·277	2·193	2·105	2·025	1·940	1·862	1·783	1·635	1·521	1·427	1·356	1·284	1·255
151	2·308	2·295	2·210	2·121	2·041	1·955	1·879	1·798	1·649	1·535	1·440	1·370	1·300	1·269
152	2·325	2·313	2·227	2·139	2·059	1·972	1·895	1·813	1·664	1·550	1·454	1·383	1·314	1·282
153	2·344	2·332	2·245	2·155	2·076	1·990	1·910	1·829	1·678	1·565	1·471	1·396	1·326	1·295
154	2·362	2·351	2·262	2·173	2·093	2·006	1·928	1·845	1·695	1·582	1·485	1·410	1·340	1·311
155	2·376	2·370	2·280	2·191	2·111	2·025	1·945	1·861	1·710	1·597	1·498	1·425	1·355	1·324
156	2·402	2·390	2·299	2·210	2·129	2·042	1·961	1·877	1·725	1·613	1·512	1·438	1·371	1·338
157	2·422	2·410	2·316	2·228	2·146	2·064	1·979	1·894	1·740	1·628	1·526	1·452	1·385	1·351
158	2·442	2·430	2·335	2·245	2·164	2·081	1·996	1·910	1·755	1·644	1·540	1·469	1·400	1·365
159	2·464	2·450	2·355	2·265	2·183	2·099	2·014	1·928	1·771	1·660	1·555	1·484	1·415	1·380

TABLE I *(Concluded)*

Extended Distances for PP

Δ°	Depth *h* =													
	Surface	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·11	0·12
160	2·484	2·472	2·374	2·284	2·200	2·117	2·031	1·948	1·786	1·675	1·572	1·498	1·430	1·395
161	2·505	2·493	2·394	2·303	2·219	2·136	2·049	1·965	1·802	1·690	1·586	1·512	1·445	1·408
162	2·527	2·515	2·413	2·322	2·237	2·155	2·066	1·983	1·818	1·709	1·601	1·525	1·460	1·421
163	2·549	2·536	2·433	2·345	2·256	2·173	2·084	2·000	1·836	1·724	1·615	1·540	1·477	1·435
164	2·570	2·557	2·453	2·365	2·275	2·191	2·102	2·018	1·852	1·740	1·630	1·555	1·493	1·449
165	2·592	2·579	2·474	2·388	2·293	2·210	2·120	2·036	1·868	1·755	1·645	1·569	1·506	1·462
166	2·614	2·600	2·494	2·403	2·311	2·229	2·138	2·055	1·884	1·771	1·660	1·584	1·521	1·480
167	2·634	2·620	2·514	2·422	2·330	2·247	2·155	2·073	1·900	1·787	1·674	1·600	1·535	1·492
168	2·655	2·641	2·535	2·441	2·348	2·265	2·175	2·091	1·916	1·802	1·689	1·614	1·548	1·505
169	2·675	2·662	2·555	2·460	2·366	2·283	2·192	2·109	1·932	1·816	1·705	1·627	1·560	1·519
170	2·695	2·682	2·575	2·480	2·384	2·300	2·210	2·125	1·949	1·830	1·720	1·640	1·573	1·533
171	2·715	2·701	2·594	2·498	2·401	2·318	2·225	2·145	1·965	1·845	1·735	1·654	1·585	1·545
172	2·734	2·721	2·615	2·516	2·419	2·334	2·242	2·160	1·981	1·859	1·749	1·666	1·597	1·557
173	2·753	2·740	2·634	2·534	2·435	2·351	2·258	2·175	1·996	1·875	1·763	1·679	1·609	1·569
174	2·771	2·759	2·650	2·551	2·455	2·366	2·276	2·189	2·010	1·888	1·776	1·691	1·622	1·580
175	2·789	2·776	2·667	2·569	2·471	2·382	2·290	2·202	2·024	1·900	1·790	1·703	1·632	1·590
176	2·805	2·793	2·684	2·585	2·486	2·396	2·305	2·215	2·036	1·913	1·803	1·714	1·642	1·600
177	2·820	2·809	2·699	2·600	2·501	2·410	2·318	2·228	2·050	1·924	1·815	1·725	1·651	1·610
178	2·835	2·824	2·714	2·616	2·515	2·424	2·332	2·240	2·061	1·934	1·829	1·735	1·660	1·619
179	2·850	2·839	2·729	2·630	2·529	2·436	2·344	2·250	2·072	1·944	1·840	1·745	1·670	1·626
180	2·865	2·854	2·742	2·645	2·542	2·449	2·355	2·261	2·083	1·954	1·850	1·756	1·678	1·634

TABLE II

Extended Distances for pP

Δ°	Depth h = 0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·11	0·12
0	....	....	....	....	....	....	....	....	....	....	....	....	....
1	....	....	....	....	....	....	....	....	....	....	....	....	....
2	....	....	....	....	....	....	....	....	....	....	....	....	....
3	....	....	....	....	....	....	....	....	....	....	....	....	....
4	−0·031	....	....	....	....	....	....	....	....	....	....	....	....
5	−0·081	....	....	....	....	....	....	....	....	....	....	....	....
6	−0·124	....	....	....	....	....	....	....	....	....	....	....	....
7	−0·155	....	....	....	....	....	....	....	....	....	....	....	....
8	−0·191	....	....	....	....	....	....	....	....	....	....	....	....
9	−0·226	....	....	....	....	....	....	....	....	....	....	....	....
10	−0·258	....	....	....	....	....	....	....	....	....	....	....	....
11	−0·290	−0·119	....	....	....	....	....	....	....	....	....	....	....
12	−0·323	−0·175	....	....	....	....	....	....	....	....	....	....	....
13	−0·359	−0·225	....	....	....	....	....	....	....	....	....	....	....
14	−0·391	−0·270	....	....	....	....	....	....	....	....	....	....	....
15	−0·422	−0·310	....	....	....	....	....	....	....	....	....	....	....
16	−0·453	−0·346	....	....	....	....	....	....	....	....	....	....	....
17	−0·491	−0·384	....	....	....	....	....	....	....	....	....	....	....
18	−0·530	−0·425	....	....	....	....	....	....	....	....	....	....	....
19	−0·581	−0·476	−0·306	....	....	....	....	....	....	....	....	....	....
20	−0·687	−0·555	−0·367	....	....	....	....	....	....	....	....	....	....
21	−0·876	−0·715	−0·455	....	....	....	....	....	....	....	....	....	....
22	−0·970	−0·873	−0·821	....	....	....	....	....	....	....	....	....	....
23	−1·022	−0·937	−0·867	....	....	....	....	....	....	....	....	....	....
24	−1·060	−0·981	−0·905	....	....	....	....	....	....	....	....	....	....
25	−1·094	−1·015	−0·940	−0·853	....	....	....	....	....	....	....	....	....
26	−1·124	−1·045	−0·971	−0·886	....	....	....	−0·067	....	....	....	....	....
27	−1·152	−1·072	−1·000	−0·917	....	....	....	−0·277	....	....	....	....	....
28	−1·180	−1·099	−1·026	−0·941	−0·851	....	....	−0·411	....	....	....	....	....
29	−1·204	−1·125	−1·050	−0·971	−0·882	....	−0·710	−0·489	....	....	....	....	....
30	−1·228	−1·148	−1·074	−0·996	−0·910	....	−0·744	−0·548	....	....	....	....	....
31	−1·249	−1·173	−1·095	−1·018	−0·936	−0·851	−0·775	−0·592	....	....	....	....	....
32	−1·270	−1·194	−1·117	−1·041	−0·960	−0·879	−0·800	−0·626	....	....	....	....	....
33	−1·289	−1·213	−1·136	−1·061	−0·979	−0·900	−0·822	−0·657	−0·500	....	....	....	....
34	−1·308	−1·232	−1·154	−1·079	−0·999	−0·920	−0·841	−0·682	−0·539	....	....	....	....
35	−1·327	−1·249	−1·172	−1·097	−1·017	−0·937	−0·862	−0·705	−0·569	....	....	....	....
36	−1·344	−1·267	−1·190	−1·114	−1·033	−0·955	−0·877	−0·726	−0·596	....	....	....	....
37	−1·360	−1·282	−1·205	−1·129	−1·050	−0·972	−0·892	−0·744	−0·618	−0·475	....	....	....
38	−1·375	−1·298	−1·221	−1·145	−1·066	−0·986	−0·906	−0·761	−0·637	−0·498	....	....	....
39	−1·389	−1·312	−1·235	−1·160	−1·081	−1·000	−0·919	−0·777	−0·656	−0·520	....	....	....

TABLE II *(Continued)*

Extended Distances for pP

Δ°	Depth h =												
	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·11	0·12
40	−1·402	−1·326	−1·249	−1·174	−1·095	−1·015	−0·934	−0·792	−0·671	−0·540	−0·412	....	....
41	−1·415	−1·341	−1·263	−1·189	−1·110	−1·028	−0·946	−0·806	−0·685	−0·558	−0·437	....	....
42	−1·430	−1·354	−1·278	−1·202	−1·124	−1·037	−0·960	−0·820	−0·700	−0·576	−0·460	....	....
43	−1·443	−1·367	−1·291	−1·215	−1·139	−1·058	−0·974	−0·833	−0·711	−0·595	−0·483	....	....
44	−1·457	−1·381	−1·306	−1·229	−1·152	−1·071	−0·987	−0·845	−0·725	−0·610	−0·501	−0·390	....
45	−1·472	−1·394	−1·321	−1·242	−1·166	−1·087	−1·001	−0·859	−0·738	−0·625	−0·520	−0·414	....
46	−1·488	−1·410	−1·336	−1·257	−1·183	−1·102	−1·017	−0·871	−0·751	−0·640	−0·535	−0·434	−0·312
47	−1·505	−1·424	−1·352	−1·273	−1·198	−1·119	−1·032	−0·886	−0·766	−0·656	−0·552	−0·451	−0·351
48	−1·523	−1·441	−1·369	−1·290	−1·215	−1·135	−1·049	−0·900	−0·780	−0·671	−0·568	−0·469	−0·383
49	−1·542	−1·458	−1·387	−1·307	−1·231	−1·152	−1·066	−0·914	−0·796	−0·690	−0·585	−0·485	−0·410
50	−1·562	−1·477	−1·406	−1·325	−1·250	−1·171	−1·085	−0·931	−0·813	−0·707	−0·605	−0·503	−0·432
51	−1·582	−1·497	−1·425	−1·345	−1·268	−1·188	−1·103	−0·948	−0·830	−0·724	−0·623	−0·522	−0·454
52	−1·605	−1·517	−1·445	−1·364	−1·289	−1·209	−1·121	−0·967	−0·850	−0·742	−0·642	−0·541	−0·475
53	−1·627	−1·539	−1·466	−1·385	−1·309	−1·230	−1·142	−0·986	−0·869	−0·760	−0·662	−0·562	−0·495
54	−1·650	−1·562	−1·489	−1·405	−1·331	−1·250	−1·164	−1·007	−0·890	−0·780	−0·682	−0·585	−0·517
55	−1·675	−1·586	−1·510	−1·429	−1·352	−1·272	−1·186	−1·029	−0·911	−0·802	−0·705	−0·607	−0·540
56	−1·699	−1·609	−1·536	−1·451	−1·376	−1·293	−1·209	−1·059	−0·935	−0·825	−0·727	−0·630	−0·565
57	−1·727	−1·634	−1·559	−1·476	−1·399	−1·316	−1·232	−1·076	−0·959	−0·846	−0·752	−0·655	−0·590
58	−1·753	−1·660	−1·583	−1·500	−1·422	−1·339	−1·254	−1·100	−0·980	−0·869	−0·775	−0·683	−0·615
59	−1·780	−1·685	−1·607	−1·524	−1·445	−1·364	−1·279	−1·124	−1·004	−0·894	−0·800	−0·709	−0·645
60	−1·807	−1·708	−1·632	−1·550	−1·471	−1·389	−1·304	−1·146	−1·027	−0·918	−0·825	−0·736	−0·670
61	−1·835	−1·740	−1·657	−1·574	−1·495	−1·411	−1·328	−1·172	−1·054	−0·941	−0·850	−0·765	−0·697
62	−1·864	−1·767	−1·685	−1·601	−1·521	−1·437	−1·354	−1·197	−1·079	−0·965	−0·876	−0·792	−0·725
63	−1·895	−1·797	−1·711	−1·626	−1·545	−1·463	−1·378	−1·224	−1·103	−0·990	−0·902	−0·819	−0·752
64	−1·925	−1·825	−1·738	−1·650	−1·571	−1·486	−1·405	−1·249	−1·127	−1·014	−0·927	−0·847	−0·780
65	−1·955	−1·858	−1·764	−1·678	−1·598	−1·510	−1·428	−1·274	−1·152	−1·039	−0·953	−0·875	−0·807
66	−1·984	−1·882	−1·792	−1·704	−1·623	−1·536	−1·454	−1·296	−1·176	−1·063	−0·978	−0·900	−0·834
67	−2·014	−1·911	−1·819	−1·733	−1·648	−1·560	−1·481	−1·321	−1·200	−1·089	−1·004	−0·925	−1·861
68	−2·043	−1·940	−1·850	−1·759	−1·676	−1·587	−1·505	−1·348	−1·225	−1·112	−1·029	−0·951	−0·888
69	−2·072	−1·970	−1·878	−1·786	−1·705	−1·612	−1·530	−1·373	−1·249	−1·139	−1·054	−0·975	−0·915
70	−2·102	−2·000	−1·907	−1·816	−1·731	−1·638	−1·557	−1·399	−1·274	−1·164	−1·077	−1·002	−0·940
71	−2·132	−2·030	−1·939	−1·844	−1·760	−1·667	−1·586	−1·425	−1·299	−1·191	−1·103	−1·026	−0·969
72	−2·162	−2·059	−1·969	−1·876	−1·790	−1·696	−1·613	−1·451	−1·326	−1·216	−1·130	−1·049	−0·995
73	−2·194	−2·093	−1·998	−1·906	−1·818	−1·723	−1·641	−1·479	−1·349	−1·243	−1·155	−1·074	−1·023
74	−2·230	−2·126	−2·031	−1·940	−1·846	−1·754	−1·671	−1·508	−1·377	−1·271	−1·180	−1·100	−1·050
75	−2·263	−2·159	−2·063	−1·975	−1·876	−1·783	−1·698	−1·534	−1·404	−1·298	−1·205	−1·124	−1·075
76	−2·299	−2·196	−2·098	−2·007	−1·910	−1·815	−1·726	−1·562	−1·431	−1·325	−1·232	−1·150	−1·102
77	−2·336	−2·230	−2·131	−2·041	−1·943	−1·846	−1·755	−1·592	−1·460	−1·352	−1·261	−1·175	−1·132
78	−2·375	−2·273	−2·166	−2·079	−1·979	−1·881	−1·789	−1·620	−1·490	−1·380	−1·289	−1·204	−1·160
79	−2·414	−2·305	−2·206	−2·114	−2·014	−1·918	−1·823	−1·651	−1·522	−1·411	−1·317	−1·233	−1·186

TABLE II *(Concluded)*

Extended Distances for pP

Δ°	Depth h =												
	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·11	0·12
80	−2·455	−2·347	−2·242	−2·153	−2·049	−1·951	−1·854	−1·684	−1·552	−1·440	−1·349	−1·263	−1·218
81	−2·497	−2·385	−2·284	−2·190	−2·089	−1·990	−1·890	−1·713	−1·585	−1·471	−1·380	−1·292	−1·245
82	−2·545	−2·425	−2·322	−2·226	−2·125	−2·028	−1·927	−1·749	−1·619	−1·500	−1·410	−1·321	−1·276
83	−2·587	−2·465	−2·360	−2·263	−2·165	−2·066	−1·962	−1·780	−1·650	−1·532	−1·440	−1·355	−1·305
84	−2·630	−2·510	−2·403	−2·300	−2·202	−2·102	−2·000	−1·814	−1·687	−1·564	−1·475	−1·386	−1·338
85	−2·670	−2·551	−2·441	−2·336	−2·239	−2·141	−2·035	−1·849	−1·720	−1·595	−1·506	−1·420	−1·370
86	−2·709	−2·589	−2·480	−2·373	−2·279	−2·177	−2·077	−1·881	−1·755	−1·627	−1·538	−1·454	−1·400
87	−2·747	−2·626	−2·516	−2·408	−2·318	−2·215	−2·115	−1·915	−1·790	−1·656	−1·570	−1·487	−1·430
88	−2·783	−2·664	−2·555	−2·445	−2·351	−2·251	−2·151	−1·951	−1·819	−1·689	−1·600	−1·518	−1·462
89	−2·815	−2·695	−2·588	−2·477	−2·382	−2·282	−2·180	−1·987	−1·850	−1·720	−1·630	−1·548	−1·492
90	−2·845	−2·725	−2·619	−2·507	−2·413	−2·313	−2·208	−2·018	−1·880	−1·751	−1·659	−1·575	−1·521
91	−2·873	−2·753	−2·647	−2·536	−2·439	−2·341	−2·235	−2·044	−1·908	−1·782	−1·684	−1·602	−1·548
92	−2·898	−2·778	−2·673	−2·561	−2·464	−2·364	−2·259	−2·070	−1·932	−1·808	−1·710	−1·626	−1·573
93	−2·920	−2·802	−2·696	−2·583	−2·484	−2·383	−2·279	−2·094	−1·951	−1·834	−1·734	−1·645	−1·595
94	−2·940	−2·823	−2·718	−2·603	−2·501	−2·400	−2·297	−2·114	−1·972	−1·857	−1·753	−1·666	−1·614
95	−2·960	−2·842	−2·735	−2·620	−2·517	−2·415	−2·313	−2·131	−1·990	−1·875	−1·770	−1·684	−1·631
96	−2·975	−2·858	−2·750	−2·635	−2·530	−2·428	−2·329	−2·147	−2·005	−1·891	−1·787	−1·700	−1·645
97	−2·989	−2·874	−2·762	−2·650	−2·541	−2·439	−2·341	−2·160	−2·019	−1·904	−1·800	−1·713	−1·657
98	−3·000	−2·886	−2·774	−2·662	−2·550	−2·449	−2·352	−2·170	−2·031	−1·915	−1·813	−1·725	−1·668
99	−3·010	−2·897	−2·783	−2·673	−2·559	−2·507	−2·361	−2·180	−2·041	−1·925	−1·823	1·737	−1·680
100	−3·017	−2·906	−2·792	−2·682	−2·565	−2·515	−2·370	−2·189	−2·050	−1·933	−1·832	−1·747	−1·690
101	−3·025	−2·913	−2·798	−2·690	−2·571	−2·520	−2·376	−2·196	−2·059	−1·940	−1·840	−1·755	−1·697
102	−3·031	−2·920	−2·805	−2·696	−2·576	−2·525	−2·381	−2·203	−2·065	−1·947	−1·846	−1·763	−1·705
103	−3·036	−2·925	−2·810	−2·701	−2·580	−2·529	−2·385	−2·209	−2·070	−1·953	−1·852	−1·770	−1·710
104	−3·041	−2·929	−2·816	−2·706	−2·583	−2·532	−2·389	−2·215	−2·075	−1·959	−1·858	−1·776	−1·716
105	−3·045	....	....	−2·712	−2·585	−2·535	−2·391	−2·219	−2·080	−1·963	−1·863	−1·783	−1·722
106	....	....	....	....	....	....	....	−2·222	−2·083	−1·967	−1·867	−1·789	−1·728
107	....	....	....	....	....	....	....	....	....	....	−1·872	−1·794	−1·733
108	....	....	....	....	....	....	....	....	....	....	....	−1·800	−1·738

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