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BY

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# Tables of Extended Distances for PKP and PcP

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## ABSTRACT

Tables of extended distances for PKP<sub>1</sub>, PKP<sub>2</sub> and PcP are presented, for surface focus and for focal depths ranging from 0.00R to 0.12R by steps of 0.01R. The tables are based on the Jeffreys-Bullen travel-time curves for the equivalent phases. They are consistent with earlier tables giving extended distances for P, so that the several phases can be used in a single solution.

## INTRODUCTION

This is the third paper of a series dealing with theoretical aspects of Byerly's method of determining the direction of faulting in an earthquake. The first paper<sup>1</sup> reviewed the method as it applies to earthquakes of normal focus, and provided some criteria for testing the geometrical validity of solutions. The second paper<sup>2</sup> gave tables of extended distances for P waves originating at various focal depths, thereby permitting application of the method to deep focus earthquakes. Since these tables do not permit use of data for stations beyond the range of P, a need was felt for tables of extended distances for the phases PKP<sub>1</sub> and PKP<sub>2</sub>, refracted through the core. In preparing these tables it has been possible, with little additional effort, to provide tables also for PcP, the phase reflected from the core.

## THE PHASES PKP<sub>1</sub> AND PKP<sub>2</sub>

In order that there may be no misunderstanding about the phase designation used in this paper, a discussion of the Jeffreys-Bullen's PKP curve will first be given. The curve is shown in Figure 1. It consists of three segments, AB, BC, and DF, and there are theoretical reasons for believing that a fourth segment, connecting C to D must exist. If we admit this segment then the curve may be conveniently regarded as a single curve with cusps.

An insert to Figure 1 defines the angle  $e$  which a ray makes with the tangent to the surface. The discussion of PKP will be in terms of this angle  $e$ . For a surface focus,  $e$  is defined by the equation

$$\cos e = v \frac{dt}{d\Delta},$$

where  $v$  is the velocity of seismic waves at the surface and  $dt/d\Delta$  is the inverse slope of the travel-time curve at the point where the ray emerges. By determining the value of  $dt/d\Delta$  at any point on a curve, one can compute the angle at which the ray, reaching that point,

<sup>1</sup> J. H. Hodgson and W. G. Milne, "Direction of Faulting in Certain Earthquakes of the North Pacific," *Bull. Seism. Soc. Am.* Vol. 41, 221-242, 1951.

<sup>2</sup> J. H. Hodgson and R. S. Storey, "Tables Extending Byerly's Fault-Plane Techniques to Earthquakes of any Focal Depth," *Bull. Seism. Soc. Am.*, Vol. 43, 49-61, 1953.

<sup>3</sup> H. Jeffreys and K. E. Bullen, *Seismological Tables*, (Brit. Assoc. Adv. Sci., 1940.)

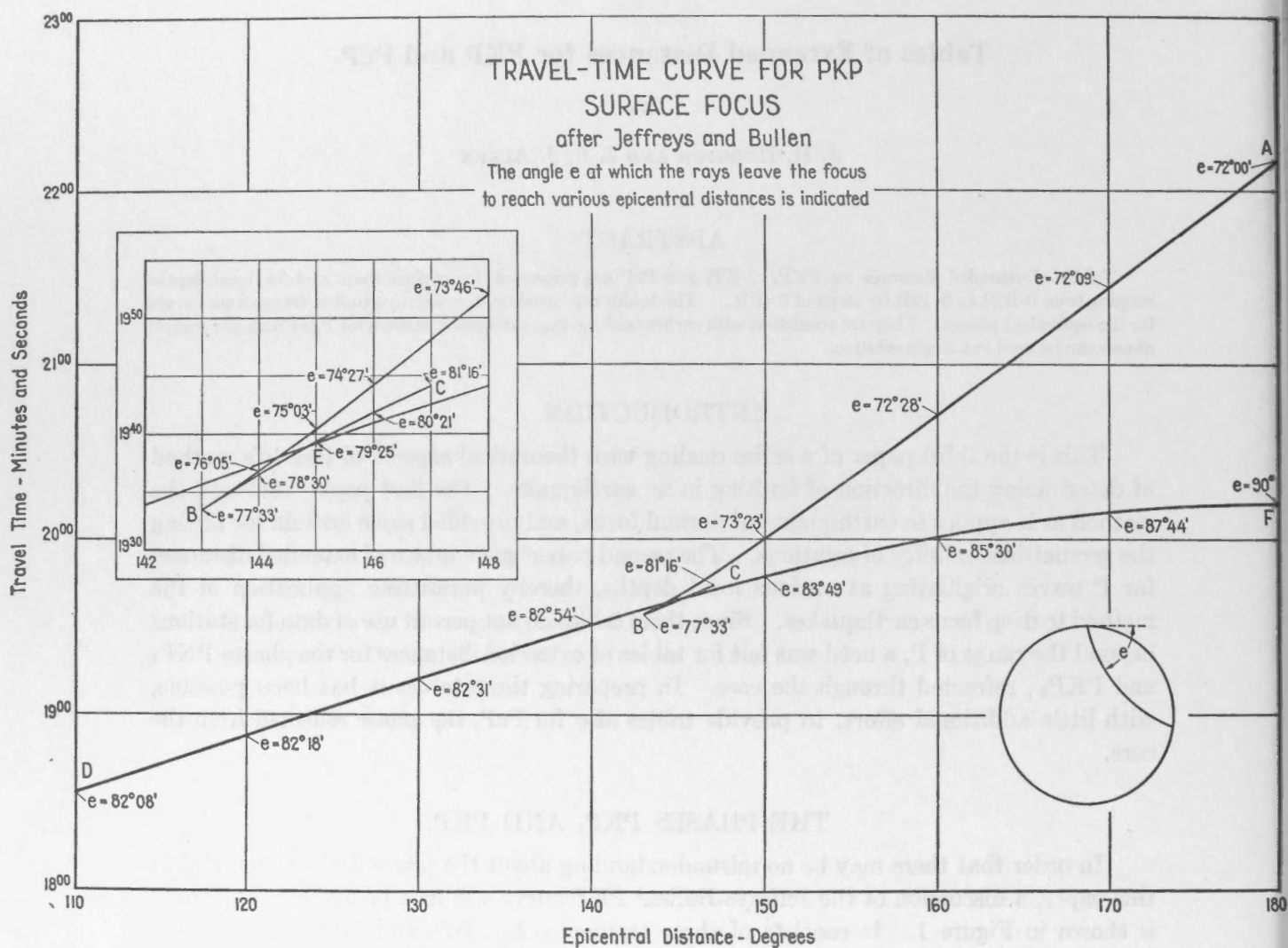


FIGURE 1

left the focus. For example, when this analysis is applied to the surface-focus P curve of the Jeffreys-Bullen tables it is found that the angle  $e$  varies from  $0^\circ$  to about  $72^\circ$  as the ray sweeps out to  $105^\circ$  epicentral distance.

When a similar analysis is applied to the core phases of Figure 1 it is found that the point A corresponds again to a value of  $e$  of about  $72^\circ$ ; for a very slight increment in the angle  $e$  the point of emergence jumps from a P at  $105^\circ$  to a PKP at  $180^\circ$ . As one moves down the upper branch of the curve from A the slope varies continuously until the point B is reached. The value of  $e$  corresponding to the point B is  $77^\circ 33'$ . Thus, while  $e$  grows from  $72^\circ$  to  $77^\circ 33'$ , the ray sweeps from  $180^\circ$  epicentral distance back to  $143^\circ$ .

Next consider the section of the curve BC. BC and AB come together tangentially at B so that there is no discontinuity in slope as we switch from one branch of the curve to the other. As the curve BC is traversed from B to C, the angle  $e$  varies from  $77^\circ 33'$  to about  $81^\circ 16'$ , while the point of emergence varies from  $143^\circ$  to  $147^\circ$  epicentral distance.

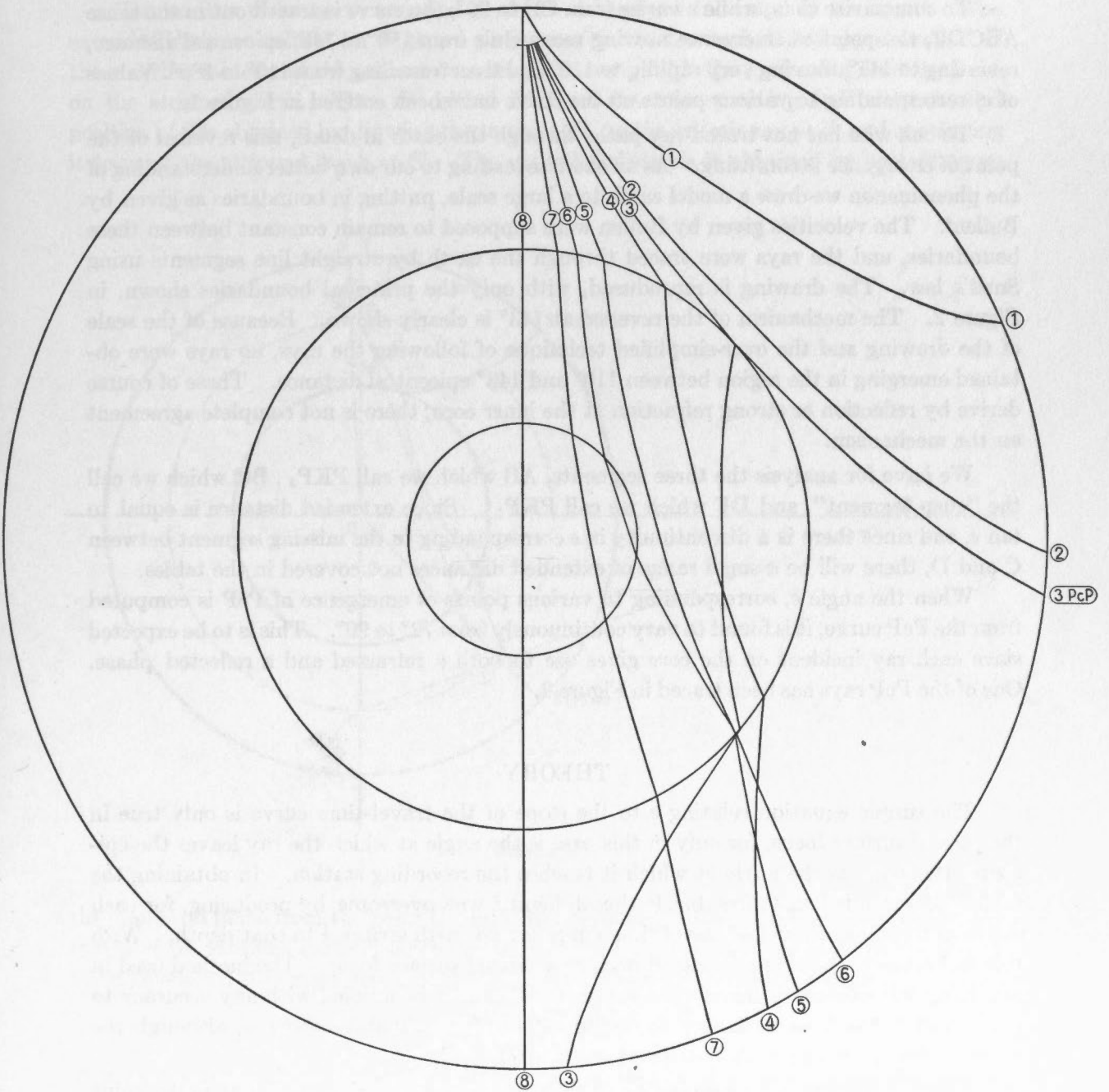


FIGURE 2.—Ray paths through the earth, showing the variation of the point of emergence with increase of the angle  $e$ .

Now, since the section CD has not been observed, it is necessary to jump to the point D. Here the corresponding value of  $e$  is  $82^{\circ} 08'$ . Traversing the curve from D to F,  $e$  varies continuously from this value to  $90^{\circ}$ . Thus the only discontinuity in  $e$  arises from the jump between the points C and D, and would be covered by the missing segment which must join BC tangentially at C and DF tangentially at D.

To summarize then, while  $e$  varies from  $72^\circ$  to  $90^\circ$ , the curve is traced out in the sense ABCDF, the point of emergence moving meanwhile from  $180^\circ$  to  $143^\circ$  epicentral distance, reversing to  $147^\circ$ , moving very rapidly to  $110^\circ$  and then travelling from  $110^\circ$  to  $180^\circ$ . Values of  $e$ , corresponding to various points on the curve have been entered in Figure 1.

To one who has not traced ray paths through the earth in detail, this reversal of the point of emergence is confusing. As an exercise leading to our own better understanding of the phenomenon we drew a model earth to a large scale, putting in boundaries as given by Bullen<sup>4</sup>. The velocities given by Bullen were supposed to remain constant between these boundaries, and the rays were traced through the earth by straight line segments using Snell's law. The drawing is reproduced, with only the principal boundaries shown, in Figure 2. The mechanism of the reversal at  $143^\circ$  is clearly shown. Because of the scale of the drawing and the over-simplified technique of following the rays, no rays were obtained emerging in the region between  $110^\circ$  and  $143^\circ$  epicentral distance. These of course derive by reflection or strong refraction at the inner core; there is not complete agreement on the mechanism.

We have for analysis the three segments, AB which we call PKP<sub>2</sub>, BC which we call the "cusp segment", and DF which we call PKP<sub>1</sub>. Since extended distance is equal to  $\tan e$ , and since there is a discontinuity in  $e$  corresponding to the missing segment between C and D, there will be a small range of extended distances not covered in the tables.

When the angle  $e$ , corresponding to various points of emergence of PcP is computed from the PcP curve, it is found to vary continuously from  $72^\circ$  to  $90^\circ$ . This is to be expected since each ray incident on the core gives use to both a refracted and a reflected phase. One of the PcP rays has been traced in Figure 2.

## THEORY

The simple equation relating  $e$  to the slope of the travel-time curve is only true in the case of surface focus, for only in this case is the angle at which the ray leaves the epicentre the same as the angle at which it reaches the recording station. In obtaining the tables<sup>2</sup> of extended distances for P this difficulty was overcome by producing, for each depth of focus considered, a travel-time curve for an earth stripped to that depth. With this technique each focus considered became a virtual surface focus. The method used in obtaining stripped-earth travel-time curves for P cannot be applied with any accuracy to PKP, and it has been necessary to devise a more fundamental approach, although the general idea of dealing with a stripped earth is retained.

Figure 3 represents a cross-section of the earth with an earthquake focus at the point F, at depth  $d$  within the earth. A ray, generating a typical PKP and its corresponding PcP, is shown leaving the focus at an angle  $e_d$ , defined in the figure. By producing the ray symmetrically to the surface a point A is defined, such that a ray starting from A would generate the same pair of phases illustrated. This ray leaves the surface at an angle  $e_s$ . The angles  $e_s$  and  $e_d$  are clearly analogous to the angle  $e$  of the previous section, the subscripts  $s$  and  $d$  indicating focus at the surface and at depth  $d$ , respectively.

<sup>4</sup> K. E. Bullen, "An Introduction to the Theory of Seismology", Cambridge University Press, 1947, p. 211.

The surface of an earth stripped to depth  $d$  has been indicated in the figure. The extended distance tables<sup>2</sup> for P give a projection on the basis of this stripped earth. If those earlier tables and the present ones are to be compatible the projection must again be on the stripped earth. Then for either PKP or the corresponding PcP the extended position of S is obtained by drawing the tangent  $FS'$  to the seismic ray at  $F$ , and continuing it to meet the stripped earth at  $S'$ . The extended distance is obtained by projecting on

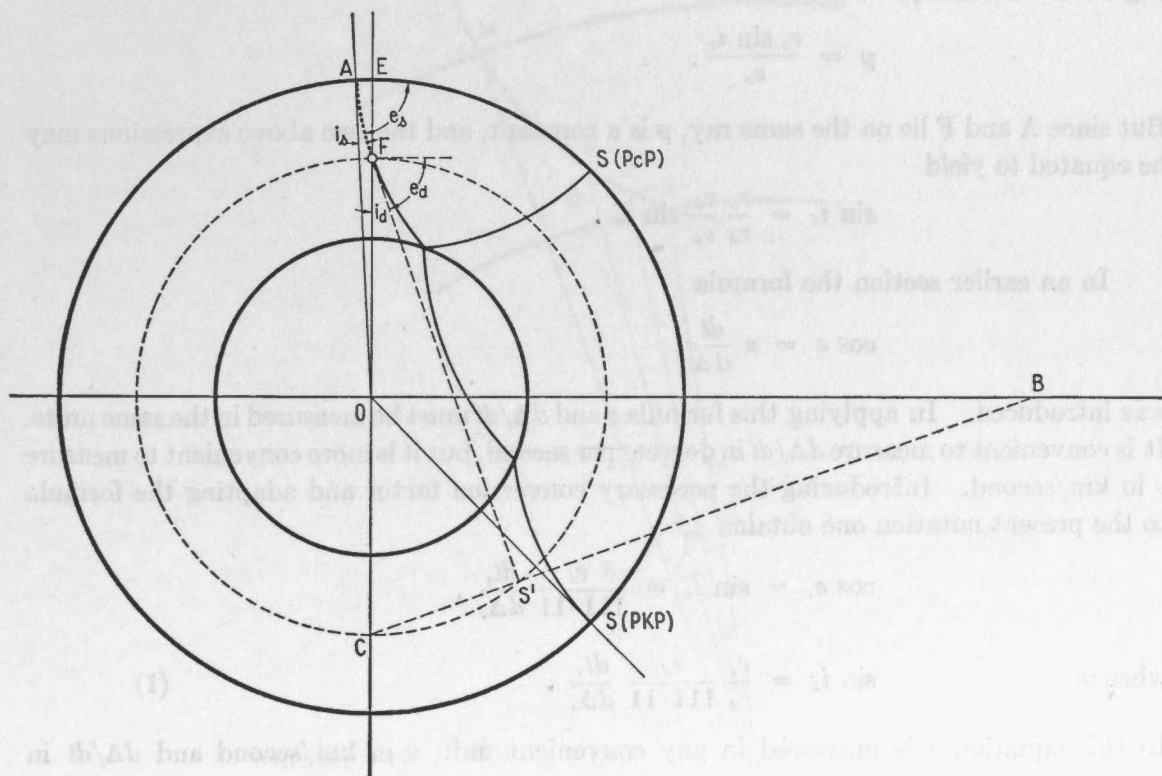


FIGURE 3

the diametral plane from the point C, and is thus equal to  $OB$ . As in the earlier paper, the radius of the stripped earth is taken equal to unity, so that  $OB = \tan e_d$ . In the theory to be developed it is more convenient to work with the complementary angle  $i_d$ , so that

$$OB = \cot i_d.$$

It is now necessary to express  $i_d$  in terms of  $e_d$ , or its complement  $i_s$ , since this can be determined readily from the surface-focus travel-time curve.

It is a well-known property of seismic rays\* in a spherically concentric earth, that for any ray the quantity

$$p = \frac{r \sin i}{v}$$

is a constant.  $p$  varies of course from ray to ray and may be regarded as a parameter defining the ray. In the above equation  $r$  is the distance from the centre of the earth to the

\* See, for example, reference 4, pp. 108-109.

point on the ray,  $v$  is the velocity at that point, and  $i$  is the angle between the radius vector and the ray. Applying this equation at the point  $F$  one obtains

$$p = \frac{r_d \sin i_d}{v_d},$$

the subscript  $d$  indicating the depth of  $F$  and  $i_d$  being identical with the angle defined in Figure 3. Similarly, at the surface,

$$p = \frac{r_s \sin i_s}{v_s}.$$

But since A and F lie on the same ray,  $p$  is a constant, and the two above expressions may be equated to yield

$$\sin i_d = \frac{r_s v_d}{r_d v_s} \sin i_s.$$

In an earlier section the formula

$$\cos e = v \frac{dt}{d\Delta}$$

was introduced. In applying this formula  $v$  and  $d\Delta/dt$  must be measured in the same units. It is convenient to measure  $d\Delta/dt$  in degrees per second, but it is more convenient to measure  $v$  in km/second. Introducing the necessary conversion factor and adapting the formula to the present notation one obtains

$$\cos e_s = \sin i_s = \frac{v_s}{111.11} \frac{dt_s}{d\Delta_s},$$

whence

$$\sin i_d = \frac{r_s}{r_d} \frac{v_d}{111.11} \frac{dt_s}{d\Delta_s}. \quad (1)$$

In this equation  $r$  is measured in any convenient unit,  $v$  in km/second and  $d\Delta/dt$  in degrees/second.

One point remains to be discussed: the position of A relative to F. Turning to Figure 3 let the arc distance of ES be called  $\Delta_d$ , that of AS  $\Delta_s$ , and that of AE  $\theta_{sd}$ . Then

$$\Delta_s = \Delta_d + \theta_{sd}. \quad (2)$$

In practice the epicentral distance of a station,  $\Delta_d$ , will be known, but to apply equation (1) it is necessary to determine the slope of the surface-focus travel-time curve at epicentral distance  $\Delta_s$ . A knowledge of  $\theta_{sd}$  is essential.

Consider the problem more generally. Let M, N, (Figure 4) be two points on a ray at depths  $m$ ,  $n$  within the earth. It is required to determine the angle subtended at the centre of the earth by this segment of the ray. Let P be a general point on the ray such that MP, of arc length  $S$ , subtends an angle  $\theta$  at the centre. Let an additional small angle  $d\theta$  cut off an additional arc  $dS$ . Let the radii to M, P and N be  $r_M$ ,  $r$  and  $r_N$  respectively. Then in the elementary triangle PQR:

$$\begin{aligned} \text{angle QPR} &= i, \text{ as previously defined,} \\ \text{RQ} &= (r - dr)d\theta \doteq r d\theta, \\ \text{PQ} &= dS. \end{aligned}$$



Hence 
$$\sin i = r \frac{d\theta}{dS} .$$

But 
$$p = \frac{r}{v} \sin i = \frac{r^2}{v} \frac{d\theta}{dS} ,$$

whence 
$$dS = \frac{r^2}{pv} d\theta .$$

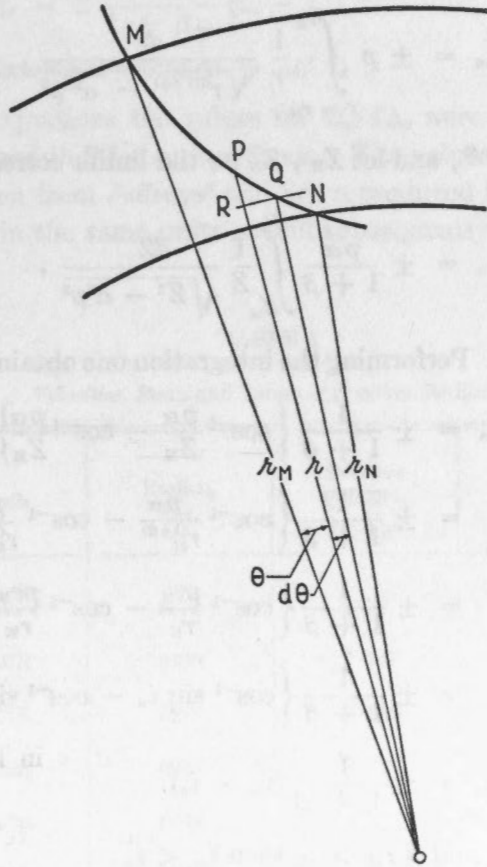


FIGURE 4

From the same triangle

$$dS^2 = dr^2 + r^2 d\theta^2 .$$

Substituting the above value for  $dS$ , and simplifying, one obtains

$$d\theta = \pm \frac{p}{r} \frac{dr}{\sqrt{\frac{r^2}{v^2} - p^2}} .$$

Then the angle subtended by the arc MN is

$$\theta_{mn} = \pm p \int_{r=r_M}^{r_N} \frac{1}{r} \frac{dr}{\sqrt{\frac{r^2}{v^2} - p^2}} .$$

Up to this point the argument has been completely general but it is now necessary to express  $v$  as some function of  $r$ . For reasons which will be outlined later we propose to assume that  $v$  may be expressed in the form

$$v = \alpha r^{-\beta},$$

where  $\alpha$  and  $\beta$  are constants. Then the integral assumes the form

$$\theta_{mn} = \pm p \int_{r=r_M}^{r=r_N} \frac{1}{r} \frac{dr}{\sqrt{r^{2(1+\beta)} - \alpha^2 p^2}}.$$

Now, substitute  $Z = r^{(1+\beta)}$ , and let  $Z_M, Z_N$  be the limits corresponding to  $r_M, r_N$ . The integral reduces to

$$\theta_{mn} = \pm \frac{p\alpha}{1+\beta} \int_{Z_M}^{Z_N} \frac{1}{Z} \frac{dZ}{\sqrt{Z^2 - \alpha^2 p^2}},$$

which is a standard form. Performing the integration one obtains

$$\begin{aligned} \theta_{mn} &= \pm \frac{1}{1+\beta} \left\{ \cos^{-1} \frac{p\alpha}{Z_N} - \cos^{-1} \frac{p\alpha}{Z_M} \right\}, \\ &= \pm \frac{1}{1+\beta} \left\{ \cos^{-1} \frac{p\alpha}{r_N^{(1+\beta)}} - \cos^{-1} \frac{p\alpha}{r_M^{(1+\beta)}} \right\}, \\ &= \pm \frac{1}{1+\beta} \left\{ \cos^{-1} \frac{pv_N}{r_N} - \cos^{-1} \frac{pv_M}{r_M} \right\}, \\ &= \pm \frac{1}{1+\beta} \left\{ \cos^{-1} \sin i_n - \cos^{-1} \sin i_m \right\}, \\ &= \pm \frac{1}{1+\beta} (i_m - i_n). \end{aligned}$$

But  $\theta_{mn}$  must be positive, and if  $r_M > r_N$  then  $i_m < i_n$ . We must write

$$\theta_{mn} = + \frac{1}{1+\beta} (i_n - i_m).$$

The law of velocity distribution assumed, viz.:

$$v = \alpha r^{-\beta},$$

is not likely to fit the facts at all depths for fixed values of  $\alpha$  and  $\beta$ . At most it may be hoped that certain values of  $\alpha$  and  $\beta$  will describe velocity over a certain range of depth; with different values another section may be described, and so on. Then the total angle subtended by an arc will be made up of a number of sections, each integral being computed with appropriate values of  $\alpha$  and  $\beta$ . This may be expressed formally by setting

$$\theta_{id} = \Sigma \frac{1}{1+\beta_{mn}} (i_n - i_m). \quad (3)$$

There is no loss of generality in such extension of the analysis.

DERIVATION OF THE TABLES

The following four equations are available from the foregoing analysis:

$$\sin i_d = \frac{r_s}{r_d} \cdot \frac{v_d}{111.11} \cdot \frac{dt_s}{d\Delta_s}, \tag{1}$$

$$\Delta_s = \Delta_d + \theta_{sd}, \tag{2}$$

$$\theta_{sd} = \Sigma \frac{1}{1 + \beta_{mn}} (i_n - i_m), \tag{3}$$

$$\text{Extended distance} = \cot i_d. \tag{4}$$

In applying these equations the values for  $dt_s/d\Delta_s$  were obtained from the Jeffreys-Bullen tables<sup>3</sup> for PKP and PcP for surface focus. The values of the P-wave velocities at various depths were taken from Jeffreys<sup>5</sup> and are reproduced in Table I. It will be noted that depth is measured in the same units as will subsequently be used in describing focal depth.

TABLE I

*Velocities, Radii and Values of  $\beta$  within the Earth*

Depth	Radius, km.	P-Wave Velocity, km/sec.	$\beta$
Surface	6371	7.72	0.6723
0.00R	6338	7.747	2.371
0.01R	6275	7.936	2.371
0.02R	6211	8.131	2.371
0.03R	6148	8.332	2.371
0.04R	6084	8.539	2.371
0.05R	6021	8.752	2.371
0.06R	5958	8.971	5.357
0.07R	5894	9.50	3.908
0.08R	5831	9.91	3.176
0.09R	5768	10.26	2.522
0.10R	5704	10.55	1.847
0.11R	5641	10.77	1.790
0.12R	5577	10.99	

<sup>5</sup> H. Jeffreys, "Times of P, S and SKS, and Velocities of P and S," *Mon. Not. Royal Astron. Soc., Geophys. Suppl.*, Vol. 4, 498-533, 1939.

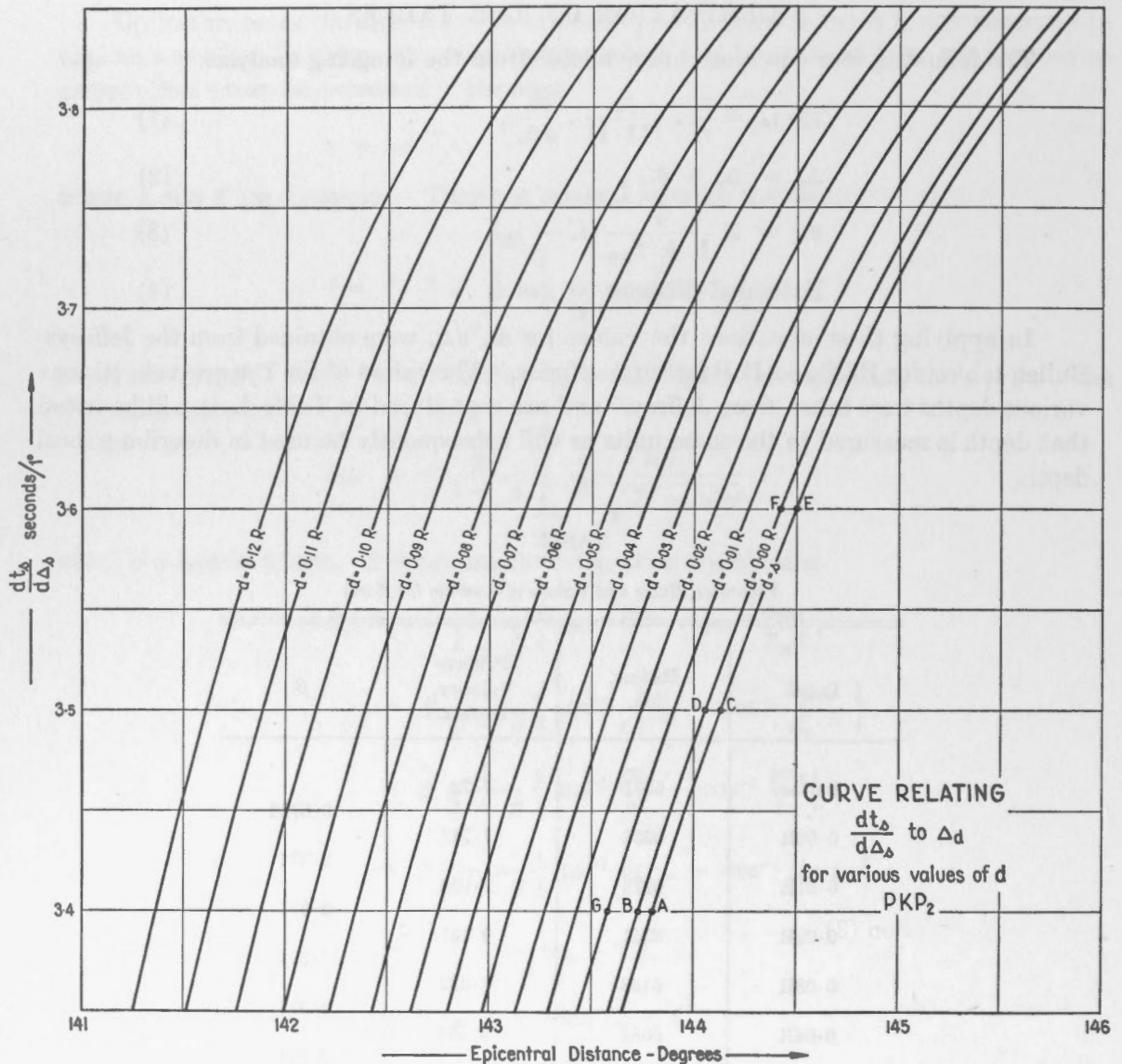


FIGURE 5

Bullen\* has pointed out that from a depth of 0.00R to a depth of 0.06R the velocity may be described by the formula.

$$v = \alpha r^{-7/3}.$$

It was this formula which suggested the form of substitution for  $v$  in terms of  $r$ . The general formula does not apply over any considerable range except this one, but we have assumed that it may be applied over each step of the range, as for example Surface to 0.00R, 0.06R to 0.07R, etc., and have computed the value of  $\beta$  appropriate to each range. These computed values of  $\beta$  are also shown in Table I.

\* Reference 4 p. 211.

In an earlier section it was shown that the slope of the PKP curve varies between the same limits as does that of PcP, or to put it another way, that a ray which gives rise to any PKP also sets up a PcP. By arranging the computation in terms of rays the work can be much reduced. This was accomplished in the following way. A curve relating  $\Delta_s$  to the inverse slope  $dt_s/d\Delta_s$  was plotted for each of PKP<sub>1</sub>, the cusp segment, PKP<sub>2</sub> and PcP. A section of this curve for PKP<sub>2</sub> is shown in Figure 5, labeled  $d = s$ . For PcP the inverse slope varies from zero to 4.5 seconds/1°, for PKP<sub>1</sub> from zero to 2.0, for the cusp segment from 2.2 to 3.1 and for PKP<sub>2</sub> from 3.1 to 4.5. The range zero to 4.5 seconds/1° therefore covers all the phases. Selecting 45 values of the inverse slope, at intervals of 0.1 sec/1°, was equivalent to selecting 45 rays for consideration.

The method of computation will be illustrated by an example corresponding to the section of the curve illustrated in Figure 5. In the section illustrated  $dt_s/d\Delta_s$  varies from 3.35 to 3.85 seconds/1°; the integral values 3.4, 3.5, 3.6, 3.7 and 3.8 are selected for computation.

Taking first of all the value 3.4 and applying equation (1) for  $d = s$  one obtains:

$$\begin{aligned} \sin i_s &= \frac{6371}{6371} \times \frac{7.72}{111.1} \times 3.4, \\ &= 0.2363, \\ i_s &= 13^\circ.66_8. \end{aligned}$$

For convenience write  $i_0$ , etc. in place of 0.00R, 0.01R etc. in subscripts. Then

$$\begin{aligned} \sin i_0 &= \frac{6371}{6338} \times \frac{7.747}{111.1} \times 3.4, \\ &= 0.2383, \\ i_0 &= 13^\circ.78_7. \end{aligned}$$

Now apply equation (3)

$$\begin{aligned} \theta_{s0} &= \frac{1}{1 + 0.6723} (13^\circ.78_7 - 13^\circ.66_8), \\ &= 0^\circ.07_2. \end{aligned}$$

Similarly, taking  $dt_s/d\Delta_s$  as 3.5, 3.6 etc., one obtains the values shown in Table II.

TABLE II

Type Computations of  $\theta_{s0}$

$dt_s/d\Delta_s$ sec/1°	$i_s$ degrees	$i_0$ degrees	$\theta_{s0}$ degrees
3.4	13.66 <sub>8</sub>	13.78 <sub>7</sub>	0.07 <sub>2</sub>
3.5	14.07 <sub>6</sub>	14.20 <sub>1</sub>	0.07 <sub>5</sub>
3.6	14.48 <sub>6</sub>	14.61 <sub>6</sub>	0.07 <sub>8</sub>
3.7	14.89 <sub>8</sub>	15.03 <sub>1</sub>	0.08 <sub>0</sub>
3.8	15.31 <sub>1</sub>	15.44 <sub>7</sub>	0.08 <sub>1</sub>

With these values of  $\theta_{s0}$  obtained, return now to Figure 5. On the ordinate corresponding to  $dt_s/d\Delta_s$  equal to 3.4, cut off AB equal to the appropriate value of  $\theta_{s0}$  ( $= 0^\circ 07_2$ ). Similarly on the ordinate 3.5 cut off CD  $= 0^\circ 07_5$ , on the ordinate 3.6 cut off EF  $= 0^\circ 07_8$ , and so on. When the series of points BDF . . . is connected, the result is a smooth curve relating epicentral distance for focal depth 0.00R to the slope of the surface travel time curve, for the new curve relates  $dt_s/d\Delta_s$  to  $\Delta_s - \theta_{s0}$  by construction, and by equation (2)

$$\Delta_s - \theta_{s0} = \Delta_d.$$

To construct the tables of extended distances enter this new curve at integral values of epicentral distance, read off the corresponding values of  $dt_s/d\Delta_s$ , apply equation (1) to obtain  $\sin i_d$ , and then equation (4) to give extended distance.

At the same time, of course, the values of  $\theta_{s0}$  obtained in Table II are applied to the PcP curve so that the one set of computations yields data on both PcP and PKP.

Returning to the computations one obtains:

$$\begin{aligned} \sin i_1 &= \frac{6371}{6275} \times \frac{7.936}{111.1} \times 3.4, \\ &= 0.2466, \\ i_1 &= 14^\circ 27_6, \\ \theta_{01} &= \frac{1}{1 + 2.371} (14^\circ 27_6 - 13^\circ 78_7), \\ &= 0^\circ 14_5. \end{aligned}$$

Then  $\theta_{s1} = \theta_{s0} + \theta_{01} = 0^\circ 07_2 + 0^\circ 14_5 = 0^\circ 21_7$ .

The distance AG is now cut off  $= 0^\circ 21_7$  on the ordinate 3.4. Similarly, taking values of  $dt_s/d\Delta_s$  equal to 3.5, 3.6 etc. a series of points is obtained; when these are connected by a smooth curve we have a graph relating epicentral distance for depth 0.01R to the slope of the surface travel-time curve. Again, using equation (1), values of  $i_1$  may be obtained for integral values of epicentral distance, and using equation (4), extended distance may be computed.

The computation thus proceeds step by step, each  $\theta$  that is computed being used to construct the curve for that particular depth, and at the same time contributing its value to the next greater depth. Between 0.00R and 0.06R the effect of accumulative error is eliminated because the same value of  $\beta$  can be used over the entire range.

## DISCUSSION

The tables will be found at the end of the paper. Table IV (pages 342 to 343) gives extended distances for PKP<sub>1</sub>, Table V (page 344) for the cusp segment, Table VI (page 345) for PKP<sub>2</sub> and Table VII (pages 346 to 348) for PcP. The tables are to be used in conjunction with the earlier ones<sup>2</sup> for P, the projections being compatible in every respect.

In an earlier section it was pointed out that, for surface focus, a ray which just misses the core and so emerges as a P phase at  $105^\circ$ , leaves the focus at nearly the same angle as a PKP<sub>2</sub> emergent at  $180^\circ$  and as a PcP emergent at  $100^\circ$ . Since extended distance is a function of this angle, the extended distance for P at  $105^\circ$  should be only very slightly smaller than that for PKP<sub>2</sub> at  $180^\circ$  and for PcP at  $100^\circ$ . The same sort of relationship should obtain at all focal depths, the last entry for P being approximately the same as the last entry for PcP and the entry for PKP<sub>2</sub> at  $180^\circ$ . This fact affords a check between the present tables and the earlier ones, which is important since the two sets have been derived by quite different methods.

The comparison is provided in Table III. It will be seen that there are no essential differences.

TABLE III

*Comparison of Limiting Values of P, PKP<sub>2</sub> and PcP*

Depth	Extended Distance		
	P	PKP <sub>2</sub>	PcP
Surface	3.066	3.08	3.08
0.00R	3.046	3.05	3.05
0.01R	2.930	2.93	2.93
0.02R	2.820	2.82	2.82
0.03R	2.713	2.71	2.71
0.04R	2.588	2.61	2.61
0.05R	2.485	2.50	2.50
0.06R	2.392	2.40	2.40
0.07R	2.223	2.22	2.22
0.08R	2.085	2.08	2.08
0.09R	1.969	1.96	1.96
0.10R	1.872	1.87	1.87
0.11R	1.800	1.79	1.79
0.12R	1.740	1.72	1.72

TABLE IV

*Extended Distances for PKP<sub>1</sub>*

$\Delta^\circ$	Depth h =													
	Surface	0-00	0-01	0-02	0-03	0-04	0-05	0-06	0-07	0-08	0-09	0-10	0-11	0-12
109	....	....	....	6.68	6.45	6.22	6.00	5.79	5.40	5.11	4.87	4.68	4.53	4.38
110	7.24	7.17	6.93	6.69	6.46	6.23	6.01	5.80	5.41	5.12	4.88	4.69	4.54	4.39
111	7.25	7.19	6.94	6.70	6.47	6.25	6.03	5.81	5.42	5.13	4.90	4.70	4.55	4.40
112	7.27	7.20	6.96	6.72	6.48	6.26	6.04	5.82	5.43	5.14	4.91	4.71	4.56	4.41
113	7.28	7.22	6.97	6.73	6.50	6.27	6.05	5.84	5.44	5.15	4.92	4.72	4.57	4.42
114	7.30	7.23	6.99	6.75	6.51	6.29	6.06	5.85	5.45	5.16	4.93	4.73	4.58	4.43
115	7.31	7.25	7.00	6.76	6.53	6.30	6.08	5.86	5.47	5.18	4.94	4.74	4.59	4.44
116	7.33	7.27	7.02	6.78	6.54	6.31	6.09	5.88	5.48	5.19	4.95	4.75	4.60	4.45
117	7.34	7.28	7.04	6.79	6.56	6.33	6.11	5.89	5.49	5.20	4.96	4.77	4.61	4.46
118	7.36	7.30	7.05	6.81	6.57	6.34	6.12	5.90	5.50	5.21	4.97	4.78	4.62	4.47
119	7.38	7.32	7.07	6.82	6.59	6.36	6.14	5.92	5.52	5.23	4.99	4.79	4.63	4.48
120	7.40	7.34	7.08	6.84	6.61	6.37	6.15	5.93	5.53	5.24	5.00	4.80	4.64	4.50
121	7.42	7.36	7.10	6.86	6.62	6.39	6.17	5.95	5.55	5.25	5.01	4.81	4.66	4.51
122	7.44	7.37	7.12	6.88	6.64	6.41	6.18	5.96	5.56	5.27	5.02	4.83	4.67	4.52
123	7.46	7.39	7.14	6.89	6.65	6.42	6.20	5.98	5.58	5.28	5.04	4.84	4.69	4.53
124	7.48	7.41	7.16	6.91	6.67	6.44	6.21	6.00	5.59	5.30	5.05	4.86	4.70	4.55
125	7.50	7.43	7.18	6.93	6.69	6.46	6.23	6.01	5.61	5.31	5.07	4.87	4.71	4.56
126	7.52	7.45	7.20	6.95	6.71	6.48	6.25	6.03	5.62	5.33	5.08	4.88	4.73	4.57
127	7.54	7.47	7.22	6.97	6.73	6.50	6.27	6.05	5.64	5.34	5.09	4.90	4.74	4.59
128	7.56	7.49	7.24	6.99	6.75	6.52	6.29	6.07	5.66	5.36	5.11	4.91	4.75	4.60
129	7.59	7.52	7.27	7.01	6.77	6.54	6.31	6.09	5.68	5.38	5.13	4.93	4.77	4.62
130	7.61	7.54	7.29	7.04	6.79	6.56	6.33	6.11	5.70	5.40	5.15	4.94	4.79	4.63
131	7.64	7.57	7.31	7.06	6.81	6.58	6.35	6.13	5.72	5.41	5.16	4.96	4.80	4.65
132	7.66	7.60	7.34	7.09	6.84	6.61	6.38	6.15	5.74	5.44	5.19	4.98	4.82	4.67
133	7.69	7.63	7.37	7.12	6.87	6.63	6.40	6.18	5.76	5.46	5.21	5.01	4.85	4.69
134	7.73	7.66	7.40	7.15	6.90	6.66	6.43	6.21	5.79	5.49	5.23	5.03	4.87	4.72
135	7.77	7.70	7.44	7.19	6.94	6.70	6.46	6.24	5.82	5.51	5.26	5.06	4.90	4.74
136	7.81	7.74	7.48	7.23	6.98	6.73	6.50	6.27	5.85	5.54	5.29	5.09	4.93	4.77
137	7.85	7.79	7.53	7.27	7.02	6.78	6.54	6.31	5.89	5.58	5.33	5.12	4.96	4.81
138	7.91	7.84	7.58	7.32	7.07	6.82	6.59	6.36	5.93	5.62	5.37	5.16	5.00	4.84
139	7.96	7.89	7.63	7.37	7.12	6.88	6.64	6.41	5.98	5.67	5.41	5.20	5.04	4.88
140	8.03	7.96	7.69	7.43	7.18	6.93	6.70	6.46	6.03	5.72	5.46	5.25	5.09	4.93
141	8.10	8.03	7.76	7.50	7.24	7.00	6.76	6.52	6.09	5.77	5.51	5.30	5.13	4.98
142	8.18	8.11	7.84	7.57	7.31	7.06	6.82	6.59	6.15	5.83	5.57	5.36	5.19	5.03
143	8.26	8.19	7.92	7.65	7.39	7.14	6.89	6.66	6.22	5.90	5.63	5.42	5.25	5.09
144	8.35	8.28	8.01	7.74	7.48	7.22	6.98	6.74	6.29	5.97	5.70	5.49	5.32	5.15
145	8.46	8.39	8.11	7.84	7.57	7.32	7.07	6.83	6.38	6.05	5.78	5.56	5.39	5.23
146	8.58	8.51	8.23	7.95	7.69	7.43	7.18	6.93	6.47	6.14	5.87	5.65	5.48	5.31
147	8.71	8.64	8.35	8.08	7.81	7.54	7.29	7.04	6.58	6.24	5.96	5.74	5.57	5.40
148	8.86	8.78	8.49	8.21	7.94	7.68	7.42	7.17	6.70	6.35	6.08	5.85	5.67	5.50
149	9.02	8.95	8.66	8.37	8.10	7.83	7.57	7.31	6.83	6.48	6.20	5.97	5.79	5.62







TABLE VI

*Extended Distances for PKP<sub>2</sub>*

$\Delta^\circ$	Depth h =													
	Surface	0-00	0-01	0-02	0-03	0-04	0-05	0-06	0-07	0-08	0-09	0-10	0-11	0-12
141	....	....	....	....	....	....	....	....	....	....	....	....	....	2-55
142	....	....	....	....	....	....	....	....	3-25	2-98	2-75	2-55	2-38	2-22
143	4-53	4-43	4-20	3-97	3-76	3-56	3-36	3-17	2-88	2-66	2-47	2-31	2-19	2-08
144	4-03	3-97	3-78	3-59	3-41	3-24	3-08	2-92	2-68	2-49	2-34	2-21	2-10	2-00
145	3-74	3-69	3-53	3-37	3-22	3-08	2-94	2-80	2-58	2-41	2-27	2-15	2-05	1-96
146	3-59	3-55	3-39	3-26	3-12	2-99	2-86	2-74	2-52	2-36	2-22	2-11	2-01	1-93
147	3-50	3-46	3-32	3-19	3-06	2-93	2-81	2-69	2-48	2-32	2-18	2-07	1-99	1-90
148	3-43	3-40	3-26	3-13	3-01	2-88	2-76	2-65	2-44	2-29	2-16	2-05	1-96	1-88
149	3-39	3-35	3-22	3-09	2-97	2-85	2-73	2-62	2-41	2-26	2-13	2-03	1-94	1-86
150	3-35	3-31	3-18	3-06	2-94	2-82	2-71	2-59	2-39	2-24	2-12	2-01	1-93	1-85
151	3-32	3-28	3-16	3-03	2-91	2-80	2-68	2-57	2-37	2-22	2-10	2-00	1-91	1-83
152	3-29	3-26	3-13	3-01	2-89	2-78	2-66	2-55	2-36	2-21	2-08	1-98	1-90	1-82
153	3-27	3-23	3-11	2-99	2-87	2-76	2-65	2-54	2-34	2-19	2-07	1-97	1-89	1-81
154	3-25	3-21	3-09	2-97	2-86	2-74	2-63	2-53	2-33	2-18	2-06	1-96	1-88	1-80
155	3-23	3-20	3-08	2-96	2-84	2-73	2-62	2-51	2-32	2-17	2-05	1-95	1-87	1-79
156	3-21	3-18	3-06	2-94	2-83	2-72	2-61	2-50	2-31	2-16	2-04	1-94	1-86	1-78
157	3-20	3-17	3-05	2-93	2-82	2-71	2-60	2-49	2-30	2-15	2-03	1-93	1-85	1-78
158	3-19	3-16	3-04	2-92	2-81	2-70	2-59	2-48	2-29	2-14	2-03	1-93	1-85	1-77
159	3-18	3-14	3-03	2-91	2-80	2-69	2-58	2-47	2-28	2-14	2-02	1-92	1-84	1-76
160	3-16	3-13	3-02	2-90	2-79	2-68	2-57	2-46	2-27	2-13	2-01	1-91	1-83	1-76
161	3-15	3-12	3-01	2-89	2-78	2-67	2-56	2-46	2-27	2-12	2-01	1-91	1-83	1-75
162	3-14	3-11	3-00	2-88	2-77	2-66	2-55	2-45	2-26	2-12	2-00	1-90	1-82	1-75
163	3-14	3-10	2-99	2-88	2-76	2-65	2-55	2-44	2-25	2-11	1-99	1-90	1-82	1-74
164	3-13	3-10	2-98	2-87	2-76	2-65	2-54	2-44	2-25	2-11	1-99	1-89	1-82	1-74
165	3-12	3-09	2-97	2-86	2-75	2-64	2-54	2-43	2-24	2-10	1-99	1-89	1-81	1-74
166	3-12	3-08	2-97	2-85	2-74	2-64	2-53	2-43	2-24	2-10	1-98	1-89	1-81	1-73
167	3-11	3-08	2-96	2-85	2-74	2-63	2-53	2-42	2-24	2-10	1-98	1-88	1-81	1-73
168	3-11	3-07	2-96	2-85	2-74	2-63	2-52	2-42	2-23	2-09	1-98	1-88	1-80	1-73
169	3-10	3-07	2-95	2-84	2-73	2-63	2-52	2-42	2-23	2-09	1-98	1-88	1-80	1-73
170	3-10	3-07	2-95	2-84	2-73	2-62	2-52	2-42	2-23	2-09	1-97	1-88	1-80	1-73
171	3-09	3-06	2-95	2-84	2-73	2-62	2-52	2-41	2-23	2-09	1-97	1-88	1-80	1-73
172	3-09	3-06	2-95	2-83	2-72	2-62	2-51	2-41	2-22	2-09	1-97	1-87	1-80	1-72
173	3-09	3-06	2-94	2-83	2-72	2-62	2-51	2-41	2-22	2-08	1-97	1-87	1-80	1-72
174	3-09	3-06	2-94	2-83	2-72	2-61	2-51	2-41	2-22	2-08	1-97	1-87	1-80	1-72
175	3-08	3-05	2-94	2-83	2-72	2-61	2-51	2-41	2-22	2-08	1-97	1-87	1-79	1-72
176	3-08	3-05	2-94	2-83	2-72	2-61	2-51	2-41	2-22	2-08	1-96	1-87	1-79	1-72
177	3-08	3-05	2-94	2-82	2-72	2-61	2-51	2-40	2-22	2-08	1-96	1-87	1-79	1-72
178	3-08	3-05	2-93	2-82	2-71	2-61	2-50	2-40	2-22	2-08	1-96	1-87	1-79	1-72
179	3-08	3-05	2-93	2-82	2-71	2-61	2-50	2-40	2-22	2-08	1-96	1-87	1-79	1-72
180	3-08	3-05	2-93	2-82	2-71	2-61	2-50	2-40	2-22	2-08	1-96	1-87	1-79	1-72

TABLE VII

*Extended Distances for PcP*

$\Delta^\circ$	Depth h =													
	Surface	0-00	0-01	0-02	0-03	0-04	0-05	0-06	0-07	0-08	0-09	0-10	0-11	0-12
0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
1	151.7	150.7	144.7	137.7	133.7	127.7	121.7	117.7	109.7	104.7	98.0	92.4	88.6	85.1
2	75.7	74.7	71.8	69.0	66.7	63.7	60.9	58.8	54.6	51.8	49.0	46.7	45.0	43.2
3	50.5	49.7	48.9	45.9	44.4	42.7	41.1	39.3	36.4	34.4	32.7	31.2	30.0	28.9
4	37.9	37.3	35.9	34.6	33.3	32.0	30.7	29.5	27.4	25.9	24.5	23.5	22.6	21.8
5	30.2	29.8	28.8	27.7	26.6	25.7	24.6	23.6	22.0	20.7	19.7	18.8	18.1	17.4
6	25.2	24.9	24.0	23.1	22.2	21.3	20.5	19.7	18.3	17.3	16.4	15.6	15.0	14.5
7	21.5	21.3	20.5	19.8	19.0	18.2	17.5	16.8	15.6	14.7	14.0	13.4	12.8	12.4
8	18.8	18.6	18.0	17.3	16.6	15.9	15.3	14.7	13.7	12.9	12.2	11.7	11.2	10.8
9	16.7	16.6	15.9	15.4	14.7	14.2	13.6	13.1	12.1	11.5	10.9	10.4	9.98	9.59
10	15.1	14.9	14.4	13.8	13.3	12.8	12.3	11.8	10.9	10.3	9.76	9.33	8.98	8.64
11	13.7	13.6	13.0	12.6	12.1	11.6	11.1	10.6	9.93	9.37	8.89	8.48	8.16	7.85
12	12.6	12.4	12.0	11.5	11.1	10.6	10.2	9.81	9.11	8.58	8.13	7.77	7.47	7.19
13	11.6	11.5	11.1	10.6	10.2	9.83	9.45	9.06	8.41	7.93	7.52	7.18	6.91	6.65
14	10.8	10.7	10.3	9.91	9.52	9.15	8.78	8.43	7.83	7.38	7.01	6.69	6.43	6.19
15	10.1	10.0	9.64	9.27	8.92	8.57	8.22	7.91	7.34	6.92	6.57	6.27	6.04	5.80
16	9.54	9.44	9.09	8.75	8.41	8.09	7.77	7.46	6.94	6.54	6.20	5.93	5.70	5.48
17	9.05	8.95	8.62	8.30	7.98	7.67	7.37	7.08	6.58	6.20	5.88	5.62	5.40	5.20
18	8.61	8.52	8.20	7.89	7.59	7.30	7.01	6.73	6.26	5.89	5.59	5.34	5.13	4.94
19	8.21	8.12	7.82	7.52	7.24	6.96	6.68	6.42	5.96	5.61	5.32	5.08	4.89	4.70
20	7.84	7.76	7.47	7.19	6.91	6.64	6.38	6.12	5.69	5.36	5.08	4.85	4.67	4.48
21	7.51	7.43	7.15	6.88	6.62	6.36	6.11	5.87	5.44	5.13	4.86	4.64	4.46	4.29
22	7.20	7.13	6.86	6.60	6.35	6.10	5.86	5.63	5.22	4.91	4.69	4.45	4.28	4.11
23	6.93	6.86	6.59	6.35	6.10	5.87	5.64	5.41	5.02	4.73	4.48	4.28	4.11	3.95
24	6.68	6.61	6.36	6.11	5.88	5.65	5.43	5.21	4.84	4.55	4.32	4.12	3.96	3.80
25	6.44	6.37	6.14	5.90	5.68	5.46	5.24	5.03	4.67	4.39	4.16	3.97	3.82	3.67
26	6.23	6.16	5.93	5.70	5.48	5.27	5.06	4.86	4.51	4.24	4.02	3.84	3.69	3.55
27	6.03	5.97	5.74	5.53	5.31	5.08	4.90	4.71	4.37	4.11	3.89	3.72	3.57	3.43
28	5.85	5.79	5.57	5.36	5.15	4.95	4.75	4.57	4.23	3.98	3.77	3.60	3.46	3.32
29	5.68	5.62	5.41	5.20	5.00	4.80	4.62	4.43	4.11	3.86	3.66	3.49	3.35	3.22
30	5.52	5.46	5.26	5.06	4.86	4.67	4.49	4.31	3.99	3.75	3.56	3.39	3.26	3.13
31	5.37	5.32	5.12	4.90	4.73	4.55	4.37	4.19	3.88	3.65	3.46	3.30	3.17	3.04
32	5.24	5.18	4.99	4.79	4.61	4.43	4.25	4.08	3.78	3.56	3.37	3.21	3.08	2.96
33	5.10	5.05	4.86	4.67	4.49	4.31	4.14	3.98	3.68	3.46	3.28	3.12	3.00	2.88
34	4.98	4.93	4.74	4.56	4.38	4.21	4.04	3.88	3.59	3.37	3.19	3.04	2.92	2.80
35	4.86	4.81	4.63	4.45	4.27	4.10	3.94	3.78	3.50	3.29	3.11	2.97	2.85	2.74
36	4.75	4.70	4.52	4.34	4.17	4.01	3.85	3.69	3.42	3.22	3.05	2.90	2.79	2.68
37	4.64	4.59	4.42	4.25	4.09	3.93	3.77	3.62	3.35	3.15	2.98	2.84	2.73	2.62
38	4.55	4.50	4.34	4.17	4.01	3.85	3.70	3.55	3.28	3.09	2.92	2.78	2.67	2.56
39	4.47	4.42	4.25	4.09	3.93	3.78	3.63	3.48	3.22	3.03	2.86	2.73	2.62	2.51

TABLE VII—Continued

*Extended Distances for PcP—Continued*

$\Delta^\circ$	Depth h =													
	Surface	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·11	0·12
40	4·38	4·34	4·18	4·02	3·86	3·71	3·56	3·42	3·16	2·97	2·81	2·68	2·57	2·47
41	4·31	4·26	4·10	3·95	3·79	3·64	3·50	3·36	3·11	2·92	2·76	2·63	2·53	2·42
42	4·24	4·19	4·03	3·88	3·73	3·58	3·44	3·30	3·05	2·87	2·72	2·59	2·48	2·38
43	4·17	4·13	3·97	3·82	3·67	3·53	3·39	3·25	3·01	2·82	2·67	2·55	2·44	2·34
44	4·11	4·07	3·91	3·76	3·62	3·47	3·34	3·20	2·96	2·78	2·63	2·51	2·41	2·31
45	4·05	4·01	3·86	3·71	3·57	3·42	3·29	3·16	2·92	2·74	2·59	2·47	2·37	2·27
46	3·99	3·95	3·80	3·66	3·52	3·38	3·24	3·11	2·88	2·70	2·56	2·43	2·34	2·24
47	3·94	3·90	3·76	3·61	3·47	3·33	3·20	3·07	2·84	2·67	2·52	2·40	2·31	2·21
48	3·89	3·85	3·71	3·57	3·43	3·29	3·16	3·04	2·81	2·63	2·49	2·37	2·28	2·18
49	3·85	3·81	3·67	3·53	3·39	3·26	3·14	3·00	2·77	2·60	2·46	2·34	2·25	2·16
50	3·81	3·77	3·62	3·49	3·35	3·22	3·09	2·97	2·74	2·57	2·43	2·32	2·22	2·13
51	3·77	3·73	3·59	3·45	3·32	3·19	3·06	2·93	2·71	2·55	2·41	2·29	2·20	2·11
52	3·73	3·69	3·55	3·41	3·28	3·15	3·02	2·90	2·68	2·52	2·38	2·27	2·17	2·08
53	3·69	3·65	3·51	3·38	3·25	3·12	2·99	2·87	2·65	2·49	2·36	2·24	2·15	2·06
54	3·65	3·62	3·48	3·35	3·22	3·09	2·97	2·84	2·63	2·47	2·33	2·22	2·13	2·04
55	3·62	3·58	3·45	3·31	3·19	3·06	2·94	2·82	2·60	2·44	2·31	2·20	2·11	2·02
56	3·58	3·55	3·41	3·28	3·16	3·03	2·91	2·79	2·58	2·42	2·28	2·17	2·08	2·00
57	3·55	3·52	3·38	3·25	3·13	3·00	2·88	2·76	2·55	2·39	2·26	2·15	2·06	1·98
58	3·52	3·49	3·35	3·22	3·10	2·98	2·86	2·74	2·53	2·37	2·24	2·13	2·05	1·96
59	3·49	3·46	3·32	3·20	3·07	2·95	2·83	2·72	2·51	2·35	2·22	2·11	2·03	1·94
60	3·46	3·43	3·30	3·17	3·05	2·93	2·81	2·69	2·49	2·33	2·20	2·10	2·01	1·93
61	3·44	3·40	3·27	3·15	3·02	2·90	2·79	2·67	2·47	2·31	2·19	2·08	1·99	1·91
62	3·41	3·37	3·25	3·12	3·00	2·88	2·77	2·65	2·45	2·30	2·17	2·06	1·98	1·90
63	3·39	3·35	3·22	3·10	2·98	2·86	2·75	2·63	2·43	2·28	2·15	2·05	1·96	1·88
64	3·36	3·33	3·20	3·08	2·96	2·84	2·73	2·62	2·41	2·26	2·14	2·04	1·95	1·87
65	3·34	3·31	3·18	3·06	2·94	2·82	2·71	2·60	2·40	2·25	2·13	2·02	1·94	1·86
66	3·32	3·29	3·16	3·04	2·92	2·81	2·70	2·59	2·39	2·24	2·11	2·01	1·93	1·85
67	3·30	3·27	3·15	3·03	2·91	2·79	2·68	2·57	2·37	2·23	2·10	2·00	1·92	1·84
68	3·29	3·25	3·13	3·01	2·90	2·78	2·67	2·56	2·36	2·22	2·09	1·99	1·91	1·83
69	3·27	3·24	3·12	3·00	2·88	2·77	2·66	2·55	2·35	2·21	2·09	1·98	1·90	1·83
70	3·26	3·23	3·11	2·99	2·87	2·76	2·65	2·54	2·34	2·20	2·08	1·98	1·90	1·82
71	3·25	3·21	3·09	2·98	2·86	2·75	2·64	2·53	2·34	2·19	2·07	1·97	1·89	1·81
72	3·24	3·20	3·08	2·97	2·85	2·74	2·63	2·52	2·33	2·18	2·06	1·96	1·88	1·81
73	3·23	3·19	3·07	2·96	2·84	2·73	2·62	2·52	2·32	2·18	2·06	1·96	1·88	1·80
74	3·22	3·18	3·07	2·95	2·84	2·72	2·62	2·51	2·32	2·17	2·05	1·95	1·87	1·80
75	3·21	3·18	3·06	2·94	2·83	2·72	2·61	2·50	2·31	2·17	2·05	1·95	1·87	1·79
76	3·20	3·17	3·05	2·94	2·82	2·71	2·60	2·50	2·31	2·16	2·04	1·94	1·87	1·79
77	3·19	3·16	3·04	2·93	2·82	2·71	2·60	2·49	2·30	2·16	2·04	1·94	1·86	1·79
78	3·19	3·16	3·04	2·92	2·81	2·70	2·59	2·49	2·30	2·15	2·03	1·94	1·86	1·78
79	3·18	3·15	3·03	2·92	2·81	2·70	2·59	2·48	2·29	2·15	2·03	1·93	1·85	1·78





