INCREMENTAL DESIGN IN ROCK MECHANICS

D. F. Coates and M. Gyenge

DEPARTMENT OF ENERGY,
MINES AND RESOURCES

Mines Branch Monograph 880
1973

ERRATA

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Mines Branch Monograph 880, 1973

by

D. F. Coates and M. Gyenge

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For three-dimensional case:

$$A = (1 - 0.5\, L/Y)\, R\, (1 + h)$$
$$B = h\,(0.8k + 0.2\, nk + 0.2\, n k_y)$$
$$C = h\, n$$
$$D = 1.8\,(1 - 0.55\, L/Y)\,(1 - r)\,(1 + h)$$

INCREMENTAL DESIGN IN ROCK MECHANICS

D. F. Coates and M. Gyenge

Mining Research Centre

**MINES BRANCH
DEPARTMENT OF ENERGY,
MINES AND RESOURCES**

A monograph on how to obtain some practical use from rock mechanics theory and research by extrapolating from past experience

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1973**

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FOREWORD

In the winning of mineral resources from the crust of the earth, the problems distinguishing mining from other industrial activities arise from the need, first, to break and remove ore rock while, second, to maintain control over the surrounding rock, i. e. rock breakage and ground control. For this reason, the basic science for mining is the subject of rock mechanics.

However, rock mechanics is a difficult scientific area; practical results from research have not come easily. The subject has required much encouragement, sustained research, and the training not only of mining engineers but also of personnel from the fields of geology, physics, mechanical and civil engineering. For this reason, the monograph "Rock Mechanics Principles", first issued by the Mines Branch in 1965 and now in its third edition, was produced; a French edition has been issued by the Branch, and a Spanish edition has been published by Litoprint, Madrid.

Now, after much research but before the subject has really had time to develop into an engineering tool, this new volume has been written in an attempt to provide the men with the problems some assistance on ground control and rock breakage. Through the novel approach of 'incremental design' some use is made of the large amount of research that has been conducted in rock mechanics. This is part of the Mines Branch's continuing effort to provide a practical outlet for the data that is accumulated and hence to ensure the optimum use for the nation of our mineral resources.

John Convey
Director

Ottawa, Canada.
July 1973

CONTENTS

INTRODUCTION

The purpose of this volume is to assist design and operating engineers to utilize the subject of rock mechanics. A large amount of research has been undertaken on theory and rock testing. To make use of some of this basic work when making practical decisions, it is necessary to recognize present limitations. A comprehensive design of a rock structure, comparable to building design, is seldom possible. However, the effect of changes in the rock formations can be predicted, which gives rise to the principle of incremental design.

Incremental design is the process of extrapolating from the known to the new, or of predicting the conditions that result from a change in the present operations. In this way, knowledge of the mechanical properties of a specific rock mass obtained from experience can be used for future design requirements. At present this approach is automatically used in the mining field where there is no comprehensive theory of stability and where a theoretical optimum design is seldom attainable because of the constantly changing conditions. Explicit incremental design changes, within practical limits, can shift the system towards a more efficient combination of unit operations.

Consider the design of an open pit wall. Suppose that the steeply dipping footwall of an iron orebody is dolomite and that a previous open pit in the same orebody exposed a natural vertical dolomite face with a slope of 72^{o} for a height of 400 ft. Also suppose that for a new pit, the average slope angle in the dolomite has to be determined for an ultimate depth of 1000 ft. Structural drilling shows that there are no major planes of weakness on which a slide could occur. It is possible to estimate the average slope angle for the new open pit by extrapolating from the data obtained from the previous slope to provide equivalent stability to that experienced in the old one. If the pre-mining stress conditions are the same in the two areas, 51^{o} is the calculated equivalent angle for the new pit.

Or consider the motion caused by blasting, which can affect structures and underground openings. Equations have been developed from field measurements that correlate acceleration, velocity and displacement with the weight of explosive, distance to the structure and rock properties. These equations can be used for extrapolating from field measurements on a particular site to a case where a different weight of explosive or a different distance applies. For example, 200 ft from the centre of gravity of a blast with 80,000 lbs of explosive in a single delay, a maximum radial velocity of 8fs was measured 5 ft below the surface. A blast of 70,000 lbs in a single delay 1000 ft from a building raises the question of possible damage; extrapolation from the previous measurements indicates that the velocity at the new location should be 0.4fs (0.2fs is considered safe).

In another context, suppose there is to be a change in the effective mining span for operations in a flat-lying orebody. The mine is intersected by a series of thick vertical dikes, striking parallel to the strike of the orebody. Stope and pillar mining proceeds between the dikes. The distance between the dikes for the new zone, which is to be developed, is 1000 ft instead of the previous 400 ft, where failure occurred in about 1 in 50 pillars. Using a hypothesis relating pillar loading, strength and the variability of these values, the probability of failure in the new zone, using the same size of pillars and stopes as before, is predicted as being close to 2 in 50 pillars. It may be judged that redesign of the mine layout, in these circumstances, would be appropriate.

By following such procedures and by assuming that the mining and geological domains are the same in the two cases, quantitative predictions can be made. In many cases, the same numbers could be obtained using the judgement of an experienced mining engineer; in some cases, they could not. Although the incremental design predictions may not be of high accuracy, they do provide a basis for planning appropriate action. In the chapters that follow an attempt has been made to illustrate the incremental design principle. However, the examples and indeed the equations are considered to be less important than the stimulation they may provide towards potential application of the principle by staff engineers who develop their own, more practical applications.

SLOPES

SYNOPSIS

Slope failure has been defined as any instability that either affects operations or requires redesign of the slope. For example, failure of a large segment of a slope may only produce a minor readjustment in its geometry, not affecting operations. On the other hand, even without an explicit failure, the working of a high steep slope could affect operations. This aspect, together with the difficulty of determining the strength of the rock mass including all the structural features, underlines the particular value of incremental design for rock slopes. Theory can then be used to make valid extrapolations, which will be less sensitive to differences between actual and assumed conditions than when attempting to make a completely new design from first principles.

For the common occurrence of instability resulting from a critically oriented structural discontinuity, relations have been established between critical height and the slope angle (1). Based on these relations, the following incremental design equation can be used to extrapolate from slopes that have been excavated to slopes that are yet to be excavated in the same formation:

$$i_2 = (i_1 - b_1)(H_1/H_2)^{1.5}(A_1/A_2) + b_2$$

where i is the slope angle, b is the dip of the critical discontinuity, H is the height of the slope, $A = b - f(1 - 0.1(D/H)^2)$, f is the angle of friction along the plane dipping at b, D is the height of the ground water level behind the crest of the slope, the subscript 1 identifies the parameters for the known slope with the acceptable degree of stability or instability, subscript 2 referring to the new slope whose angle, i_2, is to be determined for the same degree of stability.

A similar equation can be used where the mode of failure would be by rotational shear, as commonly occurs in soils:

$$i_2 = (H_1/H_2)(i_1 - B_1) + B_2$$

where H is the height, or equivalent height if there is a surcharge load of waste at the crest, and $B = f(1.2 - 0.3D/H) - 15$.

An alternate criterion for acceptable slope behaviour can be the deformation of the crest. In this case, for incremental design purposes, the following equation could be used:

$$\tan i_2 = \tan i_1 \, (K_1H_1/K_2H_2)^2$$

where K is the ratio of horizontal to vertical pre-mining rock stresses.

Consistent with the approach of extrapolating from experience is the design of support systems for rock slopes that are steeper than the acceptable angle. In some cases, it would be cheaper to use a steeper slope spending some of the savings on excavation to install a support system that will guarantee equivalent stability. A rock anchor system could then be designed using the following equations:

$$P = \frac{WS(\sin i - s \cos i \tan f)}{J(\cos(i+5) + \sin(i+5)\tan f)}$$

$$S = \frac{PJ(\cos(i+5) + \sin(i+5)\tan f)}{W(\sin i - s\cos i \tan f)}$$

$$L_j = \frac{(H - (n-1)H_b)\sin(a-1)}{\sin a \sin(i+5)} + 15$$

where P is the required capacity of the anchors, W is the weight of the supported rock per linear foot of wall, i is the stable slope angle without support, S is the spacing of the anchors along one bench, f is the angle of internal friction along the plane dipping at i, J is the number of anchors in one vertical section, H is the height of the slope in feet, a is the steeper slope angle to be achieved by the rock anchors, s is the correction factor accounting for the effects of seepage as explained in the text, L_j is the required length of an individual anchor, j is the number of the anchor and H_b is the height of the bench.

INCREMENTAL DESIGN

INSTABILITY

The conventional concept of failure is that when stress exceeds strength the material breaks down. For rock slopes, failure is classified according to the way in which stress has exceeded the strength of the rock as shown in Fig 2-1.

At the same time, the practical consequences of failure are not necessarily tied to the theoretical concepts. For example, failure according to Fig 2-1 may not affect open pit mining operations; minor rock falls might be tolerated and, more noteworthy, the case of a failure which includes a large segment of the slope may just produce a minor readjustment in the geometry of the slope. Conversely, without an explicit failure according to one of the above four classifications the obvious working of a high steep slope could still affect operations. For these reasons, it has been suggested that failure should be defined as anything that either affects operations or requires redesign of the slope, which adds a certain clarification to the subject but does not contribute to the problem of analysing and predicting failures.

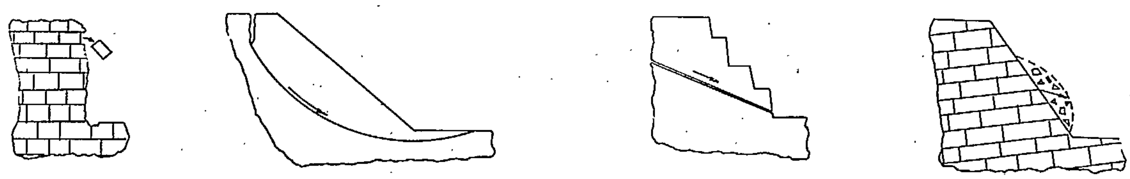


Fig 2-1 **Classification of Slope Failure by** Mechanisms: (a) failure is the consequence of loosening and separation, (b) this failure requires some yielding like in soils and then the development of a failure arc, (c) this is the most common failure mechanism - sliding on faults, joints, bedding and the like, and (d) failure is the consequence of the crushing and breakdown of the rock mass due to deformation produced by the excavation.

Another complicating factor that must be recognized is that the geological conditions are not easily represented in terms of mechanics. Whereas the strength of the rock substance, e. g. a piece of solid core, can be easily determined in the laboratory, the strength of the rock formation is very difficult to test in the field. Furthermore, the pre-mining stress conditions, which can govern the stress distribution in the slope after it has been cut, are not only expensive to determine but, in many cases, are as complex as either the mineralogical composition or the structural regime within the rock formation.

For all of these reasons, incremental design for rock slopes is particularly valuable. By extrapolating from known conditions, the testing of the mechanical properties of the rock formations has, in effect, been achieved by experience. The questions of the appropriate mechanics or practical concepts of failure can be avoided by accepting the degree of stability or instability that has been previously experienced. Furthermore, 'instability' in this context can be 'significant instability that affects operations'. Rock mechanics theory and knowledge of rock properties can be used to make valid extrapolations; however, in this case the calculations obviously will be much less sensitive to differences between actual and assumed condition than would be the case for a completely new design from first principles.

In all design analyses, including incremental design, wherever possible the variability of the various factors should be explicitly recognized. For example, the slope angle, slope height, attitude and frequency of structural features, mass density of the rock, pre-mining stress conditions and ground water levels all are subject to variations either from one station to the next or from one time to another. The method of explicitly using measurements of variability in design is in process of being worked out in the subject of structural engineering. A start has been made to apply similar analyses in rock mechanics. One of the benefits of including this aspect of the design problem is that it leads to an answer in terms of probability of failure (or probability of instability), rather than in terms of factor of safety, and therefore it can be integrated with financial analyses.

Evaluating the economics of slope design is greatly assisted if an estimate can be made of the probability of failure. In this case, recognizing the possibility of monitoring the reaction of the wallrocks to excavations that deepen and/or steepen the slope so that catastrophes can be avoided, it is possible to pursue a minimum cost criterion that includes both the cost of excavation plus the cost of slides. The cost of slides, although difficult to estimate, can be based on experience and can include the probability of failure as indicated in the following equation:

$$C_t = 0.5HXC_e + 0.5HXC_fP_f \qquad 2\text{-}1$$

where C_t is the total cost, H is the height of the slope, X is the horizontal projection of the slope, C_e is the unit cost of excavation, C_f is a unit cost which accounts for the proportion of the triangular rock prism in the slope that would fail and the cost of cleaning up the failed material, and P_f is the probability of such failures. Fig 2-2 illustrates the concepts of Eq 2-1.

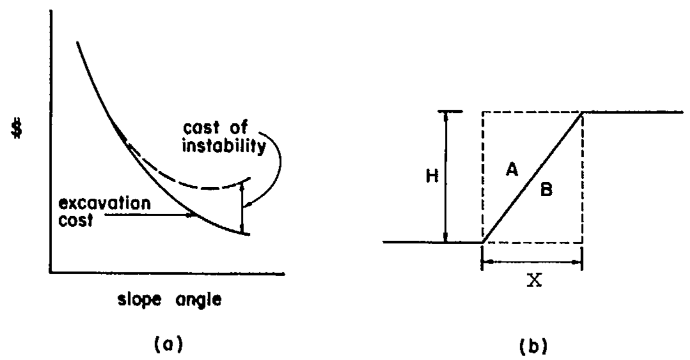


Fig 2-2 (a) total cost of excavation, which includes required excavation plus clean up of any slides, decreases then increases with slope angle, and (b) assumed required excavation prism A; and prism B from which a slide segment can develop.

The probability of failure is affected by several factors. Angle and height of the slope are obviously important. The height of the groundwater table behind the slope is also significant; this is the reason why slides occur most frequently during springtime on many mining properties. Probability of failure increases with the lateral extent of the slope, i. e. with the amount of rock exposed to the slope stress conditions. Many slides occur years after the slope has been excavated and, in some cases, do not extend over the subsequent full height of the wall, which demonstrates the importance of time.

In addition, it is possible that, for the same geometrical and structural conditions, the probability of failure can be affected by the pre-mining stress conditions. Theoretically this effect could result from the concentration of compressive stresses at the toe of the slope, which is greatly increased by tectonic forces. However, instability is more clearly promoted in those walls where the stress conditions produce some imperceptible bulging outwards and horizontal expansion parallel to the face, which permits unlocking of the geological structure; the sliding of benches and larger segments then occurs more readily. Such movement outwards does vary directly with the pre-mining horizontal stresses (3).

GROUNDWATER

Where groundwater tables are high, seepage can influence stability of slopes quite strongly. The effect is primarily the result of buoyancy. Due to this hydrostatic uplift, the normal force on the sliding surface is reduced, which in turn decreases the frictional resistance.

This mechanism is demonstrated in Fig 2-3. In Fig 2-3(a) the force, P, required to cause sliding of a block is:

$$P = F_r = N \tan f = W \tan f \qquad \text{2-2a}$$

where f is the angle of friction of the surfaces.

If this block is placed under water, as shown in Fig 2-3(b), due to the presence of the hydrostatic uplift force, U, the reaction force, N, is reduced from W to N' = W - U. Obviously, due to this reduction, the maximum frictional reaction, F_r', will also be reduced. In other words, the force, P', required to cause sliding is:

$$P' = F_r' = N' \tan f = (W - U) \tan f \qquad \text{2-2b}$$

In terms of stresses, Fig 2-3(c) shows the reduction in normal stresses while the shear stresses remain constant. Stress circle (1) moves to circle (2), which is closer to the failure envelope.

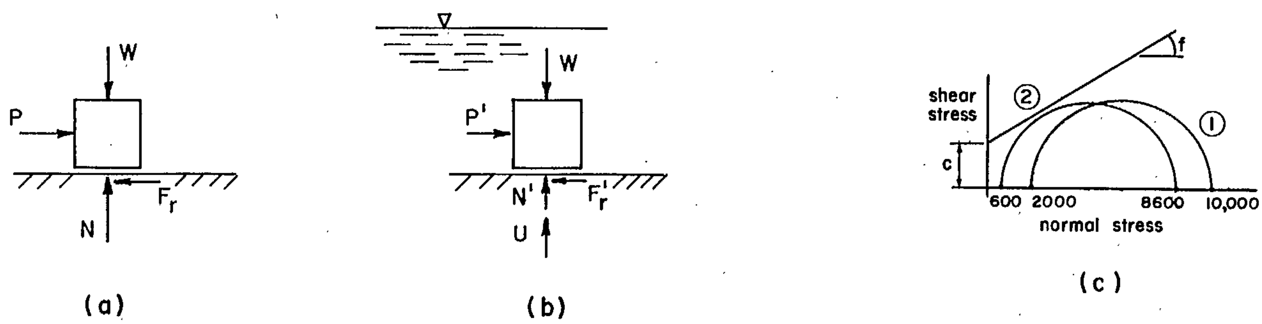
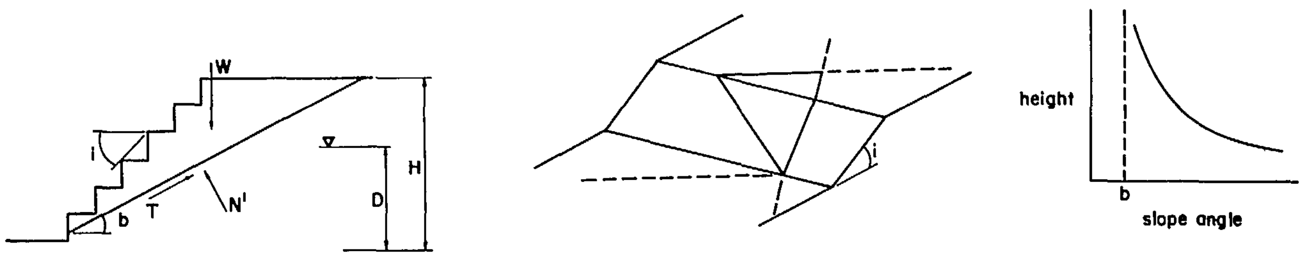


Fig 2-3 Effect of Ground Water on Shear Resistance: (a) and (b) buoyancy, U, reduces the normal force, N to N', on the failure plane, which reduces the frictional resistance, F_r to F_r', (c) the same principle applied to internal stresses: Mohr's circle ② represents the reduction in normal stresses from those shown in circle ①, a consequence of pore water pressure acting between rock grains.

PLANE SHEAR FAILURE

Plane shear describes slope failures where sliding occurs along a geological plane of weakness. In some circumstances the relatively simple case shown in Fig 2-4a will govern the stability of a slope. The stability is dependent on the dip angle of the plane of weakness, b; the cohesion on this plane, c; the angle of friction between the planes, f; and the weight of the sliding segment, W, which is determined by the slope angle, i; the density of the rock, m; and the height of the slope, H. If seepage occurs, the height of the groundwater behind the slope, D, is also an important factor. The practical difficulties encountered in evaluating the possibility of plane shear instability are in locating the surface of weakness along which the slide would occur and in determining the strength characteristics, c and f, of this plane. Consequently, at this stage incremental design is especially pertinent because previous experience, in effect, provides a measure of these factors.



(a) (b) (c)

Fig 2-4 Plane Shear Failure: (a) two-dimensional case, (b) three-dimensional case, and (c) variation of stable slope angle with height of slope.

For incipient plane shear slides curves can be developed, as shown in Fig 2-4(c), relating critical height and slope angle, i, to the above parameters(1). Alternatively, these relations can be expressed as equations, and for incremental design purposes the factors c and m can be eliminated for extrapolation in slopes in the same rock mass. The resultant design equation is as follows:

$$i_2 = (i_1 - b_1)(H_1/H_2)^{1.5}(A_1/A_2) + b_2 \qquad 2\text{-}3$$

where $A = b - f(1-0.1(D/H)^2)$, the subscript 1 identifies the parameters for the known slope with the acceptable degree of stability or instability; subscript 2 is for the new slope whose angle, i_2, is to be determined for the same degree of stability.

Fig 2-4(c) shows a typical curve of variation of stable slope angle with height of slope. Such a curve becomes asymptotic to the dip angle of the structural plane governing stability (all other factors remaining constant). Where the cohesion on this plane is small, the part of the curve close to the b-line governs all cases of practical slope heights - hence in this case the stable slope angle would have very little variation.

The underlying assumption in such extrapolations is that the structural domain of the new slope is the same as that of the previous slopes, and the same structural features govern stability. However, if i_2 is greater than i_1 the possibility should be examined of a more steeply-dipping set of fractures occurring that would now 'daylight'.

Plane shear sliding can occur on more than one plane and also on planes striking obliquely the crest of the slope, i.e. three-dimensional failure wedges can be formed as shown in Fig 2-4(b). If similar wedges are formed by the same structural sets, new slope angles, i_2 - for either changes in height of the wall, H, or in height of groundwater, D, can also be approximated by using Eq 2-3. In this case, the angle b is the steepest plunge angle of the structural boundary planes in the direction of the incipient slide.

Example: - A new open pit is being planned with 45° slopes for the walls. The limestone footwall was exposed in a previous open pit, producing a slope with a height of 500 ft at an average angle of 60°, which showed no signs of instability. The dip of the limestone bedding is 45°. Tests show that the bedding planes have an average friction angle of 39° (including roughness effects). The new open pit is to be 1000 ft deep. It is assumed that the pre-mining stress conditions around the new open pit are the same as in the area of the previous pit. Structural drilling shows that the dip of the bedding is the same and that no faults or other major structural features exist in the footwall of the projected open pit. The groundwater table is 400 ft below the surface. Using the previously gained experience and assuming that failure, if it occurs will be by plane shear on the bedding planes, the question is what would be the slope angle on the footwall of the new open pit for the same degree of stability as before.

Solution: $H_1 = 500\text{ft}$, $i_1 = 60^\circ$, $D_1 = 100\text{ft}$, $b_1 = 45^\circ$, $f = 39^\circ$, $A_1 = 45 - 39\,(1 - 0.1(100/500)^2) = 6.2$;

$H_2 = 1000\text{ft}$, $D_2 = 600\text{ ft}$, $b_2 = 45^\circ$, $A_2 = 45 - 39\,(1 - 0.1(600/1000)^2) = 7.4$

From Eq. 2-3:

$$i_2 = (60-45)\,(500/1000)^{1.5}\,(6.2/7.4) \;+\; 45 \;=\; 49.4$$

Using a slope of 49° instead of 45° would result in a large saving, e.g. at a unit direct cost of \$0.40/T for an average length of pit of 2000 ft, waste excavation would be reduced by more than \$3M.

ROTATIONAL SHEAR FAILURE

Rotational shear failure occurs when a segment of the slope fails by rotation on a more or less circular arc. Research in the field of soil mechanics has established that the failure of slopes in soils usually occurs in this way. Altered rock formations can behave in a similar manner (2).

In Fig. 2-5(a) a typical surface of failure is shown. Failure is caused by the moment of the weight of the failure segment, W, about the centre of rotation, O, i.e. Wx. The resisting moment is the product of the resisting stresses, or the strength of the ground, T_f, along the length of the arc, L, and the radius of the circle, R, i.e. RLT_f. The strength of the ground, consisting in part of friction, is dependent on the normal stresses on the failure surface and also on the buoyant effect of any seeping water as indicated in Fig 2-5(b) and explained in connection with Fig 2-3(b).

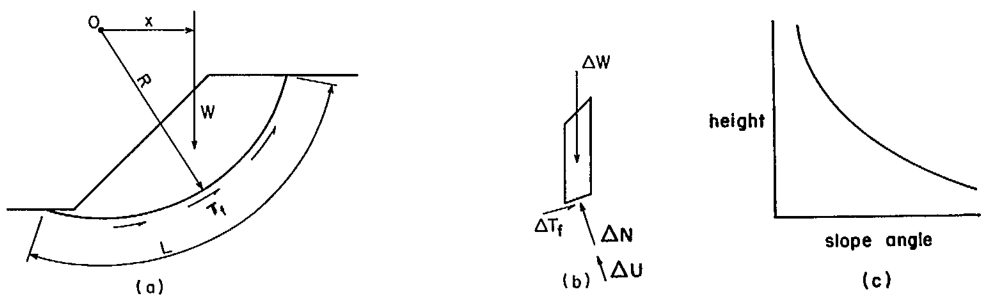


Fig 2-5 (a) Rotational shear failure in yielding ground, (b) the significant forces on a vertical slice through the failure segment, and (c) the typical variation of stable slope angle with height of slope.

Where instability arises from rotational shear, the effective strength characteristics for the rock mass are not easy to determine. However, again for purposes of extrapolation where past experience is used as a test of these properties, appropriate new slope angles can be estimated using the following equation based on approximation curves (1):

$$i_2 = (H_1/H_2)(i_1 - B_1) + B_2 \qquad 2 - 4$$

where i is the slope angle, H is the height of the slope, B = f(1.2 - 0.3D/H) - 15, f is the angle of internal friction of the rock mass, D is the height of the water table measured above the toe of the slope, and subscripts 1 and 2 refer respectively to the existing slope (that either was stable or displayed some instability due to rotational shear) and to the newly designed slope that will have the same stability or degree of instability as the existing one. Fig 2-5(c) shows the typical variation of stable slope angle with height of slope, as represented by Eq 2-4.

Sometimes walls are excavated back to the toe of previous waste dumps, and the question then arises - what is the effect of the waste surcharge on the stability of the wall? Eq 2-3 can be used for this purpose by using, instead of H, $\overline{H} = H + H_w m_w/m$; here H_w is the height of the waste embankment (which is small compared to H) m_w is the bulk density of the waste and m is the bulk density of the insitu wall rock.

Example - An existing open pit is 400 ft deep with walls of altered quartzite at a slope angle of 37°. There have been some rotational shear slides in the walls in the spring when the groundwater table rises to within 50 ft from the surface. However, the cost of the slides has been relatively small owing to either the slide segments being of small volume or else moving sufficiently slowly that the waste pushed out at the toe can be removed without disrupting operations. Consequently, it is judged that this slope angle is close to the optimum for minimum cost of excavation. The pit is now to be deepened to 800 ft. Extrapolating from the previous experience with the wallrocks, what slope angle would now be required for the same degree of stability, or instability, that has been acceptable in the past ? Laboratory tests were performed on the rock showing that it has an angle of internal friction of 34°.

Solution: $H_1 = 400$ ft, $i_1 = 37^\circ$, $D_1 = 350$ ft, $f = 34^\circ$

$H_2 = 800$ ft, $D_2 = 750$ ft

$B_1 = 34\ (1.2 - 0.3 \times 350/400) - 15 = 16.9$

$B_2 = 34\ (1.2 - 0.3 \times 750/800) - 15 = 16.2$

From Eq 2-4: $i_2 = (400/800)^{0.67}(37 - 16.9) + 16.2 = 26.3^\circ$

If the groundwater could be lowered behind the slopes to the elevation approaching the bottom of the pit, the wall slopes could be excavated at an angle of 34°, (the friction angle of the rock) which would reduce the waste excavation by 154,000 cf/LF of wall or at \$0.40/T by \$4600/LF. In other words, it would be profitable if the dewatering could be achieved with certainty at a cost of something less than \$4600/LF of wall. If, however, the drawdown system was only able to lower the water table around the pit to a depth of 300 ft below the surface (i. e. $D_2 = 500$ ft), the comparable slope angle could be calculated using Eq 2-4:

$B_2 = 34(1.2 - 0.3 \times 500/800) - 15 = 19.4$

$i_2 = (400/800)(37 - 16.9) + 19.4 = 29.5^\circ$

It is found that the ultimate crest of one of the walls for the deepened pit will end up at the toe of a 200 ft high waste dump. Whereas the bulk density of the wallrock is 160 pcf, the waste in the dump is 110 pcf. It is now necessary to determine the slope angle for this section of the wall, so that it has stability comparable to that of the existing walls.

$H_1 = 400$ ft, $i_1 = 37^\circ$, $D_1 = 350$ ft, $f = 34^\circ$

$H_2 = 800$ ft, $H_w = 200$ ft, $D_2 = 750$ ft, $m_w = 110$ pcf, $m = 160$ pcf

$\overline{H}_2 = 800 + 200\ (110/160) = 937$ ft

$B_2 = 34\ (1.2 - 0.3 \times 750/937) - 15 = 17.6$

From Eq 2-4: $i_2 = (400/937)\ (37 - 16.9) + 17.6 = 26.2^\circ$

The required slope angle for this section of the wall subject to surcharge is substantially the same as for the other sections, which seems strange. The explanation is that the ratio of the height of the groundwater to the equivalent height of slope is decreased, which means that the increased equivalent height generates almost as much increased shear resistance as increased shear stress.

BLOCK FLOW FAILURE

Block flow failure occurs in a uniform, hard rock mass that cannot fail by plane shear. Deformation causes failure by crushing the corners of the brittle rock blocks which constitute the rock mass; increased load is thus thrown onto the adjacent blocks, whose strength in turn is exceeded, thus leading to a progressive breakdown of the rock mass. This type of failure does not produce a distinct sliding segment but rather it is characterized by internal deformation. The boundary failure surface is not likely to be either circular or planar, and at this stage it cannot be predicted.

For incremental design the results of deformation studies on models can be used until better information becomes available (3). By assuming that the deflection of the crest of the slope after excavation, is a valid measure of critical conditions, the following equation has been derived which can be applied to different slopes in the same rock formation:

$$\tan i_2 = \tan i_1 \, (K_1 H_1 / K_2 H_2)^2 \qquad 2\text{-}5$$

where i is the slope angle, K is the ratio of horizontal to vertical pre-mining rock stresses, H is the height of the slope, subscript 1 refers to the existing slope which is either stable or somewhat unstable due to block flow failure, and subscript 2 refers to the new slope in the same rock with the same wall direction for which the same stability or degree of instability can be tolerated.

Example: - An existing open pit has been mined down 1000 ft using slope angles for the walls of 45°. The pit is to be extended down to 2000 ft, and primary crushers are to be placed at the elevation of the bottom of the present open pit with conveyors running to the surface either through a cut in one of the walls or through a tunnel. To provide some data for the design of the cut slopes, a survey in the same massive porphyry with joints cemented by secondary mineralization shows natural stable slopes up to 700 ft high at 63°. The section of the wall that would be cut for the conveyor belt does not contain any faults or other critical major structural features. Assuming that the field stress conditions in the rock that will contain the cut are the same as those that existed when the natural slopes were created, calculations are made to provide some limits for the cut slopes for depths of 250 ft, 500 ft and 750 ft by extrapolating from the steepest natural slope.

Solution: $H_1 = 700$ ft, $i_1 = 63^o$

$H_2 = 250$ ft, 500 ft and 750 ft, $K_2 = K_1$

From Eq 2-5

$$\tan i_{250} = \tan 63\,(700K_1/250K_2)^2 \quad \therefore \quad i_{250} = 86^o$$

$$\tan i_{500} = \tan 63\,(700K_1/500K_2)^2 \quad \therefore \quad i_{500} = 75.5^o$$

$$\tan i_{750} = \tan 63\,(700K_1/750K_2)^2 \quad \therefore \quad i_{750} = 59.5^o$$

The steepness of these calculated slopes would, however, be tempered by limitations imposed (a) by excavation operations (b) by requirement of a safety berm, and (c) by costs.

The open pit excavation causes the horizontal pre-mining stresses to be deflected around the excavation, thus raising the horizontal stresses in the walls up to approximately twice their pre-mining value. Consequently, the pre-excavation stress normal to the conveyor cut will now actually have a value of K_2 that will vary from K_1 to $2K_1$. The modification to the above computed slope angles, allowing for $K_2 = 2K_1$, is as follows:

$$\tan i_{250} = \tan 63\,(700K_1/250 \times 2K_1)^2 \quad \therefore \quad i_{250} = 75.5^o$$

$$\tan i_{500} = \tan 63\,(700K_1/500 \times 2K_1)^2 \quad \therefore \quad i_{500} = 44^o$$

$$\tan i_{750} = \tan 63\,(700K_1/750 \times 2K_1)^2 \quad \therefore \quad i_{750} = 23^o$$

The effect of the different field stress conditions is quite significant. The degree of stability in the previous slopes extrapolated to these calculated slopes might still be greater than appropriate for the mining operations, although modification of the figures at the present time would have to be substantially based on judgement.

SLOPE SUPPORT

Consistent with the incremental design approach is the design of support systems for rock slopes that are steeper than the optimum slope angle (4). In some rock formations structural features, such as joints, bedding planes or faults, govern the normal maximum slope angle beyond which plane shear failure is likely to occur. Where structural features do not dictate the critical slope angle, experience and economics usually by an evolutionary process limit the critical slope angle for any one formation. It follows that steeper slope angles can only be used if artificial support provides cost savings and the required degree of stability.

In Fig. 2-6(a) the rock is stable at a critical slope angle of i. A support system is to be designed for a greater slope angle of a. Deep rock anchors are to provide the force required to ensure that the additional prism of rock left unexcavated is held onto the slope face. Each rock anchor has a capacity of P, and there are J rock anchors distributed over the height of the slope, H. The horizontal spacing between the vertical rows of rock anchors is S. It is assumed that for ease of installation the anchors will be installed in holes inclined 5^o downwards.

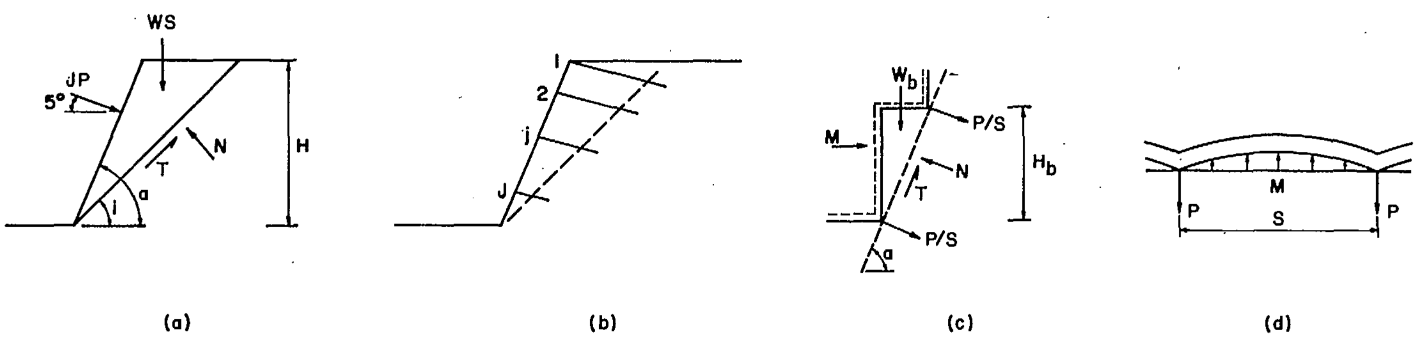


Fig 2-6 (a) The forces on a potential failure wedge including those from the rock anchors, JP, designed to prevent sliding; (b) the length of anchors required to hold a potential failure wedge; (c) the forces on a bench at the point of sliding including the mesh reaction, M, and the supporting stringers, P/S; (d) a plan view of the force from the mesh, M, acting on the stringers supported by the anchors, P.

Because it is very difficult to determine the effective cohesion acting on the plane inclined at the angle i, the conservative assumption is made that cohesion is equal to zero together with the counteracting assumption that no safety factor is required against failure of the steel in the anchor. By analyzing the stability of the wedge of rock, the required capacity of the rock anchors can be determined as follows:

$$P = \frac{WS\ (\sin i - \cos i \tan f)}{J\ (\cos (i + 5) + \sin (i + 5) \tan f)} \qquad 2\text{-}6a$$

$$W = 0.5\ H^2 m (\cot i - \cot a) \qquad 2\text{-}6b$$

where W is the weight of the wedge per linear foot of wall m is the density of the rock mass and f is the angle of internal friction along the plane inclined at i, and J is the number of anchors in one vertical section.

Alternatively, the rock anchor may be previously selected so that the capacity is known; in this case, the horizontal spacing required for these anchors can be determined as follows:

$$S = \frac{PJ\ (\cos (i + 5) + \sin (i + 5) \tan f)}{W\ (\sin i - \cos i \tan f)} \qquad 2\text{-}7$$

The design of such an anchor system must include an economic optimization analysis, the most important element of which is the sum of the lengths of the holes that must be drilled and of the anchors that must be installed. Assuming the vertical spacing of the rock anchors will be equal to the height of the bench, H_b and that the length of anchor required beyond the stable plane inclined at i is 15 feet, the length of the individual anchors, L_j, can be determined from the following equation:

$$L_j = \frac{(H - (n - 1) H_b) \sin (a - i)}{\sin a \sin (i + 5)} + 15 \qquad 2\text{-}8$$

where j is the number of the anchor, starting with 1 at the crest and finishing at the toe with J, the total number of anchors in a vertical section.

In addition to holding the extra wedge of rock above the stable slope angle, a complete support system should also include provision for holding any benches that are left on the steep face. A wire mesh, which is subjected to the forces as shown in Fig 2-6(c), can provide for this requirement. The cross-sectional area of vertical steel, A_m, that must be supplied in the mesh per linear foot of wall, assuming an ultimate capacity of 80,000 psi for this steel, is as follows:

$$A_m = M/80 \qquad 2\text{-}9a$$

where

$$M = \frac{W_b(\sin a - \cos a \tan f)}{\cos a + \sin a \tan f} \qquad 2\text{-}9b$$

and

$$W_b = 0.5\, H_b^2\, m \cot a \qquad 2\text{-}9c$$

Here W_b is the weight in kips per linear foot of wall of the wedge of rock in the bench above the average slope angle a and f is the assumed effective angle of internal friction along the surface dipping at a. According to this analysis, the horizontal steel in the mesh is not stressed; actually, some forms of instability could put these strands under stress, however, their size need only be of nominal dimensions to prevent the fall of loose rock fragments.

In making the force analysis of the wedge of rock comprising a bench, the same conservative assumption is made that the cohesion acting along the plane dipping at a is zero, and the somewhat compensating assumption is made that the ultimate capacity of the steel in the mesh can be used for this situation. In some cases, the amount of vertical steel required in the mesh exceeds that which can be practically emplaced; however, considering the conservative assumptions made in the analysis, in many cases it would not be inappropriate to reduce the calculated requirements on the basis of judgement.

To support the mesh, horizontal stringers are required between the collars of the anchors. Whereas these need only be cables, or steel bars, of requisite strength, to ensure that the mesh is kept snugly against the rock surface a poured reinforced concrete beam is recommended. Nevertheless, for analysing the steel requirements the assumption can be made that under full load the stringer will act in tension in the same way as a cable describing a catenary between the anchor collars. Assuming the catenary curve has a 2-foot sag between the points of support, the following equation can be used to determine the cross-sectional area of steel, A_s, that is required in the stringer.(which must be continuous and connected to the collars of the anchors):

$$A_s = MS/128 \qquad 2\text{-}10$$

where M is the force per linear foot in kips acting on the stringer, caused by the pressure on the wire mesh as shown in Fig 2-6(d), and S is its length.

The above analyses have been made assuming that groundwater does not affect the stability of the slope. Whereas the presence of a relatively high groundwater table behind the slope is not likely to affect the stability of individual benches and hence the design of the mesh and stringers, it could affect the calculated requirements for the anchors. To take into account the effect of seepage in the slope, Eq 2-6a and 2-7. can be written as follows:

$$P = \frac{WS (\sin i - s \cos i \tan f)}{J (\cos (i + 5) + \sin (i + 5) \tan f)} \qquad 2\text{-}11$$

$$S = \frac{PJ (\cos (i + 5) + \sin (i + 5) \tan f)}{W (\sin i - s \cos i \tan f)} \qquad 2\text{-}12$$

where s is a correction factor accounting for the effects of seepage. With detailed information on the location of the surface of the seeping water, a rigorous analysis could be made of the effects on the anchorage requirements. Without such detailed information, a conservative set of values that can be used is as follows: when the ratio of the height of the water table to the height of the slope, D/H = 1, s = 0.88; when D/H = 0.8, s = 0.92; and when D/H = 0.4, s = 0.98.

Example - Along one wall of an open pit mine it is found that the slope angle is governed by a strong set of joints dipping into the pit at 42°. Consideration is being given to steepening this wall by the use of artificial support. The ultimate plans are for the pit to go down 520 ft with benches 40 ft high. The friction angle for the joint surfaces is found to be 37°. No seepage into the pit is expected owing to the elevated location of the site. The bulk density of the rock mass is 170 pcf. The support system would consist of cable anchors with an ultimate capacity of 640 kips (12-0.6 strands) installed at the toe of each of the 13 benches and of such a length as to provide a grouted anchor 15 ft long beyond the stable slope angle of 42°. Welded-wire mesh would be used to retain the benches and loose rock. The minimum anchor length would be 30 ft. The unit costs would be as follows: anchors $175 plus $2.85/LF, drilling $5.00/LF, welded-wire mesh $0.12/lb, stringers $6.00/LF and excavation $0.40/T. What would be the design and cost of such a system for a slope of 48°?

Solution: - $i = 42^\circ$, $f = 37^\circ$, $J = 13$, $a = 48^\circ$, $m = 170$pcf, $H_b = 40$ ft

From Eq 2-6 b : $W = 0.5 \times 520^2 \times 170 (\cot 42 - \cot 48) = 4.85 \times 10^6$ lb/LF $= 4850$ kips/LF

Eq 2-7 $$S = \frac{640 \times 13 (\cos (42 + 5) + \sin (42 + 5) \tan 37)}{4850 (\sin 42 - \cos 42 \tan 37)} = 19.3 \text{ ft}$$

Eq 2-8 $$L_1 = \frac{(520 - 0) \sin (48-42)}{\sin 48 \sin 47} + 15 = 115 \text{ ft}$$

$$L_2 = \frac{(520 - 40) \sin (48 - 42)}{\sin 48 \sin 47} + 15 = 108 \text{ ft}$$

$$L_n = \frac{(520 - (n - 1) 40) \sin (48 - 42)}{\sin 48 \sin 47} + 15$$

$$L_{13} = \frac{(520 - 12 \times 40) \sin (48 - 42)}{\sin 48 \sin 47} + 15 = 30 \text{ ft}$$

Total = 905 ft

Eq 2-9c $W_b = 0.5 \times 40^2 \times 170 \times \cot 48$ = 122,000 lb
122 kips

Eq 2-9b $$M = \frac{122(\sin 48 - \cos 48 \tan 37)}{\cos 48 + \sin 48 \tan 37} = 23.8 \text{ kips}$$

Eq 2-9a $A_m = 23.8/80$ = 0.297 si/LF

The selected standard mesh in 5 ft rolls contains No. 3 wires longitudinally at 3 in. spacing with No. 8 wires transversely at 16 in. spacing, with a longitudinal cross-sectional area of 0.187 si/LF, but with a 2 ft overlap of rolls 0.312 si/LF would be provided; the weight would be 1.20 lb per square foot. The length of mesh, ignoring overlaps, would be 952 ft from crest to toe.

Cost:	Anchors (13 x 175 + 905 x 2.85)/19	= $256/LF of wall
	Drilling 905 x 5/19	= 238
	Mesh 952 x 1.20 x 0.12	= 137
	Stringers 13 x 6	= 78
		$709/LF
Excavation Saving: (4850,000/2000)0.40		$970/LF
Net Saving: (which is 37% of cost)		$261/LF

REFERENCES

1. Hoek, E. "Estimating the Stability of Excavated Slopes in Opencast Mines", Trans. IMM, London, Vol. 9, Sec. A, pp A109-A132, 1970.

2. Coates, D. F. McRorie, K. L. and Stubbins J. B. "Analyses of Pit Slides in Some Incompetent Rocks", Trans. AIME, Vol 226, pp 94-101, 1963.

3. Yu, Y. and Coates, D. F. "Analyses of Rock Slopes using the Finite Element Method", Mines Branch Research Report R229, 1970.

4. Barron, K., Coates, D. F. and Gyenge, M. "Artificial Support of Rock Slopes", Mines Branch Research Report R228 Revised 1971.

GLOSSARY

A - b - f (1-0.1(D/H)^2)

a - slope angle from toe to toe of one bench or a series of benches.

A_m - cross-sectional area of steel required in a mesh

A_s - cross-sectional area of steel required in a stringer

B - f (1.2-0.3 D/H) - 20

b - dip of structural discontinuity

c - cohesion

C_e - unit cost of excavating

C_f - unit cost of cleaning a slide

C_t - total cost

D - height of groundwater above the toe of the slope behind the slope crest

f - angle of friction

F_r - frictional resistance

H - height of slope

H_b - height of bench

H_w - height of waste embankment

$\bar{H}$ - equivalent height of slope in case of surcharge at the crest

i - dip of slope face

J - total number of anchors in one vertical section

j - number of an anchor

K - ratio of horizontal to vertical pre-mining stress

L - length of an anchor; length of the arc in a rotational shear failure

M - force acting on mesh between two lines of anchors

m - bulk density of rock mass

m_w - bulk density of waste

N - normal force

P - load or capacity of an anchor

P_f - probability of failure

R - radius of the rotational failure circle

S - spacing of the anchors along one bench

s - seepage factor

T - shear force reaction of the rock to movement on a plane

U - hydrostatic uplift

W - weight

X - horizontal projection of slope face

BLASTING

SYNOPSIS

Previous experience provides the best basis for either designing blasting patterns or evaluating the effects of blasting. Utilizing the incremental design principles, elements of blasting, such as burden and spacing, can be predicted for a future case on the basis of past experience; providing that only one factor (for example the competency of the ground) is different in the future case compared to previous cases, and providing that the functional relation between the varying factor and the design element is known. Under similar restrictions, the blasting effects, such as ground motion and failure effects on underground openings, can similarly be predicted by the incremental design principles.

If either explosive or rock properties change, the appropriate change in burden, spacing and depth to the centroid of the charge in a bench blast can be predicted by making use of the fundamental relationship governing the formation of craters:

$$Z = KW^{1/3}$$

where Z is the depth to produce either the maximum crater volume or just to break the surface, K is the site parameter for either one of these conditions and W is the weight of the explosive. By conducting crater tests where experience has evolved a design pattern, then for the future conditions design elements can be approximated from the following relations:

$$B_b = B_a Z_b / Z_a$$

$$S_b = S_a Z_b / Z_a$$

$$D_b = D_a Z_b / Z_a$$

where B is the burden, S is the spacing, D the depth to the centroid of the explosive and subscripts a and b refer respectively to the known previous case and the future case.

A similar approach can be used for predicting ground motion caused by blasting, which can affect structures and underground openings. Using the appropriate data sets, which were established by measurements in the same ground, the elements of the ground motion of a future case can be extrapolated by using the following equations:

$$A_b = A_a \, (W_b/W_a)^{0.33} (R_a/R_b)^{2.0} (C_p^b/C_p^a)^2$$

$$V_b = V_a \, (W_b/W_a)^{0.60} (R_a/R_b)^{1.80} (C_p^b/C_p^a)$$

$$D_b = D_a \, (W_b/W_a)^{0.87} (R_a/R_b)^{1.60}$$

where A is the acceleration in the ground at a distance R from the weight of explosives in one delay, W, through ground with a P-wave seismic velocity of C_p; V is the particle velocity of the ground; D is the displacement of the ground.

For failure effects on underground openings from adjacent blasts, extrapolation for the same effects in the same rock can be obtained using the following equation:

$$R_b = R_a \, (W_b/W_a)^{1/3}$$

where R is the distance measured from the centre of gravity of the explosive charge of one delay to the location where a specified failure effect occurs. The limits of the zones of specified damages can be predicted by using the following relationships:

$$R_4 = 1.6 \, R_3 = 2.1 \, R_2 = 4.2 \, R_1$$

where R_1 is the maximum radius of Zone 1 where complete collapse of the tunnel, or underground opening, has occurred, R_2 is the maximum distance of Zone 2 which is characterized by continuous spalling that increases in amount towards the explosion, R_3 is the maximum distance of Zone 3 which is characterized by light spalling, of a relatively uniform thickness, on the surface of the opening nearest to the explosion and R_4 is the maximum distance of Zone 4 where discontinuous spalling still occurs. Consequently, if the distance R_3, for example is known from experience, then in the same rock, this distance can be predicted for a different weight of explosive. Furthermore, knowing this new distance R_3, it is then possible to predict, for example, the distance R_4 to determine the limit of discontinuous spalling.

ENERGY DISTRIBUTION

An examination of current views on blasting mechanics provides a basis for applying the principles of incremental design. When an explosive is detonated in a blasthole, only part of the potential chemical energy is converted to mechanical energy, some of the energy is converted into heat and the chemical reactions may be incomplete. The initial pressure in the hole is of the order of a million psi, which quickly drops to approximately half its peak value, decreasing further with borehole expansion, and finally being eliminated by venting (Fig 3-1). Actually, the pattern varies with the type of explosive. For example, ANFO usually has a peak pressure that is little different from the equilibrium pressure.

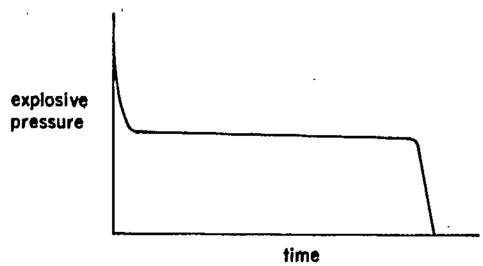


Fig 3-1 Variation of explosive pressure with time: the high detonation pressure drops in short order to about half of its peak value then, as the rock is being pushed away, this pressure decreases, and finally venting eliminates it.

The velocity with which the detonation front moves through the explosive, for various reasons, may be less than the theoretical velocity. The initial peak pressure in the gas usually varies with this detonation velocity, hence it is of some importance to know both actual and theoretical detonation velocities to be able to decide if the priming is adequate.

SHOCK ACTION

The pressure from the explosive is transmitted into the surrounding rock. The resulting dynamic stresses may be significant for hard rocks with their high density and stiffness. An element of rock adjacent to a blasthole may be subjected to radial compression and tangential tension, as shown for an elastic rock mass in Fig 3-2(a)(1). Farther out from the blasthole the pattern of stresses changes somewhat as indicated in Fig 3-2(b).

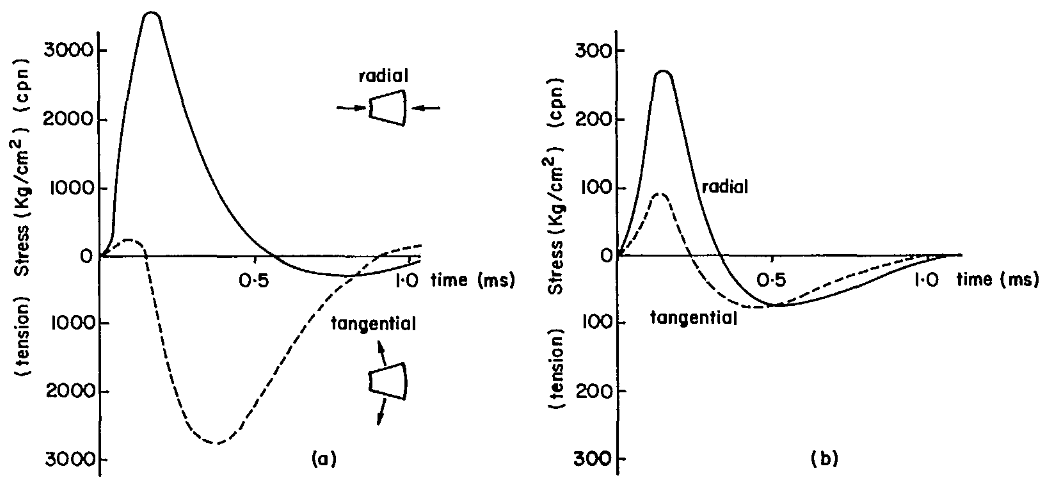


Fig 3-2 Variation of radial and tangential stresses in the rock around a blasthole: (a) close-in where the peak compressive stress can be over 3000 kg/cm^2 (40,000 psi) and the peak tensile stress may be almost as high, and (b) farther out where the peak compressive stress has diminished, due primarily to expansion of the wave front around the hole, and where the tangential stress wave shape is changed.

As a result of the high radial stresses produced in the rock around the blasthole, local crushing can occur in medium strength rocks. On the contrary, in hard rock, after a blast it is not uncommon to see the smooth wall of half of a blasthole. Although the magnitude of the compressive stress in the rock will be much greater than its static strength, the duration of the stress may not be long enough to cause crushing.

The tangential tensile stresses in the rock will cause radial cracking to occur. If such tensile stresses cannot develop owing to the jointed structure of the rock mass, radial cracking can still occur in the rock substance by the splitting action from the high radial compressive stresses (Fig 3-3). The maximum distance to which such cracks extend is called the radius of rupture, R_r.

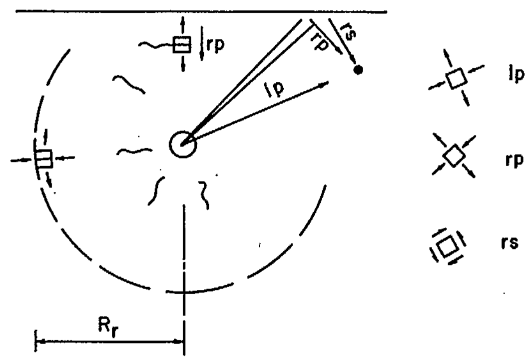


Fig 3-3 Shock wave (ip) radiating from a blasthole producing splitting from radial compressive and tangential tensile stresses; reflecting from a free face as a longitudinal wave (rp) and as a shear wave (rs); the stresses at a point in the rock can be a combination of stresses from these three waves taking into account their different times of arrival. Splitting occurs out to the radius of rupture, R_r.

There is some evidence that the radius of rupture is governed by the peak stress in the shock wave and the tensile strength of the rock substances (1). The tensile strength of the rock substance can be assumed to be proportional to either its compressive strength or to its modulus of deformation, which is proportional to the square of its seismic velocity. Other things being equal, it follows that if the rock becomes more competent, which can be predicted by measuring seismic velocities, the radius of rupture around the blastholes will decrease and the proportion of oversize muck will increase. Tensile stresses can also occur as a result of reflection at a free surface, which can contribute to fragmentation (Fig 3-3).

GAS ACTION

The momentum required for the loosening, expansion and throw of the rock mass comes from the sustained pressure in the blasthole. As this impulse is terminated by venting, the effectiveness of stemming in the blasthole will influence the momentum given to the rock. In addition, the in-place density of the rock, considering the conspicuous changes that occur from changes in grade of iron ore, can affect the magnitude of throw owing to the change in mass that must be moved.

The kinetic energy that is available for moving the rock into a muck pile is the residual amount left from the total mechanical energy of the explosive less: (a) the large losses that occur from venting of the high pressure gases, (b) the energy used up in fracturing the rock, and (c) the stored strain energy and the energy associated with the shock front that has passed through the rock before loosening occurs and translation is started. Some of the shock front energy does get trapped in slabs that are spalled from the surface producing flyrock. However, the mechanism is not important for understanding the practical effects of blasting.

FRAGMENTATION

Geologic structure substantially controls fragmentation. For example, surface slabs bounded by uncemented joints can become flyrock by trapping the momentum of the shock front, similar to slabbing produced by reflected tensile fracturing. High pressure gases from the blasthole can enter joints and bedding planes producing the commonly observed "breaking to bedding". Also, it has been found from experience that bench faces, if possible, should be oriented obliquely to any near-vertical structural features so that the least resistance to loosening and throw is provided, which also assists in minimizing backbreak, high toes and cratering at the crest (2). Another aspect of the geologic environment is the effect of high horizontal tectonic stresses; the splitting of dimension stone has been found to be easy in the direction parallel to the major principle field stress but difficult normal to this direction (3).

It would be highly desirable to be able to predict fragmentation. However, there are indications that in many formations blasting does not cause fragmentation, the rock blocks have already been created by tectonic events;. the blast merely loosens them and throws them into the muck pile. The direct effect of a blast can be relatively insignificant, e. g. the local crushing, the cracking out to the limit of the radius of rapture, the shearing of geologic blocks through which a blasthole happens to pass, and possibly the bending of the slab of rock bounded by the row of blastholes resulting from simultaneous detonations.

Bench blast design with respect to fragmentation is influenced by: the spacing between the blastholes, S; the representative spacing between joints, S_j; and the maximum acceptable size of fragment, M (oversize blocks are larger than this specification). Various combinations of these dimensions are examined as shown below in Table 1.

Table 1

Effects on Oversize Fragmentation of Blasthole Spacing, S, Joint Spacing, S_j, and Oversize Specification, M

Case	S_j:S	S_j:M	S:M	Fragmentation Sensitive to Powder Factor ?	% Oversize
1	$S_j > S$	$S_j > M$	$S > M$	Yes	Medium
2	$S_j > S$	$S_j > M$	$S < M$	Yes	Low
3	$S_j > S$	$S_j < M$	$S < M$	Yes	Low
4	$S_j < S$	$S_j > M$	$S > M$	No	High
5	$S_j < S$	$S_j < M$	$S < M$	No	Low
6	$S_j < S$	$S_j < M$	$S > M$	No	Low

In the above table Cases 2, 3 and 5 are unlikely as they indicate that the spacing of the blastholes is less than the maximum specified size of muck, M. This leaves Cases 1, 4 and 6 all of which have $S > M$.

Of the three likely cases , Case 1 is of low probability as the joint spacing, S_j, is greater than the spacing of the blastholes, S. However, if it occurred, the problem of obtaining fragmentation could be easily resolved with either the conventional use of large diameter blastholes, correspondingly large spacing and above average powder factors or, alternatively with smaller diameter blastholes, which with correspondingly smaller spacing would permit the use of a lower powder factor.

In Case 4, where the representative joint spacing, S_j, is less than that of the blastholes, S, but greater than the maximum acceptable size of muck, M, the problem of a large percentage of oversize fragments is not usually solvable by increasing the powder factor. Reduced size of hole and spacing at the same powder factor can be effective, although it might be more expensive than accepting the cost of secondary breakage with the conventional pattern.

Case 6, which represents the ideal situation of the representative joint spacing, S_j, being less than the maximum specified size of muck, M, permits the use of large diameter blastholes, large spacing and low powder factors.

The above table is somewhat superficial because it deals only with the representative joint spacing, S_j, and the maximum specified size of muck, M. In reality, the variation of spacing between joints and their degree of cementation are also very important with regard to the percentage of oversize fragments produced. Furthermore, whereas it is convenient to characterize fragmentation by the percentage of oversize blocks greater than some specified limit, the gradation of the smaller particles also affects costs, e. g. the greater the proportion of small sizes the lower will be loading, transportation and comminution costs. Also by focusing on the above three characteristics, an additional factor is ignored; the rock at elevations above the charge and particularly in the zones midway between the blastholes probably contribute a disproportionate amount of the oversize, which presents a difficult problem to resolve with current blasting methods (although bottom detonation permits a helpful build-up of the shock wave toward the collar).

The oversize problem of Case 4 can be resolved, although at a price, with reduced hole size, spacing and powder factor. Experiments have shown that not only does this result in better fragmentation, but that the total volume of the fragmented ground caused by a series of small holes is greater than the volume of fragmented ground produced by the same total weight of explosive concentrated in one hole or by the volume of broken ground produced by one isolated small hole multiplied by the number of holes (4, 5). In one mine, representing Case 1 in the above table, experiments showed that by doubling the powder factor the number of oversize fragments was reduced by a little more than half (from 47% to 21%). In the same mine, with a constant powder factor and decreasing the diameter of the hole to about 1/3 of the original size, the number of oversize fragments decreased by about 80% (from 16% to 3%).

In a second mine, representing Case 4 in the above table, when the powder factor was tripled, the number of oversize fragments decreased by less than 1/3 (from 19% to 13%), whereas by reducing the size of hole by 2/3 at a constant powder factor the number of oversize fragments was reduced by about the same amount (from 22% to 16%) (6).

From the above discussion, it can be deduced that compressive and tensile strengths of the rock substance are not normally of great importance with respect to fragmentation, although they do affect drilling costs and, in some cases, the magnitude of the radius of rupture. Joint spacing tends to be of over-riding importance producing cases where formations made up of a weak rock substance can require a greater powder factor than formations with strong rock substances.

The above discussion also indirectly points to another significant economic factor: by increasing the maximum acceptable size of fragment, such as occurs with the use of larger loading and transporting equipment, oversize problems are either eliminated or greatly diminished.

Optimum spacing is usually only determined after some experience with actual conditions. For example, the amount of unbroken toe between holes that results from a selected blasthole spacing is an important factor that can only be learned in the field. The use of delays to produce sequential detonation of blastholes can improve fragmentation (and hence influence spacing selection) by avoiding the lifting off of the face rock as one continuous slab; however, any benefits from the interaction of adjacent blastholes is eliminated. (Interaction of blastholes is significant even though simultaneous detonation for the purpose of achieving shock interaction of the shock waves is not possible, i.e. control of the two detonations is imprecise relative to the 1ms it normally takes for the shock wave to travel from one blasthole to the next).

CRATER TESTS

Situations can arise where optimum fragmentation and throw are being obtained, but conditions change, e. g. a new explosive is to be used, or the rock density or strength are conspicuously different in a new section of the mine. For the redesign of the blasting pattern, crater tests are sometimes useful. In such cases, rather than using judgment alone, guidance can be obtained on the appropriate changes that should be made in burden and spacing by using small scale field tests.

In Fig 3-4 the variations of the volume of broken rock, or crater volume, with depth of embedment of a concentrated charge is shown(7). Each combination of explosive and rock will produce such a characteristic curve. The maximum volume occurs at the optimum depth of embedment, Z_o. The explosive is completely choked and does not produce any crater at the critical depth, Z_c. These relations can be expressed in the simple equations:

$$Z_o = K_o W^{1/3} \qquad 3\text{-}1$$

$$Z_c = K_c W^{1/3} \qquad 3\text{-}2$$

where W is the weight of the concentrated charge, and K_o and K_c are parameters that vary with the rock and explosive properties. It is found that K_o is approximately equal to $0.5 K_c$ for hard rocks and approaches K_c for weak rocks and soil.

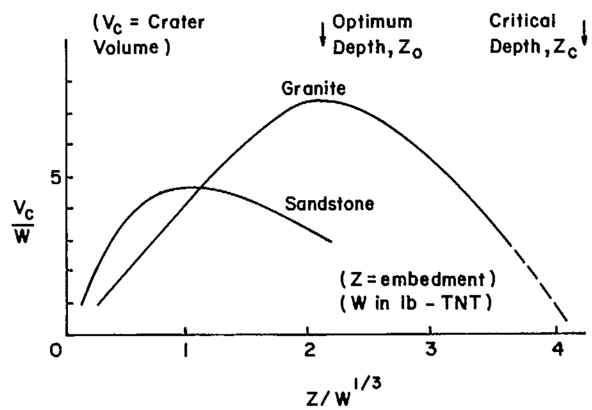


Fig 3-4 Variation of broken rock volume with the depth of embedment for a concentrated charge; a maximum is reached at optimum embedment, Z_o.

From an extensive series of crater tests with a wide variation of charge weights, i. e. from about 10 lb to 100,000 lb, when the depth of embedment was equal to the radius of rupture, it was found that K_c varied from an average value (in ft and lb weight units) of 3.6 for the hard rocks to an average of 5.1 for the soft rocks(7).

A serious deficiency in crater testing is that the size of the rock fragments do not vary with $W^{1/3}$, or any other known factor of W because they are substantially governed by the spacing of the joints. Also, the size distribution will not be the same as in the production blasts because it is affected to some extent by the size of the charge and quite significantly by the spacing of the blastholes. Consequently, cratering tests are of little value with regard to resolving problems of oversize.

To shoot a series of crater tests a level surface should either be available or created. A suitable charge size is then selected, e. g. between 10 lb and 200 lbs. This is largely determined by the size of drill that can be used because the length of the charge should not be much greater than the diameter, say a ratio of less than 3. Craters are obtained for varying depths of embedment so that a characteristic curve, as in Fig. 3-4, can be established. From $Z_o/W^{1/3}$ and $Z_c/W^{1/3}$, K_o and K_c can then be determined. Alternatively, to reduce the number of shots and the work required in mucking out craters, a few shots can be fired on either side of the anticipated critical depth from which only K_c is determined.

If the crater tests are to be used for the initial trial blasting pattern on a property, the burden is made approximately equal to Z_o, and a spacing of 1.25 Z_o can be tried and modified after observing the results (8) (the appropriate relation between burden or spacing and either Z_o or Z_c cannot be accurately predicted, nor can Z_o and Z_c themselves be accurately determined, recognising the difficulty of mucking out craters to their 'true' as opposed to their apparent boundaries). The design of blasts, of course, involves more than just the determination of burden and spacing (e.g. charge distribution, explosive type or types, required fragmentation, throw, number of rows in a blast all affect the selection of burden, B, and spacing, S). Consequently, this approach leaves many questions unanswered, which at the present time are determined from experience and judgment.

Crater testing, therefore, should be used primarily for extrapolating from known conditions to a situation where one factor is different, e.g. the explosive or the rock is different. For the two conditions, two series of craters can be shot, and then the appropriate modifications to the burden and spacing can be made for the changed conditions.

If it is assumed that the burden, B, spacing, S, and depth, D, (i.e. to the centroid of the charge) are proportional to Z_o, then it follows from the two series of crater tests, where other things are equal except the explosive or rock properties, that:

$$B_a/B_b = S_a/S_b = D_a/D_b = Z_o^a/Z_o^b = K_o^a\, W_a^{1/3}/K_o^b\, W_b^{1/3} \qquad 3\text{-}3$$

If the weight of explosives per blasthole is to be kept the same, it follows that:

$$B_a/B_b = S_a/S_b = D_a/D_b = K_o^a/K_o^b \qquad 3\text{-}4a$$

Alternatively, if conditions are such as to make it more appropriate to relate burden, spacing and depth to the critical depth, Z_c, the following equations could be used where the same weight of explosives is used in the two cases:

$$B_a/B_b = S_a/S_b = D_a/D_b = K_c^a/K_c^b \qquad 3\text{-}4b$$

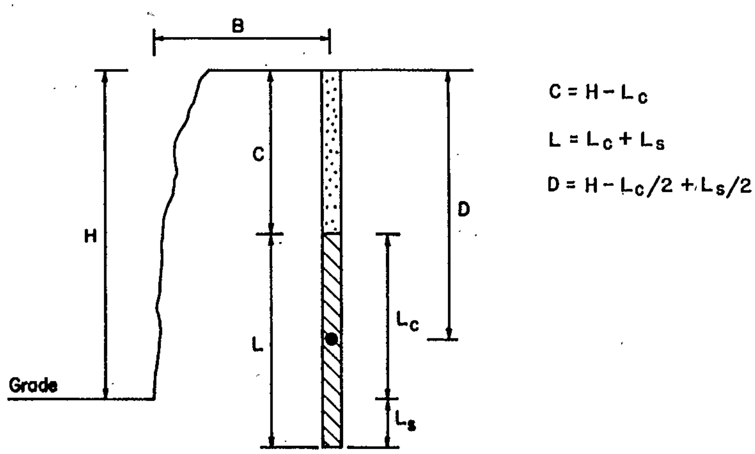


Fig 3-5 Open pit bench geometry with blasthole: D is the depth to the centroid of the charge, L_s is the subgrade drilling, L_c is the explosive column above grade and C is the collar distance.

Example: - A series of crater tests showed that the optimum depth, Z_o, using 27-lb charges of explosive A was 6.5 ft. A second series using the same weight of explosive C produced an optimum depth of 7.5 ft. The burden that is currently being used for bench blasting with 200 lb charges of explosive A is 13 ft. Determine the appropriate burden, B, when using equal weights of explosive C.

Solution: - using Eq 3-1 for $W_a = W_c = 27$ lb, $Z_o^a = 6.5$ ft., $Z_o^c = 7.5$ ft;

$$K_o^a = Z_o^a/W_a^{1/3} = 6.5/27^{1/3} = 2.17$$

$$K_o^c = Z_o^c/W_c^{1/3} = 7.5/27^{1/3} = 2.50$$

using Eq 3-4a for $B_a = 13$ ft.

$$B_c/B_a = K_o^a/K_o^a = 2.5/2.17$$

hence $B_c = 13 \times 2.5/2.17 = 15$ ft

Example: - In a hard iron formation, benches are 42 ft high, burden, B_a, is 25 ft, spacing, S_a, is 35 ft and the powder factor, PF_a, is 0.525 lb/T. At the current production rate of 2MT pa the drilling and blasting cost is $0.10/T.

Mining activity moves from (a) an area with a relatively fresh, hard phase of the formation into (b) an area with an altered, sheared phase. It is envisaged that the radius of rupture will decrease due to greater attenuation, although in a highly jointed formation the radius of rupture is not considered to be a particularly significant parameter. The optimum depth of burial, Z_o, may similarly decrease; however, it is more likely to increase owing to the probability that less energy will be used in shearing the rock during blasting. Throw and flyrock would likely increase with the same charge weight per hole, burden and spacing, and a decrease in the percent of oversize fragments could be expected owing primarily to the decrease in average joint spacing. Under these circumstances, increasing the burden and spacing would not likely increase the percent of oversize fragments and would tend to maintain the throw constant. How much can burden and spacing be increased- 1ft or 5 ft?

To resolve the question a series of crater tests are conducted in the two phases of the formation with the following results: $K_o^a = 2.3$ and $K_o^b = 2.4$ (it is of theoretical interest to see that $K_c^a = 4.3$ and $K_c^b = 3.4$). The crater tests cost $7,500.

Solution: - using Eq 3-4a $B_b = B_a K_o^b / K_o^a$

$$= 25 \times 2.4/2.3 \qquad = 26 \text{ ft}$$

$$S_b = 35 \times 2.4/2.3 \qquad = 37 \text{ ft}$$

$$PF_b = 0.525 \times 25 \times 35/(26 \times 37) \qquad = 0.477 \text{ lbs/T}$$

The saving in drilling and blasting will be (1 - 0.477/0.525) 100 = 9.1% or 2MT x 0.10 x 0.091 = $18,200 pa (which more than pays for the crater tests in the first year).

Example: - On a porphyry copper property using a bench height of 35 ft and a blasthole diameter, d, of 9 in. , the burden, B_a, is 20 ft; spacing, S_a, 25 ft; sub-grade drilling, L_s^a, 7 ft; length of explosive column, L^a, 22 ft; depth, D_a, 35 ft; and powder factor, PF_a, 0.220lb/T. Drilling costs are \$0.015/T and explosive costs are \$0.020/T.

The mix of equipment is to be changed leading to an increase in bench height to 40 ft with all other conditions remaining constant. Determine the appropriate burden, spacing and resultant powder factor. Also determine the change in costs per ton.

Solution: - It is assumed that subgrade drilling should be kept constant, and depth, burden and spacing should be changed:

from Eq 3-3 $$D_b/D_a = K_o^b\, W_b^{1/3}/K_o^a\, W_a^{1/3}$$

note, $$K_o^b = K_o^a \text{ and } W_b/W_a = L^b d_b^2/L^a d_a^2 = L^b/L^a \text{ due to } d_b = d_a$$

therefore $$D_b/D_a = {}^{1/3} = (L^b/L^a)^{1/3}$$

rewriting $$D_b/(L^b)^{1/3} = D_a/(L^a)^{1/3}$$

note, the depth, D, to the centroid of the charge is equal to the collar distance, C, plus half the length of charge, $L_s + L_c$, and the collar distance $C = H - L_c$, hence:

$$D_b = H_b - L_c^b + (L_s^b + L_c^b)/2 = H_b - L_c^b/2 + L_s^b/2$$

note, the explosive column, L, equals the length above the toe, L_c, plus the subgrade drilling, L_s, i.e. $L = L_c + L_s$; therefore:

$$\frac{D_b}{(L^b)^{1/3}} = \frac{H_b - L_c^b/2 + L_s^b/2}{(L_c^b + L_s^b)^{1/3}} = \frac{D_a}{(L^a)^{1/3}}$$

after substitution, the length above grade, L_c^b, can be calculated:

$$\frac{40 - L_c^b/2 + 7/2}{(L_c^b + 7)^{1/3}} = \frac{35}{22^{1/3}}$$

$$L_c^b = 16 \text{ ft}$$

and adding the subgrade $L^b = 23$ ft

therefore, knowing $B_b = B_a(L^b/L^a)^{1/3}$

$= 20\ (23/22)^{1/3} = 20.3$ ft

say $= 20$ ft

similarly, knowing $S_b/S_a = (L_b/L_a)^{1/3}$ $S_b = 25\ (23/22)^{1/3} = 25$ ft

hence $PF_b = 0.220(25/22)(35 \times 20 \times 25)/(40 \times 20 \times 25) = 0.219$ lb/T

and change in drilling costs $= 0.015(47/42)(35 \times 20 \times 25)/(40 \times 20 \times 25) = 0.0147$/T

change in explosive costs $= 0.020(23/22)(35 \times 20 \times 25)/(40 \times 20 \times 25) = 0.0183$/T

savings per year $= (0.015 - 0.0147 + 0.020 - 0.0183)$ 2MT = \$4,000

COLUMN CHARGES

The breaking of rock and moving it out into the muck pile at the bottom of the bench is impeded by the resistance arising from the rock below grade. Greater concentrations of explosive are thus required in this zone than at higher elevations in the bench. It has been found that column lengths of 0.3B above and below grade are fully effective in breaking out the rock at the toe; however, beyond these lengths the effectiveness of the charge for this purpose decreases, e.g. a column above grade equal in length to the burden, B, is equivalent to a concentrated charge of 0.6 Bw, where w is the weight of explosive per column length (9). For example, in establishing the burden at the toe of a bench from crater tests, assuming $B = Z_o = K_o W^{1/3}$, when $L_s = 0.3B$ and $L_c = B$, the following relation is produced:

$$B = Z_o = K_o(0.3Bw + 0.6Bw)^{1/3}$$

hence

$$B = 0.95\, K_o^{3/2}\, w^{1/2}. \qquad 3\text{-}5$$

On the other hand, when the breakage action of the column of explosive above the toe is considered, it must be recognized that the radius of rupture is larger for cylindrical dispersion around the column than for the spherical dispersion around the concentrated bottom charge. Consequently, at the upper elevations lower loading densities are required to achieve the same breakage and throw, otherwise the throw may be excessive. From tests in a granite, the charge weight per unit length of the hole for the column higher than B above grade should only be about 0.4 of that in the bottom charge (9). This leads to the consideration of such devices as the use of weaker explosives at the top of the column and the decking of the charges. The requirement for lower intensity of explosive towards the top of the column is reinforced by the presence of the inevitable slope on the bench face, which normally makes the burden at the crest much less than that at the toe (unless inclined holes are used) and the usual fracturing from the subgrade blasting associated with the previous lift.

Finally, it should be kept in mind that in the same way that the minimizing of the cost of explosions is not as significant as minimizing the total cost of drilling and blasting, so primary breakage costs are only one part of the total mining costs. Optimum operating conditions, recognizing the dependency of loading, hauling and crushing-grinding on the degree of fragmentation at the face, may not be obtained when drilling and blasting costs are a minimum.

Example: The current successful blasting pattern in a hard rock formation using 9 7/8-in diameter holes, d_a, in a 42 ft bench, H, is: burden, B_a, 25 ft; spacing, S_a, 35 ft; depth to centre of gravity of explosive, D_a, 30 ft; sub-grade drilling, L_s^a, 4 ft; explosive column above grade, L_c^a, 28 ft; weight of explosive per hole, W_a, 1617 lb and powder factor, PF_a, 0.525 lb /T. Consideration is being given to introducing larger drills that will produce 12-in diameter holes, d_b, for the same bench height. Whereas the cost of drilling now is \$3.60/LF for the 9 7/8-in holes, it is estimated that this will increase to \$4.00/LF for the 12-in holes. The cost of explosives in place is currently \$0.120/lb and is expected to be reduced with the larger holes to \$0.116/lb. The density of the ore is 12 cf/T.

Determine the blasting pattern that would be appropriate for the 12-in. diameter blastholes. How much would the cost saving be for an annual production of 2MT?

Solution: - Consideration might be given to using Eq 3-5; however, because the length of the column of explosive above grade exceeds the burden, without changing the load density the design is not consistent with the concepts behind this equation. Consequently, for extrapolation purposes the simpler criteria will be used of maintaining the scaled burden, spacing and depth constant, which is done through Eq 3-3. Because K_o is constant for the two cases, Eq 3-3 becomes:

$$B_a/B_b = S_a/S_b = D_a/D_b = (W_a/W_b)^{1/3}$$

note, $W_a = L^a d_a^2$ and $w_b = L d_b$

therefore $(W_a/W_b)^{1/3} = (L^a d_a^2/L^b d_b^2)^{1/3}$

because $d_b > d_a$, it is anticipated that $L^b < L^a$

it follows that $W_a/W_b > d_a^2/d_b^2$

then from $D_a/D_b = (W_a/W_b)^{1/3}$ $\quad D_b < D_a(d_b/d_a)^{2/3}$

$< 30\ (12/9.875)^{2/3} \quad < 34$ ft say 33 ft ,

an increase in drill hole diameter obviously involves an increase in the burden and therefore an increase in the length of subgrade drilling; assume that the subgrade drilling is increased by 1 ft.

hence $L_s^b = L_s^a + 1$

$= 4 + 1 \quad = 5$ ft

now from Fig 3 - 5 $\quad D_b = H - L_c^b + (L_s^b + L_c^b)/2$

$33 = 42 - L_c^b + (5 + L_c^b)/2$

therefore $L_c^b = 2(42 - 33 + 2.5) \quad = 23$ ft

and $L^b = 23 + 5 \quad = 28$ ft

now $W_b = 1617\ (12/9.875)^2(28/32) = 2095$ lbs

check for consistency: $D_b = D_a\ (W_b/W_a)^{1/3}$

$= 30\ (2095/1617)^{1/3} \quad = 32.6$ ft

$= 33$ ft

therefore $B_b = 25\ (2095/1617)^{1/3} \quad = 27$ ft

$S_b = 35\ (2095/1617)^{1/3} \quad = 38$ ft

$PF_b = 0.525(25 \times 35/27 \times 38)(2095/1617) = 0.580$ lb/T

The change in costs are:

drilling $\quad = \dfrac{47 \times 4.00}{42 \times 27 \times 38/12} - \dfrac{46 \times 3.60}{42 \times 25 \times 35/12} = -\$0.0017/T$

explosive $\quad = \dfrac{2095 \times 0.116}{42 \times 27 \times 38/12} - \dfrac{1617 \times 0.120}{42 \times 25 \times 35/12} = +0.0043/T$

Hence costs wi ll be increased by \$0.0026/T; it turns out that <u>the bench is not high enough to benefit from the increased size of drill.</u>

GROUND MOTION

The ground motion caused by blasting, which can affect structures and underground openings, depends on many parameters besides the size of the blast and the distance from it. The seismic velocity of the ground, the detonation velocity of the explosive, the time duration of the explosive process , the depths below ground surface of the centre of gravity of the charge and the point where the ground motion affects other structures all can be important. For purposes of guidance, the following equations have been developed from field measurements (10, 11):

$$A = e\, W^{0.33} R^{-2.0} C_p^2 \qquad 3\text{-}6$$

$$V = f\, W^{0.60} R^{-1.8}\, C_p \qquad 3\text{-}7$$

$$D = g\, W^{0.87} R^{-1.6} \qquad 3\text{-}8$$

where A is the acceleration in the ground at a distance of R from the weight, W, of explosives in one delay, through ground with a P-wave seismic velocity of C_p; V is the particle velocity of the ground; D is the displacement of the ground; and e, f and g are constants for a particular site.

Based on the above equations the following equations can be used for incremental design purposes, where field measurements have been taken on a particular site and it is desired to predict the results for some change in conditions:

$$A_b = A_a\, (W_b/W_a)^{0.33} (R_a/R_b)^{2.0} (C_p^b/C_p^a)^2 \qquad 3\text{-}9$$

$$V_b = V_a\, (W_b/W_a)^{0.60} (R_a/R_b)^{1.80} (C_p^b/C_p^a) \qquad 3\text{-}10$$

$$D_b = D_a\, (W_b/W_a)^{0.87} (R_a/R_b)^{1.60} \qquad 3\text{-}11$$

Example: - At 200 ft from the centre of gravity of a blast with 82,000 lb of explosive in a single delay, the maximum radial velocity of 8 fs was measured at a depth of 5 ft below the surface.

What will be the particle velocity in the ground motion at a building 1000 ft from a blast of 70,000 lb in a single delay which uses the same blasting system as for the above measurement and the shock passes through the same type of rock?

Solution: - Using Eq 3-10 for V_a = 8fs, W_a = 82,000 lb, W_b = 70,000 lb, R_a = 200 ft, R_b = 1,000 ft, and $C_p^a = C_p^b$:

$$V_b = V_a\, (W_b/W_a)^{0.60} (R_a/R_b)^{1.80} (C_p^b/C_p^a)$$

$$= 8(70{,}000/82{,}000)^{0.60} (200/1000)^{1.80} (1) = 0.400\text{ fs}$$

(0.2fs is considered safe)

EFFECTS ON UNDERGROUND OPENINGS

To provide some empirical information for the design of underground defence installations, an extensive series of tests was conducted using concentrated charges of 10 to 320,000lb(7). As a result of detonating charges adjacent to tunnels, four zones of damage were classified. Zone 1 (see Fig 3-6) is defined as the length of tunnel where complete collapse or break-through to the crater occurs. Zone 2 is characterized by continuous rock breakage increasing in amount towards the explosion. The failed rock originates from most of the perimeter of the tunnel in this zone. Zone 3 is characterized by continuous rock breakage of a relatively uniform thickness from the surface of the tunnel nearest to the explosion. Zone 4 is characterized by discontinuous rock failure, probably arising from the shaking down of previously loosened material.

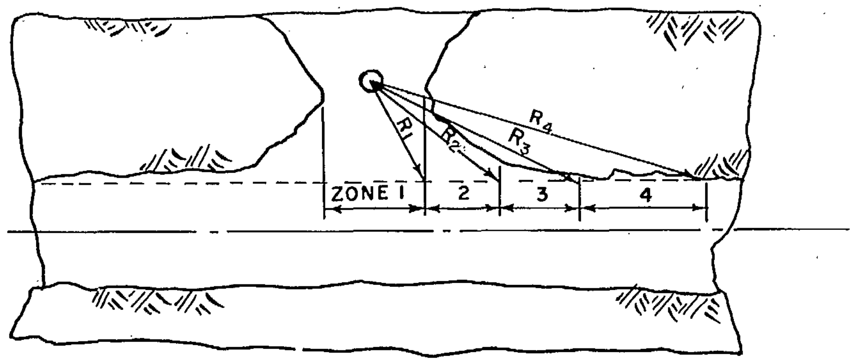


Fig. 3-6 Profile of Tunnel Damage Zones Empirically Determined from Surface Cratering Explosions (7)

Studies of these empirical results established the following outer limits for these zones:

$$R_1 = K\,W^{1/3} \qquad (3\text{-}12)$$

$$R_2 = 2K\,W^{1/3} \qquad (3\text{-}13)$$

$$R_3 = 2.6K\,W^{1/3} \qquad (3\text{-}14)$$

$$R_4 = 4.2K\,W^{1/3} \qquad (3\text{-}15)$$

where K is a constant for the site and the relative geometry of the charge and rock.

In underground and open pit mining large blasts may occur adjacent to service drifts, crusher chambers and other semi-permanent openings. To appraise the possibility of damaging such openings, extrapolation from experience can be done using Eq 3-12 to 15. The quantity W should be at most the weight of explosive detonated in one delay and in some cases less than this quantity owing to the improbability of all of the explosive contributing to the shock effect that could cause spalling and collapse. The following example demonstrates one such case.

Example: A blast in an open pit broke 250,000 tons of ore. The total amount of explosive was 120,000 lb. The blast hole pattern consisted of a burden of 21 ft, a depth to centre of charge of 23 ft, a spacing of 28 ft, and 729 lb of explosive in each 8-in diameter hole. There were 7 rows of blast holes, each detonated in a separate delay. As shown in Fig 3 - 7, 200 ft away and parallel to the nearest row of holes is a 16 ft x 16 ft inclined transportation tunnel. No damage occurred in the tunnel. How close could such a blast be before Zone 3 damage might occur ?

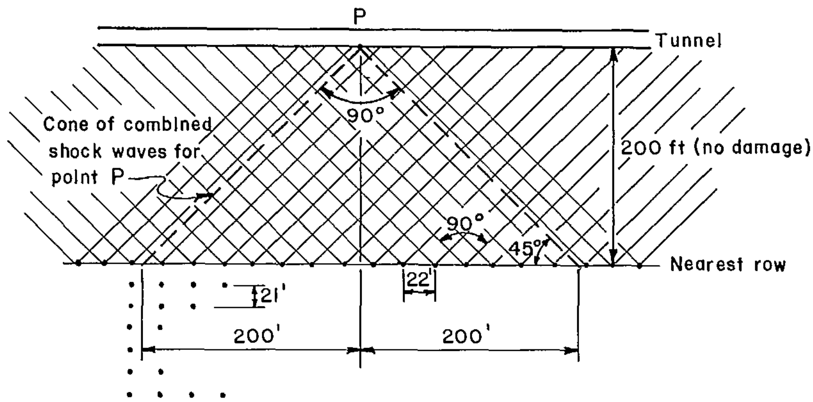


Fig. 3-7. Combined Effect of Shock Waves on Tunnel

Solution: If the shock waves from adjacent holes can combine within a cone of 90 degrees from each hole, the maximum that could have combined in the above case at any one point would have been within 200 ft in each direction along the nearest row, i. e. :

$$N = 2 \times 200/28 = 14.3, \text{ say } 14 \text{ holes}$$

Using this number of holes as the maximum possible that could have combined in a horizontal direction to produce an additive effect, K can be calculated from Eq 3-15:

$$R_4 = 4.2K(14 \times 729)^{1/3} < 200 \text{ ft}$$

Therefore $K < 2.21$

Hence $R_3 < 2.6 \times 2.21\ (14 \times 729)^{1/3} < 124$ ft

In other words, although R_3 cannot be calculated, at least an upper limit can be determined by following this procedure so that maximum use is made of the known experience. When the nearest row is 124 ft from the tunnel Zone 3 damage should not occur.

As the nearest row gets closer to the tunnel, fewer holes can combine into the effective charge weight, W; hence, the above conclusion is doubly conservative. Because the analysis is based on experience, the answer warrants some confidence. Similar evaluations could be made for underground cases such as with blast-hole stoping.

For pure extrapolation for the same effects the following equation could be used, which avoids calculating the site parameter:

$$R_b = R_a(W_b/W_a)^{1/3} \qquad \text{Eq 3-16}$$

and for extrapolating to other effects, the relative radii could be recognized:

$$R_4 = 1.6\ R_3 = 2.1\ R_2 = 4.2\ R_1 \qquad \text{Eq 3-17}$$

REFERENCES

1. Larocque, G. Sassa, K. Darling, J and Coates, D. "Field Blasting Studies", Proc. 4th Rock Mech. Symp. Mines Branch-Queen's Printer, 1968.

2. Belland, J. "Structure as a Control in Rock Fragmentation", Bull. CIMM, Vol69, p 323, 1966.

3. Nicholls, H and Duvall, W. "Pre-splitting in the Presence of a Static Stress Field", USBM RI 6843, 1966.

4. Baron, L. "Investigations on the Influence of Fracturing by the Interaction between Explosive Charges", Scientific Report of Skochinski Mining Institute, No. 11, 1961.

5. Bronnikov, D. "Influence of Natural Disturbances in the Massif on the Quality of Fragmentation", Bull. Tsvetnoi Metallurgi, No. 13 (138), 1959.

6. Demidyok, G. "Ways of Improving Fragmentation in Blast Mining of Useful Minerals", Sbornik Vzryvnye Raboty, Gosstroiizdat, 1960.

7. U.S. Corps of Engineers "Design of Underground Installations in Rock", Dept. of the Army, EM 1110-345-434, 1961.

8. Bauer, A. "Application of the Livingston Theory" Quart. Col. Sch. Mines, Vol. 56, No. 1, 1961.

9. Langefors, V. and Kihlstrom, B. "Rock Blasting", Wiley, 1963.

10. Karman, H. et al "An Empirical Ground Motion Prediction Technique for a Buried Planar Array of Explosives in Rock", Proc. 12th Symp. Rock Mech, AIME, pp 393-402, 1971.

11. Newmark, N. and Haltiwanger, J. "Principles and Practices for Design of Hardened Structures" USAF, TDR 62-138, 1962.

GLOSSARY

A - acceleration due to a shock wave

a - subscript or superscript for previous experience

B - burden

b - subscript or superscript for new case

C - length of collar

c - subscript or superscript for new case

C_p - seismic (compression) velocity

D - depth to centroid of charge or displacement due to a shock wave

d - diameter of blasthole

e - acceleration rock parameter

f - velocity rock parameter

g - displacement rock parameter

K - site parameter for tunnel damage

K_c - critical embedment rock parameter

K_o - optimum embedment rock parameter

L - length of column of explosive

L_c - length of column of explosive above grade

L_s - length of column of explosive below grade

M - oversize specification

PF - powder factor, pounds per ton

R_1 - maximum distance for collapse of a tunnel (Zone 1)

R_2 - maximum distance for continuous spalling in a tunnel increasing in volume towards the explosion (Zone 2)

R_3 - maximum distance for continuous spalling of uniform thickness in a tunnel (Zone 3)

R_4 - maximum distance for discontinuous spalling in a tunnel (Zone 4)

R_r - radius of rupture

S - spacing

S_j - spacing between joints

V - velocity in a shock wave

W - weight of explosive

w - weight of explosive in a column, pounds per foot

Z_c - critical depth of embedment, ie no crater

Z_o - optimum depth of embedment, ie maximum crater volume

UNDERGROUND OPENINGS

SYNOPSIS

Although theoretical stress analysis is likely to be more valid for the relatively simple geometry of tunnels and drifts, no well established design theory has yet been evolved for these rock structures. However, extrapolating from experience by using theoretical concepts for predicting the effects of incremental changes can, in some cases, lead to better decisions than by using intuition alone. This is the essence of incremental design.

The actual problems that can affect safety and costs are: falls, scaling of loose rock, support loads, groundwater and rockbursts.

As an example of an incremental approach consider the prediction of costs arising from falls. The probability of a fall occurring will vary with the amount of surface rock exposed. To extrapolate from experience for estimating future costs, the following equations can be used:

$$P_b/P_a = (B_b + 2H_b)/(B_a + 2H_a)$$

where subscript a refers to previous experience and subscript b to the future tunnels or drifts, P is the probability of a fall occurring, B is the breadth of the opening and H is the height of the opening. The volume of rock, V, produced by such falls could be predicted by the following equation:

$$V_b/V_a = (P_b/P_a)(B_b/B_a)$$

The total costs, C, arising from falls would then be related as follows:

$$C_b/C_a = c_b V_b/c_a V_a$$

where c is the unit cost of loading and transporting. Overbreak and scaling costs can be predicted in a similar way.

In the cases where these costs are predicted to be excessive, a basis exists for considering ways in which these costs could be reduced, e.g. by perimeter blasting both extra costs and benefits (i.e. reduced costs) could be estimated.

The prediction of support loads and costs can also be the subject of incremental design procedures, as can be the related problem of predicting the quantity of water that must be handled in a future tunnel. The design of linings for pressure tunnels, in particular, where part of the water pressure is taken by the rock must be based on incremental design principles owing to the great significance of the particular construction and rock conditions.

Rockbursts are usually caused by stress concentrations and critical structural conditions. Whereas they are more common around stopes and in pillars, they can occur in the faces of drifts and cross-cuts. Where the workings are sufficiently extensive, predictions can be made based on past experience on the probability of future occurrences.

Predicting the failure of pillars, whether by rockbursting or otherwise, can really only be done in a probability sense. For this objective, information must be gathered during previous mining on the variability of stress conditions and, if possible, of strength of the pillars. These are difficult requirements to fulfill; however, the procedures of how to use whatever information can be gathered have both immediate value and provide for future developments.

Wall stability is one of the more important problems in mining affecting safety and also costs. Such instability is difficult to predict from theory. Predicting probabilities by extrapolating from experience can be based on the occurrence of critical structural features and on the deformation and loosening, which tends to be proportional to the breadth of the opening and the pre-mining stress conditions. Similar prediction equations can be developed for dealing with the problems of inducing caving.

TUNNELS

Tunnels, drifts, cross-cuts and roadways have relatively simple geometry. Consequently, theoretical stress analysis is likely to be more valid for these cases than for the more complex geometries of other rock structures. Fig 4-1 shows the premining, or field stresses, (S_x and S_y) passing around a circular opening and creating concentrations of stress in the walls. Fig 4-2 then shows how these concentrations will be diminished if a lining exerts back-pressure (p_i) on the rock.

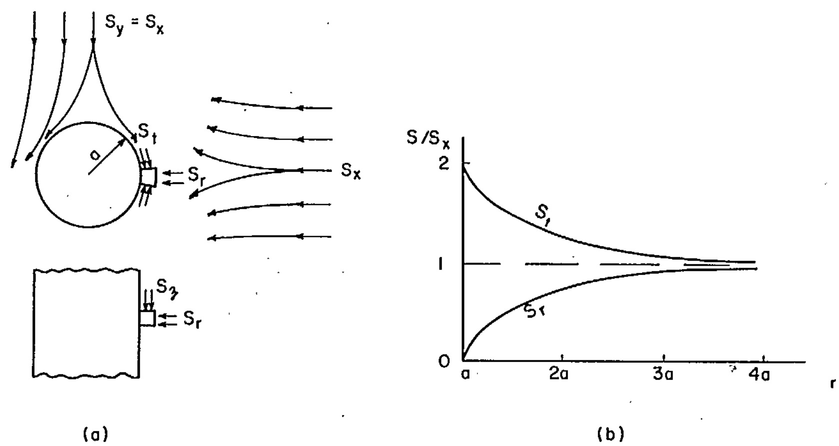


Fig 4-1 Field stresses S_x and S_y pass around an opening causing concentration of stress S_t in the walls equal, in this case, to 2 S_x (because $S_y = S_x$).

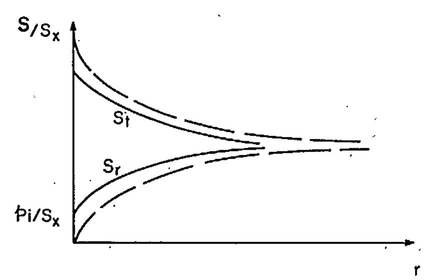


Fig 4-2 Back-pressure from a lining, p_i, decreases the stress S_t by p_i and also the shear stress by p_i

Where the field stresses (S_x and S_y) are not equal, the concentration of the stresses can be both greater and less (possibly tension) than for the above simple case. For example in Fig 4-3 the stress conditions around a circular opening is shown when $S_x = S_y/4$ and if elastic rock properties as assumed. The maximum stress concentrations, $S_t = 2.75\ S_y$, will occur at point A, as shown in Fig 4-3(b). At point B the tangential stress, S_t, will decrease (becoming tension in this case) and consequently the rock will expand.

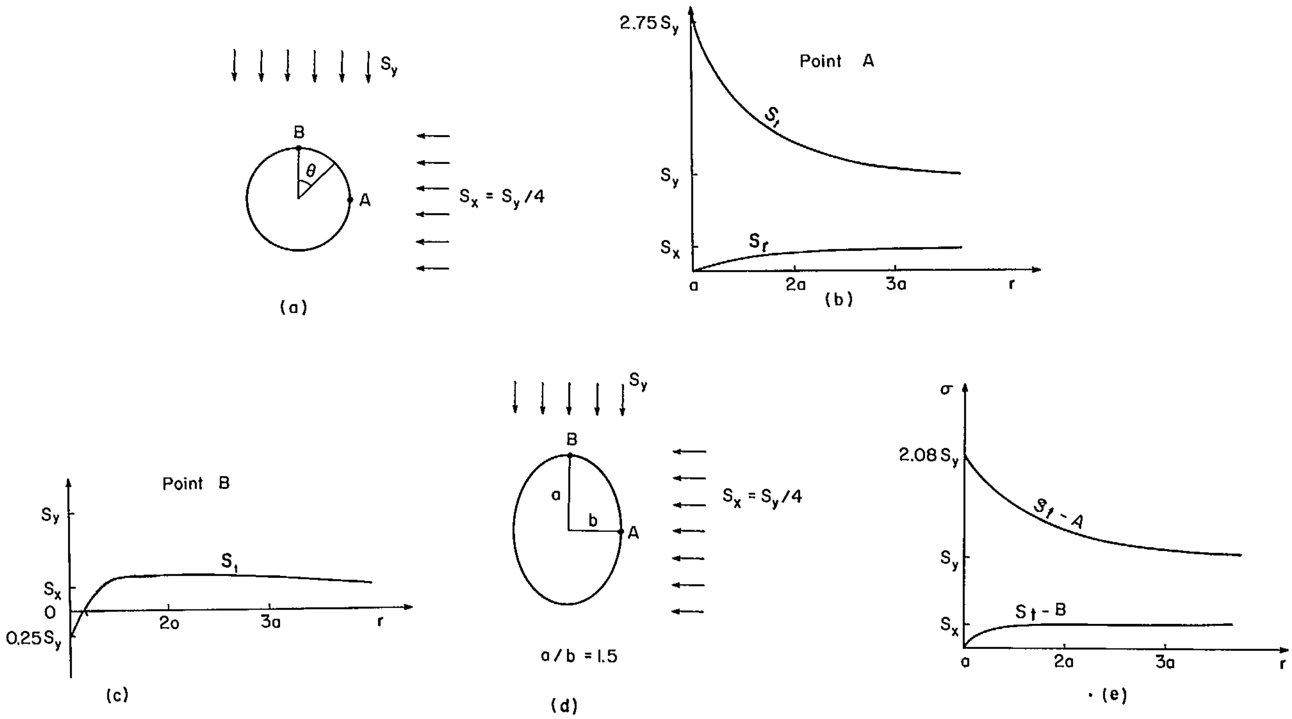


Fig 4-3 (a) Unequal field stresses ($S_x = S_y/4$) acting upon a circular opening (in elastic rock). (b) Maximum stress, $S_t = 2.75\ S_y$ occurs at point A. (c) At point B, S_t becomes tension ($S_t = -0.25\ S_y$), causing expansion in the rock. (d) Changing the shape of excavation from circular to elliptical, for the same pre-mining stress field ($S_x = S_y/4$), would (e) decrease S_t from $2.75\ S_y$ to $2.08 S_y$ at point A and eliminate the tension at point B.

Theoretically these unfavourable stress conditions, caused by the unequal field stresses, could be improved by changing the geometry of the opening. In Fig 4-3(d) the stress field is the same as before, but the opening has an elliptical shape. As shown in Fig 4-4(b) the tangential stress, S_t, would decrease at point A from $2.75 S_y$ to $2.08 S_y$ and at point B it would become zero as opposed to tensile for the circular opening before. Expansion of the rock at point B, however, still would take place, due to the elimination of compression which prevailed prior to the excavation.

The stress distribution patterns, discussed above, are applicable in the case of ideal elastic rock. These patterns can actually occur in practice. However, more commonly the effect of blasting, stress concentrations and the release of constraint will create a zone of loosened, semi-detached rock adjacent to the surface, somewhat as indicated in Fig 4-4a. The effect of the concentration of stresses around the tunnel may be as shown in Fig 4-4b, where the dotted line is a pattern that has been confirmed by field measurements. The fractured rock close to the surface sustains very little stress but through arching action builds up back-pressure away from the opening so that the stress concentrations can be sustained in the more constrained and hence more competent rock. (With increasing complexity of rock properties and of geometry of the opening, stresses and deformations can be computed for study purposes using the finite element method.)

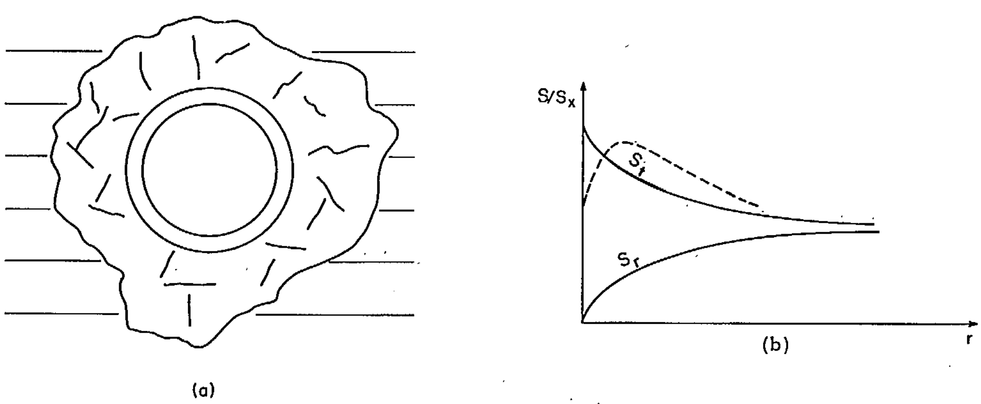


Fig 4-4(a) Blasting, stress concentrations and expansion of the surface rock around a tunnel can create a zone of loosened, semi-detached ground. (b) The zone of stress concentration moves away from the tunnel wall into the competent, undamaged rock.

The existence of the zone of loosened rock around a tunnel means that instead of a uniform lining pressure, p_o, concentrated loads, as shown in Fig 4-5a and b can occur, which can cause very large stresses in the lining. Also, a situation is shown in Fig 5c where such concentrated, oblique loads cause the, not uncommon, buckling of steel sets. Because the loosening of the surface rock takes time, these forces can develop after the tunnel support is installed. In this case, the rock pressure increases, the lining deforms, a back pressure is generated, then either equilibrium is reached or the lining fails.

Although all these concepts and theories are of value in analysing tunnel stability problems, they as yet do not lead to a well-established design theory -- rock masses are variable and these variations cannot be addquately predicted by current testing methods. Consequently, decisions must be substantially based on judgment. However, instead of extrapolating from experience by intuition, some problems can be better resolved by using theoretical concepts for predicting the effects of incremental changes, which is the basis for incremental design.

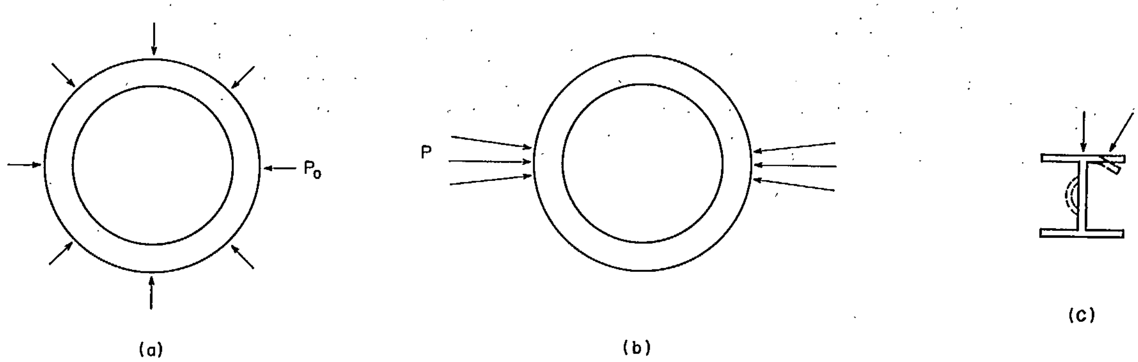


Fig 4-5 (a) A uniform rock pressure, p_o, on a lining is unlikely to occur.
(b) The surrounding damaged zone will tend to produce concentrated forces.
(c) Such forces can be directed in an oblique direction causing the buckling of flanges of steel sets.

The actual problems that can occur in tunnels, as well as other underground openings, are as follows:

1. Falls such as caused by weak bedding planes, critically oriented joints and faults, zones of alteration and brecciation, and solution or eroding of infilling material;

2. Loose Rock continually being developed and requiring scaling as a result of stress concentrations, possibly blasting effects and also just from the release of constraint;

3. Support Stresses created by concentrated rock loads, by water pressure, and by squeezing or swelling that is permitted by decreased constraint on the rock;

4. Ground water occurring either under high pressure, in large volume or at high temperatures;

5. Rockbursts from stress concentrations, possibly aggravated by geological variations, see Fig 4-6.

The nature of some of the above problems could be predicted by an intensive testing program before any rock is excavated. However, more certainty in most such predictions would be obtained if excavation in the same formations had occurred, the results measured, and then extrapolated to future excavations.

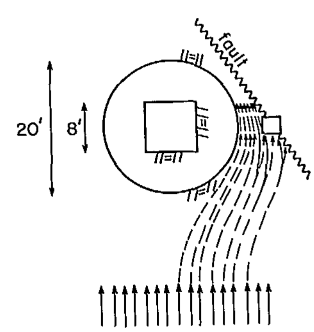


Fig 4-6 Geological features can modify theoretical stress patterns. Here a fault caused abnormal concentration of stress when a shaft was being sunk on a raise - a rockburst occurred.

FALLS AND LOOSE ROCK

Fig 4-7 suggests some of the ways that falls can originate. Weak bedding planes besides causing falls in the roof can also be the source of falls in the walls. Fracture surfaces, such as faults and joints, commonly combine to permit prisms of rock to drop out of the roof. Zones of alteration and brecciation, particularly when some of the infilling material is dissolved or washed out by ground water, can be the source of quite drastic falls.

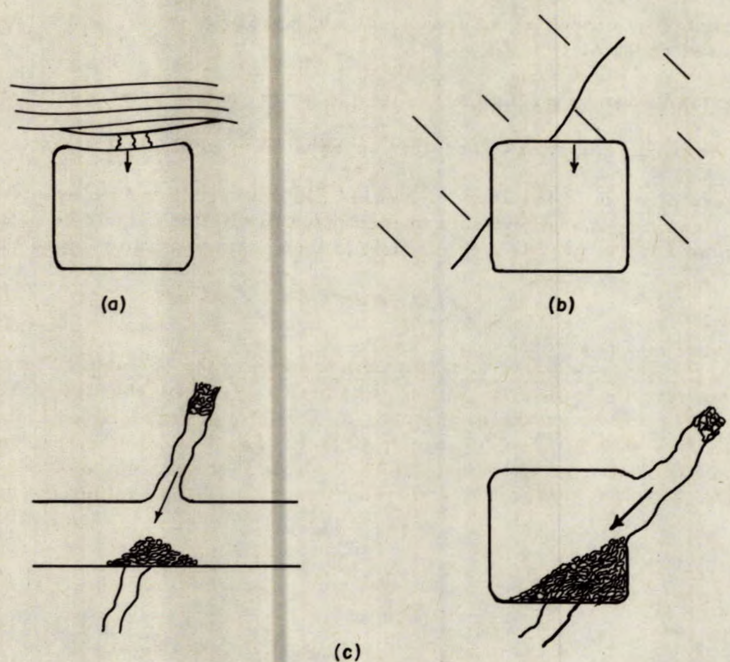


Fig 4-7 Falls can occur as a result of (a) bed separation, (b) wedges formed by faults and joints, and (c) the presence of breccia zones.

Such unique geologic features such as major faults and zones of alteration should be detectable with existing testing techniques; however, these investigations are expensive and marginal features can go unrecognized. The more widespread features such as joints and weak bedding, and their interaction, are more difficult to test for the prediction of falls.

It seems clear that the probability of a fall occurring will vary with the area of surface rock exposed in the roof and walls. Aside from foreknowledge of specific critical geological features, the probability, P, of a fault intersecting the roof or walls along a fixed length of tunnel will be proportional to the sum of the areas of the roof and walls, B+2H, where B is the breadth and H the height of the tunnel. Consequently, for purposes of extrapolating from experience the following equation can be used:

$$P_b/P_a = (B_b + 2H_b)/(B_a + 2H_a) \qquad \text{4-1a}$$

where the subscript a refers to the completed tunnel and b to the future tunnel. Then, assuming the depth of the falls is proportional to the breadth of the tunnel, the volume, V, and cost of falls for the new tunnel could be estimated by:

$$V_b/V_a = (P_b/P_a)(B_b/B_a) \qquad \text{4-1b}$$

and

$$C_b/C_a = c_b V_b/c_a V_a \qquad \text{4-1c}$$

where C is the total cost and c is the unit cost.

Example: - Estimating extra costs from delays due to falls.

A 12 ft x 12 ft pilot tunnel was driven 15,000 ft through rock at the location where two, 25 ft x 20 ft high, tunnels are to be driven (B_a = 12 ft, H_a = 12 ft, B_b = 25 ft, and H_b = 20 ft). Falls that caused significant delays in the excavation cycle of the pilot tunnel occurred at a frequency of one every 2500 feet and cost $ 22,000. Using the same excavation methods and having no more detailed geologic information, what would be the probability of falls in each of the new tunnels and their cost?

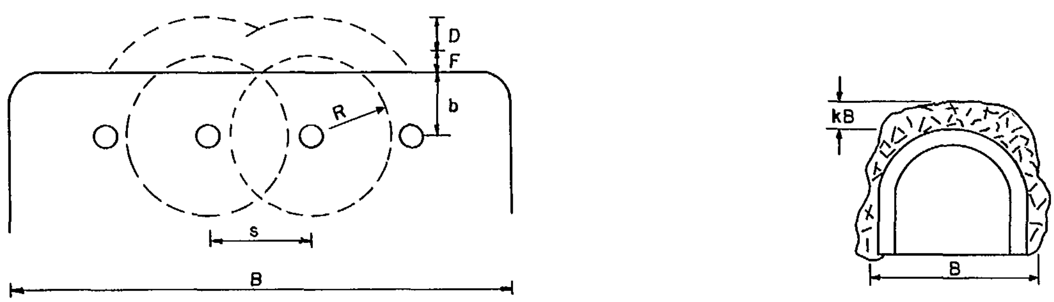
From Eq 4-1a

$$P_b/P_a = (B_b + 2H_b)/B_a + 2H_a)$$
$$= (25 + 2x20)/(12 + 2x12) = 1.81$$
$$P_b = 1.81x1/2500 = 1/1380$$
$$= 1 \text{ every } 1380 \text{ ft}$$

Combining Eq 4-1b and 1c and assuming $c_b = c_a$:

$$C_b/C_a = c_b P_b B_b / c_a P_a B_a$$
$$= (P_b/P_a)(B_b/B_a)$$
$$= (1.81)(25/12)$$
$$= 3.76$$
$$C_b = 3.76 \times 22,000 = \$83,000$$

Overbreak and/or scaling vary with the breadth of the opening. Fig 4-8a indicates how blasting causes overbreak and damages the remaining walls. The diameters of blast holes, within limits, tend to vary with the size of the opening, B; hence the radius of rupture, R, would also vary with B. Similarly, the spacing, s, and burden, b, would tend to vary with B; hence the extent of R that reaches into the roof and walls, F, as well as the damaged zone, D, would vary with B. Therefore, it can be said that the volume of overbreak and/or scaling, V, would equal the product of B by kB, where k is some constant related to either the depth, F, or F + D.



(a) (b)

Fig 4-8 (a) The depth of rock damaged by blasting will be proportional to the breadth, B, of the tunnel (R is the radius of rupture around the blast hole; D is the additional zone beyond the actual fractured rock that might be damaged due to shock, expansion and the like).
(b) The height of loosened rock that develops above sets k B, is similarly proportional to the breadth.

For purposes of extrapolating from a previous tunnel to one that is to be excavated in the same way and in the same formation, the following equation might be used:

$$V_b/V_a = B_b^2 / B_a^2 \quad \text{4-2a}$$

where V is the volume that is to be estimated, e.g. the overbreak or potential scaling. Alternatively, assuming that the previous and the new operations will have the same unit costs for either handling the overbreak or for conducting the scaling, the following equation could be used:

$$C_b/C_a = B_b^2 / B_a^2 \quad \text{4-2b}$$

where C is the cost in either \$/cy, \$/T or \$/sf.

Example: - Estimating the cost of overbreak for a new tunnel.
In a 12 ft diameter tunnel (4.2cy/LF) in granite, excavated by conventional drilling and blasting, the volume of overbreak was 22cf/LF. The unit cost of mucking, transporting and disposal was \$40/cy. A 24 ft diameter tunnel is now to be excavated in the same formation by the same methods. The unit cost of mucking, transporting and disposal is estimated to be \$30/cy. What is likely to be the extra cost arising from overbreak in this new tunnel?

From Eq 4-2a

$$V_b/V_a = B_b^2 / B_a^2$$

$$= 24^2/12^2 = 4$$

$$V_b = 4 \times 22/27 = 3.26\text{cy/LF}$$

$$C_b = 3.26 \times \$30/\text{cy} = \$98/\text{LF}.$$

PERIMETER BLASTING

Perimeter blasting, such as pre-splitting or smooth-wall techniques, can decrease the amount of loose rock that has to be scaled from an underground opening, decrease overbreak, and decrease the amount of concrete in a lining - but at a cost. The benefits arise from decreased scaling, decreased rock to be mucked and transported, and where required decreased lining costs. Clearly, the benefits should exceed the costs if such special methods are to be used, and because benefits are difficult to estimate, incremental analyses are desirable.

Example: - Smooth-wall Blasting in a Drift
In a 9 ft x 10 ft wide drift it is costing \$0.90/LF to drill and blast conventionally. From trials it is found that smooth-wall blasting costs an additional \$1.40/LF but reduces scaling costs by \$0.20/LF. From these trials, it is seen that conventional blasting causes an average overbreak of 6 in. The cost to muck and transport is \$0.70/cy. The cost of concrete in linings is \$44/cy.

A 150 ft long 40 ft x 40 ft underground crusher chamber is to be excavated. Would it pay to use smooth-wall blasting if a concrete lining were to be installed (assuming that the overbreak would be filled with concrete)? What would the answer be if rock bolt and mesh support were to be used?

Extrapolating from the experience in the drift, it can be said that the cost of smooth-wall blasting is \$1.40/(2 x 9 + 10) = 0.05 per ft of perimeter, hence:

Cost = 0.05 (2 x 40 + 40) = \$6.00/LF

Reduction in scaling costs were \$0.20/(2 x 9 + 10) = 0.007 per ft of perimeter, hence the benefit that could be expected would be:

Reduced Scaling	= 0.007 (2 x 40 + 40)	= \$0.84/LF
Reduced Muck / Transport	= 6(2 x 40 +40) 0.70/(12 x 27)	= \$1.56/LF
Reduced Concrete	= 6(2 x 40 +40) 44/(12 x 27)	= \$98.00/LF
Therefore, Total Benefit	= 0.84 + 1.56 + 98.00	= \$100.40/LF

Without concrete and ignoring the benefit of smooth-wall blasting on bolt and mesh support:

Total Benefit	= 0.84 + 1.56	= \$2.40/LF

SUPPORT

If a tunnel lining, or support, is placed before all the relaxation and detachment occurs in the roof, the loads that ultimately must be sustained can arise from the dead weight of the loose rock that develops (1). It is not unreasonable to expect that the depth of rock that becomes detached from the roof will be proportional to the breadth of the tunnel, i.e. kB as shown in Fig 4-8b. Also, a rough correlation can be expected between the structural geology of the rock mass and the coefficient k, such as:

kB	Classification
0	Massive (spacing of joints and layers greater than 6 ft)
0 to 0.7B	Blocky (spacing of joints less than 6 ft and greater than 1 ft)
2B	Broken (spacing of joints less than 1 ft)

Measurements, such as in scram drifts in iron ore (2), have shown the above table to be reasonable.

The weight of support such as blocked steel sets, which are designed to take the compressive stresses caused by the assumed loads, will be proportional to the volume of rock causing the loading, i.e. kB x B. The lengths of the steel sections, of course, will be substantially equal to B + 2H, hence the total weight of a set would be proportional to $kB^2(B + 2H)$. Consequently, to extrapolate from one tunnel, a, to another, b, for purposes of a quick estimate of the cost of support, C, in dollars per linear foot of tunnel ($/LF), the following equation might be used:

$$C_b/C_a = k_b B_b^2 (B_b + 2H_b)/k_a B_a^2 (B_a + 2H_a) \qquad \text{4-3a}$$

Whereas there might be some question about the estimates of k_a and k_b, the ratio of k_b/k_a has a greater likelihood of being relatively accurate.

Alternatively, support might be by rock bolts whose cross-section would also be proportional, theoretically, to the ultimate rock load. For temporary support, common in mining, the length of bolt would be somewhat more than kB, and hence the spacing would be equal to approximately kB (3). Therefore, the cross-section of one bolt would be proportional to the rock volume (kB x kB x kB). Accepting the length of the bolt as being approximately kB, the cost of one bolt would be proportional to $(kB)^3$ kB. The number of bolts required, assuming bolts are also needed for the upper half of the walls, would be approximately (B + 2H/2)/B . The cost per LF of tunnel would thus be proportional to: $(kB)^4((B + H)/kB)/kB$, or $(kB)^2(B + H)$. Therefore, a similar incremental design equation can be established for estimating bolting costs on a second tunnel (i.e. $/LF):

$$C_b/C_a = (k_b B_b)^2 (B_b + H_b)/(k_a B_a)^2 (B_a + H_a) \qquad \text{4-3b}$$

If bolts are only required in the roof, the equation is modified to:

$$C_b/C_c = k_b^2 B_b^3/k_a^2 B_a^3 \qquad \text{4-3c}$$

Example: - Predicting Bolting Costs

In a 12 ft x 12 ft tunnel, bolting in the roof and walls costs $15/LF of tunnel. Preliminary studies for another tunnel project in the same rock (i.e. with substantially the same k) using the same type of support are being made. What might be the cost of bolting for a 20 ft x 15 ft high tunnel?

From Eq 4-3b:

$$C_b/C_a = (k_b B_b)^2 (B_b + H_b)/(k_a B_a)^2 (B_a + H_a)$$

$$= 20^2(20 + 15)/12^2(12 + 12) = 4.0$$

$$C_b = 4.0 \times 15.0 = \$\,60/LF.$$

This estimate can be conservative owing to certain elements of bolt systems not increasing in cost with the size of the tunnel.

Squeezing and swelling ground can exert much larger lining pressures than loosened rock. Squeezing can occur from the existence of relatively large tectonic or residual stresses; these are concentrated in the rock around the excavated opening, which then creeps under the high stresses (alternatively, modest field stresses will cause creep in relatively weak rocks like salt and some shales). The associated deformation is substantially irresistible, and hence linings must be designed for the predicted deformations rather than pressures.

Swelling ground occurs either from chemical alteration of some minerals on exposure by the excavation, e.g. the oxidation of pyrite, or from a strong suction in some minerals for water which can be partially satisfied after excavation of the opening, e.g. montmorillonite clays. These swelling pressures can be very large but also can usually be controlled, e.g. by cutting off the supply of oxygen or water.

Such sources of lining pressure are difficult to predict unless incremental design principals can be used. With measurements in previous cases in the same formation, bases are obtained for extrapolating to the design of new sections (4).

Example: - Predicting Lining Stress in Squeezing Ground

In a formation of limestone, sandstone, and shales, two 51-ft diameter power tunnels are being driven with a cover of 175 ft of rock and 130 ft of soil (4). The heading and bench method is being used for the excavation. The horizontal change in diameter, d, is being measured using an invar micrometer tape. The deformation-time measurements are shown in Fig 4-9. Although steel ribs are being used for temporary support, they can not impede the rock deformation. A 3-ft thick lining of 3000 psi concrete ($E = 3 \times 10^6$ psi) is to be placed with a design life of 30 years. Assuming the concrete cannot significantly resist the rock deformation, determine how soon the lining can be placed so that the average compressive stress in the concrete will not exceed 750 psi.

The closure of the lining, d, is related to the maximum compressive stress, S, as follows:

$$d = \frac{0.552\, S R^3}{(1.332R + t)\, E\, t}$$

where R is the mean radius of the concrete lining, t is the thickness of the lining, and E is the modulus of deformation of the concrete. The maximum change in closure for S = 750 psi is thus:

$$d = \frac{0.552 \times 750 \times 24^3}{(1.332 \times 24 + 3)3 \times 10^6 \times 3} = 0.0182 \text{ ft} = 0.218 \text{ in., say } 0.22 \text{ in.}$$

or, in other words, the closure of the concrete lining must be less than 0.22 in. in order not to exceed 750 psi average compressive stress.

Assuming that the logarithmic rate of closure will continue for 30 years (1.1×10^4 days), the diameter change by that time, from Fig. 4-9, will be 1.19 in. The initial closure, d_o, which must be allowed before the concrete lining is placed, equals to 1.19-0.22 = 0.97 in. This occurs approximately at 150 days following the excavation; in other words, the concrete can be poured any day after this lapse of time.

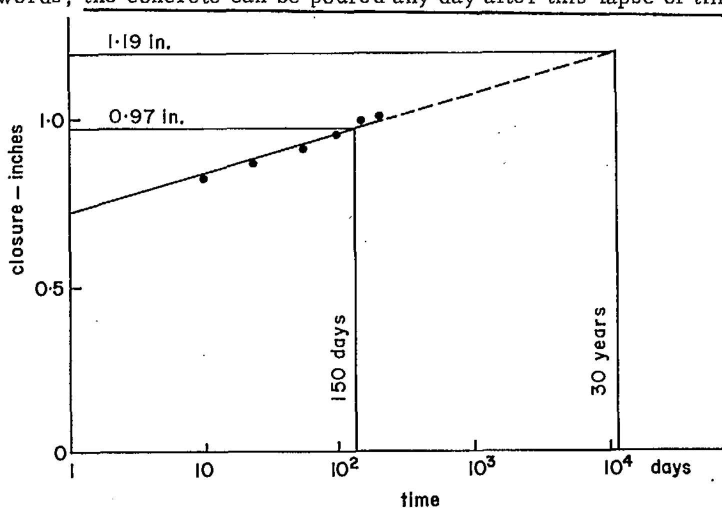


Fig 4-9 Closure creep measurements and their extrapolation for a 30-year (1.1×10^4 days) period; closure of the concrete lining must be less than 0.22 in. which means that the lining cannot be poured until after approximately 150 days after excavation of the rock.

Support requirements in soft ground tunnelling are usually more definable, although the degree of precision is still not high. Without going into the various soil mechanics theories of earth pressure, one method of support that is often used in specific circumstances is by compressed air. If the ground is too weak to sustain the stress concentrations at the surface of the tunnel, as described in Fig 4-10, compressed air can be used to create an internal pressure, p_i, to decrease the shear stresses in the ground, as indicated in Fig 4-2, until a permanent lining can be placed.

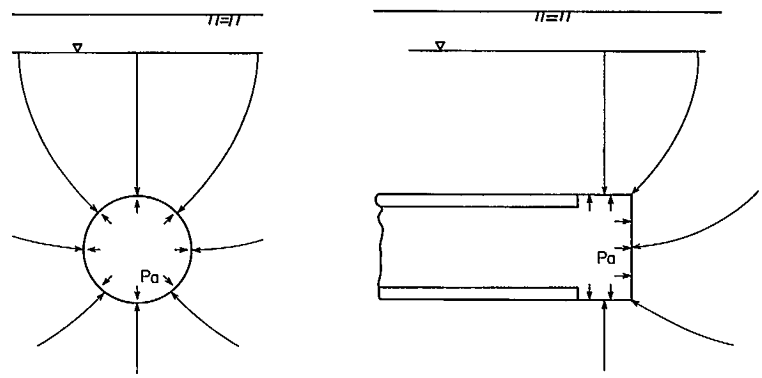


Fig 4-10 Tunnels in soft ground and under the water table frequently have to be mined under compressed air, Pa, to prevent the inflow of water with the consequent unstable face.

Example: - Using Compressed Air for Temporary Support in Soft Ground

A 10 ft diameter tunnel was driven in soft ground 58 ft below the ground surface (5), where both the vertical and the horizontal stresses were 42 psi. The concentration of stress around the tunnel initially was equal to approximately 2 x 42 = 84 psi. This was greater than the average compressive strength of 70 psi, obtained by laboratory tests. Consequently, it was specified that excavation should proceed under compressed air at a pressure of 15 psi, which would reduce the maximum compressive stresses in the walls to at least 69 psi and possibly to a lesser value if plastic yielding occurred (see Fig 4-4b).

However, recognizing the difficulties in obtaining accurate strength measurements of the ground, a test was included in the contract to determine the minimum air pressure actually required. Excavation was stopped for 72 hours and the compressed air pressure gradually reduced in stages; at 5 psi the tunnel seemed to continue to be stable; at 0 pressure the walls and roof started to fail from the stress concentration. It was clear that the laboratory testing had misrepresented the short term strength, which was closer to 75-78 psi in the tunnel environment.

Some minimum air pressure clearly was required for this tunnel, and the experience could be extrapolated to future tunnels in the same formation. In this way, costly conservatism could be avoided; however, field testing is essential for confidence in any such decisions.

Another source of soft ground instability can arise from the tunnel being below the ground water table. The flow of water towards the tunnel will add the effect of seepage stresses to that of the concentration of the field stresses. These seepage stresses can occur even though the quantity of water flowing into the tunnel is so small that it is imperceptible. The quantity of flow is primarily dependent on the permeability of the ground, whereas the seepage stresses are only dependent on the hydraulic gradient.

To counter such seepage stresses and to eliminate the inflow of water, compressed air again can be used within a limited range, e.g. beyond about 20 psi the cost becomes very high owing to limitations on the amount of time per shift a man is allowed to work. If the compressed air pressure is equal to the head of water on the tunnel, all flow will be prevented; if it is less, flow will be decreased but not prevented. Some practical compromises is this range can be found, again from field trials, and then extrapolated to future excavations in the same ground.

GROUNDWATER

The occurrence of ground water either at very high pressure or at high temperature can disrupt operations or, at least, cause costs to increase. The prediction of such specific events is substantially dependent on appropriate investigation and testing; this does not seem to be an area amenable to either probability calculations or extrapolation from past experience.

However, prediction of the quantity of water that will flow into a tunnel can be possible. Quantity is dependent on the gradient, or pressure, adjacent to the tunnel, on the permeability of the rock mass and on the perimeter area of the tunnel. The ground water level, and hence the hydraulic head driving water into the tunnel, can be determined from test borings. The permeability of the rock mass can be measured either by pressure tests or by pumping tests in borings, although such results must be used with caution as the real values for a rock mass much larger in volume than the local volume tested in the boring can be an order of magnitude different. Alternatively, extrapolations from past experience will be less subject to such variations.

The quantity of water, Q, that will flow into a tunnel under the water table, as shown in Fig 4-10, theoretically can be calculated using the following equation:

$$Q = kAH/L \qquad 4\text{-}4a$$

where k is the coefficient of permeability of the rock mass in units of velocity, A is the surface area from which the water exits, i.e. the perimeter of the tunnel, H is the head causing the water to flow, i.e. the vertical distance between the tunnel and the ground water level, and L is the length of the path of flowing water. For substantially vertical flow, as in Fig 4-10, the ratio H/L is 1; hence Eq 4-4a can be modified to:

$$Q = kA \qquad 4\text{-}4b$$

For extrapolation purposes, Eq 4-4b becomes:

$$Q_b/Q_a = k_bA_b/k_aA_a \qquad 4\text{-}4c$$

where the subscripts refer to the two different tunnels or sections of the same tunnel.

Example: - Predicting the Volume of Water to be Pumped in a New Section of a Tunnel

In one section of a tunnel that was under the ground water table, packer pressure tests in test borings gave a calculated k_a of 2×10^{-3} fpm; in a second section tests indicated k_b to be 4×10^{-4} fpm. Experience showed that for 1000 feet of tunnel in the first section, pumping of 100 cfm (625 Igpm or 748 USgpm) was required. What quantity of water can be expected to flow into 1000 feet of tunnel in the second section?

Using Eq 4-4c and recognizing that $A_b = A_a$:

$$Q_b/Q_a = k_b/k_a$$

$$= 4 \times 10^{-4}/2 \times 10^{-3} = 0.2$$

hence $Q_b = 0.2 \times 100 = 20$ cfm.

Example: - Predicting the Quantity of Water to be Pumped from a Second Tunnel in the Same Formation as the First

A pilot tunnel, 12 ft x 12 ft (A_a = 48 sf/LF), produced 100 cfm of water per 1000 LF(Q_a). It is judged that the final tunnel, 36 ft x 30 ft (A_b = 132 sf/LF), will be in substantially the same rock and under the same groundwater regime as the pilot tunnel (assuming the lined pilot tunnel will not influence the ground water regime). What quantity of water would be expected to flow into the second tunnel?

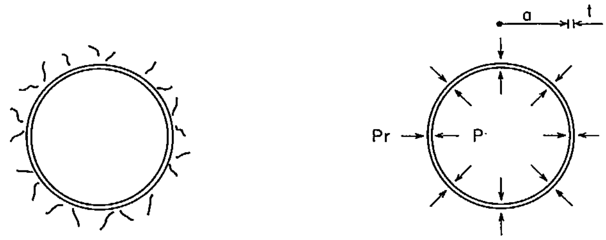
Using Eq 4-4c recognizing that $k_b = k_a$:

$$Q_b/Q_a = A_b/A_a = 132/48 = 2.75$$

and $Q_b = 2.75 \times 100 = 275$ cfm per 1000 LF

PRESSURE TUNNELS

Pressure tunnels often must carry water under high pressure, p, as shown in Fig 4-11. The lining expands under the water pressure and induces a reaction from the rock. In this way only part of the water pressure must be resisted by the lining. The greater the load taken by the rock, the thinner the lining can be.



(a) (b)

Fig 4-11 Tunnels of hydro-electric power stations may have to carry water under high pressure, P , which may be partly resisted by the rock reaction, Pr. and partly by the lining material, t.

Outside the steel lining there will normally be a concrete lining, and a zone of grouted rock. The stress in the steel lining and hence the required thickness, t, is very sensitive to any air gap that exists behind the lining. In spite of all the efforts to eliminate every gap by grouting, usually some air gaps are left behind. An equation, which relates the steel lining stress, S , to the air gap, g , and to the other significant factors, is as follows(6):

$$S = p \left(1 - \frac{1}{1 + \frac{E_s t}{E_r a}} \right) \frac{a}{t} + E_s g/a$$

or

$$S = R p a/t + E_s g/a \qquad 4\text{- }5a$$

where a is the radius of the tunnel lining, t the lining thickness, E_s and E_r are the moduli of deformation of the steel and rock, and $R = 1/(E_r a/E_s t + 1)$.

If the stresses are measured in the lining of one tunnel, or in one section of a tunnel, under water pressure, p, a test is, in effect, obtained of the effect of g and E_r - factors otherwise difficult to measure. It might then be possible to use this information following incremental design principles to determine the amount of steel required for a second tunnel, or for a subsequent section of the first tunnel, as follows:

$$t_b = p_b a_b R_b/(S_b - S_a (a_a/a_b) + p_a R_a (a_a/a_b)(a_a/t_a)) \qquad 4\text{- }5b$$

where it would still be necessary to estimate the ratio E_r/E_s, which occurs in R. For a first approximation, it might be assumed the $R_b = R_a$, which could be used to determine a trial lining thickness, t_b, for a specified stress, S_b (which is not necessarily equal to S_a). This first approximation could then be used to determine R_b and to recalcule t_b. However, it turns out that convergence is very slow owing to the usual dominant effect of g, which is independent of t. Therefore, calculating g and then using the traditional cut-and-try procedure would probably still be the more effective procedure.

Example: - Determining the Thickness of Steel Lining Required for a Second Pressure Tunnel

In a pressure tunnel with an inside lining diameter of 20 ft (a_a = 120 in.), t_a = 0.5 in., $E_s = 30 \times 10^6$ psi, $E_r = 6 \times 10^6$ psi and p = 500 feet of head, or 216 psi. The stress in the steel lining, S_a, is measured as 6600 psi. Determine the appropriate steel lining thickness, t_b, for a second tunnel 32 feet in diameter (a_b = 192 in.) to give S_b = 5000 psi under the same maximum head of water.

Assuming $R_b = R_a$ and using Eq 4-5b, $R = 1/(6 \times 10^6 \times 120/30 \times 10^6 \times 0.5 + 1) = 0.0204$

trial $t_b = 216 \times 192 \times 0.0204/(5000-6600(120/192) + 216 \times 0.0204\ (120/192)(120/0.5)) = 0.550$

check $R_b = 1/(6 \times 10^6 \times 192/30 \times 10^6 \times 0.550+ 1) = 0.0141$

and $t_b = 216 \times 192 \times 0.0141/(5000-6600(120/192) + 216 \times 0.0204\ (120/192)(120/0.5))$

$= 0.383$ in.

Another iteration of calculating an improved R_b and then t_b would be adviseable for closer convergence.

ABUTMENTS

Tunnels, drifts and stopes can be subjected to rockbursts. In driving through brittle rock containing zones of abnormally high stress, the stength of exposed rock can be exceeded and failure can take place with explosive violence. It is difficult to predict the location of rockbursts with any certainty because of the difficulty of measuring the variations of stresses and of strength throughout a rock mass. However, using previous experience in driving through a formation some predictability can be obtained for planning future work in the same rock mass.

The maximum stress around a circular tunnel, theoretically, is the same for a small tunnel as for a large tunnel. In Fig 4-12a and b the stress concentration factor, k, is the same for both cases and would typically be about 2.5, i.e., the maximum stress, S_A, in the walls would be 2.5 times the maximum field stress, S. There are reasons for believing that this theoretical deduction is valid in practice.

The difference that size does make is in the volume of rock subjected to the increased stresses. The curves in Fig. 4-12a and b indicate that the amount of rock subjected to the concentration of stress is proportional to the breadth, B. This factor can have practical consequences.

For rectangular openings, such as in Fig 4-12c, the maximum stress concentration in the abutments, k, varies somewhat with shape and can be predicted approximately with the following equation (6):

$$k = S_A/S = 0.5B/H + 1 \qquad 4\text{-}6$$

where S is the pre-mining stress perpendicular to the orebody, B is the breadth and H is the height of the opening. This equation was derived for the stresses around single openings. For several stopes, as shown in Fig 4-12d, model work has shown that with relatively rigid pillars, the abutment stresses will vary only slightly with the span of the mining zone (11, 12). Hence, even in this case Eq 4-6 can be used for a first approximation, B being the breadth of the last stope.

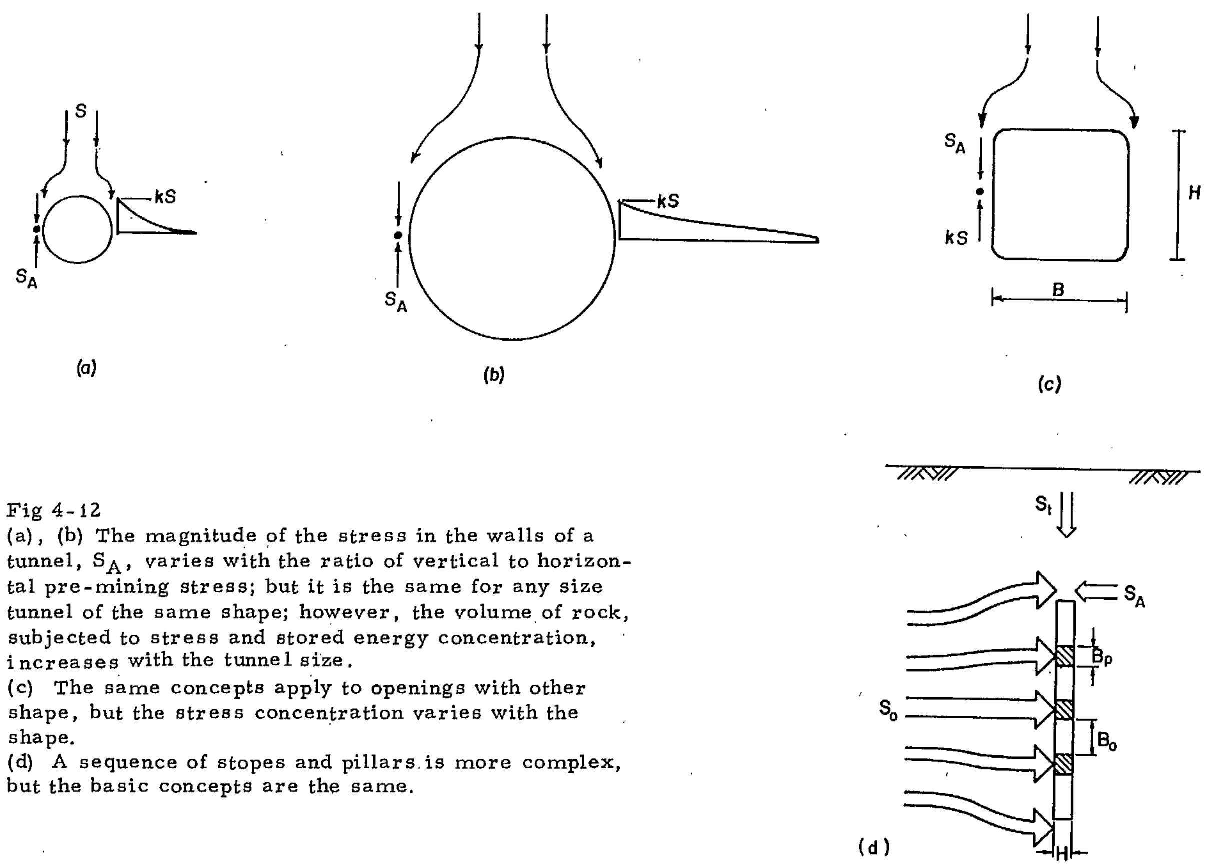


Fig 4-12
(a), (b) The magnitude of the stress in the walls of a tunnel, S_A, varies with the ratio of vertical to horizontal pre-mining stress; but it is the same for any size tunnel of the same shape; however, the volume of rock, subjected to stress and stored energy concentration, increases with the tunnel size.
(c) The same concepts apply to openings with other shape, but the stress concentration varies with the shape.
(d) A sequence of stopes and pillars is more complex, but the basic concepts are the same.

Rockbursts in the abutment zones will be triggered by the maximum stress, S_A, exceeding the strength of the near-surface rock. A calculation of the density of the total stored strain energy in this zone would provide some measure of the probable severity of any rockburst. However, based on incremental design principles such information is obtained more directly from experience. In other words, the probability of bursting for new geometry, P_b, such as the increase in the breadth of a mining panel, can be related to the frequency of bursts that has been experienced with previous geometry, P_a. Because the probability of failure is not proportionate to stress, only the trend can be stated:

$$P_b/P_a \propto S_A^b/S_A^a$$

$$\propto (S_b/S_a)(0.5B_b/H_b + 1)/(0.5B_a/H_a + 1) \qquad 4\text{-}7$$

It is known that the strength of rock tends to decrease with an increase in the volume that is subjected to maximum stresses. It is probable that there is some limit to this decrease; however, as it is undoubtedly the result of the size of critical fractures increasing with increased volume some judgment can be exercised on where the cutoff point for this mechanism is to be found. For example, the walls of a 5 ft x 6 ft dog hole may not provide a sufficiently large volume of rock to be affected by the most critical fractures in the rock mass, whereas the walls and pillars of 60 ft x 150 ft x 200 ft stopes certainly would be large enough. Where the variation with volume is valid, the correlation between the volume, V, the compressive strength of the rock mass, Q_V, and the compressive strength of a unit volume, Q_o, can be expressed by the following equation:

$$Q_V/Q_o = V^{-c} \qquad 4\text{-}8a$$

where c is a constant for the rock mass (1/3 can be assumed unless an experimental value is available). The incremental design form of the equation is:

$$Q_V^b/Q_V^a = (V_b/V_a)^{-c} = (V_a/V_b)^{1/3} \qquad 4\text{-}8b$$

The depth into the walls to which the rock is subjected to increased stress around an opening varies with its breadth, B. The affected height of rock is proportional to H. Hence, Eq 4-8b could also be written:

$$Q_V^b/Q_V^a = (B_a H_a/B_b H_b)^{1/3} \qquad 4\text{-}8c$$

Without specific information on the strength of the rock mass and its dispersion, nothing can be said with certainty about the probability, P, of either failure or bursting. However, it can be shown for small decreases in the effective rock mass strength, dQ, that the following inequality exists for constant field stresses:

$$P_b/P_a > dQ/Q + 1 \qquad 4\text{-}9$$

where P is the probability of failure or rockbursting and Q is the mean strength of the rock mass.

Where it is known that the maximum field stress has increased from S_a to S_b, only the following trend can be stated for tunnels of the same size:

$$P_b/P_a \propto S_b/S_a$$

If the mean strength of the rock and the variance of both strength and stress were known, a more complete analysis could be made.

Example: - Examining the Probability of Rockbursts in Future Tunnelling.

During the driving of a 16 ft diameter tunnel through a mountain range, rockbursts are encountered at a frequency of 1 every 1000 ft where the mean depth, Z, below the ground surface is 2000 ft. At a later date a second tunnel, 30 ft x 16 ft, is to be driven parallel to the first; what will be the probability of bursts?

Using Eq 4-8c	$Q_V^b/Q_V^a = (16 \times 16/30 \times 16)^{1/3}$	= 0.793
Using Eq 4-9	$P_b/P_a > (1-0.793) + 1$	= 1.207
hence	$P_b > 1/1000 \times 1.207$	= 1.2 per 1000 ft.

PILLARS

In room-and-pillar and stope-and-pillar mining the question often arises "why not reduce the pillar sizes and take more stope ore?" The answer to the question virtually is that the loads on the pillars would be increased and the strength of the pillars would be decreased, both increasing the probability of failure. However, if the current frequency of failure is low then the reduction of pillar sizes might be feasible providing that the increased probability of failure would still remain at a tolorable low level. (The answer also might be that subsequent pillar recover operations require that the pillars remain at some minimum dimensions.)

Using current experience, i.e. knowing the frequency of failure with present loading conditions, it is possible to extrapolate with some confidence to a proposed new layout. A basis for decision can be provided by predicting the increase in probability of failure. In this procedure, the past and present mining experience is used as a model to test the combined effects of all significant factors. Without such a large-scale, extensive test, a very expensive rock mechanics program would be required to obtain all the data necessary to predict the probability of failure for the new layout (at the present time, even such a program would not provide data that would produce results to the same level of confidence as by extrapolating from real experience).

Connected to this mining method, several other situations could arise when the incremental design procedure can be used in arriving at important technical and economic decisions. In proceeding with mining down-dip from level to level, for example, the thickness of the orebody could increase; the increased depth and the increased height to width ratio of pillars would result in unfavourable changes in pillar loading and pillar strength, producing an increase in frequency of failure of the pillars. Consequently, to maintain the existing frequency of failure the dimensions of the stopes and pillars should be changed. Similarly, the situation arises when another 1,000 ft of levels must be developed; the pillar sizes, recovery ratio and mining costs must be predicted to determine if the further development would be profitable. Another situation occurs when mining between barrier pillars. This question is: can the spacing of these barrier pillars be increased without significantly affecting the probability of failure of the stope pillars?

PILLAR LOADING

For use in examining incremental changes, the elaborate pillar loading equation previously developed can be simplified and, at the same time, modified to take into account the distance between abutments in both directions, L and Y in Fig 4-13, instead of just having a two-dimensional solution (7, 8). The following equation is based on the assumption that the loading of the pillar at the centre of the span can be used as representative of the previous conditions and of the new, changed conditions. (Also, it is accepted that the Poisson's ratios of the rock mass and of the pillars cannot be determined at the present time with any confidence and hence reasonable, arbitrary values must be selected, which fortunately do not exercise strong leverage on the answer.)

$$S_p/S_o = \frac{A + B}{C + D + E} + 1 \qquad \text{4-10a}$$

where S_p is the average stress in the pillar, S_o is the field, or pre-mining, stress in the rock normal to the orebody, and A, B, C, D, E are parameters with the values given below.

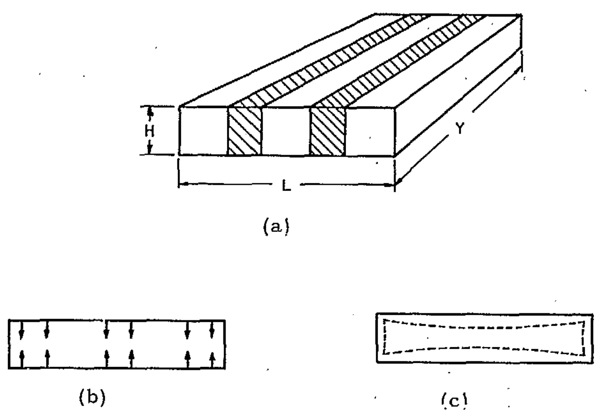


Fig 4-13 (a) Idealized room and pillar geometry.
(b) The excavation of the rooms is equivalent to applying stress downward on the roofs.
(c) The downward stress causes the roofs to deflect, which causes the increase in stress in the pillars and the expansion of the roof rock - both of which can produce problems.

A two-dimensional solution can be used when the mining zone length, Y, is more than twice its span, L. Otherwise the effect of the shape cannot be neglected and the three-dimensional solution must be used. The parameter values in Eq 4-10 are:

For two-dimensional case:

$$A = 2R(1 + h)$$
$$B = kh(0.8 + 0.2n)$$
$$C = hn$$
$$D = 1.8(1 - r)(1 + h)$$
$$E = 0.51\ Rb$$

For three-dimensional case:

$$A = (2.33 - 1.333\ L/Y)\ R(1 + h)^2$$
$$B = h(0.8k + 0.2nk + 0.2nk_y)$$
$$C = hn$$
$$D = 0.617\ (2.33 - 1.33\ L/Y)(1 - L)(1 + h/2)$$
$$E = 0.51\ Rb$$

where R is the extraction ratio for the mine, r is the local extraction ratio based on the area of the stopes adjacent to the pillar in question, h is H/L, H is the height of the pillars, L is the minimum span of the mining zone to the abutments or barrier pillars, Y is the maximum span of the mining zone to the abutments, k is S_t/S_o, S_t is the field, or pre-mining, stress parallel to the dimension L, k_y is S_y/S_o, S_y is the field, or pre-mining, stress parallel to the dimension Y, n is E/E_p, E is the modulus of deformation of the wall and abutment rocks, E_p is the modulus of deformation of the pillars, b is B_p/L, and B_p is the breadth of the pillars, i.e. the minimum dimension parallel to the walls.

Some of the above factors are easily determined, others must be based on judgement; some are relatively important and others less so. When the mining boundaries are irregular, the dimensions L and Y must be determined by judgement (it is not too difficult to obtain reasonable values, particularly if the deflection of a loaded plate, supported by such boundaries, is imagined and hence the effective span of the plate visualized). R and r are the dominant factors and should be determined as accurately as possible.
Next in order of significance are the factors h and n. In the case of n, which is a measure of the effect on the pillars. of blasting and release of constraint, some type of insitu test, such as measuring the seismic velocity across the pillar and comparing it to that of the wallrocks, could be used; otherwise its value must be judged, which is satisfactory if the incremental change in conditions does not include a change in n. Variables k and b are relatively insignificant. Consequently, if appropriate field measurements are not made, a reasonable assumption can be used for k (such as 1), which if in error would only affect the results of extrapolation slightly (as opposed to attempting to predict the loading in an entirely new situation), particularly if k actually remains the same for the previous and new conditions.

It was shown in previous research that when the span of the mining zone, L, is equal to or greater than the depth below the ground surface, Z, the structural action of the roof-rock becomes insignificant for the pillars in the central part of the mining zone, and hence the tributary area theory (i.e. assuming all the dead weight of the overburden is supported by the pillars) can be used for predicting stresses (7). Accordingly, Eq 4-10a becomes:

$$S_p/S_o = R/(1 - r) + 1 \qquad 4\text{-}10b$$

Actually, at the centreline of the mining area, the pillar stresses can be somewhat greater than this value (i.e. like 10%) and adjacent to the abutments somewhat less (i.e. like 20%).

For a large number of pillars, i.e. more than 10 across the minimum span, a further simplification can be used:

$$S_p/S_o = 1/(1 - R) \qquad 4\text{-}10c$$

PILLAR STRENGTH

The strength of any volume of a rock mass is very difficult to determine. However, the comparative strength of two volumes is easier to predict, particularly if they are in the same formation. In the case of changes in pillar sizes, the most important effect on the strength of a pillar originates from change in shape as defined by the ratio B_p/H. Although it has been shown that strength can be expected to decrease with an increase in the volume of the rock mass, it takes a large change in volume to produce a significant change in strength; consequently, in incremental design this factor can usually be ignored.

Among the many empirical equations that have been developed relating strength to the shape of rock samples or pillars, which all attempt to fit the same trend, the simplest has much to recommend it for incremental design purposes (9).

$$Q_b = Q_1\ (0.78 + 0.22\ B_p/H) \qquad 4\text{-}11$$

where Q_b is the compressive strength of the sample or pillar with a ratio $B_p/H \neq 1$, Q_1 is the compressive strength of the sample pillar with a ratio $B_p/H = 1$. It is helpful to know that the simple form of Eq 4-11 has recently been confirmed as valid, at least in one mine, by large-scale field measurements (14). It might be noted here that in incremental design it is not necessary to determine Q_1.

PROBABILITY OF FAILURE

In soil and structural design the use of safety factors, the ratio between strength and stress, has been very fruitful. However, in mining (and increasingly in structural engineering) this approach is not good enough for several reasons:

(1) large safety factors are uneconomic and unnecessary,

(2) recognising that factors like strength and stress are subject to wide variations as shown in Fig 4-14, it can be seen that safety is not guaranteed by having the average strength of the pillar, $\bar{Q}_p$, some arbitrary multiple of the average pillar stress, $\bar{S}_p$, because if the dispersions of these values are great enough a certain number of cases will occur where $S_p \geq Q_p$, which means that failure occurs, and

(3) it is important to differentiate the consequences of failure, e. g. if pillar failure only means that the pillar is cracked and fragmented but can still hold itself in place then a higher frequency of failure is acceptable than if failure is by rockbursting.

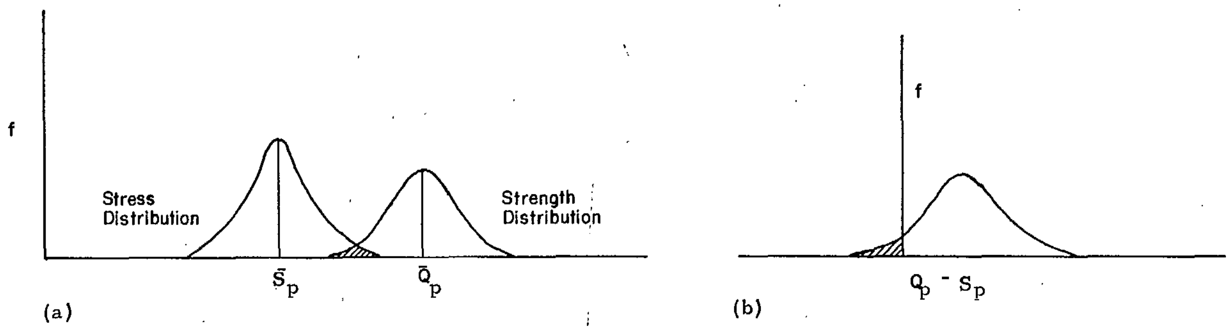


Fig 4-14 (a) Effect of dispersion of stress and strength on probability of failure (represented by the hatched area). (b) A difference diagram of frequency density distribution of $(Q_p - S_p)$ showing the probability of failure as the area to the left of the origin.

Fig 4-14a shows the dispersion of the strength of pillars about their mean value, $\bar{Q}_p$ and the dispersion of the loading of the pillars about their mean stress, $\bar{S}_p$. The strength and stress values are measured on the x-axis. The y-axis represents the frequency density, which when multiplied by some increment on the x-axis, dx, gives the frequency of occurrence of that value of Q_p.

The overlapping of the tails of the Q_p and S_p curves indicates the cases where the stress in the pillars is greater than the strength, and hence this area represents the frequency of failure. Put in another way, the area under any part of curve represents the frequency of occurrence of that range x. When describing information that is known from experience and tests, 'frequency of occurrence' is the terminology that is used. When predicting future conditions, 'probability of failure' is the appropriate terminology, and Fig 4-14a would have the y-axis labelled 'probability density, P_d'.

If both functions are standard normal distributions, which is not an unreasonable assumption unless information is provided to the contrary, their difference is also a standard normal curve, i. e. $Q_p - S_p$, as shown in Fig 4-14b. In this case, the area under the difference curve where $x < 0$ represents either the frequency of failure or the probability of failure. From statistical theory the following equations describe characteristics of this curve:

$$m = \bar{Q}_p - \bar{S}_p \qquad 4\text{-}12$$

where m is the mean of this difference curve;

$$s = (s_q^2 + s_s^2)^{\frac{1}{2}} \qquad 4\text{-}13$$

where s_q is the standard deviation of the Q_p-curve, s_s is the standard deviation of the S_p-curve and s is the standard deviation of the difference curve;

$$z = m/s \qquad 4\text{-}14$$

where z is a coupling factor that can be used to obtain the area under the tail of the curve in Fig 4-14b that would represent the probability of failure, P_f, for predicting future conditions (see Table 4-1)(10).

TABLE 4-1

Probability of Failure, P_f, Related to the Coupling Factor, z(10)

z	P_f
0.0	0.50000
0.2	0.420740
0.4	0.344578
0.6	0.274253
0.8	0.211855
1.0	0.158655
1.2	0.115070
1.4	0.080757
1.6	0.054799
1.8	0.035931
2.0	0.022750
2.1	0.017864
2.2	0.013903
2.3	0.010724
2.4	0.008197
2.5	0.006209
2.6	0.004661
2.7	0.003467
2.8	0.002505
2.9	0.001866
3.0	0.001350
3.5	0.000232
4.0	0.000032
4.5	0.000004
5.0	0.0000003

In testing brittle rock specimens with a stiff testing machine, it has been found that stress-strain curves, shown in Fig 4-15, can be produced. After the maximum stress, S_m is exceeded, further deformation of the specimen continues to cause further strain and cracking but the sample remains substantially intact. The reaction force of the rock to the deformation caused by the testing machine decreases until the maximum strain, e_m, is is reached. These tests have been of great interest because the rock samples after the maximum stress has been exceeded look very much like pillars that are working. Consequently, it is assumed that pillars in many rock formations will have similar characteristics.

Fig 4-15 also shows that during the application of stress a point S_y is reached, where the slope of the stress-strain curve starts to decrease. This can be considered as a yield point, indicating that further strains will be partially irrecoverable. Associated with such a point, measurements show that where the volume of the sample has been decreasing, Δ V, it now starts to increase suggesting serious breakdown of the rock structure, which causes the expansion. In pillars it means the start of working, i.e. internal cracking.

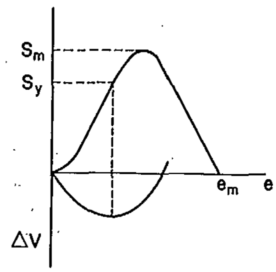


Fig 4-15 A typical stress-strain curve for rock: after the initial closing of fractures, strain, e, has a straight line variation with stress, S, and the volume of the rock decreases, ΔV; when S_y is exceeded, the rate of change of strain increases and ΔV starts to increase, i.e. internal fracturing has started and the rock is expanding; the maximum stress, S_m, is reached when through-going fractures are produced; however, the rock usually remains intact and will sustain further strain, but with decreased resistance, until some maximum strain, e_m, is reached.

The term failure, therefore, can have several meanings; it can mean exceeding the yield point, S_y, similar to the convention used in steel design. It can mean exceeding the maximum strength of the rock, S_m, which is probably the most useful definition because of the associated throughgoing vertical or diagonal fracture planes. It could also mean exceeding the maximum strain, e_m, beyond which the rock has no strength. Finally, it could mean exceeding a specified deformation, such as can occur in potash mines with respect to the clearance required for the mechanical miners to be able to exit from a panel.

In incremental design it is useful to establish a definition for pillar failures suitable for each mine, guided perhaps by the above concepts but not necessarily restricting the definition to these concepts. For example, failure can mean the point when longitudinal cracking of pillars occurs, which can seriously affect the cost of drilling blast holes. In another mine, the appropriate definition would be the point at which rockbursting occurs. The appropriate probability of failure that will be acceptable would then be related to the consequences of a failure and such practical factors as the period of time for which the pillars should provide their function. Ultimately, all decisions must be decided by optimum mining costs, or maximum profitability. Mining costs can be thought of as consisting of: operating costs + cost of a failure x probability of failure.

Example: The Ideal Case of Determining the Probability of Failure in a Room and Pillar Layout by Predicting Pillar Strengths, Stresses and Their Dispersions.

In a hard rock mine $\bar{Q}_p$ = 20,000 psi, s_q = 3000 psi, $\bar{S}_p$ = 11,000 psi, s_s = 3000 psi. The probability of failure, P_f, for these conditions is calculated as follows:

Solution: Using Eq 4-12, 13 and 14:

$$m = \bar{Q}_p - \bar{S}_p = 20{,}000 - 11{,}000 = 9{,}000 \text{ psi}$$

$$s = (s_q^2 + s_s^2)^{\frac{1}{2}} = (3000^2 + 3000^2)^{\frac{1}{2}} = 4{,}250 \text{ psi}$$

$$z = m/s \; 9000/4250 = 2.12$$

From Table 4-1, P_f = 0.017, say 0.02 or in other words, approximately two pillars in every 100 will fail.

Example: Appraising the Effect on Stability of the Pillars in Blasthole Stoping of a Large Increase in the Width of the Orebody on the Same Level

In a steeply-dipping orebody, mining is by blasthole stoping; pillars run down-dip and are 300 ft long, i.e. $L = 300$ ft, between temporary crown and sill pillars. Mining extends for 950 ft along strike, i.e. $Y = 950$ ft. The thickness of the ore is usually 50 ft, i.e. $H = 50$ ft. The breadth of the pillars is 50 ft, i.e. $B_p = 50$ ft, and the deformability of the pillars is twice that of the walls, i.e. $n = 2$. The local extraction ratio, r, adjacent to the centre of the stopes is 0.5. The general extraction ratio, R, for one level interval, i.e. for ten stopes and nine pillars, is 0.526. It is assumed that the stress ratio, k, is 1. From experience in nearby mines at similar depths and with similar geometry, it is judged that about one in twenty pillars will split, which is not drastic - it merely results in extra costs arising from the extra precautions required for the rearrangement of drilling drifts or the re-routing of supplies, etc. $P_f = 0.05$ for this type of failure.

The width of the orebody at approximately half the length of the mining zone expands to $H = 100$ ft. Should the breadth of the pillars, B_p, be changed for this section so that the frequency of failure remains the same as for the normal sections?

Solution: The effect on pillar loading is determined by using Eq 4-10a. Because $Y/L > 2$, the two-dimensional equation is used for the central pillar (because it is the most severely loaded). The parameters for the sections with H = 50 ft are:

$$h_a = H/L = 50/300 = 0.167$$

$$b = B_p/L = 50/300 = 0.167$$

$$A_a = 2R\ (1 + h) = 2 \times 0.526(1 + 0.167)$$

$$B_a = kh\ (0.8 + 0.2n) = 1 \times 0.167\ (0.8 + 0.2 \times 2)$$

$$C_a = hn = 0.167 \times 2$$

$$D_a = 1.8\ (1 - r)(1 + h) = 1.8(1 - 0.5)(1 + 0.167)$$

$$E_a = 0.51\ Rb = 0.51 \times 0.526 \times 0.167$$

and for the wide part of the orebody, where H = 100 ft

$$h_b = 100/300 = 0.333$$

$$b = 0.167$$

$$A_b = 2 \times 0.526 \times 1.333$$

$$B_b = 0.333 \times 1.2$$

$$C_b = 0.333 \times 2$$

$$D_b = 1.8 \times 0.5 \times 1.333$$

$$E_b = E_a$$

then because:

$$(S_p/S_o)_b/(S_p/S_o)_a = S_p^b/S_p^a$$

$$S_p^b/S_p^a = \frac{(A_b + B_b)/(C_b + D_b + E_b) + 1}{(A_a + B_a)/(C_a + D_a + E_a) + 1} = 0.971$$

which means that the pillar stress will be 3% lower in the wide part of the orebody. The effect on pillar strength of the widening of the orebody is examined using Eq 4-11:

$$Q_p^b/Q_p^a = (0.78 + 0.22\ B_p^b/H_b)/(0.78 + 0.22\ B_p^a/H_a)$$
$$= (0.78 + 0.22 \times 50/100)/(0.78 + 0.22 \times 50/50) = 0.890$$

which means that the pillars will have 11% less strength. Therefore, either the pillars should be increased in breadth or the extraction ratio reduced to maintain the same degree of stability as previously, otherwise the fracturing of pillars will occur with greater frequency - possibly 1 in 10 instead of 1 in 20.

Note that this extrapolation from experience, or incremental design, avoids the need of predicting actual stresses or strengths. Actual values would more likely be in error, considering the usual non-ideal conditions in the actual mine. For a comparison, the departures from the theoretical assumptions tend to affect both cases in the same way. In any event, it looks as though these pillars should be broadened.

Example: Effect of a Large Increase in Width of the Orebody Down-dip.

An increase in width of the orebody of the previous example occurs down-dip, i. e. at the depth of 2000 ft H = 50 ft and then increases so that the average width between the 2000 and 3000 ft levels is 100 ft.

Should the breadth of the pillars be changed to keep the frequency of failure the same as in the past?

Solution: From the previous example it was found that the effect of the widening of the orebody on one level was to decrease the stresses to 0.971 of the normal value. For the case where the widening occurs down-dip the field stress normal to the walls, S_o, because k = 1, is equal to the vertical stress and hence is proportioned to the depth below the surface, i. e: $S_o^b/S_o^a = 3000/2000$

The combination of both effects is:

$$S_p^b/S_p^a = 0.971 \times 3/2 = 1.456$$

which means that the pillar stresses would be 45% greater than previously, and therefore, the frequency of cracking would be much greater.

For a new design layout using Eq 4-10a, try B_p^b = 100 ft, B_o = 50 ft, hence r = 0.333 and R = 6 x 50/(5 x 100 + 6 x 50) = 0.375, b = 100/300 = 0.333 .

now

$$A_b = 2 \times 0.375 \times 1.333$$

$$B_b = 0.333 \times 1.2$$

$$C_b = 0.333 \times 2$$

$$D_b = 1.8(1 - 0.333)\ 1.333$$

$$E_b = 0.51 \times 0.375 \times 0.333$$

whence

$$\frac{S_p^b/S_o^b}{S_p^a/S_o^a} = 0.825$$

and

$$S_p^b/S_p^a = 0.825 \times 3/2 = 1.238$$

From Eq 4-11:

$$Q_p^b/Q_p^a = (0.78 + 0.22 \times 100/100)/(0.78 + 0.22 \times 50/50) = 1$$

The probability of cracking in the pillars would be greater than originally, owing to a 24% increase in mean stress while the mean strength remained constant. Hence B_p should be increased more and/or B_o decreased. However, there are practical limits to this approach; it is possible that the mining method based on rigid pillars might be modified to accept yielding pillars, which would require a different method of recovery. The above calculations would then have served their purpose of predicting the consequences of certain geological and layout changes.

Example: Change in Effective Mining Span Between Abutments

A flat-lying orebody is cut by a series of thick vertical dikes with the same strike as the orebody, between which stope and pillar mining proceeds. The height of the pillars, H, is 10 ft; the breadth of the pillars, B_p, is 10 ft; the breadth of the stopes, B_o, is 60 ft; the length of the stopes, Y_o, perpendicular to the strike is 200 ft; the span of the mining zone between Dikes No. 1 and 2, L, is 420 ft; the length of the mining zone parallel to strike, Y, is 1880 ft; adjacent to the central sections of the pillars the local extraction ratio, r, is 0.858; the general extraction ratio, R, is 0.809 (there are only sill pillars, no crown pillars): stress measurements show that the average pillar stress, $\overline{S}_p$, is 11,000 psi, and the standard deviation, s_s, is 3000 psi; for comparative purposes it is assumed that the stiffness of the pillar rock is only half of that of the walls, i.e. n = 2; and the ratio of horizontal field stresses to the vertical field stress, k, is assumed to be 1. Failure occurs in about 1 in 50 pillars, i.e. a frequency of 0.02.

Between Dikes No. 2 and 3, the span of the mining zone, L, is 1000 ft. Will the probability of failure, P_f, change, and if so, how much?

Solution: The basis parameters are:

$$h_a = H/L = 10/420 = 0.024$$
$$b_a = B_p/L = 10/420 = 0.024$$
$$h_b = 10/1000 = 0.001$$
$$b_b = 10/1000 = 0.001$$

Y/L is greater than 2 in the first case and close to 2 in the second case so that the two-dimensional version of Eq 4-10a can be used.

$$A_a = 2R(1+h) = 2 \times 0.809 \times 1.024$$
$$B_a = kh(0.8 + 0.2n) = 1 \times 0.024 \times 1.2$$
$$C_a = hn = 0.024 \times 2$$
$$D_a = 1.8(1 - r)(1 + h) = 1.8 \times 0.142 \times 1.024$$
$$E_a = 0.51 R b = 0.51 \times 0.809 \times 0.024$$
$$A_b = 2 \times 0.809 \times 1.001$$
$$B_b = 1 \times 0.001 \times 1.2$$
$$C_b = 0.001 \times 2$$
$$D_b = 1.8 \times 0.142 \times 1.001$$
$$E_b = 0.51 \times 0.809 \times 0.024$$

whence $(S_p/S_o)_b / (S_p/S_o)_a = 1.141$

and because S_o is constant $S_p^b/S_p^a = 1.141$

Hence there would be approximately a 14% increase in stress in the pillars, and the increase in probability of failure will be even greater owing to its variation with the difference between strength and stress.

To calculate the probability of failure, the mean pillar strength and its standard deviation should be known; however, this information is almost impossible to obtain through reasonable investigations. Nevertheless, using the incremental design approach, previous experience can be exploited, i.e. knowing the frequency of failure under previous conditions and the stresses in the pillars, the mean strength can be deduced that is commensurate with an assumed dispersion. Say the standard deviation of pillar strengths, s_q, is 5000 psi, which is a probable figure.

From Eq 4-13 $s = (s_q^2 + s_s^2)^{\frac{1}{2}} = (5000^2 + 3000^2)^{\frac{1}{2}} = 5830$ psi

For previous conditions, Table 4-1 gives $z_a = 2.05$ for $P_f^a = 0.02$

Hence from Eq 4-14 $m_a = z \times s = 2.05 \times 5830 = 12000$ psi

From Eq 4-12 $\bar{Q}_p = m + \bar{S}_p = 12000 + 11000 = 23000$ psi

Therefore, for the new conditions Eq 4-12 gives:

$$m_b = \bar{Q}_p - \bar{S}_p = 23000 - 1.141 \times 11000 = 10500 \text{ psi}$$

and using Eq 4-14 $z_b = m_b/s = 10500/5830 = 1.80$

from Table 4-1 $P_f^b = 0.036$ or $(0.036/0.02)\ P_f^a = 1.8P_f^a$

i. e. the probability of failure has increased by approximately 80% (this calculated change is probably more accurate than the nominal values for Q_p that have been used to extrapolate to the new conditions).

Example: A Pillar Recovery Scheme

One scheme for pillar recovery in a room and pillar operation is by retreating taking every second pillar. The span of the mining zone, L, is 1000 ft; the length, Y, is 2000 ft; the breadth of the rooms, B_o, is 50 ft; the breadth of the pillars, B_p, is 45 ft; the length of the pillars, Y_p, is 70 ft; the height of the pillars, H, is 120 ft; the general extraction ratio, R, is 0.76; the local extraction ratio, r, is 0.72; the depth below the ground surface, Z, is 1000 ft; the average density of the overlying rock is 160 pcf; and the frequency of failure has been 3 in 100, or 0.03 , pillars failing by either excessive slabbing, shearing on through-going fractures, or splitting - none of which has been ver y drastic as the roof usually remained stable.

What will likely be the probability of failure of the pillars during such a scheme of recovery?

Because the span of the mining zone is equal to the depth below ground surface, i. e. L = Z, Eq 4-10b can be used to calculate the pillar stresses for the central area:

$$S_p/S_o = R/(1 - r) + 1$$

$$S_p/(1000 \times 160/144) = 0.76/(1 - 0.72) + 1 = 3.72$$

or $S_p = 4130$ psi

By studying the actual mine plans it is found that the maximum variation of the dimensions for two-thirds of the pillars is about 5 ft, from which it follows that the standard deviation of the above calculated pillar stress, s_s, is 700 psi (obtained by calculating the maximum local extraction ratio and hence finding the difference between the maximum pillar stress and the mean stress).

The mean strength of the pillars is not known, however, a nominal figure that is consistent with experience can be calculated. It can then be used for estimating the effects of the changes caused by the recovery operation. Taking into account structural and compositional variations, it is judged that the dispersion of strength, s_q, is about 3000 psi.

From Table 4-1 and a probability of failure, P_f, of 0.030, it follows that z = 1.89 for the normal operations. The additional statistical parameters can be calculated.

From Eq 4-13 $s = (s_q^2 + s_s^2)^{\frac{1}{2}}$

$= (3000^2 + 700^2)^{\frac{1}{2}} = 3080$ psi

and Eq 4-14 $m = zs$

Therefore $m = 1.89 \times 3080 = 5820$ psi

From Eq 4-12, the nominal pillar strength is:

$$\overline{Q}_p = m + \overline{S}_p$$

$$= 5820 + 4130 = 9950 \text{ psi}$$

The effect of taking every second pillar during the recovery operation will vary. When few pillars in an area have been recovered the increase in stresses will result from an increase in the local extraction ratio, r, with the general extraction ratio, R, remaining substantially the same. When most of the recovered pillars have been mined the general extraction ratio will have been increased. For initial mining conditions, the former is the more pertinent situation. In this case, when the adjacent pillars have been recovered, the local extraction ratio, r, for one pillar is:

$$r = (A_t - A_p)/A_t = 1 - A_p/A_t$$

where A_t is the total area tributary to one pillar and A_p is the cross-sectional area of one pillar, hence:

$$r = 1 - (45 \times 70)/(190 \times 120) = 0.862$$

from Eq 4 - 9b $S_p/S_o = 0.76/(1 - 0.862) + 1 = 6.50$

and the pillar stress is: $S_p = 6.50 \times 1000 \times 160/144 = 7220$ psi

The statistical parameter for the initial pillar recovery conditions can now be calculated.

from Eq 4-12 $m = Q_p - S_p$

$= 9950 - 7220 = 2730$ psi

The composite standard deviation, s, should be substantially the same as previously, i.e. 3080 psi.

From Eq 4-14 $z = m/s = 2730/3080 = 0.89$

From Table 4-1 $P_f = 0.181$

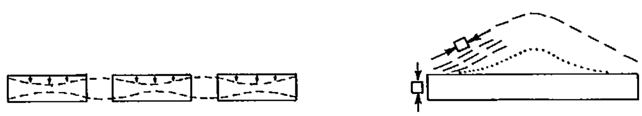
In other words, it must be expected that more than one in six of the remaining pillars will fail, which may be more serious than during the mining of the stopes because the roofs might not be stable over the increased spans.

This probability of failure of the pillars could be decreased by close inspection, recognizing the weaker zones and modifying plans so that these areas are mined out. Alternatively, these weaker pillars could be reinforced, e.g. by through-going rock anchors or by wrapping them with old cables.

WALL STABILITY

Instability of walls, particularly in open stoping, can affect mining costs and safety. Fig 4-16 shows the deformations that would normally occur in pillars and adjacent walls. Fig 4-16b shows the zone of reduced compression (or possibly tension) and, consequently, the zone of loosened rock structure. The deflection of the wall and the loosening of the rock are necessary actions together with the presence of structural elements, such as joints, oriented in the critical attitude for falls to occur.

It is difficult to predict falls from theory, although it can be said that the maximum extent initially of the loosened zone is between the quarter points of the span of the opening and for a maximum distance into the wall of about 1/4 of the span. However, with caving or working causing rock to fall away, there is no limit to the height, or depth, of fractured ground that can occur with time.



(a) (b)

Fig 4-16 (a) Wall, or roof, deflections and pillar compression resulting from mining, and (b) one stope, or room, showing the concentrated stress in the pillar arching over the stope, which can cause parallel splitting, and showing the relaxed zone (dotted line) that develops within the pressure arch.

The high intensity of stresses in the pillars will be transmitted into the walls, as indicated in Fig 4-16b, where they spread and form a pressure arch. If these stresses in the walls are high enough, splitting along their trajectories can occur. Using incremental design principles and the above mechanisms, the effect on mining costs could be estimated for a change in stope breadth from the results experienced with previous breadths.

Different types of instability can occur, but many cases will depend on the occurrence of critical structural features. The probability, P_1, of their occurrence, and the volume of waste produced, should be proportional to the breadth of the opening, B, i. e. $P_1 \propto B$.

In addition, deformation and loosening must be a contributory factor, e. g. a narrow opening even with a critically oriented fault would be more stable than a broad opening with the same fault. It is likely that the probability of falls, P_2, is more or less proportional to the sag, D, of the back. Elastic deformation at the centre of the stope, D, relative to the pillars for a long stope is proportional to the product of the breadth, B, and the field stress normal to the walls, S, i. e. $D \propto SB$ (7). Perhaps we can assume that non-elastic, or total, deformation, D, will have a similar variation.

The probability of instability, dependent on area of exposure or breadth, B, is P_1 and that due to deformation is P_2 , hence:

$$P_1^b/P_1^a = B_b/B_a \qquad \text{4-15a}$$

$$P_2^b/P_2^a = D_b/D_a \qquad \text{4-15b}$$

or

$$P_2^b/P_2^a = S_bB_b/S_aB_a \qquad \text{4-15c}$$

hence

$$P_b/P_a = S_bB_b^2/S_aB_a^2 \qquad \text{4-15d}$$

These equations are speculative but should provide a basis for estimating and for comparing with actual results.

Example: Dilution of Ore with Falls from the Walls

In a blasthole stoping operation in a steeply-dipping orebody, stopes have been 40 ft along strike, 200 ft down-dip and 120 ft wall to wall. Falls from the walls that significantly affected costs through dilution and secondary blasting at the boxholes occurred in about 1 out of every 20 stopes, i. e. $P_a = 0.05$. The stopes are to be opened up to 60 ft on strike. What effect is this likely to have on the frequency of falls ?

Solution: For the increased probability of critical joints occurring and for the effect of increased deflection use Eq 4-15d, noting that $S_b = S_a$

$$P_b/P_a = 60/40 = 1.5$$

$$P_b' = 1.5 \times 0.05 = 0.075$$

The effect of increased deflection where $S_b = S_a$ is, from Eq 4-15c:

$$P_b/P_b' = 60/40 = 1.5$$

$$P_b = 0.05 \times 60^2/40^2 = 0.112$$

Therefore, it must be expected that about 1 in 9 stopes will have significant wall falls, whose cost could then be compared with the reduced mining costs anticipated from the new plans.

Example: Roadway Falls

In a test roadway 16 ft broad, it was found that if the sag of the roof was greater than 1/4 in. in the 24 hours after exposure, there was a 75% probability that this section would produce serious roof falls sooner or later (13). What would be the approximate comparable critical sag for a 14 ft roadway?

Solution: From Eq 4-15c, when $S_b = S_a$:

$$P_2^b / P_2^a = B_b/B_a$$

$$= 14/16 = 0.877$$

and

$$P_2^b = 0.877 \times 0.75 = 0.657$$

In other words, there would be a reduced probability of falls as a result of less area being exposed.

Alternatively, for the same probability of falls, more sag could be permitted in the 14 ft roadway before extra support would be specified.

Using Eq 4-15b, where P_2^a is now 0.657 for $D_a = 1/4$ in; to have P_2^b be 0.75 the permitted D_b can be calculated as follows:

$$P_2^b/P_2^a = D_b/D_a$$

$$D_b = (0.75/0.657)\ 0.25 = 0.286 \text{ in.}$$

There is no way of knowing if this answer is really correct, but it might be, and field observations would confirm or otherwise.

CAVING

Where caving is desired, the shape and plan of the undercut is significant with respect to the stresses that will be created to cause the caving. Fig 4-17 shows the variation of the tensile stresses at the centre of the back and of the shear stresses at the abutments with the ratio of length to breadth, L/B, of the undercut. It can be seen that as L increases, the stresses increase up to the point where the ratio L/B is about 2.5 when the stresses will be approximately twice those for L/B = 1.

The practical conclusion from the above relationship is that if caving does not occur with a square undercut, consider lengthening the undercut as opposed to either increasing its span or to using the various ways of inducing caving. Mining experience confirms that lengthening can be effective.

The other important factor is the competence of the rock mass, or the occurrence of structural weaknesses (e.g. faults, critically oriented joint sets, altered zones). Unless specific detailed geological information is available, the occurrence of structural features that are significant with respect to caving can be considered a matter of probability. Clearly this probability, P_1, will be proportional to the area of the undercut, i.e.

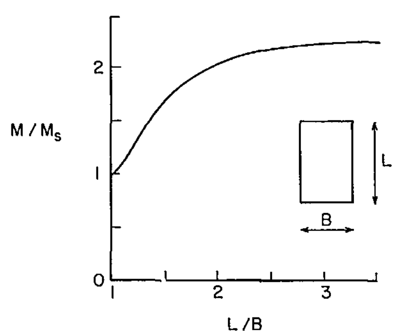


Fig 4-17 The variation of tension in the back and shear in the abutments, M, with the shape of the undercut for caving, L/B. M_s is the value when the undercut is square. Thus a rectangular undercut where L/B > 2 will produce stresses that are twice those from a square undercut, which may be required to initiate caving.

$$P_1 \propto BL \qquad \text{4-16a}$$

or

$$P_1^b/P_1^a = (BL)_b/(BL)_a \qquad \text{4-16b}$$

The probability of caving is also dependent on the caving stresses, which are proportional to the pre-mining stresses as well as to the span, B, and more or less to the ratio L/B when this ratio is less than 2. The field stresses, unless they have been measured, can be assumed to be proportional to the depth below the surface, Z. Because the probability of failure, P_2, varies with but is not proportional to stress, only a trend can be stated for ratios of L/B between 1 and 2:

$$P_2^b/P_2^a \propto Z_b B_b (L/B)_b / Z_a B_a (L/B)_a \text{ or } Z_b L_b / Z_a L_a \qquad \text{4-17}$$

Without more information on strength and its variance, this expression can be assumed to be an approximate equality.

Example: Increasing the Probability of Caving

At the start of a block caving operation using 60 ft x 60 ft panels, it is found that only one of the first four undercuts started caving within two days and continued through to the previous level. The others required assistance from fringe stopes. This means that the probability for successful caving, P_a, using 60 ft square panels is 0.25. What effect would increasing the undercut to 60 ft x 120 ft have on a block at the same depth below the surface?

From Eq 4-16b $P_1^b/P_1^a = (BL)_b/(B/L)_a$

$= (60 \times 120)/(60 \times 60) = 2$

From Eq 4-17 $P_2^b/P_2^a \propto (120/60) = 2$

Hence $P^b \simeq 2 \times 2 \times 0.25 = 1$

In other words, most of the stopes should cave.

REFERENCES

1. Terzaghi, K., "Introduction to Tunnel Geology", Rock Tunnelling with Steel Supports, Youngstown Printing, 1946.
2. Coates, D. F. and McRorie, K., "Earth Pressure on Multiple Tunnels", 15th Can. Soil Mech. Conf., NRC Tech. Mem. No. 73, 1962.
3. Coates, D. F. and Cochrane, T. S. "Development of Design Specifications for Rock Bolting from Research in Canadian Mines", Mines Branch Research Report R224, 1970.
4. Hogg, A. D., "Some Engineering Studies of Rock Movement in the Niagara Area", GSA Eng. Geol. Case Histories, No. 3, 1959.
5. Coates, D. F. "The Effect of Stress Concentrations on the Stability of Tunnels", Proc. 1st Cong. Int. Soc. of Rock Mech., Vol. 2, 1966.
6. Coates, D. F. "Rock Mechanics Principles", Mines Branch Monograph 874, 1970.
7. Coates, D. F. "Pillar Loading Part II: Model Studies", Mines Branch Research Report R170, 1965.
8. Dhar, B. "A Three-Dimensional Study of Pillar Stresses in Mines with Irregular Mining Boundaries", McGill Univ. PhD Thesis, 1970.
9. Obert, L. et al "Standardized Tests for Determining the Physical Properties of Mine Rock", USBM RI 3891, 1946.
10. National Bureau of Standards "Tables of Normal Probability Functions", Applied Math Series 23, US Govt. Printing Office, 1955.
11. Udd, J. "Studies Relating to Abutment Stresses for Mines", McGill Univ. PhD Thesis, 1970.
12. Coates, D. F. and Yu, Y. "Deformations and Stresses in the Roofs of Broad Multi-Reef Stopes", Unpub. Rpt, 1970.
13. Coates, D. F. and Dwarkin, L. M. "Roof Bolting Effectiveness at Michel" Trans. CIM, Vol. LXX, pp 32-37, 1967.
14. Bienkowski, Z. T. "Insitu Large Scale Testing of Coal" Proc. Conf. Br. Geol. Soc. on Insitu Investigation of Soils and Rocks, pp 67-74, 1969.

GLOSSARY

A - area through which water flows; a parameter in the equation for calculating pillar stresses

A_p - cross-sectional area of one pillar

A_t - total area tributary to one pillar

a - radius of a tunnel; subscript or superscript indicating a previous case

B - breadth; a parameter in the equation for calculating pillar stresses

b - subscript or superscript indicating the future case; B_p/L; burden on a blasthole

B_o - breadth of a stope

B_p - breadth of a pillar

C - total cost; a parameter in the equation for calculating pillar stresses

c - unit cost; a constant

D - blast damaged depth beyond rupture zone; deformation; a parameter in the equation for calculating pillar stresses

d - closure of a tunnel

E - modulus of deformation; a parameter in the equation for calculating pillar stresses

E_p - modulus of deformation of pillar rock

e - strain

e_m - maximum strain

F - blast rupture depth beyond rock face

g - gap between lining and rock

h - H/L

H - height of a tunnel, drift or stope; head of water

k - constant; coefficient of permeability; stress concentration factor

L - length of path over which water flows; the long dimension of an undercut; span or minimum distance between abutments of a mining zone

LF - linear feet

m - $\bar{Q}_p - \bar{S}_p$ mean of the difference curve

n - E/E_p

P - probability; a force

P_f - probability of failure

p_a - compressed air pressure above atmospheric pressure

p_i - pressure on rock from a lining or other support

p_o - pressure on a lining from the rock

p - water pressure

Q - quantity of flow; mean strength of the rock mass

Q_b - compressive strength of a pillar with a ratio $B_p/H \neq 1$

Q_1 - compressive strength of a pillar with a ratio $B_p/H = 1$

Q_o - uniaxial compressive strength of a unit volume

$\bar{Q}_p$ - average compressive strength of a number of pillars

Q_p - compressive strength of a pillar

Q_v - uniaxial compressive strength of the rock mass

R - extraction ratio for the mine; radius of rupture from blast; radius of a lining

r - extraction ratio adjacent to one pillar

S - stress; pre-mining stress perpendicular to the orebody

S_A - maximum stress in the abutment of an opening
S_m - maximum stress
S_o - pre-mining stress normal to the orebody
S_p - average compressive stress in a pillar
$\bar{S}_p$ - average compressive stress in a number of pillars
S_r - stress around a tunnel in the radial direction
S_t - stress around a tunnel parallel to the rock face (tangential stress); pre-mining stress parallel to the span of a mining zone
S_x - pre-mining rock stress in x-direction
S_y - pre-mining rock stress in y-direction; yield stress
S_z - pre-mining rock stress in z-direction
s - standard deviation; spacing of blastholes
s_q - standard deviation of strength
s_s - standard deviation of stress
t - thickness of a lining
V - volume
Y - maximum distance between abutments of a mining zone with an approximately rectangular shape
Z - depth below the ground surface
z - m/s, the statistical coupling factor

SUBSIDENCE

SYNOPSIS

To be able to predict subsidence, or surface movement, caused by underground mining, knowledge of the mechanical properties of the ground and of the mechanisms which govern the deformation are required. However, current knowledge on both of these aspects is not yet satisfactory for engineering purposes.

Alternatively, predictions can be made by extrapolating from previous experience. Field measurements of previous mining deformation provide, in effect, information on the rock properties, which can then be extrapolated using either empirical correlations or theoretical mechanisms. If the future conditions for which the extrapolation is made are not very different from those for which the measurements were obtained, the extrapolations should be fairly accurate. In any event, the method provides a best estimate, which can be useful for providing guidance in planning appropriate actions.

For flat-lying ore bodies the following equation can be used for determining the maximum subsidence that will occur over a new mining panel:

$$V_m^b / V_m^a = ((L_b/L_a)(Y_b/Y_a)(B_b/B_a)(A_b/A_a))^{1.5}(Z_a/Z_b)^3(H_b/H_a)$$

where subscripts and superscripts a and b refer to the previous and future cases, V_m is the maximum subsidence, L is the length of the panel, Y is the span of the panel, i.e. perpendicular to L, B = (2.8-L/Z), A = (2.8-Y/Z), Z is the depth below the surface to the workings, and H is the thickness of the workings for cases where 100% extraction is being obtained from the panel or is the maximum possible closure where less than 100% extraction is obtained. The equation assumes that the geological and mining conditions for the two cases are substantially the same.

The shape of the subsidence profile, which varies with the mining geometry, can also be predicted so that surface movement in the areas over the abutments of the panel can be calculated. These movements can be compared to maximum permissible amounts, which depend on the type of structure that might be affected, e.g. for some buildings 0.25 ft would be the maximum and for some pipelines 1.0 ft.

From the shape of the subsidence profile, the tilt of the ground surface, t, can be predicted using the following equation:

$$t_b = t_a (C_b/C_a)(V_b/V_a)(x_a/x_b)$$

where subscripts a and b refer to the previous and future cases, C = DE, D = (fL/Z + 0.03), f is a factor depending on the properties of the rock, L is the length of the panel, Y is the span of the panel, $E = (x/x_5)^2$, where x is the distance from the centreline of the panel to the point in question, and x_5 is the distance from the centreline of the panel to the point where the vertical subsidence is 0.5 V_m, where V_m is the maximum subsidence, V is the subsidence at the point in question.

Horizontal displacement, X, can then be obtained through its relationship with tilt, t, by using the following equation:

$$X_b = X_a(Z_b/Z_a)(t_b/t_a)$$

where Z is the depth below the ground surface of the panel.

Horizontal strain, e, which can be damaging to both structures and to the ground, can be predicted, albeit inaccurately, by using the computed horizontal displacement of points on either side of the structure or area in question, i.e. the general relationship that is used is as follows:

$$e = \Delta X/\Delta x$$

where ΔX is the difference between the horizontal displacements of the two points and Δx is the horizontal distance between these two points.

For steeply dipping orebodies, although no measurements are available for such cases, an approach is suggested for trying incremental design procedures.

INTRODUCTION

Subsidence is the ground movement, predominently vertical, above a mining excavation. It is usually related to underground excavations. Although it normally refers to moderate movement as opposed to caving, the term subsidence is often used to include these large movements. However, the mechanism of moderate movement, where strata remain substantially continuous, is different from the mechanics of drawing or shearing or flowing that are normally associated with caving (1). Therefore, for the purposes of this chapter subsidence will just refer to relatively moderate deformations.

Where surface structures - buildings, roads, railroads, pipes, waste embankments, dams, adjacent mines - occur in the area, subsidence can cause costly damages. Even without such structures, safety of the public and mine personnel makes it important to be able to predict subsidence. In addition, environmental damage in more or less civilized areas, and increasingly even in the wilderness, is making it important to control subsidence.

To be able to predict subsidence requires knowledge of the mechanical properties of the ground and the mechanisms which govern deformation. At the present time, methods for measuring rock mass properties have not been standardized. Predictions, however, can be made based on previous experience, primarily in the coal fields (2). Relations between the geometry of mining openings and surface movement have been established for the rock and mining methods of the U.K. Consequently, when used in different ground and mining conditions, the accuracy of the predictions is likely to be low, unless the different parameters applicable to the site are recognized.

Previous experience in the same ground analysed appropriately, in effect, provides a measure of the mechanical properties. Extrapolation can then be done with confidence if the ground remains substantially the same for the next mining excavation. Also, this procedure can be used for the cases of less than 100% extraction. A typical example could be: if the maximum vertical subsidence of the ground surface is 0.5 ft over an underground panel 1500 x 3000 ft with 40% extraction, what will be the vertical, horizontal and tilt movements 1000 ft to one side of an area that includes three such panels, i.e. 3000 x 5000 ft? By using the initial measurements as a test, predictions can then be made of the subsidence caused by the enlarged mining area.

Three different cases of subsidence are associated with: flat-lying orebodies, steeply-dipping veins and massive orebodies that lead to cover caving. The flat-lying orebodies, or seams, have been the subject of most study. Empirical relations have been established for the UK coal mines that may be of a form that is valid for other types of ground. Subsidence from mining in steeply-dipping veins has received very little study so that the form of the interaction of the variables is known with much less certainty. Over massive orebodies, where mining eventually leads to cover caving, the subsidence preceding caving can be treated substantially as for the flat-lying seams.

FLAT-LYING OREBODIES

Fig 5-1 shows the pattern of surface movement produced by mining a horizontal orebody. Vertical settlement, V, as shown in the figure extends beyond the abutments of the mining zone. Horizontal movement, X, can be seen to be zero on the centreline of the mining zone and a maximum over the abutments. Horizontal strain, e, then is compressive over the mining zone and tensile beyond the face of the abutments. The depth of the orebody below the surface is Z, the height or thickness of the orebody is H, one dimension of the mining zone is Y, the span of the mining zone such as the length of a longwall face (perpendicular to Y) is L, and i is the dip of the orebody.

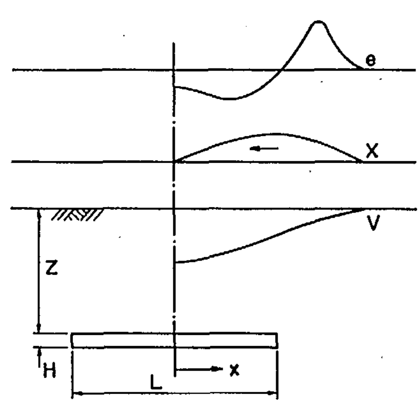


Fig 5-1 Typical subsidence over flat-lying workings; note that the settlement, V, extends beyond the abutment line; the horizontal movement, X, is a maximum over the abutment, and the horizontal strain, e, in the surface ground changes from compression over the mining excavation to tension beyond the abutment.

Fig 5-2 shows approximations to empirical curves produced by the National Coal Board based on measurements made in their coal fields covering the ranges: 18 ft $>H>$ 2 ft; 1500 ft $>L>$ 100 ft; 2600 ft $>Z>$ 100 ft; 4.0 $> L/Z >$ 0.05, and $i < 25^{\circ}$ (2). The curve relates the maximum vertical surface settlement, V_m to L, Y, and Z. V_m is expressed as a ratio of the height of the orebody, H, although a more appropriate concept is that it is related to the maximum convergence, which is only equal to H for 100% extraction and where stowing or filling is not used.

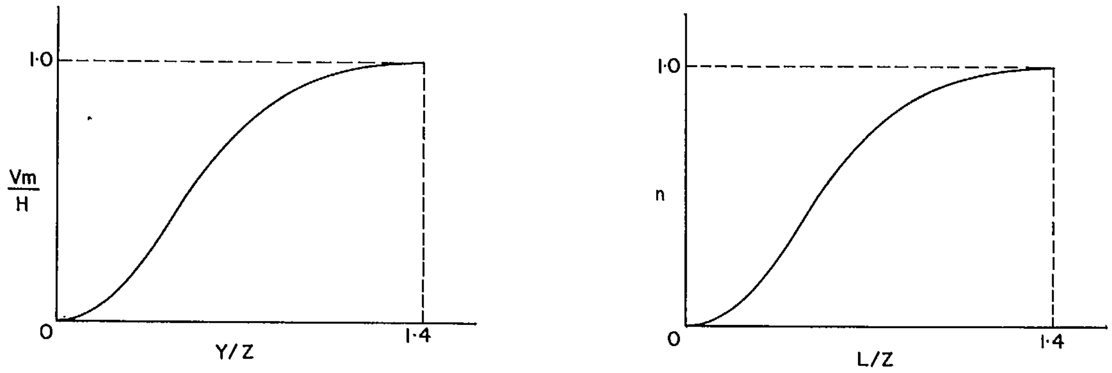


Fig 5-2 (a) Observed relation between the maximum settlement, Vm, and the span of the workings, or face advance distance, Y; Vm is expressed as the ratio of the height of the workings, H, and Y is expressed as a ratio of the depth, Z, below the ground surface. (b) A similar relation for the length of the face, L, or the dimension perpendicular to Y (after the work of the British National Coal Board).

As can be seen, V_m increases with L/Z and Y/Z up to about 1.4 when settlement no longer increases. Once the critical spans are exceeded, the vertical settlement curve, V, becomes flat in the central zone, i. e. V_m occurs over a central zone rather than just at the centreline. The following equations can be used to represent these curves for the cases where Y/Z and L/Z < 1.4:

$$V_m/H = qn(Y/Z(2.8 - Y/Z))^{1.5} \quad \text{5-1a}$$

where

$$n = 0.365(L/Z(2.8 - L/Z))^{1.5}$$

and q is a parameter that varies with the overburden rock and type of support, e. g. in UK coal mines for 100% extraction without stowing, q = 0.33 and for tight filling q = 0.16. When L/Z > 1.4 n = 1, then if Y/Z > 1.4, it follows that:

$$V_m/H = 2.74q \quad \text{5-1b}$$

For <u>incremental design</u>, when <u>measurements are taken for one case</u> and a <u>prediction</u> is required <u>for a second case</u>, Eq 5-1a can be written to eliminate the need for calculating the constants that depend on the ground conditions. Hence for the same ground and mining conditions and where both L/Z and Y/Z are less than 1.4, letting A = 2.8 - Y/Z and B = 2.8 - L/Z, it follows that:

$$V_m^b/V_m^a = ((L_b/L_a)(Y_b/Y_a)(B_b/B_a)(A_b/A_a))^{1.5}(Z_a/Z_b)^3(H_b/H_a) \quad \text{5-1c}$$

where subscripts a and b refer to the previous and future cases.

Where L/Z >1.4 in both cases, Eq. 5-1c reduces to:

$$V_m^b/V_m^a = ((Y_b/Y_a)(Z_a/Z_b)(A_b/A_a))^{1.5}(H_a/H_b) \quad \text{5-1d}$$

Clearly, there are two <u>subsidence profiles</u> - the <u>longitudinal</u> profile parallel to Y and the <u>transverse</u> profile normal to Y or parallel to L. Similar calculations can be made for each using the appropriate interaction effects of the ratios L/Z and Y/Z as expressed by Eq 5-1a. The shape of the transverse settlement profiles, as shown in Fig 5-3, varies with the rock properties and the ratio, L/Z (2,3). (Throughout the balance of the chapter, discussion will refer to the transverse profile unless otherwise specified.) An empirical equation gives approximate shapes that can be used for extrapolation in the area where V/V_m is less than 0.5, which is the area usually where horizontal strains, tilt and differential vertical settlement start to cause damage to surface structures:

$$\lg(V_m/V) = (f\ L/Z + 0.03)(x/x_5)^2 \qquad 5\text{-}2$$

where f is a constant for the site, which varies somewhat with L/Z and hence should be determined with L/Z as similar as possible to the conditions for which the prediction is required; x is the horizontal distance from the centreline to the point where V is to be predicted, and x_5 is the horizontal distance to the point where V is equal to $0.5V_m$. It has been found that when L/Z is less than 0.4 the distance, x_5, is greater than L/2, i.e. it is beyond the rib line (2).

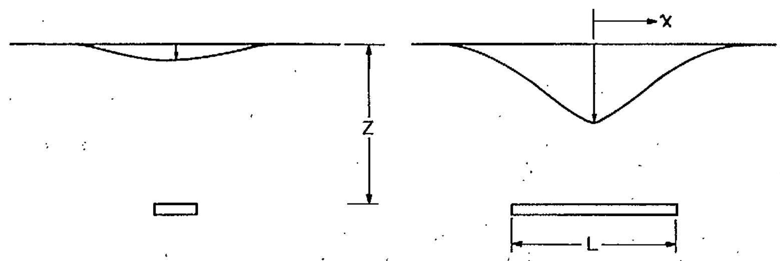


Fig 5-3 The shape of the settlement profile varies with the ratio L/Z.

When more than one panel is being mined, the vertical subsidence that is predicted at adjacent points for each panel can be added together to give the total subsidence that the combined effects will produce.

The smooth curves, of course, of Fig 5-3 become modified when distinctive structural features occur such as critically oriented faults (4).

To simplify the writing of Eq 5-2 for incremental design purposes, let $C = \lg(V_m/V)$, it follows that:

$$C_b = ((C_a(x_5/x)_a^2 - 0.03)(Z_a/L_a)(L_b/Z_b) + 0.03)(x/x_5)_b^2$$

or

$$(x_5/x)_b^2 = ((C_a(x_5/x)_a^2 - 0.03)(Z_a/Z_b)(L_b/L_a) + 0.03)(1/C_b) \qquad 5\text{-}3$$

To determine x_5 for any particular case, the field measurements can be fitted with the following empirical equation, which can then be used where $0.5 < L/Z < 2.4$ to extrapolate from measured data:

$$x_5/Z = 0.37(L/Z - 0.5)^{\frac{1}{2}} + d \qquad 5\text{-}4a$$

where d is a parameter depending on rock properties, the mining method, the degree of support, L/Z and V_m/V. Where $L/Z < 0.5$, use $L/Z = 0.5$, and where $L/Z > 2.4$, use $L/Z = 2.4$.

For incremental design purposes, the factor d can be eliminated as follows:

$$x_5^b/Z_b = x_5^a/Z_a - 0.37((L_a/Z_a - 0.5)^{\frac{1}{2}} - (L_b/Z_b - 0.5)^{\frac{1}{2}})$$

or letting $E = (L/Z - 0.5)^{\frac{1}{2}}$(where $L/Z < 0.5$, use 0.5 and where $L/Z > 2.4$, use 2.4), the equation is simplified to:

$$x_5^b/Z_b = x_5^a/Z_a - 0.37(F_a - F_b) \qquad 5\text{-}4b$$

The above expressions have been found to be applicable to seams that dip as much as 25° (2). The procedure in these cases as shown in Fig 5-4, is to draw a line perpendicular to the seam from the higher abutment or rib line, to the ground surface. The distance on the surface equal to the semi-span is then laid off to give the origin of the subsidence profile. The distances, x, are then measured horizontally from this origin.

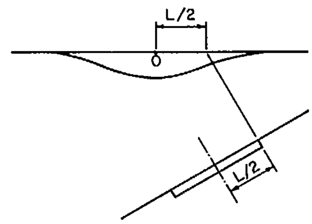


Fig 5-4 For seams dipping between 0^{o} and 25^{o} a modified procedure, as shown, can be followed that permits the use of the relations established for horizontal seams.

The maximum permissible vertical subsidence, V, depends on the structure, e. g. for some buildings it is 0.25 ft and for some pipelines 1.0 ft. To minimize structural damage, jacking points can be built into columns to compensate for differential vertical movement, and light structures can be either built in sections with flexible connections that can absorb some differential movement or can be placed on a rigid, egg-crate basement foundation.

Example: Extrapolation of Maximum Subsidence to Future Conditions

Vertical subsidence is measured during the mining of a panel, 1400 x 2800 ft, in a flat-lying seam at a depth, Z_a, of 3500 ft below the ground surface. The maximum vertical subsidence, V_m^a, was found to be 0.47 ft. The maximum convergence underground in the panel, which is equivalent to H_a, is 2 ft.

It is desired to predict the maximum vertical subsidence when the mining zone has been expanded to 2800 x 4400 ft. At this stage the maximum convergence, H_b, underground will be 7.0 ft. The depth, Z_b, is still 3500 ft.

Solution: For the previous panel $Y_a = 2800$ ft, $L_a = 1400$ ft (it does not matter for this calculation which dimension is used for Y, the same answer is obtained), $Y_a/Z_a = 0.8$, $L_a/Z_a = 0.4$. For the future mining zone $Y_b = 4400$ ft, $L_b = 2800$ ft, $Y_b/Z_b = 1.257$, $L_b/Z_b = 0.8$

Eq 5-1c is used to determine V_m for the new mining panel:

$$V_m^b = V_m^a((L_b/L_a)(Y_b/Y_a)(B_b/B_a)(A_b/A_a))^{1.5}(Z_a/Z_b)^3(H_b/H_a)$$

$$B = 2.8 - L/Z, \quad A = 2.8 - Y/Z$$

hence

$$B_a = 2.8 - 0.4 = 2.4$$

$$A_a = 2.8 - 0.8 = 2.0$$

$$B_b = 2.8 - 0.8 = 2.0$$

$$A_b = 2.8 - 1.257 = 1.543$$

$$L_b/L_a = 2800/1400 = 2.0$$

$$Y_b/Y_a = 4400/2800 = 1.57$$

$$B_b/B_a = 2.0/2.4 = 0.833$$

$$A_b/A_a = 1.543/2.0 = 0.772$$

$$Z_a/Z_b = 3500/3500 = 1.0$$

$$H_b/H_a = 7/2 = 3.5$$

and

$$V_m^b = 0.47\,(2.0 \times 1.57 \times 0.833 \times 0.772)^{1.5}\; 1^3 \times 3.5 = 4.71 \text{ ft}$$

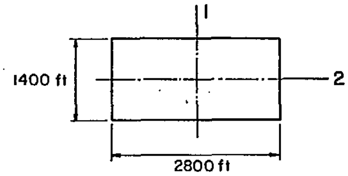


Fig 5-5 A mined out panel 1400 ft x 2800 ft for which surface settlement readings were taken along two perpendicular lines of observation points.

Example: Extrapolation of Subsidence Profile Measurements to the Effects of a Future Mining Zone on Adjacent Structures

For the 1400 x 2800 ft panel of the previous example, i.e. Z_a = 3500 ft, H_a = 2.0 ft and V^a_m = 0.47 ft, measurements provide the profiles along both lines 1 and 2, as shown in Fig 5-5. The distance to the point where V = 0.5 V_m, i.e. x_5, along Line 1 (for which L = 1400 ft and L/Z = 0.4) and along Line 2 (for which L = 2800 ft and L/Z = 0.8) is given by the profile curves. The other points of these curves, together with the corresponding horizontal displacements, X, (in brackets) are also given. They are as follows:

	Line 1	Line 2
x_5 =	700 ft (0.00 ft)	985 ft (0.00 ft)
x_4 =	810 ft (1.04 ft)	1110 ft (0.85 ft)
x_3 =	980 ft (0.82 ft)	1260 ft (0.67 ft)
x_2 =	1190 ft (0.57 ft)	1470 ft (0.45 ft)
x_1 =	1630 ft (0.27 ft)	1820 ft (0.25 ft)

It is now desired to predict for the expanded mining zone, 2800 x 4400 ft, the horizontal distance from the centre along the transverse profile (parallel to the 2800 ft side) to the point where the vertical subsidence, V^b, is 1 ft (possibly the maximum tolerance of a certain type of structure, e.g. a pipeline). For this profile Y_b = 4400 ft, L_b = 2800 ft, Y_b/Z_b = 4400/3500 = 1.257 and L_b/Z_b = 0.8.

Because the shape of a subsidence profile is sensitive to L/Z, as shown in Fig 5-3, it is preferable, if there is a choice, to use previous measurements for extrapolation purposes with an L/Z closest to the future case. Therefore, with two sets of data available, the Line 2 profile is used because L/Z = 0.8, which happens to be the same as for the future case.

Solution: Determine the point on the subsidence profile where V = 1 ft, or V/V_m = 1/4.71 = 0.21 This distance is designated x_{21}. The shape of the profile must be determined first, which is governed by the location of x_5, the distance where V/V_m = 0.5. Then the ratio x_5/x_{21} can be calculated and hence x_{21}

Eq 5-4b: $x^b_5/Z_b = x^a_5/Z_a - 0.37(F_a - F_b)$ and $F = (L/Z - 0.5)^{\frac{1}{2}}$

From the measurements along Line 2 x^a_5/Z_a = 985/3500 = 0.281. Because L_a/Z_a = 0.8, L_b/Z_b = 0.8 $F_a = F_b$,

therefore:

$$x^b_5/Z_b = 0.281 - 0.37(F_a - F_b) = 0.281$$

and

$$x^b_5 = 0.281 \times 3500 = 985 \text{ ft}$$

Eq 5-3: $(x_5/x)^2_b = ((C_a(x_5/x)^2_a - 0.03)(Z_a/Z_b)(L_b/L_a) + 0.03)/C_b$

From Line 2, x^a_{21} = 1430 ft (by interpolating between x_2 and x_1), hence $(x_5/x_{21})_a$ = 985/1430 = 0.688, $C_a = \lg(V_m/V)_a = \lg(1/0.21) = 0.676 = C_b$ because the ratio $(V_m/V)_b$ is the same; Z_a/Z_b = 1; L_b/L_a = 2800/2800 = 1, therefore:

$$(x_5/x_{21})^2_b = ((0.676 \times 0.688^2 - 0.03) + 0.03)/0.676 = 0.475$$

$$(x_5/x_{21})_b = \quad = 0.689$$

$$x^b_{21} = 985/0.689 = 1430 \text{ ft.}$$

In other words, for the same ratios of L/Z and equal values of Z, the distance to V/V_m = 0.21 will be the same.

These figures are undoubtedly only approximations, but they can provide guidance at little expense.

TILT

The slope of the subsidence curve, or tilt, as shown in Fig 5-6, can be determined by the theoretical relationship:

$$t = dV/dx \qquad 5\text{-}5a$$

Having determined the shape of one subsidence curve from field measurements, which is then used to predict the shape of a future subsidence curve, the above theoretical equation can be used to predict the associated tilt, t. In this way, a valid extrapolation can be made from the field data. It follows from Eq 5-2 (ignoring the negative sign) that:

$$t = dV/dx = 4.60(fL/Z + 0.03)(V_m x/x_5^2)10^{-DE} \qquad 5\text{-}5b$$

where $D = fL/Z + 0.03$ and $E = (x/x_5)^2$. Eq 5-5b can be written more simply as:

$$t = 4.60DE(V_m/x)/10^{DE} \qquad 5\text{-}5c$$

or recognizing that for Eq 5-3 $C = \lg(V_m/V)$ and hence from Eq 5-2 $C = DE$, further simplification yields:

$$t = 4.60\ C(V_m/x)/10^{C} \qquad 5\text{-}5d$$

or $$t = 4.60\ C\ V/x \qquad 5\text{-}5e$$

Put in incremental design form, the equation would be:

$$t_b = t_a\ (C_b/C_a)(V_b/V_a)(x_a/x_b) \qquad 5\text{-}5f$$

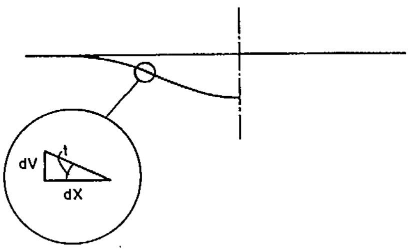


Fig 5-6 The slope of the settlement curve, or tilt, t, is related to the shape of the settlement curve.

The maximum permissible tilt depends on the type of structure and in many cases also on its length, e.g. towers and stacks are generally limited to a maximum tilt of 0.004B, where B is the width of the base. Sometimes several mining panels can be phased to produce, more or less, uniform vertical subsidence, which thus eliminates tilt.

Example: Predicting Tilt for an Expanded Mining Zone From Measurements Over a Limited Panel.

In the previous example, the distance, x, to the point where the vertical subsidence would be equal to 1.0 ft (or 0.21 of the maximum subsidence, 4.71 ft) was determined to be 1430 ft. It is now required to predict the tilt that will occur at this same distance.

Solution: Not knowing the tilt over the previous panel, extrapolation must be from the vertical measurements rather than from Eq 5-5f. To use Eq 5-5d it is necessary to know the site parameter, f. Again it is preferable to use data for ratios of L/Z and V/V_m that are closest to the future panel, hence use Line 2 data. As cited in the previous example:

for Line 2 $x_{21} = 1430$ ft and $x_5 = 985$ ft, hence:

from Eq 5-2 $\lg(1/0.21) = (0.8\ f + 0.03)(1430/985)^2$

and $f = 0.364$

For the new 2800 x 4400 x 7 ft panel L/Z = 0.8, and it was calculated in the previous examples that $V_m = 4.71$ ft, $x_{21} = 1430$ ft and $x_5 = 985$ ft for the future case

In Eq 5-5c $D = f\ L/Z + 0.03$ and $E = (x/x_5)^2$, hence for the future case

$D = 0.364 \times 0.8 + 0.03 = 0.321$ and $E = (1430/985)^2 = 2.11$, or $C = 0.321 \times 2.11 = 0.678$

hence $t = 4.60 \times 0.678 \quad (4.71/1430)/10^{0.678} = 2.1 \times 10^{-3}$

HORIZONTAL DISPLACEMENT

Laboratory experi ments have shown that the horizontal displacement of a point on the ground surface can be related to the tilt, recognizing that it is a small angle, by the relationships shown in Fig 5-7(5):

$$X = Zt \qquad 5\text{-}6a$$

It is known from theory that this relationship is only valid when the vertical motion results primarily from bending of the strata. When the depth, Z, is greater than half the span, L, a significant proportion of the vertical subsidence can come from shear strain. Furthermore, in a rock strata that is deforming from mining there is a horizontal frictional restraint at the bottom of the strata which modifies both of these actions. Consequently, Eq 5-6a should include a factor, g, that accounts for these differences from theory:

$$X = gZt \qquad 5\text{-}6b$$

where g is a constant for the particular rock and mining method (which was found to be equal to 0.66 in laboratory experiments with granular material (5)). By determining g from field measurements over one panel, predictions of X can be made for future panels.

Alternatively, put in incremental design form:

$$X_b = X_a(Z_b/Z_a)(t_b/t_a) \qquad 5\text{-}6c$$

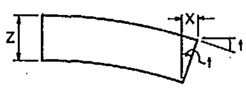


Fig 5-7 The horizontal movement of the ground surface, X, is related to the tilt and to the depth.

Absolute <u>horizontal displacement can be critical for such structures as pipes</u> carrying services such as water, gas and sewage. Maximum horizontal displacements are sometimes specified for building structures, (although horizontal strain is more appropriate) e.g. the maximum horizontal displacement for steel frame buildings has been specified as 0.005L, and to avoid the cracking of the plaster and mortar as 0.0005L, where L is the length of the building in the direction of the displacement. The effects on light structures can be minimized by placing them on a pad of compacted sand or gravel that tends to absorb the underlying ground displacement (such construction would not normally be permitted in existing building codes, which require foundations to be excavated to avoid frost heave, but a sufficient thickness of clean sand or gravel - say 4 ft - would prevent this action).

<u>Example</u>: <u>Predicting Horizontal Displacements.</u>

From the data of the previous examples it is now necessary to predict the horizontal displacement X, that would occur at the same point where the vertical displacement is 1.0 ft due to mining the future zone 2800 x 4400 x 7 ft; the depth, $Z_b = 3500$ ft. The previous example showed that the tilt, t_b, at this range would be 2.1×10^{-3} The horizontal measurements along Line 2 are:

at	$x_4 = 1110$ ft	X = 0.85 ft
	$x_3 = 1260$ ft	= 0.67 ft
	$x_2 = 1470$ ft	= 0.45 ft
	$x_1 = 1820$ ft	= 0.25 ft

<u>Solution</u>: To use Eq 5-6e, the tilt over the previous panel, t_a, must be known Analysing Line 2 data, because it is the most appropriate for extrapolation, $L_b/Z_b = 0.8$; we know that $V^a_m = 0.47$ ft, and $x_{21} = 1430$ ft. Moreover, calculations according to the previous methods show that $V_a = 0.10$ ft and by interpolation $X_a = 0.47$ ft.

To use Eq 5-5e $C_a = \lg(1/0.21) = 0.678$ hence:

$$t_a = 4.60 \times 0.678 \times 0.10/1430 = 0.217 \times 10^{-3}$$

Then from Eq 5-6c

$$X_b = X_a(Z_b/Z_a)(t_b/t_a)$$

$$= 0.47 \times 1 \times 2.1/0.217 = 4.55 \text{ ft.}$$

STRAIN

Strain in the horizontal direction at the ground surface, as mentioned above, is a more appropriate factor to consider when concerned with the effects of horizontal displacement on structures. From theory it is known that the strain, e, is related to horizontal displacement, X, by the following equation:

$$e = dX/dx \qquad \text{5-7a}$$

However, for practical purposes, it is not advisable to use Eq 5-7a(requiring, in effect, the differentiation twice of vertical subsidence). Consequently, it is better to obtain the difference in horizontal displacements, preferably from field measurements at two different points bracketing the zone of interest and then dividing by the horizontal distance between them to obtain an average strain, in which case Eq 5-7a becomes as follows:

$$e = \Delta X/\Delta x \qquad \text{5-7b}$$

Thus extrapolation from field measurements is obtained by predicting relative horizontal displacements.

If mining panels can be phased to produce approximately uniform vertical subsidence, horizontal movement, strain and the consequent damaging effects on structures will be eliminated. Otherwise, where the magnitudes of strain are 0.001 or more tension cracks in the ground will be produced.

Example: Predicting Surface Strain from Vertical Subsidence Measurements.

For the first example it is desired to predict surface strains for a mining zone measuring 2800 x 4400 x 7 ft where on the transverse profile (parallel to the 2800 ft) the vertical subsidence is 1.0 ft. Knowing from the other examples that $V^b_m = 4.71$ ft, $V/V_m = 0.21$, $x_{21} = 1430$ ft, and at this distance $X = 4.57$ ft and $t = 2.1 \times 10^3$; and also that $x_5 = 985$ ft, $x_{21}/x_5 = 1.45$ and $f = 0.364$.

Solution: In order to calculate e, it is necessary to determine two sets of X and x for the future panel so that ΔX and Δx be computed. Arbitrarily, determine second set for $V/V_m = 0.25$ to use with the set for $V/V_m = 0.21$.

To determine x^b_{25} for the new mining zone, 2800 x 4400 ft, use Eq 5-2 knowing from the previous examples $L/Z = 0.8$, $f = 0.364$ and $x_5 = 985$ ft:

$$\lg(1/0.25) = (0.8 \times 0.364 + 0.03)(x_{25}/x_5)^2$$

hence

$$(x_{25}/x_5)_b = 1.37$$

$$x^b_{25} = 1.37 \times 985 = 1350 \text{ ft}$$

For Eq 5-5e $\qquad C = \lg(V_m/V) = \lg(1/0.25) = 0.602$

hence at x_{25} $\qquad t_b = 4.60\ C\ V/x$

$$= 4.60 \times 0.602(0.25 \times 4.71)/1350 = 2.41 \times 10^{-3}$$

From Eq 5-6c the horizontal displacement can be calculated using $t_a = 0.216 \times 10^{-3}$ and $x_a = 0.47$ ft at x_{25}:

at x_{25} $\qquad X_b = X_a(Z_b/Z_a)(t_b/t_a)$

$$= 0.47(3500/3500)(2.41/0.216) = 5.24 \text{ ft}$$

From Eq 5-7b $\qquad e = \Delta X/\Delta x$

$$= (5.24 - 4.57)/(1350 - 1430) = 8000 \times 10^{-6} \text{ tension}$$

This strain is likely to produce cracking because the fracture of rocks and concrete can occur at strains of 100×10^{-6} and greater; note, however, that the strain calculation is inherently very inaccurate as it is at the end of several empirical extrapolations and it is based on two differences, which magnifies the probable error.

STEEPLY-DIPPING OREDODIES

Fig 5-8 shows the patterns of surface movement that might be produced by mining a vertical orebody. Actually, no surface measurements have been made of the surface subsidence that occurs from mining steeply-dipping veins. Consequently, this section must be based entirely on theory (ie. using the Boussinesq approach).

In many ways there is nothing more practical than a good theory; however, field measurements are required in the long-run to determine the adequacy of any theory. The approximate, simplified equations presented below would be valid for an elastic continuum; consequently, deviations will occur as a result of rock masses being fractured and possibly layered. Nevertheless, using incremental design procedures the inadequacies of the equations will be minimized because they are just being used for extrapolations.

Fig 5-8a shows the vertical movement, V, that must be expected at the ground surface over a steeply-dipping mining zone. Some settlement may occur immediately over the mining zone; however, net uplift can occur in the adjacent zones.

Fig 5-8b illustrates the theoretical concept that is being used here. The act of excavating the ore is equivalent to relieving the constraint that previously existed on the face rock or, in other words, is equivalent to forcesacting inwards at these boundaries. The vertical force, G, acting at the top of the stope will cause the ground surface to move downwards. The tilt, t, or slope, of the ground surface associated with this downward movement is simply the slope of the vertical movement, V_v, curve.

The downward force, G, will produce horizontal movement, X, of the ground surface towards the orebody. Associated with this horizontal movement will be horizontal strain, e, that will cause cracking of the surface ground and concrete in structural foundations when it is of sufficient magnitude.

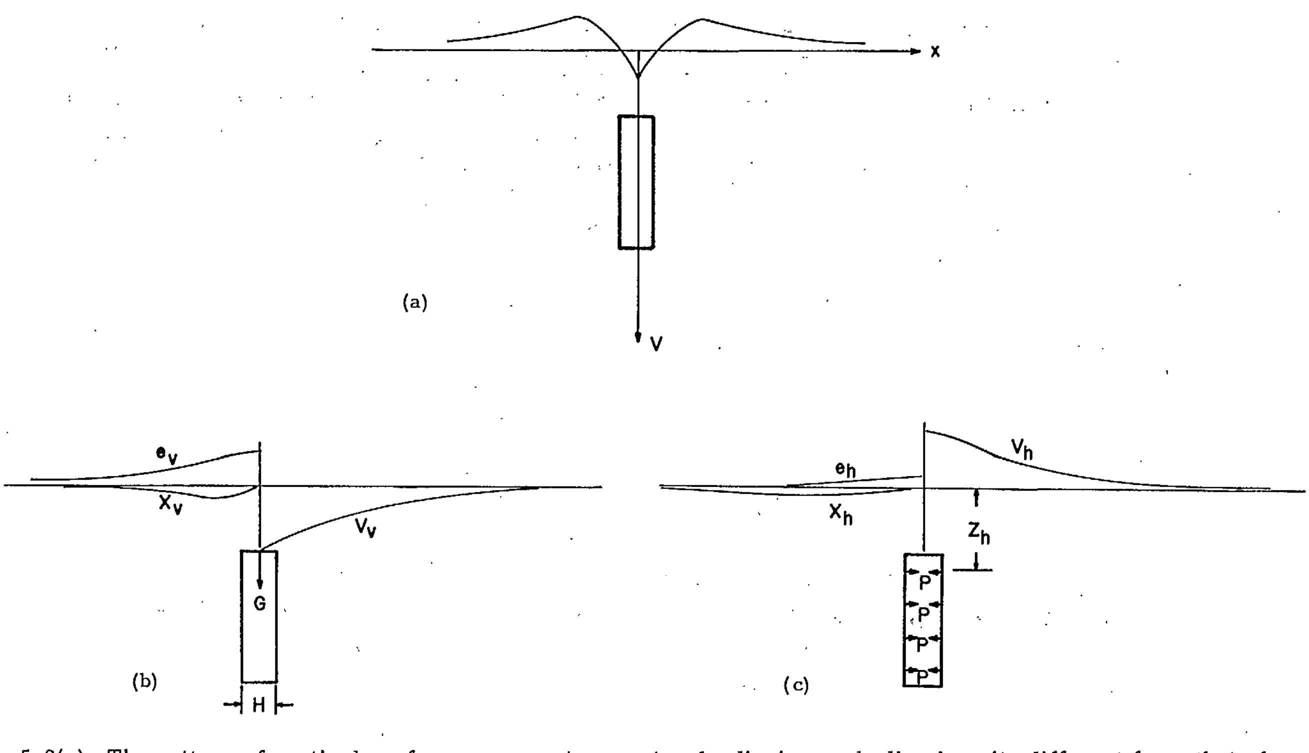


Fig 5-8(a) The pattern of vertical surface movement over steeply-dipping orebodies is quite different from that of horizontal seams; there can be both settlement and uplift. (b) The effect of releasing the constraint on the back is equivalent to applying a force, G, downward, which causes settlement, V_v, horizontal movement towards the ore zone, X_v, and horizontal compressive strain, e_v. (c) The effect on the walls is equivalent to applying forces, P, inwards, which cause uplift, V_h, horizontal movement inwards, X_h, and compressive strain, e_h. The two effects combine to produce resultant curves such as in (a) but which can vary considerably depending on the geometry of the orebody and the pre-mining stresses in the rock.

Fig 5-8c indicates that the mechanical effect of excavating the ore is also equivalent to applying horizontal forces inward on the walls. Such forces, P, will cause vertical uplift of the ground surface, which is a maximum immediately over the orebody and decreases with distance away from the vein. The horizontal movements of the ground surface vary from zero at the vein to a maximum and then decrease to zero at a large distance from the orebody. The associated surface horizontal strain can be either compression or tension.

For incremental design purposes the following theoretical approach can be tried for predicting vertical subsidence.

The total vertical movement, V, equal to the effects of G, i.e. V_v, and the effects of the P's acting on both walls, V_h, can be expressed as follows:

$$V = V_v + V_h \qquad 5\text{-}8a$$

and

$$V_v = J(3(Z/R)_v^2 + 5)(Z/R)_v \qquad 5\text{-}8b$$

$$= J\,W_v$$

where J is the parameter representing that particular rock mass and varies with the deformation properties, the pre-mining vertical stress and the horizontal area exposed on which the force G acts; Z is the depth below the ground surface to the point of application of G; R is the oblique distance to the point where V occurs, i.e. $R = (x^2 + Z^2)^{\frac{1}{2}}$, where x is the horizontal distance from the origin to the point where V is occurring, W_v represents the geometrical parameters shown in the equation and the subscript 'v' refers to the vertical force G. The effect of one P acting on each wall is as follows:

$$V_h = J\,k\,Z_h^2\,U \qquad 5\text{-}8c$$

$$= J\,W_h$$

where $k = S_h/S_v$, S_h and S_v are the pre-mining horizontal stresses (assuming this ratio is constant for the site), $U = N\text{-}Q$, $N = (x\text{-}h)/R_1^3$, $h = H/2$, H is the thickness of the orebody or width of the stopes if the full thickness is not mined, $R_1 = ((x\text{-}h)^2 + Z_h^2)^{\frac{1}{2}}$, $Q = (x+h)/R_2^3$, $R_2 = ((x+h)^2 + Z_h^2)^{\frac{1}{2}}$, W_h represents the geometrical parameters shown in the equation and subscript 'h' refers to horizontal forces P.

One of the underlying assumptions in Eq 5-8c is that the area of wall on which P is acting is the same as the area on which G is acting, e.g. the area for G could be assumed to be H x H (hence $G = S_v H^2$) and thus the area on which P would be acting would also be H x H (hence $P = S_h H^2$). If the length of the stopes down-dip was greater than H, a second set of P's would be acting on the walls and a second parameter W_h would be calculated. Eq 5-8c would then have to include the effects of this second set of forces, and the effects of a third, fourth, etc. or as many as necessary to include the full area of walls exposed, i.e.:

$$V_h = J(W_i^h + W_{ii}^h + W_{iii}^h + \ldots) \qquad 5\text{-}8d$$

$$= J \sum (W_h)$$

It follows that Eq 5-8a can be restated:

$$V = J\,W_v + J \sum (W_h)$$

$$= J(W_v + \sum (W_h))$$

$$= J\,W \qquad 5\text{-}9$$

To determine the site parameter J it is necessary to have measured for a previous case the vertical subsidence, V_a; in which case it follows that:

$$J = V_a / W_a$$

where subscript 'a' refers to the previous mine case for which V_a is measured.

Having determined the mechanical properties of the site, it is then possible to extrapolate to a new case, that is to predict the vertical subsidence, V_b, resulting from additional mining in the same formations It is assumed that the pre-mining stresses are proportional to the depth below the ground surface and that the stresses are acting on the same area of backs and walls as in the previous case. It follows that:

$$V_b = J\,W_b \qquad 5\text{-}10$$

Sign convention is that distances to the right are positive, and distances and movement downward are positive.

The corresponding tilt, or slope, of the vertical subsidence curve, can be obtained following Eq 5-5a. I n incremental design form it has the following components:

$$t_v^b = -xJ(9(Z_v^b/R)^2 + 5)/R^3 \qquad 5\text{-}10a$$

$$t_h^b = 18kJ(Z_h^b/Z_v^a)Z_h^b(xH/R^5) \qquad 5\text{-}10b$$

where t_v is the contribution by G and t_h is caused by P or a series of P's.
Note: Negative tilt is towards the mining zone. The net tilt, or slope, of the ground surface is then the algebraic sum of the two contributions:

$$t^b = t_v^b + t_h^b \qquad 5\text{-}10c$$

Similarly, the equations for determining the horizontal movement of the ground surface, X, results from the two sets of forces G and P, which is expressed by the following equations:

$$X_v^b = -xJ(Z_v^b/Z_v^a)(3Z_v^b/R^2 + 2/(R + Z_v^b))/R \qquad 5\text{-}11a$$

$$X_h^b = -2kxH\, J(Z_h^b/Z_v^a)/R^3 \qquad 5\text{-}11b$$

The net horizontal displacement for the future mining geometry is the algebraic summation of the two contributions:

$$X^b = X_v^b + X_h^b \qquad 5\text{-}11c$$

Note: negative displacement is away from the mining zone.

The associated horizontal strain at the ground surface, e, also arises from these two sets of forces, G and P. The incremental design equations based on measurements for the first mining geometry are as follows:

$$e_v^b = J(Z_v^b/Z_v^a)(4Z_v^b/R^2 - 10(Z_v^b)^3/R^4 + 5/(R + Z_v^b)3)/R \qquad 5\text{-}12a$$

$$e_h^b = 2kH\, J(Z_h^b/Z_v^a)(Z_h^b)^2/R^5 \qquad 5\text{-}12b$$

Positive strain is expansion, and the net strain is the algebraic summation of these two contributions:

$$e^b = e_v^b + e_h^b \qquad 5\text{-}12c$$

As mentioned above, it is not known how valid these equations are for actual rock masses. They should be reasonably accurate, and at least they provide a framework to plan field programs for obtaining measurements to appraise their validity. The effects of the forces P can be seen to be greater than those of G owing to the area of exposure of the walls being greater than that of the back. In addition, the equations show clearly the importance of the pre-mining stress condition as represented by the ratio $k = S_h/S_v$. In other words, if the horizontal stresses are greater than the vertical stresses, which has been shown to occur at many sites, the effects of the forces P will be even greater.

Example: To Predict Vertical Ground Surface Movement Adjacent to the Mining of a Vertical Orebody.

The orebody is 50 ft thick and has a 90° dip. Stoping started at 100 ft below the ground surface, Z_v^a. Measurements at the ground surface show that vertical settlement, V_a directly over the orebody is 0.1 ft after mining down-dip for 100 ft.

A structure on the surface is 200 ft away from the orebody. What will be the vertical movement, V_b, when mining has proceeded an additional 100 ft down-dip? It is assumed that the pre-mining horizontal rock stresses are equal to the vertical stresses, i.e. k = 1.

Solution: From Eq 5-8b with Z_v = 100 ft and x = 0 it follows that R = 100 ft, hence:

$$W_v^a = (3(Z/R)_v^2 + 5)(Z/R)_v$$
$$= (3(100/100)^2 + 5)(100/100) = 8$$

For Eq 5-8c and 5-8d two sets of forces P must be used, one set at a depth, Z_h, of 125 ft (i.e. to the centroid of the top 50 ft of the walls) and another at a depth of 175 ft (i.e. to the centroid of the bottom 50 ft of the walls); H being 50 ft, h = 25 ft; x = 0. In the first set it follows that $R_1 = ((0-25)^2 + 125^2)^{\frac{1}{2}} = 127.5 = R_2$, hence:

$$U_1 = N_1 - Q_1$$
$$= (0-25)/127.5^3 - (0+25)/127.5^3 = -2.41 \times 10^{-5}\ ft^{-2}$$

and $$W_1^h = k\, Z_h^2\, U_1$$
$$= -1 \times 125^2 \times 2.41 \times 10^{-5} = -0.377$$

Similarly for the second set $R_1 = ((0-25)^2 + 175^2)^{\frac{1}{2}} = 176.8 = R_2$, hence:

$$U_2 = (0-25)/176.8^3 - (0+25)/176.8^3 = -9.05 \times 10^{-6}\ ft^{-2}$$

and $$W_2^h = -1 \times 175^2 \times 9.05 \times 10^{-6} = -0.277$$

Following Eq 5-8d:

$$\sum(W_h) = -0.377 - 0.277 = -0.654$$

and $$W = W_v + \sum(W_h)$$
$$= 8 - 0.654 = 7.346$$

Using Eq 5-9:

$$V_a = J\, W_a$$
$$0.1 = J \times 7.346$$

or $$J = 0.0013613\ ft$$

For the future case x = 200 ft Considering first W_v: Z_v is still 100 ft, hence $R = (200^2 + 100^2)^{\frac{1}{2}}$ = 223.6 ft, and from Eq 5-8b:

$$W_v^b = (3(Z/R)_v^2 + 5)(Z/R)_v$$
$$W_v^b = (3(100/223.6)^2 + 5)(100/223.6) = 2.50$$

To determine W_h^b, the effects of two additional sets of forces must be added to the previous sets; now the depths to the four sets are 125, 175, 225 and 275 ft; the corresponding values of R_1 are $((200-25)^2 + 125^2)^{\frac{1}{2}}$ = 215.06 ft, 247.49, 285.04 and 325.96 ft; the corresponding values of R_2 are $((200+25)^2 + 125^2)^{\frac{1}{2}}$ = 257.39 ft, 285.04, 318.20 and 355.32 ft.

Using Eq 5-8c:

$$W_h = k\, Z_h^2\, U$$

or $$= k\, Z_h^2\,(N-Q)$$

hence $$W_i^h = 1 \times 125^2((200-25)/215.06^3 - (200+25)/257.39^3)$$
$$= 0.0687$$
$$W_{ii}^h = 1 \times 175^2(175/247.49^3 - 225/285.04^3)$$
$$= 0.0560$$
$$W_{iii}^h = 1 \times 225^2(175/285.04^3 - 225/318.20^3)$$
$$= 0.0290$$

$$W_{iv}^{h} = 1 \times 275^2(175/325.96^3 - 225/355.32^3)$$
$$= 0.0028$$
$$\sum W_h = 0.1565$$

and

$$W_b = (W_v + \sum W_h)_b$$
$$= 2.50 + 0.1565 \qquad = 2.6565$$

From Eq 5-10:

$$V_b = J\, W_b$$
$$= 0.0013613 \times 2.6565 \qquad = 0.00361 \text{ ft.}$$

Being positive, the movement would be downward, albeit a small amount.

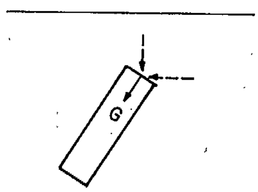


Fig 5-9 For inclined orebodies the forces G and P can be resolved into their vertical and horizontal components so that the resultant surface movement can be predicted.

For inclined orebodies, the same procedures as described above can be followed using the vertical and horizontal components of the excavation forces. In Fig. 5-9 the force on the back, G, is resolved into vertical and horizontal forces, which then can be used as in the equations above. The forces, P, on the walls can be treated similarly. Equations have been derived, however, before some empirical confirmation is obtained, it would be premature to suggest that they be used.

REFERENCES

1. Coates, D. F. "Rock Mechanics Principles", Monograph 875, Mines Branch, Revised 1970.
2. National Coal Board "Subsidence Engineers Handbook" 1966
3. Martos, F. "Concerning an Approximate Equation of the Subsidence Trough and its Time Factor" Proc. Int. Strata Control Conf. Leipzig, 1958.
4. Lee, A. J. "The Effect of Faulting on Mining Subsidence", Mining Engineer, Vol. 125, No. 71, p735, 1966.
5. Lee A. J. and Chen, C. "Horizontal Movements Related to Subsidence", Proc. ASCE V 95, No. SM1, 1969.

GLOSSARY

A - $2.8 - Y/Z$

a - a superscript or subscript for parameters relating to previous experience

B - $2.8 - L/Z$

b - a superscript or subscript for parameters relating to a future case

C - $\lg(V_m/V)$

D - $f\,L/Z + 0.03$

d - a rock parameter governing subsidence profile shape

E - $(x/x_5)^2$

e - horizontal strain at the surface

F - $(L/Z - 0.5)^{\frac{1}{2}}$

f - a rock parameter governing the shape of the subsidence profile

G - equivalent inward force on a face arising from release of constraint

g - a site parameter governing horizontal movement

H - thickness of orebody or seam, or the maximum amount of closure possible in horizontal seams

i - dip of orebody

J - $V\,R(3(Z_v/R)^2 + 5)^{-1}$

k - the ratio of S_h/S_v

L - length of a mining zone

n - $0.365(L/Z(2.8 - L/Z))^{1.5}$

P - equivalent inward force on the walls arising from release of constraint

q - a rock parameter governing V_m

R - the oblique distance from the point of application of G or P to the point on the ground surface where measurements or predictions of subsidence are being made

S_h - pre-mining horizontal stress in the ground

S_v - pre-mining vertical stress in the ground

V - vertical settlement of surface

V_m - maximum settlement

V_h - settlement due to P

V_v - settlement due to G

X - horizontal movement of surface

x - horizontal distance from the origin, e.g. the vertical centreline of the panel or orebody

x_1 - x to where $V = 0.1\,V_m$

x_5 - x to where $V = 0.5\,V_m$

Y - width of a mining zone, perpendicular to L

Z - depth below surface