

# PIT SLOPE MANUAL

supplement 4-1

## COMPUTER MANUAL FOR SEEPAGE ANALYSIS

This supplement has been prepared as part of the

PIT SLOPE PROJECT

of the

Mining Research Laboratories

Canada Centre for Mineral and Energy Technology

Energy, Mines and Resources Canada

MINERALS RESEARCH PROGRAM  
MINING RESEARCH LABORATORIES  
CANMET REPORT 77-30

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Available by mail from:

Printing and Publishing  
Supply and Services Canada,  
Ottawa, Canada K1A 0S9

CANMET  
Energy, Mines and Resources Canada,  
555 Booth St.  
Ottawa, Canada K1A 0G1

or through your bookseller.

Catalogue No. M38-14/4-1977-1 Price: Canada \$3.50  
ISBN 0-660-01007-2 Other countries: \$4.20

Price subject to change without notice.

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En vente par la poste:

Imprimerie et Édition  
Approvisionnement et Services Canada,  
Ottawa, Canada K1A 0S9

CANMET  
Énergie, Mines et Ressources Canada,  
555, rue Booth  
Ottawa, Canada K1A 0G1

ou chez votre libraire.

N° de catalogue M38-14/4-1977-1 Prix: Canada \$3.50  
ISBN 0-660-01007-2 Autres Pays: \$4.20

Prix sujet à changement sans avis préalable.

## THE PIT SLOPE MANUAL

The Pit Slope Manual consists of ten chapters, published separately. Most chapters have supplements, also published separately. The ten chapters are:

1. Summary
2. Structural Geology
3. Mechanical Properties
4. Groundwater
5. Design
6. Mechanical Support
7. Perimeter Blasting
8. Monitoring
9. Waste Embankments
10. Environmental Planning

The chapters and supplements can be obtained from the Publications Distribution Office, CANMET, Energy, Mines and Resources Canada, 555 Booth Street, Ottawa, Ontario, K1A 0G1, Canada.

Reference to this supplement should be quoted as follows:

Marlon-Lambert, J. Pit Slope Manual Supplement 4-1 - Computer Analysis of Groundwater Seepage; CANMET (Canada Centre for Mineral and Energy Technology, formerly Mines Branch, Energy, Mines and Resources Canada), CANMET REPORT 77-30; 97 p.; December 1977.

## ABSTRACT

FEFPM performs a steady-state seepage analysis to determine fluid pressures, flow quantities, rates and direction of groundwater slopes. It assumes either planar or radially symmetric flow, linear in a finite element in a porous medium. It computes the location of phreatic surface for unconfined flow problems, material type and permeability for a specific soil element, and fluid potential and stream function values. The program generates nodes and elements of a finite element mesh during iterations to locate a phreatic surface and automatically limits the flow regime to the slope profile.

## ACKNOWLEDGEMENTS

Roy Sage was responsible for the production of the Groundwater chapter and the supplement on Seepage. Address enquiries to him at: 555 Booth Street, Ottawa, Ontario K1A 0G1.

The finite element seepage computer program was developed by J. Marlon-Lambert from an original seepage program produced by the U.S. Bureau of Mines. J. Marlon-Lambert wrote the supplement and provided the sample data and results. The program was modified to run on the CDC Cyber 70 of the Department of Energy, Mines and Resources by A-S Wong and staff of the Computer Science Centre in the department. The text was edited by Sysdoc International.

Contractors: Golder Associates  
Sysdoc International

The Pit Slope Project is the result of five years' research and development cooperatively funded by the Canadian Mining Industry and the Government of Canada.

The Pit Slope Group has been led successively by D.F. Coates, M. Gyenge and R. Sage; their colleagues have been G. Herget, B. Hoare, G. Larocque, D. Murray and M. Service.

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## PROGRAM IDENTIFICATION

PROGRAM TITLE: Seepage: Finite Element analysis of Flow in  
Porous Media.

PROGRAM CODE NAME: FEFPM

WRITER: Golder Associates

ORGANIZATION: Energy, Mines and Resources  
Canadian Centre for Mineral and Energy Technology.

DATE: January, 1976

UPDATES: This program was converted in December, 1976, from  
Univac 1108 FORTRAN to Control Data CYBER FORTRAN.

SOURCE LANGUAGE: Originally written in FORTRAN<sub>V</sub> for the UNIVAC 1108,  
EXEC-8. The version supplied by CANMET has been  
converted to CDC CYBER FORTRAN.

AVAILABILITY: Pit Slope Group, Mining Research Laboratories,  
Canada Centre for Mineral and Energy Technology,  
Department of Energy, Mines and Resources,  
555 Booth Street,  
Ottawa, Canada K1A 0G1

DISCLAIMER: Neither the authors nor the Mining Research Lab-  
oratories can accept any responsibility for the  
accuracy of results generated by this program.

## ENGINEERING DOCUMENTATION

### NARRATIVE DESCRIPTION

1. The user prepares scaled drawings, based on geological samples, of the seepage area in terms of a finite element mesh. The salient features that must be included are:

- material boundaries
- boundary conditions
- an assumed initial phreatic surface.

2. The properties of the area must be related to the co-ordinates of this mesh. Units and dimensions of the co-ordinate system must be consistent and chosen carefully for a good solution. The following measurements must be used for material permeability and fluid properties:

Property	measured as
pressure (head)	length
permeability	length/time
flow rate (velocity)	length/time
flow quantity	length <sup>3</sup> /time

A knowledge of fluid and soil/rock mechanics is required to apply realistic boundary conditions.

3. The grid is usually drawn with quadrilateral elements, but triangular elements may be added. Areas of significant changes in velocity or pressure should have a finer mesh.

4. Most of the input data is obtained from this scale model. It includes:

- a. a title for the problem
- b. parameters describing the type of problem and the desired results. These are entered as a FORTRAN NAMELIST, and default values are provided to minimize input.
- c. definition of nodes; locations, boundary conditions and generation parameters for significant points are entered. The program will extrapolate from and interpolate between significant points as required.
- d. material properties: type, directional permeabilities and the extent of zoning in the problem area
- e. element definitions in terms of nodes and material types
- f. fluid inflows described along edges of elements (optional)
- g. pipes describing a string of elements linking any internal sources or sinks to a boundary (optional).

5. Intermediate results can be saved and used in a program restart. This is useful to monitor partial results before the program goes through numerous iterations with bad data that will never converge. It also permits iterated data to be restarted with different output parameters to produce different reports without changing

original data. Output includes a listing of the input data, detailed intermediate results of each iteration, including extrapolated and interpolated nodes, potential flow values defining the final phreatic surface, and optionally mesh and nodal pressures between iterations. Options are available for reducing the output printed.

## METHOD OF SOLUTION

### Finite Element Formulation of Groundwater Seepage

#### The Mathematical Problem

6. The general equation for steady-state Darcy type fluid flow in a two-dimensional porous medium, ie, wherein the rate of flow is proportional to the potential head gradient, is:

$$\frac{\partial}{\partial x} (k_x \cdot \frac{\partial \phi}{\partial x}) + \frac{\partial}{\partial y} (k_y \cdot \frac{\partial \phi}{\partial y}) = Q(x,y) = 0 \quad \text{eq 1}$$

in which  $\phi$  is the potential head function;  $k_x$ ,  $k_y$  are the material permeabilities in the x- and y- directions, respectively; and  $Q(x, y)$  is the rate of flow influx to the region per unit volume.

7. The solution to this equation must satisfy the boundary conditions for the actual porous media flow (or seepage) problem under study. The physical conditions of the particular problem and the region considered will commonly impose boundary conditions where:

- i) the value of  $\phi$  is specified, or
- ii) the rate of flow is specified across the boundary

8. Boundary condition case (ii) is governed by the following equation:

$$k_x \cdot \frac{\partial \phi}{\partial x} \cdot l_x + k_y \cdot \frac{\partial \phi}{\partial y} \cdot l_y + q = 0 \quad \text{eq 2}$$

where  $l_x$ ,  $l_y$  are the direction cosines of the outward normal to the boundary surface and  $q$  is the flow per unit area across the surface. In the case of a non-conducting boundary, the value of  $q$  is simply set to zero.

9. Equation 1, together with the boundary conditions, specifies the problem uniquely. By using the Euler conditions of the calculus of

variations it is possible to show that the problem is identical to that of determining the minimum value of a function,  $E$ , where:

$$E = \int_{\text{volume}} \frac{1}{2} (k_x \left(\frac{\partial \phi}{\partial x}\right)^2 + k_y \left(\frac{\partial \phi}{\partial y}\right)^2 + Q \cdot \phi) \cdot d(\text{volume}) \quad \text{eq 3} \\ + \int_{\text{area}} q \cdot \phi \cdot d(\text{area})$$

and with  $\phi$  specified on a boundary region. The expression within the volume integral represents the rate at which energy is being dissipated in a unit volume and thus the integral becomes the energy dissipation rate for the entire flow region.

#### The Finite Element Procedure

10. In the finite element procedure, the continuous region under analysis is divided into a number of discrete areas or elements. The shape of each element is determined, in general, by the number of nodes associated with it. The potential head at each of the nodes in the region under consideration must be defined by the vector.

$$\{\phi\} = \{\phi_1, \phi_2 \dots \phi_n\}^T \quad \text{eq 4}$$

where  $n$  is the total number of nodes. The variation of the potential head function within any particular element is defined by the nodal values of  $\phi$ , by letting the elemental potential head vector  $\{\phi\}^e$  be:

$$\{\phi\}^e = \{\phi_i, \dots \phi_m\}^T \quad \text{eq 5}$$

where  $i$  to  $m$  are the nodes associated with this element. The potential head function variation then becomes:

$$\phi(x,y) = [n] \{\phi\}^e \quad \text{eq 6}$$

11. Equation 6 now defines the function  $\phi$  uniquely within the element in terms of the elemental vector  $\{\phi\}^e$ , and the functional  $E$  can now be minimized with respect to these values by writing equations as follows:

$$\begin{aligned} \frac{\partial E}{\partial \phi_1} &= 0 \\ &\vdots \\ \frac{\partial E}{\partial \phi_m} &= 0 \end{aligned} \quad \text{eq 7}$$

12. As the function  $\phi$  is single-valued throughout each of the elemental volumes, then to minimize the total integral, it is necessary only to minimize the integral as it pertains to each element and to sum the resultants. If  $E^e$  is the contribution of an element to the total integral then:

$$E = \sum_e E^e \quad \text{eq 8}$$

13. One necessary condition is that  $E$  does not become infinite at the element interfaces, but this is obviously satisfied as  $E$  only involves the first derivatives of  $\phi$  and there is no discontinuity in  $\phi$  between elements.

$$[S]^e = \int_{\text{volume}} [k_x \cdot \frac{\partial \phi}{\partial x} \cdot \frac{\partial}{\partial \phi_1} (\frac{\partial \phi}{\partial x}) + k_y \cdot \frac{\partial \phi}{\partial y} \cdot \frac{\partial}{\partial \phi_1} (\frac{\partial \phi}{\partial y})] d(\text{volume}) \quad \text{eq 12}$$

14. Differentiating eq 3 with respect to each of the nodal values of  $\phi$  gives:

$$\begin{aligned} \frac{\partial E^e}{\partial \phi_i} &= \int_{\text{volume}} [k_x \cdot \frac{\partial \phi}{\partial x} \cdot \frac{\partial}{\partial \phi_i} (\frac{\partial \phi}{\partial x}) + k_y \cdot \frac{\partial \phi}{\partial y} \cdot \frac{\partial}{\partial \phi_i} (\frac{\partial \phi}{\partial y})] d(\text{volume}) \\ &+ \int_{\text{volume}} \frac{\partial}{\partial \phi_i} (\phi) \cdot d(\text{volume}) + \int_{\text{area}} \frac{\partial}{\partial \phi_i} (\phi) \cdot d(\text{area}) \quad \text{eq 9} \\ &= [S]^e \cdot \{\phi\}^e + \{F\}^e \end{aligned}$$

where the first integral is equal to  $[S]^e \cdot \{\phi\}^e$  and the two last one equal to  $\{F\}^e$ . It can be shown that  $\{S\}^e$  is symmetric.

15. Differentiating eq 8 then gives:

$$\frac{\partial E}{\partial \phi_i} = \sum_e \frac{\partial E^e}{\partial \phi_i} \quad \text{eq 10}$$

with the summation taken over all the elements. Thus the final set of equations becomes:

$$[S] \cdot \{\phi\} + \{F\} = \{0\} \quad \text{eq 11}$$

where typical terms are

$$S_{ij} = \sum_e S_{ij}^e$$

and

$$F_i = \sum_e F_i^e$$

in which the summation is taken over all the elements connected with node  $i$ .

16. Equation 11 is in fact a set of linear equations describing the seepage through a porous media and is directly analogous to the equation set produced for a structural analysis. As the  $[S]$  matrix and the  $\{F\}$  vector are completely known, standard linear equation solution techniques can be used to determine the potential head distribution ie,  $\{\phi\}$  throughout the flow region.

#### Finite Element Flow Matrix Development

17. The individual finite element flow matrix  $[S]^e$  has been derived previously as:

18. It is seen that the terms  $\partial\phi/\partial x$ ,  $\partial\phi/\partial y$  are needed to evaluate the integral, thus from eq 1

and introducing matrix notation

$$\left\{ \frac{\partial \phi / \partial x}{\partial \phi / \partial y} \right\} = \left[ \frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \right] [n] \{\phi\}^e = [B] \{\phi\}^e \quad \text{eq 13}$$

19. The expression for  $[S]^e$  can then be rewritten as:

$$\int_{\text{volume}} \frac{\partial}{\partial \phi_i} \left[ \frac{\partial \phi}{\partial x} \quad \frac{\partial \phi}{\partial y} \right] [K] \left\{ \frac{\partial \phi / \partial x}{\partial \phi / \partial y} \right\} \cdot d(\text{volume})$$

where  $[K]$  is the permeability matrix and thus

$$[S]^e = \left\{ \frac{\partial \phi}{\partial \phi_i} \right\} \int_{\text{volume}} [B]^T [K] [B] \cdot d(\text{volume})$$

where  $\left\{\frac{\partial \phi}{\partial \phi_i}\right\}^e$  is a unit matrix such that now

$$[S]^e = \int_{\text{volume}} [B]^T [K] [B] \cdot d(\text{volume}) \quad \text{eq 14}$$

#### Finite Element Forcing Vector

20. The individual finite element forcing vector  $\{F\}^e$  has been previously determined as:

$$\begin{aligned} \{F\}^e &= \int_{\text{volume}} Q \cdot \frac{\partial \phi}{\partial \phi_i} \cdot d(\text{volume}) + \int q \cdot \frac{\partial \phi}{\partial \phi_i} \cdot d(\text{area}) \\ &= \int_{\text{volume}} Q \cdot d(\text{volume}) + \int_{\text{area}} q \cdot d(\text{area}) \end{aligned} \quad \text{eq 15}$$

$$\begin{aligned} E &= \int_{\text{volume}} \left( \frac{1}{2} k_x \cdot \left( \frac{1}{\rho g} \cdot \frac{\partial p}{\partial x} \right)^2 + k_y \left( 1 + \frac{1}{\rho g} \cdot \frac{\partial p}{\partial y} \right)^2 + Q \left( y - y_0 + \frac{p}{\rho g} \right) \right) d(\text{volume}) \\ &\quad + \int_{\text{area}} q \left( y - y_0 + \frac{p}{\rho g} \right) \cdot d(\text{area}) \end{aligned} \quad \text{eq 19}$$

where  $Q$  represents the prescribed rate of flow of influx or efflux to the element volume, and  $q$  represents the prescribed rates of flow across the element boundaries. In physical terms,  $Q$  could represent the amount of fluid pumped out of the element.

#### Alternative Formulation Using Fluid Pressures

21. An alternative finite element formulation using fluid pressures as the variable in the energy function presents a number of advantages to the computational process. Potential function magnitudes are defined in terms of the arbitrary coordinate system assigned to a problem, whereas fluid pressure magnitudes are independent of the coordinate system. Boundary conditions are thus readily specified in terms of pressure distributions.

22. The potential head function may be expressed in terms of the fluid pressure as:

$$\phi = h + p/\rho g \quad \text{eq 16}$$

where the quantities  $h$  and  $p/\rho g$  are the elevation head and the pressure head, respectively. This definition holds for all points within the flow

region, where the elevation head is the elevation of the point in question above a certain horizontal datum level. The differentials of the potential head function, assuming that  $h = y - y_0$ , where  $y_0$  is the reference datum, then are

$$\frac{\partial \phi}{\partial x} = \frac{1}{\rho g} \cdot \frac{\partial p}{\partial x} \quad \text{eq 17}$$

and

$$\frac{\partial \phi}{\partial y} = 1 + \frac{1}{\rho g} \cdot \frac{\partial p}{\partial y} \quad \text{eq 18}$$

23. Substituting into the energy functional expression eq 3 yields

24. The pressure heads at each of the nodes in the flow region are defined by the vector  $\{p\}$  where

$$\{p\} = \{p_1, p_2, \dots, p_m\}^T \quad \text{eq 20}$$

25. The variation of the pressure head within any particular element is defined by the nodal pressures. The elemental pressure head vector is then

$$\{p\}^e = \{p_i, \dots, p_n\}^T \quad \text{eq 21}$$

where  $i$  to  $n$  are the element nodes, and the pressure head variation becomes

$$p(x, y) = [N] \{p\}^e \quad \text{eq 22}$$

26. Equation 22 defines the pressure head variation uniquely within the element in terms of the elemental vector  $\{p\}^e$  and the energy function is now minimized with respect to these values as follows:

$$\begin{aligned} \frac{\partial E}{\partial p_i} &= 0 \\ &\vdots \\ \frac{\partial E}{\partial p_m} &= 0 \end{aligned} \quad \text{eq 23}$$



27. Differentiating eq 19 with respect to each of the nodal values of  $p$  for each elemental area gives:

$$\begin{aligned} \frac{\partial E}{\partial p_i} &= \int_{\text{volume}} \left( k_x \left( \frac{1}{\rho g} \cdot \frac{\partial p}{\partial x} \right) \cdot \frac{\partial}{\partial p_i} \left( \frac{1}{\rho g} \cdot \frac{\partial p}{\partial x} \right) + k_y \left( \frac{1}{\rho g} \cdot \frac{\partial p}{\partial y} \right) \cdot \frac{\partial}{\partial p_i} \left( 1 + \frac{1}{\rho g} \frac{\partial p}{\partial y} \right) \right) \cdot d(\text{volume}) \\ &+ \int_{\text{volume}} Q \cdot \frac{\partial}{\partial p_i} \left( 1 + \frac{1}{\rho g} \frac{\partial p}{\partial y} \right) \cdot d(\text{volume}) \\ &+ \int_{\text{volume}} Q \cdot \frac{\partial}{\partial p_i} \left( y - y_o + \frac{p}{\rho g} \right) \cdot d(\text{volume}) + \int_{\text{area}} q \cdot \frac{\partial}{\partial p_i} \left( y - y_o + \frac{p}{\rho g} \right) \cdot d(\text{area}) \\ &= [S^*]^e \cdot \{p\}^e + [F^*]^e \end{aligned} \quad \text{eq 24}$$

where the first integral is equal to  $[S^*]^e \cdot \{p\}^e$  and the last three are equal to  $\{F^*\}^e$ .  $\{S^*\}^e$  can be shown to be symmetric and is known as the flow matrix while  $\{F^*\}^e$  is known as the seepage forcing vector.

28. Differentiating eq 7 with respect to the nodal pressures gives

$$\frac{\partial E}{\partial p_i} = \sum_e \frac{\partial E_e}{\partial p_i} \quad \text{eq 25}$$

with the summation taken over all the elements. The final set of equations thus becomes:

$$[S^*]^e \cdot \{p\} + \{F^*\} = \{0\} \quad \text{eq 26}$$

29. This set of equations is in fact a set of linear equations and can be solved for the unknown nodal pressure vector  $\{p\}$  by standard matrix solution techniques. The individual finite element flow matrix  $\{F\}$  has been derived above in terms of nodal pressures as:

$$\frac{\partial}{\partial \{p_i\}} \{p\}^T = \frac{\partial}{\partial \{p_i\}} \{p\}^e T [B]^T$$

$$= \begin{bmatrix} \frac{\partial p_1}{\partial p_1} & \frac{\partial p_1}{\partial p_2} & \dots & \frac{\partial p_1}{\partial p_m} \\ \frac{\partial p_2}{\partial p_1} & \frac{\partial p_2}{\partial p_2} & \dots & \frac{\partial p_2}{\partial p_m} \\ \frac{\partial p_2}{\partial p_1} & \frac{\partial p_2}{\partial p_2} & \dots & \frac{\partial p_2}{\partial p_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial p_m}{\partial p_1} & \frac{\partial p_m}{\partial p_2} & \dots & \frac{\partial p_m}{\partial p_m} \\ \frac{\partial p_1}{\partial p_1} & \frac{\partial p_1}{\partial p_2} & \dots & \frac{\partial p_1}{\partial p_m} \end{bmatrix}$$

$$[B]^T = [U] [B]^T \quad \text{eq 29}$$

$$\begin{aligned} [S^*]^e \cdot \{p\}^e &= \int_{\text{volume}} \left[ k_x \left( \frac{1}{\rho g} \cdot \frac{\partial p}{\partial x} \right) \cdot \frac{\partial}{\partial p_i} \left( \frac{1}{\rho g} \cdot \frac{\partial p}{\partial x} \right) \right. \\ &\left. + k_y \left( \frac{1}{\rho g} \cdot \frac{\partial p}{\partial x} \right) \cdot \frac{\partial}{\partial p_i} \left( 1 + \frac{1}{\rho g} \frac{\partial p}{\partial y} \right) \right] \cdot d(\text{volume}) \end{aligned}$$

30. The terms  $\partial p/\partial x$ ,  $\partial p/\partial y$  are needed to evaluate the integral, thus from eq 22 and introducing the matrix notation

$$\left\{ \frac{\partial p}{\partial x} \right\} = \left[ \frac{\partial}{\partial x} \frac{\partial}{\partial y} \right] [N] \{p\}^e = [B] \{p\}^e \quad \text{eq 27}$$

31. The expression for  $[S^*]^e \cdot \{p\}^e$  can then be written as

$$\frac{1}{(\rho g)^2} \int_{\text{volume}} \frac{\partial}{\partial p_i} \left[ \frac{\partial p}{\partial x} \frac{\partial p}{\partial y} \right] [K] \frac{\partial p}{\partial p/\partial y} \cdot d(\text{volume}) \quad \text{eq 28}$$

where  $[K]$  is the permeability matrix.

32. The partial differential operation of the above expression

$$\frac{\partial}{\partial \{p_i\}} \frac{\partial p}{\partial x} \frac{\partial p}{\partial y} = \frac{\partial}{\partial \{p_i\}} \{p\}^T$$

follows standard matrix differentiation in that the differentiation of a vector  $\{p\}$  with respect to another vector  $\{p_i\}$  yields a matrix of the following form:

since, however

$$\frac{\partial^2 p_k}{\partial p_j^2} = \begin{cases} 0 & \text{if } k \neq j \\ 1 & \text{if } k = j \end{cases} \quad \text{eq 30}$$

then [U] must be an identity matrix, therefore

$$\frac{\partial}{\partial \{p_i\}} \{p\}^T = \frac{\partial}{\partial \{p_i\}} \{p\}^{eT} [B]^T = [I] [B]^T = [B]^T \quad \text{eq 31}$$

33. Thus the element flow matrix is determined by:

$$[S^*]^e = \frac{1}{(\rho g)^2} \cdot \int_{\text{volume}} [B]^T [K] [B] \cdot d(\text{volume}) \quad \text{eq 32}$$

34. The expansions for the terms in the forcing vector  $\{F\}^e$  of eq 24 are straightforward. Letting

$$\{F^*\}^e = \{F_1\}^e + \{F_2\}^e + \{F_3\}^e \quad \text{eq 33}$$

where

$$\{F_1\}^e = \int_{\text{volume}} 2k_y \cdot \frac{\partial}{\partial p_i} \left( -\frac{1}{\rho g} \cdot \frac{\partial p}{\partial y} \right) \cdot d(\text{volume})$$

$$\{F_2\}^e = \int_{\text{volume}} Q \cdot \frac{\partial}{\partial p_i} \left( y - y_o + \frac{p}{\rho g} \right) \cdot d(\text{volume})$$

and

$$\{F_3\}^e = \int_{\text{area}} q \cdot \frac{\partial}{\partial p_i} \left( y - y_o + \frac{p}{\rho g} \right) \cdot d(\text{volume})$$

35. The first term in the series,  $\{F_1\}^e$  can be evaluated in the same manner as the flow matrix, in that

$$\frac{\partial p}{\partial y} = \frac{\partial}{\partial y} [N] \{p\}^e = [C] \{p\}^e = \{C\}^T \{p\}^e$$

and thus

$$\begin{aligned} \frac{\partial}{\partial p_i} \left( 1 + \frac{1}{\rho g} \cdot \frac{\partial p}{\partial y} \right) &= \frac{1}{\rho g} \cdot \frac{\partial}{\partial p_i} \left( \frac{\partial p}{\partial y} \right) \\ &= \frac{1}{\rho g} \cdot \left[ \frac{\partial p}{\partial p_i} \right] \{C\} \end{aligned}$$

Therefore

$$\begin{aligned} \{F_1\}^e &= \frac{2k_y}{\rho g} \int_{\text{volume}} \left[ \frac{\partial p}{\partial p_i} \right] \{C\} \cdot d(\text{volume}) \\ &= \frac{2k_y}{\rho g} \int_{\text{volume}} \{C\} \cdot d(\text{volume}) \end{aligned} \quad \text{eq 34}$$

36. The second term is simpler to evaluate in that

$$\frac{\partial}{\partial p_i} \left( y - y_o + \frac{p}{\rho g} \right) = \frac{1}{\rho g} \left[ \frac{\partial p}{\partial p_i} \right]$$

thus

$$\{F_2\}^e = \frac{1}{\rho g} \int_{\text{volume}} Q \cdot d(\text{volume}) \quad \text{eq 35}$$

where

$$Q = Q(x, y)$$

37. It therefore follows that the third term in the series is

$$\{F_3\}^e = \frac{1}{\rho g} \int_{\text{area}} q \cdot d(\text{area}) \quad \text{eq 36}$$

where

$$q = q(x, y)$$

#### Derivation of Triangular Element Flow Matrix

38. The simplest form of finite element which can be used to model a continuous region is the triangle (Fig 1).

39. The elemental pressure head vector for this element is:

$$\{p\}^e = \{p_i, p_j, p_k\}^T \quad \text{eq 37}$$

40. Assuming a linear variation of pressure head throughout the element

$$p(x, y) = A_1 + A_2 \cdot (x - x_i) + A_3 \cdot (y - y_i) \quad \text{eq 38}$$

which in matrix form becomes

$$\begin{Bmatrix} p_i \\ p_j \\ p_k \end{Bmatrix} = \begin{bmatrix} 1 & \cdot & \cdot \\ 1 & x_j - x_i & y_j - y_i \\ 1 & x_k - x_i & y_k - y_i \end{bmatrix} \cdot \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = [\Psi] \{A\} \quad \text{eq 39}$$

41. The constants  $\{A\}$  can be expressed in terms of  $\{p\}^e$  as:

$$\{A\} = [\Psi]^{-1} \{p\}^e = [N] \{p\}^e \quad \text{eq 40}$$

42. The matrix  $[N]$  can be evaluated directly as

$$[N] = \frac{1}{\Delta} \begin{bmatrix} \Delta & \cdot & \cdot \\ b_j - b_k & b_k & -b_j \\ d_k - d_j & -d_k & d_j \end{bmatrix} \quad \text{eq 41}$$

where  $\Delta = d_j b_k - d_k b_j$ , (twice the area of the triangle);

$$d_j = x_j - x_i$$

$$d_k = x_k - x_i$$

$$b_j = y_j - y_i$$

$$\text{and } b_k = y_k - y_i$$

thus

$$p = \begin{bmatrix} 1 & x - x_i & y - y_i \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix}$$

and

$$\frac{\partial p}{\partial x} = \begin{bmatrix} \cdot & 1 & \cdot \end{bmatrix} \{A\}$$

$$\frac{\partial p}{\partial y} = \begin{bmatrix} \cdot & \cdot & 1 \end{bmatrix} \{A\}$$

eq 42

thus

$$\begin{Bmatrix} v_x \\ v_y \end{Bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{Bmatrix} k_x^{-1} \\ \cdot \\ k_y^{-1} \end{Bmatrix} \cdot \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{Bmatrix} \frac{1}{\rho g} \cdot \frac{\partial p}{\partial x} \\ 1 + \frac{1}{\rho g} \cdot \frac{\partial p}{\partial y} \end{Bmatrix}$$

$$[B] = \frac{1}{\Delta} \begin{bmatrix} \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{bmatrix} \begin{bmatrix} \Delta & \cdot & \cdot \\ b_j - b_k & b_k & -b_j \\ d_k - d_j & -d_k & d_j \end{bmatrix} \quad \text{eq 43}$$

$$= \frac{1}{\Delta} \begin{bmatrix} b_j - b_k & b_k & -b_j \\ d_k - d_j & -d_k & d_j \end{bmatrix}$$

43. Due to limitations imposed by available permeability testing procedures, permeabilities are measured parallel with and perpendicular to rock strata or soil layering. As the layering directions do not coincide, in general, with the coordinate directions, a transformation of the permeability coefficients from the layer coordinate system is required. Figure 1 indicates the nature of this problem with the primed axes representing the layer coordinate system.

44. The measured directional permeabilities are  $k_x^1$  and  $k_y^1$  with the stratification angle being  $\phi$ . Darcy's Law may be expressed in terms of the layer coordinate system as

$$\begin{Bmatrix} v_x^1 \\ v_y^1 \end{Bmatrix} = - \begin{bmatrix} k_x^1 & \cdot \\ \cdot & k_y^1 \end{bmatrix} \begin{Bmatrix} \partial \phi / \partial x^1 \\ \partial \phi / \partial y^1 \end{Bmatrix} \quad \text{eq 44}$$

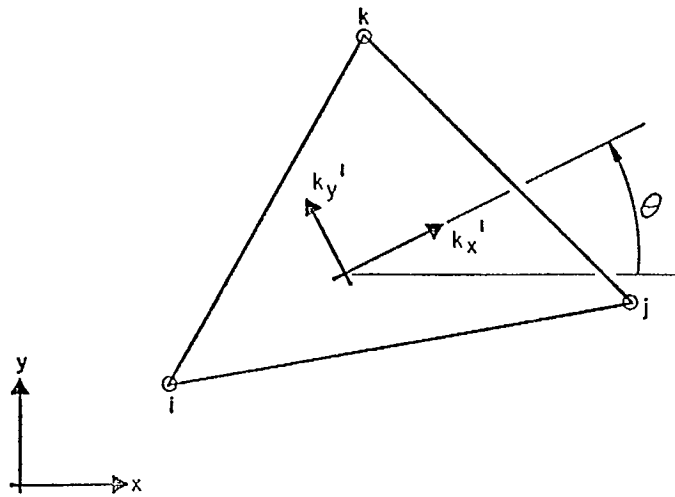
where  $v_x^1, v_y^1$  are flow velocities along the layer coordinate directions. Using standard transformation techniques it can be shown that

$$\begin{Bmatrix} v_x \\ v_y \end{Bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{Bmatrix} v_x^1 \\ v_y^1 \end{Bmatrix}$$

and that

$$\begin{Bmatrix} \frac{1}{\rho g} \frac{\partial p}{\partial x} \\ 1 + \frac{1}{\rho g} \frac{\partial p}{\partial y} \end{Bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{Bmatrix} \partial \phi / \partial x^1 \\ \partial \phi / \partial y^1 \end{Bmatrix} \quad \text{eq 45}$$

45. Combining the above transformations to give Darcy's Law in terms of the general coordinate system yields



i, j, k - Element nodes  
 $k_x^l, k_y^l$  - Element material permeabilities  
 $\epsilon$  - Element material orientation or stratification angle

Fig 1 - Triangular finite element.

or

$$\{V\} = - [L]^T [K^l] [L] \{\phi\} = - [K] \{\phi\} \quad \text{eq 46}$$

$$[S^*]^e = \frac{V_e}{(\rho g)^2} \cdot [B]^T [K] [B] \quad \text{eq 49}$$

46. Note that the permeability matrix is now

$$[K] = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix} = [L] [K^l] [L] \quad \text{eq 47}$$

where [L] is a transformation matrix containing direction cosines.

47. Now that the [B] and [K] matrices have been defined for this element, the flow matrix is determined from the equation

$$[S^*]^e = \frac{1}{(\rho g)^2} \int_{\text{volume}} [B]^T [K] [B] \cdot d(\text{volume}) \quad \text{eq 48}$$

48. However, as shown above, all the quantities in the above integral are constants within the element and integration is indicated by replacing the integral over the element by the volume  $V^e$  of the element,

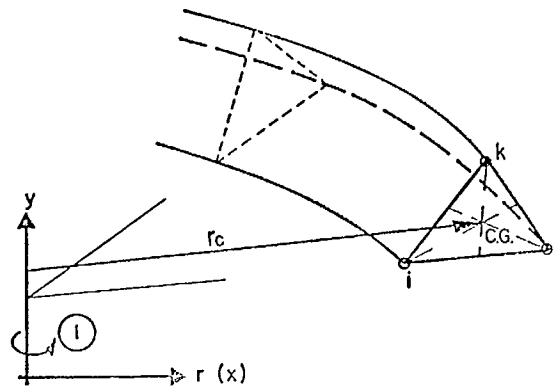
49. Both axi-symmetric, ie, radial, and plane flow conditions are described by the above expression if the basic elemental volumes are taken accordingly. For plane flow, the elemental volume utilized is the area of the basic triangular region times a unit thickness, thus

$$V_{pl}^e = \frac{\Delta}{2} = \frac{d_j b_k - d_k b_j}{2} \quad \text{eq 50}$$

where  $d_j, d_k, b_j, b_k$  have been defined previously. For an axi-symmetric or radial flow element (Fig

2) the elemental volume is that of a ring whose cross section is triangular, thus

$$V_{ax}^e = \pi \cdot r_c \cdot \Delta \quad \text{eq 51}$$



- (1) Axis of symmetry
- i, j, k - element nodal circles
- $r_c$  - radius to element centroid
- $k_x, k_y, \theta$  - same as for planar element

Fig 2 - Axi-symmetric finite element.

where  $r_c$  is the radius of the centroid of the element. Thus if the axis is to be the axis of symmetry then

$$r_c = \frac{x_i + x_j + x_k}{3} \quad \text{eq 52}$$

Formation of Quadrilateral Element Flow Matrix

50. In applications to seepage analysis problems, it is often expedient to combine four triangular elements into a basic quadrilateral shape as shown on Fig 3.

$$x_m = \frac{x_i + x_j + x_k + x_e}{4}$$

and

$$y_m = \frac{y_i + y_j + y_k + y_e}{4} \quad \text{eq 53}$$

51. The quadrilateral element flow matrix will then be assembled from the individual element flow matrices as follows:

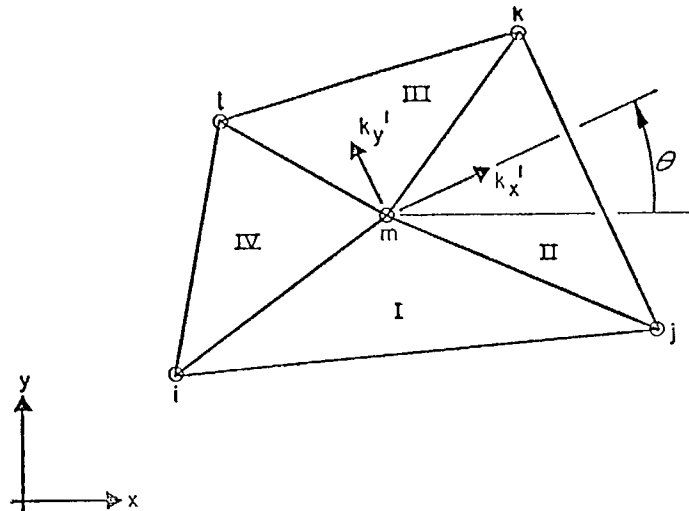
$$[S^*]^e = [S^*_{I}]^e + [S^*_{II}]^e + [S^*_{III}]^e + [S^*_{IV}]^e \quad \text{eq 54}$$

52. Each individual flow matrix is a 3 x 3 matrix and has coefficients of the following general form for a triangle with nodes n, p and q

$$[S^*]^e = \begin{bmatrix} S_{nn} & S_{np} & S_{nq} \\ S_{pn} & S_{pp} & S_{pq} \\ S_{qn} & S_{qp} & S_{qq} \end{bmatrix} \quad \text{eq 55}$$

53. The composite element flow matrix will then be a 5 x 5 matrix and will have the following general form.

$$[S^*]^e = \begin{bmatrix} S_{ii} & S_{ij} & \cdot & S_{il} & | & S_{im} \\ S_{ji} & S_{jj} & S_{jk} & \cdot & | & S_{jm} \\ \cdot & S_{kj} & S_{kk} & S_{kl} & | & S_{km} \\ S_{li} & \cdot & S_{lk} & S_{ll} & | & S_{lm} \\ \hline S_{mi} & S_{mj} & S_{mk} & S_{ml} & | & S_{mm} \end{bmatrix} = \begin{bmatrix} S_{AA} & S_{AB} \\ S_{BA} & S_{BB} \end{bmatrix}$$



- $i, j, k, l$  - element exterior nodes  
 $m$  - element interior node  
 $I, II, III, IV$  - element constituent triangles  
 $k_x^l, k_y^l$  - element material permeabilities  
 $\theta$  - element material orientation or stratification angle

Fig 3 - Quadrilateral finite element.

where the partition index A relates to those terms associated with the pressures at the exterior nodes  $i, j, k, l$  and the partition index B relates to those terms associated with the fluid pressure at the interior node  $m$ .

54. If the fluid pressure and the fluid flow at the interior node  $m$  do not constitute boundary conditions i.e., if the nodal pressure is not specified and if there is no prescribed quantity of flow through the node, it is possible to eliminate the dependence of the central node. This is done by using the static condensation algorithm which is an extension of the Gaussian elimination process. In matrix terms, this operation is

$$[S^*]^e = [S_{AA}] - [S_{AB}] \cdot [S_{BB}]^{-1} \cdot [S_{BA}] \quad \text{eq 56}$$

This reduced flow matrix is a  $4 \times 4$  matrix. The use of this form of element flow matrix reduces the overall size and bandwidth of the resulting set of equations without loss of accuracy.

#### Derivation of Seepage Forcing Vector

55. Equation 38 contains a description of the various constituents of the element seepage forcing vector. The seepage forcing vector for the problem is then simply the sum of the element forcing vectors

$$\{F^*\} = \sum_m \{F^*\}^e$$

56. For each triangular element, the constituent parts of the forcing vector are evaluated separately using eq 34, 35 and 36. Thus, the first item  $\{F_1\}^e$ , is found from

$$\{F_1\}^e = \frac{2k_y}{\rho B} \cdot \int_{\text{volume}} \{C\} \cdot d(\text{volume})$$

where  $\{C\}$  can be interpreted directly from eq 43 as

$$\{C\} = \frac{1}{\Delta} \begin{Bmatrix} d_k - d_j \\ -d_k \\ d_j \end{Bmatrix} \quad \text{eq 57}$$

57. As all the quantities in the above integral are constant within the element, the integration process becomes

$$\{F_1\}^e = \frac{2k}{\rho g} \cdot v^e \cdot \{C\} \quad \text{eq 58}$$

Thus, for planar flow elements

$$\{F_1\}^e = \frac{k}{\rho g} \begin{Bmatrix} d_k - d_j \\ -d_k \\ d_j \end{Bmatrix} \quad \text{eq 59}$$

and for axi-symmetric flow elements

$$\{F_1\}^e = \frac{2\pi \cdot k \cdot y \cdot r_c}{\rho g} \begin{Bmatrix} d_k - d_j \\ -a_k \\ a_j \end{Bmatrix} \quad \text{eq 60}$$

58. The second term in the seepage forcing vector,  $\{F_2\}^e$  is found from

$$\text{Area}_{n-p} = \pi [x_p \cdot y_p - x_p \cdot y_n + x_n \cdot y_p - x_n \cdot y_n] \left(1 + \left(\frac{x_p - x_n}{y_p - y_n}\right)^2\right)^{1/2} \quad \text{eq 64}$$

$$\{F_2\}^e = \frac{1}{\rho g} \int_{\text{volume}} Q \cdot d(\text{volume})$$

where  $Q$  represents the rate of flow influx to the element. For most practical analyses it is more convenient to apply a specific quantity of inflow per unit time to an individual node rather than to specify an influx rate over an element area. Thus, this equation reduces to

$$\{F_2\}^e = \frac{1}{\rho g} \begin{Bmatrix} Q_1 \\ \vdots \\ Q_m \end{Bmatrix} \quad \text{eq 61}$$

and as integration over the element is not required, this operation need no longer be performed as an elemental procedure, but simply as an addition to the problem seepage forcing vector.

59. The third term in the seepage forcing vector is

$$\{F_3\}^e = \frac{1}{\rho g} \int_{\text{area}} q \cdot d(\text{area})$$

where  $q$  represents the flow rate normal to the boundary surface. As both the triangular and quadrilateral elements are constant velocity elements (thus implying constant flow) only a constant rate of flow may be specified. Normally, only one side of an element is assumed to have a specified flow and thus  $\{F_3\}^e$  becomes

$$\{F_3\}^e = \frac{q}{2\rho g} \begin{Bmatrix} 1_n \\ \vdots \\ 1_p \\ \vdots \end{Bmatrix} \quad \text{area } n-p \quad \text{eq 62}$$

where  $n, p$  are the nodes at the ends of the side. For planar flow elements where a unit thickness element is presumed, the area term becomes

$$\text{Area}_{n-p} = [(x_n - x_p)^2 + (y_n - y_p)^2]^{1/2} \quad \text{eq 63}$$

For axi-symmetric elements, the area term becomes

60. It should be noted that the above derivations apply to the quadrilateral element equally well.

#### Boundary Conditions

61. Three types of boundary conditions occur in the solution of seepage problems. These are:

- surfaces with known or specified fluid pressures
- surfaces with known or specified flow rates
- free or phreatic surfaces where the atmospheric pressure is taken to be zero and the flow rate normal to the surface is known and is usually zero.

62. The first boundary condition is satisfied by computing the nodal fluid pressures at the nodes along the surface with the known or specified fluid pressure, and then modifying the equations in the flow matrix corresponding to these nodes so as to produce the nodal pressures as the

solution. The values of the other nodal pressures are obtained by solving the set of simultaneous equations represented by eq 24.

63. The second boundary condition is handled by computing the equivalent nodal flows along the surface with known or specified flow rates, then calculating the elemental contributions to the forcing vector  $\{F\}$  using eq 36 and finally adding these quantities to the forcing vector prior to solving the set of equations.

64. The inclusion of the above two types of boundary conditions permits the solution of confined flow seepage problems. Unconfined flow problems are much more difficult to solve as the location of the phreatic surface is not known. This indefiniteness is offset by the third type of boundary condition, which is, in effect, a combination of the first two boundary condition types. It is not possible, however, to solve for a simultaneous combination of these conditions on a surface by the finite element method. Instead, an iterative approach must be used in which a free surface position is estimated, one of the boundary

$$\begin{matrix} v_x^1 \\ v_y^1 \end{matrix} = -\frac{1}{\Delta} \begin{bmatrix} k_x^1 & \cdot \\ \cdot & k_y^1 \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + \frac{1}{\rho g} \begin{bmatrix} b_j - b_k & b_k - b_j \\ d_k - d_j & -d_k & d_j \end{bmatrix} \begin{Bmatrix} p_i \\ p_j \\ p_k \end{Bmatrix} \quad \text{eq 69}$$

conditions is applied, and the other is disregarded. Upon solving the set of equations, the resulting conditions along the free surface are investigated to see if the disregarded boundary conditions have been approximately satisfied. If not, then a new position of the phreatic surface is calculated from the resulting conditions and the solution process repeated. This iteration procedure continues until the disregarded boundary condition is satisfied.

#### Calculation of Flow Velocities

65. From eq 44, the flow velocities are determined from Darcy's Law as

$$\{V\} = -[K] \{\dot{\phi}'\} \quad \text{eq 65}$$

66. The vector  $V$ , however, represents the flow velocities along the general coordinate di-

rections. In general, the flows parallel with and perpendicular to the material layering are of greater interest and are calculated by

$$\{v^1\} = [K] \{\dot{\phi}'\} \quad \text{eq 66}$$

Thus

$$\begin{matrix} v_x^1 \\ v_y^1 \end{matrix} = - \begin{bmatrix} k_x^1 & \cdot \\ \cdot & k_y^1 \end{bmatrix} \begin{Bmatrix} \partial \phi / \partial x^1 \\ \partial \phi / \partial y^1 \end{Bmatrix} \quad \text{eq 67}$$

Using the transformations presented earlier (eq 45, 46)

$$\begin{aligned} v_x^1 &= - \begin{bmatrix} k_x^1 & \cdot \\ \cdot & k_y^1 \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix} \begin{Bmatrix} \frac{1}{\rho g} \cdot \frac{\partial p}{\partial x} \\ 1 + \frac{1}{\rho g} \cdot \frac{\partial p}{\partial y} \end{Bmatrix} \\ &= [K] [L]^T (\{f\} + \frac{1}{\rho g} \cdot [B] \{p\}^e) \end{aligned} \quad \text{eq 68}$$

67. For a triangular element with nodes  $i, j, k$  this becomes

68. In the case of a quadrilateral element, the pressure at the interior node is calculated as the average of the corner node pressures. The flows in each constituent triangular element are computed and averaged to provide flow values for the quadrilateral element.

#### Solution for Stream Function

69. In finite element seepage analyses, the solution is generally found in terms of the velocity potential or potential head function  $\phi$ . For two-dimensional and axi-symmetric flow situations another function exists which can be very useful for determining the amount or rate of fluid flow. This function is known as the stream function  $\psi$  and has the following properties:

- a. it is constant along a streamline, a line which is tangential at all points to the third velocity vectors at an instant



$$b. \frac{\partial \psi}{\partial y} = V_x, \quad \frac{\partial \psi}{\partial x} = V_y$$

$$(\partial \psi = V_x \partial y - V_y \partial x) \quad \text{eq 70}$$

c. the volume flow rate between any two streamlines  $i$  and  $j$  is constant and is completely defined by the differences in stream function values,

$$Q_{ij} = \psi_j - \psi_i \quad \text{eq 71}$$

d. it is a linear function of the boundary conditions.

70. Thus, it can be seen that the stream function  $\psi$  is particularly useful for calculation of seepage rates and quantities.

71. Now, if flow velocity or head potential function distributions are known exactly throughout a continuum then the stream function could readily be found, since from eq 46

$$\begin{aligned} V_x &= k_{xx} \frac{\partial \phi}{\partial x} + k_{xy} \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial y} \\ V_y &= k_{yx} \frac{\partial \phi}{\partial x} + k_{yy} \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x} \end{aligned} \quad \text{eq 72}$$

72. Unfortunately, the finite element method provides only estimates of  $V_x$  and  $V_y$  which by definition are not continuous across element boundaries. Thus, different estimates of the total fluid flow across an inter-element boundary will be found if  $\partial \psi / \partial y$  and  $\partial \psi / \partial x$  are integrated along paths on either side of the boundary. It is possible to directly formulate the finite element method in terms of the stream function  $\psi$  in much the same manner as outlined earlier. However, in this case the inverse problem occurs in that instead of  $\psi$  it is  $\phi$  that is ambiguously defined.

73. A simple least-squares technique can be utilized to obtain a set of stream function values at the nodes in a finite element mesh from the elemental flow velocities. This least-squares technique minimizes the sum of the weighted squares of the differences between the total flow across each element boundary and the flows based on the velocity within each element.

74. The least-squares technique is described as follows. Integrating eq 72 within each element for each pair of nodes  $i$  and  $j$  describing the sides of the element will yield  $m$  sets of total flow estimates  $\hat{Q}_{ij}^{(k)}$ . The flow discrepancy that occurs in the estimation for path  $k$  is then

$$\epsilon^{(k)} = Q_{ij} - \hat{Q}_{ij}^{(k)} \quad \text{eq 73}$$

which from eq 71 can be shown to be

$$\epsilon^{(k)} = \psi_j - \psi_i - \hat{Q}_{ij}^{(k)} \quad \text{eq 74}$$

(Note that at this point  $\psi_j$  and  $\psi_i$  are unknown.) The weighted squared discrepancy is then formulated as:

$$\omega^{(k)} \epsilon^{(k)2} = \omega^{(k)} (\psi_j - \psi_i - \hat{Q}_{ij}^{(k)})^2 \quad \text{eq 75}$$

75. Summing the weighted squared discrepancies for all  $m$  sets of flow estimates gives

$$Z = \sum_m \omega^{(k)} \epsilon^{(k)2} \quad \text{eq 76}$$

76. Minimizing  $Z$  with respect to each nodal stream function value  $\psi_n$  gives a series of relationships

$$\frac{\partial Z}{\partial \psi_n} = 0 \quad \text{eq 77}$$

which when expanded become

$$\begin{aligned} \frac{\partial Z}{\partial \psi_n} &= 2 \sum_m \omega^{(k)} \cdot \epsilon^{(k)} \cdot \frac{\partial \epsilon^{(k)}}{\partial \psi_n} \\ &= 2 \sum_m \omega^{(k)} \cdot (\psi_j - \psi_i - \hat{Q}_{ij}^{(k)}) \cdot \left( \frac{\partial \psi_j}{\partial \psi_n} - \frac{\partial \psi_i}{\partial \psi_n} \right) = 0 \end{aligned} \quad \text{eq 78}$$

Now, it is obvious that

$$\frac{\partial \psi_j}{\partial \psi_n} - \frac{\partial \psi_i}{\partial \psi_n} = \begin{cases} +1 & \text{if } n = j \\ -1 & \text{if } n = i \\ 0 & \text{if } n \neq i \text{ and } n \neq j \end{cases}$$

77. Putting eq 78 into matrix form yields the linear relationship

$$[W] \{\psi\} = \{\bar{Q}\} \quad \text{eq 79}$$

where  $[W]$  is a matrix of summed weighting factors,  
 $\{\psi\}$  is a vector of (unknown) nodal stream  
 function estimates, and  
 $\{\bar{Q}\}$  is a vector of summed and weighted total  
 flow estimates.

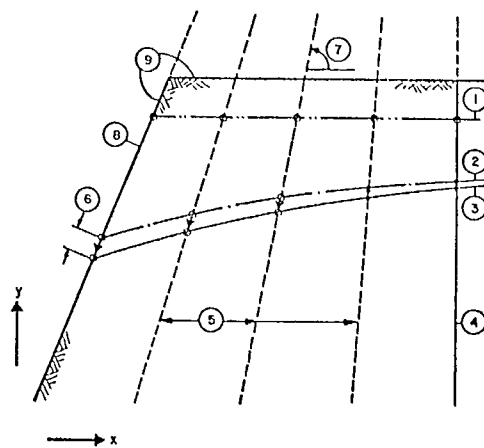
78. Equation 79 can then be solved for the  
 nodal stream function estimates  $\{\psi\}$  using a stan-  
 dard linear equation solution process once a suit-  
 able datum is provided for the stream function.

79. If a source or sink exists within the con-  
 tinuum region the stream function cannot be de-  
 fined. In mining practice, a drainage adit behind  
 a pit wall will often act as a sink. This diffi-  
 culty can be removed by connecting the source or  
 sink to the surrounding boundary by an imaginary  
 conduit.

#### Location of Phreatic Surface in Unconfined Flow Problems

80. Unconfined flow problems, in which the

location of the phreatic surface is not known at  
 the start of the analysis, are solved using an  
 iterative procedure. As outlined earlier, an ini-  
 tial position of the phreatic surface is estima-  
 ted. After applying the boundary condition of  
 known constant flow across this initial boundary  
 (most often zero flow) the nodal pressures  
 throughout the finite element mesh are calculated.  
 The pressures along the phreatic surface are then  
 interrogated and related to zero with subsequent  
 adjustment of their location. The boundary condi-  
 tions of known constant flow are then applied to  
 the distorted mesh and a new solution for nodal  
 pressures is then obtained. The pressures along  
 the phreatic surface are interrogated again and  
 another new location of the phreatic surface is  
 estimated. Figure 4 illustrates this iterative  
 procedure which continues until the total change  
 in phreatic surface location between iterations is  
 less than a specified amount.



- (1) Initial assumed phreatic surface location  $C_I^1$
- (2)  $C_I^{n-1}$  - phreatic surface location after n-1 iterations
- (3)  $C_I^n$  - phreatic surface location after n iterations
- (4) Material interface
- (5) Nodal correction directions
- (6)  $D_j^{n-1}$  - correction distance for node j
- (7)  $k$  - nodal correction angle
- (8) Slope face
- (9) Slope outline

$$\text{For convergence } d_j^{n-1} \leq \text{tolerance}$$

Fig 4 - Finite element program: phreatic surface adjustment procedure.

81. The procedure involved in phreatic surface location adjustment requires an estimate of the derivative of the third pressure gradient normal to the direction along which a node is to be moved. Two methods are available for the assessment of this quantity. The first and simplest method takes the vertical component for correction of a nodal position as being  $-p_i/\rho g$  and leads to a correction distance of

$$D_i = \frac{-p_i/\rho g}{\sin k} \quad \text{eq 80}$$

where  $p_i$  = fluid pressure at node  $i$  and  
 $k$  = nodal correction angle.

82. This method is not theoretically correct as the horizontal component of the pressure gradient has been omitted, however, it can be used safely for many practical seepage analysis problems.

83. The second method uses a least squares process to determine the fluid pressure gradient at a node and is useful for situations in which the first method fails to provide a converged solution. The difference in fluid pressure between a node and a point some distance away can be found through a Taylor series expansion, provided that the gradient is known, as

$$\Delta p = \{\nabla p\}^T \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} + \frac{1}{2} \{\nabla^2 p\}^T \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix}^2 + \dots \quad \text{eq 81}$$

where  $\{\nabla p\} = \left\{ \frac{\partial p}{\partial x}, \frac{\partial p}{\partial y} \right\}$  is the pressure gradient, and  $\Delta x, \Delta y$  are the distance components between the two points.

84. As only a linear variation of fluid pressure is allowed within the finite elements described in this report, it is reasonable to use a linear variation for fluid pressure about a nodal point. Furthermore, if a local co-ordinate system originating at the nodal point is chosen then the pore pressure at some distance from it is found through

$$p = p_i + Bx + Cy \quad \text{eq 82}$$

where  $B$  and  $C$  are parameters to be determined.

85. Equation 81 then reduces to

$$\nabla p = \{\nabla p\}^T \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} = \{\nabla p\}^T \begin{Bmatrix} x \\ y \end{Bmatrix}$$

and the pressure gradient becomes

$$\{\nabla p\} = \begin{Bmatrix} \partial p / \partial x \\ \partial p / \partial y \end{Bmatrix} = \begin{Bmatrix} B \\ C \end{Bmatrix}$$

thus giving

$$\Delta p = \begin{Bmatrix} B \\ C \end{Bmatrix}^T \begin{Bmatrix} x \\ y \end{Bmatrix} = Bx + Cy \quad \text{eq 83}$$

86. The discrepancy in fluid pressure that occurs between a set of nodes  $i$  and  $j$  is then

$$\epsilon_j = p_j - p_i - \Delta p = p_j - p_i - Bx_j - Cy_j \quad \text{eq 84}$$

The weighted squared discrepancy becomes

$$\omega_j \epsilon_j^2 = \omega_j (p_j - p_i - Bx_j - Cy_j)^2 \quad \text{eq 85}$$

which when summed over  $j$  nodal connections gives the total error as

$$Z = \sum_j \omega_j \epsilon_j^2 = \sum_j \omega_j (p_j - p_i - Bx_j - Cy_j)^2 \quad \text{eq 86}$$

Minimizing  $Z$  with respect to the unknown parameters  $B$  and  $C$  gives two relationships

$$\frac{\partial Z}{\partial B} = \sum_j \omega_j (p_j - p_i - Bx_j - Cy_j) (-x_j) \quad \text{eq 87}$$

$$\frac{\partial Z}{\partial C} = \sum_j \omega_j (p_j - p_i - Bx_j - Cy_j) (-y_j)$$

which when put in matrix form yield the linear relationships.

$$\begin{bmatrix} \sum_j \omega_j x_j^2 & \sum_j \omega_j x_j y_j \\ \sum_j \omega_j y_j x_j & \sum_j \omega_j y_j^2 \end{bmatrix} \begin{Bmatrix} B \\ C \end{Bmatrix} = \begin{Bmatrix} \sum_j \omega_j (p_j - p_i) x_j \\ \sum_j \omega_j (p_j - p_i) y_j \end{Bmatrix}$$

87. After equating the weighting function  $\omega_j$  to the distance between the two points the parameters  $B$  and  $C$  can be readily determined and thus the average fluid pressure gradient at the node has been obtained. The calculation of the

derivative of the gradient normal to the nodal correction is then a trivial matter using the chain value. The correction distance is thus found to be

$$D_i = \frac{-P_i/\rho g}{C_i \cos k - B_i \sin k} \quad \text{eq 88}$$

88. In practice, it has sometimes been found necessary to underestimate the next location of the phreatic surface. This action, known more properly as under-relaxation, generally assumes monotonic convergence of the above iteration procedures.

89. An ambiguity in the specification of the boundary conditions along the phreatic surface exists when the surface intersects the geometric boundary of the continuum. As the nodal flow at this point cannot be specified directly it becomes difficult to predict its location precisely. Retaining the finite element mesh in the region of this intersection point reduces the effect of the ambiguity. The most satisfactory practical treatment has been to fit a parabola through the three closest phreatic surface points and then extrapolate it to determine the location of the intersection point.

## PROGRAM CAPABILITIES

90. The finite element seepage analysis computer program is known acronymically as FEFPM (Finite Element analysis of Flow in Porous Media). The FEFPM computer program is used to determine fluid pressures as well as flow quantities, rates and directions in steady-state ground water regimes. These ground water regimes are vertical sections in which fluid movement patterns conform reasonably well to either planar or radially symmetric flow conditions. It is further assumed that geological conditions existing throughout the section are known.

91. The analytical capabilities of the FEFPM computer program permit the analysis of ground-water flow subject to the following conditions:

- flow is planar or radially symmetric on vertical sections
- flow is linear within a finite element (Darcy flow rules)
- material permeabilities may be isotropic or anisotropic
- flow regime may be confined or unconfined.

The program itself has been developed to the extent that it will automatically compute:

- the location of the phreatic surface for unconfined flow problems
- the material type and permeability for an element from soil region definition lines and

zones

- fluid potential and stream function values.

92. To minimize user involvement in data set-up and control of an analysis, the FEFPM computer program can:

- generate nodes and elements
- re-generate automatically the finite element mesh during phreatic surface location iterations
- automatically limit the flow regime to the slope profile.

In addition the FEFPM computer program is structured and coded to utilize available computer resources efficiently and minimize computer time required by analysis. Some means by which this efficiency is accomplished are:

- expanding and contracting computer memory (core storage) allocations as required by particular problems
- using a Choleski decomposition algorithm for the iterative solution of the flow equations
- using a bandwidth reduction process to re-order mode sequences and thus minimize the size of the global flow matrix.

93. Because the FEFPM computer program has the ability to set and adjust its internal core storage allocation, there is only one limitation to the size of the problem. This simply is that the

computer memory required to store all node, element, and material data as well as the global flow matrix, flow forcing and solution vectors does not exceed the available core storage. Most practical seepage analysis problems with about 500 nodes, 500 elements and 10 different materials are well within core storage limits on modern large high-speed computers.

94. Intermediate and final results of FEFPM analyses can also be automatically saved on computer peripheral storage units such as disks or magnetic tapes. Thus, for an unconfined seepage analysis problem in which the final location of the phreatic surface is determined by an iterative procedure, it is possible to stop the run after, say, only five iterations have been completed. The user would then check intermediate results and ascertain that the analysis was correct up to that point. He would then restart the analysis using the saved results of the fifth iteration and instruct the program to carry on iterating until the solution converged.

95. These capabilities and limits are controlled with parameters in the NAMELIST input.

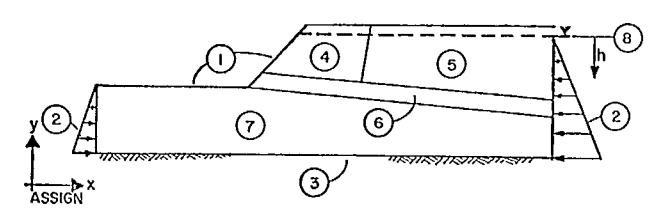
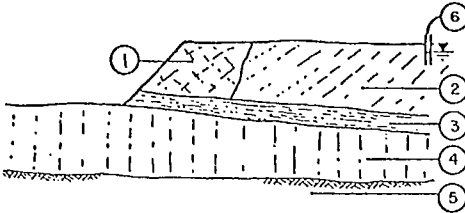
DATA INPUT

96. The seven types of possible input are coded on the forms shown in Fig 5. The order of input is approximately the order of the forms. All these inputs are assumed read from logical unit 5, the card reader (INPUT).

Setting the Analysis

97. Setting up a finite element analysis of seepage problems is a precise, yet straightforward procedure as outlined graphically in Fig 5. Initially, all data pertinent to the seepage problem must be collected. The best method is to prepare a geological section and mark the known geological and hydraulic/hydrologic data on it.

98. From this rough chart, the next step is to prepare a scaled drawing of the problem geometry including such features as material boundaries, boundary conditions and an assumed initial phreatic surface. Input units and dimensions for the co-ordinate system, fluid properties and material permeabilities must be carefully and consistently chosen. The following dimensions must be used throughout.

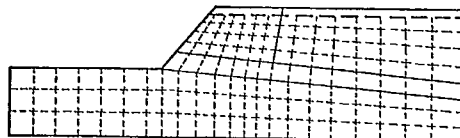


1. Seepage Problem

- (1) Fractured rock  $K$
- (2) Intact rock  $K_2$
- (3) Mineral layer  $K_3$
- (4) Underlying strata  $K_4$
- (5) Assumed impervious bedrock
- (6) Known piezometer reading

2. Layout Problem Geometry, Material Boundaries and Boundary Conditions

- (1) Set  $p = 0$
- (2) Set  $p = \gamma_w h$
- (3) Set  $q = 0$
- (4) Type 1
- (5) Type 2
- (6) Type 3
- (7) Type 4
- (B) Assumed initial phreatic surface



- 3. Develop Finite Element Mesh
- 4. Prepare Data
- 5. Check Data
- 6. Run FEFPM Computer Program
- 7. Plot Results

Fig 5 - Finite element analysis procedure.

Quantity	Measured in
pressure (head)	length (L)
permeability (velocity)	length/time (L/T)
flow rate (velocity)	length/time (L/T)
flow quantity	length <sup>3</sup> /time (L <sup>3</sup> /T)

The user must understand fluid and soil/rock mechanics sufficiently well that realistic boundary conditions are applied.

99. The finite element mesh itself must then be developed on the scaled drawing. As the FEFPM computer program permits node and element generation, nodes should be regularly spaced, wherever possible, and the node number scheme kept as systematic as possible. The use of quadrilateral elements allows most problems with irregular geometries to be modelled easily, however the user can include triangular elements into the mesh if desired. In areas where significant changes in pressure or velocities are anticipated the grid mesh should be made finer.

100. The next phase involves the coding of the input data for the program. This must be done very carefully as an error in a coordinate location or material type would likely not be detected by the program and erroneous but apparently acceptable results would be obtained. The importance of checking and re-checking the input data cannot be over-emphasized.

101. Finally, when the user is sure that his data is correct, the FEFPM computer program can be run. Should an error occur during execution, the program will print out an error message stating the cause and location, and will then stop. The user will be able to correct the situation from the error message description and re-submit the run. Upon successful completion of the analysis, the complete report of all nodal and elemental results should be studied carefully prior to plotting flow nets and fluid pressure contours.

#### Data Preparation

102. Data cards detailing regime geometric material permeabilities and boundary conditions are prepared using fixed formats. When the data cards

have been keypunched, checked and verified, the system control cards are added to the card deck. The complete run deck is then submitted to the computer through the card reader on a batch terminal.

103. There are seven distinct input items in the data for FEFPM.

- a. Title card(s) and START card. These cards are used for problem title input and job initialization.
- b. Control parameters input through Namelist \$FPM cards. These cards set values for various parameters describing the type of problem, how it is to be solved and the level of output required.
- c. Node definition cards. These cards define the locations, boundary conditions and generation parameters for nodes.
- d. Material properties cards. These define for each material, the directional permeabilities, and the extent or zoning throughout the problem geometry.
- e. Element definition cards. These define the elements in terms of their nodes and material types.
- f. Distributed inflow cards (optional). These cards define fluid inflows distributed along the edges of finite elements.
- g. Pipe card (optional). This card defines a string of elements linking any internal sources or sinks to a boundary.

Note that while the last two data input items are optional, depending on the type of problem, the first five items are always essential for initial or START runs. For RESTART runs only data items a. and b. are input.

#### Input Data Card Formats

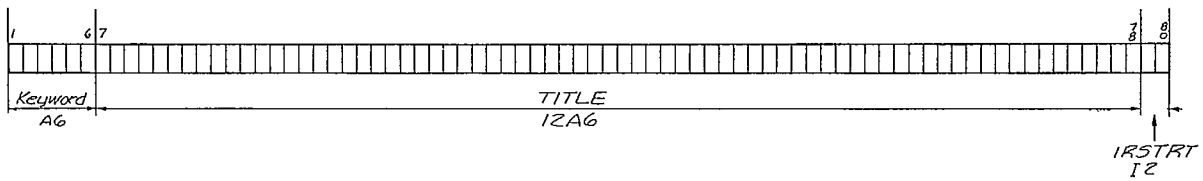
##### Title Card(s) and START Card

104. The format of the title cards and the START, RESTART or STOP card are shown in Fig 6.

##### Namelist \$FPM Cards

105. Figure 7 illustrates the manner in which the control parameters are entered through the Namelist SFPM cards. A list and description of

Title Cards and Start Card



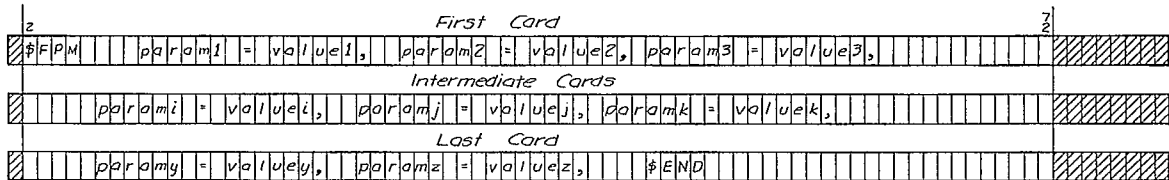
- Keyword - START means data for a new problem follow. The TITLE (72 chars) on the START card is printed on each output page.
- RESTART means an interrupted problem is to be restarted. A check point file created in a previous run must be attached to unit 13. (usually a card deck). A restarted job can add only Namelist data, no other kind.
- STOP means terminate execution.
- Anything else: the card will be printed, and the next card read.

Variable - IRSTRT is a two digit number entered only for a RESTART run. The input unit will be positioned forward of IRSTRT files.

Comment - After completion of each problem, the input stream returns to Card 1 to look for the next problem, or a STOP card.

Fig 6 - Title cards and start card.

NAMELIST \$FPM Cards



These cards contain control parameters for the problem. Each parameter itemized on the following pages has a default value so that only those having a value different from their default need be entered.

Format - free format in card columns 2 through 72 as outlined above (Card columns 1 and 73 through 80 must be blank).

- the first card must contain \$FPM in columns 2 through 5.
- the last card must contain \$END after the last parameter specification.
- each parameter is specified by a phrase of the form PARAM = VALUE, (the comma must directly follow the value).
  - for numeric parameters value is the appropriate real or integer number, and
  - for logical parameters, value is .TRUE. (or .T.) for true, or .FALSE. (or .F.) for false.
- the last card must contain a \$END.

Fig 7 - NAMELIST \$FPM cards.



all control parameters are contained in Table 1.

106. Namelist input is a convenient means to input program control parameter values as only those parameters that require to be reset from their default values need be entered. The form in which the parameter values are entered is in a phrase equivalent to:

PARAM = VALUE,

for example, to specify that the solution is to pass through ten iterations would be accomplished by including: NUMIT = 10, in the Namelist.

107. The user should note that although Namelist input is free-format there are a number of conventions that must be strictly adhered to (Fig 7).

108. For a RESTART problem only those control parameters connected with the iterative solution procedure and the form of output should be altered. The remaining Namelist parameters will have been set in the initial run and saved along with the previous results, now attached to logical unit 13.

#### Node Definition Cards

109. The general card format with which nodes are defined is shown below (Fig 11). The node definition cards are the most complicated of all the data input. Three types of nodes are used in seepage finite element analyses. These are as follows:

- 1) ordinary nodes,
- 2) phreatic surface nodes, and
- 3) pseudo-phreatic surface or linked nodes.

Variations of the illustrated format are used to input data for the different node types. The first column on the card contains a character which identifies the node type.

110. The co-ordinate system to be used for problems is an orthogonal pair of axes in which the x-axis is horizontal, increasing towards the right, and the y-axis is vertical and increasing upwards. For axi-symmetric problems the y-axis is the axis of symmetry and the x-axis values are the radial measurements (Fig 8).

Table 1: \$FPM namelist parameter description

Parameter	Type	Default	Description
* IACCEL	integer	1000	do an Aitken acceleration every IACCEL iteration (IACCEL>2)
* IDEBUG(50)	integer	0	debugging codes (see note on following page)
LUIIN	integer	5	logical unit for input of nodal and elemental information
* LUPNCH	integer	13	Logical unit to which to assign punched output
* MAXIT	integer	1	maximum number of iterations
* NEWNOD	integer	0	if reset to a positive number <NUMNP then it will be the starting node number for internal node renumbering to reduce problem bandwidth
NFLCD	integer	0	number of cards defining the distributed element boundary flow
NPIPE	integer	0	number of elements connecting internal source/sinks to the exterior boundary (see notes on PIPE cards)
NTYPE	integer	0	type of problem 0 - axi-symmetric 1 - planar flow
NUMEL	integer		number of finite elements in problem
NUMFSC	integer	0	total number of phreatic surface
NUMMAT	integer	1	number of materials and soil region lines (see material properties cards)
NUMNP	integer		number of nodal points in problem
* BETA	real	1.0	overrelaxation factor for convergence of phreatic surface
HITE	real	0.0	reference for calculating potential
QY	real	0.0	vertical infiltration into phreatic surface rare
RO	real	1.0	unit weight of fluid
SCALXY	real	1.0	scale factor for nodal co-ordinates
* TOL	real	0.1	tolerance criterion on R.M.S. movement of phreatic surface nodes. Iteration cease when this criterion is satisfied
* BRIEF	logical	.FALSE.	if BRIEF = T, a short form of printout is obtained, without nodal stream function values
ECHOIN	logical	.TRUE.	print out initial node and elemental information
* IFGRAD	logical	.FALSE.	if EFGRAD = T, use nodal fluid pressure gradients in calculating phreatic surface corrections
* NOTNSN	logical	.FALSE.	NOTNSN = T prevents continuity nodes from going into tension
* PRINT	logical	.FALSE.	PRINT = T causes intermediate printout of mesh and nodal pressures between iterations
* PUNCH	logical	.FALSE.	PUNCH = T causes final iteration data to be punched to peripheral unit LUPNCH for subsequent use in a RESTART run

Notes on Namelist \$FPM Parameters

- IDEBUG - this is an array of 50 integers normally set to zero (0). If individually reset to one (1) then specific processing and debugging information may be printed out. Not all 50 items have assigned purposes. The active parameters are:
- IDEBUG(1) - will print array MESH (in subroutine FORM).
  - IDEBUG(2) - will print out the fluid flow in each finite element (calculated in subroutine ELFLOW)
  - IDEBUG(3) - print most of the steps involved in computing the global flow matrix (in subroutine TRIFL)
  - IDEBUG(4) - print element material type chosen when automatic soil zone location is involved (in subroutine TRIFL)
  - IDEBUG(7) - print global flow matrix and the RHS vector before and after boundary conditions are applied (in subroutine MODIFY)
  - IDEBUG(11) - print the re-ordered phreatic surface nodes (in subroutine SORTPS)
  - IDEBUG(25) - print finite element properties (in subroutine BLDMAT)
  - IDEBUG(37) - print core addresses (entry points) in the expandable memory portion of the program. (Array C in COMMON block/MATRIX/ in the main routine FEFPM)
- NEWNOD - when the value of this parameter is set to a positive number less than or equal to the number of nodes in the problem (NUMNP) then a bandwidth reduction process will be initiated. This bandwidth reduction process takes the node specified by NEWNOD and uses it to start a revised node number sequence that has the minimum bandwidth. This new sequence is completely internal to the FEFPM computer and is not apparent to the user. Note that this sequence may not represent the actual absolute minimum bandwidth but it is likely to be an improvement over that resulting from the input node sequence. For problems that are to be solved iteratively this minimized bandwidth will generally result in less computer time being required for the analysis and thus will be more economic. For large confined flow problems with only one solution iteration it may still prove more economic to minimize the bandwidth than to leave it as originally specified.
- The value assigned to NEWNOD should be chosen with care. Suggested nodes to try are those at geometrical extremes (corners) of the finite element mesh.
- SCALXY - very often gridded paper is used to draw up finite element meshes. When entering nodal co-ordinates it is then more convenient to use grid square values rather than actual co-ordinate values. In such a case, SCALXY may readily be used to convert input (ie, grid) node co-ordinates to real co-ordinates.
- QY - normally in unconfined flow problems the boundary conditions on the phreatic surface are zero fluid pressure and no flow across it. In some cases, though, water, usually resulting from precipitation on the slope surface, infiltrates across the phreatic surface at a known rate. As the location of the phreatic surface changes between iterations equivalent nodal flow rates also change. Thus, if this infiltration was specified by distributed flow cards serious errors in flow quantities could arise at the end of the iterative solution. The FEFPM computer program will calculate the correct phreatic surface node infiltration quantities when QY is specified in the Namelist \$FPM cards.

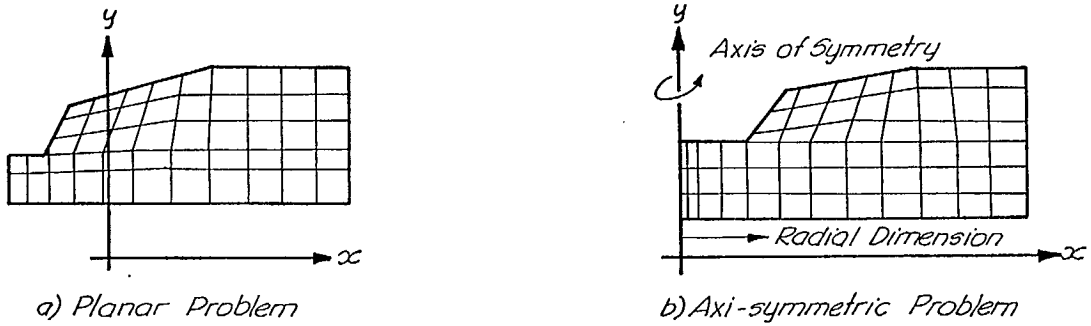


Fig 8 - Finite element co-ordinate systems.

111. All phreatic surface nodes must be defined, but ordinary or linked nodes can be automatically generated by defining only the nodes at the ends of a line of sequential nodes (Fig 9). Note that the example shown in Fig 9 indicates

node spacing of 1.0. It is possible to vary the spacing of the line using a non-unitary geometric progression ratio (RGP) as shown in Fig 10. The ratio of the distance between successive nodes is the geometric progression ratio. This ability

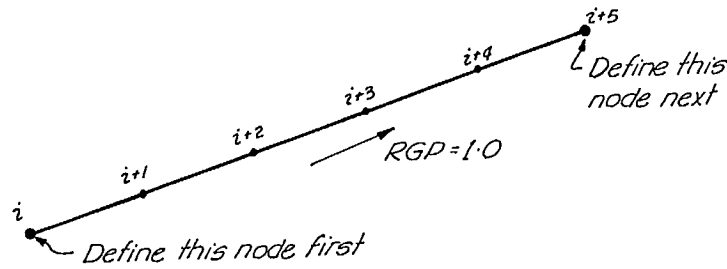


Fig 9 - Simple node generation.

that the nodes are equally spaced. This node generation line has a geometric progression ratio for

to generate non-equally spaced nodes can reduce the amount of time required to code nodal input

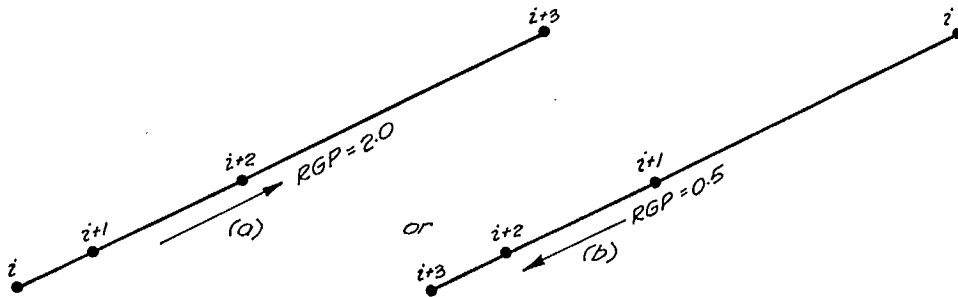
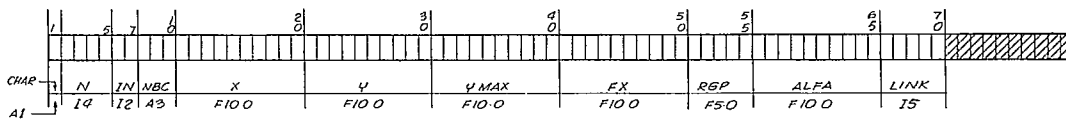


Fig 10 - Node generation with non-unitary geometric progression ratios.

data for a problem. In most problems the geometric progression ratios used to generate nodes should be limited to the range of 0.8 to 1.25 to maintain reasonably shaped and sized finite elements.

112. When nodes on finite element mesh boundaries are generated, the variation of the boundary condition values must follow in a similar manner. Note 1 of Fig 11 contains full details on the ways in which boundary conditions can be generated.

#### Node Definition Cards.



- CHAR - node type identifier - "blank" indicates an ordinary node  
 \* is used to define a phreatic surface node  
 L is for a pseudo-phreatic or linked node
- N - node number  
 IN - boundary condition generation code (see Note 1)  
 NBC - boundary condition code (see Note 1)  
 X, Y - nodal co-ordinate values  
 YMAX - upper bound for movement of a phreatic surface node (see Note 2)  
 FX - boundary condition value  
 RGP - geometric progression ratio for generation to next defined node  
 ALFA - is the correction direction for phreatic surface nodes (default = 90°) (see Note 2) and fractional movement coefficient for pseudo-phreatic nodes (default = 1.0) (see Note 3)  
 LINK - is the phreatic surface node to which pseudo-phreatic nodes are linked (see Note 3)

#### Notes on Node Definition Cards

- Boundary condition codes and generation characteristics.
  - as outlined previously in the finite element theory section, sufficient boundary conditions must be applied to the mesh to permit solution for the flow characteristics. Four variations of boundary conditions are possible for unconfined flow problems. These boundary conditions are always applied at nodes, and their type is indicated by the variable NBC on the node definition card.
  - for a node with known fluid pressure (Pressure Node), set NBC = "BPP", ie, place a P in column 10, and set FX = fluid pressure.
  - for a node with a known fluid influx (Continuity Node) set NBC = "BPPQ", ie, place a Q in column 10, and set FX = the flow influx (make FX negative if flow is out of the mesh). Most nodes in a finite element mesh will have zero influx but will still be in this category. If flow rates across a boundary are known, use distributed inflow cards to provide details of flow quantities.
  - phreatic surface nodes are defined completely by the character in the first column and thus NBC should be left blank or zero.
  - a special feature in the FEPPM program allows nodes to be set at the same fluid potential function value, which will be determined at part of the solution. To do this select one node (I) to which others will be referenced, then when one of these other nodes is defined set NBC = I on the data card. This feature is very useful when defining nodes along a well (see sample problem no. 2). The boundary condition value FX is ignored for this case.
  - boundary conditions and values can be generated along with the nodes. However, boundary condition generation must follow one of six possible forms (specified by the boundary condition generation code IN on the node definition card) as indicated in the table below.

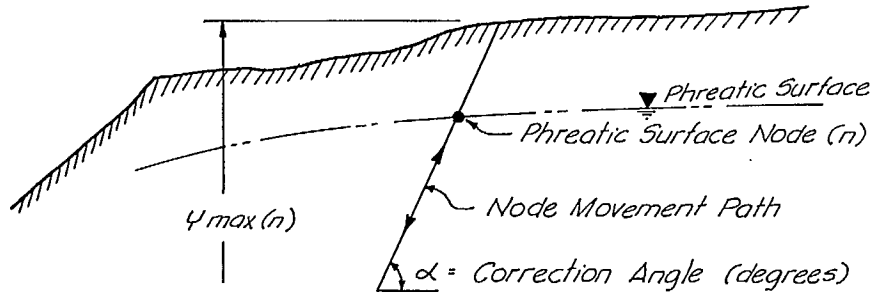
#### Boundary condition generation codes

Conditions assigned to generated nodes.

IN	Type (NBC)	Value (FX)
-1	NBC(n)	0.0
0	"Q"	0.0
1	NBC(n)	Linearly varying between specified values at the end of the line.
2	NBC(n)	Linearly varying from FX(n) to 0.0 at the end of the line.
3	NBC(n)	Linearly varying between specified values at the ends of the line. However, if the line distorts, the gradient of FX along the line is held constant, and values are generated from FX(n).
4	NBC(n)	Linearly varying as for IN = 3 but values are generated from the other end of the line if the line distorts.

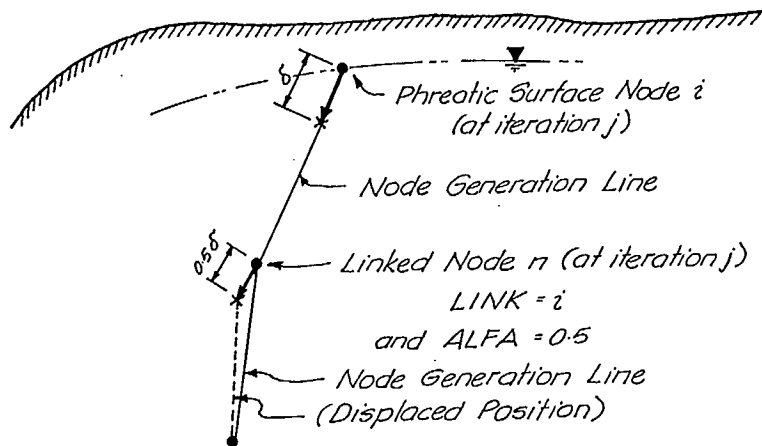
Fig 11 - Node definition cards.

2. Phreatic surface node movement directions and limits (see diagram).  
 - each phreatic surface node should have a correction direction assigned and also a limit to ensure that the flow regime stays within the slope profile.



- note if the correction angle ( $\alpha$ ) is not specified it is set so that the node movement is vertical (ie,  $\alpha = 90^\circ$ ).

3. Pseudo-phreatic surface or linked nodes.  
 - linked phreatic surface nodes are used principally to prevent severe mesh distortion during iterative solutions of unconfined flow problems. This is illustrated in the diagram below.



- in the above figure, node  $n$  is linked to phreatic surface node  $i$  and is to have half the movement of  $i$  during each iteration. If ALFA had not been specified it would have been defaulted to 1.0 and thus node  $n$  would then have the same movement as node  $i$ .

Fig 11 (cont) - Node definition cards.

#### Ordinary Node Definition

- Ordinary nodes are all nodes which are not phreatic surface or pseudo-phreatic surface card column blank.
- The following parameters are to be input:
  - the node number (N)
  - the boundary condition generation code (IN)
  - the boundary condition code for the node (NBC)
  - the co-ordinates of the node (X,Y)
  - the boundary condition value (FX)
  - the geometric progression ratio for distances

between nodes to be generated (RGP)

- The following parameters are to be input:
  - the node number (N)
  - the boundary condition generation code (IN)
  - the boundary condition code for the node (NBC)
  - the coordinates of the node (X,Y)
  - the boundary condition value (FX)
  - the geometric progression ratio for subsequent node generation (RGP)
  - the correction distance factor (ALFA), and
  - the phreatic-surface node that this node is linked to (LINK)

Phreatic Surface Node Definition

- Phreatic surface nodes define the upper limit of the groundwater regime for unconfined flow problems. The final location of the phreatic surface will be computed during the iterative procedure. Phreatic surface nodes are indicated by punching an asterisk '\*' in the first column of the card.
- The following parameters are to be input:
  - the node number (N)
  - the co-ordinates (X,Y)
  - the upper bound for phreatic surface movement (YMAX)
  - the correction direction for node movement (ALFA) (angle in degrees measured counterclockwise from the x-axis)

Material Definition Cards

113. The material definition cards define both the properties and the locations of the various material zones. The various card formats used to input material properties and locations are shown in Fig 12.

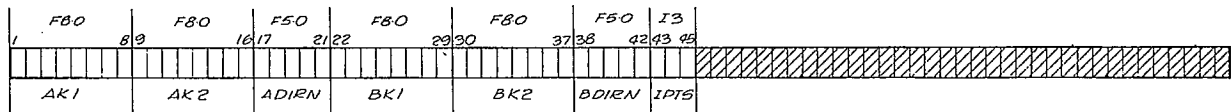
114. Individual finite element material types can be defined in one of two ways, ie,

1. by explicitly coding a material type number (MAT) on the element definition card, or
2. by having the FEFPM program do an automatic search through defined material zones and lines to find the applicable material type for the

Pseudo-Phreatic Surface or Linked Node Definition

- Pseudo-phreatic surface or linked nodes are nodes which are constrained to move in concert with specified phreatic surface nodes to prevent severe mesh distortion in unconfined flow problems. Linked nodes are identified by punching L in the first column of the card:

MATERIAL DEFINITION CARDS

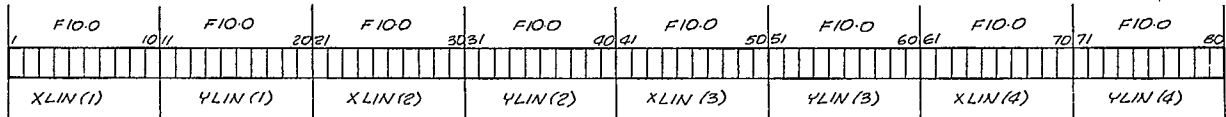


Each card represents the boundary between two materials where

- AK1 - maximum permeability just above the line
- AK2 - minimum permeability just above the line (if left blank it will be set equal to AK1)
- ADIRN - counterclockwise direction (degrees) from the X axis to the maximum permeability above the line
- BK1 } - as above for just below the line
- BK2 }
- BDIRN }
- IPTS - 0 no points entered, line is set to default values and all elements not defined by any other line will have the just above the line properties
- <0: the next card(s) contains the X, Y co-ordinates of the co-ordinate locations defining the boundary
- >0: the next card contains IPTS node numbers for nodes defining the boundary

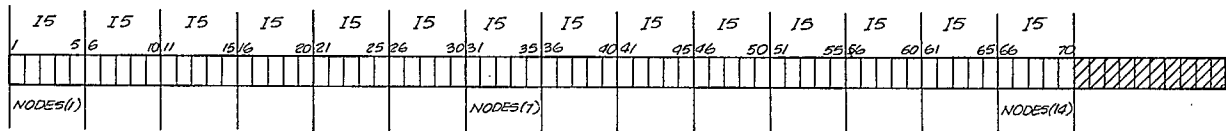
Fig 12 - Material definition cards.

MATERIAL ZONE OR LINE DEFINITION CARDS (by co-ordinate locations)  
Input only if IPTS is set negative



- Up to 14 co-ordinate locations may be used to define a material zone or line. To define a zone, ie, a closed loop, the first and last co-ordinate locations entered must be the same.
- Enter /IPTS/ co-ordinate locations in X,Y pairs as indicated above. Use as many cards as required up to a maximum of 4.

MATERIAL ZONE OR LINE DEFINITION CARDS (by defined nodes)  
Input only if IPTS is set positive



- Up to 14 nodes may be used to define a material zone or line. To define a zone, ie, a closed loop, the first and last nodes entered must be the same.
- Enter IPTS node numbers starting from the left of the card.

Fig 12 (cont) - Material definition cards.

element centroid.

115. Material type numbers are not defined explicitly on the material definition cards but are generated in the order in which they are placed in the input card deck. Care must be exercised when coding data and setting up card decks to ensure that the material definition cards are sequenced correctly.

116. The location of a particular material is defined by a series of connected line segments in the plane of the mesh. If the series of lines forms a closed loop, then all the material enclosed within the loop is considered to be the material zone. The points defining the ends of the line segments may be referenced as either:

1. finite element mesh nodes (original locations for moving phreatic surface problems), or
2. co-ordinate locations input as part of the material definition data.

Different input card formats exist for the alternate definitions of material zone or line segments.

117. All materials defined for the FEFPM computer program are defined in terms of lines to which values of permeability are set for just above and just below the line. If a line is not explicitly defined for a material type, then the FEFPM program generates a line composed of three segments as shown in Fig 13. Any element in the

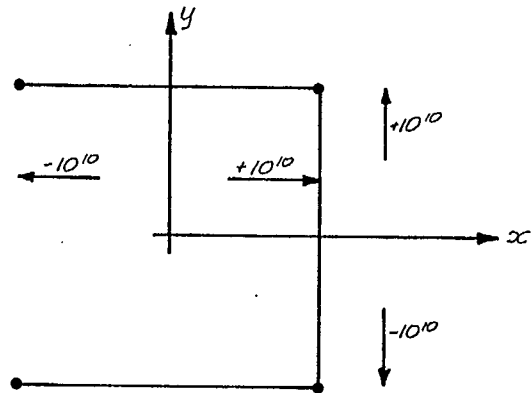


Fig 13 - Default material definition line.

mesh which is then not explicitly typed or defined by a material line or zone will have the material properties defined as those just above the default line. Thus, if there is only one material involved in the finite element mesh, a single card with the maximum permeability, AK1, the minimum permeability, AK2, and direction, ADIRN, for anisotropic problems is needed.

118. It is possible to define different material properties for just above and just below the line. This feature is very useful when a line represents the boundary between two different geological materials. The following conditions apply when the FEFPM computer program searches for material types:

- if the element centroid is found to lie within a material definition loop, then the material properties used are those that were set for just above the line. If it lies within more than one loop then the closest loop is selected.
- if the element centroid is found to lie between two material definition lines then the material properties are linearly interpreted from those set just above the closest line segment directly below and those set just below the closest line segment directly above the point. Any ambiguities about the closest line segment are settled by setting the lower sequences

lines above the others.

- if the element centroid is found to lie below the lowest line, the material properties set just below the line are used.
- if the element centroid lies above the highest line, an error message is printed and FEFPM program execution is terminated.

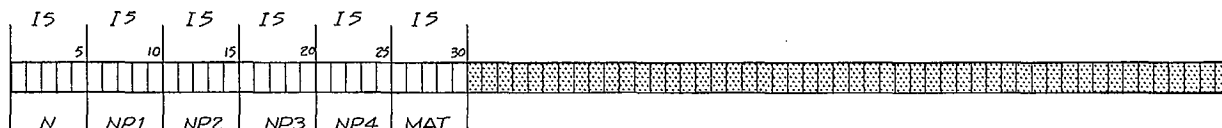
119. Users should be careful about mixing line and zone definitions for material properties. Wherever possible, it is recommended that the zone definition be used throughout a problem in which automatic material searching will be required.

#### Element Definition Cards

120. Figure 14 illustrates the format of the element definition cards. The data required to define a finite element are essentially the nodes at the corners, and the material type. As indicated below, element nodes are entered as they occur in clockwise order about the centroid of the element. If the material type is not defined, an automatic soil zone search will be initiated to determine the zone in which the element centroid lies.

121. Element input data may be automatically generated by the FEFPM program. For element data generation, the general requirement is that if a string of elements lies between two lines of sequenced nodes, as shown in Fig 15, then only the

#### Element Definition Cards



- N is the element number. Element cards must be entered in ascending order. The first and last elements must have cards. Elements which are not defined will be generated from the previous element by increasing all node numbers by 1. The material properly typed MAT will be unchanged.
- NP1, NP2, NP3, NP4 are the nodes, in counterclockwise order. For triangular elements set NP4 blank.
- MAT is the material number for the element. If zero or blank, the appropriate material will automatically be selected, based on the soil zone in which the element centroid lies.

Fig 14 - Element definition cards.



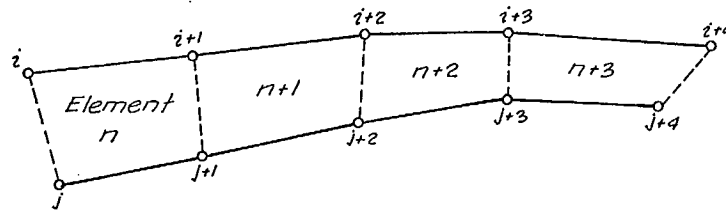


Fig 15 - Element generation procedure.

first element in the sequence need be coded.

122. Element data generation is accomplished by incrementing the element number and all attached node numbers by 1 until the end of the string is reached. The material type number remains unchanged throughout this operation. Refer to the sample problem data input lists for examples of element generation.

#### Distributed Boundary Inflow Cards

123. Figure 16 illustrates the format used for distributed boundary inflow cards. Distributed inflow occurs when fluid is injected or pumped along a boundary of a seepage element area. Most of the distributed boundary inflow occurs along the top surface of the problem area due to infiltration of precipitation. This is in contrast to a problem in which a defined rate of fluid influx or efflux can be assigned to a particular node. In the case of distributed boundary inflow, rates of flow per unit area are known and the FEFP computer program calculates equivalent nodal flow rates by integrating the unit area flow rates over

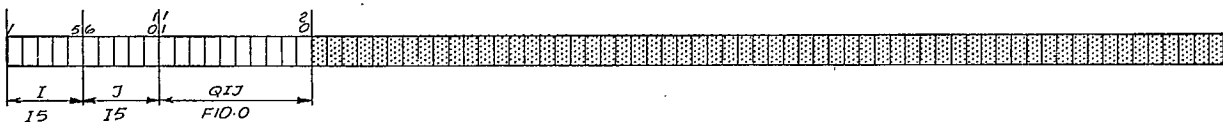
the appropriate boundary surface area. For planar problems, a unit thickness is assumed for the mesh while for axi-symmetric problems, the boundary surface area corresponds to a one-radian sector (Fig 17).

124. The number of distributed boundary inflow cards to be read is specified by the parameter NFLCD set with the Namelist \$FPM cards.

#### Pipe Cards

125. Figure 18 illustrates the format used for pipe element definition cards. Pipe cards must be input if flow sources or sinks exist within the body of the finite element mesh. The stream function can only be calculated for problems having a single outer boundary in the topological sense. Thus if an internal boundary exists, it must be connected to the outer boundary. This is accomplished for the FEFP computer program by specifying a string of elements that connect the internal source or sink to the external boundary. Figure 19 illustrates this procedure for a problem in which an adit serves as a flow sink. Up to 16

#### DISTRIBUTED BOUNDARY INFLOW CARDS (Format - 2I5,F10.0)



- I - Nodal point I  
 J - Nodal point J  
 QIJ - Fluid flow across boundary I-J. Flow/unit area.  
 - if QIJ > 0 - QIJ represents a flow influx.  
 - if QIJ < 0 - QIJ represents a flow efflux.

Note For steady-state solutions, flows across the phreatic surface must be input as a distributed boundary flow.

Fig 16 - Distributed boundary inflow cards.

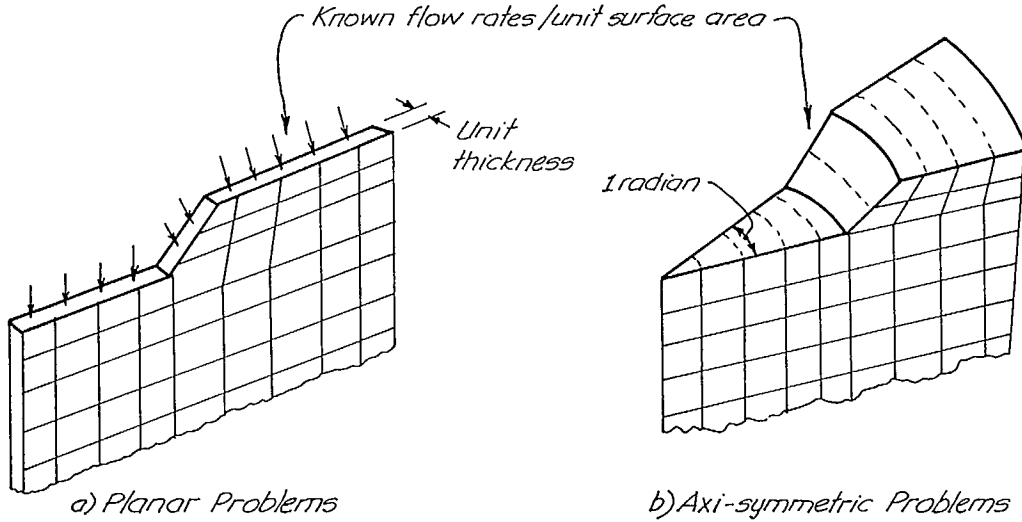
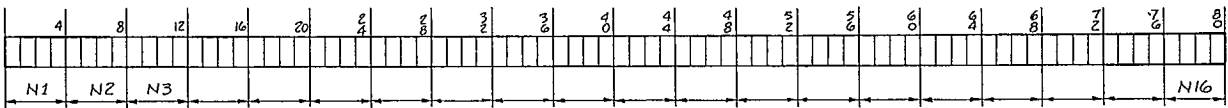


Fig 17 - Boundary surface areas.

Pipe - card. Only submitted if NPIPE>0.



The terms N1-N2 .... (NPIPE terms) are element numbers in a string that connects the internal source/sinks to the boundary. The stream-function  $\Psi$  will be discontinuous across this line of elements.

Up to 16 element numbers may be input per card. Use as many cards as required to input NPIPE numbers.

Fig 18 - Pipe card - only submitted if NPIPE > 0.

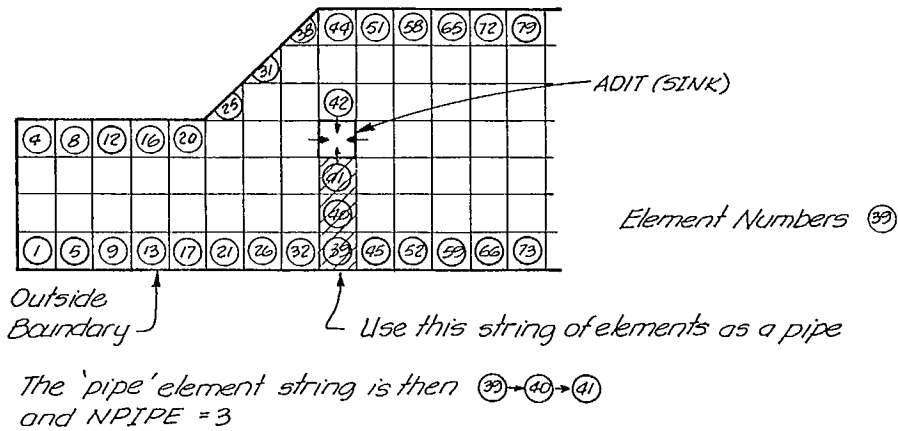


Fig 19 - Pipe elements.

pipe elements can be listed per pipe card.

#### Input Data Card Sequence

126. The card sequence must permit the following machine actions.

- a. read one card
  - if it is a START card, go to b
  - if it is a RESTART card, go to l
  - if it is a STOP card, terminate execution
  - otherwise, print the card and read next card
- b. read \$FPM cards
- c. read node cards
- d. read material cards
- e. read element cards
- f. read any distributed inflow cards
- g. read a pipe card if required

- h. do a steady-state solution
- i. read a pipe card if required
- j. do a steady-state solution
- k. go to a
- l. read in the checkpoint file produced by an earlier run
- m. read any changes to the \$FPM Namelist
- n. go to h.

#### Program Options

127. The only options available are those set by the NAMELIST parameters.

#### Sample Runs

128. Appendix A gives the input data and results for three sample runs.

## SYSTEM DOCUMENTATION

Computer Equipment

129. FEPPM, as modified for CANMET, runs on a CDC 6400 with card reader, line printer and tape drive.

Source Program

130. A source deck on magnetic tape is obtainable from: Mining Research Laboratories (Pit Slope Group), CANMET, Energy, Mines and Resources Canada, 555 Booth Street, Ottawa, Ontario, K1A 0G1, Canada.

Storage Requirements

131. The following are the core storage requirement on the CDC 6400 (see list of subroutines below).

	<u>bytes:</u>	<u>words:</u>					
Main program FEPPM	3637	1951			RENUM	730	472
	14307 (buffers)	6343		(not used)	SORTPS	310	200
CONTROL,SLINES,ELMT					FORM	2412	1290
labelled common	2712	1482			GETFLO	130	88
216-142 1260 B0					SETNOT	256	174
blank common	1	1			BLDMAT	765	501
BLKBLK common area 62 - 50					BMULT	114	76
TIME 1-1 Function NCORE	13	11			DECOMP	635	413
MESHIN	2263	1203			FINDCN	134	92
					GRADNT	172	122
					MODIFY	503	323
					TRIFL	506	326
					ELFLOW	1072	570
					FIX	41	33
					SZONE	165	117
					CROSS	50	40
					EXTINT	1177	639
					WRMESH	3y0	248
					BLK	6747	3559
					GBTTL	1070	568
					MCORE	32	26
					LCORE*	14	12
					SYFDBN	103	67
					TOD	243	163
					ETIME*	10	8
					ETIMEF*	14	12
					ERTRAN*	20	16
					NTRAN*	2061	1073

Compass :

FLD*		7
SETFLD*		11
FLDLNTH*		11
	451	297
	-----	-----
Totals:	44408.	24078.
	-----	-----

Main Program FEFPM

132. The main program is used to:

- control the data input, analysis and output sections of the system
- allocate central memory (core) locations as required
- increase as well as decrease central memory core size as required
- assign temporary scratch files (logical units 14 and 15).

Common Block Allocation

133. This main program when compiled automatically includes the labelled COMMON blocks /CONTROL/, /SLINES/ and /ELMT/ as outlined in detail in the documentation for the procedural element PDPFPM.

134. In addition, the labelled COMMON block /MATRIX/ is allocated only in the main program. It is the array C in this block which is expanded and contracted as required to provide the correct amount of memory for a particular problem.

135. COMMON BLOCK MATRIX contains the expanding array C used to store the bulk of the data in FEFPM. Each section of data is defined by its starting location value, as listed in Table 2.

Internal Calls

136. As this is the main routine many calls are made to subroutines and functions. It must be noted that all references to array storage locations in array C of the labelled COMMON block /MATRIX/ are made through defined starting locations passed in individual subroutine argument lists. See Subroutine list below.

Sequence of Steps

- Various FEFPM parameters are initialized.

- The problem title is read from cards and the problem keyword is decoded.
- The appropriate title and run description is printed (CALL GBTTL).
- The namelist \$FPM is read from cards.
- The element information block handling routine is initialized (CALL BLK).
- According to the parameters read in through the namelist \$FPM, the entry points and lengths for the various arrays required are calculated using internal function NCØRE.
- The array C in common block /MATRIX/ is expanded (CALL MCØRE).
- The node, element and material data is read in and a finite element mesh generated and checked (CALL MESHIN).
- The nodes are renumbered to minimize the problem flow matrix bandwidth (CALL RENUM).
- The nodal and elemental connectives are determined (CALL EXTINT).
- The phreatic surface nodes are sorted (CALL SØRTPS).
- The required core storage (size of array C) is checked and increased CAS002.
- The scratch drum file, logical unit 14, for element information blocks is allocated.
- The elemental and problem flow matrices are set up and solved iteratively, if necessary (CALL FØRM).
- If requested, all problem data is written to the punch file (LUPNCH).
- The matrix C is reduced to its minimum amount (CALL LCØRE).
- Minor variations in the above sequence occur if the problem to be analyzed is a RESTART from a previous run.

Common Block VariablesCOMMON BLOCK - CONTROL

GFEFPM

program description - ('GFEFPM')

HED(18)

title of problem

NUMNP

number of nodal points

NUMEL

number of elements	NDMP
NUMMAT	error return variable
number of soil region lines (not to exceed 30)	RØ
NUMFSC	unit weight of fluid
number of phreatic surface cards	BETA

Table 2: Location of variables in common block/MATRIX/

Starting location	Length		Description
N2	NUMFSC	NPFS	- phreatic surface nodes - contiguously sorted from end to end
N222	NUMFSC	MPFS	- phreatic surface nodes as entered
N3	NUMFSC	ALPHA	- the correction direction for phreatic surface nodes (measured counter-clockwise from x-axis)
N33	NUMFSC	YMAX	- upper limit for y values for phreatic surface nodes
N34	NUMFSC	ICØDFS	- phreatic surface node codes
N4	NUMNP	MESH	- input nodal numbers
N41	NUMNP	NEWNUM	- new internal node numbers
N5	NUMNP	X	- x coordinates of node points
N6	NUMNP	Y	- y coordinates of node points
N7	NUMNP	NBC	- boundary condition codes
N8	NUMNP	FX	- function determined by NBC if NBC = 0 FX = net nodal inflow NBC = 1 FX = pressure NBC < 0 FX = same potential as the linked node /NBC/
N81	NUMNP	R	- solution vector
N82	NUMNP	PSI	- flow function
N93	NUMNP	SCR	- scratch array
N96	NUMNP	PRESS	- fluid pressure at nodes
N99	$\frac{NUMNP+35}{36}$	ZERØP	- flags for no tension (NO TNSN controlled)
N10	5*NUMEL	NP NP(1,I) NP(5,I)	- element array - NP(4,I) - nodes forming the element - soil associated with element
* N122	$\frac{NUMNP+35}{36}$	NØDFLG	- flags to indicate if node is exterior (1) or interior (0)
N124	NPIPE	IPIPE	- pipe elements
* N12	3*NUMNP	NRAYS (3,1)	- node indexes connected to each node
* N13	2*NLS	LNESEG(2,1)	- line segments and related elements
N14	NUMNP*MAXBAN	C Last word of C)	Flow matrix

correction factor	IFEXT
HITE	logical variable signalling the use of 'EXTINT'
reference for potentials	NOTNSN
T/L	indicator to prevent continuity nodes from going into tension
error tolerance	N41
MAXIT	entry point (in MATRIX) for internal node numbers (NEWNUM)
maximum number of iterations	N99
MAXMSH	entry point for the flags which indicate a node is being held at zero pressure to satisfy the no tension criterion
number of node cards read	N81
NFLCD	entry point for R - solution vector
number of cards defining boundary flows	N82
PI	entry point for PSI - the flow function
constant (3.1415926/180.0)	N2
MAXBAN	entry point for NPFS - the phreatic surface nodes as input
maximum bandwidth	N222
IFGRAD	entry point for MPFS - the phreatic surface nodes as returned from SØRTPS - contiguous from one end to another (there may be a problem if SØRTPS - contiguous from one end to another (there may be a problem if SØRTPS tries to sort more than one phreatic surface)
logical variable indicating use of phreatic nodal pressures in calculating corrections	N61
PUNCH	entry points for Z - coordinates
logical variable - output to be punched upon completion of the job	NTYPE
LUPNCH	indicator of problem type - 0 Axisymmetric flow - 1 Plane flow
logical unit for output.	N3
IACCEL	entry point for ALPHA
no. of steps between Aitken accelerations	N33
QY	entry point for upper limits on Y values
vertical infiltration rate into phreatic surface	N4
LUIN	entry point for MESH - nodal numbers of input nodes
logical unit of input	N5
ECHOIN	entry point for X coordinates
logical variable signalling a printout of input mesh data	N6
SCALXY	entry point for Y coordinates
scale factor applied to x and y coordinates after input (note: this does not apply to soil zone boundaries)	N7
PRINT	entry point for NBC - boundary condition code
logical variable signalling intermediate print-out between iterations	N8
NUMIT	entry point for FX where FX is determined by NBC eg, NBC = 0 NBC eg,
number of the current iteration	
BRIEF	
logical variable signalling a short form of output is desired (no flow function data)	
ROOF	
Not used.	

NBC = 0	FX = not nodal inflow (continuity)	y-coordinates of points on region lines
NBC = 1	FX = pressure	AK1 (30)
NBC = < 0	same potential as linked node	maximum permeability above region line
N93	entry point for the scratch array (SCR)	AK2 (30)
		minimum permeability above region line
N96	entry point for the pore pressure array PRESS	ADIRN (30)
		counterclockwise direction from the x-axis to
N10	entry point for NP - element nodal conductivity and material	the maximum permeability above the region line
		AK11 (30), AK12 (30), AK22 (30)
N122	entry point for NØDFLG, the array which indicates if a node is on the boundary	permeabilities in the x-y coordinate system above the region line
		BK1 (30)
N123	not used	maximum permeability below the region line
		BK2 (30)
N124	entry point for array of elements which form a pipe and are not used in computing the flow function	minimum permeability below the region line
		BDIRN (30)
N12	entry point for NRAYS - returned from EXTINT	counterclockwise direction from the x-axis to
		the maximum permeability below the region line
N13	entry point for LNESEG - returned from EXTINT	BK11 (30), BK12 (30), BK22 (30)
		permeabilities in the x-y coordinate system below the region line
N14	entry point for flow matrix C	
N16	last word in matrix C	
NLS	number of line segments	
NPIPE	number of pipe elements	
IDEBUG(50)	debugging codes	
XXX	dummy variables	
 <u>COMMON BLOCK - SLINES</u> (MATERIAL PROPERTIES BLOCK - up to 30 different material types allowed)		
LPTS (30)	number of points defining material region	
ILØØP (30)	logical variable	
ILØØP (30)	true if the points form a closed loop	
XLIN (14,30)	x-coordinates of points on region lines	
YLIN (14,30)	y-coordinates of points on region lines	
		<u>COMMON BLOCK - ELMT</u>
		(common block retaining data to the current element)
		XX (5)
		element's nodal x-coordinates
		YY (5)
		element's nodal y-coordinates
		S (5,5)
		element flow matrix
		GG (5)
		initial R.H.S. for flow equation
		FLOW (6,4)
		real nodal flows
		XCENT (4)
		x-coordinates of centroid
		EPRESS (5)
		element nodal pressure
		ISØIL
		soil type
		NELT
		element number
		<u>Subroutines and Functions</u>
		138. Due to the conversion of this program some confusion may exist in the use of the term "field"



length".

139. Field length is used on control data systems to describe the size of central memory required to run a program. The subroutines CAS002 and FLDLNTH are concerned with evaluating and setting memory size, and refer to this as the field length, the functions (or subroutines) FLD and SETFLD are converted from or used in conjunction with the UNIVAC 1100 series function FLD, a bit manipulation function. The 'field length' described here is the number of bits in a word. Neither of these is the usual use of the word "field", meaning a subset of a computer record.

140. Below are listed the subroutines called by the main program and other subroutines. Those marked with a / use routines written in Assembler (Compass) or use CDC functions. Those marked \* use ENCODE or DECODE.

FEFPM

calls: FLD (compass)

NTRAN (FORTRAN with intrinsic CDC routines)

CAS002 (EMRLIB, compass)

SETFLD (compass)

BLK/

GBTTL/

MESHIN\*/

REMI/

SORTPS

EXTINT/

FORM/

NCORE

LCORE/

PAGBA(GBTTL)/

MESHIN\*/

calls: ETIMEF/

WRMESH/

BLKOUT(BLK)/

PAGBA(GBTTL)/

RENUM/

calls: FLD

SETFLD

ETIMEF/

ETIMEF/

SORTPS

calls: FINDCN/

FORM\*/

calls: FLD

SETFLD

ELFLOW/

ETIMEF/

GRADNT

PAGBA (GBTTL)/

SETNOT/

BLDMAT/

DECOMP

ETIME/

FINDCN/

GETFLO

MODIFY/

SUBST (DECOMP)

WRMSH (WRMESH)/

GETFLO

calls: BMULT

SETNOT/

calls: FLD

SETFLD

BLDMAT/

calls: BLKOUT (BLK)/

ETIMEF/

ETIME/

TRIFL

BMULT

calls: None.

DECOMP

FORTRAN library only

FINDCN/

SETFLD

GRADNT

FORTRAN library only

MODIFY✓

FLD

TRIFL

SZONE

ELFLOW✓

FLD

DECOMP

ETIMEF✓

BLKIN (BLK)✓

ETIME✓

FIX

PAGBA (GBTTL)✓

SUBST (DECOMP)

FIX

None

SZONE

CROSS

CROSSEXTINT✓

FLD

SETFLD

WRMESH✓

PAGBA (GBTTL)✓

BLK✓ uses CDC BUFFER statement

NTRAN

FLD

SETFLD

EXIT

GBTTL✓

ERTRAN

SETFLD

FLD

MCORE (not used)

FLDLNTH

LOCF

LCORE✓

FLDLNGTH

LOCF

SYFDBN✓

FLD

TOD✓

FLD

ERTRAN

SETFLD

ABORT✓

SYFDBN✓

ETIME✓

SECOND (CDC intrinsic function)

ETIMEF✓

SECOND

ERTRAN✓DATE } CDC intrinsic functions  
TIME }FLD✓

Compass routine

SETFLD✓

Compass

FLDLNTHCompass - same as CAS002NTRAN\*BUFFER IN } CDC FORTRAN functions  
and }  
BUFFER OUT }

READMS

WRITEMS

LENGTH

OPENMS

UNIT

Subroutine: MESHIN

PURPOSE

Subroutine MESHIN is used to read all input data describing the problem finite element mesh configuration.

CALLING SEQUENCE

CALL MESHIN (NPFS, ALPHA, YMAX, MESH, X, Y, NBC, FX, NP, PRESS, IPIPE, Z, ICØDFS).

DESCRIPTION OF ARGUMENTS

NPFS(1): Integer array - Phreatic surface nodes.

ALPHA(1): Real array - Correction directions for phreatic surface nodes.

YMAX(1): Real array - Upper limit for y-values of phreatic surface nodes. (To ensure that the phreatic surface remains within the bounds of the continuum).

MESH(1): Integer array - Node numbers as input.

X(1): Real array - x-coordinates of nodal points.

Y(1): Real array - y-coordinates of nodal points.

NBC(1): Integer array - Nodal boundary condition codes.

FX(1): Real array - Nodal function value.

NP(5,1): Integer array - Element node and material type array.

PRESS(1): Real array - Nodal fluid pressures.

IPIPE(1): Integer array - Pipe elements used in calculation of stream function for problems with a source or sink within the mesh.

Z(1): Real array.

ICØDFS(1): Integer array - Phreatic surface node codes.

COMMON BLOCK ALLOCATION

When compiled, this routine automatically includes the labelled COMMON blocks /CØNTRL/, /SLINES/ and /ELMT/ as outlined in COMMON blocks.

INTERNAL CALLS

See Subroutine list.

SEQUENCE OF STEPS

1. Read node information and generate intermediate nodal data if required.
2. Print complete nodal information (CALL WRMESH).
3. Read material properties information (including zone descriptions).

4. Print material properties data.

5. Read and write element information. Generate and write intermediate elements if required. Write out element information blocks to element file (CALL BLKØUT).

6. Read and write distributed flow input data.

7. Read and write phreatic surface nodal data.

8. Read and write pipe element data.

9. Return to calling program.

Subroutine: RENUMPURPOSE

To renumber the nodal points in a finite element mesh to give a reduced problem bandwidth.

CALLING SEQUENCE

CALL RENUM (NUMEL, NP, NEWNUM, NCØNS, IAVAIL, IHAVE, MEMJTX, NUMNP, KNEXT, MAXBAN, NBC).

DESCRIPTION OF ARGUMENTS

NUMEL: Integer - No. of finite elements in mesh.

NP(5,1): Integer array - Elemental nodal points and material type numbers.

NEWMUM(1): Integer array - Elemental nodal points and material type numbers.

NCØNS(1): Integer array - Scratch area (no. of node connections per node).

IAVAIL(1): Integer array - Scratch area (availability, numbers for a node).

IHAVE(1): Integer array - Scratch area (renumbered nodes).

MEMJTX(1): Integer array - Scratch area (nodes connected to a node).

NUMNP: Integer - No. of nodal points in mesh.

KNEXT: Integer - The number of the node from which the renumbering process is to start.

MAXBAN: Integer - Maximum half-bandwidth of renumbered finite element mesh.

NBC(1): Integer array - Nodal boundary condition indicators.

INTERNAL CALLS

See Subroutine list.

SEQUENCE OF STEPS

1. Set up arrays NOØNS and MEMJT for all nodes as

follows:

- a. NCØNS (I) is the number of nodes (included linked nodes) that are connected to node I.
  - b. MEMJT (J,I) is the Jth node connected to node I. Three node numbers are packed into each word of this array and a maximum of 30 nodes can be connected to any one node.
2. The availability numbers of all nodes are initialized to indicate that these nodes have not yet been renumbered:  
 $I\text{AVAIL}(I) = -\text{NCØNS}(I) - 100,000$
  3. If KNEXT (the starting node number) is less than or equal to zero, then return to the calling program.
  4. Start the numbering procedure with node KNEXT. Set IHAVE(1) to KNEXT and change IAVAIL) of all the nodes connected to KNEXT to indicate that they should have priority in renumbering -  $I\text{AVAIL}(J) = -50 - \text{NCØNS}(I)$   
 Note that the availability numbers for nodes connected to node KNEXT are much greater than for unconnected nodes.
  5. The only nodes considered for renumbering at this stage are those connected to KNEXT. Find the neighbour with the highest availability number. In case of a tie, pick the neighbouring node first encountered in the search. Assume it is node J, set the neighbour's availability number to zero and set IHAVE(2) to J. Then reset the availability numbers of all the unrenumbered nodes connected to J as:  
 $I\text{AVAIL}(K) = -50*(2) - \text{NCØNS}(K)$   
 Note that these availability numbers for nodes connected to new node 2 are less than those for new node 1.
  6. Continue finding the neighbours of KNEXT (new node 1), renumbering them, resetting IAVAIL for the node found and making IHAVE for the node equal to INEXT, where INEXT is the next sequence number in the new node array. Then reset the availability numbers for each of the unrenumbered nodes connected to INEXT as:  
 $I\text{AVAIL}(L) = -50*\text{INEXT} - \text{NCØNS}(L)$
  7. When all neighbouring nodes for node KNEXT have been found, move on to newnode 2 and repeat the process outlined in step 6.
  8. Repeat step 6 for each renumbered node until

all nodes have been renumbered.

9. Transfer contents of IHAVE scratch array to NEWNUM array.
10. Return to the calling program.

Note:

The basic node renumbering algorithm can be described simply as:

Number from the earliest renumbered node which has unrenumbered neighbours.

Number to the neighbour who is connected to the earliest other renumbered node. In the case of a tie number to the neighbour found first.

Subroutine: SØRTPS

PURPOSE

SØRTPS is used to sort phreatic surface nodes into a contiguous sequence from end to end.

CALLING SEQUENCE

CALL SØRTPS (NPFS, MPFS, NUMFSC, ALPHA, LNESEG, NLS, NBC, IDEBUG, YMAX, ICØDFS)

DESCRIPTION OF ARGUMENTS

NPFS(1): Integer array - Output sorted phreatic surface node array.

MPFS(1): Integer array - Input phreatic surface node array.

NUMFSC: Integer - Number of true plus dummy phreatic surface nodes in set.

ALPHA(1): Real array - Nodal correction direction array (sorted when output).

LNESEG(2,1): Integer array - Double dimensioned array containing indices of the nodal points defining each line segment present within the set of elements, along with the indexes to the elements to which they are common.

NLS: Integer - Number of line segments in finite element mesh.

NBC(1): Integer array - Array of nodal boundary condition indicators.

IDEBUG: Integer - Debug flag (pIDEBUG(11) from main program). If set >0 will cause SØRTPS to print out contents of NPFS array after sorting.

YMAX: Real array - Array of maximum permitted y-coordinates for phreatic surface nodes (sorted when output).

ICODFS(1): Integer array - Array containing indicator flags for phreatic surface nodes.

If ICDIFS (i)

= 0 - true phreatic surface node

> 0 - then ICDIFS(i) is the node to which the pseudo phreatic surface node is linked.

(Sorted when output)

#### INTERNAL CALLS

SØRTPS calls subroutine FINDCN.

#### SEQUENCE OF STEPS

1. Transfer the contents of the NPFS array to the MPFS array.
2. Search the MPFS array until a true phreatic surface node is found with only one connecting line segment. This node is then one end of the phreatic surface and is placed at the beginning of the MPFS array.
3. Search the MPFS array for the true phreatic surface node connected to the last one found. Move it to the appropriate location.
4. Repeat step 3 for all true phreatic surface nodes.
5. All pseudo-phreatic surface nodes are now located in the MPFS array after the end of the true phreatic surface nodes.
6. Return to the calling program.

#### Subroutine: FØRM

#### PURPOSE

Subroutine FØRM controls and conducts all phases of the actual seepage analysis. These phases consist of:

1. mesh generation
2. problem flow matrix construction
3. modification of flow matrix for nodal boundary conditions
4. solution of flow matrix equations
5. computation of phreatic-surface movement
6. mesh regeneration
7. solution iteration, phases 6, then 3 through 5, as required to converge results
8. calculation of nodal stream function values as well as individual finite element flows.

#### CALLING SEQUENCE

CALL FØRM (NPFS, MPFS, ALPHA, MESH, X, Y, NBC, FX, NP, R, C, RAYS, LNESEG, PSI, IFPIPE, NØDFLG, YMAX, SCR, PRESS, Z, NEWMUM, ZERØP, ICDIFS).

#### DESCRIPTION OF ARGUMENTS

NPFS(1): Integer array - Input phreatic surface nodes.

MPFS(1): Integer array - Sorted phreatic surface nodes.

ALPHA(1): Real array - Correction directions for phreatic surface nodes.

MESH(1): Integer array - Node numbers as input (used to generate F.E. geometry).

X(1): Real array - x-coordinates of nodal points.

Y(1): Real array - y-coordinates of nodal points.

NBC(1): Integer array - Nodal boundary condition codes.

FX(1): Real array - Nodal boundary condition values.

NP(5,1): Integer array - Array of elemental node and material type numbers.

R(1): Real array - Right hand side (R.H.S.) vector (used initially to store flow forcing vector).

C(MAXBAN,1): Real array - Matrix area for storage of global ie, problem, flow matrix.

RAYS(3, 1): Real array - Double-dimensioned array space for integer array containing linkage indexes for each node point.

LNESEG(2,1): Integer array - Double-dimensioned array containing the indices of the nodal points defining each line segment present within the set of elements along with the indexes to elements to which they are common.

PSI(1): Real array - Nodal stream function values.

IFPIPE(1): Integer array - Pipe element array (used in calculation of stream function values).

NØDFLG(1): Integer array - Nodal condition flags to indicate if node is exterior (=1) or interior (=0) to a finite element mesh.

YMAX(i): Real array - Upper limit of y-values to phreatic surface nodes.

SCR(1): Real array - Scratch array used to contain net nodal flows.

PRESS(1): Real array - Nodal fluid pressures.  
 Z(1): Real array - Nodal elevations.  
 NEWNUM(1): Integer array - New node position numbers.  
 ZERØP(1): Bit array - Flags for no-tension conditions.  
 ICØDES(1): Integer array - Indicator flags for phreatic surface nodes.

#### COMMON BLOCK ALLOCATION

When compiled, this routine automatically includes the labelled COMMON blocks/CØNTRL/, /SLINES/ and /ELMT/ as outlined in COMMON blocks.

#### INTERNAL CALLS

See Subroutine list.

#### SEQUENCE OF STEPS

1. Set no-tension flags for phreatic surface nodes.
2. Generate, or re-generate, finite element mesh node geometry by proceeding through the MESH array as follows:
  - a. look at the end of each node generation line
    - if it is not a phreatic surface node go to (c)
    - if this is the first iteration go to (c)
    - if the end node is not a pseudo-phreatic surface node go to (b)
    - calculate the fraction of movement of true phreatic surface node that this node is to have
    - go to (c)
  - b. retrieve the true phreatic surface node movements previously stored
  - c. check to see if nodes are to be generated along the node generation line - if not go to (e)
    - back-figure the geometric progression ratio from the first two line segments
    - calculate the sum of the geometric ratios for all line segments and then determine movements of the node at the end of the first line segment
    - if node generation line has continuity boundary condition go to (d)
  - d. start generating the nodal co-ordinates and boundary condition values along the line from the first segment
  - e. add increments (change in geometry) to the starting node on the line. If this is the first iteration go to (f), if not, and the end node is a true phreatic surface node, print the movement details
  - f. set conditions for next line segment.
3. If mesh report is to be printed, calculate the net nodal flows (S/R GETFLØ) and print the report (S/R WRMESH).
4. Build the global, ie, problem, flow matrix and the flow forcing vector (R.H.S.) (S/R BLDMAT).
  - then set correct inflows for phreatic surface nodes (S/R FINDCN)
  - then modify the global flow matrix and the flow forcing vector for specified boundary conditions (S/R MØDIFY)
5. Solve the linear equations represented by the global flow matrix and the flow forcing vector for the nodal fluid pressure (S/R DECOMP and S/R SUBST). If no-tension conditions on phreatic surface are set and phreatic surface node pressures are less than zero re-set them to zero (S/R SETNØT). If this iteration is the last one to be done, ie, = MAXIT, or if it is a confined flow problem, to to Step 13.
6. Calculate net nodal flows (S/R GETFLØ).
7. Cycle through the true phreatic surface nodes and calculate their movements as follows.
  - a. if the node is on either end of the phreatic surface node string, leave it for later.
  - b. if the node has not previously tried to leave the slope, ie, go outside it or daylight, then go to (c) - If the node then has a net third inflow its location is in error and the convergency flag for the iteration is set to false. Go back to the

start of this step for the next phreatic surface node.

- c. Set the node correction direction and vertical component. If specified by data input in NAMELIST \$FPM calculate the fluid pressure gradient at the node. Calculate and store the component movements and gradients. Also sum total phreatic surface movement.
8. If the total phreatic surface movement is less than the pre-set tolerance, the analysis has converged. Then go to Step 13.
9. Calculate solution acceleration factor.
10. Cycle through the true phreatic surface nodes again and calculate accelerated movements for the next iteration as follows.
  - a. If the node has not previously daylighted the slope, go to (b), otherwise check to see if there is a net nodal inflow. If so, switch the node to have a continuity boundary condition it should drop after the next iteration - and go back to the start of this step
  - b. Is the node moving down? If so, go to (c). Otherwise, check to ensure that the accelerated node movement will not cause it to leave or daylight the slope. If it does, then change the node to a zero pressure boundary condition and reset its co-ordinates so that it is on the slope outline.
  - c. Calculate and store the accelerated movements for the true phreatic surface node points.
11. Compute the movements of the ends of the phreatic surface. Fit a parabola to the second, third and fourth node from each end and use the slope calculated at the second node to estimate the new position of the end node. Ensure that the end nodes will not leave the slope as in Step 10.
12. Cycle through steps 2 to 11 as required to converge or finish the analysis.
13. Solve for the individual element flows and the nodal stream function values (S/R ELFLOW). Then print a nodal result report.
14. Return to the main program.

Subroutine: GETFLØ

Is used to compute net nodal outflows which are returned to FØFRM in the array SCR. To do this the original global flow matrix is retrieved from disk storage and is multiplied by the nodal potential array.

Subroutine: SETNØT

Is used to set a zero pressure flag for nodes with negative third pressures (tension) and also to indicate to FØFRM that the solution then cannot have converged.

Subroutine: BLDMAT

PURPOSE

To compute flow matrices for individual quadrilateral and triangular elements and assemble these into the global (or master) flow matrix.

CALLING SEQUENCE

CALL BLDMAT (X, Y, Z, NP, PRESS, R, C, NEWNUM, NBC).

DESCRIPTION OF ARGUMENTS

X(1): Real array - x-coordinates of nodal points.  
 Y(1): Real array - y-coordinates of nodal points.  
 Z(1): Real array - Elevations of nodal points (not used).  
 NP(5,1): Integer array - Element nodal points and material type numbers.  
 PRESS(1): Real array - Nodal pressures.  
 R(1): Real array - Solution vector  
 C(MAXBAN,1): Real array - Matrix area for storage of global flow matrix.  
 NEWNUM(1): Integer array - New internal node numbers (after bandwidth minimization).  
 NBC(1): Integer array - Nodal boundary condition codes.

COMMON BLOCK ALLOCATION

Labelled COMMON blocks /CØNTRL/, /SLINES/ and /ELMT/ are automatically included on compilation. See COMMON blocks.

INTERNAL CALLS

See Subroutine list.

SEQUENCE OF STEPS

1. Initialize solution vector R and global flow matrix C to zeros.
2. For each element in the mesh:
  - a. initialize pressure vector and flow matrix to zeros
  - b. find and store element nodal co-ordinates and determine element centroid
  - c. if element is quadrilateral, locate and define sub-elements (triangles)
  - d. build nodal position number array
  - e. if a quadrilateral, form flow matrix from triangular sub-element flow matrices (4 calls to TRIFL) and reduce the centre node
  - f. if a triangle, form flow matrix (CALL TRIFL)
  - g. add the element flow matrix using the internal node numbers carried in the NEWNUM array
3. Store the complete global flow matrix on logical unit 14 (CALL NTRAN).
4. Store the right hand side (flow forcing vector) on logical unit 14 (CALL NTRAN).
5. Return to calling program.

Subroutine: BMULT

PURPOSE

This subroutine multiplies a symmetric matrix by a vector. Only the elements of the lower half-band of the matrix, including the diagonal, are required.

CALLING SEQUENCE

CALL BMULT (A, B, C, N, M)

DESCRIPTION OF ARGUMENTS

A: - Real input one-dimensional array containing the lower half-band only including the diagonal of the symmetric multiplicand matrix of order, N, and half-bandwidth, M, stored by columns. A must be dimensioned at least  $(N-1)*M+1$ . For a diagram showing the arrangement of the elements of A, see documentation of Golder Associates library subroutine DECOMP.

B: - Real input one-dimensional array of length at least N containing the multiplier vector.

C: - Real output one-dimensional array of length at least N to contain the product vector.

N: - Integer input argument containing the order of the multiplicand matrix, which is also the length of vectors B and C.

M: - Integer input argument containing the width of the half-band of the matrix, including the diagonal.

INTERNAL CALLS

See Subroutine list.

Subroutine: DECOMP

PURPOSE

This subroutine is used to solve a system of linear equations represented by the matrix equation  $Ax=b$  for the vector x, where A is a positive-definite symmetric banded matrix. Entry DECOMP decomposes A by Cholesky's method to L, a lower triangular matrix such that  $L.L^T=A$ . Entry SUBST then solves for x by forward and backward substitution.

CALLING SEQUENCES

CALL DECOMP (A, N, M, RATIO)

ALTERNATE ENTRY POINT

CALL SUBST (A, B, N, M)

DESCRIPTION OF ARGUMENTS

A: - Real input and output one-dimensional array, dimensioned at least  $(N-1)*M+1$ . Contains, on input to entry DECOMP, the lower half-band of the positive-definite symmetric matrix A, stored columnwise. Each column is of length, M, the bandwidth, so some array locations may not be used, ie, those which protrude below the last row of the matrix.

On output from entry DECOMP, and for input to entry SUBST, A contains L, the lower triangular matrix such that  $L.L^T=A$ .

N: - Integer input argument containing the order of matrix, A, which is also the length of vectors x and b.

M: - Integer input argument containing the width of the half-band, including the diagonal.

RATIO: - Real output argument which acts as a condition word. For a normal return,  $RATIO>0$ . and is



the ratio of the smallest to the largest diagonal element of L. If  $RATIO$  is positive but extremely small - less than, say,  $1.E-7$ , matrix A is probably ill-conditioned. If matrix A is not positive-definite, or if  $M \leq 0$ ,  $RATIO$  is returned  $<0.$ , and an error message is printed.

B: - Real input and output one-dimensional array dimensioned at least N. On input, B should contain the vector b from the matrix equation  $A.x=b$ . On output, B contains vector x, the solution to the system of linear equations.

#### SEQUENCE OF STEPS

ENTRY DECOMP: DECOMP replaces each element of A with the corresponding element of L as it calculates it. First, it tests for a diagonal matrix, and if true, calculates the terms of L, calculates  $RATIO$ , and returns. For a non-diagonal matrix, the diagonal term of any empty (all-zero) row is replaced by 1. Then  $L(1)$ ,  $L(2)$  and the second diagonal term of L are calculated separately and explicitly by Cholesky's method. DECOMP then loops through the rest of the matrix row by row, each time calculating by Cholesky's method, first the left-most non-zero term of the row, then in an inner loop the next term in the row through the term immediately to the left of the diagonal, and finally, the diagonal term in the row. The minimum and maximum values for diagonals of L are carried throughout the loop. When the last row is complete,  $RATIO$  is equated to minimum over maximum diagonal term, and control is returned to the calling program.

ENTRY SUBST: Having decomposed A to  $L.L^T$ , we get  $L.L^T.x=b$ . Setting  $y \equiv L^T.x$ , we get  $L.y=b$ . Since L is lower triangular, y can be solved by forward substitution, then knowing y, we can solve  $L^T.x = y$  for x by backward substitution. Entry SUBST accomplishes this, first working downward replacing the elements of B one by one by y, then from the bottom up in turn replacing these elements by x, the solution for the system  $A.x = b$ . Control is then returned to the calling program.

#### ERROR CONDITIONS

A value of  $RATIO \leq 0.$  on return always indicates an error condition, and will always cause a printout of one of the following error messages:

"DECOMP - IMPROPER BANDWIDTH = [bandwidth]"

"\*\*SYSTEM IS NOT POSITIVE-DEFINITE\*\* ERROR OCCURRED IN ROW [row number]"

#### COMMENTS

For an explanation of Cholesky's method, see "Solving Large Systems of Linear Equations with Applications in Engineering", June 1970, by A.G. Fowler - pages 1-3.

#### Subroutine: FINDCN

#### PURPOSE

FINDCN is used to determine whether two nodes defining a line segment are attached to the same finite element.

#### CALLING SEQUENCE

CALL FINDCN (I, J, ICØN, LNESEG, NLS, \*)

#### DESCRIPTION OF ARGUMENTS

Argument: Type - Description

I: Integer - Input node number at start of line segment.

J: Integer - Input node number at end of line segment.

ICØN: Integer - Output of connection in array LNESEG if common element found.

LNESEG (2,1): Integer array - Double dimensioned array containing the indices of the nodal points defining each line segment present within the set of elements along with the indexes to elements to which they are common.

NLS: Integer - Maximum number of line segments in finite element mesh.

\*: - Error return entry if no element connection is found.

#### INTERNAL CALLS

See Subroutine list.

Subroutine: GRADNTPURPOSE

To estimate the fluid pressure gradient at any node.

CALLING SEQUENCE

CALL GRADNT (NØDE, X, Y, NP, NUMEL, PRESS, GRADX, GRADY).

DESCRIPTION OF ARGUMENTS

NØDE: Integer - No. of node at which pressure gradient is to be estimated.  
 X(1): Real array - Array of nodal x-coordinates.  
 Y(1): Real array - Array of nodal y-coordinates.  
 NP(5,1): Integer array - Array of element node and material type numbers.  
 NUMEL: Integer - Total no. of finite elements in problem.  
 PRESS: Real array - Array of nodal fluid pressures.  
 GRADX: Real - Estimated fluid pressure gradient of selected node in X-direction.  
 GRADY: Real - Estimated fluid pressure gradient of selected node in Y-direction.

SEQUENCE OF STEPS

1. Subroutine GRADNT searches through all elements to find those connected to NØDE.
2. For elements connected to NØDE, the subroutine calculates their contribution to the least square terms for the gradient estimation process.
3. The fluid pressure gradient components on the coordinate directions are estimated and returned to the calling program.

Subroutine: MØDIFYPURPOSE

To modify the global flow matrix equations remating to nodes with prescribed pressures or flows.

CALLING SEQUENCE

CALL MØDIFY (MM, NBC, FX, R, C, MPFS,

ICØDFS, NEWNUM, Z, ZERØP).

DESCRIPTION OF ARGUMENTS

MM: Integer - Maximum half-bandwidth of global flow matrix.  
 NBC(1): Integer array - Nodal boundary condition indicators.  
 FX(1): Real array - Nodal boundary condition values.  
 R(1): Real array - R.H.S. solution vector.  
 C(MM,1): Real array - Global flow matrix.  
 MPFS(1): Integer array - Phreatic surface node numbers.  
 ICØDFS(1): Integer array - Indicator flags for phreatic surface nodes.  
 NEWNUM(1): Integer array - New nodal position numbers, after bandwidth reduction).  
 Z(1): Real array - Reference elevations for linked node potential value calculations.  
 ZERØP(1): Bit array - Flags for no-tension conditions.

COMMON BLOCK ALLOCATION

When compiled, this routine automatically includes the labelled CØMMØN blocks /CØNTRL/, /SLINES/ and /ELMT/ as outlined in COMMON blocks.

INTERNAL CALLS

1. If the debug flag (IDEBUG(7)) is set greater than zero, print out
  - the renumbered nodes (NEWWUM)
  - the original global flow matrix (C)
  - the original R.H.S. (R)
2. For each node:
  - a. check if it is to be held at zero pressure, if so, go to the next element
  - b. check boundary condition indicator - if it is a continuity node, add the nodal flow to the forcing rector and go to the next node. If it is a node with fixed pressure, go the next node. For a linked potential node, calculate the new potential value and place it in the R.H.S.
3. For each node, again:
  - a. check for known pressure condition - if not, go to the next element

- b. add the term summed from the operation

$$R(n) = P(n) \cdot \sum_i S(i,n)$$

(where  $i$  = no. of equations) to the R.H.S.

- c. clear out the row and column associated with node  $n$  and set the diagonal term to one.

4. If the debug flag (IDEBUG(7)) is set greater than zero, print out
  - the modified global flow matrix (C)
  - the modified R.H.S. (R)
5. Return to the calling routine.

Subroutine: TRIFL

PURPOSE

To compute flow matrices for triangular finite elements.

CALLING SEQUENCE

CALL TRIFL (I, J., K, AREA, XC, YC)

DESCRIPTION OF ARGUMENTS

I,J,K: Integers - Node numbers for corners of triangular element, entered counter-clockwise.

AREA: Real - Area of finite element

XC: Real - x-coordinate of centroid of finite element.

YC: Real - y-coordinate of centroid of finite element.

COMMON BLOCK ALLOCATION

When compiled, this routine automatically includes by labelled COMMON blocks /CØNTRL/, /SLINES/ and /ELMT/ as outlined in COMMON blocks.

INTERNAL CALLS

See Subroutine list.

SEQUENCE OF STEPS

1. Calculate geometry and check area of finite element.
2. If material type unknown, determine region in which the finite element lies (CALL SZØNE).
3. Compute rotated permeability and flow matrices.
4. Return to calling routine.

Subroutine: ELFLØW

PURPOSE

To compute flow velocities in finite elements and stream functions at the nodes.

CALLING SEQUENCE

CALL ELFLØW (X,Y,NP,R,PSI,C,IFPIPE,NRAYS,NØDFLG,NBC,FX,NEWNUM)

DESCRIPTION OF ARGUMENTS

X(1): Real array - Nodal x-coordinates.

Y(1): Real array - Nodal y-coordinates.

NP(5,1): Integer array - Element node and material type array.

R(1): Real array - Nodal solution vector.

PSI(1): Real array - Nodal steam function array.

C(MAXBAN,1): Real array - Double-dimensioned scratch array used to contain terms in least squares matrix for calculation of stream function.

IFPIPE(1): Integer array - Pipe element array.

NRAYS(3,1): Integer array - Double-dimensioned array which provides linkage indexes for each node point to all other directly connected nodes.

NØDFLG(1): Integer array - Array containing condition flags for each node to indicate if the node is exterior (=1) or interior (=0) for a finite element mesh.

NBC(1): Integer array - Nodal boundary condition codes.

FX(1): Real array - Nodal function values.

NEWNUM\*1): Integer array - New to the program, internal node numbers after bandwidth minimization.

COMMON BLOCK ALLOCATION

When compiled, this routine automatically includes the labelled COMMON blocks /CØNTRL/, /SLINES/ and /ELMT/ as outlined in COMMON blocks.

INTERNAL CALLS

See Subroutine list.

SEQUENCE OF STEPS

1. Initialize PSI and C arrays to zero.
2. For each element in the mesh:

- a. read in element information block (CALL BLKIN)
  - b. if a quadrilateral element, back calculate fluid pressure at the central node
  - c. reset flow matrix
  - d. calculate flow across each side and add value to PSI array, then add terms to the least squares matrix
  - e. calculate average flow velocity at centroid of element and print results
3. Search for a node on an impermeable boundary - if found, set nodal stream function value to datum of 0.0 and modify the least squares array (CALL FIX). Then search for neighbouring boundary nodes and set their stream function values to the datum of 0.0 as well (CALL FIX again).
  4. Solve the least-squares matrix using the Choleski linear equation solver by:
    - a. decomposing the matrix (CALL DECØMP) and
    - b. backsubstituting to calculate the nodal stream function value vector (CALL SUBST)
  5. Return to the calling routine.

NOTE: Subroutine ELFLOW has an internal subroutine FIX which is used to modify the least squares matrix, C, so that a real datum for the stream function can be set and thus the equations solved.

Subroutine: SZØNE

PURPOSE

SZØNE determines the material region or zone in which a specified point lies, generally the centroid of a finite element.

CALLING SEQUENCE

CALL SZØNE (X, Y, LABØVE, LBELØW, FRACTN,\*)

DESCRIPTION OF ARGUMENTS

X: Real - x-coordinate of point.

Y: Real - y-coordinate of point.

LABØVE: Integer - No. of closest material zone definition line directly above the point.

LBELØW: Integer - No. of closest material zone definition line directly below the point.

FRACTN: Real - Fraction of vertical distance between LBELØW and LABØVE that exists between LBELØW and Y if point not within a looped zone.

\*: - Error return location.

COMMON BLOCK ALLOCATION

Labelled CØMMØN blocks /CØNTRL/, /SLINES/ and /ELMT/ are automatically included on compilation. (See COMMON blocks).

INTERNAL CALLS

See Subroutine list.

SEQUENCE OF STEPS

1. Cycle through all material zone definition lines to see if a segment is intersected by a vertical line drawn through the point.
2. If a segment is intersected, save the line number and compare crossing y-coordinate with previous value. If latest y-coordinate is closer to the point, store line number and y-coordinate value.
3. Continue cycling through material zone definition lines.
4. At the end of cycling operation, compare line number above and line number below. If they are the same then the point is within a material loop, FRACTN is set equal to  $1.0 \times 10^{20}$ , and control is returned to the calling program. If the line numbers are both greater than zero but different, compute the fractional vertical distance as:

$$FRACTN = \frac{Y - YBELØW}{YABØVE - YBELØW}$$

and return to the calling program. If the line number above is zero, then the point has left the continuum and the error return location is taken.

Subroutine: CROSS

PURPOSE

CROSS is used by the material region interrogation procedure (S/R SZØNE) to determine if a vertical line drawn vertically through a point,

which is usually the centroid of a finite element, cuts the line segment joining two other points.

#### CALLING SEQUENCE

CALL CRØSS (IFABØV, YCRØSS, XP, YP, X, Y)

#### DESCRIPTION OF ARGUMENTS

IFABØV: Integer - Output crossing indicator

= 0 if vertical line does not cross

= +1 if vertical line crosses above YP

= -1 if vertical line crosses below YP

YCRØSS: Real - y-coordinate of vertical line intersection

XP: Real - x-coordinate of point

YP: Real - y-coordinate of point

X(2): Real array - x-coordinates of line segment points

Y(2): Real array - y-coordinates of line segment points

#### SEQUENCE OF STEPS

1. XP is checked to ensure that it falls within the range  $XL < XP < XR$  where XL is the end of the line segment with the lower, ie, left, x-coordinate, and XR is the other end. If not, return to the calling program.
2. Calculate YCRØSS and determine if it is greater than or less than YP. Return to the calling program.

*Subroutine: EXTINT*

#### PURPOSE

To find and flag all nodal points which lie on the boundaries of a finite element mesh. These nodal points may define either the external perimeter of the mesh or any interior perimeters or holes in the mesh. In addition, subroutine EXTINT builds an array of line segments that define edges of finite elements within the mesh and also constructs an array of rays which defines all possible line segments from a given nodal point.

#### CALLING SEQUENCES

CALL EXTINT (NEL, NELM, LNESEG, MAXSEG, NLS, NRRAYS, NØDFLG, MAXNØD, NUMNØD, IERR).

#### ALTERNATE ENTRY POINT

CALL EXTIAN (NEL, NELM, LNESEG, MAXSEG, NLS, NRRAYS, NØDFLG, MAXNØD, NUMNØD, NELX, IERR).

#### DESCRIPTION OF ARGUMENTS

NEL(5,1): Integer array - Elemental nodal points and material type code.

NELM: Integer - Number of finite elements in mesh.

LNESEG(2,1): Integer array - Array returned containing indices of the nodal points defining each line segment present within the set of elements along with the indexes to the elements to which they are common.

MAXSEG: Integer - Maximum number of line segments allowed.

NLS: Integer - Number of unique line segments found by this subroutine.

NRRAYS(3,1): Integer array - Array returned containing linkages from each nodal point to all other nodes to which it is connected.

NØDFLG(1): Integer array (bit usage) - Condition bit flags for each node indicating if the node is on a boundary (=1) or not (=0).

MAXNØD: Integer - Maximum number of nodal points allowed.

NUMNØD: Integer - Number of nodal points found by this subroutine.

NELX(2,1): Integer array - Special array containing the neighbouring elements for each element in the mesh.

IERR: Integer - Returned error flag.

= 0, no errors

= 1, errors encountered.

#### SEQUENCE OF STEPS

1. Initialize NRRAYS and NØDFLG arrays.

2. For each element in the mesh:

a. determine if it is a special finite element eg, bar, joint, etc. and if so, go to the next element

b. for each side in the finite element, store the line segment in NRRAYS and LNESEG if it has not already been found. If the line segment has already been defined, check back to see that it has only been defined once

before.

3. Commence to find the exterior line segments and nodes by performing a simple sort on the data.

Subroutine: WRMESH

PURPOSE

To print nodal input data and nodal analysis results.

CALLING SEQUENCES

CALL WRMESH (X, Y, Z, NBC, FX)

ALTERNATE ENTRY POINT

CALL WRMESH (X, Y, Z, NBC, FX, PRESS, SCR)

DESCRIPTION OF ARGUMENTS

X(1): Real array - Nodal x-coordinates.  
 Y(1): Real array - Nodal y-coordinates.  
 Z(1): Real array - Nodal elevations.  
 NBC: Integer array - Nodal boundary condition indicators.  
 FX: Real array - Nodal boundary condition values.  
 PRESS: Real array - Nodal pressures.  
 SCR: Real array - Net nodal flows.

COMMON BLOCK ALLOCATION

When compiled, this routine automatically includes the labelled COMMON blocks /CØNTRL/, /SLINES/ and /ELMT/ as outlined in COMMON blocks.

Subroutine: BLK

PURPOSE

To block-buffer short records onto a data file. This allows the packing of more than one record into each block, and can substantially reduce I/O time.

CALLING SEQUENCES

CALL BLK (LU, BUFFER, NBUFF, NREC, IFTEMP)

The above call initializes the routine. Input/Output of individual records is then handed by:  
 CALL BLKOUT (IREC, FROM) to write record IREC, and  
 CALL BLKIN (IREC, TO) to read it; which are auxil-

iary entry points in BLK.

DESCRIPTION OF ARGUMENTS

LU: Integer input variable - Fortran logical unit number. If LU > 0 a scratch file called TEMPLU will be created and attached to fortran unit 15, ie, for LU=15, the file name is TEMP15. If BLK has previously been called with the same value of LU, and the file has not been deleted or freed, the previous file will be DELETED AND FREED.

If it is desired to use a file created by BLK in a previous run, this is indicated by setting the value of LU negative. The file must be assigned and attached to unit LU before BLK is called.

BUFFER: Real input or output array - Array provided for use as a buffer for the block being accessed.

NBUFF: Integer input argument - Length of array BUFFER.

NRECS: Integer input argument - Maximum number of records anticipated.

NREC: Integer input argument - Record length, in words.

NREC is also used to FREE or DELETE a file:

NREC=0 to complete and FREE the file.

NREC<0 to DELETE and FREE the file.

IFTEMP: Logical input argument - If IFTEMP is •TRUE• a scratch file will be used; otherwise a permanent file will be used. A permanent file will not be readable by a subsequent run unless BLK is called with NREC=0 at the end of the original run.

IREC: Integer input argument - The record number to be read/written.

FROM: Real input array - Array from which the record to be written is to come.

TO: Real output array - Array to which the record to be read is to be copied.

INTERNAL CALLS

See Subroutine list.

INTERNAL DOCUMENTATION

Internal variables and logic are described by comment cards within the program.

OUTPUT

CALL BLK for NREC>0 creates a new file, destroying any previous existing file assigned to LU. CALL BLKOUT eventually causes a write of the block containing IREC to the file assigned to LU.

LIMITATIONS

No more than 1080 records may be used.

ERROR MESSAGES

1. XXX FATAL ERROR IN S/R BLK - TOO MANY BLOCKS - nnnnnn  
- the program will stop.
2. ERROR IN S/R BLK: IREC=nnnn IERR=nn IN=nnnnmm IM=nnnnmm IFOUT=nn IPACK=nnn  
- an NTRAN operation has failed.  
- the program will stop.
3. WARNING - YOU HAVE ATTEMPTED TO READ BLOCK nnnn WHICH WAS NOT PREVIOUSLY WRITTEN  
- the record in question will be zero-filled.
4. There are several other self-explanatory and fatal messages.

Notes: BLK will only handle one logical unit at a time. If an attempt is made to read a record which has not previously been written, a record of 0.0's will be sent.

Subroutine: GBTTLPURPOSE

This is a heading routine which creates a title block and page headers containing title and page numbers.

CALLING SEQUENCES

CALL GBTTL (NAME, ILEN) - for title block

ALTERNATE ENTRY POINTS

CALL PAGBA (IPAGE) - to print a page header

CALL SPAGA (IPAGE) - CSTS and DIGITECH only - to print a shorter page header

CALL SGBTTL (PNAME) - for a short title block (<=6 characters).

DESCRIPTION OF ARGUMENTS

NAME: Alphanumeric input argument. Array con-

taining the title in A6 format. Up to 72 characters may be printed.

ILEN: Integer input argument. This controls the combination of header record, title, and trailer record to be printed.

The title block consists of three separable parts:

HEADER - This contains the title.

TITLE - A printout of the title starting at NAME.

TRAILER - This contains the day of year and time of day.

ILEN controls what is printed thus:

ILEN	HEADER PRINTED?	TITLE PRINTED? (N CHARACTERS)	TRAILER PRINTED?
n	yes	yes	yes
-1000	yes	no	no
-(1000+N)	yes	yes	no
-N	no	yes	no
0	no	no	yes
1000	no	no	no

A CALL GBTTL for ILEN=1000 prints nothing, but merely sets up the title for future calls to PAGBA & SPAGA.

Note: The number of characters printed is limited or truncated to 72 characters. If N is greater than or equal to 90, the run date and time are returned to the calling program in characters 73 to 90 (words 13-15 of NAME) as:

MO/DA/YR-HH:MM:SS

However, the title printed will still be truncated at 72 characters.

IPAGE: Integer input argument containing the page number to be printed at the top of the new page, or may be input as 0 which causes an automatic increment by 1 of the page number, starting with 1, eg, if for the first call to PAGBA or SPAGA, IPAGE is input as 0, a page number of '2' will be printed.

PNAME: Alphanumeric input argument which is a one-word 'short' title (of 1-6 characters) to be printed by SGBTTL in a title block.

OUTPUT: The title block produced by GBTTL is 79

print positions wide +1 for carriage control. Calls to GBTTL for all in-range values of ILEN except 1000 will produce, printed at the top of a new page, some or all of the title block as indicated in the above table. If ILEN=1000, nothing is printed, but the title for future calls to PAGBA and SPAGA is changed to contain up to 72 characters beginning at NAME. A call to PAGBA will cause a page header to be printed at the top of the next page containing the current title, time and date. This page header is 122 print positions wide (+1 for carriage control). Entry point SPAGA is identical to PAGBA except that the page header is split into two lines, only 79 print positions wide (+1 for carriage control). If the title input starting at NAME contains no more than 36 characters ignoring (trailing blanks), it will be double-spaced horizontally in both title block and page header. The title block produced by SGBTTL is 59 print positions wide (+1 for carriage control) and contains the heading desired, followed by run time and date.

#### INTERNAL CALLS

See Subroutine list.

MOD - FORTRAN V library function  
 MINO - FORTRAN V library function  
 IABS - FORTRAN V library function  
 DOY - CSCX library utility subroutine: return day of year  
 DOY - DIGITECH system library routine: return day of year  
 DOYF - CSTS library utility entry point: return day of year  
 TOD - CSCX library utility subroutine: return time of day in hours, minutes and seconds  
 TOD - DIGITECH system library routine: return time of day in hours, minutes and seconds  
 TODF - CSTS library utility entry point: return time of day in hours, minutes and seconds.

Subroutine: MCORE

#### SOURCE LANGUAGE

UNIVAC 1100 series SLEUTH II Assembler on

Digitech's UNIVAC 1106 with EXEC 8 operating system.

#### PURPOSE

This routine performs dynamic expansion and contraction of a user's DBANK.

#### ENTRY POINTS

MCORE - gets more core  
 LCORE - releases core previously obtained by MCORE

#### CALLING SEQUENCE

CALL MCORE (ADR, N, ISTAT)  
 CALL LCORE (ADR)

#### COMMON BLOCK

Each pre-processor must define a labelled COMMON block which is forced by collector directives to be positioned at the end of the systems DBANK. It is this COMMON block which is expanded and contracted by MCORE/LCORE.

#### DESCRIPTION OF ARGUMENTS

ADR - Input argument of any type giving, for MCORE, the starting location within the labelled COMMON block from which expansion is to begin. For LCORE, ADR is the highest location to be retained after core contraction.  
 N - Integer input argument - number of core words of DBANK to be added to ADR.  
 ISTAT - Integer output argument - acts as a status word. For a successful expansion, ISTAT is returned = 0. If ISTAT = 1, core has not been expanded due to size limitations.

#### LIMITATIONS

MCORE will not allow DBANK to exceed 65K.  
 LCORE can only release core acquired by MCORE. If LCORE is called with ADR lower than the highest collection time DBANK address, DBANK will be reduced to its original size.

Since LCORE\$, the executive request used by LCORE, releases core in blocks of 512 words, DBANK will probably extend past ADR after a call to LCORE by an amount no greater than 511 words.



ERROR CONDITIONS

If a call to MCODE requests expansion of DBANK over 65K, argument ISTAT is returned = 1, and no expansion is performed.

Subroutine: TOD

PURPOSE

This subroutine retrieves the current time of day and day of year.

CALLING SEQUENCES

CALL TOD (ITIME)  
DOY (IDAY) (alternate entry point)

DESCRIPTION OF ARGUMENTS

ITIME - Alphanumeric 2-word output array which contains on output the current time in the format. HH.MM.SS, where HH represents hours (00-24), MM represents minutes (00-59) and SS represents seconds (00-59)

IDAY - Alphanumeric 2-word output array containing on output the date in the format DD,MM,YY where DD represents day of month (0-31), MON represents month (first three letters) YY represents year (last two digits)

INTERNAL CALLS

FLD - FORTRAN V library function  
ERTRAN - FORTRAN V library subroutine  
SYFDBN - Golder Associates library subroutine

ERROR CONDITIONS

The only possible error return is a RETURN 0, which will occur only if the system data word containing the date is in error.

Subroutine: SYFDBN

SOURCE LANGUAGE

FORTRAN V on Digitech's UNIVAC 1106 with EXEC 8 operating system. SYFDBN uses compiler option FLD=R.

PURPOSE

SYFDBN converts a field data character string to an integer. Leading blanks in the

string are considered as zero; all other characters encountered which are not 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9 cause an error return.

ENTRY POINT

SYFDBN

CALLING SEQUENCE

CALL SYFDBN (INP, CHAR, BIN, \$)

DESCRIPTION OF ARGUMENTS

INP - Alphanumeric input word or array containing character string to be converted. The first character is considered to be the first digit.

CHAR - Integer input argument giving the number of characters, beginning with the first character in the first word of INP, to be included in the conversion.

BIN - Integer output argument to contain the integer equivalent of the character string.

\$ - Statement number in calling program to which control is transferred in case of an error condition ie, if an invalid character is encountered.

LIMITATIONS

The maximum value of the number to be converted is  $2^{35}-1$ . Negative numbers are not allowed, as characters other than '0' - '9' cause an error return.

INTERNAL CALLS

FLD - FORTRAN V library function

This subroutine uses DEFINE procedures, a feature of FORTRAN V.

Subroutine: ERTRAN

PURPOSE

Returns the date and time and accepts a dummy value.

CALLING SEQUENCE

CALL ERTRAN(DUMMY,DAT,TM)

DAT returns the data in the format: dd/mm/yy

TM returns the time in the format: hh.mm.ss.

Comments: The original routine is a UNIVAC 1100 series FORTRAN interface with the operating sys-

tem. In FEFPM, only ABORT\$ (operation code 1) and DATE\$ (operation code 9) are used. Thus, the Control Data FORTRAN intrinsic routines DATE, TIME and ABORT are used to convert it. Most other systems have means of returning this information from FORTRAN.

Subroutine: NTRAN

PURPOSE

NTRAN performs binary reading and writing of tape or disk and permits I/O buffering.

CALLING SEQUENCE

CALL NTRAN(LU,OP,LEN,BUF,STAT)

LU is the logical unit to be read or written, the default is 15.

OP is one of the operation codes:

Code	Meaning
1	write
2	read
8	position-sequential file
9	write End-of-file
10 or 11	rewing
6	position random file
22	does nothing

LEN Length of the data to be read or written, in words.

BUF is the name of the variable to be read or written

STAT is a variable that NTRAN uses to return a device status:

- 2 end of tape or disk file
- 3 device error

or STAT will return a positive value which should be the number of words transmitted.

Comments: This, although written in FORTRAN, performs I/O with routines written in Compass, since they are Control Data intrinsic functions (BUFFER IN, BUFFER OUT, LOCF). The original of this routine is a special UNIVAC subroutine that handles the access of a UNIVAC drum with 28 word sectors,

and it permits the queuing of I/O. Operation 22 is intended to cause a wait until the stack is empty. The routine documented here cannot handle I/O queuing or all of the operations of the original NTRAN. The conversion of this subroutine would depend upon the I/O devices of a particular system and would require assembler or operating system routines on most systems.

Subroutine: FLDLNTH

PURPOSE

This routine returns the current memory being used (field length), requests a new field length or requests the maximum field length available.

CALLING SEQUENCE

CALL FLDLNTH(LENGTH)

If LENGTH = 0 the current memory size is returned in the variable.

If LENGTH = -1 the maximum available field length is returned (this is the value available on the job card as the CM specification).

otherwise the value specified as LENGTH will be the number of words that will become the new field length.

Comments: This program is written in Compass, the assembler language of the CNC CYBER series computers. It uses a system macro to allocate the new central memory storage or to determine the amount already in use.

Conversion of this program requires assembler level reprogramming.

Subroutine/Function: SETFLD

PURPOSE

To move a bit string from one word to another without affecting the rest of the output word. The input word (WORD 2) contains the bit string right justified. The specified number of bits (N) are moved to the specified position (starting at bit IPOS) of the output word (WORD 1). The value of the final value of the output word (WORD 1) is also returned in an

arithmetic register (X6) so that this routine can be used as either a Subroutine or a function.

#### CALLING SEQUENCE

CALL SETFLD (IPOS,N,WORD1,WORD2) or

JJ = SETFLD (IPOS,N,WORD1,WORD2)

Function: FLD

Type: Integer

#### PURPOSE

To perform bit string manipulation. Specifically, FLD is used to access the bytes of a computer word. The position and width of the byte and the word to be accessed are the arguments of the function.

#### CALLING SEQUENCE

FLD(IPOS,N,WORD)

IPOS is an integer, between 0 and 35 (the length of an 1108 word, hence the upper limit in these programs) and is the first bit to be accessed in the word.

N is an integer between 1 and 36 (see note above), and are the number of bits to be accessed.

WORD is an integer, real, logical or (on the 1108) typeless and is the word to be accessed.

Bits are counted from left to right in the word. The first bit is zero. The contents of the byte extracted from the word are right-adjusted in the arithmetic register and the remaining bits are set to zero. The result is considered an integer.

Examples of use: NI = FLD(35,1,STRING)

NI then contains one bit, bit 35, of the contents of variable STRING. That one bit is right justified in the variable NI, the rest of which is zero.

Comments: This is a routine written in Compass to replace an intrinsic function of UNIVAC 1100 series FORTRAN V. For conversion of this routine, a knowledge of assembly language of a computer will probably be necessary, or the use of the ANSI standard (1977) function.

#### Subroutine: CAS002

#### PURPOSE

To query the system to determine the amount of core taken by the program (field length). It can be used to change the core allocated to the program, as long as the program stays within the upper limit set on the job card (CM).

#### CALLING SEQUENCE

CALL CAS002(IFL)

If IFL = 0, the current field length is returned in IFL. (IFL will be number of words.)

If IFL is unsigned and greater than zero, the field length is set to the value of IFL

If IFL is a signed integer, the current field length will be increased or decreased by the value of IFL.

Comments: This FORTRAN callable subroutine is written in Compass. CDC's assembler language for the CYBER series computers. It is part of the Department of Energy, Mines and Resources Library (EMRLIB). It performs the same functions as MCORE and LCORE routines in the original 1108 version of these programs, and functions similarly to FLDLNTH.

Programs of this kind usually require an interface with the operating system, so that program conversion for another computer system will require an expert programmer.

#### Subroutine: LCORE

#### PURPOSE

To contract core to the specified address (a variable name)

#### CALLING SEQUENCE

CALL LCORE (ADR)

ADR - this is the last variable to have core.

Comments: This routine uses the CDC intrinsic function LOCF, finds the location of the variable

(ADR), and then calls FLDLNTH to change the size of the core allocated to the program. See FLDLNTH for additional comments.

Subroutine: ETIMEF

PURPOSE

Used to return the CPU time since the last call to subroutine ETIME.

CALLING SEQUENCE

CALL ETIMEF(SEC)

Comments: This subroutine picks up the variable TO in the labelled COMMON block TIME, and calls

the Control Data intrinsic function SECOND.

Subroutine: ETIME

PURPOSE

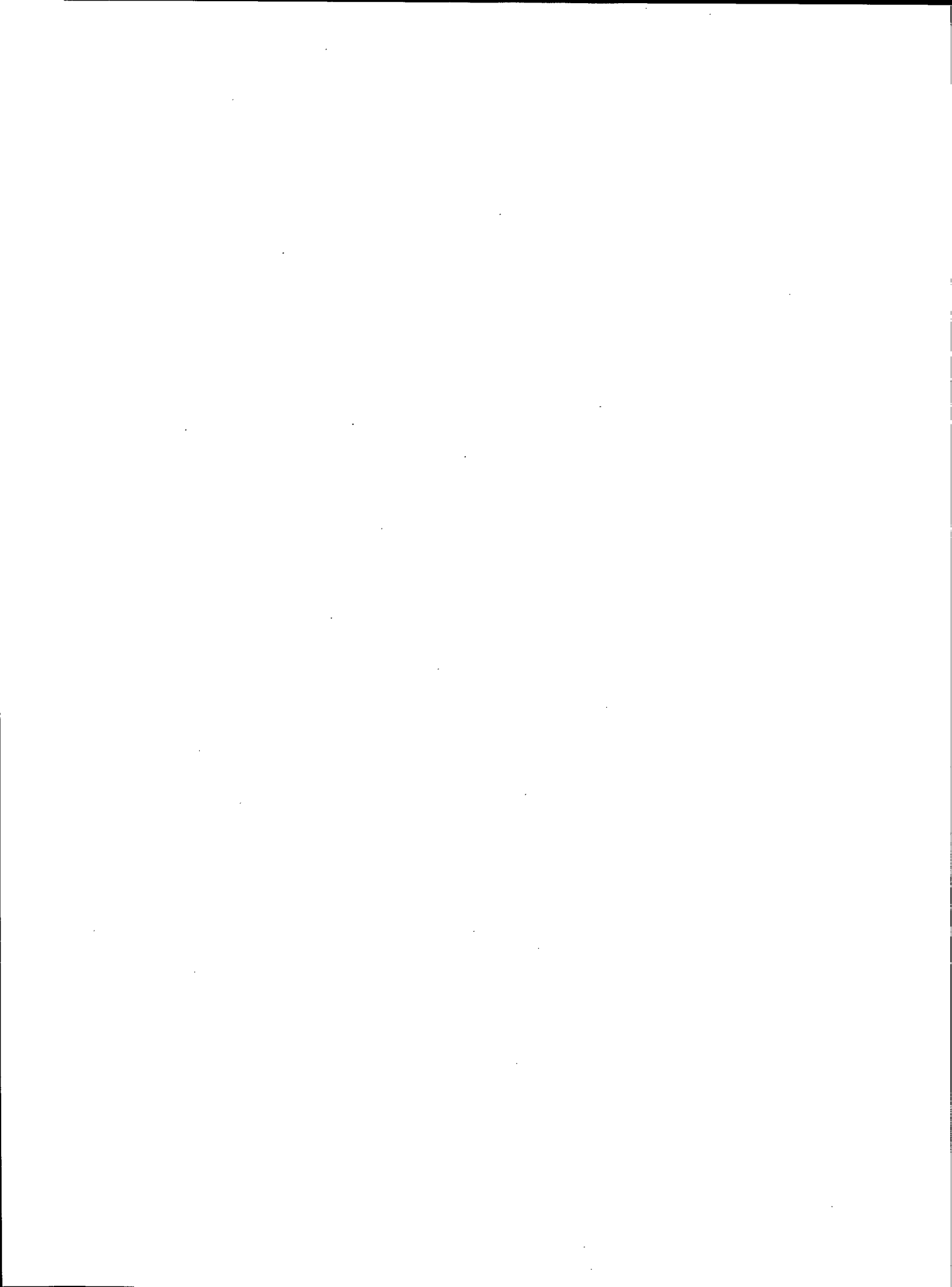
Obtains the number of elapsed CPU seconds (as a real number) since the start of the job.

The time is stored in a variable TO in a labelled COMMON block called TIME.

CALLING SEQUENCE

CALL ETIME

Comments: This subroutine uses the Control Data FORTRAN intrinsic function SECOND.



**APPENDIX A**

**SAMPLE PROBLEMS**

1. Three sample problems illustrating typical use of most features of the FEFPM computer program are presented. Drawings of each problem showing finite element mesh layouts, complete listings of input data and the entire FEFPM computer program output are included for each sample problem.

#### SAMPLE PROBLEM 1 - DUPUIT PARABOLA

2. The first sample problem presented is one of the program validation examples reported earlier in this supplement. The problem geometry and finite element mesh used in the analysis are shown in Fig A-1. The purpose of this problem was to check the capabilities and accuracy of the FEFPM computer program; geometries and permeability values used are not necessarily realistic.

3. The problem involves planar flow through an isotropic material zone with vertical sides situated on an impermeable base. Constant hydrostatic pressure conditions with different surface levels exist on the vertical sides. The object of

the analysis is to determine the steady-state groundwater flow regime. This involves determining location of the steady state phreatic surface.

4. The nodes and elements used to define the finite element mesh are illustrated in Fig A-1. Note that these nodes and elements are sequentially numbered in the vertical direction, thus facilitating data generation. The actual data cards used for this problem are reproduced below with annotations and the data generation procedures for both nodes and elements are obvious. The outlined areas on the input data listing are the bounds of the respective formats for each type of data card.

5. The printout from the FEFPM program for this sample problem follows the input data card listing. The headings for each output section describe the printout but some additional annotations have been made to facilitate understanding. The final phreatic surface location determined in the analysis is shown in Fig A-1.

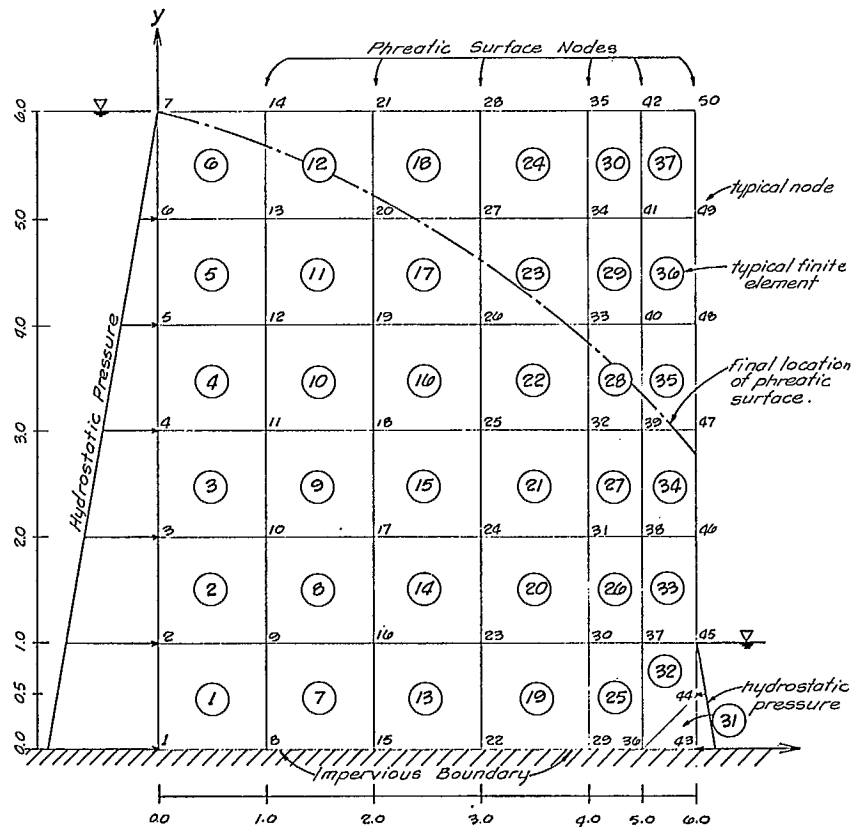


Fig A-1 - Sample problem 1 - geometry and mesh.

SAMPLE PROBLEM 1 - INPUT DATA

START								SAMPL 1, DUPUIT PARABOLA								Title Card	
SFPM								TOL#0,01,MAXIT#10,NTYPE #1,NUMMAT#1,NUMEL#37,NUMNP#50,NUMFSC#6,								} Namelist & FPM Cards	
BETA#0,8,								SENO									
1	2	P															
7		P												6	1.0		
8		Q													1.		
14		Q													6		
15		Q													1.0		
21		Q													6		
22		Q													1.0		
28		Q													6		
29		Q													1.0		
35		Q													6		
36		Q													1.0		
42		Q													6		
43		P													1.0		
44		P													0.5		
45	2	Q													1		
50		Q													6		
STOP																} Material Card	
1	1	1	8	9	2	1											
7	8	15	16	9	1												
13	15	22	23	16	1												
19	22	29	30	23	1												
25	29	36	37	30	1												
31	36	43	44	1													
32	36	44	45	37	1												
37	41	49	50	42	1												
STOP																} Element Cards	

Column 1

FEFPM - SAMPLE PROBLEM NO. 1 - OUTPUT - 1

```

*****
*
*          S E E P A G E   A N A L Y S I S
*
*    S A M P L E   1 ,   D U P U I T   P A R A B O L A
*
*    01/30/76-14157:23
*
*****

```

```

NUMBER OF NODAL POINTS----- 50
NUMBER OF ELEMENTS----- 37
NUMBER OF DIFF. MATERIALS--- 1
UNIT WEIGHT OF FLUID----- 1.0000+00
REFERENCE FOR POTENTIALS---- 0.0000
CORRECTION FACTOR----- .R0000
MAX. NO. ITERATIONS----- 10
ERROR TOLERANCE----- .01000

```

} Input Control Parameters

PLANE FLOW PROBLEM

```

ASG10 DYNAMIC E.R. - *ASG,1    TEMP15,017/ 3410/TRK/ 3410 .
ASG11 DYNAMIC E.R. - *MISE    15,TEMP15 .

```

← Assignment of temporary file (logical unit 15)



FEPFM - SAMPLE PROBLEM NO. 1 - OUTPUT - 2

\* SAMPLE 1, DUPUIT PARABOLA \*\* 01/30/76-14:57123 \*\* PAGE 2

*Initial Node Geometry  
and Boundary Conditions*

NODE	H.C.	X=ORD	Y=ORD	F	
1	PRESS.	.0000	.0000	6.0000+00	0.0000 *
2	PRESS.	.0000	1.0000	5.0000+00	0.0000 *
3	PRESS.	.0000	2.0000	4.0000+00	0.0000 *
4	PRESS.	.0000	3.0000	3.0000+00	0.0000 *
5	PRESS.	.0000	4.0000	2.0000+00	0.0000 *
6	PRESS.	.0000	5.0000	1.0000+00	0.0000 *
7	PRESS.	.0000	6.0000	0.0000	0.0000 *
8	CUNT	1.0000	.0000	0.0000	0.0000 *
9	CUNT	1.0000	1.0000	0.0000	0.0000 *
10	CUNT	1.0000	2.0000	0.0000	0.0000 *
11	CUNT	1.0000	3.0000	0.0000	0.0000 *
12	CUNT	1.0000	4.0000	0.0000	0.0000 *
13	CUNT	1.0000	5.0000	0.0000	0.0000 *
14	CUNT	1.0000	6.0000	0.0000	0.0000 *
15	CUNT	2.0000	.0000	0.0000	0.0000 *
16	CUNT	2.0000	1.0000	0.0000	0.0000 *
17	CUNT	2.0000	2.0000	0.0000	0.0000 *
18	CUNT	2.0000	3.0000	0.0000	0.0000 *
19	CUNT	2.0000	4.0000	0.0000	0.0000 *
20	CUNT	2.0000	5.0000	0.0000	0.0000 *
21	CUNT	2.0000	6.0000	0.0000	0.0000 *
22	CUNT	3.0000	.0000	0.0000	0.0000 *
23	CUNT	3.0000	1.0000	0.0000	0.0000 *
24	CUNT	3.0000	2.0000	0.0000	0.0000 *
25	CUNT	3.0000	3.0000	0.0000	0.0000 *
26	CUNT	3.0000	4.0000	0.0000	0.0000 *
27	CUNT	3.0000	5.0000	0.0000	0.0000 *
28	CUNT	3.0000	6.0000	0.0000	0.0000 *
29	CUNT	4.0000	.0000	0.0000	0.0000 *
30	CUNT	4.0000	1.0000	0.0000	0.0000 *
31	CUNT	4.0000	2.0000	0.0000	0.0000 *
32	CUNT	4.0000	3.0000	0.0000	0.0000 *
33	CUNT	4.0000	4.0000	0.0000	0.0000 *
34	CUNT	4.0000	5.0000	0.0000	0.0000 *
35	CUNT	4.0000	6.0000	0.0000	0.0000 *
36	CUNT	4.5000	.0000	0.0000	0.0000 *
37	CUNT	4.5000	1.0000	0.0000	0.0000 *
38	CUNT	4.5000	2.0000	0.0000	0.0000 *
39	CUNT	4.5000	3.0000	0.0000	0.0000 *
40	CUNT	4.5000	4.0000	0.0000	0.0000 *
41	CUNT	4.5000	5.0000	0.0000	0.0000 *
42	CUNT	4.5000	6.0000	0.0000	0.0000 *
43	PRESS.	5.0000	.0000	1.0000+00	0.0000 *
44	PRESS.	5.0000	.5000	5.0000-01	0.0000 *
45	PRESS.	5.0000	1.0000	0.0000	0.0000 *
46	PRESS.	5.0000	2.0000	0.0000	0.0000 *
47	PRESS.	5.0000	3.0000	0.0000	0.0000 *
48	PRESS.	5.0000	4.0000	0.0000	0.0000 *
49	PRESS.	5.0000	5.0000	0.0000	0.0000 *
50	CUNT	5.0000	6.0000	0.0000	0.0000 *

FEPFM - SAMPLE PROBLEM NO. 1 - OUTPUT - 3

\* SAMPLE 1, DUPUIT PARABOLA \*\* 01/30/76-14:57123 \*\* PAGE 3

LINE	PROPERTIES	MATERIAL	PROPERTIES	BELOW	DELIMITING NODE STRING
1	1.00-01	1.00-01	.0	1.00-01	1.00-01
					1.000+10 1.000+20 1.000+10 1.000+20
					1.000+10 -1.000+10 -1.000+10 -1.000+10

FEFPM - SAMPLE PROBLEM NO. 1 - OUTPUT - 4

\* SAMPLE 1. DUPUIT PARABOLA \*\* 01/30/76-14:57:23 \*\* PAGE 4

ELMT	1	J	K	L	MAT
1	1	8	9	2	1
2	2	9	10	3	1
3	3	10	11	4	1
4	4	11	12	5	1
5	5	12	13	6	1
6	6	13	14	7	1
7	8	15	16	9	1
8	9	16	17	10	1
9	10	17	18	11	1
10	11	18	19	12	1
11	12	19	20	13	1
12	13	20	21	14	1
13	15	22	23	16	1
14	16	23	24	17	1
15	17	24	25	18	1
16	18	25	26	19	1
17	19	26	27	20	1
18	20	27	28	21	1
19	22	29	30	23	1
20	23	30	31	24	1
21	24	31	32	25	1
22	25	32	33	26	1
23	26	33	34	27	1
24	27	34	35	28	1
25	29	36	37	30	1
26	30	37	38	31	1
27	31	38	39	32	1
28	32	39	40	33	1
29	33	40	41	34	1
30	34	41	42	35	1
31	36	43	44	44	1
32	36	44	45	37	1
33	37	45	46	38	1
34	38	46	47	39	1
35	39	47	48	40	1
36	40	48	49	41	1
37	41	49	50	42	1

*Finite Element List*

\* \* \* \* \* FREE SURFACE \* \* \* \* \*

NODE	CORR. ANGLE	Y(MAX)
14	90.0	*****
21	90.0	*****
28	90.0	*****
35	90.0	*****
42	90.0	*****
50	90.0	*****

*Phreatic surface node details.*

CPU TIME USED IN S/R MESHIN = 1.358 SECONDS

FEFPM - SAMPLE PROBLEM NO. 1 - OUTPUT - 5

TOTAL SPACE ACQUIRED FOR /MATRIX/ IS 1541

MAXIMUM HANDWIDTH = 10  
 ASGIO DYNAMIC L.R. = #ASG,1      TELP14,F17/      1/TRK/      1.  
 ASGIO DYNAMIC F.R. = #USE      14,TEMP14.

← *Assignment of temporary file (Logical unit 14)*

## FEPPM - SAMPLE PROBLEM NO. 1 - OUTPUT - 6

\* SAMPLE 1, DUPUIT PARABOLA \*\* 01/30/76-14157123 \*\* PAGE 5  
 TOTAL MESH AREA= 3.000000+01 CPU TIME TO BUILD FLOW MATRIX = 3.300 SECONDS.

\*\*\*\*\* ITERATION 1

*Details of phreatic surface node  
 movements during iterative solution.*

TIME TO SOLVE MATRIX = .109 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 8.85-01  
 PHREATIC SURFACE END-NODE 14 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -.2985 DX,DY= 1.4901-08 -3.6493-01  
 PHREATIC SURFACE END-NODE 50 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .0536 DX,DY= 0.0000 -9.4278-01

NODE	X=ORD	Y=ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
14	1.0000	0.0000	-.43816+00	.55618+01	.0000	-.3649	.0000	.0000
21	2.0000	0.0000	-.82929+00	.51707+01	-.0000	-.6634	.0000	-1.0000
28	3.0000	0.0000	-.11144+01	.48856+01	-.0000	-.8915	.0000	-1.0000
35	4.0000	0.0000	-.12235+01	.47765+01	-.0000	-.9788	.0000	-1.0000
42	4.5000	0.0000	-.12120+01	.47880+01	-.0000	-.9696	.0000	-1.0000
50	5.0000	0.0000	-.12482+01	.47518+01	.0000	-.9428	.0000	.0000

TOTAL MESH AREA= 2.6625503+01 CPU TIME TO BUILD FLOW MATRIX = 3.140 SECONDS.

\*\*\*\*\* ITERATION 2

TIME TO SOLVE MATRIX = .127 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 5.24-01  
 PHREATIC SURFACE END-NODE 14 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -.5507 DX,DY= 7.4506-09 5.4345-02  
 PHREATIC SURFACE END-NODE 50 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .0047 DX,DY= 0.0000 -7.0533-01

NODE	X=ORD	Y=ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
14	1.0000	5.6351	-.18158-01	.56169+01	.0000	.0543	.0000	.0000
21	2.0000	5.3366	-.24730+00	.50893+01	-.0000	-.1978	.0000	-1.0000
28	3.0000	5.1085	-.54217+00	.45663+01	-.0000	-.4337	.0000	-1.0000
35	4.0000	5.0212	-.79633+00	.42244+01	-.0000	-.6371	.0000	-1.0000
42	4.5000	5.0504	-.85106+00	.41793+01	-.0000	-.6808	.0000	-1.0000
50	5.0000	5.0572	-.41312+00	.41041+01	.0000	-.7053	.0000	.0000

TOTAL MESH AREA= 2.5053712+01 CPU TIME TO BUILD FLOW MATRIX = 3.170 SECONDS.

\*\*\*\*\* ITERATION 3

TIME TO SOLVE MATRIX = .100 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 3.17-01  
 PHREATIC SURFACE END-NODE 14 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -.6729 DX,DY= 7.4506-09 1.2761+01  
 PHREATIC SURFACE END-NODE 50 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -.1688 DX,DY= 0.0000 -5.6637-01

## FEFPM - SAMPLE PROBLEM NO. 1 - OUTPUT - 7

NODE	X=ORD	Y=ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
14	1.0000	5.6894	-.15253+01	.56742+01	.0000	.1276	.0000	.0000
21	2.0000	5.1387	-.67882+02	.51455+01	.0000	.0054	.0000	-1.0000
28	3.0000	4.6747	-.19175+00	.44830+01	-.0000	-.1534	.0000	-1.0000
35	4.0000	4.3841	-.48192+00	.39022+01	-.0000	-.3855	.0000	-1.0000
42	4.5000	4.3496	-.59952+00	.37500+01	-.0000	-.4796	.0000	-1.0000
50	5.0000	4.3519	-.65831+00	.36936+01	.0000	-.5664	.0000	.0000

TOTAL MESH AREA= 2.4322404+01 CPU TIME TO BUILD FLOW MATRIX = 2.850 SECONDS.

\*\*\*\*\* ITERATION 4

TIME TO SOLVE MATRIX = .101 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 1.91-01  
 PHREATIC SURFACE END-NODE 14 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -.6670 DX,DY= 7.4506-09 5.1275-02  
 PHREATIC SURFACE END-NODE 50 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -.4390 DX,DY= 0.0000 -4.5727-01

NODE	X=ORD	Y=ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
14	1.0000	5.8170	-.11162+00	.57054+01	.0000	.0513	.0000	.0000
21	2.0000	5.1442	-.71430+01	.52156+01	.0000	.0571	.0000	-1.0000
28	3.0000	4.5213	-.42333+02	.45171+01	-.0000	-.0034	.0000	-1.0000
35	4.0000	3.9966	-.24590+00	.37527+01	-.0000	-.1967	.0000	-1.0000
42	4.5000	3.8699	-.40274+00	.34672+01	-.0000	-.3222	.0000	-1.0000
50	5.0000	3.7855	-.43944+00	.33461+01	.0000	-.4573	.0000	.0000

TOTAL MESH AREA= 2.4000886+01 CPU TIME TO BUILD FLOW MATRIX = 3.050 SECONDS.

\*\*\*\*\* ITERATION 5

TIME TO SOLVE MATRIX = .105 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 1.02-01  
 PHREATIC SURFACE END-NODE 14 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -.5912 DX,DY= 7.4506-09 -3.2808-02  
 PHREATIC SURFACE END-NODE 50 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -.7233 DX,DY= 0.0000 -3.2412-01

NODE	X=ORD	Y=ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
14	1.0000	5.8683	-.15765+00	.57106+01	.0000	-.0328	.0000	.0000
21	2.0000	5.2013	-.53748+01	.52551+01	.0000	.0430	.0000	-1.0000
28	3.0000	4.5179	-.68678+01	.45866+01	.0000	.0549	.0000	-1.0000
35	4.0000	3.8019	-.76049+01	.37258+01	-.0000	-.0608	.0000	-1.0000
42	4.5000	3.5477	-.22743+00	.33203+01	-.0000	-.1819	.0000	-1.0000
50	5.0000	3.3283	-.24832+00	.30799+01	.0000	-.3241	.0000	.0000

TOTAL MESH AREA= 2.3892340+01 CPU TIME TO BUILD FLOW MATRIX = 2.750 SECONDS.

\*\*\*\*\* ITERATION 6

## FEPPM - SAMPLE PROBLEM NO. 1 - OUTPUT - 8

TIME TO SOLVE MATRIX = .102 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 4.55-02  
 PHREATIC SURFACE END-NODE 14 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -.5098 DX,DY= 7.4506-09 -6.7055-02  
 PHREATIC SURFACE END-NODE 50 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -.9446 DX,DY= 0.0000 -1.7733-01

NODE	X-ORD	Y-ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
14	1.0000	5.8355	-.13336+00	.57021+01	.0000	-.0671	.0000	.0000
21	2.0000	5.2443	.17957-01	.52623+01	.0000	.0144	.0000	-1.0000
28	3.0000	4.5729	.70841-01	.46437+01	.0000	.0567	.0000	-1.0000
35	4.0000	3.7410	.25941-01	.37670+01	.0000	.0208	.0000	-1.0000
42	4.5000	3.3658	-.83382-01	.32824+01	.0000	-.0667	.0000	-1.0000
50	5.0000	3.0041	-.11010+00	.28940+01	.0000	-.1773	.0000	.0000

TOTAL MESH AREA= 2.3834255+01 CPU TIME TO BUILD FLOW MATRIX = 2.860 SECONDS.

\*\*\*\*\* ITERATION 7

TIME TO SOLVE MATRIX = .094 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 2.86-02  
 PHREATIC SURFACE END-NODE 14 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -.4599 DX,DY= 7.4506-09 -5.7462-02  
 PHREATIC SURFACE END-NODE 50 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -1.0713 DX,DY= 0.0000 -6.1587-02

NODE	X-ORD	Y-ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
14	1.0000	5.7644	-.78301-01	.56901+01	.0000	-.0575	.0000	.0000
21	2.0000	5.2587	-.94781-02	.52492+01	-.0000	-.0076	.0000	-1.0000
28	3.0000	4.6296	.38844-01	.46684+01	.0000	.0311	.0000	-1.0000
35	4.0000	3.7618	.59110-01	.38209+01	.0000	.0473	.0000	-1.0000
42	4.5000	3.2941	.22316-02	.33013+01	.0000	.0018	.0000	-1.0000
50	5.0000	2.8268	-.36515-01	.27905+01	.0000	-.0616	.0000	.0000

TOTAL MESH AREA= 2.3821244+01 CPU TIME TO BUILD FLOW MATRIX = 2.500 SECONDS.

\*\*\*\*\* ITERATION 8

TIME TO SOLVE MATRIX = .091 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 2.36-02  
 PHREATIC SURFACE END-NODE 14 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -.4462 DX,DY= 7.4506-09 -2.9894-02  
 PHREATIC SURFACE END-NODE 50 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -1.1177 DX,DY= 0.0000 6.8914-04

NODE	X-ORD	Y-ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
14	1.0000	5.7110	-.29497-01	.56811+01	.0000	-.0299	.0000	.0000
21	2.0000	5.2511	-.20284-01	.52308+01	-.0000	-.0162	.0000	-1.0000
28	3.0000	4.6606	.49334-02	.46656+01	.0000	.0039	.0000	-1.0000
35	4.0000	3.8091	.46017-01	.38555+01	.0000	.4371	.0000	-1.0000
42	4.5000	3.3009	.29856-01	.33407+01	.0000	.0239	.0000	-1.0000
50	5.0000	2.7652	-.94854-02	.27557+01	.0000	.0007	.0000	.0000

TOTAL MESH AREA= 2.3819036+01 CPU TIME TO BUILD FLOW MATRIX = 2.690 SECONDS.

## FEPPM - SAMPLE PROBLEM NO. 1 - OUTPUT - 9

\*\*\*\*\* ITERATION 9

TIME TO SOLVE MATRIX = .099 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 1.55-02  
 PHREATIC SURFACE END-NODE 14 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -.4551 DX,DY= 7.4506-09 -4.6220-03  
 PHREATIC SURFACE END-NODE 50 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= -1.1180 DX,DY= 0.0000 1.9601-02

NODE	X-ORD	Y-ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
14	1.0000	5.6811	-.36742-02	.56774+01	.0000	-.0046	.0000	.0000
21	2.0000	5.2349	-.16905-01	.52180+01	-.0000	-.0135	.0000	-1.0000
28	3.0000	4.6606	-.13087-01	.46515+01	-.0000	-.0105	.0000	-1.0000
35	4.0000	3.8462	.20623-01	.38868+01	.0000	.0165	.0000	-1.0000
42	4.5000	3.3248	.24693-01	.33495+01	.0000	.0198	.0000	-1.0000
50	5.0000	2.7659	-.57666-02	.27601+01	.0000	.0196	.0000	.0000

TOTAL MESH AREA= 2.3817574+01 CPU TIME TO BUILD FLOW MATRIX = 2.500 SECONDS.

\*\*\*\*\* ITERATION 10

TIME TO SOLVE MATRIX = .133 SECONDS.

MAX. NUMBER OF ITERATIONS REACHED

← Note that the solution has not fully converged. However the phreatic surface node movements (DY above) are now quite small and thus the results of iteration 10 are presumed to be accurate enough for program validation purposes.



FEFPM - SAMPLE PROBLEM NO. 1 - OUTPUT - 12

CONTINUITY NODES: SPECIFIED FLOW 0.000 0.000  
 OTHER FLOW 6.584-07 1.956-08

ASG10 DYNAMIC E.H. = #FREE TEMP14. :  
 ASG10 DYNAMIC E.H. = #FREE TEMP15. :

← Release of temporary files.

#FIN

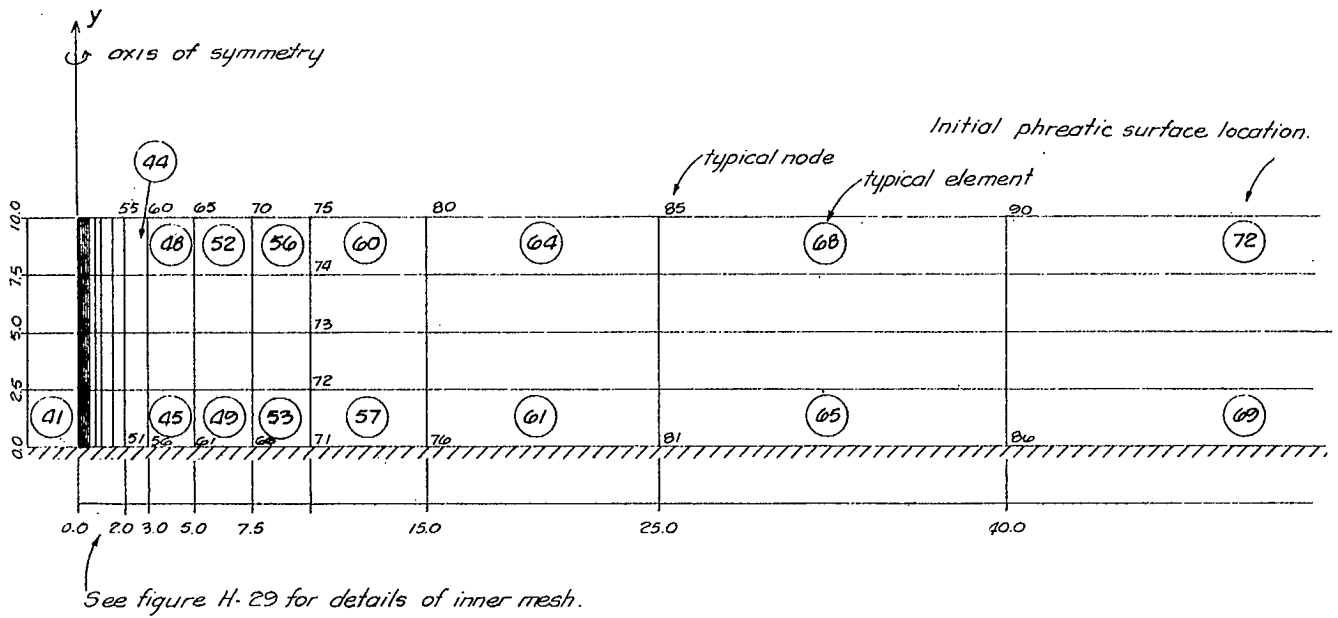


Fig A-2 - Sample problem 2 - finite element mesh.

### SAMPLE PROBLEM 2 - WELL DRAWDOWN

6. The second sample problem is a simulation of a well of known flow in a homogeneous water-table type aquifer. For this problem, finite element modelling of the flow close to the well requires careful analysis if accurate results are to be produced. Fortunately, when the well flow is specified, even a poor model of the behaviour close to the well does not cause bad results further away from the well. When the well water level rather than the flow is specified, this is not the case and a good model must be constructed. Fig A-2 and A-3 illustrate the geometry of the problem and also the finite element mesh used. Note that node and element data are generated along vertical lines.

7. Figure A-4 depicts schematically the behaviour of a well. Sketches (a) and (b) show the effect of low and high pumping rates. For the low capacity pump, the phreatic surface exits just above the level of the water in the well. For the high capacity pump, the phreatic surface exits high above the well water level. The FEFPM program could be used to compute formation characteristic curves which, combined with the pump characteristic curves as shown in Fig A-4 (a), (d) and (e), would give a prediction of the well's performance. Such an analysis could be misleading if the head loss due to existing the formation and passing through the well casing were significant.

8. In sample problem 2, there is a fairly low pumping rate and therefore it is assumed that the phreatic surface exits at the well water level. As discussed above, minor errors near the well do not affect accuracy of results further away, so the error due to this assumption is negligible.

9. A problem arises when designing the mesh for this case - the node at the intersection of the phreatic surface and the well is both zero-pressure and has an unknown net outflow. The FEFPM program is designed only to handle phreatic surface nodes with a known net flow based on  $QY$ , the surface infiltration rate. The solution is to insert a layer of finite elements within the well and withdraw the well outflow from the centre. Provided these elements have a high vertical permeability, the potentials of the nodes on the well-soil interface will all be the same, and the solution will be valid. In this problem the dummy elements are numbers 1-4, and a high effective vertical permeability is ensured by giving nodes 1-5 at the centre identical potentials. The line of dummy nodes is linked to the movements of node 10 at the intersection of the phreatic surface and the well. The inner portion of the mesh is shown schematically in Fig A-5.

10. A listing of the data input cards used in the analysis and a complete printout follows. The final phreatic surface location in the region of the well is shown in Fig A-5.

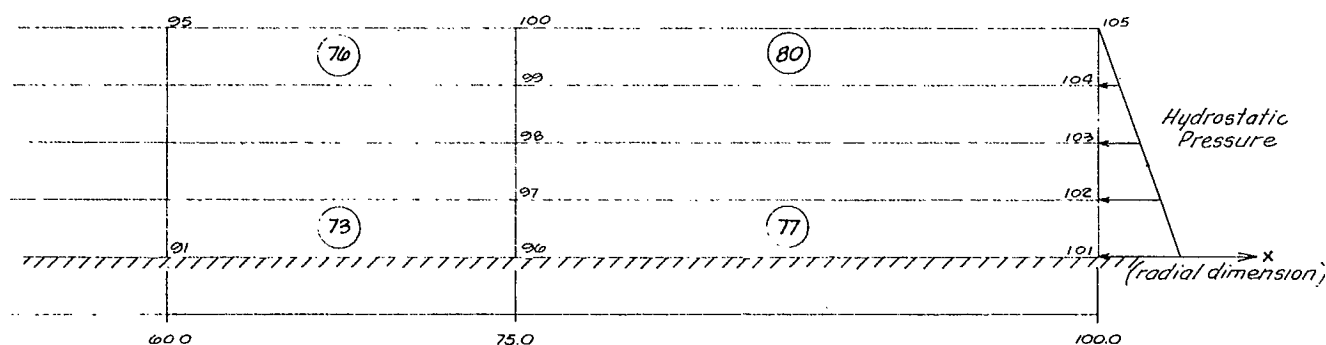


Fig A-2 (cont) - Sample problem 2 - finite element mesh.



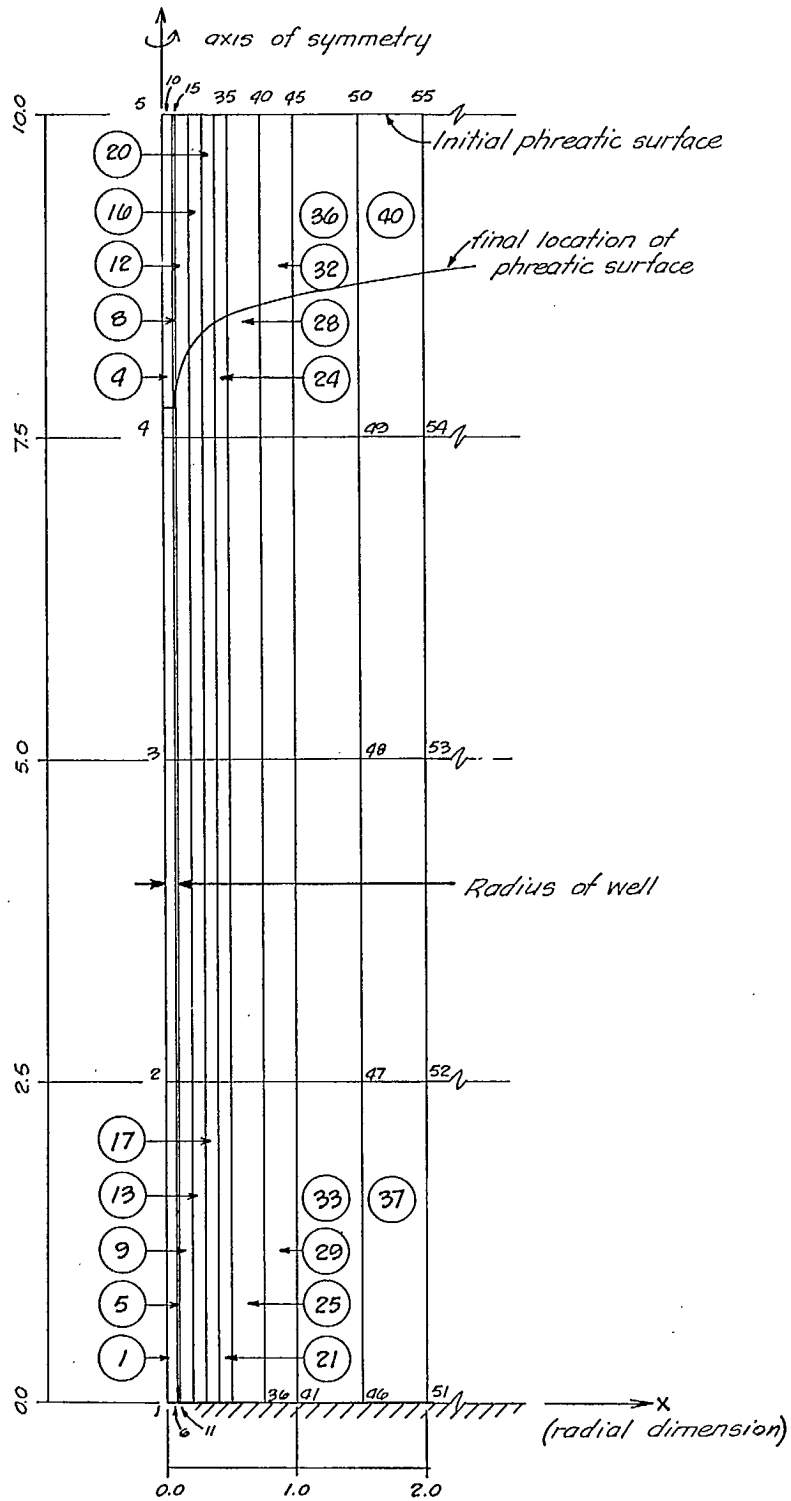
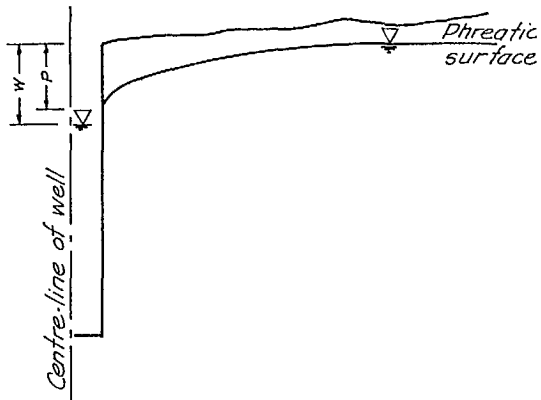
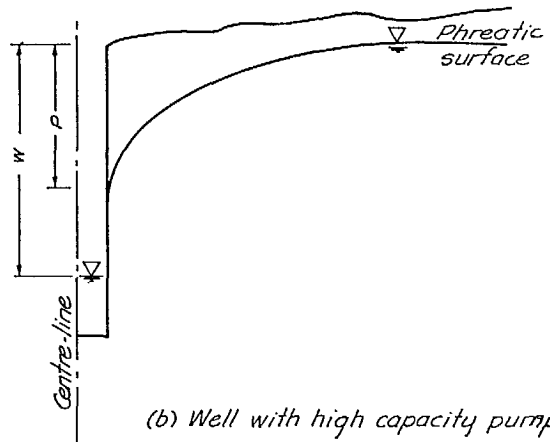


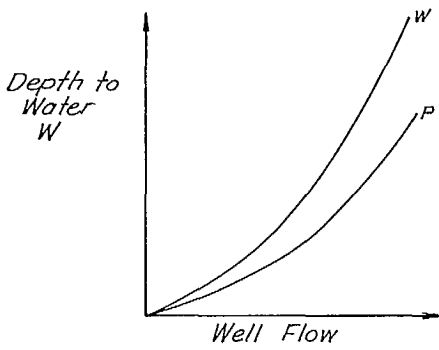
Fig A-3 - Sample problem 2 - inner mesh.



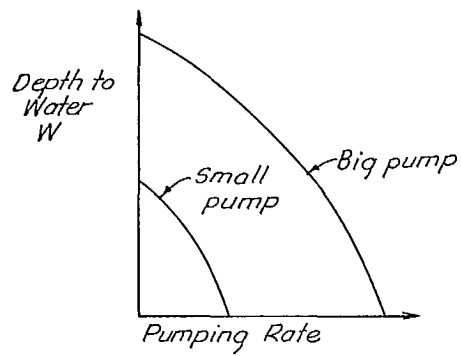
(a) Well with low capacity pump.



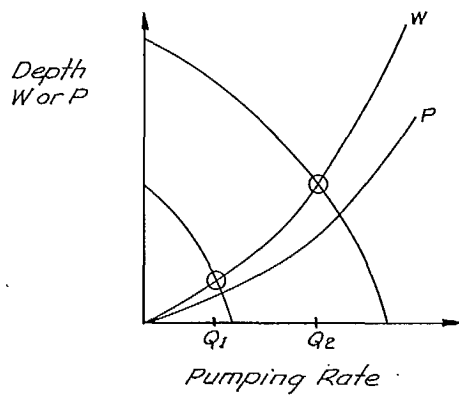
(b) Well with high capacity pump



(c) Formation characteristic curves.



(d) Pump characteristic curves



(e) Superimposed characteristic rate.

Fig A-4 - Well behaviour.

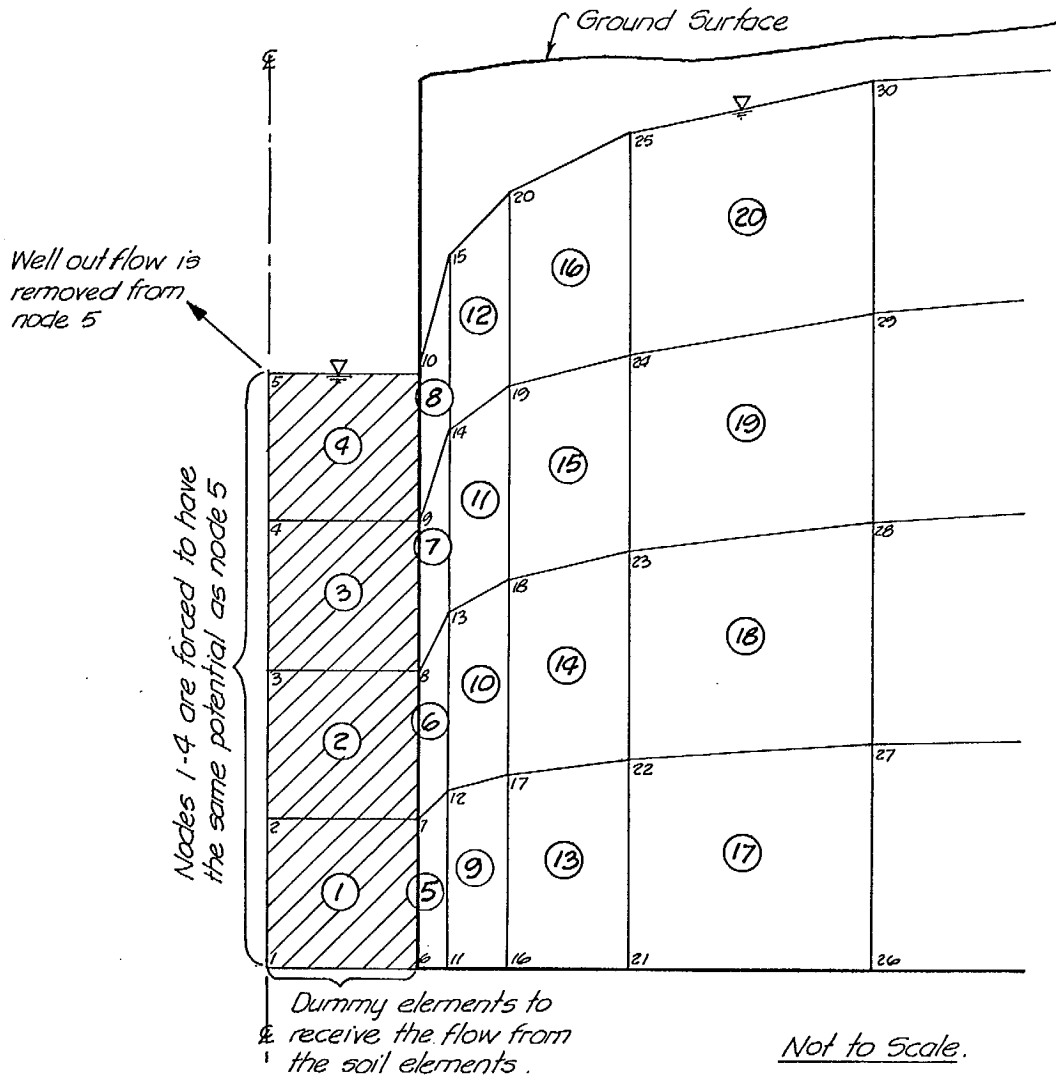


Fig A-5 - Finite element and boundary conditions used to model characteristics of the well.

SAMPLE PROBLEM 2 - INPUT DATA

START		SAMPLE 2		WELL DRAWDOWN WITH PHREATIC SURFACE						
SFPM	NUMFSC#23	NUMEL#80	NUMNP#105	TOL#0.01	MAXIT#10	NTYPE#0	Name list \$ FPM Cards			
SEND										
1	5	0	0							
2	5	0	2.5				0.25	10	} Node Cards	
3	5	0	5				0.5	10		
4	5	0	7.5				0.75	10		
5	0	0	10				1	10		
6	0	.0762								
10	0	.0762								
11	0	.1								
15	0	.1								
16	0	.2								
20	0	.2								
21	0	.3								
25	0	.3								
26	0	.4								
30	0	.4								
31	0	.5								
35	0	.5								
36	0	.75								
40	0	.75								
41	0	1.0								
45	0	1.0								
46	0	1.5								
50	0	1.5								
51	0	2.0								
55	0	2.0								
56	0	3.0								
60	0	3.0								
61	0	5.0								
65	0	5.0								
66	0	7.5								
70	0	7.5								
71	0	10.0								
75	0	10.0								
76	0	15.0								
80	0	15.0								
81	0	25.0								
85	0	25.0								
86	0	40.0								
90	0	40.0								
91	0	60.0								
95	0	60.0								
96	0	75.0								
100	0	75.0								
101	2	P	100			10				
105	P	100	10							
.000001		.000001		Material Properties Card						
1	1	6	7	2	1					} Element Cards
5	6	11	12	7	1					
9	11	16	17	12	1					
13	16	21	22	17	1					
17	21	26	27	22	1					
21	26	31	32	27	1					
25	31	36	37	32	1					
29	36	41	42	37	1					
33	41	46	47	42	1					
37	46	51	52	47	1					
41	51	56	57	52	1					
45	56	61	62	57	1					
49	61	66	67	62	1					
53	66	71	72	67	1					
57	71	76	77	72	1					
61	76	81	82	77	1					
65	81	86	87	82	1					
69	86	91	92	87	1					
73	91	96	97	92	1					
77	96	101	102	97	1					
80	99	104	105	100	1					

FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 1

```

*****
*                                     *
*             S E E P A G E   A N A L Y S I S             *
*                                     *
*             SAMPLE 2   WELL DRAWDOWN WITH PHREATIC SURFACE   *
*                                     *
*                   01/26/76-15:13:59                   *
*                                     *
*****
    
```

```

NUMBER OF NODAL POINTS-----105
NUMBER OF ELEMENTS----- 80
NUMBER OF DIFF. MATERIALS--- 1
UNIT WEIGHT OF FLUID----- 1.0000+00
REFERENCE FOR POTENTIALS---- 0.0000

CORRECTION FACTOR----- 1.00000
MAX. NO. ITERATIONS----- 10
ERROR TOLERANCE----- .01000
    
```

```

AXISYMMETRIC FLOW PROBLEM
ASGIO DYNAMIC E.R. = #ASG,T   TEMP15,017/ 6770/TRK/ 6770 .
ASGIO DYNAMIC E.R. = #USE    15,TEMP15 .
    
```

FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 2

* SAMPLE 2 WELL DRAWDOWN WITH PHREATIC SURFACE				** 01/26/76-15:13:59 **				PAGE 2
NODE B.C.	X=DRD	Y=DRD	F	NODE B.C.	X=DRD	Y=DRD	F	
1 ( 5)	.0000	.0000	0.0000	* 51 CONT	2.0000	.0000	0.0000	
2 ( 5)	.0000	2.5000	0.0000	* 52 CONT	2.0000	2.5000	0.0000	
3 ( 5)	.0000	5.0000	0.0000	* 53 CONT	2.0000	5.0000	0.0000	
4 ( 5)	.0000	7.5000	0.0000	* 54 CONT	2.0000	7.5000	0.0000	
5 CONT	.0000	10.0000	-3.1830-06	* 55 CONT	2.0000	10.0000	0.0000	
6 CONT	.0762	.0000	0.0000	* 56 CONT	3.0000	.0000	0.0000	
7 CONT	.0762	2.5000	0.0000	* 57 CONT	3.0000	2.5000	0.0000	
8 CONT	.0762	5.0000	0.0000	* 58 CONT	3.0000	5.0000	0.0000	
9 CONT	.0762	7.5000	0.0000	* 59 CONT	3.0000	7.5000	0.0000	
10 CONT	.0762	10.0000	0.0000	* 60 CONT	3.0000	10.0000	0.0000	
11 CONT	.1000	.0000	0.0000	* 61 CONT	5.0000	.0000	0.0000	
12 CONT	.1000	2.5000	0.0000	* 62 CONT	5.0000	2.5000	0.0000	
13 CONT	.1000	5.0000	0.0000	* 63 CONT	5.0000	5.0000	0.0000	
14 CONT	.1000	7.5000	0.0000	* 64 CONT	5.0000	7.5000	0.0000	
15 CONT	.1900	10.0000	0.0000	* 65 CONT	5.0000	10.0000	0.0000	
16 CONT	.2000	.0000	0.0000	* 66 CONT	7.5000	.0000	0.0000	
17 CONT	.2000	2.5000	0.0000	* 67 CONT	7.5000	2.5000	0.0000	
18 CONT	.2000	5.0000	0.0000	* 68 CONT	7.5000	5.0000	0.0000	
19 CONT	.2000	7.5000	0.0000	* 69 CONT	7.5000	7.5000	0.0000	
20 CONT	.2000	10.0000	0.0000	* 70 CONT	7.5000	10.0000	0.0000	
21 CONT	.3000	.0000	0.0000	* 71 CONT	10.0000	.0000	0.0000	
22 CONT	.3000	2.5000	0.0000	* 72 CONT	10.0000	2.5000	0.0000	
23 CONT	.3000	5.0000	0.0000	* 73 CONT	10.0000	5.0000	0.0000	
24 CONT	.3000	7.5000	0.0000	* 74 CONT	10.0000	7.5000	0.0000	
25 CONT	.3000	10.0000	0.0000	* 75 CONT	10.0000	10.0000	0.0000	
26 CONT	.4000	.0000	0.0000	* 76 CONT	15.0000	.0000	0.0000	
27 CONT	.4000	2.5000	0.0000	* 77 CONT	15.0000	2.5000	0.0000	
28 CONT	.4000	5.0000	0.0000	* 78 CONT	15.0000	5.0000	0.0000	
29 CONT	.4000	7.5000	0.0000	* 79 CONT	15.0000	7.5000	0.0000	
30 CONT	.4000	10.0000	0.0000	* 80 CONT	15.0000	10.0000	0.0000	
31 CONT	.5000	.0000	0.0000	* 81 CONT	25.0000	.0000	0.0000	
32 CONT	.5000	2.5000	0.0000	* 82 CONT	25.0000	2.5000	0.0000	
33 CONT	.5000	5.0000	0.0000	* 83 CONT	25.0000	5.0000	0.0000	
34 CONT	.5000	7.5000	0.0000	* 84 CONT	25.0000	7.5000	0.0000	
35 CONT	.5000	10.0000	0.0000	* 85 CONT	25.0000	10.0000	0.0000	
36 CONT	.7500	.0000	0.0000	* 86 CONT	40.0000	.0000	0.0000	
37 CONT	.7500	2.5000	0.0000	* 87 CONT	40.0000	2.5000	0.0000	
38 CONT	.7500	5.0000	0.0000	* 88 CONT	40.0000	5.0000	0.0000	
39 CONT	.7500	7.5000	0.0000	* 89 CONT	40.0000	7.5000	0.0000	
40 CONT	.7500	10.0000	0.0000	* 90 CONT	40.0000	10.0000	0.0000	
41 CONT	1.0000	.0000	0.0000	* 91 CONT	60.0000	.0000	0.0000	
42 CONT	1.0000	2.5000	0.0000	* 92 CONT	60.0000	2.5000	0.0000	
43 CONT	1.0000	5.0000	0.0000	* 93 CONT	60.0000	5.0000	0.0000	
44 CONT	1.0000	7.5000	0.0000	* 94 CONT	60.0000	7.5000	0.0000	
45 CONT	1.0000	10.0000	0.0000	* 95 CONT	60.0000	10.0000	0.0000	
46 CONT	1.5000	.0000	0.0000	* 96 CONT	75.0000	.0000	0.0000	
47 CONT	1.5000	2.5000	0.0000	* 97 CONT	75.0000	2.5000	0.0000	
48 CONT	1.5000	5.0000	0.0000	* 98 CONT	75.0000	5.0000	0.0000	
49 CONT	1.5000	7.5000	0.0000	* 99 CONT	75.0000	7.5000	0.0000	
50 CONT	1.5000	10.0000	0.0000	* 100 CONT	75.0000	10.0000	0.0000	

FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 3

\* SAMPLE 2 WELL DRAWDOWN WITH PHREATIC SURFACE \*\* 01/26/76-15:13:59 \*\* PAGE 3

NODE B.C.	X-DRD	Y-DRD	F
101 PRESS.	100.0000	.0000	1.0000+01 0.0000 *
102 PRESS.	100.0000	2.5000	7.5000+00 0.0000 *
103 PRESS.	100.0000	5.0000	5.0000+00 0.0000 *
104 PRESS.	100.0000	7.5000	2.5000+00 0.0000 *
105 PRESS.	100.0000	10.0000	0.0000 0.0000 *

FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 4

\* SAMPLE 2 WELL DRAWDOWN WITH PHREATIC SURFACE \*\* 01/26/76-15:13:59 \*\* PAGE 4

LINE *	PROPERTIES	MATERIAL ABOVE	PROPERTIES BELOW	DELIMITING NODE STRING
1 *	1.00-06	1.00-06 .0	1.00-06 1.00-06 .0	* -1.000+10 1.000+20 1.000+10 1.000+20 * 1.000+10 -1.000+10 -1.000+10 -1.000+10 *

FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 5

\* SAMPLE 2 WELL DRAWDOWN WITH PHREATIC SURFACE \*\* 01/26/76-15:13:59 \*\* PAGE 5

ELMT	I	J	K	L	MAT
1	1	6	7	2	1
2	2	7	8	3	1
3	3	6	9	4	1
4	4	9	10	5	1
5	6	11	12	7	1
6	7	12	13	8	1
7	8	13	14	9	1
8	9	14	15	10	1
9	11	16	17	12	1
10	12	17	18	13	1
11	13	18	19	14	1
12	14	19	20	15	1
13	16	21	22	17	1
14	17	22	23	18	1
15	18	23	24	19	1
16	19	24	25	20	1
17	21	26	27	22	1
18	22	27	28	23	1
19	23	28	29	24	1
20	24	29	30	25	1
21	26	31	32	27	1
22	27	32	33	28	1
23	28	33	34	29	1
24	29	34	35	30	1
25	31	36	37	32	1
26	32	37	38	33	1
27	33	38	39	34	1
28	34	39	40	35	1
29	36	41	42	37	1
30	37	42	43	38	1
31	38	43	44	39	1
32	39	44	45	40	1
33	41	46	47	42	1
34	42	47	48	43	1
35	43	48	49	44	1
36	44	49	50	45	1
37	46	51	52	47	1
38	47	52	53	48	1
39	48	53	54	49	1
40	49	54	55	50	1
41	51	56	57	52	1
42	52	57	58	53	1
43	53	58	59	54	1
44	54	59	60	55	1
45	56	61	62	57	1
46	57	62	63	58	1
47	58	63	64	59	1
48	59	64	65	60	1
49	61	66	67	62	1
50	62	67	68	63	1

FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 6

```

*          SAMPLE 2      WELL DRAWNDOWN WITH PHREATIC SURFACE          ** 01/26/76-15:13:59      ** PAGE 6
ELMT      I      J      K      L      MAT
51      63      68      69      64      1
52      64      69      70      65      1
53      66      71      72      67      1
54      67      72      73      68      1
55      68      73      74      69      1
56      69      74      75      70      1
57      71      76      77      72      1
58      72      77      78      73      1
59      73      78      79      74      1
60      74      79      80      75      1
61      76      81      82      77      1
62      77      82      83      76      1
63      78      83      84      79      1
64      79      84      85      80      1
65      81      86      87      82      1
66      82      87      88      83      1
67      83      88      89      84      1
68      84      89      90      85      1
69      86      91      92      87      1
70      87      92      93      88      1
71      88      93      94      89      1
72      89      94      95      90      1
73      91      96      97      92      1
74      92      97      98      93      1
75      93      98      99      94      1
76      94      99      100     95      1
77      96      101     102     97      1
78      97      102     103     98      1
79      98      103     104     99      1
80      99      104     105     100     1
    
```

\*\*\* FREE SURFACE \*\*\*

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NODE      CORR, ANGLE      Y(MAX)
2          WILL HAVE .25 TIMES THE MOVEMENTS OF NODE 10
3          WILL HAVE .50 TIMES THE MOVEMENTS OF NODE 10
4          WILL HAVE .75 TIMES THE MOVEMENTS OF NODE 10
5          WILL HAVE 1.00 TIMES THE MOVEMENTS OF NODE 10
10         90.0      *****
15         90.0      *****
20         90.0      *****
25         90.0      *****
30         90.0      *****
35         90.0      *****
40         90.0      *****
45         90.0      *****
50         90.0      *****
55         90.0      *****
60         90.0      *****
65         90.0      *****
    
```

FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 7

```

70      90.0      *****
75      90.0      *****
80      90.0      *****
85      90.0      *****
90      90.0      *****
95      90.0      *****
100     90.0      *****

CPU TIME USED IN S/R MESHIN = 2.234 SECONDS
TOTAL SPACE ACQUIRED FOR /PATRIX/ IS 2989
MAXIMUM BANDWIDTH = 7
ASGIO DYNAMIC E.R. = #ASG,T      TEMPI4,F17/      1/TRK/      1.
ASGIO DYNAMIC E.R. = #USE      14,7EMPI4.
    
```

## FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 8

\* SAMPLE 2 WELL DRAWN WITH PHREATIC SURFACE \*\* 01/26/76-15113159 \*\* PAGE 7  
 TOTAL MESH AREA= 9.999999+02 CPU TIME TO BUILD FLOW MATRIX = 6.030 SECONDS.

\*\*\*\*\* ITERATION 1

TIME TO SOLVE MATRIX = .187 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 1.32+00  
 PHREATIC SURFACE END-NODE 100 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .0044 DX,OY= -1.1921-07 -9.5604-02  
 PHREATIC SURFACE END-NODE 10 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= 2.6212 DX,OY= 0.0000 -2.2410+00

NODE	X-ORD	Y-ORD	PRESSURE	POTENTIAL	DX	OY	DP/DX	DP/OY
10	.0762	10.0000	-.22649+01	.77351+01	.0000	-2.2410	.0000	.0000
15	.1000	10.0000	-.21786+01	.78214+01	-.0000	-2.1786	.0000	-1.0000
20	.2000	10.0000	-.19610+01	.80390+01	-.0000	-1.9610	.0000	-1.0000
25	.3000	10.0000	-.18325+01	.81675+01	-.0000	-1.8325	.0000	-1.0000
30	.4000	10.0000	-.17412+01	.82588+01	-.0000	-1.7412	.0000	-1.0000
35	.5000	10.0000	-.16703+01	.83297+01	-.0000	-1.6703	.0000	-1.0000
40	.7500	10.0000	-.15418+01	.84582+01	-.0000	-1.5418	.0000	-1.0000
45	1.0000	10.0000	-.14505+01	.85495+01	-.0000	-1.4505	.0000	-1.0000
50	1.5000	10.0000	-.13220+01	.86780+01	-.0000	-1.3220	.0000	-1.0000
55	2.0000	10.0000	-.12307+01	.87693+01	-.0000	-1.2307	.0000	-1.0000
60	3.0000	10.0000	-.11024+01	.88976+01	-.0000	-1.1024	.0000	-1.0000
65	5.0000	10.0000	-.94189+00	.90581+01	-.0000	-.9419	.0000	-1.0000
70	7.5000	10.0000	-.81400+00	.91860+01	-.0000	-.8140	.0000	-1.0000
75	10.0000	10.0000	-.72285+00	.92772+01	-.0000	-.7228	.0000	-1.0000
80	15.0000	10.0000	-.59530+00	.94047+01	-.0000	-.5953	.0000	-1.0000
85	25.0000	10.0000	-.43602+00	.95640+01	-.0000	-.4360	.0000	-1.0000
90	40.0000	10.0000	-.28906+00	.97109+01	-.0000	-.2891	.0000	-1.0000
95	60.0000	10.0000	-.16171+00	.98383+01	-.0000	-.1617	.0000	-1.0000
100	75.0000	10.0000	-.90965-01	.99090+01	-.0000	-.0956	.0000	.0000

TOTAL MESH AREA= 9.6807415+02 CPU TIME TO BUILD FLOW MATRIX = 6.230 SECONDS.

\*\*\*\*\* ITERATION 2

TIME TO SOLVE MATRIX = .205 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 5.42-02  
 PHREATIC SURFACE END-NODE 100 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .0044 DX,OY= -1.1921-07 -8.8401-04  
 PHREATIC SURFACE END-NODE 10 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= 4.0286 DX,OY= 2.3283-10 -6.5304-02

NODE	X-ORD	Y-ORD	PRESSURE	POTENTIAL	DX	OY	DP/DX	DP/OY
10	.0762	7.7590	-.11491+00	.76441+01	.0000	-.0653	.0000	.0000
15	.1000	7.8214	-.131807-01	.77896+01	-.0000	-.0318	.0000	-1.0000
20	.2000	8.0390	-.70980-01	.81100+01	.0000	.0710	.0000	-1.0000
25	.3000	8.1675	-.97856-01	.82653+01	.0000	.0979	.0000	-1.0000
30	.4000	8.2588	-.10299+00	.83618+01	.0000	.1030	.0000	-1.0000
35	.5000	8.3297	-.99909-01	.84296+01	.0000	.0999	.0000	-1.0000
40	.7500	8.4582	-.81662-01	.85398+01	.0000	.0817	.0000	-1.0000
45	1.0000	8.5495	-.61920-01	.86115+01	.0000	.0619	.0000	-1.0000
50	1.5000	8.6780	-.31163-01	.87091+01	.0000	.0312	.0000	-1.0000
55	2.0000	8.7693	-.10334-01	.87796+01	.0000	.0103	.0000	-1.0000
60	3.0000	8.8976	-.11877-01	.88857+01	-.0000	-.0119	.0000	-1.0000
65	5.0000	9.0581	-.24033-01	.90341+01	-.0000	-.0240	.0000	-1.0000
70	7.5000	9.1860	-.24680-01	.91613+01	-.0000	-.0247	.0000	-1.0000
75	10.0000	9.2772	-.22369-01	.92548+01	-.0000	-.0224	.0000	-1.0000
80	15.0000	9.4047	-.16648-01	.93881+01	-.0000	-.0166	.0000	-1.0000
85	25.0000	9.5640	-.91413-02	.95548+01	-.0000	-.0091	.0000	-1.0000
90	40.0000	9.7109	-.39960-02	.97069+01	-.0000	-.0040	.0000	-1.0000
95	60.0000	9.8383	-.12180-02	.98371+01	-.0000	-.0012	.0000	-1.0000
100	75.0000	9.9044	-.42938-02	.99087+01	-.0000	-.0009	.0000	.0000

TOTAL MESH AREA= 9.6761249+02 CPU TIME TO BUILD FLOW MATRIX = 6.440 SECONDS.

\*\*\*\*\* ITERATION 3

TIME TO SOLVE MATRIX = .175 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 2.08-02  
 PHREATIC SURFACE END-NODE 100 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .0044 DX,OY= -1.1921-07 -2.1281-05  
 PHREATIC SURFACE END-NODE 10 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= 4.2710 DX,OY= 2.3283-10 4.8677-02

NODE	X-ORD	Y-ORD	PRESSURE	POTENTIAL	DX	OY	DP/DX	DP/OY
10	.0762	7.6937	-.76215-02	.76861+01	.0000	.0487	.0000	.0000
15	.1000	7.7896	-.54046-01	.78401+01	.0000	.0544	.0000	-1.0000
20	.2000	8.1100	-.66951-01	.81769+01	.0000	.0670	.0000	-1.0000
25	.3000	8.2653	-.55987-01	.83213+01	.0000	.0560	.0000	-1.0000
30	.4000	8.3618	-.44251-01	.84060+01	.0000	.0443	.0000	-1.0000
35	.5000	8.4296	-.34004-01	.84640+01	.0000	.0344	.0000	-1.0000
40	.7500	8.5398	-.18108-01	.85580+01	.0000	.0181	.0000	-1.0000
45	1.0000	8.6115	-.89613-02	.86204+01	.0000	.0090	.0000	-1.0000
50	1.5000	8.7091	-.10011-02	.87101+01	.0000	.0010	.0000	-1.0000
55	2.0000	8.7796	-.16115-02	.87780+01	-.0000	-.0016	.0000	-1.0000
60	3.0000	8.8857	-.23312-02	.88834+01	-.0000	-.0023	.0000	-1.0000
65	5.0000	9.0341	-.15695-02	.90325+01	-.0000	-.0016	.0000	-1.0000
70	7.5000	9.1613	-.90988-03	.91604+01	-.0000	-.0009	.0000	-1.0000
75	10.0000	9.2548	-.58652-03	.92542+01	-.0000	-.0006	.0000	-1.0000
80	15.0000	9.3881	-.31970-03	.93877+01	-.0000	-.0003	.0000	-1.0000
85	25.0000	9.5548	-.12157-03	.95547+01	-.0000	-.0001	.0000	-1.0000
90	40.0000	9.7069	-.37218-04	.97069+01	-.0000	-.0000	.0000	-1.0000
95	60.0000	9.8371	-.77802-05	.98371+01	-.0000	-.0000	.0000	-1.0000
100	75.0000	9.9035	-.51776-02	.99087+01	-.0000	-.0000	.0000	.0000

TOTAL MESH AREA= 9.6763348+02 CPU TIME TO BUILD FLOW MATRIX = 6.220 SECONDS.



FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 9

\*\*\*\*\* ITERATION 4

TIME TO SOLVE MATRIX = .173 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 5.64-03

FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 10

CONVERGENCE.

FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 11

SAMPLE 2 WELL DRAWDOWN WITH PHREATIC SURFACE						** 01/26/76-15113159 **			PAGE 8
ELMT	X=ORO	Y=ORD	Z=ORO	X=FLOW	Y=FLOW	TOTAL FLOW	DIRN	MATERIAL	
1	.038	.968	4.355	-1.3324-05	-3.9676-10	1.3324-05	-180.00	1	
2	.038	2.903	5.323	-1.3387-05	-2.0686-09	1.3387-05	-179.99	1	
3	.038	4.839	6.291	-1.3340-05	3.8923-09	1.3340-05	179.98	1	
4	.038	6.775	7.259	-1.5417-05	-8.5652-08	1.5418-05	-179.68	1	
5	.068	.974	.974	-4.5058-06	-9.0889-10	4.5058-06	-179.99	1	
6	.088	2.922	2.922	-4.5033-06	-4.0322-09	4.5033-06	-179.95	1	
7	.088	4.871	4.871	-4.6256-06	-6.4574-09	4.6256-06	179.92	1	
8	.088	6.819	6.819	-4.9824-06	-1.8174-07	4.9858-06	-177.91	1	
9	.150	1.001	1.001	-2.7079-06	-1.3313-09	2.7079-06	-179.97	1	
10	.150	3.004	3.004	-2.7008-06	-3.3512-09	2.7008-06	-179.93	1	
11	.150	5.007	5.007	-2.8553-06	-3.2293-09	2.8553-06	-179.94	1	
12	.150	7.009	7.009	-2.5921-06	-1.9459-07	2.5994-06	-175.71	1	
13	.250	1.031	1.031	-1.5954-06	-1.7046-09	1.5954-06	-179.94	1	
14	.250	3.093	3.093	-1.5992-06	-2.9207-09	1.5992-06	-179.90	1	
15	.250	5.156	5.156	-1.5819-06	-1.5763-08	1.6819-06	-179.46	1	
16	.250	7.218	7.218	-1.3530-06	-1.8562-07	1.3656-06	-172.19	1	
17	.350	1.045	1.045	-1.1338-06	-1.7943-09	1.1338-06	-179.91	1	
18	.350	3.136	3.136	-1.1398-06	-3.3410-09	1.1398-06	-179.83	1	
19	.350	5.227	5.227	-1.1852-06	-2.2341-08	1.1854-06	-178.92	1	
20	.350	7.318	7.318	-9.1065-07	-1.6317-07	9.2515-07	-169.84	1	
21	.450	1.054	1.054	-8.7991-07	-1.8164-09	8.7991-07	-179.88	1	
22	.450	3.163	3.163	-8.8605-07	-3.9292-09	8.8606-07	-179.75	1	
23	.450	5.272	5.272	-9.1223-07	-2.5847-08	9.1260-07	-178.38	1	
24	.450	7.381	7.381	-8.8543-07	-1.4247-07	7.0008-07	-168.26	1	
25	.625	1.064	1.064	-6.3725-07	-1.8231-09	6.3725-07	-179.84	1	
26	.625	3.192	3.192	-6.4250-07	-4.8800-09	6.4252-07	-179.56	1	
27	.625	5.319	5.319	-6.5164-07	-2.8025-08	6.5224-07	-177.54	1	
28	.625	7.447	7.447	-4.8302-07	-1.1505-07	4.9712-07	-166.62	1	
29	.875	1.074	1.074	-4.5279-07	-1.8332-09	4.5279-07	-179.77	1	
30	.875	3.221	3.221	-4.5057-07	-5.9895-09	4.5061-07	-179.25	1	
31	.875	5.368	5.368	-4.5477-07	-2.8616-08	4.5567-07	-176.40	1	
32	.875	7.516	7.516	-3.3769-07	-8.5235-08	3.4828-07	-165.83	1	
33	1.250	1.083	1.083	-3.1774-07	-1.6683-09	3.1774-07	-179.66	1	
34	1.250	3.249	3.249	-3.1963-07	-6.9152-09	3.1971-07	-178.76	1	
35	1.250	5.416	5.416	-3.1147-07	-2.6324-08	3.1258-07	-175.17	1	
36	1.250	7.582	7.582	-2.3672-07	-5.8874-08	2.4393-07	-166.03	1	
37	1.750	1.093	1.093	-2.2523-07	-1.9125-09	2.2524-07	-179.51	1	
38	1.750	3.279	3.279	-2.2542-07	-7.4459-09	2.2554-07	-178.11	1	
39	1.750	5.465	5.465	-2.1529-07	-2.2225-08	2.1643-07	-174.11	1	
40	1.750	7.651	7.651	-1.6975-07	-3.6680-08	1.7367-07	-167.81	1	
41	2.500	1.104	1.104	-1.5694-07	-1.8890-09	1.5695-07	-179.31	1	
42	2.500	3.312	3.312	-1.5369-07	-7.0495-09	1.5585-07	-177.41	1	
43	2.500	5.519	5.519	-1.4645-07	-1.6569-08	1.4739-07	-173.55	1	
44	2.500	7.727	7.727	-1.2191-07	-2.1296-08	1.2375-07	-170.09	1	
45	4.000	1.120	1.120	-9.5974-08	-1.5710-09	9.5987-08	-179.06	1	
46	4.000	3.359	3.359	-9.4104-08	-5.2500-09	9.4250-08	-176.81	1	
47	4.000	5.599	5.599	-8.8411-08	-8.0909-09	8.8916-08	-173.89	1	
48	4.000	7.838	7.838	-7.9492-08	-9.4938-09	8.0057-08	-173.19	1	
49	6.250	1.137	1.137	-5.9099-08	-9.8248-10	5.9108-08	-179.05	1	
50	6.250	3.411	3.411	-5.7852-08	-2.8875-09	5.7924-08	-177.14	1	

## FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 12

* ELMT	SAMPLE 2 X=ORD	WELL Y=ORD	DRAWDOWN Z=ORD	WITH PHREATIC SURFACE X=FLOW	Y=FLOW	TOTAL FLOW	DIRN	MATERIAL	** 01/26/76-15:13:59 ** PAGE 9
51	6.250	5.685	5.685	-5.5342-06	-4.1694-09	5.5499-08	-175.69	1	
52	6.250	7.959	7.959	-5.2498-08	-3.7004-09	5.2629-08	-175.97	1	
53	8.750	1.151	1.151	-4.0872-08	-4.9715-10	4.0875-08	-179.30	1	
54	8.750	3.453	3.453	-4.0250-08	-1.3649-09	4.0273-08	-178.06	1	
55	8.750	5.755	5.755	-3.9179-08	-1.8218-09	3.9221-08	-177.34	1	
56	8.750	8.056	8.056	-3.8044-08	-1.6737-09	3.8081-08	-177.48	1	
57	12.500	1.165	1.165	-2.7762-08	-2.0075-10	2.7763-08	-179.58	1	
58	12.500	3.495	3.495	-2.7558-08	-5.5739-10	2.7563-08	-178.84	1	
59	12.500	5.826	5.826	-2.7219-08	-7.5440-10	2.7229-08	-178.41	1	
60	12.500	8.156	8.156	-2.6867-08	-7.4991-10	2.6878-08	-178.40	1	
61	20.000	1.184	1.184	-1.6867-08	-5.3916-11	1.6867-08	-179.82	1	
62	20.000	3.552	3.552	-1.6838-08	-1.5244-10	1.6838-08	-179.48	1	
63	20.000	5.929	5.929	-1.6786-08	-2.2628-10	1.6788-08	-179.23	1	
64	20.000	8.287	8.287	-1.6726-08	-2.7059-10	1.6728-08	-179.07	1	
65	32.500	1.204	1.204	-1.0177-08	-1.4552-11	1.0173-08	-179.92	1	
66	32.500	3.612	3.612	-1.0174-08	-4.3372-11	1.0173-08	-179.76	1	
67	32.500	6.019	6.019	-1.0154-08	-7.1119-11	1.0153-08	-179.60	1	
68	32.500	8.427	8.427	-1.0152-08	-9.7529-11	1.0153-08	-179.45	1	
69	50.000	1.221	1.221	-0.5155-09	-5.8975-12	6.5156-09	-179.95	1	
70	50.000	3.664	3.664	-0.5145-09	-1.7676-11	6.5145-09	-179.84	1	
71	50.000	6.107	6.107	-0.5124-09	-2.9104-11	6.5124-09	-179.74	1	
72	50.000	8.550	8.550	-0.5093-09	-4.0245-11	6.5094-09	-179.65	1	
73	67.500	1.234	1.234	-4.7740-09	-3.0269-12	4.7790-09	-179.96	1	
74	67.500	3.701	3.701	-4.7786-09	-9.1660-12	4.7786-09	-179.89	1	
75	67.500	6.169	6.169	-4.7770-09	-1.5230-11	4.7777-09	-179.82	1	
76	67.500	8.636	8.636	-4.7763-09	-2.1245-11	4.7763-09	-179.75	1	
77	87.500	1.244	1.244	-3.6565-09	-1.1795-12	3.6565-09	-179.98	1	
78	87.500	3.732	3.732	-3.6561-09	-3.5243-12	3.6561-09	-179.94	1	
79	87.500	6.220	6.220	-3.6551-09	-5.8549-12	3.6551-09	-179.91	1	
80	87.500	8.708	8.708	-3.6537-09	-8.2565-12	3.6537-09	-179.87	1	

CPU TIME IN S/H ELFLOW = 1.478 SECONDS.

## FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 13

* NODE	B.C.	SAMPLE 2 X=ORD	WELL Y=ORD	DRAWDOWN Z=ORD	WITH PHREATIC SURFACE PRESSURE	POTENTIAL	FLOW FN	OUTFLOW	** 01/26/76-15:13:59 ** PAGE 10
1( 5)		.00	.00	.00	6.3502+00	6.3502+00	-9.6112-08	0.0000	
2( 5)		.00	1.94	1.94	4.4146+00	6.3502+00	-7.3677-07	0.0000	
3( 5)		.00	3.87	3.87	2.0790+00	6.3502+00	-1.4155-06	0.0000	
4( 5)		.00	5.81	5.81	5.4343+01	6.3502+00	-2.0895-06	0.0000	
5 CONT.		.00	7.74	7.74	-1.3922+00	6.3502+00	-2.7818-06	3.1830-06	
6 CONT.		.08	.00	.00	7.3648+00	7.3648+00	0.0000	-8.5265-14	
7 CONT.		.08	1.94	1.94	5.4367+00	7.3663+00	-7.1898-07	0.0000	
8 CONT.		.08	3.87	3.87	3.5031+00	7.3743+00	-1.4170-06	3.4106-13	
9 CONT.		.08	5.81	5.81	1.5524+00	7.3593+00	-2.1146-06	-5.6843-14	
10 CONT.		.08	7.74	7.74	-5.1588-02	7.6908+00	-2.8478-06	1.4211-13	
11 CONT.		.10	.00	.00	7.4718+00	7.4718+00	0.0000	0.0000	
12 CONT.		.10	1.96	1.96	5.5127+00	7.4738+00	-7.3133-07	-1.0232-12	
13 CONT.		.10	3.92	3.92	3.5594+00	7.4815+00	-1.4561-06	-1.0860-12	
14 CONT.		.10	5.88	5.88	1.5843+00	7.4714+00	-2.2062-06	-1.4779-12	
15 CONT.		.10	7.84	7.84	4.0506+03	7.8481+00	-2.9612-06	0.0000	
16 CONT.		.20	.00	.00	7.7413+00	7.7413+00	0.0000	2.2737-13	
17 CONT.		.20	2.04	2.04	5.7604+00	7.7447+00	-7.7253-07	-5.1159-13	
18 CONT.		.20	4.09	4.09	3.6619+00	7.7503+00	-1.5447-06	-6.5370-13	
19 CONT.		.20	6.13	6.13	1.6410+00	7.7737+00	-2.3636-06	-6.5370-13	
20 CONT.		.20	8.18	8.18	-4.7520+04	8.1764+00	-3.0870-06	-1.7053-13	
21 CONT.		.30	.00	.00	7.9006+00	7.9006+00	0.0000	-1.1369-13	
22 CONT.		.30	2.08	2.08	5.8240+00	7.9043+00	-7.9484-07	-1.2506-12	
23 CONT.		.30	4.16	4.16	3.7500+00	7.9107+00	-1.5946-06	1.7053-12	
24 CONT.		.30	6.24	6.24	1.7114+00	7.9524+00	-2.4390-06	-4.5475-13	
25 CONT.		.30	8.32	8.32	-6.2535-03	8.3150+00	-3.1155-06	1.2506-12	
26 CONT.		.40	.00	.00	8.0139+00	8.0139+00	0.0000	2.2737-13	
27 CONT.		.40	2.10	2.10	5.9162+00	8.0177+00	-8.0939-07	0.8212-13	
28 CONT.		.40	4.20	4.20	3.8223+00	8.0254+00	-1.6260-06	9.6634-13	
29 CONT.		.40	6.30	6.30	1.7725+00	8.0771+00	-2.4773-06	1.7053-12	
30 CONT.		.40	8.41	8.41	-9.3864-03	8.3967+00	-3.1302-06	-1.3074-12	
31 CONT.		.50	.00	.00	8.1019+00	8.1019+00	0.0000	0.8212-13	
32 CONT.		.50	2.12	2.12	5.9497+00	8.1059+00	-8.1909-07	-1.1369-12	
33 CONT.		.50	4.23	4.23	3.8827+00	8.1147+00	-1.6465-06	2.2737-13	
34 CONT.		.50	6.35	6.35	1.8240+00	8.1729+00	-2.4966-06	1.0232-12	
35 CONT.		.50	8.46	8.46	-1.0813-02	8.4532+00	-3.1367-06	0.8212-13	
36 CONT.		.75	.00	.00	8.2612+00	8.2612+00	0.0000	-2.2737-13	
37 CONT.		.75	2.14	2.14	6.1256+00	8.2651+00	-8.2997-07	2.2737-13	
38 CONT.		.75	4.28	4.28	3.9979+00	8.2769+00	-1.6681-06	-1.7053-13	
39 CONT.		.75	6.42	6.42	1.9204+00	8.3388+00	-2.5134-06	-1.8190-12	
40 CONT.		.75	8.56	8.56	-1.0926-02	8.5070+00	-3.1427-06	-1.5348-12	
41 CONT.		1.00	.00	.00	8.3743+00	8.3743+00	0.0000	-4.5475-13	
42 CONT.		1.00	2.16	2.16	6.2232+00	8.3783+00	-8.3616-07	-2.0464-12	
43 CONT.		1.00	4.31	4.31	4.0526+00	8.3922+00	-1.6795-06	-2.6843-13	
44 CONT.		1.00	6.47	6.47	1.9676+00	8.4531+00	-2.5134-06	-6.2528-13	
45 CONT.		1.00	8.62	8.62	-9.5365-03	8.6109+00	-3.1418-06	-6.2528-13	
46 CONT.		1.50	.00	.00	8.5431+00	8.5431+00	0.0000	-4.5475-13	
47 CONT.		1.50	2.18	2.18	6.3597+00	8.5472+00	-8.4207-07	5.6843-14	
48 CONT.		1.50	4.36	4.36	4.1942+00	8.5533+00	-1.6870-06	-4.5475-13	
49 CONT.		1.50	6.53	6.53	2.0738+00	8.6064+00	-2.5037-06	-1.4779-12	
50 CONT.		1.50	8.71	8.71	-6.6056-03	8.7055+00	-3.1370-06	-1.4779-12	

## FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 14

* SAMPLE 2	WELL DRAWDOWN WITH PHREATIC SURFACE						** 01/26/76-15113159	** PAGE 11
NODE B.C.	X-ORD	Y-ORD	Z-ORD	PRESSURE	POTENTIAL	FLOW FN	OUTFLOW	
51 CONT.	2.00	.00	.00	8.6456+00	8.6456+00	0.0000	-2.2737-13	
52 CONT.	2.00	2.19	2.19	6.4554+00	8.6099+00	-8.4445+07	-9.0949-13	
53 CONT.	2.00	4.39	4.39	4.2773+00	8.6664+00	-1.6857+06	-1.2506-12	
54 CONT.	2.00	6.58	6.58	2.1269+00	8.7104+00	-2.4831+06	-2.8422-12	
55 CONT.	2.00	8.78	8.78	-4.4725+03	8.7735+00	-3.1258+06	-1.7053-12	
56 CONT.	3.00	.00	.00	8.8026+00	8.8026+00	0.0000	-6.8212-13	
57 CONT.	3.00	2.22	2.22	8.5959+00	8.8067+00	-8.4376+07	-2.2737-12	
58 CONT.	3.00	4.44	4.44	4.3796+00	8.8213+00	-1.6751+06	-3.1632-12	
59 CONT.	3.00	6.66	6.66	2.1879+00	8.8504+00	-2.4530+06	-1.9327-12	
60 CONT.	3.00	8.88	8.88	-2.1993+03	8.8812+00	-3.1125+06	-1.6085-12	
61 CONT.	5.00	.00	.00	8.9951+00	8.9951+00	0.0000	-1.0232-12	
62 CONT.	5.00	2.26	2.26	6.7399+00	8.9980+00	-8.3272+07	-1.3642-12	
63 CONT.	5.00	4.52	4.52	4.4906+00	9.0069+00	-1.6077+06	-2.7285-12	
64 CONT.	5.00	6.77	6.77	2.2458+00	9.0201+00	-2.4189+06	-1.2506-12	
65 CONT.	5.00	9.03	9.03	-7.2116+04	9.0318+00	-3.1287+06	-1.7953-12	
66 CONT.	7.50	.00	.00	9.1435+00	9.1435+00	0.0000	2.2737-13	
67 CONT.	7.50	2.29	2.29	6.8550+00	9.1451+00	-8.1697+07	-4.0227-12	
68 CONT.	7.50	4.58	4.58	4.5691+00	9.1493+00	-1.6190+06	-3.4106-12	
69 CONT.	7.50	6.87	6.87	2.2847+00	9.1550+00	-2.3928+06	-4.6612-12	
70 CONT.	7.50	9.16	9.16	-2.3943+04	9.1602+00	-3.1392+06	-1.3642-12	
71 CONT.	10.00	.00	.00	9.2461+00	9.2461+00	0.0000	-2.5011-12	
72 CONT.	10.00	2.31	2.31	6.9553+00	9.2468+00	-8.0700+07	-1.1369-12	
73 CONT.	10.00	4.63	4.63	4.6216+00	9.2489+00	-1.6024+06	-2.2737-13	
74 CONT.	10.00	6.94	6.94	2.3109+00	9.2516+00	-2.3757+06	-1.8190-12	
75 CONT.	10.00	9.25	9.25	-9.3784+05	9.2541+00	-3.1180+06	-1.1369-12	
76 CONT.	15.00	.00	.00	9.3852+00	9.3852+00	0.0000	3.2969-12	
77 CONT.	15.00	2.35	2.35	7.0384+00	9.3854+00	-7.9916+07	-9.0949-13	
78 CONT.	15.00	4.69	4.69	4.6921+00	9.3859+00	-1.5927+06	2.2737-12	
79 CONT.	15.00	7.04	7.04	2.3460+00	9.3868+00	-2.3744+06	-1.0459-11	
80 CONT.	15.00	9.39	9.39	-2.6250+05	9.3877+00	-3.1353+06	-1.3168-11	
81 CONT.	25.00	.00	.00	9.5539+00	9.5539+00	0.0000	4.5475-12	
82 CONT.	25.00	2.39	2.39	7.1653+00	9.5540+00	-7.9064+07	4.5475-12	
83 CONT.	25.00	4.78	4.78	4.7768+00	9.5541+00	-1.5964+06	4.5475-12	
84 CONT.	25.00	7.17	7.17	2.3843+00	9.5544+00	-2.3921+06	2.7285-12	
85 CONT.	25.00	9.55	9.55	-1.8531+05	9.5547+00	-3.1854+06	-3.0013-11	
86 CONT.	40.00	.00	.00	9.7066+00	9.7066+00	0.0000	-1.4552-11	
87 CONT.	40.00	2.43	2.43	7.2799+00	9.7066+00	-8.0290+07	-6.1846-11	
88 CONT.	40.00	4.85	4.85	4.8532+00	9.7067+00	-1.6075+06	-5.4570-11	
89 CONT.	40.00	7.28	7.28	2.4266+00	9.7068+00	-2.4141+06	-1.6735-10	
90 CONT.	40.00	9.71	9.71	-1.7361+05	9.7069+00	-3.2236+06	-7.2760-11	
91 CONT.	60.00	.00	.00	9.8369+00	9.8369+00	0.0000	-8.0036-11	
92 CONT.	60.00	2.46	2.46	7.3776+00	9.8369+00	-8.0556+07	1.4552-11	
93 CONT.	60.00	4.92	4.92	4.9194+00	9.8369+00	-1.6166+06	-2.1826-10	
94 CONT.	60.00	7.38	7.38	2.4592+00	9.8370+00	-2.4336+06	-7.2760-12	
95 CONT.	60.00	9.84	9.84	-1.5110+05	9.8370+00	-3.2503+06	-1.4552+10	
96 CONT.	75.00	.00	.00	9.9086+00	9.9086+00	0.0000	-1.8190-11	
97 CONT.	75.00	2.48	2.48	7.4327+00	9.9086+00	-8.0299+07	-3.6380-11	
98 CONT.	75.00	4.95	4.95	4.9549+00	9.9086+00	-1.6206+06	-1.3097-10	
99 CONT.	75.00	7.43	7.43	2.4810+00	9.9086+00	-2.4422+06	-1.8917-10	
100 CONT.	75.00	9.90	9.90	5.1672+03	9.9087+00	-3.2574+06	-1.5280-10	

## FEFPM - SAMPLE PROBLEM NO. 2 - OUTPUT - 15

* SAMPLE 2	WELL DRAWDOWN WITH PHREATIC SURFACE						** 01/26/76-15113159	** PAGE 12
NODE B.C.	X-ORD	Y-ORD	Z-ORD	PRESSURE	POTENTIAL	FLOW FN	OUTFLOW	
101 PRESS	100.00	.00	.00	1.0000+01	1.0000+01	8.1812+08	-3.9948+07	
102 PRESS	100.00	2.50	2.50	7.5000+00	1.0000+01	-7.8919+07	-7.9896+07	
103 PRESS	100.00	5.00	5.00	5.0000+00	1.0000+01	-1.6272+06	-7.9875+07	
104 PRESS	100.00	7.50	7.50	2.5000+00	1.0000+01	-2.4678+06	-7.9839+07	
105 PRESS	100.00	10.00	10.00	0.0000	1.0000+01	-3.3438+06	-3.8810+07	
SUMMARY OF EXTERNAL FLOWS				INFLOW	OUTFLOW			
PRESSURE NODES:				3.184+06	0.000			
CONTINUITY NODES:				SPECIFIED FLOW	0.000	3.183+06		
				OTHER FLOW	1.454+09	6.438+11		
ASGEO DYNAMIC E.R. - #FREE TEMP14.								
ASGEO DYNAMIC E.R. - #FREE TEMP15.								

#FIN

SAMPLE PROBLEM 3 - OPEN PIT INFLOW

11. This sample case illustrates the application of the FEFPM program to a hypothetical open pit mine inflow problem. The general cross section geometry of the pit and the finite element mesh used in the analysis are illustrated in Fig A-6. Flow is assumed to be planar through this vertical cross section. The pit outline and the geology involved are typical of actual mines.

12. The size of this sample problem and the number of nodes and elements in the mesh should not be considered typical for seepage analyses. This sample problem was purposely set up to illustrate FEFPM program features and capabilities and was limited in size so that it could be included in this appendix. The node and element data is generated along vertical lines in the same manner as for the first two sample problems. The nodes, however, are no longer equally spaced to refine the element size close to the phreatic surface.

13. The geological domains are specified with material definition lines and thus the appropriate material type for each element is automatically determined by the location of its centroid. A material region with anisotropic permeabilities runs as an inclined band across the flow regime. The y-axis is the centre-line of the pit and is presumed to be a streamline, ie, a non-conducting boundary. The base of the problem is at some distance below the pit and is assumed to be impermeable. Also included in this analysis is the effect of horizontal drains at the toe of the slope.

14. A listing of the complete set of input data cards as well as a copy of the printout from the analysis are included. The final phreatic surface location is indicated in Fig A-6 and the marked effect on it of the anisotropic material zone should be noted. It should also be noted that the horizontal toe drains do not contribute significantly to drainage of the pit slope.

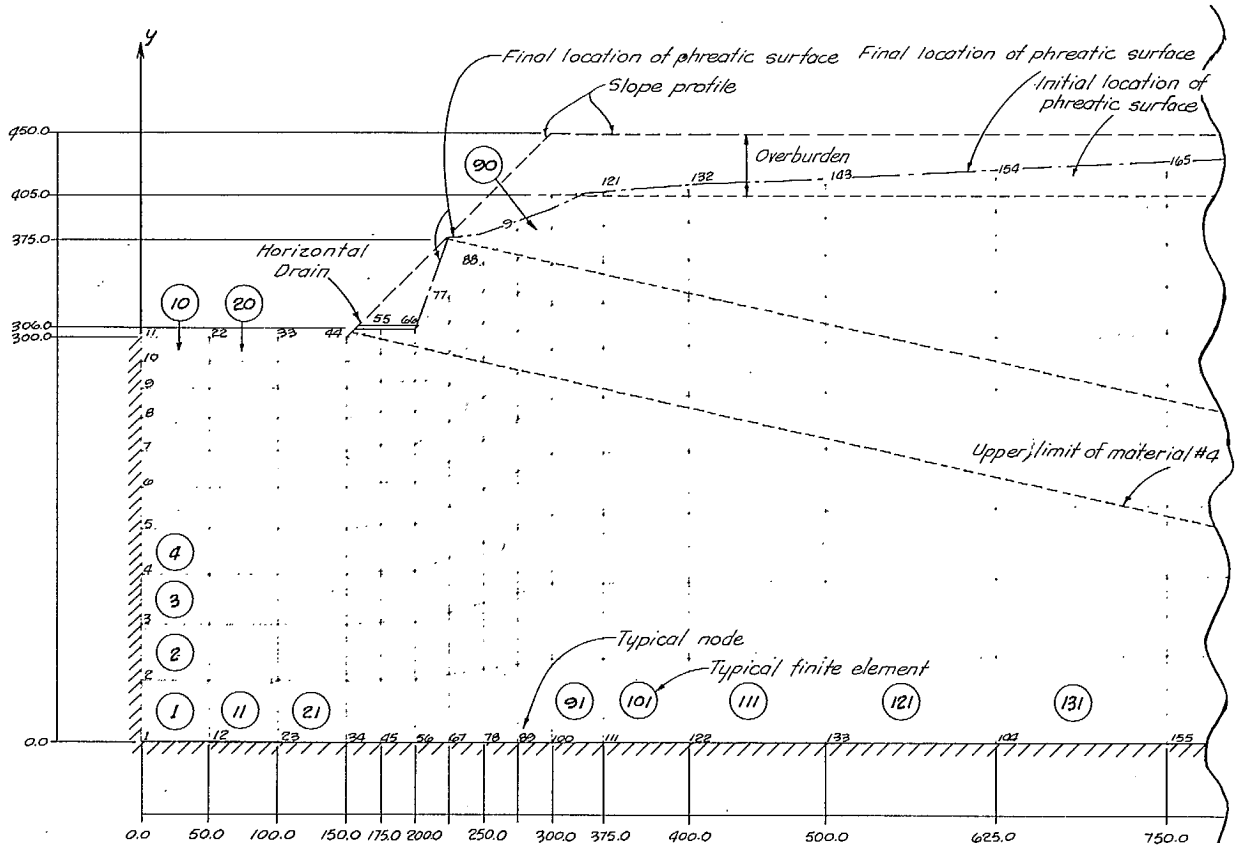


Fig A-6 - Sample problem 3 - mesh geometry and boundary conditions.

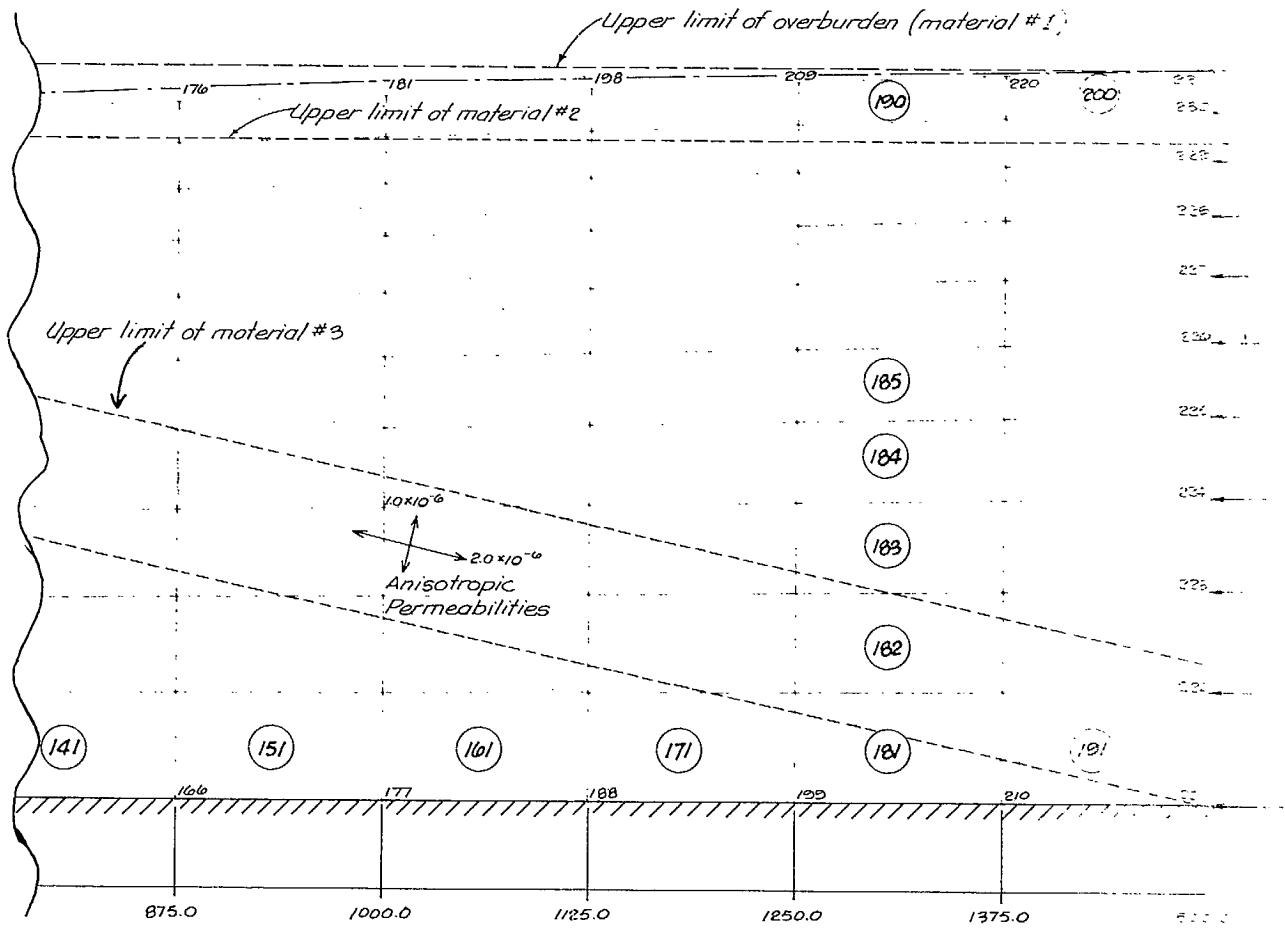


Fig A-6 (cont) - Sample problem 3 - mesh geometry and boundary conditions.

SAMPLE PROBLEM 3 - INPUT DATA

START	SAMPLE 3		OPEN PIT INFLOW							
BFPM	TOL#0,	1,MAXIT#10,	NUMFSC#15,	NUMNP#231,	NUMEL#200,	NUMMAT#4,	NTYPE#0,			
SEND										
1	0	0	0			0.9				
11	P	0	300							
12	Q	50	0			0.9				
22	P	50	300							
23	Q	100	0			0.9				
33	P	100	300							
34	Q	150	0			0.9				
44	P	150	300							
45	Q	175	0			0.9				
55	Q	175	306	325						
56	Q	200	0			0.9				
66	P	200	306							
67	Q	225	0			0.9				
* 77	Q	225	330.75	375						
78	Q	250	0			0.9				
* 88	Q	250	355.55	400						
89	Q	275	0			0.9				
* 99	Q	275	380.25	425						
100	Q	300	0			0.9				
* 110	Q	300	405	450						
111	Q	337.5	0			0.9				
* 121	Q	337.5	406.41	450						
122	Q	400	0			0.9				
* 132	Q	400	408.75	450						
133	Q	500	0			0.9				
* 143	Q	500	412.50	450						
144	Q	625	0			0.9				
* 154	Q	625	417.19	450						
155	Q	750	0			0.9				
* 165	Q	750	421.88	450						
166	Q	875	0			0.9				
* 176	Q	875	426.56	450						
177	Q	1000	0			0.9				
* 187	Q	1000	431.25	450						
188	Q	1125	0			0.9				
* 198	Q	1125	435.94	450						
199	Q	1250	0			0.9				
* 209	Q	1250	440.63	450						
210	Q	1375	0			0.9				
* 220	Q	1375	445.31	450						
221	2 P	1500	0		450	0.9				
231	P	1500	450							
		255	405	2.0-05	2.0-05	-3	1500	450		
		2.0-05		300	450	-3	1500	450		
		275	375	2.5-06	2.5-06	-3	1500	405		
		2.5-06	2.5-06	375	405	-3	1500	405		
		155	305	2.0-06	1.0-06	167.2	-3	1500	85.87	
		2.0-06	1.0-06	167.2	5.0-07	5.0-07	1.0-8	-4		
		0	300	150	300	155	305	1500	0	
1	1	12	15	2						
11	12	23	24	13						
21	23	34	35	24						
31	34	45	46	35						
41	45	56	57	46						
51	56	67	68	57						
61	67	78	79	68						
71	78	89	90	79						
81	89	100	101	90						
91	100	111	112	101						
101	111	122	123	112						
111	122	133	134	123						
121	133	144	145	134						
131	144	155	156	145						
141	155	166	167	156						
151	166	177	178	167						
161	177	188	189	178						
171	188	199	200	189						
181	199	210	211	200						
191	210	221	222	211						
200	219	230	231	220						

Name list  
\$FPM Cards

Node  
Cards

Material  
definition  
(Line)  
Cards

Element  
Cards

FEFPM - SAMPLE PROBLEM NO. 3 - OUTPUT - 1

```

*****
*
*           S E T P A G E   A N A L Y S I S
*
*   S A M P L E   3           O P E N   P I T   I N F L O W
*
*           01/28/76-11:52:12
*
*****

NUMBER OF NODAL POINTS-----231
NUMBER OF ELEMENTS-----200
NUMBER OF DIFF. MATERIALS---- 4

UNIT WEIGHT OF FLUID----- 1.0000+00
REFERENCE FOR POTENTIALS---- 0.0000

CORRECTION FACTOR----- 1.00000
MAX. NO. ITERATIONS----- 10
ERROR TOLERANCE----- .10000

AXISYMMETRIC FLOW PROBLEM

ASG10 DYNAMIC E.M. = #ASG,T      IFMP15,017/ 16850/THK/ 16850 .
ASG10 DYNAMIC F.M. = #USE      15,TEMP15 .
    
```

FEFPM - SAMPLE PROBLEM NO. 3 - OUTPUT - 2

* NODF	B.C.	X=ORD	Y=ORD	F	NODE	B.C.	X=ORD	Y=ORD	F	**	PAGE 2
1	CONT	.0000	.0000	0.0000	* 51	CONT	175.0000	220.1356	0.0000	0.0000	
2	CONT	.0000	46.0602	0.0000	* 52	CONT	175.0000	245.1034	0.0000	0.0000	
3	CONT	.0000	87.5144	0.0000	* 53	CONT	175.0000	267.5745	0.0000	0.0000	
4	CONT	.0000	124.8231	0.0000	* 54	CONT	175.0000	287.7484	0.0000	0.0000	
5	CONT	.0000	158.4010	0.0000	* 55	CONT	175.0000	306.0000	0.0000	0.0000	
6	CONT	.0000	188.6211	0.0000	* 56	CONT	200.0000	.0000	0.0000	0.0000	
7	CONT	.0000	215.8192	0.0000	* 57	CONT	200.0000	46.9814	0.0000	0.0000	
8	CONT	.0000	240.2975	0.0000	* 58	CONT	200.0000	89.2647	0.0000	0.0000	
9	CONT	.0000	262.3279	0.0000	* 59	CONT	200.0000	127.3196	0.0000	0.0000	
10	CONT	.0000	282.1553	0.0000	* 60	CONT	200.0000	161.5690	0.0000	0.0000	
11	PRESS.	.0000	300.0000	0.0000	* 61	CONT	200.0000	192.3935	0.0000	0.0000	
12	CONT	50.0000	.0000	0.0000	* 62	CONT	200.0000	220.1356	0.0000	0.0000	
13	CONT	50.0000	46.0602	0.0000	* 63	CONT	200.0000	245.1034	0.0000	0.0000	
14	CONT	50.0000	87.5144	0.0000	* 64	CONT	200.0000	267.5745	0.0000	0.0000	
15	CONT	50.0000	124.8231	0.0000	* 65	CONT	200.0000	287.7484	0.0000	0.0000	
16	CONT	50.0000	158.4010	0.0000	* 66	PRESS.	200.0000	306.0000	0.0000	0.0000	
17	CONT	50.0000	188.6211	0.0000	* 67	CONT	225.0000	.0000	0.0000	0.0000	
18	CONT	50.0000	215.8192	0.0000	* 68	CONT	225.0000	50.5890	0.0000	0.0000	
19	CONT	50.0000	240.2975	0.0000	* 69	CONT	225.0000	96.4846	0.0000	0.0000	
20	CONT	50.0000	262.3279	0.0000	* 70	CONT	225.0000	137.6175	0.0000	0.0000	
21	CONT	50.0000	282.1553	0.0000	* 71	CONT	225.0000	174.6371	0.0000	0.0000	
22	PRESS.	50.0000	300.0000	0.0000	* 72	CONT	225.0000	207.9548	0.0000	0.0000	
23	CONT	100.0000	.0000	0.0000	* 73	CONT	225.0000	237.9407	0.0000	0.0000	
24	CONT	100.0000	46.0602	0.0000	* 74	CONT	225.0000	264.9280	0.0000	0.0000	
25	CONT	100.0000	87.5144	0.0000	* 75	CONT	225.0000	289.2165	0.0000	0.0000	
26	CONT	100.0000	124.8231	0.0000	* 76	CONT	225.0000	311.0763	0.0000	0.0000	
27	CONT	100.0000	158.4010	0.0000	* 77	CONT	225.0000	330.7500	0.0000	0.0000	
28	CONT	100.0000	188.6211	0.0000	* 78	CONT	250.0000	.0000	0.0000	0.0000	
29	CONT	100.0000	215.8192	0.0000	* 79	CONT	250.0000	50.5890	0.0000	0.0000	
30	CONT	100.0000	240.2975	0.0000	* 80	CONT	250.0000	103.7191	0.0000	0.0000	
31	CONT	100.0000	262.3279	0.0000	* 81	CONT	250.0000	147.9362	0.0000	0.0000	
32	CONT	100.0000	282.1553	0.0000	* 82	CONT	250.0000	187.7316	0.0000	0.0000	
33	PRESS.	100.0000	300.0000	0.0000	* 83	CONT	250.0000	223.5475	0.0000	0.0000	
34	CONT	150.0000	.0000	0.0000	* 84	CONT	250.0000	255.7817	0.0000	0.0000	
35	CONT	150.0000	46.0602	0.0000	* 85	CONT	250.0000	284.7926	0.0000	0.0000	
36	CONT	150.0000	87.5144	0.0000	* 86	CONT	250.0000	310.9023	0.0000	0.0000	
37	CONT	150.0000	124.8231	0.0000	* 87	CONT	250.0000	334.4011	0.0000	0.0000	
38	CONT	150.0000	158.4010	0.0000	* 88	CONT	250.0000	355.5500	0.0000	0.0000	
39	CONT	150.0000	188.6211	0.0000	* 89	CONT	275.0000	.0000	0.0000	0.0000	
40	CONT	150.0000	215.8192	0.0000	* 90	CONT	275.0000	58.3813	0.0000	0.0000	
41	CONT	150.0000	240.2975	0.0000	* 91	CONT	275.0000	110.9245	0.0000	0.0000	
42	CONT	150.0000	262.3279	0.0000	* 92	CONT	275.0000	158.2133	0.0000	0.0000	
43	CONT	150.0000	282.1553	0.0000	* 93	CONT	275.0000	200.7133	0.0000	0.0000	
44	PRESS.	150.0000	300.0000	0.0000	* 94	CONT	275.0000	239.0773	0.0000	0.0000	
45	CONT	175.0000	.0000	0.0000	* 95	CONT	275.0000	273.5508	0.0000	0.0000	
46	CONT	175.0000	46.9814	0.0000	* 96	CONT	275.0000	300.5771	0.0000	0.0000	
47	CONT	175.0000	89.2647	0.0000	* 97	CONT	275.0000	332.5007	0.0000	0.0000	
48	CONT	175.0000	127.3196	0.0000	* 98	CONT	275.0000	357.6150	0.0000	0.0000	
49	CONT	175.0000	161.5690	0.0000	* 99	CONT	275.0000	380.2500	0.0000	0.0000	
50	CONT	175.0000	192.3935	0.0000	* 100	CONT	300.0000	.0000	0.0000	0.0000	



FEFPM - SAMPLE PROBLEM NO. 3 - OUTPUT - 3

* SAMPLE 3 OPEN PIT INFLOW ** 01/28/76-11:52:12 ** PAGE 3											
NODE	H.C.	X=ORD	Y=ORD	F		NODE	H.C.	X=ORD	Y=ORD	F	
101	C0NT	300.0000	62.1813	0.0000	0.0000	*	151	C0NT	625.0000	334.1657	0.0000
102	C0NT	300.0000	118.1444	0.0000	0.0000	*	152	C0NT	625.0000	364.8020	0.0000
103	C0NT	300.0000	168.5112	0.0000	0.0000	*	153	C0NT	625.0000	392.3746	0.0000
104	C0NT	300.0000	213.8414	0.0000	0.0000	*	154	C0NT	625.0000	417.1900	0.0000
105	C0NT	300.0000	254.6385	0.0000	0.0000	*	155	C0NT	750.0000	.0000	0.0000
106	C0NT	300.0000	291.3559	0.0000	0.0000	*	156	C0NT	750.0000	64.7729	0.0000
107	C0NT	300.0000	324.4016	0.0000	0.0000	*	157	C0NT	750.0000	123.0686	0.0000
108	C0NT	300.0000	354.1427	0.0000	0.0000	*	158	C0NT	750.0000	175.5346	0.0000
109	C0NT	300.0000	380.9097	0.0000	0.0000	*	159	C0NT	750.0000	222.7541	0.0000
110	C0NT	300.0000	405.0000	0.0000	0.0000	*	160	C0NT	750.0000	265.2516	0.0000
111	C0NT	337.5000	.0000	0.0000	0.0000	*	161	C0NT	750.0000	303.4944	0.0000
112	C0NT	337.5000	62.3978	0.0000	0.0000	*	162	C0NT	750.0000	337.9223	0.0000
113	C0NT	337.5000	118.5557	0.0000	0.0000	*	163	C0NT	750.0000	368.9030	0.0000
114	C0NT	337.5000	169.0979	0.0000	0.0000	*	164	C0NT	750.0000	396.7856	0.0000
115	C0NT	337.5000	214.5459	0.0000	0.0000	*	165	C0NT	750.0000	421.8800	0.0000
116	C0NT	337.5000	255.5250	0.0000	0.0000	*	166	C0NT	875.0000	.0000	0.0000
117	C0NT	337.5000	292.3703	0.0000	0.0000	*	167	C0NT	875.0000	65.4915	0.0000
118	C0NT	337.5000	325.5510	0.0000	0.0000	*	168	C0NT	875.0000	124.4338	0.0000
119	C0NT	337.5000	355.3757	0.0000	0.0000	*	169	C0NT	875.0000	177.4819	0.0000
120	C0NT	337.5000	382.2358	0.0000	0.0000	*	170	C0NT	875.0000	225.2251	0.0000
121	C0NT	337.5000	406.4100	0.0000	0.0000	*	171	C0NT	875.0000	268.1941	0.0000
122	C0NT	400.0000	.0000	0.0000	0.0000	*	172	C0NT	875.0000	306.8661	0.0000
123	C0NT	400.0000	62.7570	0.0000	0.0000	*	173	C0NT	875.0000	341.6710	0.0000
124	C0NT	400.0000	119.2383	0.0000	0.0000	*	174	C0NT	875.0000	372.9953	0.0000
125	C0NT	400.0000	170.0715	0.0000	0.0000	*	175	C0NT	875.0000	401.1673	0.0000
126	C0NT	400.0000	215.8214	0.0000	0.0000	*	176	C0NT	875.0000	426.5600	0.0000
127	C0NT	400.0000	256.9963	0.0000	0.0000	*	177	C0NT	1000.0000	.0000	0.0000
128	C0NT	400.0000	294.8537	0.0000	0.0000	*	178	C0NT	1000.0000	66.2115	0.0000
129	C0NT	400.0000	327.4053	0.0000	0.0000	*	179	C0NT	1000.0000	125.8019	0.0000
130	C0NT	400.0000	357.0218	0.0000	0.0000	*	180	C0NT	1000.0000	179.4333	0.0000
131	C0NT	400.0000	384.4367	0.0000	0.0000	*	181	C0NT	1000.0000	227.7015	0.0000
132	C0NT	400.0000	408.7500	0.0000	0.0000	*	182	C0NT	1000.0000	271.1429	0.0000
133	C0NT	500.0000	.0000	0.0000	0.0000	*	183	C0NT	1000.0000	310.2801	0.0000
134	C0NT	500.0000	63.3328	0.0000	0.0000	*	184	C0NT	1000.0000	345.4276	0.0000
135	C0NT	500.0000	120.3423	0.0000	0.0000	*	185	C0NT	1000.0000	377.0564	0.0000
136	C0NT	500.0000	171.8318	0.0000	0.0000	*	186	C0NT	1000.0000	405.5983	0.0000
137	C0NT	500.0000	217.8014	0.0000	0.0000	*	187	C0NT	1000.0000	431.2500	0.0000
138	C0NT	500.0000	259.4540	0.0000	0.0000	*	188	C0NT	1125.0000	.0000	0.0000
139	C0NT	500.0000	296.7514	0.0000	0.0000	*	189	C0NT	1125.0000	66.9316	0.0000
140	C0NT	500.0000	330.4090	0.0000	0.0000	*	190	C0NT	1125.0000	127.1701	0.0000
141	C0NT	500.0000	360.7009	0.0000	0.0000	*	191	C0NT	1125.0000	181.3847	0.0000
142	C0NT	500.0000	387.4636	0.0000	0.0000	*	192	C0NT	1125.0000	230.1778	0.0000
143	C0NT	500.0000	412.5000	0.0000	0.0000	*	193	C0NT	1125.0000	274.0916	0.0000
144	C0NT	625.0000	.0000	0.0000	0.0000	*	194	C0NT	1125.0000	313.6141	0.0000
145	C0NT	625.0000	64.0528	0.0000	0.0000	*	195	C0NT	1125.0000	349.1843	0.0000
146	C0NT	625.0000	121.2704	0.0000	0.0000	*	196	C0NT	1125.0000	381.1975	0.0000
147	C0NT	625.0000	173.5832	0.0000	0.0000	*	197	C0NT	1125.0000	410.8093	0.0000
148	C0NT	625.0000	220.2777	0.0000	0.0000	*	198	C0NT	1125.0000	435.9400	0.0000
149	C0NT	625.0000	262.5028	0.0000	0.0000	*	199	C0NT	1250.0000	.0000	0.0000
150	C0NT	625.0000	300.1254	0.0000	0.0000	*	200	C0NT	1250.0000	67.6517	0.0000

FEFPM - SAMPLE PROBLEM NO. 3 - OUTPUT - 4

* SAMPLE 5 OPEN PIT INFLOW ** 01/28/76-11:52:12 ** PAGE 4										
NODE	H.C.	X=ORD	Y=ORD	F						
201	C0NT	1250.0000	128.5382	0.0000	0.0000	*				
202	C0NT	1250.0000	183.5361	0.0000	0.0000	*				
203	C0NT	1250.0000	232.6542	0.0000	0.0000	*				
204	C0NT	1250.0000	277.0904	0.0000	0.0000	*				
205	C0NT	1250.0000	316.9881	0.0000	0.0000	*				
206	C0NT	1250.0000	352.9409	0.0000	0.0000	*				
207	C0NT	1250.0000	385.2985	0.0000	0.0000	*				
208	C0NT	1250.0000	414.4204	0.0000	0.0000	*				
209	C0NT	1250.0000	440.6300	0.0000	0.0000	*				
210	C0NT	1375.0000	.0000	0.0000	0.0000	*				
211	C0NT	1375.0000	68.3702	0.0000	0.0000	*				
212	C0NT	1375.0000	124.9034	0.0000	0.0000	*				
213	C0NT	1375.0000	185.2853	0.0000	0.0000	*				
214	C0NT	1375.0000	235.1252	0.0000	0.0000	*				
215	C0NT	1375.0000	279.9829	0.0000	0.0000	*				
216	C0NT	1375.0000	320.3548	0.0000	0.0000	*				
217	C0NT	1375.0000	356.6896	0.0000	0.0000	*				
218	C0NT	1375.0000	389.3908	0.0000	0.0000	*				
219	C0NT	1375.0000	418.8220	0.0000	0.0000	*				
220	C0NT	1375.0000	445.3100	0.0000	0.0000	*				
221	PRESS.	1500.0000	.0000	4.5000+02	0.0000	*				
222	PRESS.	1500.0000	69.0903	3.8091+02	0.0000	*				
223	PRESS.	1500.0000	131.2716	3.1873+02	0.0000	*				
224	PRESS.	1500.0000	187.2347	2.6277+02	0.0000	*				
225	PRESS.	1500.0000	237.6015	2.1240+02	0.0000	*				
226	PRESS.	1500.0000	282.9317	1.6707+02	0.0000	*				
227	PRESS.	1500.0000	323.7288	1.2627+02	0.0000	*				
228	PRESS.	1500.0000	360.4462	8.4554+01	0.0000	*				
229	PRESS.	1500.0000	393.0919	5.8508+01	0.0000	*				
230	PRESS.	1500.0000	423.2330	2.6767+01	0.0000	*				
231	PRESS.	1500.0000	450.0000	0.0000	0.0000	*				

FEFPM - SAMPLE PROBLEM NO. 3 - OUTPUT - 5

\* SAMPLE 3 OPEN PIT INFLOW \*\* 01/28/76-11:52:12 \*\* PAGE 5

LINE	PROPERTIES	ABOVE	MATERIAL	PROPERTIES	RELOW	DELIMITING	NODE	STRING
1	0.00	0.00	.0	2.00-05	2.00-05	.0	2.550+02	4.050+02 1.500+03 4.500+02
2	2.00-05	2.00-05	.0	2.50-06	2.50-06	.0	2.250+02	3.750+02 1.500+03 4.050+02
3	2.50-06	2.50-06	.0	2.00-06	1.00-06	167.2	1.550+02	3.050+02 1.500+03 4.587+01
4	2.00-06	1.00-06	167.2	5.00-07	5.00-07	.0	0.000	3.000+02 1.550+02 3.050+02

FEFPM - SAMPLE PROBLEM NO. 3 - OUTPUT - 6

\* SAMPLE 3 OPEN PIT INFLOW \*\* 01/28/76-11:52:12 \*\* PAGE 6

ELMT	I	J	K	L	MAT
1	1	12	13	2	0
2	2	13	14	3	0
3	3	14	15	4	0
4	4	15	16	5	0
5	5	16	17	6	0
6	6	17	18	7	0
7	7	18	19	8	0
8	8	19	20	9	0
9	9	20	21	10	0
10	10	21	22	11	0
11	11	22	23	12	0
12	12	23	24	13	0
13	13	24	25	14	0
14	14	25	26	15	0
15	15	26	27	16	0
16	16	27	28	17	0
17	17	28	29	18	0
18	18	29	30	19	0
19	19	30	31	20	0
20	20	31	32	21	0
21	21	32	33	22	0
22	22	33	34	23	0
23	23	34	35	24	0
24	24	35	36	25	0
25	25	36	37	26	0
26	26	37	38	27	0
27	27	38	39	28	0
28	28	39	40	29	0
29	29	40	41	30	0
30	30	41	42	31	0
31	31	42	43	32	0
32	32	43	44	33	0
33	33	44	45	34	0
34	34	45	46	35	0
35	35	46	47	36	0
36	36	47	48	37	0
37	37	48	49	38	0
38	38	49	50	39	0
39	39	50	51	40	0
40	40	51	52	41	0
41	41	52	53	42	0
42	42	53	54	43	0
43	43	54	55	44	0
44	44	55	56	45	0
45	45	56	57	46	0
46	46	57	58	47	0
47	47	58	59	48	0
48	48	59	60	49	0
49	49	60	61	50	0
50	50	61	62	51	0
51	51	62	63	52	0
52	52	63	64	53	0
53	53	64	65	54	0
54	54	65	66	55	0

## FEFPM - SAMPLE PROBLEM NO. 3 - OUTPUT - 7

* ELMT	S A M P L E 3			O P E N P I T I N F L O W		** 01/28/76-11:52:12 **	PAGE 7
	I	J	K	L	HAT		
51	56	67	68	57	0		
52	57	68	69	58	0		
53	58	69	70	59	0		
54	59	70	71	60	0		
55	60	71	72	61	0		
56	61	72	73	62	0		
57	62	73	74	63	0		
58	63	74	75	64	0		
59	64	75	76	65	0		
60	65	76	77	66	0		
61	67	78	79	68	0		
62	68	79	80	69	0		
63	69	80	81	70	0		
64	70	81	82	71	0		
65	71	82	83	72	0		
66	72	83	84	73	0		
67	73	84	85	74	0		
68	74	85	86	75	0		
69	75	86	87	76	0		
70	76	87	88	77	0		
71	78	89	90	79	0		
72	79	90	91	80	0		
73	80	91	92	81	0		
74	81	92	93	82	0		
75	82	93	94	83	0		
76	83	94	95	84	0		
77	84	95	96	85	0		
78	85	96	97	86	0		
79	86	97	98	87	0		
80	87	98	99	88	0		
81	89	100	101	90	0		
82	90	101	102	91	0		
83	91	102	103	92	0		
84	92	103	104	93	0		
85	93	104	105	94	0		
86	94	105	106	95	0		
87	95	106	107	96	0		
88	96	107	108	97	0		
89	97	108	109	98	0		
90	98	109	110	99	0		
91	100	111	112	101	0		
92	101	112	113	102	0		
93	102	113	114	103	0		
94	103	114	115	104	0		
95	104	115	116	105	0		
96	105	116	117	106	0		
97	106	117	118	107	0		
98	107	118	119	108	0		
99	108	119	120	109	0		
100	109	120	121	110	0		
101	111	122	123	112	0		
102	112	123	124	113	0		
103	113	124	125	114	0		
104	114	125	126	115	0		
105	115	126	127	116	0		
106	116	127	128	117	0		
107	117	128	129	118	0		
108	118	129	130	119	0		
109	119	130	131	120	0		
110	120	131	132	121	0		
111	122	133	134	123	0		
112	123	134	135	124	0		
113	124	135	136	125	0		
114	125	136	137	126	0		
115	126	137	138	127	0		
116	127	138	139	128	0		
117	128	139	140	129	0		
118	129	140	141	130	0		
119	130	141	142	131	0		
120	131	142	143	132	0		
121	133	144	145	134	0		
122	134	145	146	135	0		
123	135	146	147	136	0		
124	136	147	148	137	0		
125	137	148	149	138	0		
126	138	149	150	139	0		
127	139	150	151	140	0		
128	140	151	152	141	0		
129	141	152	153	142	0		
130	142	153	154	143	0		
131	144	155	156	145	0		
132	145	156	157	146	0		
133	146	157	158	147	0		
134	147	158	159	148	0		
135	148	159	160	149	0		
136	149	160	161	150	0		
137	150	161	162	151	0		
138	151	162	163	152	0		
139	152	163	164	153	0		
140	153	164	165	154	0		
141	155	166	167	156	0		
142	156	167	168	157	0		
143	157	168	169	158	0		
144	158	169	170	159	0		
145	159	170	171	160	0		
146	160	171	172	161	0		
147	161	172	173	162	0		
148	162	173	174	163	0		
149	163	174	175	164	0		
150	164	175	176	165	0		

PAGE 8

## FEFPM - SAMPLE PROBLEM NO. 3 - OUTPUT - 8

\*                    S A M P L E   3                    O P E N   P I T   I N F L O W                    \*\*   01/28/76-11:52:12                    \*\*   P A G E   9

ELMT	I	J	K	L	MAT
151	166	177	178	167	0
152	167	178	179	168	0
153	168	179	180	169	0
154	169	180	181	170	0
155	170	181	182	171	0
156	171	182	183	172	0
157	172	183	184	173	0
158	173	184	185	174	0
159	174	185	186	175	0
160	175	186	187	176	0
161	177	188	189	178	0
162	178	189	190	179	0
165	179	190	191	180	0
164	180	191	192	181	0
165	181	192	193	182	0
166	182	193	194	183	0
167	183	194	195	184	0
168	184	195	196	185	0
169	185	196	197	186	0
170	186	197	198	187	0
171	188	199	200	189	0
172	189	200	201	190	0
173	190	201	202	191	0
174	191	202	203	192	0
175	192	203	204	193	0
176	193	204	205	194	0
177	194	205	206	195	0
178	195	206	207	196	0
179	196	207	208	197	0
180	197	208	209	198	0
181	199	210	211	200	0
182	200	211	212	201	0
183	201	212	213	202	0
184	202	213	214	203	0
185	203	214	215	204	0
186	204	215	216	205	0
187	205	216	217	206	0
188	206	217	218	207	0
189	207	218	219	208	0
190	208	219	220	209	0
191	210	221	222	211	0
192	211	222	223	212	0
193	212	223	224	213	0
194	213	224	225	214	0
195	214	225	226	215	0
196	215	226	227	216	0
197	216	227	228	217	0
198	217	228	229	218	0
199	218	229	230	219	0
200	219	230	231	220	0

## FEFPM - SAMPLE PROBLEM NO. 3 - OUTPUT - 9

\*\*\*\*\* FREE SURFACE \*\*\*\*\*

NODE	CURR. ANGLE	Y(MAX)
77	90.0	375.000
88	90.0	400.000
99	90.0	425.000
110	90.0	450.000
121	90.0	450.000
132	90.0	450.000
143	90.0	450.000
154	90.0	450.000
165	90.0	450.000
176	90.0	450.000
187	90.0	450.000
198	90.0	450.000
209	90.0	450.000
220	90.0	450.000

CPU TIME USED IN S/R MESHIN = 4.961 SECONDS

TOTAL SPACE ACQUIRED FOR /MATRIX/ IS 7955

MAXIMUM BANDWIDTH = 13

ASGID DYNAMIC E,R. = #ASG,T                    TEMP14,F17/                    2/TRK/                    2.

ASGID DYNAMIC E,R. = #USE                    14,TEMP14.

FEFPM - SAMPLE PROBLEM NO. 3 - OUTPUT - 10

\* SAMPLE 3 OPEN PIT INFLOW \*\* 01/28/76-11:52:12 \*\* PAGE 10

TOTAL MESH AREA= 6.0877680+05 CPU TIME TO BUILD FLOW MATRIX = 38.370 SECONDS.

\*\*\*\*\* ITERATION 1

TIME TO SOLVE MATRIX = 1.292 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 1.55+01  
 PHREATIC SURFACE END-NODE 77 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .4578 DX,DY= 2.1257-07 4.4250+01  
 PHREATIC SURFACE END-NODE 220 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .0116 DX,DY= -9.5367-07 3.3710+00

NODE	X-ORD	Y-ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
77	225.0000	330.7500	.38556+02	.36931+03	.0000	44.2500	.0000	.0000
88	250.0000	355.5500	.36276+02	.39183+03	.0000	36.2757	.0000	-1.0000
99	275.0000	380.2500	.29444+02	.40074+03	.0000	20.4684	.0000	-1.0000
110	300.0000	405.0000	-.41412+00	.40459+03	-.0000	-.4141	.0000	-1.0000
121	325.0000	406.4100	.18242+01	.40874+03	.0000	1.8282	.0000	-1.0000
132	400.0000	408.7500	.84417+01	.41719+03	.0000	8.4417	.0000	-1.0000
143	500.0000	412.5000	.11772+02	.42427+03	.0000	11.7720	.0000	-1.0000
154	625.0000	417.1900	.15479+02	.43267+03	.0000	15.4794	.0000	-1.0000
165	750.0000	421.8800	.14980+02	.45666+03	.0000	14.9799	.0000	-1.0000
176	875.0000	426.5600	.13973+02	.44053+03	.0000	13.9730	.0000	-1.0000
187	1000.0000	431.2500	.11929+02	.44318+03	.0000	11.9289	.0000	-1.0000
198	1125.0000	435.9400	.95515+01	.44549+03	.0000	9.5515	.0000	-1.0000
209	1250.0000	440.6300	.65998+01	.44723+03	.0000	6.5998	.0000	-1.0000
220	1375.0000	445.3100	.34752+01	.44479+03	-.0000	3.3710	.0000	.0000

TOTAL MESH AREA= 6.2287634+05 CPU TIME TO BUILD FLOW MATRIX = 36.880 SECONDS.

\*\*\*\*\* ITERATION 2

TIME TO SOLVE MATRIX = 1.225 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 5.42+00  
 PHREATIC SURFACE END-NODE 77 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .2657 DX,DY= 9.9288-13 3.8147-06  
 PHREATIC SURFACE END-NODE 220 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .0153 DX,DY= -9.5367-07 -1.8325-01

NODE	X-ORD	Y-ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
77	225.0000	375.0000	.00000	.37500+03	.0000	.0000	.0000	.0000
88	250.0000	391.8257	-.92677+01	.38256+03	-.0000	-9.2677	.0000	-1.0000
99	275.0000	400.7384	-.11640+02	.38905+03	-.0000	-11.6898	.0000	-1.0000
110	300.0000	404.5854	-.93501+01	.39524+03	-.0000	-9.3501	.0000	-1.0000
121	325.0000	408.2382	-.15290+01	.40651+03	-.0000	-1.5299	.0000	-1.0000
132	400.0000	417.1917	-.22531+01	.41494+03	-.0000	-2.2531	.0000	-1.0000
143	500.0000	424.2267	-.20445+01	.42223+03	-.0000	-2.0445	.0000	-1.0000
154	625.0000	432.6698	-.38445+01	.42882+03	-.0000	-3.8445	.0000	-1.0000
165	750.0000	436.8599	-.25977+01	.43426+03	-.0000	-2.5977	.0000	-1.0000
176	875.0000	440.5336	-.23828+01	.43815+03	-.0000	-2.3828	.0000	-1.0000
187	1000.0000	443.1789	-.17100+01	.44147+03	-.0000	-1.7100	.0000	-1.0000
198	1125.0000	445.4415	-.11402+01	.44435+03	-.0000	-1.1402	.0000	-1.0000
209	1250.0000	447.2298	-.64543+00	.44659+03	-.0000	-.6454	.0000	-1.0000
220	1375.0000	448.6810	-.30315+00	.44838+03	-.0000	-.3032	.0000	.0000

TOTAL MESH AREA= 6.2001770+05 CPU TIME TO BUILD FLOW MATRIX = 36.660 SECONDS.

\*\*\*\*\* ITERATION 3

TIME TO SOLVE MATRIX = 1.214 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 1.34+00  
 PHREATIC SURFACE END-NODE 77 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .3050 DX,DY= 2.3842-07 -3.5710+00  
 PHREATIC SURFACE END-NODE 220 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .0180 DX,DY= -9.5367-07 4.7039-01

NODE	X-ORD	Y-ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
77	225.0000	375.0000	.00000	.37500+03	.0000	-3.5710	.0000	.0000
88	250.0000	382.5581	-.35052+01	.37905+03	-.0000	-3.5052	.0000	-1.0000
99	275.0000	389.0405	-.23350+01	.38671+03	-.0000	-2.3350	.0000	-1.0000
110	300.0000	395.2358	-.78805+00	.39445+03	-.0000	-.7885	.0000	-1.0000
121	325.0000	400.9083	-.87590+00	.40603+03	-.0000	-.8759	.0000	-1.0000
132	400.0000	414.9346	-.14004+01	.41354+03	-.0000	-1.4004	.0000	-1.0000
143	500.0000	422.2267	-.26888+00	.42196+03	-.0000	-.2689	.0000	-1.0000
154	625.0000	428.8249	.52806+00	.42935+03	.0000	.5281	.0000	-1.0000
165	750.0000	434.2622	.30530+00	.43457+03	.0000	.3053	.0000	-1.0000
176	875.0000	438.1503	.16524+00	.43832+03	.0000	.1652	.0000	-1.0000
187	1000.0000	441.4689	.51557+01	.44152+03	.0000	.0516	.0000	-1.0000
198	1125.0000	444.3513	-.61996+01	.44429+03	-.0000	-.0620	.0000	-1.0000
209	1250.0000	446.4864	.12825+00	.44671+03	.0000	.1283	.0000	-1.0000
220	1375.0000	448.4978	-.54532-01	.44844+03	-.0000	-.4704	.0000	.0000

TOTAL MESH AREA= 6.1976854+05 CPU TIME TO BUILD FLOW MATRIX = 39.270 SECONDS.

FEFPM - SAMPLE PROBLEM NO. 3 - OUTPUT - 11

\*\*\*\*\* ITERATION 4

TIME TO SOLVE MATRIX = 1.282 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 2.95+00  
PHREATIC SURFACE END=NODE 77 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .2386 DX,DY= 1.0359-07 3.5710+00  
PHREATIC SURFACE END=NODE 220 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .0141 DX,DY= -9.5367-07 -3.4272-01

NODE	X=ORD	Y=ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
77	225.0000	371.4290	.12802+02	.38423+03	.0000	3.5710	.0000	.0000
88	250.0000	379.0529	.45590+01	.38561+03	.0000	6.5590	.0000	-1.0000
99	275.0000	380.7135	.50999+01	.39181+03	.0000	5.0999	.0000	-1.0000
110	300.0000	390.4474	.40424+01	.39849+03	.0000	4.0423	.0000	-1.0000
121	337.5000	400.0320	.27552+01	.40879+03	.0000	2.7552	.0000	-1.0000
132	400.0000	413.5382	.21990+01	.41574+03	.0000	2.1994	.0000	-1.0000
143	500.0000	429.9593	.16948+01	.42365+03	.0000	1.6948	.0000	-1.0000
154	625.0000	429.3530	.12651+01	.43062+03	.0000	1.2651	.0000	-1.0000
165	750.0000	430.5675	.97503+00	.43554+03	.0000	.9754	.0000	-1.0000
176	875.0000	430.3155	.78420+00	.43910+03	.0000	.7842	.0000	-1.0000
187	1000.0000	441.5200	.84510+00	.44218+03	.0000	.8352	.0000	-1.0000
198	1125.0000	440.2893	.52040+00	.44481+03	.0000	.5204	.0000	-1.0000
209	1250.0000	440.7147	.15220+00	.44687+03	.0000	.1522	.0000	-1.0000
220	1375.0000	448.4082	-.45720+00	.44851+03	-.0000	-.3427	.0000	.0000

TOTAL MESH AREA= 6.2128151+05 CPU TIME TO BUILD FLOW MATRIX = 36.940 SECONDS.

\*\*\*\*\* ITERATION 5

TIME TO SOLVE MATRIX = 1.277 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 2.47+00  
PHREATIC SURFACE END=NODE 77 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .3117 DX,DY= 2.3842+07 -2.8748+00  
PHREATIC SURFACE END=NODE 220 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .0150 DX,DY= -9.5367-07 -9.6918-02

NODE	X=ORD	Y=ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
77	225.0000	375.0000	.00000	.37500+03	.0000	-2.8748	.0000	.0000
88	250.0000	385.0114	-.56947+01	.37992+03	-.0000	-5.6947	.0000	-1.0000
99	275.0000	391.8135	-.42026+01	.38761+03	-.0000	-4.2026	.0000	-1.0000
110	300.0000	398.4897	-.33824+01	.39511+03	-.0000	-3.3824	.0000	-1.0000
121	337.5000	400.7876	-.22036+01	.40658+03	-.0000	-2.2036	.0000	-1.0000
132	400.0000	415.7376	-.14883+01	.41425+03	-.0000	-1.4883	.0000	-1.0000
143	500.0000	425.6541	-.78596+00	.42287+03	-.0000	-.7860	.0000	-1.0000
154	625.0000	430.8180	-.14285+01	.42919+03	-.0000	-1.4285	.0000	-1.0000
165	750.0000	430.5029	-.10000+01	.43454+03	-.0000	-1.0000	.0000	-1.0000
176	875.0000	439.0997	-.73618+00	.43836+03	-.0000	-.7362	.0000	-1.0000
187	1000.0000	442.1556	-.52626+00	.44163+03	-.0000	-.5263	.0000	-1.0000
198	1125.0000	440.8097	-.30919+00	.44468+03	-.0000	-.3092	.0000	-1.0000
209	1250.0000	440.8684	-.21287+00	.44665+03	-.0000	-.2129	.0000	-1.0000
220	1375.0000	448.6255	-.21502+00	.44801+03	-.0000	-.0969	.0000	.0000

TOTAL MESH AREA= 6.1999328+05 CPU TIME TO BUILD FLOW MATRIX = 38.010 SECONDS.

\*\*\*\*\* ITERATION 6

TIME TO SOLVE MATRIX = 1.247 SECONDS.

RMS PHREATIC SURFACE NODE MOVEMENT = 2.22+00  
PHREATIC SURFACE END=NODE 77 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .2065 DX,DY= 1.1178-07 2.8748+00  
PHREATIC SURFACE END=NODE 220 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE= .0171 DX,DY= -9.5367-07 4.9008-01

NODE	X=ORD	Y=ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
77	225.0000	372.1252	.12145+02	.38427+03	.0000	2.8748	.0000	.0000
88	250.0000	379.9173	.49921+01	.38441+03	.0000	4.9921	.0000	-1.0000
99	275.0000	387.6108	.39696+01	.39158+03	.0000	3.9696	.0000	-1.0000
110	300.0000	395.1073	.31592+01	.39827+03	.0000	3.1592	.0000	-1.0000
121	337.5000	400.5840	.20098+01	.40859+03	.0000	2.0098	.0000	-1.0000
132	400.0000	410.2514	.13012+01	.41555+03	.0000	1.3012	.0000	-1.0000
143	500.0000	422.8681	.57738+00	.42345+03	.0000	.5774	.0000	-1.0000
154	625.0000	429.1896	.12125+01	.43040+03	.0000	1.2125	.0000	-1.0000
165	750.0000	430.5426	.79644+00	.43534+03	.0000	.7964	.0000	-1.0000
176	875.0000	430.3636	.53113+00	.43899+03	.0000	.5311	.0000	-1.0000
187	1000.0000	441.6294	.30926+00	.44194+03	.0000	.3093	.0000	-1.0000
198	1125.0000	440.4606	.10940+00	.44457+03	.0000	.1094	.0000	-1.0000
209	1250.0000	440.6540	.22170+00	.44688+03	.0000	.2217	.0000	-1.0000
220	1375.0000	448.5286	-.89154-02	.44852+03	-.0000	.4901	.0000	.0000

TOTAL MESH AREA= 6.2111791+05 CPU TIME TO BUILD FLOW MATRIX = 37.630 SECONDS.

FEFPM - SAMPLE PROBLEM NO. 3 - OUTPUT - 12

\*\*\*\*\* ITERATION 7

TIME TO SOLVE MATRIX = 1.243 SECONDS.

HMS PHREATIC SURFACE NODE MOVEMENT = 2.27+00  
 PHREATIC SURFACE END-NODE 77 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE = .3208 DX,DY= 2.3842-07 -3.2894+00  
 PHREATIC SURFACE END-NODE 220 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE = .0150 DX,DY= -9.5367-07 -4.8538-01

NODE	X-ORD	Y-ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
77	225.0000	375.0000	.00000	.37500+03	.0000	-3.2894	.0000	.0000
88	250.0000	384.9094	-.51789+01	.37973+03	-.0000	-5.1789	.0000	-1.0000
99	275.0000	391.5805	-.40044+01	.38758+03	-.0000	-4.0044	.0000	-1.0000
110	300.0000	398.2665	-.31935+01	.39507+03	-.0000	-3.1935	.0000	-1.0000
121	337.5000	404.9938	-.20293+01	.40656+03	-.0000	-2.0293	.0000	-1.0000
132	400.0000	415.5526	-.13155+01	.41424+03	-.0000	-1.3155	.0000	-1.0000
143	500.0000	425.4455	-.58644+00	.42286+03	-.0000	-.5865	.0000	-1.0000
154	625.0000	430.4020	-.12177+01	.42918+03	-.0000	-1.2177	.0000	-1.0000
165	750.0000	435.3390	-.79494+00	.43454+03	-.0000	-.7949	.0000	-1.0000
176	875.0000	438.8947	-.55203+00	.43836+03	-.0000	-.5320	.0000	-1.0000
187	1000.0000	441.9386	-.30875+00	.44163+03	-.0000	-.3087	.0000	-1.0000
198	1125.0000	444.5700	-.10681+00	.44444+03	-.0000	-.1068	.0000	-1.0000
209	1250.0000	446.8757	-.21785+00	.44666+03	-.0000	-.2178	.0000	-1.0000
220	1375.0000	449.0186	-.60707+00	.44841+03	-.0000	-.4854	.0000	.0000

TOTAL MESH AREA= 6.1997352+05 CPU TIME TO BUILD FLOW MATRIX = 34.080 SECONDS.

\*\*\*\*\* ITERATION 8

TIME TO SOLVE MATRIX = 1.268 SECONDS.

HMS PHREATIC SURFACE NODE MOVEMENT = 2.53+00  
 PHREATIC SURFACE END-NODE 77 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE = .2349 DX,DY= 9.5481-08 3.2894+00  
 PHREATIC SURFACE END-NODE 220 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE = .0171 DX,DY= -9.5367-07 4.8767-01

NODE	X-ORD	Y-ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
77	225.0000	371.7106	.12586+02	.38430+03	.0000	3.2894	.0000	.0000
88	250.0000	379.7305	.00668+01	.38580+03	.0000	6.0668	.0000	-1.0000
99	275.0000	387.5761	.43254+01	.39190+03	.0000	4.3254	.0000	-1.0000
110	300.0000	395.0731	.33950+01	.39847+03	.0000	3.3950	.0000	-1.0000
121	337.5000	402.5645	.21537+01	.40872+03	.0000	2.1537	.0000	-1.0000
132	400.0000	414.2371	.14085+01	.41584+03	.0000	1.4085	.0000	-1.0000
143	500.0000	422.8540	.65281+00	.42351+03	.0000	.6528	.0000	-1.0000
154	625.0000	424.1804	.12584+01	.43005+03	.0000	1.2584	.0000	-1.0000
165	750.0000	429.5401	.85384+00	.43537+03	.0000	.8539	.0000	-1.0000
176	875.0000	434.3627	.55831+00	.43842+03	.0000	.5583	.0000	-1.0000
187	1000.0000	438.1629	.32777+00	.44196+03	.0000	.3278	.0000	-1.0000
198	1125.0000	444.4652	.11960+00	.44458+03	.0000	.1196	.0000	-1.0000
209	1250.0000	446.6579	.22520+00	.44688+03	.0000	.2252	.0000	-1.0000
220	1375.0000	448.5333	-.10151+01	.44852+03	-.0000	.4877	.0000	.0000

TOTAL MESH AREA= 6.2119444+05 CPU TIME TO BUILD FLOW MATRIX = 38.050 SECONDS.

\*\*\*\*\* ITERATION 9

TIME TO SOLVE MATRIX = 1.211 SECONDS.

HMS PHREATIC SURFACE NODE MOVEMENT = 2.48+00  
 PHREATIC SURFACE END-NODE 77 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE = .3088 DX,DY= 2.3842-07 -2.7601+00  
 PHREATIC SURFACE END-NODE 220 EXTRAPOLATED FROM NEIGHBOUR AT SLOPE = .0150 DX,DY= -9.5367-07 -4.8780-01

NODE	X-ORD	Y-ORD	PRESSURE	POTENTIAL	DX	DY	DP/DX	DP/DY
77	225.0000	375.0000	.00000	.37500+03	.0000	-2.7601	.0000	.0000
88	250.0000	385.7973	-.58375+01	.37996+03	-.0000	-5.8375	.0000	-1.0000
99	275.0000	391.9015	-.43012+01	.38760+03	-.0000	-4.3012	.0000	-1.0000
110	300.0000	398.4681	-.33862+01	.39508+03	-.0000	-3.3862	.0000	-1.0000
121	337.5000	404.7182	-.21531+01	.40657+03	-.0000	-2.1531	.0000	-1.0000
132	400.0000	415.6455	-.14086+01	.41424+03	-.0000	-1.4086	.0000	-1.0000
143	500.0000	423.5118	-.65325+00	.42286+03	-.0000	-.6532	.0000	-1.0000
154	625.0000	430.4094	-.12656+01	.42918+03	-.0000	-1.2656	.0000	-1.0000
165	750.0000	435.3739	-.85450+00	.43454+03	-.0000	-.8345	.0000	-1.0000
176	875.0000	438.9210	-.55890+00	.43836+03	-.0000	-.5589	.0000	-1.0000
187	1000.0000	441.9576	-.32827+00	.44163+03	-.0000	-.3283	.0000	-1.0000
198	1125.0000	444.5828	-.11999+00	.44444+03	-.0000	-.1200	.0000	-1.0000
209	1250.0000	446.8831	-.22548+00	.44666+03	-.0000	-.2255	.0000	-1.0000
220	1375.0000	449.0209	-.60950+00	.44841+03	-.0000	-.4878	.0000	.0000

TOTAL MESH AREA= 6.1999298+05 CPU TIME TO BUILD FLOW MATRIX = 36.870 SECONDS.

\*\*\*\*\* ITERATION 10

TIME TO SOLVE MATRIX = 1.216 SECONDS.

MAX. NUMBER OF ITERATIONS REACHED









FEPPM - SAMPLE PROBLEM NO. 3 - OUTPUT - 16

Table with columns: \* SAMPLE 3, OPEN PIT IN FLOW, \*\* 01/28/76=11:52:12, \*\* PAGE 17. Rows include NODE, H.C., x=0R0, y=0R0, 7=0R0, PRESSURE, POTENTIAL, FLOW FN, and OUTFLOW. The table contains numerical data for 200 nodes.

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FEFPM - SAMPLE PROBLEM NO. 3 - OUTPUT - 17

* SAMPLE 3		OPEN PIT INFLOW				** 01/28/76=11:52:12 ** PAGE 19		
NODE	B.C.	X=ORD	Y=ORD	Z=ORD	PRESSURE	POTENTIAL	FLOW FN	OUTFLOW
201	CONT.	1250.00	130.30	130.30	3.1657+02	4.4687+02	-3.4583-03	-3.1665-08
202	CONT.	1250.00	185.84	185.84	2.6096+02	4.4681+02	-6.0602-03	-2.7940-08
203	CONT.	1250.00	235.84	235.84	2.1093+02	4.4677+02	-8.4652-03	-3.7253-09
204	CONT.	1250.00	280.83	280.83	1.6593+02	4.4676+02	-1.0667-02	-2.9802-08
205	CONT.	1250.00	321.32	321.32	1.2505+02	4.4677+02	-1.2666-02	-7.4506-09
206	CONT.	1250.00	357.77	357.77	8.9055+01	4.4682+02	-1.4469-02	-7.4506-08
207	CONT.	1250.00	390.57	390.57	5.6338+01	4.4691+02	-1.6084-02	8.1956-08
208	CONT.	1250.00	420.09	420.09	2.6794+01	4.4688+02	-2.1781-02	1.0282-06
209	CONT.	1250.00	446.66	446.66	2.1735-01	4.4687+02	-3.2365-02	7.7486-07
210	CONT.	1375.00	.00	.00	4.4849+02	4.4849+02	0.0000	1.0747-08
211	CONT.	1375.00	68.87	68.87	3.7970+02	4.4856+02	-1.8916-03	-6.3330-08
212	CONT.	1375.00	130.84	130.84	3.1775+02	4.4859+02	-3.4932-03	5.2154-08
213	CONT.	1375.00	186.82	186.82	2.6191+02	4.4853+02	-6.4452-03	-4.0978-08
214	CONT.	1375.00	236.83	236.83	2.1168+02	4.4851+02	-8.6658-03	7.4506-09
215	CONT.	1375.00	282.01	282.01	1.6684+02	4.4850+02	-1.0655-02	1.4901-08
216	CONT.	1375.00	322.67	322.67	1.2583+02	4.4850+02	-1.2400-02	5.2154-08
217	CONT.	1375.00	359.27	359.27	8.9232+01	4.4850+02	-1.3866-02	-6.7055-08
218	CONT.	1375.00	392.21	392.21	5.6295+01	4.4850+02	-1.4994-02	1.6391-07
219	CONT.	1375.00	421.85	421.85	2.6659+01	4.4851+02	-2.4027-02	-6.7055-07
220	CONT.	1375.00	448.53	448.53	-1.3855-02	4.4852+02	-3.3290-02	-7.1526-07
221	PRFSS	1500.00	.00	.00	4.5000+02	4.5000+02	1.5499-04	-8.7135-04
222	PRFSS	1500.00	69.09	69.09	3.8091+02	4.5000+02	-2.0944-03	-2.1092-03
223	PRFSS	1500.00	131.27	131.27	3.1873+02	4.5000+02	-4.3945-03	-2.3114-03
224	PRFSS	1500.00	187.23	187.23	2.6277+02	4.5000+02	-6.6586-03	-2.2091-03
225	PRFSS	1500.00	237.60	237.60	2.1240+02	4.5000+02	-8.7783-03	-2.0334-03
226	PRFSS	1500.00	282.93	282.93	1.6707+02	4.5000+02	-1.0697-02	-1.8460-03
227	PRFSS	1500.00	323.73	323.73	1.2627+02	4.5000+02	-1.2383-02	-1.6668-03
228	PRFSS	1500.00	360.05	360.05	8.9540+01	4.5000+02	-1.3792-02	-1.5068-03
229	PRFSS	1500.00	393.49	393.49	5.6508+01	4.5000+02	-1.4817-02	-1.7195-03
230	PRFSS	1500.00	423.23	423.23	2.6767+01	4.5000+02	-2.4486-02	-9.0160-03
231	PRFSS	1500.00	450.00	450.00	0.0000	4.5000+02	-3.3732-02	-4.4227-03

SUMMARY OF EXTERNAL FLOWS  
 PRESSURE NODES: INFLOW 1.231-02 OUTFLOW 3.231-02  
 CONTINUITY NODES: SPECIFIC FLOW 0.000 0.000  
 OTHER FLOW 4.842-06 4.596-06

ASG10 DYNAMIC E.R. = #FMFE TEMP14.  
 ASG10 DYNAMIC E.R. = #FMFE TEMP15.

#FIN