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CANMET
Canada Centre for Mineral and Energy Technology
Centre canadien de la technologie des minéraux et de l'énergie

ESTIMATION OF UPPER BOUNDS TO ROCK SLOPES BY ANALYSIS OF EXISTING SLOPE DATA

WALL STABILITY IN THE SOUTH ROBERTS PIT - An example of the use of previous slopes

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FOREWORD

The Pit Slope Project was initiated in 1972 to ensure optimum use of Canada's mineral resources. Its objective is to reduce the cost of open pit mining and thereby to increase reserves of ore that can be mined at a profit. The cost reduction is to be realized by the advancement of slope design methods and by the judicious use of new support methods.

Output from the project in 1977 will be an engineering manual for use by staff engineers on the mining properties. Developement studies were conducted under contract in the field utilizing previous research findings. Other work was concerned with evolving a comprehensive design procedure. One approach of continuing interest since the earlier studies at Steep Rock and Knob Lake, is the utilization of field data on both stable and unstable slopes to obtain, in effect, a measure of the actual strength properties of the pertinent rock formations. One of the papers herein, by R.O. Stark, gives a final account of the pay-off of the studies at Steep Rock.

With this orientation, a somewhat novel analytical procedure was developed by T. Shuk of Bogota, Colombia. In communication with Mr. Shuk, a basic report on the development of this approach was obtained from him. This presentation was translated by CANMET and is issued herein together with a reprint of another article by Shuk and a report by Dr. B. McMahon, who was commissioned to make a detailed examination of this approach.

These papers are being made available to encourage mining companies to conduct similar studies on their properties. Such practical research would within a few years have a distinct effect on the confidence that planning engineers would have in designing pit walls.

R. Sage,
Project Officer,
Pit Slope Project

AVANT-PROPOS

Le projet Recherche sur les pentes des exploitations à ciel ouvert a été créé en 1972 dans le but d'assurer une utilisation maximale des ressources minérales canadiennes. Son objectif est de réduire le coût de l'exploitation à ciel ouvert par l'amélioration des plans des pentes et par l'usage judicieux de nouvelles méthodes de soutènement, et par conséquent augmenter les réserves d'or pouvant être extraites à profit.

Le projet, devant être terminé en 1977, donnera suite à un manuel technique destiné à l'usage des ingénieurs miniers. Des études sur le terrain, exécutées à forfait et pour lesquelles on s'est servi de découvertes précédentes, sont présentement en cours. De plus, une étude exhaustive de l'élaboration des plans est en marche. Par l'utilisation des données obtenues à partir d'enquêtes sur le terrain sur les pentes stables et instables, étude qui a toujours été d'un certain intérêt depuis les études antérieures de Steep Rock et Knob Lake, on obtient les propriétés réelles de résistance des formations rocheuses concernées. L'un des présents documents, écrit par R.O. Stark, donne un compte rendu final du succès des études effectuées à Steep Rock.

Dans la même direction, M.T. Shuk de Bogota en Colombie, a développé une nouvelle procédure analytique. CANMET en a obtenu un rapport de base, l'a ensuite traduit et publié avec un autre article de M. Shuk. L'analyse de cette nouvelle découverte, effectuée par le Dr. B. McMahon, a été publiée par la même occasion.

Ces documents sont mis à la disposition des compagnies minières afin de les encourager à entreprendre de semblables études chez-eux. De telles recherches pourraient avoir, d'ici quelques années, un effet certain sur la confiance des ingénieurs de la plannification dans leur conception des plans des murs de mine.

R. Sage,
Le projet Recherche
sur les pentes des exploitations à ciel ouvert

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ESTIMATION OF UPPER BOUNDS TO ROCK SLOPES BY ANALYSIS OF EXISTING SLOPE DATA

by

B.K. McMahon*

ABSTRACT

This report applies a method developed by Shuk to the investigation of existing rock slope data. The study is incomplete. It is presented in this form in the hope of drawing additional data and explanations from other workers.

The rock slopes studied tend to confirm Shuk's observation that data from existing rock slopes form straight lines on log-log graphs when the slope height is plotted against the slope "length". The term "length" is defined by Shuk to be the horizontal component of the slope on a log-log plot, and this definition has been retained throughout this compilation of papers. However, it is also shown that Shuk's procedure for estimation of probability of failure by regression analysis of existing slopes is likely to yield trivial results unless all slopes analysed are known to be close to limiting equilibrium. This emphasises Shuk's requirements that the sample must be carefully defined and bounded if this procedure is to be used. It does raise concern that the requirements may be too restrictive for the procedure to be widely applied in practice.

Of the data studied, only the slopes in shale are known to be close to limiting equilibrium. The remainder are conservative to an unknown degree. However, it was assumed that the steepest existing slopes in any category would approach the upper limit for stable slopes in that category. A first attempt at drawing upper bounds for various broadly defined rock mass categories has been made on this basis.

The proposed "upper bounds" are straight lines which, when extrapolated, converge at a point equivalent to a slope angle of 8° at a slope height of 10,000 ft. If they are confirmed and more precisely defined by further study, such upper bounds may provide a criterion for design of rock slopes in stable orientations with respect to the geologic structure. Design of slopes with orientations that are critical with respect to structure should be based on analysis of the fractures.

Theoretical curves derived from the Mohr failure criteria are a poor fit to most of the empirical data. Shale slopes fit a straight line or a Mohr-Coulomb curve equally well.

Besides the poor fit, the deduced values of the Mohr strength parameters are unexpectedly low and appear inconsistent with published values based on laboratory tests. Possible explanations are either that: (a) steepness is limited by surface failures due to processes such as ravelling, over-toppling and exfoliation of small surface blocks and not by the strength of the underlying rock mass; (b) strength of the underlying rock mass is subject to a "power failure law" from which it is tentatively deduced that slope height would bear an exponential relationship to a slope length; or (c) information for slopes in rocks other than shale merely represents accidental variation of slopes which could all be much steeper.

It would follow from the first explanation that open pit slopes with stable orientations

* Partner, Australian Rock Engineering Consultants

could be excavated to very steep angles using surface reinforcement such as rock bolts. The second explanation would imply that very high rock slopes may need to be designed at slope angles less than residual angles of friction determined in the laboratory. The third explanation assumes that the rather impressive fit of the data is due to coincidence.

INTRODUCTION

This is a preliminary report compiled from data available to the writer. It is presented in this form to suggest possible directions for further study.

The study was inspired by papers by Shuk (Ref 1, 2 and 3) who showed that a wide variety of rock slopes tended to form straight line patterns when the slope height was plotted against the slope "length" (as defined in Plate 1) on a log-log plot. Shuk has suggested that regression analysis of these plots provides a means of obtaining a design estimate of probability of failure of any rock slope given sufficient data from existing slopes when:

a. the populations of height and length of the slopes follow log-normal (or other transforms of the normal distribution);
b. the sample is random; and
c. the population studied is properly defined and bounded.
d. the slopes must all belong to the same time scale.

(The (c) requirement is interpreted by the present writer to mean that the sampled slopes must form a unimodal, statistically homogeneous population from which it would normally follow that they must all be formed by the same processes.

The (a) requirement derives from the theory of the commonly used methods of regression analysis (Ref 4). It could be overcome by using more general methods of analysis.

The above requirements result in some obvious limitations to the application of Shuk's procedures to actual design of excavated slopes. Estimates of probabilities of failure computed from analysis of natural slope data would be expected to be conservatively high when applied to excavated slopes, due to the effects of erosion and long-term progressive failure. Estimates of probability of failure based on existing excavated slopes will also be conservatively high, unless all slopes used in the analysis were at the point of critical equilibrium.

However, although it is clear that most existing stable rock slopes are probably less steep than they could be, it is likely that at least a few approach the maximum stable slope. The upper bound to the steepest existing slopes in any rock mass category is, therefore, likely to be a reasonably valid estimate of the true upper bound for that category.

This upper bound would possibly have value in design. It could be a first estimate of the slope design for slopes excavated in rock masses where the fractures are either discontinuous or in stable orientations, so that the slope is controlled by the average strength of the rock mass. Slopes where the fractures are relatively continuous and unfavourably oriented, should be designed by analysis of the fractures, as described in References 7 to 10. In addition, it could be a method of back-calculating the maximum strength exhibited by various rock mass categories. This would provide insight into the "reduction factors" which should be applied to rock substance strengths to estimate rock mass strengths. It might be an approach to identification of the "failure law" applicable to rock masses on a large scale.

In this report, actual slope data is presented on plates together with theoretical curves useful for interpretation and back-calculation.

ANALYSIS OF OPEN PIT SLOPES

A scatter diagram obtained by plotting measurements of 233 rock slopes compiled by Kley and Lutton is shown in Plate 2 (Ref 8). These data were obtained from a total of 153 open pit mines and quarries in a wide range of rock types.

The degree of conservatism in the design of these slopes is not known.

The data tend to form an elongated two-dimensional log-normal distribution, as observed elsewhere by Shuk. The regression line-of-best-fit and 90% confidence limits are as shown. The correlation coefficient of height upon "length" is 0.78 and regression equation is

$$\text{Log } H = 1.187 + 0.521 \log L \qquad \text{eq 1}$$

where H and L are the slope height and slope "length" respectively.

Although impressive at first sight, this result is in fact trivial. The apparently high correlation coefficient is due to the fact that L and H are not independent variables, but are restricted to a narrow belt in the middle of the plot. Slopes 300 ft high, for example, cover a range from 15° to 76°.

Additional studies in which the data were segregated into smaller rock type categories, did not result in significant reduction in the scatter. It is clear this scatter is probably due to factors which are not related to the average strength of the rock mass, such as the presence of individual weak fractures in unstable orientations and varying degrees of design conservatism.

ESTIMATION OF UPPER BOUNDS

The steepest existing rock slopes known to the writer for several categories of rock slopes are shown in Plates 3 to 6. The following categories have been considered:

a. Strong granitic rocks

The steepest slopes considered were those of El Capitan and Half Dome in Yosemite National Park, California. These are monolithic granites with few joints other than exfoliation fractures. The slopes shown in Plate 3 were measured from cross-sections drawn by Hansen through the steepest part of the Yosemite Valley (Ref 9).

b. Horizontally layered sandstones and shales

The slopes considered were in the thickly bedded sandstones and stable shales of the Grand Canyon of the Colorado River, Lodore Canyon of the Green River and the steepest slopes in Zion National Park. These slopes have also been measured from cross-sections provided by Hansen (Ref 9) and are shown on Plate 4. Slopes from the Day-Loma and Frazier-Lemac mines in Wyoming are also shown on Plate 4.

c. Strong but jointed granite and gneiss

The steepest slopes measured in the Precambrian gneiss of the Big Thomson and Clear Creek Canyons in Colorado are shown in Plate 3 (Ref 5). The major defect in the gneiss was the foliation which had a strike approximately at right angles to the direction of the canyon in all examples shown. Slopes in similar but more broadly jointed gneiss in the Black Canyon of the Gunnison are also shown.

d. Jointed and altered crystalline rocks

This category includes the typical porphyries of the open pit mines in Utah, Arizona and Nevada, and similar intensively jointed, partially altered crystalline rocks. The slopes shown in Plate 5 are the steepest of this category listed by Kley and Lutton (Ref 11).

e. Stable clay shales

Published information on slopes in paint rock by Coates (Ref 10) is shown in Plate 6. Paint rock is described as a Keewatin sedimentary rock, mainly composed of kaolin, quartz and pyrolusite with a void ratio of 0.4. The slopes shown are all unstable, but are indicated by Coates to be close to critical equilibrium. The higher slopes of this group lie remarkably close to a straight line on the log-log plot.

f. Swelling clay shales

Examples of stable and unstable slopes in weathered and "firm" Bearpaw shale have been published by Lane (Ref 11). This is a well known swelling bentonitic clay shale. This information was originally plotted by Lane on slope charts of height vs slope cotangent on arithmetic scales. When the data is replotted on log-log scales, as shown in Plate 6, the curves separating stable from unstable slopes are seen to approximate straight lines.

Straight lines shown on these plates represent first estimates of the upper bounds for the slopes at each locality or formation. These upper bounds are most reliable for the clay shales, where information on both stable and unstable slopes controlled by the average properties of the rock mass was available.

Some data on unstable rock slopes in the porphyry copper mines was also available (Plate 5), but in all cases these failures were associated with underlying weak fractures and occurred on slopes much flatter than the steepest slopes in the mine (Ref 8). However, it is possibly significant that two of the larger slides recorded at Chuquicamata and Bingham Canyon lie close to the upper bound for the stable slopes studied, as shown in Plate 5.

THEORETICAL CURVES

Mohr Strength Relationships

To compare the empirical upper bound estimates with predictions based on theoretical failure laws, sets of curves have been developed for a dry material with failure law:

$$\tau = c + \sigma \tan \phi \qquad \text{eq 2}$$

where τ = shear stress at failure, c = cohesion, σ = normal stress at failure, and σ = angle of friction.

The sets of curves shown as Plates 9 and 10 are derived from the simplified concept of limiting equilibrium in which the slopes are assumed to be rectilinear and infinitely extended and failure occurs along a critical circular arc. These curves have been developed from dimensionless relationships derived by Hoek (Ref 12). The second set of curves, on Plate 11 are derived from the general concept of limiting equilibrium in which the whole of the rock mass in the lower part of the slope is considered to be in a state of limiting equilibrium and failure occurs along a slip-line network. These curves have been derived from tables given by Sokolovskii (Ref 13).

Both sets of curves cover the range $\phi = 10°$ to $40°$, $c/\gamma = 1$ to 1000 ft (where γ is the density of the rock mass).

As shown, both sets of curves become asymptotic to the angle of friction with increasing height of slope and, for any slope height, the predicted slope angles are slightly lower for the exact theory than for the simplified theory. For purposes of this preliminary report, the curves have not been completed beyond the ranges of values given by Hoek and Sokolovskii.

Exponential Relationships

A straight line on a log-log plot of slope height vs slope length, indicates that the two variables have an exponential relationship of the form:

$$H = aL^b \qquad \text{eq 3}$$

where H is the slope height, L is the slope length (i.e. the horizontal component of the slope, as shown in Plate 1), a is a constant (equal to the value of H when L = 1), and b is a constant equal to the slope of the line.

As shown in Plate 9, slopes which maintain the same angle regardless of height, plot as straight lines with the constant b equal to one and constant a equal to the tangent of the slope angle. Slopes whose average inclinations become flatter with increasing slope height, are characterized by a value of the constant b less than one.

A possible explanation for the observed straight-line relationships derives from the similarity in form between eq 3 and the empirical power failure law:

$$\tau = K\sigma^n \qquad \text{eq 4}$$

This is suggested by Jaeger (Ref 14) on the basis of results of triaxial testing of rock cores and crushed rock. Similar relationships have been suggested by Hobbs (Ref 15) on the basis of triaxial testing of crushed rock and Maurer (Ref 16) on the basis of direct shear testing of sawn rock surfaces. As pointed out by Jaeger "there is some justification for this since Archard (Ref 17) points out that a power law ... should apply to irregular surfaces in place of Amonton's laws." This requires that points of contact are subject to elastic deformation instead of the plastic deformation that is implicit in the classical theory (Ref 18).

It follows from eq 4, when n is less than 2 that, the average coefficient of friction decreases exponentially with increasing normal stress:

$$\mu = \tan \phi = \frac{\tau}{\sigma} = \frac{K\sigma^n}{\sigma} = K\sigma^{n-1} \qquad \text{eq 5}$$

Since the average normal stress across a potential failure surface is also likely to increase as a power function of the height of the slope, it would follow that the average coefficient of friction decreases exponentially with increasing height of the slope.

An alternative to the elastic deformation mechanism to explain this phenomena is the theory proposed by Byerlee (Ref 19). As explained by Brace and Byerlee (Ref 23):

> " One of the puzzling results of previous work was the consistent difference between the coefficient of friction, μ, of rocks and μ of rock-forming minerals. For rocks, μ is usually around 0.8 (Jaeger, Ref 20), whereas for minerals it is often about 0.1 (Horn and Deere, Ref 21). Byerlee showed that this could be explained by differences in roughness of the surfaces of the specimens used. Since roughness should not be an important factor according to Tabor, Ref 22) this led to a reexamination of this theory, particularly with regard to brittle materials. Byerlee proposed a theory which differed from the classical in that failure of asperities on a sliding surface occurred through brittle rather than ductile processes. The theory predicted μ of about 0.1 for finely polished surfaces; this seemed to correspond with the values commonly obtained for minerals (mineral specimens used in friction experiments are usually polished).
>
> Experiments showed ... that, as roughness of either a rock or a mineral specimen was increased, μ also increased, approaching an upper limit of 0.6 to 0.8. Most samples of rock in friction experiments are sawcuts which are ground rather than polished. Thus, in previous studies probably corresponded to the rough end of the scale in Byerlee's experiments."
>
> The apparent difference between μ of rocks and minerals could be explained therefore by differences in roughness rather than by an inherent difference in these two classes of materials."

Byerlee's concept, which was applied to laboratory sized rocks and minerals, suggests to the writer that the roughness factor contributing to the total angle of internal friction in a fractured rock mass is dependent on scale and shows an exponential relationship to the height of the slope.

It is intriguing at this stage that all the attempted upper bounds for various rock categories shown in Plates 7 and 8 converge at a value:

$$H/_L \simeq 0.1$$

which, if the jointed rock masses are regarded as purely frictional materials, is the limiting value predicted by Byerlee.

COMPARISON BETWEEN THE EMPIRICAL SLOPE DATA AND THE THEORETICAL CURVES

As shown in Plates 3 to 7, the fit of straight line segments to the empirical data is impressively good. With the exception of the data pertaining to clay shale slopes, which can be fitted equally well by straight lines or the curves derived from the Mohr strength relationship, the overall fit of the Mohr curves is remarkably poor. However, it is possible to fit

Mohr curves to short segments of the data. The procedure for fitting these curves is described in Appendix 1.

Values of "first attempt" estimates of upper-bound values of the constants a and b in eq 3 for the various categories of rock slopes shown in Plate 8 are given in Table 1.

TABLE 1 "FIRST-ATTEMPT" ESTIMATES of the UPPER-BOUND VALUES of EXPONENTIAL EQUATION CONSTANTS for INDICATED ROCK CATEGORIES

Rock mass category	Constant a	Constant b
Massive granite with few joints	139	0.28
Horizontally layered sandstone	85	0.42
Strong but jointed granite and gneiss	45	0.47
Jointed partially altered crystalline rocks	16	0.58
Stable shales	8.5	0.62
Swelling shales	2.4	0.75

Note: a and b are the constants in eq 3 corresponding to the lines shown in Plate 8 for each rock category.

For the purposes of investigation, values of the Mohr strength parameters were fitted to selected groups of rock slopes as shown in Table 2. The curve fitting followed the procedure outlined in Appendix 1, but for rock types other than shale required some license as the slope data tends to lie on lines tangential to the theoretical curves rather than along the curves. Assumptions were then made regarding the specific gravity of the materials, the groundwater levels and the unconfined compressive strength of the rock substance, so that the order of mangnitude of the rock mass unconfined compressive strength and reduction factors could be computed from the following relations:

$$Cu = \frac{2c \cos\phi}{1 - \sin\phi} \qquad \text{eq 6}$$

$$R.F. = \frac{Cu \text{ (rock mass)}}{Cu \text{ (rock substance)}}$$

where Cu = the unconfined compressive strength and R.F. = Reduction Factor.

DISCUSSION

The results shown in Table 2 appear rather strange. The low values of ϕ obtained for the granites, gneisses and sandstones compared with the relatively high value of ϕ obtained for the paint rock do not agree with results of laboratory tests. Even if the magnitudes of friction obtained from laboratory tests were wrong, the relative order of values for different rock types would be expected to remain much the same.

With the exception of the value for the Grand Canyon the reduction factors calculated are mostly two orders of magnitude lower than values given by Deere et al (Ref 24). These values however were based on plate bearing tests which could conceivably be far too small in scale to be correct for the large scale of a rock slope. Nevertheless, the values calculated here seem surprisingly low.

These observations suggest that factors, other than the average strength of the underlying rock mass, control the steepness of rock slopes, even in those cases where unfavourably oriented weak fractures are not present.

In the writer's opinion, a likely explanation is that steep rock slopes are controlled by small surface failures due to loosening, ravelling, overtoppling and exfoliation. These phenomena limit the steepness of both excavated and natural slopes and, in the case of natural slopes, are compounded over long periods of time by weathering and erosion.

It is perhaps significant that all rock slides known to the writer, except those in shale, have been associated with faults or other weak defects behind the slope. It appears that slopes in which these weak defects are absent or in stable orientations, have been prevented by

TABLE 2 - Fitted Mohr Strength Parameters and Derived Rock Mass Compressive Strengths and Reduction Factors for Selected Rock Categories

Rock group or locality	Data shown on Plate No.	Fitted Mohr Strength Parameters ϕ degrees	Fitted Mohr Strength Parameters c/γ feet	Assumed groundwater level	Assumed specific gravity p.c.f.	Corrected Mohr-Strength parameters ϕ degrees	Corrected Mohr-Strength parameters c p.s.f.	Calculated rock mass unconfined compressive strength p.s.i.	Assumed rock substance unconfined compressive strength p.s.i.	Calculated reduction factor
Yosemite	3	8	1000	Low	165	9	165000	2700	35000	.077
Grand Canyon	4	20	750	Low	124	22	93000	1900	7000	.27
Lodore Canyon	4	8	220	Low	124	9	27000	430	7000	.062
Bingham Canyon Mine	5	17	100	High	165	22	16500	340	18000	.018
Clear Creek Canyon	3	10	100	High	165	13	16500	290	35000	.008
Big Thomson Canyon	3	10	75	High	165	13	12375	216	35000	.006
Esperanza Mine	5	20	35	Low	165	22	5800	118	18000	.006
Bagdad Mine	5	23	20	Low	165	25	3800	82	18000	.004
Paint rock	6	30	7	Medium	110	34	770	20	2500	.008
Bearpaw shale	6	20	1.8	Low	120	20	216	4	1600	.002

surface failures, (or possibly human intuition in the case of excavated slopes) from approaching a combination of steepness and height which would result in failure through the underlying rock mass.

It also appears that the cumulative effect of these surface failures, or human intuition, results in an exponential flattening of the slope with increasing height.

If the above rationalization is correct, it would follow that slopes which are stable in the orientation with respect to the underlying rock structure could be excavated very steeply with the addition of such surface treatment as smooth wall blasting and rock bolting.

An alternative to this theory, however, is that rock mass strength is subject to a power failure law, so that it decreases exponentially with the size of the slope. As previously noted, there is some experimental evidence that such power failure laws do exist. Also, some field observations, such as the obvious stress relief exfoliation at Yosemite and other high rock slopes, could be interpreted as support for this concept, especially where the slopes have the curved "bathtub shaped" profile predicted by Sokolovskii. This explanation would have serious consequences for the design of very high rock slopes, as it suggests that they may have to be excavated at angles shallower than the residual angle of friction obtained by laboratory tests.

TENTATIVE CONCLUSIONS

The following tentative conclusions have been drawn from this study.

a. The observation of Shuk that natural and excavated slopes tend to form straight lines when slope height is plotted against slope "length" on log-log plots, has been independently confirmed.
b. When extrapolated, most of these straight lines converge at a point equivalent to a slope of 8° with a height of 10,000 ft. The slope angles do not appear to become asymptotic to the friction angle, as predicted by theories based on the Mohr strength failure criterion.
c. With the exception of slopes in shale, the fitted values of the Mohr strength parameters and estimates of rock mass strength deduced therefrom, are surprisingly low even for the steepest slopes known to the writer.
d. A possible explanation is that steepness of high rock slopes appears to be limited by either small surface failures such as ravelling, overtoppling or exfoliation of small blocks. This suggests that rock slopes with stable orientations with respect to the underlying fractures can be excavated to very steep angles if such surface treatment as smooth wall blasting and rock bolting are applied.
e. An alternative explanation is that the average strength of a fractured rock mass is subject to a power failure law, so that it decreases exponentially with the height of the slope. This suggests that very high rock slopes may have to be excavated at angles less than the residual angle of friction obtained by laboratory direct shear tests.
f. A third possible explanation is that only the information for shale slopes is sufficiently well bounded to be meaningful and the other data represents accidental variation of rock slopes which could all be much steeper. This explanation presumes that the rather impressive fit of the data to converging straight lines is mere coincidence.

SUGGESTIONS FOR FURTHER STUDIES

The rock mass categories used in this report are rather vague and generalized. They should be more quantitatively defined.

Additional data should be collected and plotted, particularly any data which would change the "first-attempt" upper bounds presented here.

Any information regarding slope failures of the slip-circle type, and not bounded by major defects in slopes other than shales, should be plotted on the appropriate graphs.

Additional work is required to refine the possible explanations given here or to suggest other explanations. Suggestions for independent methods of evaluating conflicting explanations are required.

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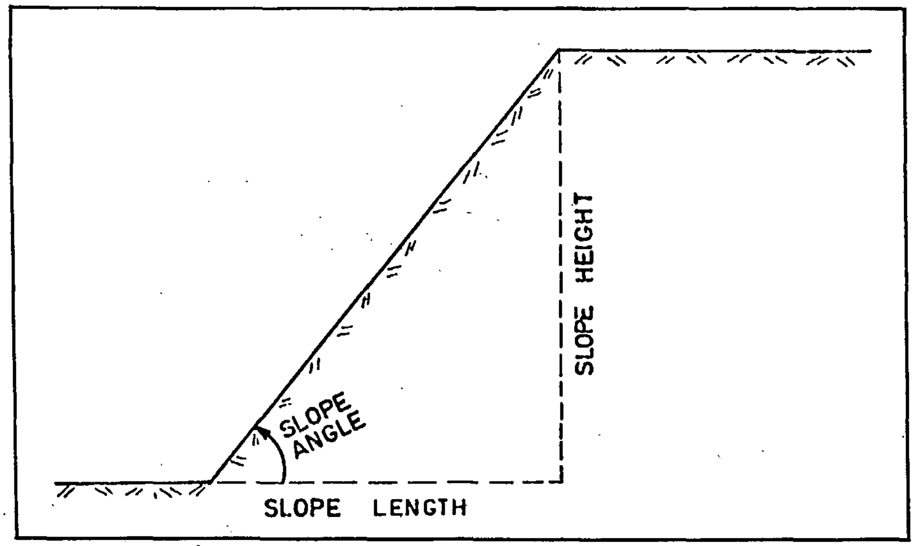

Plate 1 - Cross-section through slope showing terminology (after Shuk)

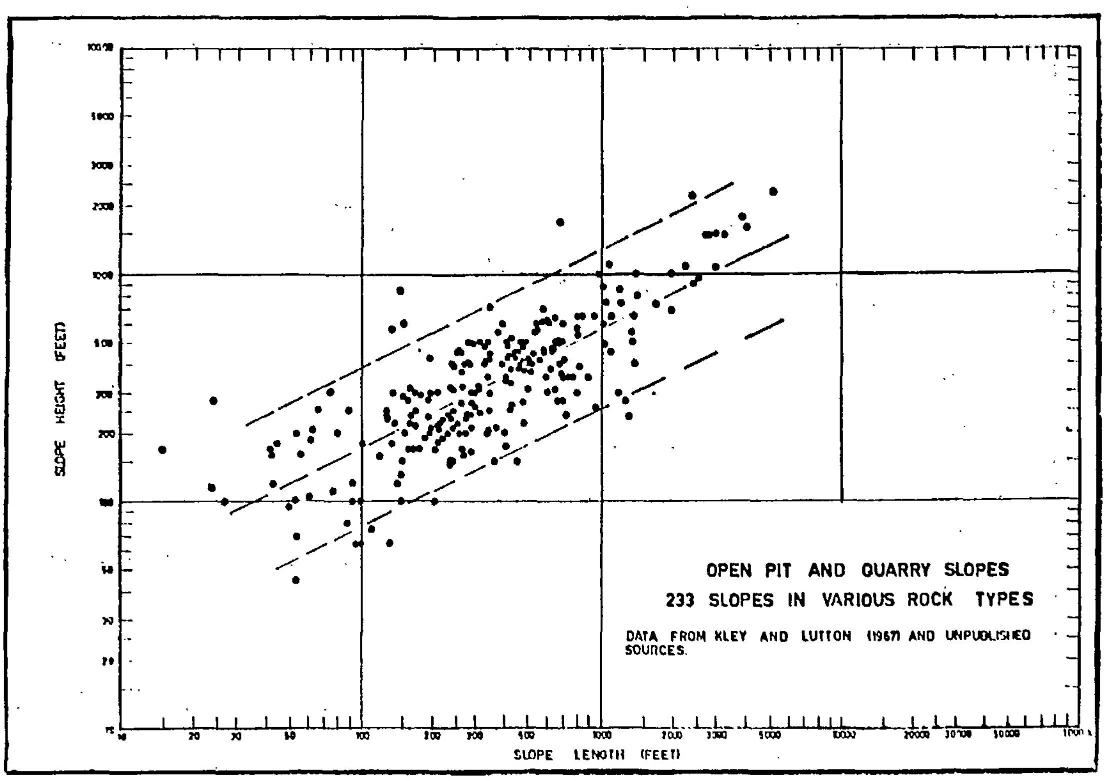

Plate 2 - Scatter diagram of 233 slopes in various rock types

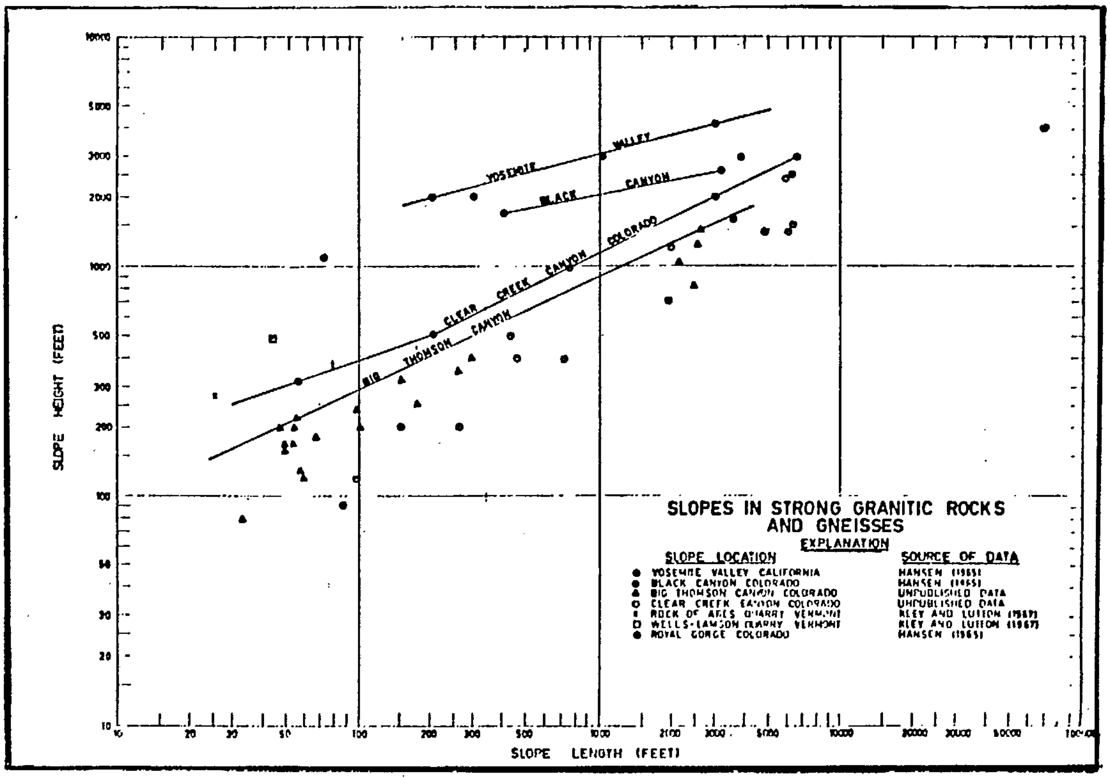

Plate 3 - Slopes in strong granitic rocks and gneisses

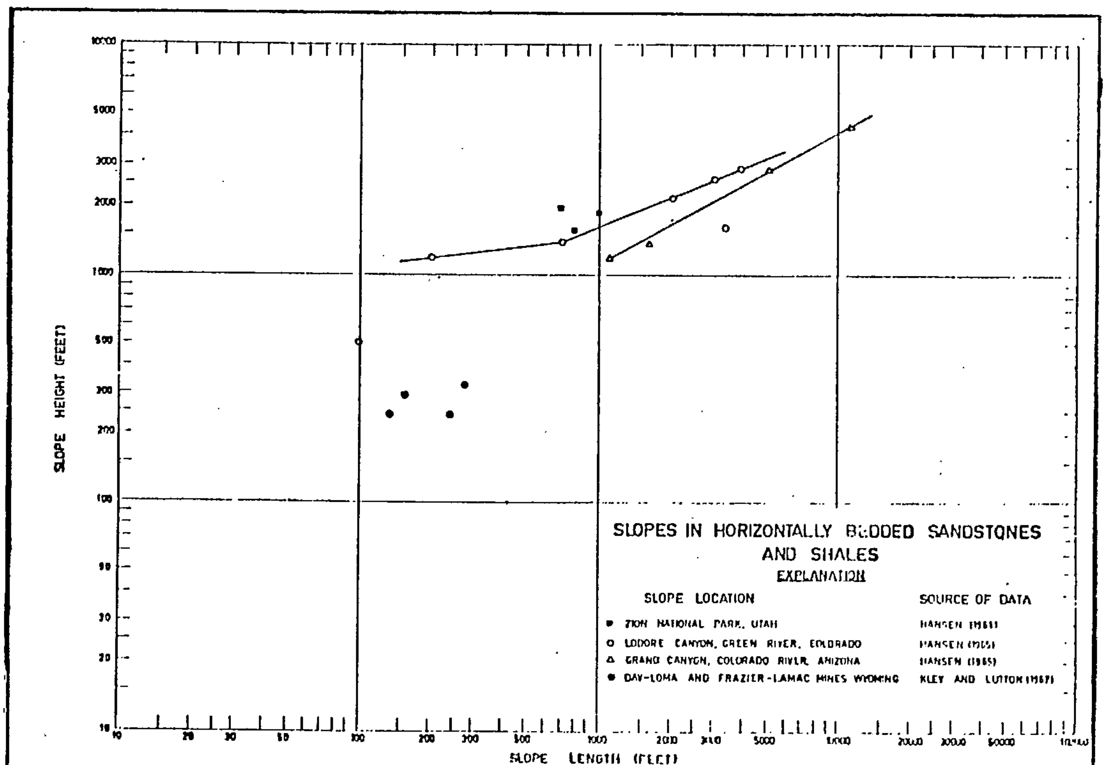

Plate 4 - Slopes in horizontally bedded sandstones and shales

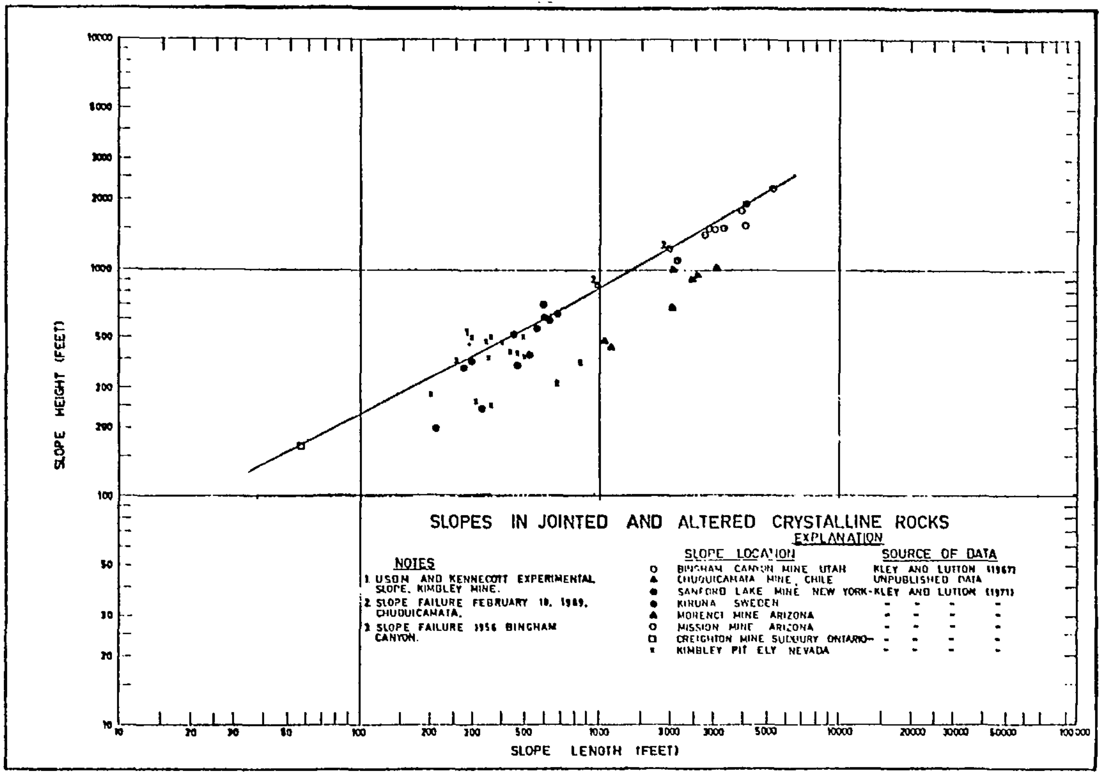

Plate 5 - Slopes in jointed and altered crystalline rocks

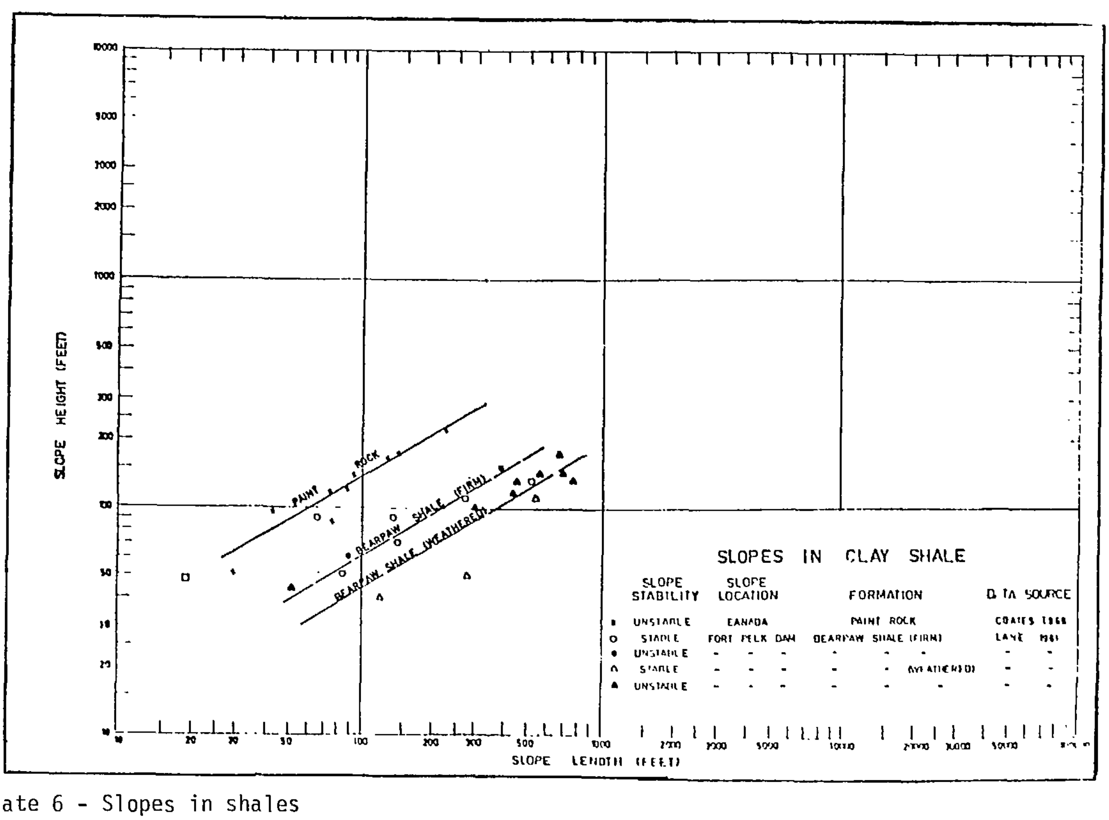

Plate 6 - Slopes in shales

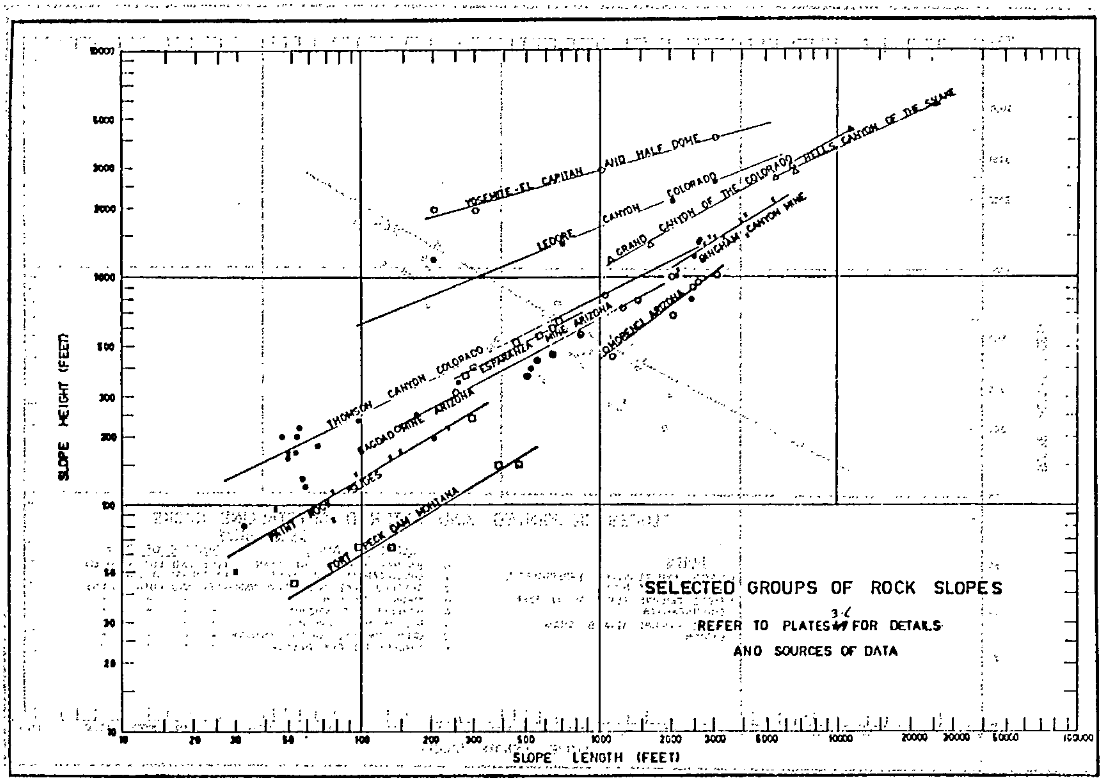

Plate 7 - Selected groups of rock slopes

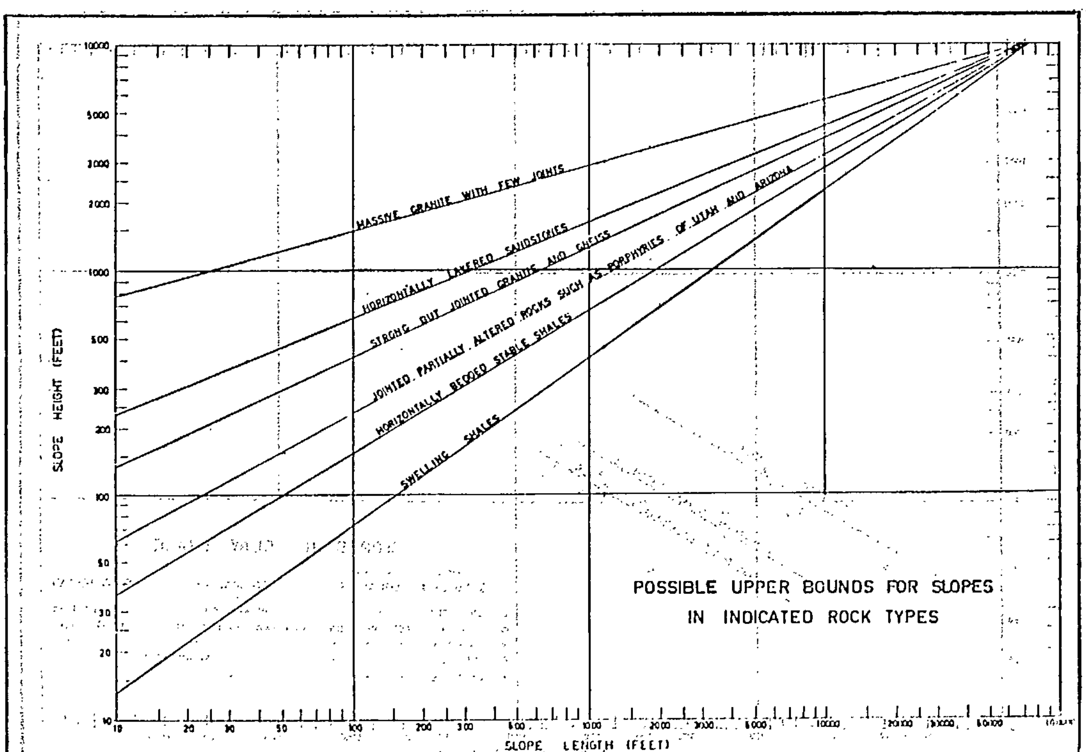

Plate 8 - Possible upper bounds for slopes in indicated rock types

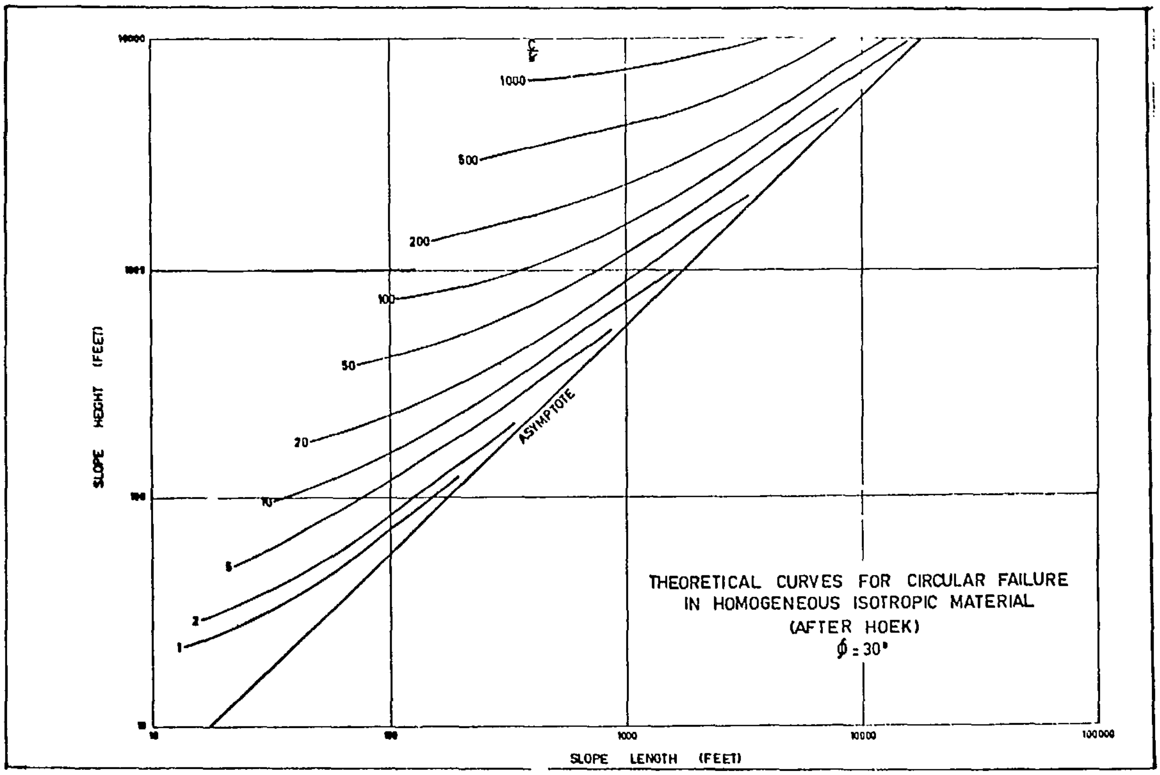

Plate 9 - Theoretical curves for circular failure in homogeneous isotropic material 30^0 (after Hoek)

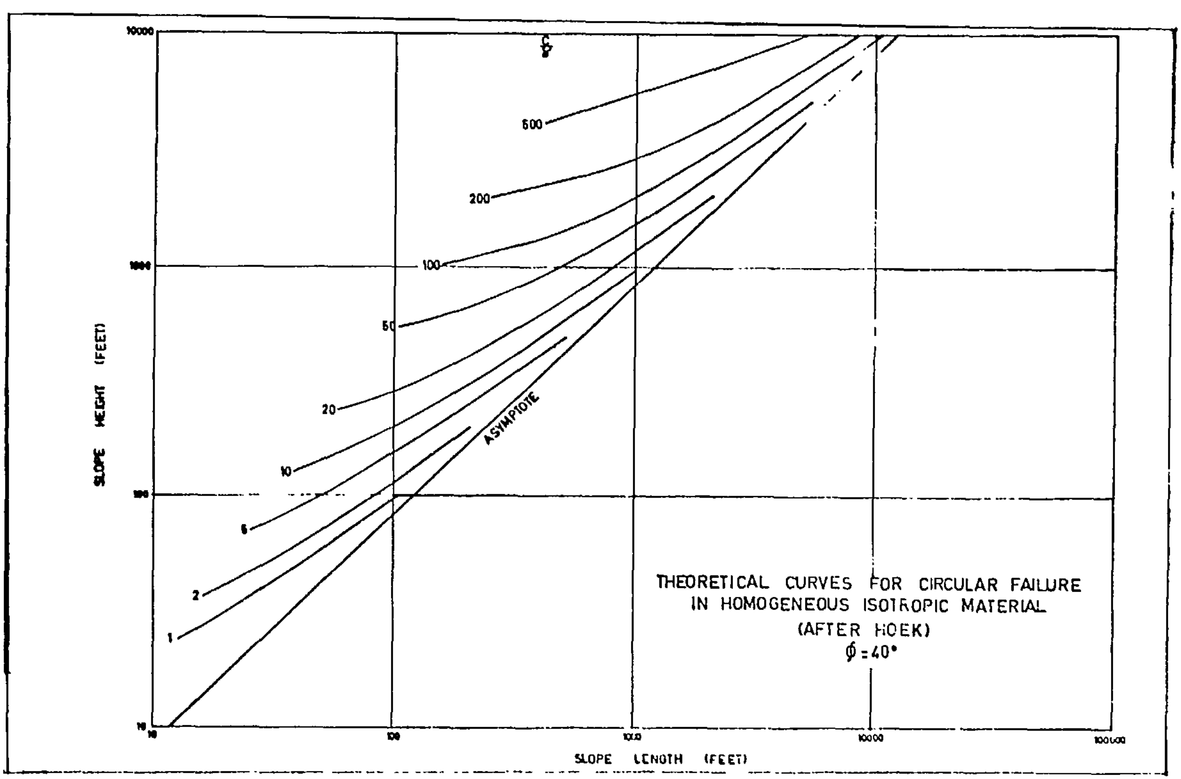

Plate 10 - Theoretical curves for circular failure in homogeneous isotropic material 40^0 (after Hoek)

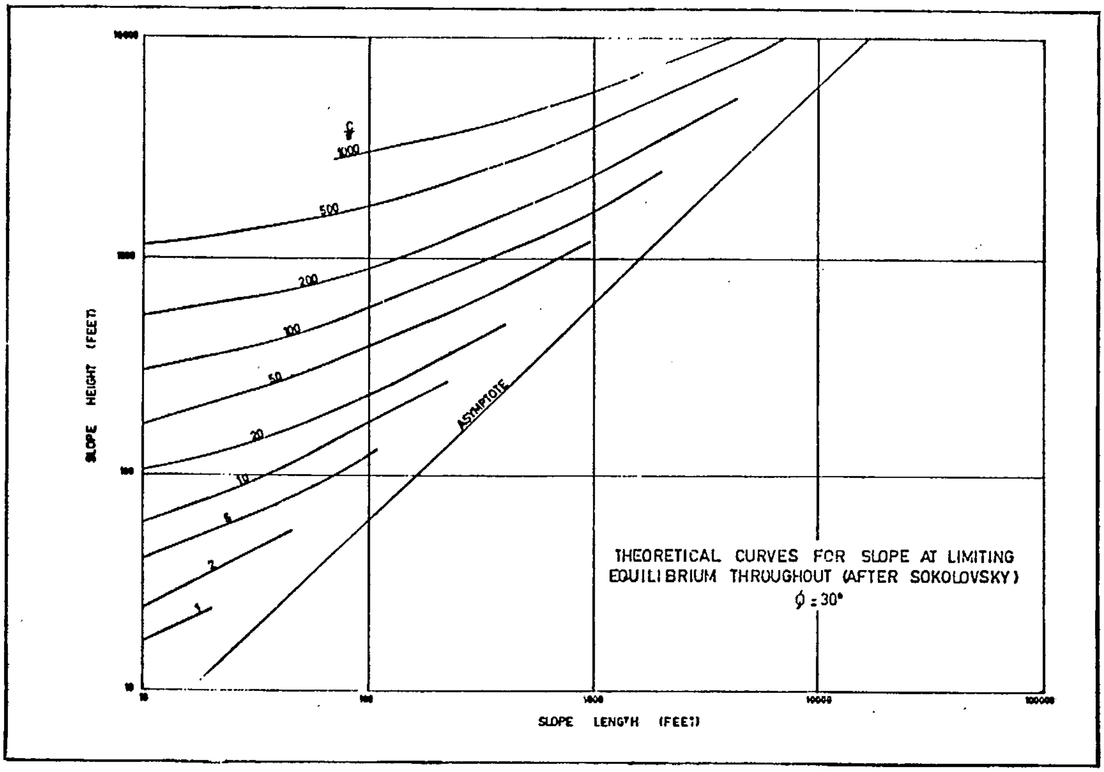

Plate 11 - Theoretical curves for slope at limiting equilibrium throughout 30^0 (after Sokolovsky)

APPENDIX A

USE OF OVERLAY CHARTS FOR BACK-CALCULATION OF MOHR STRENGTH PARAMETERS FROM SLOPE DATA

The use of these charts for fitting strength parameters to natural rock slopes can be demonstrated by the example of the data for paint rock on Plate 6. Superposition of Plates 9 and 10 shows that the slope of the line of best fit coincides most closely to the interpolated theoretical curve (for dry slopes) for a material $\phi = 30°$, $c/\gamma = 7$. As the majority of the slopes were subject to groundwater levels equal to half the height of the slope (Ref 10), it is necessary to apply a correction factor (after Hoek, Ref 12) as follows:

$$\phi w = (1 + d \frac{Hw}{H}) \phi \qquad \text{eq A1}$$

where ϕw = the average angle of fricton necessary to maintain equilibrium in a wet slope, ϕ = the average angle of friction necessary to maintain equilibrium in a dry slope, d = a coefficient suggested by Hoek to be 0.3 if normal drawdown is present and 0.5 if no drawdown is present, Hw = height of groundwater table above the toe of the slope, and H = height of the slope.

The value of ϕ for paint rock would become 34° for normal drawdown, or 37° if no drawdown occurred. These values may be compared with average values of $\phi = 37°$ and c = 1000 psf estimated by Coates (Ref 13) on the basis of laboratory tests to determine the angle of friction and back-calculation of individual slopes, assuming $\phi = 37°$ to obtain the cohesion.

The procedure described here has a possible advantage over the procedure used by Coates, as both the values c and ϕ can be estimated from the field data, thereby eliminating the need to extrapolate the value of ϕ from the laboratory to the field.

APPENDIX B

A SIMPLE DESIGN METHOD TO MINIMIZE COST OF ROCK SLOPES

THOMAS E. SHUK*

SUMMARY

A method of designing slopes in cuts is proposed, based on research analysis of a set of 22 natural slopes in this country. This method utilizes inference and statistical analysis in conjunction with postulates given by present theory on economic decision. Such a method is simple to apply as it does not require full knowledge of the mechanical characteristics of the rock mass, or of the forces which may act on the slope itself. As a result of this analysis, several questions have arisen concerning various practices now commonly used in rock slope design.

INTRODUCTION

Until recent years, the Civil Engineering department utilized tables giving a great variety of procedures for the design of soil and rock slopes, especially in designing road cross-sections.

Technology developed in soil mechanics has slowly introduced a more rational method of designing slopes based on a knowledge of more realistic mechanical and physico-chemical characteristics of soils as well as a knowledge of deformation and forces to which slopes may be subjected. A stable slope is designed in combination with stability analysis and by means of a safety factor.

While this method is more rational than the earlier, the safety factor which varies in terms of other factors is difficult to apply as it does not possess any logical basis and, in most cases, varies considerably.

For the following reasons the day is yet to come when one can apply the same method of designing slopes in rock as in soils:

a. While the mechanical characteristics correctly interpreted from laboratory results with soil samples are in most cases representative of the mechanical characteristics of the continuous mass, the rock mass is not considered continuous, thus giving nonrepresentative laboratory results. For example, mass resistance is generally significantly lower than that of a common laboratory sample. Even though emphasis has been on the use of rock masses for in situ experiments on a large scale, the results of such experiments only constitute an initial approximation. In some cases of road rock cuts, the cost of carrying out such an in situ experiment, with results representative of the mechanical characteristics of the rock mass itself, would far exceed potential benefits.

b. While in the case of soils it is not difficult

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*Associate Civil Engineer and Head of the Geotechnical Department - GEOCOLOMBIA Ltda., Bogota.

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to determine within a certain degree of confidence the stresses which may act on a slope, this is practically impossible when dealing with rock masses. This is because the instrumentation employed often destroys the system of forces that may already exist where measurements are being taken.

Due to the above reasons, the highway engineer finds himself obliged to continue using tables to design "stable" rock slopes.

Purpose

The purpose of this paper is to devise a rational method of designing a rock slope based on investigation results. The problem is to provide a slope at an overall minimum cost. The proposed method is simple to apply and does not require a knowledge of the mechanical characteristics of the rock mass or of the forces to which the slope may be subjected. To apply this method, however, one requires adequate topographical and geological information of the area and a firm knowledge of the theory and application of statistical analysis. The method is based primarily on the hypothesis that Nature itself provides the best possible in situ tests for design purposes.

PRELIMINARY INVESTIGATIONS

Using the above hypothesis, several engineers in the past have utilized natural slope analysis to design rock slopes. The first analysis of this type was published in 1949 by Binger and Thompson. The authors described the first systematic attempt at organizing a great quantity of data consisting of precise topographical information on natural slopes, on measurements and other pertinent data on rock slides, and on geological information - all of these obtained during and after construction of the Panama Canal. This data was compiled and then correlated in a system of "curves for excavating rock slopes". These curves were subsequently utilized with complete success in the Cucaracha and Culetra formations for the Third Locks Project of Canal de Panama in 1939 to 1942.

In 1961 Lane used basically the same method in the Bearpaw (Fort Peck Dam), Fort Union (Garrison Dam), and in Lutita-calize (Tuttle Creek Dam). Based on curves of slope height vs angle cotangents of slope, complemented by stability analyses and the results of laboratory tests, slope designs were achieved with great success.

It should be pointed out in the cases described above, that the formations behaved as soils and not as rock masses, and that Binger and Thompson, like Lane, applied safety factors to the curves derived from the geometrical relationship of the natural slopes to obtain design parameters.

In 1962, some criteria were postulated for the design of slopes in rock masses based only on the geometrical characteristics of the principal systems of discontinuities such as stratification planes, cracks, etc. (Terzaghi, 1962). Unfortunately, such criteria apply only to hard, sound rock masses with few cracks where both the apparent cohesion effects of discontinuous rather than continuous systems and the water pressure effects are insignificant. Such conditions are rarely found in Colombia.

In 1963, curves of natural slope geometrical relations in conjunction with a detailed knowledge of the characteristics of the principal system of discontinuities were utilized for the first time to obtain design involute curves for rock masses without in situ trials (Geocolombia, 1963).

In 1965, also for the first time, a probabilistic-statistical method was utilized, based on curves of natural slope geometric relationships to determine the probability of failure of a rock slope whose height and tangent were previously fixed (Geocolombia).

Later in the same year, the results of an investigation were presented which claimed that by applying a probabilistic approach - the only one which permits a direct relation to the theory of economic decision - there is always a point for a given probability of failure where the cost of a slope is a minimum (Langejan, 1965). Langejan's investigation was limited exclusively to soils and his method was based on laboratory tests in conjunction with stability analysis.

At the same time in 1965, the present author planned to apply Langejan's method to rock slopes, basing his method, however, on the hypothesis that design results (Shuk, 1965). In other words the geometric relationship of natural slopes are utilized.

The results, analysis and conclusions derived from the above method are presented here.

PHASES OF NATURAL SLOPE ANALYSIS

This present paper shows results of a 4-phase investigation into the necessary requirements for slope design as follows:

1. Correlation analysis between height "H" (See nomenclature, Fig. 1) and length "L" based on samples obtained from a population set of "L" and "H".
2. Regression analysis of the equation which relates "H" to "L".
3. Conditional probability of failure analysis of a given length "L" for a given height "H".
4. Results of the above analysis as applied to the theory of economic decision.

Correlation Analysis Between L and H

The object of the analysis of the variables L and H is to determine their relationship. For such an analysis to be valid as far as results and inferences are concerned, one must meet the following requirements:

a. the population boundaries from where the samples are obtained must be clearly defined
b. sampling of variables must be strictly random
c. the number of samples must be statistically significant
d. the variances must be statistically homogenous with a high degree of certainty
e. if there is a relationship between the variables, the correlation coefficients must be statistically significant with a high degree of certainty.

The population of natural slopes comprise all those characterized by the following:

1. rectilinear faces (see Fig. 1) not determined by erosional movements or accumulation of erosional products, and
2. must be within a geological unit characterized by the same lithology, identical origin and of the same age.

The population of the variables L and H was defined as the measurements of L carried out - either consistently perpendicular to a plane formed by a system of discontinuities (faults or bedding planes) from which these could be determined, or consistently perpendicular to the slopes themselves, or consistently perpendicular to a predetermined direction in sets of natural slopes uninterrupted by a significant fault producing geomorphological divisions such as a river crossing a geological layer having different slopes on the two parts.

It is clear that in a population of natural slopes, various populations of L and H may occur depending on the direction in which L is measured. For example, for designing road cuts the number of populations of L and H is given by the number of different orientations of the longitudinal axes of the slopes resulting from the map work.

Random sampling was carried out based on a system of three coordinates (X, Y, H) and by using random numbers, measuring L with a scale on an adequate topographical map. The remaining conditions for the analysis will be verified by the usual statistical methods. (See Dixon and Massey, 1957, Spiegel, 1961).

Until now, correlation analysis from a population of 22 natural slopes in various regions of the country have been carried out, and the above requirements have been met in all.

For the sake of brevity, correlation curves are presented for a reduced number of variables L and H. Figure 2 contains curves for just six of these populations containing variables L and H. These are characterized by sedimentary rocks, metamorphic rocks, igneous rocks, and rocks from different geological ages and three different regions.

The following deductions are made from such correlation analyses performed on 22 natural slopes:

a. regression equations for the sample population of L and H are of the exponential type ($H=B.L^C$ where B and C are constants). This type of equation has been examined in geomorphological analysis (Leopold, Wolman, and Miller, 1964, P 335)

b. the variances are statistically homogeneous
c. variance does not change with magnitude of the variable
d. the correlation coefficient is high and statistically significant.

It must be emphasized that a high and statistically significant correlation coefficient does not necessarily imply a cause and effect relationship between variables; it only implies a close relationship between the "explainable" variation of the variable and the total variation.

When determining resistance parameters of mass rocks based on natural slopes, it must be noted that within a population of L and H there exists maximum values and, that within the same population of natural slopes the regression equations of the sample populations of L and H have the same C coefficient and a different value of B coefficient. (Curves 3, 4, 5 in Fig 2)

Regression Analysis

The objective of regression analysis is to infer correlations about certain statistical parameters of a given population. By using such an analysis, the outside limits of that population may be found. For example, in the equation $H = B.L^C$, the constants have limited values (high or low) that are also the boundaries within which the correlation between L and H may lie.

For the sake of brevity, the results of this correlation analysis are not given here. It is thus assumed that the parameter values of the sample correlations are the same as those of the population parameters.

Some Probability Considerations

Recognizing that some probabilistic concepts are indeed confusing, some general considerations are noted before presenting the probability analysis results.

Natural slopes are among many of the geological forms subject to analysis from a probabilistic point of view. (Miller and Kahn, 1962; Krumbein and Graybill, 1965). From the analysis of marginal distributions of certain values of L and H, it follows that:

a. each L and H has minimum and maximum values (e.g. H max. Fig. 2), and therefore, the distribution curve (probability distribution) must have its boundaries
b. even though the above is true, the marginal distributions of L and H do have statistical parameters in agreement with normal distributions at a confidence level of 95%, if the variables are transformed by means of logarithms.

In practice these conclusions imply that the marginal distribution of the variables L and H may be represented by a distribution of the log normal type even though this cannot be strictly true. Such a procedure facilitates the analysis of probabilities enormously.

Given the validity of the previous conclusions for the 22 populations of natural slopes and given that the same analysis is performed on any of the 22 populations of L and H studied, only the results corresponding to the population of L and H from the APTIANO de EL COLEGIO formation (Curves 1 and 2) are shown.

Figure 3 shows the points corresponding to values of L and H from the APTIANO de EL COLEGIO. If the correlation analysis is statistically valid, the curve corresponding to the regression equation represents the 50% probability of failure line for values of L and H represented on the curve.

In the same manner, knowing the type of probability distribution of the variables, one can establish curves corresponding to different failure probabilities on the correlation curve. Figure 3 indicates the curves corresponding to a 0.1% and 99.9% probability of failure for given L and H values of the APTIAN del EL COLEGIO. These curves are parallel to that of the regression equation only in the case where the variance does not change with the magnitude of the variables.

Once the probabilities of variables L and H are established it is possible to determine the conditional distribution of a variable for a known value of the other variable. For instance, Fig 4 shows the conditional distribution of failure probabilities of L given that H = 100 meters. The conditional distribution is also log-normal.

Based on curves of this type, it is possible for a given height of slope to determine the failure probability for any angle of slope, or for any length of slope; vice versa, the failure probabilities of H can likewise be determined for a given fixed L.

Economic Analysis

Once the probability of failure for L for a given H is determined, the next step is to determine the criteria of choosing a certain failure probability for design purposes.

The method used to establish such criteria is based on this economic decision theory. This assumes that total cost of any work project be a minimum. Total cost is equal to the initial cost plus capitalized value of failure cost times the probability of failure. For purposes of simplification in this paper it is assumed that the initial cost and the failure cost apply solely to the volume per metre of slope (HL/2) and are proportional to it. The simple equation, which for comparative and illustrative purposes adequately describes the cost of a slope as a function of the probability failure, is as follows:

$$\mathrm{COST} = \frac{HL\ (100 + P)}{100} \quad \text{where}$$

P = failure probability (in %)

L = function of P for a constant H.

Obviously, the complete equation should include the appropriate unit prices in conjuction with the adequate factors of initial cost and capitalized failure cost that are a function of the volume per longitudinal metre of slope and some factors that are not, such as works of art, size of land parcels needed for roads affected by the slope, etc.

Figure 5 gives results obtained by means of the above equation. Figure 5A represents the relation between cost of a slope and the length of slope (or angle of slope) for a given H of 100 metres; Fig. 5B shows it for H of 50 metres. Figures 5C and 5D show the same relation for H of 100 metres but varying the standard estimation error, which in turn is a measure of the variance. In Fig. 5C the standard estimation error is half that of the original correlation curve. In Fig. 5D, it is 25% greater than the original.

In these curves it can be observed that:

a. a probability of failure exists where the cost of slope is a minimum
b. the probability of failure for a minimum cost is the same for a different slope height within a population of L and H
c. the probability of failure for a minimum cost increases proportionally with increasing variance. In other words, the optimum failure probability is a function of the population variance.

The observations mentioned above are valid, thus meeting the requirements previously mentioned under the heading "Correlation Analysis". For example, if the correlation variance is high, and thus the correlation coefficient is not statistically significant, no minimum exists in the curve of costs. If the variance changes with the size of the variables L and H, the failure probabilities for minimum costs shall be different for each slope height.

Again, it must be warned that the equation used to determine such cost curves is adequate only for comparative purposes within the general and illustrative nature of this work. If one utilized the complete equation, one finds it would vary for each case of slope design due to variations in original topography before commencing the cut, and that, for example, the failure probability for minimum cost cannot be the same for different heights of slopes.

RELATION BETWEEN PROBABILITY OF FAILURE AND FACTOR OF SAFETY

Once a minumum cost is determined for a given slope, and consequently for a slope design, it is not necessary to determine a safety factor. Nevertheless, it is interesting to explore the relation between these two factors - the failure probability and the safety factor (since the safety factor is used so indiscriminately in the field of engineering). In broad terms, one can define the safety factor as the relation that exists between resistive forces and acting forces.

$$\text{Safety factor} = \frac{\text{Resistive forces}}{\text{Acting forces}}$$

One can deduce, in some cases rigorously, the probability distribution of such a relation (Wadsworth and Bryan, 1960). If resistance is independent of the acting forces one needs only to know the marginal probability distribution of such a resistance and acting forces. For resistance, it is necessary to obtain a representative mean from a number of statistically significant tests; for acting forces, it is necessary to determine various possibilities that might occur in the ground. In the case of rock masses, and with presently available technology, both are practically impossible to obtain as explained under "Introduction".

The equation in Fig. 1, giving the relation between safety factor and probability of failure was derived. This equation is, at the same time, a function of a failure probability for a given H. This equation was derived for comparative purposes and based on a simple hypothesis of slope failure. The results of the equation for the APTIANO de EL COLEGIO formation are shown in Fig. 6.

Before discussing such results, it is necessary to clarify an important point. A structure whose safety factor is 1 has an equal probability of success or failure. In other words, a safety factor of 1 corresponds to a 50% probability of failure.

Figure 6A shows curves relating the factor of safety and failure probability for a slope height of 100 meters for the original population and for the same population but with different hypothetical variances. Figure 6B shows the same variables for heights of 100 metres and 50 metres.

Figure 6A shows that:

a. for a given factor of safety, the greater the variance, the greater the probability of failure;

b. for a given probability of failure, the greater the variance, the greater the factor of safety.

These observations are in accord with other observations (for example, Lumb, 1966).

The above results suggest that a safety factor not be used in the geotechnical field. For example Fig. 6A, comparing the minimum variance curve with that of the maximum variance, shows that with a safety factor of 2, (which is very common in the design of "stable" slopes), the probability of failure for the population of slopes characterized by the minimum variance is 1.7%, while the probability of failure of those with high variance is a dangerous 30%. It is also important to observe in Fig. 6B, that for a 14% probability of failure (corresponding to a minimum cost) for any slope height from a population of L and H, the safety factor corresponding to a height of 50 metres is 1.65 while that applicable to a slope height of 100 metres is 2.00. Therefore, using the same safety factor for any slope height from the same population of L and H produces a design whose inefficiency from an economic point of view increases as the height of slope decreases.

A good illustration of the above is found in Fig. 7 where the cost of three different solutions has been calculated for the same slope height cut. The figure reveals that solution by means of "terracing" is in general more economic. If the regression equation of the population of L and H is of the exponential type, this will always be the case.

DESIGN OF MINIMUM COST SLOPES IN ROCK MASSES

Applying the results of the foregoing

analysis, the steps of such a method for designing minimum cost slopes for road planning are as follows:

1. The boundaries and nature of each population of natural slopes are defined, based on an adequate geological map (see "Correlation Analysis").
2. In each population of natural slopes, the rectilinear faces are outlined, eliminating those that are accumulations of erosion talus. In this step it is indispensible to utilize geological photo interpretation.
3. Within each population of natural slopes, the populations of L and H are found. For the purpose of road construction, the number of different orientations of the longitudinal axis of the slope (given by the design plan) within a population of natural slopes is made equal to the number of populations of L and H.
4. A system for the random sampling of L and H is designed.
5. Applying the selected system of random sampling, the values of L and H are measured to scale on a suitable topographic plan. If H is determined by the coordinates of the system then only L is measured. L is measured perpendicular to the orientation of the longitudinal axis of the slopes with a given population of L and H. Do not sample in the areas eliminated in Step 2.
6. Once the values of L and H are obtained, the correlation analysis is carried out. (See section "Correlation Analysis"). If the correlation requirements are not met, one cannot continue with the following steps.
7. Regression analysis is carried out (see section "Regression Analysis"). The parameters of the population are defined by using adequate criteria.
8. The distribution of the marginal probabilities of the variables L and H are determined, and the degree of fit to the normal distribution (or to any of its direct transformations, for example, log-normal transformations) is next determined to confirm a high confidence level. If such a fit is not satisfactory, the following steps cannot be continued.
9. Based on the results of steps 6, 7, and 8, the conditional distribution or probabilities of L are calculated for the different heights of slope H of the populations of L and H.
10. The complete cost equation is found as a function of the failure probability.
11. Based on the results of steps 9 and 10, the relation curve between cost and the probability of failure, is calculated and the probability of failure is determined for a minimum cost for each height of slope H.
12. Knowing the failure probability of minimum cost for each height H of slope, the length L or angle of slope is determined.

Figure 8 could be the manner in which such results are represented.

The design graph is shown in Fig. 3 (probability of failure = 14%). Please note for the case of minimum cost probability of failure, that regardless of the slope height, the design curve is parallel to the regression surve. If the minimum cost probability is based on the <u>complete</u> cost equation, such parallelism cannot exist.

It is also interesting to note in Fig 8 that the design angle decreases as the height increases. This will be true as long as the equation is of an exponential nature. This presents doubts about the actual practice of slope design - of utilizing slope angle for any given height within the same type of rock - because the regression equation of an exponential nature occurs frequently in Colombia.

CONCLUSIONS

If in a population of natural slopes there is a correlation between height and length, and such correlation meets the requirements imposed by a statistical analysis, then:

a. one can predict the probability of failure of a given slope; however, the exact time of failure cannot be foretold.
b. one can use a method based on the hypothesis that nature provides the best possible in situ practice. This relates statistical and prob-

ability analysis with the theory on economic decision to achieve a minimum cost slope design. Such a method has the following advantages:

- it is simple to apply as it does not require knowledge of the mechanical characteristics of a rock mass nor of the applied forces. This knowledge is almost practically impossible to obtain with the present available technology.
- eliminates the safety factor concept which in many cases provides unfavourable results. Among other reasons, where the variance is high, a safety factor which may indeed be considered satisfactory may well be equivalent to a dangerous probability of failure.

c. If such correlation is represented by an exponential type of equation which is frequently the case in Colombia, then there are doubts as to the common practices of using the same safety factor for any height H, and utilizing the same angle of slope for any height H in a given rock type.

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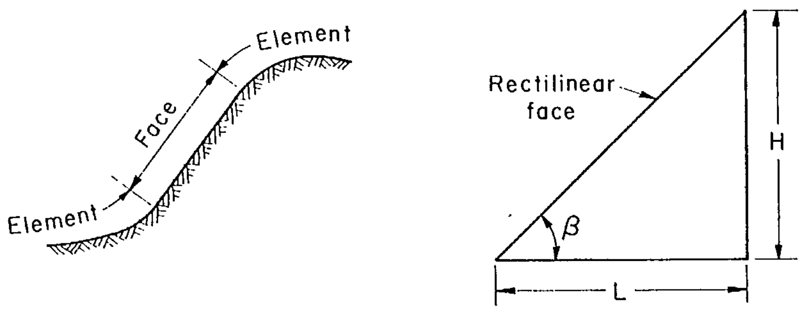
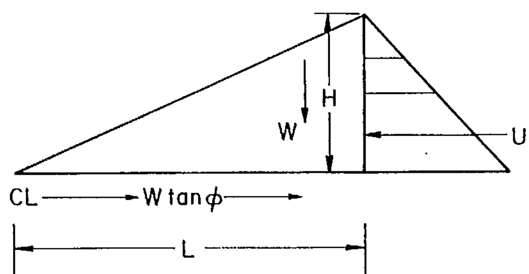


(a)



W – weight of block $= \frac{HL}{2}\gamma_s$

γ_s – unit weight of mass

ϕ – friction angle

C – cohesion

U – water pressure $= \frac{\gamma_w H^2}{2}$

γ_w – unit weight of water

(b)

Note: For the curves in Fig. 6, the following values were utilized:

$\gamma_s = 2.3\ T/m^3$

$\phi = 30°$

$C = 3.0\ T/m^2$

$\gamma_w = 1.0\ T/m^3$

$H = 100$ meters

f(p) obtained from Fig. 4

Safety factor $F_s = \dfrac{CL + \frac{HL}{2}\gamma_s \tan\phi}{\frac{\gamma_w H^2}{2}}$

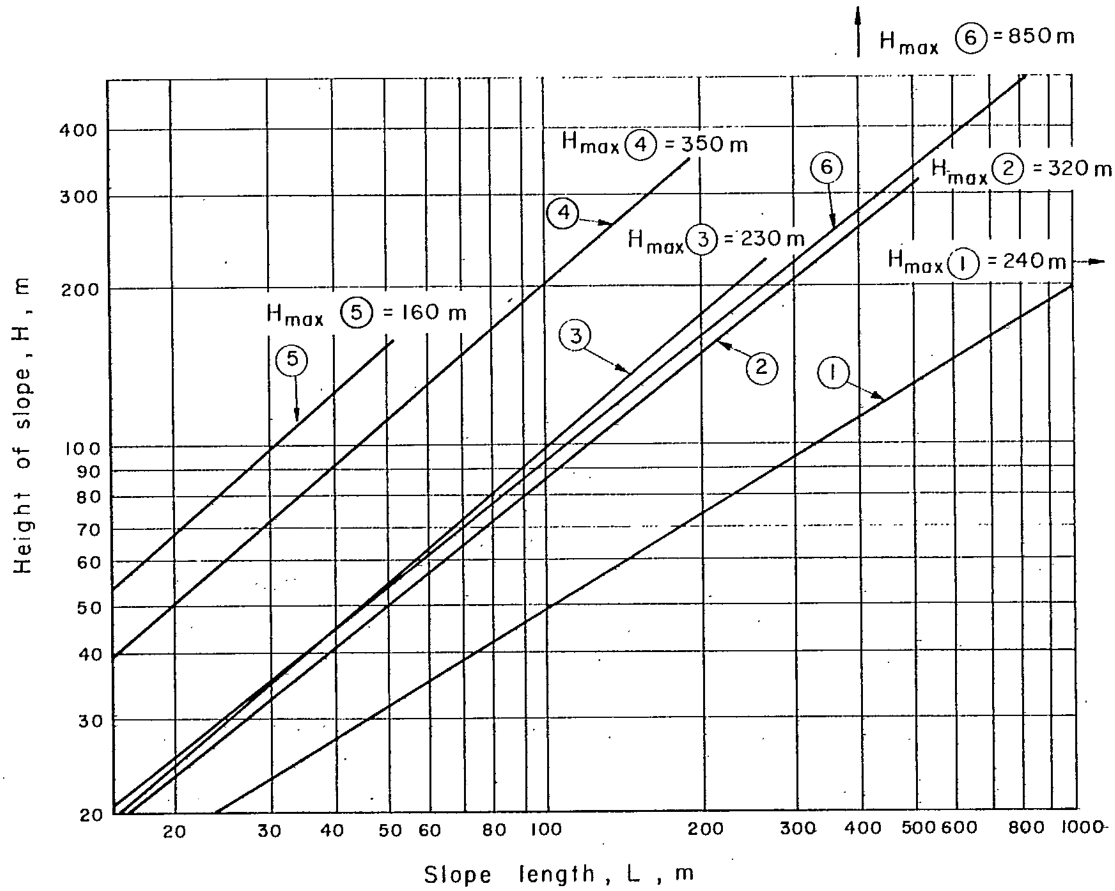
$$\frac{\partial F_s}{\partial L} = \frac{2\left(C + \frac{H}{2}\tan\phi\,\gamma_s\right)}{H^2\gamma_w} = A$$

$\Delta F_s = A \cdot \Delta L$

$F_s = 1.0 + \Delta F_s = 1.0 + A \cdot \Delta L$, where ΔL is a function of probability of failure for H, constant.

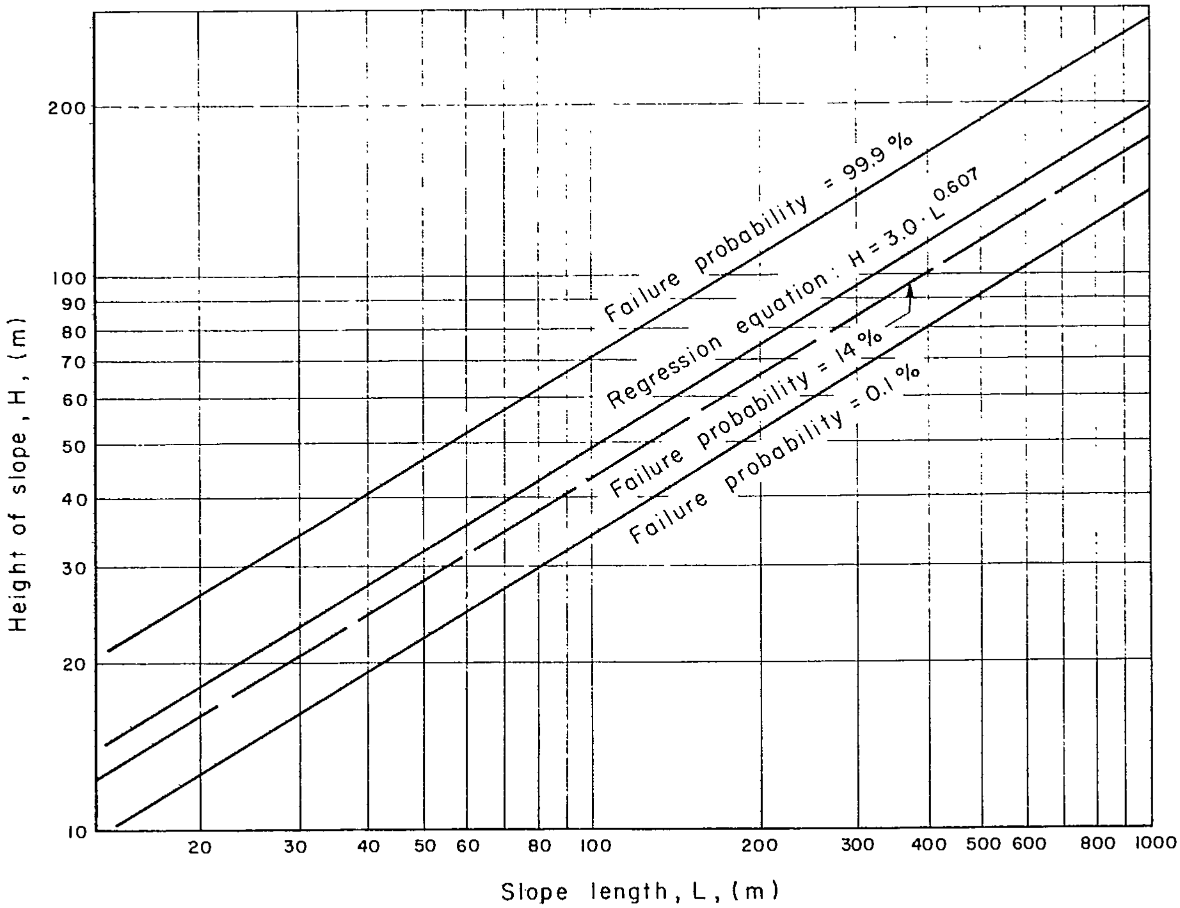
$F_s = 1.0 + A \cdot f(p)$

Fig. 1 - Nomenclature and derivation of the relation between safety factor and failure probability.



Curve	Project
①	Tubería de Carga - Central Hidroeléctrica "El Coleglo."
②	
③	Presa "La Esmeralda" Rio Batá.
④	
⑤	
⑥	Carretera Panamericana. Sector Pasto - Rio Patía.

Fig. 2 - Correlation curves for samples of various "L" and "H" populations.



Correlation coefficient "r" = 0.987   Limits of r : 0.987 ± 0.012 (95% confidence)

Estimated standard error "$S_{L\,H.}$" : 0.200

Fig. 3 - Correlation curve.

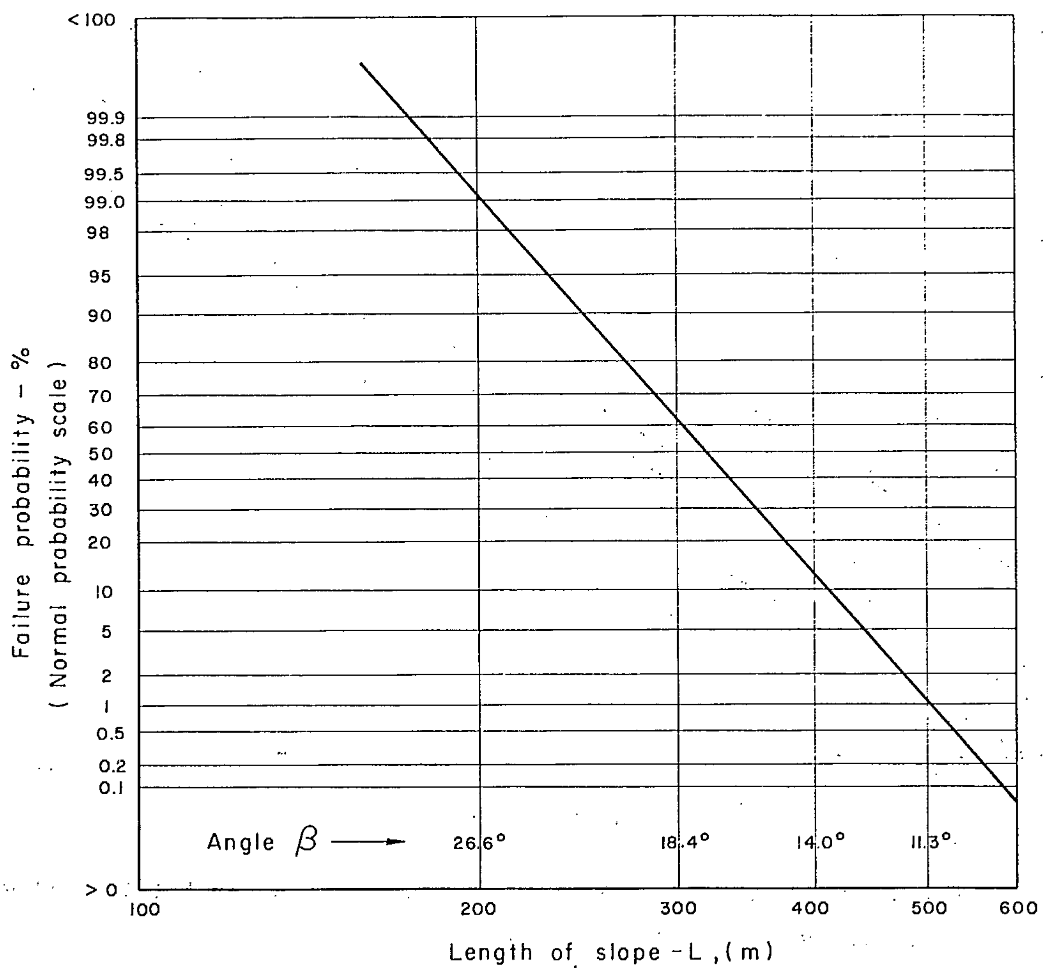
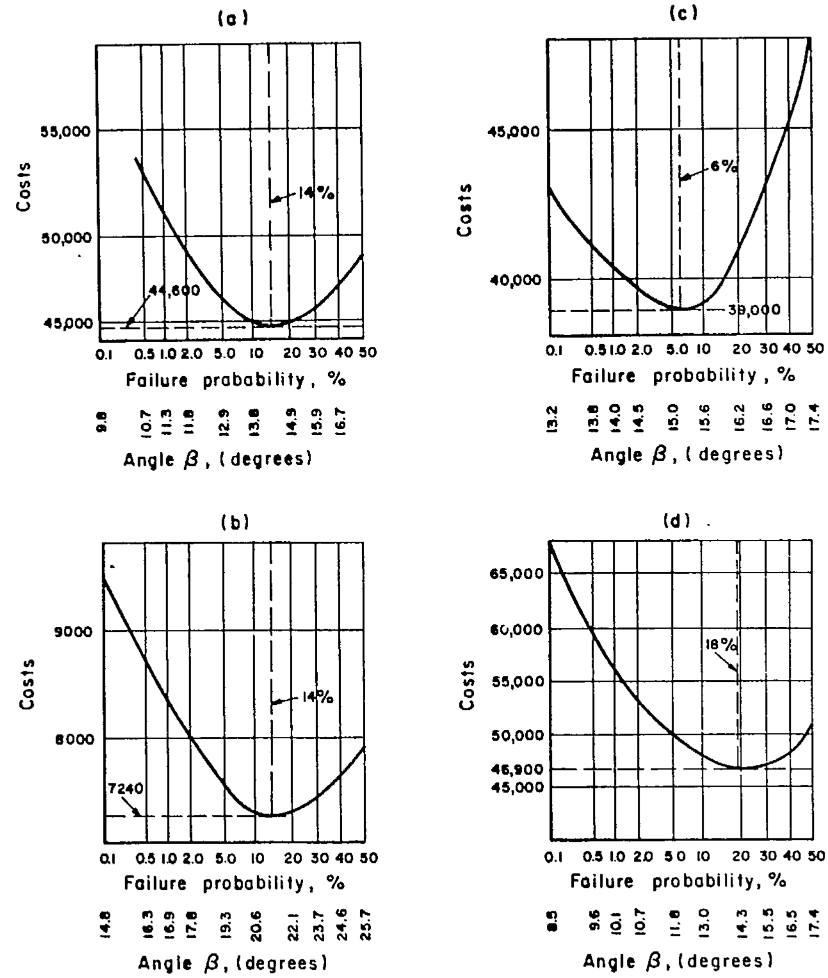
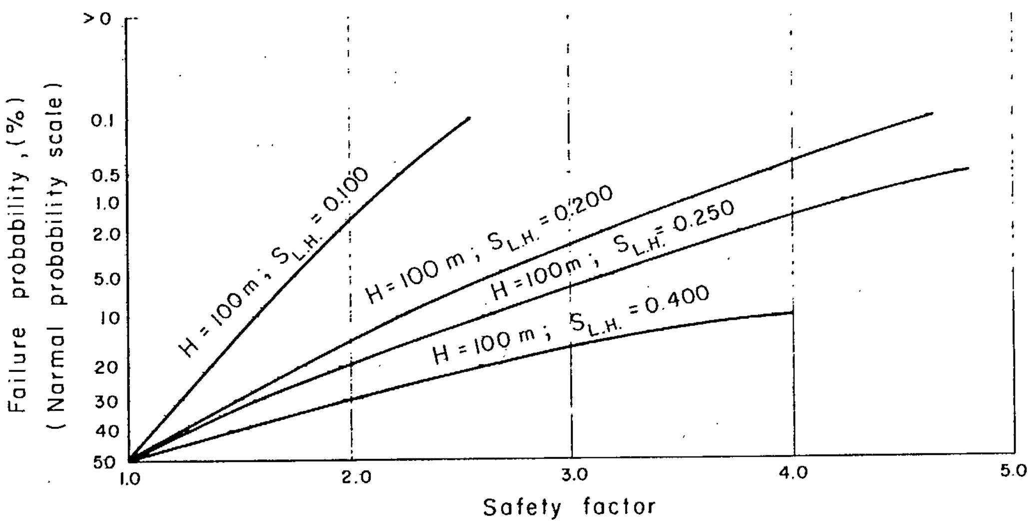


Fig. 4 - Conditional probability distribution of "L" for H = 100 metres.

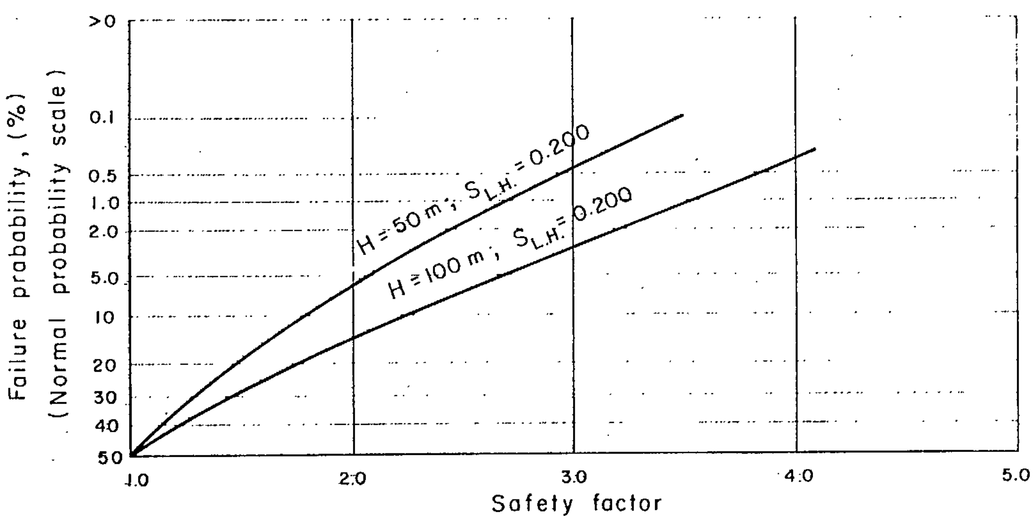


Population of natural slopes	Population of L and H	Fig.	Slope height (H)	Estimated standard error	Failure probability at minimum cost (%)	Cost
Artiano El Colegio	Measurements perpendicular to direction of strata	a	100	0.200	14	44,600
		b	50	0.200	14	7240
		c	100	0.100	6	39,000
		d	100	0.250	18	46,900

Fig. 5 - Curves relating cost of slope and its failure probability or slope angle.



(a)



(b)

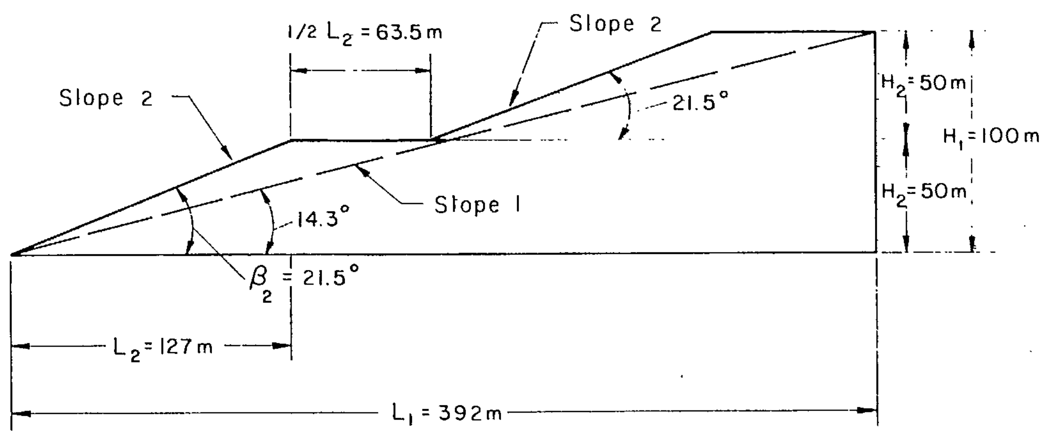
$S_{L.H.}$ = Estimated standard error (function of the variance)

Fig. 6 - Curves showing relation between failure probability and factor of safety for a) different variances; b) different slope height.

Solution 1: H = 100 m — Design using maximum probability of failure (Fig. 5a)

Solution 2: Terracing using optimum probability of failure (Fig. 5b)

Solution 3: Terracing using probability of failure corresponding to safety factor obtained in solution 1 (Figs. 5b and 6b) not shown.



Solution	H	L	β	Probability of failure	Safety factor	Cost
1	100	392	14.3°	14 %	2.00	44,600
2	50	127	21.5°	14 %	1.65	35,200
3	50	141	19.5°	6 %	2.00	37,250

Fig. 7 - Comparative economic analysis of three different solutions for a slope with 100 metres.

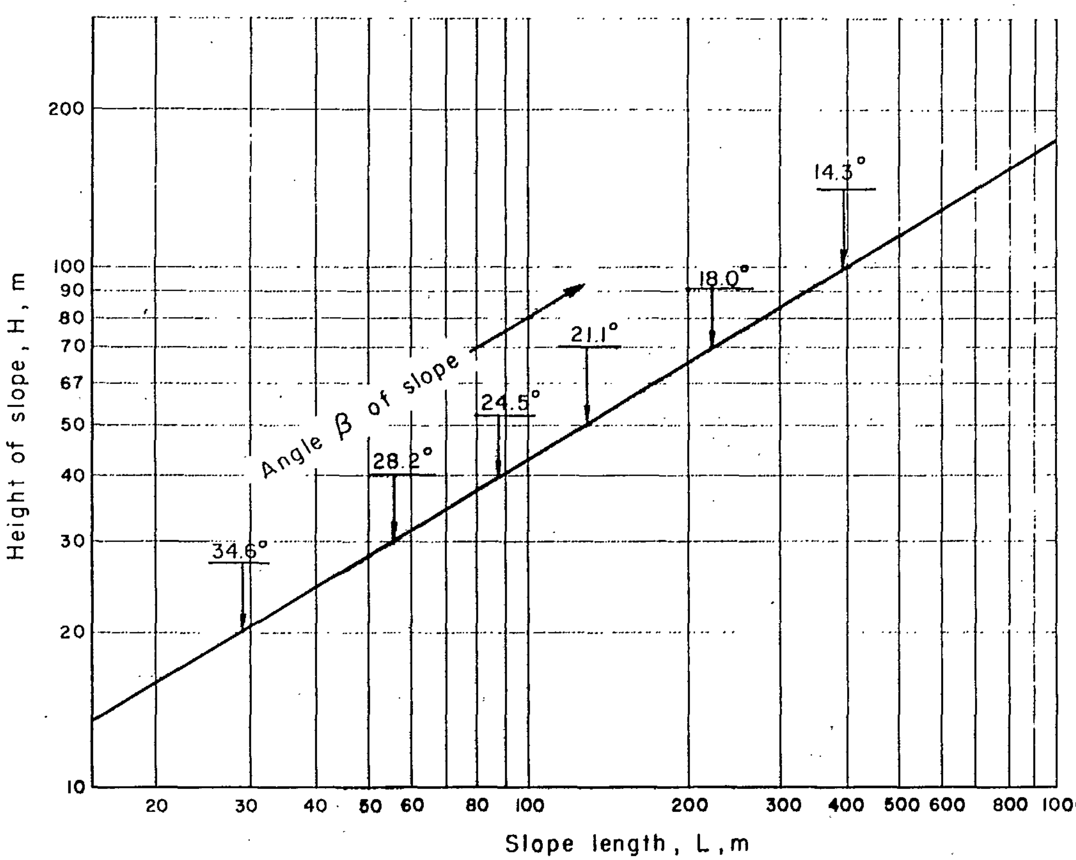


Fig. 8 - Example of a graphical design for slopes.

APPENDIX C

OPTIMIZATION OF SLOPES DESIGNED IN ROCK

THOMAS SHUK*

SUMMARY

The probability of failure as a function of the geometry of a slope designed in rock can be obtained through the use of statistical analysis and inference, and of probability analysis. Three different methods to obtain the probability of failure are described: the safety factor method, the risk method and the natural slope method. The total expected cost of the design slope is a function of its probability of failure, and it has a minimum cost point which corresponds to the optimum design point. The present day conceptual and mathematical limitations implied in an optimization analysis for slopes do not reduce its value as a rational design tool. The approach presented in this paper allows the use of the full range of values of design parameters, and it has the advantage or reducing subjective factors in the choice of values to be used out of a number of possible values of parameters, as well as that of obtaining the most economical cost for the slope.

INTRODUCTION

Slopes are usually designed on the basis of determining beforehand, and usually arbitrarily, a so-called adequate factor of safety, by deciding on a unique available shear strength either through laboratory or field tests, and by computing the acting stresses. The computations are made along several possible failure planes or lines in two-dimensional analysis. The factor of safety of the critical failure plane or line so obtained is compared with the previously decided value. The geometrical configuration which conforms to the adequate safety factor is then the design slope.

It has been shown that a probability of failure can be determined for a given slope (Beirnatowski, 1969; Langejan, 1965; Shuk, 1968). This probability of failure, if so desired, can be related to the factor of safety (Lumb, 1966; Shuk, 1968), and can be used within the framework of economic decision theory to obtain the minimum cost slope.

Such an approach gives the designer the opportunity of obtaining a more rational design resulting in the minimum cost slope, and of placing his analysis within the proper framework of the variability and random nature of the properties of the rock mass. In addition, it has the advantage of reducing subjective factors in the choice of unique design parameter values which have been traditionally based on so-called "judgement" and "experience", and allows him to use the full range of values of these parameters.

PROBABILITY OF FAILURE

In practice, the relative frequency definition of probability has to be used. In mathematical notation (symbols are defined in Appendix - Notation):

$$P(E) \equiv \lim_{n \to \infty} \left(\frac{r}{n}\right)$$

* Civil Engineer, Partner and General Manager, Geocolombia, Bogota, Colombia, S. America.

With "r" occurrence of the event "E" in "n" trials of a specified experiment, the probability of event "E" is defined as the limit of the relative frequency $\frac{r}{n}$ as the number of trials is increased indefinitely.

In most design situations the operational concept of probability of failure is clear for the problem at hand if the relative frequency definition is used; the operational definition in the case of slopes is not clearly understood. This situation arises in part due to the fact that a 100% probability of failure determined on the basis of a relative frequency defintion does not necessarily imply failure. Three main categories of operational definitions are proposed for slopes:

a. The percentage probability of failure represents the percentage of slope which fails (Coates, 1965).
b. The probability of failure represents the percentage of slopes which would fail out of a number of slopes with equal geometries (Shuk, 1968).
c. Since all slopes fail on a geological time scale, the probability of failure is a relative measure of the time to failure with respect to the other slopes within the population, and for a large slope life-period. In this respect, the probability of failure could possibly represent a <u>stochastic</u> variable of a Bernouilli (yes or no) process (Barrera, 1969).

The choice of operational definition of probability of failure does not affect the mathematical analysis to obtain this probability but does influence the optimization analysis.

It has to be remembered that in the case of continuous variables such as slope design parameters, the probability of a given value of the variable is undefined, and either alternative of the probability being "less than..." or "more than...." has to be used for a given value of the variable. In this paper the first alternative will be implied whenever the word probability is used.

For the design of slopes, there are three general approaches to obtain the probability of failure. These methods will be explained below.

<u>Safety Factor Method</u>

Using the usual general definition of the safety factor,

$$F_s = \frac{S}{s} \quad \text{or} \quad F_s = \frac{M_r}{M_a}$$

If the variables which enter in the division for the safety factor equation are random and their probability densities can be determined, the marginal probability density or distribution of the safety factor can be obtained. The details of the process are now within the scope of this paper, and are explained in standard reference texts (for example: Haugen, 1968; Wadsworth & Bryan, 1960). In this case:

$$P_f = P(F_s < 1.0)$$

That is, the probability of failure is the probability of the safety factor being less than one. This concept is made clearer in Fig. 1.

Figure 1A shows probability densities (with the assumption that the distributions are normal) for two different standard deviations (or variances) of an average safety factor of 1.5, which under normal circumstances is considered adequate. It is clear that if the variability of the data is large, the probability of failure for this adequate safety factor is not tolerable. This is stressed in Fig. 1B, where the probability distribution of the high variance population gives a probability of failure of 27% for a factor of safety of 1.5.

This is why it is always important to determine, if possible, the probability distribution of the safety factor for design slopes (Shuk, 1969). Figure 1B shows also a real design case, where due to the large variability of the strength data an unusually large safety factor of approximately 3.5 would have had to be used so as to have a probability of failure below 5%. For this particular case a benching solution was adopted.

Risk Method

This method is widely used in structural and aeronautical design. In this type of approach, the probability of failure becomes:

$$P_f = P(S - s < 0)$$

That is, the probability of failure is the probability that the difference between the available strength and the acting stress is less than zero. This concept is illustrated by the Warner diagram (Haugen, 1968) shown on Fig. 2A, and by the strength minus stress density shown on Fig. 2B.

In general, if strength and stress are statistically independent (which is not necessarily the case for slopes), then:

$$P_f = \iint_R f(S)f(s)\, dSds$$

With this method, a relationship between the safety factor and probability of failure can be determined. Figure 2C shows, for the case of normal probability distributions, the results of the relationship between safety factor and probability of failure as determined by the safety factor method and by the risk method.

Natural Slope Method

This approach has developed since 1949 (Binger & Thompson, 1949; Lane, 1961; Terzaghi, 1962; Shuk, 1965; 1968, 1970), and is specially well adapted to rock masses. In this case the probability of failure results directly from a function obtained through a regression analysis relating geometrical variables of a slope.

Figure 3A shows such a function for a population of geometrical variables of slopes in a rock mass composed of diabase; the regression equation is exponential. Equations of this type have been found to exist with significant correlation coefficients, and with a high degree of statistical confidence for the correlation coefficient, for all of a large number of rock mass populations studied in Colombia. These rock mass populations include all types of lithologies, and all types of origins (igneous, metamorphic, sedimentary), and are mostly moderately to highly jointed. Such types of functions have also been found by geomorphologists (Leopold & Wolman & Miller, 1964). They can also be derived from theoretical slope stability analyses (Shuk, 1970).

If the variables are normally distributed, or can be normalized by appropriate transformations, the regression analysis provides the means to obtain the probabilities of failure. The methods used for regression analysis are described in standard reference texts (for example: Dixon & Massey, 1957; Krumbein & Graybill, 1965; Miller & Kahn, 1962; Spiegel, 1961), and are not within the scope of this paper. A detailed explanation of the sampling and measurement of the variables is given elsewhere (Shuk, 1968).

In the case of the natural slopes studied in Colombia, the random variables L and H (see Fig. 3) have been found to fit a normal distribution with a high level of statistical significance as indicated by the chi-square test, if the variates are transformed by logarithms; that is, they have log-normal distributions. This occurs provided the population is carefully defined and bounded, and that sampling is random. The log-normal distribution did not fit the data in the case of natural or excavated rock slopes in jointed gneiss studied in the central region of the state of Colorado, U.S.A. (McMahon, 1968), but there is some doubt as to the validity of the author's population bounds.

Figure 3A also shows several probability of failure curves obtained by regression analysis on the basis of log-normal distribution of the variables. These curves are parallel to the regression curve if the variance is statistically homogeneous for the range of values of the variables.

From the above it is possible to obtain a relationship between the probability of failure and the value of one of the variables for a given constant value of the other variable. Figure 3B shows this relationship for the variable L of the diabase slopes given a value of 100 metres for the variable H.

Once this probability of failure is e

blished for the range of values of L, a relationship between the safety factor and probability of failure can be computed, from the starting point of a factor of safety of one being equivalent to a probability of failure of 50%. One method of obtaining such a relationship is described elsewhere (Shuk, 1968), and the results for an H of 100 metres are shown on Fig. 3C.

OPTIMIZATION ANALYSIS

By any of the three methods described previously it is possible to establish a function relating L (for a given H) to the probability of failure such as the one shown on Fig. 3B, obtained by the use of the natural slope method.

The second operational definition of probability of failure, as described in the previous discussion, is used to derive the expected total cost function. Furthermore, the total cost function is assumed independent of time, because at present no method is available to predict the probability of failure within a given amount of time. The inadequacy of this hypothesis is reflected in the fact that present day capitalized costs cannot be calculated for special maintenance costs (failure of the slope at any time) within the economic life-span of the slope. Accordingly, a zero rate of interest is implied in the derivation which follows. The expected total cost is given by the sum of three costs:

$$\overline{C}_T = C_o + C_m + C_o \cdot P_f \qquad \text{eq 1}$$

C_o: initial construction cost; this cost is made up of a part which is proportional to the area of the slope (the slope construction cost), and a part which is not proportional to the area of the slope but which is affected by its stability, such as the cost of culverts, roadways, rail-guards, minor or major structures, etc.

C_m: normal maintenance costs during the economic life of the slope.

C_oP_f: special maintenance costs during the economic life of the project due to failure of the slope, affecting not only the slope but also the works within its influence area.

From equation 1:

$$\overline{C}_T = C_m + C_o\,(1 + P_f) \qquad \text{eq 2}$$

Where:

$$P_f = F(L) \text{ for } H = \text{constant}$$

and:

$$C_o = k\left(\frac{H_oL_o}{2} + \frac{H_o + H}{2} \cdot L\right) + C_b$$

(see Fig. 4A)

The value of L for minimum cost can be obtained from the positive roots of:

$$\frac{\partial \overline{C}_T}{\partial L} = 0 \qquad \text{eq 3}$$

In most cases equation 3 cannot be solved explicitly to obtain the optimum L. In this case equation 1 or 2 have to be solved numerically or graphically, or equation 3 solved by the use of iterative computer programs. Figure 4C shows the results for two real cases.

In equation 1, $C_o + C_m$ is a monotonically increasing function with increasing L, while $C_o \cdot P_f$ is a monotonically decreasing function with increasing L, as shown on Fig. 4B, for most relationships between L and the probability of failure. The existence of a minimum point depends on the rate of decrease of the function $C_o \cdot P_f$ with respect to the rate of increase of $C_o + C_m$. If the variance of L for a given value of H is high, and the rate of decrease of $C_o \cdot P_f$ is low, the minimum point might not exist in the total expected cost function.

The previous analysis does not impose any restrictions on the value of the optimum probability of failure, and it is possible to obtain a high probability of failure for minimum expected cost, such as shown by curve 2 on Fig. 4C. If it is desired to introduce a restriction on P_f so that it is always less than a certain value, it is simple to introduce analytically a similar restriction on L.

CONCLUSIONS

By means of methods based on statistical and probabilistic techniques it is possible to determine the probability of failure for a design slope in rock as a function of the parameters of geometry of the slope.

The present lack of an adequate operational definition of the probability of failure for slopes, implies that the expected total cost function has to be set up independent of time, and consequently the optimization analysis presented should be considered as first approximation of a cost function which in the future should include the present day capitalized cost of failure. The previous statement points out the urgent need for research in determining an operational definition of probability of failure for slopes, and for establishing a time-dependent model of slope failure.

The present conceptual and mathematical limitations of optimization analyses for slopes do not reduce their value as tools for a more rational design of slopes in rock masses. Furthermore, they permit the designer to use the full range of his results towards an economically efficient objective.

ACKNOWLEDGEMENTS

The author is grateful for the help received from Eng. Antonio Barrera (Director of the Public Investment Group, National Department of Planning, Bogota) during the course of the preparation of this paper.

SYMBOLS

C_b: initial cost of the works in the area of influence of the stability of the design slope.

C_m: normal maintenance cost during the economic life of the design slope.

C_o: initial construction cost of design slope and of the works in the area of influence of its stability.

C_T: total expected cost during the economic life of the design slope and of the works affected by its stability.

E: a given event.

f( ): probability density of the variable in parenthesis.

F( ): function of the variable in parenthesis.

F_s: safety factor.

H: height of slope (see Fig. 3A).

H_o: height of slope corresponding to L_o (see Fig. 4A).

k: unit price per unit volume of initial construction cost of the slope.

L: length of slope (see Fig. 3A).

L_o: length of slope needed as space for engineering work (see Fig. 4A).

M_a: acting moment.

M_r: resisting moment.

μ: average value.

n: number of trials of a specified experiment.

P( ): probability of the event specified in parenthesis.

P_f: probability of failure.

r: number of occurrences of an outcome of a given type.

ρ: correlation coefficient.

R: appropriate region of integration.

s: applied stress.

$\bar{s}$: average applied stress.

S: available strength.

$\bar{S}$: average available strength.

σ( ): standard deviation of the variable in parenthesis.

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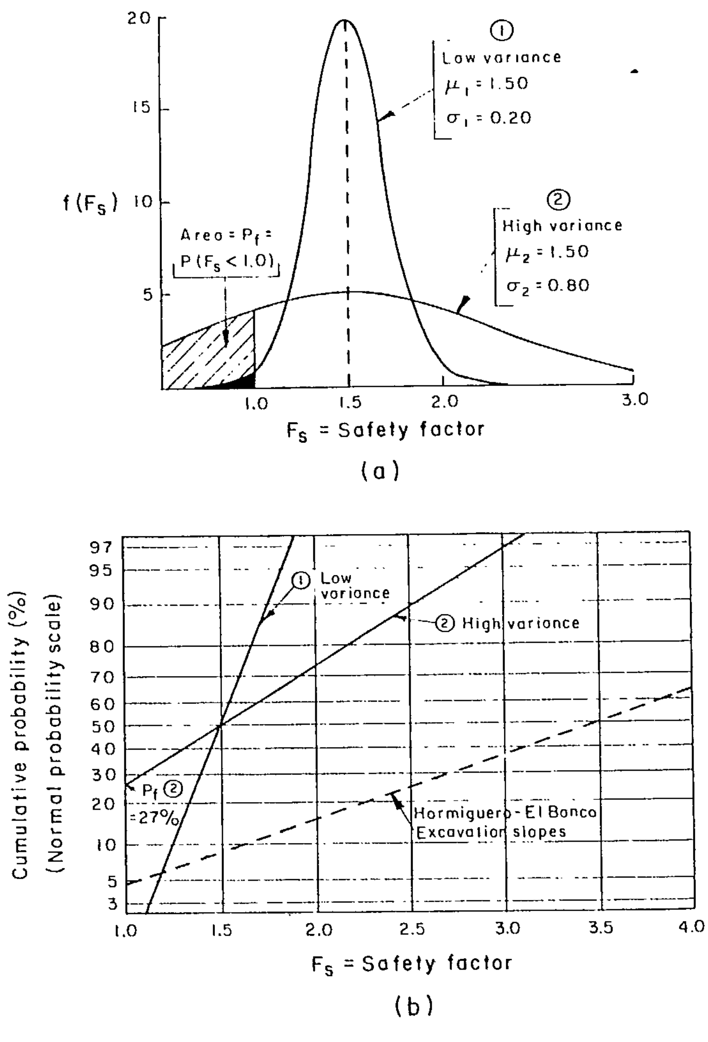


Fig. 1 - Safety factor method

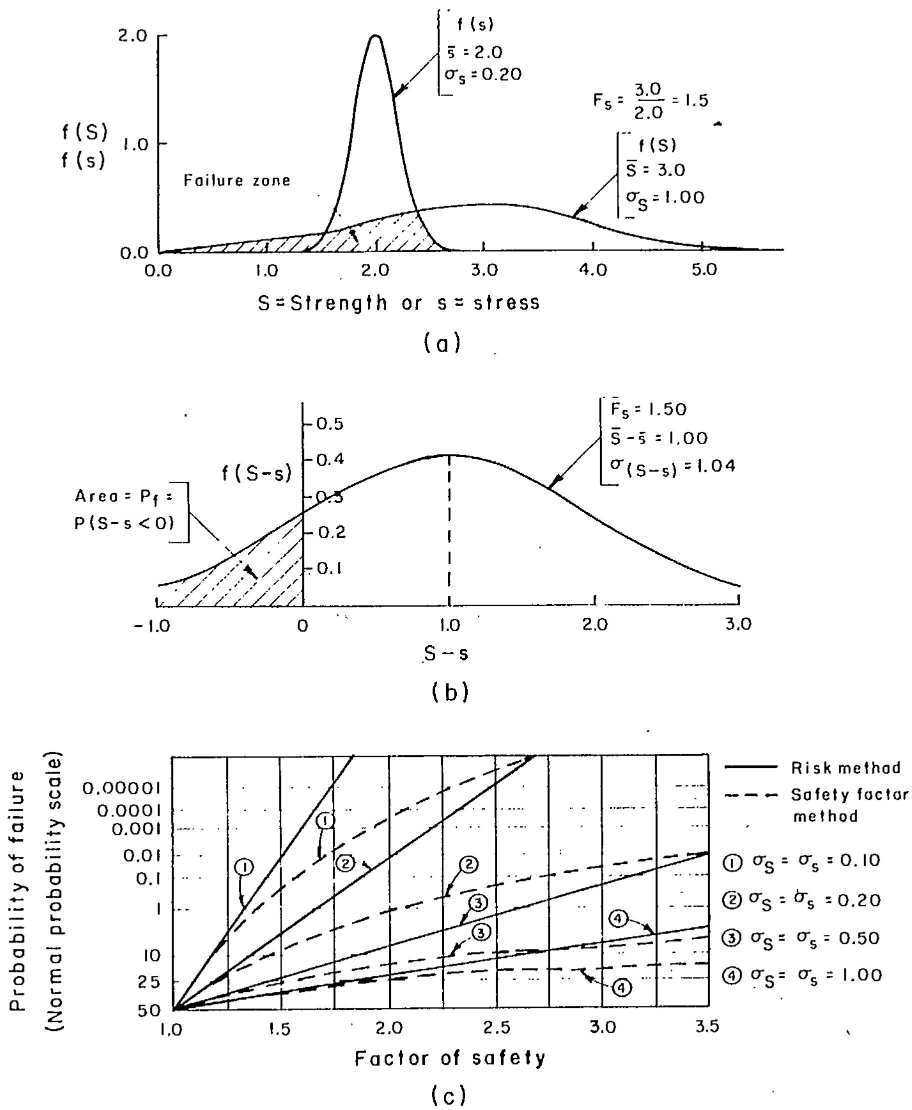
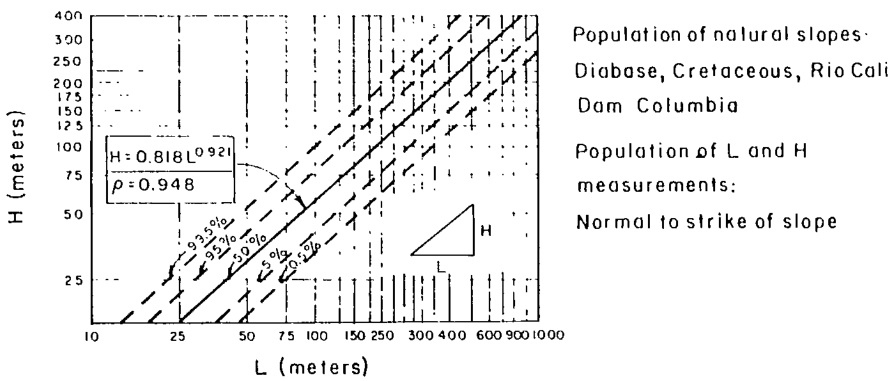
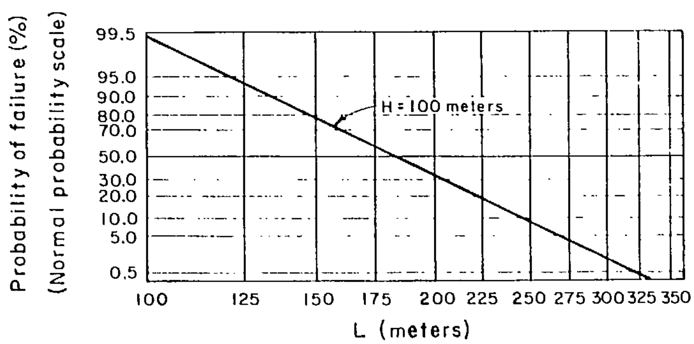


Fig. 2 - Risk method



(a)



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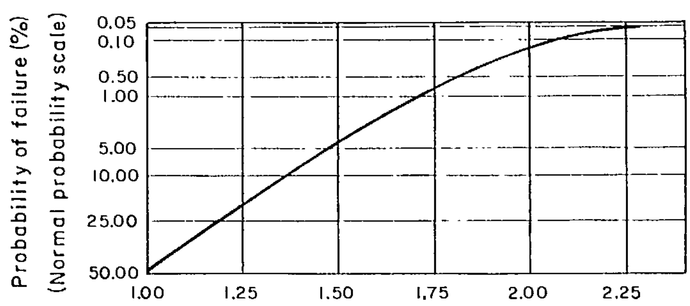

(c)

Fig. 3 - Natural slope method

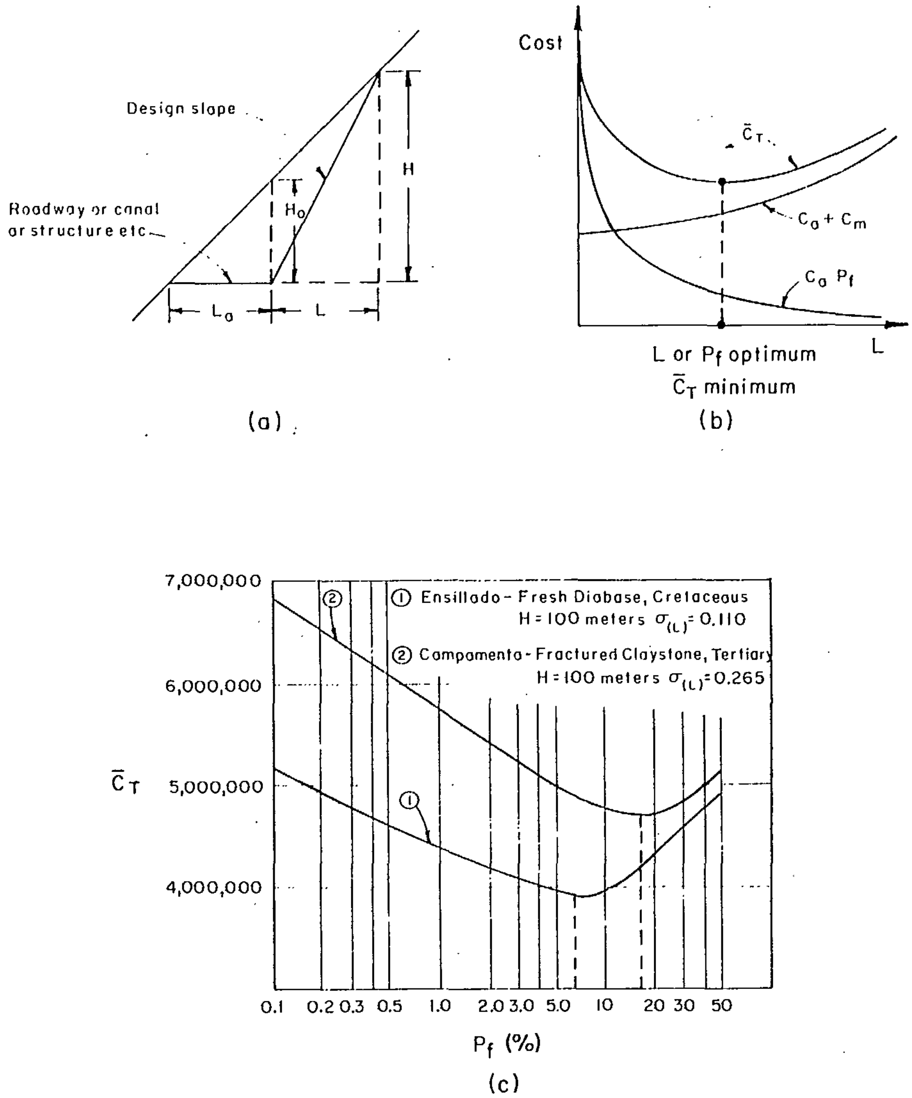

Fig. 4 - Optimization of slopes

WALL STABILITY IN THE SOUTH ROBERTS PIT - AN EXAMPLE OF THE USE OF PREVIOUS SLOPES

by R.O. Stark*

ABSTRACT

The South Roberts pit was started in 1961 and completed in 1972 by Steep Rock Iron Mines Limited. Mine design was based substantially on the experience obtained in the previous Errington, Hogarth and Barrier pits in the same formations.

Whereas the walls in the prior pits were designed largely by trial and error, in the case of the South Roberts pit a request was made to Mines Branch, Department of Mines and Technical Surveys, to assist in establishing a rock mechanics program for both underground and open pit operations. Previous slides as well as stable walls were analyzed. The major rock types in the walls were tested in the laboratory. Slope angles were then recommended that could result in no more than 10% of the total walls being subject to instability, thereby providing an optimum between waste excavation and cost of slide clean-up.

On completion of mining, a review of the accumulated experience indicated that the wall design had actually been close to the economic optimum. The carbonate slope, excavated at 58° for a height of more than 1000 ft, remained stable except at the contacts with the altered paint rock where some ravelling occurred. The paint rock slopes within the designed height limitations were substantially stable at 42.5°; however, when excessive digging at the toe created a steeper slope, instability almost invariably occurred. In the ash rock excavated to a depth of 760 ft, slides occurred in some 13% of the wall although 61/2% of the instability took place after mining had ceased.

*Senior mining engineer, Steep Rock Iron Mines Limited, Atikokan, Ont.

INTRODUCTION

History was made on July 29, 1972, when Steep Rock Iron Mines Limited completed one of the deepest open pits in North America. The South Roberts Pit, in operation since 1961, was finally abandoned after it had been stripped and mined to designed limits. A total of 75,000,000 tons of stripping was moved in that period to mine 15,900,000 tons of ore. In addition, many millions of tons of clay were dredged from the original lake bottom before stripping could begin. The successful completion of the program was dependent on stability of the walls. This paper is a description of the problems encountered and the steps taken to solve them in a safe and economical manner.

Steep Rock Iron Mines Limited is currently mining a high grade hematite-goethite deposit at Atikokan, Ontario (Fig. 1). The orebody was located on the bottom of Steep Rock Lake and its existence was suspected as far back as 1897; however, it was not until 1938 that it was confirmed by drilling. Development of the property was accelerated because of the shortage of iron ore during the Second World War, and in late 1944 the first ore was mined. The development of the mine was a major engineering feat which included the diversion of the Seine River system around Steep Rock Lake, the pumping of 118 billion gallons of water, and the dredging of 120 million cubic yards of clay.

The first orebody mined was the "B" or the Errington Pit, located at the south end of the middle arm. This pit was mined to an elevation of 725 ft or 535 ft below the original lake level.

The next area to be mined was the Hogarth Pit at the extreme north end of the middle arm.

This was mined to the 600 ft elevation, 660 ft below the original lake level.

When the Hogarth was finished, development of the Barrier Pit began, eventually becoming part of the South Roberts Pit. This was mined in a series of operating phases and was completed to a depth of over 1,000 ft below the initial stripping limit, 940 ft below the original lake level.

In the beginning, pit slopes were designed on a trial-and-error basis. Very little work had been done up to that time to optimize slopes, and there were also very few pits mined to any depth in similar types of rock. Generally, the walls were steeper that now considered safe, thus slumping, ravelling, and minor sliding of the walls were common.

As the pits deepened and wall failure became more frequent and severe, Steep Rock Iron Mines approached the Department of Mines and Technical Surveys in Ottawa for assistance. This was the beginning of a new era when overall pit slopes were no longer based on the ore contacts being intercepted by benches or by the quantity of ore required in a year.

RESEARCH PROGRAM

In 1960, a program of compiling information on ground pressures had been undertaken by the Mines Branch, Department of Mines and Technical Surveys, through an external contract under the direction of Dr. D.F. Coates, research engineer. The prime purpose of this program was to establish strength parameters of the ore and waste materials for underground mining. Many of the tests completed for this purpose, however, were equally useful for establishing parameters for slope stability in the open pit. It was judged that since some of the ore and the paint rock materials were soft and earthy in their natural state, the strength of these materials, when compacted to their original density and moisture content, should be similar to their strength in place. Laboratory tests of the various rock types included the use of drill cores and re-compacted samples.

In 1961, within this research program, a compilation of all the experience in the open pits mined to that time was undertaken. This was the first effort to establish safe pit slope parameters for the open pit operations. This project included the gathering of information on all wall failures including the material type, height and breadth of failure, slope angle before failure occurred, contour of the wall in the slide area, and the presence of water. The slide data were then grouped by material type.

In addition to the compilation of information on failures, a program was undertaken to study the same parameters on slopes which had not failed. This information was for comparison with data collected from the slide areas. Where the basic information appeared similar for a stable and an unstable wall, more consideration was given to side resistance, groundwater conditions, and any other factors which might affect stability.

The two major recommendations resulting from this study were that (1) the water table is a major factor and must not be ignored, and (2) the optimum slope angle is a function of slope height.

Recommendations were submitted considering the above parameters for optimum slopes in the footwall carbonate, paint rock, and the hangingwall ashrock. It was estimated that by optimizing the recommended slopes, only 10% of the total wall would show signs of instability.

Geology

The three most important rock types encountered in the mine area are the carbonate rock, paint rock and ashrock, all of which are from the Archean Pre-Cambrian era. Figure 2 is a typical cross-section through the orebody, showing the relative location of these rocks.

The carbonate is described as a brecciated dolomite cemented together with calcite and quartz. Traces of hematite are associated with the calcite and quartz, and small crystal-lined voids occur.

The paint rock is a transition zone between the carbonate and the main orebody and is a soft, fine-grained mass of quartz, pyrolucite, and kaolin with subangular fragments of chert, hematite, and goethite. Dykes associated with the orebody are treated as paint rock because of their similar physical characteristics.

The ashrock is a pyroclastic of an unusual basic type. It occurs both as an unaltered hard, blackish material, and as a soft, altered, schistose material. Alteration is generally characterized by a decrease in the silica content and an increase in the iron content, mainly in the form of pyrite.

Test Results

On the basis of laboratory tests performed on core removed from structural drilling and the analysis of the steep cliffs which existed after the water and clay were removed from the lake, it was apparent that the carbonate was very competent and was not affected by groundwater, natural erosion or weak joints. A 58° angle provided sufficient room to leave a 20-ft bench every 50 ft of height to control erosion (1).

The paint rock is generally an incompetent, earthy material. Its stability is adversely affected by groundwater, erosion, and by the height of the slope. On the basis of the laboratory tests and the data which had been accumulated from previous pits, a graph was plotted using the two main variables of slope height and groundwater (Fig. 3). As mentioned previously, the laboratory tests were based on re-compacted samples assumed to have the same strength characteristics as material in place.

Similarly, a graph (Fig. 4) was plotted for the ashrock using the same parameters. The laboratory tests on drill hole cores, however, were not as significant since the weakness in the ashrock was generally in the joints and highly-altered areas (2, 3).

Design

The South Roberts Pit was to be mined to a depth of 900 ft below the old lake bottom. Based on recommendations resulting from the research program, a set of criteria was established for the various rock types.

The carbonate was designed at an overall slope of 58°, the maximum angle to leave a 20-ft berm every 50 ft.

The paint rock slopes were designed using Curve III of Fig. 3 because seepage was not a problem and the excavation would take place in two embayments where side resistance was maximized. As only a small portion of the total wall in paint rock would exceed 400 ft in height, the slope angle chosen was 42.5° the maximum permissible angle for that height.

Although stability of the ashrock varies considerably with seepage and alteration, it was decided to choose an overall slope for the total area as the degree of alteration in any particular area was not known until excavation took place. Using Curve II of Fig. 4, an overall slope of 42.5° was chosen, based on the worst condition of seepage and side resistance.

One other problem was having to mine into an end of the pit that had previously been backfilled. It was decided to use 37.5°, the approximate natural slope of dumped material.

A section of clay was encountered in the southwest corner of the pit. Here the slopes were designed with the assistance of Dr. R.M. Hardy, consulting engineer, based on a program of shear tests. This included pit design for the South Roberts orebody (Fig. 5).

A relatively shallow pit was designed for the North Roberts orebody between the South Roberts and the Hogarth Pits. It was designed to a maximum depth of 400 ft, with the same slopes as the South Roberts Pit.

Monitoring

It was recognized that some form of instrumentation was required to record the acceleration of movement in unstable ground and to record the total movement in the same area. Because of the physical characteristics of the rock, surface monitoring was considered the best method of control. This was accomplished by one or more of three basic types of monitors (Fig. 6).

Monitor type "A" was used when movement was localized and when it was desirable that anyone should be able to check the movement at any time. The construction of this type of monitor is relatively simple. One-inch square steel pins were made and bars welded to one side of each. These pins were then driven into the ground such that the bars were lined up vertically and horizontally and were end on, tight to each other. Any movement could thus be measured in three planes.

Monitor type "B" was used when movement occurred over a large area. A control station was installed on stable ground on one side of the area, a back-site on the opposite side, and scales were installed between. Readings were taken along the line of sight. In some cases, elevations were also established on the top of each monitor pin.

The triangulation type monitor was used when movement encompassed a very large area and other forms of control were inadequate.

Actual Experience

The carbonate slope established at 58° was definitely stable. The carbonate in the South Roberts was relatively free of joints and weak areas and no problems were encountered except near contacts with paint rock. On these contacts, some ravelling did occur. The main reasons were that the carbonate material at the contact is highly altered and paint rock-filled cracks were common. It was not possible to maintain consistent berm width in the carbonate because of the backbreak, especially in the South Roberts. When the design was changed to a 30-ft berm every 75 ft, better conditions were experienced but this did not solve the problem completely. However, in the new Hogarth Pit presently being mined, a greater degree of blasting control at the wall is being used, and considerable improvement is being experienced.

One problem which did not occur in the South Roberts but which has affected stability in the new Hogarth Pit, is the presence of large mud-filled voids. These appear to be the result of former sub-level water courses or solution cavities which have since filled with earthy materials. The location, direction, and extent of these voids in unpredictable. To date, they have not been of sufficient size to affect overall stability of the pit wall but have caused some problems on individual benches.

The paint rock slope of 42.5° established in the South Roberts Pit proved to be very close to optimum. Even though the ultimate pit was designed at 42.5°, some interim walls were mined steeper in an effort to recover ore or to establish more working room. In virtually every case in the South Roberts where the slope angle in paint rock exceeded 45°, cracking and/or slumping occurred, and in many cases, actual wall failure took place. A typical example of this occurred when a pocket of ore was encountered in the paint rock and the ore was taken behind limits. When an overall angle exceeding 45° was established for a vertical height of 100 ft, failure occurred. The preceding benches were then established to the design limits and the slide and wall again became stable. It is significant that the tests based on recompacted samples of paint rock rather than in situ samples were accurate and dependable.

Groundwater was not a problem in the paint rock as topography behind the footwall was more favourable for gathering and disposing of runoff water. The rock, however, did have a fairly high natural moisture content, and it was discovered that the overall slope could be increased if the material had been exposed to weathering for some time.

Experience showed that slope design was the most critical in ashrock. More than half of the wall in the South Roberts pit was in ashrock or altered ashrock materials. The hard unaltered ashrock generally was quite stable and probably could have been designed at a significantly steeper slope. Altered zones, which inevitably appeared as the ore zone was approached, would not stand or support a slope angle greater than that designed. The three major wall failures were all experienced in ashrock slopes.

The failure in 2 Zone, South Roberts, is shown in Fig. 7. A secondary ore lens was located to the west of the primary ore body. It widened out over a distance of 500 ft, and, because of the closeness to the scarp wall on the old shoreline, presented unique problems. The scarp condition was found to be the result of a fault through the area and the ashrock between the fault and the orebody is highly altered, described locally as "tuffs and grits". In late 1968, as the zone was nearing completion, a slide occurred which resulted in a fatality. The wall in this area was designed and completed to an overall slope of 42.5°. However, the presence of the relatively unstable "tuffs and grits" and the high water table in the fault area caused sudden failure, similar to one in clays or other wet soils. The slide was cleaned up later and the overall slope of 42.5° was re-established after the zone had been dewatered. With the exception of minor settling along the scarp, no signs of instability occurred up to the time the area was backfilled for further development.

In the North Roberts Pit, a failure on the hangingwall affected the extent to which the pit could be developed. Failure started as large blocks and slabs began tipping and settling to the pit bottom. The wall was developed on a 42.5° slope as designed. However, the presence of incompetent iron pyrites at the base of the wall (Fig. 8) created a foundation failure which caused block flow in the relatively competent ashrock above. Groundwater in this area was not measured and was not considered a major factor.

The last, and by far the most spectacular wall failure occurred in the 4 Zone of the hangingwall of the South Roberts Pit one month after the pit was completed (Fig. 9). The possibility of a failure in this area became evident in the spring of 1971, when tension cracks appeared at the crest of the pit on an old access road. These cracks were first monitored on Type "A" monitors (Fig. 6), which were later replaced with the more common Type "B" when more massive movement was suspected. When the movement had accelerated to a point where it registered in inches per day and involvement of control points was anticipated, a triangulation type monitor using control points on the other side of the pit was initiated.

In the meantime, pit development progressed and the original pit design was completed, resulting in a 42.5° slope from elevation 1150 to the pit road at elevation 470. A final mining operation was then carried out by ramping down with waste and removing another 80 ft of the road established in ore (Fig. 10). During both of these operations, a constant visual inspection of the wall was maintained along with the various monitor readings. When movement accelerated, the operation was governed accordingly, and for the last six months equipment usage in the area was intermittent.

It is interesting to examine the experience in the hangingwall ashrock for the three pits as shown in the new type of correlation graph of Fig. 11. Three of the factors which had a direct effect on movement in the unstable zones were time, surface water, and height of face.

Time was a substantial factor because of the peculiar characteristics of the hangingwall rock types. The wall was composed of "tuffs and grits" on the upper half, and unaltered ashrock on the lower half. The forms decomposed quite rapidly upon exposure to the air and moisture. As a result, this material was continually losing its strength and the weight of the material exerted increasing stress on the ashrock below. Eventually, the stress exerted by the "tuffs and grits" exceeded the shear strength of the ashrock and total failure occurred.

Surface water was a contributing factor as the amount of rainfall could be related to accelerated movement with a 24- to 48-hour lapse of time. After a heavy rainfall and accelerated movement, slight negative movement had been recorded from time to time, indicating that the pressure exerted by the water running into the cracks was released (Fig. 12).

The height of face was certainly a contributing factor as the movement recorded between completion of the design pit and removal of the ore under the ramp was minimal. When the mining operations resumed, movement accelerated. Figure 13 is an evaluation of these factors.

All of these factors were not only considered in the research program, but were emphasized in the recommendations submitted. Where possible, surface water was diverted and tension cracks were filled in attempts to minimize seepage. It is the opinion of the author that the slide was delayed to some extent because of these actions.

The factors can be studied further as they are compared with the graph for this material type in Fig. 4. Following Curve II, used for pit design, the maximum slope of 42.5° is reached at approximately 550 ft of height. This slope checks with the 580 ft mined. When ore under the ramp was mined a further 80 ft, the optimum was exceeded and a failure was inevitable as tension cracks were present and groundwater could not be completely controlled.

Another interesting observation is that the slope of 37.5° in the old pit fill was relatively stable but adversely affected by ground and surface water. Slope angle of 37.5° is certainly the maximum in a case such as this, as one bench was dug 30 ft behind the toe and ravelling continued in that area until the pit was completed. It is significant that the slope was over 300 ft high and did not fail. All wastes which have since been deposited in the South Roberts maintain a slope of approximately 37°. At a point about 250 ft from the crest, however, extreme changes occur and the waste creeps out at an angle of between 15° and 20°.

The reasons could be due to the breakdown of ashrock blocks and the extrusion of the resulting mass under the pressure of the embankment. Figure 14 shows a photograph of the dump advancement.

CONCLUSIONS

It is evident that the work completed in the early 1960's was of inestimable value to Steep Rock Iron Mines. Without doubt, the designed slopes were very close to optimum and, as a result, the stripping ratio was minimized. Considering all factors, the failures which did occur, did so because the nature of the particular factors was not completely known.

The results of experiences in the South Roberts Pit have been applied to the new Hogarth Pit. The only modifications were to reduce the slope in the paint rock where the vertical distance exceeds 200 ft, and to reduce the slope on the hanging wall if significant quantities of "tuffs and grits" are encountered.

The estimate made by Dr. Coates of 10% instability in the pit slopes compares favourably with 13% actually experienced in slides, considering that approximately one-half of the 13% occurred one month after the pit excavation was completed.

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2. Larocque, G.E.; Determination of Physical Properties of Geologic Samples From Steep Rock Iron Mines; unpublished Mines Branch report; Nov. 1961.

3. Coates, D.F., McRorie, K.L., Stubbins, J.B.; Analyses of Pit Slides in Some Incompetent Rocks; Trans. AIME; Vol. 226; pp 94-100; 1963.

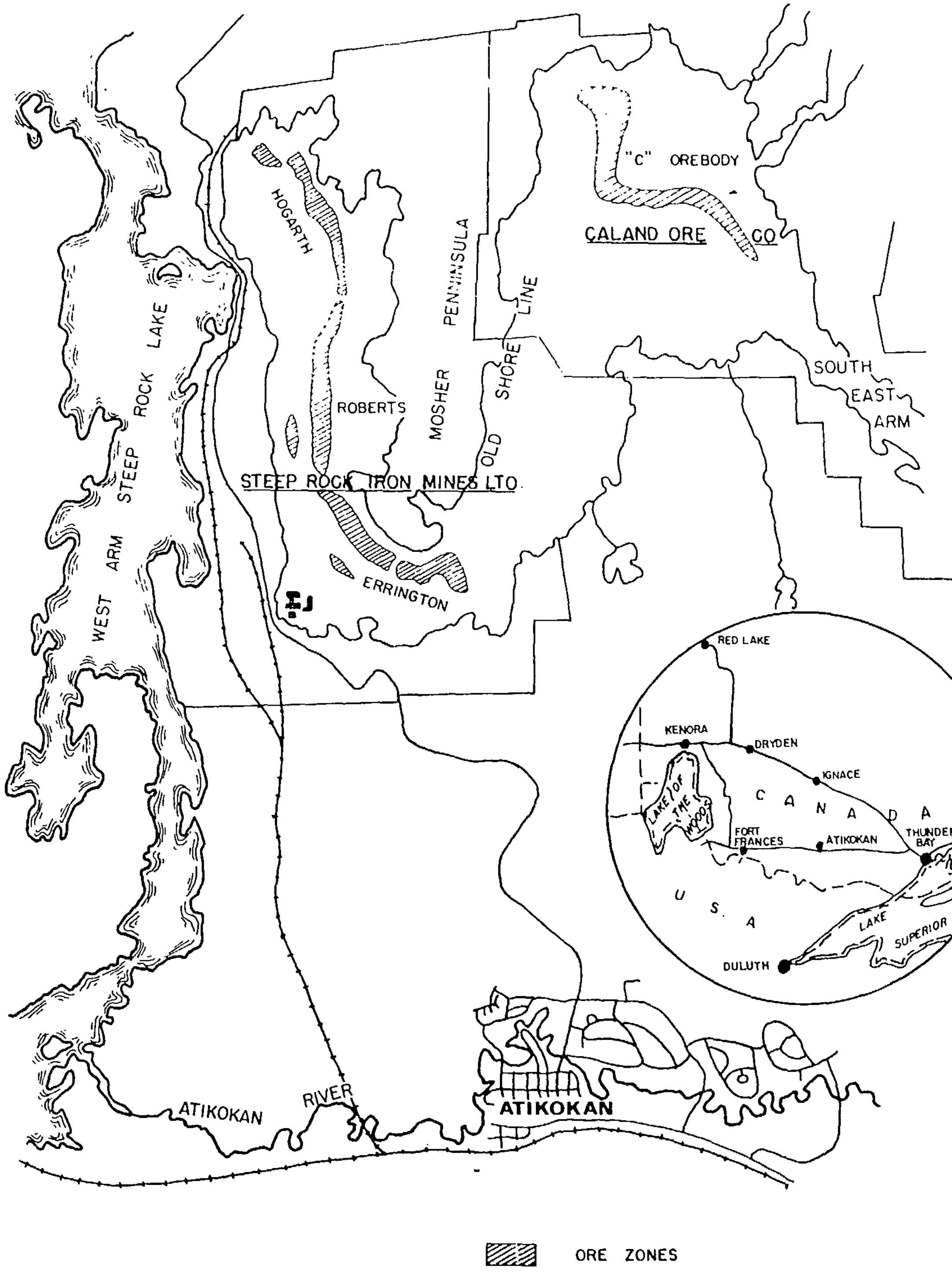

Fig. 1 - General plan of Steep Rock Lake area

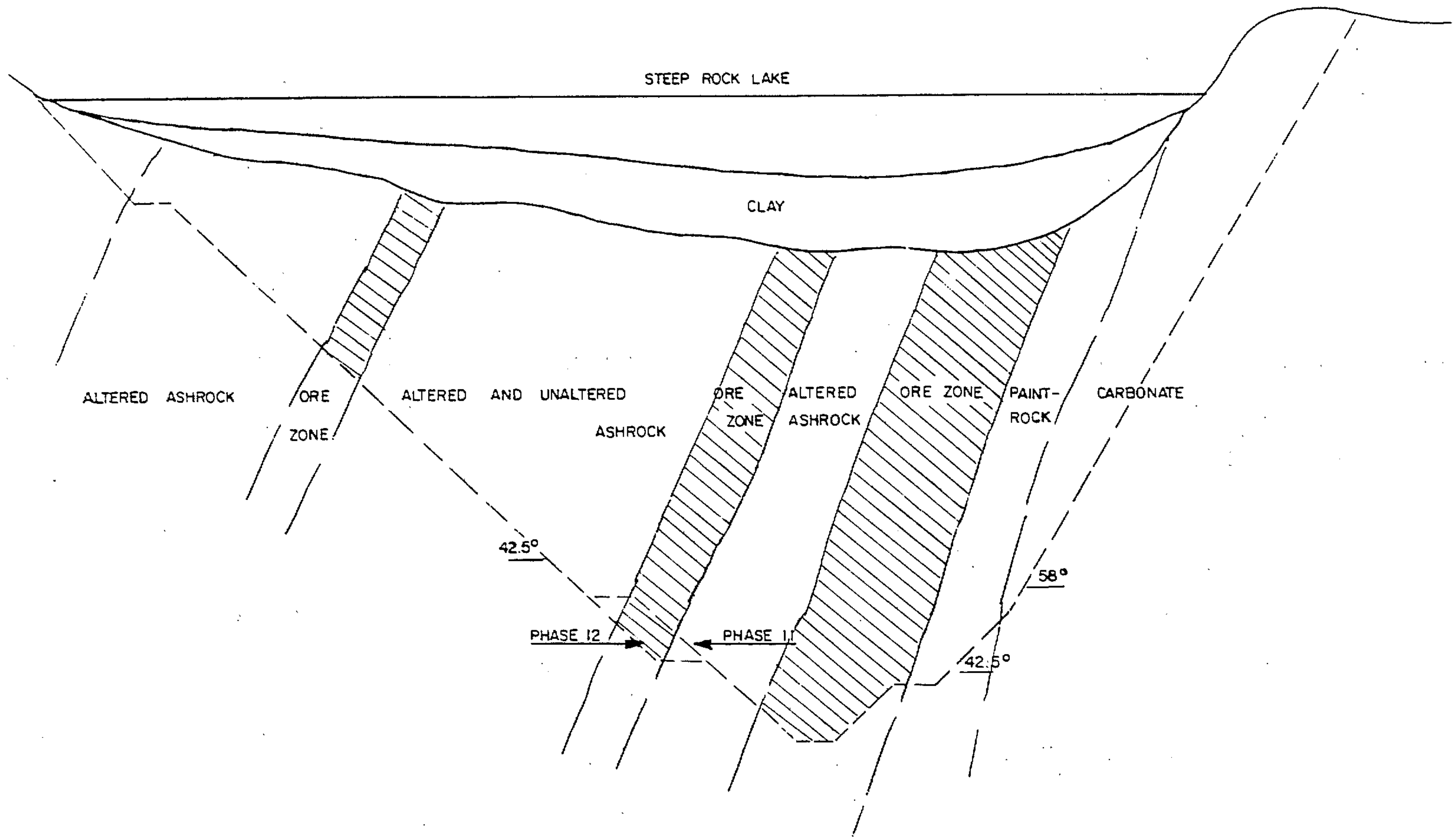

Fig. 2 - South Roberts pit, typical section looking north (scale 1" = 200'

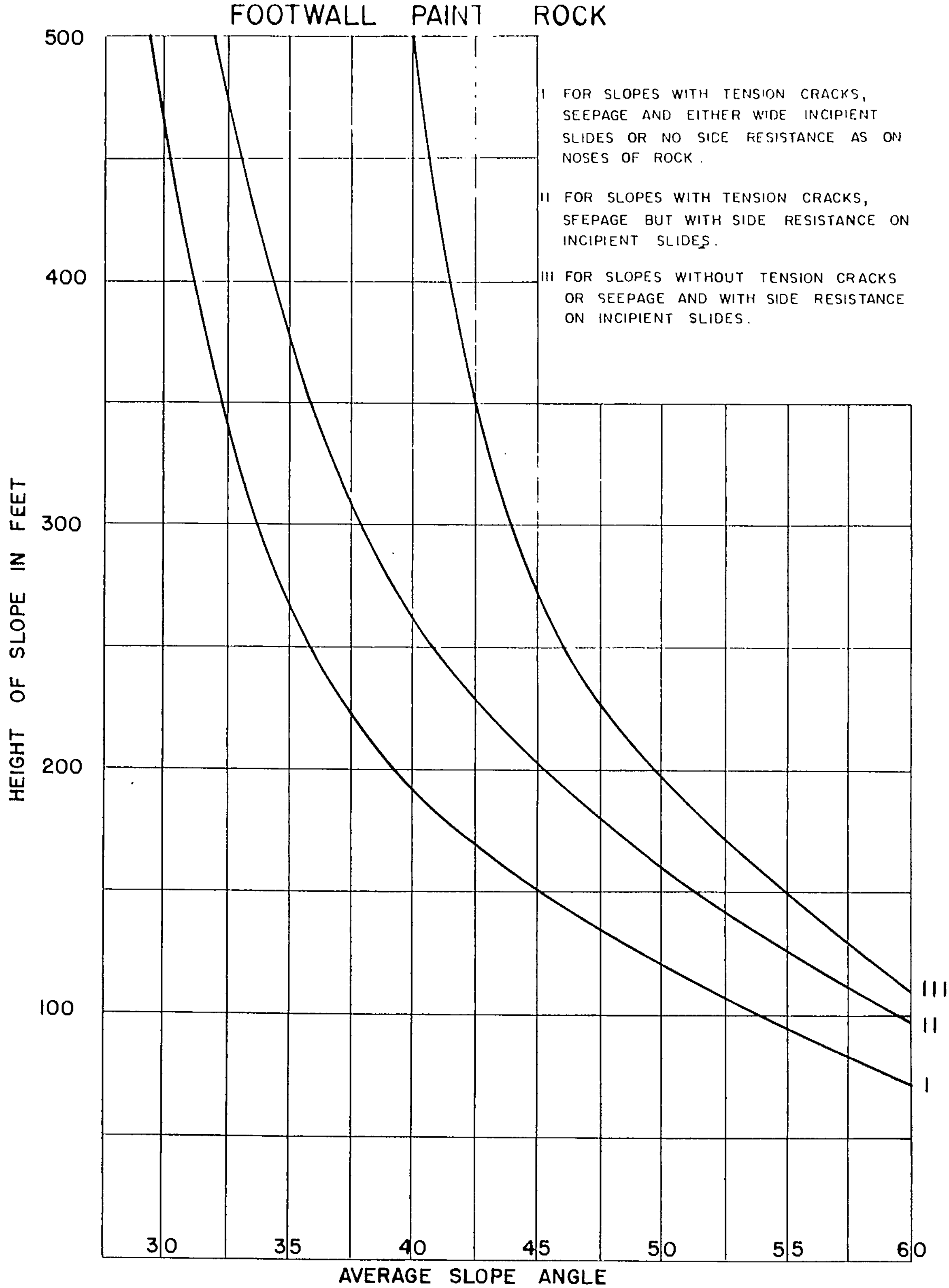

Fig. 3 - Slope angle versus slope height for footwall paint rock

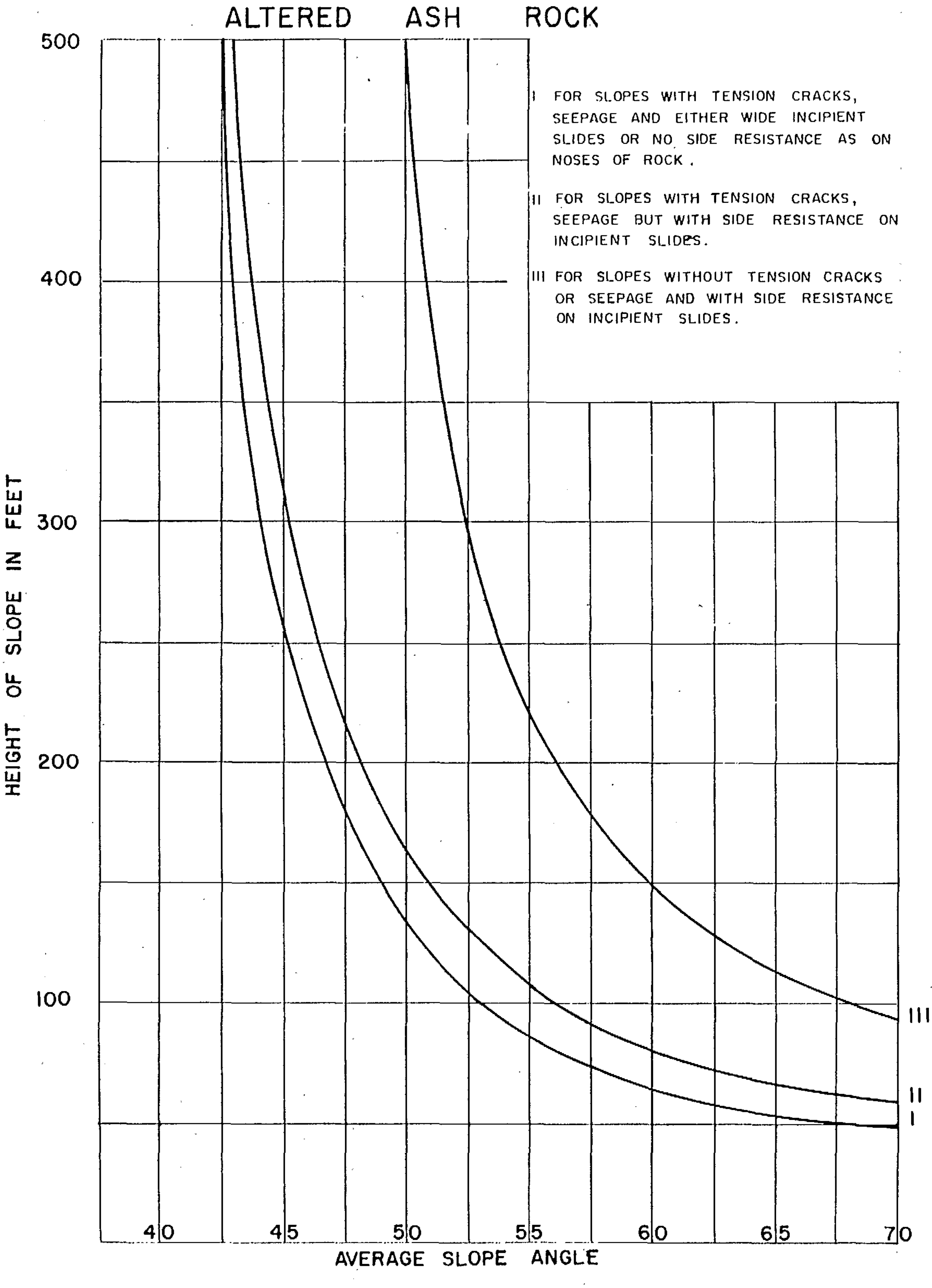

Fig. 4 - Slope angle versus height for altered ash rock

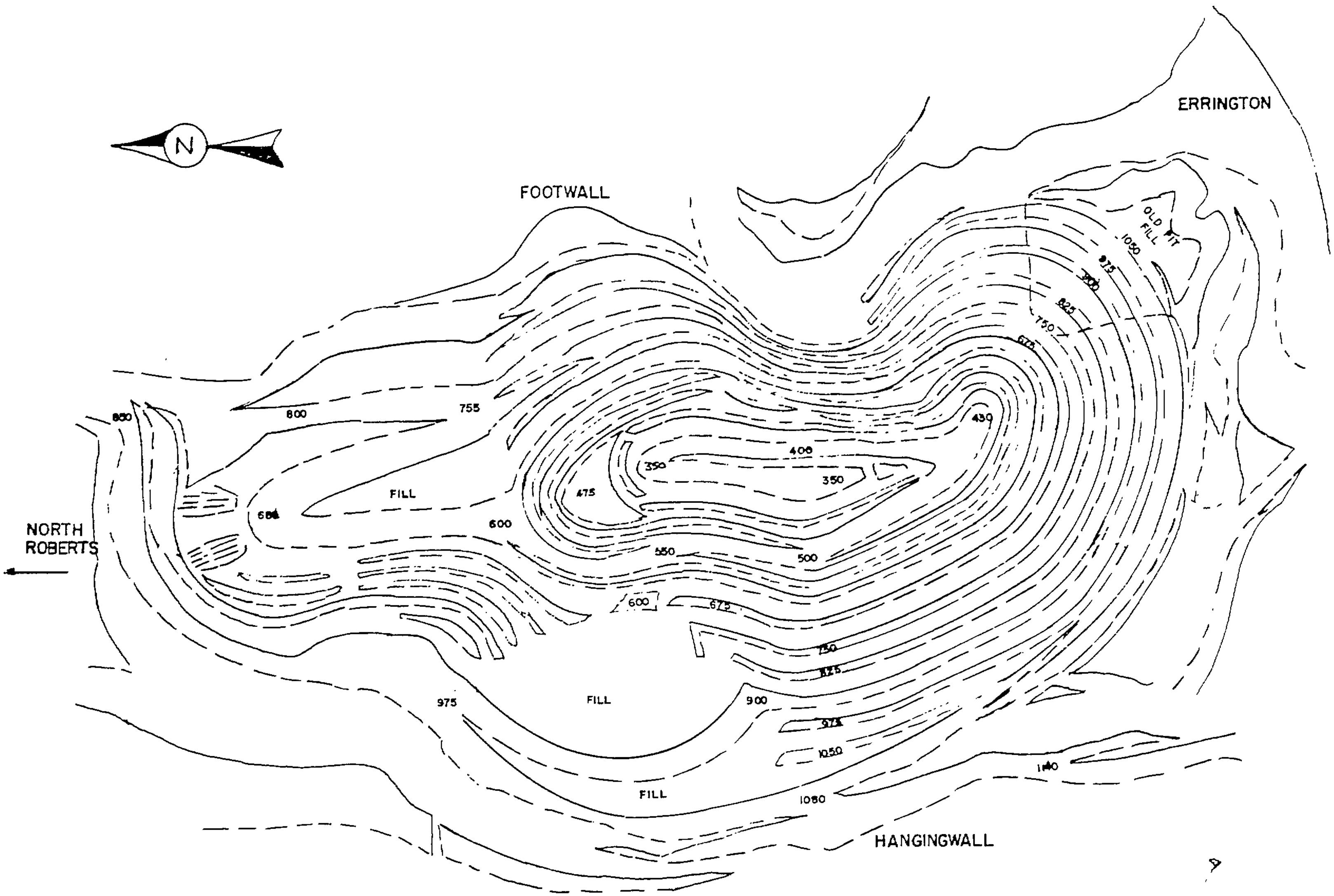

Fig. 5 - South Roberts pit design (scale 1" = 400')

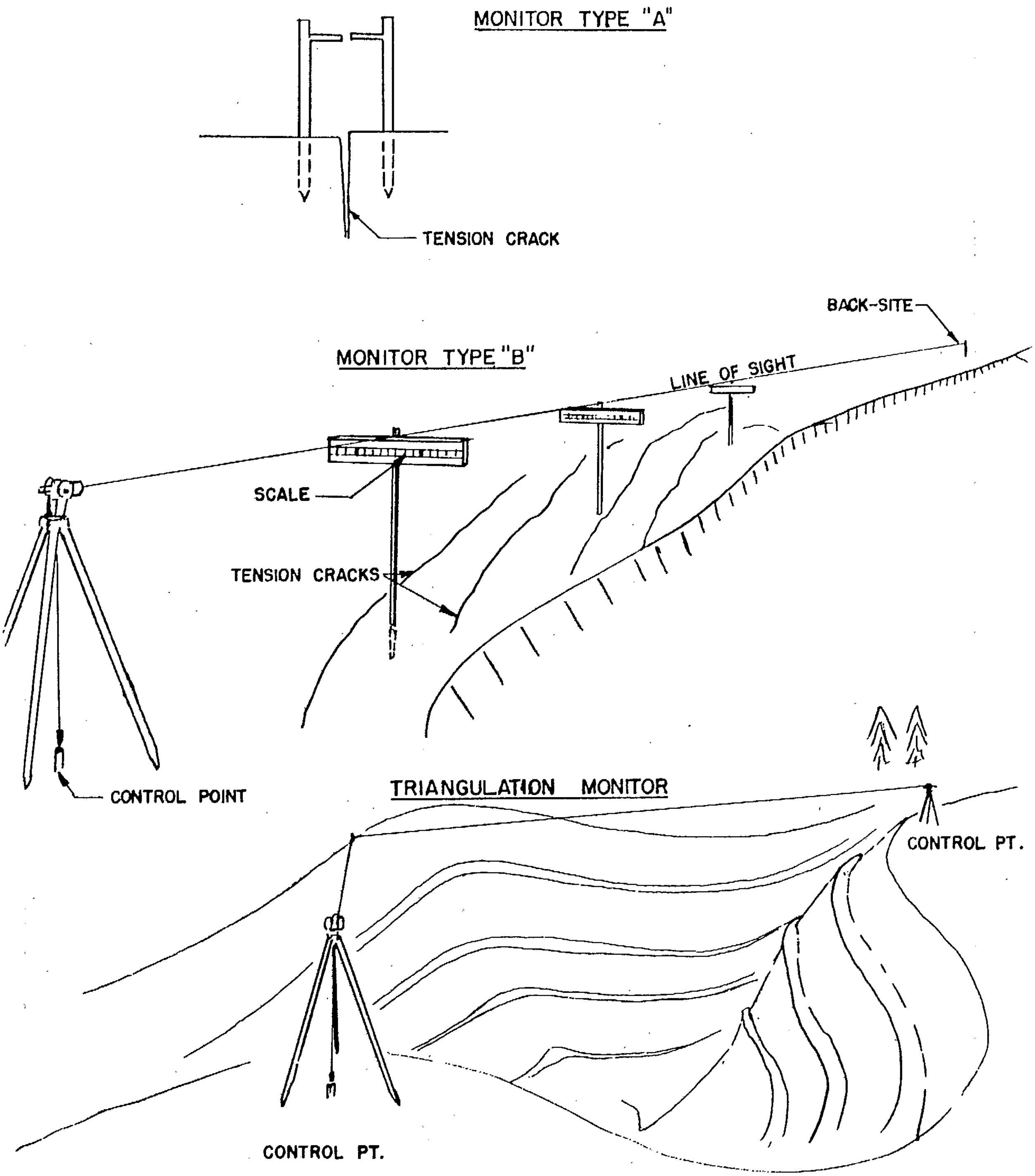

Fig. 6 - Three types of monitors

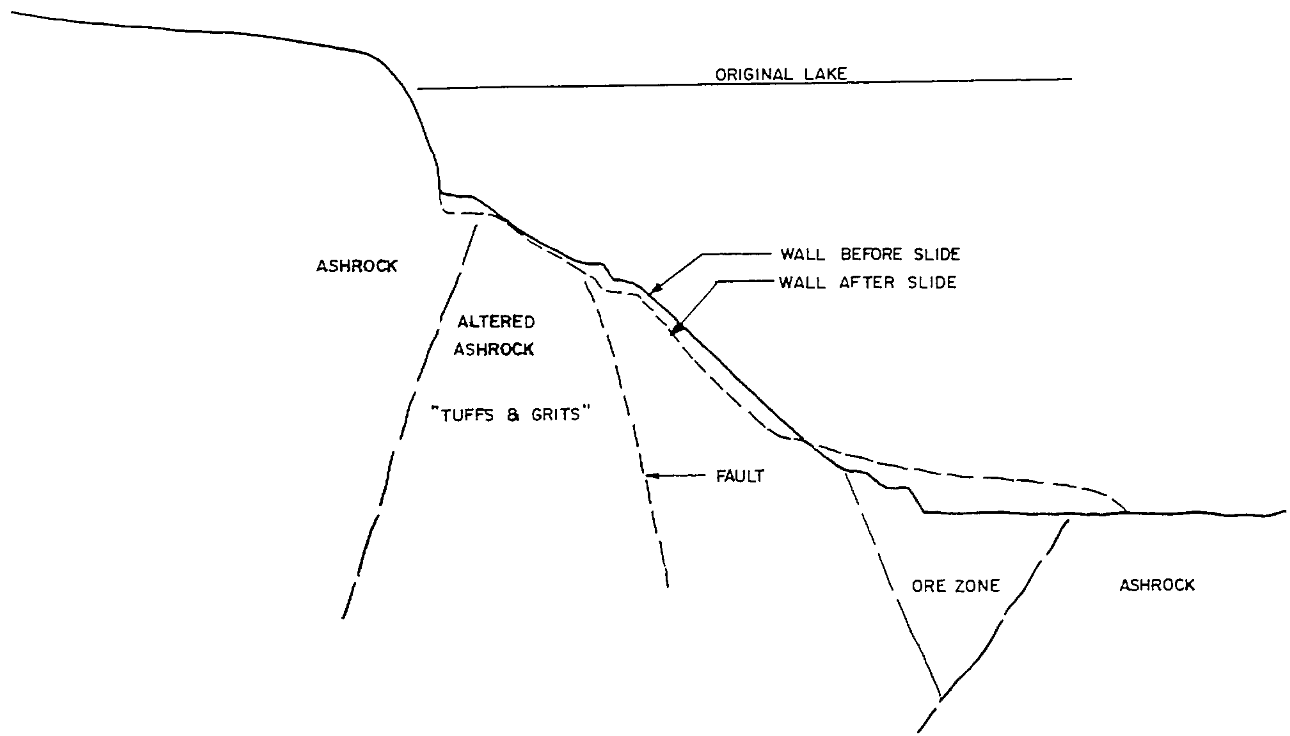

Fig. 7 - South Roberts pit, section looking north - two zone slide
(scale 1" = 200')

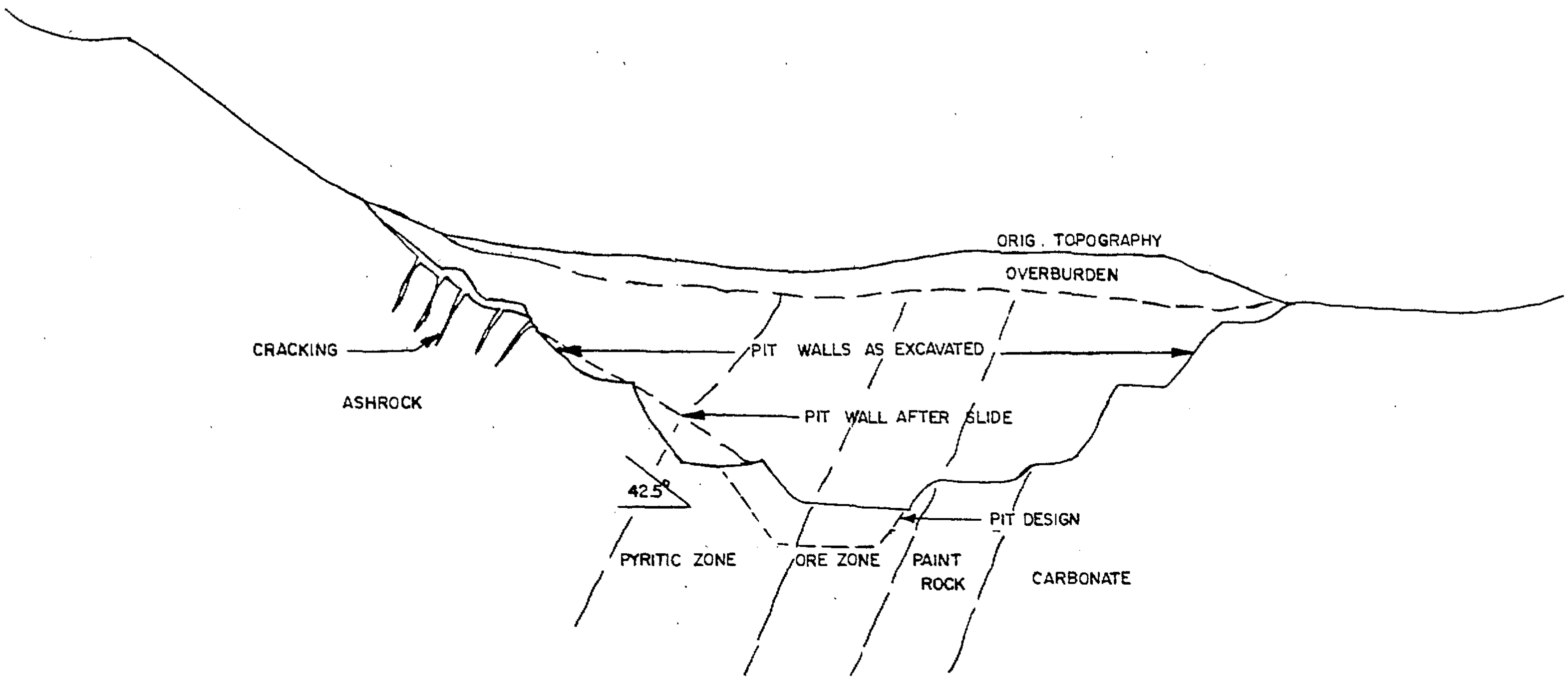

Fig. 8 - North Roberts pit slide, looking north (scale 1" = 100')

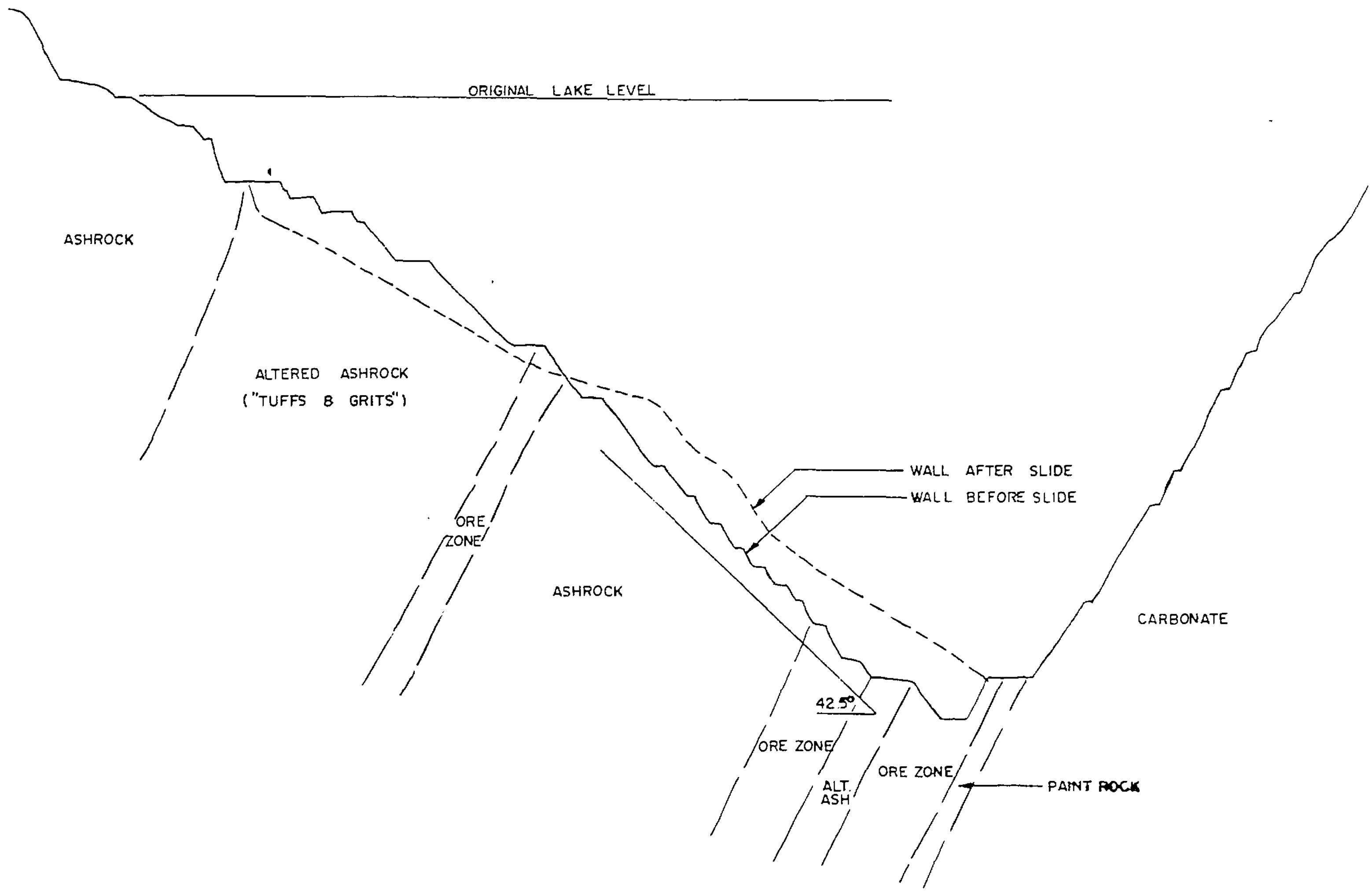

Fig. 9 - South Roberts pit, section looking north - four zone slide area (scale 1" = 200')

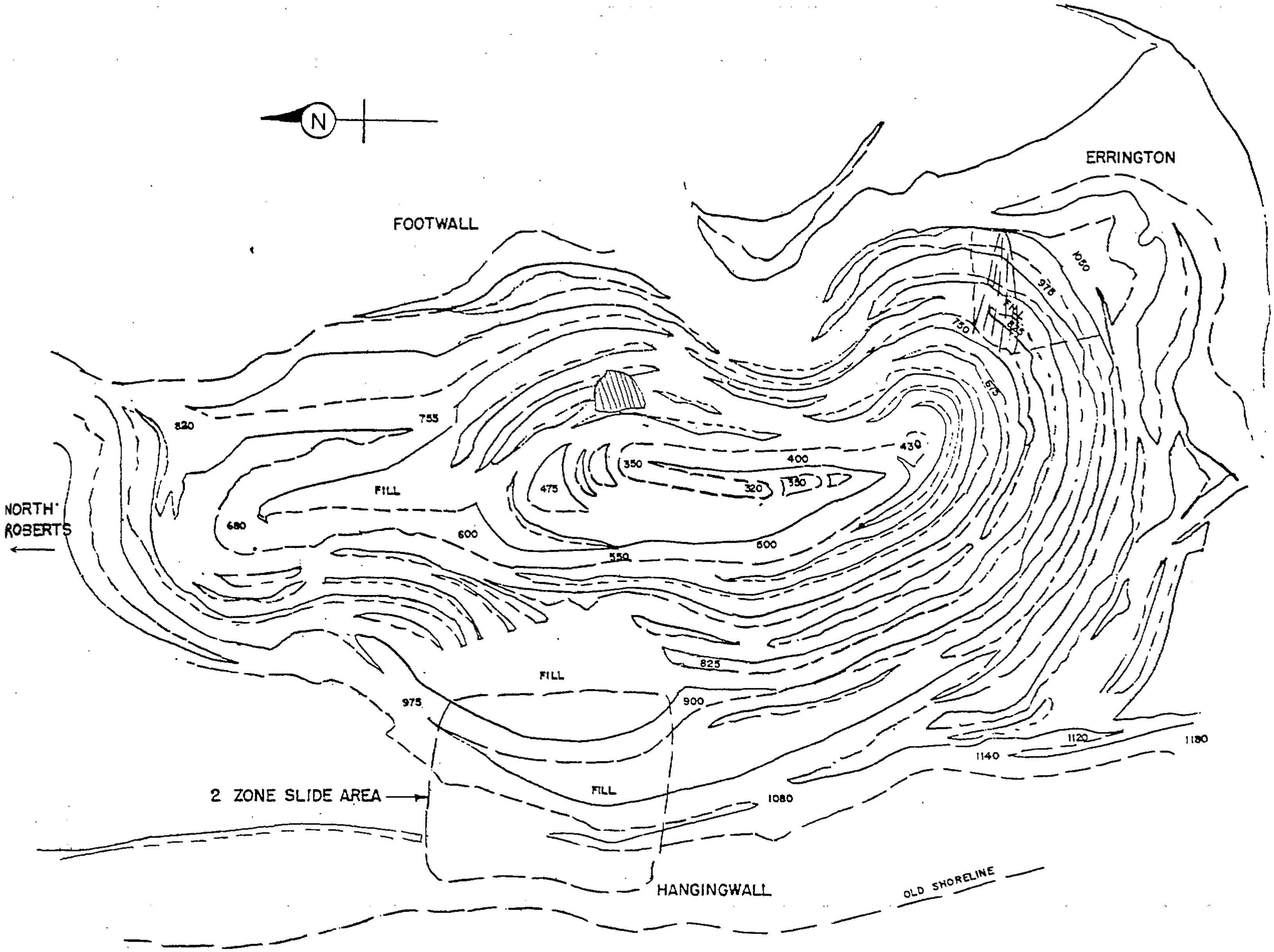

Fig. 10 - South Roberts pit, phase 11 (scale 1" = 400')

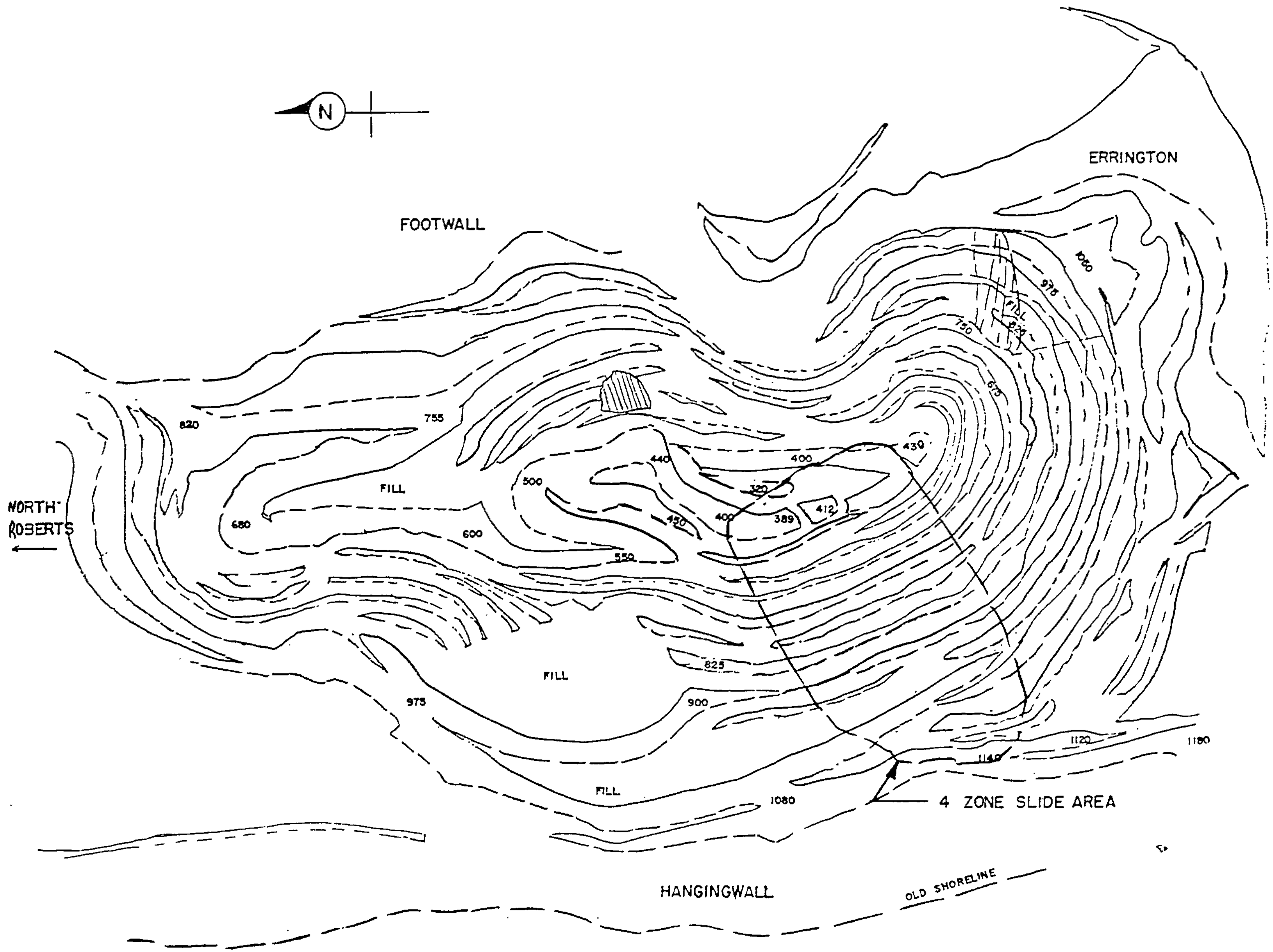

Fig. 11 - South Roberts pit, phase 12 (scale 1" = 400')

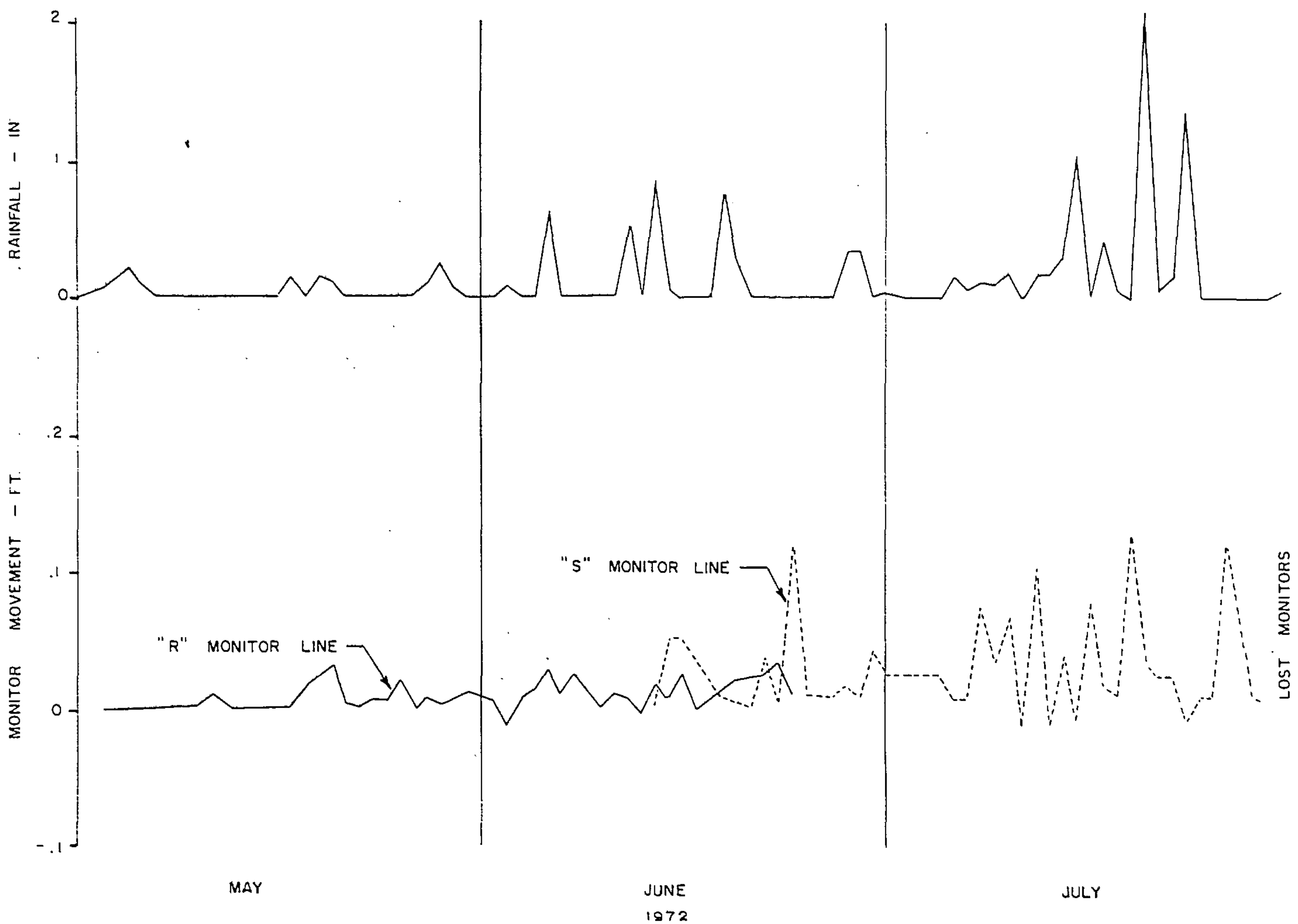

Fig. 12 - Ground movement related to rainfall

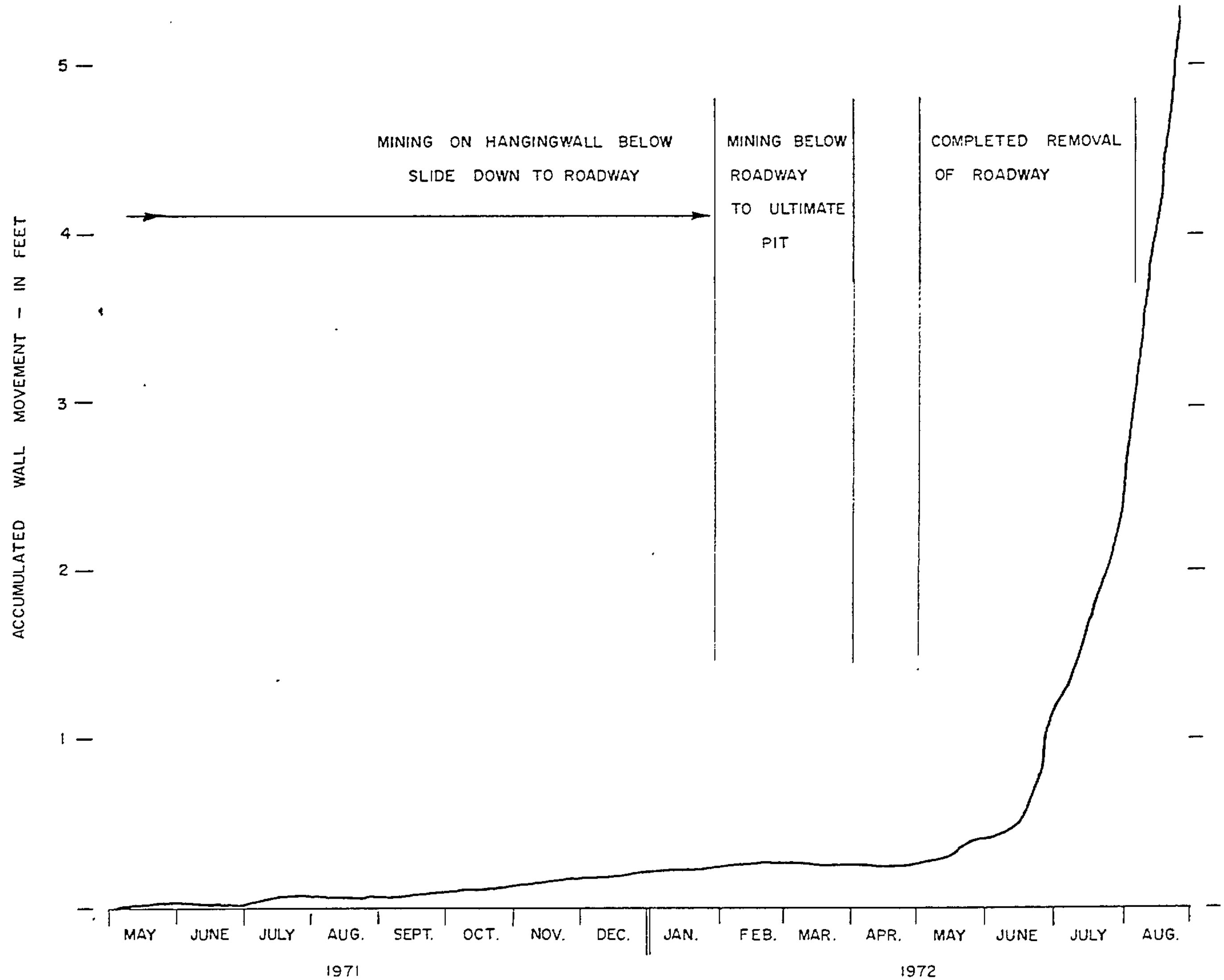

Fig. 13 - Ground movement related to mining activity and increased height of face

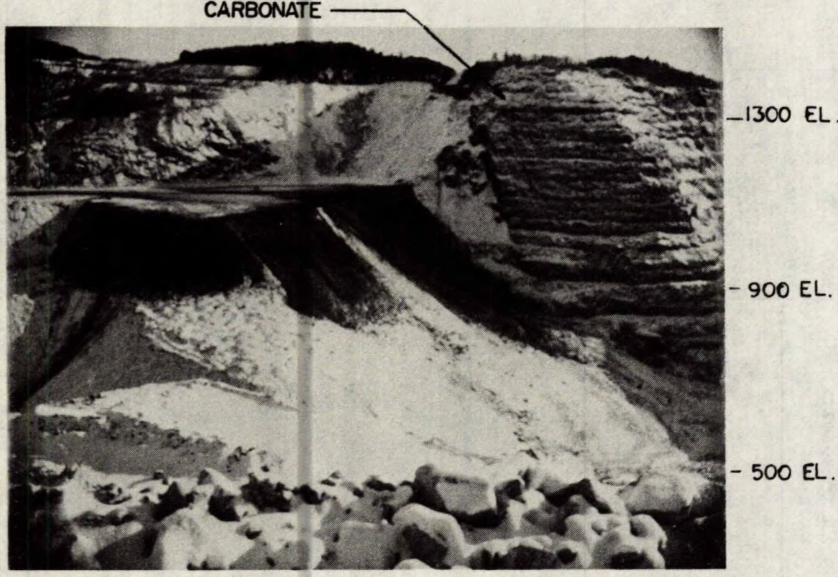

Fig. 14 - South Roberts waste disposal looking east

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