CANADA

RESOLUTION OF THE OPERATING VARIABLES OF A SMALL HYDROCYCLONE [1]

DEPARTMENT OF MINES AND TECHNICAL SURVEYS, OTTAWA

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RESEARCH REPORT

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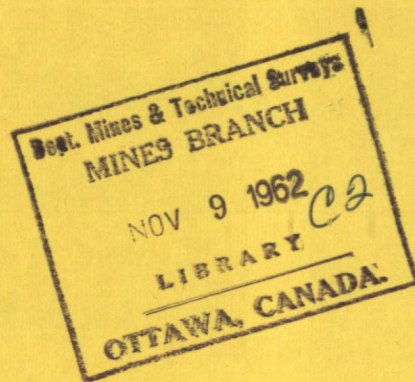
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Resolution of the Operating Variables of a Small Hydrocyclone[1]

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Application of dimensional analysis and experiment to the operation of a hydrocyclone yielded an equation of the form:

$$Q = K_1 P^{(d-1)/2} \delta^{(d-3)/2} \mu^{(2-d)}$$

This equation shows that the volume throughput Q is a function of the pressure differential P, the density δ and the viscosity μ of the medium. The constant K_1 and the coefficients of the pressure differential, density and viscosity were shown by statistical analysis to be relatively simple functions of the dimensions of the body and orifices of small glass hydrocyclones used with liquid media.

L'application de l'analyse dimensionnelle et de mesures expérimentales au fonctionnement d'un hydrocyclone conduit à une équation de la forme:

$$Q = K_1 P^{(d-1)/2} \delta^{(d-3)/2} \mu^{(2-d)}$$

Cette expression montre que la capacité volumétrique Q est une fonction de la différence de pression δ, de la densité δ et de la viscosité μ du milieu. On trouve, à partir de l'analyse statistique, que la constante K_1 et les coefficients respectifs de la différence de pression, de la densité et de la viscosité sont des fonctions plutôt simples des dimensions du corps et des orifices des petits hydrocyclones en verre utilisés dans les milieux liquides.

Hydrocyclones are used extensively in the mineral industry and in the pulp and paper industry for operations such as classification, thickening and dewatering. This wide acceptance has been achieved because the hydrocyclone is efficient and has no moving parts. However, despite the apparent simplicity of the hydrocyclone, the principles governing its operation are not well understood. This is not because of lack of effort; much work has been done using both theoretical hydrodynamic[1,2] and empirical approaches[3,4,5].

The relationship between the throughput of the hydrocyclone, and the pressure drop across it, has been found to have the form

$$Q = kP^m.$$

The generally accepted value of m is 0.50, and this is supported by gas cyclone theory[4] and by the theoretical hydrodynamic treatments of Banerji and Roy[1], Bradley[6] and Trawinski[7]. In addition, Chaston[8] analyzed the results of hydrocyclone tests done by several investigators and concluded that the equation, $Q = kP^{0.5}$, was adequate to represent the relationship between these two variables.

However, the results of experimental hydrocyclone tests by Kelsall[5], Haas et al.[9] and Matschke and Dahlstrom[10] indicate that for both large and small hydrocyclones the value of m lies in the range 0.42 to 0.46.

The work reported in this submission was begun with the object of resolving the factors responsible for causing the variation in the power to which the pressure term is raised. The first attempts made to resolve the results were empirical, and when this work was reviewed for publication it was suggested by a reviewer[11] that dimensional analysis might be used to resolve a relationship for correlating the results. The development of this relationship, and its use in correlating the results of pressure variation on the throughput of a small glass cyclone operated with liquid only, will be discussed.

Development of Relationships

Preliminary experiments, done with a small glass hydrocyclone using a water medium to which various quantities of sugar had been added to increase the medium density, indicated that the variables influencing the throughput of the hydrocyclone were the pressure drop across it and the density and viscosity of the medium. In addition, it seemed reasonable to believe that the geometry of the hydrocyclone, as represented by the diameter of the inlet, overflow and underflow openings, as well as the diameter of the hydrocyclone itself, would influence the throughput.

The functional relationship between these variables was expressed by the equation:

$$Q = f(P, \delta, \mu, D_1, D_2, \ldots D_n).$$

To permit application of dimensional analysis, the diameters $D_1 \ldots D_n$ were replaced by a single variable, D, having the dimension of length, and the result was:

$$Q = f(P, \delta, \mu, D).$$

Assuming a simple power relationship between the variables, the following equation was obtained:

$$Q = K P^a \delta^b \mu^c D^d \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

When the appropriate mass, length, and time units[12] were substituted in Equation (1) the following relationship was obtained:

$$(L^3 T^{-1}) = (ML^{-1}T^{-2})^a (ML^{-3})^b (ML^{-1}T^{-1})^c L^d.$$

Equating the indices of the three basic dimensions, M, L and T, on both sides of this equation yielded:

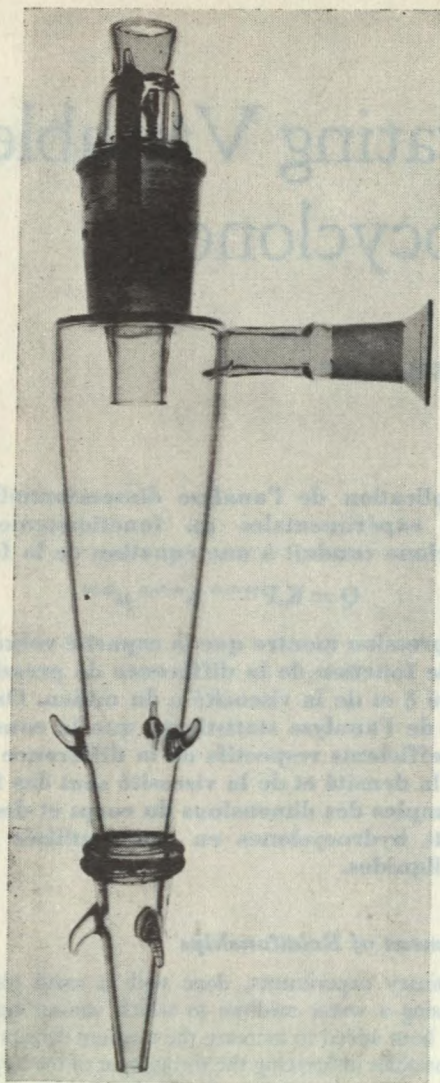$$a = \frac{d-1}{2}, \quad b = \frac{d-3}{2}, \quad c = 2 - d.$$

Figure 1—Photograph of one of the hydrocyclones used in the experiments.

Substituting in Equation (1)

$$Q = K P^{(d-1)/2} \delta^{(d-3)/2} \mu^{(2-d)} D^d \dots \dots \dots (2)$$

and

$$Q = \frac{K\mu^2}{(P\delta^3)^{0.5}} \left[\frac{(P\delta)^{0.5} D}{\mu} \right]^d \dots \dots \dots (2a)$$

When Equation (2a) was rearranged and KD^d was equated with a new constant K_1, the equations

$$\frac{Q\delta}{\mu} = K_1 \left[\frac{(P\delta)^{0.5}}{\mu} \right]^{d-1} \dots \dots \dots (3)$$

and

$$Q = K_1 P^{(d-1)/2} \delta^{(d-3)/2} \mu^{(2-d)} \dots \dots \dots (4)$$

were obtained.

For a hydrocyclone of fixed geometry and opening sizes, a graph of $\log \dfrac{Q\delta}{\mu}$ against $\log \left[\dfrac{(P\delta)^{0.5}}{\mu} \right]$ should be a straight line of slope $(d - 1)$, having an intercept of $\log K_1$. From the slope and intercept, the exponents of P, δ and μ in Equation (4) can be calculated.

It should be noted that for the special case when the slope $(d - 1)$ is 1, the equation for Q becomes

$$Q = K_1 (P/\delta)^{0.5};$$

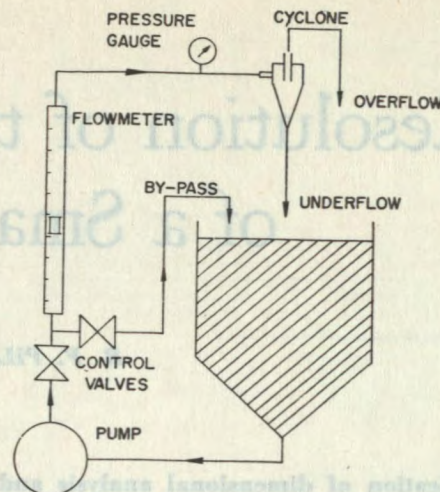that is, Q is directly proportional to $P^{0.5}$ and is independent of the liquid viscosity, μ.

Figure 2—Diagram showing the arrangement of the apparatus used in the experiments.

Apparatus and Procedure

The experimental work was done using two small Pyrex glass cyclones with internal diameters of 15 and 30 mm. respectively. These are part of a Laboratory Hydrocyclone Test Set purchased from Liquid-Solid Separations Limited, of London, England. A photograph of one of the cyclones is shown in Figure 1.

The hydrocyclones were supplied with a series of interchangeable underflow and overflow openings of different sizes. However, as is evident from Figure 1, the size of the inlet opening of the hydrocyclone was fixed; to obtain a variation in the size of this orifice, short pieces of Pyrex tubing of the appropriate internal diameter were inserted in the inlet and cemented in place. To ensure reproducibility and to prevent leakage, both the overflow and underflow openings were carefully centered and cemented in place. The sizes of all of the openings were determined microscopically, using a calibrated eyepiece graticule.

For the experiments, the hydrocyclone was mounted in closed circuit with a 15-litre glass tank, a small circulating pump and a rotameter flowmeter. The pressure differential across the hydrocyclone was measured by a 0-30 p.s.i. gauge in the cyclone feed line, and was controlled by valves in the feed and by-pass lines. The temperature of the liquid in the tank was regulated by a Bronwill Constant Temperature Circulator and an auxiliary cooling coil. The arrangement of the essential parts of the apparatus is shown in Figure 2.

Water was used as the hydrocyclone medium for the investigation of liquids of low density and viscosity. Aqueous sucrose solutions were used to simulate denser and more viscous media. Liquid densities were calculated from the weight of a known volume of the medium, and an Ostwald pipette was used to determine liquid viscosities. The flowmeter was calibrated for each of the media by measuring the volume of fluid delivered during a specified period of time. The pressure gauge was calibrated with a mercury manometer.

Results and Discussion

The throughput in millilitres per minute, was determined for each hydrocyclone at pressure differentials from 2 to 20 p.s.i.g. with liquids having densities from 0.988 to 1.236 g./ml. and viscosities from 0.0054 to 0.137 poise. The various combinations of inlet, overflow and underflow openings used in the experiments are shown in Table 1.

TABLE 1
Hydrocyclone Dimensions for Various Tests

Test No.	Cyclone Size D_c(mm.)	Inlet Opening D_i(mm.)	Overflow Opening D_o(mm.)	Underflow Opening D_u(mm.)
1	30	6.00	8.40	2.07
2	30	2.67	8.40	3.05
3	30	6.00	5.87	3.05
4	30	2.67	5.87	2.07
5	15	3.00	3.96	1.28
6	15	1.46	3.96	1.57
7	15	3.00	2.97	1.57
8	15	1.46	2.97	1.28

Detailed results for Test No. 1 are shown in Table 2. The parameters $\dfrac{Q\delta}{\mu}$ and $\dfrac{(P\delta)^{0.5}}{\mu}$, as required by Equation (3), are included in Table 2. The logarithms of these quantities are plotted in Figure 3 for Tests No. 1, 2, 5 and 6. Similar results for Tests No. 3, 4, 7 and 8 are shown in Figure 4.

It is evident from Figures 3 and 4 that the relationship between $\log \dfrac{Q\delta}{\mu}$ and $\log \dfrac{(P\delta)^{0.5}}{\mu}$ is linear for each test, in accordance with the prediction of Equation (3). The assumption of a simple power relationship between the variables, which underlies Equation (1), is thus justified. To evaluate the constants of Equation (3), the slope and intercept of each line were calculated. These are shown in Table 3.

The exponents of P, δ and μ were determined by the substitution in Equation (4) of the constants shown in Table 3. For example, for Test No. 1,

$$Q = 3970 \ P^{0.452} \ \delta^{-0.548} \ \mu^{0.096}$$

TABLE 2
Experimental Results used to Calculate the Parameters $\dfrac{Q\delta}{\mu}$ and $\dfrac{(P\delta)^{0.5}}{\mu}$ for Test No. 1

δ g./ml.	μ poise	P p.s.i.g.	Q ml./min.	$\dfrac{(P\delta)^{0.5}}{\mu}$	$\dfrac{Q\delta}{\mu} \times 10^{-4}$
1.199	0.0846	2	3550	18.4	5.03
		4	5030	25.9	7.13
		6	6150	31.8	8.72
		8	7110	36.6	10.1
		10	7910	41.0	11.2
		11	8230	42.9	11.7
1.164	0.0498	4	5070	43.4	11.9
		6	6130	53.2	14.3
		8	6940	61.2	16.2
		10	7680	68.5	18.0
1.122	0.0288	4	5060	73.5	19.7
		6	6060	90.1	23.6
		8	6970	104	27.2
		10	7620	116	29.7
		11	8040	122	31.3
1.000	0.0127	4	5000	157	39.4
		6	6060	193	47.7
		8	6760	223	53.2
		10	7470	249	58.8
		12	8180	273	64.4
0.989	0.0054	4	4540	369	83.1
		6	5480	451	100
		8	6240	520	114
		10	6850	583	125
		12	7540	638	138

Similar substitutions show that the exponent of P varies from a minimum of 0.420 in Test No. 7 to a maximum of 0.475 in Test No. 6.

The results shown in Tables 1 and 3 constitute a statistical design in which the quantities $(d - 1)$ and K_1 were obtained at two levels of each of the four geometric variables. By

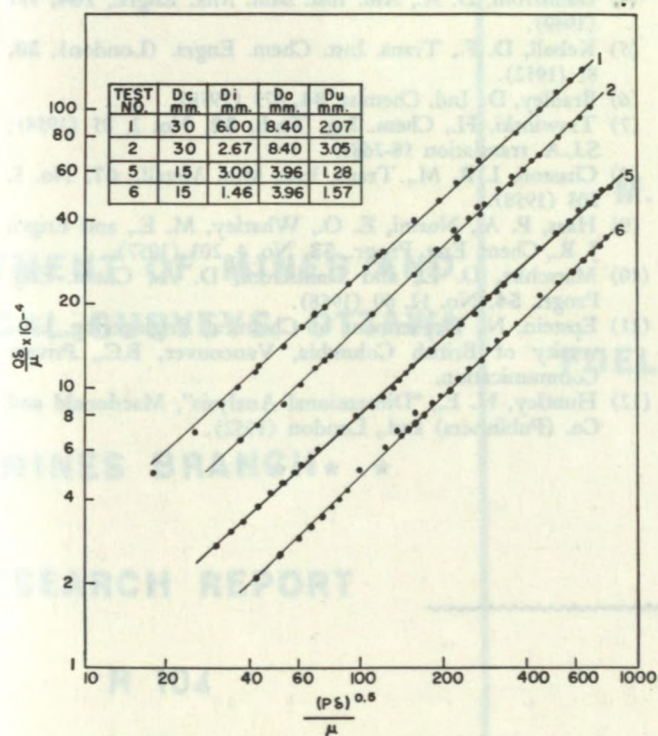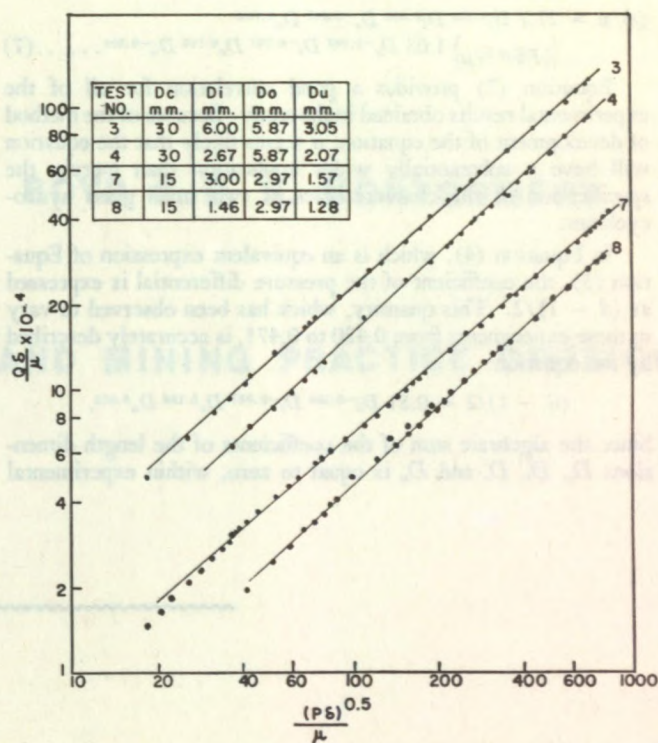| TEST NO.	Dc mm.	Di mm.	Do mm.	Du mm.
1	30	6.00	8.40	2.07
2	30	2.67	8.40	3.05
5	15	3.00	3.96	1.28
6	15	1.46	3.96	1.57

Figure 3—Graphs of $\log\left(\dfrac{Q\delta}{\mu}\right)$ against $\log\left[\dfrac{(P\delta)^{0.5}}{\mu}\right]$ for Tests No. 1, 2, 5 and 6.

TEST NO.	Dc mm.	Di mm.	Do mm.	Du mm.
3	30	6.00	5.87	3.05
4	30	2.67	5.87	2.07
7	15	3.00	2.97	1.57
8	15	1.46	2.97	1.28

Figure 4—Graphs of $\log\left(\dfrac{Q\delta}{\mu}\right)$ against $\log\left[\dfrac{(P\delta)^{0.5}}{\mu}\right]$ for Tests No. 3, 4, 7 and 8.

TABLE 3

SLOPES AND INTERCEPTS

OF THE LOG $\dfrac{Q\delta}{\mu}$ VS. LOG $\dfrac{(P\delta)^{0.5}}{\mu}$ RELATIONSHIP

Test No.	Slope, $(d-1)$	Intercept, K_1
1	0.904	3970
2	0.939	2210
3	0.863	4280
4	0.900	2500
5	0.900	1290
6	0.950	604
7	0.840	1460
8	0.904	569

assuming that each quantity can be expressed as a power function as follows:

$$(d - 1) = A D_c^{a_1} D_i^{b_1} D_o^{c_1} D_u^{d_1}$$

and

$$K_1 = B D_c^{a_2} D_i^{b_2} D_o^{c_2} D_u^{d_2},$$

the coefficients and constants can be evaluated by statistical analysis of the first degree equations:

$$\log(d - 1) = \log A + a_1 \log D_c + b_1 \log D_i + c_1 \log D_o + d_1 \log D_u$$

and

$$\log K_1 = \log B + a_2 \log D_c + b_2 \log D_i + c_2 \log D_o + d_2 \log D_u.$$

When the results of the eight tests were analyzed, the two equations:

$$(d - 1) = 1.03 \, D_c^{-0.082} \, D_i^{-0.067} \, D_o^{0.156} \, D_u^{-0.014} \quad \ldots \ldots (5)$$

and

$$K_1 = 27.7 \, D_c^{1.128} \, D_i^{0.908} \, D_o^{-0.217} \, D_u^{0.053} \quad \ldots \ldots \ldots (6)$$

were obtained.

When the quantities shown in Equations (5) and (6) were substituted in Equation (3), the following composite expression was obtained:

$$Q\delta/\mu = 27.7 \, D_c^{1.128} \, D_i^{0.908} \, D_o^{-0.217} \, D_u^{0.053}$$
$$\left[(P\delta)^{0.5}/\mu \right] \, 1.03 \, D_c^{-0.082} \, D_i^{-0.067} \, D_o^{0.156} \, D_u^{-0.014} \quad \ldots \ldots (7)$$

Equation (7) provides a good correlation for all of the experimental results obtained in the study. Because of the method of development of the equation, it seems likely that the equation will have a substantially wider application than merely the specification of the characteristics of two small glass hydrocyclones.

In Equation (4), which is an equivalent expression of Equation (3), the coefficient of the pressure differential is expressed as $(d - 1)/2$. This quantity, which has been observed to vary in these experiments from 0.420 to 0.475, is accurately described by the equation:

$$(d - 1)/2 = 0.51 \, D_c^{-0.105} \, D_i^{-0.067} \, D_o^{0.156} \, D_u^{0.014}.$$

Since the algebraic sum of the coefficients of the length dimensions D_c, D_i, D_o and D_u is equal to zero, within experimental

error, the quantity $(d-1)/2$ is dimensionless. It does vary however, and the deviation from 0.51 is due to the magnitudes of the geometric variables of the particular hydrocyclone used in the experiments.

Conclusions

Dimensional analysis and the statistical treatment of experimental results have been used to develop a general relationship whereby the throughput of a hydrocyclone can be quantitatively described as a function of the pressure differential, the density and viscosity of the medium, and the dimensions of the body and orifices of the hydrocyclone. It seems likely that the coefficient of the pressure differential is entirely dependent upon the dimensions of the body and orifices of the hydrocyclone.

Nomenclature

Q = hydrocyclone volume throughput
P = pressure drop across cyclone
δ = density of hydrocyclone medium
μ = viscosity of hydrocyclone medium
k,K,K_1,A,B = constants
$D_1, D_2, \ldots . D_n$ = characteristic dimensions of the hydrocyclones and their openings
D = single variable having the dimension of length, equivalent to combined effect of $D_1, D_2, \ldots . D_n$
D_c = hydrocyclone inside diameter
D_i = diameter of inlet pipe
D_o = diameter of overflow opening (vortex finder)
D_u = diameter of underflow opening (apex)
a,b,c,d = powers to which P, δ, μ and D are raised
$a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2$ = powers to which D_c, D_i, D_o and D_u are raised
M,L,T = fundamental units of mass, length and time, respectively

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