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INTERRELATIONSHIP OF DEFORMATION AND FRACTURE CONTOURS

L. P. TRUDEAU

PHYSICAL METALLURGY DIVISION

JANUARY 1970

Research Report R 220

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Ottawa, 1970

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INTERRELATIONSHIP OF DEFORMATION AND FRACTURE CONTOURS

by

L.P. Trudeau*

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ABSTRACT

Analytical work on the aspects of the stress field that cause a slant fracture to start forming is described. The "build-up" distance for transverse stress and the orientation of the stress field both point to the importance of the quantity $(\sqrt{2} - 1)R$ for a circular contour. It is shown, further, that a unit zero isoclinic beginning from the location defined by this quantity is practically an osculating curve of the unit circle. Using this interrelationship, it is demonstrated that fractures, in a number of sections from crack-notch toughness specimens, follow the elastic zero isoclinic precisely even though considerable plastic flow preceded the fracture. For cup-cone fractures in round tensile-test specimens, this same quantity appears to define the interrelationship between surface contour and the width of the cone.

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Direction des mines
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RELATION ENTRE LA DÉFORMATION ET LES PROFILS DE FRACTURE

par
L. P. Trudeau*

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RÉSUMÉ

L'auteur décrit les travaux analytiques effectués sur les aspects du champ de contrainte qui provoquent l'amorce d'une fracture inclinée. La distance de développement de la contrainte transversale et l'orientation du champ de contrainte mettent toutes deux en relief l'importance de la valeur $(\sqrt{2} - 1)R$ pour un profil circulaire. L'auteur montre ultérieurement qu'un isocline zéro unitaire partant de l'endroit déterminé par cette valeur est pratiquement une courbe osculatrice du cercle unitaire. A l'aide de cette relation, il montre que dans un certain nombre de coupes provenant d'éprouvettes servant aux essais de résistance à l'effet d'entaille, les fractures suivent très exactement l'isocline zéro élastique, même lorsqu'une déformation plastique considérable a précédé la fracture. Dans le cas des fractures en forme de cônes dans les éprouvettes cylindriques soumises à des essais de traction, il semble que cette même valeur détermine la relation entre le profil de surface et la largeur du cône.

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INTRODUCTION

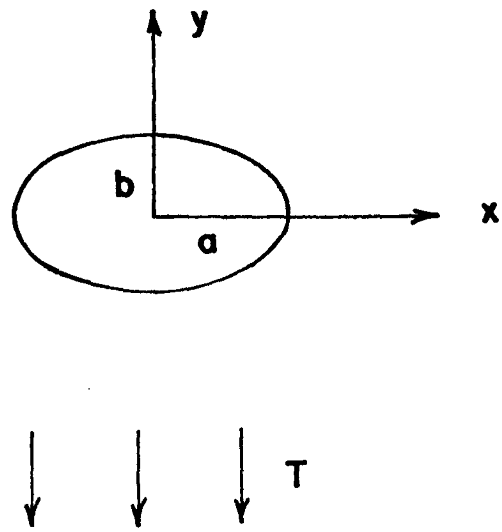
In a previous publication[1], evidence was presented to show that "shear lips" formed in the process of tensile fracture are actually zero isoclinic surfaces characterized by purely normal displacements and zero shear. Investigations on the aspects of the stress field that cause the slant fracture to start forming are now reported. An interrelationship between the change in surface contour caused by plastic flow and the form of the fracture has been found. Fracture contours are shown that follow an elastic zero isoclinic precisely, even though several per cent of plastic flow preceded fracture.

PLANAR THEORY

Two aspects of stress fields were considered as possible influences on the form of a tensile fracture. One was the "build-up" distance of transverse or radial stress arising from a curved cavity or surface contour, and the second was the effect of such a contour on the orientation of the stress field. At the symmetry plane surface of a cavity in a plate under simple tension, the transverse stress will, of course, be zero, but it will increase to a maximum at some distance away from the cavity and this distance is referred to here as the "build-up" distance.

The question of the "build-up" distance of transverse stress was approached through the use of an N.I. Muskhelishvili complex potential for the plane problem of an elliptical hole in a plate under tension. The calculations involved are outlined in Appendix I. Figure 1 shows the coordinates used and the "build-up" distance for an ellipse of any eccentricity. For the limiting cases of a circle ($m = o$) and a crack ($m = 1$), the expression derived yields results known to be correct. As will be shown subsequently, the case of a circular contour is of most immediate importance and the "build-up" distance from the surface is $(\sqrt{2} - 1)R$, or $0.414R$.

As further background for studies of the form of a tensile fracture, information was desired on the orientation of the stress field in the presence of a cavity or surface contour of finite radius. In a previous report(1) it was shown that in the presence of a crack there are three zero isoclinics with one, of course, being the symmetry plane containing the crack. The other two isoclinics, which are symmetrically disposed with respect to the crack plane, begin at the crack tip at an angle of 60 degrees to the symmetry plane and gradually bend over to a lower angle as the distance from the crack tip increases. If now one considers the case of a cavity with a finite radius of curvature, there will still be a zero isoclinic forming the primary symmetry plane. But the question arises as to whether the two other zero isoclinics will be coincident with the symmetry plane for some distance away from the surface of the cavity or whether they immediately diverge as in the case of a crack. If they are coincident, over what distance does this coincidence persist?



$$X = R\left(\rho + \frac{m}{\rho}\right) \cos \theta$$
$$Y = R\left(\rho - \frac{m}{\rho}\right) \sin \theta$$

Semi-axes of elliptic hole $a = R(1+m)$, $b = R(1-m)$

"Build up" distance for σ_x stress with $\theta = 0$ is;

$$2\rho^2(-2m^3 + m^2 - 3) = (m^2 - 2m - 3)(\rho^4 + m^2)$$

For a circle $m = 0$ and

$$\rho = \frac{X}{R} = \sqrt{2}$$

Figure 1. Coordinate system and equation for "build-up" distance of transverse stress.

These questions can be resolved most expeditiously if one has a solution for the rectangular coordinate XY shear stress. An A.C. Stevenson complex potential was considered to be most convenient for this purpose. In Appendix II the shear stress and zero isoclinics are given for a circular hole in a plate under simple tension. It turns out that the auxiliary zero isoclinics are coincident with the symmetry plane for a distance away from the surface of the hole of 0.414R.

This distance of 0.414R is the same as the "build-up" distance determined above. This quantity is the relevant one for the examples to be discussed, but the isoclinic result is not without further significance for fracture. For instance, in stress corrosion tests with cracked specimens the fracture often follows an inclined isoclinic, whereas the same material will break flat when tested dry. This behaviour is rationalized on the basis that the inclined isoclinic carries a higher normal stress but a lower hydrostatic tension than the symmetry plane. The present result indicates that if the defect has a finite root radius, the fracture will tend to start flat because all three isoclinics are coincident.

The next aspect investigated was the possible inter-relationship between a circular contour causing a certain "build-up" distance and an isoclinic originating from that point. Figure 2 shows the result.

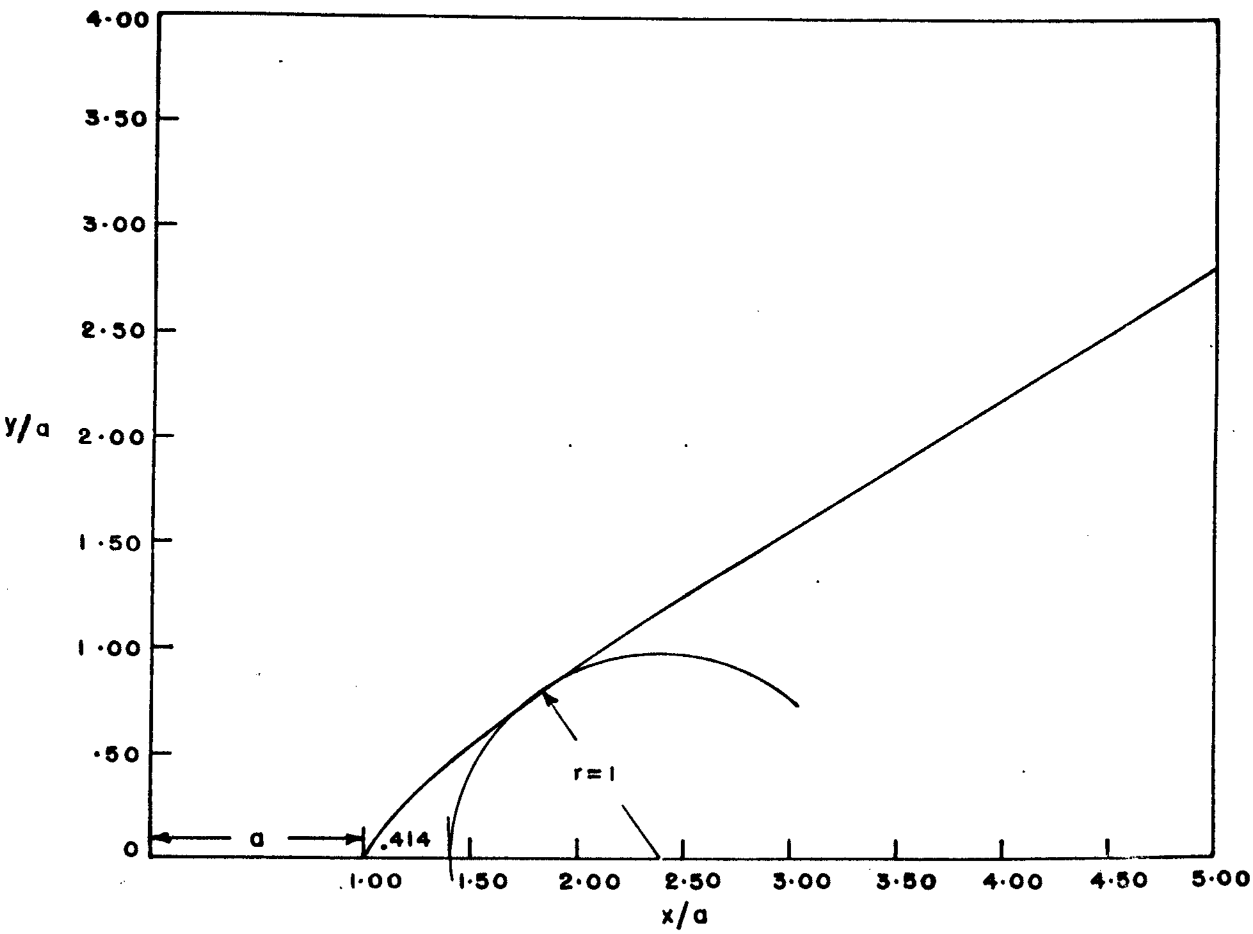


Figure 2. A unit circle has a "build-up" distance such that a unit isoclinic originating from that point is almost exactly an osculating curve of the circle.

It is found that a unit circle has a "build-up" distance such that a unit isoclinic originating at that point is practically an osculating curve of the circle. When a search was made for the physical significance of this inter-relationship, the results shown in Figures 3, 4 and 5 were obtained. The fracture contours in sections from plate-type, crack-notch toughness specimens follow the elastic zero isoclinic precisely, even though considerable plastic flow preceded fracture. This leads to the expectation that surface dimpling from plastic flow preceding fracture should have a circular contour. Initial measurements support this, and further measurements are planned.

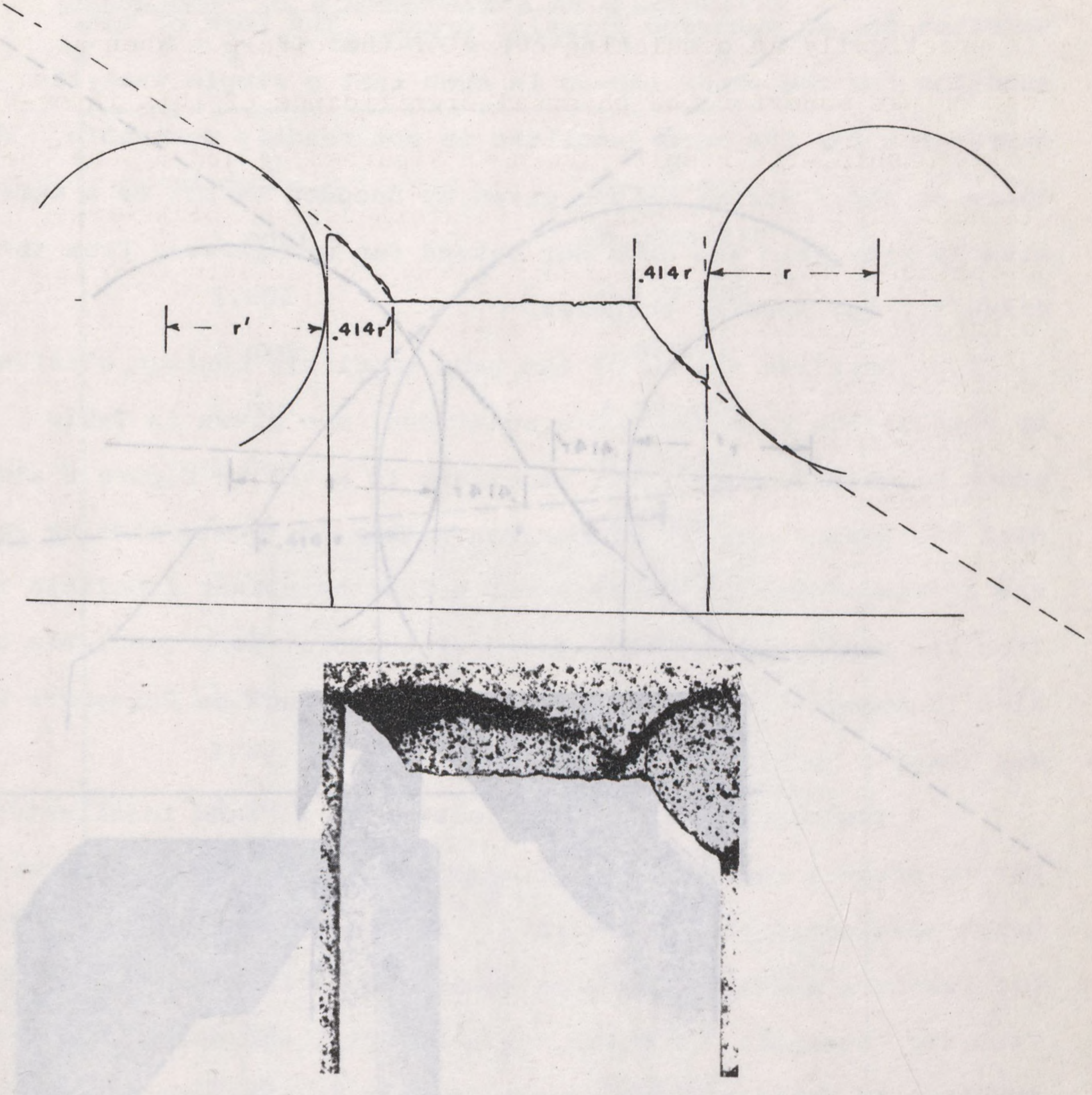


Figure 3. Section of fracture in $\frac{1}{4}$-inch-thick 250-grade maraging steel specimen near start of fracture.

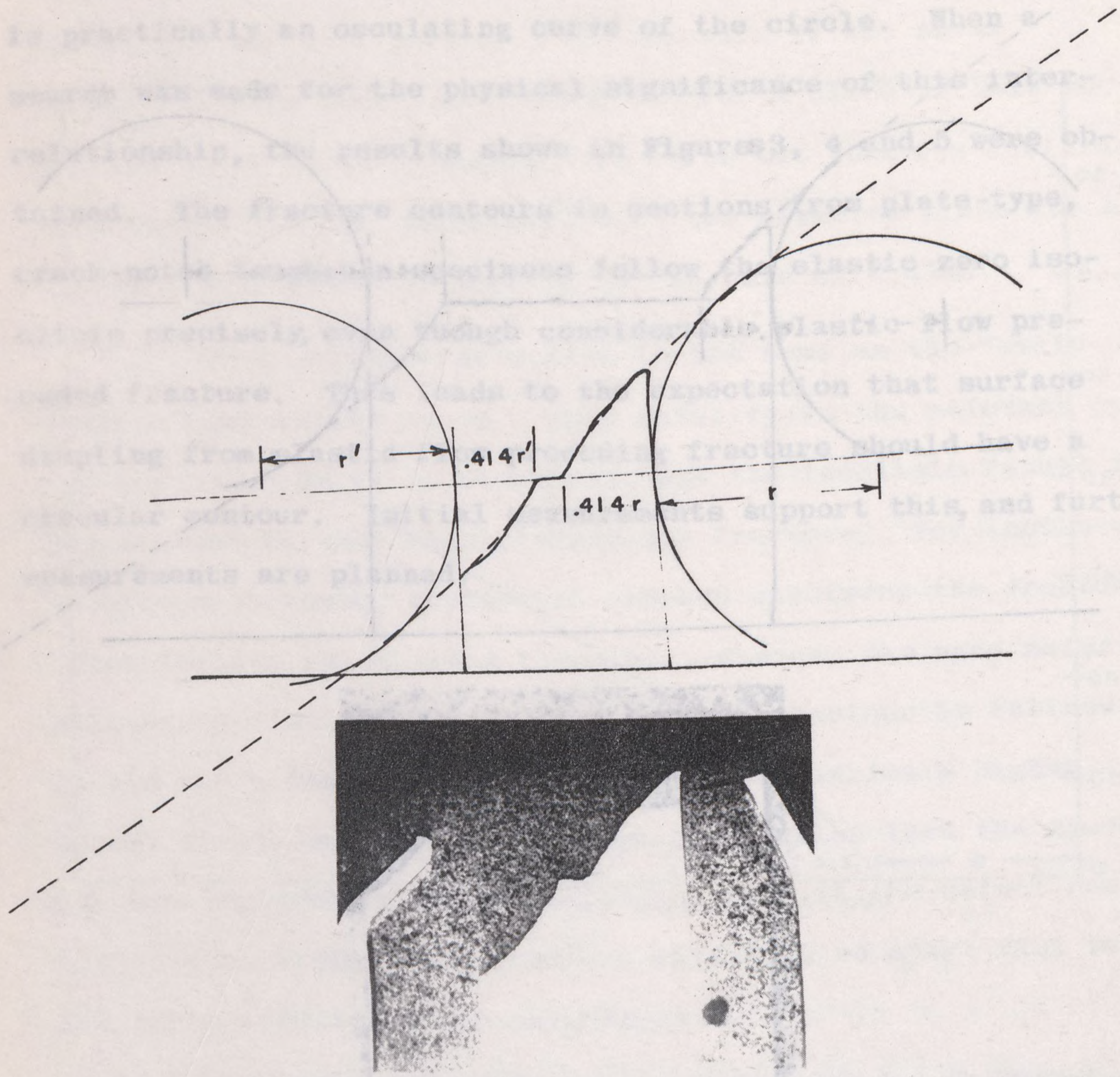


Figure 4. Section of fracture from same specimen as in Figure 3, but farther away from start of fracture.

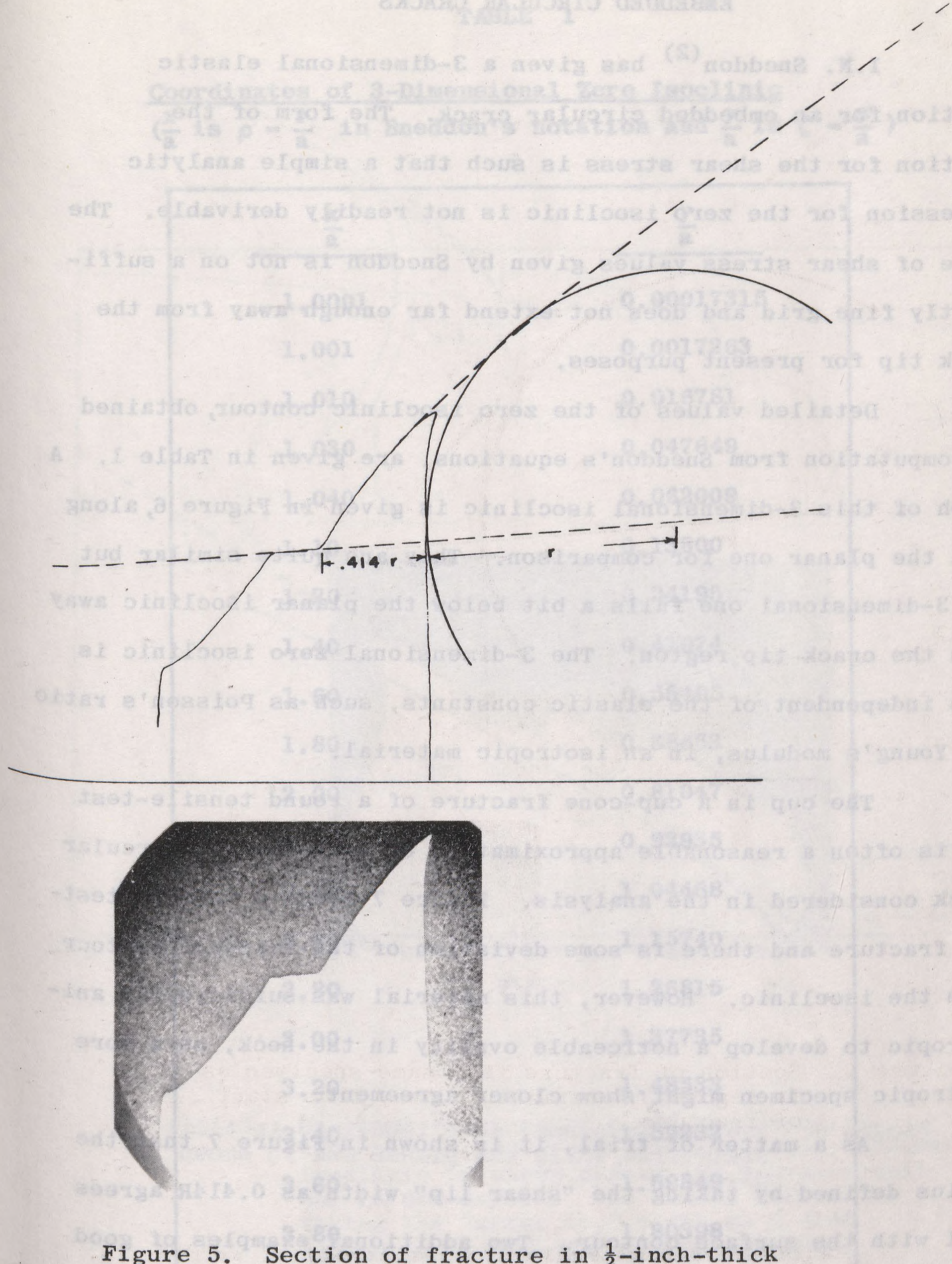


Figure 5. Section of fracture in $\frac{1}{2}$-inch-thick HP 9-4-25 steel specimen.

EMBEDDED CIRCULAR CRACKS

I.N. Sneddon[2] has given a 3-dimensional elastic solution for an embedded circular crack. The form of the solution for the shear stress is such that a simple analytic expression for the zero isoclinic is not readily derivable. The table of shear stress values given by Sneddon is not on a sufficiently fine grid and does not extend far enough away from the crack tip for present purposes.

Detailed values of the zero isoclinic contour, obtained by computation from Sneddon's equations, are given in Table 1. A graph of this 3-dimensional isoclinic is given in Figure 6, along with the planar one for comparison. They are quite similar but the 3-dimensional one falls a bit below the planar isoclinic away from the crack tip region. The 3-dimensional zero isoclinic is also independent of the elastic constants, such as Poisson's ratio and Young's modulus, in an isotropic material.

The cup in a cup-cone fracture of a round tensile-test bar is often a reasonable approximation of the embedded circular crack considered in the analysis. Figure 7 shows a tensile-test-bar fracture and there is some deviation of the fracture contour from the isoclinic. However, this material was sufficiently anisotropic to develop a noticeable ovality in the neck, so a more isotropic specimen might show closer agreement.

As a matter of trial, it is shown in Figure 7 that the radius defined by taking the "shear lip" width as 0.414R agrees well with the surface contour. Two additional examples of good agreement are given in Figure 8 for copper specimens partially broken in a stiff testing machine[3].

TABLE 1

Coordinates of 3-Dimensional Zero Isoclinic
($\frac{x}{a}$ is $\rho = \frac{r}{a}$ in Sneddon's notation and $\frac{y}{a}$ is $\zeta = \frac{z}{a}$)

$\frac{x}{a}$	$\frac{y}{a}$
1.0001	0.00017315
1.001	0.0017263
1.010	0.016781
1.030	0.047649
1.040	0.062009
1.10	0.13800
1.20	0.24195
1.40	0.41074
1.60	0.55495
1.80	0.68653
2.00	0.81047
2.20	0.92935
2.40	1.04468
2.60	1.15740
2.80	1.26815
3.00	1.37735
3.20	1.48533
3.40	1.59232
3.60	1.69849
3.80	1.80398
4.00	1.90891

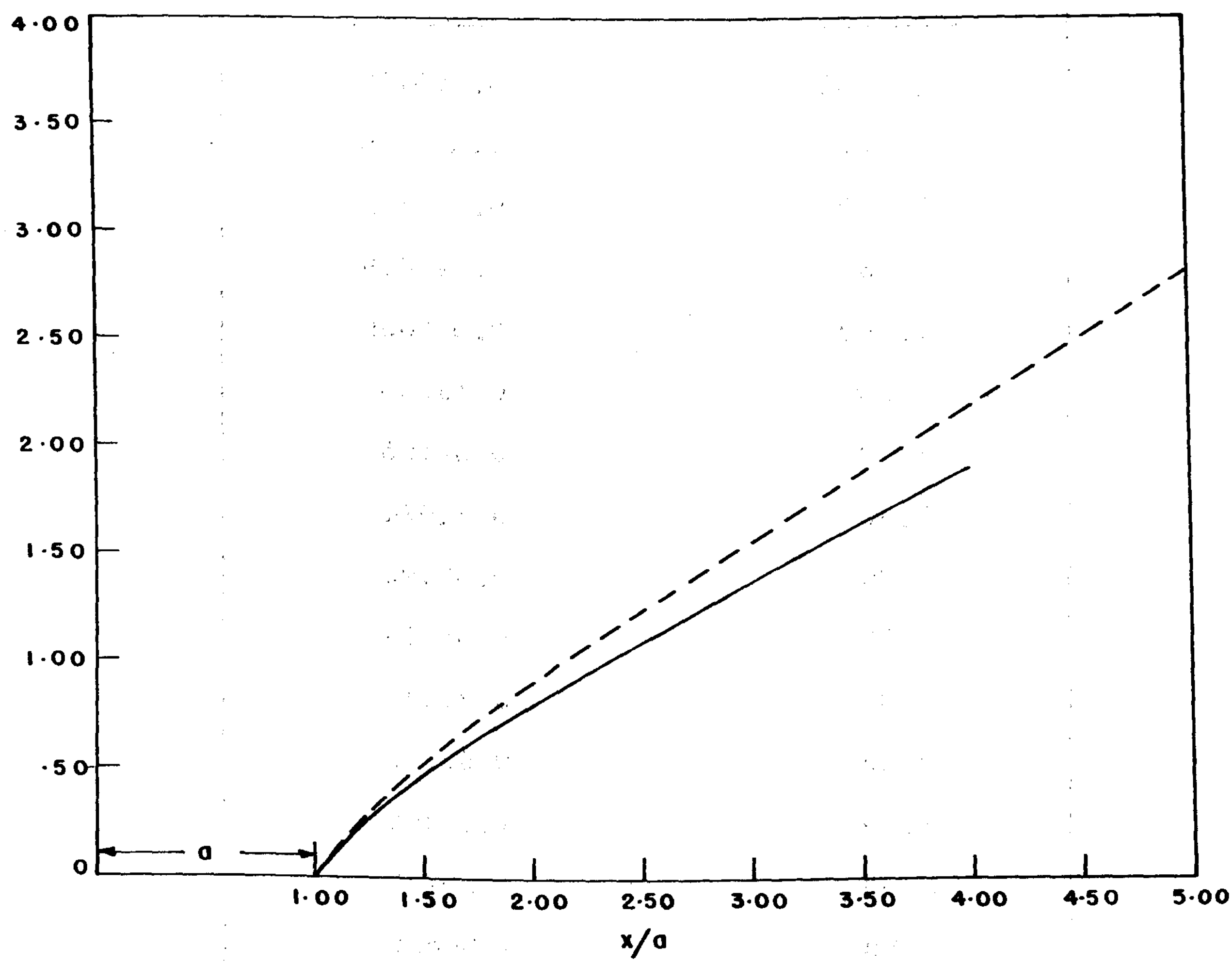


Figure 6. The 3-dimensional zero isoclinic is the solid line, while the planar one is shown dashed.

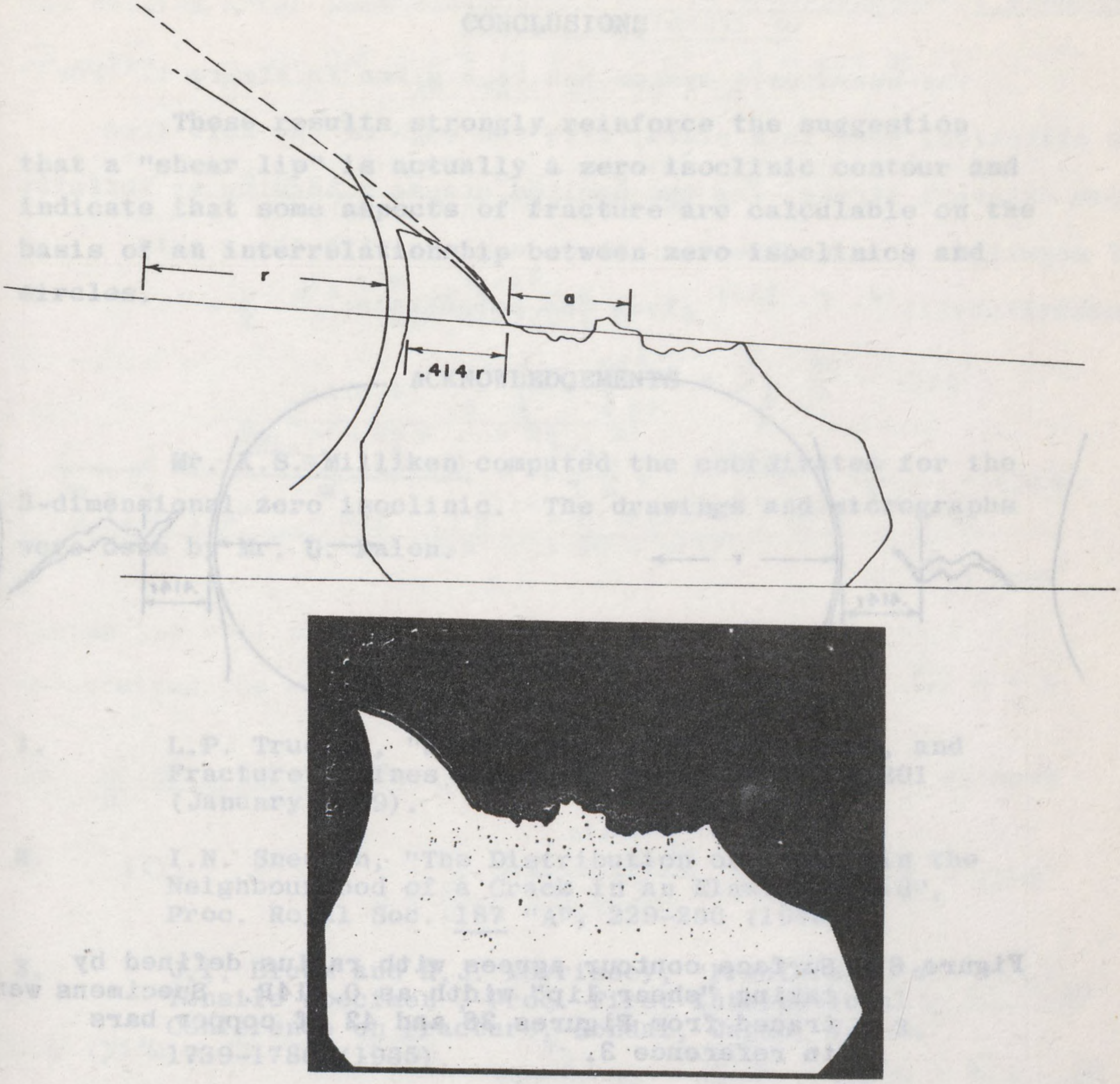


Figure 7. Cup-cone fracture of 0.505-inch tensile-test bar of 200-grade maraging steel. Planar isoclinic is shown dashed and 3-dimensional one is dot-dashed. The radius defined by taking the "shear lip" width as 0.414R agrees well with the surface contour.

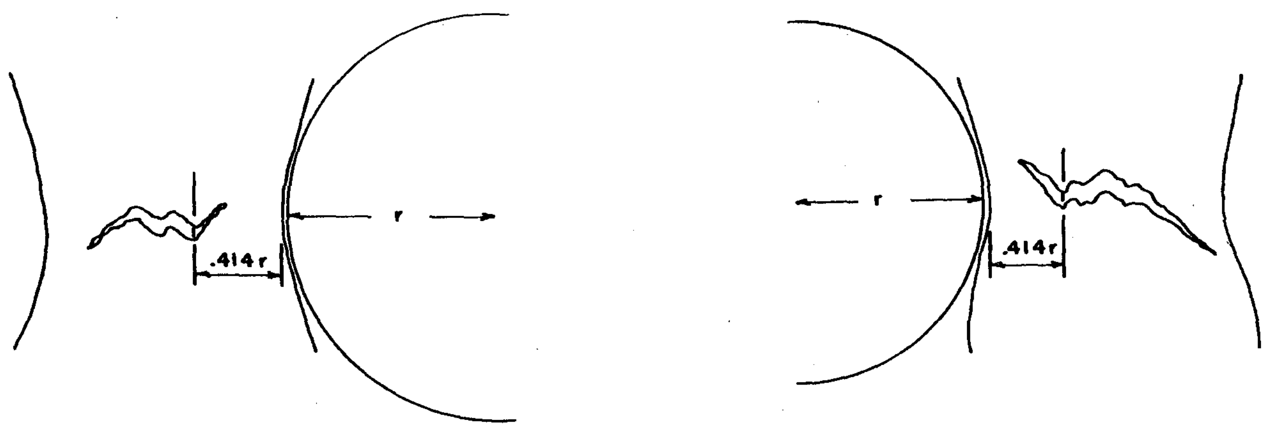


Figure 8. Surface contour agrees with radius defined by taking "shear lip" width as 0.414R. Specimens were traced from Figures 38 and 42 of copper bars in reference 3.

CONCLUSIONS

These results strongly reinforce the suggestion that a "shear lip" is actually a zero isoclinic contour and indicate that some aspects of fracture are calculable on the basis of an interrelationship between zero isoclinics and circles.

ACKNOWLEDGEMENTS

Mr. K.S. Milliken computed the coordinates for the 3-dimensional zero isoclinic. The drawings and micrographs were done by Mr. G. Palen.

REFERENCES

1. L.P. Trudeau, "Shear Lips, Zero Isoclinics, and Fracture", Mines Branch Research Report R 201 (January 1969).

2. I.N. Sneddon, "The Distribution of Stress in the Neighbourhood of a Crack in an Elastic Solid", Proc. Royal Soc. 187 "A", 229-260 (1946).

3. J.I. Bluhm and R.J. Morrissey, "Fracture in a Tensile Specimen", Proc. First International Conference on Fracture, Sendai, Japan, Vol. 3, 1739-1780 (1965).

4. N.I. Muskhelishvili, "Some Basic Problems of the Mathematical Theory of Elasticity", P. Noordhoff Ltd., Groningen, Holland (1953).

5. A.C. Stevenson, "Tension of Semi-Infinite Plate With Notched Boundary", Phil. Mag., 36, 178-183 (1945).

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APPENDIX I - Calculation of "Build-up" Distance of Transverse Stress

The coordinate system has been given in Figure 1. For an elliptical hole in a plate, with the edge of the hole free from external stress, and the applied stress a tension at infinity of magnitude p in a direction at an angle α with the x axis, Muskhelishvili (4, p. 338) gives the potentials:

$$\phi(\zeta) = \frac{pR}{4}\left(\zeta + \frac{2e^{2i\alpha} - m}{\zeta}\right)$$

$$\psi(\zeta) = -\frac{pR}{2}\left\{e^{-2i\alpha}\zeta + \frac{e^{2i\alpha}}{m\zeta} - \frac{(1+m^2)}{m}\frac{(e^{2i\alpha} - m)\zeta}{\zeta^2 - m}\right\}$$

From p. 333,

$$z = \omega(\zeta) = R\left(\zeta + \frac{m}{\zeta}\right) \text{ and } \zeta = \rho e^{i\theta}$$

From pp. 182-3,

$$\Phi(\zeta) = \frac{\phi'(\zeta)}{\omega'(\zeta)} = \frac{p}{4}\,\frac{\zeta^2 - 2e^{2i\alpha} + m}{\zeta^2 - m} \tag{1}$$

$$\Psi(\zeta) = \frac{\psi'(\zeta)}{\omega'(\zeta)}$$

$$\widehat{\rho\rho} - i\,\widehat{\rho\theta} = \Phi(\zeta) + \overline{\Phi(\zeta)} - \frac{\zeta^2}{\rho^2\,\overline{\omega'(\zeta)}}\left\{\overline{\omega(\zeta)}\,\Phi'(\zeta) + \omega'(\zeta)\,\Psi(\zeta)\right\}$$

Thus the $\rho\rho$ stress that is desired is the real part of this expression. Performing the required operations, one obtains:

$$\Phi'(\zeta) = \frac{p\,\zeta(e^{2i\alpha} - m)}{(\zeta^2 - m)^2}$$

$$\overline{\omega(\zeta)}\,\Phi'(\zeta) = \frac{Rp\,\zeta\,(\overline{\zeta^2} + m)(e^{2i\alpha} - m)}{\bar{\zeta}\,(\zeta^2 - m)^2} \tag{2}$$

$$\omega'(\zeta)\Psi(\zeta) = \psi'(\zeta) = -\frac{pR}{2}\left\{e^{-2i\alpha} - \frac{e^{2i\alpha}}{m\zeta^2} - \frac{(1+m^2)(e^{2i\alpha} - m)}{m}\left[\frac{(\zeta^2 - m) - 2\zeta^2}{(\zeta^2 - m)^2}\right]\right\}$$

(3)

Substituting (1), (2) and (3) in the expression for the stresses, one obtains, after some reduction:

$$\widehat{\rho\rho} - i\widehat{\rho\theta} = \frac{p}{2}\,\frac{\rho^4 - 2\rho^2 \cos 2(\theta - \alpha) - m^2 + 2m\cos 2\alpha}{\rho^4 - 2m\rho^2 \cos 2\theta + m^2}$$

$$- \frac{p\,\rho^2 e^{2i\theta}\,(e^{2i\alpha} - m)(\rho^4 e^{-4i\theta} - m^2)}{(\rho^4 - 2m\rho^2\cos 2\theta + m^2)^2}$$

$$+ \frac{p}{2}\,\frac{\rho^2 e^{-2i\alpha}\,(\rho^2 e^{2i\theta} - m)}{\rho^4 - 2m\rho^2\cos 2\theta + m^2}\,m^2$$

$$- \frac{p}{2m}\,\frac{e^{2i(\alpha-\theta)}\,(\rho^2 e^{2i\theta} - m)}{\rho^4 - 2m\rho^2\cos 2\theta + m^2}$$

$$- \frac{p}{2}\,\frac{(1+m^2)(e^{2i\alpha} - m)\rho^2\left[(\rho^2 e^{2i\theta} - m) - 2\rho^2 e^{2i\theta}\right](\rho^2 e^{-2i\theta} - m)}{m(\rho^4 - 2m\rho^2\cos 2\theta + m^2)^2}$$

Taking the real part of this expression with $\alpha = \frac{\pi}{2}$ and $\theta = o$ we obtained the stress:

$$\widehat{\rho\rho} = \frac{p}{2}\,\frac{\rho^4 + 2\rho^2 - m^2 - 2m}{(\rho^2 - m)^2} + p\,\frac{\rho^2(1+m)(\rho^2+m)}{(\rho^2 - m)^3}$$

$$- \frac{p}{2}\,\frac{\rho^2}{\rho^2 - m} + \frac{p}{2m(\rho^2 - m)}$$

$$- \frac{p\,\rho^2}{2m}\,\frac{(1+m^2)(1+m)(\rho^2+m)}{(\rho^2 - m)^3}$$

As a check on the correctness of this result, the $\widehat{\rho\rho}$ stress should be zero at the surface of the hole where $\rho = 1$, and this is the case.

To obtain the "build-up" distance we need the location where the $\widehat{\rho\rho}$ stress is a maximum. Taking the derivative of the $\widehat{\rho\rho}$ stress and equating it to o, one finds, after some reduction, that the "build-up" distance along the x-axis is defined by:

$$2\rho^2(-2m^3+m^2-3) = (m^2-2m-3)\ (\rho^4+m^2)$$

As a partial check, the limiting conditions are given correctly by this equation, because for a circle m = o and $\rho = \pm\sqrt{2}$, and for a crack m = 1 and $\rho = \pm 1$.

This general equation may prove helpful in fractographic and other studies. Part of the "stretch zone" observed in fractography may be a "shear lip".

APPENDIX II - Calculation of Orientation of Stress Field with Circular Hole

Stevenson[5] gives the following potentials for an elliptical hole in a plate under a simple tension T at infinity at an angle β to the x-axis:

$$\Omega(\sigma) = Tc\left[\sigma + \frac{(2e^{2i\beta} - \lambda)}{\sigma}\right]$$

$$\omega(\sigma) = -TC^2\left[e^{-2i\beta}\sigma^2 + e^{2i\beta}\sigma^{-2} + 2(1-2\lambda\cos 2\beta + \lambda^2)\log\sigma\right]$$

where $Z = C(\sigma + \frac{\lambda}{\sigma})$, $2C = a + b$ and $2\lambda C = a-b$

For a circle $\lambda = o$ and with $\beta = \frac{1}{2}\pi$, these potentials reduce to:

$$\Omega(\sigma) = TC\left[\sigma - \frac{2}{\sigma}\right]$$

$$\omega(\sigma) = TC^2\left[\sigma^2 + \frac{1}{\sigma^2} - 2\log\sigma\right]$$

Stevenson gives the stress relation:

$$-2(\widehat{xx} - \widehat{yy} + 2i\widehat{xy}) = Z\,\bar{\Omega}''(\bar{Z}) + \bar{\omega}''(\bar{Z})$$

Differentiating:

$$\Omega''(Z) = -\frac{4T}{C\sigma^3}$$

$$\omega''(Z) = T\left[2 + \frac{2}{\sigma^2} + \frac{6}{\sigma^4}\right].$$

With $Z = r\,e^{i\theta} = C\sigma$, then $\sigma = \frac{r\,e^{i\theta}}{C}$

Substituting:

$$Z\,\bar{\Omega}''(\bar{Z}) = -\frac{4TC^2}{r^2}e^{i4\theta}$$

$$\bar{\omega}''(\bar{Z}) = T\left[2 + 2\frac{C^2}{r^2}e^{i2\theta} + 6\frac{C^4}{r^4}e^{i4\theta}\right]$$

Thus $\widehat{xx}-\widehat{yy}+2i\widehat{xy} = 2T\,\frac{c^2}{r^2}\,e^{i4\theta} - T\left[1+\frac{c^2}{r^2}\,e^{i2\theta} + \frac{3c^4}{r^4}\,e^{i4\theta}\right]$

As a check on the correctness, consider first the real part of this equation:

$$\widehat{xx}-\widehat{yy} = 2T\,\frac{c^2}{r^2}\cos 4\theta - T - T\,\frac{c^2}{r^2}\cos 2\theta - 3T\,\frac{c^4}{r^4}\cos 4\theta$$

On the edge of the hole for $r = c$ and $\theta = o$, the transverse stress $\widehat{xx}$ will be zero.

Substituting,

$$-\widehat{yy} = 2T-T-T-3T,$$
$$\widehat{yy} = 3T,$$

which is the correct stress concentration factor for a hole.

The $\widehat{xy}$ shear stress is $\frac{1}{2}$ the imaginary part,

$$\widehat{xy} = \frac{T}{2}\,\frac{c^2}{r^4}\,(2r^2 \sin 4\theta - 3c^2 \sin 4\theta - r^2 \sin 2\theta)$$

For the zero isoclinic the $\widehat{xy}$ shear stress is zero. Therefore:

$$2r^2 \sin 4\theta - r^2 \sin 2\theta = 3c^2 \sin 4\theta$$

$$r^2 = \frac{3c^2 \sin 4\theta}{2 \sin 4\theta - \sin 2\theta}$$

For θ small, $\sin \theta = \theta$

$$r = \pm c\sqrt{\frac{12\theta}{6\theta}} = \pm\sqrt{2}\,c$$

All three zero isoclinics are coincident over the same distance as the "build-up" distance for the $\widehat{xx}$ stress.

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