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ROCK ANCHOR DESIGN MECHANICS

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ROCK ANCHOR DESIGN MECHANICS

by

D. F. Coates* and Y. S. Yu**

ABSTRACT

Rock anchors are useful and, in many cases, important structural elements. However, little research work has been conducted on the mechanics of their behaviour. For these reasons, some finite element studies were conducted to clarify certain aspects of the stress distributions in the rock surrounding typical anchors.

The nature of the transmission of load from a compression anchor to rock is shown to vary, as had been predicted qualitatively, with the ratio of the modulus of deformation of the anchor to that of the rock mass. The proportion of the load transmitted through bearing on the bottom of the anchor, although it is not likely to be more than 15%, increases with the ratio of the moduli (anchor/rock).

Tension anchors produce large tensile stresses in the rock, which may account for some of the commonly obtained initial creep.

Shear stresses induced by both tension and compression anchors are large and can be accompanied by diagonal tensile stress. These tensile stresses might cause some local fracturing in both the rock and the concrete (or grout) of the anchor.

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KEY WORDS: Rock Anchor, Design, Mechanics, Stress, Finite Element.

Direction des mines

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ÉLÉMENTS DE MÉCANIQUE POUR L'ÉTUDE DES ANCRAGES AU ROCHER

par

D. F. Coates* et Y. S. Yu**

RÉSUMÉ

En construction, les ancrages au rocher sont des éléments utiles, et dans bien des cas, importants. Cependant, peu d'études ont été consacrées aux problèmes mécaniques posés par leur comportement. C'est pour ces raisons que les auteurs ont entrepris des études en vue d'élucider, par la méthode des éléments finis, certains aspects de la distribution des contraintes dans la roche, au voisinage de différents types d'ancrage.

Les auteurs démontrent que la nature de la transmission de la charge entre un ancrage travaillant à la compression et la roche varie, comme on pouvait prévoir qualitativement, avec le rapport des modules d'élasticité du massif d'ancrage et du massif rocheux. La proportion de la charge transmise par la poussée du fond de l'ancrage, bien que ne dépassant probablement pas les 15%, augmente avec le rapport des modules (ancrage/rocher).

Les ancrages travaillant à la traction provoquent des contraintes d'extension importantes dans la roche, et ces contraintes peuvent être responsables en partie du fluage initial que l'on obtient généralement.

Les contraintes de cisaillement provoquées par les ancrages travaillant à la compression et à la traction sont importantes et peuvent être accompagnées de contraintes diagonales d'extension. Ces contraintes d'extension pourraient causer quelques fractures, aussi bien dans la roche que dans le béton (ou le coulis) du massif d'ancrage.

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Mots clefs: ancrage au rocher, étude, élément de mécanique, problèmes mécaniques, contrainte, méthode des éléments finis.

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INTRODUCTION

Many uses are being found for rock anchors. In large cantilever hangers, normally the weight of the roof is counterbalanced with a large mass of concrete. Where bedrock conditions are favourable and reliability can be guaranteed, a cheaper solution is to use rock anchors to provide the reaction against the upward pull. Cantilever bridges have the same force system and anchorage requirements.

In dam design, rock anchors have been used to reduce the cross-section of a normal gravity dam, with considerable savings in concrete costs. The reduced base section gives rise to tension at the heel of the dam, which must then be resisted by rock anchors. The reliability of these anchors is extremely important. Tall buildings subjected to wind and earthquake forces might produce uplift on the foundations on one side and hence require a similar reaction.

In building excavations, an elaborate system of struts may be required to hold the banks of soil until the structure is constructed. These struts complicate the pouring of foundations and walls; consequently, both construction time and costs are reduced when struts can be eliminated by rock anchors. The hanging wall in a mining stope is often similarly supported by a series of struts that impede the operation; rock bolts can be used to eliminate the use of struts and to improve such operations.

Poor rock in the abutment areas of dams is often supported with rock anchors. Rock slopes for highway cuts or open-pit mining operations might also be similarly stabilized by rock anchors.

Other, less common uses of rock anchors include the recompression of foundation soils and soft rocks, either to minimize the expansion and deterioration that would normally follow the relief of pressure from excavating the overlying ground or to precompress such compressible layers so that the settlement under the structural loads will not be excessive. Dolphins, which provide mooring posts for ships, can be cheaply constructed if good bedrock provides anchorage for both compressive and tensile forces arising from the impact of the ship. Where the force being transmitted into the ground is compressive, the structural element is a pile socketed into bedrock, which is used for other types of structures as well. Because compression anchors, or piles, can be constructed with a greater sense of reliability in their performance over time, they are probably of greater economic importance than tension anchors. Indeed, when they are in competition with caissons, which are expensive structural elements, the importance of work on their design details is enhanced.

The design of compression anchors or of socketed piles is currently based, owing to the lack of experimental data, on an assumption regarding the proportion of load transmitted into the rock through shear on the sides and through bearing on the end, regardless of the varying deformation properties of the material (1). However, a theoretical examination has shown that, when the moduli of deformation of the rock and pile are equal, the shear stresses will vary widely over the length of the anchor (2). Very high values of shear stress would occur near the surface of the rock and only about 4% of the load would be transmitted in bearing at the bottom. In addition, it was reasoned that tensile stresses could occur as a result of the high shear stresses induced at the contact between the anchor and the rock. Where the deformation modulus of the anchor is less than that of the rock and if no shrinkage occurs in the concrete, the proportion of load transmitted to the bottom of the pile should be even less when the moduli are equal. Alternatively, with the anchor material having a modulus of deformation greater than that of the rock, the proportion of load transmitted to the bottom would be greater than in the previous cases, although this would not normally be too serious as rock, especially in the confined state at the bottom of the hole, would usually have adequate bearing strength unless open joints or beds of soft material were located immediately under the bottom.

Some analytical work had been done previously on this problem; however, the mechanics affecting design had not been completely established (3, 4, 5). For this reason, a series of finite element models were analysed using a previously developed program (6). The model shown in Figure 1, with the length of the anchor three times its diameter, was examined for ratios of moduli of deformation of the anchor to that of the rock, E_e/E_r, equal to 0.1, 1 and 10.

COMPRESSION ANCHORS

Figure 2 shows the variation of shear stress along the interface for various cases of compression anchors. In addition, the calculated curve using Boussinesq's equation is included (5). For moduli ratios of 0.1 and 1, the patterns of variation are similar to each other and to that obtained from the Boussinesq calculation. When the moduli ratio is 10 the pattern changes somewhat, the maximum shear stress at the top of the anchor being significantly decreased. Recent tests show similar patterns (9).

In Figure 3(a) the variation of vertical stress (as a ratio of the pressure, p, on the end of the anchor) at the centre of the anchor over its embedded length is shown for a moduli ratio of 10. An almost linear variation is obtained, which is consistent with the minor variation of shear stress shown in Figure 2. Figure 3(b) shows that the stress (or load) transmitted in bearing on the end of the anchor varies from approximately 3.5% to 10.5% for E_a/E_r from 0.1 to 10.

The stresses at the interface produce diagonal tensile stresses as shown in Figure 4. These are greater than 0.01 p only in the elements within a distance equal to the anchor radius, r_i, of the top. It might be thought that these tensile stresses would be suppressed by horizontal residual stresses, which seem to be quite common even at the surface of the bedrock (2). However, owing to the insignificant vertical stress in the rock in this problem, these tensions could not be suppressed. For a typical 24-in. (60-cm)-diameter anchor with a load of about 600 tons, the average loading pressure would be about 2,600 psi (185 ksc, kg/cm^2), which would produce, according to these models, a maximum tension of 300 psi (20 kg/cm^2).

TENSION ANCHORS

While an anchor is sustaining a tensile force, it is possible that the induced tensile stresses in the rock will be serious with regard to instability. The shear stress distribution at the interface for the models was found to be substantially the same as in Figure 2 (owing to the small amount of load transmission in end bearing for the compression anchors). These stresses in turn produce tensile stresses in the rock, with the maximum intensity occurring at the ground surface.

At the bottom of the anchor, the patterns of variation of these tensile stresses out from the anchor are shown in Figure 5; the reduction with distance from the anchor is very rapid. For a 150-ton anchor in a 3-in. (7.5-cm)-diameter hole, the maximum tensile stress for these cases would be about 20,000 psi (1,500 ksc) in the rock at the top of the anchor and about 7,000 psi (500 ksc) at the bottom (if the ground would sustain tensile stresses). It is probable that cracking would occur and that the tension would decrease as it moves out to larger radial distances, reaching an equilibrium position if pullout does not occur. Such crack propagation possibly accounts for some of the observed anchor creep that frequently occurs for a period of time after installation (although recent work suggests that time-dependent compression of joints might account for some such creep (10)). The superposition of the field stress concentrations would not diminish these tensile stresses significantly, as the maxima are in the vertical direction.

DESIGN MECHANICS

A common type of rock anchor consists of a steel structural shape, rod or cable anchored into rock with Portland cement grout or concrete. The ultimate strength of such a rock anchor is governed by several different modes of failure. First, the shear stress arising at the interface of the steel and grout might fail. It has been found that the shear stress, as shown in Figure 2, is fairly evenly distributed along the length of the anchor if the modulus of deformation of the anchor is much greater than that of the ground. This distribution would thus not be valid for hard rock. However, where valid, the average shear stress can be calculated according to the equation:

$$\tau = P/(2\pi RL), \qquad \text{Eq. 1}$$

where R is the radius of the steel rod, L is the length of the anchor, and P is anchor load.

Based on experimental work done in the field of concrete construction, the allowable average stress for smooth rods can be determined according to the equation:

$$\tau_a = 2.4\sqrt{f'_c}\ (160 \text{ psi max}), \qquad \text{Eq. 2}$$

where τ_a is the permissible stress in shear and f'_c is the uniaxial compressive strength of the concrete or grout. For deformed bars or any structural elements, such as lugs, that, in effect, throw the failure surface into the concrete, the permissible shear stress can be calculated according to the equation:

$$\tau_a = 0.1 f'_c \text{ (350 psi max).} \qquad \text{Eq. 3}$$

These equations incorporate a safety factor against the mean strength of the concrete of the order of 2 to 2.5. If we assume the safety factor to be 2, then the maximum capacity of the rock anchor would be calculated from the following equation:

$$P_f = 4\pi RL\tau_a, \qquad \text{Eq. 4}$$

where P_f is the anchor load at failure.

In addition to the shear stress created at the interface of the steel and grout, there will be shear stress also at the interface of the grout and rock. Assuming that the strength of the rock is greater than that of the grout, Equations 3 and 4 would be used for the determination of the capacity of the anchor with respect to failure at this interface. In this case, as the radius, R, would be that of the hole and thus greater than the radius of the steel element, the capacity of the anchor would not be governed by this mode of failure. On the other hand, the possibility of significant shrinkage in the grout diminishing the intimate contact between the grout and rock might actually provide a weaker surface here than that adjacent to the steel, although recent tests using a conventional Portland cement grout mix not only showed no sign of shrinkage but did show surprisingly high interface shear strength (9). However, it might be helpful if the grout included a small amount of expanding agent.

For concrete compression anchors with a steel shell and core, the appropriate length of socket, L, to transfer the load into the rock is usually determined by using the following formula (1):

$$L = \frac{Q - 0.35f'_c\, A_c}{0.05\; f'_c\; C_c}, \qquad \text{Eq. 5}$$

where Q is the design load for the pile, f'_c is the specified uniaxial compressive strength of the concrete, A_c is the cross-sectional area of the concrete, and C_s is the circumference of the concrete or the inside circumference of the shell. Equation 5 implies that $0.35f'_c$, is the appropriate bearing pressure between the bottom of the pile and the rock. This is consistent with the maximum permissible bearing stress on concrete, such as occurs on the top of footings, of $0.375\ f'_c$ where the area on which the bearing stress occurs is less than 1/3 of the total horizontal area. The formula also implies that $0.05\ f'_c$ is the appropriate, allowable shear or bond stress between the concrete and the rock walls of the socket. Furthermore, there is a presumption that the distribution of the load between shear stresses on the sides and bearing stresses on the bottom will be according to these allowable stresses.

As is shown in Figure 2, the pile load, Q, will give rise to shear stresses, τ, on the sides of the socket, and to bearing stresses, q, on the bottom of the socket. But the load transmitted to the rock in bearing at the bottom of an anchor is of the order of 3% to 10% of the total load. Therefore, unless the bond between the anchor and the rock is purposely eliminated it should be assumed that all the load is taken by the sides of the socket, and the required length should be analysed using such model results and Equations 1 to 4.

The effect of the shear stresses will be as shown in Figure 6(a). The element on the left side shows that the shear stresses will induce both compressive stresses and tensile stresses in the rock. The element on the right side shows that the corresponding stresses would also be created in the concrete. With the tensile strength of concrete normally being about 0.1 f'_c, it would seem that tensile failure in the concrete at least, if not in the rock, might occur. In Figure 6(b) the Mohr circle 1 shows the stress conditions at failure for a condition of pure shear. The nominal failure envelope shown in the figure includes an arbitrary inclination of 45°. Circle 1 hence represents failure in tension. However, there will also be normal, compressive stresses acting on vertical and horizontal planes. The effect of these stresses would be to move Circle 1 to the right.

With the specification that the permissible compressive stresses in the concrete part of the pile be 0.225 f'_c, the allowable envelope of stresses could be considered to be as shown by the dashed line in Figure 6(b). Circle 2 represents the permissible uniaxial compressive stresses. The maximum shear stresses associated with this stress circle would be 0.1125 f'_c. Because the compressive stress on horizontal planes in the concrete below the top of the socket is likely to be less than the value above the socket in the pile, and as the shear stress will be greater than the average value along the socket, the stress circle is moved to the left and its radius is increased. The need for a safety factor based on average stresses can be appreciated.

If shrinkage in the concrete is greater than the roughness of the socket, 100% transmission of load to the bottom of the pile will occur. The bearing pressure will be much higher than the normal maximum allowable pressure cited in various building codes, 50 to 60 tsf. However, the ultimate bearing capacity of a brittle rock substance will be equal to at least three, and more probably 8 to 21, times its uniaxial compressive strength (2, 8). If a safety factor of three were used against bearing failure, then the allowable bearing pressure would be equal to the uniaxial compressive strength of the rock substance. As it is a very poor rock that does not have a uniaxial compressive strength of more than 5,000 psi, the lack of shear resistance in the socket, and the transmission of the full load to the bottom of the pile, should not produce a serious condition unless structural features such as open joints or beds of soft material are located immediately under the bottom of the socket. The possible occurrence of these types of features, especially in near-surface bedrock, accounts for the conservatism in the tabulated values for permissible bearing pressures in most building codes. However, it would seem that, in many cases, capacity will be governed by the permissible stresses in the concrete.

From the above, a rational procedure for designing such a high-capacity pile would be to drill a socket down from the surface of the bedrock to a depth sufficient to eliminate from the bearing zone any significant open joints, altered rock, or soft layers. The construction procedure could include the drilling of a small-diameter hole below the bottom of the socket to establish absolutely the absence of such structural features. Then there would seem to be an advantage not only in not counting on the transfer of some load through shear into the sides of the socket, but also in preventing the development of such shear stresses, by using a liner of thin steel or cardboard, so that the occurrence of possibly very high, damaging, shear stresses near the top of the socket would be avoided.

For a tension anchor, failure due to tensile stresses in the rock may be initiated at the bottom of the anchor, and will likely create a cone of rock bounded by planes on which diagonal tensile stresses will be acting. Experimental work on anchor bolts in concrete (5) can be analysed to show that the maximum load on the anchors at the time of failure, which produced a cone of concrete, could have been closely predicted by assuming a cone of failure bounded by planes at 45 degrees to the surface and using the average tensile strength of the concrete. Thus, we may calculate the capacity of the anchor when governed by this mode of failure by the following equation:

$$P_f = \sqrt{2}\pi L^2 T_s,$$

$$P_f = 4.45 L^2 T_s, \qquad \text{Eq. 6}$$

where T_s is the tensile strength of the rock mass in the direction of the stress.

CONCLUSIONS

Well-bonded compression anchors will transfer more than 85% of their load into the rock through shear, rather than approximately half in shear and half in end bearing as implied by current design equations. Furthermore, the variation of these shear stresses can be quite large and can induce tensile stresses in both the rock and the anchor. Recognizing the enhanced bearing strength due to confinement of the rock at the bottom of such an anchor, consideration should be given to designing these anchors so that the load is transmitted entirely in end bearing.

Tension anchors produce both tensile and compressive stresses in the rock. Creep, commonly experienced, may occur either as a result of crack propagation in the rock, induced by large tensile stresses acting parallel to and at the end of the anchor, or by compression of joints in the rock along the anchor. Deformation measurements adjacent to such anchors, which could be compared to elastic deformation in a finite element model, would provide useful information on these possibilities.

Current methods should be modified to include the additional information that has been contributed to this subject. However, such important factors as determining the effective tensile strength of a rock mass, and the techniques for monitoring behaviour over time of important installations, still must be matters to be decided by the engineer's judgement for each particular structure and site.

ACKNOWLEDGEMENTS

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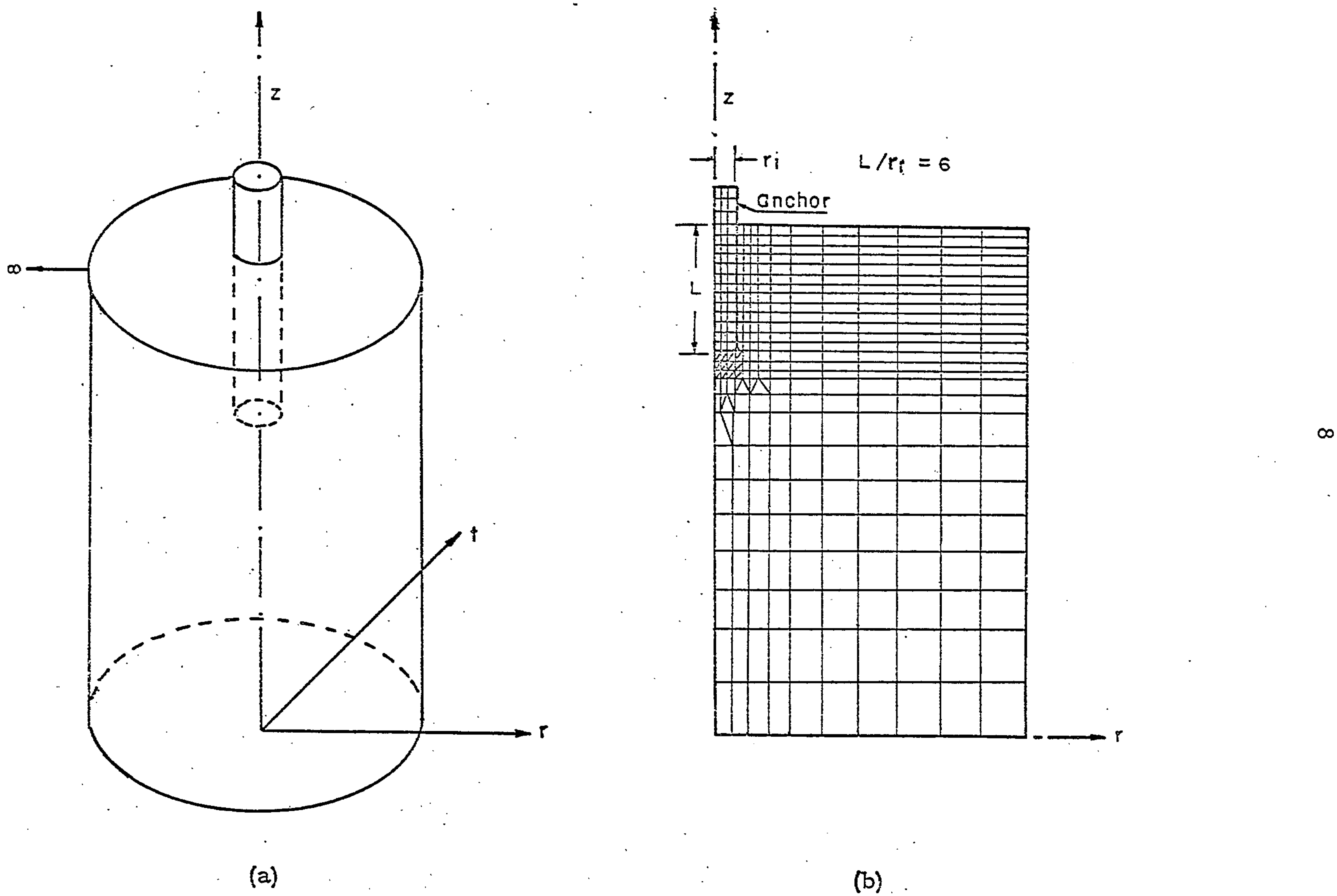


Figure 1. The geometry of the rock anchor model: (a) definition of axes; (b) finite element model.

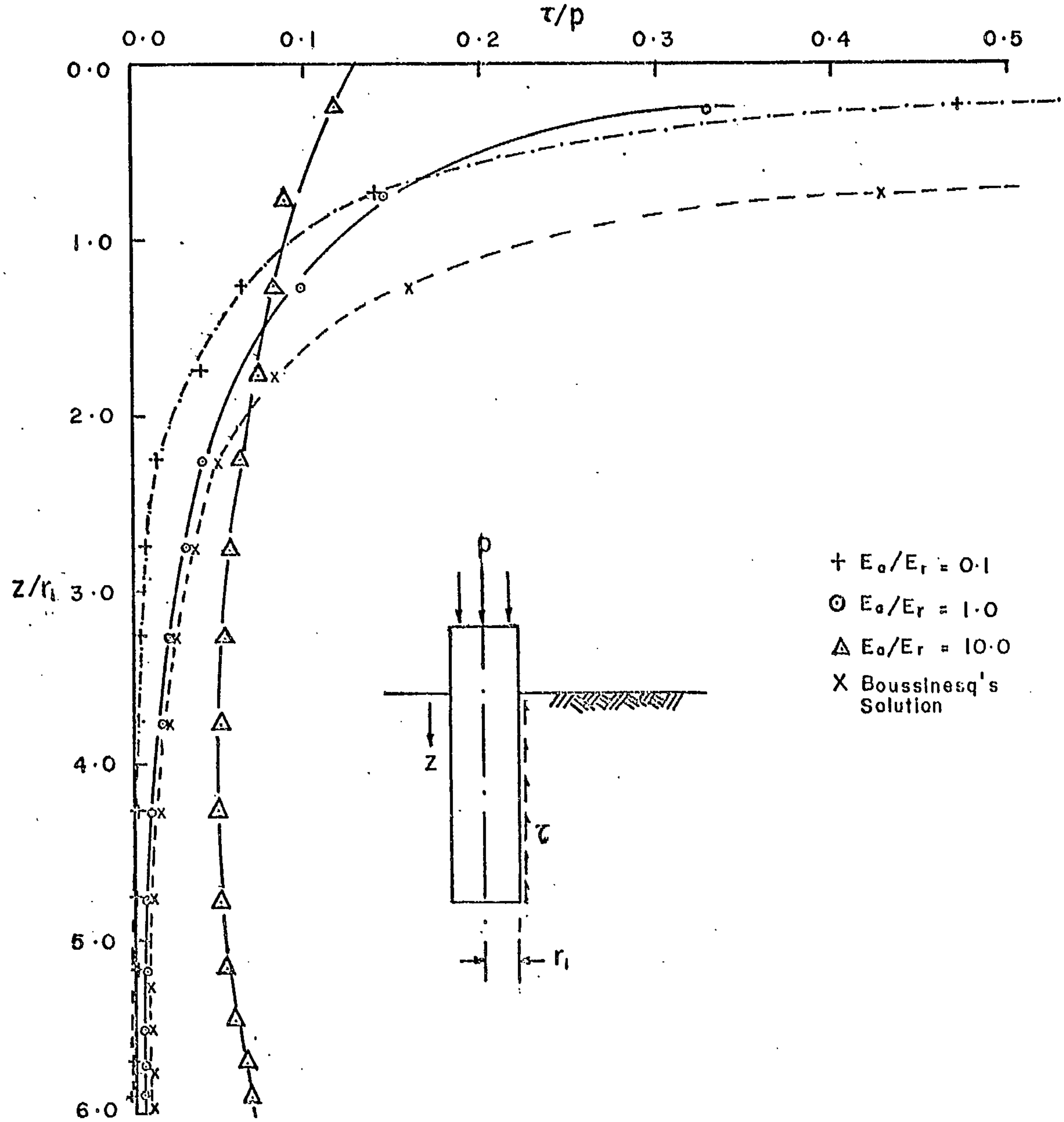


Figure 2. Variation of shear stress with depth along the rock interface for a compression anchor.

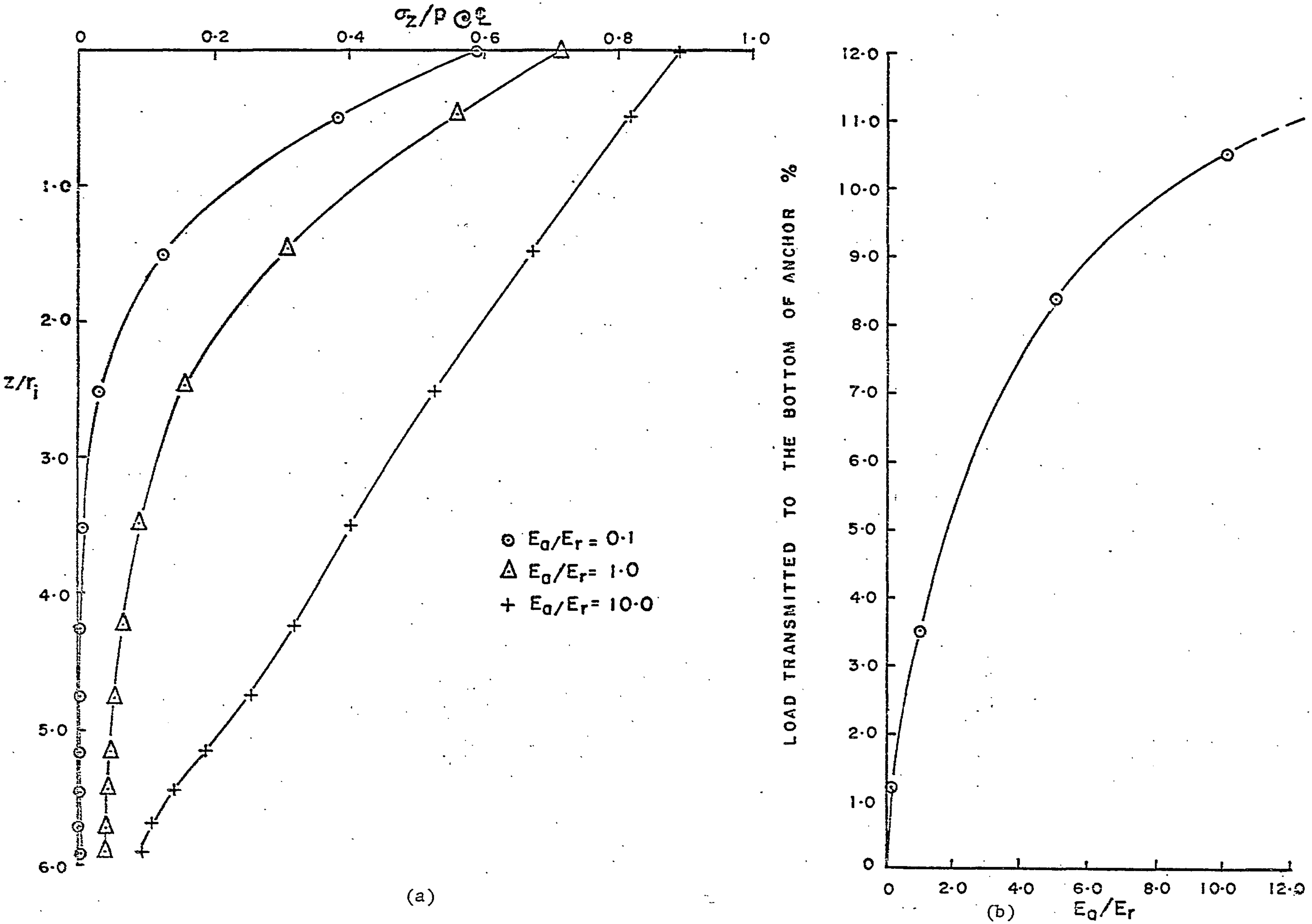


Figure 3. (a) Variation of vertical compressive stress in the anchor with depth; (b) variation of load transmitted to the bottom of the anchor with the moduli ratio E_a/E_r.

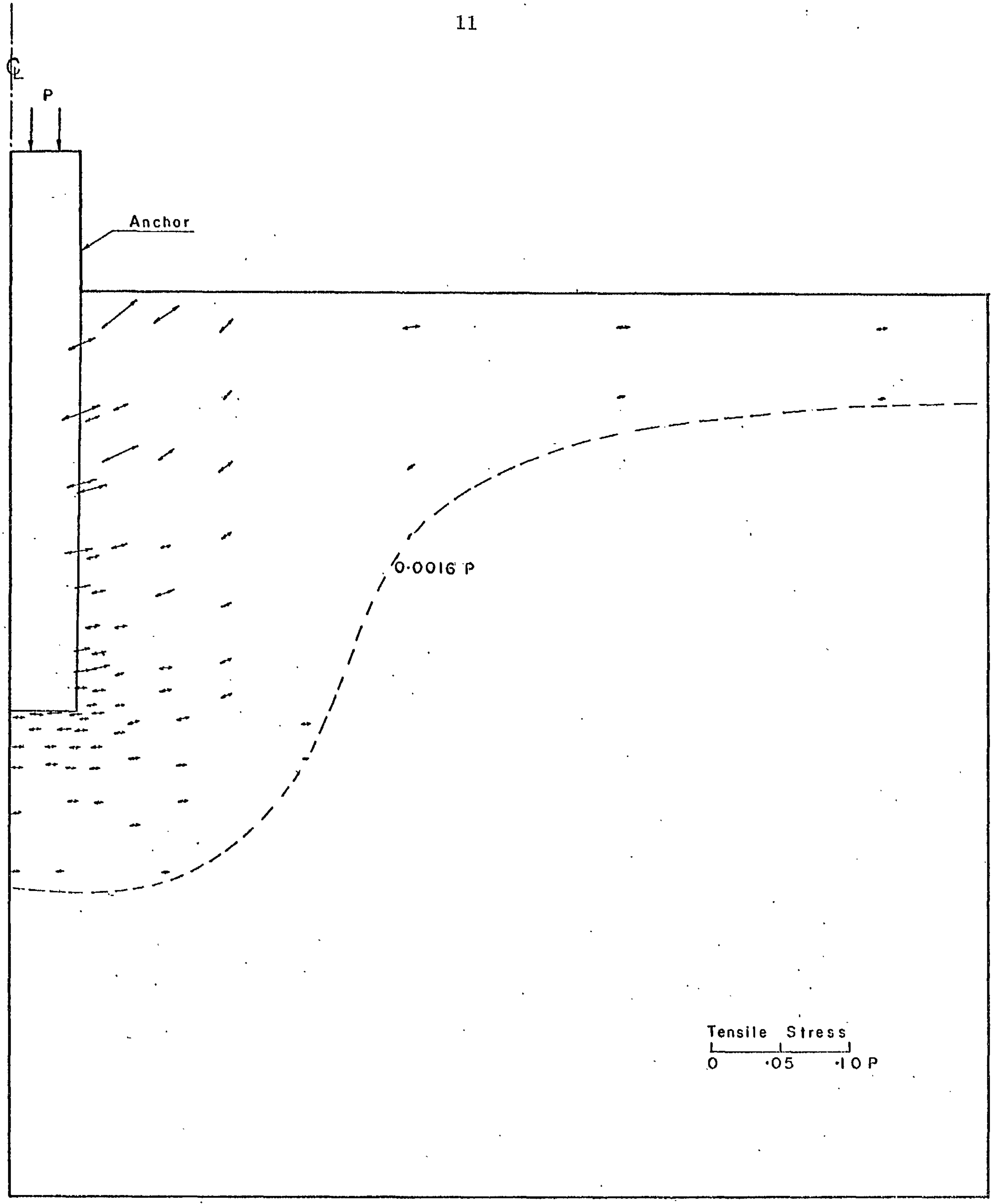


Figure 4. Trajectories of tensile stresses produced by a compression anchor.

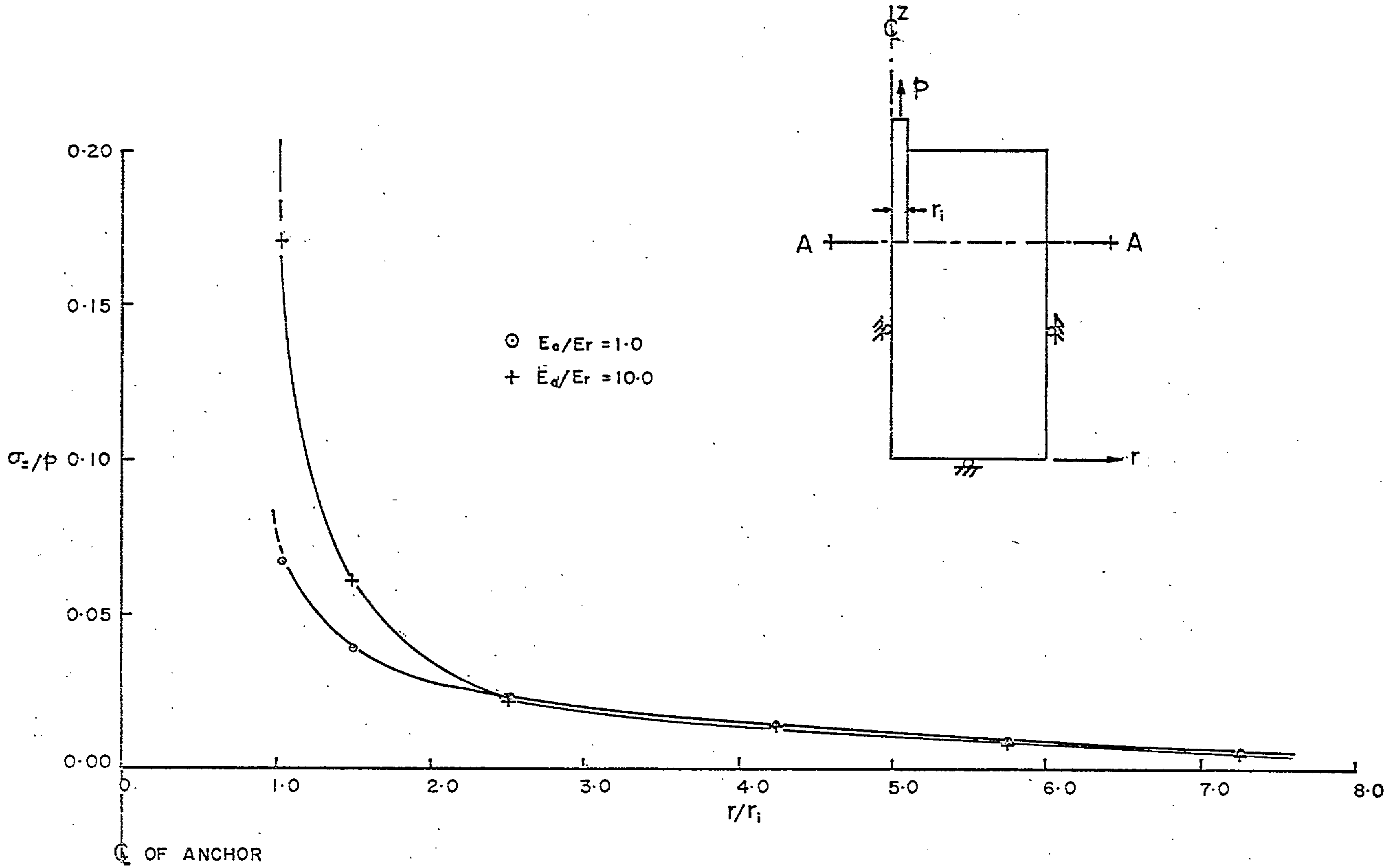
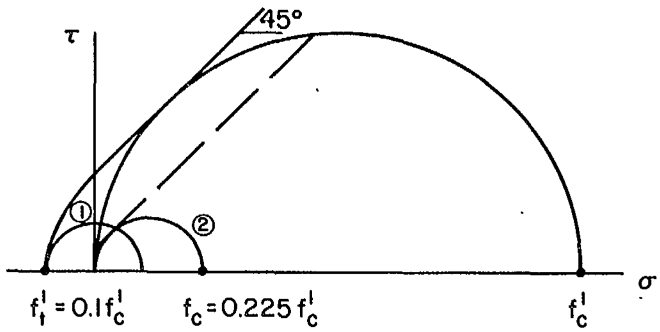


Figure 5. Variation of tensile stress on a horizontal plane in the rock adjacent to the end of a tension anchor.

(a)



(b)

Figure 6. (a) Stresses on an interface element;
(b) Mohr diagram for concrete in the pile.