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SHEAR LIPS, ZERO ISOCLINICS, AND FRACTURE

L. P. TRUDEAU

PHYSICAL METALLURGY DIVISION

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SHEAR LIPS, ZERO ISOCLINICS, AND FRACTURE

by

L. P. Trudeau*

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ABSTRACT

Examination of "shear lip" contours formed in the process of fracture under tensile stress shows that they regularly start at 60 degrees to the flat part of the fracture. These well-defined contours might be expected to have a mechanical cause that predicts their form more closely than the common assertion that they are 45-degree shear fractures. It is suggested that the "shear lips" are actually zero isoclinic contours characterized by zero shear. This hypothesis is physically plausible because the oblique part of the fracture could occur with purely normal displacements in the same way as the flat part of the fracture is formed. An equation for the elastic zero-isoclinic contour is presented, and also experimental evidence, in support of the proposal, obtained from tensile tests and tests with birefringent plastic on steel crack-notch toughness specimens.

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Direction des mines

Rapport de recherches R 201

LÈVRES DE CISAILLEMENT, ISOCLINES ZÉRO, ET CASSURE

par

L. P. Trudeau*

RÉSUMÉ

L'étude des isoclines des "lèvres de cisaillement" formées par la cassure sous contrainte par traction indiquent qu'elles commencent d'ordinaire à un angle de 60° par rapport à la partie plate de la cassure. On pourrait penser que ces lignes bien définies sont le résultat d'une cause mécanique permettant de prévoir plus exactement leur forme que l'affirmation bien connue, à savoir, qu'il s'agit de cassures de cisaillement à 45°. L'auteur pense que les "lèvres de cisaillement" sont en fait des isoclines zéro caractérisés par un cisaillement nul. Cette hypothèse est physiquement plausible parce que la partie oblique de la cassure pourrait se présenter avec des déplacements absolument normaux, de la même façon que la partie plate de la cassure est formée. Il présente une équation de l'isocline zéro élastique, ainsi que des preuves obtenues par expérience à l'aide d'essais de traction et d'essais faits avec du plastique biréfringent afin de déterminer la résistance au criquage des échantillons d'acier.

* Chercheur scientifique, Section de la physique appliquée, Division de la métallurgie physique, Direction des mines, ministère de l'Énergie, des Mines et des Ressources, Ottawa, Canada.

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1. INTRODUCTION

On examining fractures produced under a wide range of conditions, from unnotched tensile test-pieces to crack-notch toughness specimens, one is struck by the regularity of the "shear lips" that form. This is particularly so if the fractures are sectioned and examined, because it is found that the "shear lips" have a well-defined contour and are not 45-degree shear fractures. This contour, in a number of cases examined, starts at 60 degrees to the flat part of the fracture and gradually bends over to a lower angle (see, for example, Figures 4, 5 and 6). One suspects that such regular and well-defined contours should have a well-defined mechanical cause. In the present report it is suggested that so-called "shear lips" are actually zero-isoclinic surfaces characterized by purely normal displacements and zero shear.

2. THEORY

Opening mode fractures will be considered where the load is perpendicular to the crack plane. If we examine first the usual Mode I equations used in fracture mechanics[1], we find that the stress perpendicular to the crack plane, σ_y, is:

$$\sigma_y = \frac{K_1}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \qquad (1)$$

and the shear stress referred to the x and y directions is:

$$\tau_{xy} = \frac{K_1}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \qquad (2)$$

where (r, θ) are, of course, polar coordinates from the tip of the crack.

Because of symmetry, the crack plane is a principal plane and thus is a zero-isoclinic surface with respect to the direction of loading. This fact is also evident from Equation 2, because $\sin\frac{\theta}{2}$ is zero. This is not the only zero-isoclinic direction in the plane of the specimen, however, because the shear stress is also zero at 60 degrees to the crack plane, where $\cos\frac{3\theta}{2}$ is zero. For the latter direction the local principal stresses are lined up with the primary symmetry directions, and substitution in Equation 1 shows that σ_y in the 60-degree direction is 30% higher than σ_y perpendicular to the x direction for the same "r" value. These zero isoclinics for the "one term" stress solution are shown in Figure 1.

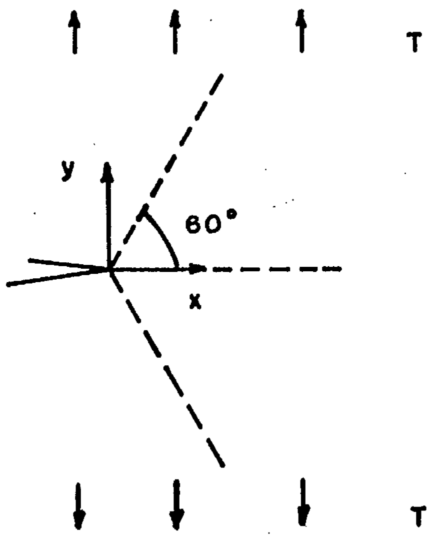

Figure 1. Zero isoclinics from "one term" stress solution are shown as dashed straight lines.

Photoelastic studies of cracks indicate that the zero isoclinics, which are shown in Figure 1 as straight lines at 60 degrees to the crack plane, actually bend over to a smaller angle as one moves away from the immediate crack-tip region. A more complete elastic solution is needed and this is provided in a convenient form by M. Rothman and D. S. Ross[2], using a complex potential method developed by A. C. Stevenson. For a wide two-dimensional plate containing a central crack of length 2a under a simple tension T at infinity, with the notation shown in Figure 2, the complete solution for the shear stress is:

$$\tau_{xy} = \frac{T r_3 a^2}{(r_1 r_2)^{3/2}} \sin\theta_3 \cos\tfrac{3}{2}(\theta_1 + \theta_2) \qquad (3)$$

I. N. Sneddon[3] used this notation earlier and his solution for the shear stress, using an H. M. Westergaard stress function, for the case of a crack in a plate opening under the action of a uniform pressure in the crack, is exactly the same as Equation 3 with T replaced by the pressure P.

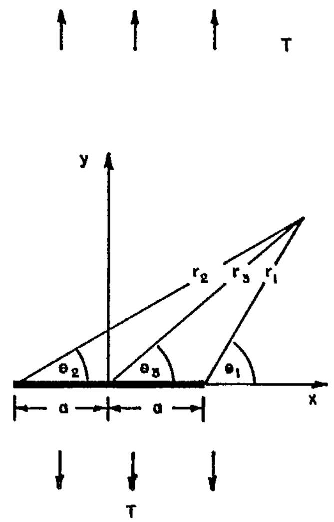

Figure 2. Notation used in complete solution for shear stress in Equation 3.

The zero isoclinics will occur where τ_{xy} is zero. In the present case, they are unique everywhere except at the singular point at the crack tip. The crack plane (y = o) is a zero isoclinic surface, both because of symmetry and because sin θ_3 is zero. The isoclinics at an angle to the crack plane will be defined by:

$$\cos \frac{3}{2} \ (\theta_1 + \theta_2) = o \qquad (4)$$

For this condition, $(\theta_1 + \theta_2) = \frac{\pi}{3}$ and, in terms of coordinates $(r_1, \ \theta_1)$ from the tip of the crack,

$$\tan \theta_2 = \tan \left(\frac{\pi}{3} - \theta_1\right) = \frac{r_1 \sin \theta_1}{2a + r_1 \cos \theta_1} \qquad (5)$$

With $\theta_1 = \theta$, and a reduced coordinate $r = \frac{r_1}{a}$, the equation for the zero isoclinic is:

$$\tan\left(\frac{\pi}{3} - \theta\right) = \frac{r \sin \theta}{2 + r \cos \theta} \qquad (6)$$

This curve is plotted in Figure 3.

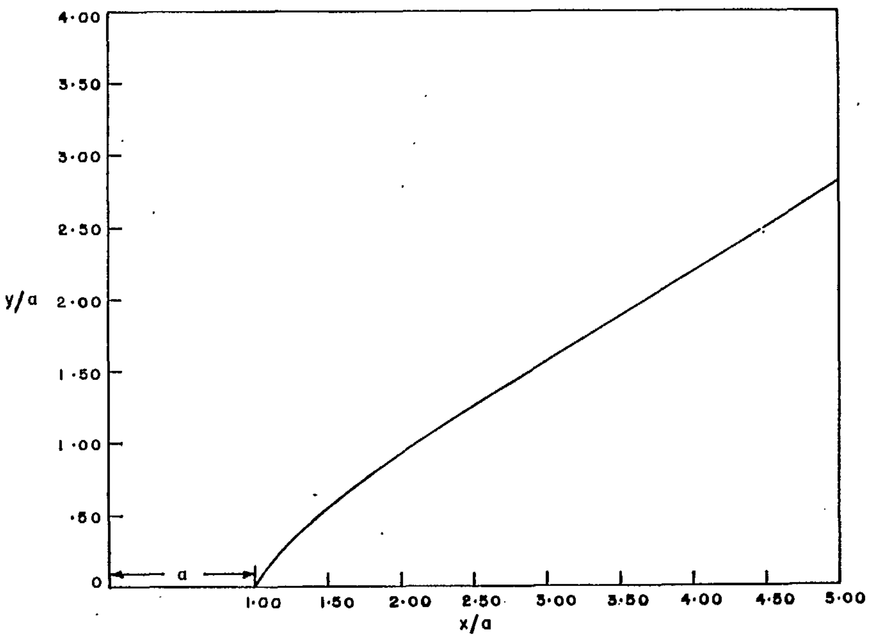

Figure 3. The zero-isoclinic contour for a crack in an infinite two-dimensional plate.

We might note, incidentally, that this result is independent of the elastic constants, such as Poisson's ratio and Young's modulus, in an isotropic material. Figure 3 shows that, while the isoclinic begins at 60 degrees to the crack plane, it gradually bends over, becoming 45 degrees at a distance into the body, along the crack plane, of about $\frac{1}{3}$ of the crack length "a". Further out in the stress field, this angle decreases till the limit value of 30 degrees is reached. Since the equation for the zero isoclinic is normalized with respect to the crack length "a", the isoclinic for short cracks bends over to smaller angles at shorter distances out into the stress field than is the case for longer cracks.

The resemblance of this zero isoclinic contour to the "shear lip" contour is striking. It is also plausible, on physical grounds, that the zero isoclinic should define the "shear lip" formed on fracture, because the separation could occur with purely normal displacements in the same fashion as the flat part of the fracture is formed. This is further reinforced when we recall that the σ_y stress is about 30% higher at 60 degrees to the crack plane than it is on the crack plane.

Consider, first, the case of the ordinary cup-cone fracture in a cylindrical tension test bar of a material assumed to be approximately isotropic and homogeneous. Sneddon(3) has compared stresses in the immediate vicinity of the crack tip for two- and three-dimensional cases, and has found that they differed only by a numerical factor. Further, he gives a table of τ_{zr} values for the three-dimensional case, and the zero isoclinic defined by these tabular values agrees well with the planar isoclinic for distances out into the stress field of 0.8 of the crack length "a". Evidently, the same isoclinic may be used for this three-dimensional case. Planar fracture may be assumed to start in the centre of the neck and spread outward, carrying a "crack-like" stress field with it if the material does not have extremely high ductility. The radial component of stress will build up from zero at the surface to the constrained value in the interior of the neck, and this "build up" distance can be expected to vary with the radius of curvature of the neck. Then, one might hypothesize that when the crack has progressed outward to the location where the radial stress starts to decrease, the stress situation for the remaining part of the fracture approaches a higher ductility plane stress condition and the remaining fracture occurs on the higher-stress zero-isoclinic contour at an angle to the crack plane. Figure 4 shows a central section of a tension-test-bar fracture in 200-grade maraging steel, and it may be noted that the "shear lip" contour is quite an accurate zero isoclinic when the radius of the flat fracture is equal to the crack length "a". The agreement with the elastic isoclinic is remarkable, and a possible explanation may be that the specimen did not stretch much plastically after the flat part of the fracture was completed.

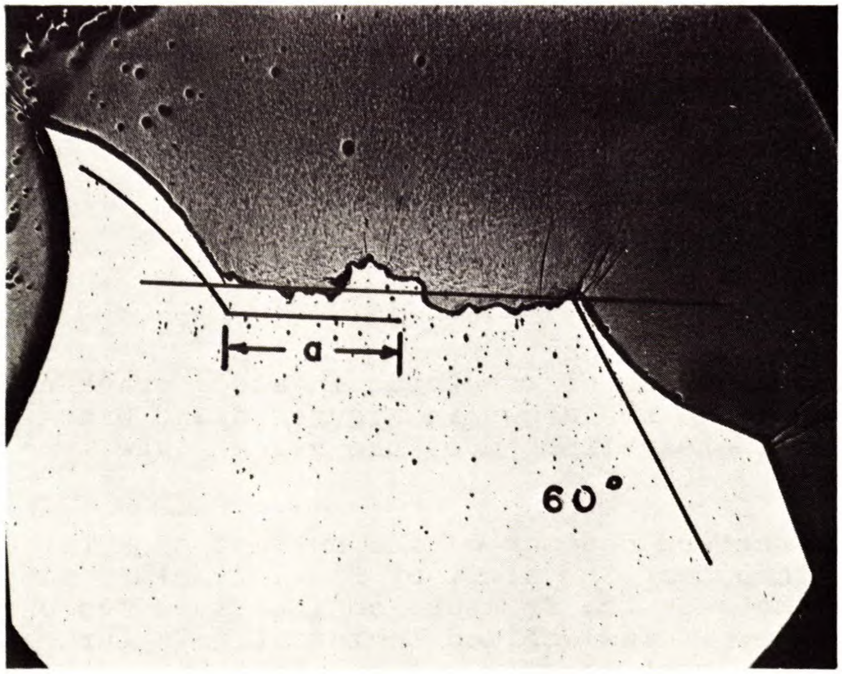

Figure 4. Cup-cone fracture of 0.505-inch tensile-test-bar of 200-grade maraging steel. The zero isoclinic reproduces the "shear lip" contour. Magnification 12.

P. W. Bridgman, in his book "Studies in Large Plastic Flow and Fracture" (McGraw-Hill Book Company, Inc., New York, 1952), gave data on specimens of steel that had been pulled to various reductions of area under pressure and then subsequently broken at atmospheric pressure. He noted that, for a tempered martensite steel with cup-cone fractures, the ratio of the area of flat fracture to total area was constant. From results for specimens 9-8-1, 9-8-5 and 9-8-13 on pp 298-299, and assuming a specimen diameter of 0.170 inch (p 44), one can calculate the "shear lip" widths. Then from a graph for this steel, of the ratio of neck radius to neck curvature versus strain, on p 27, one can find the radius of curvature of the neck at fracture. It turns out that the ratio of the "shear lip" width to the square root of the neck curvature is approximately constant. This suggests that the "build up" distance for the radial stress in the neck is directly proportional to the square root of the radius of curvature of the neck.

It is tempting to speculate that zero isoclinics may also play a role in the development of oblique flow layers and fractures that sometimes occur in tensile tests of thin specimens of material with a low work-hardening rate[4]. These layers occur at an angle of approximately 30 degrees to the horizontal in a vertical tensile specimen. Some small notch effects at the edge would be expected to give rise to isoclinics that would be predominantly at 30 degrees. The conventional explanation in terms of plasticity theory predicts an angle of 35°16', but measurements on the illustrations in Nadai's book[4] indicate 30 degrees.

3. EXPERIMENTS

Cross-sections of fractures in steel crack-notch toughness specimens are shown in Figures 5 and 6 and it may be noted that the "shear lips" meet the crack plane ($y = o$) at 60 degrees.

The fracture contour on the surface of a fairly thick cracked specimen near the start of final fracture also tends to be similar. Because the fracture contour shows up on the surface of the specimen, it was decided to investigate further, using a layer of birefringent plastic. The material used, "Type S Photostress Plastic", 0.071 inch thick, was cemented to the specimen with "Photostress Adhesive Type RCT", both supplied by the Budd Company, Phoenixville, Pa.

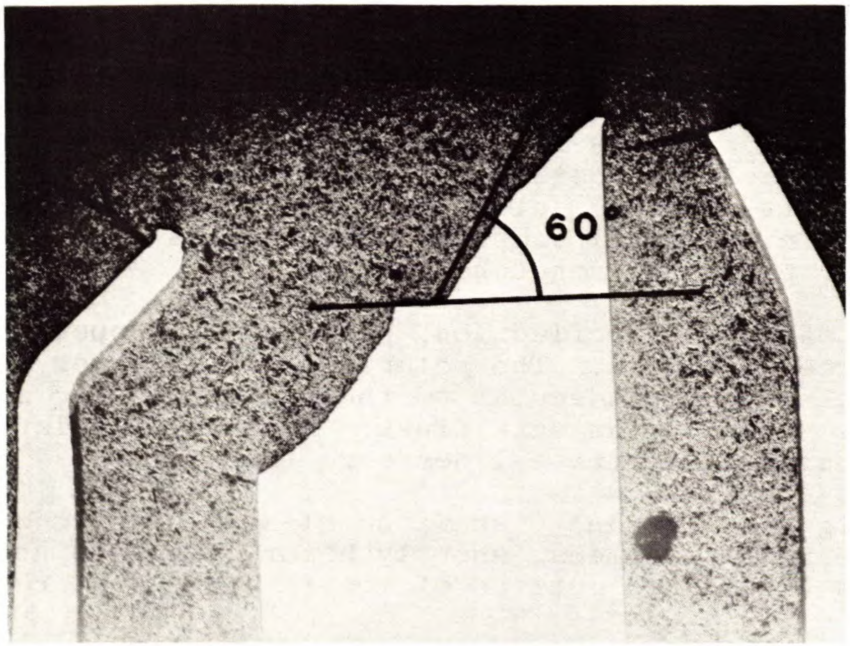

Figure 5. Section of fracture in $\frac{1}{4}$-inch-thick 250-grade maraging steel specimen.

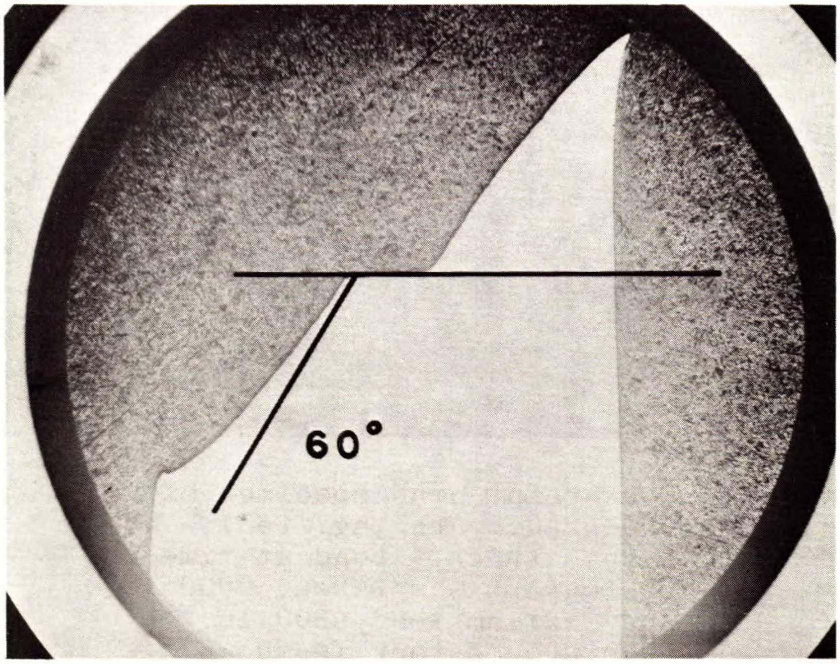

Figure 6. Section of fracture in $\frac{1}{2}$-inch-thick HP9-4-25 steel specimen.

It was thought that the strains at the crack tip were probably too intense for the plastic to follow, so the edge of the plastic was close to, but usually not at, this point. In some early tests a slot with a 1/8-inch root radius was machined in the plastic and this slot enveloped the pre-crack. It was found that isoclinics developed in the plastic on loading as a result of this machined notch and these tended to obscure the isoclinics in the metal sample. Best results were obtained with rectangular pieces of unnotched plastic.

A dark-field, reflection, plane polariscope, with a white light source, was used. The polariser and analyser were lined up parallel and perpendicular to the specimen axis, so the zero isoclinics showed up as dark lines. A circular polariscope was used to confirm that these lines were isoclinics.

Figures 7, 8 and 9 show the elastic-plastic zero isoclinics in 3 specimens, shortly before fracture, and superimposed is the dashed outline of the fracture that resulted.

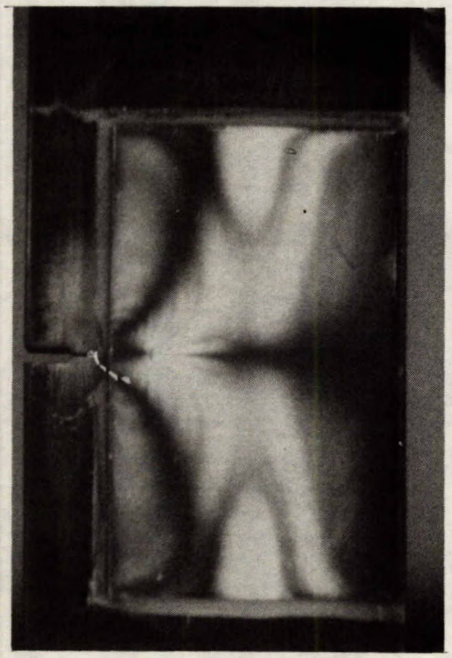

Figure 7. Four-point bend specimen of 250-grade maraging steel, $\frac{3}{8}$ inch thick. Load at time photograph was taken, 9000 lb, and maximum load, 9800 lb. Fracture contour is shown dashed. Magnification 1.

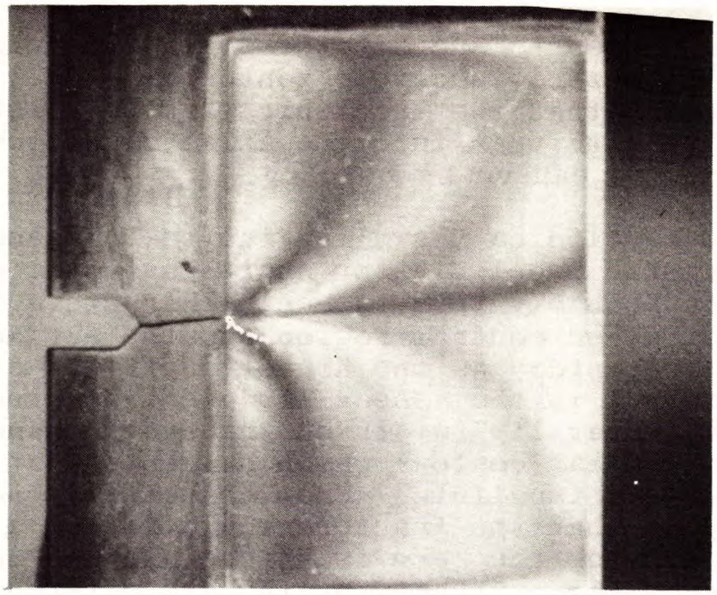

Figure 8. Single-edge-notch tension specimen of 250-grade maraging steel, $\frac{1}{2}$ inch thick. Load at time photograph was taken, 42,000 lb, and fracture load, 46,500 lb. Fracture contour is shown dashed. Magnification 1.

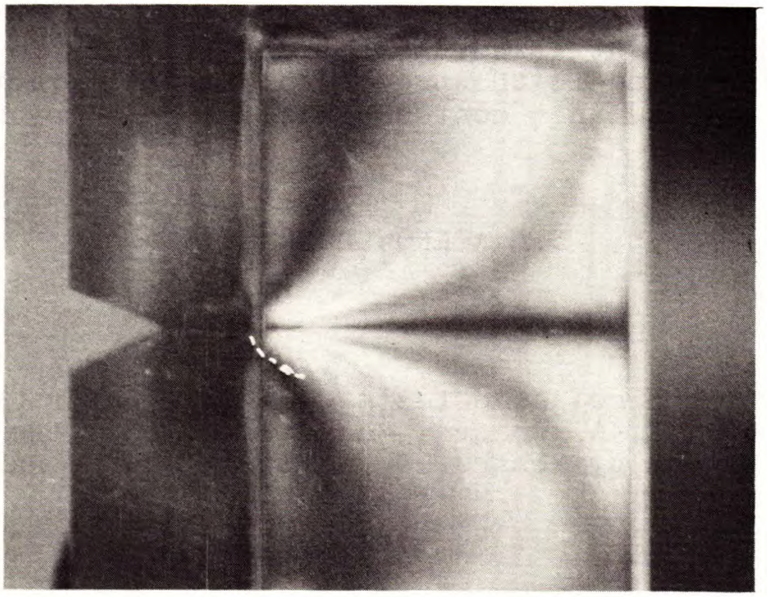

Figure 9. Single-edge-notch tension specimen of HP9-4-25 steel, 3/4 inch thick. Load at time photograph was taken, 107,000 lb, and fracture load, 114,000 lb. Fracture contour shown dashed is from a $\frac{1}{2}$-inch-thick specimen of same material that broke at a load equivalent to 140,000 lb. Magnification 1.

These results support the suggestion that fracture tends to follow a zero-isoclinic contour. It may be noted in these photographs that the isoclinics at some distance into the stress field tend to turn into the axial direction of the specimen. This is caused by bending.

The isoclinics shown in Figures 7, 8 and 9 only follow the elastic isoclinic for short distances out into the stress field. Probably this is because fracture then starts in the more constrained central region and the surface contour bends over more rapidly as the higher ductility of the "plane stressed" material in the "shear lip" is exhausted. The "build up" of "shear lip" width on the crack plane ($y = o$) again shows a similar contour, which suggests that it may also result from a zero isoclinic. From continuity considerations, one might expect that the fracture would follow a zero-isoclinic contour running from the zero-isoclinic visible on the surface to the zero-isoclinic surface forming the primary symmetry plane $y = o$. The sections of "shear lip" contours in Figures 5 and 6 are consistent with this conjecture.

4. CONCLUSION

Experimental and theoretical evidence indicates that "shear lips" formed in the process of tensile fracture are actually zero-isoclinic contours characterized by zero shear.

5. ACKNOWLEDGEMENTS

The photoelastic tests were done by Mr. F. W. Marsh assisted by Mr. K. A. Rocque. Assistance with the rest of the work was provided by Messrs. J. A. Ellis and G. Palen.

6. REFERENCES

1. See, for example, G. R. Irwin, "Fracture", in Encyclopaedia of Physics, Vol. VI (Springer Publishing Co. Inc., New York, 1958), p 551.

2. M. Rothman and D. S. Ross, "Stresses in Plates with Cracks and Notches", Engineering (London), 179, 175-180 (1955).

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4. A. Nadai, "Theory of Flow and Fracture of Solids", Vol. 1 (McGraw-Hill Book Company Inc., New York, 1950), pp 316-327.

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