SOURCE-LOCATION TECHNIQUES USING P-WAVE ARRIVALS

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CANADA AWARDS FOR EXCELLENCE 1986

SILVER AWARD IN TECHNOLOGY TRANSFER

CANMET Special Publication
SP 86-15E

Available in Canada through

Associated Bookstores
and other booksellers

or by mail from

Canadian Government Publishing Centre
Supply and Services Canada
Ottawa, Canada K1A 0S9

Catalogue No. M38-15-86-15E
Canada: $3.75

ISBN 0-660-12755-5
Other Countries: $4.50

Price subject to change without notice

Foreword

The Canada – Ontario – Industry Rockburst Project was initiated through a "Memorandum of Understanding" signed in September, 1985. The goals of the project are to develop improved Canadian capabilities in rockburst monitoring, analysis, and prevention. To accomplish these, a team of specialists is being assembled at the Mining Research Laboratories facilities at Elliot Lake, Ontario. The team will be available on a national basis to serve industry needs.

This report is one of the first outputs of the project. In the interests of technology transfer, it, and subsequent reports, will be published in a format which will encourage the broadest possible distribution.

John E. Udd
Director
Mining Research Laboratories

Avant-propos

Le projet Canada – Ontario – Industries sur les coups de toit a été entrepris grâce à un "mémoire d'entente" qui a été signé en septembre 1985. Ce projet a pour but de développer une expertise canadienne accrue dans les domaines du contrôle, de l'analyse et de la prévention des coups de toit. Pour atteindre cet objectif, une équipe de spécialistes a été rassemblée au Laboratoire d'Elliot Lake des Laboratoires de recherche minière. Cette équipe sera au service de l'industrie pour répondre à leurs besoins à travers le pays.

Le présent rapport est un des premiers comptes rendus du projet. Dans le but de promouvoir un transfert de technologie, ce rapport ainsi que ceux qui suivront seront publiés dans un format qui encouragera la plus vaste distribution possible.

John E. Udd
Directeur
Laboratoires de recherche minière

SOURCE-LOCATION TECHNIQUES USING P-WAVE ARRIVALS

*J. Niewiadomski**

Abstract

Short descriptions are given for each of four source-location techniques used in Ontario mines. For the two linear methods (least-square and USBM-ISA), a simple geometrical interpretation is outlined which may be used as a tool for fast evaluation of the conditioning of the linear system of equations created by these methods.

For the direct, non-linear methods (block and simplex methods), the forms of the objective functions are presented.

Keywords: Seismology; Location techniques.

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TECHNIQUES DE LOCALISATION DES FOYERS FONDÉES SUR L'ARRIVÉE DES ONDES P

*J. Niewiadomski**

Résumé

Le présent rapport est une courte description des quatre techniques de localisation des foyers utilisées dans des mines de l'Ontario. Les deux méthodes linéaires (méthode des moindres carrés et méthode USBM-ISA) font l'objet d'une interprétation géométrique simple qui peut être utilisée pour évaluer rapidement le conditionnement du système linéaire d'équations créé par ces méthodes.

En ce qui concerne les méthodes non linéaires directes (méthode du simplexe et méthode des blocs), on en présente les fonctions objectives résultantes.

Mots-clés: Séismologie; techniques de localisation.

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INTRODUCTION

In Ontario, eleven mines operate microseismic networks. All use the Electrolab MP-250 system with either velocity gauges or accelerometers as the sensors, with the number of channels ranging from 16 to 64.

Output from the Electrolab MP-250, based on P-wave arrivals, is used to calculate the co-ordinates of the source location. In most cases, this calculation is done using the least-square method, with unknown velocity. Groups of six geophones are usually used. In some mines, use is made of the least-square method, the block method, and the graphical method with input data from more than six geophones.

The four source-location techniques described in this report are:

- the least-square method;
- the source-location technique written for Mount Isa Mine (USBM-ISA);
- the block method; and
- the simplex method.

LEAST-SQUARE METHOD

The least-square method was presented by Blake et al. (1) of the U.S. Bureau of Mines (USBM). They created a system of linear algebraic equations in which the unknown values are three source co-ordinates (or also velocity). This system was achieved by simple mathematical operations on the set of basic equations. The basic equation for the i-th geophone can be written in the form:

$$d_i = [(X - a_i)^2 + (Y - b_i)^2 + (Z - c_i)^2]^{1/2} = V_i t_i \qquad \text{Eq 1}$$

where:

d_i is the distance measured from the seismic source to the i-th geophone;
X, Y, and Z are the source co-ordinates;
a_i, b_i, and c_i (i = 1,2,...) are co-ordinates of geophones;
V_i is the seismic wave velocity; and
t_i is the travel time of the impulse from a seismic source to the i-th geophone.

The system of linear algebraic equations obtained from Blake et al.(1) can be written thus:

$$a_{i1}X + a_{i2}Y + a_{i3} = b_i \qquad \text{Eq 2}$$

or $$a_{i1}X + a_{i2}Y + a_{i3}Z + a_{i4}V = b_i;\ i = 1,2,\ldots,n$$

The coefficients a_i and b_i are composed of the measured arrival times and geophone co-ordinates, for which the right-hand side of the equations for both cases (with and without velocity) are different. To solve Equation 2, at least five geophones are needed if the velocity is known, and at least six if the velocity is unknown.

Equation 2 can be solved using different methods, but the chosen method should provide stability, because we are dealing with an inverse problem. Generally all inverse problems are sensitive to errors in input data and to errors created during calculations; for example, as a result of rounding or truncation in computers, and the interpretation of P-waves as the other phases. To diminish the errors in source location, Blake et al. decided to use the least-square method. This method, when it is used with the number of equations equal to the number of unknowns, is sometimes called "the direct-solution method," but, from the mathematical point of view, there is no difference between the "direct-solution" and the "least-square" methods. In both cases a final system of linear equations is obtained by the multiplication of both sides of the equation by the transposed basic matrix. To explain this, let the basic system of Equation 2 be written in the matrix form:

$$\underset{(n,m)}{A} \cdot \underset{(m,1)}{\overline{X}} = \underset{(n,1)}{\overline{Y}};\ \overline{X} = \begin{bmatrix} X \\ Y \\ Z \\ \text{or also } V \end{bmatrix}, \overline{Y} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \text{or also } b_4 \end{bmatrix} \qquad \text{Eq 3}$$

where:

$\underline{A}$ is the matrix of the system (Eq. 2),
$\overline{Y}$ is the right-hand side vector, and
$\overline{X}$ is the vector we are looking for.

Below, in parentheses, are the dimensions of the proper matrices and vectors (number of rows and number of columns). The X and Y vectors are simply one column matrices. If both sides of Equation 3 are pre-multiplied by the transpose of the matrix A, A^T, then we obtain for n = m ("direct-solution") or n > m ("least-square") the system of equations in which the main matrix of the system is square (and also symmetric), and has the dimension equal to the number of unknowns, m:

$$\underset{(m,m)}{(A^TA)} \cdot \underset{(m,1)}{\overline{X}} = \underset{(m,1)}{\overline{Y}} \qquad \text{Eq 4}$$

In the case of inverse problems, matrix A is generally poorly conditioned. Conditioning of the matrix can be checked by calculating the singular values of the matrix A. If the relationship of the highest singular value to the smallest one is greater than some number N (usually N = 5 ÷ 10), then the matrix is ill-conditioned. When solving such a system of linear equations, even tiny changes in the input data can produce big errors in the solution as a result of the system instability. To check the matrix conditioning one can use the geometric interpretation of the system of linear equations, which gives some qualitative measure of the matrix conditioning and the possibility of comparing the two systems of equations. To explain this let us say we deal with the system:

$$\begin{bmatrix} S_{11}, S_{12}, \ldots, S_{1m} \\ S_{21}, S_{22}, \ldots, S_{2m} \\ \cdot \\ \cdot \\ \cdot \\ S_{m1}, \ldots\ldots, S_{mm} \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ \\ \\ \\ X_m \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ \\ \\ \\ Y_m \end{bmatrix} \qquad \text{Eq 5}$$

Every row of the matrix in Equation 5 may be treated as the set of the parameters that describes some representative hyperplane in an m-dimensioned space (in the case of three dimensions they are simply called "planes"). System 5 is expected to be stable, insensitive to errors, if the planes are inclined to one another under angles that are not small.

If the hyperplanes are close to being parallel, the system is evidently unstable. (In the case of two dimensions the same interpretation may be used to describe the system of two equations; instead of hyperplanes we then have two straight lines.) The angles α_{ik} between the 'i' and 'k' hyperplanes may be calculated in terms of their cosines:

$$\mathrm{COS}(\alpha_{ik}) = \frac{\sum_{j=1}^{m} S_{ij} * S_{kj}}{\left(\sum_{j=1}^{m} S_{ij}^2 * \sum_{j=1}^{m} S_{kj}^2\right)^{1/2}} \qquad \text{Eq 6}$$

For illustration let us consider six geophones with Y, Y, and Z co-ordinates as follows:

No.	X	Y	Z
1	(0,	0,	0)
2	(0,	0,	1000)
3	(1000,	0,	0)
4	(1000,	0,	1000)
5	(0,	1000,	0)
6	(0,	1000,	1000)

Let the velocity, which is used for calculation of time arrivals, be v = 6000 m/s. Let a seismic source be located inside the geophone array with co-ordinates X = 300, Y = 400, Z = 800. The final matrix that is created by the least-square program (transposed original matrix multiplied by original matrix) is:

(1): 2.98 E−01, −6.59 E−01, −6.39 E−02, 2.15 E−08,
(2): −6.59 E−01, 3.00 E−00, 9.65 E−01, −1.21 E−07,
(3): −6.39 E−02, 9.65 E−01, 3.82 E−01, −3.74 E−08,
(4): 2.15 E−08, −1.21 E−07, −3.74 E−08, 4.55 E−15.

The angles of intersection for the different hyperplanes (shown in parentheses) are as follows:

for (1 and 2): the angle is 16.75°,
for (1 and 3): the angle is 25.78°,
for (1 and 4): the angle is 18.33°,
for (2 and 3): the angle is 9.03°.

If the seismic source is located outside the geophone array with co-ordinates X = 3000, Y = 4000, Z = 8000, then the matrix is changed, as well as the angles. The matrix is as follows:

(1):	2.20 E−00,	−1.96 E−00,	1.92 E−01,	−4.19 E−08,
(2):	−1.96 E−00,	1.77 E−00,	−1.87 E−01,	3.73 E−08,
(3):	1.92 E−01,	−1.87 E−01,	2.47 E−02,	−3.73 E−09,
(4):	−4.19 E−08,	3.73 E−08,	−3.73 E−09,	7.99 E−16.

The angles between the specified hyperplanes are:

for (1 and 2): the angle is 0.55°,
for (1 and 3): the angle is 3.02°,
for (1 and 4): the angle is 0.04°,
for (2 and 3): the angle is 2.30°.

The angles in this case are smaller, because the seismic source is located outside the geophone array. It is noteworthy that the changes in the matrix elements seem to be relatively small, but the importance of these changes can be verified only when comparing the changes in the angles between the planes. This simplified method of matrix evaluation for the given source-location problem provides the opportunity to evaluate the sensitivity of the source location to errors. The examples indicate that the least-square method is reasonably reliable for sources located inside the geophone array, but that it is error-sensitive for sources outside the array.

USBM-ISA METHOD

The USBM-ISA method was developed for cases in which the potential source locations are in, or close to, a tabular orebody. The method was described in detail by Godson et al. (3).

The algorithm for source location is based on the USBM least-square method with some modifications. First, the importance of the first two hits are no longer treated as a basic assumption. Secondly, the normalization of equations present in the USBM algorithm is removed. The velocity is assumed to be known and constant. Godson et al. introduced a new variable, T_o, for the travel time from the source to the first geophone hit, which is treated as an additional unknown value.

The basic system of linear algebraic equations may then be written as follows:

$$a_{i1} X + a_{i2} Y + a_{i3} Z + a_{i4} T_o = b_i \; ; \; i = 2,3, ..., n \qquad \text{Eq 7}$$

where:

$$a_{i1} = 2 \cdot (a_i - a_{i-1})$$
$$a_{i2} = 2 \cdot (b_i - b_{i-1})$$
$$a_{i3} = 2 \cdot (c_i - c_{i-1})$$
$$a_{i4} = 2 \cdot (t_i - t_{i-1}) * V^2$$
$$b_i = (a_i^2 - a_{i-1}^2) + (b_i^2 - b_{i-1}^2) + (c_i^2 - c_{i-1}^2) - V^2 \cdot (t_i^2 - t_{i-1}^2).$$

In these formulae, a_i, b_i, and c_i stand for geophone co-ordinates, and t_i is the travel time from the source to the i-th geophone diminished by t_o. The system of linear algebraic equations (Eq 7), may be solved in many different methods, for example, by the least-square method.

This method has a better performance than the USBM least-square method for dealing with events outside the geophone array, especially when geophones used for source location are unfavorably distributed, and when the T_o variable is introduced. This variable is able partially to compensate potential errors in the time readings.

To compare the performance of the USBM-ISA method and the least-square method let us consider the same two source locations as before, one inside the geophone array with the co-ordinates X = 300, Y = 400, and Z = 800.

The final matrix of the system (i.e., the product of the matrix described directly by Equation 7, pre-multiplied by the same, but transposed, matrix) is, for the source inside the array:

$$\begin{array}{lrrrr} (1): & 1.2\ E+07, & -8.0\ E+06, & 4.0\ E+06, & 8.8\ E+09, \\ (2): & -8.0\ E+06, & 1.6\ E+07, & 0.0\ E+00, & 3.2\ E+09, \\ (3): & 4.0\ E+06, & 0.0\ E+00, & 4.0\ E+06, & -9.0\ E+09, \\ (4): & -8.8\ E+09, & 3.2\ E+09, & -9.0\ E+09, & 1.2\ E+14, \end{array} \qquad \text{Eq 8}$$

which gives the next set of angles between the hyperplanes:

for (1 and 2): the angle is 0.40°,
for (1 and 3): the angle is 0.04°,
for (1 and 4): the angle is 0.09°,
for (2 and 3): the angle is 0.22°,
for (2 and 4): the angle is 0.32°,
for (3 and 4): the angle is −0.05°.

The computed angles are extremely small, which means that direct use of the system for the source location is not recommended. The system first needs to be "scaled." When inspecting the matrix (Eq 8), one can see that the last column and the last row are large in comparison with the rest of the matrix elements. This inequality stems from the a(i,4) elements of the original matrix of the system, where we have v*v multipliers. If, in the original matrix, described in Equation 7, we use v rather than v*v, then instead of unknown value T_o, we will have the system of equations built for $T_o * v$, instead of T_o. Such a change will significantly improve the conditioning of the matrix. In its final form, improved by scaling, the matrix elements are:

$$\begin{array}{lrrrr} (1): & 1.2\ E+07, & -8.0\ E+06, & 4.0\ E+06, & 4.4\ E+05, \\ (2): & -8.0\ E+06, & 1.6\ E+07, & 0.0\ E+00, & 1.6\ E+05, \\ (3): & 4.0\ E+06, & 0.0\ E+00, & 4.0\ E+06, & -4.5\ E+05, \\ (4): & -8.8\ E+09, & 3.2\ E+09, & -9.0\ E+09, & 1.2\ E+09. \end{array}$$

Once again, this matrix may be understood as a set of hyperplanes with increased angles of intersection compared to the matrix without scalings:

for (1 and 2): the angle is 33.21°,
for (1 and 3): the angle is 41.35°,
for (1 and 4): the angle is 77.59°,
for (2 and 3): the angle is 71.35°,
for (2 and 4): the angle is 85.97°,
for (3 and 4): the angle is 87.38°.

Let us now, also with scaling, calculate the same angles for the case when the seismic source is located outside the geophone array (X = 3000, Y = 4000, Z = 8000). The final matrix is:

$$\begin{array}{lrrrr} (1): & 1.6\ E+07, & -8.0\ E+06, & 0.0\ E+00, & -1.3\ E+06, \\ (2): & -8.0\ E+06, & 1.2\ E+07, & -4.0\ E+06, & 8.9\ E+05, \\ (3): & 0.0\ E+00, & -4.0\ E+06, & 4.0\ E+06, & -1.8\ E+06, \\ (4): & -1.3\ E+06, & 8.9\ E+05, & -1.8\ E+06, & 1.6\ E+06, \end{array}$$

and the angles of the hyperplane intersections are:

for (1 and 2): the angle is 33.07°,
for (1 and 3): the angle is 71.19°,
for (1 and 4): the angle is 53.75°,
for (2 and 3): the angle is 42.48°,
for (2 and 4): the angle is 46.13°,
for (3 and 4): the angle is 37.11°.

Comparison of the angles for both source locations shows that the deterioration of the matrix for the source outside the geophone array is very small for the USBM-ISA method with scaling; far smaller than in the case of the least-square method.

Both methods have one useful common feature: they treat the source location problem as a linear one, allowing use of linear algebraic methods. On the other hand, algebraic operations (such as squaring and subtracting) involved in the derivation of the linear system of the equation used for source location enlarge the influence of errors present in the input data, often creating an extremely unstable system of equations.

BLOCK AND SIMPLEX METHODS

The block and simplex methods are based on another approach in that they look for the minimum of a non-linear function, which, in optimization theory, is called the objective function. The objective function for the aims of source-location technique may be easily created from readings of arrival times from the geophone network. Let t_i be the arrival time for the i-th geophone; a_i, b_i, and c_i, the i-th geophone co-ordinates; X, Y, and Z the co-ordinates of the trial point; and F(X,Y,Z) the objective function, then:

$$F(X,Y,Z) = \sum_{i=1}^{n} \left[(t_i - t_1) - (t_i^c - t_1^c)\right]^2 \qquad \text{Eq 9}$$

where,

$$t_i^c = \left\{(X-a_i)^2 + (Y-b_i)^2 + (Z-c_i)^2\right\}^{1/2}/V_i$$

and V_i is the average velocity from the source to the i-th geophone. If the objective function is calculated with Equation 9, the number of geophones used should be at least four.

In the block method the program calculates the value of objective function at the vertices of a cube and at the centre of its sides around a given point, with co-ordinates X, Y, Z. In the next step the cube is transferred to the next location, one for which the objective function is smaller. The dimension of the cube may be changed, but its shape is not changed. The creation of a very narrow valley by the objective function may cause some difficulties, especially when the dimension of the cube is too big in comparison with the area occupied by the miniumum of the objective function.

This problem can be solved by using a simplex instead of a cube, as is done in the simplex method. The shape of the simplex automatically accommodates to the shape of the valleys of the objective function. This flexibility of the simplex is the main advantage of the simplex method. In both cases the objective function may be such as described by Equation 9, or may be built in a different way, with introduction of another variable, T_o (see the USBM-ISA method).

The alternative objective function may then be as follows:

$$F(X,Y,Z) = \sum_{i=1}^{n} \left[(t_i - t_1 + T_o) - \sqrt{(X-a_i)^2 + (Y-b_i)^2 + (Z-c_i)^2/V_i} \right]^2 \qquad \text{Eq 10}$$

where:

t_i is the arrival time to the i-th geophone,
V_i is the average velocity from the source to the i-th geophone, and
T_o is the time needed for the signal to travel from the source to the first geophone.

This value is not known at the beginning of the calculation and may be included into the algorithm as a true unknown, making the number of unknowns equal four. Alternatively, it may first be assumed as equal to 0, then be enlarged with a given increment, and finally be chosen as a number, which gives the lower value for the objective function. Such a procedure makes the source location a little longer, but in this case only three geophones are generally needed for source location. The co-ordinates of the source are calculated at the same time as the parameter T_o. In both cases the velocity correction can be made by comparing calculated and measured time arrivals.

Some graphical examples of the objective functions are shown in Figures 1 and 2. To show this function properly one would need a four-dimensional space: X, Y, Z co-ordinates and fourth dimension for F(X,Y,2). To avoid complications, we may keep one of the coordinates constant, for example Z, and then we are able to use the three-dimensional space: X,Y,F(X,Y).

Figure 3 shows the objective function calculated with Equation 9 for the X and Y co-ordinates in the range 0 to 1000, and for Z = 800 (constant), when the seismic source was located at the point with co-ordinates X = 300, Y = 400, Z = 800 (the geophone co-ordinates are specified in the tables shown on Figs. 1 and 2). This is a theoretical example, and the arrival times are calculated without errors, so the function F(X,Y,Z = 800) reaches its minimum at the point where the source is located. In this case the minimum is equal to 0. The projection of this function on the X,Y plane (isolines) is shown on Figure 3 (marks on X and Y axes are done every 50 units).

If the objective function is calculated for the plane X,Y, which is "below" the source (see Fig. 2 where Z = 500, and the source co-ordinates are: X = 300, Y = 400, Z = 800), then the shape of the objective function is a little different, and its minimum does not equal 0. It is also not vertically under the real source (as shown on Fig. 4). This example illustrates that by forcing the solution to be on the given elevation (as in this case, when we calculated the function for different X and Y co-ordinates and keeping Z constant, but below the source) we may obtain wrong source co-ordinates. It follows from Figure 4 that the minimum of the objective function has the coordinates: X = 150, Y = 250 (the marks on the X and Y axes on Fig. 4 are spaced by 50 units), instead of X = 300 and Y = 400.

A useful algorithm of the simplex optimization technique is given by Nelder and Mead (4), and a similar one is implemented in the simplex method prepared by CANMET. Another of the possible implementations of the simplex method to the source-location technique is described by Gendzwill and Prugger (2).

The block and simplex methods are iterative methods because they look for the minimum of non-linear functions. The theory of optimization provides several different methods for finding the minimum of such functions. They are well known and FORTRAN codes are available. If we cannot supply the computer with correct input data, we have no reason to expect correctly calculated locations, even though we use a mathematically correct algorithm. Sometimes, if the input data are erroneous, the proper statistical approach may offer some remedy. However, in the case of most source locations, the amount of data is too scarce for efficient implementation of statistical methods.

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FIGURES

I	XG	YG	ZG	V(OLD)	V(NEW)	T(REAL)
1	0.	0.	1000.	20000.	20000.	0.00000
2	0.	1000.	1000.	20000.	20000.	0.00808
3	1000.	0.	1000.	20000.	20000.	0.01461
4	0.	0.	0.	20000.	20000.	0.02024
5	0.	1000.	0.	20000.	20000.	0.02528
6	1000.	0.	0.	20000.	20000.	0.02986

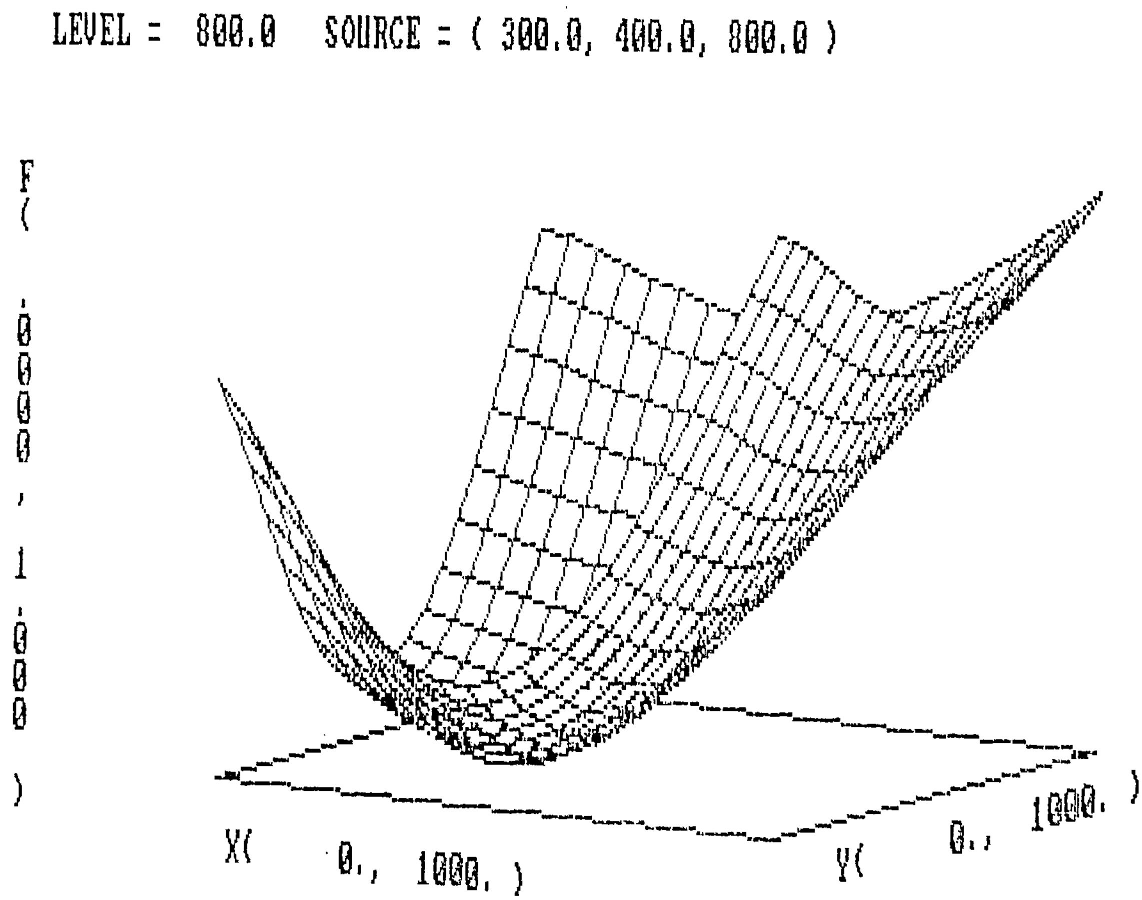

Fig. 1 – The objective function F(X,Y,Z = 800), (Eq 9)

I	XG	YG	ZG	V(OLD)	V(NEW)	T(REAL)
1	0.	0.	1000.	20000.	20000.	0.00000
2	0.	1000.	1000.	20000.	20000.	0.00808
3	1000.	0.	1000.	20000.	20000.	0.01461
4	0.	0.	0.	20000.	20000.	0.02024
5	0.	1000.	0.	20000.	20000.	0.02528
6	1000.	0.	0.	20000.	20000.	0.02986

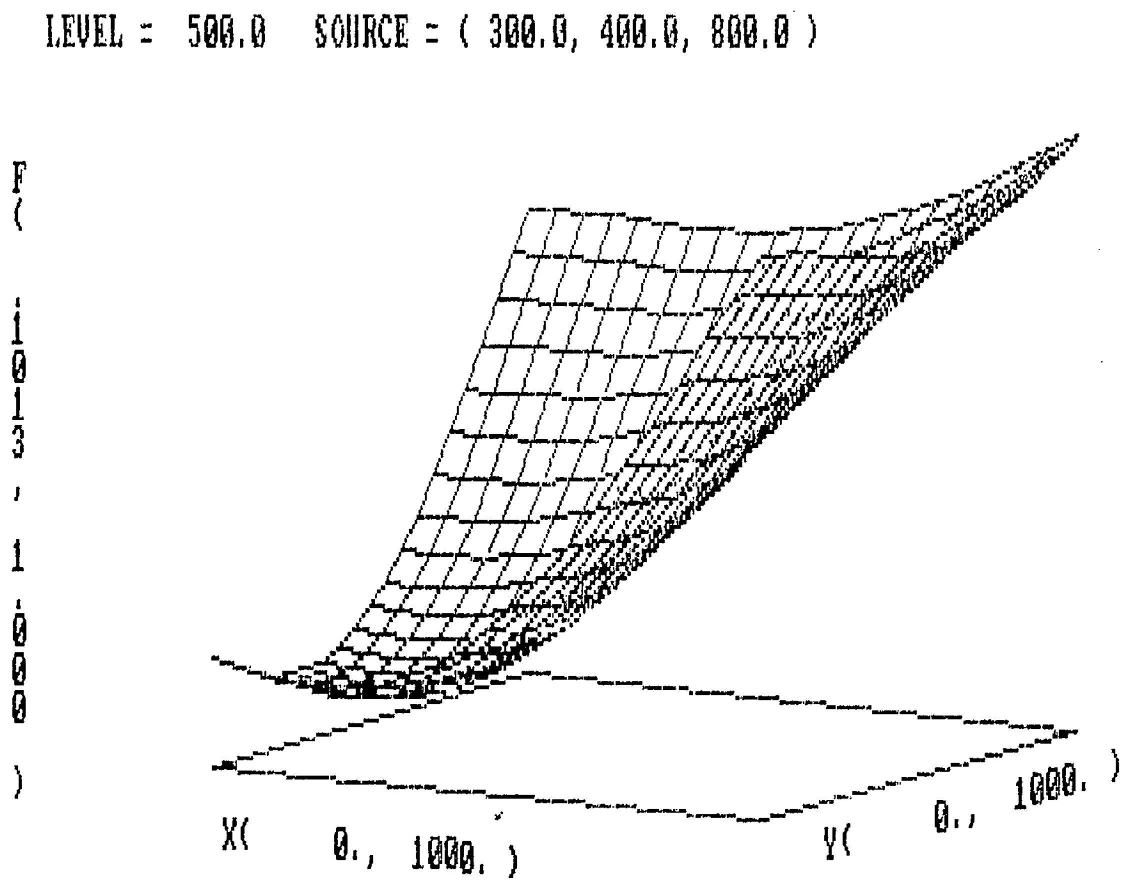

Fig. 2 – The objective function F(X,Y,Z = 500), (Eq 9)

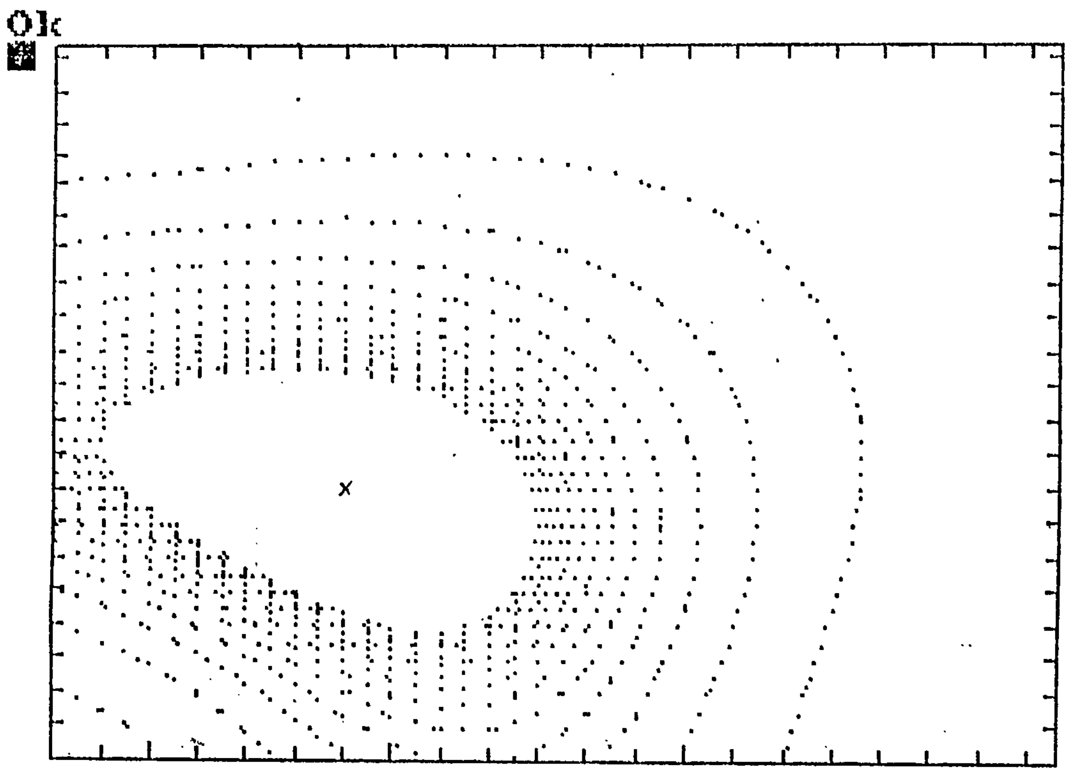

Fig. 3 – The isolines of the objective function from Figure 1

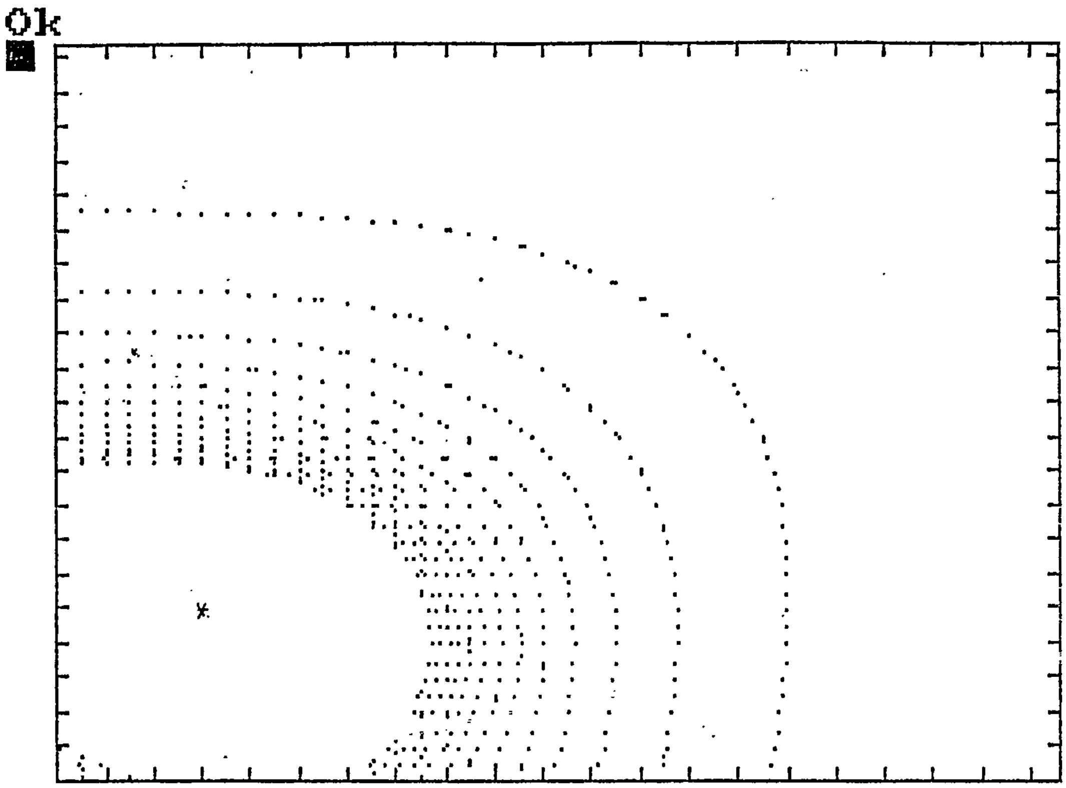

Fig. 4 – The isolines of the objective function from Figure 2