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SPOC

Simulated Processing of Ore and Coal

Chapter 7.3 RTD and MIXERS Computer Programs

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The SPOC Manual

Chapter 7.3 RTD and MIXERS Computer Programs

RTD and MIXERS — Computer Programs for Residence Time Determination of Process Units by Tracer Experiments

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Editor: D. Laguitton

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THE SPOC MANUAL

The **SPOC*** manual consists of eighteen chapters, published separately. Their numbers and short titles are as follows:

1. Summary
2. Sampling Methodology
2.1 SAMBA Computer Program
2.2 Grinding Circuit Sampling
3. Material Balance
3.1 BILMAT Computer Program
3.2 MATBAL Computer Program
4. Modelling and Simulation
4.1 Industrial Ball Mill Modelling
5. Unit Models: Part A
5.1 Unit Models: Part B
5.2 Unit Models: Part C
6. Flowsheet Simulators
7. Model Calibration
7.1 STAMP Computer Program
7.2 FINDBS Computer Program
7.3 RTD and MIXERS Computer Programs
8. Miscellaneous Computer Programs

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*S**imulated **P**rocessing of **O**re and **C**oal

FOREWORD

High energy costs and depleting ore reserves combine to make process evaluation and optimization a challenging goal in the 80's. The spectacular growth of computer technology in the same period has resulted in widely available computing power that can be distributed to the most remote mineral processing operations. The SPOC project, initiated at CANMET in 1980, has undertaken to provide Canadian industry with a coherent methodology for process evaluation and optimization assisted by computers. The SPOC Manual constitutes the written base of this methodology and covers most aspects of steady-state process evaluation and simulation. It is expected to facilitate industrial initiatives in data collection and model upgrading.

Creating a manual covering multidisciplinary topics and involving contributions from groups in universities, industry and government is a complex endeavour. The reader will undoubtedly notice some heterogeneities resulting from the necessary compromise between ideals and realistic objectives or, more simply, from oversight. Critiques to improve future editions are welcomed.

D. Laguitton
SPOC Project Leader
Canada Centre for Mineral and Energy Technology

AVANT-PROPOS

La croissance des coûts de l'énergie et l'appauvrissement des gisements ont fait de l'évaluation et de l'optimisation des procédés un défi des années 80 au moment même où s'effectuait la dissémination de l'informatique jusqu'aux concentrateurs les plus isolés. Le projet SPOC, a été lancé en 1980 au CANMET, en vue de développer pour l'industrie canadienne, une méthodologie d'application de l'informatique à l'évaluation et à l'optimisation des procédés minéralurgiques. Le Manuel SPOC constitue la documentation écrite de cette méthodologie et en couvre les différents éléments. Les retombées devraient en être une vague nouvelle d'échantillonnages et d'amélioration de modèles.

La rédaction d'un ouvrage couvrant différentes disciplines et rassemblant des contributions de groupes aussi divers que les universités, l'industrie et le gouvernement est une tâche complexe. Le lecteur notera sans aucun doute des ambiguïtés ou contradictions qui ont pu résulter de la diversité des sources, de la traduction ou tout simplement d'erreurs. La critique constructive est encouragée afin de parvenir au format et au contenu de la meilleure qualité possible.

D. Laguitton
Chef du projet SPOC,
Centre canadien de la technologie des minéraux et de l'énergie

ABSTRACT

The transport properties of material through various ore and coal processing units are important factors which control the performance of those units. This manual describes the basic tools for determining the flow pattern for a piece of equipment. The residence time distribution is defined, as well as three approaches to represent it. Then, several experimental methods based on tracers are presented and compared. The FORTRAN programs used to process the tracer data are fully documented, as are the mathematics on which they are based. All the methods and programs are illustrated with actual data from industrial grinding and flotation circuits.

This manual is directed to plant process engineers. All the necessary definitions are given, and only limited mathematical ability is required to apply the methods and use the programs. For those more familiar with process modelling, extended appendices give details of the mathematics. This should allow ongoing improvements and modifications to the packages as well as independent programming of the methods for users who want to have their own program in a language other than FORTRAN.

RÉSUMÉ

Les caractéristiques de l'écoulement des matériaux dans les unités de traitement des minerais et des charbons affectent de façon importante l'efficacité de ces unités. Ce chapitre décrit une méthodologie de base pour déterminer les caractéristiques de l'écoulement d'une unité de traitement. On définit la distribution du temps de séjour ainsi que trois différentes méthodes pour la représenter. Par la suite, on présente et compare plusieurs méthodes expérimentales utilisant des traceurs. Les programmes en FORTRAN utilisés pour traiter l'information par traceur, ainsi que les mathématiques nécessaires, sont présentés avec une documentation détaillée. On illustre toutes les méthodes et tous les programmes à l'aide de données provenant de circuits industriels de broyage et de flottation.

Ce manuel s'adresse aux ingénieurs d'usine. Toutes les définitions nécessaires sont fournies et seules des connaissances limitées en mathématiques sont requises pour mettre en application les méthodes et utiliser les programmes. Pour les familiers de la mise en modèle de procédé, on explique en annexe les détails des mathématiques. Ceci devrait permettre d'apporter des améliorations et des modifications au logiciel, ainsi que la programmation individuelle des méthodes pour les utilisateurs qui désirent avoir leur programme écrit dans un autre langage que le FORTRAN.

ACKNOWLEDGEMENTS

The SPOC project has benefited from such a wide range of contributions throughout the industry, the university, and the government sectors that a nominal acknowledgement would be bound to make unfair omissions. The main groups that contributed are: the various contractors who completed project elements; the Industrial Steering Committee members who met seven times to provide advice to the project leader; the various users of project documents and software who provided feedback on their experience; the CANMET Mineral Sciences Laboratories staff members who handled the considerable in-house task of software development, maintenance, and documentation; the EMR Computer Science Centre staff who were instrumental in some software development; and the CANMET Publications Section. Inasmuch as in a snow storm, every flake is responsible, their contributions are acknowledged.

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1. CONCEPT OF RESIDENCE TIME DISTRIBUTION

In the mineral industry, the transport of material through process units such as comminution machines (1), flotation (2), or lixiviation cells and thickeners affects the efficiency of the transformation that takes place in the unit. Two idealized flow patterns are generally considered: plug flow and perfectly mixed flow (to be defined in Section 1.1). However, real processes never exhibit these behaviours, rather they exhibit a combination of plug flow, mixing, channeling, stagnant zones, and short-circuiting. It is important to have a good model of the actual flow pattern occurring within a process unit in order to predict the characteristics of the product delivered by the unit. This manual describes the residence time distribution determination in *non-ideal* flows and proposes four techniques to represent it by models.

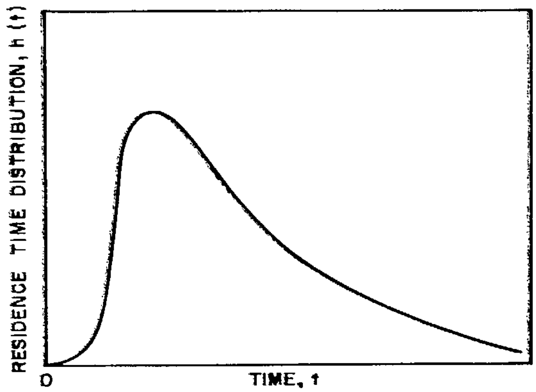

Fig. 1 – Typical residence time distribution

1.1 DEFINITION OF RESIDENCE TIME DISTRIBUTION

The various elements of material which enter a unit can follow very different paths to the discharge. As a result, different ore particles (or chemical elements) can reside for different lengths of time in a given process unit. The distribution of these times for the stream of material leaving the unit is called the **residence time distribution** (commonly referred to as RTD) (3). By definition the RTD, h(t), is such that h(t)dt is the fraction of the feed which stays a time between t and t + dt in the process equipment. This distribution is normalized such that the area under the curve is unity:

$$\int_0^\infty h(t)dt = 1 \qquad \text{Eq 1}$$

Figure 1 gives a usual shape of h(t). The mean of this distribution (called the **mean residence time** τ) is physically related to the volume V (or weight) of material retained in the machine compared to the volumetric (or weight) throughput Q. So the following equation can be written:

$$\tau = \frac{V}{Q} = \int_0^\infty h(t)\, t\, dt \qquad \text{Eq 2}$$

It has been frequently observed that over the normal range of operating conditions for a piece of equipment, the RTD expressed in dimensionless time $\theta = t/\tau$ remains practically unchanged (see Reference 4 for grinding mills). It is therefore convenient to define a **dimensionless** RTD $H(\theta)$ by the following function:

$$H(\theta) = \tau h(t) \qquad \text{Eq 3}$$

the mean value of which is one.

The dispersion of the residence times around their mean value τ is related to the magnitude of the mixing and drag forces acting within the machine. It can be quantified by the standard deviation of the h(t) distribution or its **variance**:

$$\sigma^2 = \int_0^\infty h(t)\,(t-\tau)^2 dt \qquad \text{Eq 4}$$

For a dimensionless RTD, the dimensionless variance is equal to σ^2/τ^2 (5).

Two limiting conditions can be identified (3). If no back mixing occurs in the machine, all particles or fluid elements entering a time 0 will discharge at time τ, which leads to a zero value of σ^2. This behaviour is termed **plug flow**.

The other extreme is **perfect mixing**, which means that the mill contents are perfectly homogeneous and consequently the discharge has exactly the same composition as the contents. In this situation the standard deviation is equal to the mean residence time (i.e., $\sigma^2 = \tau^2$). Figure 2 shows these two extreme behaviours.

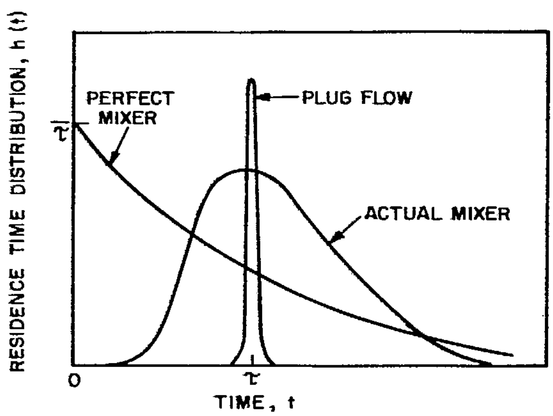

Fig. 2 – Idealized and actual RTD's

Actual flow properties lead to intermediate variances between the two limiting conditions. Generally, the variance is controlled by the importance of the mixing action relative to transport by convection. The dimensionless **Peclet number**, Pe, expresses this (3).

It is defined as vL/D, where v is the convective velocity of the material, L is the length of the device, and D is a dispersion coefficient similar to a diffusion coefficient for atomic or molecular mixing processes. When Pe is high, the flow behaviour is close to plug flow and the variance small; when Pe is low, the flow behaviour is close to perfect mixing (large D value) and the variance large.

The Peclet number is a function of the size of the equipment, of the pulp throughput and viscosity, and of the magnitude of the mixing forces (produced by the rotating ball load in a grinding mill or by the impeller and air bubbles in a flotation cell) (7,8).

1.2 MATHEMATICAL MODELLING OF FLOW PATTERNS

As explained above, the RTD characterizes the transport and mixing properties in the equipment. It relates the discharge concentration y(t) of any component to the feed concentration u(t).

$$u(t) \xrightarrow[\text{Flow model}]{} y(t)$$

A model of the flow properties of any process is a set of equations which predicts the discharge signal y(t) for a given feed signal u(t). Three types of models are used in this manual: the unit impulse response model, the perfect mixers-in-series model, and the time-discrete model.

1.2.1 Unit Impulse Response Model

A very useful type of input concentration signal u(t) is the **impulse function** $A\delta(t)$, where $\delta(t)$ is zero for any value of t except for t = 0, and A is the amplitude of the impulse (3). The concentration u(t), assumed to be zero before the impulse, increases instantaneously at the time of the impulse and returns instantaneously to zero after the impulsion. Figure 3a represents this type of signal. It can be approximated by a very short duration injection of an amount T of a component (a **tracer**) into the unit feed stream of flow rate Q. The representation of the approximation of the ideal impulse is given in Figure 3b.

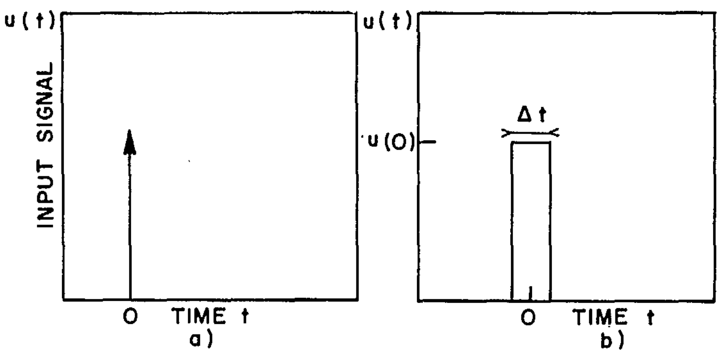

Fig. 3 – The impulse signal

The initial concentration u(0) is given by:

$$u(o) = \frac{T}{Q\Delta t} \qquad \text{Eq 5}$$

and the impulse magnitude by:

$$A = \frac{T}{Q} \qquad \text{Eq 6}$$

and, because the tracer is conserved, it follows that:

$$A = \int_0^\infty y(t)dt \qquad \text{Eq 7}$$

Using Equations 1 and 7, we have:

$$y(t) = Ah(t) \qquad \text{Eq 8}$$

When A is 1 (unit impulse), the concentration curve y(t) is the **unit impulse response** and is equivalent to the RTD model, h(t). For any input signal u(t), the output signal y(t) can be calculated by the convolution integral (see Appendix A):

$$y(t) = \int_{z=0}^{z=t} u(z)h(t-z)dz \qquad \text{Eq 9}$$

1.2.2 Perfect Mixers-in-Series Model

It is possible to approximate the response of a process unit using a perfect mixers-in-series model (3). The mixers can be identical or of different volumes.

Let us consider a single perfect mixer. In this case, the concentration of tracer in the mixer is equal to the concentration of tracer in the discharge y(t). Using the above definition and writing the mass conservation of any component during a time interval dt, we have:

$$Vdy = Qu(t)\,dt - Qy(t)\,dt$$

$$Qu(t)\,dt \rightarrow \boxed{V} \rightarrow Qy(t)\,dt \qquad \text{Eq 10}$$

A particular solution to this differential equation for an impulse input signal [$u(t) = A\delta(t)$] is:

$$y(t) = Ah(t) = \frac{A}{\tau} e^{-t/\tau} \qquad \text{Eq 11}$$

where $\tau = V/Q$ is the mean residence time. This concentration curve, y(t), is shown in Figure 2. With a supplementary plug flow component d, this equation becomes:

$$y(t) = \frac{A}{\tau} e^{-(t-d)/\tau} \qquad \text{Eq 12}$$

The solution can be derived for more complex situations with more than one perfect mixer (see Appendix B). When the number of mixers increases, the variance of the RTD decreases and the mixing properties tend towards a plug flow behaviour.

Other flow models including dead volumes, by-pass and recycle flows, have also been described (5,9,10), but are not used in the present package.

1.2.3 Time-Discrete Model

This model can be obtained by using a recursive equation. If we divide time into equal intervals Δt (a situation which necessarily occurs with digital computers), the input and output signals u and y can be represented by the following sequences:

$$\begin{aligned} &\textit{input signal} \quad \{u\} = \{u(1), u(2), ...u(i), ...u(N)\} \\ &\textit{output signal} \ \{y\} = \{y(1), y(2), ...y(i), ...y(N)\} \end{aligned} \qquad \text{Eq 13}$$

where u(i) is the signal after i time intervals and similarly for y(i). If the sequence {u} is known, the sequence {y} can be generated by the recursive equation:

$$\begin{aligned} &y(i) + a_1y(i-1) + a_2y(i-2) + ... + a_ny(i-n) \\ &= b_0u(i) + b_1u(i-1) + ... + b_mu(i-m) \end{aligned} \qquad \text{Eq 14}$$

The number of ***a*** parameters (n) is the order of the model and the number of ***b*** parameters (m+1) is smaller or equal to ***n***. When {u} is an impulse, the sequence {y} is a discretized representation of the RTD.

```
RRRRRR    TTTTTTT   DDDDDD
RRRRRRR   TTTTTTT   DDDDDDD
RRRRRRR   TTTTTTT   DDDDDDD
RRR  RRR    TTT     DDD  DDD
RRRRRRR     TTT     DDD  DDD
RRRRRR      TTT     DDD  DDD
RRRRRRR     TTT     DDD  DDD
RRR  RRR    TTT     DDD  DDD
RRR  RRR    TTT     DDDDDDD
RRR  RRR    TTT     DDDDDDD
RRR  RRR    TTT     DDDDDD

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  &&&    &&

MMMMMMMMMM   III  XXX XXX  EEEEEEE  RRRRRR    SSSSSS
MMMMMMMMMMM  III  XXX XXX  EEEEEEE  RRRRRRR  SSSSSSS
MMMMMMMMMMM  III  XXX XXX  EEEEEEE  RRRRRRR  SSSSSSS
MMM MMM MMM  III  XXX XXX  EEE      RRR RRR  SSS
MMM MMM MMM  III  XXXXXXX  EEEEEEE  RRRRRRR  SSSSSS
MMM MMM MMM  III   XXXXX   EEEEEEE  RRRRRR   SSSSSSS
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MMM MMM MMM  III  XXX XXX  EEE      RRR RRR      SSS
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2. RESIDENCE TIME DISTRIBUTION DETERMINATION METHODS

As presented in Section 1, the RTD function is an important factor in process simulation. First, it gives an indirect measurement of the hold-up weight by the mean residence time τ. Secondly, it gives a usable quantitative description of the transport through the process unit.

This section describes the experimental aspect of tracer measurements, and four data-processing techniques: two for the unit impulse model, one for the time-continuous model, and one for the time-discrete model.

2.1 EXPERIMENTAL TECHNIQUE

The tracer should ideally have the same mixing properties as the flowing material; it should not affect transport phenomena in the equipment; it should be easily detectable and should not react with other components in the flowing material. The following discussion summarizes experimental points of interest in tracer selection, tracer addition, and tracer sampling. Further details are given in Chapter 2 of the "SPOC Manual" (11).

2.1.1 The Tracer

The frequently-used simplifying assumption for the experimental technique is that all the components of the flowing material (i.e., slurry) have the same mixing behaviour independent of their other properties, i.e., particle size, particle specific gravity, solids or liquids, chemical composition.

This assumption is, for instance, usually valid for ball mills if the particles are sufficiently fine. Fine particles exhibit the same mean residence time as the water. The coarsest particles, however, may have as great as 10% longer mean residence times (12).

Since the liquid is generally simpler to trace than the solids, the above assumption allows the use of water-soluble tracers, i.e., tritiated water (4,13), LiCl, KBr, NaCl (14) or dyes (10). The solid can be traced by another solid component (15,16), by irradiation (5), or by fluorescein impregnation.

The total quantity of tracer to be injected can be determined from an evaluation of the water volume in the piece of equipment and the circuit, from the sensitivity threshold of the analytical procedure, and from the background of tracer present in the material to be traced. The quantity (500 times the low limit of concentration times the rough estimate of traced material hold-up) normally gives good results (14).

2.1.2 The Test

Different types of tracer injection can be used to generate the input tracer signal u(t) in the feed stream. The most commonly used is the *impulse injection* which can be performed directly in the feed or anywhere else in the circuit, providing that it produces a suitable u(t) signal in the feed.

The *step test* has also been used (13). It requires the continuous addition of tracer at a constant rate. This technique generally requires much more tracer than the impulse technique.

These types of tests are simple. However, in some circumstances they do not produce discharge signals containing sufficient information to accurately calculate the model parameters. In those cases, more general input signals must be used (see Section 3.1.1).

2.1.3 The Sampling Procedure

The sampling procedure depends on the circuit configuration, the mean retention time of the units, the type of test performed, and the method of computation available. The interaction between the calculation method and the type of experimental data available is discussed in Section 2.2.4. However, some general guidelines can be given here.

Generally, it is recommended that samples be taken from the feed and the discharge of the process unit in order to have measured values of both u(t) and y(t). The sampling time sequence must be adapted to the rate of variation of the signal. When the signal varies rapidly, a ten- or fifteen-second sampling interval may be necessary, whereas a one- or two-minute sampling interval suffices when the concentration varies only slowly. It is, therefore, important to know approximately the shape of the signal to be sampled.

This can be found by a rough calculation or a preliminary test. The sampling should continue for a period of time long enough to measure the tail of the output signal. This is about 4τ for an open circuit and 6τ for a closed circuit. It is not unusual to require 50 to 100 samples for a test.

For fast-varying signals, the time measurement must be done carefully to avoid large errors. Sometimes the tracer concentration varies considerably from sample to sample and it is important to guard against cross-contamination during analysis. Finally, it should be emphasized that the RTD **must** be measured under steady-state conditions.

2.2 TRACER DATA PROCESSING

When the process is operated in open-circuit, the measured concentration in the discharge following a perfect impulse gives the RTD, h(t) directly according to Equation 8. This function can be converted afterwards to a time-continuous or time-discrete model, in order to facilitate unit simulation.

However, the problem of RTD determination is usually complicated by the following factors:

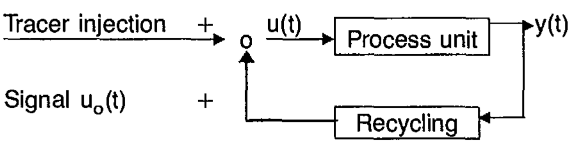
- the inaccuracy on the magnitude of the impulse; (It can be calculated from the area under the output signal curve (see Eq 7). However, if the signal has a long tail and if the analytical procedure is inaccurate at low concentrations, this is not always suitable.);
- the experimental errors in the sampling of the stream and in the sample manipulation and analysis;
- the natural disturbances which are always present in the flowing behaviour of a unit;
- the practical difficulty of generating a true impulse, especially in closed circuit, where recycling streams return some tracer to the feed stream.

The methods presented in this manual have been developed to cope with these difficulties. Least-squares modelling and data-filtering techniques are designed to resolve the first three types of difficulties listed above.

In the case of tracer recycling, two different approaches have been followed:

- mathematical methods based on the measurement of the whole input signal u(t);
- simplifying assumptions on the concentration of the recycled tracer.

A typical tracer recycling situation can be schematically described as follows:



where the true input signal is the sum of the tracer injection signal $u_o(t)$ (an impulse or anything else) plus the recycled tracer signal.

In some circumstances, the recycling stream can be assumed to be a pure delay (also called plug flow) and to return to the process a constant fraction u of the tracer present in the discharge. For this simplified situation only:

$$u(t) = u_o(t) + \alpha y(t-d) \qquad \text{Eq 15}$$

Let us consider the grinding circuit arrangements of Figure 4. For the situation depicted in Figures 4a and 4b, a pure recycling delay can be assumed because of the small volume of the sump compared to the hold-up in the mill. If a liquid tracer is used, the u coefficient is the fraction of mill-discharge water returning to the mill feed, i.e., the ratio of the cyclone underflow to the cyclone feed-water flow rates. If a solid tracer is used, it is the fraction of discharged solids returned to the mill feed. In Figures 4a and 4b, α is the ratio CLR/(1+CLR) where CLR is the circulating-load ratio (cyclone underflow solids/circuit feed solids).

For the Figure 4c arrangement, the assumptions of Equation 15 are not valid because the second ball mill introduces a non-plug flow element in the recycle. Similarly, in Figure 5, Equation 15 cannot be used when the RTD of the rougher is being measured since the scavenger cell does not behave as a plug flow.

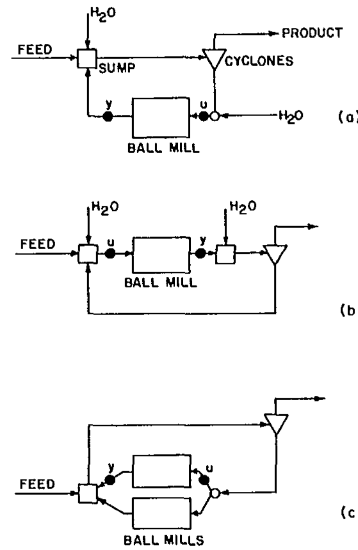


Fig. 4 – Grinding circuits with recycling tracer

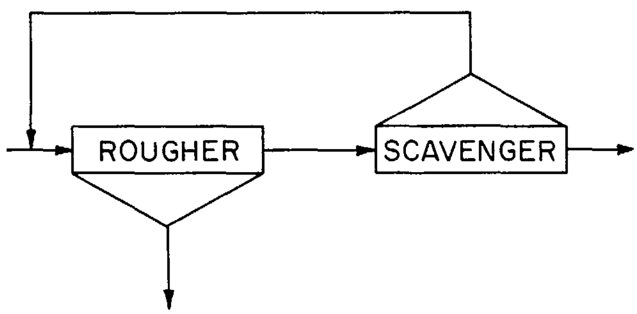


Fig. 5 – A flotation circuit

2.2.1 Determination of the Unit Impulse Response

In this approach, the mixing model is the RTD computed at discrete time intervals (see Section 1.2.1). The two proposed methods are based on the deconvolution of the integral given by Equation 9; that is, the computation of h(t) when y(t) is measured and u(t) measured or computable from mass balance data.

2.2.1.1 Austin method of correction for recycling

Principle

This method was initially proposed by Austin (17). The output signal y(t) is considered as the superimposition of the signals produced by the original tracer and the tracer that has recycled one, two, or more times. The amount of tracer which is recycled more than four to five times is generally negligible. Using the assumption that the proportion of tracer recycled is constant and simply delayed by a time d (simplifying the assumption of Section 2.2), the response y(t) is progressively corrected for the tracer recycled to finally generate the open-circuit response to the initial impulse. The mathematics of this method are given in Appendix C.

Data requirements

The data requirements are:

- The test must be an impulse.
- The recycling simplifying assumption of Section 2.2 must be valid, and the recycling coefficient α – as well as the recycling delay d – must be known*.
- y(t) must be sampled a sufficient number of times (but not necessarily equally spaced) up to its vanishing value.

Options

The options are:

- An estimate of the impulse magnitude (A) is useful to the program to shorten the calculations.
- When the flow rate through the equipment is measured, the program calculates the amplitude, A, of the impulse signal.
- The convergence criterion is: x% (relative error on A).

Calculation technique

From an initial estimate of the impulse magnitude, the open-circuit impulse response y(t) is calculated step by step. If the impulse magnitude calculated using Equation 7 is different from the initial estimate, the new value replaces the initial estimate and the procedure is iterated. The procedure stops when the impulse magnitude does not vary more than x% in two successive iterations. Then, h(t) is generated by dividing the y(t) obtained at the last iteration by the impulse magnitude at convergence.

2.2.1.2 Direct deconvolution method

Principle

The convolution integral (Eq 9) is discretized (6) as follows:

$$y_i = \sum_{j=0}^{i} u_j h_{i-j} \Delta t \qquad \text{Eq 16}$$

where Δt is the sampling interval and indices i and j correspond to the number of Δt time intervals elapsed. Since $u(t) = 0$ when $t \leq 0$, it is possible to calculate h step-by-step solving Equation 16 written for each i value from 0 to N, where N corresponds to a time such that h becomes negligible (18).

Data requirements

The data requirements are:

- The test requires an impulse.
- y(t) and u(t) must be measured accurately at the same time (but not necessarily equally spaced) until their values vanish.

Options

The options are the same as for the Austin method (see Section 2.2.1.1).

Calculation technique

The resolution of Equation 16 with respect to h is based on a straight-forward, step-by-step method (see Appendix E). Due to inaccuracies in the initial estimates of the impulse magnitude, it may occur that the area under h is not one. The impulse magnitude is changed accordingly and the calculation restarted.

2.2.2 Perfect Mixers-in-Series Model

Principle

A time-continuous model, based on the mixers-in-series representation, is selected prior to calculation. Then its parameters are determined by the minimization of the sum of the squared differences between the predicted output $\hat{y}(t)$ and the measured output y(t):

$$J = \Sigma\, [y(t) - \hat{y}(t)]^2 \qquad \text{Eq 17}$$

Data requirements

The data requirements are:

- The test can be an impulse or any other known tracer feed.
- If the test is an impulse, u(t) must be measured when the recycling assumptions of Equation 15 are not valid.
- If the test is not an impulse, u(t) must always be measured.
- When the test is an impulse and u(t) is not measured, the recycling delay d must be evaluated and the recycling coefficient α given an initial estimate.
- The number of measured y(t) values must be greater than the number of the model parameters to be evaluated. The more y(t) values, the better the parameters.

*When u(t) is also measured, Appendix D gives a method to estimate α and d.

- The model parameters must be given initial estimates.

Options
The options are:

- Four models are available;
  - n perfect mixers of equal values (n = 1 to 9),
  - 2 mixers of different volume,
  - 3 mixers of different volume,
  - 2 equal mixers plus one mixer of different volume,
  - (in all the above options a plug flow component is included).
- The impulse magnitude can be refined, or not refined, by the program.
- Open- or closed-circuit calculations can be performed.
- The minimization algorithm can be controlled.
- Several printout options are available.

Calculation technique
The squared residuals are minimized by the Powell algorithm (19,20) with respect to the model parameters (plug flow time, mean residence time of each perfect mixer, recycling coefficient α, impulse magnitude). The predicted y(t) is calculated by the convolution product of h(t) and u(t), h(t) being generated by the analytical expressions available for each model option (see Appendix B).

2.2.3 Time-Discrete Model

Principle
The method is based on the general discrete-time model expressed by Equation 14 whose parameters $\{a\}$ and $\{b\}$ are estimated by the minimization of the squared difference between y(t) measured and $\hat{y}(t)$ predicted by the model. Furthermore, the method filters the data, u(t) and y(t), to eliminate the correlated errors on the measurements. This procedure, named the generalized least-squares procedure (GLS method) (18,21,22), eliminates biases in the parameters $\{a\}$ and $\{b\}$ (see details in Appendix F). This method also provides standard deviations for the calculated model parameters.

Data requirements
The data requirements are:

- u(t) can have any form, but should be well defined by the sampling.
- u(t) and y(t) must be measured during a total time at least equal to the mean residence time.
- The duration of the plug flow component of the model must be known.

Options
The options are:

- the minimum and maximum number of $\{a\}$ and $\{b\}$ parameters.
- the convergence criterion for filtering (see calculation technique below).

Calculation technique
The calculation technique involves the minimization of the squared residuals $(y_i - \hat{y}_i)^2$ with respect to coefficients a and b. All possible combinations of numbers of a and b coefficients are tested to a user-defined maximum. The best model is then selected, either by choosing the lowest number of parameters producing an acceptable minimum or the highest number of parameters having an acceptable precision. Further details on this are given in Appendix G.

The calculation of the time-discrete model involves the use of a technique called data filtering, which itself is a very specialized topic. For the occasional user, a brief narrative on the subject will illustrate the principles of the model which are described in more detail in Appendix F. Natural process perturbations, as well as perturbations introduced in the tracer signal by sampling and sample analysis, can be considered as a noise superimposed on the actual signal. The part of this noise that introduces correlation in the data can be studied through the difference between the true value of the output signal, estimated by a first model application, and the measured value y_i.

If a relationship between the residuals $(y_i - \hat{y}_i)$, at time t_i, and those $(y_j - \hat{y}_j)$, at times t_j, is observed, it is used to recalculate a better estimate of the coefficients of the time-discrete model.

This procedure is called "filtering the data", i.e., removing internal correlations due to perturbations. It is repeated until the filter does not modify the squared residuals.

2.2.4 Selection and Comparison of the Four Methods

The data can be acquired with the intention of using a given method or, inversely, data may already be available and the best calculation method has to be selected.

The *availability* and *nature* of the input signal are the first criteria used to select a method.

If the input signal u(t) is **not** available, the Austin method (see Section 2.2.1.1) can be used. However, it is applicable only if: (i) the performed test is an impulse; (ii) the recycling assumption of Equation 15 is valid; and the recycling delay d and coefficient α are known. If α is not accurately known, the perfect-mixers-in-series method (see Section 2.2.2) is recommended instead of the Austin method.

If the input signal is available, one can use the direct deconvolution (see Section 2.2.1.2), mixers-in-series (see Section 2.2.2), and time-discrete (see Section 2.2.3) methods.

The various methods can also be used successively. For instance, since the Austin and direct deconvolution methods generate tables of RTD values, their result can in turn be modelled by the mixers-in-series or the time-discrete methods. The advantage of this two-step data processing is that an RTD function is easier to use than an RTD table (23). The time-discrete model results can also be converted into a mixers-in-series model if subsequent use requires it; i.e., kinetic model of ball mills (23).

Table 1 gives some qualitative characteristics of the four methods in order to compare their range of application and performance.

It is obvious that the accuracy of each method increases with the amount and accuracy of the available data. However, the sensitivity of the calculations to measurement errors also depends on the method. Due to error propagation, the direct deconvolution method is the most delicate method to use. The least sensitive one is the time-discrete method, since the filtering technique eliminates some correlated errors and provides an estimation of the results reliability.

Since the time-discrete and time-continuous mixers methods are both based on mathematical modelling, they require more user-intervention for the model selection and interpretation of results than the two other methods. In the time-continuous approach, it is the user's responsibility to test several model structures. In the time-discrete approach, the program can explore various orders of the model because the form of the equation is unique. For both methods, the user has to select the best model from among the tested ones (see Appendix G).

Another element of comparison between the methods is the amount of data needed. The two methods which produce h(t) directly require that the tracer concentrations be measured very frequently over a long period of time (up to the vanishing concentrations in the tail). The Austin method requires only the output signal; however, flow data are needed to determine the recycling coefficient. For the mixers and time-discrete methods, the sampling for tracer concentrations measurement can be performed over shorter periods, since they involve models.

The computer requirements vary depending on the method used. The time-continuous method uses a nonlinear programming procedure which requires significant CPU time. The other methods use analytical solutions which are generally very fast. The memory capacity required by the time-discrete routine increases rapidly when the model order and the number of data points increase.

Table 1 – Comparison of the four proposed methods

	Austin method	Direct method	Time-discrete method	Mixers method
Sensitivity to data inaccuracy	Sensitive	Very sensitive	Eliminates the effect of some noise	Sensitive
Results reliability estimation	Not provided	Not provided	Provided	Not provided
Utilization	Simple	Simple	Requires user's judgement	Requires user's judgement
Data requirement	Concentration y(t) must be measured for at least four mean residence times. Circuit flow rates must be known to correct for recycle	u(t) and y(t) must be sampled accurately especially at the beginning of the test (for an impulse)	u(t) and y(t) must be available. Both should show strong but well-defined variation	y(t) must be available and u(t) or circuit mass balance
Computer memory capacity	Low	Low	High	Low
CPU time	Low	Low	Very dependent on number of coefficients	High
Nature of program output	Discrete	Discrete	Time-discrete model parameters	Time-continuous model parameters

3. ILLUSTRATION OF THE METHODS

The flow properties of a slurry have been measured for a closed grinding circuit and a bank of flotation cells in the Brenda Mines concentrator. In both cases only the water is traced, assuming that the solids behave as the liquid phase. Various tests have been performed and the different calculation methods are illustrated.

3.1 GRINDING BALL-MILL FLOW PROPERTIES

3.1.1 Experimental Procedure

The configuration of the circuit containing the ball mill is depicted in Figure 6. A sampling campaign was performed on the circuit to determine the circuit mass balance. Two tracer tests were performed; one just preceding the sampling campaign, and one just following the campaign. During the two tracer tests and the sampling campaign, the grinding circuit was in a fairly stable steady-state with the exception of a sudden cut of the pump-house water near the end of the first residence time test. When this was noticed, the water flow was quickly restored and the circuit allowed to settle back to steady-state before starting the mass balance sampling campaign.

The first tracer test used eight kilograms of powdered crystalline NaCl added to the ball-mill feed. Ball-mill feed and discharge water samples were collected every fifteen seconds for eight minutes; then less frequently until thirty minutes had elapsed.

In the second tracer test, twenty kilograms of salt were added to the rod-mill discharge. Four minutes later, another twenty kilograms were added. Because of mixing in the pump box, this procedure produced a smoothly-varying, but distinctive, double-peaked feed signal to the ball mill. As in the first experiment, feed and discharge water samples were collected every fifteen seconds.

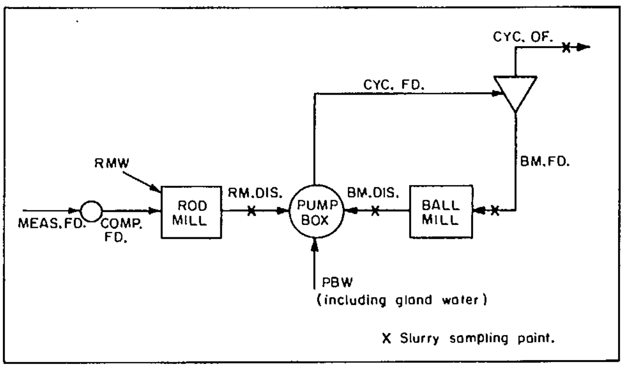


Fig. 6 – Brenda Mines grinding circuit

The tracer samples were analyzed immediately to minimize evaporation effects. A Varian 575 atomic absorption analyzer was used. In emission mode, atomic absorption is a very sensitive technique for the detection of Na^+. With care, the supernatant liquid could be analyzed directly from the sample bottles. This reduced sample preparation time significantly. Two hundred tracer samples were analyzed in four hours.

3.1.2 Residence Time Distribution Calculation

For the first tracer test an impulse was generated in the mill feed stream. Figure 7 shows the measured concentrations in the mill feed and in the mill discharge just after the impulse. The recycling delay was determined using the method described in Appendix D.

For the second test, two impulse injections were made in the rod-mill discharge. Figure 8 shows the resulting tracer concentration variations as measured in the ball-mill feed and ball-mill discharge. Since, in addition to those signals, a mass balance computation was performed to determine the recycling coefficient (20), all required data were available for use with any of the four RTD calculation methods.

An example data file for the RTD/MIXERS programs* and an example of the interpolation routine output (in full output mode) are given in Tables 2 and 3.

Using data from tracer experiment No. 1 (impulse), four program options are illustrated below.

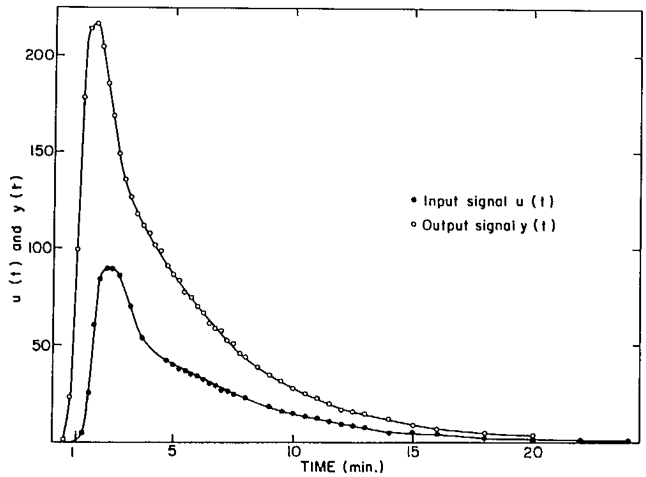


Fig. 7 – Input and output signals for the first ball-mill tracer test

*MIXERS is a stand-alone option of the RTD program to save memory (see Section 4).

Table 2 – Example data file for RTD or MIXERS packages

TRACER TEST #1 - INPUT SIGNAL	
43	0.25
0.00	2240.
0.25	0.0
0.50	0.0
0.75	0.0
1.00	0.0
	4.0
	25.0
	61.
	83.
	88.
	89.
	86.
3.25	70.
3.75	53.
4.75	42.
5.00	40.
5.25	38.
	37.
	35.
	34.
	32.
	30.
	29.
	27.
	26.
7.50	24.
8.00	23.
9.00	18.
9.50	16.
10.00	15.
	13.
	12.
	11.
	9.
	8.
	7.
14.00	5.
15.00	5.
	4.
18.00	2.
20.00	1.
23.00	0.0
26.00	0.0

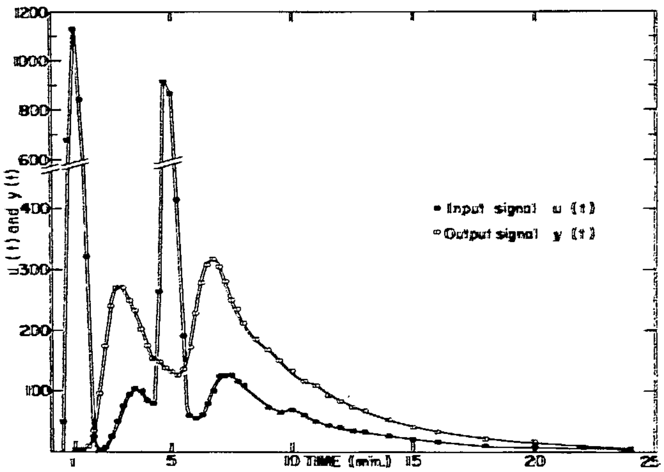


Fig. 8 – Input and output signals for the second ball-mill tracer test

Table 3 – Example of output from RTDINT subroutine for full output mode

```
ENTERED RTD INTERPOLATION ROUTINE
READING     TRACER TEST #1 - INPUT SIGNAL
0.          2240.
 .2500    0.
 .5000    0.
 .7500    0.
 1.000    0.
 1.250     4.000
 1.500     25.00
 1.750     61.00
 2.000     83.00
 2.250     88.00
 2.500     89.00
 2.750     86.00
 3.000     79.05      INTERPOLATED
 3.250     70.00
 3.500     61.22      INTERPOLATED
 3.750     53.00
 4.000     48.41      INTERPOLATED
 4.250     45.51      INTERPOLATED
 4.500     43.61      INTERPOLATED
 4.750     42.00
 5.000     40.00
 5.250     38.00
 5.500     37.00
 5.750     35.00
 6.000     34.00
 6.250     32.00
 6.500     30.00
 6.750     29.00
 7.000     27.00
 7.250     26.00
 7.500     24.00
 7.750     23.18      INTERPOLATED
 8.000     23.00
 8.250     21.94      INTERPOLATED
 8.500     20.67      INTERPOLATED
 8.750     19.31      INTERPOLATED
 9.000     18.00
 9.250     16.91      INTERPOLATED
 9.500     16.00
 9.750     15.50      INTERPOLATED
 10.00     15.00
 10.25     14.00      INTERPOLATED
 10.50     13.00
 10.75     12.44      INTERPOLATED
 11.00     12.00
 11.25     11.56      INTERPOLATED
 11.50     11.00
 11.75     10.00      INTERPOLATED
 12.00     9.000
 12.25     8.438      INTERPOLATED
 12.50     8.000
 12.75     7.500      INTERPOLATED
 13.00     7.000
 13.25     6.444      INTERPOLATED
 13.50     5.900      INTERPOLATED
 13.75     5.406      INTERPOLATED
 14.00     5.000
 14.25     4.930      INTERPOLATED
 14.50     4.938      INTERPOLATED
 14.75     4.977      INTERPOLATED
 15.00     5.000
 15.25     4.814      INTERPOLATED
 15.50     4.578      INTERPOLATED
 15.75     4.303      INTERPOLATED
 16.00     4.000
 16.25     3.736      INTERPOLATED
 16.50     3.472      INTERPOLATED
 16.75     3.209      INTERPOLATED
 17.00     2.950      INTERPOLATED
 17.25     2.697      INTERPOLATED
 17.50     2.453      INTERPOLATED
 17.75     2.220      INTERPOLATED
 18.00     2.000
 18.25     1.833      INTERPOLATED
 18.50     1.681      INTERPOLATED
 18.75     1.542      INTERPOLATED
 19.00     1.414      INTERPOLATED
 19.25     1.298      INTERPOLATED
 19.50     1.191      INTERPOLATED
 19.75     1.092      INTERPOLATED
 20.00     1.000
 20.25     .8895      INTERPOLATED
 20.50     .7830      INTERPOLATED
 20.75     .6809      INTERPOLATED
 21.00     .5833      INTERPOLATED
 21.25     .4907      INTERPOLATED
 21.50     .4031      INTERPOLATED
 21.75     .3210      INTERPOLATED
 22.00     .2444      INTERPOLATED
 22.25     .1738      INTERPOLATED
 22.50     .1094      INTERPOLATED
 22.75     .5135E-01  INTERPOLATED
 23.00    0.
 23.25    0.          INTERPOLATED
 23.50    0.          INTERPOLATED
 23.75    0.          INTERPOLATED
 24.00    0.          INTERPOLATED
 24.25    0.          INTERPOLATED
 24.50    0.          INTERPOLATED
 24.75    0.          INTERPOLATED
 25.00    0.          INTERPOLATED
 25.25    0.          INTERPOLATED
 25.50    0.          INTERPOLATED
 25.75    0.          INTERPOLATED
 26.00    0.
INTERPOLATION SUCCESSFULL
READ  43 RAW DATA POINTS
NOW HAVE 105 DATA POINTS IN TOTAL
```

3.1.2.1 Austin method, Sample run 1

The ball-mill discharge data for tracer concentrations are read, missing data are interpolated, and the short output mode warns the user that 51 data points have been expanded to 105 data points. The user returns to the full output mode and selects the Austin method, then enters the first approximations of search parameters. The iterative computation unfolds until a final table of results is displayed as shown in Table 4.

Table 4 – Example using AUSTIN subroutine on grinding mill test #1 data, sample run 1

```
RESIDENCE TIME DISTRIBUTION PROGRAM

COMMAND MENU

1 - SWITCH TO SHORT OUTPUT MODE
2 - SWITCH TO FULL OUTPUT MODE
3 - READ/INTERPOLATE DISCHARGE DATA ON TAPE8
4 - READ/INTERPOLATE FEED DATA ON TAPE7
5 - ANALYSIS USING AUSTIN TECHNIQUE PROGRAM
6 - ANALYSIS USING DIRECT DECONVOLUTION
7 - ANALYSIS USING GRAAIM LEAST SQUARES METHOD
8 - ANALYSIS USING MIXERS IN SERIES
9 - END PROGRAM


COMMAND NUMBER: 3
ENTERED RTD INTERPOLATION ROUTINE
READING    TRACER TEST #1 - OUTPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  51 RAW DATA POINTS
NOW HAVE 105 DATA POINTS IN TOTAL


COMMAND NUMBER: 2
FULL OUTPUT MODE


COMMAND NUMBER: 5
                         RECYCLE DELAY TIME: .9
                        RECYCLE COEFFICIENT: .426
       TRACER MEDIUM FLOW RATE THROUGH UNIT: .238
        QUANTITY OF TRACER ADDED AS IMPULSE: 0
                     FINISH ACCURACY FACTOR: 0


ENTERED AUSTIN METHOD ROUTINE


STARTING PARAMETERS
                         RECYCLE DELAY TIME:   1.00
                     SAMPLING TIME INTERVAL:   .250
                        RECYCLE COEFFICIENT:   .426
       TRACER MEDIUM FLOW RATE THROUGH UNIT:   .238
               INITIAL QUANTITY OF TRACER:   0.


ITERATION 1
       AREA UNDER CURVE:   480.
 AVERAGE RESIDENCE TIME:   2.28
        VARIANCE OF RTD:   .752


ITERATION 2
       AREA UNDER CURVE:   533.
 AVERAGE RESIDENCE TIME:   2.51
        VARIANCE OF RTD:   1.20
```

Table 4 (cont'd)

ITERATION 3

AREA UNDER CURVE:	553.
AVERAGE RESIDENCE TIME:	2.60
VARIANCE OF RTD:	1.46

ITERATION 4

AREA UNDER CURVE:	560.
AVERAGE RESIDENCE TIME:	2.64
VARIANCE OF RTD:	1.57

....

AUSTIN ANALYSIS FINAL RESULTS

TRACER RECOVERED:	134.0
AVERAGE RESIDENCE TIME:	2.651
VARIANCE OF RTD:	1.601

TIME	SAMPLE CONCENTRATION	CORRECTED CONCENTRATION	NON-DIMENSIONAL RTD TIME	NON-DIMENSIONAL RTD CONCENTRATION
0.00	0.	0.	0.000	0.000
.25	0.	0.	.094	0.000
.50	1.000	1.000	.189	.005
.75	23.00	23.00	.283	.108
1.00	99.00	99.00	.377	.466
1.25	178.0	178.0	.472	.838
1.50	214.0	214.0	.566	1.008
1.75	216.0	216.0	.660	1.017
2.00	204.0	204.0	.754	.961
2.25	185.0	185.0	.849	.871
2.50	168.0	167.9	.943	.791
2.75	148.0	147.1	1.037	.693
3.00	135.0	131.5	1.132	.619
3.25	126.0	117.3	1.226	.553
3.50	117.0	101.0	1.320	.476
3.75	111.0	86.54	1.415	.408
4.00	107.0	74.32	1.509	.350
4.25	101.0	61.14	1.603	.288
4.50	98.00	52.36	1.697	.247
4.75	90.00	40.08	1.792	.189
5.00	86.00	33.14	1.886	.156
5.25	83.00	28.28	1.980	.133
5.50	77.00	21.29	2.075	.100
5.75	74.00	18.00	2.169	.085
6.00	69.00	13.25	2.263	.062
6.25	66.00	10.93	2.358	.051
6.50	61.00	6.962	2.452	.033
6.75	58.00	5.325	2.546	.025
7.00	57.00	5.953	2.640	.028
7.25	52.00	2.730	2.735	.013
7.50	50.00	2.605	2.829	.012
7.75	45.00	0.	2.923	0.000

END OF AUSTIN METHOD ROUTINE

COMMAND NUMBER: 9

3.1.2.2 Direct deconvolution method, Sample run 2

The ball-mill feed and discharge data for tracer concentrations are used, interpolated as for the first sample run (see Section 3.1.2.1), and the iterative computation is performed in a format very similar to the previous example (see Table 5).

Table 5 – Example using DIRECT subroutine on grinding mill test #1 data, sample run 2

```
RESIDENCE TIME DISTRIBUTION PROGRAM

COMMAND MENU

1 - SWITCH TO SHORT OUTPUT MODE
2 - SWITCH TO FULL OUTPUT MODE
3 - READ/INTERPOLATE DISCHARGE DATA ON TAPE8
4 - READ/INTERPOLATE FEED DATA ON TAPE7
5 - ANALYSIS USING AUSTIN TECHNIQUE PROGRAM
6 - ANALYSIS USING DIRECT DECONVOLUTION
7 - ANALYSIS USING GRAAIM LEAST SQUARES METHOD
8 - ANALYSIS USING MIXERS IN SERIES
9 - END PROGRAM


COMMAND NUMBER: 3
ENTERED RTD INTERPOLATION ROUTINE
READING    TRACER TEST #1 - OUTPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  51 RAW DATA POINTS
NOW HAVE 105 DATA POINTS IN TOTAL


COMMAND NUMBER: 4
ENTERED RTD INTERPOLATION ROUTINE
READING    TRACER TEST #1 - INPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  43 RAW DATA POINTS
NOW HAVE 105 DATA POINTS IN TOTAL
COMMAND NUMBER: 2
FULL OUTPUT MODE
```

Table 5 (cont'd)

```
COMMAND NUMBER: 6
       TRACER MEDIUM FLOW RATE THROUGH UNIT: .238
        QUANTITY OF TRACER ADDED AS IMPULSE: O
                     FINISH ACCURACY FACTOR: O


ENTERED DIRECT METHOD ROUTINE


STARTING PARAMETERS
              SAMPLING TIME INTERVAL:    .250
TRACER MEDIUM FLOW RATE THROUGH UNIT:    .238
          INITIAL QUANTITY OF TRACER:  0.


ITERATION  1
     QUANTITY OF TRACER:   107.
 AVERAGE RESIDENCE TIME:   3.12
        VARIANCE OF RTD:   2.19


ITERATION  2
     QUANTITY OF TRACER:   121.
 AVERAGE RESIDENCE TIME:   2.82
        VARIANCE OF RTD:   1.69


ITERATION  3
     QUANTITY OF TRACER:   128.
 AVERAGE RESIDENCE TIME:   2.78
        VARIANCE OF RTD:   1.94


ITERATION  4
     QUANTITY OF TRACER:   131.
 AVERAGE RESIDENCE TIME:   2.76
        VARIANCE OF RTD:   2.08


............................................................................


DIRECT METHOD FINAL RESULTS


       TRACER RECOVERED:  131.9
 AVERAGE RESIDENCE TIME:  2.754
        VARIANCE OF RTD:  2.138
```

Table 5 (cont'd)

TIME	RECYCLE CONCENTRATION	DISCHARGE CONCENTRATION	CORRECTED CONCENTRATION	NON-DIMENSIONAL RTD TIME	CONCENTRATION
0.00	2216.	0.	0.	0.000	0.000
.25	0.	0.	0.	.091	0.000
.50	0.	1.000	1.006	.182	.005
.75	0.	23.00	23.14	.272	.115
1.00	0.	99.00	99.59	.363	.495
1.25	4.000	178.0	179.1	.454	.890
1.50	25.00	214.0	215.3	.545	1.070
1.75	61.00	216.0	217.3	.636	1.080
2.00	83.00	204.0	205.2	.726	1.020
2.25	88.00	185.0	185.6	.817	.923
2.50	89.00	168.0	166.9	.908	.829
2.75	86.00	148.0	142.8	.999	.710
3.00	79.05	135.0	123.3	1.090	.613
3.25	70.00	126.0	106.2	1.180	.528
3.50	61.22	117.0	88.79	1.271	.441
3.75	53.00	111.0	74.80	1.362	.372
4.00	48.41	107.0	63.91	1.453	.318
4.25	45.51	101.0	52.51	1.543	.261
4.50	43.61	98.00	45.80	1.634	.228
4.75	42.00	90.00	35.66	1.725	.177
5.00	40.00	86.00	30.85	1.816	.153
5.25	38.00	83.00	27.99	1.907	.139
5.50	37.00	77.00	22.75	1.997	.113
5.75	35.00	74.00	20.90	2.088	.104
6.00	34.00	69.00	17.28	2.179	.086
6.25	32.00	66.00	15.83	2.270	.079
6.50	30.00	61.00	12.47	2.361	.062
6.75	29.00	58.00	11.13	2.451	.055
7.00	27.00	57.00	11.76	2.542	.058
7.25	26.00	52.00	8.358	2.633	.042
7.50	24.00	50.00	7.957	2.724	.040
7.75	23.18	45.00	4.518	2.815	.022
8.00	23.00	43.00	4.080	2.905	.020
8.25	21.94	40.55	3.178	2.996	.016
8.50	20.67	38.00	2.149	3.087	.011
8.75	19.31	35.88	1.498	3.178	.007
9.00	18.00	34.00	1.040	3.269	.005
9.25	16.91	32.50	.9062	3.359	.005
9.50	16.00	31.00	.7570	3.450	.004
9.75	15.50	29.00	.1009	3.541	.001
10.00	15.00	27.00	0.	3.632	0.000

END OF DIRECT METHOD ROUTINE

COMMAND NUMBER: 9

3.1.2.3 Time-discrete method, Sample run 3

The time-discrete method, called GRAAIM in the program menu, uses the interpolated feed and discharge signals for the ball mill. Two computations are shown, one in which the best number of parameters for Equation 14 is selected in short output mode. Best results are obtained when NA=2 and NB=3. The second computation in full-output mode is with these two numbers of parameters (see Table 6).

Table 6 – Example using GRAAIM subroutine on grinding mill test #1 data, sample run 3

```
RESIDENCE TIME DISTRIBUTION PROGRAM

COMMAND MENU

1 - SWITCH TO SHORT OUTPUT MODE
2 - SWITCH TO FULL OUTPUT MODE
3 - READ/INTERPOLATE DISCHARGE DATA ON TAPE8
4 - READ/INTERPOLATE FEED DATA ON TAPE7
5 - ANALYSIS USING AUSTIN TECHNIQUE PROGRAM
6 - ANALYSIS USING DIRECT DECONVOLUTION
7 - ANALYSIS USING GRAAIM LEAST SQUARES METHOD
8 - ANALYSIS USING MIXERS IN SERIES
9 - END PROGRAM

COMMAND NUMBER: 3
ENTERED RTD INTERPOLATION ROUTINE
READING    TRACER TEST #1 - OUTPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  51 RAW DATA POINTS
NOW HAVE 105 DATA POINTS IN TOTAL

COMMAND NUMBER: 4
ENTERED RTD INTERPOLATION ROUTINE
READING    TRACER TEST #1 - INPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  43 RAW DATA POINTS
NOW HAVE 105 DATA POINTS IN TOTAL

COMMAND NUMBER: 2
FULL OUTPUT MODE

COMMAND NUMBER: 7
                   PLUG FLOW PURE TIME DELAY: .8
                      FINISH ACCURACY FACTOR: .001
     MINIMUM NUMBER OF PARAMETERS NA AND NB: 2 3
   *MAXIMUM* NUMBER OF PARAMETERS NA AND NB: 3 3


*** ENTERED GRAAIM ROUTINE ***      NA:NB=2 3


ITERATION  1
 AVERAGE RESIDENCE TIME:   2.74
        VARIANCE OF RTD:   2.19


ITERATION  2
 AVERAGE RESIDENCE TIME:   2.75
        VARIANCE OF RTD:   2.26


ITERATION  3
 AVERAGE RESIDENCE TIME:   2.76
        VARIANCE OF RTD:   2.30


ITERATION  4
 AVERAGE RESIDENCE TIME:   2.76
        VARIANCE OF RTD:   2.32
```

Table 6 (cont'd)

```
 ITERATION  5
  AVERAGE RESIDENCE TIME:   2.76
         VARIANCE OF RTD:   2.32

GRAAIM LEAST SQUARES METHOD FINAL RESULTS


 AVERAGE RESIDENCE TIME:  2.759       INCLUDING PLUG FLOW OF:   .75
        VARIANCE OF RTD:  2.326
   PREDICTION CRITERION:  62.86
 STD. DEV. OF RESIDUALS:  1.167

    FUNCTION PARAMETERS      ABS.S.D.   REL.S.D.
    -------------------      --------   --------

         A1:   -1.457         .4151E-02      0.
         A2:     .5188        .3937E-02      1.
         B1:     .1083E-01    .3247E-03      3.
         B2:     .2752E-01    .4527E-03      2.
         B3:     .2303E-01    .4871E-03      2.



*** ENTERED GRAAIM ROUTINE ***      NA:NB=3  3
ITERATION  1
 AVERAGE RESIDENCE TIME:   2.80
        VARIANCE OF RTD:   2.48


ITERATION  2
 AVERAGE RESIDENCE TIME:   2.79
        VARIANCE OF RTD:   2.49


ITERATION  3
 AVERAGE RESIDENCE TIME:   2.79
        VARIANCE OF RTD:   2.47


ITERATION  4
 AVERAGE RESIDENCE TIME:   2.78
        VARIANCE OF RTD:   2.45


ITERATION  5
 AVERAGE RESIDENCE TIME:   2.78
        VARIANCE OF RTD:   2.45


GRAAIM LEAST SQUARES METHOD FINAL RESULTS


 AVERAGE RESIDENCE TIME:  2.784       INCLUDING PLUG FLOW OF:   .75
        VARIANCE OF RTD:  2.448
   PREDICTION CRITERION:  55.39
 STD. DEV. OF RESIDUALS:  1.156

    FUNCTION PARAMETERS      ABS.S.D.   REL.S.D.
    -------------------      --------   --------

         A1:   -1.535         .2068E-01      1.
         A2:     .6445        .3326E-01      5.
         A3:   -.5198E-01     .1380E-01     27.
         B1:     .1040E-01    .3222E-03      3.
         B2:     .2760E-01    .4163E-03      2.
         B3:     .1960E-01    .9840E-03      5.
```

Table 6 (cont'd)

```
COMMAND NUMBER: 2
FULL OUTPUT MODE


COMMAND NUMBER: 7
                    PLUG FLOW PURE TIME DELAY: .8
                       FINISH ACCURACY FACTOR: .001
     MINIMUM NUMBER OF PARAMETERS NA AND NB: 2 3
   *MAXIMUM* NUMBER OF PARAMETERS NA AND NB: 2 3


*** ENTERED GRAAIM ROUTINE ***      NA:NB=2 3


STARTING PARAMETERS
   PLUG FLOW PURE DELAY:    .750
 FINISH ACCURACY FACTOR:    .100E-02
ITERATION  1
 AVERAGE RESIDENCE TIME:    2.74
        VARIANCE OF RTD:    2.19


ITERATION  2
 AVERAGE RESIDENCE TIME:    2.75
        VARIANCE OF RTD:    2.26


ITERATION  3
 AVERAGE RESIDENCE TIME:    2.76
        VARIANCE OF RTD:    2.30


ITERATION  4
 AVERAGE RESIDENCE TIME:    2.76
        VARIANCE OF RTD:    2.32


ITERATION  5
 AVERAGE RESIDENCE TIME:    2.76
        VARIANCE OF RTD:    2.32


----------------------------------------------------------------------------


GRAAIM LEAST SQUARES METHOD FINAL RESULTS


 AVERAGE RESIDENCE TIME:   2.759       INCLUDING PLUG FLOW OF:    .75
        VARIANCE OF RTD:   2.326
   PREDICTION CRITERION:   62.86
 STD. DEV. OF RESIDUALS:   1.167

    FUNCTION PARAMETERS       ABS.S.D.    REL.S.D.
    -------------------       --------    --------

         A1:   -1.457         .4151E-02       0.
         A2:    .5188         .3937E-02       1.
         B1:    .1083E-01     .3247E-03       3.
         B2:    .2752E-01     .4527E-03       2.
         B3:    .2303E-01     .4871E-03       2.
```

Table 6 (cont'd)

TIME	FEED SIGNAL	DISCHARGE SIGNAL	MODEL DISCHARGE	IMPULSE RESPONSE	NON-DIMENSIONAL RTD TIME	CONCENTRATION
0.00	2240.	0.	0.	0.	0.00	0.000
.25	0.	0.	0.	0.	.09	0.000
.50	0.	1.000	0.	0.	.18	0.000
.75	0.	23.00	24.26	.4332E-01	.27	.120
1.00	0.	99.00	97.00	.1732	.36	.478
1.25	4.000	178.0	180.4	.3221	.45	.889
1.50	25.00	214.0	212.6	.3796	.54	1.047
1.75	61.00	216.0	216.2	.3861	.63	1.065
2.00	83.00	204.0	204.9	.3658	.72	1.009
2.25	88.00	185.0	186.8	.3328	.82	.918
2.50	89.00	168.0	167.4	.2953	.91	.815
2.75	86.00	148.0	150.2	.2577	1.00	.711
3.00	79.05	135.0	136.7	.2223	1.09	.613
3.25	70.00	126.0	126.6	.1904	1.18	.525
3.50	61.22	117.0	119.0	.1621	1.27	.447
3.75	53.00	111.0	113.0	.1375	1.36	.379
4.00	48.41	107.0	107.9	.1162	1.45	.321
4.25	45.51	101.0	103.0	.9811E-01	1.54	.271
4.50	43.61	98.00	98.06	.8267E-01	1.63	.228
4.75	42.00	90.00	92.85	.6959E-01	1.72	.192
5.00	40.00	86.00	87.49	.5853E-01	1.81	.162
5.25	38.00	83.00	82.18	.4920E-01	1.90	.136
5.50	37.00	77.00	77.08	.4134E-01	1.99	.114
5.75	35.00	74.00	72.30	.3472E-01	2.08	.096
6.00	34.00	69.00	67.86	.2916E-01	2.17	.080
6.25	32.00	66.00	63.76	.2448E-01	2.26	.068
6.50	30.00	61.00	59.99	.2055E-01	2.36	.057
6.75	29.00	58.00	56.54	.1725E-01	2.45	.048
7.00	27.00	57.00	53.36	.1448E-01	2.54	.040
7.25	26.00	52.00	50.43	.1215E-01	2.63	.034
7.50	24.00	50.00	47.69	.1020E-01	2.72	.028
7.75	23.18	45.00	45.12	.8562E-02	2.81	.024
8.00	23.00	43.00	42.71	.7186E-02	2.90	.020
8.25	21.94	40.55	40.44	.6030E-02	2.99	.017
8.50	20.67	38.00	38.28	.5061E-02	3.08	.014
8.75	19.31	35.88	36.26	.4247E-02	3.17	.012
9.00	18.00	34.00	34.39	.3564E-02	3.26	.010
9.25	16.91	32.50	32.66	.2991E-02	3.35	.008
9.50	16.00	31.00	31.04	.2510E-02	3.44	.007
9.75	15.50	29.00	29.50	.2107E-02	3.53	.006
10.00	15.00	27.00	28.01	.1768E-02	3.62	.005
10.25	14.00	25.38	26.58	.1484E-02	3.71	.004
10.50	13.00	24.00	25.20	.1245E-02	3.81	.003
10.75	12.44	23.00	23.89	.1045E-02	3.90	.003
11.00	12.00	22.00	22.67	.8769E-03	3.99	.002
11.25	11.56	20.56	21.51	.7359E-03	4.08	.002
11.50	11.00	19.00	20.41	.6176E-03	4.17	.002
11.75	10.00	17.38	19.36	.5183E-03	4.26	.001
12.00	9.000	16.00	18.36	.4350E-03	4.35	.001
12.25	8.438	15.38	17.43	.3650E-03	4.44	.001

END OF GRAAIM ROUTINE

COMMAND NUMBER: 9

3.1.2.4 Mixers-in-series method, Sample run 4

The mixers-in-series option, when activated in program RTD, results in a single message: USE SEPARATE MIXERS PROGRAM. A user, aware of this division of the program into two parts, directly uses the MIXERS program. Four models are offered (see Section 2.2.2). Sample run 4 illustrates the results obtained after interpolation of the ball-mill feed and discharge tracer data and selection of a plug flow plus two mixers-in-series models (see Table 7).

Table 7 – Example using MIXERS with raw feed and discharge data of the first grinding mill test, sample run 4

```
MIXERS IN SERIES MODELLING

ENTERED RTD INTERPOLATION ROUTINE
READING    TRACER TEST #1 - OUTPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  51 RAW DATA POINTS
NOW HAVE 105 DATA POINTS IN TOTAL

                        MODEL TYPE: 1
                  NUMBER OF MIXERS: 2

     ENTER ESTIMATES OF
                   PLUG FLOW DELAY: .75
              MEAN RESIDENCE TIME 1: .9

   SEARCH IMPULSE AMPLITUDE ?(Y/N) : Y

       INITIAL FEED CONCENTRATION: 2240

      FEED SIGNAL AVAILABLE ?(Y/N) : Y

ENTERED RTD INTERPOLATION ROUTINE
READING    TRACER TEST #1 - INPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  43 RAW DATA POINTS
NOW HAVE 105 DATA POINTS IN TOTAL

               CONTROL DIRECTIVE: 3

ITERATION    4            36 FUNCTION VALUES          F =   .91278946E+03
  .8019      .9127       2126.

FINAL RESULTS FOR MODEL TYPE 1

  STD. DEV. OF RESIDUALS:  4.361

        PLUG FLOW DELAY:  .8019
   MEAN RESIDENCE TIME 1:  .9127
       NUMBER OF MIXERS: 2

OPEN CIRCUIT AVERAGE RESIDENCE TIME:  2.627
                           VARIANCE:  1.664
```

Table 7 (cont'd)

TIME	FEED SIGNAL	DISCHARGE SIGNAL	MODEL DISCHARGE	IMPULSE RESPONSE	DIMENSIONLESS IMPULSE RESPONSE	
0.00	2126.	0.	0.	0.	0.000	0.000
.25	0.	0.	0.	0.	.095	0.000
.50	0.	1.000	0.	0.	.190	0.000
.75	0.	23.00	0.	0.	.285	0.000
1.00	0.	99.00	101.7	.191	.381	.503
1.25	4.000	178.0	174.9	.329	.476	.865
1.50	25.00	214.0	207.2	.390	.571	1.025
1.75	61.00	216.0	214.0	.403	.666	1.058
2.00	83.00	204.0	205.7	.387	.761	1.017
2.25	88.00	185.0	189.2	.356	.856	.935
2.50	89.00	168.0	170.1	.317	.952	.833
2.75	86.00	148.0	152.4	.277	1.047	.727
3.00	79.05	135.0	138.0	.237	1.142	.624
3.25	70.00	126.0	126.7	.201	1.237	.528
3.50	61.22	117.0	118.0	.168	1.332	.443
3.75	53.00	111.0	111.2	.140	1.427	.368
4.00	48.41	107.0	105.5	.115	1.522	.303
4.25	45.51	101.0	100.2	.947E-01	1.618	.249
4.50	43.61	98.00	95.01	.772E-01	1.713	.203
4.75	42.00	90.00	89.67	.627E-01	1.808	.165
5.00	40.00	86.00	84.25	.507E-01	1.903	.133
5.25	38.00	83.00	78.92	.408E-01	1.998	.107
5.50	37.00	77.00	73.83	.328E-01	2.093	.086
5.75	35.00	74.00	69.08	.263E-01	2.188	.069
6.00	34.00	69.00	64.68	.210E-01	2.284	.055
6.25	32.00	66.00	60.62	.167E-01	2.379	.044
6.50	30.00	61.00	56.92	.133E-01	2.474	.035
6.75	29.00	58.00	53.55	.106E-01	2.569	.028
7.00	27.00	57.00	50.48	.836E-02	2.664	.022
7.25	26.00	52.00	47.67	.662E-02	2.759	.017
7.50	24.00	50.00	45.04	.523E-02	2.855	.014
7.75	23.18	45.00	42.61	.412E-02	2.950	.011
8.00	23.00	43.00	40.33	.325E-02	3.045	.009
8.50	20.67	38.00	36.17	.201E-02	3.235	.005
9.00	18.00	34.00	32.51	.124E-02	3.425	.003
9.50	16.00	31.00	29.39	.759E-03	3.616	.002
10.00	15.00	27.00	26.56	.464E-03	3.806	.001
10.50	13.00	24.00	23.91	.283E-03	3.996	.001
11.00	12.00	22.00	21.54	.172E-03	4.187	.000
11.50	11.00	19.00	19.40	.104E-03	4.377	.000
12.00	9.000	16.00	17.46	.631E-04	4.567	.000
12.50	8.000	15.00	15.76	.381E-04	4.758	.000
13.00	7.000	14.00	14.17	.230E-04	4.948	.000
14.00	5.000	11.00	11.20	.832E-05	5.328	.000
15.00	5.000	8.000	8.650	.299E-05	5.709	.000
16.00	4.000	7.000	6.761	.107E-05	6.090	.000
18.00	2.000	4.000	4.588	.135E-06	6.851	.000
20.00	1.000	2.000	2.673	.169E-07	7.612	.000
22.00	.2444	1.000	1.370	.209E-08	8.373	.000
24.00	0.	0.	.5141	.255E-09	9.135	.000
26.00	0.	0.	.1076	.310E-10	9.896	.000

MODEL TYPE: 0

END OF MIXERS ROUTINE

3.1.2.5 Complex tracer input, Sample runs 5 and 6

Using tracer test No. 2 data (i.e., a complex feed signal), two methods can be used; namely, the time-discrete method (see Section 2.2.3) and the mixers-in-series model (see Section 2.2.2). The results of these two sample runs are presented as sample runs 5 and 6, respectively, in Tables 8 and 9.

Table 8 – Example using GRAAIM subroutine on grinding mill test #2 data, sample run 5

```
RESIDENCE TIME DISTRIBUTION PROGRAM

COMMAND MENU

1 - SWITCH TO SHORT OUTPUT MODE
2 - SWITCH TO FULL OUTPUT MODE
3 - READ/INTERPOLATE DISCHARGE DATA ON TAPE8
4 - READ/INTERPOLATE FEED DATA ON TAPE7
5 - ANALYSIS USING AUSTIN TECHNIQUE PROGRAM
6 - ANALYSIS USING DIRECT DECONVOLUTION
7 - ANALYSIS USING GRAAIM LEAST SQUARES METHOD
8 - ANALYSIS USING MIXERS IN SERIES
9 - END PROGRAM


COMMAND NUMBER: 3
ENTERED RTD INTERPOLATION ROUTINE
READING    TRACER TEST #2 - OUTPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  51 RAW DATA POINTS
NOW HAVE 105 DATA POINTS IN TOTAL


COMMAND NUMBER: 4
ENTERED RTD INTERPOLATION ROUTINE
READING    TRACER TEST #2 - INPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  51 RAW DATA POINTS
NOW HAVE 105 DATA POINTS IN TOTAL


COMMAND NUMBER: 7
                  PLUG FLOW PURE TIME DELAY: .8
                     FINISH ACCURACY FACTOR: .001
     MINIMUM NUMBER OF PARAMETERS NA AND NB: 2 2
   *MAXIMUM* NUMBER OF PARAMETERS NA AND NB: 3 3


*** ENTERED GRAAIM ROUTINE ***      NA:NB=2 2


ITERATION  1
 AVERAGE RESIDENCE TIME:   2.69
        VARIANCE OF RTD:   1.45


ITERATION  2
 AVERAGE RESIDENCE TIME:   2.68
        VARIANCE OF RTD:   1.41
```

Table 8 (cont'd)

```
ITERATION  3
 AVERAGE RESIDENCE TIME:   2.68
        VARIANCE OF RTD:   1.42


GRAAIM LEAST SQUARES METHOD FINAL RESULTS


 AVERAGE RESIDENCE TIME:  2.683       INCLUDING PLUG FLOW OF:   .75
        VARIANCE OF RTD:  1.426
   PREDICTION CRITERION:  1171.
 STD. DEV. OF RESIDUALS:  9.124

    FUNCTION PARAMETERS       ABS.S.D.   REL.S.D.
    -------------------       --------   --------

         A1:   -1.603         .1869E-01       1.
         A2:     .6485        .1821E-01       3.
         B1:   -.2734E-02     .2487E-02      91.
         B2:    .4854E-01     .2829E-02       6.



*** ENTERED GRAAIM ROUTINE ***       NA:NB=2 3


ITERATION  1
 AVERAGE RESIDENCE TIME:   2.84
        VARIANCE OF RTD:   2.50


ITERATION  2
 AVERAGE RESIDENCE TIME:   2.80
        VARIANCE OF RTD:   2.32


ITERATION  3
 AVERAGE RESIDENCE TIME:   2.80
        VARIANCE OF RTD:   2.32


GRAAIM LEAST SQUARES METHOD FINAL RESULTS


 AVERAGE RESIDENCE TIME:  2.799       INCLUDING PLUG FLOW OF:   .75
        VARIANCE OF RTD:  2.320
   PREDICTION CRITERION:  545.9
 STD. DEV. OF RESIDUALS:  5.034

    FUNCTION PARAMETERS       ABS.S.D.   REL.S.D.
    -------------------       --------   --------

         A1:   -1.459         .1649E-01       1.
         A2:     .5203        .1540E-01       3.
         B1:    .1124E-01     .1977E-02      18.
         B2:    .1648E-01     .3369E-02      20.
         B3:    .3352E-01     .2965E-02       9.
```

Table 8 (cont'd)

```
*** ENTERED GRAAIM ROUTINE ***       NA:NB=3 2


ITERATION  1
 AVERAGE RESIDENCE TIME:   2.85
        VARIANCE OF RTD:   2.30


ITERATION  2
 AVERAGE RESIDENCE TIME:   2.97
        VARIANCE OF RTD:   3.10


ITERATION  3
 AVERAGE RESIDENCE TIME:   2.98
        VARIANCE OF RTD:   3.18


ITERATION  4
 AVERAGE RESIDENCE TIME:   2.98
        VARIANCE OF RTD:   3.19


ITERATION  5
 AVERAGE RESIDENCE TIME:   2.98
        VARIANCE OF RTD:   3.19


ITERATION  6
 AVERAGE RESIDENCE TIME:   2.97
        VARIANCE OF RTD:   3.19


ITERATION  7
 AVERAGE RESIDENCE TIME:   2.97
        VARIANCE OF RTD:   3.19


GRAAIM LEAST SQUARES METHOD FINAL RESULTS


 AVERAGE RESIDENCE TIME:  2.974       INCLUDING PLUG FLOW OF:   .75
        VARIANCE OF RTD:  3.184
   PREDICTION CRITERION:  604.9
 STD. DEV. OF RESIDUALS:  4.102

    FUNCTION PARAMETERS      ABS.S.D.   REL.S.D.
    -------------------      --------   --------

         A1:   -1.974        .2800E-01        1.
         A2:    1.330        .4876E-01        4.
         A3:   -.3210        .2223E-01        7.
         B1:    .3596E-02    .1262E-02       35.
         B2:    .3113E-01    .1947E-02        6.
```

Table 8 (cont'd)

```
*** ENTERED GRAAIM ROUTINE ***       NA:NB=3 3


ITERATION  1
 AVERAGE RESIDENCE TIME:    2.75
        VARIANCE OF RTD:    2.07


ITERATION  2
 AVERAGE RESIDENCE TIME:    2.79
        VARIANCE OF RTD:    2.28


ITERATION  3
 AVERAGE RESIDENCE TIME:    2.83
        VARIANCE OF RTD:    2.46


ITERATION  4
 AVERAGE RESIDENCE TIME:    2.85
        VARIANCE OF RTD:    2.58


ITERATION  5
 AVERAGE RESIDENCE TIME:    2.87
        VARIANCE OF RTD:    2.65


ITERATION  6
 AVERAGE RESIDENCE TIME:    2.87
        VARIANCE OF RTD:    2.69


ITERATION  7
 AVERAGE RESIDENCE TIME:    2.88
        VARIANCE OF RTD:    2.72


ITERATION  8
 AVERAGE RESIDENCE TIME:    2.88
        VARIANCE OF RTD:    2.73


GRAAIM LEAST SQUARES METHOD FINAL RESULTS


 AVERAGE RESIDENCE TIME:   2.882       INCLUDING PLUG FLOW OF:   .75
        VARIANCE OF RTD:   2.739
   PREDICTION CRITERION:   490.3
 STD. DEV. OF RESIDUALS:   4.141

    FUNCTION PARAMETERS      ABS.S.D.    REL.S.D.
    -------------------      --------    --------

         A1:   -1.737        .6455E-01       4.
         A2:    .9668        .1015          10.
         A3:   -.1817        .4067E-01      22.
         B1:    .8335E-02    .1812E-02      22.
         B2:    .2137E-01    .3292E-02      15.
         B3:    .1792E-01    .4564E-02      25.
```

Table 8 (cont'd)

```
COMMAND NUMBER: 2
FULL OUTPUT MODE


COMMAND NUMBER: 7
                    PLUG FLOW PURE TIME DELAY: .8
                       FINISH ACCURACY FACTOR: .001
     MINIMUM NUMBER OF PARAMETERS NA AND NB: 2 3
   *MAXIMUM* NUMBER OF PARAMETERS NA AND NB: 2 3


*** ENTERED GRAAIM ROUTINE ***      NA:NB=2 3


STARTING PARAMETERS
   PLUG FLOW PURE DELAY:    .750
 FINISH ACCURACY FACTOR:    .100E-02


ITERATION  1
 AVERAGE RESIDENCE TIME:    2.84
        VARIANCE OF RTD:    2.50


ITERATION  2
 AVERAGE RESIDENCE TIME:    2.80
        VARIANCE OF RTD:    2.32


ITERATION  3
 AVERAGE RESIDENCE TIME:    2.80
        VARIANCE OF RTD:    2.32

..........................................................................

GRAAIM LEAST SQUARES METHOD FINAL RESULTS


 AVERAGE RESIDENCE TIME:   2.799       INCLUDING PLUG FLOW OF:    .75
        VARIANCE OF RTD:   2.320
   PREDICTION CRITERION:   545.9
 STD. DEV. OF RESIDUALS:   5.034

    FUNCTION PARAMETERS      ABS.S.D.   REL.S.D.
    -------------------      --------   --------

         A1:   -1.459        .1649E-01       1.
         A2:     .5203       .1540E-01       3.
         B1:     .1124E-01   .1977E-02      18.
         B2:     .1648E-01   .3369E-02      20.
         B3:     .3352E-01   .2965E-02       9.
```

Table 8 (cont'd)

TIME	FEED SIGNAL	DISCHARGE SIGNAL	MODEL DISCHARGE	IMPULSE RESPONSE	NON-DIMENSIONAL RTD TIME	CONCENTRATION
0.00	0.	0.	0.	0.	0.00	0.000
.25	0.	0.	0.	0.	.09	0.000
.50	53.00	0.	0.	0.	.18	0.000
.75	680.0	0.	0.	.4496E-01	.27	.126
1.00	1130.	1.000	0.	.1315	.36	.368
1.25	840.0	1.000	.5957	.3026	.45	.847
1.50	320.0	8.000	9.385	.3731	.54	1.044
1.75	25.00	33.00	39.07	.3869	.63	1.083
2.00	2.000	96.00	103.0	.3704	.71	1.037
2.25	5.000	173.0	185.3	.3391	.80	.949
2.50	25.00	242.0	250.4	.3021	.89	.846
2.75	50.00	270.0	280.2	.2643	.98	.740
3.00	75.00	270.0	279.4	.2285	1.07	.640
3.25	95.00	248.0	262.3	.1958	1.16	.548
3.50	105.0	233.0	238.5	.1669	1.25	.467
3.75	100.0	202.0	214.0	.1416	1.34	.396
4.00	85.00	176.0	192.2	.1197	1.43	.335
4.25	80.00	156.0	174.3	.1010	1.52	.283
4.50	265.0	149.0	160.3	.8512E-01	1.61	.238
4.75	915.0	139.0	149.4	.7163E-01	1.70	.200
5.00	865.0	132.0	140.2	.6022E-01	1.79	.169
5.25	415.0	127.0	134.0	.5060E-01	1.88	.142
5.50	190.0	137.0	139.9	.4249E-01	1.96	.119
5.75	60.00	172.0	168.0	.3567E-01	2.05	.100
6.00	55.00	229.0	222.0	.2994E-01	2.14	.084
6.25	60.00	279.0	274.5	.2512E-01	2.23	.070
6.50	80.00	309.0	302.7	.2108E-01	2.32	.059
6.75	100.0	318.0	306.8	.1768E-01	2.41	.049
7.00	125.0	304.0	293.7	.1483E-01	2.50	.042
7.25	125.0	279.0	272.7	.1244E-01	2.59	.035
7.50	125.0	250.0	249.5	.1043E-01	2.68	.029
7.75	115.0	235.0	227.9	.8751E-02	2.77	.024
8.00	110.0	211.0	209.5	.7340E-02	2.86	.021
8.25	102.3	195.4	194.7	.6156E-02	2.95	.017
8.50	93.00	185.0	182.7	.5163E-02	3.04	.014
8.75	83.56	175.9	172.5	.4330E-02	3.13	.012
9.00	75.00	168.0	163.5	.3631E-02	3.22	.010
9.25	68.56	160.1	155.2	.3045E-02	3.30	.009
9.50	65.00	152.0	147.3	.2554E-02	3.39	.007
9.75	67.50	143.1	139.5	.2142E-02	3.48	.006
10.00	70.00	134.0	131.7	.1796E-02	3.57	.005
10.25	65.94	124.9	124.0	.1507E-02	3.66	.004
10.50	60.00	117.0	116.5	.1263E-02	3.75	.004
10.75	54.69	112.4	109.5	.1060E-02	3.84	.003
11.00	50.00	108.0	103.3	.8886E-03	3.93	.002
11.25	47.19	100.5	97.91	.7452E-03	4.02	.002
11.50	45.00	93.00	92.90	.6250E-03	4.11	.002
11.75	42.44	88.19	88.08	.5242E-03	4.20	.001
12.00	40.00	84.00	83.37	.4396E-03	4.29	.001
12.25	37.94	78.94	78.77	.3687E-03	4.38	.001

END OF GRAAIM ROUTINE

COMMAND NUMBER: 9

Table 9 – Example using MIXERS and raw feed and discharge data of the second grinding mill test, sample run 6

```
MIXERS IN SERIES MODELLING

ENTERED RTD INTERPOLATION ROUTINE
READING     TRACER TEST #2 - OUTPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  51 RAW DATA POINTS
NOW HAVE 105 DATA POINTS IN TOTAL

                       MODEL TYPE: 3

     ENTER ESTIMATES OF
                  PLUG FLOW DELAY: .75
            MEAN RESIDENCE TIME 1: 1.5
            MEAN RESIDENCE TIME 2: .5
            MEAN RESIDENCE TIME 3: .2

   SEARCH IMPULSE AMPLITUDE ?(Y/N) : N

      FEED SIGNAL AVAILABLE ?(Y/N) : Y

ENTERED RTD INTERPOLATION ROUTINE
READING     TRACER TEST #2 - INPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  51 RAW DATA POINTS
NOW HAVE 105 DATA POINTS IN TOTAL

                CONTROL DIRECTIVE: 3

ITERATION     5            55 FUNCTION VALUES          F =   .16336798E+04
  .7611      1.641      .3671      .1674




FINAL RESULTS FOR MODEL TYPE 3

  STD. DEV. OF RESIDUALS:  5.896

         PLUG FLOW DELAY:  .7611
   MEAN RESIDENCE TIME 1:  1.641
   MEAN RESIDENCE TIME 2:  .3671
   MEAN RESIDENCE TIME 3:  .1674

OPEN CIRCUIT AVERAGE RESIDENCE TIME:  2.937
                           VARIANCE:  2.856
```

Table 9 (cont'd)

TIME	FEED SIGNAL	DISCHARGE SIGNAL	MODEL DISCHARGE	IMPULSE RESPONSE	DIMENSIONLESS	IMPULSE RESPONSE
0.00	0.	0.	0.	0.	0.000	0.000
.25	0.	0.	0.	0.	.085	0.000
.50	53.00	0.	0.	0.	.170	0.000
.75	680.0	0.	0.	0.	.255	0.000
1.00	1130.	1.000	0.	.139	.340	.410
1.25	840.0	1.000	0.	.299	.426	.877
1.50	320.0	8.000	1.848	.371	.511	1.090
1.75	25.00	33.00	27.67	.382	.596	1.123
2.00	2.000	96.00	95.08	.362	.681	1.063
2.25	5.000	173.0	181.8	.328	.766	.963
2.50	25.00	242.0	248.6	.290	.851	.853
2.75	50.00	270.0	276.6	.254	.936	.745
3.00	75.00	270.0	273.7	.220	1.021	.647
3.25	95.00	248.0	254.6	.190	1.107	.559
3.50	105.0	233.0	229.7	.164	1.192	.481
3.75	100.0	202.0	205.4	.141	1.277	.414
4.00	85.00	176.0	184.6	.121	1.362	.356
4.25	80.00	156.0	168.4	.104	1.447	.306
4.50	265.0	149.0	156.3	.895E-01	1.532	.263
4.75	915.0	139.0	147.2	.769E-01	1.617	.226
5.00	865.0	132.0	139.5	.660E-01	1.702	.194
5.25	415.0	127.0	132.2	.567E-01	1.788	.167
5.50	190.0	137.0	132.0	.487E-01	1.873	.143
5.75	60.00	172.0	162.3	.418E-01	1.958	.123
6.00	55.00	229.0	220.9	.359E-01	2.043	.105
6.25	60.00	279.0	274.8	.308E-01	2.128	.091
6.50	80.00	309.0	303.5	.265E-01	2.213	.078
6.75	100.0	318.0	306.0	.227E-01	2.298	.067
7.00	125.0	304.0	291.4	.195E-01	2.383	.057
7.25	125.0	279.0	269.1	.168E-01	2.468	.049
7.50	125.0	250.0	245.6	.144E-01	2.554	.042
7.75	115.0	235.0	224.3	.124E-01	2.639	.036
8.00	110.0	211.0	207.1	.106E-01	2.724	.031
8.50	93.00	185.0	183.0	.783E-02	2.894	.023
9.00	75.00	168.0	165.8	.577E-02	3.064	.017
9.50	65.00	152.0	150.7	.426E-02	3.235	.013
10.00	70.00	134.0	135.5	.314E-02	3.405	.009
10.50	60.00	117.0	120.3	.231E-02	3.575	.007
11.00	50.00	108.0	107.2	.171E-02	3.745	.005
11.50	45.00	93.00	96.69	.126E-02	3.916	.004
12.00	40.00	84.00	86.98	.928E-03	4.086	.003
12.50	36.00	74.00	77.71	.684E-03	4.256	.002
13.00	32.00	66.00	69.29	.505E-03	4.426	.001
14.00	26.00	52.00	55.10	.274E-03	4.767	.001
15.00	20.00	41.00	43.91	.149E-03	5.107	.000
16.00	15.00	33.00	35.02	.811E-04	5.448	.000
18.00	10.00	21.00	21.52	.240E-04	6.129	.000
20.00	7.000	14.00	13.57	.709E-05	6.810	.000
22.00	5.000	8.000	9.057	.210E-05	7.491	.000
24.00	4.000	4.000	6.317	.620E-06	8.172	.000
26.00	2.000	2.000	4.692	.183E-06	8.852	.000

MODEL TYPE: 0

END OF MIXERS ROUTINE

3.1.2.6 Discrete vs continuous RTD, Sample run 7

As discussed in Section 2.2, the Austin (see Section 2.2.1.1), direct deconvolution (see Section 2.2.1.2) and time-discrete (see Section 2.2.3) methods produce a discrete RTD table rather than a time-continuous function. Since the latter is required to use the kinetic ball-mill model as described in Chapter 7.2 of the SPOC Manual (23), a second step is often necessary to convert the RTD table into a mixers-in-series model. This is illustrated in Table 10 where results from sample run 5 (see Section 3.1.2.5) have been processed by the plug flow plus three different mixers options of the MIXERS program.

For comparison, Figure 9 shows the dimensionless RTD curve obtained from the four different methods applied to the two tracer tests. The agreement is excellent.

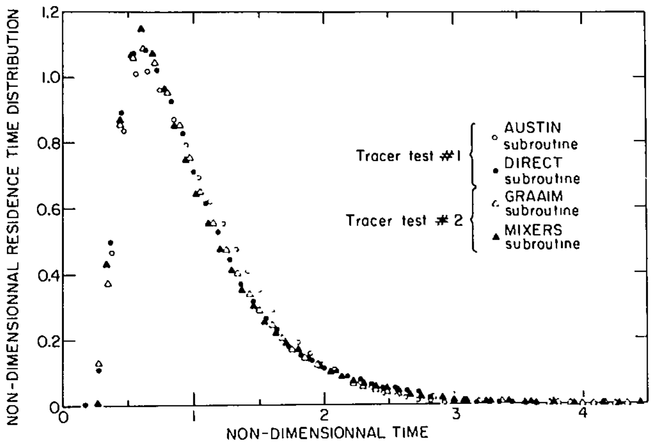


Fig. 9 – Comparison of the results of the four methods for the two ball-mill tracer tests

Table 10 – Example using MIXERS and GRAAIM output from example 5, sample run 7

```
MIXERS IN SERIES MODELLING

ENTERED RTD INTERPOLATION ROUTINE
READING  IMPULSE RESPONSE FROM GRAAIM PROGRAM
INTERPOLATION SUCCESSFULL
READ  51 RAW DATA POINTS
NOW HAVE  51 DATA POINTS IN TOTAL

                       MODEL TYPE: 3

     ENTER ESTIMATES OF
                    PLUG FLOW DELAY: .75
            MEAN RESIDENCE TIME 1: 1.5
            MEAN RESIDENCE TIME 2: .5
            MEAN RESIDENCE TIME 3: .2

   SEARCH IMPULSE AMPLITUDE ?(Y/N) : N

      FEED SIGNAL AVAILABLE ?(Y/N) : N

   DATA IS FOR OPEN CIRCUIT ?(Y/N) : Y

       INITIAL FEED CONCENTRATION: 4

                CONTROL DIRECTIVE: 3

ITERATION    8              92 FUNCTION VALUES          F =  .17328062E-
  .6610      1.455      .3498       .3471

FINAL RESULTS FOR MODEL TYPE 3

  STD. DEV. OF RESIDUALS:   .6072E-02

         PLUG FLOW DELAY:   .6610
   MEAN RESIDENCE TIME 1:   1.455
   MEAN RESIDENCE TIME 2:   .3498
   MEAN RESIDENCE TIME 3:   .3471

OPEN CIRCUIT AVERAGE RESIDENCE TIME:   2.806
                           VARIANCE:   2.300
```

Table 10 (cont'd)

TIME	FEED SIGNAL	DISCHARGE SIGNAL	MODEL DISCHARGE	IMPULSE RESPONSE	DIMENSIONLESS IMPULSE RESPONSE	
0.00	4.000	0.	0.	0.	0.000	0.000
.25	0.	0.	0.	0.	.089	0.000
.50	0.	0.	0.	0.	.178	0.000
.75	0.	.4496E-01	.1855E-01	.185E-01	.267	.052
1.00	0.	.1315	.1598	.160	.356	.448
1.25	0.	.3026	.2917	.292	.445	.819
1.50	0.	.3731	.3647	.365	.534	1.023
1.75	0.	.3869	.3859	.386	.624	1.083
2.00	0.	.3704	.3735	.373	.713	1.048
2.25	0.	.3391	.3431	.343	.802	.963
2.50	0.	.3021	.3053	.305	.891	.857
2.75	0.	.2643	.2663	.266	.980	.747
3.00	0.	.2285	.2293	.229	1.069	.643
3.25	0.	.1958	.1958	.196	1.158	.550
3.50	0.	.1669	.1664	.166	1.247	.467
3.75	0.	.1416	.1409	.141	1.336	.395
4.00	0.	.1197	.1191	.119	1.425	.334
4.25	0.	.1010	.1005	.101	1.514	.282
4.50	0.	.8512E-01	.8476E-01	.848E-01	1.603	.238
4.75	0.	.7163E-01	.7144E-01	.714E-01	1.693	.200
5.00	0.	.6022E-01	.6020E-01	.602E-01	1.782	.169
5.25	0.	.5060E-01	.5071E-01	.507E-01	1.871	.142
5.50	0.	.4249E-01	.4272E-01	.427E-01	1.960	.120
5.75	0.	.3567E-01	.3598E-01	.360E-01	2.049	.101
6.00	0.	.2994E-01	.3030E-01	.303E-01	2.138	.085
6.25	0.	.2512E-01	.2552E-01	.255E-01	2.227	.072
6.50	0.	.2108E-01	.2149E-01	.215E-01	2.316	.060
6.75	0.	.1768E-01	.1810E-01	.181E-01	2.405	.051
7.00	0.	.1483E-01	.1524E-01	.152E-01	2.494	.043
7.25	0.	.1244E-01	.1284E-01	.128E-01	2.583	.036
7.50	0.	.1043E-01	.1081E-01	.108E-01	2.672	.030
7.75	0.	.8751E-02	.9106E-02	.911E-02	2.762	.026
8.00	0.	.7340E-02	.7668E-02	.767E-02	2.851	.022
8.25	0.	.6156E-02	.6458E-02	.646E-02	2.940	.018
8.50	0.	.5163E-02	.5439E-02	.544E-02	3.029	.015
8.75	0.	.4330E-02	.4580E-02	.458E-02	3.118	.013
9.00	0.	.3631E-02	.3857E-02	.386E-02	3.207	.011
9.25	0.	.3045E-02	.3249E-02	.325E-02	3.296	.009
9.50	0.	.2554E-02	.2736E-02	.274E-02	3.385	.008
9.75	0.	.2142E-02	.2304E-02	.230E-02	3.474	.006
10.00	0.	.1796E-02	.1940E-02	.194E-02	3.563	.005
10.25	0.	.1507E-02	.1634E-02	.163E-02	3.652	.005
10.50	0.	.1263E-02	.1376E-02	.138E-02	3.741	.004
10.75	0.	.1060E-02	.1159E-02	.116E-02	3.831	.003
11.00	0.	.8886E-03	.9761E-03	.976E-03	3.920	.003
11.25	0.	.7452E-03	.8220E-03	.822E-03	4.009	.002
11.50	0.	.6250E-03	.6923E-03	.692E-03	4.098	.002
11.75	0.	.5242E-03	.5830E-03	.583E-03	4.187	.002
12.00	0.	.4396E-03	.4910E-03	.491E-03	4.276	.001
12.25	0.	.3687E-03	.4135E-03	.414E-03	4.365	.001
12.50	0.	.3092E-03	.3482E-03	.348E-03	4.454	.001

MODEL TYPE: 0

END OF MIXERS ROUTINE

3.2 BANK OF FLOTATION CELLS RTD

3.2.1 Experimental Procedure

The third cleaners of the molybdenum flotation circuit at Brenda Mines were tested in 1978 using fluorescein dye as the tracer. The circuit flowsheet and the test conditions are given in Figure 10.

The tracer was added as a pulse to the feed to the third cleaners (point 1). Samples were cut from the cell tails (point 3) and from each of the two feed streams (points 2 and 4), since both contained recycled tracer. In this case, the recycle simplifying assumption does not hold since the recycling flows do not behave as plug flow. As a consequence, only the methods using the input signal can be used. Prior to RTD calculation, this input signal had to be calculated by combining points 3 and 4 recorded signals in the right proportions.

Figure 11 gives the raw data for this test and the reconstructed input signal as explained in the next section.

3.2.2 Calculation Procedure

3.2.2.1 Preliminary calculations

Since the raw data do not include any percentage of solid in the pulp, it is not possible to calculate either the impulse amplitude in ppm, or the ratios of the water flow rates of streams 4 and 2 to stream 1.

The direct deconvolution model is used to estimate the impulse magnitude and the water flow-rate ratios can be estimated from the volumetric pulp flow rates of the corresponding streams. If U_1, U_2, and U_4, are the fluorescein concentrations at points 1, 2, and 4, respectively, U_1 is defined with data from Figure 10, by:

$$U_1 = \frac{3}{8.5} U_2 + \frac{5.5}{8.5} U_4 \qquad \text{Eq 18}$$

To sample streams 2, 3 and 4, three different time sequences were used. To calculate U_1, using Equation 18, it was necessary to define a common time scale for U_2 and U_4 signals. This was done by a graphical interpolation of both signals according to the U_3 time sequence.

The RTDINT outputs of Tables 11 and 12 summarize the data used for calculation.

3.2.2.2 Residence time distribution calculation

Since the recycle simplifying assumption is not valid, the Austin method cannot be used. The following three examples describe the calculation by the other three methods.

Example 1
Using an arbitrary value of the impulse magnitude (approximately 60% of the area under y(t) curve), the discrete deconvolution method is used. The results are printed in Table 13.

Example 2
Using the impulse magnitude computed in Example 1 above, the time-discrete method is used. The results are printed in Table 14.

Example 3
Using input and output signals already used in Example 2, a mixers-in-series model is used. The results are printed in Table 15.

Figure 12 shows the three dimensionless RTD curves. The agreement is less satisfactory than for the grinding tests (Fig. 9), probably because of the inaccuracy in the calculated input signal (see Section 3.2.2.1).

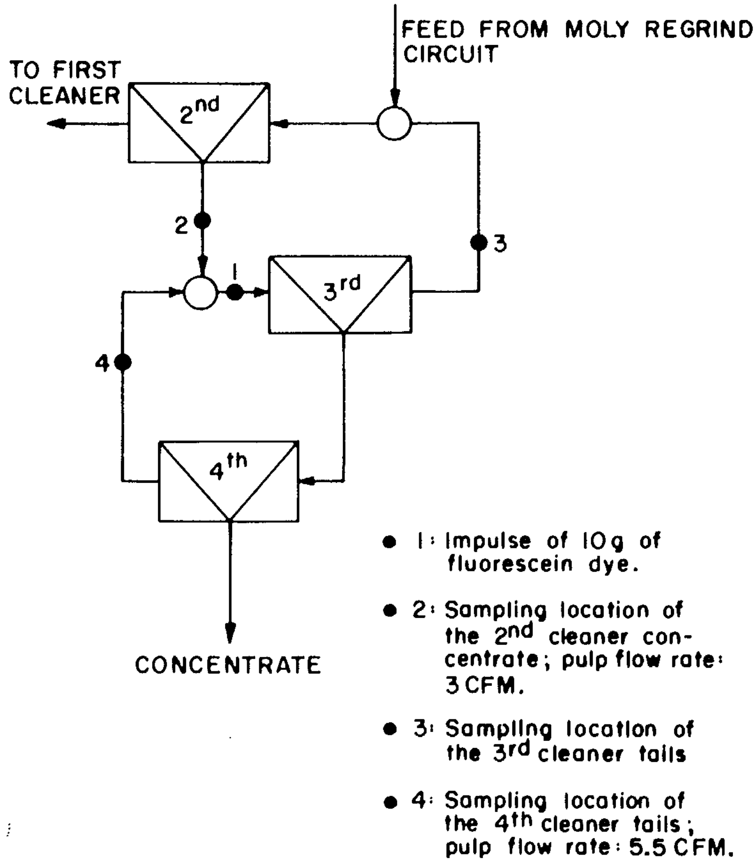


Fig. 10 – Flowsheet of the molybdenum flotation circuit cleaning stage

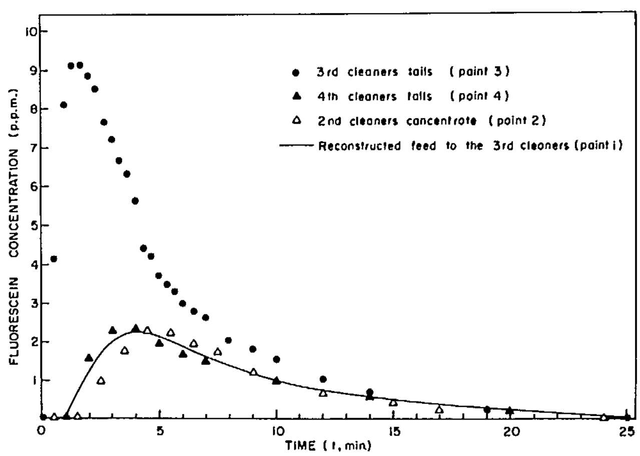


Fig. 11 – Input and output signals for the flotation cell tracer test

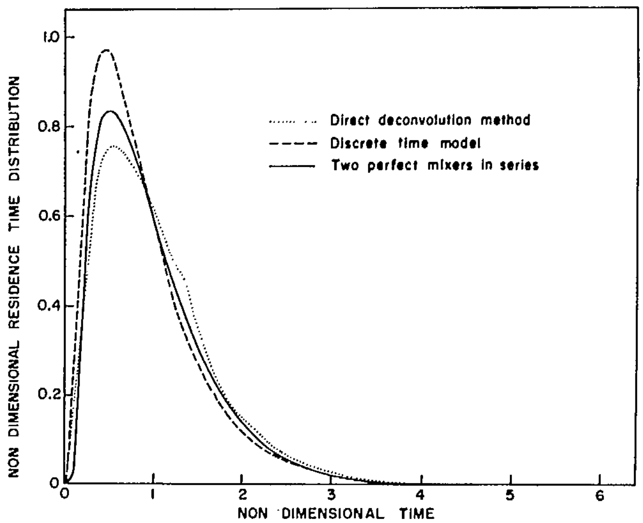


Fig. 12 – Results of RTD calculations for the flotation cell test

Table 11 – RTDINT full output for flotation cell output signal

```
ENTERED RTD INTERPOLATION ROUTINE
READING FLOTATION CELLS --- OUTPUT SIGNAL
0.       0.
 20.00    3.460    INTERPOLATED
 40.00    6.190    INTERPOLATED
 60.00    8.100
 80.00    9.100
 100.0    9.100
 120.0    8.830
 140.0    8.470
 160.0    7.650
 180.0    7.200
 200.0    6.660
 220.0    6.300
 240.0    5.580
 260.0    4.410
 280.0    4.180
 300.0    3.680
 320.0    3.470
 340.0    3.320
 360.0    2.960
 380.0    2.761    INTERPOLATED
 400.0    2.661    INTERPOLATED
 420.0    2.600
 440.0    2.407    INTERPOLATED
 460.0    2.210    INTERPOLATED
 480.0    2.030
 500.0    1.931    INTERPOLATED
 520.0    1.856    INTERPOLATED
 540.0    1.790
 560.0    1.702    INTERPOLATED
 580.0    1.611    INTERPOLATED
 600.0    1.520
 620.0    1.432    INTERPOLATED
 640.0    1.345    INTERPOLATED
 660.0    1.262    INTERPOLATED
 680.0    1.181    INTERPOLATED
 700.0    1.103    INTERPOLATED
 720.0    1.030
 740.0    .9665    INTERPOLATED
 760.0    .9074    INTERPOLATED
 780.0    .8525    INTERPOLATED
 800.0    .8015    INTERPOLATED
 820.0    .7541    INTERPOLATED
 840.0    .7100
 860.0    .6661    INTERPOLATED
 880.0    .6245    INTERPOLATED
 900.0    .5853    INTERPOLATED
 920.0    .5483    INTERPOLATED
 940.0    .5135    INTERPOLATED
 960.0    .4806    INTERPOLATED
 980.0    .4497    INTERPOLATED
 1000.    .4206    INTERPOLATED
 1020.    .3933    INTERPOLATED
 1040.    .3676    INTERPOLATED
 1060.    .3434    INTERPOLATED
 1080.    .3206    INTERPOLATED
 1100.    .2992    INTERPOLATED
 1120.    .2790    INTERPOLATED
 1140.    .2600
 1160.    .2359    INTERPOLATED
 1180.    .2128    INTERPOLATED
 1200.    .1906    INTERPOLATED
 1220.    .1694    INTERPOLATED
 1240.    .1493    INTERPOLATED
 1260.    .1303    INTERPOLATED
 1280.    .1124    INTERPOLATED
 1300.    .9567E-01 INTERPOLATED
 1320.    .8013E-01 INTERPOLATED
 1340.    .6582E-01 INTERPOLATED
 1360.    .5278E-01 INTERPOLATED
 1380.    .4104E-01 INTERPOLATED
 1400.    .3064E-01 INTERPOLATED
 1420.    .2162E-01 INTERPOLATED
 1440.    .1400E-01 INTERPOLATED
 1460.    .7840E-02 INTERPOLATED
 1480.    .3160E-02 INTERPOLATED
 1500.   0.
 1520.   0.         INTERPOLATED
 1540.   0.         INTERPOLATED
 1560.   0.
INTERPOLATION SUCCESSFULL
READ  26 RAW DATA POINTS
NOW HAVE  79 DATA POINTS IN TOTAL
```

Table 12 – RTDINT full output for flotation cell input signal

```
ENTERED RTD INTERPOLATION ROUTINE
READING FLOTATION CELL --- INPUT SIGNAL
0.          100.0
 20.00     0.
 40.00     0.
 60.00     0.
 80.00      .4550
 100.0      .7800
 120.0      1.168
 140.0      1.509
 160.0      1.791
 180.0      2.019
 200.0      2.152
 220.0      2.220
 240.0      2.250
 260.0      2.210
 280.0      2.170
 300.0      2.100
 320.0      2.030
 340.0      1.950
 360.0      1.860
 380.0      1.780
 400.0      1.710
 420.0      1.650
 440.0      1.540
 460.0      1.470
 480.0      1.390
 500.0      1.310
 520.0      1.240
 540.0      1.170
 560.0      1.120
 580.0      1.060
 600.0      1.000
 620.0      .9600
 640.0      .9100
 660.0      .8600
 680.0      .8300
 700.0      .7900
 720.0      .7500
 740.0      .7200
 760.0      .6900
 780.0      .6600
 800.0      .6400
 820.0      .6200
 840.0      .6000
 860.0      .5700
 880.0      .5500
 900.0      .5400
 920.0      .5100
 940.0      .4800
 960.0      .4600
 980.0      .4400
 1000.      .4200
```

Table 12 (cont'd)

```
 1020.      .3900
 1040.      .3700
 1060.      .3500
 1080.      .3300
 1100.      .3100
 1120.      .2900
 1140.      .2700
 1160.      .2500
 1180.      .2300
 1200.      .2100
 1220.      .1900
 1240.      .1800
 1260.      .1600
 1280.      .1500
 1300.      .1300
 1320.      .1100
 1340.      .1000
 1360.      .9000E-01
 1380.      .8500E-01
 1400.      .7500E-01
 1420.      .6600E-01
 1440.      .5700E-01
 1460.      .4700E-01
 1480.      .3800E-01
 1500.      .2900E-01
 1520.      .1900E-01
 1540.      .1000E-01
 1560.     0.
INTERPOLATION SUCCESSFULL
READ  79 RAW DATA POINTS
NOW HAVE  79 DATA POINTS IN TOTAL
```

Table 13 – Example using DIRECT subroutine on flotation cell test data

```
RESIDENCE TIME DISTRIBUTION PROGRAM

COMMAND MENU

1 - SWITCH TO SHORT OUTPUT MODE
2 - SWITCH TO FULL OUTPUT MODE
3 - READ/INTERPOLATE DISCHARGE DATA ON TAPE8
4 - READ/INTERPOLATE FEED DATA ON TAPE7
5 - ANALYSIS USING AUSTIN TECHNIQUE PROGRAM
6 - ANALYSIS USING DIRECT DECONVOLUTION
7 - ANALYSIS USING GRAAIM LEAST SQUARES METHOD
8 - ANALYSIS USING MIXERS IN SERIES
9 - END PROGRAM


COMMAND NUMBER: 3
ENTERED RTD INTERPOLATION ROUTINE
READING FLOTATION CELLS --- OUTPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  26 RAW DATA POINTS
NOW HAVE  79 DATA POINTS IN TOTAL


COMMAND NUMBER: 4
ENTERED RTD INTERPOLATION ROUTINE
READING FLOTATION CELL --- INPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  79 RAW DATA POINTS
NOW HAVE  79 DATA POINTS IN TOTAL


COMMAND NUMBER: 2
FULL OUTPUT MODE


COMMAND NUMBER: 6
       TRACER MEDIUM FLOW RATE THROUGH UNIT: 1.
        QUANTITY OF TRACER ADDED AS IMPULSE: 2000.
                     FINISH ACCURACY FACTOR: .001


ENTERED DIRECT METHOD ROUTINE


STARTING PARAMETERS
              SAMPLING TIME INTERVAL:   20.0
TRACER MEDIUM FLOW RATE THROUGH UNIT:   1.00
          INITIAL QUANTITY OF TRACER:   .200E+04


ITERATION  1
     QUANTITY OF TRACER:   .203E+04
 AVERAGE RESIDENCE TIME:   163.
        VARIANCE OF RTD:   .954E+04
```

Table 13 (cont'd)

```
ITERATION  2
     QUANTITY OF TRACER:    .204E+04
 AVERAGE RESIDENCE TIME:    162.
        VARIANCE OF RTD:    .967E+04

.........................................................................

DIRECT METHOD FINAL RESULTS

       TRACER RECOVERED:  2037.
 AVERAGE RESIDENCE TIME:  162.3
        VARIANCE OF RTD:  9701.
```

TIME	RECYCLE CONCENTRATION	DISCHARGE CONCENTRATION	CORRECTED CONCENTRATION	NON-DIMENSIONAL RTD TIME	NON-DIMENSIONAL RTD CONCENTRATION
0.00	101.8	0.	0.	0.000	0.000
20.00	0.	3.460	3.463	.123	.276
40.00	0.	6.190	6.196	.247	.494
60.00	0.	8.100	8.108	.370	.646
80.00	.4550	9.100	9.109	.493	.726
100.00	.7800	9.100	9.093	.616	.724
120.00	1.168	8.830	8.784	.740	.700
140.00	1.509	8.470	8.355	.863	.666
160.00	1.791	7.650	7.432	.986	.592
180.00	2.019	7.200	6.850	1.109	.546
200.00	2.152	6.660	6.155	1.233	.490
220.00	2.220	6.300	5.623	1.356	.448
240.00	2.250	5.580	4.725	1.479	.376
260.00	2.210	4.410	3.376	1.602	.269
280.00	2.170	4.180	2.978	1.726	.237
300.00	2.100	3.680	2.322	1.849	.185
320.00	2.030	3.470	1.975	1.972	.157
340.00	1.950	3.320	1.711	2.095	.136
360.00	1.860	2.960	1.259	2.219	.100
380.00	1.780	2.761	.9922	2.342	.079
400.00	1.710	2.661	.8470	2.465	.067
420.00	1.650	2.600	.7593	2.588	.060
440.00	1.540	2.407	.5571	2.712	.044
460.00	1.470	2.210	.3674	2.835	.029
480.00	1.390	2.030	.2076	2.958	.017
500.00	1.310	1.931	.1399	3.081	.011
520.00	1.240	1.856	.1047	3.205	.008
540.00	1.170	1.790	.8708E-01	3.328	.007
560.00	1.120	1.702	.5277E-01	3.451	.004
580.00	1.060	1.611	.2066E-01	3.574	.002
600.00	1.000	1.520	0.	3.698	0.000

```
END OF DIRECT METHOD ROUTINE

COMMAND NUMBER: 9

NORMAL RTD PROGRAM COMPLETION
```

Table 14 – Example using GRAAIM subroutine on flotation cell test data

```
RESIDENCE TIME DISTRIBUTION PROGRAM

COMMAND MENU

1 - SWITCH TO SHORT OUTPUT MODE
2 - SWITCH TO FULL OUTPUT MODE
3 - READ/INTERPOLATE DISCHARGE DATA ON TAPE8
4 - READ/INTERPOLATE FEED DATA ON TAPE7
5 - ANALYSIS USING AUSTIN TECHNIQUE PROGRAM
6 - ANALYSIS USING DIRECT DECONVOLUTION
7 - ANALYSIS USING GRAAIM LEAST SQUARES METHOD
8 - ANALYSIS USING MIXERS IN SERIES
9 - END PROGRAM


COMMAND NUMBER: 3
ENTERED RTD INTERPOLATION ROUTINE
READING FLOTATION CELLS --- OUTPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  26 RAW DATA POINTS
NOW HAVE  79 DATA POINTS IN TOTAL


COMMAND NUMBER: 4
ENTERED RTD INTERPOLATION ROUTINE
READING FLOTATION CELL --- INPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  79 RAW DATA POINTS
NOW HAVE  79 DATA POINTS IN TOTAL


COMMAND NUMBER: 7
                  PLUG FLOW PURE TIME DELAY: 20.
                     FINISH ACCURACY FACTOR: .001
     MINIMUM NUMBER OF PARAMETERS NA AND NB: 2 2
   *MAXIMUM* NUMBER OF PARAMETERS NA AND NB: 3 4


*** ENTERED GRAAIM ROUTINE ***      NA:NB=2 2


ITERATION  1
 AVERAGE RESIDENCE TIME:   160.
        VARIANCE OF RTD:   .847E+04


ITERATION  2
 AVERAGE RESIDENCE TIME:   186.
        VARIANCE OF RTD:   .147E+05


ITERATION  3
 AVERAGE RESIDENCE TIME:   185.
        VARIANCE OF RTD:   .145E+05


ITERATION  4
 AVERAGE RESIDENCE TIME:   183.
        VARIANCE OF RTD:   .141E+05
```

Table 14 (cont'd)

```
ITERATION  5
 AVERAGE RESIDENCE TIME:   182.
        VARIANCE OF RTD:   .138E+05


ITERATION  6
 AVERAGE RESIDENCE TIME:   182.
        VARIANCE OF RTD:   .137E+05


GRAAIM LEAST SQUARES METHOD FINAL RESULTS


 AVERAGE RESIDENCE TIME:  181.4       INCLUDING PLUG FLOW OF: 20.00
        VARIANCE OF RTD:  .1370E+05
   PREDICTION CRITERION:  1.049
 STD. DEV. OF RESIDUALS:  .4484

    FUNCTION PARAMETERS       ABS.S.D.   REL.S.D.
    -------------------       --------   --------

         A1:    -1.619        .2184E-01       1.
         A2:     .6582        .2141E-01       3.
         B1:     .2639E-01    .4407E-02      17.
         B2:     .1270E-01    .3295E-02      26.



*** ENTERED GRAAIM ROUTINE ***      NA:NB=2 3


ITERATION  1
 AVERAGE RESIDENCE TIME:   197.
        VARIANCE OF RTD:   .213E+05


ITERATION  2
 AVERAGE RESIDENCE TIME:   168.
        VARIANCE OF RTD:   .122E+05


ITERATION  3
 AVERAGE RESIDENCE TIME:   166.
        VARIANCE OF RTD:   .114E+05


ITERATION  4
 AVERAGE RESIDENCE TIME:   166.
        VARIANCE OF RTD:   .113E+05


GRAAIM LEAST SQUARES METHOD FINAL RESULTS


 AVERAGE RESIDENCE TIME:  165.7       INCLUDING PLUG FLOW OF: 20.00
        VARIANCE OF RTD:  .1122E+05
   PREDICTION CRITERION:  .9606
 STD. DEV. OF RESIDUALS:  .1911
```

Table 14 (cont'd)

FUNCTION PARAMETERS		ABS.S.D.	REL.S.D.
A1:	-1.584	.2600E-01	2.
A2:	.6309	.2438E-01	4.
B1:	.3448E-01	.8721E-02	25.
B2:	.6727E-02	.5729E-02	85.
B3:	.5480E-02	.3062E-02	56.

```
*** ENTERED GRAAIM ROUTINE ***      NA:NB=3 2


ITERATION  1
 AVERAGE RESIDENCE TIME:   300.
        VARIANCE OF RTD:  -.100E+05


ITERATION  2
 AVERAGE RESIDENCE TIME:   171.
        VARIANCE OF RTD:   .926E+04


ITERATION  3
 AVERAGE RESIDENCE TIME:   167.
        VARIANCE OF RTD:   .966E+04


ITERATION  4
 AVERAGE RESIDENCE TIME:   169.
        VARIANCE OF RTD:   .106E+05


ITERATION  5
 AVERAGE RESIDENCE TIME:   171.
        VARIANCE OF RTD:   .115E+05


ITERATION  6
 AVERAGE RESIDENCE TIME:   173.
        VARIANCE OF RTD:   .122E+05


ITERATION  7
 AVERAGE RESIDENCE TIME:   174.
        VARIANCE OF RTD:   .126E+05

GRAAIM LEAST SQUARES METHOD FINAL RESULTS


 AVERAGE RESIDENCE TIME:  174.2       INCLUDING PLUG FLOW OF: 20.00
        VARIANCE OF RTD:  .1276E+05
   PREDICTION CRITERION:  .9618
 STD. DEV. OF RESIDUALS:  .2881
```

Table 14 (cont'd)

FUNCTION	PARAMETERS	ABS.S.D.	REL.S.D.
A1:	-1.759	.1044	6.
A2:	.9008	.1731	19.
A3:	-.1055	.7473E-01	71.
B1:	.2884E-01	.2248E-01	78.
B2:	.7644E-02	.1047E-01	137.

*** ENTERED GRAAIM ROUTINE ***   NA:NB=3 3

ITERATION 1
AVERAGE RESIDENCE TIME: 224.
VARIANCE OF RTD: .105E+05

ITERATION 2
AVERAGE RESIDENCE TIME: 155.
VARIANCE OF RTD: .152E+05

GRAAIM LEAST SQUARES METHOD FINAL RESULTS

AVERAGE RESIDENCE TIME: 164.6   INCLUDING PLUG FLOW OF: 20.00
VARIANCE OF RTD: .1541E+05
PREDICTION CRITERION: .8972
STD. DEV. OF RESIDUALS: .7536

FUNCTION	PARAMETERS	ABS.S.D.	REL.S.D.
A1:	-1.253	.1092	9.
A2:	.1361	.1745	128.
A3:	.1782	.7697E-01	43.
B1:	.8839E-01	.2969E-01	34.
B2:	-.5382E-01	.2749E-01	51.
B3:	.2716E-01	.8600E-02	32.

*** ENTERED GRAAIM ROUTINE ***   NA:NB=3 4

ITERATION 1
AVERAGE RESIDENCE TIME: 39.1
VARIANCE OF RTD: 146.

ITERATION 2
AVERAGE RESIDENCE TIME: 45.4
VARIANCE OF RTD: .128E+04

ITERATION 3
AVERAGE RESIDENCE TIME: 65.5
VARIANCE OF RTD: .429E+04

Table 14 (cont'd)

```
ITERATION  4
 AVERAGE RESIDENCE TIME:   102.
        VARIANCE OF RTD:   .870E+04
DIVERGING VALUE OF C


COMMAND NUMBER: 2
FULL OUTPUT MODE


COMMAND NUMBER: 7
                  PLUG FLOW PURE TIME DELAY: 20.
                     FINISH ACCURACY FACTOR: .001
     MINIMUM NUMBER OF PARAMETERS NA AND NB: 2 3
   *MAXIMUM* NUMBER OF PARAMETERS NA AND NB: 2 3


*** ENTERED GRAAIM ROUTINE ***      NA:NB=2 3


STARTING PARAMETERS
   PLUG FLOW PURE DELAY:   20.0
 FINISH ACCURACY FACTOR:   .100E-02


ITERATION  1
 AVERAGE RESIDENCE TIME:   197.
        VARIANCE OF RTD:   .213E+05


ITERATION  2
 AVERAGE RESIDENCE TIME:   168.
        VARIANCE OF RTD:   .122E+05


ITERATION  3
 AVERAGE RESIDENCE TIME:   166.
        VARIANCE OF RTD:   .114E+05


ITERATION  4
 AVERAGE RESIDENCE TIME:   166.
        VARIANCE OF RTD:   .113E+05


............................................................................


GRAAIM LEAST SQUARES METHOD FINAL RESULTS


 AVERAGE RESIDENCE TIME:  165.7       INCLUDING PLUG FLOW OF: 20.00
        VARIANCE OF RTD:  .1122E+05
   PREDICTION CRITERION:  .9606
 STD. DEV. OF RESIDUALS:  .1911
```

Table 14 (cont'd)

FUNCTION PARAMETERS		ABS.S.D.	REL.S.D.
A1:	-1.584	.2600E-01	2.
A2:	.6309	.2438E-01	4.
B1:	.3448E-01	.8721E-02	25.
B2:	.6727E-02	.5729E-02	85.
B3:	.5480E-02	.3062E-02	56.

TIME	FEED SIGNAL	DISCHARGE SIGNAL	MODEL DISCHARGE	IMPULSE RESPONSE	NON-DIMENSIONAL RTD TIME	CONCENTRATION
0.00	100.0	0.	0.	0.	0.00	0.000
20.00	0.	3.460	3.460	.1724E-02	.12	.286
40.00	0.	6.190	6.154	.3068E-02	.24	.508
60.00	0.	8.100	8.114	.4046E-02	.36	.670
80.00	.4550	9.100	8.972	.4474E-02	.48	.741
100.00	.7800	9.100	9.110	.4536E-02	.60	.752
120.00	1.168	8.830	8.802	.4363E-02	.72	.723
140.00	1.509	8.470	8.244	.4050E-02	.84	.671
160.00	1.791	7.650	7.572	.3663E-02	.97	.607
180.00	2.019	7.200	6.872	.3248E-02	1.09	.538
200.00	2.152	6.660	6.200	.2835E-02	1.21	.470
220.00	2.220	6.300	5.584	.2442E-02	1.33	.405
240.00	2.250	5.580	5.037	.2080E-02	1.45	.345
260.00	2.210	4.410	4.561	.1754E-02	1.57	.291
280.00	2.170	4.180	4.151	.1467E-02	1.69	.243
300.00	2.100	3.680	3.801	.1217E-02	1.81	.202
320.00	2.030	3.470	3.501	.1003E-02	1.93	.166
340.00	1.950	3.320	3.245	.8209E-03	2.05	.136
360.00	1.860	2.960	3.024	.6677E-03	2.17	.111
380.00	1.780	2.761	2.832	.5399E-03	2.29	.089
400.00	1.710	2.661	2.663	.4340E-03	2.41	.072
420.00	1.650	2.600	2.514	.3470E-03	2.53	.057
440.00	1.540	2.407	2.380	.2759E-03	2.66	.046
460.00	1.470	2.210	2.258	.2181E-03	2.78	.036
480.00	1.390	2.030	2.146	.1715E-03	2.90	.028
500.00	1.310	1.931	2.041	.1341E-03	3.02	.022
520.00	1.240	1.856	1.942	.1042E-03	3.14	.017
540.00	1.170	1.790	1.848	.8054E-04	3.26	.013
560.00	1.120	1.702	1.759	.6182E-04	3.38	.010
580.00	1.060	1.611	1.673	.4713E-04	3.50	.008
600.00	1.000	1.520	1.592	.3566E-04	3.62	.006
620.00	.9600	1.432	1.514	.2676E-04	3.74	.004
640.00	.9100	1.345	1.440	.1989E-04	3.86	.003
660.00	.8600	1.262	1.369	.1463E-04	3.98	.002
680.00	.8300	1.181	1.301	.1063E-04	4.10	.002
700.00	.7900	1.103	1.238	.7609E-05	4.22	.001

END OF GRAAIM ROUTINE

COMMAND NUMBER: 9

NORMAL RTD PROGRAM COMPLETION

Table 15 – Example using MIXERS package on flotation cell test data

```
MIXERS IN SERIES MODELLING

ENTERED RTD INTERPOLATION ROUTINE
READING FLOTATION CELLS --- OUTPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  26 RAW DATA POINTS
NOW HAVE  79 DATA POINTS IN TOTAL

                       MODEL TYPE: 3

     ENTER ESTIMATES OF
                  PLUG FLOW DELAY: 20
            MEAN RESIDENCE TIME 1: 80
            MEAN RESIDENCE TIME 2: 50
            MEAN RESIDENCE TIME 3: 30

   SEARCH IMPULSE AMPLITUDE ?(Y/N) : N

      FEED SIGNAL AVAILABLE ?(Y/N) : Y

ENTERED RTD INTERPOLATION ROUTINE
READING FLOTATION CELL --- INPUT SIGNAL
INTERPOLATION SUCCESSFULL
READ  79 RAW DATA POINTS
NOW HAVE  79 DATA POINTS IN TOTAL

                CONTROL DIRECTIVE: 3

ITERATION    4             53 FUNCTION VALUES           F =  .17629991E+01
  .8260E-02  98.23      49.13      26.92




FINAL RESULTS FOR MODEL TYPE 3

  STD. DEV. OF RESIDUALS:  .2831

         PLUG FLOW DELAY:  .8260E-02
   MEAN RESIDENCE TIME 1:  98.23
   MEAN RESIDENCE TIME 2:  49.13
   MEAN RESIDENCE TIME 3:  26.92

OPEN CIRCUIT AVERAGE RESIDENCE TIME:  175.0
                           VARIANCE:  .1290E+05
```

Table 15 (cont'd)

TIME	FEED SIGNAL	DISCHARGE SIGNAL	MODEL DISCHARGE	IMPULSE RESPONSE	DIMENSIONLESS	IMPULSE RESPONSE
0.00	100.0	0.	0.	0.	0.000	0.000
60.00	0.	8.100	7.499	.375E-02	.343	.656
80.00	.4550	9.100	8.856	.443E-02	.457	.775
100.00	.7800	9.100	9.298	.464E-02	.571	.813
120.00	1.168	8.830	9.108	.453E-02	.686	.794
140.00	1.509	8.470	8.546	.422E-02	.800	.740
160.00	1.791	7.650	7.809	.381E-02	.914	.667
180.00	2.019	7.200	7.027	.336E-02	1.028	.588
200.00	2.152	6.660	6.280	.291E-02	1.143	.509
220.00	2.220	6.300	5.608	.249E-02	1.257	.436
240.00	2.250	5.580	5.026	.211E-02	1.371	.369
260.00	2.210	4.410	4.534	.177E-02	1.485	.310
280.00	2.170	4.180	4.123	.148E-02	1.600	.259
300.00	2.100	3.680	3.781	.123E-02	1.714	.216
320.00	2.030	3.470	3.496	.102E-02	1.828	.179
340.00	1.950	3.320	3.255	.845E-03	1.942	.148
360.00	1.860	2.960	3.050	.697E-03	2.057	.122
420.00	1.650	2.600	2.573	.387E-03	2.399	.068
480.00	1.390	2.030	2.217	.213E-03	2.742	.037
540.00	1.170	1.790	1.918	.116E-03	3.085	.020
600.00	1.000	1.520	1.653	.636E-04	3.428	.011
720.00	.7500	1.030	1.219	.189E-04	4.113	.003
840.00	.6000	.7100	.9065	.560E-05	4.799	.001
1140.00	.2700	.2600	.4629	.268E-06	6.513	.000
1500.00	.2900E-01	0.	.1268	.696E-08	8.569	.000
1560.00	0.	0.	.9149E-01	.379E-08	8.912	.000

MODEL TYPE: 0

END OF MIXERS ROUTINE

4. DESCRIPTION AND USE OF RTD/MIXERS PROGRAM

4.1 INTRODUCTION

There are essentially six components in this package: a main program which interacts with the user and facilitates execution of the subroutines; an interpolation routine which reads data, interpolates when necessary, and stores the data in vectors for later processing; and finally, four different calculation methods which operate independently to extract the residence time distribution from the data. For reasons of space, the MIXERS option is a stand-alone program.

Interactive program execution requires that the remote terminal be associated with the FORTRAN logical unit numbers 5 for input and 6 for output. The routines to do this are installation dependent. Discharge data need to be associated with the FORTRAN logical unit number 8. The direct deconvolution and time-discrete options also require feed data which must be associated with logical unit number 7. The Austin, direct and discrete methods can generate a file (number 9) of open-circuit tracer concentration data for use with the time-continuous model (MIXERS option).

4.2 ALGORITHM

I. Print command menu (9 options).

II. Read integer command and perform one of the following options:

1. Set flag for abbreviated output.
2. Set flag for complete output.
3. Call RTDINT to read and interpolate discharge data.
4. Call RTDINT to read and interpolate feed data.
5. Read parameters for AUSTIN, then call AUSTIN.
   - Do AUSTIN calculations.
   - Print calculated concentration data on disk file (if full output mode).
6. Read parameters for DIRECT, then call DIRECT.
   - Do DIRECT calculations.
   - Print corrected concentration data on disk file (if full output mode).
7. Read parameters for GRAAIM, then call GRAAIM (time-discrete model).
   - Do GRAAIM calculations.
   - Print impulse response on disk file (if full output mode).
8. Refer to MIXERS program (time-continuous model).
9. Stop.

III. Go back to step II.

4.2.1 List of Options

The user is asked to select from the following list of options:

1. Switch to short output mode.
2. Switch to full output mode.
3. Read/interpolate discharge data on file 8.
4. Read/interpolate feed data on file 7.
5. Use Austin model.
6. Use direct deconvolution model.
7. Use time-discrete model.
8. Use MIXERS model.
9. End program.

All options are self-explanatory and are handled by separate routines as described below.

4.2.2 Options 1 and 2: Output Mode

By default, the output mode is set for abbreviated output when the program is started. In order to change the output mode, commands 1 and 2 can be used. (Use of option 7 with more than one parameter set will change the mode to abbreviated output.)

4.2.3 Options 3 and 4: Interpolation (RTDINT Program)

RTDINT reads tracer-test data from sequential disk files. Input for options 5, 6, and 7 must be evenly spaced in time. If the raw data are not evenly spaced, then RTDINT interpolates any necessary points (Appendix H). Even when the raw data are evenly spaced, RTDINT must be used to read the raw data.

4.2.3.1 RTDINT input (files 7 and 8)

RTDINT reads tracer-test data in a batch mode from files connected to logical units 7 and 8. The format of these files are given in Table 16.

Table 16 – Description of the input file of the RTD program

Record No.		Data description	FORTRAN format
1.	Alphanumeric title for file		6A10
2.	Number of sample points (n)	Desired sampling interval	13,2X,G10.4
3.	0.00	Sample concentration at time = 0.00	2G10.4
4.	Time after first sampling interval	Sample concentration	2G10.4
.	.	.	
.	.	.	.
x	Sample time if it defines a new sampling interval	Sample concentration	2G10.4
.	.	.	
.	.	.	.
2+n	Last sample time	Sample concentration	2G10.4

Notes

- The models assume no background of tracer. Any background should be subtracted from the feed and discharge samples prior to entry in the files.
- G10.4 format tells the user to use ten or fewer characters including the decimal point (mandatory) and the power of ten exponent (optional) (i.e., 2.56 or 2.56 E1).
- The desired sampling interval defined in both the feed and discharge data files should be the same.
- The first and last sampling times should be the same for both files.
- The number of sampling points (n) in each file must be at least four.
- When the feed includes an impulse it could result in poor values being interpolated near the impulse.
- When a record contains a sample concentration and a blank value for the time, the program assumes that the current time interval is the same as the last time interval. Therefore, times need only be explicitly entered in the file when they define a new sampling interval.
- Current program memory space limits the total number of sample concentration values (after interpolation) to 200.

An example data file appears in Section 3.1.2 (Table 2).

4.2.3.2 RTDINT output

Detection of an error will result in an appropriate error message and a request for a new command number. If this happens, it is often necessary to stop the program and correct the data file before resuming.

If the file is successfully read and interpolated, then the number of data points before (n) and after interpolation will be printed. Note that options 5, 6, and 7 require equispaced data. Therefore, RTDINT generates data points as necessary to create a set of points evenly separated by the desired sampling interval. In addition, because negative concentration values are meaningless, any interpolated point which would be negative is assigned values of zero.

In full output mode, the complete final data set is printed out. All interpolated points are labelled.

4.2.3.3 Error messages for RTDINT

There are five error messages for the RTDINT program. These are named and described below.

BAD NUMBER OF DATA POINTS, N = n
This occurs when n<4, to allow the interpolation. Note that a sound RTD measurement should involve at least 20 points.

TIME SEQUENCE ERROR DETECTED NEAR RECORD n
When the sampling time does not explicitly appear in the data file, it is calculated by adding the current sampling interval to the last sampling time (explicit or calculated). The current sampling interval is the time between the last two explicitly entered sampling times. It is therefore possible that a calculated sampling time can post-date the next sampling time explicitly entered in the file. The time values should be corrected accordingly.

TOO MANY POINTS TO INTERPOLATE
A maximum of 20 points (based on the desired sampling interval) can be interpolated between any two raw data points. If this error occurs, more intermediate data points should be supplied or the desired sampling interval should be made longer.

SAMPLING INTERVAL TOO SMALL, DT = x
The smallest sampling interval entered in record 2 of the data file is less than 0.01. Check that the desired sampling interval is entered properly.

TOO MANY INTERVALS
The maximum memory space available for tracer data after interpolation (= 200 points) has been exceeded. Reduce the number of time intervals or increase the space available in the program code.

4.2.4 Option 5: Austin Technique Calculations

The AUSTIN subroutine requires two parameters and three optional parameters. The main program automatically requests these parameters before proceeding with the calculations. All parameters should be entered free format, from a terminal.

4.2.4.1 Input required by the Austin method

(free format terminal entry)

The following prompts are issued to the user by the Austin method.

RECYCLE TIME DELAY
This is the time required for the tracer to travel from the unit discharge back to the unit input. This time must be at least as long as one sampling interval. Since the calculations use time-discrete functions, the program rounds the delay to the nearest integer number of sampling intervals.

RECYCLE COEFFICIENT
This is the fraction α of the tracer in the unit discharge which is returned to the unit feed (see Eq 15).

TRACER MEDIUM FLOW RATE THROUGH UNIT
This is the absolute tracer medium flow rate through the unit. Note that the units of flow must be such that when the absolute quantity of tracer is divided by this flow rate, the resulting concentration and time units agree with those in the data files. For example, if the concentrations are given in ppm, and the quantity of tracer is known in moles, then the flow rate must be in millions of moles per unit time. If unknown, enter zero.

QUANTITY OF TRACER ADDED AS IMPULSE
The Austin method requires an initial estimate of the area under the open-circuit impulse response curve (impulse amplitude). If both the quantity of tracer (T) added as an impulse and the medium flow rate (Q) are known, then this area is estimated as the quotient T/Q. If neither is known, then the program assumes an area equal to twice the area under the closed-circuit RTD curve from time zero to the peak concentration. If unknown, enter zero.

FINISH ACCURACY FACTOR
The program finishes when a stable value is found for the area A under the open-circuit impulse response curve. This occurs when (1-A'/A) is smaller than the entered value. A zero entry activates default of 0.01.

4.2.4.2 Output of the Austin method

After each iteration, the integrated area under the open circuit impulse response A is printed along with the average residence time and the variance. When a stable value of A is determined, the final results are printed. In this case, the absolute quantity of tracer recovered in the discharge is given, rather than the integral A. Unfortunately, unless the tracer medium flow rate was supplied, the absolute quantity of tracer cannot be calculated.

In the full output mode, the listing also includes an echo of the interactive parameters and a table containing the following:

TIME	– the sampling time.
SAMPLE CONCENTRATION	– the measured or interpolated discharge tracer concentration.
CORRECTED CONCENTRATION	– the discharge tracer concentrations as they would be if the equipment operated in open circuit.
NON-DIMENSIONAL RTD	– the time is the real time divided by the average residence time, and the concentration is the corrected concentration multiplied by the average residence time.

In full output mode, the time and the normalized corrected concentration are also copied to a sequential file (logical unit 9) suitable for use with the program MIXERS. The normalized corrected concentration has an integrated area of one.

4.2.4.3 Error messages for option 5

There are two error messages for option 5 which are named and described below.

NO CONVERGENCE
The calculated impulse amplitude (the integrated area under the corrected concentration curve) is not tending towards a stable value. Check data or try other model.

TOO MANY ITERATIONS
A stable value of the impulse strength has not been found after ten iterations. Check terminal entry values or try other model.

4.2.5 Option 6: Direct Deconvolution Calculations

The direct deconvolution subroutine presents many similarities to the Austin algorithm from a user's point of view.

4.2.5.1 Input required by option 6

(free format terminal entry)

The direct deconvolution routine issues the following three prompts:

1. TRACER MEDIUM FLOW RATE THROUGH UNIT
2. QUANTITY OF TRACER ADDED AS IMPULSE
3. FINISH ACCURACY FACTOR

All these optional parameters are as described for option 5 (see Section 4.2.4).

4.2.5.2 Option 6: Output

The output for this routine is similar to the output for option 5, except that the feed tracer data used are also printed in the table.

4.2.5.3 Option 6: Error messages

There are two error messages for option 6 which are named and described below.

NO CONVERGENCE
The calculated impulse amplitude (the integrated area under the corrected concentration curve) is not tending towards a stable value. Check data or try other model.

TOO MANY ITERATIONS
A stable value of the impulse strength has not been found after ten iterations. Check terminal entry data or try other model.

4.2.6 Option 7: Time-Discrete Method Calculations

The time-discrete method searches for the best values of two sets of parameters involved in the definition of a recursive model. The complexity of the model depends on the number of these parameters. The program is written so that it can test all possible models, lying between a minimum and a maximum number of parameters in a sequential order.

4.2.6.1 Input required by option 7
(free format terminal entry)

The following prompts must be answered by the user of option 7.

PLUG FLOW PURE DELAY
This is the time elapsed between when the tracer first enters the equipment and when it first appears in the discharge. The value is automatically rounded to the nearest integer number of sampling intervals.

FINISH ACCURACY FACTOR
Option 7 calculates the sum of squared differences (C) between the previous set of filtered data and the current set of filtered data. The program finishes when the change in C between successive iterations becomes insignificant. This is when (1-C′/C) is smaller than the entered value. A zero entry activates default of 0.01.

MINIMUM NUMBER OF PARAMETERS NA AND NB
Commonly, NA and NB range between two and five. However, to facilitate the initial choice, the main program iteratively tests an entire range of numbers of parameters. Enter the minimum desired number of ***a*** and ***b*** parameters. Remember that NB cannot be larger than NA+1. The program checks that the minimum desired number of ***b*** parameters is not larger than the minimum number of ***a*** parameters plus one. Bad input causes the minimum number of ***a*** and ***b*** parameters to be requested again.

MAXIMUM NUMBER OF PARAMETERS NA AND NB
If the calculations are desired for only one set of parameters, then the maximum and minimum number of parameters can be made equal. (**Warning**: If the calculations are done for a range of numbers of parameters, then the output flag will automatically be set for abbreviated output).

Remember that NB cannot exceed NA+1. Also, because of limited program space the maximum value of neither NA nor NB can exceed nine. In Appendix G, two criteria are given to help in the selection of the number of ***a*** and ***b*** parameters.

4.2.6.2 Option 7: Output

Option 7 causes a message to be issued and the current number of parameters NA and NB to be printed. After each iteration the average residence time and the variance are shown. If a stable set of filtered data is calculated, then the final results, which include the following, are printed.

AVERAGE RESIDENCE TIME
This is the open-circuit residence time as calculated from the impulse response discharge concentration curve.

VARIANCE OF RTD
This is the variance of the residence time distribution about the average.

PREDICTION CRITERION
This is the sum of squared differences between the last two sets of filtered data (Appendix F).

STANDARD DEVIATION OF RESIDUALS
This is the average of the sum of squared differences between the model discharge and the measured discharge.

FUNCTION PARAMETERS
A table shows the final values of the model parameters with their absolute and relative per cent standard deviations.

If the calculations are done for only one set of parameters and the full output flag is set, then a second table showing the following is also printed.

TIME	–the sampling time.
FEED SIGNAL	–the measured tracer concentration values for the feed.
DISCHARGE SIGNAL	–the measured tracer concentration values for the discharge.
MODEL DISCHARGE	–the discharge signal generated by the function parameters and the measured feed signal. A negative noise bias may result in negligibly small negative concentration values here.
IMPULSE RESPONSE	–the discharge signal that would be generated by the function parameters for a unit impulse feed signal.
NON-DIMENSIONAL RTD	–the time is the real time divided by the average residence time, and the concentration is the impulse response multiplied by the average residence time.

In full output mode, the time and unit impulse responses are also printed to a sequential file suitable for use with the program MIXERS.

4.2.6.3 Option 7: Error messages

There are three error messages for option 7 which are named and described below.

X-TRANSPOSE * X NOT INVERTIBLE IN MINV, DET = 0
The matrix inversion subroutine MINV (part of the IBM Scientific Subroutine Library) cannot invert the matrix. Check data only.

NOT ABLE TO MEET FINISH CRITERION
After 2 (NA+NB+1) iterations, the change in the prediction criterion is still not small enough to satisfy the FINISH ACCURACY FACTOR.

DIVERGING VALUE OF C
The value of the prediction criterion is diverging. Check data entry.

4.2.7 Option 8: Mixers-in-Series Model

Option 8 of the RTD program issues the self-explanatory message: USE SEPARATE MIXERS PROGRAM.

Although it is a separate program for reasons of size, the MIXERS program is described here in a format similar to that used for previous options 5, 6, and 7.

4.2.7.1 Input required by the MIXERS program
(free format terminal entry)

The data files contain the tracer data, while the user specifies the desired and estimated time constants from the terminal.

In general, the concentration of tracer in the feed and in the discharge is known. However, for the special case of an impulse of tracer in the feed, followed by tracer recycling from the discharge, it is not necessary to explicitly know the feed signal, since it can be internally generated by the program.

There are usually two data files: one for the feed, and one for the discharge. They should be associated with the FORTRAN logical unit numbers 7 and 8, respectively. Both files are identical to those used by the program (see Section 4.2.3).

When the program is started, its first operation is the reading and interpolation of the discharge tracer data file. When this step is completed, the file title, number of sampling times (n), and total number of points after interpolation are printed. Later, if the user indicates that a feed tracer data file is available, that file is similarly read and interpolated.

All interactive input and output are handled with the FORTRAN logical unit numbers 5 and 6, respectively. The program prints prompts for all the information it requires as described below.

MODEL TYPE
In the present program form, four different models are implemented. Enter 1, 2, 3 or 4 as desired. Entering 0 is the normal method of stopping the program.

Model type	Description
0	Normal program stop
1	One to nine identical mixers-in-series
2	Two differently-sized mixers
3	Three differently-sized mixers
4	Two identical and one different mixer

NUMBER OF MIXERS
This prompt occurs only with model type 1.
Enter 1, 2, or 3, etc. as desired, to a maximum of 9.

PLUG FLOW DELAY
Estimate the plug flow delay time in units consistent with the units of time in the tracer data files. The delay time is approximately the time between the first significant tracer concentration in the feed and the first significant tracer concentration in the discharge.

MEAN RESIDENCE TIME i
Estimate the mean residence time for the mixer i in the model. Note that the sum of the residence times plus the plug flow time is the total average residence time.

Bear in mind that the residence times appear in exponential expressions. Therefore, avoid large numbers that would result in numerical overflow. The capabilities of computers vary, but entries that result in numbers larger than 10^{10} are dangerously large. Also, residence times for differently-sized mixers should not be equal, or even nearly equal. If this happens, the program automatically uses a more appropriate model to avoid numerical errors.

SEARCH FOR IMPULSE AMPLITUDE (Y/N)?
Entering Y causes the program to search for a better value of the initial concentration of tracer in the feed. This is desirable if the feed signal is an impulse (see Appendix I).

N implies that the feed signal is well known and/or does not include an impulse.

INITIAL FEED CONCENTRATION
This prompt occurs only if feed includes an impulse.

When the program must search for the initial tracer concentration in the feed, a starting estimate must be supplied by the user. The initial concentration may be estimated by dividing the amplitude of the impulse by the smallest sampling interval (Eq 5,6). If the flow rate is not known, then use a value slightly less than the area under the discharge concentration curve.

When the feed signal includes an impulse that is so well known that it need not be a search variable, then enter the known value. This could occur, for example, when the discharge signal corresponds to a unit impulse response generated by another method (Options 5, 6 or 7). In that case, the initial feed concentration is one divided by the smallest sampling interval.

FEED SIGNAL AVAILABLE (Y/N)?
Entering Y causes a feed signal to be read from file and used in the calculations. If an initial feed concentration value is entered interactively, the entered value will be used with the feed signal.

Entering N implies either that the unit is operated in closed circuit and the feed signal can be generated by superimposing the initial impulse and the discharge signal after dilution and delay; or that the discharge signal is the device's open-circuit response to an impulse.

DATA IS FOR OPEN CIRCUIT (Y/N)?
The calculations and resultant computer time can be significantly reduced if the discharge signal results from a simple impulse in open circuit. Entering Y allows the program to take advantage of this simple case. This implies the entry of a known initial feed concentration, if an estimated value has not already been entered.

Entering N means that the recycle signal must be internally generated.

RECYCLE COEFFICIENT
This prompt occurs only if there is no feed signal and the circuit is closed.

When generating the feed signal, the discharge signal is multiplied by this fraction. An estimate suffices because the program searches for a better value.

RECYCLE DELAY
This prompt occurs only if there is no feed signal and the circuit is closed.

When generating the feed signal, the discharge signal is delayed from reaching the feed by this time. The recycle time must be greater than the smallest sampling interval. This number is necessarily rounded to the nearest integer number of smallest sampling intervals. Unfortunately, this integer cannot be adjusted by the search method and remains as observed. Any error in this quantity may result in an error in the average residence time of the same magnitude.

CONTROL DIRECTIVE
This allows the user to tailor the search procedure. Enter 4 to redefine the search routine parameters. Enter 1, 2, or 3 as desired to control the printout of intermediate search results. Enter 0 to reenter all terminal input starting from the model type.

Directive	Effect
0	Go back and get new input starting with the model type.
1	Print initial model estimates and intermediate results after a new minimum is found in each search direction.
2	Print intermediate results after each iteration.
3	Print results of last iteration only.
4	Permit user to enter own search parameters.

ESCALE:
This prompt occurs only if the control directive is 4.

The maximum step size is set as this fraction of the convergence limit. Enter 0 to retain the default value which is 0.3.

E1
This prompt occurs only if the control directive is 4.

This is the convergence limit (in % of the estimated value) for the first variable (plug flow delay). The limits for all other search variables (mean residence times, amplitude, recycle coefficient, and delay) are also subsequently requested. The search stops when no variable changes by more than this limit during an iteration. Enter 0 to retain the default value which is 0.01 + 10% of the initial estimate.

MAXIT
This prompt occurs only if the control directive is 4.

Specify the maximum number of iterations to be performed before aborting if no objective function minimum is found. If this limit is reached, the intermediate ***final*** results will still be printed. Enter 0 to retain the default value which is ten.

4.2.7.2 MIXERS program output

If the reading and interpolation of the data files are successful, a summary of the results for both files is printed. Then, the control directive varies the amount of output generated by the search procedure. The final results show the following:

STANDARD DEVIATION OF RESIDUALS
This is the average of the square root of the sum of squared difference between the measured sample concentration and the concentrations predicted by the model. It can be used as a crude figure of merit for comparing models, and for distinguishing local objective function minima from the absolute minimum.

PLUG FLOW DELAY
This is the final best value determined by the search.

MEAN RESIDENCE TIME i
This is the final best value determined by the search. (Other time constants appropriate to the model are also printed.)

NUMBER OF MIXERS:
(only if model type 1)

This is the number chosen by the user at the start of the program.

RECYCLE COEFFICIENT:
(only if no feed signal and closed circuit)

The final best value as determined by the search.

RECYCLE DELAY:
(only if no feed signal and closed circuit)

This value was entered by the user at the start of the program (rounded to the nearest integral number of sampling intervals).

OPEN-CIRCUIT AVERAGE RESIDENCE TIME
This is the average once-through residence time of the tracer injected in the feed. It is calculated from the analytical expression of the impulse response and can be used as a check for the residence time as calculated from the time constants and plug flow time.

VARIANCE
This is the variance of the residence time around the mean. It can be used as a check for the variance calculated from the time constants.

Finally, a table is printed with the following information.

TIME	– the time at which each discharge sample was taken.
FEED SIGNAL	– the feed signal (either from file or generated) used for the calculations.
DISCHARGE SIGNAL	– the discharge output signal from file.
MODEL DISCHARGE	– the discharge signal as produced by the model.
RESIDENCE TIME DISTRIBUTION	– this is the open-circuit response of the model to a unit impulse (normalized concentration curve).
DIMENSIONLESS IMPULSE RESPONSE	– Column 1: dimensionless time coordinate equal to the real time divided by the average residence time. Column 2: dimensionless transfer function equal to the transfer function multiplied by the average residence time.

The program then restarts by asking for another model type. This is useful because the objective function can have many minima. To find absolute minimum, it is advisable to run the same model type with several different sets of starting estimates.

4.2.7.3 MIXERS program: Error messages

The response to data entered with the wrong format depends not only on the program, but also on the computer.

Abnormal terminal entries for the following prompts will result in their being requested again:

- MODEL TYPE
- NUMBER OF MIXERS
- RECYCLE COEFFICIENT
- RECYCLE DELAY
- CONTROL DIRECTIVE.

Error messages originating in RTDINT can also be issued (see Section 4.2.3).

The following error messages originate in the search routine BOTM.

MAXIMUM CHANGE DOES NOT ALTER FUNCTION

The present set of search variables is in an area where the objective surface is so flat the search program cannot detect a slope or direction to move. Check that the arguments for the exponential terms of the transfer function are not numerically zero within the accuracy of the computer.

The program which calculates the objective function (CALCFX) arbitrarily assigns a value of 10^{20} whenever it detects an error in any of the parameters. This could occur, for example, if the plug flow time or initial tracer concentration were given negative values by the search program (BOTM). It could also occur if the time constant of two supposedly different mixers became nearly equal.

n ITERATIONS COMPLETED BY BOTM

A minimum has not been found after the maximum number of allowable iterations defined by MAXIT. Rerun starting with the intermediate estimates and/or increase the value of MAXIT using control directive 4.

ACCURACY LIMITED BY ERRORS IN F

Due to numerical errors the values returned by CALCFX are inconsistent. Try different starting estimates.

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APPENDIX A

CONVOLUTION INTEGRAL

CONVOLUTION INTEGRAL

Let us consider the simple example of a perfect mixer. The output concentration following an impulse of magnitude A is:

$$y(t) = \frac{A}{\tau}(e^{-t/\tau}) \qquad \text{Eq A.1}$$

where $\tau = W/Q$ is a time constant for the process (W – mill hold-up; Q – mill throughput).

If two consecutive impulses of an amplitude A_1 and A_2, each of duration Δt, are added to the feed, then the concentration of tracer in the feed is:

$$u = 0,\ t < 0 \text{ and } t > 2\Delta t$$

$$u_1 = \frac{A_1}{\Delta t},\ 0 < t < \theta t \qquad \text{Eq A.2}$$

$$u_2 = \frac{A_2}{\Delta t},\ \Delta t < t < 2\Delta t$$

The response of the mixer to each impulse is the same, but the second response is delayed. Therefore, the next response is:

$$y(t) = \frac{A_1}{\tau} e^{-Qt/W} + \frac{A_2}{\tau} S(t-\Delta t)\ e^{-Q(t-\Delta t)/W} \qquad \text{Eq A.3}$$

$$y(t) = \frac{u_1}{\tau} e^{-t/\tau} \Delta t + \frac{u_2}{\tau} S(t-\Delta t)\ e^{-(t-\Delta t)/\tau} \Delta t \qquad \text{Eq A.4}$$

where S is a unit step function (called Heaviside's):

$$S(t-\Delta t) = 0,\ t<\Delta t$$
$$S(t-\Delta t) = 1,\ t>\Delta t \qquad \text{Eq A.5}$$

In general, the response to n consecutive impulses is:

$$y(t) = \sum_{i=1}^{n} \frac{u_i}{\tau} S(t-(i-1)\Delta t)\ e^{-(t-(i-1)\Delta t)/\tau} \Delta t \qquad \text{Eq A.6}$$

In the limit, as Δt tends to dt, any feed concentration curve can be represented by a series of increasingly-narrower impulses. Therefore:

$$y(t) = \int_{z=0}^{z=t} \frac{u(z)}{\tau} s(t-z)\ e^{-(t-z)/\tau} dz \qquad \text{Eq A.7}$$

For convenience, define the function $h(t) = \frac{S(t)}{\tau} e^{-t/\tau}$.

This is called the unit impulse response to succinctly relate the discharge concentration to the feed concentration curve. The integral is called the convolution integral:

$$y(t) = \int_{z=0}^{z=t} u(z)h(t-z)dz \qquad \text{Eq A.8}$$

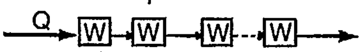
APPENDIX B

MIXERS-IN-SERIES IMPULSE RESPONSE

MIXERS-IN-SERIES IMPULSE RESPONSE

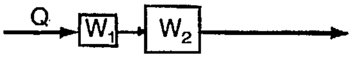
Equation 11 (see Section 1.2.2) was developed using a single perfect mixer as an example, but any combination of mixers can be used. The only effect is that the unit impulse response h(t) changes. Four different h(t) functions are programmed. They are:

1. n identical perfect mixers-in-series:



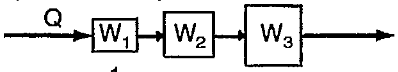
$$h(t) = \left(\frac{n}{\tau}\right) n \frac{t^{n-1} e^{-nt/\tau}}{(n-1)!} \qquad \text{Eq B.1}$$

2. Two mixers of different sizes:



$$h(t) = \frac{1}{\tau_1 - \tau_2}\left(e^{-t/\tau_1} - e^{-1/\tau_2}\right) \qquad \text{Eq B.2}$$

3. Three mixers of different sizes:



$$h(t) = \frac{1}{\tau_1 - \tau_2}\frac{\tau_1}{\tau_1 - \tau_3} e^{-t/\tau_1} - \frac{\tau_2}{\tau_2 - \tau_3} e^{t/\tau_2} + \left(\frac{\tau_2}{\tau_2 - \tau_3} - \frac{\tau_3}{\tau_1 - \tau_3}\right) e^{-t/\tau_3} \qquad \text{Eq B.3}$$

4. Two identical and one different mixer:

$$h(t) = \frac{1}{\tau_1 - \tau_2}\left[\frac{t}{\tau_1} e^{-t/\tau_1} + \frac{\tau_2}{\tau_2 - \tau_1}\left(e^{-t/\tau_1} - e^{-t/\tau_2}\right)\right] \qquad \text{Eq B.4}$$

All the models can be extended to include a plug flow component (d) by substituting (t − d) for t in the unit impulse response h(t).

APPENDIX C

MATHEMATICS OF THE AUSTIN METHOD

MATHEMATICS OF THE AUSTIN METHOD

The Austin method corrects the measured output signal $y(t)$ to eliminate the recycle effect.

The process open loop response $y(t)$ to an impulse can be expressed by a convolution integral (see Section 1.2 and Appendix A):

$$y_1(t) = \int_{z=0}^{z=t} u(z)\, h(t-z)\, dz \qquad \text{Eq C.1}$$

When $u(t)$ is an impulse $A\delta(t)$ Eq C.1 leads to:

$$h(t) = \frac{y_1(t)}{A} \qquad \text{Eq C.2}$$

substituting C.2 into C.1:

$$y_1(t) = \frac{1}{A} \int_{z=0}^{z=t} u(z)\, y_1(t-z)\, dz \qquad \text{Eq C.3}$$

Let us consider next the first recycle. Only a fraction u of the output signal (see Section 2.2) is returned to the mill input and this after a delay ***d***. The new input signal to the mill is:

$$u_1(t) = \alpha y_1(t-d) \qquad \text{if } t \geq d$$

$$u_1(t) = 0 \qquad \text{if } t < d \qquad \text{Eq C.4}$$

The process response to $u_1(t)$ can be expressed using the integral convolution:

$$y_2(t) = \frac{1}{A} \int_{z=0}^{z=t} u_1(z)\, y_1(t-z)\, dz \qquad \text{Eq C.5}$$

or

$$y_2(t) = \begin{cases} 0 & , \text{if } t<d \\ \dfrac{\alpha}{A} \displaystyle\int_{z=0}^{z=t-d} y_1(z)\, y_1(t-d-z)\, dz, & \text{if } t \geq d \end{cases} \qquad \text{Eq C.6}$$

Since this new signal $y_2(t)$ will be recycled too, it is possible, following the same scheme, to build a recursive function $y_n(t)$ expressing the successive recycles:

$$y_n(t) = \begin{cases} 0 & , \text{if } t<d \\ \dfrac{\alpha}{A} \displaystyle\int_{z=(n-2)d}^{z=t-d} y_{n-1}(z)\, y_1(t-d-z)\, dz, & \text{if } t \geq d \end{cases} \qquad \text{Eq C.7}$$

The measured closed loop $y(t)$ response to an impulse can thus be expressed by the sum of all these $y_n(t)$:

$$y(t) = \sum_{n=1}^{\infty} y_n(t) \qquad \text{Eq C.8}$$

The problem is to extract $y_1(t)$ from Equation C.8 where $y(t)$ is known. Then $y_1(t)$ leads directly to the residence time distribution $h(t)$ (Eq C.2).

The numerical methods consist in computing $y_1(t)$ step-by-step, using Equation C.7 and C.8. For t varying between 0 and d, the first recycle has no effect on the output so that $y_1(t)$ is equal to $y(t)$, the measured output signal. For t varying from d to 2d, $y_2(t)$ is computed using Equation C.7 and $y_1(0 \leq t \leq d)$. During this time interval all other $y_n(t)$ have zero values since other recycles have no effect on the output due to delay time. Therefore, Equation C.8 can be used to compute $y_1(t)$ for t between d and 2d:

$$y_1(t) = y(t) - y_2(t), \text{ if } d < t < 2d \qquad \text{Eq C.9}$$

This scheme can be iteratively processed until $y_1(t)$ is completely known.

As can be seen, all these equations require a value for A. A is the tracer impulse magnitude, that is the total amount T of tracer added as an impulse, divided by the tracer medium flow rate Q. A is unknown since Q is generally unknown, and the whole calculation process has to be iterated.

Starting with an estimated value of A, a new A value can be calculated by:

$$A = \int_{0}^{\infty} y_1(t)\, dt \qquad \text{Eq C.10}$$

at the end of each iteration and used to start the next iteration. The calculation is stopped when the difference between the A values of two successive iterations is less than an accuracy factor. The residence time distribution $h(t)$ is then computed using Equation C.2 and the last values of A and $y_1(t)$.

APPENDIX D

DETERMINATION OF RECYCLE DELAY OR RECYCLE COEFFICIENT FROM INPUT AND OUTPUT SIGNALS VALUES

DETERMINATION OF RECYCLE DELAY OR RECYCLE COEFFICIENT FROM INPUT AND OUTPUT SIGNALS VALUES

When the recycle assumption of Equation 15 (see Section 2.2) is valid, the input and output signals are coupled by the following equation:

$$u(t) = u_o(t) + \alpha y(t-d) \qquad \text{Eq D.1}$$

where: $u(t)$ is the total input signal at time t;
$u_o(t)$ is the generated input signal at time t;
α is the recycle coefficient;
d is the recycle delay;
$y(t)$ is the output signal at time t.

Generally, it has been observed that, to a first approximation, a mixing process can be modelled by a series of n perfect mixers. The RTD of such a model is:

$$h(t) = \left(\frac{n}{\tau}\right)^n \frac{t^{n-1} e^{-nt/\tau}}{(n-1)!} \qquad \text{Eq D.2}$$

where τ is the global mean residence time. On a log scale, log h(t) would be almost proportional to t for t greater than τ, as can be seen in Equation D.3:

$$\log h(t) = \log\frac{(n/\tau)^n}{(n-1)!} + (n-1)\log t - \frac{nt}{\tau} \qquad \text{Eq D.3}$$

According to Equation 8 (Section 1.2.1) and Eq D.1, by plotting the logarithm of $u(t)-u_o(t)$ versus t, a straight line is expected for $t > \tau$ and when $u_o(t)$ is an impulse. If α is known, by plotting log $[\alpha y(t)]$ on the same sheet, a second straight line should be obtained. In this case, the recycle delay d can be evaluated as shown in Figure D.1.

In the opposite situation of d known and α unknown, the latter can be estimated by plotting together the logarithms of $u(t)-u_o(t)$ and $y(t-d)$ as shown in Figure D.2.

If both d and α are unknown, by accurately sampling the feed stream, it is possible to evaluate the recycling delay d as the difference in abscisses of the peaks of u(t) and y(t) drawn on a linear scale sheet (Fig. D.3) and then to determine α as described just above.

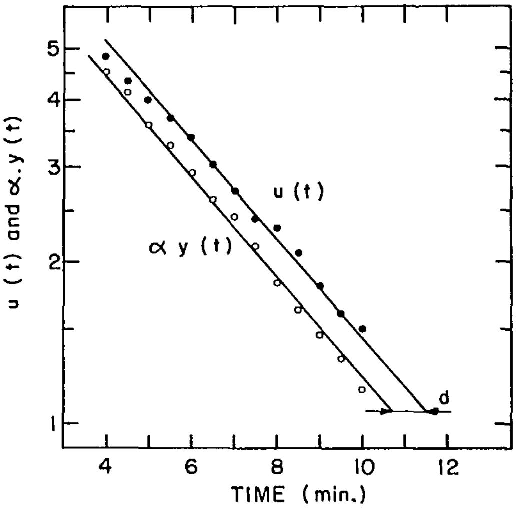


Fig. D.1 – Evaluation of the recycling delay (d) when the recycling coefficient (π) is known

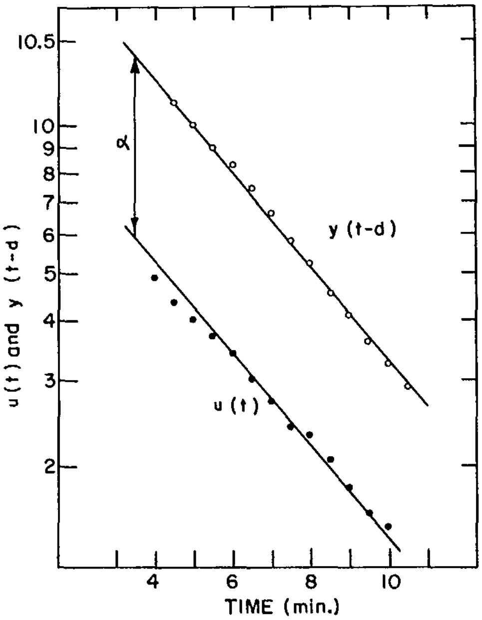


Fig. D.2 – Evaluation of the recycling coefficient (π) when the recycling delay (d) is known

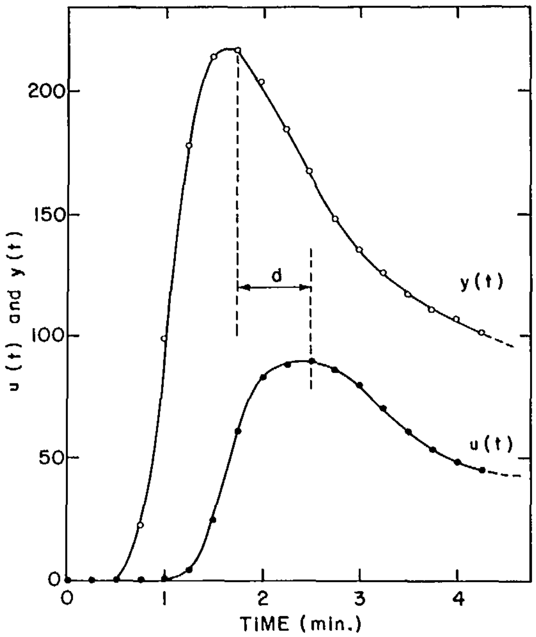


Fig. D.3 – Evaluation of the recycling delay (d) from input and output signal peaks

APPENDIX E

DIRECT DECONVOLUTION METHOD

DIRECT DECONVOLUTION METHOD

This method consists in discretizing the convolution integral (see Section 1.2 and Appendix A):

$$y(t) = \int_{z=0}^{z=t} u(z)\, h(t-z)\, dz \qquad \text{Eq E.1}$$

using the following expression:

$$y_i = \sum_{j=0}^{i} u_j\, h_{i-j}\, \Delta t \qquad \text{Eq E.2}$$

where: y_i is the measured output signal at instant i;
h_j is the residence time distribution value at instant j;
u_i is the measured input signal at instant i;
Δt is the time interval between two samples.

When {y}, {u}, and Δt are known, it is possible to calculate {h}, by solving step-by-step Equation E.2 written for all i values:

$$y_o = h_o u_o \Delta t$$

$$y_1 = (h_o u_1 + h_1 u_o)\Delta t$$

$$y_2 = (h_o u_2 + h_1 u_1 + h_2 u_o)\Delta t$$

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.

$$y_i = (h_o u_i + h_1 u_{i-1} + \ldots + h_j u_{i-j} + \ldots h_i u_o)\Delta t$$

$$y_n = (h_o u_n + h_1 u_{n-1} + \ldots + h_j u_{n-j} + \ldots + h_n u_o)\Delta t \qquad \text{Eq E.3}$$

The h_i values are calculated by successive eliminations:

$$h_i = (y_i - \sum_{j=0}^{i=1} h_j u_{i-j}\, \Delta t)/u_o \Delta t \qquad \text{Eq E.4}$$

Since the initial input signal is an impulse, $u_o \Delta t$ is the impulse magnitude A. This value is generally unknown, so that the process has to be iterated. An estimated value of A is used to compute h_i values. These should verify the following property (see Section 1.1):

$$\int_0^{\infty} h(t)\, dt = 1 \qquad \text{Eq E.5}$$

which is equivalent to the following discretized form:

$$\sum_{i=0}^{n} h_i \Delta t = 1 \qquad \text{Eq E.6}$$

Thus {h} should verify:

$$\sum_{i=0}^{n} A h_i \Delta t = A \qquad \text{Eq E.7}$$

A new estimated value of A is obtained from Equation E.7 and used to start a new iteration. The iterative process stops when the difference between two successive values of A is less than a given accuracy factor.

APPENDIX F

THE TIME-DISCRETE MODEL

THE TIME-DISCRETE MODEL

General Principles

This method is based on the general time-discrete model expressed by the following recursive equation:

$$v(i) + a_1 v(i-1) + a_2 v(i-2) + \ldots + a_n v(i-n) = b_o u(i) + b_1 u(i-1) + \ldots + b_m u(i-m) \quad \text{Eq F.1}$$

where: a_i (i = 1 to n) are parameters, the series of which is hereafter denoted {a};
b_i (i = 0 to m) are parameters, the series of which is hereafter denoted {b}; (m<n);
u(i) is the ith element of the input signal, {u};
v(i) is the ith element of the deterministic output signal, {v}; as opposed to the observed stocastic output signal {y}.

The objective is to find the {a} and {b} parameters. Since the measured values of {u} and {v} contain some measurement errors and since the model is linear, a noise can be introduced in the model by writing the following equation which expresses the resulting noise superimposed on the output signal {v}.

$$y(i) = v(i) + z(i) \quad \text{Eq F.2}$$

where z(i) is the resulting noise so that Equation F.1 becomes:

$$y(i) + \sum_{j=1}^{n} a_j y(i-j) = \sum_{j=0}^{m} b_j u(i-j) + z(i) + \sum_{j=1}^{n} a_j z(i-j) \quad \text{Eq F.3}$$

For the sake of simplicity, the following notation will be used:

$$Ay(i) = Bu(i) + e(i) \quad \text{Eq F.4}$$

where e(i) is the imbalance of Equation F.1.

The objective is, therefore, to find the mathematical operators A and B which minimize e(i) values.

Note 1

When a plug flow delay of k time intervals within the unit is observed, the measured output at time i is only dependent on the input signal from time $i' = i-k$ back to $i' = 0$. The simplest way to process this delay is to translate the input signal by k sampling time intervals:

$$u'(i) = u(i') \quad \text{Eq F.5}$$

and to use this signal u' instead of u in Equation F.4. This translation is automatically done by the program when a non-zero plug time is entered in option 7 of the RTD program.

Note 2

State Error and Prediction Error: It is important to introduce the distinction between these two kinds of errors. From Equation 14 (see Section 1.2.3):

$$\hat{y}(i) = -a_1 y(i-1) - a_2 y(i-2) - \ldots - a_n y(i-n) + b_o u(i) + b_1 u(i-1) + \ldots + b_m u(i-m) \quad \text{Eq F.6}$$

$\hat{y}_i$ is the output signal value predicted at time i by the model ({a} and {b}) using all the measured values from time 0 to $i-1$. Then it follows:

$$\hat{y}(i) - y(i) = e(i) \quad \text{Eq F.7}$$

which means that e(i) is the difference between the measured and predicted output signals after the ith time increment: it is called the **prediction error**.

The following value $y_M(i)$ calculated by:

$$y_M(i) = -a_1 y_M(i-1) - a_2 y_M(i-2) - \ldots - a_n y_M(i-n) + b_o u(i) + b_1 u(i-1) + \ldots + b_m u(i-m) \quad \text{Eq F.8}$$

is the output signal value at time i predicted by the model using output signal values $y_M(j)$ predicted by the model at time prior to i. Therefore:

$$e_M(i) = y_M(i) - y(i) \quad \text{Eq F.9}$$

is the difference between the measured signal and a signal depending only upon initial conditions and model parameters: it is called the **state error**.

The following section presents a method to compute the values of {a} and {b} parameters which minimize, in a least-squares sense, the **prediction error**.

The Least-Squares Method

Let us suppose that N couples of samples are available to describe input and output signals; if Equation F.3 is written for N-n of these couples, it follows:

$$\begin{bmatrix} y(N) \\ y(N-1) \\ \vdots \\ y(N-n-1) \\ y(N-n-2) \\ \vdots \\ y(n+1) \end{bmatrix} = \begin{bmatrix} -y(N-1)\ldots & -y(N-n) & u(N)\ldots & u(N-m) \\ -y(N-2)\ldots & -y(N-n-1) & u(N-1)\ldots & u(N-m-1) \\ \vdots & \vdots & \vdots & \vdots \\ -y(N-n-2)\ldots & -y(N-2n-1) & u(N-n-1)\ldots & u(N-m-n-1) \\ -y(N-n-3)\ldots & -y(N-2n-2) & u(N-n-2)\ldots & u(N-m-n-2) \\ \vdots & \vdots & \vdots & \vdots \\ -y(n)\ldots & -y(1) & u(n+1)\ldots & u(n+1-m) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ an \\ b_o \\ \vdots \\ b_m \end{bmatrix} + \begin{bmatrix} e(N) \\ e(N-1) \\ \vdots \\ e(N-n-1) \\ e(N-n-2) \\ \vdots \\ e(n+1) \end{bmatrix}$$

which can be written using a matrix form: $Y = X\,\theta + E$ Eq F.10

Y and X are known values, since they are directly derived from measurements; θ is the sequence of searched parameters; and E is the prediction error. In order to estimate θ, the sum C of squared prediction errors can be minimized:

$$C = \sum_{I=n+1}^{N} e^2(I) = E^T E \qquad \text{Eq F.11}$$

where E^T stands for the transposed matrix of E. C can be written as:

$$C = (Y - X\theta)^T \ (Y - X\theta) \qquad \text{Eq F.12}$$

or

$$C = Y^TY - \theta^TX^TY - Y^TX\theta + \theta^TX^TX\theta \qquad \text{Eq F.13}$$

C is an extremum with respect to θ, if its first derivative has a zero value:

$$\frac{\delta C}{\delta\theta} = -2X^TY + 2X^TX\theta = 0 \qquad \text{Eq F.14}$$

The solution of this equation is directly:

$$\theta = (X^TX)^{-1} X^TY \qquad \text{Eq F.15}$$

These θ values minimize C, if the second derivative is not negative, which is always verified since:

$$\frac{\delta^2 C}{\delta\theta^2} = 2X^TX \geq 0 \qquad \text{Eq F.16}$$

The θ values calculated from Equation F.15 are those which minimize the prediction error in a least-squares sense.

The least-squares method gives the best estimates of parameters, provided the prediction errors are white, i.e., uncorrelated or independent of each other. Actually, it can be shown that the estimates of {a} and {b} are biased because disturbances in the mixing process produce correlated errors. To take this problem into account, it is necessary to introduce a mathematical tool called a filter. A filter is a model capable of predicting the residuals e(i) of Equation F.7 from a white noise* ξ: $F^{-1}\xi(i) = e(i)$.

Generalized Least-Squares Method

The proposed generalized least-squares method uses an autoregressive filter of the following form:

$$\xi(i) = p(i) + f_1p(i-1) + \ldots + f_qp(i-q) \qquad \text{Eq F.17}$$

or

$$\xi(i) = F\,p(i) \qquad \text{Eq F.18}$$

where:

{f} are the filter parameters;
p(i) is the residual value;
$\xi(i)$ is the white noise value;
F is the filter operator.

The filter is applied to Equation F.4:

$$A\,F\,y(i) = B\,F\,u(i) + F\,e(i) \qquad \text{Eq F.19}$$

which can then be written:

$$A\,F\,y(i) = B\,F\,u(i) + \xi(I) \qquad \text{Eq F.20}$$

since:

$$\xi(i) = F\,e(i) \qquad \text{Eq F.21}$$

*A white noise is a sequence of independent random events, here gaussian, with zero mean and constant variance.

The products F y(i) and F u(i), are called the filtered output y* and input u*, respectively. From Equation F.20:

$$A\,y^*(i) = B\,u^*(i) + \xi(i) \qquad \text{Eq F.22}$$

By writing Equation F.21 for all i values from i = N to i = q+1, the following system or equation is obtained:

$$\begin{bmatrix} e(N) \\ e(N-1) \\ \cdot \\ \cdot \\ \cdot \\ e(q+1) \end{bmatrix} = \begin{bmatrix} -e(N-1) & \ldots -e(N-q) \\ -e(N-2) & \ldots -e(N-q-1) \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ -e(q) & -e(1) \end{bmatrix} \times \begin{bmatrix} f_1 \\ f_2 \\ \cdot \\ \cdot \\ \cdot \\ f_q \end{bmatrix} + \begin{bmatrix} \xi(N) \\ \xi(N-1) \\ \cdot \\ \cdot \\ \cdot \\ \xi(q+1) \end{bmatrix} \qquad \text{Eq F.23}$$

or in the matrix notation:

$$E = X_E\ f + \xi \qquad \text{Eq F.24}$$

The matrix f which produces the minimum sum of squared white noise ξ is obtained by a solution similar to Equation F.15:

$$f = (X_E^T X_E)^{-1}\, X_E^T E \qquad \text{Eq F.25}$$

Since the first estimates of θ (Eq F.15) are biased, the E values (Eq F.10) are also biased, and so are the f values. In order to solve this problem, it is necessary to iterate the process, according to the following steps:

1. Form matrix X (Eq F.10).
2. Compute θ values (Eq F.15).
3. Compute E values and Criterion C value (Eq F.10 and F.11).
4. Form matrix X_E (Eq F.24).
5. Compute f values (Eq F.25).
6. Apply the filter F to u and y values (Eq F.17).

   $$y^*(i) = y(i) + f_1 y(i-1) + \ldots + f_q y(i-q)$$

   $$u^*(i) = u(i) + f_1 u(i-1) + \ldots + f_q u(1-q)$$

7. Return to step 1 using u* and y* values until C does not decrease significantly.
8. When C is minimum, compute parameters accuracies and RTD curve, and from parameters and input signal values rebuild the output signal.
9. Print results.

APPENDIX G

TWO CRITERIA TO SELECT THE BEST NUMBERS OF *A* AND *B* PARAMETERS OF THE TIME-DISCRETE MODEL

TWO CRITERIA TO SELECT THE BEST NUMBERS OF *A* AND *B* PARAMETERS OF THE TIME-DISCRETE MODEL

The GLS method is an attempt to couple input and output discrete signals through a recursive equation (see Section 1.2). For a given number of *a* parameters (n) and *b* parameters (m), the GLS method calculates the best values of these parameters. Unfortunately, n and m are not known prior to calculations. To settle this problem, the technique consists in performing calculations for several combinations of n and m, and then selecting the best set of parameters. For selection purposes, two criteria are given here.

By plotting the prediction criterion, computed by the program versus n (when m is constant) and versus m (when n is constant), a curve is obtained. A typical example is given in Figure G.1. The n (or m) value corresponding to the elbow should be selected as the best one, since it gives a low criterion value for the lowest number of parameters. In the case shown, $n=3$ would be the answer of the problem.

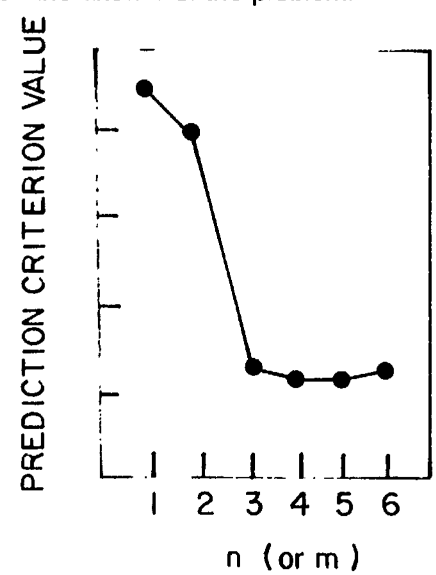


Fig. G.1 – An example of the decrease of the prediction criterion value with the model order

The second proposed criterion to select n and m numbers consists in examining the parameter values and inaccuracies calculated by the program. The following table gives a typical result. The test model has three *a* parameters and four *b* parameters.

Table G.1 – Parameter values and inaccuracies

	Parameter value	Accuracy (%)
a_1	−1.60087	5.82
a_2	0.77918	17.34
a_3	0.12135	40.08
b_0	0.00736	25.70
b_1	0.02535	17.34
b_2	0.01712	26.86
b_3	0.00709	64.88

We can see that a_3, the last parameter *a*, has a low value which is not very accurately determined compared to a_1 and a_2. Among *b* parameters, b_3 is low and not accurately determined, and b_0 is low but much more accurate. So it would be advisable to select ($n=3$, $m=3$) or ($n=2$, $m=3$) model, provided the prediction criteria for these cases are comparable.

With respect to the b_0 parameter, it must be pointed out that if b_0 is small and not accurately determined, the plug flow delay is under-evaluated. In this case, it is necessary to restart calculations with a plug flow delay increased by one sampling-time increment.

APPENDIX H

INTERPOLATION METHOD

INTERPOLATION METHOD

The model calculations require concentration values at evenly-spaced time intervals. If the data were not collected at evenly-spaced times, then the program must interpolate some values. Given any set of n distinct data pairs x_i and $f(x_i)$ (not necessarily evenly spaced), there is a polynomial of degree $(n-1)$ which exactly passes through them all.

The polynomial $P_{n-1}(x)$ can be expressed in numerous forms, one of which is the Lagrangian form:

$$P_{n-1}(x) = \sum_{j=1}^{n} \frac{\prod_{i \neq j} (x - x_i)}{\prod_{i \neq j} (x_j - x_i)} f(x_j)$$

In subroutine RTDINT, whenever an interpolated value is required, it is calculated by evaluating the cubic Lagrangian polynomial formed by using the two closest raw data points on each side of the point to be interpolated. At the ends of the data set, where the point to be interpolated cannot be centered among four points, the end four points are used.

APPENDIX I

CALCULATION OF THE INITIAL IMPULSE MAGNITUDE

CALCULATION OF THE INITIAL IMPULSE MAGNITUDE

If the feed signal u(t) consists of a strong initial impulse followed by a weak, smoothly-varying concentration curve, the calculation of y(t) can depend heavily on the magnitude of the initial impulse. Unfortunately, this magnitude is often poorly known in RTD experiments. Therefore, the program can adjust the initial impulse magnitude as necessary. The initial impulse is isolated from the convolution integral by decomposing the feed signal into two parts: the impulse and the remaining signal.

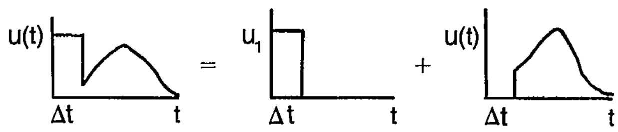
$$u_1 = \frac{T}{Q\Delta t},\ 0 < t < \Delta t$$

$$u(t) \text{ for } t > \Delta t$$

In this case Equation A.8 becomes:

$$y(t) = u_1 h(t)\Delta t + \int_{z=\Delta t}^{z=t} u(z)\, h(t-z)\, dz \qquad \text{Eq I.1}$$

The initial tracer concentration u_1 can be allowed to vary as a search variable. This is only important when the first term in Equation I.1 is significant compared to the second term.



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