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## Chapter 7.2 FINDBS Computer Program

## CANMET

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## The



Chapter 7.2 FINDBS Computer Program FINDBS - Program for Breakage and Selection Functions Determination in the Kinetic Model of Ball Mills

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## THE SPOC MANUAL

The SPOC* manual consists of eighteen chapters, published separately. Their numbers and short titles are as follows:

1. Summary
2. Unit Models: Part A
3. Sampling Methodology
2.1 SAMBA Computer Program
2.2 Grinding Circuit Sampling
5.1 Unit Models: Part B
4. Material Balance
3.1 BILMAT Computer Program
3.2 MATBAL Computer Program
5. Modelling and Simulation
4.1 Industrial Ball Mill Modelling
5.2 Unit Models: Part C
6. Flowsheet Simulators
7. Model Calibration
7.1 STAMP Computer Program
7.2 FINDBS Computer Program
7.3 RTD and MIXERS Computer Programs
8. Miscellaneous Computer Programs

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## FOREWORD

High energy costs and depleting ore reserves combine to make process evaluation and optimization a challenging goal in the 80's. The spectacular growth of computer technology in the same period has resulted in widely available computing power that can be distributed to the most remote mineral processing operations. The SPOC project, initiated at CANMET in 1980, has undertaken to provide Canadian industry with a coherent methodology for process evaluation and optimization assisted by computers. The SPOC Manual constitutes the written base of this methodology and covers most aspects of steady-state process evaluation and simulation. It is expected to facilitate industrial initiatives in data collection and model upgrading.
Creating a manual covering multidisciplinary topics and involving contributions from groups in universities, industry and government is a complex endeavour. The reader will undoubtedly notice some heterogeneities resulting from the necessary compromise between ideals and realistic objectives or, more simply, from oversight. Critiques to improve future editions are welcomed.
D. Laguitton

SPOC Project Leader
Canada Centre for Mineral and Energy Technology

## AVANT-PROPOS

La croissance des coûts de l'énergie et l'appauvrissement des gisements ont fait de l'évaluation et de l'optimisation des procédés un défi des années 80 au moment même où s'effectuait la dissémination de l'informatique jusqu'aux concentrateurs les plus isolés. Le projet SPOC, a été lancé en 1980 au CANMET, en vue de développer pour l'industrie canadienne, une méthodologie d'application de l'informatique à l'évaluation et à l'optimisation des procédés minéralurgiques. Le Manuel SPOC constitue la documentation écrite de cette méthodologie et en couvre les différents éléments. Les retombées devraient en être une vague nouvelle d'échantillonnages et d'amélioration de modèles.
La rédaction d'un ouvrage couvrant différentes disciplines et rassemblant des contributions de groupes aussi divers que les universités, l'industrie et le gouvernement est une tâche complexe. Le lecteur notera sans aucun doute des ambiguïtés ou contradictions qui ont pu résulter de la diversité des sources, de la traduction ou tout simplement d'erreurs. La critique constructive est encouragée afin de parvenir au format et au contenu de la meilleure qualité possible.
D. Laguitton

Chef du projet SPOC,
Centre canadien de la technologie des minéraux et de l'énergie

## ABSTRACT

This manual describes the various procedures involved in the development of a ball-mill simulator. The major task is the estimation of the breakage rate and distribution parameters which are central to the model. That is why the bulk of this volume is devoted to describing the methods, mathematics and computer programs used to calculate model parameters. Section 1 outlines the general structure of the kinetic bail-mill model. Section 2 focuses on the laboratory tests and computation techniques required to estimate the breakage distribution and rate parameters. Finally, Section 3 illustrates the methods using data from an operating industrial mill.

The computer programs are interactive and fully documented and can therefore be used without a detailed knowledge of the principles. However, it must be emphasized that a correct interpretation of the results depends on a thorough understanding of the kinetic model. For that reason, program users should at least read Section 1 describing the basis of the method. Further explanations and analysis appear in the remaining sections; the mathematics have been detailed in the appendices. The program structure should allow easy modifications.

## RÉSUMÉ

Ce manuel décrit les divers procédés en jeu dans la mise au point d'un simulateur d'un broyeur à boulets. La tâche principale consiste à évaluer la fonction de sélection et les paramètres de distribution des fragments qui constituent le centre du modèle. C'est pourquoi la majeure partie de ce volume est dédiée à la description des méthodes, des mathématiques et des programmes d'informatique utilisés pour calculer les paramètres du modèle. La section 1 présente les grandes lignes de la structure générale du modèle cinétique du broyeur à boulets. La section 2 porte sur les essais en laboratoire et sur les techniques de calcul utilisées pour évaluer la vitesse de broyage et les paramètres de distribution. Finalement, la section 3 illustre les méthodes à l'aide de données fournies par un broyeur industriel en opération.
Les programmes de calcul sont conversationnels et accompagnés d'une documentation complète et peuvent en conséquent être utilisés par quelqu'un qui ne possède pas de connaissance approfondie des principes. Cependant, il est clair que l'interprétation correcte des résultats dépend de la compréhension en profondeur du modèle cinétique. Pour cette raison, les utilisateurs du programme devraient au moins lire la première section qui décrit les fondements de la méthode. Les applications et les analyses plus détaillées se trouvent dans les sections deux et trois, alors que les détails des mathématiques se situent dans les annexes. La structure du programme devrait permettre de les modifier facilement.

## ACKNOWLEDGEMENTS

The SPOC project has benefited from such a wide range of contributions throughout the industry, the university, and the government sectors that a nominal acknowledgement would be bound to make unfair omissions. The main groups that contributed are: the various contractors who completed project elements; the Industrial Steering Committee members who met seven times to provide advice to the project leader; the various users of project documents and software who provided feedback on their experience; the CANMET Mineral Sciences Laboratories staff members who handled the considerable in-house task of software development, maintenance, and documentation; the EMR Computer Science Centre staff who were instrumental in some software development; and the CANMET Publications Section. Inasmuch as in a snow storm, every flake is responsible, their contributions are acknowledged.

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## 1. BASIC CONCEPTS OF THE BALL-MILL MATHEMATICAL MODEL

The mathematical model of the ball-mill described in this chapter is the phenomenological model derived from population balance considerations (1-3). In this model all breakage events which prevail in a mill under a given set of operating conditions are lumped together, and only an average of all the individual breakage events is considered to characterize the grinding performance of the mill. Two types of information are required when this model is used to predict the performance of a ballmill: the kinetics of breakage of the ore particles; and the flow pattern of the particles through the mill.

The breakage kinetics is conveniently described by dividing the entire population of particles into a series of discrete size-intervals and considering the rates at which each size-interval loses material due to breakage, and also the rates at which each size-interval receives the product of breakage from coarser size-intervals.
The rates of breakage and formation of particles, coupled with the time of grinding, determine the size distribution of the mill product. In the batch mode of operation, all the feed particles are subjected to the grinding action of the mill for the same length of time. However, in the continuous mode of operation, due to the random nature of the mixing action generated by the tumbling balls, all the particles that enter the mill together do not leave the mill at the same time. In fact, a wide variation is observed in their time of residence in the mill. A quantitative description of this variation is, therefore, also required to calculate the size distribution of the product of a continuous mill.

Based on the approach outlined above, in the following sections mathematical expressions are derived for the size distribution of the mill product for batch and continuous modes of operation. These derivations are preceded by the definitions of the basic elements of the mathematical model.

### 1.1 DISCRETE SIZE-INTERVALS

A discrete size-interval is defined as the size range bounded by the size of the mesh openings of two adjacent sieves in a standard series, normally characterized by a constant sieve-size ratio $\delta(=\sqrt{2}$ or 2 ). The entire size range of interest is divided into $n$ size-intervals. For the $i^{\text {th }}$ size-interval the size of the mesh openings of the upper sieve is denoted by $x_{1-1}$ and that of the lower one by $x_{1}$. The upper-most size-interval is represented by $i=1$, and the last size-interval by $i=n$. The mass fraction of the particulate solids in the $i^{\text {th }}$ size-interval is denoted by $\mathrm{M}_{\mathrm{i}}$. The fraction reporting to the pan corresponds to the finest size-interval and is denoted by $M_{n}$ (Fig. 1).


Flg. 1 - Illustration of the notations used in the mathematical model

### 1.2 BREAKAGE RATE PARAMETERS

For homogeneous materials, in a dry batch operation, the rate of loss of material from a size-interval $j$ can be described by the following first-order expression $(1,4,5)$ :

$$
\begin{equation*}
\frac{d M_{j}(t)}{d t}=-S_{j} M_{j}(t) \tag{Eq 1}
\end{equation*}
$$

where $S_{j}$ is a constant (similar to the reaction rate constant for a first-order chemical reaction), and is known as the rate parameter for size-interval $j$, also called the selection function in the grinding literature. From Equation 1 it follows that $\boldsymbol{S}_{\boldsymbol{J}}$ can also be defined as the specific rate of disappearance of material in size-interval $j$, and it has the dimension of time ${ }^{-1}$.

For particles of a given size, the magnitude of the rate parameter is determined by two main factors: the rate of occurrence of the breakage events in the mill for particles of this size; and the nature (shear or impact) and intensity of various breakage events in relation to the strength of the particles. The role of the first factor is obvious. The second set of factors determines the size distribution of the fragments of the broken particles (6), which in turn determines the fraction of the broken material that leaves the parent size-interval (3). Obviously, the greater the intensity of the breakage events, the higher the value of the rate parameter.

The rate parameter values for different sieve-size fractions can be plotted as a function of some characteristic
size associated with each size-interval*. Figure 2 shows a typical variation of the $S$ values with size. A log-log plot has been used in order to obtain uniform spacing between the size-intervals. For a given set of operating conditions, the $S$ values in general increase with particle size up to a certain limiting size, and then decrease continuously $(4,8)$. This behaviour results from the interaction between two basic factors which have opposite effects on the value of the rate parameter. The positive factor - the probability of being caught between two colliding surfaces $1 / \mathrm{N}$ as well as the negative factor $1 / \mathrm{N}$ the strength of the particles $1 / \mathrm{N}$ both tend to become more effective with increase in particle size $(7,8)$. Depending upon the intensity of the collisions (which is determined mainly by ball and mill diameter and mill speed), the positive factor dominates up to a certain size, beyond which increase in particle strength becomes more important and the particles cannot be broken effectively.
In addition to particle size and particle strength, particle load ( 5,8 ), ball size (8), ball load (9), mill speed (9), mill diameter and per cent solids in the pulp $(10,11,12)$ can be listed as important variables that determine the magnitude of the rate parameters, and also their relative variation with particle size (i.e., the shape of the $S_{1}$ vs $x_{1}$ curve). A detailed discussion on the effect of these variables can be found in the literature.

In the wet-grinding operation, the values of the rate parameters are also influenced by the overall size dis-


Fig. 2-Use of the log-log graph paper to show variation of the rate parameters with particle size
tribution of the particles in the mill $(13,14)$. In general, the finer the contents of the mill, the higher the $S$ values for each size-interval. However, the variation in $S$ values is not uniform for all size-intervals, and it becomes less pronounced as the particle size decreases (14).
It should be pointed out that as the last size-interval retains all the product of breakage, the value of the rate parameter is always zero for this size-interval.

### 1.3 BREAKAGE DISTRIBUTION PARAMETERS

A set of parameters $\boldsymbol{b}_{1, \text { is }}$ is used to describe the distribution of the material leaving a size-interval $j$, over the finer size-intervals $i=j+1, j+2, \ldots . n$. The distribution is represented as weight fractions in different size-intervals and, therefore, $b_{i, j}$ is a dimensionless parameter.
Conservation of mass requires that:

$$
\sum_{k=j+1}^{n} b_{k, j}=1
$$

Using this set of parameters, it becomes possible to write a mathematical expression for the rate of appearance of material in different size-intervals. For example, a size-interval $j$ loses material at a rate of $\mathrm{S}_{\mathrm{j}} \mathrm{M}_{\mathrm{j}}$ and a fraction $b_{i, j}$ of this material reports to a finer sizeinterval $i$. Therefore, for the size-interval $i$, the rate of appearance of material originating from the size-interval $j$ is given by: $b_{i, j} S_{j} M_{j}$.
Using the concept of distribution function (15), another set of parameters $\mathrm{B}_{\mathrm{i}, \mathrm{j}}$ is defined as follows:

$$
\begin{equation*}
B_{1, j}=\sum_{k=1+1}^{n} b_{k, j} \tag{Eq 3}
\end{equation*}
$$

Thus, $\mathrm{B}_{1, J}$ denotes the weight fraction of the material leaving size-interval j which is finer than size $\mathrm{x}_{\mathrm{i}}$. It follows that:

$$
\begin{array}{cc}
b_{i, j}=B_{i-1, \mathrm{~J}}-\mathrm{B}_{\mathrm{i}, \mathrm{~J}} & \text { Eq } 4 \\
\mathrm{~B}_{\mathrm{j}, \mathrm{~J}}=1 & \text { Eq } 5 \tag{Eq 5}
\end{array}
$$

and

$$
B_{n, j}=0
$$

Eq 6
The two sets of parameters $b_{1, j}$ and $B_{1, j}$ are often called distribution parameters and cumulative distribution parameters, respectively, or breakage and cumulative breakage functions.

[^0]Sometimes the values of all the individual parameters $\mathrm{B}_{\mathrm{i}, \mathrm{j}}$ and $\mathrm{b}_{\mathrm{i}, \mathrm{j}}$ depend only on the difference ( $\left.\mathrm{i}-\mathrm{j}\right)$. For example:

$$
\begin{aligned}
& B_{3,1}=B_{4,2}=B_{5,3}= \\
& B_{5,1}=B_{6,2}=B_{7,3}=
\end{aligned}
$$

In such a case, the set of distribution parameters is said to be difference-similar (3) or normalizable (4).
For a given size-fraction of a given material, the values of the distribution parameters are determined by the strength and nature of the breakage forces experienced by the particles. Some forces result in abrasion, some in chipping, and others in fracture of the particles (6). The proportions of the different types of forces depend mainly on mill speed (9), shape (16), size (8), and load (9) of the grinding media. Several investigators have reported that the distribution parameters are fairly independent of the ball and mill diameter ( $11,12,17$ ). Also, over the range of $30-40$ volume per cent solids in the pulp, the values of these parameters are found to be practically the same as those obtained under dry-grinding conditions $(11,13,14)$.

Figure 4 shows some typical values of the $\boldsymbol{B}$ parameters in the form of a distribution curve. This representation is convenient for graphical display of the $B$ values.
Figure 3 gives two examples of difference-similar and non-difference-similar sets of distribution parameters in the matrix form. The $\mathrm{j}^{\text {th }}$ column gives the distribution parameters for particles originating from the $j^{\text {th }}$ sizeinterval. A "-" mark in the matrix indicates that the corresponding $\mathrm{B}_{\mathrm{i}, \mathrm{j}}$ or $\mathrm{b}_{\mathrm{i}, \mathrm{j}}$ parameter is not defined, or is physically not meaningful. In Fig. 3a, only one curve has been drawn corresponding to $\mathrm{B}_{\mathrm{i}, 1}$ values. Because the distribution parameters are difference-similar, all other $\mathrm{B}_{\mathrm{i}, \mathrm{j}}$ values are also equally well represented by this curve. In Fig. 3b, separate curves have been drawn for different $j$ values because the $B$ parameters are non-difference-similar.
Generally, for macrocrystalline materials the distribution parameters are observed to be difference-similar and for polycrystalline and multicomponent materials these are observed to be non-difference-similar (13).


Fig. 3 - Examples of difference-similar and non-difference-similar distribution parameters


Fig. 4 - Representation of the $B$ values in the form of a distributlon curve

### 1.4 RESIDENCE TIME DISTRIBUTION

The variation in the time of residence of the particles in the mill is described by a function $\boldsymbol{H}(t)$, called the residence time distribution (RTD) (15). $\boldsymbol{H}\left(t^{\prime}\right)$ represents the weight fraction of the feed particles that leave the mill by a certain time $\mathrm{t}^{\prime}$. Thus, the difference $\left[\mathrm{H}\left(\mathrm{t}_{2}\right)-\mathrm{H}\left(\mathrm{t}_{1}\right)\right]$ gives the weight fraction of the feed for which the time of residence is in the $t_{1}$ to $t_{2}$ range. The time derivative of the function $\mathrm{H}(\mathrm{t})$ is represented by $\mathrm{h}(\mathrm{t})$, and is called the residence time density function. The quantity $h(t) d t$ can be identified as the weight fraction of the feed that leaves the mill during the time interval $t, t+d t$. The following two equations express the conservation of mass:

$$
\begin{array}{cc}
\int_{0}^{\infty} h(t) d t=1 & \text { Eq } 8 a \\
H(\infty)=1 & \text { Eq } 8 b
\end{array}
$$

It is obvious that $\mathrm{H}(0)$ is always zero, and $\mathrm{h}(\mathrm{t})$ is a positive-definite function. The relationship between $h(t)$ and $H(t)$ can also be expressed as:

$$
H(t)=\int_{0}^{t} h(t) d t
$$

Eq 9
The mean residence time of the particles, $\tau$, is given by:

$$
\begin{equation*}
\tau=\int_{0}^{\infty} \mathrm{th}(\mathrm{t}) \mathrm{dt} \tag{Eq 10}
\end{equation*}
$$

For comparison of the basic shapes of H or h functions, a dimensionless variable $\theta(=t / \tau)$ can be used. The corresponding functions $\mathrm{H}^{\star}(\theta)$ can be defined as:

$$
\begin{array}{ll}
H^{*}(\theta)=H(\tau \theta) & \text { Eq 11a } \\
h^{*}(\theta)=\tau h(\tau \theta) & \text { Eq 11b }
\end{array}
$$

The residence time distribution function can be regarded to a first approximation as independent of particle size ( $11,18-20$ ), and the basic shape of the RTD function can be considered to be independent of the operating conditions for a given mill, over the range of practical interest for different operating variables $(11,18)$.
A typical example of the residence time distribution function is shown in Figure 5.


Flg. 5 - A typlcal example of the residence time distributlon function in the density form

### 1.5 BATCH-GRINDING OPERATION

Using the concepts of rate and distribution parameters, for a dry batch-grinding operation the overall rate of change in the mass fraction $M_{i}(t)$ for a size interval $i$ can be expressed as:

$$
\frac{d M_{i}(t)}{d t}=-S_{i} M_{i}(t), i=1
$$

$$
\frac{d M_{i}(t)}{d t}=-S_{i} M_{i}(t)+\sum_{j=1}^{i-1} b_{i, j} S_{j} M_{j}(t), i=2,3, \ldots n
$$

In Equation 12b the summation term corresponds to the contributions from all size-intervals coarser than size interval i .
Various analytical solutions have been developed for the set of coupled differential equations in Equation 12 (1,21-23). Appendix A gives the details of the solution adopted in the computer program.
For the first size-interval, the solution to Equation 12a is very simple:

$$
\begin{equation*}
M_{1}(t)=M_{1}(0) \exp \left(-S_{1} t\right) \tag{Eq 13}
\end{equation*}
$$

This equation can be used to calculate the value of the rate parameter $S_{1}$ from the experimental data.

### 1.6 CONTINUOUS-GRINDING OPERATION

Using the concept of the residence time distribution, the mathematical model of the batch-grinding operation can easily be extended to describe the continuous-grinding operation. The mill product size distribution in the continuous mode of operation, in principle, should be the same as the one obtained by carrying out a weighted summation of the size distributions of the batch-ground products obtained from the same mill corresponding to time values ranging from zero to infinity, with $h(t)$ - the density of the residence time distribution - as the weighting factor. Mathematically this can be expressed as $(2,11,24,25)$ :

$$
\begin{equation*}
p_{1}=\int_{0}^{\infty} h(t) M_{1}(t) d t \tag{Eq 14}
\end{equation*}
$$

where $p_{1}$ is the mass fraction of the particulate solids in the $i^{\text {th }}$ size-interval of the mill discharge (or product). Substituting for $\mathrm{M}_{\mathrm{i}}(\mathrm{t})$ from the solution to Equation 12, and replacing $M_{i}(0)$ by $f_{i}$, the equivalent notation for representing the size distribution of the feed to the continuous mill, an appropriate expression can be obtained for the mill product size distribution $p_{1}$ as shown in Appendix A.
In the derivation of Equation 14 it has been assumed that the values of the rate parameters do not change along the length of the mill, although the size distribution of the mill contents changes. It has been shown that for the usual extent of variation in particle-size distribution across the mills in the plants, this assumption is valid (11).

Using the approach outlined above for the derivation of Equation 14, it is not possible to describe the size distribution of the mill contents along the length of the mill. For this reason, except for some special cases, it is not possible to write a general population balance equation similar to Equation 12 for the continuous-grinding operation for the batch-grinding operation. One special case is when the mill acts like a perfect mixer. In this case, the size distribution of the mill holdup of the solids $\left(w_{i}\right)$ is the same as that of the mill discharge $\left(p_{i}\right)$. At any instant of time for size-interval $i$ the rate of change of the mill
contents is equal to the net result of the feed and discharge rates, and of the breakage and production of size i particles. Let $\boldsymbol{W}, \boldsymbol{F}$ and $\boldsymbol{P}$ be the total hold-up weight of the solids in the mill, feed rate, and discharge rate of the solids, respectively; then the population balance can be mathematically expressed as:

$$
\begin{align*}
\frac{d w_{i}}{d t} & =F f_{i}-P p_{i}-W S_{i} w_{i} \\
& +W \sum_{j=1}^{i-1} b_{i, j} S_{j} w_{j}, i=2,3, \ldots n \tag{Eq 15}
\end{align*}
$$

Note that for steady state operation:

$$
\begin{equation*}
\frac{d w_{1}}{d t}=0 ; F=P ; W_{i}=p_{i} \tag{Eq 16}
\end{equation*}
$$

we obtain the following solution to the set of Equation 15:

$$
\begin{equation*}
p_{i}=\frac{f_{i}+\tau \sum_{j=1}^{i-1} b_{i, j} S_{j} p_{j}}{1+S_{i} \tau} \tag{Eq 17}
\end{equation*}
$$

where

$$
\begin{equation*}
\tau=W / F \tag{Eq 18}
\end{equation*}
$$

is the mean residence time.
In general, the residence time distribution does not correspond to that of a perfect mixer. Therefore, Equation 14 must be used. As the RTD information is usually not available in terms of an explicit mathematical functional form, use of Equation 14 involves evaluation of ( $\mathrm{n}-1$ ) integrals by numerical methods (Appendix A). It has been shown that for industrial mills, the mixing characteristics as characterized by the RTD function, can be closely approximated by considering the mill as two or more perfect mixers in series followed by a small pure-delay element (18-20). For this case and some others, explicit expressions can be obtained for the various integrals. These are given in Appendix A.

# 2. DETERMINATION OF THE BREAKAGE RATE AND DISTRIBUTION PARAMETERS 

From Equation 12 it follows that the mill-product size distribution is related to the mill feed size distribution through $\boldsymbol{n}$ rate and $\mathrm{n}(\mathrm{n}-1) / 2$ distribution parameters. Generally, the number $n$ is of the order 12-15. In practice, therefore, the total number of the unknown parameters would range from 78 to 120 , or more.
At present, it is not possible to predict the values of the $S$ and $B$ parameters from first principles using only fundamental physical properties of the mill-material system. The estimates of the model parameters must, therefore, be obtained from the experimental feed-product size distribution data.

An examination of the basic structure of the feed-product size distribution relationships would show that these expressions are highly non-linear in the S and B parameters (Appendix A). It is well known that even for a set of equations with moderate degree of non-linearity and only 10-20 unknown parameters, it is quite difficult to obtain accurate estimates of the parameters (26). Obviously, in our case, determination of one hundred or so rate and distribution parameters by usual methods is practically impossible.

Two main approaches to the problem have emerged from research in this area: direct estimation of the $S$ and B parameters using single-size feeds (such as 2 mm feed) for generating the size distribution data $(27,28)$, and indirect estimation of $S$ and $B$ using functional forms for describing the variation of these parameters with particle size (11,29-31). Although direct methods give more reliable results, these are not very attractive because an enormous effort is required for the preparation of the single-size feeds. In this manual, the more practical, indirect, functional form approach (also called back-calculation method) has been used.
It has been shown $(11,31)$ that several sets of feedproduct size distribution data corresponding to a wide range of mill feed size distributions are required for simultaneous estimation of all the S and B parameters by the functional form method (Appendix B). In the actual plant operation, the mill feed size distribution can be varied to only a very limited extent. Therefore, both sets of parameters cannot be obtained from the plant data alone. However, as the $B$ parameters are practically independent of the mill size and the mode of operation (wet/dry), these can be determined first from laboratory dry-batch-grinding tests (11-14). Thus, only the Sparameters need to be calculated from plant data. It will be shown in Section 3 that for most of the size intervals of interest, good estimates of the S parameters can be obtained from the plant data. This outlines the overall approach to the parameter estimation problem.

The following parts of this chapter are devoted to the details of the required laboratory batch-grinding tests,
the continuous tests in the plant, and the functional form method for parameter estimation.

### 2.1 CALCULATION TECHNIQUE

In using the functional form approach for the estimation of the $S$ and $B$ parameters, it is assumed that both the sets of parameters vary continuously with particle size and therefore their variation can be described in terms of mathematical functions. The two functions for the two sets of parameters can be called as rate function and breakage distribution function ( $11,29,30$ ). The problem of the estimation of one hundred or so model parameters therefore reduces to finding the values of only a few unknown constants in the chosen functions for the two sets of parameters.

### 2.1.1 The Data Requirements

Even though the $S$ and $B$ values generally vary systematically with particle size, the exact forms of the $S$ and $B$ functions cannot be known a priori. For this reason, in order to obtain the real values of the $S$ and $B$ parameters it is important that the breakage characteristics of all size-intervals of interest be well represented in the size distribution data used for the estimation of the parameters (31). The data requirements for the correct estimation of the $S$ and $B$ parameters are discussed in Appendix $B$.

### 2.1.2 Choice of Functional Forms

The correct choice of the functional forms for the S and B parameters is also very important for the success of this method. The accuracy of the parameter estimates depends on the closeness with which the assumed functional forms can describe the size variation of the parameters. Certainly, a very flexible form, such as a sixth-order polynomial, can describe complex variations in the parameter values quite closely. However, functions with too many unknown constants are not desirable. The choice of the functional form for one set of parameters should be consistent with that for the other set of parameters, because both types of parameters are interrelated through the basic size continuous functions $S\left(v_{s}\right)$ and $B(x, v)(3,32)$. Take for example, when $\mathrm{S}_{1}=\mathrm{s}_{1} \mathrm{x}_{1} \mathrm{~s}_{2}$ (where $\mathrm{s}_{1}$ and $\mathrm{s}_{2}$ are constants), the functional form for the B parameters must generate differencesimilar distribution parameters (33). However, if reasonably flexible functional forms are chosen for both S and B parameters, and if good data are used, then these functional forms can be confidently expected to provide a very close approximation to the real values. The only disadvantage in this case is that the values of the constants may not be unique. Depending upon the initial
values of the constants used in the optimization exercise (discussed later in this chapter), one can obtain different sets of values which will generate practically identical $S$ and $B$ values (31).

## Functional forms for B

The following three functional forms have been found to be quite suitable for representing the breakage distribution function for various materials (11,29-31):

$$
\begin{align*}
& B_{i, j}=b_{1}\left(\frac{x_{i}}{x_{j}}\right)^{b_{2}}+\left(1-b_{1}\right)\left(\frac{x_{i}}{x_{j}}\right)^{b_{3}} \quad \text { Eq } 19  \tag{Eq 19}\\
& B_{i, j}=b_{1}\left(\frac{1 .}{x_{j}}\right)^{b_{4}}\left(\frac{x_{i}}{x_{j}}\right)^{b_{2}}+\left(1-b_{1}\left(\frac{1 .}{x_{i}}\right)^{b_{4}}\right)\left(\frac{x_{i}}{x_{j}}\right)^{b_{3}} \text { Eq } 20  \tag{Eq 20}\\
& B_{i, j}=b_{1}\left(\frac{1 .}{x_{j}}\right)^{b_{4}}\left(\frac{x_{i}}{x_{j}}\right)^{b_{2}+b_{5} j} \\
&+\left(1-b_{1}\left(\frac{1 .}{x_{j}}\right)^{b_{4}}\right)\left(\frac{x_{1}}{x_{j}}\right)^{b_{3}+b_{6} j} \tag{Eq 21}
\end{align*}
$$

where $b_{1}, b_{2}, b_{3} \ldots b_{6}$ are constants and $R$ is the sievesize ratio. Equation 19 generates a difference-similar set of breakage distribution parameters. For small deviations from the difference-similar form, Equation 19 is good. For larger deviations from the difference-similar form, Equation 21 should be used ( $11,30,31$ ).

## Functional forms for S

The following two functional forms have been found to be quite suitable for the rate function:

$$
\begin{gather*}
S_{i}=\frac{s_{1} x_{i}^{s_{2}}}{1+s_{3} x_{i}^{s_{4}}}  \tag{Eq 22}\\
\ln S_{i}=s_{1}+s_{2}\left(\ln x_{i}\right)+s_{3}\left(\ln x_{i}\right)^{2}+s_{4}\left(\ln x_{i}\right)^{3}+\ldots \tag{Eq 23}
\end{gather*}
$$

where $\mathrm{s}_{1}, \mathrm{~s}_{2}, \mathrm{~s}_{3}$, etc. are constants. The functional form in Equation 23 is the most flexible one. Depending on the complexity of the shape of the rate function curve, higher order terms can be included in the polynomial. The functional form (23) is implemented in the program up to the third degree, but it could easily be extended to higher degrees, if needed.

### 2.1.3 Estimation of the Unknown Constants

A weighted least-squares method can be used to calculate the best values of the unknown constants in the $S$ and $B$ functions. A squared errors function is defined which provides a measure of the differences in the experimental and the calculated product-size distributions:

$$
\text { Er }=\sum_{\text {test }}^{\Sigma} \sum_{\substack{\text { size } \\ \text { intervals }}}\left(M_{i, j}-\hat{M}_{\mathrm{i}, \mathrm{j}}\right)^{2} \frac{1}{\sigma_{\mathrm{ij}}{ }^{2}} \quad \text { Eq } 24
$$

where $\sigma_{i j}^{2}$ is the variance associated with the $i^{\text {th }}$ sizeinterval mass fraction of the $j^{\text {th }}$ test. The statistics of the experimental errors are generally not known, except if the same test is repeated many times, which is improbable. This is the reason why, generally, Er is not weighted.

When the error Er is not weighted, it is in fact implicitly assumed that $\sigma_{i j}^{2}$ is independent of $i$ and $j$. This means that the measurement standard deviation $\sigma$ is constant. In this situation the results of the model parameter calculation are mainly influenced by the size-intervals containing significant amounts of material. Another limiting condition is the assumption of constant relative accuracy of the measured values. This implies that $\sigma_{i \mid}$ is proportional to the amount of material $M_{i, j}$ of size $i$ in the product. In this situation the results of the model parameter calculation can be strongly influenced and possibly biased by the size-intervals containing very little material.
Generally, it is acceptable to use an unweighted criterion; however, the following simple structure of the standard deviation of the measurements might be better:

$$
\begin{equation*}
\sigma=\mathrm{P}_{\mathrm{o}}+\alpha \mathrm{M}_{\mathrm{i}, \mathrm{j}} \tag{Eq 25}
\end{equation*}
$$

where $P_{0}$ is the threshold of sensitivity of the size analysis method. It can be taken as the smallest measureable mass fraction, for instance, the fraction which exhibits a $100 \%$ error ( $0.2 \%$ for a standard sieving technique). The $\alpha$ coefficient is the mean relative accuracy for the sizeinterval in which the weight percentage is very significant ( $10 \%$ for example). Generally, it can be taken between 5 to 10\%.
Another recommended weighted least-squares criterion is inspired from chi-square ( $\mathrm{x}^{2}$ ) statistics (34). The error function is given by:

$$
\begin{equation*}
E r=\Sigma \Sigma\left(M_{\mathrm{i}, \mathrm{j}}-\hat{M}_{\mathrm{i}, \mathrm{j}}\right)^{2} \frac{1}{\hat{M}_{\mathrm{i}, \mathrm{j}}} \tag{Eq 26}
\end{equation*}
$$

Table 1 gives the effect of this weighting strategy for three typical levels of agreement between calculated and experimental size distributions. Three different constant values are given to the ratio $(\mathrm{M}-\hat{\mathrm{M}})^{2} / \hat{M}$ corresponding to satisfactory, good, and excellent fit. For increasing $\hat{M}$ value, the allowed adjustment $\hat{M}$ - $\hat{M}$ increases but the relative adjustment (\% error in table) decreases. This is in agreement with the usual error structure of a size analysis: poor relative accuracy for small weights and better for large weights.
The best values of the unknown constants in the functional forms of $S$ and $B$ are then determined by a nonlinear programming method which minimizes Er.
In a first step, both sets of parameters can be obtained from the batch-grinding size distribution data. As
pointed out earlier, the same B parameters can then be used to estimate the S parameters for the mill in the plant.
The usual technique to minimize a function with respect to some variables is to solve the set of equations obtained by differentiating the function with respect to each variable and equating the derivatives to zero.
For the present case, the values of the unknown constants can be obtained by solving the following system of equations:

$$
\begin{array}{ll}
\frac{\delta E r}{\delta b_{k}}=0 ; k=1,2 \ldots . & \begin{array}{l}
\text { depending on the } \\
\text { functional form }
\end{array} \\
\frac{\delta E r}{\delta s_{k}}=0 ; k=1,2 \ldots & \text { selected }
\end{array}
$$

In fact, this system is strongly non-linear and very difficult to solve. When the breakage distribution function is known, it has been shown (35) that this system can be solved very efficiently by the Gauss-Newton method starting from a reasonable initial value of the rate parameters. However, when B and S parameters are simultaneously calculated, this method becomes very unstable. So it is preferable to select a method which does not involve the formal calculation of the derivatives. The Powell algorithm is based on an approximation of the conjugate gradient technique (36) and is one of the most powerful non-linear programming methods avallable which do not require derivatives.

### 2.2 BATCH-GRINDING TESTS

The main purpose of carrying out the laboratory batchgrinding tests is to obtain the estimates of the B parameters for the plant operation. It is, therefore, important that the laboratory tests be carried out under conditions which will simulate the breakage behaviour of the particles in the industrial mill. The various recommendations made in the following sections are based on this general criterion. Depending upon the actual operating conditions in a plant, suitable variations can be made by the user.

### 2.2.1 Apparatus

The tests should be carried out in a mill of at least 25 cm in diameter. The ball charge should consist of approximately equal numbers of 1.27 cm ( 0.5 in. ), 1.91 cm ( 0.75 in. ), and 2.5 cm ( 1 in. ) balls ( 8,27 , and $65 \mathrm{wt} \%$, respectively). In case balls of several different intermediate sizes are available (for example, from the mill in the plant), these can be used to duplicate continuous grinding ball size distributions. The weight of the ball charge should correspond to $35-40 \%$ apparent filling of the mill volume. The important considerations in the selection of the values of these parameters are: the profile of the tumbling ball charge; the number of layers of ball in the tumbling charge (at least 6-7 layers of ball should be there to simulate the actual grinding action of the plant mill); and the porosity of the ball charge. All these factors along with the mill speed determine the proportions of the different types of breakage events in the mill.
The mill should run at $70 \%$ of the critical speed. The critical speed can be calculated from the following expression:

$$
\begin{equation*}
\text { critcal speed }=\frac{423}{(D-\bar{d})^{0.5}} \mathrm{rpm} \tag{Eq 27}
\end{equation*}
$$

where $\boldsymbol{D}$ is the mill diameter in cm and $\boldsymbol{d}$ is the average ball diameter, which can be taken as 2 cm .

### 2.2.2 Preparation of the Test Feeds

Actual ball-mill feed obtained from the plant is best for the breakage tests. In case the rod mill discharge or the crushed run-of-mine ore is used for preparing the test feeds, this material should first be ground in the ball-mill for one to two minutes before the final samples are prepared. This procedure ensures, to a certain extent, that the material used in the laboratory tests is representative of the actual ball-mill feed in the plant in terms of the particle shape and strength distribution (31).
In the plant, the feed to the ball-mill may contain proportionately greater amounts of heavier minerals due to preferential classification by the hydrocyclones. This

Table 1 -Differences between the experimental and calculated values of M corresponding to three different values of the chi-square term, $\mathrm{Er}=(\mathbf{M}-\hat{\mathrm{M}})^{2} / \mathbf{M}$

|  | Relative error $[\mathrm{M}-\hat{\mathrm{M}}] \times 100$ |  |  |  | Relative error $[\mathrm{M}-\hat{\mathrm{M}}] \hat{\mathrm{M}}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\hat{\mathrm{M}} \times 100$ | $\mathrm{Er}=0.02500$ | $\mathrm{Er}=0.00625$ | $\mathrm{Er}=0.00156$ |  | $\mathrm{Er}=0.02500$ | $\mathrm{Er}=0.00625$ | $\mathrm{Er}=0.00156$ |
| 40 | 1.000 | 0.500 | 0.250 |  | 2.50 | 1.25 | 0.62 |
| 30 | 0.866 | 0.433 | 0.216 |  | 2.89 | 1.44 | 0.72 |
| 20 | 0.707 | 0.353 | 0.176 |  | 3.53 | 1.76 | 0.88 |
| 15 | 0.612 | 0.306 | 0.153 |  | 4.08 | 2.04 | 1.02 |
| 10 | 0.500 | 0.250 | 0.125 |  | 5.00 | 2.50 | 1.25 |
| 5 | 0.353 | 0.176 | 0.088 |  | 7.06 | 3.52 | 1.76 |
| 3 | 0.274 | 0.137 | 0.068 |  | 9.13 | 4.57 | 2.27 |
| 2 | 0.224 | 0.112 | 0.056 |  | 11.20 | 5.60 | 2.80 |
| 1 | 0.158 | 0.079 | 0.039 |  | 15.80 | 7.90 | 3.95 |
| Fit | Satisfactory | Good | Excellent |  | Satisfactory | Good | Excellent |

factor cannot be taken into account, if the actual plant ball-mill feed is not used for the tests.
Three or four feeds of different top size need to be prepared (Appendix B). The top size of the first feed should correspond to approximately the $90 \%$ passing size of the plant ball-mill feed. The top size of the last (the finest) feed should correspond to one or maximum two size-intervals above the last sieve-size interval used in the analysis of the plant data. For example, if the $90 \%$ passing size of the plant feed is 8 mesh and the last screen used in the size analysis is 270 mesh, then the top size-interval of the first feed would be 8/10 mesh and the top size-interval of the last feed would be 150/200 mesh. The typical size distributions of the test feeds are indicated in Table 2. The general guidelines are: the top-most size-interval should contain $50 \%$ or more material; the next size-interval should contain less than $35 \%$ of the amount in the top-most size-interval; and the remaining material should be well distributed over the finer size-intervals.

It is relatively difficult to prepare large amounts of the finer size fractions such as a 100/150 mesh size fraction. Fortunately, for these size fractions the values of the rate parameters are quite small. It is, therefore, possible to maintain a reasonably high weight ratio for the top-most size-interval with respect to the next size-interval for a longer period of time, even when the starting weight ratio is relatively smaller than the general recommended value of three. However, as indicated in Table 2, it is recommended that even in case of the finest feed, this ratio should not be less than two.
For each grinding experiment the weight of the test feed should correspond to $80-100 \%$ apparent filling of the void volume of the static ball charge in the mill. This
weight can be calculated by multiplying the void volume of the ball charge by the bulk density of the ground ore. The void volume of the balls can be measured by adding water to the mill until the water level is just at the top of the balls. The bulk density of the ore can be obtained by measuring the volume of a known weight of the ore sample with a measuring cylinder under loose packing condition.

### 2.2.3 Size Analysis

For size analysis, standard sieves of the Tyler series (size ratio $\sqrt{2}$ ) must be used. Standard sampling and screening procedures should be used. Good sampling, uniform screening time and uniform efficiency of screening for particles of all sizes are very important. It should be emphasized that as the percentage of fines in the sample increases, the sample weight cut for screening should be reduced. Sometimes, it may therefore be necessary to divide the original sample into two or three small samples and screen each one of them separately.

### 2.2.4 Details of Grinding Tests

It is preferable to grind each test feed twice. This way it is possible to check if the rate parameter of the top-most size fraction remains reasonably constant with grinding time - which is the main basis of the mathematical model. Moreover, with two sets of feed-product size distribution data for each starting feed, more reliable estimates of the S and B parameters can be obtained.
For an accurate estimation of all the B parameters, it is important that each size-interval receives a significant amount of the broken material. For this reason it is recommended that in each one of the two stages of

Table 2 - A typical set of starting feed size distributions

|  | Set of four feeds |  |  | Set of three feeds |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Size } \\ \text { (mesh) } \end{gathered}$ | interval micrometres* | Weight \% | $\begin{gathered} \text { Size } \\ \text { (mesh) } \end{gathered}$ | interval micrometres* | Weight \% |
| Feed 1 | 8/10 | 2000 | 65 | 8/10 | 2000 | 60 |
|  | 10/14 | 1400 | 20 | 10/14 | 1400 | 15 |
|  | -14 | -1180 | 15 | -14 | -1180 | 25 |
| Feed 2 | 20/28 | 710 | 75 | 28/35 | 500 | 68 |
|  | 28/35 | 500 | 18 | 35/48 | 355 | 15 |
|  | -35 | -425 | 7 | -48 | -300 | 17 |
| Feed 3 | 48/65 | 250 | 58 | 100/150 | 125 | 55 |
|  | 65/100 | 180 | 22 | 150/200 | 90 | 20 |
|  | -100 | -150 | 20 | -200 | -75 | 25 |
| Feed 4 | 150/200 | 90 | 50 |  |  |  |
|  | 200/270 | 63 | 25 |  |  |  |
|  | -270 | -53 | 25 |  |  |  |

[^1]grinding, the top-most size fraction should be reduced by 30-50 per cent. Certainly, an initial guess of grinding time is required for the first stage of grinding of each feed. In most cases a time period of 1.5 min will be found to be quite suitable for the coarsest feed. After carrying out the first experiment, the time period for the second experiment can be fixed more accurately with the help of the $S_{1}$ value obtained from the first experiment sizedistribution data. For $50 \%$ reduction, the expected duration of the second experiment can be calculated as (Eq 13):
\[

$$
\begin{equation*}
t_{2}=\frac{\ln 0.50}{S_{1}} \tag{Eq 28}
\end{equation*}
$$

\]

where

$$
\begin{equation*}
S_{1}=-\frac{\ln \left[M_{1}\left(t_{1}\right) / M_{1}(0)\right]}{t_{1}} \tag{Eq 29}
\end{equation*}
$$

For successively finer feeds the grinding time of the first experiment can be increased by $25 \%$. Depending on the result obtained in the first experiment, the grinding time for the second period can be adjusted in the same way as explained above.

### 2.3 CONTINUOUS TESTS

To be able to model the continuous mill, the minimum data requirements are the mill feed and product size distributions during one steady-state sampling campaign. However, in this case, only the product of the mean residence time $\tau$ and the rate parameters can be calculated ( $\tau \mathrm{S}_{\mathrm{j}}$ for size interval j ). This can be understood by looking at the mathematical solution of the continuous grinding equation given in Appendix $A$. When the $\mathrm{S}_{\mathrm{j}}$ parameters are multiplied by any constant factor the matrix $Z$ is not changed; consequently, the predicted product of a mill depends only upon the variables $\tau S_{j}$ present in the diagonal matrix V .
The product $\tau \mathrm{S}_{\mathrm{j}}$ varies strongly with the operating conditions of the mill. As a result it is not a suitable parameter to build a grinding mill model. The mean residence time $\tau$ is by definition the mass of ore retained in the mill divided by the ore throughput (Eq 18). Consequently, we have:

$$
\begin{equation*}
\tau S_{\mathrm{j}}=\frac{W S_{\mathrm{j}}}{F} \tag{Eq 30}
\end{equation*}
$$

When the throughput $F$ is measured, the value of only the product $S W$ is needed for the calculation of the mill product size distribution. This parameter can be named the absolute rate of breakage and denoted by $\overline{\mathrm{S}}_{\mathrm{j}}$. It is generally expressed in units of tons per minute.
Some simulation studies performed on a complex sulphide ore from New Brunswick at a pilot scale (11) and at
full scale $(37,38)$ indicated clearly that the $\bar{S}_{j}$ values are relatively insensitive to operating conditions in the normal range of operation. For a $2.7 \mathrm{~m} \times 3 \mathrm{~m}$ ball-mill $\overline{\mathrm{S}}$ was found almost constant for throughput varying from 100 to $200 \mathrm{t} / \mathrm{h}$. For a $40 \mathrm{~cm} \times 40 \mathrm{~cm}$ mill, $\overline{\mathrm{S}}$ was found almost constant for throughput varying from 70 to $110 \mathrm{~kg} / \mathrm{h}$ and for hold-up weight varying from 5 to 8 kg . The function $\overline{\mathrm{S}}$ is, therefore, a definitely better modelling parameter than the product $\mathrm{S}_{\tau}$.
When the true rate parameter $S$ is needed, the mean residence time $\tau$ must be known. The value of $\tau$ can be obtained by measuring the ore hold-up weight of the mill and the mill throughput. The measurement of the ore hold-up weight of the mill is very difficult to perform. The power drawn by the mill motor is influenced by the mill hold-up. However, the variations due to the ore hold-up are too small compared to those due to the ball charge. The sound emitted by the mill is related to the hold-up of pulp; however, this method needs a calibration. In addition, other factors - such as pulp rheology and ball load - influence the noise level. A suitable technique, although not perfect, is the tracer test.
Ideally, the best way to perform this kind of test is to tag the ore itself by irradiation (39). But in wet grinding, a simpler way is to trace water. Twelve experiments in a $40 \mathrm{~cm} \times 40 \mathrm{~cm}$ pilot mill, with a complex sulphide ore, Indicated that the mean residence time measured with a liquid tracer differs by less than $10 \%$ from the value obtained by emptying the mill and weighing the ore holdup (40).
Some tracer experiments performed with unbreakable particles have shown that the mean residence time (MRT) of small particles is the same as the water MRT, and that coarse particles show a trend to stay longer in the mill (41). Some authors (19) have observed that the MRT of water is $10 \%$ lower than the MRT of solids. As a conclusion, it is quite acceptable in the relatively fine size range of ball-mill grinding operation to measure the MRT of the water. Another Chapter 7.3 of the SPOC Manual (18) gives the experimental methods and the programs needed for such tracer tests.
It should be noted that the accuracy of the calculated MRT is affected by the choice of the mathematical representation used for the RTD function. However, the S calculation (when $\tau$ is known) and the $\bar{S}$ calculation (when only $F$ is known) is not very sensitive to the assumed model of the dimensionless RTD. A comparison of $\overline{\mathrm{S}}$ calculated for a $2.7 \mathrm{~m} \times 3.6 \mathrm{~m}$ mill, assuming a series of perfect mixers and a log normal representation of the RTD with the same variance, indicated a $10 \%$ variation (42).
In summary, the following table shows what can be calculated given the available data:

| Available data | Calculations possible |
| :---: | :---: |
| Feed and product size <br> distributions <br> and |  |
| reasonable approximation <br> of the dimensionless <br> RTD | $\left(\tau \mathrm{S}_{\mathrm{j}}\right)$ |
| All of the above plus <br> mill throughput | $\overline{\mathrm{S}}_{\mathrm{j}}$ |
| All of the above plus <br> the mean residence <br> time | $\mathrm{S}_{\mathrm{j}}$ and W <br> (the mill hold-up <br> mass) |

Generally, one set of data is largely insufficient to reliably determine the rate parameters because of: the natural experiment errors and disturbances which occur in an industrial environment; and the variation of the rate function with the operating conditions.
Before planning any experiments, the objectives must be clearly established in terms of the more important variables which should be studied for a given grinding circuit. The rate function of a mill of given size and speed of rotation depends on various factors, such as particlesize distribution, \% solids, pulp viscosity, pulp flowrate, ball-load, ore grindability, ore holdup.

A complete mathematical model based on this analysis requires the quantitative evaluation of the relationships between those factors. This procedure would require a voluminous set of data.
However, in a normal range of operation, where $\bar{S}$ is almost constant as well as the bail load, the main contributing factors to the value of $\bar{S}$ are:

- ore grindability
- particle-size distribution
- per cent solids throughput.
The experimental design must be performed according to the variables which are assumed to have prominent effect. In any circumstance, it is highly recommended to record the measurements of the following operating variables:
- feed and product size distribution;
- throughput;
- per cent solids;
- ore grindability;
- mean residence time;
- noise level;
- level of balls when the mill is stopped;
- size distribution of the balls;
- pulp viscosity.

The ore grindability is of prime importance in situations where it varies frequently, since by neglecting this factor
the effect of the other factors on the grinding kinetics may be hidden. The ore grindability can be inferred from data on comminution machines preceding the ball milling circuit (rod mill or crushing stage), or it can be directly measured by a standard Bond test or by the simplified batch-grinding procedure described in Appendix C.

In summary, the model will be as good as the data from which it is derived. In addition, the performance of the model outside the normal operating range will be better if a semi-empirical model is used (with utilization of the concept of transport phenomena analyzed in terms of rheology and ore hold-up) rather than a fully empirical model relating the kinetics to the input variables (feed rate, \% solids, size distribution).
For the execution of a sampling campaign, it is recommended to follow the procedures described in other Chapter 2 of the "SPOC Manual" (43). However, the following points must be emphasized:

- Check and maintain the steadystate operation of the circuit.
- Take samples during approximately a one-hour period.
- Take at least six samples from each location and make a composite sample at each location.
- Take as many samples as possible to have a redundant set of data.
- Carefully perform the size analysis.

Then, the raw data have to be adjusted by a mass balance program (44). It has been demonstrated (45) that the use of consistent data, which satisfy mass conservation laws, produces more reliable estimates of the model parameters.
In the computer program for continuous operation described in Section 4, a mixers-in-series model has been used for calculating the mill product size distribution. The mean residence time in each mixer should, therefore, be known for the calculation of the mill product size distribution as is shown in Appendix A. For details related to this aspect see reference (18).

### 2.4 GUIDELINES FOR THE ESTIMATION OF THE MODEL PARAMETERS

The following recommendations are made regarding the procedures to be followed for the estimation of the model parameters.

## Recommended Procedures

1. Calculate the S values for the top size-intervals of each feed from the batch-grinding data (use Eq 29 with appropriate modifications in time values for each period of grinding). Check the $S$ values for time-independence. The proposed method cannot be used if the $S$ values change significantly with grinding time.
2. In Equation 19 through 21, the values of the constants $b_{1}, b_{2}$ and $b_{3}$ depend on the sieve-size ratio (R). We recommend that a root-two sieve series always be used. In that case:

$$
R=x_{i} / x_{i+1}=\sqrt{2} .
$$

Also, the values of the constants $b_{4}, b_{5}$ and $b_{6}$ depend on the magnitude of the $x_{i}$ values. We suggest $x_{1}$ be the geometric mean size in millimetres (although it could also be the upper or lower limit of each size function). In the computer program described in the next section, it is only necessary to provide the $x_{i}$ value for the top size ( $i=1$ ). All other sizes are calculated as $\mathrm{x}_{\mathrm{i}}=\mathrm{X}_{1} / \mathrm{R}^{(i-1)}$.
3. The values of the constants in the functional forms for the rate parameters (Eq 22 and 23) depend on the magnitude of the $x_{i}$ values. Therefore, there is an obvious need for being consistent in the units of $\mathrm{X}_{1}$ values. The user can use any suitable units such as micron or millimetre. However, based on our experience, we recommend the use of millimetres.
4. Use a logarithmic graph paper to plot log $S$ vs log x , as shown in Figure 2. Draw the best-fit straight line on this graph and calculate the corresponding values of the constants $s_{1}$ and $s_{2}$ in Equation 23. These values can be used as the initial estimates of the two constants in the optimization exercise.
5. Start with the simplest functional forms for the B and S parameters (i.e., use Eq 19 and a first-order Eq 23). The initial values for the constants $b_{1}, b_{2}$ and $b_{3}$ can, in general, be taken as $0.4,1.0$ and 4.0 , respectively (a recommendation based on our experience).
In case these functional forms do not provide a satisfactory simulation of the experimental data, progressively higher order forms should be tried for the S parameters.
In case even a third-order S polynomial does not provide a satisfactory simulation of the experimental data, Equation 20 should be tried for the B parameters in combination with at least a secondorder polynomial for the $S$ parameters. The initial estimate of the constant $b_{4}$ can be taken as 0.1.

If increasing the order of polynomial to a value of four does not help, Equation 21 should be tried in combination with at least a third-order S polynomial. In most cases a fourth-order polynomial will be found to be quite satisfactory. The initial estimates of the constants $b_{5}$ and $b_{6}$ can be taken as 0.01 and 0.1 , respectively.
It should be emphasized that the order of S polynomial must increase with the complexity of the functional form for the B parameters, otherwise meaningless results can be obtained. For example, some of the $S$ and $B$ values can become negative, or some of the $B$ values may become greater than 1.0.
6. At the end of each estimation exercise, the error distribution should be studied for the predicted mass-fraction values. This can help in deciding the suitability of the functional forms and also provide hints for improving them.
It is also recommended to calculate the standard estimate of error, SE, which is defined as $(29,30)$ :

$$
S E=\sqrt{E r /(n a-n b)}
$$

where na is the total number of the mass-fraction values in the data set without the pan, and nb is the total number of constants in the two functional forms. It is expected that not only the error function Er will decrease, but simultaneously the SE value will also go down each time a new constant is added. If this does not happen, the basic structure of the functional forms should be modified.
7. The computer program also provides an additional option for minimizing the unweighted leastsquares error-function corresponding to cumulative mass-fraction passing values denoted by $P_{i}$ :

$$
P_{1}=\sum_{j=i-1}^{n} M_{j}=1-\sum_{k=1}^{i} M_{k}
$$

The error structure of $P_{i}$ values is not well understood, despite their general use in industrial data. This option should therefore be used with circumspection.

## 3. ILLUSTRATION OF THE METHOD

### 3.1 DATA COLLECTION

The simulation of a ball-mill requires knowledge of the rate and distribution parameters and mill-mixing properties. The following experiments provided data for determining these parameters. First a sampling campaign was performed around the grinding circuit of interest. Then batch-grinding tests were performed on rod mill discharge collected during the sampling campaign.

### 3.1.1 Industrial Ball-Mill Sampling Experiment

The ball-mill was operated in reverse closed circuit as shown in Figure 6.

To determine the flow rate through the ball-mill a complete mass balance sampling campaign was necessary. Enough data were collected to over-define the circuit balance so that a least-squares adjustment procedure could be used to obtain a reliable set of results despite unavoidable sampling, screening errors, etc.

Figure 6 shows the four locations accessible for slurry sampling. At each location a sample was cut every ten minutes for one hour and the six samples combined for composite location samples. In the case of the cyclone overflow and underflow, the design of the cyclone cluster did not permit sampling of combined streams. Each cyclone had to be sampled individually and the samples combined manually. The rod and mill discharge samples were obtained by reaching into the mill and scooping samples off the rotating trunnion.
In addition to the slurry samples, circuit instrumentation provided the rod mill and pump box water flow rates, the cyclone feed pulp density and the integrated rod mill


Fig. 6 - Industrial grinding circuit used for methodology demonstration
feed tonnage. The rod mill feed rate was then calculated as the integrated tonnage divided by the elapsed time. A monitoring computer program calculated signal variances for many of the process variables and provided a second estimate of the rod mill feed rate. Finally, a rod mill feed sample was taken to correct the feed rate for per cent moisture.
For demonstrative purposes, two different tracer experiments were performed for determining the residence time distribution function. The first immediately preceded the sampling campaign, and the second immediately followed the sampling campaign. Details of these experiments are given elsewhere (18).
Immediately after the sampling campaign, the slurry samples were weighted for density measurements, pressure filtered, and left to dry before screen analysis.

### 3.1.2 Mass Balance Calculations

The sampling campaign provided flow rates, pulp per cent of solids, and size distribution data. The BILMAT program (46) was used to make them self-consistent from a mass balance point of view. A relative standard deviation (accuracy of the measurement) was associated with each measured value. The flow rates and pulp per cent of solids accuracies were supplied by a computer program which monitored these variables during the sampling campaign. The standard deviations of the screen analysis results were estimated by assuming a minimum threshold variance ( $\sigma_{t h}^{2}$ ) plus a term proportional to the weight per cent retained on each screen $\left(w_{i}\right)$ :

$$
\begin{aligned}
& \text { Relatlve standard } \\
& \text { devlation (\%) }
\end{aligned}=100 \times \frac{\sqrt{\sigma_{t}^{2}\left(1+\frac{w_{1}}{40}\right)}}{w_{i}}
$$

The 1190 and 1680 micron screens were arbitrarily given a high standard deviation since it was observed that the 1680 micron screen was damaged.
Table 3 shows the program results. The RELAT. ST. $D E V$. column summarizes the percentage of accuracy given to each measured value. A 0.00 value in the MEAS. VALUES and the RELAT. ST. DEV. columns corresponds to an unmeasured variable. The values calculated by BILMAT are given in the ESTIM. VALUES columns, while the RESID. VALUES are the percentage adjustments performed by the program to make the data self-consistent.

Table 3 - BILMAT output of the mass balance results
Relative ore flow rates values:
Streams Relative flow rates

| MEAS.FD | 1.0000 |
| :--- | :--- |
| COMP.FD | 1.0000 |
| RM.DIS. | 1.0000 |
| CYC.FD. | 3.4902 |
| CYC.OF. | 1.0000 |
| BM.DIS. | 2.4902 |
| BM.FD. | 2.4902 |

## Mass balance results

| Streams | Pulp flow rates |  |  |  | Water flow rates |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Meas. values | Estim. values | Resid. values | Relat. St.dev. | Meas. values | Estim. values | Resid. values | Relat. St.dev. |
| MEAS.FD | 348.18 | 348.53 | . 10 | . 29 | 0.00 | 8.38 | 100.00 | 0.00 |
| COMP.FD | 349.90 | 348.53 | . 39 | . 57 | 0.00 | 8.38 | 100.00 | 0.00 |
| RM.DIS. | 0.00 | 438.89 | 100.00 | 0.00 | 0.00 | 98.73 | 100.00 | 0.00 |
| CYC.FD. | 0.00 | 1854.09 | 100.00 | 0.00 | 0.00 | 666.88 | 100.00 | 0.00 |
| CYC.OF. | 0.00 | 723.18 | 100.00 | 0.00 | 0.00 | 383.03 | 100.00 | 0.00 |
| BM.DIS. | 0.00 | 1130.91 | 100.00 | 0.00 | 0.00 | 283.86 | 100.00 | 0.00 |
| BM.FD. | 0.00 | 1130.91 | 100.00 | 0.00 | 0.00 | 283.86 | 100.00 | 0.00 |
| RMW | 0.00 | 90.36 | 100.00 | 0.00 | 82.00 | 90.36 | 10.19 | 15.00 |
| PBW | 0.00 | 284.29 | 100.00 | 0.00 | 291.00 | 284.29 | 2.31 | 6.87 |
| Streams | Solid flow rates |  |  |  | Pulp per cent solids |  |  |  |
|  | Meas. values | Estim. values | Resid. values | Relat. St.dev. | Meas. values | Estim. values | Resid. values | Relat. St.dev. |
| MEAS.FD | 0.00 | 340.15 | 100.00 | 0.00 | 97.60 | 97.60 | . 00 | . 20 |
| COMP.FD | 0.00 | 340.15 | 100.00 | 0.00 | 97.60 | 97.60 | . 00 | . 20 |
| RM.DIS. | 0.00 | 340.15 | 100.00 | 0.00 | 77.40 | 77.50 | . 13 | . 65 |
| CYC.FD. | 0.00 | 1187.20 | 100.00 | 0.00 | 63.90 | 64.03 | . 21 | 15.65 |
| CYC.OF. | 0.00 | 340.15 | 100.00 | 0.00 | 47.10 | 47.04 | . 14 | 1.06 |
| BM.DIS. | 0.00 | 847.05 | 100.00 | 0.00 | 75.00 | 74.90 | . 13 | . 67 |
| BM.FD. | 0.00 | 847.05 | 100.00 | 0.00 | 74.80 | 74.90 | . 13 | . 67 |

Table 3 - continued

| Streams | RM. DIS. |  |  |  | CYC. OF. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Meas. values | Estim. values | Resid. values | Relat. St.dev. | Meas. values | Estim. values | Resid. values | Relat. St.dev. |
| +6730 | . 04 | . 04 | 7.20 | 127.00 | 0.00 | . 00 | 100.00 | . 00 |
| 4750 | . 31 | . 31 | 1.01 | 18.13 | 0.00 | 0.00 | 100.00 | . 00 |
| 3360 | 1.18 | 1.18 | . 30 | 6.24 | 0.00 | . 00 | 100.00 | . 00 |
| 2380 | 4.37 | 4.29 | 1.91 | 3.14 | 0.00 | . 00 | 100.00 | . 00 |
| 1680 | 6.86 | 6.87 | . 17 | 27.29 | 0.00 | . 00 | 100.00 | . 01 |
| 1190 | 13.17 | 12.99 | 1.34 | 11.90 | . 28 | . 30 | 8.65 | 99.29 |
| 841 | 11.02 | 11.02 | . 00 | 2.45 | . 59 | . 59 | . 16 | 10.47 |
| 600 | 8.80 | 8.84 | . 47 | 2.57 | 2.10 | 2.10 | . 23 | 4.38 |
| 425 | 7.01 | 7.02 | . 13 | 2.71 | 5.31 | 5.31 | . 00 | 2.94 |
| 300 | 6.76 | 6.74 | . 27 | 2.74 | 9.62 | 9.67 | . 47 | 2.52 |
| 212 | 5.94 | 5.92 | . 32 | 2.84 | 12.03 | 12.11 | . 63 | 2.42 |
| 150 | 4.45 | 4.43 | . 42 | 3.12 | 10.03 | 10.11 | . 76 | 2.50 |
| 106 | 4.01 | 4.00 | . 20 | 3.25 | 9.03 | 9.07 | . 42 | 2.55 |
| 75 | 3.84 | 3.83 | . 18 | 3.30 | 8.45 | 8.48 | . 38 | 2.59 |
| 53 | 2.15 | 2.13 | . 76 | 4.33 | 4.58 | 4.62 | . 93 | 3.09 |
| 45 | 1.31 | 1.30 | . 99 | 11.63 | 2.62 | 2.65 | 1.28 | 7.82 |
| -45 | 18.78 | 19.08 | 1.60 | 4.61 | 35.36 | 35.00 | 1.03 | 4.28 |


| Streams | BM. DIS. |  |  |  | BM. FD. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Meas. values | Estim. values | Resid. values | Relat. St.dev. | Meas. values | Estim. values | Resid. values | Relat. St dev. |
| +6730 | . 09 | . 08 | 8.41 | 57.56 | . 09 | . 10 | 8.15 | 57.56 |
| 4750 | . 11 | . 12 | 6.02 | 47.45 | . 25 | . 24 | 3.05 | 22.00 |
| 3360 | . 34 | . 33 | 1.60 | 16.71 | . 80 | . 81 | . 87 | 8.25 |
| 2380 | 1.07 | 1.01 | 5.26 | 6.67 | 2.62 | 2.74 | 4.39 | 3.91 |
| 1680 | 1.22 | 1.20 | 1.63 | 60.98 | 4.05 | 3.96 | 2.23 | 32.35 |
| 1190 | 5.59 | 5.44 | 2.60 | 14.47 | 10.32 | 10.54 | 2.13 | 12.42 |
| 841 | 7.66 | 7.66 | . 02 | 2.65 | 11.85 | 11.85 | . 03 | 2.42 |
| 600 | 10.64 | 10.78 | 1.28 | 2.47 | 13.70 | 13.49 | 1.57 | 2.36 |
| 425 | 12.20 | 12.25 | . 41 | 2.41 | 13.00 | 12.94 | . 49 | 2.38 |
| 300 | 13.47 | 13.33 | 1.03 | 2.37 | 12.05 | 12.16 | . 89 | 2.41 |
| 212 | 10.96 | 10.84 | 1.12 | 2.46 | 8.28 | 8.35 | . 89 | 2.60 |
| 150 | 7.17 | 7.08 | 1.28 | 2.70 | 4.75 | 4.80 | 1.04 | 3.05 |
| 106 | 5.42 | 5.39 | . 56 | 2.92 | 3.34 | 3.36 | . 46 | 3.50 |
| 75 | 4.50 | 4.48 | . 49 | 3.11 | 2.60 | 2.61 | . 42 | 3.92 |
| 53 | 2.33 | 2.29 | 1.90 | 4.15 | 1.26 | 1.29 | 2.09 | 5.97 |
| 45 | 1.31 | 1.28 | 2.53 | 11.63 | . 71 | . 73 | 3.11 | 18.08 |
| -45 | 15.92 | 16.45 | 3.31 | 4.67 | 10.33 | 10.06 | 2.66 | 4.97 |

Table 4 - Batch-grinding tests data

| Mesh | Feed | Weight per cent retained on size |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 2.0-min product | 4.0-min product |  |
| 10 | 62.96 | 30.50 | 15.38 |  |
| 14 | 36.72 | 40.39 | 34.32 |  |
| 20 | 0.16 | 11.58 | 15.87 |  |
| 28 | 0.00 | 6.00 | 10.39 |  |
| 35 | 0.00 | 3.03 | 5.78 |  |
| 48 | 0.00 | 2.24 | 4.43 |  |
| 65 | 0.00 | 1.46 | 3.08 |  |
| 100 | 0.00 | 1.09 | 2.28 |  |
| 150 | 0.00 | 0.85 | 1.85 |  |
| 200 | 0.00 | 0.73 | 1.66 |  |
| 270 | 0.00 | 0.55 | 1.17 |  |
| -270 | 0.16 | 1.58 | 3.81 |  |
| 20 | 59.26 | 33.45 | 19.38 |  |
| 28 | 25.30 | 30.89 | 27.29 |  |
| 35 | 7.80 | 14.33 | 17.47 |  |
| 48 | 4.26 | 8.70 | 11.98 |  |
| 65 | 1.97 | 5.12 | 7.57 |  |
| 100 | 0.55 | 2.39 | 4.41 |  |
| 150 | 0.24 | 1.54 | 3.00 |  |
| 200 | 0.24 | 1.02 | 2.50 |  |
| 270 | 0.16 | 0.68 | 1.66 |  |
| -270 | 0.24 | 1.88 | 4.74 |  |
| 35 | 48.41 | 31.28 | 19.95 |  |
| 48 | 32.51 | 33.37 | 29.63 |  |
| 65 | 10.60 | 16.38 | 19.71 |  |
| 100 | 5.06 | 8.37 | 11.00 |  |
| 150 | 3.42 | 5.42 | 7.98 |  |
| 200 | 0.00 | 1.72 | 3.87 |  |
| 270 | 0.00 | 1.11 | 2.54 |  |
| -270 | 0.00 | 2.34 | 5.32 |  |
| Mesh | Feed | 2.0-min product | 4.0-min product | 6.0-min product |
| 65 | 41.20 | 28.83 | 22.52 | 18.36 |
| 100 | 38.01 | 38.70 | 35.61 | 33.04 |
| 150 | 18.54 | 21.78 | 23.00 | 23.74 |
| 200 | 2.02 | 5.34 | 8.01 | 9.94 |
| 270 | 0.00 | 1.78 | 3.49 | 4.49 |
| -270 | 0.23 | 3.56 | 7.38 | 10.43 |

### 3.1.3 Characterization of the Flow Pattern of the Particles Through the Grinding Mill

The results of the tracer experiments were analyzed using "mixers-in-series" model (18) for obtaining a convenient representation of the flow pattern of the particles through the production ball-mill. The total number of the mixers to be used and the mean residence time values for each mixer were obtained using a sophisticated calculation technique (18). Three unequal mixers and a pure-delay unit were required for representing the mill. In terms of fractions, the distribution of the overall mean residence time was found to be:

| Mixer 1 | Mixer 2 | Mixer 3 | Pure-delay |
| :---: | :---: | :---: | :---: |
| 0.5597 | 0.0980 | 0.0965 | unit |

### 3.1.4 Batch-Grinding Tests

The material collected from the rod mill discharge was used to prepare four different feeds corresponding to top sizes of $8 / 10,14 / 20,28 / 35$, and $48 / 65$ mesh. The size distributions of the feeds are given in Table 4 along with the size distributions of the batch-ground products. It is
seen that although the feeds do not have the ideal size distribution*, all size-intervals up to 100 mesh size are well represented by these four feeds. The grinding tests were carried out in a 28 cm diameter mill under the recommended conditions for these tests (Section 2.2).
Table 5 shows the variation of the rate parameters with grinding time for the four top size-intervals. Although the $48 / 65$ mesh size fraction exhibits a pronounced decrease in grinding rate with time, in general, the deviations are not very significant. This data can therefore by analyzed using the mill model described above which supposes that $S$ does not vary with time.

Tahle 5 - Variation of rate parameters with grinding time in the batch-grinding operation

| Size <br> interval | Rate parameters, $\mathrm{min}^{-1}$ |  |  |
| :--- | :---: | :---: | :---: |
|  | time interval, min |  |  |
|  | $0-2$ | $2-4$ | $4-6$ |
| $8 / 10$ | 0.3624 | 0.3423 | - |
| $14 / 20$ | 0.2859 | 0.2729 | - |
| $28 / 35$ | 0.2184 | 0.2249 | - |
| $48 / 65$ | 0.1785 | 0.1235 | 0.1021 |

[^2]
### 3.2 ESTIMATION OF THE S AND B PARAMETERS FROM THE BATCHGRINDING DATA

Figure 7 shows the shapes of the $S$ function for the $0-2$ and 2-4 min periods of grinding. For the first period the variation of $S$ parameters can be very closely approximated by a first-order Equation 23 with a $s_{2}$ value of 0.333 . The second set of $S$ values can be described only by a polynomial of higher order. For this reason, it was decided to estimate the parameters for the two timeperiods separately.


Fig. 7-The shapes of the S function for $0-2$ and $2-4 \mathrm{~min}$ periods of grinding in the batch mill

As suggested in the general guidelines in Section 2, Equation 19 was initially used for the $B$ parameters, and a first-order Equation 23 for the $S$ parameters. The initial estimates of the unknown constants for the $S$ function were obtained from the experimental $S$ values reported in Table 5. For the constants of the B function the suggested general values were used. Tables 6 and 8 give the estimated values of the constants in the functional forms and Tables 7 and 9 give the corresponding calculated values of some of the $S$ and $B$ parameters. In Tables 6 and 8, the values of the error function Er (the sum of squared differences), and the standard estimate of error, SE, are also given.
Two important observations can be made from these results. First, the assumed first-order functional form for the $S$ function provides a poor approximation of the experimental S values for the 2-4 minute period. This was expected, however, because of the complex shape of the experimental $S$ curve. Second, the $b_{i, j}$ values for the 0-2 minute data correspond to a coarser product of breakage as compared to that obtained during the 2-4
minute period of grinding (i.e., $b_{i+1, i}$, value is greater and $\mathrm{b}_{i+\mathrm{g}, \mathrm{l}}$ value is smaller for the $0-2$ minute data).
For the data set corresponding to the 2-4 minute period, even with a third-order polynomial form for the $S$ function, good estimates of the $S$ parameters could not be obtained, although the Er and SE values improved significantly (Tables 8 and 9 ). In fact, for the finer sizeintervals (such as 100/150 and 200/270 mesh), the S values became unrealistically small. The improvement in Er and SE values can be attributed to a change in the $b_{i, j}$ values.
In the next step, Equation 20 was used in combination with the first-, second- and third-order forms of Equation 23 to process the 2-4 minute data. The results are given in Tables 10 and 11. As mentioned earlier, with a first-order form for S , the B function must necessarily be difference-similar. This is why in the first case, the estimated value of the constant $b_{4}$ which introduces non-difference-similar behaviour in the $\mathrm{B}_{\mathrm{i}, \mathrm{J}}$ values, is estimated as practically zero. As the order of the polynomial for the $S$ function was increased, $b_{4}$ values became more significant, Er and SE values improved although the $S$ values did not improve in every case.
At this step, an error analysis was carried out for all $M_{i}$ values predicted by various functional forms. It was found that in all the cases reported so far, the predicted $\mathrm{M}_{10 / 14}$ and $\mathrm{M}_{14 / 20}$ values for the -8 mesh feeds were highly erroneous. Out of the total thirty-six $\mathrm{M}_{\mathrm{i}}$ values in each data set, these two $M_{i}$ values alone contributed nearly $30-60 \%$ of the total Er value. The predicted $M_{10 / 14}$ values were found to be always lower and the predicted $\mathrm{M}_{14 / 20}$ values were found to be always higher than the corresponding experimental values by nearly the same amount. Two explanations are possible: the 14 mesh screen used in the sieve-analysis had openings smaller than the standard size (or some of the openings were blocked); and the $8 / 10$ mesh size fraction actually exhibited a different breakage behaviour.
With a view to studying the influence of the $\mathrm{M}_{10 / 14}$ and $\mathrm{M}_{14 / 20}$ values (in the product of a -8 mesh feed) on the estimates of the $S$ and $B$ parameters, in the next exercise the weighting factors for these two $M_{i}$ values were made zero (i.e., these terms did not contribute at all to the error function, Er). The results of this exercise for 0-2 minute data are given in Tables 6 and 8; those for 2-4 minute data in Tables 8 and 9 . It can be seen that in both cases, along with the Er and SE values, the Sestimates also improved considerably for most of the size-intervals. No attempt was made to use more complex functional forms for the B parameters, because the differ-ence-similar form provided a very satisfactory simulation of the experimental data.
It should also be pointed out that the size-intervals finer than 100 mesh size were not adequately represented in the experimental data used. For this reason it is likely that the breakage properties of the particles finer than 100 mesh sizes are not accurately described by the estimated $B$ and $S$ functions.

It is important to note that for the 2-4 minute data set, the estimated $b_{i, j}$ values, in general, correspond to a finer product of breakage than that represented by the $\mathrm{b}_{\mathrm{i}, \mathrm{j}}$ values for the 0-2 minute data set. Also, an improvement in the Er value for the 2-4 minute data set is always found to be associated with a change in $\mathrm{b}_{\mathrm{i}, \mathrm{j}}$ values corresponding to a finer product of breakage.

Two more detailed illustrations of the method can be found in references 30 and 31 . In these two cases, the six-parameter functional form (Eq 21) was required for the B parameters.

Table 6 - Estimated values of the constants in the $S$ and $B$ functions for 0-2 min period (difference-similar B)

| Constant | All data 1st order Eq 23 | Two $\mathrm{M}_{\mathrm{i}}$ values ignored |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 1st order Eq 23 | 2nd order Eq 23 | 3rd order Eq 23 |
| $\mathrm{b}_{1}$ | 0.329 | 0.357 | 0.370 | 0.370 |
| $\mathrm{b}_{2}$ | 0.845 | 0.956 | 0.939 | 0.947 |
| $\mathrm{b}_{3}$ | 20.0 | 20.0 | 20.0 | 20.0 |
| $\mathrm{s}_{1}$ | 0.262 | 0.272 | 0.283 | 0.289 |
| $\mathrm{s}_{2}$ | 0.426 | 0.452 | 0.412 | 0.447 |
| $\mathrm{s}_{3}$ | - | - | -0.080 | -0.149 |
| $\mathrm{s}_{4}$ | - | - | - | -0.054 |
| Er | 35.45 | 8.32 | 5.97 | 5.26 |
| SE | 1.15 | 0.56 | 0.48 | 0.46 |

Table 7 - Estimated values of the $S$ and $B$ parameters for $0-2$ min period (difference-similar $B$ )

| Parameters | All data | The $\mathrm{M}_{\mathrm{i}}$ values ignored |  |  | Experimental |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{array}{r} \hline \text { 1st order } \\ \text { Eq } 23 \end{array}$ | 1st order Eq 23 | 2nd order Eq 23 | 3rd order Eq 23 |  |
| $\mathrm{S}_{8 / 10}$ | 0.353 | 0.348 | 0.364 | 0.360 | 0.362 |
| $\mathrm{S}_{14 / 20}$ | 0.263 | 0.266 | 0.284 | 0.290 | 0.286 |
| $\mathrm{S}_{28 / 35}$ | 0.196 | 0.198 | 0.206 | 0.202 | 0.218 |
| $\mathrm{S}_{48 / 65}$ | 0.146 | 0.143 | 0.138 | 0.135 | 0.179 |
| $\mathrm{S}_{100 / 150}$ | 0.108 | 0.100 | 0.086 | 0.098 | - |
| $\mathrm{S}_{\text {200/270 }}$ | 0.081 | 0.068 | 0.049 | 0.084 | - |
| $b_{i+1, i}$ | 0.754 | 0.752 | 0.732 | 0.733 | - |
| $b_{i+2, i}$ | 0.063 | 0.064 | 0.075 | 0.075 | - |
| $\mathrm{b}_{1+5, \mathrm{i}}$ | 0.026 | 0.026 | 0.028 | 0.028 |  |
| $\mathrm{b}_{1+7, \mathrm{i}}$ | 0.014 | 0.014 | 0.015 | 0.014 |  |
| $\mathrm{b}_{1+10,1}$ | 0.006 | 0.006 | 0.005 | 0.005 | - |

Table 8 - Estimated values of the constants in the $S$ and $B$ functions for 2-4 min period (difference-similar B)

| Constant | All data |  |  | Two $\mathrm{M}_{\mathrm{i}}$ values ignored |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1st order Eq 23 | 2nd order Eq 23 | 3rd order Eq 23 | 1st order Eq 23 | 2nd order Eq 23 | 3rd order Eq 23 |
| $\mathrm{b}_{1}$ | 0.418 | 0.403 | 0.438 | 0.407 | 0.425 | 0.426 |
| $\mathrm{b}_{2}$ | 0.832 | 0.775 | 0.783 | 0.830 | 0.799 | 0.792 |
| $\mathrm{b}_{3}$ | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 |
| $\mathrm{s}_{1}$ | 0.229 | 0.243 | 0.225 | 0.257 | 0.274 | 0.267 |
| $\mathrm{S}_{2}$ | 0.428 | 0.295 | 0.207 | 0.500 | 0.371 | 0.348 |
| $\mathrm{s}_{3}$ | - | -0.134 | 0.213 | - | -0.156 | -0.067 |
| $\mathrm{S}_{4}$ | - | - | 0.255 | - | - | 0.059 |
| Er | 35.51 | 31.17 | 23.86 | 10.96 | 5.97 | 5.56 |
| SE | 1.15 | 1.09 | 0.98 | 0.64 | 0.48 | 0.47 |

Table 9 - Estimated values of the S and B parameters for 2-4 min period (difference-similar B)

| Parameters | All data |  |  | Two $\mathrm{M}_{\mathrm{i}}$ values ignored |  |  | Experimental |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1st order Eq 23 | 2nd order Eq 23 | 3rd order Eq 23 | 1st order Eq 23 | 2nd order Eq 23 | 3rd order Eq 23 |  |
| $\mathrm{S}_{8 / 10}$ | 0.309 | 0.280 | 0.316 | 0.366 | 0.330 | 0.337 | 0.342 |
| $\mathrm{S}_{14 / 20}$ | 0.230 | 0.244 | 0.225 | 0.259 | 0.275 | 0.268 | 0.272 |
| $\mathrm{S}_{28 / 35}$ | 0.171 | 0.187 | 0.199 | 0.183 | 0.198 | 0.200 | 0.224 |
| $\mathrm{S}_{48 / 65}$ | 0.127 | 0.126 | 0.130 | 0.129 | 0.122 | 0.125 | 0.124 |
| $\mathrm{S}_{100 / 150}$ | 0.094 | 0.074 | 0.038 | 0.091 | 0.065 | 0.058 | - |
| $\mathrm{S}_{100 / 270}$ | 0.070 | 0.039 | 0.003 | 0.065 | 0.030 | 0.018 | - |
| $\mathrm{b}_{1+1,1}$ | 0.686 | 0.691 | 0.666 | 0.694 | 0.677 | 0.676 | - |
| $\mathrm{b}_{1+2,1}$ | 0.079 | 0.073 | 0.080 | 0.077 | 0.079 | 0.078 |  |
| $\mathrm{b}_{1+5,1}$ | 0.033 | 0.032 | 0.035 | 0.032 | 0.034 | 0.034 | - |
| $\mathrm{b}_{i+7,1}$ | 0.019 | 0.019 | 0.020 | 0.018 | 0.020 | 0.020 | - |
| $\mathrm{b}_{1+10,1}$ | 0.008 | 0.008 | 0.009 | 0.008 | 0.009 | 0.009 | - |

Table 10 - Estimated values of the constants in the $S$ and $B$ functions for 2-4 min period (non-difference-similar B)

|  | All 2-4 min data |  |  |
| :--- | ---: | ---: | ---: |
| Constant | 1st order <br> Eq 23 | 2nd order <br> Eq 23 | 3rd order <br> Eq 23 |
| $\mathrm{b}_{1}$ | 0.417 | 0.416 | 0.436 |
| $\mathrm{~b}_{2}$ | 0.813 | 0.864 | 0.901 |
| $\mathrm{~b}_{3}$ | 20.0 | 20.0 | 20.0 |
| $\mathrm{~b}_{4}$ | -0.071 | 0.152 | 0.343 |
| $\mathrm{~s}_{1}$ | 0.228 | 0.248 | 0.234 |
| $\mathrm{~s}_{2}$ | 0.411 | 0.279 | 0.140 |
| $\mathrm{~s}_{3}$ | - | -0.184 | 0.162 |
| $\mathrm{~s}_{4}$ | - | - | 0.318 |
| Er | 35.16 | 30.41 | 20.54 |
| SE | 1.16 | 1.10 | 0.93 |

Table 11 - Estimated values of the S and B parameters for 2-4 min period (non-difference-similar B)

|  | All 2-4 min data |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
| Parameter | 1st order | 2nd order | 3rd order | Experimental |
| $\mathrm{S}_{8 / 10}$ | 0.305 | 0.275 | 0.313 | 0.342 |
| $\mathrm{~S}_{14 / 20}$ | 0.229 | 0.249 | 0.234 | 0.272 |
| $\mathrm{~S}_{28,35}$ | 0.173 | 0.188 | 0.207 | 0.224 |
| $\mathrm{~S}_{48,65}$ | 0.130 | 0.119 | 0.115 | 0.124 |
| $\mathrm{~S}_{100 / 150}$ | 0.098 | 0.063 | - |  |
| $\mathrm{S}_{200 / 270}$ | 0.073 | 0.028 | - |  |
| $\mathrm{b}_{2,1}$ | 0.668 | 0.722 | 0.001 | - |
| $\mathrm{b}_{5,4}$ | 0.692 | 0.675 | 0.749 | - |
| $\mathrm{b}_{11,10}$ | 0.734 | 0.554 | 0.642 | - |
| $\mathrm{b}_{3,1}$ | 0.082 | 0.072 | 0.270 | - |
| $\mathrm{b}_{6,1}$ | 0.035 | 0.029 | 0.068 | - |
| $\mathrm{b}_{10,1}$ | 0.009 | 0.007 | 0.026 | - |

### 3.3 ESTIMATION OF S PARAMETERS FOR THE PRODUCTION MILL

In addition to the sampling campaign data shown in Table 3, data was available from seven other similar sampling campaigns around the same ball-mill. Use of many sets of production mill data is recommended because it increases the reliability and range of the estimated S parameters. Table 12 summarizes the ballmill feed and product size distributions for all eight data sets.

The circulating load for the production ball-mill is about 250 per cent. This means that nearly $70 \%$ of the feed to the ball-mill consists of particles which have passed through the mill at least once. It can therefore be argued that the breakage characteristics of these particles will be better represented by those particles which have been ground in the laboratory ball-mill for some time, and not by those coming directly from the rod mill. For this reason, it was decided that the B values obtained for the 2-4 minute data set of the batch tests should be used for the analysis of the plant data (i.e., Set 0 in Tables 8 and 9 ).

It was pointed out earlier that for all size-intervals the absolute rate of breakage $\bar{S}(=S x W)$ remains constant over the usual range of operating conditions in the plant. This means that for the calculation of the mill-product size distribution an estimate of the mean residence time is not required. However, if an estimate of $\tau$ is available, the $\mathrm{S}_{\mathrm{i}}$ values can be easily calculated from $\mathrm{S}_{\mathrm{i}}=\mathrm{S}_{\mathrm{i}} / \mathrm{F} \tau$. It was therefore decided to estimate the parameters $\bar{S}_{i}$ simultaneously for eight pairs of feed-product size distribution data available for the industrial mill, assuming that these were not functions of the operating conditions over the range covered by the data. It was further assumed that the dimensionless RTD did not change appreciably due to variation in the operating conditions and the fractional mean residence time values reported in Section 3.1.3 applied to all the eight sets of data.
Two sets of estimates were obtained corresponding to two different error-minimization criteria: the difference square; and the difference square divided by the predicted $M_{i}$ value (called $X^{2}$ or chi-square). As all $\bar{S}_{i}$ values for different size-intervals i differ from their corresponding $\mathrm{S}_{\mathrm{i}}$ values by a constant factor $\mathrm{F}_{\mathrm{T}}$, the functional forms used for the $S_{i}$ parameters can also be used for the estimation of the $\bar{S}_{i}$ parameters.
The estimated values of the $\bar{S}_{i}$ parameters corresponding to different orders of the polynomial functional form are given in Table 13 and Figure 8, along with the error function values $\operatorname{Er}$ and $\operatorname{Er}\left(\mathrm{X}^{2}\right)$. The estimates of the $s$ constants are given in Table 14.

An error analysis of the predicted size distribution data showed that the error distribution was quite random with


Fig. 8-Variation of the absolute rates of breakage, $\bar{s}_{j}$, with particle size for the industrial mill (B vaiues used were obtained from batch-grinding tests - corresponding to Set 0 in Table 8)
respect to the variations in the operating conditions. The overall simulation of the product size distribution data was good, as is indicated by the error function values. In this way it was confirmed that the absolute rates of breakage for different size-intervals remained practically independent of the operating conditions.
It must be pointed out that for the size-intervals finer than 65 mesh, reliable estimates of the $\overline{\mathrm{S}}_{1}$ parameters could not be obtained - even when a third-order polynomial form was used to describe the size-variation of these parameters. The reasons for the observed abnormal variations in this size range are not precisely known, though several possibilities can be identified: a change in the breakage distribution function for particles in this size range due to the liberation of the various constituent minerals in the ore; insensitivity of the error function to the size distribution in this size range; and a genuine need for using a higher-order polynomial form for describing the size-variation of the $\overline{\mathrm{S}}_{\mathrm{i}}$ parameters.
In this context, it is also noteworthy that, between 10 and 65 mesh, practically the same $\overline{\mathrm{S}}_{1}$ estimates were obtained irrespective of the complexity of the functional form used for describing the function $\overline{\mathrm{S}}_{\mathrm{j}}$ (Fig. 8). An inspection of the experimental feed-product size distribution data (Table 12) shows that these size-intervals contained most of the material.

The $M_{i}$ predictions for the top four size-intervals were observed to be reasonably accurate only when a thirdorder polynomial form was used. The reason for this is that to obtain the real $\overline{\mathrm{S}}_{\mathrm{i}}$ values for these size-intervals, the function $\overline{\mathrm{S}}_{1}$ must be flat in the $6-10$ mesh size range and then its value should decrease as the particle size increases. In our case, this type of natural shape of the $\bar{S}_{i}$ function (which follows from the shape of the function $\mathrm{S}_{\mathrm{i}}$ ) could be generated only by a more flexible thirdorder polynomial form.
In general, the $\bar{S}_{i}$ values obtained using the $X^{2}$ (chisquare) criterion are practically the same as those obtained using the difference-square criterion. The only exceptions observed are those corresponding to the top three size-intervals. For these size-intervals, predicted $M_{1}$ values were found to be closer to the experimental values when the chi-square criterion was used. Moreover, as shown in Table 13 the difference-square Er value obtained for the $X^{2}$ set of predicted $M_{i}$ values, is practically the same as that obtained using the differ-ence-square criterion. These two observations show that the chi-square is a better error-criterion.

It was stated earlier that correct estimates of the $S$ and $B$ parameters cannot be obtained if both the sets of parameters are simultaneously estimated from the plant
data. With a view to demonstrating this fact, an exercise was carried out using the available size distribution data. The difference-similar form for the B function and the second-order polynomial form for the $S$ function were used. The usual initial estimates were used for all the six unknown constants in the two functional forms. During the course of the error minimization program, several sub-optimal (or intermediate) sets of values of the six unknown constants and the corresponding difference square error-function were printed.
It was found that for each one of these sub-optimal sets, the value of the error-function Er was considerably lower than that obtained with a third-order polynomial for the $\overline{\mathrm{S}}_{1}$ function with fixed batch mill B values (Table 13). A study of the results showed that these apparently superior sub-optimal sets corresponded to widely different $\bar{S}_{i}$ and $b_{i, j}$ values, some of which were negative, which is physically meaningless.
Finally, it may be mentioned that when any one of these intermediate sets of $b_{1, j}$ values was used for simulation of the batch-grinding results, the Er values obtained were 50 to 100 times higher than those obtained earlier. This observation further supports the statement that S and B parameters cannot be simultaneously estimated from plant data.

Table 12 - Production ball-mill feed and product size distribution and throughput

| Particle size | Test 1 |  | Test 2 |  | Test 3 |  | Test 4 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Feed | Product | Feed | Product | Feed | Product | Feed | Product |
| +4 | . 34 | . 20 | . 55 | . 31 | . 01 | . 00 | . 08 | . 04 |
| 4/6 | . 81 | . 33 | 1.06 | . 52 | . 12 | . 07 | . 22 | . 12 |
| 6/8 | 2.74 | 1.01 | 2.53 | 1.12 | . 50 | . 08 | 1.00 | . 40 |
| 8/10 | 3.96 | 1.20 | 3.84 | 1.99 | 2.04 | . 34 | 3.02 | . 94 |
| 10/14 | 10.54 | 5.44 | 6.71 | 3.87 | 4.30 | . 94 | 6.11 | 2.21 |
| 14/20 | 11.85 | 7.66 | 9.57 | 6.88 | 7.50 | 2.80 | 9.50 | 4.60 |
| 20/28 | 13.49 | 10.78 | 11.16 | 9.08 | 11.02 | 6.10 | 14.53 | 9.96 |
| 28/35 | 12.94 | 12.25 | 12.66 | 11.26 | 14.33 | 10.71 | 18.39 | 15.19 |
| 35/48 | 12.16 | 13.33 | 14.95 | 15.05 | 17.03 | 16.37 | 15.11 | 15.25 |
| 48/65 | 8.35 | 10.84 | 11.08 | 12.73 | 10.84 | 12.65 | 8.83 | 11.01 |
| 65/100 | 4.80 | 7.08 | 6.23 | 8.08 | 6.85 | 10.16 | 5.90 | 8.86 |
| 100/150 | 3.36 | 5.39 | 4.29 | 6.02 | 4.90 | 7.70 | 3.98 | 6.70 |
| 150/200 | 2.61 | 4.48 | 3.17 | 4.74 | 3.40 | 5.80 | 2.51 | 4.62 |
| 200/270 | 1.29 | 2.29 | 1.63 | 2.43 | 2.40 | 3.73 | 1.50 | 2.78 |
| -270 | 10.76 | 17.72 | 10.57 | 16.00 | 14.76 | 22.53 | 9.30 | 17.33 |
| Throughput st/h | 847 |  | 1090 |  | 558 |  | 679.4 |  |


|  | Test 5 |  | Test 6 |  | Test 7 |  | Test 8 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Particle size | Feed | Product | Feed | Product | Feed | Product | Feed | Product |
| +4 | . 47 | . 16 | . 47 | . 29 | . 14 | . 03 | . 21 | . 04 |
| 4/6 | . 52 | . 16 | 1.08 | . 49 | . 30 | . 05 | . 48 | . 11 |
| 6/8 | 2.40 | . 70 | 3.00 | 1.00 | 2.00 | . 45 | 2.00 | . 50 |
| 8/10 | 5.50 | 2.01 | 5.65 | 2.63 | 4.03 | 1.13 | 4.04 | 1.33 |
| 10/14 | 9.00 | 4.40 | 9.59 | 5.37 | 7.00 | 2.55 | 6.65 | 2.70 |
| 14/20 | 11.75 | 7.30 | 12.00 | 8.50 | 10.12 | 5.00 | 9.60 | 5.02 |
| 20/28 | 14.60 | 11.42 | 14.62 | 11.96 | 12.80 | 8.70 | 12.42 | 8.72 |
| 28/35 | 14.31 | 13.43 | 14.95 | 14.16 | 14.93 | 12.82 | 15.27 | 13.44 |
| 35/48 | 11.51 | 13.36 | 11.46 | 13.17 | 12.72 | 14.03 | 14.24 | 15.04 |
| 48/65 | 7.03 | 9.38 | 6.84 | 8.91 | 7.31 | 9.98 | 8.33 | 10.70 |
| 65/100 | 5.10 | 7.68 | 5.35 | 7.50 | 5.60 | 8.42 | 5.70 | 8.30 |
| 100/150 | 3.59 | 5.85 | 3.50 | 5.70 | 4.20 | 6.75 | 4.10 | 6.47 |
| 150/200 | 2.45 | 4.22 | 2.19 | 3.71 | 3.17 | 5.12 | 2.96 | 4.72 |
| 200/270 | 1.67 | 2.90 | 1.46 | 2.39 | 2.11 | 3.51 | 1.97 | 3.28 |
| -270 | 10.07 | 17.01 | 7.83 | 14.24 | 13.58 | 21.46 | 12.03 | 19.64 |
| Throughput st/h | 844.6 |  | 1001.9 |  | 636.2 |  | 693.6 |  |

Table 13 - Estimated values of the absolute rates of breakage ( $\left.\bar{S}_{\mathrm{i}} \times 10^{-3} \mathrm{st} / \mathrm{h}\right)$, for the 14 sizeintervals listed in Table 12, using two different error-minimization criteria

|  | Difference square minimization |  |  | $\mathrm{X}^{2}$ minimization |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Parameter set 1 1 st order Eq 23 | Parameter set 2 2nd order Eq 23 | Parameter set 3 3rd order Eq 23 | Parameter set 4 3rd order Eq 23 | Parameter set 5 2nd order Eq 23 |
|  | 2.737 | 5.409 | 0.976 | 0.782 | 1.558 |
|  | 2.201 | 3.603 | 1.273 | 1.112 | 1.463 |
|  | 1.770 | 2.469 | 1.409 | 1.311 | 1.340 |
|  | 1.423 | 1.740 | 1.363 | 1.326 | 1.198 |
|  | 1.145 | 1.262 | 1.187 | 1.187 | 1.045 |
|  | 0.920 | 0.942 | 0.958 | 0.973 | 0.889 |
|  | 0.740 | 0.723 | 0.738 | 0.753 | 0.738 |
|  | 0.595 | 0.571 | 0.559 | 0.570 | 0.598 |
|  | 0.479 | 0.463 | 0.429 | 0.435 | 0.472 |
|  | 0.385 | 0.387 | 0.343 | 0.346 | 0.364 |
|  | 0.310 | 0.333 | 0.295 | 0.296 | 0.274 |
|  | 0.249 | 0.294 | 0.280 | 0.283 | 0.201 |
|  | 0.200 | 0.268 | 0.304 | 0.310 | 0.144 |
|  | 0.161 | 0.250 | 0.387 | 0.404 | 0.100 |
| Er | 45.25 | 38.68 | 31.45 | - | - |
| $\operatorname{Er}\left(\mathrm{X}^{2}\right)$ | - | - |  | 3.77 | 7.41 |

Table 14 - Estimated values of the constants in the $\overline{\mathbf{S}}$ function corresponding to Equation 23

|  | Difference square minimization |  |  | $\mathrm{X}^{2}$ minimization |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Parameter set 1 1st order Eq 23 | Parameter set 2 2nd order Eq 23 | Parameter set 3 3rd order Eq 23 | Parameter set 4 3rd order Eq 23 | Parameter set 5 2nd order Eq 23 |
| $S_{1}$ | 0.9147 | 0.9341 | 0.9509 | 0.9661 | 0.8847 |
| $\mathrm{S}_{2}$ | 0.6286 | 0.8019 | 0.7036 | 0.6767 | 0.5034 |
| $\mathrm{S}_{3}$ | - | . 1178 | -0.1893 | -0.2302 | -0.1027 |
| $\mathrm{S}_{4}$ | - | - | -0.1180 | -0.1305 | - |

## 4. DESCRIPTION AND USE OF THE FINDBS PROGRAM

### 4.1 INTRODUCTION

The program is capable of processing both batchgrinding data and data taken from a continuously operating mill modelled as a series of perfect mixers. All the functional forms for the rate and distribution parameters previously mentioned in Section 1 are available. It is possible to have any combination of the $S$ and $B$ functional forms, or just one functional form for one set of parameters and known values of the other set of parameters; known values of both parameters can also be used. The general program structure is given in Appendix D.
The input data can be divided into three categories:

1. feed and product size distributions (on logical file No. 7);
2. known values, if any, of $S_{j}$ and $b_{i, j}$ (on logical file No. 8); and
3. other data, such as initial estimates of search variables, RTD parameters, and miscellaneous program options (on logical file No. 5).
The output data can vary according to user-controlled options and are written on logical file No. 6. Logical unit No. 9 is always used to write computed values of the B and $S$ parameters and can be used as file No. 8 in subsequent program runs.

### 4.2 SIZE DISTRIBUTION FILE FORMAT

This file format is for Logical Unit No. 7.
When batch-grinding data are used, up to three different product size distributions corresponding to progressively longer grinding times can be entered with a single feed size distribution. If data sets with different feeds are available (as is often the case with single-size fraction batch-grinding data), up to five sets may be processed together. But whatever the extent of the data, the screen sizes must be in a complete geometric series, i.e., every screen size must be a fraction $R$ of the next larger size. An example data file is shown in the sample run.

| Record No. | Data description | FORTRAN format |
| :---: | :---: | :---: |
| 1 | Alphanumeric title for data file | 6A10 |
| 2 | NSETS, NSIZES, R. TOPSIZ | 11,1X,12,2(1X,F5.3) |
| 3 | NTIMES,[TIME( i ), $\mathrm{i}=1, \mathrm{NTIMES}]$ | 11,3(1X,F5.3) |
| 4 | Alphanumeric name for | A10,4G10.4 |
|  | largest screen size, product |  |
|  | size distribution(s) in wt \%, |  |
|  | feed size distribution(s) in wt \% |  |
| 3+NSIZES | Alphanumeric name for the pan | . |

## Notes

1. NSETS is the number of data sets or feed size distributions in the file ( $\leq 5$ ).
2. NSIZES is the number of screens including the pan ( $\leq 20$ ).
3. $R$ is the screen-size ratio.
4. TOPSIZ is the size of the coarsest size fraction of interest (Section 2.4).
5. NTIMES is the number of product size distributions. Must be one for batch mills.
6. Repeat record 3 through ( $3+$ NSIZES) inclusive for each data set.
7. The number of screen sizes (NSIZES) must be between 2 and 20 inclusive and be the same for all data sets.
8. The number of product size distributions (NTIMES) must be between 1 and 3 inclusive.
9. When using batch-grinding data the values of TIME (i) are the grinding times. Otherwise, TIME (1) is the total mean residence time of the solids in the mill, or (1.0/flow rate of solids) for calculation of SW. (Use consistent units).
10. The screen-size ratio ( $R$ ) must be the same for all data sets.

### 4.3 B AND S MATRIX FILE FORMAT

This file format is for Logical Unit 8.
The program can search for both the distribution and rate functions simultaneously, or just one at a time. In the latter case, one of the functions must be fixed by supplying the values of all the parameters in a second data file or by supplying the function constants interactively.
To use a fixed distribution parameter matrix, create a file using the following format.


## Notes

1. If NSIZES is greater than 13, continue the line when necessary onto a second line with the same format.
2. $b(p a n, j)=1-\sum_{i} b_{i, j}, i=(j+1)$ to (NSIZES-1)

To use a fixed rate parameter matrix create a file of only one record with the following format:

| Record <br> No. | Data description | FORTRAN <br> format |
| :--- | :--- | :--- |
| 1 | $\mathrm{S}(1,1) \mathrm{S}(2,2) . \ldots$ <br> $\mathrm{S}(\mathrm{NSIZES}, \mathrm{NSIZES})$ | 13 F 6.3 |

## Notes

1. The program assumes that all off-diagonal elements are zero. Therefore, enter the main diagonal as a single vector in Record 1.
2. If NSIZES is greater than 13, continue onto a second record with the same format.
3. $S($ NSIZES,NSIZES $)=S($ pan,pan $)=0.0$.
4. No two S elements can be equal because then the equations used by this program lead to undefined values (Appendix A, Eq A.8).

For a direct ball-mill simulation, it is possible to fix both the distribution and rate matrix elements; then the rate matrix main diagonal should immediately follow the distribution matrix in the same file.

### 4.4 TERMINAL INPUT

A program sample of terminal input with all entries in free-field format is shown in Appendix E .

Interactive data entry is requested by the appropriate combination of the messages listed in the following subsection.

### 4.4.1 Options

Options are integer codes which direct the flow of the program. They are IBOPT, IBFIX, ISOPT, ISFIX, and MIXERS

IBOPT
There are five options for IBOPT presently implemented.

| IBOPT | Effect |
| :--- | :--- |
| 0 | Stop the program. <br> 1 |
| Read the function using supplied param- <br> eters or values (as determined by IBFIX). |  |
| 3 | Calculate a three-parameter normalized <br> function. |

$$
B_{i, j}=b_{1}\left(x_{i} / x_{j}\right)^{b_{2}}+\left[1-b_{1}\right]\left(x_{i} / x_{j}\right)^{b_{3}}
$$

4
Calculate a four-parameter non-normalized distribution function.

$$
\begin{aligned}
& B_{i, j}=b_{1}\left(1 . / x_{j}\right)^{b_{4}}\left(x_{i} / x_{j}\right)^{b_{2}} \\
&+ {\left[1-b_{1}\left(1 . / x_{j}\right)^{b_{4}}\right]\left(x_{i} / x_{j}\right)^{b_{3}} }
\end{aligned}
$$

6
Calculate a six-parameter non-normalized distribution function.

$$
\begin{aligned}
B_{i, j} & =b_{1}\left(1 . / x_{j}\right)^{b_{4}}\left(x_{i} / x_{j}\right)^{a_{1}} \\
& +\left[1-b_{1}\left(1 . / x_{j}\right)^{a_{4}}\right]\left(x_{i} x_{j}\right)^{e_{2}}
\end{aligned}
$$

where $e_{1}=b_{2}+b_{5}{ }^{j}$

$$
e_{2}=b_{3}+b_{6}{ }^{j}
$$

Other entries produce a repeat of the question.

During the calculation of the breakage function parameters, the program does not allow values of $b_{3}$ greater than 20 and absolute values of $b_{6}$ smaller than two. If either parameter reaches these limiting values, the program should be restarted with better initial estimates.

## IBFIX

The IBFIX value is ignored when IBOPT $\neq 1$. If IBOPT=1, IBFIX is used to control the input of the breakage function as follows:

| IBFIX | Effect |
| :--- | :--- |
| 1 | Read matrix of breakage function values on <br> Unit 8. |
| $3,4,6$ | Read 3, 4, or 6 parameters for the breakage <br> function interactively. |

Other entries produce a repeat of the prompt.

## ISOPT

The selection function values on parameters are read or calculated according to the user entry for variables ISOPT and ISFIX that work on the same principle as IBOPT and IBFIX.

## ISOPT Effect

1 Read the selection function values or parameters (as determined by ISFIX).
$2 \quad$ Calculate a two parameter log-polynomial rate function.

$$
s_{i, 1}=s_{1}\left(x_{i}\right)^{s_{2}}
$$

or equivalent

$$
\begin{gathered}
\ln \left(s_{\left.i_{i}\right)}\right)=\ln \left(s_{1}\right) \\
+s_{2} \ln \left(x_{i}\right)
\end{gathered}
$$

3 Calculate a second-order log-polynomial rate function.

$$
\begin{align*}
\ln \left(\mathrm{s}_{\mathrm{i}, \mathrm{i}}\right) & =\ln \left(\mathrm{s}_{1}\right)+\mathrm{s}_{2} \ln \left(\mathrm{x}_{\mathrm{i}}\right) \\
& +\mathrm{s}_{3}\left[\ln \left(\mathrm{x}_{\mathrm{i}}\right)\right]^{2} \tag{4}
\end{align*}
$$

Calculate a third-order log-polynomial rate function.

$$
\begin{aligned}
\ln \left(\mathrm{s}_{\mathrm{i}, \mathrm{i}}\right) & =\ln \left(\mathrm{s}_{1}\right)+\mathrm{s}_{2} \ln \left(\mathrm{x}_{\mathrm{i}}\right) \\
& +\mathrm{s}_{3}\left[\ln \left(\mathrm{x}_{\mathrm{i}}\right)\right]^{2}+\mathrm{s}_{4}\left[\ln \left(\mathrm{x}_{\mathrm{i}}\right)\right]^{3}
\end{aligned}
$$

5
Calculate a four-parameter rate function.

$$
S_{i, i}=\frac{s_{1}\left(x_{i}\right)^{s_{2}}}{1+\left[x_{i} / s_{3}\right]^{s_{4}}}
$$

## ISFIX

The ISFIX values are ignored when ISOPT $\neq 1$. If ISOPT = 1, ISFIX is used to control the input of the selection function as follows:

| ISFIX | Effect |
| :--- | :--- |
| 1 | Read the selection function diagonal matrix <br> on Unit 8. |
| $2,3,4,5$ | Read parameters corresponding to cases <br> $2,3,4,5$ of ISOPT interactively. |

Other entries produce a repeat of the prompt.

## MIXERS

The MIXERS option specifies the number of perfect mixers used to represent the RTD of the mill. The options are:

| MIXERS | Effect |
| :--- | :--- |
| 0 | Used for batch mill data. |
| 1 to 9 | Indicate the number of perfect mixers-in- <br> series with a plug flow component. |

### 4.4.2 Entry of B, S, and MIXER Data

## Estimate of BConstant i

An estimate of parameters $b_{1}$ to $b_{6}$ of the breakage function is required when IBOPT $>1$ and as controlled by IBOPT. The better the estimate, the faster the final solution.

## Known B Constant i

The fixed values of parameters $b_{1}$ to $b_{6}$ of the breakage function are required when IBOPT $=1$ and as controlled by IBFIX.

## Estimate of $S$ Constant $i$

An estimate of parameters $s_{1}$ to $s_{4}$ of the selection function is required when ISOPT $>1$ and as controlled by ISOPT.

## Known S Constant i

The fixed values of parameters $s_{1}$ to $s_{4}$ of the selection function are required when ISOPT $=1$ and as controlled by ISFIX.

## Known RT Fraction for MIXER i

If MIXERS $\neq 0$, the fractional mean residence time of each mixer must be supplied ( $\tau_{1} / \tau, \tau_{2} / \tau, \ldots$ ).

### 4.4.3 Criterion Type

This allows the user to select the type of criterion to be minimized.

| Option | Criterion |
| :--- | :--- |
| 1 | $E r=\sum_{i}\left(M_{i}-\hat{M}_{i}\right)^{2}$ |
| 2 | $E r=\sum_{i} W E I G H_{i} \times\left(M_{i}-\hat{M}_{i}\right)^{2}$ (weighted form) |
| 3 | $E r=\sum_{i}\left(M_{i}-\hat{M}_{i}\right)^{2} / \hat{M}_{i} \quad\left(X^{2}\right.$ form) |

4 Same as option 1, except $M$ and $\hat{M}$ are expressed as cumulative weight \% passing.

If option 2 is selected, the user is asked to enter the error model as follows:

## ENTER K AND X (SIGMA $=K+X^{\star} M$ ) VALUES.

$K$ and $X$ are used to build the general error model WEIGH $H_{i}=1 /\left(K+X_{*} M_{i}\right)_{* *} 2$ (Eq 25).
Since the user may desire to give more weight to some data points, the following prompts are also issued:

DO YOU WISH TO ALTER THE GENERAL ERROR MODEL FOR A PARTICULAR DATA POINT (Y/N)? If $\boldsymbol{Y}$ is entered, the data point location and its specific error model are requested.

## enter point coordinates i,J,K in data file.

1 is the number of the grinding experiment in which the point is located.
$J \quad$ is the number of the size-interval in that experiment.
$\boldsymbol{K} \quad$ is the number of the product size distribution in which the print is located (i.e., 2,4,2 indicates second data set, fourth size-interval, second product of size distribution).

## ENTER K AND X.

## ANOTHER CHANGE (Y/N)?

This allows the user to modify the general error model for another point.

### 4.4.4 SEARCH Option

This option allows the user to control the SEARCH routine.

| Option | Effect |
| :--- | :--- |
| 1 | Prints results after a minimum is found for a <br> variable in every search direction. |
| 2 | Prints results after a minimum is found for all <br> the variables after a search in all directions. |
| 3 | Prints results at the end of the minimization <br> process. |
| Allows user to change default parameters. In <br> this case the user is asked to enter a value <br> for: ESCALE |  |

## ESCALE

The larger ESCALE, the larger the step-size. The default is 0.9 .

## CONVERGENCE CRITERION $E_{I}$

This is the convergence limit for the SEARCH variables. The search routine stops when no variable changes by more than this value during a single iteration. The default is ( $20 \%$ of the absolute value of the initial estimate) +.001 .

## MAXIT

Specify the maximum number of iterations to be performed before terminating if no error-function minimum is found. The default is three times the number of SEARCH variables.

### 4.5 PROGRAM OUTPUT

### 4.5.1 Normal Results

Following the request for a SEARCH option the program searches for the best set of constants. The quantity of intermediate results printed by the search routine depends on the SEARCH option, but the results of the last iteration are always printed. This includes the last iteration number, the total number of error-function evaluations, the last error-function value, and the final set of the constants.

A table comparing the observed and predicted size distribution(s) is always printed. The table is labelled using the screen-size names and grind times supplied in the size distribution data file.
Finally, the selection and breakage matrices are printed. They are the ones in the prediction of the final results.

These values are also copied to Unit 9 for possible use in subsequent runs (to be renamed Unit 8).

The program then loops back to ask for new starting options. Enter 0 to stop the program. When this is done the message NORMAL PROGRAM TERMINATION is printed.

### 4.5.2 Diagnostics

The response of the program to data entered with the wrong format also depends on the computer.

The program reads and checks the file definition parameters in record 2 of the size distribution data file. Should any of these values be illegal, the message NTIMES $=n$ NSIZES $=m$ SIEVE RATIO $=x$ TOP SIZE $=y$ will be printed and the program will stop. The program will also stop after printing the message NTIMES $=\mathrm{n}$ IN SET m, when more than three product size distributions are entered in the file. In either case, it is necessary to correct the file. The rest of the file is never checked. Data file No. 8 ( B and S values) is not checked.
Terminal input is checked whenever possible. For instance, illegal IBOPT or ISOPT values result in a repeated prompt to enter them. If a non-zero MIXERS option is entered, the size distribution data are necessarily from a continuous mill and the program checks that the number of product size distributions is one. Should this test fail, the message NTIMES MUST EQUAL 1 FOR CONTINUOUS MILL DATA is printed and the program stops. And if any RT fractions are negative, or if their sum is greater than one, they are rerequested.
The following three error messages originate in the SEARCH routine:

## maximum change does not alter function.

This can result, for example, when the constant $b_{3}$ becomes so large that the term $\left(x_{i} / x_{j}\right)_{3}{ }_{3}$ is numerically zero within the accuracy of the computer. Further attempts by the SEARCH procedure to find a minimum by varying $b_{3}$ would be futile and result in this error message.

## n ITERATIONS COMPLETED BY BOTM.

A minimum has not been found after the maximum allowable number of iterations defined by MAXIT. If desired, the program can be rerun using as estimates the last set of SEARCH variable values. Or, the SEARCH option can be used to increase the value of MAXIT, or starting estimates can be changed.

## ACCURACY LIMITED BY ERRORS IN F.

The values returned by the program are inconsistent. Try different starting estimates.

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APPENDIX A

## SOLUTIONS OF THE GRINDING EQUATION

## SOLUTIONS OF THE GRINDING EQUATION

## A Solution to the Batch-Grinding Equation

The set of simultaneous first-order differential equations (Eq 12) of the batch-grinding operation has been solved by Reid (1) using the following form:

$$
M_{i}(t)=\sum_{j=1}^{1} \alpha_{i j} e^{-S_{j} t}
$$

where:

$$
\begin{align*}
& \alpha_{i j}=\sum_{k=j}^{i-1} \frac{s_{k} b_{i, k}}{S_{i}-s_{j}} \alpha_{k j} \\
& \alpha_{i l}=M_{i}(0)-\sum_{k=1}^{i-1} \alpha_{i k} \\
& \alpha_{i 1}=M_{i}(0)
\end{align*}
$$

The expansion of this expression in terms of distribution and rate parameters becomes progressively more tedious for $\mathrm{i}>2$. Thus, up to $\mathrm{i}=3$, the solution is:

$$
\begin{array}{rl}
M_{1}(t)=M_{1}(0) e^{-S_{1} t} & E q A .3 \\
M_{2}(t) & =M_{2}(0) e^{-S_{2} t}+M_{1}(0) \frac{S_{1} b_{21}}{S_{2}-S_{1}}\left(e^{-S_{1} t}-e^{-S_{2} t}\right) \\
M_{3}(t) & =M_{3}(0) e^{-S_{3} t}+M_{2}(0) \frac{S_{2} b_{3,2}}{S_{3}-S_{2}}\left(e^{-S_{2} t}-e^{-S_{3} t}\right) \\
& +M_{1}(0) \frac{S_{1} b_{3,1}}{S_{3}-S_{1}}\left(e^{-S_{1} t}-e^{-S_{3} t}\right) \\
& +M_{1}(0) S_{1} S_{2} b_{2,1} b_{3,2}\left[\frac{e^{-S_{1} t}}{\left(S_{3}-S_{1}\right)\left(S_{2}-S_{1}\right)}\right. \\
& \left.+\frac{e^{-S_{2} t}}{\left(S_{3}-S_{2}\right)\left(S_{1}-S_{2}\right)}+\frac{e^{-S_{3} t}}{\left(S_{1}-S_{3}\right)\left(S_{2}-S_{3}\right)}\right] \quad \text { Eq A.5 }
\end{array}
$$

It can be seen from these equations that this set of equations is highly nonlinear in the $S$ and $B$ parameters.

## A General Matrix Formalism of the Grinding Mill Model

The size distribution vector of the mill product, $P$, of any grinding operation - batch or continuous - can be obtained by multiplying the feed vector $F$ by a grinding matrix $\mathbf{G}$ :

$$
\mathrm{P}=\mathrm{GF}
$$

Eq A. 6
The matrix can be diagonalized as follows (23):

$$
G=Z V Z^{-1}
$$

where $\boldsymbol{V}$ is a diagonal matrix and $\mathbf{Z}$ the eigen vectors matrix of $(B-I) S$. Here $B$ is the matrix of the distribution parameters $\mathrm{b}_{\mathrm{i}, \mathrm{j}}$ and S is the diagonal matrix of the rate parameters. It can be shown that $Z$ is generated by the formulae:

$$
\begin{array}{ll}
z_{i j}=0 & , j>i \\
z_{i j}=1 & , j=i \\
z_{i j}=\sum_{k=1}^{i-1} \frac{b_{i k} s_{k}}{S_{i}-S_{j}} z_{k j} & , j<i
\end{array}
$$

The matrix $V$ depends only upon $S$ and the mixing properties of the mill. For a plug flow continuous mill or a batch test, V is defined by:

$$
V_{i i}=e^{-\tau S_{i}}
$$

where $\tau$ is the mean residence time or the time of grinding for batch operation.
For a perfect mixer, $\mathrm{V}_{\mathrm{il}}$ is given by:

$$
V_{\mathrm{ii}}=\frac{1}{1+\tau S_{i}}
$$

For a model of $p$ perfect mixers in series, plus some delay, the V matrix is:
$V_{i i}=e^{-S_{i}\left(\tau-\sum_{p} \tau_{p}\right)} \frac{1}{\left(1+\tau_{1} S_{i}\right)\left(1+\tau_{2} S_{i}\right) \ldots\left(1+\tau_{p} S_{i}\right)}$
Eq A. 11
where $\tau_{\mathrm{p}}$ is the mean residence time of the $\mathrm{p}^{\text {th }}$ mixer and $\tau$ the total mean residence time.

For a mixing model described by its unit impulse response $h(t)$, matrix $V$ is calculated by:

$$
v_{i f}=\int_{0}^{\infty} e^{-S_{1} t} h(t) d t \quad E q A .12
$$

Or using the dimensionless RTD:

$$
V_{i I}=\int_{0}^{\infty} e^{-S_{1} \pi \theta} h(\theta) d \theta \quad E q A .13
$$

For a discrete-time model described by a recursive equation of parameters $a$ and $b$ (18), the diagonal terms
of V can be calculated by Equation A .14 , where d is a pure delay, and T the sampling time interval.
This option is not presently available in the FINDBS program. However, it can be easily added when necessary.
It should be noted that the matrix $Z$ does not change when $S$ is multipled by any constant factor. Furthermore, $V$ depends only upon the product $\tau S$, a dimensionless number. So, the rate function S can be calculated from grinding data only if $\tau$ is known. Otherwise, only the product $\tau S$ is calculable.

$$
V_{i I}=e^{-S_{1} d b_{0}+b_{1} e^{-S_{1} T}+b_{2} e^{-2 S_{i} T}+\ldots+b_{p} e^{-p S_{i} T}} \frac{1+a_{1} e^{-S_{i} T}+a_{2} e^{-2 S_{1} T}+\ldots+a_{n} e^{-n S_{1} T}}{1 . \ldots}
$$

Eq A. 14

## APPENDIX B

## THE ROLE OF SIZE DISTRIBUTION IN THE ESTIMATION OF MODEL PARAMETERS

## THE ROLE OF SIZE DISTRIBUTION IN THE ESTIMATION OF MODEL PARAMETERS*

In order to assess the influence of particle size distribution on the accuracy of the back-calculation method, let us recall the basic batch-grinding equation:

$$
\frac{d M_{i}(t)}{d t}=-S_{i} M_{i}(t)+\sum_{j=1}^{i-1} \bar{B}_{i, j} S_{j} M_{j}(t) \quad \text { Eq B. } 1
$$

It can be seen that for an accurate estimation of the parameter $\mathrm{S}_{\mathrm{i}}$, the term $\mathrm{S}_{\mathrm{i}} \mathrm{M}_{\mathrm{i}}(\mathrm{t})$ should be dominant or at least comparable to the summation term, otherwise the rate of change of $M_{i}$ would not be significantly affected by the value of the parameter $\mathrm{S}_{\mathrm{i}}$.
Similarly, for an accurate estimation of the parameter $\mathrm{B}_{\mathrm{i}, \mathrm{k}}$ (where k is any number in the range 1 to $\mathrm{i}-1$ ), the term $\mathrm{B}_{\mathrm{i}, \mathrm{k}} \mathrm{S}_{\mathrm{k}} \mathrm{M}_{\mathrm{k}}$ should be predominant or at least comparable to the total of the remaining terms on the right-hand side of Equation B.1.

Obviously, the parameter $\mathrm{S}_{\mathrm{i}}$ and the set of parameters $\bar{B}_{m, i}$ are best represented in the set of equations under consideration (i.e., Eq B. 1 for $\mathrm{i}=1,2 \ldots \mathrm{n}$ ) when $\mathrm{M}_{\mathrm{i}}=1$.
However, if only one single-size feed of size-fraction $i$ is used to generate the size distribution data, no matter how long this feed is ground, it will not be possible to have more than 8-10 wt \% material in the finer sizeintervals (for example, $i+7, i+8, i+9$, etc.) at any time. Under these conditions, except for the parameters associated with just a few size-fractions next to the starting size-interval i , it will not be possible to satisfy the conditions outlined above for the correct estimation of the parameters.
On the other hand, if a single-size feed is ground for each size-interval of interest, the back-calculation method will no longer be required because all the parameters can be easily calculated using a direct method. In fact, the main objective of developing the
back-calculation method has been to eliminate the cumbersome preparation of single-size feeds, which is especially difficult in the case of fine size-fractions such as 150/200 mesh.
Taking all these factors into consideration, the authors arrived at a design for the grinding experiments, which has been found to be quite satisfactory for all the materials studied in the laboratory. This design of experiments has been indicated in Table 2 and corresponds to the case in which the size range of interest is 8 to 270 mesh.

There are two principal features of this design: more than one (for example, $30-40 \%$ of the total number of size-intervals under consideration) distributed feeds of different fineness are used; each feed starts from a different top size and these sizes are equally spaced in terms of the number of sieves over the size range of interest; and in each feed most of the material ( $60-80 \%$ ) is concentrated in the top-most size-interval, about 15-25\%.

In the next size-interval the remaining $10-20 \%$ is distributed over the remaining finer size-intervals. The size distribution data are generated by grinding each feed twice for a suitable duration of time, such that in each step the top-size undergoes $25-40 \%$ reductions.
This type of size distribution data forces the functional forms to give precise estimates of the $S$ and $B$ parameters for at least those size-intervals which contain the bulk of the material. If the $S$ and $B$ parameters really vary systematically with particle size, the functional forms are expected to provide good estimates for the remaining size-intervals also, which lie between, or close to, the selected size-intervals (i.e., those containing $60-80 \%$ of the material in the starting feeds). It seems that placing $15-25 \%$ material in the size-interval second from top helps in this direction.

[^3]
## APPENDIX C

## SIMPLIFIED BATCH-GRINDING PROCEDURE

## SIMPLIFIED BATCH-GRINDING PROCEDURE

In this procedure a sample taken directly from the production mill feed is ground dry in the laboratory mill for different times ( $2-4-8 \mathrm{~min}$, for instance). The breakage distribution parameters, $\mathrm{B}_{\mathrm{i}}$, are assumed to remain the same as those obtained from the batch tests. The breakage rate parameters are assumed to change only by a constant factor, g:

$$
S_{i}=g S_{i}^{o}
$$

where $\mathrm{S}_{\mathrm{i}}^{\rho}$ is the rate parameter obtained from the batch tests done for the estimation of the B parameters.
The constant g is estimated from new size distribution data using an estimation program, and it is taken as a grindability index, the ore used to measure $B$ being taken as a reference.

## APPENDIX D

## GENERAL STRUCTURE OF THE FINDBS COMPUTER PROGRAM

## GENERAL STRUCTURE OF THE FINDBS COMPUTER PROGRAM

## Program FINDBS Algorithm

1. Start FINDBS.
2. Read size distribution data file.
3. Print out data file title.
4. Read IBOPT, IBFIX, ISOPT, ISFIX, MIXERS:

If IBOPT $=0$, then stop.
If IBOPT $=1$, then read $B$ matrix elements or constants.
If ISOPT $=1$, then read $S$ matrix elements or constants.
If IBOPT $\neq 1$, then get estimates of $b$ constants.
If ISOPT $\neq 1$, then get estimates of $s$ constants.
If MIXERS $\neq 0$, then get fractional mean residence time values.
5. Read minimization criterion option.

If IBOPT $\neq 1$ then assign default values to or search routine parameters.
ISOPT $\neq 1$,
6. Read SEARCH option.

If
SEARCH
option $=4$, then get new search routine parameters
7. Call BOTM to search for best $B$ and/or $S$ function constants.
8. Print out final estimates.
9. Calculate final predicted size distribution(s).
10. Print out size distribution table.
11. Call SBOUT to print $B$ and $S$ matrices.
12. Go back to step 4.
13. End.

## Function CALCFX Algorithm*

1. If IBOPT $\neq 1$, call BREAK to calculate distribution matrix elements.
If ISOPT $\neq 1$, call SELECT to calculate rate matrix elements.
2. Call MAKEZ to calculate ( $Z$ ) and (Z-inverse), (Eq A.7).
3. Do for all mill residence times:

Call MAKEV to calculate the matrix (V).
*CALCFX computes the objective function.

Call MAKEG to calculate the grinding matrix (G).

Call GRIND to calculate the predicted size distributions.
Calculate the sum of square differences (error function).
4. Return.

## Subroutine GRIND Algorithm

1. Calculate predicted size distribution

$$
p=(G) F .
$$

2. Return.

## Subroutine MAKEZ Algorithm

1. Calculate the diagonalized grinding matrix $(Z)$.
2. Calculate ( $Z$ inverse) and store ( $Z$ inverse) transposed in (Z).
3. Return.

## Subroutine MAKEG Algorithm

1. Calculate the grinding matrix $(\mathrm{G})=(\mathrm{Z})_{*}(\mathrm{~V})_{*}(\mathrm{Z}$ inverse).
2. Return.

## Subroutine BREAK Algorithm

1. Calculate distribution matrix elements according to option IBOPT, using different ENTRY points.
2. Return.

## Subroutine SELECT Algorithm

1. Calculate matrix elements according to option ISOPT, using different ENTRY points.
2. Return.

## Subroutine SBOUT Algorithm

1. Entry SBOUT.
2. Print $B$ matrix elements.
3. Print S matrix elements.
4. Return.
5. Entry BRKIN.
6. Read distribution matrix elements.
7. Return.
8. Entry SELIN.
9. Read rate parameters.
10. Return.

## APPENDIX E

FINDBS PROGRAM EXAMPLES

| BATCH GRINDING DATA TEST |  |  |  |
| :---: | :---: | :---: | :---: |
| 4121.4142 .022 |  |  |  |
| 22.4 . |  |  |  |
| 8/10M | 62.96 | 30.5 | 15.38 |
| 10/14M | 36.72 | 40.39 | 34.32 |
| 14/20M | . 16 | 11.58 | 15.87 |
| 20/28M | 0.0 | 6.0 | 10.39 |
| 28/35M | 0.0 | 3.03 | 5.78 |
| 35/48M | 0.0 | 2.24 | 4.43 |
| 48/65M | 0.0 | 1.46 | 3.08 |
| 65/100M | 0.0 | 1.09 | 2.28 |
| 100/150M | 0.0 | 0.85 | 1.85 |
| 150/200M | 0.0 | 0.73 | 1.66 |
| 200/270M | 0.0 | 0.55 | 1.17 |
| PAN | 0.16 | 1. 58 | 3.81 |
| 22.4. |  |  |  |
| 8/10M |  |  |  |
| 10/14M |  |  |  |
| 14/20M | 59.26 | 33.45 | 19.38 |
| 20/28M | 25.3 | 30.89 | 27.29 |
| 28/35M | 7.8 | 14.33 | 17.47 |
| 35/48M | 4.26 | 8.7 | 11.98 |
| 48/65M | 1.97 | 5.12 | 7.57 |
| 65/100M | 0.55 | 2.39 | 4.41 |
| 100/150M | 0.24 | 1.54 | 3.0 |
| 150/200M | 0.24 | 1.02 | 2.5 |
| 200/270M | 0.16 | 0.68 | 1.66 |
| PAN | . 24 | 1.88 | 4.74 |
| 22.4. |  |  |  |
| 8/10M |  |  |  |
| 10/14M |  |  |  |
| 14/20M |  |  |  |
| 20/28M | 0.8 | 0.2 | 0.0 |
| 28/35M | 47.61 | 31.08 | 19.95 |
| 35/48M | 32.51 | 33.37 | 29.63 |
| 48/65M | 10.6 | 16.38 | 19.71 |
| 65/100M | 5.06 | 8.37 | 11.0 |
| 100/150M | 3.42 | 5.42 | 7.98 |
| 150/200M | 0.0 | 1.72 | 3.87 |
| 200/270M | 0.0 | 1.11 | 2.54 |
| PAN | 0.0 | 2.34 | 5.32 |
| 22.4 . |  |  |  |
| 8/10M |  |  |  |
| 10/14M |  |  |  |
| 14/20M |  |  |  |
| 20/28M |  |  |  |
| 28/35M |  |  |  |
| 35/48M |  |  |  |
| 48/65M | 28.83 | 22.52 | 18.36 |
| 65/100M | 38.7 | 35.61 | 33.04 |
| 100/150M | 21.78 | 23.0 | 23.74 |
| 150/200M | 5.34 | 8.01 | 9.94 |
| 200/270M | 1.78 | 3.49 | 4.49 |
| PAN | 3.56 | 7.38 | 10.43 |

## EXAMPLE DATA FILE CONTAINING B MATRIX (Unit 8)

```
0.000
    .737 0.000
    .072 .724 0.000
    .052 .076 .711 0.000
    .038 .055 .079 .697 0.000
    .028 .040 .057 .083 .682 0.000
    .020 .029 .042 .060 .087 . .067 0.000
    .015 .021 .030 .044 .063 .091 .050 0.000
    .011 .015 .022 .032 .046 .066 .096 . .033 0.000
    .008 .011 .016 .023 .033 .048 .069 . 100 . 616 0.000
    .006 .008 .012 .017 .024 .035 .050 .073 . .05 .597 0.000
    .015 .022 .031 .045 .065 .093 .134 .193 .279 .403 1.000 0.000
    .337 .293 .261 . 233 .204 .171 .134 .096 .061 .034 .016 0.000
```


## EXECUTION OF FINDBS WITH BATCH GRINDING DATA

FINDBS

DISTRIBUTION AND RATE FUNCTION DETERMINATION

BATCH GRINDING DATA TEST

OPTIONS: IBOPT, IBFIX, ISOPT, ISFIX, MIXERS: 40400 ESTIMATE OF B CONSTANT 1:.5 ESTIMATE OF B CONSTANT 2: 1. ESTIMATE OF B CONSTANT 3: 20 ESTIMATE OF B CONSTANT 4: . 2 ESTIMATE OF S CONSTANT 1:.5 ESTIMATE OF S CONSTANT 2:.2 ESTIMATE OF S CONSTANT 3: . 1 ESTIMATE OF S CONSTANT 4:. 1 CRITERION TYPE: 3
SEARCH OPTION: 3
236 FUNCTION VALUES F = .57410502E+01

$\mathrm{Bl}=.3980$
$\mathrm{B} 2=.9202$
$\mathrm{B} 3=20.00$
$B 4=.1367$
Sl $=.2605$
$\mathrm{S} 2=.3178$
S3 = .2075E-02
S4 = . $9039 \mathrm{E}-01$
FEED PRODUCT - OBSERVED/PREDICTED
SIZE 2.00 4.00

| 8/10M | 62.96 | 30.50 | 1 | 32.11 | 15.38 | / | 16.38 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10/14M | 36.72 | 40.39 | 1 | 37.07 | 34.32 | 1 | 29.11 |
| 14/20M | . 16 | 11.58 | 1 | 14.40 | 15.87 | 1 | 20.30 |
| 20/28M | 0.00 | 6.00 | 1 | 5.15 | 10.39 | / | 10.44 |
| 28/35M | 0.00 | 3.03 | / | 2.93 | 5.78 | 1 | 6.07 |
| 35/48M | 0.00 | 2.24 | 1 | 2.12 | 4.43 | 1 | 4.33 |
| 48/65M | 0.00 | 1.46 | 1 | 1.59 | 3.08 | 1 | 3.32 |
| 65/100M | 0.00 | 1.09 | 1 | 1.19 | 2.28 | 1 | 2.56 |
| 100/150M | 0.00 | . 85 | 1 | . 89 | 1.85 | 1 | 1.95 |
| 150/200M | 0.00 | . 73 | 1 | . 65 | 1.66 | 1 | 1.46 |
| 200/270M | 0.00 | . 55 | 1 | . 48 | 1.17 | 1 | 1.07 |
| PAN | . 16 | 1.58 | 1 | 1.43 | 3.81 | 1 | 3.02 |

FEED PRODUCT - OBSERVED/PREDICTED

| 8/10M | 0.00 | 0.00 | 1 | 0.00 | 0.00 | 1 | 0.00 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10/14M | 0.00 | 0.00 | 1 | 0.00 | 0.00 | 1 | 0.00 |
| 14/20M | 59.26 | 33.45 | 1 | 35.13 | 19.38 | 1 | 20.82 |
| 20/28M | 25.30 | 30.89 | 1 | 29.29 | 27.29 | 1 | 26.32 |
| 28/35M | 7.80 | 14.33 | / | 14.29 | 17.47 | 1 | 17.90 |
| 35/48M | 4.26 | 8.70 | 1 | 7.81 | 11.98 | 1 | 11.04 |
| 48/65M | 1.97 | 5.12 | 1 | 4.65 | 7.57 | 1 | 7.19 |
| 65/100M | . 55 | 2.39 | 1 | 2.61 | 4.41 | 1 | 4.67 |
| 100/150M | . 24 | 1.54 | 1 | 1. 69 | 3.00 | 1 | 3.24 |
| 150/200M | . 24 | 1.02 | 1 | 1.29 | 2.50 | 1 | 2.44 |
| 200/270M | . 16 | . 68 | 1 | . 94 | 1.66 | 1 | 1.79 |
| PAN | . 24 | 1.88 | 1 | 2.31 | 4.74 | 1 | 4.60 |



## EXECUTION OF FINDBS WITH CONTINUOUS GRINDING DATA



|  | FEED | PRO | DUCT - | OBSERVED / PREDICTED |
| :---: | :---: | :---: | :---: | :---: |
| SIZE | 1.79 |  |  |  |
|  |  |  |  |  |
| 4/6M | . 12 | . 08 | 1.03 |  |
| 6/8M | . 50 | . 08 | / . 10 |  |
| 8/10M | 2.04 | . 34 | / . 41 |  |
| 10/14M | 4.30 | . 94 | / 1.22 |  |
| 14/20M | 7.50 | 2.80 | 13.00 |  |
| 20/28M | 11.02 | 6.10 | / 6.11 |  |
| 28/35M | 14.33 | 10.71 | / 10.25 |  |
| 35/48M | 17.03 | 16.37 | / 14.41 |  |
| 48/65M | 10.84 | 12.65 | / 13.23 |  |
| 65/100M | 6.85 | 10.16 | $/ 9.98$ |  |
| 100/150M | 4.90 | 7.70 | / 7.41 |  |
| 150/200M | 3.40 | 5.80 | 15.36 |  |
| 200/270M | 2.40 | 3.73 | / 3.75 |  |
| PAN | 14.76 | 22.53 | / 24.75 |  |
|  | FEED | PRODUCT - OBSERVED/PREDICTED |  |  |
| SIZE | 1.47 |  |  |  |
| $+4 \mathrm{M} \quad .09 \quad .04 / .03$ |  |  |  |  |
| 4/6M | . 22 | . 12 | 1.07 |  |
| 6/8M | 1.00 | . 40 | 1.26 |  |
| 8/10M | 3.02 | . 94 | 1.83 |  |
| 10/14M | 6.11 | 2.21 | / 2.21 |  |
| 14/20M | 9.50 | 4.60 | 14.75 |  |
| 20/28M | 14.53 | 9.96 | 19.08 |  |
| 28/35M | 18.39 | 15.19 | / 14.35 |  |
| 35/48M | 15.11 | 15.25 | / 15.82 |  |
| 48/65M | 8.83 | 11.01 | / 12.44 |  |
| 65/100M | 5.90 | 8.86 | 19.06 |  |
| 100/150M | 3.98 | 6.70 | 16.51 |  |
| 150/200M | 2.51 | 4.62 | / 4.52 |  |
| 200/270M | 1.50 | 2.78 | 12.99 |  |
| PAN | 9.30 | 17.33 | / 17.06 |  |
|  | FEED | PRO | DUCT - | OBSERVED / PREDICTED |
| SIZE |  |  | . 18 |  |
| $+4 M$ | . 48 | . 16 | $; \quad .21$ |  |
| 4/6M | .53 | . 16 | 1.27 |  |
| 6/8M | 2.40 | .70. | 1.80 |  |
| 8/10M | 5.50 | 2.01 | 12.07 |  |
| 10/14M | 9.00 | 4.40 | 14.35 |  |
| 14/20M | 11.75 | 7.30 | / 7.54 |  |
| 20/28M | 14.60 | 11.42 | / 11.43 |  |
| 28/35M | 14.31 | 13.43 | / 13.93 |  |
| $35 / 48 \mathrm{M}$ | 11.51 | 13.36 | / 13.45 |  |
| 48/65M | 7.03 | 9.38 | / 10.15 |  |
| 65/100M | 5.10 | 7.68 | 17.57 |  |
| $100 / 150 \mathrm{M}$ | 3.59 | 5.85 | 15.56 |  |
| 150/200M | 2.45 | 4.22 | 13.97 |  |
| 200/270M | 1.67 | 2.90 | 12.75 |  |
| PAN | 10.07 | 17.01 | / 15.95 |  |


|  | FEED | PRODUCT - OBSERVED/PREDIGTED |  |  |
| :---: | :---: | :---: | :---: | :---: |
| SIZE | 1.00 |  |  |  |
| +4M .................................... |  |  |  |  |
| 4/6M | 1.08 | $.49 / .50$ |  |  |
| 6/8M | 3.00 | $1.00 / 1.24$ |  |  |
| 8/10M | 5.65 | $2.63 / 2.63$ |  |  |
| 10/14M | 9.59 | $5.37 / 5.22$ |  |  |
| 14/20M | 12.00 | 8.50 / 8.49 |  |  |
| 20/28M | 14.62 | 11.96 / 12.21 |  |  |
| 28/35M | 14.95 | $14.16 / 14.68$ |  |  |
| 35/48M | 11.46 | 13.17 / 13.53 |  |  |
| 48/65M | 6.84 | $8.91 / 9.76$ |  |  |
| 65/100M | 5.35 | $7.49 / 7.46$ |  |  |
| 100/150M | 3.50 | $5.69 / 5.30$ |  |  |
| 150/200M | 2.19 | $3.71 / 3.60$ |  |  |
| 200/270M | 1.46 | $2.39 / 2.45$ |  |  |
| PAN | 7.83 | $14.24 / 12.70$ |  |  |
|  | FEED | PRODUCT - OBSERVED/PREDICTED |  |  |
| SIZE |  | 1.57 |  |  |
| $+4 M \quad .14 \quad .03 / .05$ |  |  |  |  |
| 4/6M | . 30 | $.05 / .10$ |  |  |
| 6/8M | 2.00 | $.45 / .44$ |  |  |
| 8/10M | 4.03 | $1.13 / 1.16$ |  |  |
| 10/14M | 7.00 | $2.55 / 2.62$ |  |  |
| 14/20M | 10.12 | 5.00 | / 5.17 |  |
| 20/28M | 12.80 | $8.70 / 8.68$ |  |  |
| 28/35M | 14.93 | 12.82/12.45 |  |  |
| 35/48M | 12.72 | $14.03 / 13.65$ |  |  |
| 48/65M | 7.31 | $9.98 / 10.83$ |  |  |
| 65/100M | 5.60 | $8.42 / 8.38$ |  |  |
| 100/150M | 4.20 | $6.75 / 6.44$ |  |  |
| 150/200M | 3.17 | $5.12 / 4.85$ |  |  |
| 200/270M | 2.11 | $3.51 / 3.36$ |  |  |
| PAN | 13.58 | $21.46 / 21.84$ |  |  |
|  | FEED | PRODUCT - OBSERVED/PREDICTED |  |  |
| SIZE |  | 1.44 |  |  |
| $+4 \mathrm{M}$ | . 21 | . 04 | / .08 |  |
| 4/6M | . 48 | .11 | 1.17 |  |
| 6/8M | 2.00 | . 50 | 1.53 |  |
| 8/10M | 4.04 | 1.33 | / 1.31 |  |
| 10/14M | 6.65 | 2.70 | / 2.78 |  |
| 14/20M | 9.60 | 5.02 | 15.27 |  |
| 20/28M | 12.42 | 8.72 | / 8.71 |  |
| 28/35M | 15.27 | 13.44 | / 12.69 |  |
| 35/48M | 14.24 | 15.04 | / 14.52 |  |
| 48/65M | 8.33 | 10.70 | / 11.59 |  |
| 65/100M | 5.70 | 8.30 | 18.55 |  |
| 100/150M | 4.10 | 6.47 | 16.34 |  |
| 150/200M | 2.96 | 4.72 | 14.65 |  |
| 200/270M | 1.97 | 3.28 | 13.20 |  |
| PAN | 12.03 | 19.64 | / 19.60 |  |

```
B MATRIX
0.000
    .771 0.000
    .063 .760 0.000
    .045 .066 .749 0.000
    .033 .047 .069 . .737 0.000
    .024 .034 .050 .072 .724 0.000
    .017 .025 .036 .052 .076 . . 711 0.000
    .013 .018 .026 .038 .055 .079 . .097 0.000
    .009 .013 .019 .028 .040 .057 .083 . .082 0.000
    .007 .010 .014 .020 .029 .042 .060 .087 .066 0.000
    .005 .007 .010 .015 .021 .030 .044 .063 .091 . .050 0.000
    .004 .005 .007 .011 .015 .022 .032 .046 .066 .096 . .033 0.000
    .003 .004 .005 .008 .011 .016 .01023 .033 .048 .069 . .000 . . .016 0.000
    .002 .003 .004 .006 .008 .012 .017 .024 .035 .050 .073 . . .05 . .0797
0.000
    .005 .007 .010 .015 .022 .031 .045 .065 .093 . . . .04 . . 194 . . 279 .403
1.000 0.000
RATE CONSTANTS
    .803 1.143 1.352 1.373 1.236 1.020 .797 . 608 .469 . 377 . . 326 . 314 . 347
    .456 0.000
    OPTIONS: IBOPT, IBFIX, ISOPT, ISFIX, MIXERS: 0 0 0 0 0
NORMAL PROGRAM TERMINATION
```

Canadä
anacia


[^0]:    *Lower sieve size, upper sieve size, arithmetic or geometric mean of the lower and upper sieve sizes - any one of these can be used to represent a size-interval. In each case the ratio of the characteristic sizes for adjacent sizeintervals is the same as the ratio for the size of the openings of two successive screens (i.e., $\sqrt{2}$ ).

[^1]:    *mean size

[^2]:    *Due to a limited amount of sample received, it was not possible to prepare feeds of the desired size distribution.

[^3]:    * See also Reference 31.

