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Centre canadien de la technologie des minéraux et de l'énergie

## SPOC <br> Simulated Processing of Ore and Coal

## Chapter 3.2 MATBAL Computer Program

> SER
> $622(21)$
> C2129P
> $85-1 / 3.2 E$
> c. 2


Energy, Mines and Resources Canada

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## CANMET

Canada Centre for Mineral and Energy Technology

Centre canadien de la technologie des minéraux et de l'énergie

## The



Manual
Chapter 3.2 MATBAL Computer Program MATBAL - Computer Program for Material Balance Data Adjustment
D. Laguitton

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Minister of Supply and Services Canada 1985
Available in Canada through
Authorized Book Agents and other bookstores or by mail from 13

Canadian Government Publishing Centre (11) Ottawa, Canada K1A 0S9

Price subject to change without notice
Disponible en français

## THE SPOC MANUAL

The SPOC* manual consists of eighteen chapters, published separately. Their numbers and short titles are as follows:

1. Summary
2. Sampling Methodology
2.1 SAMBA Computer Program
2.2 Grinding Circuit Sampling
3. Material Balance
3.1 BILMAT Computer Program
3.2 MATBAL Computer Program
4. Modelling and Simulation
4.1 Industrial Ball Mill Modelling
5. Unit Models: Part A
5.1 Unit Models: Part B
5.2 Unit Models: Part C
6. Flowsheet Simulators
7. Model Calibration
7.1 STAMP Computer Program
7.2 FINDBS Computer Program
7.3 RTD and MIXERS Computer Programs
8. Miscellaneous Computer Programs

These chapters are available from: CANMET, Energy, Mines and Resources Canada Technology Information Division 555 Booth Street Ottawa, Ontario

[^0]
## FOREWORD

High energy costs and depleting ore reserves combine to make process evaluation and optimization a challenging goal in the 80 's. The spectacular growth of computer technology in the same period has resulted in widely available computing power that can be distributed to the most remote mineral processing operations. The SPOC project, initiated at CANMET in 1980, has undertaken to provide Canadian industry with a coherent methodology for process evaluation and optimization assisted by computers. The SPOC Manual constitutes the written base of this methodology and covers most aspects of steady-state process evaluation and simulation. It is expected to facilitate industrial initiatives in data collection and model upgrading.
Creating a manual covering multidisciplinary topics and involving contributions from groups in universities, industry and government is a complex endeavour. The reader will undoubtedly notice some heterogeneities resulting from the necessary compromise between ideals and realistic objectives or, more simply, from oversight. Critiques to improve future editions are welcomed.
D. Laguitton

SPOC Project Leader
Canada Centre for Mineral and Energy Technology

## AVANT-PROPOS

La croissance des coûts de l'énergie et l'appauvrissement des gisements ont fait de l'évaluation et de l'optimisation des procédés un défi des années 80 au moment même où s'effectuait la dissémination de l'informatique jusqu'aux concentrateurs les plus isolés. Le projet SPOC, a été lancé en 1980 au CANMET, en vue de développer pour l'industrie canadienne, une méthodologie d'application de l'informatique à l'évaluation et à l'optimisation des procédés minéralurgiques. Le Manuel SPOC constitue la documentation écrite de cette méthodologie et en couvre les différents éléments. Les retombées devraient en être une vague nouvelle d'échantillonnages et d'amélioration de modèles.

La rédaction d'un ouvrage couvrant différentes disciplines et rassemblant des contributions de groupes aussi divers que les universités, l'industrie et le gouvernement est une tâche complexe. Le lecteur notera sans aucun doute des ambiguiltés ou contradictions qui ont pu résulter de la diversité des sources, de la traduction ou tout simplement d'erreurs. La critique constructive est encouragée afin de parvenir au format et au contenu de la meilleure qualité possible.
D. Laguitton

Chef du projet SPOC,
Centre canadien de la technologie des minéraux et de l'énergie

## ABSTRACT

MATBAL is a FORTRAN program to calculate data adjustments required in order to satisfy mass conservation equations valid for a mineral process flowsheet. The calculation is done by minimizing the sum of squares of assay and flow rate adjustments weighted by their reciprocal variance. The available data and the mass conservation equations are coded in a suitable form and the derivatives of a Lagrangian function are internally calculated and cancelled to obtain the adjustments and flow rates which are a solution to the problem. The precision of calculated flow rates can also be obtained.
Writing a program to perform such calculations is only part of the overall task if the method is to be transferred to the end user in the mineral industry. Program documentation has traditionally been the weak point which jeopardizes and often cancels all efforts made to develop a program. This user's manual is an attempt to avoid such a dead-end by fully explaining both the method and the program so that the program can be installed in different computers or translated if FORTRAN is not supported by a given machine. The documentation follows the standards of the A.S.C.E.*

## RÉSUMÉ

MATBAL est un programme en FORTRAN pour calculer les ajustements nécessaires afin que des données expérimentales satisfassent les équations de conservation de masse valides pour un circuit donné. Le calcul consiste à minimiser la somme des carrés des ajustements, pondérés par l'inverse des variances associées à chaque mesure. Les équations de conservation de masse, ainsi que les données disponibles, sont traitées à l'aide d'un formalisme propre au programme afin de calculer automatiquement les dérivées partielles d'un Lagrangien. L'annulation simultanée de ces dérivées conduit aux ajustements et aux variables calculées qui sont les solutions du problème. MATBAL permet aussi d'estimer la précision sur les variables calculées.
Le développement d'un programme de calcul n'est qu'une partie du travail requis sil'on veut que la méthode soit disponible a l'utilisateur intéressé. La documentation des programmes est toujours l'enfant pauvre du travail scientifique ce qui compromet ou annulle souvent les efforts protés au stade du developpement de la méthode. Ce manuel essaie d'éviter cette impasse en expliquant à la fois la méthode et le programme qui la met en application. De cette façon on peut espérer que le programme puisse être activé sur différents ordinateurs sans problèmes majeurs ou qu'il puisse être traduit si le langage FORTRAN ne convient pas. La documentation suit les normes de la Société Américaine des Ingénieurs Civils, A.S.C.E.*

## ACKNOWLEDGEMENTS

The SPOC project has benefited from such a wide range of contributions throughout the industry, the university, and the government sectors that a nominal acknowledgement would be bound to make unfair omissions. The main groups that contributed are: the various contractors who completed project elements; the Industrial Steering Committee members who met seven times to provide advice to the project leader; the various users of project documents and software who provided feedback on their experience; the CANMET Mineral Sciences Laboratories staff members who handled the considerable in-house task of software development, maintenance, and documentation; the EMR Computer Science Centre staff who were instrumental in some software development; and the CANMET Publications Section. Inasmuch as in a snow storm, every flake is responsible, their contributions are acknowledged.

[^1]
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## 1. PROGRAM IDENTIFICATION



[^2]
## 2. ENGINEERING DOCUMENTATION

### 2.1 NARRATIVE DESCRIPTION

### 2.1.1 Introduction

The need for computer programs to calculate material balances in mineral processing circuits has long been recognized and several such programs have been developed (1-7). They differ by the mathematics and the computer languages they use, but they often have in common a lack of availability to the outside user or a lack of documentation which makes them difficult to transfer to a different computer site.
The present manual attempts to provide a remedy to this situation by promoting, documenting, and making available a material balance program that displays increased versatility and new features. This work was performed in the Mineral Processing Laboratory of CANMET. It was at first a necessary step in the study of computer simulation as a way to devise new processing schemes for New Brunswick fine, complex, sulphide ores and it subsequently developed into the SPOC project (Simulated Processing of Ore and Coal).

### 2.1.2 History of MATBAL

MATBAL was a FORTRAN program originally developed at the Mineral Resources Research Centre of the University of Minnesota by R.L Wiegel and coworkers (1). It had been designed to calculate solid flow rates in a circuit where only chemical and physical analyses (referred to as "assays" in the text) had been collected. A later modification* enabled the program to handle measured solid flow rates in the input data, as well.
From the very beginning, MATBAL was meant as a computer substitute to the two- or three-product formulae which until then had been the only mathematical methods of estimating mass flow rates from experimental assays (1).
In 1978, CANMET's Ore Dressing Section obtained a source listing of the program from the University of Minnesota and began testing it as part of a study of mineral process simulation (8). The program was found to perform well on several flowsheets and raised interest among potential users. It was then decided that a new version should be developed and written prior to further distribution. This led to MATBAL2 which gave the user more freedom in formulating the problem and contained a method to estimate the precision of the calculated flow rates $(9,10)$.
Feedback from several users of MATBAL2 underlined the requirement for several improvements to facilitate usage and to reduce required memory and execution time. These changes were implemented in the MATBAL3 version and further refined in this present version of MATBAL.

### 2.1.3 The Material Balance Problem

### 2.1.3.1 Principle

The evaluation of a mineral processing circuit is often required for metallurgical inventories in order to keep track of the total mass of valuable species processed weekly or monthly. More recently, process simulation has become a new incentive for rigorous investigation of process flowsheets.
In both cases, restrictions arise from the available instrumentation, which frequently does not permit accurate measurement of high solid flow rates. The only available data come from assaying grab samples or using on-line analyzers. Such assays suffer, of course, from various errors such as sample non-representativity, manipulation losses, and limited precision of analytical methods. Ideally, available data could be used to calculate the true value of the mass flow rates and of the assays. In practice, one can but approach this ideal and only obtain estimates of the true values.
The quality of the estimates, as well as the quality of the sampling method, depends on the degree of refinement of the calculation method. The simplest method is the two- or three-product formula, in which the experimental assays are considered as the best estimates of the true assays, and are used to solve the mass conservation equations for the solid flow rates (1).
The limitations of this method are threefold. First, the values of the calculated flow rates depend largely on the precision of the experimental assays. Secondly, all available assays are not used if they are redundant with respect to the number of mass conservation equations. Thirdly, recirculation streams and large flow sheets are very difficult to balance by this method.
With the assistance of a computer, the effort involved in using a more sophisticated method is reduced to properly coding the method into a computer program and providing the program with suitable input data. This modern approach is the basis for the material balance programs announced in the mineral processing literature since the early 70 's.
Using these computer methods, the user makes an assumption about the type of errors involved in sampling and assaying. The experimental assays are then processed in order to make them satisfy the mass conservation equations and fit this assumption. The program output is a best estimate of the true assay values and of the mass flow rates. Furthermore, several units can be balanced at the same time and more experimental data can be taken into consideration. The availability of computers makes this latter method more and more accessible even to non-computer trained personnel.

[^3]In this manual, the method is described both in general terms and in detailed mathematical terminology. The reader who finds it difficult to understanding either form will find consolation in the fact that a material balance program can be installed on a given site and operate entirely as a 'black box' for routine inventories, provided that a sampling strategy is defined to produce the appropriate input data.

### 2.1.3.2 Glossary of terms

A few words or expressions are used throughout this Chapter with a specific meaning in a material balance context.
Flowsheet is used to designate the schematic representation of a circuit to be balanced.
It is composed of units which represent the individual pieces of machinery used in the schematized process.
Process units are interconnected by streams which are usually pipes, or belts, or any other device, for the transportation of the processed materials from one unit to the other.

A stream is characterized by its flow rate or weight which can be a slurry, solid, liquid or gas flow rate; and by assays which can be any chemical or physical measurement performed on the circulating material. Very often an assay type is the chemical content of a solid, e.g., $\mathrm{Fe}, \mathrm{Cu}, \mathrm{Zn}$. In that case, Fe is also an assayed species.
Imbalance refers to the deviation of a mass conservation equation from perfect balance. By extension it is used to designate the quantity by which the equation differs from zero, also called residual.
Error model is used to designate the information describing the nature of the errors on the experimental data. Such information is necessary in order for the program to distribute the data adjustments according to the data precision. In other words, an assay which is known to be inaccurate must be allowed to be adjusted by a larger amount than an accurate assay.
In MATBAL, the error model is partly built-in by the assumption that all errors are gaussian, unbiased, uncorrelated, and partly input by associating a standard deviation to each experimental value.

### 2.2 METHOD OF SOLUTION

### 2.2.1 Material Balance Through Optimization

It is well known that in order to calculate, for example, $N$ independent unknowns, it is necessary to have at least $N$ independent equations involving these unknowns. Therefore, if N independent flow rates are to be calculated in a processing flowsheet, $N$ independent equations must be found between them. Using as an example, the simple unit shown in Figure 1, around which streams 1, 2, and 3 have been assayed for species $R$, one can write two mass conservation equations expressing the total mass conservation and the conservation of species $R$ :

$$
\begin{align*}
& W 1-W 2-W 3=0  \tag{Eq 1}\\
& W 1 . R 1-W 2 . R 2-W 3 . R 3=0 \tag{Eq 2}
\end{align*}
$$

where $W 1, W 2$, and $W 3$ are the solid flow rates.


Fig. 1-A two-product process unit
Having two equations, it is only possible to calculate two unknowns. For instance, if W1, R1, R2, and R3 are known, the solutions to Equations 1 and 2 are

$$
\begin{align*}
& W_{2}=W 1 \frac{R 1-R 3}{R 2-R 3}  \tag{Eq 3}\\
& W 3=W 1 \frac{R 2-R 1}{R 2-R 3}
\end{align*}
$$

Eq 4

This constitutes the two-product formula applied to Figure 1. The values of W2 and W3 calculated in Equation 3 and 4 are directly influenced by the precision with which W1, R1, R2, and R3 are known. Also, if the same flowsheet is sampled for another species $R^{\prime}$, this other data can at best be used by this method as a duplicate to confirm the values of W2 and W3. In most cases, it will lead to conflicting results, complicating the interpretation of the calculation.

The main reasons for developing a computer method of material balance calculation are illustrated by in the above example. All redundant data should be used, and information on the type and the magnitude of the errors on the assays should be part of the data and, thus, influence the solution. The precision of the solution should also be estimated.

To perform a material balance calculation it is therefore necessary to:

1. Collect suitable experimental data.
2. Measure or estimate the precision of experimental data.
3. Analyze the flowsheet and create the data to fit the computer program requirements.
These three important steps are reviewed in the following sections.

### 2.2.1.1 Data collection

Data collection is, of course, the first step in a material balance calculation. It consists of performing chemical or physical measurements on the streams connected to the units of a process flowsheet in order to be able to use mass conservation equations across these units. The type of data that can be collected depends on the type of operation and on the available manpower or instrumentation.
Particle-size distributions are important parameters in studying a grinding circuit. However, they can only satisfy mass conservation across classification units and not across grinding units. The same is true for the liberation of specific minerals.
Chemical composition is used to balance flotation circuits. Instrument readouts, such as feed rates from weightometers; per cent solid from gamma gauges, and water flow rates from flowmeters, provide another source of data.
Experimental data collected in mineral processing circuits can be described as belonging to several families, or levels, in a hierarchy. The data can characterize a stream considered as a slurry without distinction of solid or liquid phases. They can also refer to sub-streams of solid only, or liquid only. They can also refer to specific fractions of the solid, or of the liquid.
This distinction is better illustrated by giving examples of data of each level. Slurry data can include slurry flow rates and pulp densities, often called per cent solids. Solid data can include such factors as chemical or mineralogical assays, particle-size distributions, density distribution, solid flow rate. Liquid data can include such factors as pure liquid stream flow rates, chemical composition.

Data in a lower level could be the mineralogical, or chemical, assay of a size interval or a density fraction. This distinction is important since the mass conservation equations used in MATBAL allow the mass balance of only one level of data.
For instance, if the streams are considered as slurry streams, the calculated flow rates are slurry flow rates and a material balance calculation can only be made on assays expressed as slurry assays. A particle size distribution in such a case would have to be multiplied by the solid to slurry ratio (\% solid/100) before being considered for adjustment by MATBAL.
Similarly, if the solid flow rates are to be considered, any water flow rate that is available must be transformed into solid flow rate by assuming a small, but non-null, solid-to-water ratio in all water streams. The quantity which allows such a transformation from one level of data to the other is usually the pulp density and the following relations are useful:

$$
\text { Pulp density }=\frac{S}{S+L}
$$

where $S$ and $L$ are the solid and liquid flow rates:

$$
\begin{aligned}
& \% \text { Solid }=P S=\frac{S}{S+L} \times 100 \\
& \text { Solid-to-liquid ratio }=\frac{S}{L}=\frac{P S}{100-P S} \\
& \text { Liquid-to-solid ratio }=\frac{L}{S}=\frac{100}{P S}-1
\end{aligned}
$$

The user can assess whether the data transformation is justified or not, depending on the amount and quality of data of a second level that is available for adjustment. The transformation must be made manually and can be lengthy and a source of numerical ill-conditioning by introducing data of very different magnitude.
To obtain the best material balance on a given circuit, several sampling strategies must be evaluated in order to produce the most precise flow rate evaluation. A rule of thumb is to sample around units showing the largest split between output stream compositions. More on sampling methodology can be found in Chapter 2 of the SPOC Manual.

### 2.2.1.2 Precision of the experimental data

As mentioned before, a material balance calculation requires a model of the errors involved in sampling and assaying the samples. In practice, it is only possible to measure the standard deviation of each individual assay, or of each assay type, and to make all possible efforts to avoid biased results through improper calibration of an instrument, faulty procedures, non-representative sampling, or analytical methods.

The error model acceptable by MATBAL assumes that errors are gaussian (normal), unbiased, and uncorrelated. In addition to trying to satisfy these criteria, the user must provide the value of the standard deviation of the measurements. If repeats are possible the classical formula can be used:

$$
\begin{equation*}
\sigma_{\mathrm{R}}=\sqrt{\frac{\sum(\mathrm{R}-\overline{\mathrm{R}})^{2}}{\mathrm{~N}-1}} \tag{Eq 5}
\end{equation*}
$$

where $\sigma_{\mathrm{R}}$ is the standard deviation of assay R , and $\overline{\mathrm{R}}$ is the mean of the $N$ measurements $R$.
Very often it is not possible to repeat sampling of a large circuit when the same steady-state conditions cannot be reached, or the amount of work is too large. The user must then compensate for this by replacing experimentation with sound judgement. By evaluating the difficulty of sampling at a specific point or any other source of error, the user must make a quantitative estimate of the standard deviation, remembering that, in a first approximation, proper ranking of data according to their precision is most important.
The user may prefer to categorize each assay, or each assay type, as very good, good, fair, poor, or very poor, and let the program handle the internal numerical equivalents of these qualifiers. In any case, it can never be over-emphasized that the superiority of the material balance data adjustment by computer over the three-product formula is in its ability to take into consideration experimental errors. Information on these must, therefore, appear as an input to the problem and not as a result of the calculation.

### 2.2.1.3 Flowsheet analysis for data preparation

The following discussion is illustrated by Figure 2. It includes four units: a ball mill, a sump box, a hydrocyclone, and a flotation cell. They are connected by seven streams. Depending on the degree of sophistication needed, one may want to calculate only the output streams 5 and 6 or, alternatively, to investigate the circuit in detail and calculate all streams.

Let us consider this latter general case and assume that three assays $R, R^{\prime}$, and $R^{\prime \prime}$ have been obtained in every stream from 1 to 7 . To be more specific, R is the copper content of the solid; $\mathrm{R}^{\prime}$ is the zinc content; and $\mathrm{R}^{\prime \prime}$ is the per cent passing $700 \mu \mathrm{~m}$.
An estimate of the precision of these assays is also available. In order to feed this information into the program, it is necessary to analyze qualitatively the different mass conservation equations that must be satisfied in this circuit. To do so, MATBAL uses the concept of normal and non-normal (or special) mass conservation equations (also called node equations). There are as many nodes as units in a circuit, including hypothetical mixers represented by the convergence point of assayed streams (Fig. 2).
For each node, two kinds of mass conservation equations can be satisfied. One expresses the total mass conservation through the node, the other expresses the mass conservation of an assayed species. For instance, the mass conservation equation of the first type across the hydrocyclone is:

$$
\begin{equation*}
W 2-W 3-W 4=0 \tag{Eq 6}
\end{equation*}
$$

Wi is the solid flow rate of stream i .
One mass conservation equation of the second type across the hydrocyclone is:

$$
\begin{equation*}
\text { W2.R2-W3.R3-W4.R4 = } 0 \tag{Eq 7}
\end{equation*}
$$

Several mass conservation equations of the second type may be valid across a unit. Equation 7 could also be written with $\mathrm{R}^{\prime}$ or $\mathrm{R}^{\prime \prime}$ replacing R .
A normal node is a unit or a group of units across which all available assays can be used in mass conservation equations of the second type. It is important to understand that the samples taken restrict what is, or is not, a node. In the case of Figure 2, the cyclone, the sump box, and the flotation cell qualify as normal nodes. The ball mill does not qualify, since $R^{\prime \prime}$ (per cent passing $700 \mu \mathrm{~m}$ ) cannot be used in Equation 7. It is, therefore, a nonnormal node.


Fig. 2 - Schematic circuit used to Illustrate flowsheet analysis for MATBAL

The user must provide the program with the necessary information to define the flowsheet and the type of node equations to be satisfied by the assays and the flow rates. This is done by filling two tables of node coefficlents. These are now described using the same symbols as in the program.
Let NMAX be the number of normal nodes, NNMAX the number of non-normal nodes, MMAX the number of sampled streams, and OMAX the number of assay types. First, all assay results can be gathered in an array R, which contains MMAX rows and OMAX columns.
In the example, MMAX $=7$ and $\mathrm{OMAX}=3$. The number of normal nodes is NMAX $=3$ since the sump box, cyclone, and flotation cell satisfy the definition given above for normal nodes. The number of nonnormal nodes is NNMAX = 1 (ball mill), according to our definition. In MATBAL, all measured or fixed flow rates are internally converted into non-normal nodes, but the user must not count them as such in determining NNMAX. For instance, if flow rates 1,5 , and 6 had been measured, they would internally be converted into nonnormal nodes expressed by the equations:

$$
\begin{array}{ll}
\hat{W}_{1}=W_{1} \text { obs }+ \text { adj. } & \text { Eq } 8 \\
\hat{W}_{5}=W_{5} \text { obs }+ \text { adj. } & \text { Eq } 9 \\
\hat{W}_{6}=W_{6} \text { obs }+ \text { adj. } & \text { Eq } 10
\end{array}
$$

where the symbol " indicates a calculated estimate, adj. is an adjustment calculated by the program.
In this example, for the user, NNMAX is therefore equal to one, even if this value is internally modified due to measured flow rates.
In all cases, one flow rate must be known and calculations made with reference to this chosen stream. The feed rate (stream 1 in our example) is usually taken as a reference of, for example, 1 or 100, and treated as a measured flow rate of 1 or 100, assumed to be known with very high accuracy (i.e., very small standard deviation).
Having determined these important numbers, the user must fill the table of normal node coefficients A , with NMAX rows and MMAX columns. The user must also fill the table of non-normal node coefficients B , with NNMAX rows and MMAX columns. Each row symbolizes a node equation and contains the coefficients of all flow rates in Equation 6. These coefficients can only take the values $+1,0$, or -1 . For instance, the row corresponding to Equation 6 (hydrocyclone) would be $0,1,-1,-1,0,0,0$. The reader will easily verify that the whole matrix A describing Figure 2 is:

$$
\mathrm{A}=\begin{array}{rrrrrrrl}
1 & -1 & 0 & 0 & 0 & 0 & 1 & \text { Sump box } \\
0 & 1 & -1 & -1 & 0 & 0 & 0 & \\
0 & 0 & 0 & 1 & -1 & -1 & 0 & \text { Cyclone } \\
\text { Flotation cell }
\end{array}
$$

Matrix $B$ is obtained in the same manner for the NNMAX non-normal nodes:

$$
\mathrm{B}=0 \begin{array}{lllllll}
0 & 0 & 1 & 0 & 0 & 0-1
\end{array} \quad \text { Ball mill }
$$

In MATBAL, however, the user simply enters the number of streams connected to each node and their numbers with a negative sign, if they are output streams.
In the example above, $A$ and $B$ would be read as:

$$
A=\begin{array}{llll}
3 & 1 & -2 & 7 \\
3 & 2 & -3 & -4 ; \\
3 & 4 & -5 & -6
\end{array} \quad B=2 \quad 3-7
$$

From the definition of the non-normal nodes, it is easily understood that one must identify the special assays that can be used in equations similar to Equation 7. This is done by setting a table of numerical 'switches' -1 or - 0 containing a numerical Yes or No for each special node and each assay type. This matrix has, therefore, NNMAX rows and OMAX columns and is designated as BNNO.
In our example:

$$
\begin{array}{llll}
\text { BNNO }=1 & 1 & 0 & \begin{array}{l}
\text { Assays } 1 \text { and } 2 \text { balanced } \\
\text { across ball mill }
\end{array}
\end{array}
$$

In MATBAL, BNNO is entered by reading the number of assays for which there is a balance for a given node, followed by their numbers. In the example above, $\mathrm{BNNO}=212$. Note that if no assay applies, one must enter BNNO = 10 for the program to insert a full row of zeros in BNNO.
The total number of special assays (non-null BNNO values) is $\operatorname{IMAX}(\operatorname{IMAX}=2)$.
All necessary tables to be filled with data before executing the program have been defined above. The exact format of each table is described later in the manual, as well as the order in which they appear in the input data file. Extra data required in the input for special problems are discussed next.
Sometimes the user is unable to make an estimate of the precision of some assays, or the program is employed for predictive material balance calculation, using pre-set assay values at given locations and letting other guessed assays bear the adjustment. In such a case, the user may want to remove selected assays from the adjustment process by 'freezing' them to their observed value.
To do so, it is necessary to identify individually all assays which must be kept constant. This can be done by assigning a very high accuracy (i.e., very small standard deviation) to those assays. The user must, therefore, enter the total number NNNMAX of such assays and the stream and type subscripts, $M$ and $O$, of each fixed assay.

### 2.2.1.4 Error estimation for the calculated flow rates

The error estimation for the calculated flow rates is a good way to compare numerical results obtained from different data on the same circuit. As shown above, the user is free to enter as many assay types as desired, and to balance a circuit above the minimum requirement of one assay type per node equation. On the other hand,
since the material balance calculation is an optimization technique, its result depends on the data fed to the program including the assumed error model used to define the optimum.
Therefore, two users balancing the same circuit may end up with different results, unless they both used exactly the same assays and the same error model (which is often subjective input). Confronted with inconsistent results, such as widely different circulating load values, they can argue indefinitely over which calculation is better and which flow rates more precise, or they can reject the results entirely.
This situation can be avoided or at least tempered by using an error estimator, the function of which is to quantify the impact of the input data-including the error model - on the calculated flow rates. Two error estimators have already been described ( 2,9 ). One is referred to as the analytical estimator, the other as the Monte Carlo estimator which involves random numbers generation.
The Monte Carlo estimator can be automatically activated, and requires no further explanation in this section. When the material balance program is used on a routine basis to process data of constant precision, the Monte Carlo simulator can very advantageously be used to estimate the precision of the calculated flow rates. The estimated precision will be valid as long as the precision of the sampling and the circuit flowsheet are not altered. The analytical estimator has been discussed by Smith (2) for the node imbalance minimization method and is implemented in the SAMBA program (Chapter 2.1, SPOC Manual). It is not used in MATBAL.

### 2.2.2 The Mathematics of MATBAL

### 2.2.2.1 Outline of the method

A typical material balance problem consists of using mass conservation equations, applicable to the units of a flowsheet, to estimate the flow rates and chemical or physical assays for which raw experimental values are known. The number of unknowns frequently being larger than the number of equations, the problem is an optimization problem and a criterion must be defined to select the 'best' among all possible solutions.
This is done by assuming that the adjustments to be made to all experimental data to obtain their best estimate follow a given model. This model is, therefore, a model for the errors of the experimental data.
Considering the large number of operations performed during the collection and analysis of grab samples, this model assumes that all errors are gaussian (normal), unbiased, and uncorrelated. Consequently, the weighted sum of the squares of the errors must be minimum and the adjusted data must satisfy the mass conservation equations. The problem is one constrained minimization for which several methods have been proposed.

Let $R$ be a set of MMAX experimental assays and $\hat{R}$ the corresponding set of adjusted assays. $\hat{R}$ is the best estimate of the true assay values, $\mathrm{R}^{*}$. MMAX is the number of streams in the flowsheet. Similarly, $\hat{W}$ is a set of MMAX estimates of the true flow rates, $\mathrm{W}^{*}$. The mass conservation equations have the general form:

$$
A \cdot \hat{W}=0
$$

and

$$
\begin{equation*}
A \cdot \hat{R} \cdot \hat{W}=0 \tag{Eq 11}
\end{equation*}
$$

where $A$ is a matrix of coefficients ( $-1,0$ or +1 ), R is a diagonal matrix, and $\hat{W}$ a column vector.
The weighted sum of squares defined above can be written as:

$$
\Sigma \frac{(\hat{R}-R)^{2}}{\sigma_{R}{ }^{2}}+\Sigma \frac{(\hat{W}-W)^{2}}{\sigma_{W}{ }^{2}}=\text { minimum } \quad E q 12
$$

where $\sigma_{\mathrm{R}}{ }^{2}$ and $\sigma_{\mathrm{w}}{ }^{2}$ are the diagonal covariance matrices of the assays and flow rates, respectively. Minimizing Equation 12 and satisfying Equation 11 will provide the $\hat{\mathbf{R}}$ and $\hat{W}$ solutions to the problem.
MATBAL uses the Lagrange multipliers method (11), in which the constraints of Equation 11 are incorporated into the objective function after multiplication by unknown coefficients. The solution is obtained for a stationary point of the new function:

$$
\begin{aligned}
F(\hat{R}, \hat{W}, L 1, L 2) & =\Sigma \frac{(\hat{R}-R)^{2}}{\sigma_{R}{ }^{2}}+\Sigma \frac{(\hat{W}-W)^{2}}{\sigma_{W^{2}}} \\
& +L 1 \cdot A \cdot \hat{R} \cdot \hat{W}+L 2 \cdot A \cdot \hat{W} \quad E q 13
\end{aligned}
$$

where L1 and L2 are diagonal matrices of the Lagrange coefficients. The stationary points of $F$ are obtained by cancelling all partial derivatives of $F$ with respect to the unknowns.
The problem has thus been brought to a resolution of a set of as many equations as unknowns. It can be seen from Equation 13 that the derivatives with respect to L1 are non-linear since they contain the product $\hat{R} . \hat{W}$. Therefore, an iterative solution is necessary in which the product $\hat{\mathrm{R}} . \hat{W}$ is linearized by:

$$
\begin{equation*}
\hat{R} . \hat{W}=\operatorname{Ro} . \hat{W}+\hat{R} . W o-R o . W o \tag{Eq 14}
\end{equation*}
$$

where Ro and Wo are successive approximations of $\hat{R}$ and $\hat{W}$, treated as constants.
Having defined starting values for Ro and Wo and a convergence criterion for the iteration, the material balance problem has become a conventional solution of a set of linear equations for which standard routines are available.
The main difficulty is in defining a suitable algebraic form to allow the coding of a general method to generate the partial derivative coefficients from input data for a variety of flowsheet configurations. This is described in full detail in the next section.

### 2.2.2.2 Solution of a general material balance problem

The equations of the material balance problem are greatly simplified when written in a matrix form. However, matrix algebra cannot be handled as such in FORTRAN. Therefore, conventional scalar algebra is used throughout this section. The meanings of most symbols appear in Table 1.
$E R(M, O)$ is the value of the adjustment to assay $R(M, O)$

$$
\begin{equation*}
E R(M, O)=R(M, O)-\hat{R}(M, O) \tag{Eq 15}
\end{equation*}
$$

where $\hat{R}$ is the adjusted assay. Similarly, ERC(NN) is the value of the adjustment to the measured or estimated flow rate C(NN). WFM(M,O) and WFC(NN), which initially contain the assay and flow rate standard deviations are internally converted into weighting factors (i.e., inverse square of standard deviations).

## Objective function

The objective function to be minimized is:

$$
\begin{aligned}
S S= & \sum_{O M} \Sigma E R(M, O)^{2} W F M(M, O)+ \\
& \sum_{N N} \operatorname{ERC}(N N)^{2} W F C(N N)
\end{aligned}
$$

Eq 16

## Constraints

The constraints to be satisfied are:

$$
\sum_{M} A(M, N) W(M)=0 \quad N M A X \text { equations } \quad E q 17
$$

$$
\begin{array}{r}
\sum_{M} A(M, N) W(M)[R(M, O)-E R(M, O)]=0 \\
N M A X \cdot O M A X \text { equations }
\end{array}
$$

Eq 18

$$
\begin{gathered}
\sum_{M}^{E B(M, N N) W(M)-[C(N N)-E R C(N N)]}=0 \\
N N M A X \text { equations } \quad E q 19
\end{gathered}
$$

$$
\begin{gathered}
B N N O(N N, O) \sum_{M} B(M, N N) W(M)[R(M, O)-E R(M, O)]=0 \\
\text { IMAX equations } \quad \text { Eq } 20
\end{gathered}
$$

## Lagrange multipliers

The minimum of Equation 16 satisfying Equations 17 to 20 is obtained for the same values of ER, ERC, and W as the stationary point of the new function:
$F=S S L+\Sigma L 1 \cdot[17]+\Sigma L 2 \cdot[18]+\Sigma L 3 \cdot[19]+\Sigma L 4 \cdot[20]$
L1 to L4 are the Lagrange multipliers which are introduced as new variables but whose values are not explicitly used in the output.

## Equations to be solved

The stationary point of $F$ is obtained by cancelling all partial derivatives with respect to the unknowns.

$$
\begin{aligned}
& \frac{\delta F}{\delta E R(I, K)}=2 \cdot E R(I, K) \cdot W F M(I, K)-\sum_{N} L 2(N, K) \cdot A(I, N) \cdot W(I) \\
&- \sum_{N N} L 4(N N, K) \cdot B N N O(N N, K) \cdot B(I, N N) \cdot W(I)=0 \\
& E q 22
\end{aligned}
$$

$\frac{\delta F}{\delta E R C(J J)}=2 \cdot E R C(J J) \cdot W F C(J J)+L 3(J J)=0 \quad E q 23$

$$
\begin{align*}
\frac{\delta F}{\delta W(I)} & =\sum_{N} L 1(N) \cdot A(I, N)+\sum_{O N} L 2(N, O) \cdot A(I, N) \\
& \cdot[R(I, O)-E R(I, O)]+\sum_{N N} L 3(N N) \cdot B(I, N N) \\
& +\sum_{N N} \sum_{O} L 4(N N, O) \cdot B N N O(N N, O) \cdot B(I, N N) \\
& \cdot[R(I, O)-E R(I, O)]=0
\end{align*}
$$

$$
\begin{equation*}
\frac{\delta F}{\delta L 1(J)}=\sum_{M} A(M, J) \cdot W(M)=0 \tag{Eq 25}
\end{equation*}
$$

$$
\frac{\delta F}{\delta L 2(J, K)}=\sum_{M} A(M, J) \cdot W(M) \cdot[R(M, K)-E R(M, K)] \underset{E q}{=0} 26
$$

$$
\frac{\delta F}{\delta L 3(J J)}=\sum_{M} B(M, J J) \cdot W(M)-[C(J J)-E R C(J J)]=\underset{E q}{=0} 27
$$

$$
\frac{\delta F}{\delta L 4(J J, K)}=B N N O(J J, K) \cdot \sum_{M} B(M, J J) \cdot W(M)
$$

$$
\begin{equation*}
\cdot[R(M, K)-E R(M, K)]=0 \tag{Eq 28}
\end{equation*}
$$

Two observations can be made about Equation 22 to 28. First, $\mathrm{ER}(\mathrm{I}, \mathrm{K})$ can be expressed as a function of other unknowns (W,L2,L4). This can considerably reduce the size of the set of equations to be solved since there are MMAX-OMAX equations of type 22. Secondly, most partial derivative equations contain cross products of two variables, such as $W(M) \cdot E R(M, K)$ in Equation 26. They cannot, therefore, be written in the form:

$$
\begin{equation*}
D \cdot X=E \tag{Eq 29}
\end{equation*}
$$

where $D$ is a matrix of coefficients, $X$ a column vector of unknowns, and E a column vector of constants. In MATBAL, linearization is made by using the approximation:

$$
\begin{aligned}
W(M) \cdot E R(M, O) & \simeq W O(M) \cdot E R(M, O)+W(M) \cdot E R O(M, O) \\
& -W O(M) \cdot E R O(M, O)
\end{aligned}
$$

Eq 30
where WO and ERO are successive approximations of W and ER, treated as constant in each step of an iterative calculation.

Table 1 - Variable names, dimensions and definitions

| Name | Dimension | Definition | Name | Dimension | Definition |
| :---: | :---: | :---: | :---: | :---: | :---: |
| TITLE | 15 | Label used to identify the problem | ERO | MMAX,OMAX | Successive approximations of ER |
| MMAX | 1 | Number of streams | ERI | MMAX,OMAX | Initial values of ER |
| OMAX | 1 | Number of assay types | IVC | MMAX | Number of the measured |
| MOAX | 1 | MMAX, OMAX |  |  | flow rates |
| NMAX | 1 | Number of normal nodes | W | MMAX | Stream weight values |
| NNMAX | 1 | Number of non-normal nodes | WO | MMAX | Successive approximations |
| NNNMAX | 1 | Number of fixed assays |  |  | of W |
| IMAX | 1 | Number of special assays (non-null BNNO) | WI <br> WS | MMAX NMC,MMAX | Initial values of $W$ Stored flow rate values |
| NNXC | 1 | Number of measured flow rates | NI | 1 | in Monte Carlo simulation Number of the input file |
| NMXC | 1 | NMAX + NNXC | NO | 1 | Number of the output file |
| NNP3 | 1 | NNMAX + NNXC + 3 | D | MAX1** | Vector of Lagrangian |
| MRECOV | 1 | Number of the reference stream |  |  | derivative coefficients for subroutine COEF |
| IPRINT | 8 | Output control vector | E | MAX*** | Vector of solutions from |
| IWF | 1 | Type of error model entered |  |  | subroutine COEF in that order: W. Lagrange |
| ICON | 1 | Type of error estimator |  |  | coefficient, ERC |
| NMC | 1 | used <br> Number of Monte Carlo reports | DR | MAX1** | Vector of Lagrangian derivative coefficients for subroutine SOLV |
| ISEED | 1 | Seed of the random number generator (internally calculated) | ER | MAX** | Vector of solutions from subroutine SOLV in that order: W. Lagrange |
| WFEQ | 1 | Uniform relative standard deviation | ISKIP | 1 | coefficients, ERC <br> Option of initial stream |
| ANAME | $\text { MMAX, } 5$ | Label identifying streams |  |  | weights input |
| R | MMAX,OMAX | Assay values | AMFGR | NMXC,MMAX | Normal node and mea- |
| ONAME | OMAX, 5 | Label identifying assays |  |  | sured flow rate coefficient |
| WFM | MMAX, OMAX* | Assay standard deviations |  |  | used in subroutine MFGR |
| A | MMAX,NMAX | Normal node coefficients | IROW | MMXC | Vector of independent |
| B | MMAX,NNP3 | Non-normal node coefficients | ICOL | MMAX | rows in subroutine MFGR Vector of independent |
| C | MMAX | Measured flow rates |  |  | columns in subroutine |
| RS | NMC, MOAX | Stored adjusted assays in Monte Carlo simulation | AUX | MAXM1 | MFGR <br> Working area in subrou- |
| WFC | MMAX | Measured flow rate relative standard deviations | MAXM1 | 1 | tine SOLV MAX-1 |
| BNNO | NNP3, OMAX | Special assays coefficients | ITMAX | 1 | Maximum number of iterations for convergence |
| 10 | OMAX | Mask for assay-types |  |  | test |
| MIMAX | 1 | Number of independent Flow rates | TOL1 | 1 | Convergence criterion for flow rates |
| AI WRK | MMAX,MMAX OMAX | Work area Work area | TOL2 | 1 | Convergence criterion for assays |
| $\begin{aligned} & \text { * } \text { Not ne } \\ & \text { ** MAX1 } \\ & \text { ** MAX }= \\ & \text { + } \text { MAX + } \end{aligned}$ | cessarily entere $=$ MAX (MAX + $=$ MMAX + NMA NNXC in SOLV | d with these dimensions. <br> 1)/2 in SOLV <br> X(OMAX + 1) + NNMAX | $\begin{gathered} \text { ** MA) } \\ \text { *** MA } \end{gathered}$ | 1 $=$ MAX $(\mathrm{MAX}$ $=$ MMAX +N | $+1) / 2$ in COEF MAX + NNMAX + NNXC in |

If $E R(1, K)$ is substituted from Equation 22 into Equations 24 to 28 and the product W.ER replaced by Equation 30, all elements of matrices $D$ and $E$ of Equation 29 can be

$$
\begin{equation*}
\frac{\delta F^{\prime}}{\delta E R C(J J)}=2 \cdot \operatorname{ERC}(J J) \cdot W F C(J J)+L 2(J J) \tag{Eq 35}
\end{equation*}
$$ calculated. (The detailed structure of these matrices is given In the Appendices.) X is the column vector of the unknowns: W, L1 to L4, ERC. Such a linear system can be solved by conventional methods. MATBAL uses a

$$
\begin{equation*}
\frac{\delta F^{\prime}}{\delta L 1(J)}=\sum_{M} A(M, J) \cdot W(M) \tag{Eq 36}
\end{equation*}
$$ Gauss elimination technique with largest pivotal divisors (12). The calculation is started with an arbitrary value of $\sigma_{\mathrm{P}}$ for $\mathrm{ERO}(\mathrm{M}, \mathrm{O})$ while $\mathrm{WO}(\mathrm{M})$ is initialized by a simplified least-squares method which minimizes squared

$$
\frac{\delta F^{\prime}}{\delta L 2(J J)}=\sum_{M} B(M, J J) \cdot W(M)+E R C(J J)-C(J J)
$$ node imbalances of Equations 18 and 20, constrained by Equations 17 and 19.

The experimental assays $R$ are considered as constant in this first approximation which can be represented as follows:

## Node imbalance

$$
\begin{equation*}
\sum_{M} A(M, N) \cdot W(M) \cdot R(M, O)=D A(N, O) \tag{Eq 31}
\end{equation*}
$$

$B N N O(N N, O) \cdot \sum_{M}^{B(M, N N) \cdot W(M) \cdot R(M, O)=} \underset{E q 3}{D B(N N, O)}$

Objective function, using Lagrange multipliers

$$
\begin{aligned}
& F^{\prime}=\sum_{O}\left[\sum_{N} D A(N, O)^{2}+\sum_{N N} D B(N N, O)^{2}\right] \\
& +\sum_{N N} E R C(N N)^{2} \cdot W F C(N N)+\sum_{N} L 1(N) \cdot \sum_{M} A(M, N) \cdot W(M) \\
& +\sum_{N N} L 2(N N) \cdot\left[\sum_{M} B(M, N N) \cdot W(M)-[C(N N)-E R C(N N)]\right]
\end{aligned}
$$

## Equations to be solved

The first approximation of W is obtained by cancelling the partial derivatives of $F^{\prime}$ with respect to W , ERC, L1 and L 2 .

$$
\begin{aligned}
\frac{\delta F^{\prime}}{\delta W(I)}= & \sum_{N} L 1(N) \cdot A(I, N)+\sum_{N N} L 2(N N) \cdot B(I, N N) \\
+ & \sum_{O} \sum_{N} 2 \cdot A(1, N) \cdot R(I, O) \sum_{M} A(M, N) \cdot W(M) \cdot R(M, O) \\
+ & \sum_{O} \sum_{N N} 2 \cdot B N N O(N N, O)^{2} \cdot B(I, N N) \cdot R(I, O) \\
& \cdot \sum_{M} B(M, N N) \cdot W(M) \cdot R(M, O) \quad \text { Eq } 34
\end{aligned}
$$

Equations 34 to 37 are linear and can therefore be solved directly in the form of Equation 29. Detailed calculation of matrices $D$ and $E$ is given in the Appendices.
A general material balance problem has been shown to be amenable to a set of linearized equations. Before describing the implementation of the method in MATBAL, the estimation of the error on the calculated flow rates is described in the following section.

### 2.2.2.3 The confidence problem

Calculating the best estimates of the flow rates and the assays corresponding to a set of experimental data is the main thrust of a material balance problem. The solution is, however, incomplete without the ability to also estimate the precision of the calculated values.
MATBAL uses the error model imposed on the assays to simulate new pseudo-experimental assays belonging to the same population and to repeat the material balance as many times as desired, until a sufficient number of flow rate estimates is obtained to calculate their covariance matrix (13).

## The Monte Carlo error estimator

The Monte Carlo error estimator uses the result R of the material balance calculation as the true value of the assays and simulates artificial experimental data belonging to the same population. These simulated values are normally distributed around the true value, are uncorrelated, and have the same standard deviation as the original data used to calculate the true value.
The pseudo-random numbers used in the simulation are produced by subroutine URAND (14). After each generation of simulated experimental data, a material balance is performed. The only user-intervention is to preset the number of simulations (NMC) in the initial input file of MATBAL. The number of simulations must be at least ten, and preferably 50 to 100 . The random numbers seed can be any integer and is internally generated, using a computer clock function which may need to be changed on a non-CDC computer.

### 2.3 PROGRAM CAPABILITIES

The MATBAL computer program is used to calculate mineral process flowsheet data which satisfy given mass balance constraints. The results are calculated by adjusting given experimental data in order to minimize the weighted sum of squares of adjustments.
All experimental data must relate to a unique carrier medium; for example, if solid flow rates are to be adjusted, all adjusted data must be relative to the solid phase (e.g., solid composition or size distribution).
The number of streams and constraint equations (node equations) that can be handled by MATBAL has one limitation: the output subroutine MATOUT imposes restrictions on the maximum number of streams and assays that can be handled by the program. These restrictions are $O M A X \leq 16$, NMAX $<16$, and NNMAX < 16. These limits should accommodate most material balance problems.
Due to the unlimited variety of data that can be submitted for material balance calculation, no guarantee of proper program functioning can be given. Some safeguards have been established to detect abnormal data behaviour and runaway computation. The common sources of failure are:

- errors on the data file;
- ill-conditioned data.

Poor input data can either produce a run termination or meaningless results. III-conditioned data usually arise from assay values for which the experimental value, or the adjusted value, does not show a pattern of concentration or dilution across a unit as most assays should.
For example, whereas assay $\hat{R}$ of the three streams in Figure 1 may be split from a value of $50 \%$ in stream 1 to values of $75 \%$ and $25 \%$ in streams 2 and 3 , respectively. A second assay type, $\mathrm{R}^{\prime}$, may show non-significant variations, such as $0.10 \%$ in stream 1 , to $0.11 \%$ and $0.12 \%$ in streams 2 and 3 . Even if, in principle, the standard deviation of $R^{\prime}$ allows the three values to be adjusted to, for example, $0.15,0.16$, or 0.07 , respectively, assay $\mathrm{R}^{\prime}$ will be a source of instability in the results and may have to be eliminated from the data.

Finally, coding bugs may remain in the program and will be detected only as the number of users increases. The author would appreciate receiving these observations in order to upgrade this present version of MATBAL.

### 2.4 DATA INPUT

The data are input in free format (i.e. blank or comma separators) except for alphanumeric strings. The following is a data file description record-per-record. Note that some data can be continued on two records and repeated several times (one record $=$ one card) (See also Fig. 3).

| SECTION | DATA |
| :---: | :---: |
| Rec. 1- | Title (up to 60 characters) |
| Rec. 2- | MMAX,OMAX,NMAX,NNMAX, |
|  | NNNMAX,IMAX,NNXC,MRECOV, |
| Rec. 3- | IPRINT ( 1 to 7 ), IWF,ICON, ISKIP,WFEQ,ITMAX,TOL1,TOL2 |
| 2 | M,ANAME (Format 12, 1x,5A4) |
| 3 | M, R(M,O) |
| 4 | O,ONAME (Format l2,1x,5A4) |
| 5 | W (if ISKIP = 1) |
| 6 | WFM (Format 16(A2,1X) if $W W=5$ or 6) |
| 7 | N,K,A(J,N) : K values ex: 131-2-3 |
| 8 | NN,K,B(J,NN) : $K$ values ex: $131-2-3$ |
| 9 | IVC, C, WFC |
| 10 | NN,K,BNNO(NN,O) (if IMAX $\neq 0$ ) $K$ values ex: $1212(\mathrm{~K}>0)$ |
| 11 | $\mathrm{M}, \mathrm{O}$ (if NNNMAX $\neq 0$ ) |
| 12 | K, IO: K values ex: 3124 |

Fig. 3 - Data input file
The conversational editor EDIBAL, described in Appendix $E$, allows the user to create a data file or modify an existing input file in a separate session. The data file is entered or modified section by section as follows:

| Section 1 | Record 1 | Title Record |
| :--- | :--- | :--- |
|  | TITLE | 60 columns of <br> alphanumeric data to <br> identify problem |


| Section 1 | Record 2 | Dimensions |
| :--- | :--- | :--- |
|  | MMAX | Number of streams |
|  | OMAX | Number of assays |
|  | NMAX | Number of normal nodes |
|  | NNMAX | Number of non-normal <br> nodes |
|  | IMAMAX | Number of fixed assays <br> Number of special assays <br> (non-null BNNO) (assays <br> that do not satisfy all <br> normal nodes) |
|  | MNXC | Number of measured flow <br> rates |
|  | MMECOV | Number of the reference <br> stream for recovery <br> calculation |
|  | Number of Monte Carlo <br> repeats |  |


| Section 1 | Record 3 | Options |
| :--- | :--- | :--- |
|  | IPRINT | Control of seven output <br> options |
|  | IWF | Option of error model input |
|  | ICON | Option of error estimator |
| ISKIP | Option to enter first <br> approximation of stream <br> weights |  |

WFEQ Uniform value of all assay standard deviations (if IWF = 7)
ITMAX Maximum number of iterations
TOL1 Convergence criterion for flow rate values (\% variation between two iterations)
TOL2 Convergence criterion for assay values (\% variation between two iterations)

| Section 2 | Stream Names (MMAX records) |
| :---: | :---: |
| M,ANAME | Stream number and name in format 1 |
|  | 1X, 5A4 (observe format: xx xxx |
|  | $M$ is right justified: e.g. 01 STREAM |

$\frac{\text { Section } 3}{\mathbf{M}, \mathbf{R}(\mathbf{M}, \mathbf{0})} \quad$ Assays (MMAX records) $\quad$ Assay values, using continuation records if necessary

Section 4 Assay Names (OMAX records)
O,ONAME Assay type number, assay name in format I2, 1X, 5A4 (observe format: xx xxx e.g. 01 assay 1

| Section 5 | Initial Flow-Rate Values (if ISKIP $=1$ ) |
| :--- | :--- |
| $\mathbf{W}$ | Flow-rate values (MMAX Values) |

Section 6 Error Model for Assays
Data on this record vary with option IWF
IWF = 1 or 2
WFM Assay type standard deviation
IWF $=3$ or 4 (MMAX records)
WFM Assay standard deviation
IWF = 5
WFM Assay type rating (VG, G, F, P, VP) in format A2,1X, right justified, separated by one blank (observe format: $\mathrm{xx} x \mathrm{xx}$

IWF $=6$ (MMAX records)
WFM Assay type rating as above
IWF = 7 No record

Section $7 \quad$ Normal Node Coefficients (NMAX records)

| $\mathbf{N , K , A}$ | Normal node number, number of <br> streams, normal node coefficients <br> e.g. $131-2-3$ |
| :--- | :--- |
| Section 8 | Non-Normal Node Coefficients <br> (NNMAX records) |
| NN,K,B | Non-normal node number, number of <br> streams, non-normal node coefficients <br> e.g. $131-2-3$ |


| Section 9 | Measured Flow Rates (NNXC records) |
| :---: | :---: |
| IVC | Number of the measured flow rate |
| C | Measured flow rate |
| WFC | Standard deviation of the measured flow rate (relative \%) |
| Section 10 | Special Assay Mask (NNMAX records) if $\operatorname{IMAX} \neq 0$ |
| NN,K,BNNO | Number of non-normal node, number of entries, mask of assays for non normal node NN $\text { e.g. } 1225$ |
| Section 11 | Fixed Assay Record (NNNMAX records) |
| M | Stream subscript of fixed assay |
| 0 | Type subscript of fixed assay |
| Section 12 | General Assay Type Mask |
| K,IO | Mask for assay types, number of assay types used followed by assay type numbers $\text { e.g. } 3145$ |
| 2.5 PROGRAM OPTIONS |  |
| Data input or calculation path options can be controlled by setting code variables to the proper value in the input file. |  |
| Variable IWF is an in standard dev WFM). Seven value of IWF | teger indicating in what form the assay iations are entered in Section 6 as (variable options are available depending on the for which ten values are possible: |
| 1. OMAX values of the absolute standard deviation assigned to each assay type are read. <br> 2. OMAX values of the relative standard deviation assigned to each assay type are read, in percentage. <br> 3. MMAX. OMAX values of the absolute standard deviations are read; one per assay. <br> 4. MMAX.OMAX values of the relative standard deviations are read; one per assay, in percentage. <br> 5. OMAX alphabetic qualifiers are read; one per assay type: <br> VG for very good (internally converted into a $0.3 \%$ relative standard deviation); <br> G for good (1\%); <br> F for fair (10\%); <br> P for poor (50\%); <br> VP for very poor (100\%); blank is interpreted as $F$. |  |

6. MMAX.OMAX alphabetic qualifiers are read, one per assay.
7. All relative standard deviations are equal to $1 \%$ by default, or to the value coded in WFEQ.
8. Internal option used to convert weighting factors into standard deviations in WFM and WFC.
9. Internal option used to convert standard deviations into weighting factors in WFM and WFC.
10. Internal option used to convert WFC from relative standard deviation percentage into weighting factors.
ICON is an integer controlling the type of error estimation to be performed for calculated flow rates; can have two values:
11. No error estimation.
12. The Monte Carlo estimator is activated and the number of simulated data-sets specified by NMC is produced.
ISKIP is an integer controlling the user's entry of approximated flow-rate values:

0 . Values calculated by COEF.

1. Values entered.

IO is a vector of numerical switches to activate, or mask, assay types present in the input file. For instance, $10=3145$ activates the three assay types, numbers 1 , 4, and 5.

### 2.6 PRINTED OUTPUT (IPRINT OPTIONS)

In order to fit CRT and hard copy terminals, as well as direct report insertion, the output tables generated by MATBAL contain fewer than 80 columns.
Subroutine MATOUT controls the output and it can be selectively modified to satisfy special output requirements, without altering the overall program structure. Seven output options are available depending on the value ( 0 or 1 ) stored in the eight-position vector IPRINT. The output resulting from a value of one in each of the IPRINT positions is:

1. Header;
2. Input data;
3. First approximation of flow rates calculated by minimizing the node imbalances (used for debugging);
4. Assay adjustments and flow rates calculated at each iteration (used for debugging):
5. Final result of material balance;
6. Analysis of calculated adjustments;
7. Echo of the input file;
8. If $I C O N=1$, this option is internally preset to print the results of the Monte Carlo simulation. The input file defines the first seven options only.

### 2.7 FLOWSHEET

A general flowsheet of MATBAL is shown in Figure 4, which illustrates the modular structure of the program. The various input and output files used by MATBAL are described in Figure 5.
MATBAL includes a FORTRAN preprocessor, called DIMSET, which reads the problem dimensions in cards 1 and 2 and writes a FORTRAN driver in which all arrays are preset at the exact problem dimensions. This driver calls the material balance subroutines, in which all dimensions are transmitted as arguments. The main advantages of this feature are that only the required memory is used, and only the small driver program needs to be compiled for a new problem.


Fig. 5 - File interaction flowsheet for MATBAL package


Fig. 4 - General flowsheet of MATBAL

## 3. SYSTEM DOCUMENTATION

### 3.1 COMPUTER EQUIPMENT

MATBAL runs on a CDC-CYBER computer, operating under NOS/BE, with a card reader, line printer, and disk drive. An IBM-PC version has also been developed.

### 3.2 SOURCE PROGRAM

A complete listing, or a source card deck, can be obtained from CANMET at a nominal cost (see Section 1).

### 3.3 PROGRAM DIMSET

### 3.3.1 Purpose

The program DIMSET is used to:

- Assign required logical files for input and output.
- Read necessary input data for dimensioning.
- Create a 'driver' MATBAL which will call subroutine MAIN, containing the MATBAL method. This program is a preprocessor used to relieve the user from the tedious task of array dimensioning. It is a stand-alone program.


### 3.4 PROGRAM MATBAL

### 3.4.1 Purpose

MATBAL, which is also the driver, is used to:

- Allocate central memory locations, as required.
- Re-assign required logical files for input and output.
- Rewind input file and read the array dimensions.
- Set random number generator seed.
- Call subroutine MAIN, which is equivalent to the main program of the old program MATBAL2.


### 3.5 SUBROUTINES AND FUNCTIONS

### 3.5.1 Subroutine MAIN

### 3.5.1.1 Purpose

Subroutine MAIN is used to:

- Call the other subroutines.
- Control iteration towards solution.
- Add the NNXC measured flow rates to matrix $B$.
- Perform various checking functions such as freezing negative results to small positive values and restarting the program, and controlling error estimation by Monte Carlo simulation of NMC sampling data sets.


### 3.5.1.2 Calling sequence

CALL MAIN (list of arguments).

### 3.5.1.3 Description of arguments

All arguments transmitted by subroutine MAIN as well as others, are described in Table 1. The list of arguments can also be found in the program listing itself.

### 3.5.1.4 Sequence of steps

1. Input data are read, (CALL MATIN).
2. Matrices of weighting factors are calculated.
3. The NNXC measured flow rates are added to matrix B of non-normal nodes.
4. First approximation of flow rates is calculated (CALL COEF) if option ISKIP $=0$.
5. Approximation of assay adjustments is calculated.
6. Matrix of gradient coefficients of Lagrangian is set (CALL SOLV) and the solution of linear set of equations is obtained.
7. Solution obtained at step 6 is transformed in assay and flow-rate adjustments.
8. Convergence criterion is applied; if not satisfied, return to step 5 and acquire better approximation.
9. Occurrence of negative flow rates or assays is checked. The most negative value is frozen to a small positive value; intermediate results are printed; and calculation restarted at step 4.
10. If the Monte Carlo simulation method is to be used (ICON = 1), artificial sets of experimental data are generated and calculation restarted at step 1, NMC times.

### 3.5.2 Subroutine MATIN

### 3.5.2.1 Purpose

MATIN is used to read all input data in a logical file.

### 3.5.2.2 Calling sequence

CALL MATIN (List).

## 3-5.2.3 Description of arguments

See Table 1 and program listing.

### 3.5.2.4 Sequence of steps

1. Title and problem dimensions are read.
2. Variables are initialized.
3. Rest of input data is read.
4. Print echo of input file.
5. Return to the calling program..

### 3.5.3 Subroutine INIT

### 3.5.3.1 Purpose

INIT is used to initialize most arrays of Table 1.

### 3.5.3.2 Calling sequence

CALL INIT (List).

### 3.5.3.3 Description of arguments

See Table 1 and program listing.
$K$ and FLOAT(K) are used to initialize integer and floating point arrays, respectively. In normal use, $\mathrm{K}=0$.

### 3.5.3.4 Sequence of steps

1. $\operatorname{Set} Z=\operatorname{FLOAT}(\mathrm{K})$.
2. Initialize arrays to $K$ or $Z$.
3. Return to calling program.

### 3.5.4 Subroutine DIAGNOS

### 3.5.4.1 Purpose

DIAGNOS is used to print diagnostics on the output file when critical conditions are encountered during calculation.

### 3.5.4.2 Calling sequence

CALL DIAGNOS (List).

## 3-5.4.3 Description of arguments

See Table 1 and program listing.
$I$ is the number of the desired diagnostic. It is used in a computed GO TO statement to issue the proper message as listed in Table 4.

### 3.5.5 Subroutine WFSET

### 3.5.5.1 Purpose

WFSET is used to set the tables of weighting factors to the desired value. It can convert the standard deviations read by MATIN into inverse of variances, or conversely transform inverse variances back to standard deviations.

### 3.5.5.2 Calling sequence

CALL WFSET (List).

### 3.5.5.3 Description of arguments

(See Table 1 and program listing.)
Variable IWF is the code which directs calculations performed in WFSET. The various options are described in Section 2.5 . WFM is the array containing the assay standard deviations or inverse variances; WFC is the array containing the flow rate standard deviations or inverse variances.

### 3.5.6 Subroutine COEF

### 3.5.6.1 Purpose

COEF is used to calculate the matrix of derivative coefficients needed to solve the set of linear equations whose solutions are the first approximations of the flow rates (Eq 34 to 37 ).
Since matrix D is symmetric, a substantial saving in memory is achieved by storing only the upper triangular portion of $D$. The symmetric system is then solved by subroutine GELS.

### 3.5.6.2 Calling sequence

CALL COEF (List).

### 3.5.6.3 Description of internal arguments

D: array of coefficients of linear equations $D \cdot X=E$.
E: vector of constants, returns values of solutions of Eq 34 to 37 .
MAX: dimension of the set of equations $M A X=$ MMAX + NMAX + NNMAX + NNXC.
IER: output digit; 0 for a normal solution, 1 for a singular set of equations.
MAX1: storage dimension $M A X 1=M A X \cdot(M A X+1) / 2$.

### 3.5.6.4 Sequence of steps

1. Initialize D and E .
2. Calculate non-null values in $D$ and $E$ tables.
3. Call GELS to solve set of equations.
4. Return to calling program.

### 3.5.7 Subroutine SOLV

### 3.5.7.1 Purpose

SOLV is used to calculate the matrix of derivative coefficients needed to solve the set of linear equations whose solutions are the successive approximations of the flow rates (Eq 22 to 29). DR is also stored as a symmetric matrix.

### 3.5.7.2 Calling sequence

CALL SOLV (List).

### 3.5.7.3 Description of internal arguments

DR: array of coefficients of linear equations DR. $\mathrm{X}=\mathrm{ER}$.
ER: vector of constants, returns values of solutions of Eq 22 to 29.
MAX: dimension of the set of equations. MAX $=$ MMAX + NMAX. (OMAX +1 ) + NNMAX + IMAX + NNXC.
IER: output digit; 0 for a normal solution, 1 for a singular set of equations.
MAX1: storage dimension. MAX $1=$ MAX. $($ MAX +1$) / 2$.

### 3.5.7.4. Sequence of steps

1. Initialize DR and ER.
2. Calculate non-null values in $D R$ and ER.
3. Call GELS to solve set of equations.
4. Return to calling program.

### 3.5.8 Subroutine GELS

### 3.5.8.1 Purpose

GELS is used to obtain a solution of a set of simultaneous linear equations, $A . X=B$. when $A$ is a symmetric matrix.

### 3.5.8.2 Calling sequence

CALL GELS (List).

### 3.5.8.3 Description of internal arguments

R: $\quad \mathrm{M}$ by N right-hand side matrix (destroyed).
A: upper triangular part of the symmetric MxM coefficient matrix (destroyed).
M: the number of equations in the system.
$\mathbf{N}$ : the number of right-hand side vectors.
EPS: an input constant which is used as relative tolerance for test on loss of significance: default $=10^{-4}$.

IER: resulting error parameter coded as follows:
IER = 0-no error;
IER $=-1-$ no result because of $M$ less than 1 or pivot element at any elimination step equal to 0 ;
IER $=\mathrm{K}$ - warning due to possible loss of significance indicated at elimination step $\mathrm{K}+1$, where pivot element was less than or equal to the internal tolerance EPS times absolutely greatest main diagonal element of matrix $A$.
AUX: an auxiliary storage array with dimension MMAX-1.
Note: Upper triangular part of matrix $A$ is assumed to be stored columnwise in $M(M+1) / 2$ successive storage locations.

### 3.5.8.4 Sequence of steps

1. Search for greatest main diagonal element.
2. Pivot row reduction and row interchange in right hand side R.
3. Row and column interchange and pivot row reduction in matrix $A$.
4. Element reduction and search for next pivot.
5. Back substitution and back interchange.
6. Return to calling program.

### 3.5.9 Subroutine MATOUT

### 3.5.9.1 Purpose

MATOUT is used to print results of material balance problem on unit NO.

### 3.5.9.2 Calling sequence

CALL MATOUT (List).

### 3.5.9.3 Description of arguments

IP: option digit used in a computed GO TO to control the output to logical unit NO.
See also Table 1 and listing.

### 3.5.9.4 Sequence of steps

1. Go to entry point corresponding to digit IP.
2. Print results on logical unit NO, if IPRINT $(\operatorname{IP})=1$.
3. Return to calling program.

### 3.5.10 Function GNOISE

### 3.5.10.1 Purpose

To calculate a simulated variable belonging to a gaussian distribution of given mean and standard deviation.

### 3.5.10.2 Calling sequence

X = GNOISE (IY, AM, STD).

### 3.5.10.3 Description of arguments

IY: arbitrary integer used as a seed to the pseudorandom number generator URAND. Should be initialized prior to the first call and not altered later by the calling program.
AM: value of the gaussian distribution mean.
STD: Value of the gaussian distribution standard deviation.

### 3.5.10.4 Sequence of steps

1. Calculate two random numbers in interval 0,1 .
2. Calculate corresponding member of Gaussian population with mean AM and standard deviation STD.
3. Return to calling program.

### 3.5.11 Function URAND

### 3.5.11.1 Purpose

URAND is a uniform random number generator returning values in the interval 0,1 (14).

### 3.5.11.2 Calling sequence

$\mathrm{X}=$ URAND (IY).

### 3.5.11.3 Description of arguments

IY: arbitrary integer used as a seed. Should be initialized prior to the first call and not altered later by the calling program.

### 3.6 ERROR ESTIMATION METHOD

### 3.6.1 Monte Carlo Estimator: Subroutine CORRE

### 3.6.1.1 Purpose

To calculate the covariance matrix of flow rates when the covariance matrix of experimental data is known.
Note: The Monte Carlo simulations are done in subroutine MAIN after printing the final results. In MATBAL, no temporary disk file storage is used and the statistics on calculated flow rates are obtained directly by applying subroutine CORRE to array WS within subroutine MAIN.

### 3.6.1.2 Sequence of steps

1. Cancel output during simulations.
2. Store stream weight values of the NMC simulations in array WS.
3. Store the calculated assay analyses in RS.
4. Call subroutine CORRE.
5. Print results.
6. End program.

### 3.6.2 Subroutine: CORRE

### 3.6.2.1 Purpose

Compute means standard deviation, sum of cross products of deviations, and correlation coefficients (12).

Note: In order to save memory, the following equivalences are made:

WFC $\rightarrow$ XBAR
AI $\rightarrow \mathrm{RX}$
D $\quad \rightarrow \mathrm{R}$
$\mathrm{E} \quad \rightarrow \mathrm{B}$
W0 $\rightarrow$ D
C $\quad \rightarrow \mathrm{T}$
WS $\rightarrow \mathrm{X}$.

### 3.6.2.2 Calling sequence

CALL CORRE ( $\mathrm{N}, \mathrm{M}, \mathrm{IO}, \mathrm{X}, \mathrm{XBAR}, \mathrm{STD}, \mathrm{RX}, \mathrm{R}, \mathrm{B}, \mathrm{D}, \mathrm{T}$ ).

### 3.6.2.3 Description of arguments

N : number of observations.
M : number of variables.
10: option code for input data: $10=0$ if data are to be read in from input device in the special subroutine named DATA $=1$, if all data are already in core.
X: $\quad$ if $I O=0$, the value of $X$ is 0.0 ; if $I O=1, X$ is the input matrix ( N by M ) containing data.
XBAR: output vector of length $M$ containing means.
STD: output vector of length $M$ containing standard deviations.
RX: output matrix (M×M) containing sums of crossproducts of deviations from means.
R: output matrix (only upper triangular portion of the symmetric matrix of $M$ by $M$ ) containing correlation coefficients.
B: output vector of length $M$ containing the diagonal of the matrix of sums of crossproducts of deviations from means.
D,T: working vectors of length M.

### 3.6.2.4 Sequence of steps

1. Initialization.
2. Read observations and calculate temporary means from data in $T(J)$.
3. Calculate sums of cross-products of deviations from temporary means for M observations.
4. Calculate means.
5. Calculate correlation coefficients.
6. Calculate standard deviations.
7. Return to calling program.

### 3.6.2.5 Dimension statements

Table 2 lists variables for subroutine CORRE.

### 3.7 DATA STRUCTURES

All input data are in free-field format on logical unit 5 except for the alphanumeric data which must appear within a specified field as described in Section 2.4. All output data are tabulated on logical unit 6 to fit an 80column display device. No intermediate disk storage is used.

### 3.8 STORAGE REQUIREMENT

Table 3 gives the length of all programs. A substantial part of the storage is used for the arrays containing the problem variables. The size of these arrays can vary considerably from one problem to the other. The arrays are automatically dimensioned to the problem size by preprocessor DIMSET. All dimensions are then transmitted by arguments at execution time.

### 3.9 MAINTENANCE AND UPDATES

The author cannot provide any formal maintenance of the program. Several updates are expected to evolve from the source program distributed to users. They will be dictated by various considerations, such as inputoutput formats, bypassing of options, and code optimization. The author would appreciate receiving comments on modifications which have substantially improved overall program performance.

Table 2 - Variable names and dimensions in CORRE

| Name | Dimensions | Definition |
| :--- | :--- | :--- |
| XBAR | MMAX | Returns means of variables in CORRE |
| STD | MMAX | Returns standard deviations in CORRE |
| RX | MMAX,MMAX | Returns sum of cross products of deviations from means in CORRE |
| D, T | MMAX | Working vectors |
| X | NMC,MMAX | Input matrix containing data |
| R | 1/2(MMAX).(MMAX +1) | Output matrix containing correlation coefficients, stored as a vector (only <br> upper triangular portion) |

Table 3 - Storage requirement for MATBAL

| Name | CDC Words |  |
| :--- | ---: | ---: |
|  | Octal | Decimal |
| MAIN | 3362 | 1778 |
| MATIN | 1313 | 715 |
| INIT | 164 | 116 |
| DIAGNOS | 155 | 109 |
| WFSET | 372 | 250 |
| MFGR | 302 | 194 |
| COEF | 314 | 204 |
| SOLV | 636 | 414 |
| GELS | 253 | 171 |
| GNOISE | 57 | 47 |
| URAND | 117 | 79 |
| CORRE | 372 | 250 |
| DATA | 6 | 6 |
| MATOUT | 5520 | 2896 |
| Sub Total | 16075 | 7229 |
| MATBAL* | 23250 | 9896 |
| Arrays | variable with each problem |  |
|  | (see Tables 1 and 2) |  |

[^4]
## 4. OPERATING DOCUMENTATION

### 4.1 OPERATOR INSTRUCTIONS

This present version of MATBAL has been developed and tested on the CYBER 730 computer at the Computer Science Centre of Energy, Mines and Resources Canada. The Network Operating System/Batch Environment (NOS/BE 1), was used whenever the problem size precluded the use of time-sharing access through the INTERCOM system.

### 4.2 OPERATING MESSAGES

Normal system messages are produced by the NOS/BE 1 and INTERCOM systems. Special diagnostics, issued on the output file when abnormal conditions occur during the program execution, are listed in Table 4.

Table 4 - MATBAL diagnostic messages

| Message | Significance | Action |
| :---: | :---: | :---: |
| MORE THAN 3 ATTEMPTS TO FIX NEGATIVE WEIGHTS. CHECK DATA | MATBAL has been restarted 3 times after fixing most negative flow rates at 0.01 . Answer still contains negative flow rate. | Check input data, use IO mask. |
| MORE THAN 3 ATTEMPTS TO FIX NEGATIVE ASSAYS. CHECK DATA | As above, for negative assay. | Check input data, use IO mask. |
| SINGULAR MATRIX IN COEF, RUN TERMINATED | GELS called by COEF returned $\operatorname{IER}=1$, set of equations was found singular. | Check node equations used, for their independence. |
| SINGULAR MATRIX IN SOLV, RUN TERMINATED | GELS called by SOLV returned IER $=1$, set of equations was found singular. | Check node equations used, for their independence. |
| ITMAX ITERATIONS COMPLETED, RUN TERMINATED | Results did not meet convergence criteria after ITMAX iterations. | Check input data, change criteria or maximum number of iterations permissible. |
| ERROR ESTIMATION CANCELLED, NEGATIVE WEIGHT, RUN RESTARTED | A negative flow rate was found. ICON was set to zero and run restarted. | Resubmit data which do not lead to negative weights if an error estimation is needed. |
| ERROR ESTIMATION CANCELLED, NEGATIVE ASSAY, RUN RESTARTED | As above for negative assay. | As above for negative assay. |
| R(M,O) NEGATIVE AT SIMULATION I | During Monte Carlo simulation, number I, assay $R(M, O)$ was found negative. Run continues. | Verify that negative values are compatible with standard deviation and 'true' value of $\mathrm{R}(\mathrm{M}, \mathrm{O})$. |
| W(M) NEGATIVE AT SIMULATION I | During Monte Carlo simulation, number I, flow rate W(M) was found negative. Run continues. | Verify that negative values are compatible with standard deviation and 'true' value of W(M). |
| $\begin{aligned} & \text { COEFFICIENT COUNT OFF } \\ & \text { NP }=X X, \text { MAX1 }=X X, \\ & \text { NMAX }=X X, \text { OMAX }=X X, \\ & \text { IMAX }=X X, \text { NNNMAX }=X X, \\ & \text { NNXC }=X X \end{aligned}$ | Array DR in SOLV has not been set properly due to an error in the listed variables. | Check values of listed variables in input file. |
| ANALYSIS R(M,O) ADJUSTED BY MORE THAN 3 SIGMA | Analysis $R(M, O)$ is biased or has been assigned too small a standard deviation. | Check value of $R(M, O)$, modify standard deviation assigned to $R(M, O)$. |
| WEIGHT W(M) ADJUSTED BY MORE THAN 3 SIGMA | Flow rate $W(M)$ is biased or has been assigned too small a standard deviation. | Check value of C[IVC(M)], modify standard deviation assigned to C[IVC(M)]. |

### 4.3 CONTROL CARDS

### 4.3.1 NOS/BE 1

NOS/BE 1 control cards are as follows:
1 - JOBxx, CM77000, IO1000, T100, P2.
2 - ACCOUNT, xxxxx.
3 - ATTACH, TAPE5, (PFN,ID), MR=1.
4 - ATTACH, DIMSET, (PFN,ID), MR=1.
5 - DIMSET.
6 - FTN, I=TAPE1, B = LGO.
7 - ATTACH, MATBAL, (PFN, ID), MR=1.
8 - LOAD(MATBAL).
9 -LGO.
10 - REWIND, TAPE6.
11 - COPYCF, TAPE6, OUTPUT.
The following describes the control cards:

1. This is the job card, defining core requirement, input-output time, execution time and priority.
2. This is the account card.
3. Attaches input data file.
4. Attaches the compiled program DIMSET.
5. Executes DIMSET.
6. Compiles "driver".
7. Attaches the compiled program MATBAL.
8. Loads MATBAL program.
9. Executes the "driver" program.
10. Rewinds unit 6 containing results.
11. Copies unit 6 on output file.

### 4.3.2 INTERCOM

INTERCOM commands to perform the same calculations are as follows:

1. ATTACH, TAPE5, (PFN,ID), MR=1.
2. ATTACH, DIMSET, (PFN,ID), MR=1.
3. DIMSET.
4. $\mathrm{FTN}, \mathrm{I}=$ TAPE1, $\mathrm{B}=\mathrm{LGO}, \mathrm{L}=0$.
5. PFN, ID.
6. ATTACH, MATBAL4, (PFN,ID), MR=1.
7. XEQ.
8. LOAD MATBAL4.
9. LGO.
10. REWIND, TAPE6.
11. COPYCF, TAPE6, OUTPUT.

### 4.4 ERROR RECOVERY

Program must be restarted on error.

### 4.5 RUN TIME

Run time depends on the problem size, and on the chosen options. Monte Carlo simulations multiply run time by the value of variable NMC (or less).
Sample run 1 with option ICON $=1$ and assays 6 to 10 masked by option 10 required 68.2 seconds and 133500 words of memory with the MATBAL2 version. The same problem processed with MATBAL3 gave identical results in 35.7 second and used 61400 words of memory.
The need to redimension MATBAL2 arrays and to recompile the entire program required an extra 20 sec onds of CP time, while the preprocessor method of MATBAL3 required only 0.5 seconds to write and compile a tailored driver. The current version of MATBAL has approximately the same performance as MATBAL3.

## 5. SAMPLE RUN 1: BALANCE OF A FLOTATION CIRCUIT

### 5.1 PROBLEM DEFINITION

The circuit considered in this problem is described by the flowsheet shown in Figure 6. It consists of six flotation cells and a ball mill used to regrind cleaner scavenger tailings. To calculate a metallurgical inventory for this circuit, the plant metallurgist has collected 15 grab samples at different locations of the flowsheet, as indicated by the numbers in Figure 6. (The numbering sequence is arbitrary).
No flow rates have been measured and their value relative to a feed rate of $100 \mathrm{t} / \mathrm{h}$ is expected from a material balance calculation using available data. An error estimation is also needed to quantify the precision of the calculated flow rates. The estimated error on the assays is a uniform $5 \%$ relative standard deviation. Available data are summarized in Table 5.


Fig. 6 - Flotatlon circuit flowsheet for sample run 1

Table 5 - Experimental data for flotation circuit

| Stream | Cu <br> $\%$ | Pb <br> $\%$ | Zn <br> $\%$ | Fe <br> $\%$ | Ag <br> PPM | Sb <br> PPM | In <br> PPM | Bi <br> PPM | Sn <br> PPM |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 0.18 | 4.49 | 9.03 | 28.71 | 105 | 500 | 70 | 60 | 980 |
| 2 | 0.81 | 23.15 | 14.20 | 23.58 | 497 | 1320 | 100 | 300 | 1150 |
| PPM |  |  |  |  |  |  |  |  |  |

[^5]
### 5.2 FLOWSHEET ANALYSIS

In order to create an input data file for MATBAL, it is necessary to analyze the flowsheet in terms of normal and non-normal node equations. Using Figure 6 as a guide, we can successively define the following variables:
TITLE:
CIRCUIT

| MMAX: | 15 | Fifteen streams have been <br> sampled. |
| :--- | :---: | :--- |
| OMAX: | 10 | Ten assay types have been <br> obtained. |
| NMAX: | 8 | Eight conservation equations <br> valid for all assay types can be <br> used;* they are: |

$$
1-10-13=0 \text { Rougher }
$$

$$
10+11-12-14=0
$$

Scavenger

$$
13+14-15=0 \quad \text { Connecting }
$$ streams

$8+5+7+15-3-2=0$ 1st cleaner
3-9-8 = 0 Cleaner scavenger
$2-5-4=0$ 2nd cleaner 4-7-6=0 3rd cleaner $9-11=0 \quad$ Ball mill

NNMAX: $0 \quad$| No node equation valid for |
| :--- |
| selected assays. |

NNNMAX: $0 \quad$ No fixed assay.
IMAX: $0 \quad$ No special assay valid for certain node equations only.
NNXC: 1 Feed stream is used as a reference of 100 and is considered as measured with high precision.
MRECOV: 1 Stream number 1 used to calculate recoveries.
IWF: 7 All assays known with same accuracy (5\%).

| ICON: | 1 | Monte Carlo option is active. <br> WFEQ: |
| :--- | :---: | :--- |
| 5.0 | Uniform relative standard <br> deviation of all assays. |  |
| ISKIP: | 0 | Use subroutine COEF. |
| ITMAX: | 99 | Maximum number of <br> iterations. |
| TOL1: | 0.01 | Convergence criterion for <br> flow rates ( $0.01 \%$ ). |
| TOL2: | 0.01 | Convergence criterion for <br> assays ( $0.01 \%)$. |
| A. | The normal ner |  |

B: $\quad$ The non-normal node equation coefficients are obtained from the conservation equations corresponding to the non-normal nodes. There is no such node.
C: $\quad 100 \quad$ The measured flow rate.
WFC:
0.0001

Assigning a very small relative standard deviation to the measured stream guarantees that it will keep the value of 100 used as reference.

BNNO: Does not apply since $\operatorname{IMAX}=0$ and NNMAX $=0$.
IO: All assays are used, therefore 10 is entered as $10,1,2,3,4,5,6,7,8,9,10$.

### 5.3 PROBLEM SOLUTION

Having completed the flowsheet analysis of the circuit, we can create a card deck or a disk file containing the input data to be processed by MATBAL. The only variables which have not been defined in the flowsheet analysis are user's options controlling the amount of output to be printed and other general purpose variables described earlier in the manual.
A typical file ready for execution is shown in Table 6. IPRINT has been set at $0,0,1,0,1,1,0$ in order to display the first approximation of flow rates, the final results and the adjustment analysis. ICON is set at +1 in order to calculate an error estimation by the Monte Carlo method. NMC is set at ten. 10 is set to use only 5 assays listed in the file.

[^6]Table 6 －Input data for sample run 1

```
SAMPLE RUN 1: BALANCE OF A FLOTATIDN CIRCUIT
15,10,8,0,0,0,1,1,10
0,0,1*0;1,1,0+7,1%0+5,9日*0,1,0+1
    1,ROUGHER FEED
    2,FIRST CLEANER CONC.
    3,FIRST CLEANER TAIL
    4,SEC + CLEANER CONC.
    5,SECOND CLEANER TAIL
    G,THIRD CLEANER CONC.
    7,THIRD CLEANER TAIL
    8,CL, SCU, CDNC.
    日,CL, SCU, TAIL.
10,ROUGHER TAIL
11,MILL DISCHARGE
12,SCAV, TAIL
13,ROUGHER CONC.
14,SCAV, CONC,
15:ROUGHER+SCAV. CONC.
1;+18,4,4日,9,03,28,71,1,05,5,0,7,0,0,0,0,8,9,0
2,+81,23,15,14,2,23,58,4,97,13,2,10,30,11,5,7
3,59,5,49,13,73,29,84,3,03,9,5,7,14,10,4,6
4,1,34,87,11,12,18,GG,G+28,1G,5,7,31,日,日,5
5,,G5,15,84,14,G4,2G,5G,4,03,13,10,27,12,8
G,1,1,3G,G日,10,42,1日,35,G,G5,15,G,G+34,日,4,G
7, 50,17,29,15,33,25,43,3,93,11+8,10,20,11,4,8
8;,9,15,2G,16,4,25,53,4,45,15,1,12,24,14,8,5
日,49,5+21,11+17,33+94,1+88,8+3,7,12,8,3,5
10,,14,1,99,9,48,29,44,59+3,5,日,3,8,5,7
11,.51,7.02,10,72,32,70,2,02,8,3,G,16,8,4,G
12,,13+1,55,9,54,27,58,+5,3,6,4,7,8,8
13,.61,25,1,10,32,25,48,4,8,13,7,7,23,9,7,6
14,.73,10,89,13,31,20,86,3,1,11,4,8,19,12+2,7
15,,67,16,88,12,07,30,14,3,93,12+8,8,20,12,6
    1,CDPPER
    2,LEAD
    3.ZINC
    4,IRON
    5,SILUER
    G,ANTIMDNY
    7:INDIUM
    B,BISMUTH
    G.TIN
10,MERCURY
1,3+1,-10;-13
2,4,10,11,-12,-14
3,3,13+14,-15
4,6,8,5,7,15,-3,-2
5,3,3,-9,-8
G,3,2,-5,-4
7,3,4,-7,-6
8,2,日,-11
1,100,.0001
5,1,2,3+4,5
```

The user will perform the balance calculation with different values of $I O$ in order to test the effect of eliminating some assay types without altering the rest of the input file.
Final results and adjustment analysis for the flotation circuit example are given in Appendix A. If the Monte Carlo estimator is used during the execution of MATBAL, IPRINT (8) is set to print the errors on the flow
rates. Table 7 shows the results of the method when all ten assay types are used in the material balance solution.
Table 8 presents the error estimates obtained when only the first five assay types are used in the material balance. It must be noted that since the seed to the random numbers is generated internally, no two runs of the same problem will give exactly identical results for the estimated precision of the flow rates.

Table 7 - Error estimates for sample run 1 when all assays are used

| Error Estimates for Calculated Flow Rates Monte Carlo Simulation Method with 10 Repeats |  |  |  |
| :---: | :---: | :---: | :---: |
| Sample run 1: Balance of a Flotation Cirouit |  |  |  |
| Statistical test on calculated flomt rates |  |  |  |
| M | MEAN |  | TRUE |
|  | FLOW RATE | SIGMA | FLOW RATE |
| 1 | 100.0000 | . 0001 | 100.0000 |
| 2 | 25.6125 | 2,3760 | 25.8820 |
| 3 | 16.0480 | +9257 | 16.5314 |
| 4 | 8.6621 | . 6074 | 8.6078 |
| 5 | 16.9504 | 2.2953 | 17.2741 |
| 6 | 8.3763 | . 5600 | 8.5402 |
| 7 | . 2859 | . 3107 | . 0676 |
| 8 | 3.9877 | + 3939 | 4.4152 |
| 9 | 12.0603 | -8868 | 12.1161 |
| 10 | 88.1181 | , 9649 | 87.7887 |
| 11 | 12.0603 | + 8868 | 12.1161 |
| 12 | 91.6237 | . 5609 | 91.4598 |
| 13 | 11.8818 | . 9649 | 12.2113 |
| 14 | 8.5547 | . 8180 | 8.4450 |
| 15 | 20.4366 | 1.2676 | 20.6564 |

Table 8 - Error estimates for sample run 1 when the first five assays are used

| Error Estimates for Calculated Flow Rates Monte Carlo Simulation Method with 10 Repeats |  |  |  |
| :---: | :---: | :---: | :---: |
| Sample run 1: Balance of a Flotation Circuit |  |  |  |
| Statistical test on calculated flow rates |  |  |  |
| M | MEAN |  | TRUE |
|  | FLOW RATE | SIGMA | FLOW RATE |
| 1 | 100.0000 | . 0001 | 100.0000 |
| 2 | 24.9976 | 1.9948 | 23.0300 |
| 3 | 21.9912 | 1.9353 | 21.560G |
| 4 | 9.4789 | . 6824 | 8.8016 |
| 5 | 15.5187 | 1.9953 | 14.2285 |
| G | 8.1671 | . 3477 | 7.8692 |
| 7 | 1.3118 | . 6770 | . 9324 |
| 8 | 8.5333 | 1.1941 | 8.1700 |
| 5 | 13.4579 | 1.0599 | 13.3906 |
| 10 | 89,2879 | . 4801 | 89,6787 |
| 11 | 13.4579 | 1.0599 | 13.3906 |
| 12 | 91.8329 | . 3477 | 92.1308 |
| 13 | 10.7121 | . 4802 | 10.3213 |
| 14 | 10.9130 | 1.0218 | 10.9385 |
| 15 | 21.6250 | 1.2407 | 21,2598 |

## 6. SAMPLE RUN 2: BALANCE OF A GRINDING CIRCUIT

### 6.1 PROBLEM DEFINITION

The circuit under consideration in this example is described in Figure 7. It consists of a rod mill and two stages of closed grinding with a ball mill, a sump box, and a hydrocyclone classifier in each stage.
The complete data available for evaluating this circuit are given in Table 9. The size distribution and the pulp density (\% solid) are available for streams 2 to 8; whereas streams 9 to 13 are water addition lines for which the flow rate has been measured. The measured or assumed precision of this data is also given in Table 9.
The problem consists in adjusting experimental data within the given error model to satisfy known mass conservation equations in the circuit.


Fig. 7 - Grinding circuit flowsheet for sample run 2

### 6.2 FLOWSHEET ANALYSIS

The first step in solving this problem is to express all data on the same material basis: slurry, solid, or water. As indicated in Section 2.2.1.1, MATBAL requires all data to refer to only one stream or substream.
Four options are available to process data from Table 9.

## Options

1. All data can be converted into slurry data: the size distribution values must be multiplied by the solid-to-slurry ratio (\% solid/100) to make it relative to the slurry. The water streams are considered as slurry stream with a fixed solid-to-slurry ratio of 0.001. Transformed data are given in Table 10.
2. All data can be expressed as solid data: the \% solid must be transformed into water-to-solid ratio and the water flow rates into solid flow rates, assuming a fixed solid-to-liquid ratio of 0.001 in the water streams. Table 11 contains this data. The slurry flow rate in the feed must be multiplied by the solid-to-slurry ratio.
3. All data can be referred to water: all values of the size distribution must be multiplied by the solid-towater ratio in each stream. The slurry flow rate in the feed must be multiplied by the water-to-slurry ratio. The \% solid in the slurry must be transformed into solid to water ratio in each stream. This transformation is quite tedious and inappropriate for the specific problem under consideration and will not be covered.
4. All water and slurry data can be left out, i.e., \% solid and water flow rates from Table 9 are ignored.

Table 9 - Experimental data for sample run 2

| Stream | $+420 \mu \mathrm{~m}$ | $\begin{aligned} & -420 \\ & +297 \end{aligned}$ | $\begin{aligned} & -297 \\ & +210 \end{aligned}$ | $\begin{aligned} & -210 \\ & +149 \end{aligned}$ | -149 | Flow rate t/h | \% Solid | $\begin{aligned} & \sigma \text { Flow rate } \\ & \% \end{aligned}$ | $\begin{gathered} \sigma \% \text { Solid } \\ \% \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  | 87.10* | 97.08 | 1.5 | 1.0 |
| 2 | 45.05 | 6.9 | 5.45 | 5.2 | 37.4 |  | 84.4 |  | 1.0 |
| 3 | 35.15 | 14.15 | 14.05 | 10.4 | 26.25 |  | 86.0 |  | 1.6 |
| 4 | 22.85 | 12.35 | 13.5 | 12.0 | 39.30 |  | 83.3 |  | 1.0 |
| 5 | 1.90 | 2.35 | 6.05 | 11.55 | 78.15 |  | 67.9 |  | 1.0 |
| 6 | 1.20 | 1.65 | 5.30 | 17.45 | 74.4 |  | 79.6 |  | 1.6 |
| 7 | 0.65 | 0.75 | 3.25 | 14.0 | 81.35 |  | 77.1 |  | 1.0 |
| 8 | 0.10 | 0.10 | 0.30 | 0.85 | 98.65 |  | 30.5 |  | 0.75 |
| 9 |  |  |  |  |  | 17.6** |  | 2.5 |  |
| 10 |  |  |  |  |  | 17.8 |  | 5.0 |  |
| 11 |  |  |  |  |  | 13.8 |  | 2.5 |  |
| 12 |  |  |  |  |  | 8.6 |  | 2.5 |  |
| 13 |  |  |  |  |  | 118.9 |  | 5.0 |  |

[^7]The problem variables can then be defined as in sample run 1. The data from Table 10 will be used for the example. The reader is invited to treat data from Table 11 as an exercise. In all cases of experimental data transformation, the error model must be modified accordingly since a standard deviation has the same unit as the measurement to which it applies.

In such a case, the following formulae can be used:

$$
\begin{gathered}
\sigma_{x y}^{2}=x^{2} y^{2}\left[\frac{\sigma_{x}^{2}}{x^{2}}+\frac{\sigma_{y}^{2}}{y^{2}}\left(+\frac{2 C V_{x y}}{x y}\right)\right] \\
\sigma_{x+y}^{2}=\sigma_{x}^{2}+\sigma_{y}^{2}\left(+2 C V_{x y}\right)
\end{gathered}
$$

where $x$ and $y$ are two random variables, and $\mathrm{CV}_{\mathrm{xy}}$ is the covariance between $x$ and $y$, which is zero in our assumption of uncorrelated errors.

Table 10 - Sample run 2: data converted to slurry data

| Stream | $+420 \mu \mathrm{~m}$ | -420 <br> +297 | -297 <br> +210 | -210 <br> +149 | -149 | Flow rate <br> t/h | $\%$ Solid |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 |  |  |  |  |  | 87.10 | 97.08 |
| 2 | 38.40 | 5.82 | 4.60 | 4.39 | 31.57 |  | 84.4 |
| 3 | 30.23 | 12.17 | 12.08 | 8.94 | 22.57 |  | 86.0 |
| 4 | 19.03 | 10.29 | 11.24 | 10.00 | 32.74 |  | 83.3 |
| 5 | 1.29 | 1.60 | 4.11 | 7.84 | 53.06 |  | 67.9 |
| 6 | 0.95 | 1.31 | 4.22 | 13.89 | 59.22 |  | 79.6 |
| 7 | 0.50 | 0.58 | 2.51 | 10.79 | 62.72 |  | 77.1 |
| 8 | 0.03 | 0.03 | 0.09 | 0.26 | 30.01 |  | 30.5 |
| 9 |  |  |  |  |  | 17.6 | 0.1 |
| 10 |  |  |  |  |  | 17.8 | 0.1 |
| 11 |  |  |  |  |  | 13.8 | 0.1 |
| 12 |  |  |  |  |  | 11.6 | 0.1 |
| 13 |  |  |  |  |  |  | 0.0 |

${ }^{*} \sigma$ should be converted accordingly.

Table 11 - Sample run 2: data converted to solid data

| Stream | + $420 \mu \mathrm{~m}$ | $\begin{aligned} & \hline-420 \\ & +297 \\ & \hline \end{aligned}$ | $\begin{array}{r} -297 \\ +210 \end{array}$ | $\begin{array}{r} -210 \\ +149 \\ \hline \end{array}$ | -149 | Flow rate t/h | $\frac{\text { Water }}{\text { Solid }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  | 84.57 | 0.03 |
| 2 | 45.05 | 6.9 | 5.45 | 5.20 | 37.40 |  | 0.18 |
| 3 | 35.15 | 14.15 | 14.05 | 10.40 | 26.25 |  | 0.16 |
| 4 | 22.85 | 12.35 | 13.5 | 12.00 | 39.30 |  | 0.20 |
| 5 | 1.90 | 2.35 | 6.05 | 11.55 | 78.15 |  | 0.47 |
| 6 | 1.20 | 1.65 | 5.30 | 17.45 | 74.40 |  | 0.26 |
| 7 | 0.65 | 0.75 | 3.25 | 14.00 | 81.35 |  | 0.30 |
| 8 | 0.10 | 0.10 | 1.30 | 0.85 | 98.65 |  | 2.28 |
| 9 |  |  |  |  |  | 0.0176 | 1000 |
| 10 |  |  |  |  |  | 0.0178 | 1000 |
| 11 |  |  |  |  |  | 0.0138 | 1000 |
| 12 |  |  |  |  |  | 0.0086 | 1000 |
| 13 |  |  |  |  |  | 0.1189 | 1000 |

* $\sigma$ should be converted accordingly.

TITLE: SAMPLE RUN 2: BALANCE OF A GRINDING CIRCUIT

| MMAX: | 13 | Thirteen streams have been sampled. |  |
| :---: | :---: | :---: | :---: |
| OMAX: | 5 | Five assay types are available: four size distribution values, one pulp density. (one of the five size distribution values must be left out since it is equal to $100-\%$ solid and does not add any information). |  |
| NMAX: | 2 | Normal nodes: $2-3+4-5+12=0$ <br> Primary cyclone $5-6+7-8+13=0$ <br> Secondary cyclone. | IWF: A: |
| NNMAX: | 3 | Three non-normal nodes valid for \% solid: <br> $1-2+9=0$ Rod mill $3-4+10=0$ Primary ball mill <br> $6-7+11=0$ Secondary ball mill. | B: |
| NNNMAX: | 13 | The assays for all water streams used in normal node equations must be fixed at zero $[R(12, O)$ and $R(13,0)$, $\mathrm{O}=1$ to 5]. The assays used for all water streams appearing in non-normal node equations must be fixed at zero $[R(9,5), R(10,5)$, $R(11,5)]$. Thus a total of 13 assays must be fixed at zero. | C: WFC: BNNO: IO: <br> WFM,WFC: |
| IMAX: | 3 | The three non-null values of BNNO. Select the \% solid value as the only assay to be balanced for the three nonnormal node equations. |  |
| NNXC: | 6 | There are six measured flow rates. |  |

MRECOV:
1 Recoveries are calculated with respect to the feed rate. In the present problem, recoveries for species which are generated within the circuit (size distribution values) have no interest and one should expect some very high values which may overflow the output format specifications. The recoveries calculated for assay number 5 (\% solid) include the circulating load ratio for each closed circuit.
4 All standard deviations are read as relative (in \%).
Matrix A can easily be entered by considering the normal node equations: $1,5,2,-3,4,-5,121$ st row 2,5,5, -6,7, - 8, 13 2nd row Matrix $B$ has three rows and thirteen columns. It is filled in the same manner as A .
Six values of $C$ are read in positions 4 to 9 of array $C$.
One value is read for each value of C .
The three rows are identical, i.e., NN,1,5.

All assays 1 to 5 are used; therefore, $I O=5,1,2,3,4,5$.
All assays and measured flow rates must be assigned a standard deviation. Zero is not permitted in these arrays; by default it is converted to 0.0001 internally.

### 6.3 PROBLEM SOLUTION

The input file ready for execution is shown in Table 12. ICON is set at +1 in order to activate the error estimation by the Monte Carlo method. NMC is set to 10 for ten Monte Carlo simulations.
The program output and the error estimation results are given in Appendix B. As expected, one obtains very high (and non-significant) values of the recoveries for particle-size intervals. The circulating load ratio can be found in the recovery table for solids in the slurry.

## Table 12 - Input data for sample run 2

```
COMMAND- COPYSBF,TAPE5
SAMPLE RUN2: BALANCE OF A GRINDING CIRCUIT
13
0
    1 ROD MILI FEED
    R ROD MILI DISCHARGE
    3 PRIM, CYCLONE U/F
    4 PRIM. BAII MIII D.
    5 PRIM. CYGLONE O/F
    6 SEC. GYGLONE U/F
    7 SEG. BALL MILI D.
    8 SEC. CYGLONE O/F
    9 WATER TO ROD MILL
10 WATER TO P.B.M
11 WATER TO S.B.M
12 WATER TO P. SUMP
13 WATER TO S. SUMP
    10 0 0 0 97.08
    2 5.82 4.6 4.39 31.57 84.4
    3 12.17 12.08 8.94 22.57 86.0
    4 10.29 11.24 10.0 32.74 83.3
    5 1.6 4.11 7.84 53.06 67.9
    6 1.31 4.22 13.89 59.22 79.6
    7 0.58 2.51 10.79 62.72 77.1
    80.03 0.09 0.26 30.01 30.5
    9 0 0 0 0 0
1000000
11 0}00000
12 0}0000
130}0000
    1 -420 +297 MIGRONS
    3-210 +149 MICRONS 124
    4-149 MICRONS 12 5
```

    - 12
    \(297+210\) MICRONS
    131
    132
    133
    134
    135
    95
    105
    115
    \(5,1,2,3,4,5\)
    
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## APPENDIX A

## OUTPUT FROM SAMPLE RUN 1

| COMMAND- REWIND, TAPEG |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| COMMAND- COPYCF, TAPE6 |  |  |  |  |  |
| FIRSt APPROXIMATION OF Stream fuow-rates |  |  |  |  |  |
| M FLOW |  |  |  |  |  |
| 1100.0000 |  |  |  |  |  |
| 216.8684 |  |  |  |  |  |
| $\begin{array}{ll}2 & 16.8684 \\ 3 & 15.8499\end{array}$ |  |  |  |  |  |
| 4 7.7729 |  |  |  |  |  |
| $5 \quad 9.0956$ |  |  |  |  |  |
| 68.4152 |  |  |  |  |  |
| $7{ }_{7} \quad-.6423$ |  |  |  |  |  |
| $8 \quad 9.4904$ |  |  |  |  |  |
| 6.3595 |  |  |  |  |  |
| $10 \quad 90.8554$ |  |  |  |  |  |
| 11 6.3595 |  |  |  |  |  |
| 1291.5848 |  |  |  |  |  |
| 13 9.1446 |  |  |  |  |  |
| 14 5.6302 |  |  |  |  |  |
| 15 14.7747 |  |  |  |  |  |
|  |  |  |  |  |  |
| FINAL RESULTS AFTER ll It erations |  |  |  |  |  |
| calculated material balances |  |  |  |  |  |
| Sample run l: balance of a flotation circuit |  |  |  |  |  |
| Stream flow-rates |  |  |  |  |  |
| M | Stream | OBSERVED | Calculated | WT.PCT. |  |
| 1 | ROUGHER FEED | 100.0000 | 0100.0000 | 100.0000 |  |
| 2 | first cleaner conc. | . $\mathrm{N}-\mathrm{O}$ | 23.0299 | 23.0299 |  |
| 3 | first cleaner tail | $\mathrm{N}-\mathrm{O}$ | 21.5608 | 21.5608 |  |
| 4 | SEC.cleaner conc. | $\mathrm{N}-\mathrm{O}$ | 8.8016 | 8.8016 |  |
| 5 | SECOND Cleaner tail | - $\quad \mathrm{N}-\mathrm{O}$ | 14.2284 | 14.2284 |  |
| 6 | third cleaner conc. | - $\mathrm{N}-\mathrm{O}$ | 7.8690 | 7.8690 |  |
| 7 | third Cleaner tail | $\mathrm{N}-0$ | . 9326 | . 9326 |  |
| 8 | CL. SCV. CONC. | $\mathrm{N}-\mathrm{O}$ | 8.1699 | 8.1699 |  |
| 9 | CL. SCV. TAIL. | $\mathrm{N}-\mathrm{O}$ | 13.3909 | 13.3909 |  |
| 10 | ROUGHER TAIL | $\mathrm{N}-\mathrm{O}$ | 89.6789 | 89.6789 |  |
| 11 | MILL discharge | N-O | 13.3909 | 13.3909 |  |
| 12 | SCAV. TAIL | $\mathrm{N}-\mathrm{O}$ | 92.1310 | 92.1310 |  |
| 13 | ROUGHER CONC. | $\mathrm{N}-\mathrm{O}$ | 10.3211 | 10.3211 |  |
| 14 | SCAV. CONC. | N-0 | 10.9388 | 10.9388 |  |
| 15 | ROUGHER+SCAV. CONC. | . $\mathrm{N}-\mathrm{O}$ | 21.2599 | 21.2599 |  |
| ANALYSES |  |  |  |  |  |
| COPPER |  |  |  |  |  |
| M | Stream | OBSERVED C | calculated | Recovery | Wt Units |
| 1 | ROUGHER FEED | . 1800 | . 1930 | 100.00 | . 19298 |
| 2 | First cleaner conc. | .8100 | . 7843 | 93.598 | . 18063 |
| 3 | First cleaner tail | . 5900 | . 6285 | 70.218 | . 13551 |
| 4 | SEC.Cleaner conc. | 1.0000 | . 9849 | 44.921 | . $86689 \mathrm{E}-01$ |
| 5 | SECOND CLEANER TAIL | . 6500 | . 6602 | 48.677 | . $93938 \mathrm{E}-01$ |
| 6 | third Cleaner conc. | 1.1000 | 1.0352 | 42.210 | . $81457 \mathrm{E}-01$ |
| 7 | third cleaner tail | . 5600 | . 5610 | 2.7110 | . $52317 \mathrm{E}-02$ |
| 8 | CL. SCV. CONC. | . 9000 | . 8661 | 36.665 | . $70757 \mathrm{E}-01$ |
| 9 | CL. SCV. TAIL. | . 4900 | . 4835 | 33.553 | . $64751 \mathrm{E}-01$ |
| 10 | ROUGHER TAIL | . 1400 | . 1431 | 66.481 | . 12830 |
| 11 | MILI DISCHARGE | . 5100 | . 4835 | 33.553 | . $64751 \mathrm{E}-01$ |
| 12 | SCAV. TAIL | . 1300 | . 1210 | 57.790 | . 11152 |
| 13 | ROUGHER CONC. | . 6100 | . 6267 | 33.519 | . $64686 \mathrm{E}-01$ |
| 14 | SCAV. CONC. | .7300 | . 7453 | 42.243 | .81522E-01 |
| 15 | ROUGHER+SCAV. CONC. | . 6700 | . 6877 | 75.763 | . 14621 |


| LEAD |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| M | STREAM | OBSERVED | CALCULATED | RECOVERY | WT UNITS |
| 1 | ROUGHER FEED | 4.4900 | 4.3477 | 100.00 | 4.3477 |
| 2 | FIRST CLEANER CONC. | 23.1500 | 23.1671 | 122.72 | 5.3354 |
| 3 | FIRST CLEANER TAIL | 9.4900 | 9.4462 | 46.845 | 2.0367 |
| 4 | SEC.CLEANER CONC. | 34.8700 | 35.0198 | 70.895 | 3.0823 |
| 5 | SECOND CLEANER TAIL | 15.8400 | 15.8351 | 51.822 | 2.2531 |
| 6 | THIRD CLEANER CONC. | 36.6900 | 37.1215 | 67.187 | 2.9211 |
| 7 | THIRD CLEANER TAIL | 17.2900 | 17.2857 | 3.7077 | .16120 |
| 8 | CL. SCV. CONC. | 15.2600 | 15.3029 | 28.756 | 1.2502 |
| 9 | CL. SCV. TAIL. | 5.2100 | 5.8730 | 18.089 | . 78644 |
| 10 | ROUGHER TAIL | 1.9900 | 2.0176 | 41.616 | 1.8093 |
| 11 | MILL DISCHARGE | 7.0200 | 5.8730 | 18.089 | . 78644 |
| 12 | SCAV. TATL | 1.5500 | 1. 5484 | 32.813 | 1.4266 |
| 13 | ROUGHER CONC. | 25.1000 | 24.5939 | 58.384 | 2.5384 |
| 14 | SCAV. CONC. | 10.8900 | 10.6883 | 26.892 | 1.1692 |
| 15 | ROUGHER+SCAV. CONC. | 16.8800 | 17.4391 | 85.276 | 3.7075 |
| ZINC |  |  |  |  |  |
| M | STREAM | OBSERVED | CALCULATED | RECOVERY | WT UNITS |
| 1 | ROUGHER FEED | 9.0300 | 9.4081 | 100.00 | 9.4081 |
| 2 | FIRST CLEANER CONC. | 14.2000 | 13.6163 | 33.331 | 3.1358 |
| 3 | FIRST CLEANER TAIL | 13.7300 | 13.4233 | 30.762 | 2.8942 |
| 4 | SEC.CLEANER CONC. | 11.1200 | 11.3416 | 10.610 | . 99824 |
| 5 | SECOND CLEANER TAIL | 14.6400 | 15.0233 | 22.720 | 2.1376 |
| 6 | THIRD CLEANER CONC. | 10.4200 | 10.8710 | 9.0925 | . 85544 |
| 7 | THIRD CLEANER TAIL | 15.3300 | 15.3129 | 1. 5179 | . 14280 |
| 8 | CL. SCV. CONC. | 16.4000 | 16.5658 | 14.386 | 1.3534 |
| 9 | CL. SCV. TAIL. | 11. 1700 | 11.5059 | 16.377 | 1. 5407 |
| 10 | ROUGHER TAIL | 9.4800 | 9.3531 | 89.154 | 8.3877 |
| 11 | MILL DISCHARGE | 10.7200 | 11.5059 | 16.377 | 1. 5407 |
| 12 | SCAV. TAIL | 9.5400 | 9.2832 | 90.907 | 8.5527 |
| 13 | ROUGHER CONC. | 10.3200 | 9.8868 | 10.846 | 1.0204 |
| 1.4 | SCAV. CONC. | 13.3100 | 12.5768 | 14.623 | 1.3758 |
| 15 | ROUGHER+SCAV. CONC. | 12.0700 | 11.2709 | 25.469 | 2.3962 |
| IRON |  |  |  |  |  |
| M | STREAM | OBSERVED | CALCULATED | RECOVERY | WT UNITS |
| 1 | ROUGHER FEED | 28.7100 | 28.1376 | I00.00 | 28.138 |
| 2 | FIRST CLEANER CONC. | 23.5800 | 23.6483 | 19.356 | 5.4462 |
| 3 | FIRST CLEANER TAIL | 29.8400 | 30.5134 | 23.381 | 6.5789 |
| 4 | SEC.CLEANER CONC. | 18.6600 | 19.0280 | 5.9521 | 1.6748 |
| 5 | SECOND CLEANER TAIL | 26.5600 | 26.5064 | 13.404 | 3.7714 |
| 6 | THIRD CLEANER CONC. | 18.3500 | 18.2783 | 5.1117 | 1. 4383 |
| 7 | THIRD CLEANER TAIL | 25.4300 | 25.3544 | . 84032 | . 23645 |
| 8 | CL. SCV. CONC. | 25.5300 | 25.3432 | 7.3586 | 2.0705 |
| 9 | CL. SCV. TAIL. | 33.9400 | 33.6677 | 16.023 | 4.5084 |
| 10 | ROUGHER TAIL | 29.4400 | 28.4274 | 90.602 | 25.493 |
| 11 | MILL DISCHARGE | 32.7900 | 33.6677 | 16.023 | 4.5084 |
| 12 | SCAV. TAIL | 27.5800 | 28.9797 | 94.888 | 26.699 |
| 13 | ROUGHER CONC. | 25.4800 | 25.6198 | 9.3975 | 2.6442 |
| 14 | SCAV. CONC. | 29.8600 | 30.1905 | 11.737 | 3.3025 |
| 15 | ROUGHER+SCAV. CONC. | 30.1400 | 27.9716 | 21.134 | 5.9467 |



err estimates for caloulated flow rates
MONTE CARLO SIMULATION METHOD WITH 10 REPEATS
SAMPLE RUN 1: BALANCE OF A FLOTATION CIRCUIT
Statistical test on calculated flow rates
$M$ MEAN SIGMA TRUE
FLOW RATE FLOW RATE
$100.0000 .0001 \quad 100.0000$
$23.3521 \quad 23.0299$
$22.8624 \quad 3.3729 \quad 21.5608$

| 8.9752 | 8.7438 | 8.816 |
| ---: | ---: | ---: |

$14.3770 \quad 2.2291 \quad 14.2284$
$7.8996 \quad 7.8690$
1.0756 .5997 .9326
$9.0508 \quad 2.2086 \quad 8.1699$
$13.8116 \quad 1.3688 \quad 13.3909$
$89.5484 \quad 1.1416 \quad 89.6789$
$13.8116 \quad 1.368813909$
$92.1004 \quad .4004 \quad 92.1310$
$10.4516 \quad 1.1415 \quad 10.3211$
$11.2596 \quad 10.9388$
21.2599

## APPENDIX B

## OUTPUT FROM SAMPLE RUN 2

```
COMMAND- IGET,DIMSET.BIN
COMMAND- DIMSET
    STOP
    O26100 MAXIMUM EXECUTION FL.
        0.289 GP SEGONDS EXECUTION TIME.
COMMAND- FTN,I=TAPE1,B=DRIVER,I=O
            .261 CP SECONDS COMPILATION TIME
COMMAND- IGET,MATBAL4.BIN
COMMAND- XEQ
OPTION=LOAD=MATBAL4
OPTION=DRIVER
    STOP
    044000 MAXIMUM EXECUTION FL.
    11.734 CP SECONDS EXECUTION TIME.
```

| FINAL RESULTS AFTER 9 ITERATIONSCALCULATED MATERIAL BALANCES |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| SAMPLE RUNZ: BALANCE OF A GRINDING GIRGUIt |  |  |  |  |
| Stream flow-rates |  |  |  |  |
| M | Stream | observed | calculated | WT.PCT. |
| 1 | Rod Midu feed | 87.1000 | 87.0296 | 100.0000 |
| 2 | ROD MILL DISCHARGE | N -o | 104.1228 | 119.6407 |
| 3 | PRIM. CYCLONE U/F | $\mathrm{N}-\mathrm{O}$ | 307.8821 | 353.7671 |
| 4 | PRIM. BALL MILL D. | $\mathrm{N}-\mathrm{O}$ | 324.8581 | 373.2731 |
| 5 | PRIM. CYCLONE O/F | N-O | 129.6521 | 148.9747 |
| 6 | SEC. CYCLONE U/F | $\mathrm{N}-\mathrm{O}$ | 301.0730 | 345.9432 |
| 7 | SEC. BALL MILL D. | N-0 | 314.8979 | 361.8285 |
| 8 | SEC. CYCLONE O/F | N-0 | 280.9551 | 322.8271 |
| 9 | WATER TO ROD MILL | 17.6000 | 17.0932 | 19.6407 |
| 10 | WAter to p.b.m | 17.8000 | 16.9760 | 19.5060 |
| 11 | WATER TO S.B.M | 13.8000 | 13.8249 | 15.8853 |
| 12 | WATER TO P. SUMP | 8.6000 | 8.5532 | 9.8280 |
| 13 | WATER TO S. SUMP | 119.0000 | 137.4782 | 157.967 |


| M | Stream | OBSERVED | CALCULATED | RECOVERY | WT UNITS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | ROD MILL FEED | . 0001 | . 0001 | 100.00 | .87030E-04 |
| 2 | ROD MILL DISCHARGE | 5.8200 | 5.8211 | . $69644 \mathrm{E}+07$ | 6.0611 |
| 3 | PRIM. CYCLONE U/F | 12.1700 | 12.1559 | . $43003 \mathrm{E}+08$ | 37.426 |
| 4 | PRIM. BALL MILL D. | 10.2900 | 10.3007 | . $38450 \mathrm{E}+08$ | 33.463 |
| 5 | PRIM. CYCLONE O/F | 1.6000 | 1.6181 | . $24106 \mathrm{E}+07$ | 2.0979 |
| 6 | SEC. CYCLONE U/F | 1.3100 | 1.2816 | . $44336 \mathrm{E}+07$ | 3.8586 |
| 7 | SEC. BALL MILL D. | . 5800 | . 5858 | . $21197 \mathrm{E}+07$ | 1.8447 |
| 8 | SEC. CYCLONE O/F | . 0300 | . 0300 | 96803. | . $84248 \mathrm{E}-01$ |
| 9 | WATER TO ROD MILI | . 0001 | . 0001 | 19.641 | . $17093 \mathrm{E}-04$ |
| 10 | WATER TO P.B.M | . 0001 | . 0001 | 19.506 | .16976E-04 |
| 11 | WATER TO S.B.M | . 0001 | . 0001 | 15.885 | .13825E-04 |
| 12 | Water to p. Sump | . 0001 | . 0001 | 9.8280 | . $85532 \mathrm{E}-05$ |
| 13 | WATER TO S. SUMP | . 0001 | . 0001 | 158.27 | . $13774 \mathrm{E}-03$ |
| -297 +210 MICRONS |  |  |  |  |  |
| M | StREAM | OBSERVED | calculated | RECOVERY | WT UNITS |
| 1 | ROD MILL FEED | . 0001 | . 0001 | 100.00 | .87030E-04 |
| 2 | ROD MILL DISCHARGE | 4.6000 | 4.6095 | . $55148 \mathrm{E}+07$ | 4.7995 |
| 3 | PRIM. CYCLONE U/F | 12.0800 | 11.8872 | . $42053 \mathrm{E}+08$ | 36.599 |
| 4 | PRIM. BALI MILL D. | 11.2400 | 11.4161 | . $42613 \mathrm{E}+08$ | 37.086 |
| 5 | PRIM. CYCLONE O/F | 4.1100 | 4.0778 | . $60749 \mathrm{E}+07$ | 5.2870 |
| 6 | SEC. CYCLONE U/F | 4.2200 | 4.2758 | . $14792 \mathrm{E}+08$ | 12.873 |
| 7 | SEC. BALI MILI D. | 2.5100 | 2.4894 | $.90072 \mathrm{E}+07$ | 7.8390 |
| 8 | SEC. CYCLONE O/F | . 0900 | . 0900 | . $29062 \mathrm{E}+06$ | .25293 |
| 9 | WAter to rod milu | . 0001 | . 0001 | 19.641 | . $17093 \mathrm{E}-04$ |
| 10 | WATER TO P.B.M | . 0001 | . 0001 | 19.506 | . $16976 \mathrm{E}-04$ |
| 11 | WATER TO S.B.M | . 0001 | . 0001 | 15.885 | .13825E-04 |
| 12 | WATER TO P. SUMP | . 0001 | . 0001 | 9.8281 | . 85533E-05 |
| 13 | WATER TO S. SUMP | . 0001 | . 0001 | 157.91 | .13743E-03 |
| -210 +149 MICRONS |  |  |  |  |  |
| M | Stream | OBSERVED | calculated | Recovery | WT UNITS |
| 1 | ROD MILL FEED | . 0001 | . 0001 | 100.00 | .87030E-04 |
| 2 | ROD MILI DISCHARGE | 4.3900 | 4.3960 | . $52594 \mathrm{E}+07$ | 4.5772 |
| 3 | PRIM. CYCLONE U/F | 8.9400 | 8.8665 | . $31367 \mathrm{E}+08$ | 27.298 |
| 4 | PRIM. BALL MILL D. | 10.0000 | 10.0971 | . $37690 \mathrm{E}+08$ | 32.801 |
| 5 | PRIM. CYCLONE O/F | 7.8400 | 7.7748 | . $11582 \mathrm{E}+08$ | 10.080 |
| 6 | SEC. CYClone U/F | 13.8900 | 14.1917 | . $49095 \mathrm{E}+08$ | 42.727 |
| 7 | SEC. BALL MILL D. | 10.7900 | 10.5996 | . $38352 \mathrm{E}+08$ | 33.378 |
| 8 | SEC. CYCLONE O/F | . 2600 | . 2601 | .83967E+06 | . 73076 |
| 9 | WATER TO ROD MILI | . 0001 | . 0001 | 19.641 | . $17093 \mathrm{E}-04$ |
| 10 | WATER TO P.B.M | . 0001 | . 0001 | 19.506 | . $16976 \mathrm{E}-04$ |
| 11 | WATER TO S.b.M | . 0001 | . 0001 | 15.885 | .13825E-04 |
| 12 | WATER TO P. SUMP | . 0001 | . 0001 | 9.8280 | . $85533 \mathrm{E}-05$ |
| 13 | WATER TO S. SUMP | . 0001 | . 0001 | 157.94 | . $13745 \mathrm{E}-03$ |
| -149 MICRONS |  |  |  |  |  |
| M | Stream | ObSERVED | calculated | RECOVERY | WT UNITS |
| 1 | ROD MILL FEED | . 0001 | . 0001 | 100.00 | .87030E-04 |
| 2 | ROD MILI DISCHARGE | 31.5700 | 31.5145 | . $37704 \mathrm{E}+08$ | 32.814 |
| 3 | PRIM. CYCLONE U/F | 22.5700 | 22.6538 | . $80142 \mathrm{E}+08$ | 69.747 |
| 4 | PRIM. BALL MILI D. | 32.7400 | 32.5539 | . $12151 \mathrm{E}+09$ | 105.75 |
| 5 | PRIM. CYCLONE O/F | 53.0600 | 53.0810 | . $79077 \mathrm{E}+08$ | 68.821 |
| 6 | SEC. CYGLONE U/F | 59.2200 | 59.7235 | . $20661 \mathrm{E}+09$ | 179.81 |
| 7 | SEC. BALL MILL D. | 62.7200 | 62.1293 | . $22480 \mathrm{E}+09$ | 195.64 |
| 8 | SEC. CYCLONE O/F | 30.0100 | 30.1306 | .97270E+08 | 84.654 |
| 9 | WAter to rod mill | . 0001 | . 0001 | 19.641 | .17093E-04 |
| 10 | WATER TO P.B.M | .0001 | . 0001 | 19.506 | .16976E-04 |
| 11 | WATER TO S.b.M | . 0001 | . 0001 | 15.885 | .13825E-04 |
| 12 | WATER TO P. SUMP | . 0001 | . 0001 | 9.8279 | . $85532 \mathrm{E}-05$ |
| 13 | WATER TOS. SUMP | . 0001 | . 0001 | 157.96 | .13748E-03 |


| \% SOLID IN SLURRY |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| M | Stream | OBSERVED | calculated | RECOVERY | WT UNITS |
| 1 | ROD MILI FEED | 97.0800 | 99.2102 | 100.00 | 86.342 |
| 2 | ROD MILI DISCHARGE | 84.4000 | 82.9235 | 100.00 | 86.342 |
| 3 | PRIM. CYCLONE U/F | 86.0000 | 87.3444 | 311.46 | 268.92 |
| 4 | PRIM. BALI MILI D. | 83.3000 | 82.7801 | 311.46 | 268.92 |
| 5 | PRIM. CYCLONE O/F | 67.9000 | 66.5953 | 100.00 | 86.342 |
| 6 | SEC. CYGLoNe U/F | 79.6000 | 80.3426 | 280.15 | 241.89 |
| 7 | SEC. BALI MILI D. | 77.1000 | 76.8154 | 280.15 | 241.89 |
| 8 | SEC. CYGLONE O/F | 30.5000 | 30.7317 | 100.00 | 86.342 |
| 9 | WATER TO ROD MILL | . 0001 | . 0001 | .19798E-04 | .17094E-04 |
| 10 | WATER TO P.B.M | . 0001 | . 0001 | .19662E-04 | .16976E-04 |
| 11 | WATER TO S.b.M | . 0001 | . 0001 | .16011E-04 | . $13825 \mathrm{E}-04$ |
| 12 | WATER TO P. SUMP | . 0001 | . 0001 | .99063E-05 | . $85533 \mathrm{E}-05$ |
| 13 | WATER TO S. SUMP | . 0001 | . 0001 | . $15919 \mathrm{E}-03$ | . $13745 \mathrm{E}-03$ |
|  | ** | ** | ***** | ******** |  |
| WARNING: WEIGHT W(13) ADJUSTED BY MORE THAN 3 SIGMA |  |  |  |  |  |
| ANALYSIS OF ASSAY ADJUSTMENTS |  |  |  |  |  |
| node imbalance calculated with observed assays and calculated weights |  |  |  |  |  |
| NORMAL NODES |  |  |  |  |  |
| N | $0=1 \begin{array}{ll}1 & \end{array}$ | 3 | $4 \quad 5$ |  |  |
| 1 | -5.6-121.7 | -63.3 9 | . 8567.4 |  |  |
| 2 | -12.7 27.5 | 159.336 | . 747.5 |  |  |
| SPEGIAL NODES, (0.0 WHEN BNNO=0.). |  |  |  |  |  |
| N N | $0=1 \begin{array}{lll}1 & 2\end{array}$ | 3 | 45 |  |  |
| 1 | 0.00 .0 | 0.0 | .0-339.1 |  |  |
| 2 | 0.00 .0 | 0.0 | .0-582.8 |  |  |
| 3 | 0.00 .0 | 0.0 | .0-313.2 |  |  |
| 4 | 0.00 .0 | 0.0 | . 00.0 |  |  |
| 5 | 0.00 .0 | 0.0 | . 00.0 |  |  |
| 6 | 0.000 | 0.0 | . 00.0 |  |  |
| 7 | 0.000 | 0.0 | . 00.0 |  |  |
| 8 | 0.00 .0 | 0.0 | . 00.0 |  |  |
| 9 | 0.00 .0 | 0.0 | . 00.0 |  |  |
| Standard analysis residuals |  |  |  |  |  |
| M | $0=12$ | 3 | 45 |  |  |
| 1 | 0.00 .0 | 0.0 | .0-2.2 |  |  |
| 2 | -. 0 -. 1 | -. 1 | . 1.7 |  |  |
| 3 | . 1 . 8 | . 4 | . $2-1.0$ |  |  |
| 4 | . 1 -. 8 | -. 5 | .3 . 6 |  |  |
| 5 | -. 6 . 4 | . 4 | . 01.9 |  |  |
| 6 | $1.1-7$ | -1.1 | . 4 -. 6 |  |  |
| 7 | -. 5 . 4 | . 9 | .5 . 4 |  |  |
| 8 | . 0 -. 0 | -. 0 | . $2-1.0$ |  |  |
| 9 | 0.00 .0 | 0.0 | . $0-.0$ |  |  |
| 10 | 0.00 .0 | 0.0 | . $0-.0$ |  |  |
| 11 | 0.00 .0 | 0.0 | .0 . 0 |  |  |
| 12 | -. 0 -. 0 | -. 0 | $.0-.0$ |  |  |
| 13 | -. 0 . 0 | . 0 | .0 .0 |  |  |
| average s.s. of Standard residuals Per Stream |  |  |  |  |  |
| M Stream (M) Std.res. |  |  |  |  |  |
| 1 ROD MILL FEED - |  |  |  |  |  |
| 2 ROD MILL DISCHARGE |  |  | 67 |  |  |
| 3 PRIM. GYCLONE U/F |  |  | 97 |  |  |
| 4 PRIM. BALL MILL D. . |  |  | 45 |  |  |
| 5 PRIM. CYCLONE 0/F . |  |  | 79 |  |  |
| 6 SEC. CYCLONE U/F - |  |  | 22 |  |  |
|  | SEC. ball mill d. |  | 15 |  |  |
|  | SEC. CYCLONE 0/F |  | 36 |  |  |
|  | Water to rod milu |  | 00 |  |  |
|  | WATER TO P.b.M |  | 00 |  |  |
| 10 | Water to s.b.M |  | 00 |  |  |
| 111 | WATER TO P. SUMP |  | 00 |  |  |
|  | WATER TO S. SUMP |  | 00 |  |  |




APPENDIX C

## GRADIENT MATRIX IN SUBROUTINE SOLV

## GRADIENT MATRIX IN SUBROUTINE SOLV

Equation 29 is schematically represented in Figure C1, where the general system of linear equations to be solved is tabulated in numbered boxes for easy identification in the coding of subroutine SOLV.
Matrix D (called DR in SOLV) consists of up to 36 blocks whose dimensions appear along the $X$ and $Y$ coordinates in Figure C1, with an identification number at the upper left corner of each box. For instance, block D17 has NMAX.OMAX rows and IMAX columns. Each block contains the coefficients of the derivatives of the objective function as shown in Figure C1.
Whenever a box is zero-filled, this is indicated by a zero at the upper right corner of the box. For instance, D1, D6 and D7 are zero-filled, as are E1, E2, and E6.
The actual values of the coefficients calculated by SOLV in arrays $D$ and $E$ prior to calling subroutine GELS to solve the system, are obtained as follows:
by replacing W.ER by its approximation of Eq 30 into Eq 18 and Eq 20:

$$
\begin{aligned}
& \sum_{M} A(M, N) W(M) R(M, O) \\
& -\sum_{M} A(M, N)[W O(M) E R(M, O) \\
& \quad+W(M) E R O(M, O)-W O(M) E R O(M, O)] E q 18^{\prime}
\end{aligned}
$$

$$
B N N O(N N, O) \sum_{M}^{B}(M, N N) W(M) R(M, O)-B N N O(N N, O)
$$

$$
\sum_{\mathrm{M}}^{\mathrm{B}}(\mathrm{M}, \mathrm{NN})[\mathrm{WO}(\mathrm{M}) \mathrm{ER}(\mathrm{M}, \mathrm{O})
$$

$$
\begin{array}{r}
\mathrm{M} \\
+\mathrm{W}(\mathrm{M}) \mathrm{ERO}(\mathrm{M}, \mathrm{O})-\mathrm{WO}(\mathrm{M}) \mathrm{ERO}(\mathrm{M}, \mathrm{O})] \\
\mathrm{Eq} 20^{\prime}
\end{array}
$$

Equation 21 becomes:

$$
\left.F=S S+\Sigma L 1[17]+\Sigma L 2\left[18^{\prime}\right]+\Sigma L 3\left[19^{\prime}\right]+\Sigma L 4\left[20^{\prime}\right] \quad E q 21^{\prime}\right]
$$

The Lagrangian derivative with respect to $\mathrm{ER}(1, \mathrm{~K})$ is:

$$
\begin{align*}
\frac{\delta F}{\delta E R(I, K)}= & 2 \cdot E R(I, K) \cdot W F M(1, K) \\
& -\sum_{N} L 2(N, K) A(I, N) W O(I) \\
- & \sum_{N N} \mathrm{~L} 4(\mathrm{NN}, \mathrm{~K}) \mathrm{BNNO}(\mathrm{NN}, \mathrm{~K}) \mathrm{B}(\mathrm{I}, \mathrm{NN}) \mathrm{WO}(\mathrm{I}) \tag{Eq 22}
\end{align*}
$$

From which, at the stationary point:
$E R(1, K)=\frac{0.5}{W F M(1, K)}\left(\sum_{N}^{L L 2 . A . W O}+\sum_{N N} L 4 . B N N O . B . W O\right)$
The adjustments $\mathrm{ER}(1, \mathrm{~K})$ to assays $\mathrm{R}(1, \mathrm{~K})$ can therefore be calculated as a function of the Lagrange multipliers and coefficients $A, B N N O, B, W F M$ and WO, thus reducing the size of the problem by MMAX.OMAX equations!

The other derivatives are:

| $\frac{\delta F}{\delta W(M)}=\sum_{N} L 1(N) \cdot A(M, N)+\sum_{O N} \sum_{N} L 2(N, O)$ |  |
| :---: | :---: |
|  |  |
| $\overline{\delta W(M)}$$+$ | [ $[\mathrm{A}(\mathrm{M}, \mathrm{N}) . \mathrm{R}(\mathrm{M}, \mathrm{O})-\mathrm{A}(\mathrm{M}, \mathrm{N}) . \mathrm{ERO}(\mathrm{M}, \mathrm{O})]$ |
|  | $+\sum_{N N} L 3(N N) \cdot B(M, N N)+\sum_{O N} \sum_{N N} L 4(N N, O)$ |
|  | . $\mathrm{BNNO}(\mathrm{NN}, \mathrm{O}$ ) B ( $\mathrm{M}, \mathrm{NN}$ ) |
|  | [ $\mathrm{R}(\mathrm{M}, \mathrm{O})-\mathrm{ERO}(\mathrm{M}, \mathrm{O})]$ |
| ¢F | $=\sum_{M} A(M, N) \cdot W(M)$ |
| $\overline{\delta L 1(N)}$ |  |
| ¢F | - $=\sum_{M} A(M, N) \cdot W(M) \cdot[R(M, O)-E R O(M, O)]$ |
| 8L2( $\mathrm{N}, \mathrm{O}$ ) |  |
| 8L4(NN,O) | $N, O)-\sum_{M} A(M, N) \frac{1}{2 . W F M(M, O)} \sum_{J} L 2(J, O)$ |
|  | . $\mathrm{A}(\mathrm{M}, \mathrm{J}) . \mathrm{WO}(\mathrm{M})^{2}$ |
|  | $-\sum_{M} A(M, N) \frac{1}{2 . W F M(M, O)} \quad \sum_{N N} L 4(N N, O)$ |
|  | . $\mathrm{BNNO}\left(\mathrm{NN}, \mathrm{O}\right.$ ). $\mathrm{B}(\mathrm{M}, \mathrm{NN}) . \mathrm{WO}(\mathrm{M})^{2}$ |
|  | $+\sum_{M} A(M, N)-[W O(M) \cdot E R O(M, O)]$ |
| 8F | $=\sum_{M} B(M, N N) \cdot W(M)+E R C(N N)$ |
| סL3(NN) |  |
|  | - C(NN) |
| ¢F | $=\mathrm{BNNO}(\mathrm{NN}, \mathrm{O}) \cdot \mathrm{M} \mathrm{~B}(\mathrm{M}, \mathrm{NN}) \cdot \mathrm{W}(\mathrm{M})$ |
| ¢L4(NN,O) |  |
|  | . $[\mathrm{R}(\mathrm{M}, \mathrm{O})-\mathrm{ERO}(\mathrm{M}, \mathrm{O})]$ - BNNO( $\mathrm{NN}, \mathrm{O}$ ) |
|  | $\cdot{ }_{M} \mathrm{~B}(\mathrm{M}, \mathrm{NN}) \frac{1}{2 \cdot \mathrm{WFM}(\mathrm{M}, \mathrm{O})} \cdot \sum_{N} \mathrm{~L} 2(\mathrm{~N}, \mathrm{O})$ |
|  | . $\mathrm{A}(\mathrm{M}, \mathrm{N}) . \mathrm{WO}^{2}(\mathrm{M})-\mathrm{BNNO}(\mathrm{NN}, \mathrm{O})$ |
|  | $\sum_{M} \mathrm{~B}(\mathrm{M}, \mathrm{NN})$ |
|  | $1 . \Sigma \mathrm{L} 4(\mathrm{JJ}, \mathrm{O}) \cdot \mathrm{BNNO}(\mathrm{JJ}, \mathrm{O})$ |
|  | 2.WFM(M,O) ${ }^{\text {JJ }}$ |
|  | . $\mathrm{B}(\mathrm{M}, \mathrm{JJ}) . \mathrm{WO}^{2}(\mathrm{M})+\mathrm{BNNO}(\mathrm{NN}, \mathrm{O})$ |
|  | . $\mathrm{E}_{\mathrm{M}} \mathrm{B}(\mathrm{M}, \mathrm{NN})$ |
|  | .WO(M).ERO(MO) |
|  | $=2 \cdot \mathrm{WFC}(\mathrm{NN}) \cdot \mathrm{ERC}(\mathrm{NN})+\mathrm{L3}(\mathrm{NN})$. |
| $\delta E R C(N N)$ |  |

The value of all coefficients in D and E (called respectively, DR and ER in SOLV) can be found from these equations.
For instance, blocks D7 to D12 contain the coefficients of $\frac{8 F}{\delta \mathrm{~F}}$ decomposed on variables W, L1 to L4 and ERC. All values are zero except $\frac{\delta \mathrm{FF}}{8 \mathrm{Lt}}(\mathrm{W})$ contained in D7 which have the value $A(M, N)$. Only the upper triangular portion of DR is stored, columnwise.


Fig. C1 - Gradient matrix in subroutine SOLV

## APPENDIX D

## GRADIENT MATRIX IN SUBROUTINE COEF

## GRADIENT MATRIX IN SUBROUTINE COEF

Equation 29 can be schematically represented as in Figure D1, where the general system of linear equations to be solved is tabulated in numbered boxes for easy identification in the coding of subroutine COEF.
Matrix D consists of up to 16 blocks whose dimensions appear along the X and Y coordinates in Figure D1, with an identification number at the upper left corner of each box. For instance, block D10 has NNMAX rows and NMAX columns. Each block contains the coefficient of the derivative of the objective function. Whenever a box is zero-filled, this is indicated by a zero at the upper right corner of the box (e.g., D4, D6, E1, E2).

The actual values of the coefficients calculated by COEF in D and E prior to calling GELS to solve the system, are obtained directly from Eq 34 to 37. For instance, blocks D5 to D8 contain the coefficients of $\frac{8 \mathrm{~F}}{8 L 1}$, decomposed on variables $W, L 1, L 2$, and ERC.
Equation 36 shows that all these coefficients are zero, except $\frac{8 F}{\delta L 1}(W)$ contained in D5, which have the values $\mathrm{A}(\mathrm{M}, \mathrm{J})$. D and E are represented by variables D and E in subroutine COEF. Only the upper triangular portion of $D$ is stored, columnwise.


Fig. D1 - Gradient matrix in subroutine COEF

## APPENDIX E <br> INPUT FILE EDITOR EDIBAL

## INPUT FILE EDITOR EDIBAL

As discussed in Section 2.4, "DATA INPUT", the data file for submission of a problem to MATBAL can be described in twelve sections, the first of which contains all the problem dimensions in records 2 and 3 .
Figure 3 of the main text summarizes the structure of the input data file. A conversational program called EDIBAL permits the interactive creation of a new file or the modification of an existing data file in a separate session.
The program displays the sections in sequential order and allows their modification before copying them on disk file No. 5 as expected by the MATBAL program. If a disk file containing previous data is being modified, it must be attached as logical unit No. 8.

The user can enter HELP whenever a description of the section content is required. The addition or deletion of records required by a change in the key variables of section one is automatically requested.
In the current version, no check of the actual number of items in each record is performed. It is the user's responsibility to ensure that the content of logical unit 5 is adequate and stored on a permanent device prior to activating MATBAL.
The very simple use of EDIBAL is illustrated below by two examples in modification mode.

## Example 1

```
COMMAND- IUSE,MATBAL4,DLAIPF,SPOC
COMMAND- IGET,TAPE8.FFL
COMMAND- IGET,EDIBAL.BIN
COMMAND- EDIBAL
```

    INTERACTIVE PROGRAM EDIBAL TO EDIT
    INPUT FOR MATBAL
    CHOOSE OPTION :
$\mathrm{C}=$ CREATE NEW FILE 5 ,
M MODIFY FILE 8 INTO FILE 5
OPTION (C/M) =M
SECTION NO. 1 HAS 3 RECORD (S)
VALUES STORED FOR SECTION : 1
REC NO. 1
SAMPLE RUN 1: BALANCE OF A FLOTATION CIRCUIT
REC NO. 2
$15,10,8,0,0,0,1,1,10$
REC NO. 3
$0,0,0,0,1,1,1,7,1,0,5,99,0.1,0.1$
WOULD YOU LIKE TO MODIFY THIS DATA (Y/N/HELP/END) ?HELP
SECTION 1

1. TITLE RECORD
2. DIMENSIONS
3. OPTIONS
YOU WANT THE DESCRIPTION OF WHICH RECORD ( 1 TO 3 , 'O' TO EXIT ) ? 2
RECORD 2 : DIMENSIONS
MMAX : NUMBER OF STREAM
OMAX : NUMBER OF ASSAYS
NMAX : NUMBER OF NORMAL NODES
NNMAX : NUMBER OF NON-NORMAL NODES
NNNMAX : NUMBER OF FIXED ASSAYS
IMAX : NUMBER OF SPECIAL ASSAYS (NON NULL BNNO)
NNXC : NUMBER OF MEASURED FLOW RATES
MRECOV : NUMBER OF REFERENCE STREAM FOR RECOVERY CALCULATION
NMC : NUMBER OF MONTE CARLO REPEATS
USE BLANK OR COMMA AS DELIMITER BETWEEN TWO VALUES
ONE RECORD NUMBER TO BE CHANGED (O TO QUIT): 3
NEW VALUES FOR REC NO. 3 (OR DELETE)
$0,0,1,0,1,1,0,7,1,0,5,99,0.1,0.1$
ONE RECORD NUMBER TO BE CHANGED (O TO QUIT): 0
VALUES STORED FOR SECTION: 1
REC NO. 1
SAMPLE RUN 1: BALANCE OF A FLOTATION CIRCUIT
REC NO. 2
$15,10,8,0,0,0,1,1,10$
REC NO. 3
$0,0,1,0,1,1,0,7,1,0,5,99,0.1,0.1$
WOULD YOU LIKE TO MODIFY THIS DATA (Y/N/HELP/END) ?N
SECTION NO. 2 HAS 15 RECORD(S)
VALUES STORED FOR SECTION: 2
REC NO. 1
1, ROUGHER FEED
REC NO. 2
2,FIRST CLEANER CONC.
REO NO. 3
3, FIRST CLEANER TAIL
REC NO. 4
4, SEC. CLEANER CONC.

```
    REC NO. 5
    5,SECOND ClEANER TAIL
    REC NO. 6
    6,THIRD CLEANER CONC.
    REC NO. 7
    7,THIRD CLEANER TAIL
    REC NO. 8
    8,CL. SCV. CONC.
    REC NO. 9
    9,Cl. SCV. TAIL.
    REC NO.lo
10,ROUGHER TAIL
    REC NO.ll
11,MILL DISCHARGE
    REC NO.l2
12,SCAV. TAIL
    REC N0.l3
    13,ROUGHER CONC.
    REC NO.l4
14,SCAV. CONC.
    REC NO.l5
15,ROUGHER+SCAV. CONC.
    WOULD YOU LIKE TO MODIFY THIS DATA (Y/N/HELP/END) PN
SECTION NO. 3 HAS l5 RECORD(S)
    VALUES STORED FOR SECTION : 3
    REC NO. l
1,.18,4.49,9.03,28.71,1.05,5.0,7.0,6.0,9.8,9.0
    REC NO. 2
2,.81,23.15,14.2,23.58,4.97,13.2,10,30,11.5,7
    REC NO. 3
3,.59,9.49,13,73,29.84,3.03,9.5,7,14,10.4,6
    REC NO. 4
4,1,34.87,11.12,18.66,6.28,16.5,7,31,9.8,5
    REC NO. 5
5,.65,15.84,14.64,26.56,4.03,13,10,27,12,8
    REC NO. 6
6,1.1,36.69,10.42,18.35,6.65,15.9,6,34,8.4,6
    REC NO. 7
7,.56,17.29,15.33,25.43,3.93,11.8,10,20,11.4,8
    REC NO. 8
8,.9,15.26,16.4,25.53,4.45,15.1,12,24,14.8,5
    REC NO. 9
9,.49,5.21,11.17,33.94,1.88,8.3,7,12,8.3,5
    REC NO.10
    10,.14,1.99,9.48,29.44,.59,3.5,6,3,8.5,7
    REC NO.11
11,.51,7.02,10.72,32.79,2.02,8.3,6,16,8.4,6
    REC NO.12
12,.13,1.55,9.54,27.58,.5,3,6,4,7.8,8
    REC NO.13
13,.61,25.1,10.32,25.48,4.8,13.7,7,23,9.7,6
    REC N0.14
    14,.73,10.89,13.31,29.86,3.1,11.4,8,19,12.2,7
        REC NO.15
    15,.67,16.88,12.07,30.14,3.93,12.8,8,20,12,6
        WOULD YOU LIKE TO MODIFY THIS DATA (Y/N/HELP/END) PY
        ONE RECORD NUMBER TO BE CHANGED (O TO QUIT) : }
        NEW VALUES FOR REC NO. 3 (OR DELETE)
    3,.59,9.49,13.73,29.84,3.03,9.5,7,14,10.4,6
        ONE RECORD NUMBER TO BE CHANGED (O TO QUIT) : O
        VALUES STORED FOR SECTION : 3
        REC NO. l
    1,.18,4.49,9.03,28.71,1.05,5.0,7.0,6.0,9.8,9.0
    REC NO. 2
```

```
2,.81,23.15,14.2,23.58,4.97,13.2,10,30,11.5,7
    REC NO. 3
3,.59,9.49,13.73,29.84,3.03,9.5,7,14,10.4,6
    REC NO. 4
4,1,34.87,11.12,18.66,6.28,16.5,7,31,9.8,5
    REC NO. 5
5,.65,15.84,14.64,26.56,4.03,13,10,27,12,8
    REC NO. }
6,I.1,36.69,10.42,18.35,6.65,15.9,6,34,8.4,6
    REC NO. 7
7,.56,17.29,15.33,25.43,3.93,11.8,10,20,11.4,8
    REC NO. 8
8,.9,15.26,16.4,25.53,4.45,15.1,12,24,14.8,5
    REC NO. }
9,.49,5.21,11.17,33.94,1.88,8.3,7,12,8.3,5
    REC NO.10
10,.14,1.99,9.48,29.44,.59,3.5,6,3,8.5,7
    REC NO.II
11,.51,7.02,10.72,32.79,2.02,8.3,6,16,8.4,6
    REC NO.12
12,.13,1.55,9.54,27.58,.5,3,6,4,7.8,8
    REC NO.I3
13,.61,25.1,10.32,25.48,4.8,13.7,7,23,9.7,6
    REC NO.14
14,.73,10.89,13.31,29.86,3.1,11.4,8,19,12.2,7
    REC NO.15
15,.67,16.88,12.07,30.14,3.93,12.8,8,20,12,6
    WOULD YOU LIKE TO MODIFY THIS DATA (Y/N/HELP/END) ?END
SECTION NO. }4\mathrm{ HAS 1O RECORD(S)
SECTION NO. }7\mathrm{ HAS 8 RECORD(S)
SECTION NO. }9\mathrm{ HAS I RECORD(S)
SECTION NO.I2 HAS I RECORD(S)
    YOUR NEW FILE IS READY ON DISK FILE NUMBER 5
    IT IS ADVISABLE TO LIST IT BEFORE EXECUTING MATBAL
            STOP
            O33600 MAXIMUM EXECUTION FL.
                1.016 CP SECONDS EXECUTION TIME.
COMMAND- COPYSBF,TAPE5
SAMPLE RUN 1: BALANCE OF A FLOTATION CIRCUIT
15,10,8,0,0,0,1,1,10
0,0,1,0,1,1,0,7,1,0,5,99,0.1,0.1
    1,ROUGHER FEED
    2,FIRST CLEANER CONC.
    3,FIRST CLEANER TAIL
    4,SEC.CLEANER CONC.
    5,SECOND CLEANER TAIL
    6,THIRD CLEANER CONC.
    7,THIRD CLEANER TAIL
    8,CL. SCV. CONC.
    9,CL. SCV. TAIL.
10,ROUGHER TAIL
11,MILL DISCHARGE
12,SCAV. TAIL
13,ROUGHER CONC.
14,SCAV. CONC.
15, ROUGHER+SCAV , CONC.
1,.18,4.49,9.03,28.71,1.05,5.0,7.0,6.0,9.8,9.0
2,.81,23.15,14.2,23.58,4.97,13.2,10,30,11.5,7
3,.59,9.49,13.73,29.84,3.03,9.5,7,14,10.4,6
4,1,34.87,l1.12,18,66,6.28,16.5,7,31,9.8,5
5,.65,15.84,14.64,26.56,4.03,13,10,27,12,8
```

```
6,1.1,36.69,10.42,18.35,6.65,15.9,6,34,8.4,6
7,.56,17.29,15.33,25.43,3.93,11.8,10,20,11.4,8
8,.9,15.26,16.4,25.53,4.45,15.1,12,24,14.8,5
9,.49,5.21,11.17,33.94,1.88,8.3,7,12,8.3,5
10,.14,1.99,9.48,29.44,.59,3.5,6,3,8.5,7
11,.51,7.02,10.72,32.79,2.02,8.3,6,16,8.4,6
12,.13,1.55,9.54,27.58,.5,3,6,4,7.8,8
13,.61,25.1,10.32,25.48,4.8,13.7,7,23,9.7,6
14,.73,10.89,13.31,29.86,3.1,11.4,8,19,12.2,7
15,.67,16.88,12.07,30.14,3.93,12.8,8,20,12,6
    1, COPPER
    2,LEAD
    3, ZINC
    4, IRON
    5,SILVER
    6,ANTIMONY
    7, INDIUM
    8, BISMUTH
    9,TIN
1O,MERCURY
1,3,1,-10,-13
2,4,10,11,-12,-14
3,3,13,14,-15
4,6,8,5,7,15,-3,-2
5,3,3,-9,-8
6,3,2,-5,-4
7,3,4,-7,-6
8,2,9,-11
1,100,.0001
5,1,2,3,4,5
COMMAND- IGET,DIMSET.BIN < MATBAL EXECUTION
COMMAND- DIMSET
    STOP
    O26100 MAXIMUM EXECUTION FL.
        0.265 CP SECONDS EXECUTION TIME.
COMMAND-FTN,I=TAPEI,B=DRIVER,L=O
        .272 CP SECONDS COMPILATION TIME
COMMAND- IGET,MATBALA.BIN
COMMAND - XEQ
OPTION=IOAD=MATBALA
OPTION=DRIVER
    STOP
    060700 MAXIMUM EXECUTION FL.
    48.979 CP SECONDS EXECUTION TIME.
```


## Example 2

```
COMMAND- IUSE,MATBAL4,DLAIPF,SPOC
COMMAND- IGET,TAPE8=RUN2.DAT
COMMAND- IGET,EDIBAL.BIN
COMMAND- EDIBAL
INTERACTIVE PROGRAM EDIBAL TO EDIT
INPUT FOR MATBAL
CHOOSE OPTION :
    C m CREATE NEW FILE 5,
    M mODIFY FILE 8 INTO FILE 5
OPTION (C/M) =M
SEGTION NO. 1 HAS 3 RECORD(S)
    VALUES STORED FOR SECTION : l
    REC NO. l
SAMPLE RUN2: BALANCE OF A GRINDING CIRCUIT
    REC NO. 2
13}5
    REC NO. 3
l 11 1 1 1 1. 1 1 1 4 4 0
    WOULD YOU LIKE TO MODIFY THIS DATA (Y/N/HELP/END) ?Y
    ONE RECORD NUMBER TO BE CHANGED (O TO QUIT): 
    NEW VALUES FOR REC NO. 3 (OR DELETE)
```



```
    ONE RECORD NUMBER TO BE CHANGED (O TO QUIT) : 0
    VALUES STORED FOR SECTION : l
    REC NO. l
SAMPLE RUN2: BALANCE OF A GRINDING GIRCUIT
    REC NO. 2
```



```
    REC NO. 3
0
    WOULD YOU LIKE TO MODIFY THIS DATA (Y/N/HELP/END) ?END
SECTION NO. 2 HAS 13 RECORD(S)
SECTION NO. 3 HAS 13 RECORD(S)
SECTION NO. }4\mathrm{ HAS 5 RECORD(S)
SECTION NO. 6 HAS 13 RECORD(S)
SECTION NO. }7\mathrm{ HAS 2 RECORD(S)
SECTION NO. }8\mathrm{ HAS 3 RECORD(S)
SECTION NO. }9\mathrm{ HAS 6 RECORD(S)
SECTION NO.1O HAS 3 RECORD(S)
SECTION NO.ll HAS 13 RECORD(S)
SECTION NO.12 HAS 1 RECORD(S)
    YOUR NEW FILE IS READY ON DISK FILE NUMBER 5
    IT IS ADVISABLE TO LIST IT BEFORE EXECUTING MATBAL
        STOP
        033600 MAXIMUM EXECUTION FL.
                0.807 CP SECONDS EXECUTION TIME.
```

SER

622(21) | $85-1 / 3.2 E ~ c .2 ~$ |
| :---: |
| c.2 |$\quad$ C212sp

Laguitton, D.
MATBAL, computer program for material balance data adjustment. -- (SPOC manual. Chapter 3.2, MATBAL computer program)


[^0]:    *Simulated Processing of Ore and Coal

[^1]:    * Engineering Computer Program Documentation Standards; Journal of the Soil Mechanics and Foundations Division SM3, March / Mars 1973, pp 249-266

[^2]:    * Original MATBAL source was obtained from the University of Minnesota (1).

[^3]:    * K. Smith, private communication.

[^4]:    *Dimensions set for sample run 1

[^5]:    *Precision: All assays assumed to have a $5 \%$ standard deviation.

[^6]:    * For the sake of clarity, only stream numbers are represented. The first equation should be read $\mathrm{W} 1-\mathrm{W} 10-\mathrm{W} 13=0$, where Wi is the flow rate of stream i .

[^7]:    Precision: All size distribution values have a $2 \%$ standard deviation
    *Puip flow rate; **water flow rates.

